The Contagion Box: Measuring Financial Market Co-movements by Regression Quantiles*

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Abstract

This paper develops an econometric framework to investigate the structure of co-dependences across markets and to test whether it changes over time or across market conditions. Our approach is based on the computation, over both a test and a benchmark period, of the conditional probability that the returns on one market are lower than a given quantile, when returns on the other market are also lower than their corresponding quantile, for any set of prespecified quantiles. Quantiles are allowed to vary over time using the CAViaR methodology developed by Engle and Manganelli (2004). Graphically, the conditional probabilities can be represented in what we call "the contagion box", which is a square of unit side. Since a 45° line represents the case of independence, the presence of co-movements is indicated when the conditional probability plots above this line. Differences in the intensity of co-movements can be identified directly from the conditional probability plots for test and benchmark periods. From this insight, rigorous econometric tests of contagion are derived and implemented. In the process we obtain a new result in the regression quantile literature. We illustrate the methodology by investigating the impact of the "tequila" (1994/95), Asian (1997) and Russian (1998) crises on the major Latin American equity markets. Our results suggest significant presence of contagion.

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1 Introduction

The financial crises which characterised the second half of the 1990s have stirred a hot debate about the stability of the international financial system. One of the key questions still in search of an answer is whether the Tequila crisis, the Asian flu and the Russian worm were episodes of financial contagion. Contagion is broadly defined as an increase in financial market co-movement over crisis periods. The issue is particularly important because under contagion the likelihood that financial crises spread over from one country to another increases. Measuring co-dependences across financial markets, though, remains an open issue. Policy intervention would have different scope whether one detects contagion or simple interdependence. An accurate measure of financial co-movements therefore constitutes an indispensable instrument in the policy maker toolbox. Precise measures of asset co-movements are also important for a broad spectrum of applications, which range from portfolio allocation, risk management, and monitoring financial stability.

In the empirical literature several methodologies to measure co-dependence among asset returns are available. Extensive surveys are provided by Dungey, Fry, González-Hermosillo, and Martin (2003), Pericoli and Sbracia (2003), and de Bandt and Hartmann (2000). In essence, one can distinguish between two different approaches: modelling first and/or second moments of returns (see, for instance, Forbes and Rigobon, 2002, King, Sentana and Wadhwani, 1994, Ciccarelli and Rebucci, 2003), and estimating the probability of co-exceedance[1] (see, among others, Longin and Solnik, 2001, Hartmann, Straetmans and de Vries, 2003, and Bae, Karolyi and Stulz, 2003). Each of these methodologies suffers from several drawbacks. Correlation-based models do not account for asymmetries in the joint distribution. GARCH-type approaches assume that negative and positive extremes follow the same process as the other returns. Probability models generally analyse only single points of the support of the distribution and adopt a two-step estimation procedure without correcting the standard errors.

This paper provides a common econometric framework to investigate the problem at hand. The cornerstone of our approach is the estimation of the conditional probability that returns on market Y are lower than a given quantile, when returns on market X are also lower than their corresponding quantile. Quantiles are modelled through the Conditional Autoregressive Value at Risk (CAViaR) approach of Engle and Manganelli (2004) and estimated via regression quantile (Koenker and Bassett, 1978). In general, the stronger the co-dependence between X and Y, the higher the

[1] Co-exceedance occurs when both market returns on X and Y exceed some pre-specified thresholds.

conditional probability of co-movement. Comparing these probabilities in crisis and tranquil periods allows one to directly identify contagion.

Our methodology possesses several advantages. First, casting the econometric framework in term of regression quantiles permits to make proper inference. Second, we are able to measure co-dependence over any subset of the support of the joint distribution. In particular, asymmetries in co-movement in the positive and negative parts of the distribution can be tested for. Third, one can test whether economic variables significantly increase the probability of co-movement. Fourth, since regression quantile is a semi-parametric technique, there is no need to impose any distributional assumption on returns. Fifth, the results can be easily visualised in what we call "the contagion box". The contagion box is a square of unit side, where, for any set of θ-quantiles, $\theta \in (0,1)$, the conditional probabilities are plotted against θ. When the plot of the conditional probability lies above the 45° line, which represents the case of independence between two markets, there is evidence of positive co-movements. When the conditional probability of co-movements for the crisis and tranquil periods are plotted in the same graph, differences in the intensity of co-movements can be identified directly. From this insight, rigorous econometric tests for contagion are derived and implemented. In the process we obtain a new result in the regression quantile literature. We show that the asymptotic covariance matrix of the estimated probabilities depends on the joint bivariate distribution evaluated at the quantiles. This can be interpreted as the bivariate extension of the height of the density function that typically appears in the standard errors of regression quantiles. We illustrate our methodology by investigating the impact of the major crises of the Nineties on the main Latin American equity markets.

The focus of this study is mostly methodological, and its applications are not limited to the specific issue of testing for contagion. For instance, for strategic allocation purposes, risk-averse investors could use the contagion box to select those asset classes which exhibit lowest co-movements. Hedging hinges on a similar principle: investors search hedge and underlying assets which move into opposite directions. Finally, policy makers are interested in measuring dependence among asset returns: if economies are largely interconnected through financial markets and crises spill over despite sound fundamentals, there would be limited scope for intervention. As a result, financial stability could be in danger and alternative strategies need to be implemented.

The paper proceeds as follows. In Section 2 we describe our empirical framework, provide some intuition and compare our tests to the alternatives in the literature. The formal econometrics of the tests is developed in Section 3. Section 4 describes the data. Section 5 reports the results of the analysis. Section 6 concludes.

2 The contagion box

In this section we first develop a formal framework to measure co-movements between two random variables and then show how it can be used to test for contagion. The probability of co-movements will be conveniently represented in a square with unit side, the "contagion box". After defining a benchmark against which our measure of co-dependence can be compared, we derive an analytical definition of contagion. Finally, we show that the contagion box can include as special cases other methodologies commonly used to detect contagion, such as Extreme Value Theory (EVT), the logit/probit approach and the correlation framework proposed by Forbes and Rigobon (2002).

2.1 The analytical framework

Let y_t and x_t denote two different random variables. Let $q_t^Y(\beta_{\theta Y}^0) \equiv q^Y(\beta_{\theta Y}^0, \Omega_t)$ be the time t θ-quantile of the conditional distribution of y_t, where $\beta_{\theta Y}^0$ is a vector of unknown true parameters that characterise the θ-quantile and Ω_t the information set which includes all variables observed up to the beginning of time t. Analogously, for x_t, we define $q_t^X(\beta_{\theta X}^0) \equiv q^X(\beta_{\theta X}^0, \Omega_t)$. These quantiles can depend on any variable that belongs to the information set at time t. If Ω_t is the empty set (i.e., $\Omega_t = \varnothing$, $\forall t$), then the θ-quantiles are constant for all t. On the other hand, if Ω_t contains all the available information up to time t, the θ-quantiles are not necessarily constant.

Denote the conditional cumulative joint distribution of the two random variables by $F_t(y, x)$. Define $F_t^-(y|x) \equiv \Pr(y_t \leq y \mid x_t \leq x) = \frac{\Pr(y_t \leq y, x_t \leq x)}{\Pr(x_t \leq x)}$ and $F_t^+(y|x) \equiv \Pr(y_t \geq y \mid x_t \geq x) = \frac{\Pr(y_t \geq y, x_t \geq x)}{\Pr(x_t \geq x)}$. Our basic tool of analysis is the following conditional probability:

$$p_t(\theta) \equiv \begin{cases} F_t^-\left(q_t^Y(\beta_{\theta Y}^0) | q_t^X(\beta_{\theta X}^0)\right) & \text{if } \theta \leq 0.5 \\ F_t^+\left(q_t^Y(\beta_{\theta Y}^0) | q_t^X(\beta_{\theta X}^0)\right) & \text{if } \theta > 0.5 \end{cases}. \tag{1}$$

This conditional probability represents an effective way to summarise the characteristics of $F_t(y, x)$[2,3].

[2] We could study both $F_t^-(y|x)$ and $F_t^+(y|x)$ for the whole range of θ between 0 and 1, $0 \leq \theta \leq 1$. However for $\theta = 1$, $F_t^-(y|x) = 1$ and for $\theta = 0$, $F_t^+(y|x) = 1$. Hence most of the interesting information about the co-movements of x_t and y_t is provided by $F_t^-(y|x)$ for $\theta \leq 0.5$ and by $F_t^+(y|x)$ for $\theta. > 0.5$.

[3] For hedging purposes, we could as well have defined $G_t^-(y|x) \equiv \frac{\Pr(y_t \leq y, x_t \geq x)}{\Pr(x_t \geq x)}$ and $G_t^+(y|x) \equiv$

If we think of $\{x_t\}_{t=1}^T$ and $\{y_t\}_{t=1}^T$ as the time series returns of two different markets, for each quantile θ, $p_t(\theta)$ measures the probability that on market Y the return will fall below (or above) its θ-quantile, conditional on the same event occurring in market X.

The characteristics of $p_t(\theta)$ can be conveniently analysed in what we call the "**contagion box**" (see Figure 1). The contagion box is a square with unit side, where $p_t(\theta)$ is plotted against θ. The shape of $p_t(\theta)$ will generally depend on the characteristics of the joint distribution of the random variables x_t and y_t, and therefore for generic distributions it can be derived only by numerical simulation. There are, however, three important special cases that do not require any simulation: 1) perfect positive correlation, 2) independence and 3) perfect negative correlation. If two markets are independent, which implies $\rho_{YX} = 0$, $p_t(\theta)$ will be piece-wise linear, with slope equal to one, if $\theta \in (0, 0.5)$, and slope equal to minus one, if $\theta \in (0.5, 1)$. When there is perfect positive correlation between x_t and y_t (i.e. $\rho_{YX} = 1$), $p_t(\theta)$ is a flat line that takes on unit value. Under this scenario, the two markets essentially reduce to one. The polar case occurs for a perfect but negative correlation, i.e. $\rho_{YX} = -1$. In this case $p_t(\theta)$ is always equal to zero. The reason is that if y_t falls in one half of its distribution, x_t will not, because it will take on diametrically opposite values.

The above discussion suggests that the shape of $p_t(\theta)$ might provide key insights about the dependence between two random variables x_t and y_t. Indeed, $p_t(\theta)$ satisfies some basic desirable properties, as summarised in the following theorem (all proofs can be found in Appendix B):

Theorem 1 *$p_t(\theta)$ for $\theta \in (0,1)$ satisfies the following properties:*

1. $F_t^-\left(q_t^Y(\beta_{\theta Y}^0)|q_t^X(\beta_{\theta X}^0)\right) = F_t^-\left(q_t^X(\beta_{\theta X}^0)|q_t^Y(\beta_{\theta Y}^0)\right)$,
   $F_t^+\left(q_t^Y(\beta_{\theta Y}^0)|q_t^X(\beta_{\theta X}^0)\right) = F_t^+\left(q_t^X(\beta_{\theta X}^0)|q_t^Y(\beta_{\theta Y}^0)\right)$ *(Symmetry),*
2. $p_t(\theta) = 1$ *for* $\theta \in (0,1) \Longleftrightarrow$ *Co-monotonicity,*
3. $p_t(\theta) = 0$ *for* $\theta \in (0,1) \Longleftarrow$ *Counter-monotonicity,*

$\frac{\Pr(y_t \geq y, x_t \leq x)}{\Pr(x_t \leq x)}$ as well as

$$s_t(\theta) \equiv \begin{cases} G_t^-\left(q_t^Y(\beta_{\theta Y}^0)|q_t^X(\beta_{1-\theta X}^0)\right) & \text{if } \theta \leq 0.5 \\ G_t^+\left(q_t^Y(\beta_{\theta Y}^0)|q_t^X(\beta_{1-\theta X}^0)\right) & \text{if } \theta > 0.5 \end{cases}.$$

Similar results and tools as those developed below for $p_t(\theta)$ can be derived to study the changes in $s_t(\theta)$.

