

# The Tail that Wags the Economy: Belief-driven Business Cycles and Persistent Stagnation

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## Introduction

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The “Great Recession” spawned two major lines of business cycle research

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- Secular stagnation: Long-lived adverse effects from large shocks

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- Most belief-driven theories have no internal propagation
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**Can belief changes explain persistent responses to transitory shocks ?**

*Yes, when agents are **learning about distributions** (as opposed to hidden states)*

## This paper

A new approach to beliefs in business cycles

### **Agents estimate the distribution of aggregate shocks using real time data**

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Results:

- **Tail events** have a large, permanent effect on beliefs
- **Leverage** amplifies belief revisions from left-tail shocks
- A calibrated model predicts a **permanent 13% drop** in US GDP

## Contribution to the Literature

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Secular stagnation: Summers (2014), Eggertsson and Mehrotra (2014), Gordon (2015)

- We add : *new mechanism, acting through belief revisions*

Belief-driven business cycles

- Belief shocks: Gourio (2012), Angeletos and La'O (2013), Bloom (2009)...
  - We add: *endogenous belief revisions, persistence*
- Learning models: Johannes et. al. (2012), Cogley and Sargent (2005)...
  - We add: *production, flexible non-parametric distributions*
- Endogenous uncertainty: Fajgelbaum et.al. (2014), Straub and Ulbricht (2013)...
  - We add: *empirical discipline, larger effects*

**Preferences:** Representative household

$$U_t = \left[ (1 - \beta) \left( C_t - \zeta \frac{L_t^{1+\gamma}}{1+\gamma} \right)^{1-\psi} + \beta \mathbb{E}_t \left( U_{t+1}^{1-\eta} \right)^{\frac{1-\psi}{1-\eta}} \right]^{\frac{1}{1-\psi}}$$

- $M_{t+1} \equiv \left( \frac{dU_t}{dC_t} \right)^{-1} \frac{dU_t}{dC_{t+1}}$ : Stochastic discount factor

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**Technology:** A continuum of firms, indexed by  $i$

- Production:  $y_{it} = A k_{it}^\alpha l_{it}^{1-\alpha}$
- Aggregate capital quality shocks:  $k_{it} = \phi_t \hat{k}_{it}$      $\phi_t \sim G(\cdot)$     *iid*
- Idiosyncratic shocks,  $\Pi_{it} = v_{it} [y_{it} + (1 - \delta)k_{it}]$
- $v_{it} \sim F(\cdot)$ , common knowledge, *iid*  $\int v_{it} di = 1$

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**Beliefs:**

- $\mathbb{E}_t(\cdot) \equiv \mathbb{E}[\cdot | \mathcal{I}_t]$  : More on  $\mathcal{I}_t$  later

### Labor markets

- Hired in advance, i.e. before observing aggregate/idiosyncratic shocks
- **Non-contingent** wages  $\rightarrow$  workers subject to default risk
- Economy-wide wage rate (in period  $t$  consumption )  $\mathcal{W}_t \equiv \left( \frac{dU_t}{dC_t} \right)^{-1} \frac{dU_t}{dL_{t+1}}$

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### Default

- Firm assets sold to a identical new firm at a discount of  $1 - \theta$
- Proceeds distributed pro-rata among bondholders and workers

## The firm's problem

$$V(\Pi_{it}, B_{it}, S_t) = \max \left[ 0, \max_{d_{it}, \hat{k}_{it+1}, b_{it+1}, w_{it+1}, l_{it+1}} d_{it} + \mathbb{E}_t M_{t+1} V(\Pi_{it+1}, B_{it+1}, S_{t+1}) \right]$$

Dividends:  $d_{it} \leq \Pi_{it} - B_{it} - \hat{k}_{it+1} + \chi q_{it} b_{it+1}$

Discounted wages:  $\mathcal{W}_t \leq w_{it+1} q(\hat{k}_{it+1}, l_{it+1}, B_{it+1}, S_t)$

