Unemployment (Fears), Precautionary Savings, and Aggregate Demand

Wouter J. Den Haan (LSE & CEPR), Pontus Rendahl (University of Cambridge & CEPR), and Markus Riegler (LSE)

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Overview

1. Model
   - interaction between goods and labor market
   - precautionary savings *could* end up in productive investment
2. Algorithm: XPA
   - laws of motion for aggregate variables are obtained by *explicit* aggregation of individual policy functions
   - correct firm value when firm owners are heterogeneous and markets are incomplete
3. Model properties
   - fear of unemployment exacerbates (dampens) downturn when nominal wages are (are not) sticky

Model: Key ingredients

1. Search frictions in labor market
2. Heterogeneous agents and incomplete markets
3. (Some) nominal wage stickiness

Individual agent

unemployed and employed agents

- unemployed search for work
- employed get nominal wage W_t
- exogenous job loss probability, ρ_x
- agents can invest in
  - money, $M_{i,t}$
  - firm ownership (equity), $q_{i,t}$

First-order conditions

$$C_{i,t} + J_t q_{i,t} + M_{i,t}$$
$$=$$
$$e_{i,t} W_t + (1 - e_{i,t}) U_t + q_{i,t-1} (D_t + (1 - \rho_x) J_t) + M_{i,t-1}$$

$$q_{i,t} \geq 0$$

$$C_{i,t} = P_t c_{i,t}$$
$$D_t = P_t d_t$$
$$J_t = P_t j_t$$

First-order conditions

$$c_{i,t}^{-\nu} = \beta \mathsf{E}_t \left[\frac{P_t}{P_{t+1}} c_{i,t+1}^{-\nu} \right] + \zeta_0 \left(\frac{M_{i,t}}{P_t} \right)^{-\zeta_1}$$

$$\frac{J_t}{P_t} = \beta \mathsf{E}_t \left[\left(\frac{c_{i,t+1}}{c_{i,t}} \right)^{-v} \left(\frac{D_{t+1}}{P_{t+1}} + (1 - \rho_x) \frac{J_{t+1}}{P_{t+1}} \right) \right]$$

Job/Firm creation

Standard free-entry condition:

$$
\begin{aligned}
P_t \psi &= \pi_{f,t} J_t \\
\pi_{f,t} &= \phi_o \left(\frac{v_t}{1 - n_{t-1}} \right)^{\phi_1 - 1} \\
n_t &= (1 - \rho_x) n_{t-1} + \phi_o v_t^{\phi_1} (1 - n_{t-1})^{1 - \phi_1}
\end{aligned}
$$

Existing firm

$$D_t = P_t z_t - W_t$$

Wage setting

$$W_t = \omega_0 z_t^{\omega_1} P_t^{\omega_2}$$

- $\omega_1 = 0, \omega_2 = 1$: sticky real wages
- $\omega_1 > 0, \omega_2 = 0$: sticky nominal wages

Equilibrium

- demand for money = (constant) money supply
- demand for firm ownership = number of firms

Algorithm

1. Correctly dealing with firm value

2. XPA
   - explicit aggregation to get aggregate variables right
   - surprisingly few state variables

Firm value

$$\frac{J_t}{P_t} \stackrel{?}{=} \mathsf{E}_t \left[MRS_{i,t+1} \left(\frac{D_{t+1}}{P_{t+1}} + (1-\rho_x) \frac{J_{t+1}}{P_{t+1}} \right) \right]$$

Which $MRS_{i,t+1}$ to use?

Firm value

$$\frac{J_t}{P_t} \stackrel{?}{=} \mathsf{E}_t \left[MRS_{i,t+1} \left(\frac{D_{t+1}}{P_{t+1}} + (1-\rho_x) \frac{J_{t+1}}{P_{t+1}} \right) \right]$$

Literature:

- representative agent: $MRS_{t+1} = (c_{t+1}/c_t)^{-\nu}$
- heterogeneous agents:
  - Krusell, Mukoyama, Sahin (2010): two assets and two outcomes for aggregate state $\Longrightarrow$ use prices of the two Arrow-Debreu securities
  - dinky "solution": assume risk neutral firm manager, which is inconsistent with risk averse firm owners

This paper: Get $J(\cdot)$ by imposing equilibrium

Solving for firm value

$$J_t = J\left(s_t\right)$$

- solve for $J\left(s_t\right)$ by imposing equilibrium

$$\int_i q_{i,t} di = n_t$$

- LHS: demand for firm ownership from individual problem
- RHS: supply of firm ownership comes from free-entry condition

