

# The Fiscal Multiplier in a Small Open Economy: The Role of Liquidity Frictions

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## A Online Appendix

### A.1 The SOE-DEFK Model

#### A.1.1 Households

The home economy consists of a continuum of identical households. Each household consists of a continuum of members  $j \in [0, 1]$ . In each period, members have an i.i.d. opportunity  $\varkappa$  to invest in new capital. Household members ( $j \in [0, \varkappa)$ ) who receive the opportunity to invest are “entrepreneurs”, whereas those who do not ( $j \in [\varkappa, 1]$ ) are “workers”. Entrepreneurs invest and do not work. Workers work to earn labour income. Each household’s assets are divided equally among its own members at the beginning of each period. After members realise whether they are entrepreneurs or workers, households cannot reallocate assets among them. If any household members need extra funds, they need to obtain them from external sources. This assumption is important as it gives rise to liquidity constraints. At the end of each period, household members return all their assets plus any income that they earn during the period to the household.<sup>15</sup>

The representative household’s utility depends on the aggregate consumption  $C_t \equiv \int_0^1 C_t(j) dj$  as consumption goods are jointly utilised by its members. Each member seeks to maximise the utility of the household as a whole, which is given by:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ \frac{C_s^{1-\sigma}}{1-\sigma} - \frac{1}{1+v} \int_{\varkappa}^1 H_s(j)^{1+v} dj \right], \quad (\text{A.1})$$

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<sup>15</sup>The assumption that entrepreneurs and workers belong to the same household is based on Shi (2015). This is different from the setting in Kiyotaki and Moore (2008), in which entrepreneurs and workers are two separate entities. As noted by DEFK (2012), adopting this assumption increases the flexibility of the model to incorporate various modifications.

where  $\beta$  is the discount factor,  $\sigma$  is the coefficient of relative risk aversion, and  $v$  is the inverse Frisch elasticity of labour supply. Labour supply  $H_t(j) = 0$  for entrepreneurs. Each period, household members choose optimally among non-durable consumption, saving in bonds or equity and, if they are entrepreneurs, investment in capital. Details of their investment/saving options are as follows: (i) *Investment in new capital*. Entrepreneurs have the opportunity to invest in new capital  $I_t$ , which costs  $p_t^I \equiv \frac{P_t^I}{P_t^C}$  per unit, where  $P_t^C$  is the consumption price index. Each unit of capital goods generates a rental income of  $r_t^k$ , depreciates at a rate of  $\delta$  and has a market value of  $q_t$ . The return on new capital is therefore  $\frac{r_{t+1}^k + (1-\delta)q_{t+1}}{p_t^I}$ . Entrepreneurs can borrow to invest. Borrowing is in the form of issuing equity,  $N_t^I$ , that entitles the holder to receive the future returns on the underlying capital goods. (ii) *Saving in private equity*. Household members can purchase the equity issued by other households,  $N_t^O$ , at the market price of  $q_t$ . As equity holders receive income from the underlying capital goods, the return on equity over  $t$  to  $t + 1$  is  $\frac{r_{t+1}^k + (1-\delta)q_{t+1}}{q_t}$ . The household's net equity is defined as its equity holdings plus capital stocks minus any equity it has issued:  $N_t \equiv N_t^O + K_t - N_t^I$ . (iii) *Saving in government bonds*. Household members may save in a risk-free domestic government bond,  $L_t$ , which has a unit face value and pays a gross nominal interest rate,  $R_t$ , over the period  $t$  to  $t + 1$ . In addition, savers have access to an international government bond,  $L_t^*$ , which is denominated in foreign currency and pays a gross nominal interest rate of  $R_t^*$  over  $t$  to  $t + 1$ .

International asset markets are incomplete. There is a risk premium  $\Gamma(\cdot)$  for trading foreign bonds so that the actual nominal yield received by a bond holder in the home country is  $R_t^*(1 - \Gamma(\cdot))$ . The size of the risk premium depends on the net foreign asset position in the following way:  $\Gamma\left(\frac{s_t L_t^*}{Y_t}\right) = \eta \left[ \exp\left(\frac{s_t L_t^*}{Y_t}\right) - 1 \right]$ , where  $\eta$  is the risk premium parameter;  $s_t \equiv S_t \frac{P_t^{C*}}{P_t^C}$  is the real exchange rate, defined as the ratio of foreign to domestic consumption prices expressed in the same currency;  $S_t$  is the nominal exchange rate, defined as the price of foreign currency in terms of home currency. The net foreign asset position is expressed relative to  $Y_t$ , the GDP of the home economy. It is assumed that domestic government bonds are not traded in foreign markets.

At the beginning of each period, the representative household also receives dividends from intermediate-goods firms and capital-goods firms amounting to  $D_t$  and  $D_t^K$  re-

spectively. The household pays lump-sum taxes,  $\tau_t$ , to the government. Taxes are lump-sum so that they are non-distortive. The intertemporal budget constraint is:<sup>16</sup>

$$\begin{aligned}
& C_t + p_t^I I_t + q_t [N_t - I_t] + L_t + s_t L_t^* \\
= & [r_t^k + (1 - \delta) q_t] N_{t-1} + \frac{R_{t-1}}{\pi_t^C} L_{t-1} + \frac{R_{t-1}^* [1 - \Gamma(\cdot)]}{\pi_t^{C^*}} s_t L_{t-1}^* + \int_{\varkappa}^1 \frac{W_t(j)}{P_t^C} H_t(j) dj \\
& + D_t + D_t^K - \tau_t
\end{aligned} \tag{A.2}$$

where  $W_t(j)$  is the nominal wage for type- $j$  workers,  $\pi_t^C \equiv \frac{P_t^C}{P_{t-1}^C}$  and  $\pi_t^{C^*} \equiv \frac{P_t^{C^*}}{P_{t-1}^{C^*}}$  are the gross CPI inflation rates in the home economy and the rest of the world respectively. Entrepreneurs and workers face different problems as explained below.

### A.1.2 Entrepreneurs

In the steady state and the post-shock equilibria, the market price of equity  $q_t$  is always greater than the investment cost of new capital  $p_t^I$ . Hence, the return on new capital ( $\frac{r_{t+1}^k + (1-\delta)q_{t+1}}{p_t^I}$ ) is strictly greater than the return on equity ( $\frac{r_{t+1}^k + (1-\delta)q_{t+1}}{q_t}$ ) and the real return on government bonds. Entrepreneurs are rational and would invest all their available resources in new capital. However, they are subject to liquidity constraints which limit their ability to invest. As in DEFK, entrepreneurs are precluded from taking short positions on government bonds so that both  $L_t(j)$  and  $L_t^*(j)$  must be non-negative for  $j \in [0, \varkappa]$ .<sup>17</sup> To obtain funds for investment, rational entrepreneurs would sell all their domestic and foreign government bond holdings, leading to  $L_t(j) = L_t^*(j) = 0$ . If entrepreneurs want to obtain funds through equity, they are bound by borrowing and resaleability constraints: Entrepreneurs can borrow by issuing equity of only up to  $\theta \in (0, 1)$  fraction of their new investment. Also, in each period, entrepreneurs can sell only up to  $\phi_t \in (0, 1)$  fraction of their net equity holdings. Since both borrowing and resaleability constraints are binding, entrepreneurs' net equity evolves according to  $N_t(j) = (1 - \phi_t)(1 - \delta)N_{t-1}(j) +$

<sup>16</sup>In this paper, stock variables at  $t$  show the amounts of stocks at the *end* of the period. This is different from the timing convention in DEFK (2011). In their paper, stock variables at  $t$  are defined as the amounts at the *beginning* of the period.