Future obligations:  $B_{it+1} = b_{it+1} + w_{it+1} l_{it+1}$

Resources:  $\Pi_{it+1} = v_{it+1} \left[ A(\phi_{t+1} \hat{k}_{it+1})^\alpha l_{it+1}^{1-\alpha} + (1-\delta)\phi_{t+1} \hat{k}_{it+1} \right]$

Bond price:  $q(\hat{k}_{it+1}, l_{it+1}, B_{it+1}, S_t) = \mathbb{E}_t M_{t+1} \left[ r_{it+1} + (1-r_{it+1}) \frac{\theta \tilde{V}_{it+1}}{B_{it+1}} \right]$

- Dividends  $d_{it}$  can be negative, i.e. no financing constraints
- Default policy  $r_{it+1} \in \{0, 1\}$  and value  $\tilde{V}_{it+1} \equiv V(\Pi_{it}, 0, S_t)$
- Aggregate state:  $S_t$  (includes information)

## Information and learning

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- Distribution  $G$  of aggregate shocks unknown to agents
  - $\mathcal{I}_t$ : (Finite) History of aggregate variables  $\rightarrow \{\phi_{t-s}\}_{s=0}^T$
- Agents construct an estimate  $\hat{G}_t$  from observed data
  - Use a standard Gaussian kernel density estimator

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- Agents construct an estimate  $\hat{G}_t$  from observed data
  - Use a standard Gaussian kernel density estimator
- Equilibrium concept: anticipated utility
  - Agents myopic with respect to belief changes, but otherwise rational

## The mechanism

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$$\begin{aligned} & \max_{\hat{k}_{t+1}, l_{t+1}, lev_{t+1}} && - \hat{k}_{t+1} - \chi \mathcal{W}_t l_{t+1} \\ & + \underbrace{\mathbb{E}_t [M_{t+1} \Pi_{t+1}]}_{\text{Output + Undep capital}} & + & \underbrace{(\chi - 1) q_t \cdot lev_{t+1} \cdot \hat{k}_{t+1}}_{\text{Tax advantage of debt}} - \underbrace{(1 - \theta) \mathbb{E}_t [M_{t+1} (1 - r_{t+1}) \Pi_{t+1}]}_{\text{Cost of default}} \end{aligned}$$

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A negative shock  $\rightarrow$  More pessimistic beliefs

- $\mathbb{E}_t [M_{t+1} \Pi_{t+1}]$  declines (also present without debt)
- Tax advantage goes down (because  $q_t$  declines)
- Default costs rise

$\Rightarrow$  **Lower incentives to invest and hire**

## Calibration

Strategy: Match aggregate and cross-sectional moments of the US economy

Parameter	Value	Description
$\beta$	0.91	Discount factor
$\eta$	10	Risk aversion
$\psi$	0.50	1/Intertemporal elasticity of substitution
$\gamma$	0.50	1/Frisch elasticity
$\zeta$	1	Labor disutility
$\alpha$	0.40	Capital share
$\delta$	0.03	Depreciation rate
$A$	1	TFP
$\chi$	1.06	Tax advantage of debt
$\theta$	0.70	Recovery rate
$\hat{\sigma}$	0.33	Idiosyncratic volatility
$lev^{\text{Target}}$	0.70	Leverage ratio

## Measuring capital quality shocks

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$$\phi_t = \frac{K_t}{\hat{K}_t} = \frac{\text{value of capital}}{\text{yesterday's capital} + \text{investment}}$$

### Observables

$NFA_t^{RC}$  = Replacement cost of non-financial assets (Flow of Funds)

$NFA_t^{HC}$  = Historical cost of non-financial assets (Flow of Funds)

$PINDX_t^k$  = Investment price index (BEA)