Idea behind XPA

Suppose individual policy rules are linear in *individual* state variables:

$$k_{i,t} = \alpha_0 (s_t) + \alpha_1 (s_t) k_{i,t-1}$$

$\Longrightarrow$ aggregation trivial, namely

$$K_t = \alpha_0 (s_t) + \alpha_1 (s_t) K_{t-1}$$

Idea behind XPA

Suppose individual policy rules are quadratic:

$$k_{i,t} = \alpha_0(s_t) + \alpha_1(s_t)\,k_{i,t-1} + \alpha_2(s_t)\,k_{i,t-1}^2$$

$\Longrightarrow$ aggregation gives

$$\begin{aligned} K_t &= \alpha_0(s_t) + \alpha_1(s_t)\,K_{t-1} + \alpha_2(s_t)\,K_{t-1}(2) \\ K_{t-1}(2) &= \int_i k_{i,t-1}^2 di \end{aligned}$$

$\Longrightarrow$ we need a law of motion for $K_t(2) = \int_i k_{i,t}^2 di$

Idea behind XPA

Approach #1:

use $k_{i,t}^2 = \left(\alpha_0(s_t) + \alpha_1(s_t)k_{i,t-1} + \alpha_2(s_t)k_{i,t-1}^2\right)^2$

$$K_t(2) = \int_i k_{i,t}^2 di = \int_{ii,t}^{\infty} \left(\alpha_0(s_t) + \alpha_1(s_t)k_{i,t-1} + \alpha_2(s_t)k_{i,t-1}^2\right)^2 di$$

$\Longrightarrow K_t(3)$ and $K_t(4)$ become state variables, etc.

Idea behind XPA

Approach #2:

approximate $k_{i,t}^2$ with

$$k_{i,t}^2 = \widetilde{\alpha}_0(s_t) + \widetilde{\alpha}_1(s_t)\,k_{i,t-1} + \widetilde{\alpha}_2(s_t)\,k_{i,t-1}^2$$

which gives

$$K_t(2) = \widetilde{\alpha}_0(s_t) + \widetilde{\alpha}_1(s_t)\,K_{t-1} + \widetilde{\alpha}_2(s_t)\,K_{t-1}(2)$$

$\Longrightarrow$ set of state variables does not increase

Implementation

- Individual problem is solved accurately with a global method and piecewise linear policy functions
- For aggregation a linear approximation of this nonlinear policy function is used

State variables

- Individual state variables
  - cash on hand: $q_{t-1}\left(D_t + \left(1-\rho_x\right)J_t\right) + M_{i,t-1}$
  - employment status
- Aggregate state variables
  - aggregate productivity
  - number of firms = equity shares

Precautionary savings

How to get precautionary savings in a model?

- typically done through $\Delta\beta$
- this paper through Δunemployment

Typical precautionary savings story

Households want to save more

- $\Longrightarrow$ demand for consumption $\downarrow$ & prices do not adjust
- $\Longrightarrow$ demand for labor $\downarrow$, etc.

Where do savings end up?

- typically not allowed to end up in investment because
  - there is no physcial investment
  - or incorrect discounting of firm profits

Precautionary savings in this paper

We do have something like the standard channel:

- unemployment $\uparrow \Longrightarrow$ demand for money $\uparrow$
- $\Longrightarrow P_t \downarrow \Longrightarrow$ real profits $\downarrow$ (because of sticky nominal wages)
- $\Longrightarrow$ firm/job creation $\downarrow$
  - but in this paper !!!

Precautionary savings in this paper

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  - but in this paper !!!
  - precautionary savings could end up in productive investment since $MRS_{i,t} \uparrow$ when precautionary savings $\uparrow$

Precautionary savings and productive investment

- This paper: investment in firm/job creation *could* ↑ when precautionary savings ↑

Reasons why it *could* ↓:

- agents less willing to hold firm equity when profits ↓
- agents less willing to hold risky assets when unemployment ↑

Idiosyncratic risk & investment portfolio

- simple example

Idiosyncratic risk & investment portfolio

$$\max_{c_1,c_2,m,a} c_t^{1-\nu} + \beta c_{t+1}^{1-\nu}$$

s.t.

$$c_1 = y_1 - m - a$$

$$c_2 = y_2 + m(1 + r_m) + a\,(1 + r_a)$$

$$\begin{aligned} y_1 &= \mathsf{E}\,[y_2] = 1 \\ r_a &= \begin{cases} +0.060 \text{ with prob. } \frac{1}{2} \\ -0.039 \text{ with prob. } \frac{1}{2} \end{cases},\ \mathsf{E}\,[r_a] > r_m \end{aligned}$$

Case 1 no idiosyncratic risk

- no idiosyncratic risk: $y_2 = 1$
- $m = -0.0408$ and $a = 0.0408$
- no savings, $m + a = 0$
  realizations of r_a chosen to get this outcome