4. $p_t(\theta) = \theta$ *for* $\theta \in (0,1) \Longleftarrow$ *Independence.*

According to Theorem 1 our measure of conditional probability will allow us to recognise joint random variables characterised by co-monotonicity, which includes the case of perfect positive correlation. For independence and counter-monotonicity (of which perfect negative correlation is a special case), we can only derive a necessary condition. This is the price we have to pay for looking only at co-movements associated to the same quantiles. Of course, one could look at different quantiles simultaneously, thus recovering the entire information contained in the joint distribution of the two random variables. Such information, however, could not be displayed in the simple contagion box illustrated above. Our measure aims at striking a reasonable compromise between simplicity and completeness.

2.2 Measuring Contagion

While $p_t(\theta)$ can be used to measure the dependence between different markets, the interest of the researcher often lies in testing whether this dependence has changed over time. Contagion is an important case in point.

In epidemiology contagion is associated to any disease which is easily transmitted by contact. In statistical terms, the presence of contagion can be tested by identifying a "control group" and an "experimental group." In the experimental group, unlike in the control group, patients are exposed to the potentially contagious disease. Next, one would compute the conditional probability that one patient contracts the disease, provided that another one is already sick. The presence of contagion would imply that this conditional probability would be higher in the experimental than in the control group.

The analogy with economics is straightforward: "patients" can be replaced by "markets" and "sick" by "quantile exceedance". The control group is given by the set of returns in "tranquil times", while the experimental group by the set of returns in "crisis periods". Testing for financial contagion is equivalent to testing if the conditional probability of co-movements between two markets increases over crisis periods versus tranquil times. This is indeed the spirit of the "very restrictive" definition of the World Bank.[4]

The framework of the contagion box can be used to formalise this intuition. Let $p^C(\theta) \equiv C^{-1} \sum_{t \in \{crisis\ times\}} p_t(\theta)$ and $p^N(\theta) \equiv N^{-1} \sum_{t \in \{tranquil\ times\}} p_t(\theta)$, where

[4] See the web site http://www1.worldbank.org/economicpolicy/managing%20volatility/contagion/definitions.html

C and N denote the number of crisis and tranquil times, respectively. We adopt the following working definition of contagion:

Definition 1 ***(Contagion)*** *- There exists* ***contagion*** *in a given interval* $[\underline{\theta}, \bar{\theta}]$ *if* $\delta\left(\underline{\theta}, \bar{\theta}\right) = \int_{\underline{\theta}}^{\bar{\theta}} [p^C(\theta) - p^N(\theta)] d\theta > 0.$

$\delta\left(\underline{\theta}, \bar{\theta}\right)$ measures the area between the average conditional probabilities $p^C(\theta)$ and $p^N(\theta)$ over the interval $\left[\underline{\theta}, \bar{\theta}\right]$. Unlike correlation-based measures, $\delta\left(\underline{\theta}, \bar{\theta}\right)$ permits to analyse changes in co-dependence over specific parts of the distribution. For instance, it may occur that $\delta(0,1)$ is quite small just because of positive co-dependence on the left tail of the distribution and negative on the right tail, so that the two values tend to offset each other.

We can describe existing contributions to the contagion literature in terms of the contagion box. First, our approach has direct ties with the EVT. Indeed, $\lim_{\theta \to 0} p_t(\theta)$ is exactly the definition of "tail dependence" for the lower tail used in the EVT literature (similar result holds for the upper tail). Existing contributions (e.g., Longin and Solnik, 2001 and Hartmann, Straetmans and de Vries, 2003) differ from ours under two important aspects. First, they only look at one (extreme) point of the distribution. Second, in the light of Definition 1, they fail to compare this point to some benchmark against which contagion can be measured. Moreover, it is not obvious how these approaches can be modified to control for economic variables.

Our methodology is also close to the logit/probit literature (e.g., Eichengreen, Rose and Wyplosz, 1996, Bae, Karolyi and Stulz, 2003, and Gropp and Moerman, 2004). The value of $p_t(\theta)$ in the contagion box can in principle be computed through the logit/probit approach. The main problem with this methodology is that it adopts a two-step procedure and it is not clear how correct inference can be made. In the next section we propose a more coherent econometric framework based on regression quantile.

Finally, previous research (see, for instance, Longin and Solnik, 1995, Karolyi and Stulz, 1996, De Santis and Gerard, 1997, and Ang and Bekaert, 2002) suggests that correlation increases when returns are large in absolute value, and in particular over bear markets. However, as pointed out by Longin and Solnik (2001) and Forbes and Rigobon (2002), among others, the difference in estimated correlation between volatile and tranquil periods could be spurious and due to heteroskedasticity. By modelling conditional probability with regression quantiles, our approach is robust to this problem.

It is instructive to see how the contagion box fits the framework used by Forbes and Rigobon (2002). They propose the following model for contagion:

$$\begin{aligned} y_t &= \beta x_t + \varepsilon_t, \\ x_t &= u_t. \end{aligned}$$

According to this model, an increase in β would induce a higher degree of co-movements between the two markets X and Y. In terms of the contagion box, this requires that the conditional probability $p[y_t > q^\theta_Y\left(\beta^0_\theta, \Omega_t\right) \mid x_t > q^\theta_X\left(\delta^0_\theta, \Omega_t\right), \Omega_t]$ is increasing in β. If ε_t and u_t are independent, the θ-quantile of y_t can be written as $q^\theta_{tY} = \bar{\varepsilon}_t + \beta q^\theta_{tX}$, where $\bar{\varepsilon}_t$ is a suitable constant independent of β. This conditional probability can be rewritten as follows:

$$\begin{aligned} & \theta^{-1} p\left[y_t > q^\theta_{tY},\ u_t > q^\theta_{tX} | \Omega_t\right] = \\ = \; & \theta^{-1} p\left[\beta u_t + \varepsilon_t > \bar{\varepsilon}_t + \beta q^\theta_{tX}, u_t > q^\theta_{tX} | \Omega_t\right] \\ = \; & \theta^{-1} p\left[u_t > q^\theta_{tX} + (\bar{\varepsilon}_t - \varepsilon_t)/\beta,\ u_t > q^\theta_{tX} | \Omega_t\right] \\ = \; & \theta^{-1}\{p\left[u_t > q^\theta_{tX} + (\bar{\varepsilon}_t - \varepsilon_t)/\beta\right] p[\varepsilon_t < \bar{\varepsilon}_t] + p\left[u_t > q^\theta_{tX}\right] p[\varepsilon_t > \bar{\varepsilon}_t]\}. \end{aligned}$$

The derivative of the above expression with respect to β is positive for all θ.

3 The Econometrics of the Contagion Box

Constructing the contagion box and testing for differences in the probability of co-movement requires several steps. First, we estimate the univariate quantiles associated to the return series of interest. Second, we construct, for each series and for each quantile, indicator variables which are equal to one if the observed return is lower than this quantile and zero otherwise. Finally, we regress the θ–quantile indicator variable of country Y on the θ–quantile indicator variable of country X, interacted with crisis dummies. These regression coefficients will provide a direct estimate of the conditional probabilities of co-movements.

In this section we briefly review the CAViaR model of Engle and Manganelli (2004) that is used to estimate time-varying quantiles, and derive their joint distribution. Next, in section 3.2 we discuss the estimation of the conditional probabilities and their asymptotic properties.

3.1 CAViaR and Regression Quantiles

The CAViaR model parametrises directly a time-varying quantile, using an autoregressive structure. Let z_t be the random variable of interest. The evolution of the time-varying quantiles is specified as follows:

$$q_t(\beta_\theta) = \beta_{\theta 0} + \sum_{i=1}^{q} \beta_{\theta i} q_{t-i} + \sum_{j=1}^{p} l(\beta_{\theta j}, z_{t-j}, \Omega_t). \tag{2}$$

The autoregressive terms $\beta_{\theta i} q_{t-i}(\beta_\theta)$ ensure that the quantile changes slowly over time. The rationale is to capture the volatility clustering typical of financial variables. $l(\cdot)$, which is a function of a finite number of lagged values of observables that belong to the information set at time t, establishes a link between these predetermined variables and the quantile. This is the means by which variables characterizing the financial and economic conditions of the market under scrutiny are allowed to affect the characteristics of the returns distribution.

The unknown parameters of the CAViaR model are estimated via the regression quantiles loss function, first introduced by Koenker and Bassett (1978). Define $\rho_\theta(\lambda) \equiv [\theta - I(\lambda \leq 0)]\,\lambda$, where $I(\cdot)$ denotes an indicator function that takes on value one if the expression in parenthesis is true and zero otherwise. The unknown parameters of the quantile specification can be consistently estimated by solving the following minimisation problem:

$$\min_{\beta_\theta} T^{-1} \sum_{t=1}^{T} \rho_\theta \left(z_t - q_t(\beta_\theta)\right).$$

Engle and Manganelli (2004) provide sufficient conditions for consistency and asymptotic normality results.

For the purpose of the present paper, we need to derive the joint distribution of the regression quantile estimators of the two different time series, y_t and x_t. Let $\beta_{\theta_i} \equiv [\beta'_{\theta_i Y}, \beta'_{\theta_i X}]'$ denote the vector containing the θ_i-quantile regression parameters for y_t and x_t, and $\beta \equiv [\beta'_{\theta_1}, ..., \beta'_{\theta_m}]'$, where $0 < \theta_1 < \ldots < \theta_m < 1$. Define also the following matrices:

$$D_{\theta Z} \equiv E\left[T^{-1} \sum_{t=1}^{T} h_t^{\theta Z}(q_t^Z(\beta^0_{\theta Z})|\Omega_t) \nabla q_t^Z(\beta^0_{\theta Z}) \nabla' q_t^Z(\beta^0_{\theta K})\right] \qquad (Z = Y, X), \tag{3}$$

$$D_\theta \equiv diag[D_{\theta Y}, D_{\theta X}],$$

where $h_t^{\theta Y}(q_t^Y(\beta^0_{\theta Y})|\Omega_t)$ and $h_t^{\theta X}(q_t^X(\beta^0_{\theta X})|\Omega_t)$ are the value of the density functions of y_t and x_t evaluated at the θ-quantile and $\nabla q_t^K(\beta^0_{\theta j})$ is the gradient of the quantile function. Finally, let $\nabla q_t(\beta^0_{\theta_i}) \equiv [\nabla' q_t^Y(\beta^0_{\theta_i Y}),\ \nabla' q_t^X(\beta^0_{\theta_i X})]'$. The following corollary derives the joint asymptotic distribution of the regression quantile estimators.

Corollary 1 *Under assumptions C0-C7 and AN1-AN4 in Appendix A,* $\sqrt{T}A^{-1/2}D(\hat{\beta}-\beta^0)\xrightarrow{d}N(0,I)$*, where*

$$D\equiv diag(D_{\theta_i})\qquad i=1,...,m,$$
$$A\equiv\left[\left(\min\{\theta_i,\theta_j\}-\theta_i\theta_j\right)A^{ij}\right]_{i,j=1}^{m},$$
$$A^{ij}\equiv E\left[T^{-1}\sum_{t=1}^{T}\nabla q_t(\beta^0_{\theta_i})\nabla' q_t(\beta^0_{\theta_j})\right]\qquad i,j=1,...,m.$$

Engle and Manganelli (2004) provide asymptotically consistent estimators of the variance-covariance matrix (see their theorem 3).