### Model objects

$$P_t^k K_t = NFA_t^{RC}$$

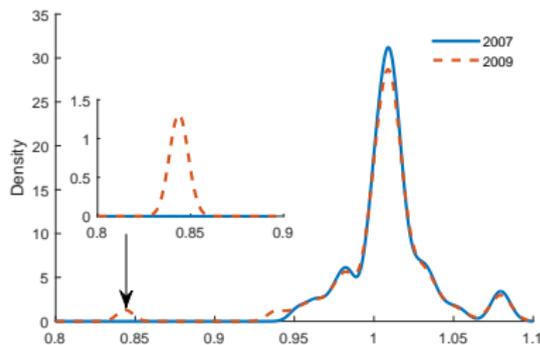
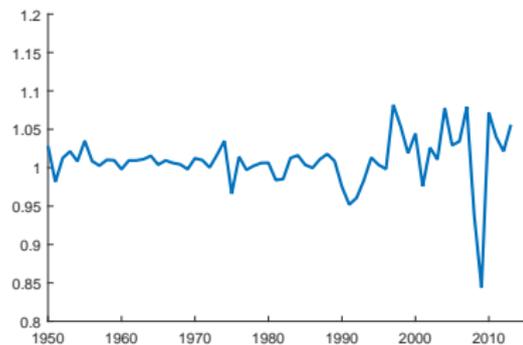
$$P_{t-1}^k \hat{K}_t = (1 - \delta) NFA_{t-1}^{RC} + P_{t-1}^k X_{t-1}$$

$$= (1 - \delta) NFA_{t-1}^{RC} + NFA_t^{HC} - (1 - \delta) NFA_{t-1}^{HC}$$

$$\Rightarrow \phi_t = \left( \frac{P_t^k K_t}{P_{t-1}^k \hat{K}_t} \right) \left( \frac{PINDX_{t-1}^k}{PINDX_t^k} \right)$$

## Capital quality shocks

- Between 1950-2007,  $\phi_t$  in a relatively tight range around 1
- Large negative shocks in 2008-09  $\rightarrow$  significant rise in tail risk



## Effect of a transitory shock

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Experiment:

- Start with beliefs estimated on 1950-2007 data, add '08 and '09 shocks
- Simulate aggregate variables, *holding beliefs fixed*
- (For now, leverage is also held fixed - relaxed later).

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### Baseline results:

- Compare to de-trended data  
*GDP close to the data, overshoot on capital and undershoot on labor*

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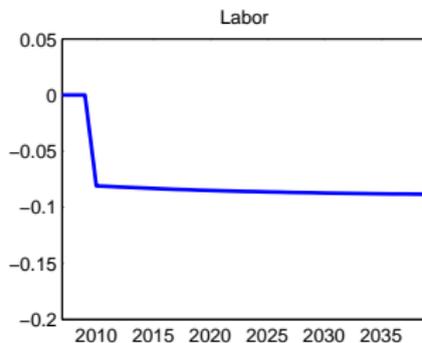
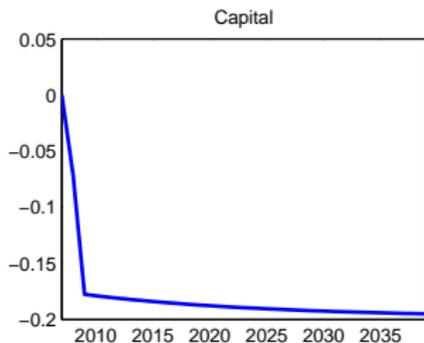
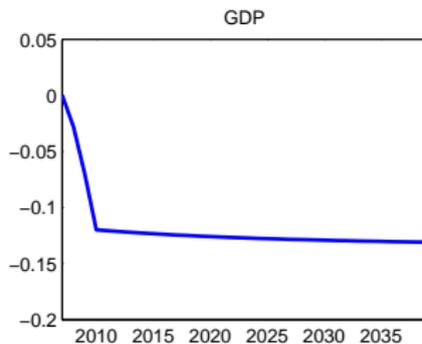
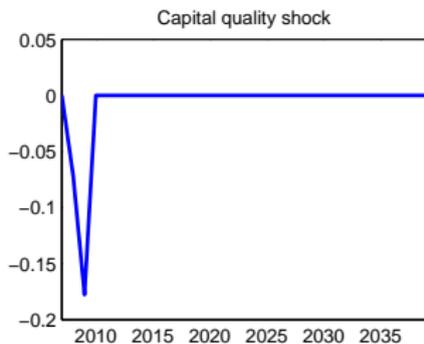
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### Decomposition:

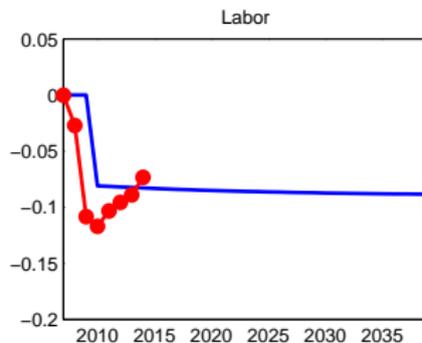
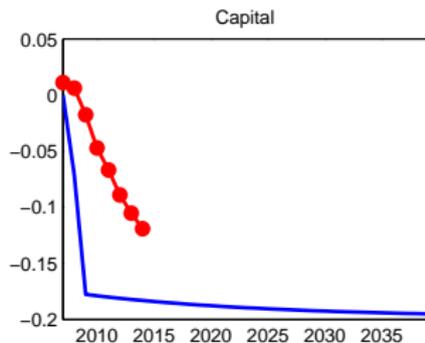
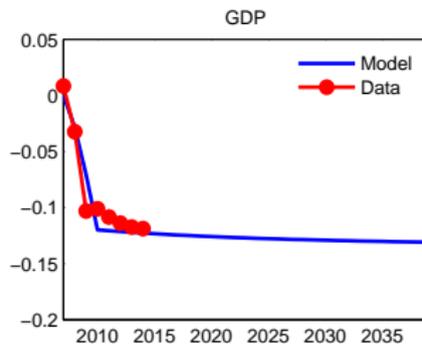
- Role of shock size: Contrast 2008-09 shocks ( $5\sigma$ ) to 2001 shock ( $1\sigma$ ).  
*Small shocks have transitory effects*
- Role of learning: Use distribution implied by full sample throughout  
*Without learning, initial impact similar, but less persistence*
- Role of leverage: Assume no debt ( $\chi = 1, Lev = 0$ )  
*Debt accounts for a third of the long-run effects*
- Role of higher moments: Assume  $\mathbb{E}(\phi_t) = 1$  throughout  
*Higher moments account for more than half of total effect*
- Role of risk-aversion: Assume  $\psi = \eta = 0$ , i.e. preferences are quasi-linear  
*Risk aversion doubles effects, both in the short run and long run*

## Results: Baseline



- A permanent drop in output of 13%

## Results: Model vs Data



- Data: Deviations from log-linear, pre-crisis trend

## Persistent vs Permanent ?

What would temper our long-run effects?

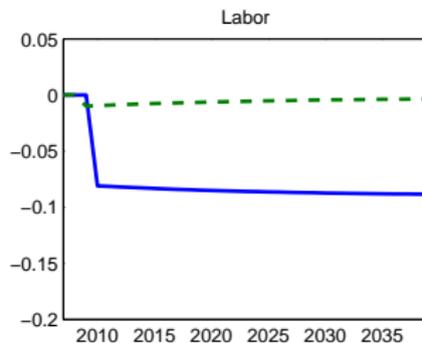
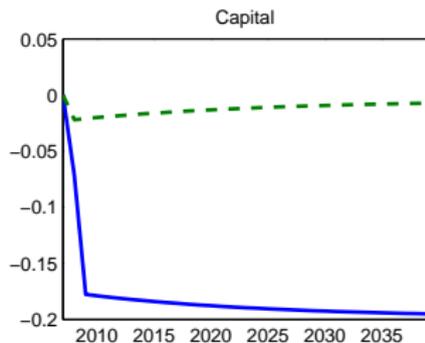
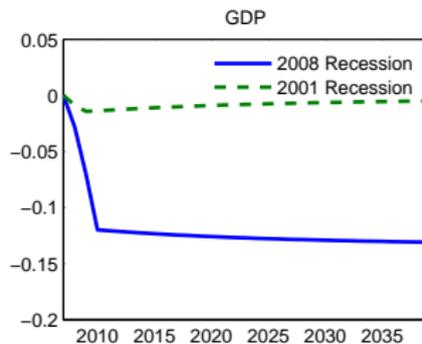
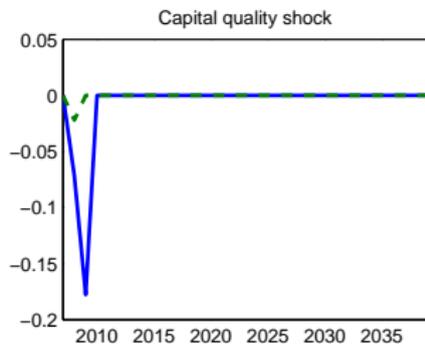
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What would temper our long-run effects?