<sup>17</sup>This assumption is important in ensuring determinacy in the model. Without this assumption, liquidity-constrained entrepreneurs would be able to obtain infinite liquidity by taking negative positions on government bonds. Note that although entrepreneurs are precluded from taking short positions on foreign bonds, the net foreign asset position for the home country as a whole can be negative.

$(1 - \theta)I_t(j)$ . To spare more funds for investment, entrepreneurs choose not to spend on consumption goods, giving  $C_t(j) = 0$  for  $j \in [0, \varkappa)$ . Combining entrepreneurs' first order conditions for  $C_t(j)$ ,  $L_t(j)$ ,  $L_t^*(j)$  and  $N_t(j)$  with the intertemporal budget constraint (A.2) gives the expression for aggregate investment  $I_t = \int_0^{\varkappa} I_t(j) dj$ :

$$I_t = \varkappa \frac{[r_t^k + (1 - \delta) q_t \phi_t] N_{t-1} + \frac{R_{t-1}}{\pi_t^C} L_{t-1} + \frac{R_{t-1}^* [1 - \Gamma(\cdot)]}{\pi_t^{C^*}} s_t L_{t-1}^* + D_t + D_t^K - \tau_t}{p_t^I - \theta q_t} \quad (\text{A.3})$$

Aggregate investment depends on the abundance of households' liquidity. By contrast, in a standard DSGE model without liquidity frictions, investment opportunities are not scarce. Investment in new capital simply provides the same rate of return as other forms of assets. In such models, investment expenditure is not affected by liquidity conditions.

### A.1.3 Workers

After solving the entrepreneurs' problem, workers' consumption and saving decisions can be derived by considering the household as a whole. Workers choose  $C_t$ ,  $L_t$ ,  $L_t^*$  and  $N_t$  to maximise the household's utility (A.1), subject to the intertemporal budget constraint (A.2) and entrepreneurs' investment decision (A.3). The first-order conditions give the respective Euler equations for domestic government bonds, private equity and foreign government bonds:

$$C_t^{-\sigma} = \beta E_t \left\{ C_{t+1}^{-\sigma} \left[ \frac{R_t}{\pi_{t+1}^C} + \frac{\varkappa (q_{t+1} - p_{t+1}^I)}{p_{t+1}^I - \theta q_{t+1}} \frac{R_t}{\pi_{t+1}^C} \right] \right\} \quad (\text{A.4})$$

$$C_t^{-\sigma} = \beta E_t \left\{ C_{t+1}^{-\sigma} \left[ \frac{r_{t+1}^k + (1 - \delta) q_{t+1}}{q_t} + \frac{\varkappa (q_{t+1} - p_{t+1}^I)}{p_{t+1}^I - \theta q_{t+1}} \frac{r_{t+1}^k + (1 - \delta) q_{t+1} \phi_{t+1}}{q_t} \right] \right\} \quad (\text{A.5})$$

$$C_t^{-\sigma} = \beta E_t \left\{ C_{t+1}^{-\sigma} \left[ \frac{R_t^* [1 - \Gamma(\cdot)] s_{t+1}}{\pi_{t+1}^{C^*} s_t} + \frac{\varkappa (q_{t+1} - p_{t+1}^I)}{p_{t+1}^I - \theta q_{t+1}} \frac{R_t^* [1 - \Gamma(\cdot)] s_{t+1}}{\pi_{t+1}^{C^*} s_t} \right] \right\} \quad (\text{A.6})$$

These Euler equations reduce to the standard ones if  $\varkappa = 0$ . In the DEFK model, holders of bonds or equity receive a premium on top of the standard returns because

households are liquidity-constrained. By choosing to hold one extra unit of government bonds instead of consumption at  $t$ , a holder of domestic bonds gains  $\frac{R_t}{\pi_{t+1}^C}$  extra units of liquidity at  $t + 1$ , whereas a holder of foreign bonds gets  $\frac{R_t^*[1-\Gamma(\cdot)]s_{t+1}}{\pi_{t+1}^{C*}s_t}$  over the same period. By choosing to hold one extra unit of equity at  $t$  instead of spending, an equity holder gains  $\frac{r_{t+1}^k+(1-\delta)q_{t+1}\phi_{t+1}}{q_t}$  extra units of liquidity at  $t + 1$ . The extra liquidity allows them to profit from an investment opportunity when it arrives at  $t + 1$  if they become entrepreneurs. Euler equations (A.4) and (A.6) give rise to the uncovered interest parity:

$$\frac{R_t}{E_t(\pi_{t+1}^C)} = E_t\left(\frac{R_t^*[1-\Gamma(\cdot)]s_{t+1}}{\pi_{t+1}^{C*}}\frac{s_{t+1}}{s_t}\right) \quad (\text{A.7})$$

The presence of a risk premium  $\Gamma(\cdot)$  for trading foreign bonds creates a wedge between the domestic and the foreign risk-free interest rates. The size of this risk premium determines the home country's ease of access to international capital markets.

Workers' wage-setting assumptions are standard New Keynesian. Differentiated workers  $j \in [\varkappa, 1]$  supply labour  $H_t(j)$  to the production sector through the arrangement of employment agencies. Employment agencies combine  $H_t(j)$  into homogeneous units of labour input,  $H_t$ , according to:<sup>18</sup>

$$H_t = \left[ \left( \frac{1}{1-\varkappa} \right)^{\frac{\lambda_\omega}{1+\lambda_\omega}} \int_{\varkappa}^1 H_t(j)^{\frac{1}{1+\lambda_\omega}} dj \right]^{1+\lambda_\omega} \quad (\text{A.8})$$

They choose the profit-maximising amount of  $H_t(j)$  to hire, taking nominal wages  $W_t(j)$  as given. Accordingly, the demand for type- $j$  labour is:

$$H_t(j) = \frac{1}{1-\varkappa} \left[ \frac{W_t(j)}{W_t} \right]^{-\frac{1+\lambda_\omega}{\lambda_\omega}} H_t, \quad (\text{A.9})$$

where  $\lambda_\omega > 0$  and  $W_t = \left[ \frac{1}{1-\varkappa} \int_{\varkappa}^1 W_t(j)^{-\frac{1}{\lambda_\omega}} dj \right]^{-\lambda_\omega}$  is the aggregate nominal wage index. Each type- $j$  labour is represented by a labour union who sets their nominal wage  $W_t(j)$  optimally on a staggered basis. Each period, there is a history-independent probability of  $(1 - \zeta_\omega)$  for a union to reset their wage. Otherwise, unions keep their