3.2 Estimation of the Conditional Probability

We estimate the average conditional probability $p_t(\theta)$ by running the following regression:

$$I_t^Y(\beta^0_{\theta_i})=\alpha_{\theta_i}I_t^X(\beta^0_{\theta_i})+\epsilon_t\qquad i=1,...,m,\tag{4}$$

where $I_t^Y(\beta^0_{\theta_i})\equiv I\left(y_t\leq q_t^Y(\beta^0_{\theta_i Y})\right)$ and $I_t^X(\beta^0_{\theta_i})\equiv I\left(x_t\leq q_t^X(\beta^0_{\theta_i X})\right)$. In case one is interested in testing whether this conditional probability changes during crisis times, a dummy variable D_t^C indicating the crisis period can be included in the regression.

The econometrics is complicated by the fact that we observe only estimated quantities. In practice, we can run only the following regression:

$$I_t^Y(\hat{\beta}_{\theta_i})=\tilde{\alpha}_{\theta_i}I_t^X(\hat{\beta}_{\theta_i})+\tilde{\epsilon}_t\qquad i=1,...,m,\tag{5}$$

where the hat indicates that the expression is evaluated at the estimated regression quantile parameters. To incorporate crisis dummies, it is convenient to rewrite this regression in a more general form:

$$I_t^Y(\hat{\beta}_{\theta_i})=\hat{W}_{\theta_i t}\tilde{\alpha}_{\theta_i}+\tilde{\epsilon}_t.\tag{6}$$

Without crisis dummies, $\hat{W}_{\theta_i t}\equiv I_t^X(\hat{\beta}_{\theta_i})$. When crisis dummies $\left(D_t^C\right)$ are included, we have $\hat{W}_{\theta_i t}\equiv[I_t^X(\hat{\beta}_{\theta_i}),\ I_t^X(\hat{\beta}_{\theta_i})D_t^C]$.

Let $\alpha^0\equiv[\alpha^0_{\theta_1}{}',...,\alpha^0_{\theta_m}{}']'$ be the vector of true unknown parameters to be estimated. Similarly, define $\hat{\tilde{\alpha}}\equiv[\hat{\tilde{\alpha}}_{\theta_1}{}',...,\hat{\tilde{\alpha}}_{\theta_m}{}']'$, where $\hat{\tilde{\alpha}}_{\theta_i}$ is the OLS estimator of (6). We need to derive the asymptotic distribution of $\sqrt{T}\left(\hat{\tilde{\alpha}}-\alpha^0\right)$. Note that $\hat{\tilde{\alpha}}\equiv T^{-1}\hat{Q}^{-1}\hat{R}$, where $\hat{Q}\equiv T^{-1}diag(\hat{W}'_{\theta_i}\hat{W}_{\theta_i})$, $\hat{R}\equiv[\hat{W}'_{\theta_i}I^Y(\hat{\beta}_{\theta_i})]_{i=1}^{m}$, $\hat{W}_{\theta_i}\equiv[\hat{W}_{\theta_i t}]_{t=1}^{T}$ and $I^Y(\hat{\beta}_{\theta_i})\equiv[I_t^Y(\hat{\beta}_{\theta_i})]_{t=1}^{T}$.

The following theorem shows that the OLS estimators of regression (6) are asymptotically consistent estimators of the average conditional probability $p_t(\theta)$ in tranquil and crisis periods.

Theorem 2 ***(Consistency)*** - *Assume that* $C/T \overset{T\to\infty}{\longrightarrow} k$, *where* $k \in (0,1)$ *is the asymptotic ratio between the number of observations in crisis periods (C) and the total number (T) of periods. Under the same assumptions of Corollary 1,*

$$\hat{\hat{\alpha}}^1_{\theta_i} \xrightarrow{p} E[p_t(\theta_i)|\text{ no crisis}] \equiv p^N(\theta_i) \qquad i = 1, ..., m,$$

$$\hat{\hat{\alpha}}^1_{\theta_i} + \hat{\hat{\alpha}}^2_{\theta_i} \xrightarrow{p} E[p_t(\theta_i)|\text{crisis}] \equiv p^C(\theta_i) \qquad i = 1, ..., m.$$

$\hat{\hat{\alpha}}^1_{\theta_i}$ is the parameter associated with $I^X_t(\hat{\beta}_{\theta_i})$ and, as such, it converges to the average probabilities of no crisis. Similarly, since $\hat{\hat{\alpha}}^2_{\theta_i}$ is the coefficient of $I^X_t(\hat{\beta}_{\theta_i})D^C_t$, the sum of $\hat{\hat{\alpha}}^1_{\theta_i} + \hat{\hat{\alpha}}^2_{\theta_i}$ converges in probability to the average probabilities of a crisis. According to this theorem, testing for an increase in the conditional probability during crisis periods is equivalent to testing for the null that $\alpha^2_{\theta_i}$ is equal to zero. Indeed, it is only when $\alpha^2_{\theta_i} = 0$ that the two conditional probabilities coincide. Otherwise, if $\alpha^2_{\theta_i}$ is less than zero, the conditional probability over crisis times will be lower than the conditional probability during no crisis. By the same token, if $\alpha^2_{\theta_i}$ is greater than zero, the conditional probability over crisis periods will be higher than the conditional probability estimated during tranquil times.

The asymptotic distribution of the OLS estimators is derived in the following theorem.

Define

$$W^0_{\theta_i} \equiv \left[I^X_t(\beta^0_{\theta_i}), I^X_t(\beta^0_{\theta_i})D^C_t\right]^T_{t=1},$$

$$R \equiv [R_t(\beta^0)]^T_{t=1},$$

$$R_t(\beta^0) \equiv \left[I^{YX}_t(\beta^0_{\theta_i}) - E[I^{YX}_t(\beta^0)], \quad I^{YX}_t(\beta^0_{\theta_i})D^C_t - E[I^{YX}_t(\beta^0)\ |\text{crisis}]\right]^m_{i=1},$$

$$I^{YX}_t(\beta^0_{\theta_i}) \equiv I^X_t(\beta^0_{\theta_i})I^Y_t(\beta^0_{\theta_i})$$

$$\Psi \equiv [\psi_t(\beta^0)]^T_{t=1}$$

$$\psi_t(\beta^0) \equiv [\psi_t(\beta^0_{\theta_i})]^m_{i=1}$$

$$\psi_t(\beta^0_{\theta_i}) \equiv \left[(\theta_i - I^Y_t(\beta^0_{\theta_i Y}))\nabla' q^Y_t(\beta^0_{\theta_i Y}),\ (\theta_i - I^X_t(\beta^0_{\theta_i X}))\nabla' q^X_t(\beta^0_{\theta_i X})\right]'$$

Theorem 3 ***(Asymptotic Normality)*** - *Under the same assumptions of Corollary 1,*

$$\sqrt{T}M^{-1/2}Q\left(\hat{\hat{\alpha}} - \alpha^0\right) \xrightarrow{d} N(0, I), \tag{7}$$

where

$$Q \equiv E[T^{-1}diag(W^{0\prime}_{\theta_i}W^0_{\theta_i})], \tag{8}$$

$$M \equiv E[T^{-1}(R + GD^{-1}\Psi)'(R + GD^{-1}\Psi)], \tag{9}$$

$$G \equiv diag(G_{\theta_i}), \tag{10}$$

$$\begin{aligned} G_{\theta_i} \quad \equiv \quad & E\left\{T^{-1}\sum_{t=1}^{T} U_t \left[\nabla' q_t^X(\beta_{\theta_i}^0) \int_{-\infty}^{q_t^Y(\beta_{\theta_i}^0)} h_t(q_t^X(\beta_{\theta_i}^0), y)dy + \right.\right. \\ & \left.\left. +\nabla' q_t^Y(\beta_{\theta_i}^0) \int_{-\infty}^{q_t^X(\beta_{\theta_i}^0)} h_t(x, q_t^Y(\beta_{\theta_i}^0))dx\right]\right\}, \end{aligned} \tag{11}$$

$$U_t \equiv [1, D_t^C]',$$

and $h_t(x, y)$ is the joint pdf of (x_t, y_t).

This result is new in the regression quantile literature. Without the correction term $GD^{-1}\Psi$ in the matrix M, we would get the standard OLS variance-covariance matrix. The correction is needed in order to account for the estimated regression quantile parameters that enter the OLS regression. This correction term is similar to the one derived by Engle and Manganelli (2004) for the in-sample Dynamic Quantile test. The main difference is related to the composition of the matrix G. Since two different random variables (x_t and y_t) enter the regression, G contains the terms $\int_{-\infty}^{q_t^Y(\beta_{\theta_i}^0)} h_t(q_t^X(\beta_{\theta_i}^0), y)dy$ and $\int_{-\infty}^{q_t^X(\beta_{\theta_i}^0)} h_t(x, q_t^Y(\beta_{\theta_i}^0))dx$, which can be interpreted as the bivariate analogue of the height of the density function evaluated at the quantile that typically appears in standard errors of regression quantiles.

The variance-covariance matrix can be consistently estimated using plug-in estimators. The only non-standard term is G_{θ_i}, whose estimator is provided by the following theorem.

Theorem 4 ***(Variance-Covariance Estimation)*** **-** *Under the same assumptions of Theorem 3 and assumptions VC1-VC3 in Appendix A, $\hat{G}_{\theta_i} \xrightarrow{p} G_{\theta_i}$, where*

$$\begin{aligned} \hat{G}_{\theta_i} \equiv (2T\hat{c}_T)^{-1} \sum_{t=1}^{T} & \left\{ I(|x_t - q_t^X(\hat{\beta}_{\theta_i})| < \hat{c}_T) I(y_t - q_t^Y(\hat{\beta}_{\theta_i}) < 0) U_t \nabla'_\beta q_t^X(\hat{\beta}_{\theta_i}) \right. \\ & \left. + I(|y_t - q_t^Y(\hat{\beta}_{\theta_i})| < \hat{c}_T) I(x_t - q_t^X(\hat{\beta}_{\theta_i}) < 0) U_t \nabla'_\beta q_t^Y(\hat{\beta}_{\theta_i}) \right\}, \end{aligned}$$

and $\hat{c}_T$ is defined in assumption VC1.

4 Data

The empirical analysis is carried out on returns on equity indices for four Latin American countries, Brazil, Mexico, Chile and Argentina. We choose these equity markets for several reasons. First, they are considered to be emerging markets and therefore believed to be less robust to externals shocks than fully developed markets. Second, at least one of the countries in the sample (Mexico) has experienced in the recent past a severe financial crisis that is widely recognized to have affected other Latin American economies. Third, the four equity markets are open over the same hours during the day. Hence the daily returns we investigate are synchronous, avoiding the confounding effects that non synchronous returns can have on the measurement of co-movements (see Martens and Poon, 2001, and Sander and Kleinmeier, 2003). Equity returns are continuously compounded and computed from Morgan Stanley Capital International (MSCI) world indices, which are market-value-weighted and do not include dividends. The data set covers the period from December 31st, 1987 to June 3rd, 2004 for a total of 4226 days on which at least one of the market is open. Although the four equity markets in our sample are almost always open simultaneously, there are instances in which markets are closed in one country and opened in the other, as national holidays and administrative closure do not fully coincide. To adjust for these non-simultaneous closures, for each pair of country, we include only the returns for the days on which both markets were open that day and had been open the day before.[5]

Descriptive statistics for the asset data and the sample characteristics are given in Table 1. In Panel A the overall sample univariate statistics are reported. There is strong evidence of excess skewness and leptokurtosis at 1% significance level, a clear sign of non-normality. This is confirmed by the Jarque-Bera normality test. The second part of Panel A reports, for each pair of countries, sample correlations on the first line and sample size on the second line. When considering each market individually (diagonal elements), we have a maximum of 3,975 valid daily returns for Chile and a minimum of 3,883 returns for Brazil. The off-diagonal report bivariate correlations and sample size. For example, over the whole period, there are 3,718 days for which both the Argentinian and Mexican equity markets were open simultaneously, and neither was closed on the preceding day. Bivariate sample sizes vary from a maximum of 3,749 for Chile and Argentina to a minimum of 3,682 for Brazil and Argentina. Over those days on which both market in each pair was open, the average

[5]We also implemented an alternate way to adjust for non–simultaneous market closures. We retained the returns on the day after the market closure for the market that did close. However, since the return on the day after a market closure is in fact a multi–day return, we adjusted the returns on the market that did not close by cumulating the daily returns over the period the other market closed plus the day it reopened. This procedure added between 10 and 25 observations to the different pairs and did not materially affect the results.

correlation of daily returns is 0.25.