**Answer:** *if long-run beliefs differ significantly from current, e.g. because of*

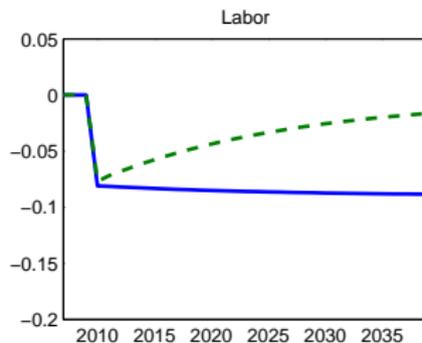
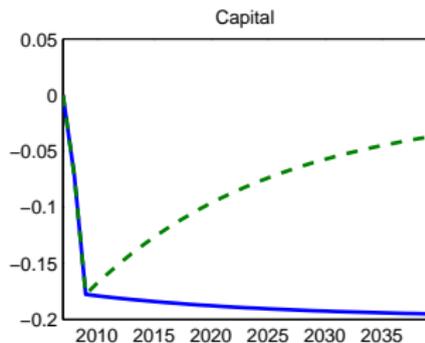
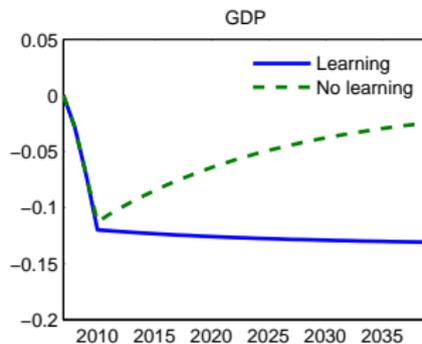
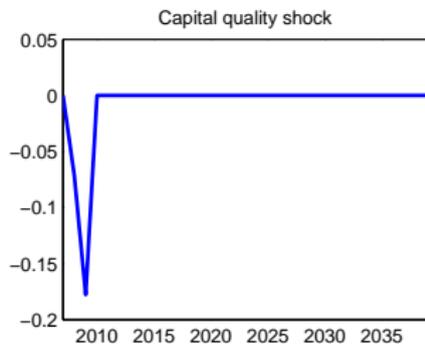
- *New data, e.g. a long period without crises or with very good shocks*
- *Agents discount (or forget) past data*
- *Agents perceive regime changes (the distribution  $g$  changes over time)*

## Results: Role of shock size



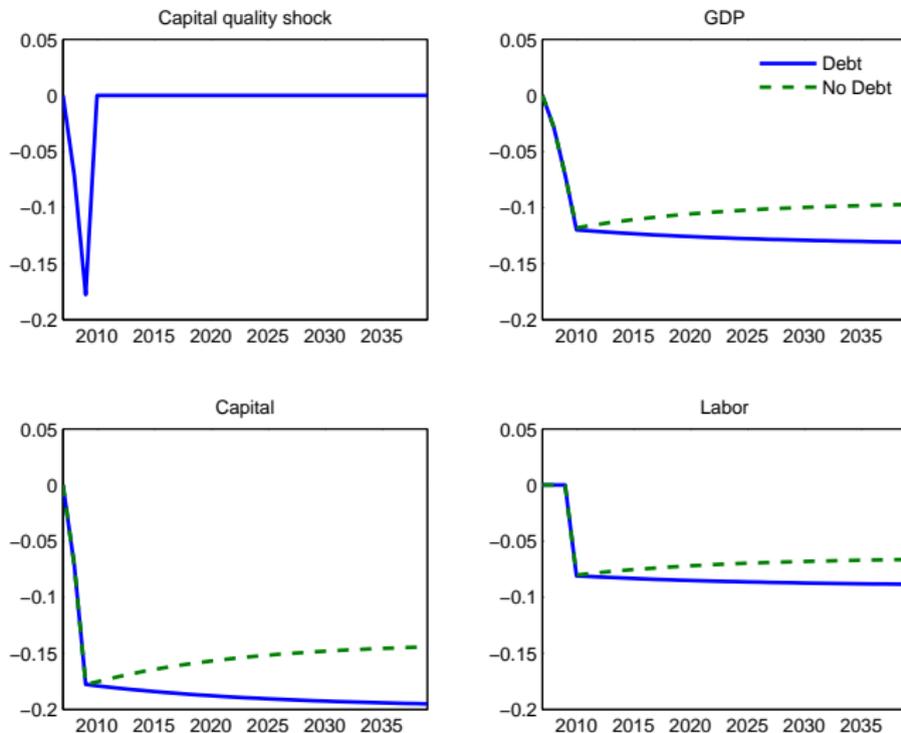
- Small shocks  $\rightarrow$  small belief revisions  $\rightarrow$  negligible long-run effects