Case 2 idiosyncratic risk

- $y_2 = 1$ when r_a takes on high value
- $y_2 \in \{0, 2\}$ $\mathrm{E}y_2 = 1$ when r_a takes on low value
  - higher spread in recession
  - but mean not affected (for transparency)
- not surprisingly, $m + a \uparrow$ to 0.226

Case 2 idiosyncratic risk

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    - higher spread in recession
    - but mean not affected (for transparency)

- not surprisingly, $m + a \uparrow$ to 0.226
- but $m \uparrow$ to 9.2872 and $a \downarrow$ to -9.0610

Model properties

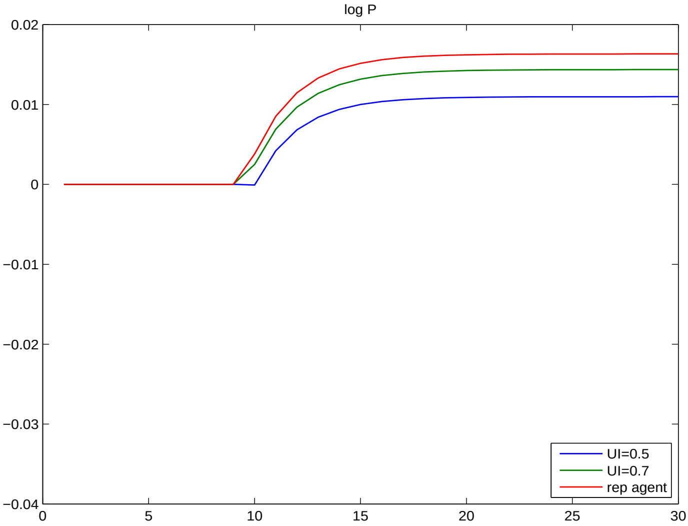
1. Model 1: no nominal wage stickyness
   precautionary savings **dampen** downturn

2. Model 2: with nominal wage stickyness
   precautionary savings **worsen** downturn

No nominal wage stickyness

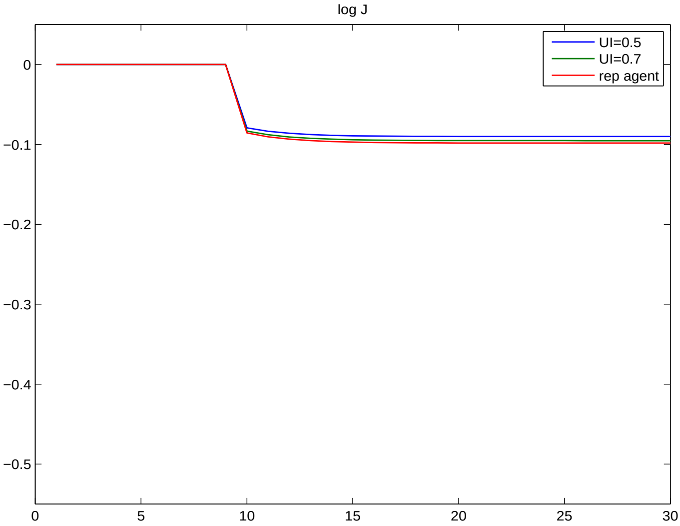
- productivity $\downarrow$
- $\Longrightarrow$ profits $\downarrow \Longrightarrow$ firm value $\downarrow \Longrightarrow$ unemployment $\uparrow$
- $\Longrightarrow$ precautionary savings $\uparrow$
    - $\Longrightarrow$ demand for firm ownership may $\uparrow \Longrightarrow$ unemployment $\downarrow$
    - $\Longrightarrow$ demand for money $\uparrow \Longrightarrow P \downarrow \not\Rightarrow \Delta$ profits since nominal wages adjust

No nominal wage stickyness



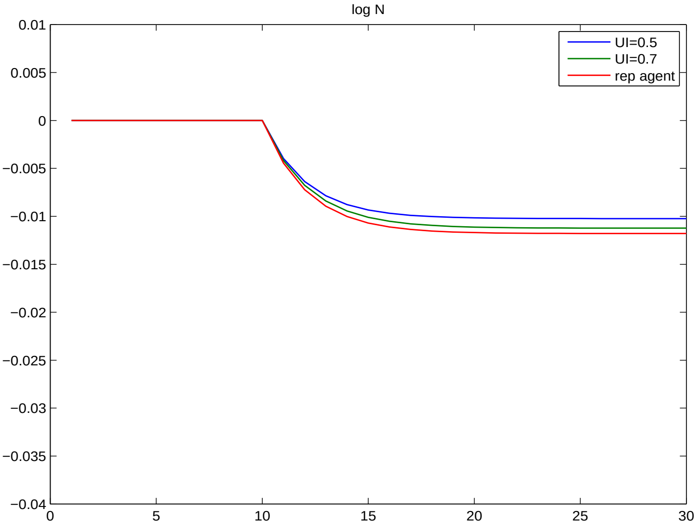
Precautionary demand for M reduces price increase