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<sup>18</sup>Following DEFK, the term  $\frac{1}{1-\varkappa}$  is added to the labour aggregate to simplify the notations without changing the substance.

nominal wages constant. Workers' optimal wage-setting equation, in real terms, is:

$$E_t \sum_{s=t}^{\infty} (\beta \zeta_{\omega})^{s-t} C_s^{-\sigma} \left\{ \frac{\tilde{w}_t}{\pi_{t,s}^C} - (1 + \lambda_{\omega}) \frac{H_s(j)^v}{C_s^{-\sigma}} \right\} H_s(j) = 0, \quad (\text{A.10})$$

where  $\tilde{w}_t \equiv \frac{\tilde{W}_t}{\tilde{P}_t^C}$  is the optimal wage chosen by a wage-setting labour union at  $t$  and  $\pi_{t,s}^C \equiv \begin{cases} 1, & \text{for } s = t \\ \pi_{t+1}^C \pi_{t+2}^C \dots \pi_s^C, & \text{for } s \geq t + 1 \end{cases}$ .  $H_s(j) = \frac{1}{1-\varkappa} \left( \frac{\tilde{w}_t}{\pi_{t,s}^C w_s} \right)^{-\frac{1+\lambda_{\omega}}{\lambda_{\omega}}}$   $H_s$  is the labour supply by type- $j$  labour, where  $w_t \equiv \frac{W_t}{P_t^C}$  is the aggregate real wage index. The zero-profit condition for employment agencies gives rise to the dynamics of  $w_t$ :

$$w_t^{-\frac{1}{\lambda_{\omega}}} = (1 - \zeta_{\omega}) \tilde{w}_t^{-\frac{1}{\lambda_{\omega}}} + \zeta_{\omega} \left( \frac{w_{t-1}}{\pi_t^C} \right)^{-\frac{1}{\lambda_{\omega}}} \quad (\text{A.11})$$

#### A.1.4 Intermediate-Goods Firms

The home country consists of a continuum of monopolistically competitive intermediate-goods firms  $i \in [0, 1]$ . Each intermediate-goods firm produces according to the production function  $Y_t(i) = A_t K_t(i)^{\gamma} H_t(i)^{1-\gamma}$ , where  $A_t$  is productivity and  $\gamma \in [0, 1]$  is the capital share. Firms maximise their profits by choosing the optimal capital and labour inputs, taking the wage and rental rate of capital as given. The cost-minimising conditions imply that their real marginal cost,  $mc_t \equiv \frac{MC_t}{P_t^C}$ , is:

$$mc_t = mc_t(i) = \frac{1}{A_t} \left( \frac{w_t}{1-\gamma} \right)^{1-\gamma} \left( \frac{r_t^k}{\gamma} \right)^{\gamma}, \quad (\text{A.12})$$

which is the same across firms despite differences in their output levels. Intermediate-goods firms also set nominal prices for their goods. In each period, each firm has a probability of  $(1 - \zeta_p)$  to reset their prices. Otherwise, they keep their prices unchanged. They charge different prices at home and abroad. Firms that reset their prices in the domestic market maximise their expected future profits:

$$E_t \sum_{s=t}^{\infty} (\beta \zeta_p)^{s-t} \left( \frac{C_s}{C_t} \right)^{-\sigma} \left[ \tilde{P}_t^H(i) - MC_s \right] Y_s^H(i), \quad (\text{A.13})$$

where  $\tilde{P}_t^H(i)$  is the optimal nominal price chosen by price-resetting firm  $i$  at  $t$ . Similarly, firms that reset their prices in the foreign market maximise:

$$E_t \sum_{s=t}^{\infty} (\beta \zeta_p)^{s-t} \left( \frac{C_s}{C_t} \right)^{-\sigma} \left[ S_s \tilde{P}_t^{H*}(i) - MC_s \right] X_s(i), \quad (\text{A.14})$$

where  $\tilde{P}_t^{H*}(i)$  is the export price in foreign currency that firm  $i$  optimally chooses to charge at  $t$ ;  $X_t(i)$  is the amount that it exports to the foreign market.

In the home market, the demand for firm  $i$ 's intermediate good is given by  $Y_t^H(i) = \left[ \frac{P_t^H(i)}{P_t^H} \right]^{-\frac{1+\lambda_f}{\lambda_f}} Y_t^H$ , where  $\lambda_f > 0$ ,  $P_t^H(i)$  is the price charged by firm  $i$  and  $P_t^H$  is the aggregate domestic price index for aggregate domestic demand,  $Y_t^H \equiv \left[ \int_0^1 Y_t^H(i)^{\frac{1}{1+\lambda_f}} di \right]^{1+\lambda_f}$ .

In the foreign market, the demand for firm  $i$ 's exports is:  $X_t(i) = \left[ \frac{P_t^{H*}(i)}{P_t^{H*}} \right]^{-\frac{1+\lambda_f}{\lambda_f}} X_t$ , where  $P_t^{H*}$  is the aggregate price index for home country's exports,  $X_t$ .

Accordingly, intermediate-goods firms' optimal price-setting equations at home and abroad, respectively, are:

$$E_t \sum_{s=t}^{\infty} (\beta \zeta_p)^{s-t} C_s^{-\sigma} \left\{ \frac{\tilde{P}_t^H}{P_t^H \pi_{t,s}^H} - (1 + \lambda_f) \frac{P_s^C m c_s}{P_s^H} \right\} \left( \frac{\tilde{P}_t^H}{P_t^H \pi_{t,s}^H} \right)^{-\frac{1+\lambda_f}{\lambda_f}} Y_s^H = 0 \quad (\text{A.15})$$

$$E_t \sum_{s=t}^{\infty} (\beta \zeta_p)^{s-t} C_s^{-\sigma} \left\{ \frac{\tilde{P}_t^{H*}}{P_t^{H*} \pi_{t,s}^{H*}} - (1 + \lambda_f) \frac{P_s^C m c_s}{P_s^{H*} S_s} \right\} \left( \frac{\tilde{P}_t^{H*}}{P_t^{H*} \pi_{t,s}^{H*}} \right)^{-\frac{1+\lambda_f}{\lambda_f}} X_s = 0 \quad (\text{A.16})$$

where  $\pi_t^H \equiv \frac{P_t^H}{P_{t-1}^H}$  and  $\pi_t^{H*} \equiv \frac{P_t^{H*}}{P_{t-1}^{H*}}$ . The evolution of domestic price index and export price index are:

$$1 = (1 - \zeta_p) \left( \frac{\tilde{P}_t^H}{P_t^H} \right)^{-\frac{1}{\lambda_f}} + \zeta_p \left( \frac{1}{\pi_t^H} \right)^{-\frac{1}{\lambda_f}} \quad (\text{A.17})$$

$$1 = (1 - \zeta_p) \left( \frac{\tilde{P}_t^{H*}}{P_t^{H*}} \right)^{-\frac{1}{\lambda_f}} + \zeta_p \left( \frac{1}{\pi_t^{H*}} \right)^{-\frac{1}{\lambda_f}} \quad (\text{A.18})$$