We use the definitions of Forbes and Rigobon (2002) to determine the crisis periods in our sample. Turbulent times in our sample cover three sub-periods: November 1, 1994 to March 31, 1995 (Tequila crisis); June 2, 1997 to December 31, 1997 (Asian crisis); and August 3, 1998 to December 31, 1998 (Russian crises). The crisis sample includes 371 potential trading days. Excluding market closures and the subsequent day, we have a maximum of 347 valid crisis daily returns for Argentina and a minimum of 343 returns for Brazil. Panel B and C report univariate sample size and volatilities (diagonal elements) and bivariate sample size and correlations (off-diagonal elements) for both tranquil and crisis periods. What is striking from Panel B and C is that correlations increase dramatically between tranquil and crisis periods: the average correlation is approximately 0.19 over tranquil days and approximately 0.68 for days of turbulence. Based on this type of evidence traditional tests of correlation would have indicated the presence of contagion. However, the table also documents that for all countries, except Argentina, returns volatility increased dramatically in crisis over tranquil periods. This highlights the heteroskedasticity problem identified by Forbes and Rigobon (2002) and casts doubts on the reliability of the correlation evidence.

In the following section we investigate these issues with the contagion box and provide a more robust and nuance answer to the question.

5 Empirical Results: an Application to Latin America

In this section, we report the results of the contagion box methodology to the analysis of co-movements across some Latin American equity markets. We investigate if the probability of co-movement over crisis times versus tranquil periods increases for Brazil, Mexico, Chile and Argentina. To illustrate the methodology, we first plot the conditional probability of tail events, $p(\theta)$, estimated using unconditional and conditional quantile regressions against the benchmark of independence. Next, we compare these probabilities to those obtained from simulations of typical bivariate returns distributions calibrated to match sample moments. Finally, in a second group of charts, we report estimated conditional probabilities of co-movements between equity return pairs during tranquil and crisis times, and provide tests of the difference in co-movement incidence between the two periods. Crisis periods are first determined exogenously and then in terms of high volatility.

To characterize the shape of $p(\theta)$ it would be necessary to have knowledge about the joint distribution of security returns. Natural benchmarks are the normal or

Student$-t$ distribution, in the case fat tails need to be accommodated. Therefore, in the simulation exercise, we assume that returns are either bivariate normal or Student$-t$ with five degrees of freedom. The distributions are calibrated with the unconditional correlation and volatility of the relevant sample returns. In the same set of charts we also report a conditional probability estimated according to equation (5) where constant and time-varying quantiles are used. When estimating this probability we utilise the whole sample period, which includes both crisis and tranquil times. More importantly, no assumption about the distribution of returns is needed. A visual comparison allows to detect whether estimated probabilities deviate from what would be expected if the true data generating process followed a normal or a Student$-t$ distribution. Take as an example the country pair Brazil-Argentina (see figure 2). For $\theta \leqslant 0.5$, that is, for returns below the median, the simulated probabilities tend to underestimate the estimated conditional probability of co-movements. As for the right tail, i.e. for $\theta > 0.5$, the probability curve obtained with regression quantiles approximately coincide with the co-movement probability generated by the simulation. If co-movements were analysed through correlation estimates, it would not be possible to distinguish between right and left tails of a distribution.

We estimate the time-varying quantiles of the returns, z_t, using the following CAViaR specification:

$$q_t(\beta_\theta) = \beta_{\theta 0} + \beta_{\theta 1} D_t^C + \beta_{\theta 2} z_{t-1} + \beta_{\theta 3} q_{t-1}(\beta_\theta) - \beta_{\theta 2}\beta_{\theta 3} z_{t-2} + \beta_{\theta 4} |z_{t-1}| . \qquad (12)$$

The rational behind this parametrisation lies in the strong autocorrelation (both in levels and squares) exhibited by our sample returns. This CAViaR model would be correctly specified if the true DGP were as follows:

$$z_t = \gamma_0 + \gamma_1 z_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim i.i.d. \left(0, \sigma_t^2\right), \qquad (13)$$

$$\sigma_t = \alpha_0 + \alpha_1 |z_{t-1}| + \alpha_2 \sigma_{t-1}.$$

We add the dummy variable D_t^C to the CAViaR specification to ensure that we have exactly the same proportion of quantile exceedances in both tranquil and crisis periods. This will guarantee that $\Pr\left(y_t \leq q_t^Y(\beta_{\theta Y}^0) | x_t \leq q_t^X(\beta_{\theta X}^0)\right) = \Pr(x_t \leq q_t^X(\beta_{\theta X}^0) | y_t \leq q_t^Y(\beta_{\theta Y}^0))$ as per Theorem 1.[6] For each market we estimate model (12)

[6] Asymptotically, correct specification would imply the same number of exceedances in crisis and tranquil periods. However, in finite samples, this need not to be the case. Failure to account for this fact would affect the estimation of the conditional probabilities.

for 99 quantile probabilities ranging from 1% to 99%.

To check whether the parametrization we propose is sensible, we carry out the in-sample Dynamic Quantile (DQ) test of Engle and Manganelli (2004). The DQ statistic tests the null hypothesis of no autocorrelation in the exceedances of the quantiles as correct specification would require. The DQ test is implemented with 20 lags of the "hit" function (see Theorem 4 of Engle and Manganelli, 2004, for details). We report in figures 3A-3B the p-values of the DQ test statistic for the 99 estimated quantiles of Argentinian and Brazilian returns. For comparison, we show in the same picture the DQ test associated to the unconditional quantiles. Unconditional quantile specifications are rejected most of the times, while CAViaR models are not.

Figure 4 and figures 5A-5E represent the estimated conditional probabilities of co-movement over crisis and tranquil times for all the country pairs. Notice that conditional probabilities are represented over the whole distribution and not only for lower and upper quantiles. Our approach permits to explore how and if the conditional probability of co-movements changes for any interval in the support of the distribution. The attractiveness of inspecting all the quantiles lies in the fact that one does not need to arbitrarily specify a large absolute value return as a symptom of a crisis.

In figures 4 and 5A-5E two solid lines are plotted together with the case of independence. The thin line indicates the conditional probability of co-movements under the benchmark or, equivalently, over tranquil times. This line is the graphical representation of $p^N(\theta)$ in Definition 1. The thick line, instead, shows the conditional probability of co-movements during crisis times and plots $p^C(\theta)$. The confidence bands associated to plus or minus twice the standard errors are reported as dotted lines. When the bold line lies above the benchmark, this can be interpreted as evidence for increased co-movements or contagion. When the two lines approximately coincide, there is no difference in co-movements between the two periods. Finally, if the thick line lies below the benchmark, during crises time the co-movements between two different markets actually decrease.

The results for Argentina and Brazil show striking evidence of contagion for most quantiles. Only in the extreme upper and lower parts of the distribution, where standard errors become wider due to the limited number of exceedances, the probability of co-movement in crisis time is not statistically different from the probability of co-movement in tranquil times. The increase in probability is not only statistically but also economically significant. For instance, the probability of co-movement associated to the 10%-quantile jumps from about 24% in tranquil times to about 60% in crisis times. This implies that in quiet periods one should expect Brazilian and Argentinian equity returns to simultaneously exceed the 10%-quantile only one day out of four. In crisis periods, instead, this event will occur on average two days out of three. Similar patterns characterise the other country pairs, although the increases in probabilities are less impressive.

The interest may lie in testing whether specific parts of the distribution are subject to contagion. Rigorous joint tests for contagion which follow from the Definition 1 can be constructed as follows:

$$\begin{aligned} \widehat{\delta}\left(\underline{\theta}, \bar{\theta}\right) &= \sum_{\theta \in \left[\underline{\theta}, \bar{\theta}\right]} \left[p^C(\theta) - p^N(\theta)\right] \\ &= \sum_{\theta \in \left[\underline{\theta}, \bar{\theta}\right]} \hat{\hat{\alpha}}_{\theta_i}^2, \end{aligned} \tag{14}$$

where $\hat{\hat{\alpha}}_{\theta_i}^2$ is defined in Theorem 2. For each country pair, table 2 contains the standard errors associated with the sum of $\hat{\hat{\alpha}}_{\theta_i}^2$ over θ. Panels A, B, and C report the test statistics computed over different intervals of θ.

Three interesting points emerge from a close examination of the table. First, the country pair Mexico-Chile is the only one for which we never identify contagion. For all the others there is evidence of contagion for most parts of the distribution. Second, there are instances where one part of the distribution is subject to contagion, while others are not. This is the case for Mexico and Brazil when $\theta \in (0, 0.5]$ and $\theta \in [0.5, 1)$, and for the couples Brazil-Chile and Argentina-Chile when $\theta \in (0, 0.1]$ and $\theta \in [0.9, 1)$. Notice that this analysis could not be carried out with tests based on the estimation of correlation coefficients (Forbes and Rigobon, 2002). Third, the tests get weaker as the values of θ are restricted to be closer to the tails (see Panel C). This suggests that using only single quantiles may diminish the possibility of finding significant contagion and that a wider spectrum of quantiles is needed.

Overall, the table indicates that the distributions are characterised by strong asymmetries, which cannot be detected by simple correlation. Interestingly, the overall picture which emerges from table 2 is *not* in line with that of Forbes and Rigobon (2002), who never found evidence of contagion between Mexico and the other Latin American countries.

Finally, in figure 6 we present an example of how to introduce economic variables in the contagion box. Instead of using the historical crisis times as in Forbes and Rigobon (2002), we define crisis and tranquil periods in terms of high and low volatility, respectively. We compute the volatility of the average returns on Argentinian and

Brazilian stock markets as an exponentially weighted moving average (EWMA) with decay coefficient equal to 0.97. Next we identify as crisis periods the 10% number of observations with highest EWMA volatility, i.e. $D_t^C \equiv I\left(\sigma^2_{EWMA,t} > q^{0.90}_{\sigma^2_{EWMA}}\right)$. Contrary to the findings of Bae, Karolyi and Stulz (2003), figure 6 shows that volatility crises do not significantly increase the probability of co-movement and therefore cannot be responsible for the contagion effects we found in figure 3.

6 Summary of Results and Conclusions

In this study we propose a new methodology to measure co-dependence across distinct asset classes and financial markets. Our approach is based on the CAViaR model of Engle and Manganelli (2004) and permits to investigate whether co-dependence across securities increases during turbulent times relative to calm periods. We compute the conditional probability that returns on a certain market fall in the left (or right) tail of their own distribution provided that returns on a different market have fallen in the same tail of their own distribution. Probabilities are computed not only for extreme quantiles, but span the whole distribution. These conditional probabilities are visualised in "the contagion box", which is a square of unit side. As an illustration, we utilise our methodology to detect possible presence of contagion across the most important Latin American equity markets. Our results show that, on average, over turbulent times, co-movements in equity returns across national markets tend to increase significantly, both in the left and in the right tails of the distributions.