## Results: Role of learning



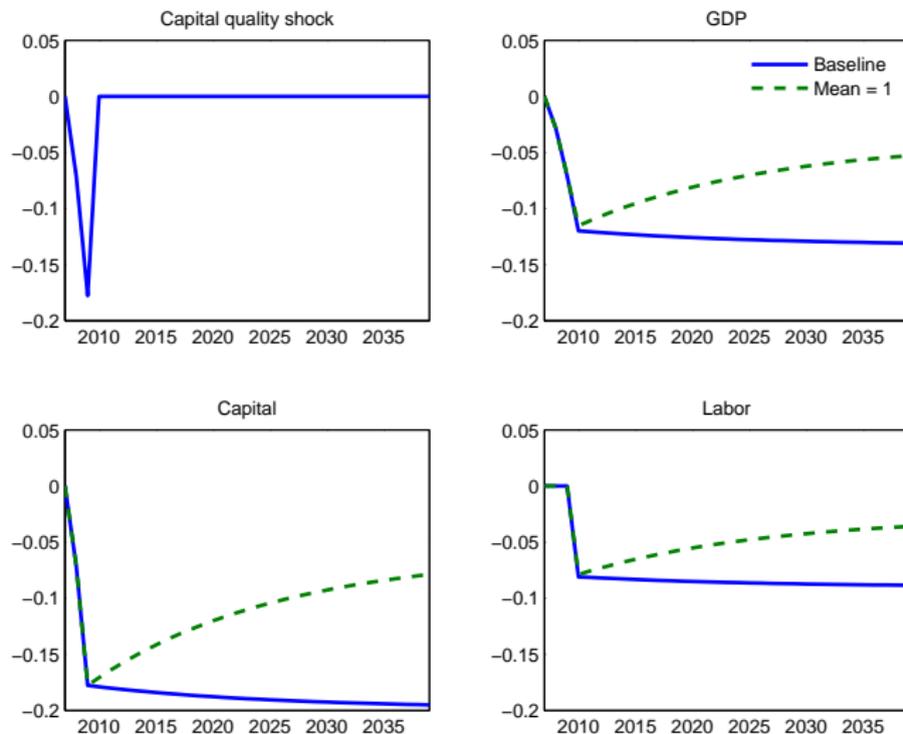
- No learning  $\rightarrow$  effects are transitory