### A.1.5 Final-Goods Firms

Final-goods firms combine differentiated intermediate goods into final goods. They are classified according to the type of final goods that they produce: private consumption goods, investment goods and public consumption goods. Private consumption goods are produced according to:

$$C_t = \left[ (1 - \alpha)^{\frac{1}{\mu}} (C_t^H)^{\frac{\mu-1}{\mu}} + \alpha^{\frac{1}{\mu}} (C_t^F)^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}} \quad (\text{A.19})$$

where  $C_t^H = \left[ \int_0^1 C_t^H(i)^{\frac{1}{1+\lambda_f}} di \right]^{1+\lambda_f}$  and  $C_t^F = \left[ \int_0^1 C_t^F(i^*)^{\frac{1}{1+\lambda_f}} di \right]^{1+\lambda_f}$  are the CES aggregators of home-produced and imported intermediate goods respectively.  $\alpha$  is the inverse of home bias in preferences, which can be interpreted as the trade openness index for the home economy.  $\mu > 0$  measures the elasticity of substitution between home and foreign goods. Final-goods firms first choose the optimal amounts of each intermediate good from firms  $i$  and  $i^*$ , and then choose the optimal allocation between home-produced and imported goods to maximise their profits. The respective quantities demanded for home-produced intermediate good  $i$  and for imported intermediate good  $i^*$  by private consumption final-goods firms are:

$$C_t^H(i) = \left[ \frac{P_t^H(i)}{P_t^H} \right]^{-\frac{1+\lambda_f}{\lambda_f}} C_t^H, \quad C_t^F(i^*) = \left[ \frac{P_t^F(i^*)}{P_t^F} \right]^{-\frac{1+\lambda_f}{\lambda_f}} C_t^F \quad (\text{A.20})$$

where  $P_t^F$  is the aggregate price index for the home country's imports. The optimal allocation between home-produced and imported goods demanded by private con-



sumption final-goods firms is:

$$C_t^H = (1 - \alpha) \left( \frac{P_t^H}{P_t^C} \right)^{-\mu} C_t, \quad C_t^F = \alpha \left( \frac{P_t^F}{P_t^C} \right)^{-\mu} C_t \quad (\text{A.21})$$

where

$$P_t^C = \left[ (1 - \alpha) (P_t^H)^{1-\mu} + \alpha (P_t^F)^{1-\mu} \right]^{\frac{1}{1-\mu}} \quad (\text{A.22})$$

Similarly, investment goods are produced according to:

$$I_t [1 + S(\cdot)] = \left[ (1 - \alpha)^{\frac{1}{\mu}} (I_t^H)^{\frac{\mu-1}{\mu}} + \alpha^{\frac{1}{\mu}} (I_t^F)^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}} \quad (\text{A.23})$$

where  $I_t^H = \left[ \int_0^1 I_t^H(i)^{\frac{1}{1+\lambda_f}} di \right]^{1+\lambda_f}$  and  $I_t^F = \left[ \int_0^1 I_t^F(i^*)^{\frac{1}{1+\lambda_f}} di \right]^{1+\lambda_f}$ . There is an adjustment cost  $S(\cdot)$  in the production of investment goods. The adjustment cost is quadratic in aggregate investment in a way that  $S(\frac{I_t}{I}) = \frac{\kappa}{2} \left( \frac{I_t}{I} - 1 \right)^2$ , where  $I$  is the steady-state investment and  $\kappa > 0$  is the adjustment cost parameter. Under this equation,  $S(1) = S'(1) = 0$  and  $S''(1) > 0$ . Capital-goods firms choose the amount of  $I_t$  to produce which maximises their profits,  $D_t^K = [p_t^I - (1 + S(\cdot))] I_t$ . Their first-order condition is:

$$p_t^I = 1 + S(\cdot) + S'(\cdot) \frac{I_t}{I} \quad (\text{A.24})$$

The profit-maximising conditions of the capital-goods firms yield the respective quantities demanded for home-produced intermediate good  $i$  and for imported intermediate good  $i^*$ :

$$I_t^H(i) = \left[ \frac{P_t^H(i)}{P_t^H} \right]^{-\frac{1+\lambda_f}{\lambda_f}} I_t^H, \quad I_t^F(i^*) = \left[ \frac{P_t^F(i^*)}{P_t^F} \right]^{-\frac{1+\lambda_f}{\lambda_f}} I_t^F \quad (\text{A.25})$$

The optimal allocation between home-produced and imported goods demanded by capital-goods firms is:

$$I_t^H = (1 - \alpha) \left( \frac{P_t^H}{P_t^C} \right)^{-\mu} I_t [1 + S(\cdot)], \quad (\text{A.26})$$

$$I_t^F = \alpha \left( \frac{P_t^F}{P_t^C} \right)^{-\mu} I_t [1 + S(\cdot)] \quad (\text{A.27})$$

The government consumes only home-produced goods. Public consumption-goods firms choose the optimal amounts of each intermediate good  $i$  to use in the production of public consumption goods, which are bundled according to  $G_t = \left[ \int_0^1 G_t(i)^{\frac{1}{1+\lambda_f}} di \right]^{1+\lambda_f}$ . The demand for intermediate good  $i$  by public consumption final-goods firms is:

$$G_t(i) = \left[ \frac{P_t^H(i)}{P_t^H} \right]^{-\frac{1+\lambda_f}{\lambda_f}} G_t \quad (\text{A.28})$$

The demand for intermediate good  $i$  in the home market,  $Y_t^H(i)$ , thus equals  $C_t^H(i) + I_t^H(i) + G_t(i)$ . The home country's demand for imported good type- $i^*$ ,  $M_t(i^*)$ , is  $C_t^F(i^*) + I_t^F(i^*)$ .

### A.1.6 Government Policies

The government's budget constraint is:

$$G_t + \frac{R_{t-1}}{\pi_t^C} L_{t-1} = \tau_t + L_t, \quad (\text{A.29})$$

so that the government needs to finance its consumption and interest expenditures either by taxes or bond issues. In addition, the fiscal rule requires taxes to be proportional to the government's debt at the beginning of each period:

$$\tau_t - \tau = \psi_\tau \left[ \left( \frac{R_{t-1}}{\pi_t^C} L_{t-1} - \frac{R}{\pi^C} L \right) \right], \quad (\text{A.30})$$

where the policy parameter  $\psi_\tau > 0$ . The variables without a time subscript represent steady-state values. The value of  $\psi_\tau$  is low to reflect that the adjustment on taxes is slow compared to bond issue, so the government has to obtain funds for fiscal expansion mainly by issuing bonds.