The approach we propose is quite general and can find application for portfolio allocation, risk management and financial stability. Our methodology permits to estimate the probability of co-movements for different ranges of the return distribution and for different market conditions. Crisis periods may be defined exogenously or endogenously as a function of information variables. Further our methodology allows us to take into account local and global economic forces that may drive the returns distribution and their co-movements.

A number of questions can be addressed, which leaves ample room for future research. For instance, it would be possible to test if a crisis spills over across markets, independently of how sound fundamentals are. Contagion is often divided into two categories (Karolyi, 2003). The first category refers to the so-called "fundamental-based contagion", which occurs when co-movements in financial asset prices result from real and financial linkages among market economies. The second category of contagion involves co-movements that cannot be explained by fundamentals, but are rather the results of investors' behaviour. Financial panic, herd behaviour and increase in risk aversion are examples of the so-called "irrational contagion". A possible strategy to implement such a test would be to define the crisis periods in terms of a set of economic variables and then testing whether the associated coefficient is significantly different from zero.

Other issues related to market linkages can be addressed as well. In the context of the European Union, for instance, there is strong interest in investigating how the inter-relations among "New" and "Old" Member States financial markets have evolved after accession.

Appendix A - Assumptions

Consistency Assumptions

C0. (Ω, F, P) is a complete probability space, and $\{y_t, x_t, \omega_t\}$, $t = 1, 2, ...$ are random variables on this space.

C1. The functions $q_t^Z(\beta_{\theta_i Z})$, $Z = Y, X$, $i = 1, ..., m$, a mapping from B (a compact subset of $\Re^p$) to $\Re$ are measurable with respect to the information set Ω_t and continuous in B, for any given choice of explanatory variables $\{z_{t-1}, \omega_{t-1}, ..., z_1, \omega_1\}$, where $z_t = y_t, x_t$ and $\omega_t \in \Omega_t$.

C2. $h_t^Z(z|\Omega_t)$ - the conditional density of z_t - is continuous.

C3. There exists $h > 0$ such that, for all t and for all $i = 1, ..., m$, $h_t^{\theta_i Z}(q_t^Z(\beta^0_{\theta_i Z})|\Omega_t) \geq h$.

C4. $|q_t^Z(\beta_{\theta_i Z})| < K(\Omega_t)$ for all $\beta_{\theta_i Z} \in B$ and for all t, where $K(\Omega_t)$ is some (possibly) stochastic function of variables that belong to Ω_t, such that $E[K(\Omega_t)] \leq K_0 < \infty$.

C5. $E[|z_t|] < \infty$ for all t.

C6. $\{\rho_{\theta_i}(z_t - q_t^Z(\beta_{\theta_i Z}))\}$ obeys the uniform law of large numbers.

C7. For every $\xi > 0$, there exists a $\tau > 0$ such that if $||\beta - \beta^0_{\theta_i Z}|| \geq \xi$, then $\liminf_{T\to\infty} \sum P[|q_t^Z(\beta_{\theta_i Z}) - q_t^Z(\beta^0_{\theta_i Z})| > \tau] > 0$.

Asymptotic Normality Assumptions

AN1. $q_t^Z(\beta_{\theta_i Z})$ is differentiable in B and for all β and γ in a neighbourhood υ_0 of $\beta^0_{\theta_i Z}$, such that $||\beta_{\theta_i Z} - \gamma_{\theta_i Z}|| \leq d$ for d sufficiently small and for all t:

(a) $||\nabla q_t^Z(\beta_{\theta_i Z})|| \leq F(\Omega_t)$, where $F(\Omega_t)$ is some (possible) stochastic function of variables that belong to Ω_t and $E[F(\Omega_t)^3] \leq F_0 < \infty$, for some constant F_0.

(b) $||\nabla q_t^Z(\beta_{\theta_i Z}) - \nabla q_t^Z(\gamma_{\theta_i Z})|| \leq M(\Omega_t, \beta_{\theta_i Z}, \gamma_{\theta_i Z}) = O(||\beta_{\theta_i Z} - \gamma_{\theta_i Z}||)$, where $M(\Omega_t, \beta_{\theta_i Z}, \gamma_{\theta_i Z})$ is some function such that $E[M(\Omega_t, \beta_{\theta_i Z}, \gamma_{\theta_i Z})^2] \leq M_0||\beta_{\theta_i Z} - \gamma_{\theta_i Z}|| < \infty$ and $E[M(\Omega_t, \beta_{\theta_i Z}, \gamma_{\theta_i Z})F(\Omega_t)] \leq M_1||\beta_{\theta_i Z} - \gamma_{\theta_i Z}|| < \infty$ for some constants M_0 and M_1.

AN2. (a) $h_t^Z(z|\Omega_t) \leq H < \infty\ \forall t$.

(b) $h_t^Z(z|\Omega_t)$ satisfies the Lipschitz condition $|h_t^Z(\lambda_1|\Omega_t) - h_t^Z(\lambda_2|\Omega_t)| \leq L|\lambda_1 - \lambda_2|$, $\forall t$, for some constant $L < \infty$.

AN3. The matrices A^{ij} and $D_{\theta_i Z}$ have smallest eigenvalue bounded below by a positive constant for T sufficiently large.

AN4. The sequences $\{T^{-1/2}\sum_{t=1}^{T}[\theta_i - I(z_t \leq q_t^Z(\beta^0_{\theta_i Z}))]\nabla q_t^Z(\beta^0_{\theta_i Z})\}$ obey the central limit theorem.

Variance-Covariance Matrix Estimation Assumptions

VC1. $\hat{c}_T/c_T \xrightarrow{p} 1$, where the non-stochastic positive sequence c_T satisfies $c_T = o(1)$ and $c_T^{-1} = o(T^{1/2})$.

VC2. $E[F(\Omega_t)^4] \leq F_1 < \infty$, $\forall t$, where $F(\Omega_t)$ was defined in assumption AN1(a).

VC3. (a) $T^{-1}\sum_{t=1}^{T}\nabla q_t^Z(\beta^0_{\theta_i Z})\nabla' q_t^Z(\beta^0_{\theta_j Z}) \xrightarrow{p} A^{ij}$

(b)$T^{-1}\sum_{t=1}^{T} h_t^{\theta_i Z}(q_t^Z(\beta^0_{\theta_i Z})|\Omega_t)\nabla q_t^Z(\beta^0_{\theta_i Z})\nabla' q_t^Z(\beta^0_{\theta_i Z}) \xrightarrow{p} D_{\theta_i K}$

(c) $T^{-1}\sum_{t=1}^{T} U_t \left[\nabla' q_t^X(\beta^0_{\theta_i})\int_{-\infty}^{0} h_t(q_t^X(\beta^0_{\theta_i}), y)dy + \nabla' q_t^Y(\beta^0_{\theta_i})\int_{-\infty}^{0} h_t(x, q_t^Y(\beta^0_{\theta_i}))dx\right] \xrightarrow{p}$ G_{θ_i}

Appendix B - Proofs of theorems in the text

Proof of Theorem 1

1. Symmetry: $F_t^-\left(q_t^Y(\beta^0_{\theta Y}) \mid q_t^X(\beta^0_{\theta X})\right) \equiv \frac{\Pr\left(y_t \leq q_t^Y(\beta^0_{\theta Y}),\, x_t \leq q_t^X(\beta^0_{\theta X})\right)}{\Pr\left(x_t \leq q_t^X(\beta^0_{\theta X})\right)} = \frac{\Pr\left(y_t \leq q_t^Y(\beta^0_{\theta Y}),\, x_t \leq q_t^X(\beta^0_{\theta X})\right)}{\Pr\left(y_t \leq q_t^Y(\beta^0_{\theta Y})\right)} \equiv F_t^-\left(q_t^X(\beta^0_{\theta X}) \mid q_t^Y(\beta^0_{\theta Y})\right)$, because $\Pr\left(x_t \leq q_t^X(\beta^0_{\theta X})\right) = \Pr\left(y_t \leq q_t^Y(\beta^0_{\theta Y})\right) = \theta$.

2. Co-monotonicity

$\Longleftarrow$ Co-monotonicity requires that $F_t(y^1, x^2) = \min\{F_t^Y(y^1), F_t^X(x^2)\}$, where $F_t^Y(y)$ and $F_t^X(x)$ are the distribution functions of y_t and x_t, respectively. Let $y^1 = q_t^Y(\beta^0_{\theta Y})$ and $x^2 = q_t^X(\beta^0_{\theta X})$ and suppose first that $\theta < 0.5$. We get $F_t(q_t^Y(\beta^0_{\theta Y}), q_t^X(\beta^0_{\theta X})) = F_t^Y(q_t^Y(\beta^0_{\theta Y})) = F_t^X(q_t^X(\beta^0_{\theta X})) = \theta$. Therefore, $p(\theta) \equiv \frac{F_t(q_t^Y(\beta^0_{\theta Y}), q_t^X(\beta^0_{\theta X}))}{F_t^X(q_t^X(\beta^0_{\theta X}))} = \frac{F_t^X(q_t^X(\beta^0_{\theta X}))}{F_t^X(q_t^X(\beta^0_{\theta X}))} = 1$. For $\theta > 0.5$, simply note that $\Pr(y_t \geq y^1, x_t \geq x^2) = 1 - \Pr(y_t \leq y^1) - \Pr(x_t \leq x^2) + \Pr(y_t \leq y^1, x_t \leq x^2) = 1 - \theta$.

$\Longrightarrow$ Suppose, without loss of generality, that $\Pr(y_t \leq y^1) = \Pr(x_t \leq x^1) \leq \Pr(x_t \leq x^2)$. Suppose first that $\Pr(y_t \leq y^1) = \Pr(y_t \leq y^2) < 0.5$. Then, $\Pr(y_t \leq y^1, x_t \leq x^2) = \Pr(y_t \leq y^1)\Pr(x_t \leq x^2 \mid y_t \leq y^1)$. But $\Pr(x_t \leq x^2 \mid y_t \leq y^1) = \Pr(x_t \leq x^1 + (x^2 - x^1) \mid y_t \leq y^1) = \Pr(x_t \leq x^1 \mid y_t \leq y^1) + \Pr(x^1 \leq x_t \leq x^2 \mid y_t \leq y^1) \geq \Pr(x_t < x^1 \mid y_t < y^1) = 1$, so $\Pr(x_t \leq x^2 \mid y_t \leq y^1) = 1$. Therefore $\Pr(y_t \leq y^1,\ x_t \leq x^2) = \Pr(y_t \leq y^1) = \min\{F_t^Y(y^1), F_t^X(x^2)\}$. Suppose now that $\Pr(y_t \leq y^1) > 0.5$. Then, $\Pr(y_t \geq y^1, x_t \geq x^2) = \Pr(x_t \geq x^2)\Pr(y_t \geq y^1|x_t \geq x^2)$. But $\Pr(y_t \geq y^1|x_t \geq x^2) \geq \Pr(y_t \geq y^2|x_t \geq x^2) = 1$. So $\Pr(y_t \geq y^1, x_t \geq x^2) =$

$\Pr(x_t \geq x^2) = 1 - \Pr(x_t \leq x^2) - \Pr(y_t \leq y^1) + \Pr(y_t \leq y^1, x_t \leq x^2)$, which implies $\Pr(y_t \leq y^1, x_t \leq x^2) = \Pr(y_t \leq y^1) = \min\{F_t^Y(y^1), F_t^X(x^2)\}$.