## Results: Role of debt



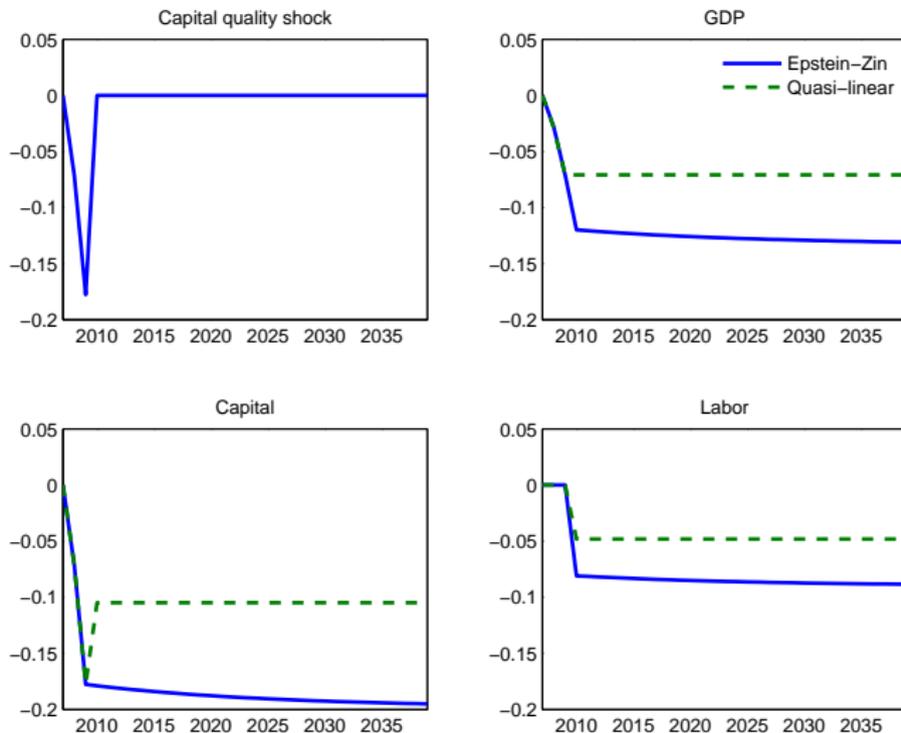
- Debt accounts for one-third of long-run effects

## Results: Role of higher moments



- Higher moments account for half of the long-run effects

## Results: Role of risk aversion



- Risk aversion amplifies effects of belief revisions

## Conclusion

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- A simple, tractable framework of investment and hiring under learning
- Debt and large belief changes combine to generate significant - and *persistent* - declines in economic activity
- A potential explanation for the recent prolonged stagnation ?

## The quasi-linear case

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- $\psi = \eta = 0 \quad \Rightarrow \quad M_{t+1} = \beta$
- Isolates the effect of belief revisions on returns
- Results presented for endogenous leverage

## Optimality conditions

$$\begin{aligned}(1 - \theta) \mathbb{E}_t [M_{t+1} \underline{v} f(\underline{v})] &= \left( \frac{\chi - 1}{\chi} \right) \mathbb{E}_t [M_{t+1} (1 - F(\underline{v}))] \\ 1 &= \mathbb{E}_t \left[ M_{t+1} R_{t+1}^k J^k(\underline{v}) \right] - \chi \mathcal{W}_t \frac{l_{t+1}}{\hat{k}_{t+1}} \\ \chi \mathcal{W}_t &= \mathbb{E}_t \left[ M_{t+1} (1 - \alpha) A \phi_{t+1}^\alpha \left( \frac{\hat{k}_{t+1}}{l_{t+1}} \right)^\alpha J'(\underline{v}) \right]\end{aligned}$$