The central bank adopts the Taylor (1993) rule in conducting monetary policy, such that the nominal interest rate responds to both CPI inflation and output:

$$R_t = \max \left\{ R (\pi_t^C)^{\psi_\pi} \left( \frac{Y_t}{Y} \right)^{\psi_Y}, 1 \right\} \quad (\text{A.31})$$

where  $\psi_\pi > 1$ , and  $\psi_Y \in (0, 1)$ . The zero lower bound on the nominal interest rate requires that  $R_t$  cannot be lower than 1.

### A.1.7 Aggregation and Resource Constraint

The market clears for both labour and capital so that  $H_t = \int_0^1 H_t(i) di$  and  $K_{t-1} = \int_0^1 K_t(i) di$ . Firms' optimal capital-labour ratio implies:

$$\frac{K_{t-1}}{H_t} = \frac{\gamma}{(1-\gamma)} \frac{w_t}{r_t^k} \quad (\text{A.32})$$

and the aggregate production function is:

$$A_t K_{t-1}^\gamma H_t^{1-\gamma} = \int_0^1 Y_t(i) di \quad (\text{A.33})$$

All capital in the domestic economy is owned by households through their private equity holdings:

$$K_t = N_t \quad (\text{A.34})$$

Capital evolves according to:

$$K_t = (1 - \delta) K_{t-1} + I_t \quad (\text{A.35})$$

The resource constraint of the economy requires that:

$$Y_t = C_t^H + I_t^H + G_t + X_t \quad (\text{A.36})$$

where the aggregate domestic demand for home-produced goods is:

$$Y_t^H = C_t^H + I_t^H + G_t \quad (\text{A.37})$$

The aggregate demand for imports by the home country is:

$$M_t = C_t^F + I_t^F \quad (\text{A.38})$$

Net foreign asset holdings evolve according to:

$$s_t L_t^* = \frac{R_{t-1}^* [1 - \Gamma(\cdot)]}{\pi_t^{C^*}} s_t L_{t-1}^* + s_t p_t^{H^*} X_t - p_t^F M_t \quad (\text{A.39})$$

The profits for intermediate-goods and capital-goods firms are wholly distributed to households as dividends. Replacing  $D_t$  and  $D_t^K$ , (A.3) becomes:

$$I_t = \varkappa \frac{\left\{ \begin{array}{l} [r_t^k + (1 - \delta) q_t \phi_t] N_{t-1} + r_{t-1} L_{t-1} + r_{t-1}^* [1 - \Gamma(\cdot)] s_t L_{t-1}^* \\ + p_t^H Y_t^H + s_t p_t^{H^*} X_t - w_t H_t - r_t^k K_{t-1} + p_t^I I_t - [1 + S(\cdot)] I_t - \tau_t \end{array} \right\}}{p_t^I - \theta q_t} \quad (\text{A.40})$$

where  $r_t = \frac{R_t}{E_t(\pi_{t+1}^C)}$  and  $r_t^* = \frac{R_t^*}{E_t(\pi_{t+1}^{C^*})}$  are the gross real interest rates on domestic and foreign government bonds respectively.

Further, the real prices of home-produced and imported goods are expressed relative to the consumption price index in the home country:

$$p_t^H \equiv \frac{P_t^H}{P_t^C}, \quad p_t^F \equiv \frac{P_t^F}{P_t^C}, \quad (\text{A.41})$$

whereas the real price of exports is expressed relative to the CPI in the foreign economy:

$$p_t^{H^*} \equiv \frac{P_t^{H^*}}{P_t^{C^*}} \quad (\text{A.42})$$

### A.1.8 Rest of the World

The rest of the world is modelled as one large economy, populated with a continuum of monopolistically competitive firms  $i^* \in [0, 1]$ . Prices are set in a similar manner to that in the small open economy. Each period, there is a Calvo probability  $(1 - \zeta_p)$

for firm  $i^*$  to reset the prices of their goods. Firms resetting the price of their exports at  $t$  choose the one that maximises their expected future profits:

$$E_t \sum_{s=t}^{\infty} (\beta \zeta_p)^{s-t} \left( \frac{C_s^*}{C_t^*} \right)^{-\sigma} \left[ \frac{\tilde{P}_t^F(i^*)}{S_s} - MC_s^* \right] M_s(i^*), \quad (\text{A.43})$$

where  $\tilde{P}_t^F(i^*)$  is the nominal price in domestic currency that is optimally chosen by firm  $i^*$  at  $t$ ;  $MC_t^*$  is the world nominal marginal cost. Activities of the small open economy have no impacts on the rest of the world. Therefore, the world variables  $Y_t^*$ ,  $R_t^*$ ,  $\pi_t^{C^*}$  and  $MC_t^*$  are taken as exogenous by the small open economy.<sup>19</sup> The small open economy's exports,  $X_t$ , are increasing with the world economy's aggregate output,  $Y_t^*$ :

$$X_t = \alpha \left( \frac{P_t^{H^*}}{P_t^{C^*}} \right)^{-\mu} Y_t^* \quad (\text{A.44})$$

## A.2 Steady State

We assume there is no inflation at steady state so that  $\pi^C = \pi^{C^*} = 1$ ,  $R = r$  and  $R^* = r^*$ . The uncovered interest rate parity implies  $r = r^*$ . We normalise the relative prices for home-produced and imported goods by setting  $p^H = p^{H^*} = p^F = 1$  and  $s = 1$ . In addition, we assume that the home country's foreign bond holdings  $L^*$  is zero at steady state, so that  $\Gamma\left(\frac{sL^*}{Y}\right) = 0$ .

Since prices are the same at steady state,  $C^H = (1 - \alpha)C$  and  $C^F = \alpha C$ . The capital adjustment cost  $S(1) = S'(1) = 0$ , implying  $I^H = (1 - \alpha)I$  and  $I^F = \alpha I$ . Total imports,  $M = C^F + I^F = \alpha(C + I)$ . Following Galí and Monacelli (2005), we assume zero net exports at steady state, i.e.  $X - M = 0$ . The resource constraint in the home country therefore becomes:

$$Y = C + I + G \quad (\text{A.45})$$

The first order condition for capital-goods producers (A.24) implies  $p^I = 1$ . Euler

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<sup>19</sup>The rest of the world is modelled as one large closed economy. The linearised equations for the world economy are included in the Appendix.

equations (A.4) and (A.5) reduce to:

$$\frac{1}{\beta} = r \left( 1 + \varkappa \frac{q-1}{1-\theta q} \right) \quad (\text{A.46})$$

$$\text{and } \frac{1}{\beta} = \frac{r^k + (1-\delta)q}{q} \left[ 1 + \frac{\varkappa(q-1)}{1-\theta q} \right] - \frac{\varkappa(q-1)(1-\delta)(1-\phi)}{1-\theta q} \quad (\text{A.47})$$

respectively. Equation (A.10) suggests that the real wage is simply a markup over the marginal rate of substitution between labour and consumption:

$$w = (1 + \lambda_\omega) \frac{\left[ \frac{1}{1-\varkappa} H \right]^v}{C^{-\sigma}} \quad (\text{A.48})$$

The optimal capital-labour ratio (A.32) becomes:

$$\frac{K}{H} = \frac{\gamma}{(1-\gamma)} \frac{w}{r^k} \quad (\text{A.49})$$

The price-setting equation (A.15) and the marginal cost function (A.12) suggest that:

$$mc = \frac{1}{1 + \lambda_f} = \frac{1}{A} \left( \frac{w}{1-\gamma} \right)^{1-\gamma} \left( \frac{r^k}{\gamma} \right)^\gamma \quad (\text{A.50})$$

Combining (A.49) and (A.50) with the aggregate production function gives:

$$\frac{Y}{K} = \frac{(1 + \lambda_f) r^k}{\gamma} \quad (\text{A.51})$$

which implies that firms would choose the amount of  $K$  so that the marginal product of capital  $\gamma \frac{Y}{K}$  equals to a markup over the capital rental rate  $r^k$ .