3. Counter-monotonicity

$\Longleftarrow$ Counter-monotonicity requires that $F_t(y^1, x^2) = \max\{F_t^Y(y^1) + F_t^X(x^2) - 1, 0\}$. Assume, without loss of generality, that $F_t^Y(y^2) = F_t^X(x^2) \geq F_t^X(x^1) = F_t^Y(y^1)$. Consider first the case $F_t^Y(y^1) + F_t^X(x^2) \leq 1$. Then $F_t(y^1, x^2) \geq F_t(y^1, x^1) = 0$ implying $p(\theta) = 0$. Consider now the case $F_t^Y(y^1) + F_t^X(x^2) \geq 1$. Note that $\Pr(y_t \geq y^1, x_t \geq x^2) = 1 - F_t^X(x^2) - F_t^Y(y^1) + F_t(y^1, x^2) = 0$. Therefore, $0 = \Pr(y_t \geq y^1, x_t \geq x^2) \geq \Pr(y_t \geq y^2, x_t \geq x^2)$ implying $p(\theta) = 0$.

4. Independence:

$\Longleftarrow$ By independence $F_t(y^1, x^1) = F_t^Y(y^1) F_t^X(x^1)$. So $\Pr(y_t \leq y^1 \mid x_t \leq x^1) = \frac{\Pr(y_t \leq y^1)\Pr(x_t \leq x^1)}{\Pr(x_t \leq x^1)} = \theta$. Q.E.D.

Proof of Corollary 1 - Rewrite equation (B2) in the proof of theorem 2 of Engle and Manganelli (2004) for y_t, x_t and all θ_i:

$$\begin{array}{c}
D_{\theta_1 Y} T^{1/2}(\hat{\beta}_{\theta_1 Y} - \beta^0_{\theta_1 Y}) \xrightarrow{d} T^{-1/2} \sum_{t=1}^T \psi_t(\beta^0_{\theta_1 Y}) \\
D_{\theta_1 X} T^{1/2}(\hat{\beta}_{\theta_1 X} - \beta^0_{\theta_1 X}) \xrightarrow{d} T^{-1/2} \sum_{t=1}^T \psi_t(\beta^0_{\theta_1 X}) \\
\vdots \\
D_{\theta_m Y} T^{1/2}(\hat{\beta}_{\theta_m Y} - \beta^0_{\theta_m Y}) \xrightarrow{d} T^{-1/2} \sum_{t=1}^T \psi_t(\beta^0_{\theta_m Y}) \\
D_{\theta_m X} T^{1/2}(\hat{\beta}_{\theta_m X} - \beta^0_{\theta_m X}) \xrightarrow{d} T^{-1/2} \sum_{t=1}^T \psi_t(\beta^0_{\theta_m X})
\end{array}$$

where $\psi_t(\beta^0_{\theta_i Y}) \equiv [\theta_i - I(y_t \leq q_t^Y(\beta^0_{\theta_i Y}))] \nabla q_t^Y(\beta^0_{\theta_i Y})$, $i = 1, ..., m$ and $\psi_t(\beta^0_{\theta_i X})$ is defined analogously. Defining $\psi_t(\beta^0_{\theta_i}) \equiv [\psi_t(\beta^0_{\theta_i Y})', \psi_t(\beta^0_{\theta_i X})']'$ and stacking every pair Y and X together:

$$\begin{array}{c}
D_{\theta_1} T^{1/2}(\hat{\beta}_{\theta_1} - \beta^0_{\theta_1}) \xrightarrow{d} T^{-1/2} \sum_{t=1}^T \psi_t(\beta^0_{\theta_1}) \\
\vdots \\
D_{\theta_m} T^{1/2}(\hat{\beta}_{\theta_m} - \beta^0_{\theta_m}) \xrightarrow{d} T^{-1/2} \sum_{t=1}^T \psi_t(\beta^0_{\theta_m})
\end{array}$$

Stacking once again these relationships together, we get:

$$D\, T^{1/2}(\hat{\beta} - \beta^0) \xrightarrow{d} T^{-1/2} \sum_{t=1}^T \begin{bmatrix} \psi_t(\beta^0_{\theta_1}) \\ \vdots \\ \psi_t(\beta^0_{\theta_m}) \end{bmatrix}$$

The result follows from application of the central limit theorem (assumption AN4). Q.E.D.

Proof of Theorem 2 - We denote with $\sum_C$ and $\sum_N$ the summation over the observations in crisis and non crisis periods. The OLS estimators for a generic θ_i are:

$$\hat{\hat{\alpha}}^1_{\theta_i} = \frac{\sum_N I_t^{YX}(\hat{\beta}_{\theta_i})}{\sum_N I_t^{X}(\hat{\beta}_{\theta_i})}$$

and

$$\hat{\hat{\alpha}}^2_{\theta_i} = \frac{\sum_C I_t^{YX}(\hat{\beta}_{\theta_i})}{\sum_C I_t^{X}(\hat{\beta}_{\theta_i})} - \frac{\sum_N I_t^{YX}(\hat{\beta}_{\theta_i})}{\sum_N I_t^{X}(\hat{\beta}_{\theta_i})}$$

Next, we show that both numerators and denominators, when appropriately re-scaled, converge to well defined probabilities. We consider only one case, as the others can be obtained similarly. We show first that $C^{-1}\left\{\sum_C [I_t^X(\hat{\beta}_{\theta_i}) - I_t^X(\beta^0_{\theta_i})]\right\} = o_p(1)$. Define $\epsilon^X_{\theta_i t} \equiv x_t - q_t^X(\beta^0_{\theta_i})$, $\hat{\epsilon}^X_{\theta_i t} \equiv x_t - q_t^X(\hat{\beta}_{\theta_i})$ and $\delta_t(\hat{\beta}_{\theta_i}) \equiv q_t^X(\beta^0_{\theta_i}) - q_t^X(\hat{\beta}_{\theta_i})$. Suppose that $\delta_t(\hat{\beta}_{\theta_i}) > 0$. The same reasoning goes through for $\delta_t(\hat{\beta}_{\theta_i}) < 0$. Then:

$$\begin{aligned} |I_t^X(\hat{\beta}_{\theta_i}) - I_t^X(\beta^0_{\theta_i})| &= |I(\epsilon^X_{\theta_i t} \leq \delta_t(\hat{\beta}_{\theta_i})) - I(\epsilon^X_{\theta_i t} \leq 0)| \\ &\leq I(0 \leq \epsilon^X_{\theta_i t} \leq \delta_t(\hat{\beta}_{\theta_i})) \end{aligned}$$

Therefore, applying the mean value theorem:

$$\begin{aligned} E|I_t^X(\hat{\beta}_{\theta_i}) - I_t^X(\beta^0_{\theta_i})| &\leq E|\textstyle\int_0^{\delta_t(\hat{\beta}_{\theta_i})} \tilde{h}_t^{\theta_i X}(\epsilon)\, d\epsilon| \\ &= E|\tilde{h}_t^{\theta_i X}(\delta_t(\hat{\beta}_{\theta_i}))\nabla q_t^X(\beta^*_{\theta_i})(\hat{\beta}_{\theta_i} - \beta^0_{\theta_i})| \end{aligned}$$

where $\tilde{h}_t^{\theta_i X}(\epsilon)$ is the pdf of $(x_t - q_t^X(\beta^0_{\theta_i}))$ and $\beta^*_{\theta_i}$ lies between $\hat{\beta}_{\theta_i}$ and $\beta^0_{\theta_i}$. Now choose $d > 0$ arbitrarily small and T sufficiently large such that $||\hat{\beta}_{\theta_i} - \beta^0_{\theta_i}|| < d$. This, together with assumptions AN1(a) and AN2(a), implies that

$$\begin{aligned} E|I_t^X(\hat{\beta}_{\theta_i}) - I_t^X(\beta^0_{\theta_i})| &\leq E|H||\hat{\beta}_{\theta_i} - \beta^0_{\theta_i}||F(\Omega_t)| \\ &\leq E|HdF(\Omega_t)| \\ &\leq E|HdF_0| = O(d) \end{aligned}$$

Since d can be chosen arbitrarily small, this result implies that:

$$\begin{aligned} E\left|C^{-1}\left\{\textstyle\sum_C [I_t^X(\hat{\beta}_{\theta_i}) - I_t^X(\beta^0_{\theta_i})]\right\}\right| &\leq C^{-1}\left\{\textstyle\sum_C E|I_t^X(\hat{\beta}_{\theta_i}) - I_t^X(\beta^0_{\theta_i})|\right\} \\ &= O(d) = o_p(1) \end{aligned}$$

Now we need to show that $C^{-1}\sum_C \left[I_t^X(\beta^0_{\theta_i}) - \Pr(x_t \leq q_t^X(\beta^0_{\theta_i}))\right] = o_p(1)$. This term has expectation 0 and variance equal to:

$$C^{-2}\sum_C E[I_t^X(\beta^0_{\theta_i}) - \Pr(x_t \leq q_t^X(\beta^0_{\theta_i}))]^2 = C^{-1}\theta_i(1-\theta_i) \overset{T\to\infty}{\to} 0$$

because all the cross products have expectation 0. Exactly the same reasoning is valid for the other terms. Q.E.D.

Proof of Theorem 3 - Consider first the case $m = 1$ and drop the subscript θ for notational convenience. Define:

$$g_t(\hat{\beta}) \equiv R_t(\hat{\beta}) - \bar{\alpha}^0$$

where $\bar{\alpha}^0 \equiv \left[E[I_t^{YX}(\beta^0)],\ E[I_t^{YX}(\beta^0) \mid \text{crisis}]\right]'$. We show first that $T^{-1/2}\sum_{t=1}^T g_t(\hat{\beta}) = T^{-1/2}\sum_{t=1}^T\{g_t(\beta^0) + \tilde{G}_t(\hat{\beta}-\beta^0)\} + o_p(1)$, where $\tilde{G}_t \equiv U_t[\nabla' q_t^X(\beta^0)\int_{-\infty}^0 \tilde{h}_t(0,\eta)d\eta + \nabla' q_t^Y(\beta^0)\int_{-\infty}^0 \tilde{h}_t(\nu,0)d\nu]$and $\tilde{h}_t(\nu,\eta)$ is the joint pdf of $\left(x_t - q_t^X(\beta^0), y_t - q_t^Y(\beta^0)\right)$. Define $r_t(\hat{\beta}) \equiv \left[g_t(\hat{\beta}) - g_t(\beta^0)\right] - \tilde{G}_t(\hat{\beta}-\beta^0)$. We need to show that $r_T(\hat{\beta}) \equiv T^{-1/2}||\sum_{t=1}^T r_t(\hat{\beta})||$ converges to zero in probability, that is, $\forall \xi > 0$, $\lim_{T\to\infty} P\left(r_T(\hat{\beta}) > \xi\right) = 0$, or, by the Chebyschev inequality, that $\lim_{T\to\infty} E[r_T(\hat{\beta})] = 0$

First note that

$$\begin{aligned} g_t(\hat{\beta}) - g_t(\beta^0) &= U_t\left[I_t^{YX}(\hat{\beta}) - I_t^{YX}(\beta^0)\right] \\ &= U_t\left[I(\eta_t \le \delta_t^Y(\hat{\beta}))I(\nu_t \le \delta_t^X(\hat{\beta})) - I(\eta_t \le 0)I(\nu_t \le 0)\right] \end{aligned}$$

where $\delta_t^Y(\hat{\beta}) \equiv q_t^Y(\hat{\beta}) - q_t^Y(\beta^0)$ and $\delta_t^X(\hat{\beta}) \equiv q_t^X(\hat{\beta}) - q_t^X(\beta^0)$.