No nominal wage stickyness



Precautionary savings has small upward effect on firm value

No nominal wage stickyness

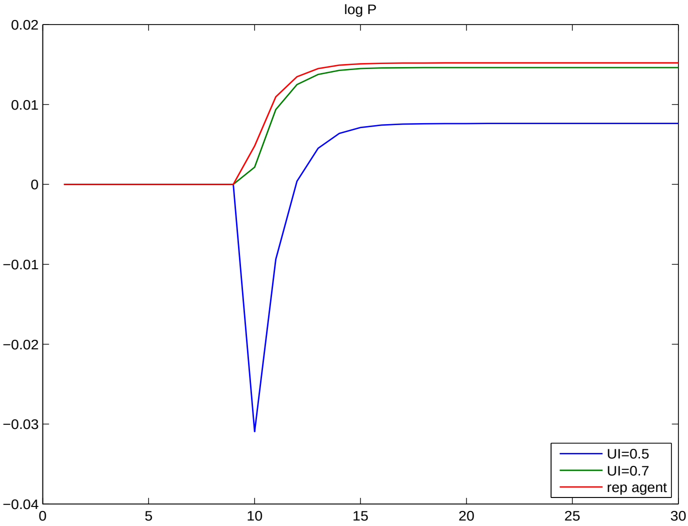


Precautionary savings has small upward effect on employment

With nominal wage stickyness

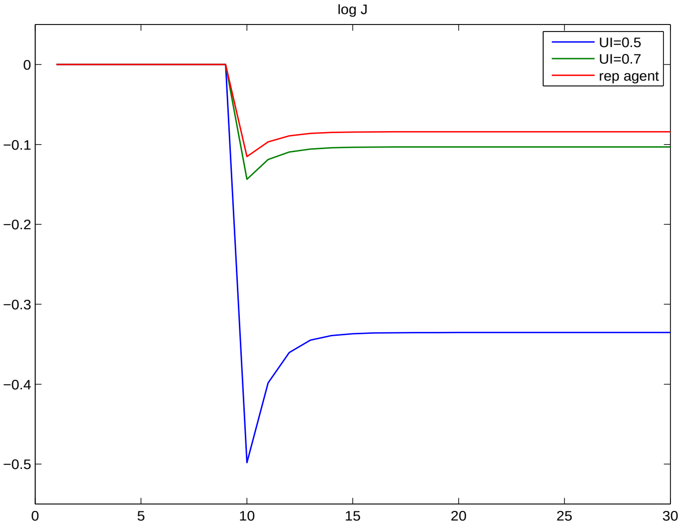
- productivity $\downarrow$
- $\Longrightarrow$ profits $\downarrow$ $\Longrightarrow$ firm value $\downarrow$ $\Longrightarrow$ unemployment $\uparrow$
- $\Longrightarrow$ precautionary savings $\uparrow$
  - $\Longrightarrow$ demand for firm ownership may $\uparrow \Longrightarrow$ unemployment $\downarrow$
  - $\Longrightarrow$ demand for money $\uparrow \Longrightarrow P \downarrow \Longrightarrow$ profits $\downarrow$ unemployment $\uparrow \Longrightarrow$ downward spiral

With nominal wage stickyness



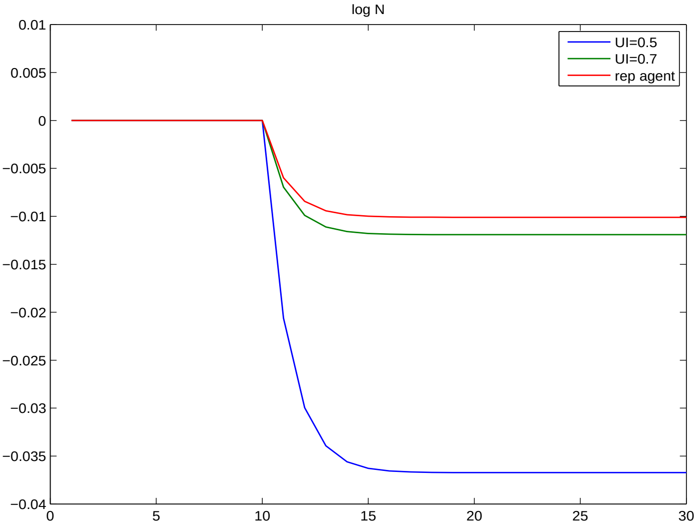
Precautionary demand for M strongly reduces prices

With nominal wage stickyness



Precautionary demand for M strongly reduces firm value

With nominal wage stickyness



Precautionary demand for M strongly reduces firm value