The government's budget constraint (A.29) suggests:

$$\tau = (r-1)L + G \quad (\text{A.52})$$

Using  $K = N$ , aggregate investment (A.40) at steady state is:

$$I = \varkappa \frac{\left[ r^k + (1-\delta)q\phi \right] K + rL + Y - wH - r^k K - \tau}{1-\theta q} \quad (\text{A.53})$$

where the aggregate profits of intermediate-goods firms  $(Y - wH - r^k K)$  is simply  $(Y - \frac{1}{1+\lambda_f} Y) = \frac{\lambda_f}{1+\lambda_f} Y$  because of the fact that  $mc = \frac{1}{1+\lambda_f}$ . Since the evolution of capital at steady state is  $I = \delta K$ , we can rewrite equation (A.53) as:

$$\delta - [\delta\theta + \varkappa(1 - \delta)\phi]q = \varkappa \left\{ 1 + \frac{(1 + \lambda_f)L}{\gamma Y} + \frac{\lambda_f}{\gamma} \right\} r^k - \varkappa \frac{G}{K} \quad (\text{A.54})$$

where  $\frac{L}{Y}$  is the steady-state public debt-to-GDP ratio, which is calibrated.

### A.3 The Log-linear Model

The log-deviation of variable  $x_t$  from steady state is denoted as  $\hat{x}_t \equiv \ln(x_t/x)$ . After log-linearization, the Euler equation for domestic bonds (A.4) becomes:

$$\begin{aligned} -\sigma\hat{C}_t &= -\sigma E_t(\hat{C}_{t+1}) + \hat{R}_t - E_t(\hat{\pi}_{t+1}^C) \\ &\quad + \beta r \varkappa \frac{1 - \theta}{(1 - \theta q)^2} q E_t(\hat{q}_{t+1}) - \beta r \varkappa \frac{1 - \theta}{(1 - \theta q)^2} q E_t(\hat{p}_{t+1}^I) \end{aligned} \quad (\text{A.55})$$

The Euler equation for equity (A.5) becomes:

$$\begin{aligned} \hat{q}_t - \sigma\hat{C}_t &= -\sigma E_t(\hat{C}_{t+1}) + \beta \frac{r^k}{q} \left[ 1 + \frac{\varkappa(q-1)}{1-\theta q} \right] E_t(\hat{r}_{t+1}^k) + \beta \frac{\varkappa(q-1)(1-\delta)\phi}{1-\theta q} E_t(\hat{\phi}_{t+1}) \\ &\quad + \beta \left[ (1-\delta) + \frac{\varkappa(q-1)(1-\delta)\phi}{1-\theta q} + \varkappa [r^k + (1-\delta)q\phi] \frac{1-\theta}{(1-\theta q)^2} \right] E_t(\hat{q}_{t+1}) \\ &\quad - \beta \varkappa [r^k + (1-\delta)q\phi] \frac{1-\theta}{(1-\theta q)^2} E_t(\hat{p}_{t+1}^I) \end{aligned} \quad (\text{A.56})$$

Define the home country's net foreign asset position,  $\hat{F}_t \equiv \frac{s_t L_t^*}{Y_t}$ . The uncovered interest rate parity (A.7) after log-linearisation becomes:

$$\hat{R}_t - E_t(\hat{\pi}_{t+1}^C) = \hat{R}_t^* - E_t(\hat{\pi}_{t+1}^{C*}) + E_t(\hat{s}_{t+1}) - \hat{s}_t - \eta \hat{F}_t \quad (\text{A.57})$$

The wage-setting condition (A.10) is:

$$\begin{aligned} & \left(1 + v \frac{1 + \lambda_\omega}{\lambda_\omega}\right) \widehat{w}_t - v(1 - \beta\zeta_\omega) \frac{1 + \lambda_\omega}{\lambda_\omega} \widehat{w}_t \\ = & (1 - \beta\zeta_\omega) \left(\sigma \widehat{C}_t + v \widehat{H}_t\right) + \beta\zeta_\omega \left(1 + v \frac{1 + \lambda_\omega}{\lambda_\omega}\right) E_t \left(\widehat{w}_{t+1} + \widehat{\pi}_{t+1}^C\right) \end{aligned} \quad (\text{A.58})$$

The evolution of real wages (A.11) becomes:

$$\widehat{w}_t = (1 - \zeta_\omega) \widehat{w}_t + \zeta_\omega (\widehat{w}_{t-1} - \widehat{\pi}_t^C) \quad (\text{A.59})$$

The marginal cost (A.12) of the intermediate-goods firms is:

$$\widehat{m}c_t = (1 - \gamma) \widehat{w}_t + \gamma \widehat{r}_t^k - \widehat{A}_t \quad (\text{A.60})$$

The price-setting condition (A.15) together with (A.17) yields the Phillip's curve for home-produced goods:

$$\widehat{\pi}_t^H = \frac{(1 - \zeta_p)(1 - \beta\zeta_p)}{\zeta_p} (\widehat{m}c_t - \widehat{p}_t^H) + \beta E_t (\widehat{\pi}_{t+1}^H) \quad (\text{A.61})$$

Analogously, the first order condition for  $\widehat{p}_t^{H*}$  (A.16) together with (A.18) yields the Phillip's curve for the home country's exports:

$$\widehat{\pi}_t^{H*} = \frac{(1 - \zeta_p)(1 - \beta\zeta_p)}{\zeta_p} (\widehat{m}c_t - \widehat{s}_t - \widehat{p}_t^{H*}) + \beta E_t (\widehat{\pi}_{t+1}^{H*}) \quad (\text{A.62})$$

Foreign firms' price-setting behaviour gives rise to the Phillip's curve for the small open economy's imports:

$$\widehat{\pi}_t^F = \frac{(1 - \zeta_p)(1 - \beta\zeta_p)}{\zeta_p} (\widehat{m}c_t^* + \widehat{s}_t - \widehat{p}_t^F) + \beta E_t (\widehat{\pi}_{t+1}^F) \quad (\text{A.63})$$

In the sensitivity analysis, under the law of one price, firms charge the same price in the rest of the world as in the domestic market, such that  $P_t^F = S_t P_t^{C*}$  and  $P_t^{H*} = \frac{P_t^H}{S_t}$ . These equations replace the Phillips curves for the home country's exports and imports, (A.62) and (A.63).