Assume now, without loss of generality, that both $\delta_t^Y(\hat{\beta})$ and $\delta_t^X(\hat{\beta})$ are greater than zero. The same reasoning goes through in the other cases.

$$\begin{aligned} g_t(\hat{\beta}) - g_t(\beta^0) = U_t\Big[\Big[I(\eta_t \le 0) + I(0 < \eta_t \le \delta_t^Y(\hat{\beta})\Big]\,[I(\nu_t \le 0)+ \\ +I(0 < \nu_t \le \delta_t^X(\hat{\beta})\Big] - I(\eta_t \le 0)I(\nu_t \le 0)\Big] \end{aligned}$$

$$\begin{aligned} &= U_t\Big[I(\eta_t \le 0)I(\nu_t \le 0) + I(\eta_t \le 0)I(0 < \nu_t \le \delta_t^X(\hat{\beta}))+ \\ &+I(\nu_t \le 0)I(0 < \eta_t \le \delta_t^Y(\hat{\beta})) + I(0 < \eta_t \le \delta_t^Y(\hat{\beta}))\cdot \\ &\cdot I(0 < \nu_t \le \delta_t^X(\hat{\beta})) - I(\eta_t \le 0)I(\nu_t \le 0)\Big] \end{aligned}$$

Putting these results together, we get:

$$\begin{aligned} E[r_T(\hat{\beta})] \;\le\; & T^{-1/2}\sum_{t=1}^T E||U_t[I(\eta_t < 0)I(0 < \nu_t < \delta_t^X(\hat{\beta})) + & (15)\\ & + I(\nu_t < 0)I(0 < \eta_t < \delta_t^Y(\hat{\beta})) + & (16)\\ & + I(0 < \eta_t < \delta_t^Y(\hat{\beta}))I(0 < \nu_t < \delta_t^X(\hat{\beta})) - & (17)\\ & -[\nabla' q_t^X(\beta^0)\int_{-\infty}^0 \tilde{h}_t(0,\eta)d\eta + \\ & +\nabla' q_t^Y(\beta^0)\int_{-\infty}^0 \tilde{h}_t(\nu,0)d\nu](\hat{\beta}-\beta^0)]|| \end{aligned}$$

For the expectation in (15), applying Holder's inequality ($E||Y|| \leq ||E(Y)||$), we have:

$$
\begin{aligned}
&E\left[U_t I(\eta_t < 0) I(0 < \nu_t < \delta_t^X(\hat{\beta}))\right] \\
&\quad = E\left[U_t E_t\left[I(\eta_t < 0) I(0 < \nu_t < \delta_t^X(\hat{\beta}))\right]\right] \\
&\quad = E\left[U_t \int_{-\infty}^0 \int_0^{\delta_t^X(\hat{\beta})} \tilde{h}_t(\nu, \eta) d\nu d\eta\right] \\
&\quad = E\left[U_t \int_{-\infty}^0 \tilde{h}_t(0, \eta) \nabla' q_t^X(\beta^*)(\hat{\beta} - \beta^0) d\eta\right]
\end{aligned}
$$

where β^* lies between $\hat{\beta}$ and β^0. For (16):

$$
\begin{aligned}
&E\left[U_t I(\nu_t < 0) I(0 < \eta_t < \delta_t^Y(\hat{\beta}))\right] \\
&\quad = E\left[U_t E_t\left[I(\nu_t < 0) I(0 < \eta_t < \delta_t^Y(\hat{\beta}))\right]\right] \\
&\quad = E\left[U_t \int_{-\infty}^0 \int_0^{\delta_t^Y(\hat{\beta})} \tilde{h}_t(\nu, \eta) d\eta d\nu\right] \\
&\quad = E\left[U_t \int_{-\infty}^0 \tilde{h}_t(\nu, 0) \nabla' q_t^Y(\beta^{**})(\hat{\beta} - \beta^0) d\nu\right]
\end{aligned}
$$

where β^{**} lies between $\hat{\beta}$ and β^0. For (17):

$$
\begin{aligned}
&E\left[U_t I(0 < \eta_t < \delta_t^Y(\hat{\beta})) I(0 < \nu_t < \delta_t^X(\hat{\beta}))\right] \\
&\quad = E\left[U_t E_t\left[I(0 < \eta_t < \delta_t^Y(\hat{\beta})) I(0 < \nu_t < \delta_t^X(\hat{\beta}))\right]\right] \\
&\quad = E\left[U_t \int_0^{\delta_t^Y(\hat{\beta})} \int_0^{\delta_t^X(\hat{\beta})} \tilde{h}_t(\nu, \eta) d\nu d\eta\right] \\
&\quad = E\left[U_t \int_0^{\delta_t^Y(\hat{\beta})} \tilde{h}_t(0, \eta) \nabla' q_t^X(\beta^*)(\hat{\beta} - \beta^0) d\eta\right] \\
&\quad = E\left\{\left[U_t \tilde{h}_t(0,0) \nabla' q_t^Y(\beta^{**}) \nabla' q_t^X(\beta^*)(\hat{\beta} - \beta^0) + \right.\right. \\
&\qquad\qquad \left.\left. + \int_0^{\delta_t^Y(\hat{\beta})} \tilde{h}_t(0, \eta) \nabla' q_t^X(\beta^*) d\eta\right](\hat{\beta} - \beta^0)\right\} \\
&\quad = E\left\{\left[2U_t \tilde{h}_t(0,0) \nabla' q_t^Y(\beta^{**}) \nabla' q_t^X(\beta^*)(\hat{\beta} - \beta^0)\right](\hat{\beta} - \beta^0)\right\}
\end{aligned}
$$

where β^* and β^{**} lie between $\hat{\beta}$ and β^0. This last term is $O(\|\hat{\beta} - \beta^0\|^2)$. So:

$$
\begin{aligned}
Er_T(\hat{\beta}) \leq\ & T^{-1/2} \textstyle\sum_{t=1}^T ||U_t E[\int_{-\infty}^0 \tilde{h}_t(0, \eta) d\eta \nabla' q_t^X(\beta^*)(\hat{\beta} - \beta^0) + \\
& + \textstyle\int_{-\infty}^0 \tilde{h}_t(\nu, 0) d\nu \nabla' q_t^Y(\beta^{**})(\hat{\beta} - \beta^0) + \\
& + 2\tilde{h}_t(0,0) \nabla' q_t^Y(\beta^{**}) \nabla' q_t^X(\beta^*)(\hat{\beta} - \beta^0)(\hat{\beta} - \beta^0) \\
& - (\nabla' q_t^X(\beta^0) \textstyle\int_{-\infty}^0 \tilde{h}_t(0, \eta) d\eta + \nabla' q_t^Y(\beta^0) \int_{-\infty}^0 \tilde{h}_t(\nu, 0) d\nu)(\hat{\beta} - \beta^0)]|| \\
\\
& = T^{-1/2} \textstyle\sum_{t=1}^T ||U_t E[\int_{-\infty}^0 \tilde{h}_t(0, \eta)[\nabla' q_t^X(\beta^*) - \nabla' q_t^X(\beta^0)](\hat{\beta} - \beta^0) d\eta + \\
& + \textstyle\int_{-\infty}^0 \tilde{h}_t(\nu, 0)[\nabla' q_t^Y(\beta^{**}) - \nabla' q_t^Y(\beta^0)](\hat{\beta} - \beta^0) d\nu + \\
& + 2\tilde{h}_t(0,0) \nabla' q_t^Y(\beta^{**}) \nabla' q_t^X(\beta^*)(\hat{\beta} - \beta^0)(\hat{\beta} - \beta^0)]|| \\
\\
& \leq T^{-1/2} \textstyle\sum_{t=1}^T ||E[M(\Omega_t, \beta^*, \beta^0)(\hat{\beta} - \beta^0) + \\
& + M(\Omega_t, \beta^{**}, \beta^0)(\hat{\beta} - \beta^0) + \\
& + 2HF(\Omega_t)^2 ||\hat{\beta} - \beta^0||]
\end{aligned}
$$

$= T^{-1/2}\sum_{t=1}^{T} O_p(||\hat{\beta}-\beta^0||^2)$

$= o_p(1)$

because $\hat{\beta}-\beta^0 = o_p(T^{-1/2})$. Therefore:

$$T^{-1/2}\sum_{t=1}^{T} g_t(\hat{\beta}) = T^{-1/2}\sum_{t=1}^{T} g_t(\beta^0) + \tilde{G}\sqrt{T}(\hat{\beta}-\beta^0) + o_p(1) \tag{18}$$

Furthermore, from the proof of corollary 2 we have:

$$DT^{1/2}(\hat{\beta}-\beta^0) \xrightarrow{d} T^{-1/2}\sum_{t=1}^{T} \psi_t(\beta^0) \tag{19}$$

Combining these two relations we get:

$$\begin{aligned} T^{-1/2}\textstyle\sum_{t=1}^{T} g_t(\hat{\beta}) &= T^{-1/2}\textstyle\sum_{t=1}^{T} g_t(\beta^0) + \tilde{G}D^{-1}T^{-1/2}\textstyle\sum_{t=1}^{T}\psi_t(\beta^0) \\ &= T^{-1/2}\textstyle\sum_{t=1}^{T}\left[g_t(\beta^0) + \tilde{G}D^{-1}\psi_t(\beta^0)\right] \end{aligned}$$

Since $\tilde{G} \xrightarrow{p} G$, application of the central limit theorem yields the result.

For the case $m \geqslant 2$, simply stack the above relationships together for each θ_i to get:

$$T^{-1/2}\sum_{t=1}^{T} g_t(\hat{\beta}) = T^{-1/2}\sum_{t=1}^{T}\left[[g_t(\beta^0_{\theta_i})]_{i=1}^{m} + diag(G_{\theta_i})D^{-1}[\psi_t(\beta^0_{\theta_i}))]_{i=1}^{m}\right] \tag{20}$$

The result follows. Q.E.D.

Proof of Theorem 4 - The proof is similar to the proof of Theorem 3 of Engle and Manganelli (2004). Drop the subscript θ for notational convenience and define

$$\tilde{G}_X \equiv (2Tc_T)^{-1}\sum_{t=1}^{T} I(|\nu_t| < c_T)I(\eta_t < 0)U_t\nabla'_\beta q_t^X(\beta^0) \tag{21}$$

The other term of G can be estimated analogously. We first show that $\hat{G}_X - \tilde{G}_X = o_p(1)$ and then that $\tilde{G}_X - G_X = o_p(1)$. Define $\hat{\nu}_t \equiv x_t - q_t^X(\hat{\beta})$ and $\hat{\eta}_t \equiv y_t - q_t^Y(\hat{\beta})$. Then:

$||\hat{G}_X - \tilde{G}_X|| = \frac{c_T}{\hat{c}_T}||(2c_T T)^{-1}$

$\sum_{t=1}^{T} I(|\hat{\nu}_t| < \hat{c}_T)I(\hat{\eta}_t < 0)-$

$-I(|\nu_t| < c_T)I(\eta_t < 0)U_t\nabla'_\beta q_t^X(\hat{\beta})+$

$$+I(|\nu_t| < c_T)I(\eta_t < 0)U_t\left[\nabla'_\beta q_t^X(\hat{\beta}) - \nabla'_\beta q_t^X(\beta^0)\right] +$$
$$+\tfrac{c_T-\hat{c}_T}{c_T}I(|\nu_t| < c_T)I(\eta_t < 0)U_t\nabla'_\beta q_t^X(\beta^0)||$$