where

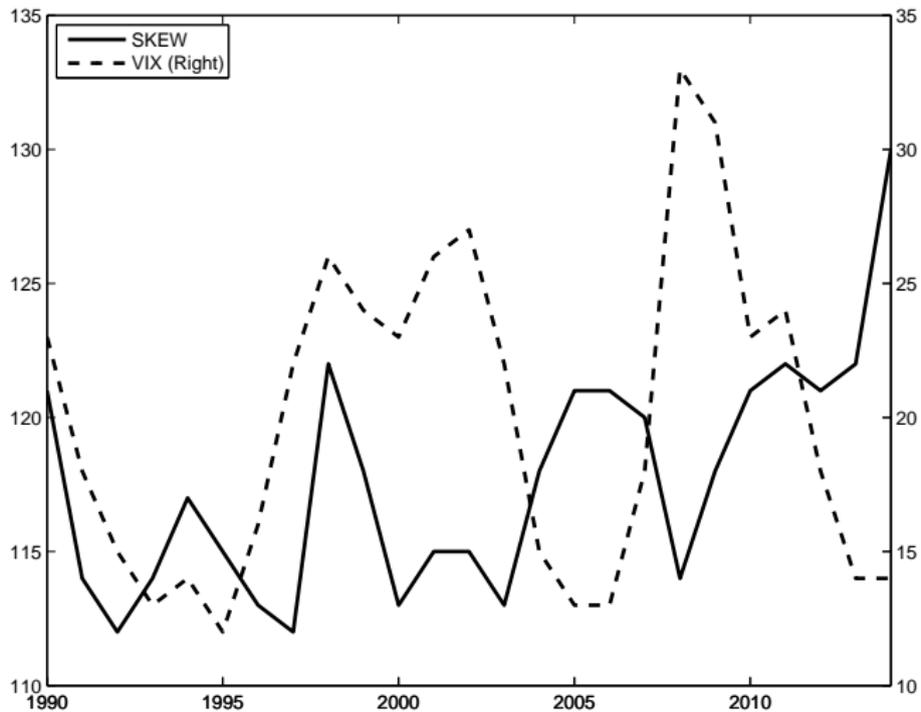
$$\begin{aligned}R_{t+1}^k &= \frac{A \phi_{t+1}^\alpha \hat{k}_{t+1}^\alpha l_{t+1}^{1-\alpha} + (1 - \delta) \phi_{t+1} \hat{k}_{t+1}}{\hat{k}_{t+1}} \\ J^k(\underline{v}) &= 1 + \underline{v} (\chi - 1) (1 - F(\underline{v})) + (\theta \chi - 1) h(\underline{v}) \\ J'(\underline{v}) &= 1 + h(\underline{v}) (\theta \chi - 1) - \underline{v}^2 f(\underline{v}) \chi (\theta - 1)\end{aligned}$$

Now,

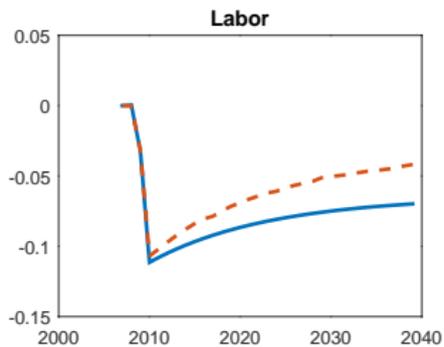
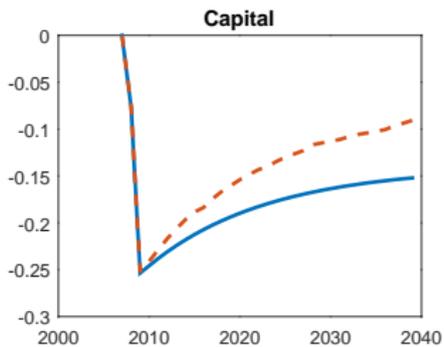
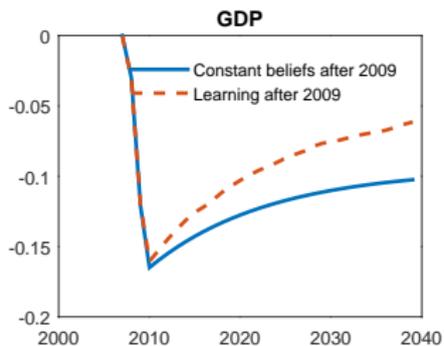
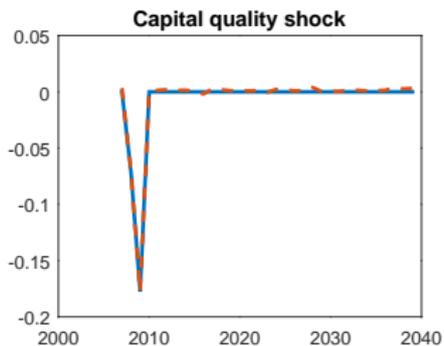
$$\chi = 1 \quad \Rightarrow \quad \underline{v} = 0 \quad \Rightarrow \quad J^k = J' = 1$$

## Variance vs Tail Risk

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## Simulation with belief revisions post-2009



## With belief revisions post-2009

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