Equation (A.21) suggests that the demand for home-produced and imported consumption goods, respectively, are:

$$\widehat{C}_t^H = -\mu\widehat{p}_t^H + \widehat{C}_t \quad (\text{A.64})$$

$$\widehat{C}_t^F = -\mu\widehat{p}_t^F + \widehat{C}_t \quad (\text{A.65})$$

The consumption price index (A.22) becomes:

$$(1 - \alpha)\widehat{p}_t^H + \alpha\widehat{p}_t^F = 0 \quad (\text{A.66})$$

The profit-maximising condition for capital-goods producers (A.24) becomes:

$$\widehat{p}_t^J = \kappa\widehat{I}_t \quad (\text{A.67})$$

(A.26) and (A.27) are log-linearised to give:

$$\widehat{I}_t^H = -\mu\widehat{p}_t^H + \widehat{I}_t \quad (\text{A.68})$$

$$\widehat{I}_t^F = -\mu\widehat{p}_t^F + \widehat{I}_t \quad (\text{A.69})$$

The government's budget constraint (A.29) becomes:

$$\tau\widehat{\tau}_t = rL \left[ \widehat{R}_{t-1} + \widehat{L}_{t-1} - \widehat{\pi}_t^C \right] - L\widehat{L}_t + G\widehat{G}_t \quad (\text{A.70})$$

The fiscal rule (A.30) is:

$$\tau\widehat{\tau}_t = \psi_\tau \left[ rL \left( \widehat{R}_{t-1} + \widehat{L}_{t-1} - \widehat{\pi}_t^C \right) \right] \quad (\text{A.71})$$

The monetary policy rule (A.31) becomes:

$$\widehat{R}_t = \max \left[ \left( \psi_\pi \widehat{\pi}_t^C + \psi_Y \widehat{Y}_t \right), -\log(R) \right] \quad (\text{A.72})$$

The optimal capital-labour ratio (A.32) implies:

$$\widehat{K}_{t-1} = \widehat{w}_t - \widehat{r}_t^k + \widehat{H}_t \quad (\text{A.73})$$

The aggregate production function (A.33) becomes:

$$\widehat{Y}_t = \widehat{A}_t + \gamma \widehat{K}_{t-1} + (1 - \gamma) \widehat{H}_t \quad (\text{A.74})$$

The ownership of capital (A.34) suggests:

$$\widehat{K}_t = \widehat{N}_t \quad (\text{A.75})$$

The law of motion of capital (A.35) is simply:

$$\widehat{K}_t = (1 - \delta) \widehat{K}_{t-1} + \delta \widehat{I}_t \quad (\text{A.76})$$

The resource constraint (A.36) is:

$$\widehat{Y}_t = (1 - \alpha) \frac{C}{Y} \widehat{C}_t^H + (1 - \alpha) \frac{I}{Y} \widehat{I}_t^H + \frac{G}{Y} \widehat{G}_t + \frac{X}{Y} \widehat{X}_t \quad (\text{A.77})$$

The home demand for domestic-produced goods (A.37) becomes:

$$(Y - X) \widehat{Y}_t^H = (1 - \alpha) C \widehat{C}_t^H + (1 - \alpha) I \widehat{I}_t^H + G \widehat{G}_t \quad (\text{A.78})$$

The aggregate demand for imports by the small open economy (A.38) becomes:

$$(C + I) \widehat{M}_t = C \widehat{C}_t^F + I \widehat{I}_t^F \quad (\text{A.79})$$

The aggregate demand for the small open economy's exports (A.44) becomes:

$$\widehat{X}_t = -\mu \widehat{p}_t^{H*} + \widehat{Y}_t^* \quad (\text{A.80})$$

The evolution of net foreign assets (A.39) becomes:

$$\widehat{F}_t = R^* \widehat{F}_{t-1} + \frac{X}{Y} \left( \widehat{s}_t + \widehat{p}_t^{H*} + \widehat{X}_t \right) - \frac{M}{Y} \left( \widehat{p}_t^F + \widehat{M}_t \right) \quad (\text{A.81})$$

The aggregate investment function (A.40) is:

$$\begin{aligned}
& \delta(1-\theta q)\widehat{I}_t + \delta(1-\varkappa)\widehat{p}_t^I - [\delta\theta + \varkappa(1-\delta)\phi]q\widehat{q}_t - \varkappa(1-\delta)q\phi\widehat{\phi}_t \\
= & \varkappa[r^k + (1-\delta)q\phi]\widehat{N}_{t-1} + \varkappa r\frac{L}{K}\left[\widehat{R}_{t-1} + \widehat{L}_{t-1} - \widehat{\pi}_t^C\right] + \varkappa r^*\frac{Y}{K}\widehat{F}_{t-1} + \varkappa\frac{Y^H}{K}\left(\widehat{p}_t^H + \widehat{Y}_t^H\right) \\
& + \varkappa\frac{X}{K}\left(\widehat{s}_t + \widehat{p}_t^{H*} + \widehat{X}_t\right) - \frac{\varkappa(1-\gamma)r^k}{\gamma}\left[\widehat{w}_t + \widehat{H}_t\right] - \varkappa r^k\widehat{K}_{t-1} - \varkappa\frac{\tau}{K}\widehat{\tau}_t
\end{aligned} \tag{A.82}$$

The definition of relative prices (A.41) and (A.42) gives:

$$\widehat{\pi}_t^H = \widehat{\pi}_t^C + \widehat{p}_t^H - \widehat{p}_{t-1}^H \tag{A.83}$$

$$\widehat{\pi}_t^F = \widehat{\pi}_t^C + \widehat{p}_t^F - \widehat{p}_{t-1}^F \tag{A.84}$$

$$\widehat{\pi}_t^{H*} = \widehat{\pi}_t^{C*} + \widehat{p}_t^{H*} - \widehat{p}_{t-1}^{H*} \tag{A.85}$$

In the world economy, the Euler equation gives:

$$-\sigma\widehat{Y}_t^* = -\sigma E_t\left(\widehat{Y}_{t+1}^*\right) + \widehat{R}_t^* - E_t\left(\widehat{\pi}_{t+1}^{C*}\right) \tag{A.86}$$

The Phillip's curve for the world economy is derived in the same way as for a closed economy:

$$\widehat{\pi}_t^{C*} = \frac{(1-\zeta_p)(1-\beta\zeta_p)}{\zeta_p}\widehat{m}c_t^* + \beta E_t\left(\widehat{\pi}_{t+1}^{C*}\right) \tag{A.87}$$

As in Gali and Monacelli (2005),  $\widehat{m}c_t^*$  is increasing with world output in the following way:

$$\widehat{m}c_t^* = (\sigma + v)\widehat{Y}_t^* \tag{A.88}$$

The world nominal interest rate is set according to the standard Taylor (1993) rule:

$$\widehat{R}_t^* = \psi_\pi\widehat{\pi}_t^{C*} + \psi_Y\widehat{Y}_t^* \tag{A.89}$$

The linearised uncovered interest rate parity (A.57) is defined in the text. The Fisher equation suggests:

$$\widehat{r}_t = \widehat{R}_t - E_t\left(\widehat{\pi}_{t+1}^C\right) \tag{A.90}$$

$$\widehat{r}_t^* = \widehat{R}_t^* - E_t\left(\widehat{\pi}_{t+1}^{C*}\right) \tag{A.91}$$

Table 2. Calibration

<i>Structural parameters:</i>		
$\beta$	0.989	Discount factor
$\sigma$	1.1515	Relative risk aversion
$\delta$	0.025	Depreciation rate
$\gamma$	0.36	Capital share
$\kappa$	2.4	Capital goods adjustment cost parameter
$\nu$	1.9697	Inverse Frisch elasticity of labour supply
$\alpha$	0.3368	Degree of trade openness
$\mu$	1	Elasticity of substitution between home and foreign goods
$\lambda_f$	0.11	Price mark-up
$\lambda_w$	0.11	Wage mark-up
$\zeta_p$	0.7191	Price Calvo probability
$\zeta_w$	0.7923	Wage Calvo probability
<i>Parameters related to liquidity constraints:</i>		
$\varkappa$	0.05	Probability of investment opportunity
$\theta$	0.19	Borrowing constraint at steady state
$\phi$	0.19	Equity resaleability constraint at steady state
<i>Policy parameters:</i>		
$\psi_\pi$	1.5	Taylor rule coefficient on inflation
$\psi_Y$	0.125	Taylor rule coefficient on output
$\rho_G$	0.80	Persistence of government spending
$\psi_\tau$	0.1	Fiscal rule parameter

## A.4 Calibration

I calibrate the parameters of the model to match the key features of UK macroeconomic data for the period from 1992, when the inflation targeting regime started, until 2007, just before the onset of the financial crisis. The calibrated values are summarised in Table 2. Two important parameters, the borrowing constraint  $\theta$  and the resaleability constraint  $\phi_t$ , jointly determine the amount of liquidity in the economy. I set the steady-state values of  $\theta$  and  $\phi$  to 0.19, which is slightly higher than the value in DEFK (0.185) to reflect the higher steady-state real interest rate in the UK (around 3.5% on 3-months T-bills). This means that entrepreneurs can sell up to 57% ( $= 1 - (1 - 0.19)^4$ ) of their equity holdings in the course of a year.

Other parameters related to capital investment are  $\varkappa$ ,  $\kappa$ ,  $\gamma$  and  $\delta$ . Consistent with DEFK, I calibrate the i.i.d. opportunity to invest in each quarter ( $\varkappa$ ) to 0.05, which

equals to a 19% ( $= 1 - (1 - 0.05)^4$ ) opportunity to invest in one year.<sup>20</sup> The capital adjustment cost parameter ( $\kappa$ ) is set to 2.4 following the estimation by Groth (2008). The capital share in the production function ( $\gamma$ ) and the quarterly depreciation rate ( $\delta$ ) take on the conventional values of 0.36 and 0.025 respectively.

For the parameters that are standard in a DSGE model, I assign values mainly by referring to the posterior mode obtained by Cogley, De Paoli, Matthes, Nikolov and Yates (2011) using UK data over the same period. The coefficient of relative risk aversion ( $\sigma$ ) is 1.1515, and the inverse Frisch elasticity of labour supply ( $\nu$ ) is 1.9697. The Calvo probabilities for prices ( $\zeta_p$ ) and wages ( $\zeta_w$ ) are 0.7191 and 0.7923 respectively. As in Cogley et al. (2011), the curvature parameters of the Dixit-Stiglitz aggregators in goods and labour markets are assumed to be 10, implying a markup of 0.11 in both goods and labour markets.

$\alpha$  and  $\mu$  are the parameters related to the model's open-economy features. Following Cogley et al. (2011), I calibrate the elasticity of substitution between home-produced and imported goods,  $\mu$ , to 1, implying that the two kinds of goods are substitutes in utility. The trade openness index,  $\alpha$ , is set to 0.3368 which is the posterior mode obtained by Cogley et al. (2011).

I assume that the central bank conducts monetary policy by adopting the Taylor (1993) rule. The coefficients of inflation ( $\psi_\pi$ ) and output ( $\psi_Y$ ) in the monetary policy rule are 1.5 and 0.125 respectively.<sup>21</sup> As in DEFK, I assume the fiscal rule parameter ( $\psi_\tau$ ) to be 0.1, implying that the adjustment of taxes to the government's debt position is gradual. To obtain the fiscal multiplier, I follow Christiano, Eichenbaum and Rebelo (2011) to set the persistence of government spending ( $\rho_G$ ) to 0.8.

With the exogenous parameters calibrated as in Table 2, the steady-state values of the endogenous variables are calculated and reported in Table 3. Two steady-state

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<sup>20</sup>As noted by DEFK, 5% is a conservative estimate of the investment opportunity in the literature. I thus carried out numerical experiments to increase the value of  $\varkappa$  and found that even a slight increase of  $\varkappa$  to 5.5% would cause the condition that  $q_t > p_t^I$  not to hold. Since such condition is crucial in deriving the first order conditions of entrepreneurs, I stick with DEFK's calibration to set  $\varkappa$  at 5%.

<sup>21</sup>Taylor (1993) proposes the coefficients of inflation and output to be 1.5 and 0.5 respectively, based on a policy rule with annualised inflation and interest rates. In my model,  $R_t$  and  $\pi_t$  are quarterly rates, so the coefficient of output is 0.125 ( $=0.5/4$ ).

Table 3. Steady-state values of endogenous variables

Consumption-to-GDP ratio	$C/Y$	0.58
Investment-to-GDP ratio	$I/Y$	0.22
Government spending share	$G/Y$	0.20
Import-to-GDP ratio	$M/Y$	0.27
Export-to-GDP ratio	$X/Y$	0.27
Quarterly GDP	$Y$	3.06
Quarterly labour	$H$	0.90
Capital stock	$K$	26.79
Public debt-to-GDP ratio	$L/4Y$	0.40
Tax-to-GDP ratio	$\tau/Y$	0.21
Aggregate real wage	$w$	1.95
Capital rent	$r^k$	3.70%
Investment cost of new capital	$p^I$	1
Market price of equity	$q$	1.04
Real marginal cost	$mc$	0.90
Real interest rate (annualised)	$4(R - 1)$	3.43%

ratios are exogenous: the public debt-to-GDP ratio ( $L/4Y$ ) and the government consumption share in GDP ( $G/Y$ ).  $L/4Y$  shows the amount of Treasury bonds issued as a share of annual GDP. I set it to 40% to match the UK data (42% on average) for the period 1992-2007.  $G/Y$  takes the average value of 20% observed in the UK over the same period. Inflation is assumed zero at steady state, so  $\pi = 1$ .