Note that $\hat{\eta}_t \equiv \eta_t - \delta_t^Y(\hat{\beta})$ and $\hat{\nu}_t \equiv \nu_t - \delta_t^X(\hat{\beta})$. We can rewrite the indicator functions in the first line as:

$$|I(|\hat{\nu}_t| < \hat{c}_T)I(\hat{\eta}_t < 0) - I(|\nu_t| < c_T)I(\eta_t < 0)| =$$
$$= |I(|\nu_t - \delta_t^X(\hat{\beta})| < \hat{c}_T)I(\eta_t - \delta_t^Y(\hat{\beta}) < 0) - I(|\nu_t| < c_T)I(\eta_t < 0)|$$
$$= |I(|\nu_t - \delta_t^X(\hat{\beta})| < \hat{c}_T)[I(\eta_t < 0) + I(0 < \eta_t < \delta_t^Y(\hat{\beta}))I(\delta_t^Y(\hat{\beta}) > 0) -$$
$$-I(\delta_t^Y(\hat{\beta}) < \eta_t < 0)I(\delta_t^Y(\hat{\beta}) < 0)] - I(|\nu_t| < c_T)I(\eta_t < 0)|$$
$$\leq \left|\left[I(|\nu_t - \delta_t^X(\hat{\beta})| < \hat{c}_T) - I(|\nu_t| < c_T)\right] I(\eta_t < 0)\right| +$$
$$+I(|\nu_t - \delta_t^X(\hat{\beta})| < \hat{c}_T)I\left(-\left|\delta_t^Y(\hat{\beta})\right| < \eta_t < \left|\delta_t^Y(\hat{\beta})\right|\right)$$

Next note that:

$$\left|\left[I(|\nu_t - \delta_t^X(\hat{\beta})| < \hat{c}_T) - I(|\nu_t| < c_T)\right] I(\eta_t < 0)\right| \leq$$
$$\leq I\left(|\nu_t + c_T| < |\delta_t^X(\hat{\beta})| + |c_T - \hat{c}_T|\right) +$$
$$+I\left(|\nu_t - c_T| < |\delta_t^X(\hat{\beta})| + |c_T - \hat{c}_T|\right)$$

Therefore:

$$||\hat{G}_X - \tilde{G}_X|| \leq \tfrac{c_T}{\hat{c}_T}(2c_T T)^{-1}$$
$$\textstyle\sum_{t=1}^T\{[I\left(|\nu_t + c_T| < |\delta_t^X(\hat{\beta})| + |c_T - \hat{c}_T|\right) +$$
$$+I\left(|\nu_t - c_T| < |\delta_t^X(\hat{\beta})| + |c_T - \hat{c}_T|\right)]F(\Omega_t)+$$
$$+I(|\nu_t - \delta_t^X(\hat{\beta})| < \hat{c}_T)I\left(-\left|\delta_t^Y(\hat{\beta})\right| < \eta_t < \left|\delta_t^Y(\hat{\beta})\right|\right)F(\Omega_t)+$$
$$+I(|\nu_t| < c_T)I(\eta_t < 0)M(\Omega_t, \hat{\beta}, \beta^0)+$$
$$+\left|\tfrac{c_T-\hat{c}_T}{c_T}\right|I(|\nu_t| < c_T)I(\eta_t < 0)F(\Omega_t)\Big\}$$
$$\equiv \tfrac{c_T}{\hat{c}_T}(A_1 + A_2 + A_3 + A_4)$$

where $M(\Omega_t, \hat{\beta}, \beta^0)$ and $F(\Omega_t)$ are defined in assumptions AN1 of Engle and Manganelli (2004).

Now note that for any arbitrarily small $d > 0$, there always exists $\bar{T}$ sufficiently large such that $\forall T > \bar{T}$, $\left|\frac{c_T-\hat{c}_T}{c_T}\right| < d$ and $c_T^{-1}||\hat{\beta} - \beta^0|| < d$. Next we show that $E(A_i) = O(d)$, $i = 1, 2, 3, 4$, which implies that $||\hat{G}_X - \tilde{G}_X||$ can be made arbitrarily small by choosing d sufficiently small.

$$E(A_1) \equiv (2c_T T)^{-1}E\textstyle\sum_{t=1}^T[I\left(|\nu_t + c_T| < |\delta_t^X(\hat{\beta})| + |c_T - \hat{c}_T|\right) +$$
$$+I\left(|\nu_t - c_T| < |\delta_t^X(\hat{\beta})| + |c_T - \hat{c}_T|\right)]F(\Omega_t)$$
$$= (2c_T T)^{-1}E\textstyle\sum_{t=1}^T[I\left(|\nu_t + c_T| < c_T|\nabla q_t^X(\beta^*)(\hat{\beta} - \beta^0)/c_T| + c_T|c_T - \hat{c}_T|/c_T\right) +$$
$$+I\left(|\nu_t - c_T| < c_T|\nabla q_t^X(\beta^*)(\hat{\beta} - \beta^0)/c_T| + c_T|c_T - \hat{c}_T|/c_T\right)]F(\Omega_t)$$
$$\leq (2c_T T)^{-1}E\textstyle\sum_{t=1}^T E\{[I\left(|\nu_t + c_T| < c_T d[F(\Omega_t) + 1]\right) +$$
$$+I\left(|\nu_t - c_T| < c_T d[F(\Omega_t) + 1]\right)]F(\Omega_t)|\Omega_t\}$$

$= (2c_T T)^{-1} E \sum_{t=1}^{T} [\int_{-c_T d[F(\Omega_t)+1]-c_T}^{c_T d[F(\Omega_t)+1]-c_T} h_t^X(\nu) d\nu +$
$+ \int_{-c_T d[F(\Omega_t)+1]+c_T}^{c_T d[F(\Omega_t)+1]+c_T} h_t^X(\nu) d\nu] F(\Omega_t)$
$\leq (2c_T T)^{-1} E \sum_{t=1}^{T} [2c_T d[F(\Omega_t)+1]H + 2c_T d[F(\Omega_t)+1]H] \, F(\Omega_t)$
where H is the maximum height of the density function (defined in $AN2$)
$= (2c_T T)^{-1} E \sum_{t=1}^{T} \left[4c_T d[F(\Omega_t)^2 + F(\Omega_t)]H\right]$
$\leq (2c_T T)^{-1} \sum_{t=1}^{T} [4c_T d[F_0 + F_0]H]$
$= T^{-1} \sum_{t=1}^{T} [2d[F_0 + F_0]H]$
$= [2d[F_0 + F_0]H]$
$= O(d)$

$E(A_2) \equiv (2c_T T)^{-1} E \sum_{t=1}^{T} I(|\nu_t - \delta_t^X(\hat{\beta})| < \hat{c}_T) I\left(-\left|\delta_t^Y(\hat{\beta})\right| < \eta_t < \left|\delta_t^Y(\hat{\beta})\right|\right) F(\Omega_t)$
$\leq (2c_T T)^{-1} E \sum_{t=1}^{T} I\left(-\left|\nabla q_t^Y(\beta^*)(\hat{\beta}-\beta^0)\right| < \eta_t < \left|\nabla q_t^Y(\beta^*)(\hat{\beta}-\beta^0)\right|\right) F(\Omega_t)$
$\leq (2c_T T)^{-1} E \sum_{t=1}^{T} I\left(-\left|c_T F(\Omega_t)(\hat{\beta}-\beta^0)/c_T\right| < \eta_t < \left|c_T F(\Omega_t)(\hat{\beta}-\beta^0)/c_T\right|\right) F(\Omega_t)$
$\leq (2c_T T)^{-1} E \sum_{t=1}^{T} I\left(-c_T F(\Omega_t) d < \eta_t < c_T F(\Omega_t) d\right) F(\Omega_t)$
$= (2c_T T)^{-1} E \sum_{t=1}^{T} \int_{-c_T F(\Omega_t) d}^{c_T F(\Omega_t) d} h_t^Y(\eta) d\eta F(\Omega_t)$
$\leq (2c_T T)^{-1} E \sum_{t=1}^{T} 2c_T F(\Omega_t) d H F(\Omega_t)$
$\leq T^{-1} E \sum_{t=1}^{T} F_0 d H$
$= F_0 d H$
$= O(d)$

$E(A_3) \equiv (2c_T T)^{-1} E \sum_{t=1}^{T} I(|\nu_t| < c_T) I(\eta_t < 0) M(\Omega_t, \hat{\beta}, \beta^0)$
$\leq (2c_T T)^{-1} E \sum_{t=1}^{T} M(\Omega_t, \hat{\beta}, \beta^0)$
$\leq (2c_T T)^{-1} \sum_{t=1}^{T} c_T M_0 ||\hat{\beta} - \beta^0|| / c_T$
$\leq (2T)^{-1} \sum_{t=1}^{T} M_0 d$
$= M_0 d/2$
$= O(d)$

$E(A_4) \equiv (2c_T T)^{-1} E \sum_{t=1}^{T} \left|\frac{c_T - \hat{c}_T}{c_T}\right| I(|\nu_t| < c_T) I(\eta_t < 0) F(\Omega_t)$
$\leq (2c_T T)^{-1} E \sum_{t=1}^{T} d I(|\nu_t| < c_T) F(\Omega_t)$
$= (2c_T T)^{-1} E \sum_{t=1}^{T} d \int_{-c_T}^{c_T} h_t^X(\nu) d\nu F(\Omega_t)$
$\leq (2c_T T)^{-1} E \sum_{t=1}^{T} d 2c_T H F(\Omega_t)$
$\leq T^{-1} \sum_{t=1}^{T} d H F_0$
$= d H F_0$
$= O(d)$

It remains to show that $\tilde{G}_X - G_X = o_p(1)$.
$\tilde{G}_X - G_X = (2Tc_T)^{-1} \sum_{t=1}^{T} I(|\nu_t| < c_T) I(\eta_t < 0) U_t \nabla'_\beta q_t^X(\beta^0) -$

$-(2Tc_T)^{-1}\sum_{t=1}^{T}E\left[I(|\nu_t|<c_T)I(\eta_t<0)|\Omega_t\right]U_t\nabla'_{\beta}q_t^X(\beta^0)+$
$+(2Tc_T)^{-1}\sum_{t=1}^{T}E\left[I(|\nu_t|<c_T)I(\eta_t<0)|\Omega_t\right]U_t\nabla'_{\beta}q_t^X(\beta^0)-$
$-E\left\{T^{-1}\sum_{t=1}^{T}U_t\nabla' q_t^X(\beta_\theta^0)\int_{-\infty}^{0}h_t(0,\eta)d\eta\right\}$

The term in the first two lines has expectation equal to 0 and variance equal to:

$(2Tc_T)^{-2}E\left\{\sum_{t=1}^{T}\left(I(|\nu_t|<c_T)I(\eta_t<0)-E\left[I(|\nu_t|<c_T)I(\eta_t<0)|\Omega_t\right]\right)U_t\nabla'_{\beta}q_t^X(\beta^0)\right\}^2$
$=(2Tc_T)^{-2}E\left\{\sum_{t=1}^{T}E\left(I(|\nu_t|<c_T)I(\eta_t<0)-E\left[I(|\nu_t|<c_T)I(\eta_t<0)|\Omega_t\right]\right)^2\left(U_t\nabla'_{\beta}q_t^X(\beta^0)\right)^2\right\}$
$\leq(2Tc_T)^{-2}\sum_{t=1}^{T}E\left(F(\Omega_t)\right)^2$
$\leq(2Tc_T)^{-2}\sum_{t=1}^{T}F_0$
$=(4Tc_T^2)^{-1}F_0$
$=o(1)$

For the term in the last two lines, instead, note that:

$\left|(2c_T)^{-1}E\left[I(|\nu_t|<c_T)I(\eta_t<0)|\Omega_t\right]-\int_{-\infty}^{0}h_t(0,\eta)d\eta\right|$
$=\left|(2c_T)^{-1}\int_{-c_T}^{c_T}\int_{-\infty}^{0}h_t(\nu,\eta)d\eta d\nu-\int_{-\infty}^{0}h_t(0,\eta)d\eta\right|$
$\leq\left|(2c_T)^{-1}2c_T\int_{-\infty}^{0}h_t(c^*,\eta)d\eta-\int_{-\infty}^{0}h_t(0,\eta)d\eta\right|$
where $c^*\equiv\max_{\nu\in[-c_T,c_T]}\int_{-\infty}^{0}h_t(\nu,\eta)d\eta$
$=\left|\int_{-\infty}^{0}h_t(c^*,\eta)d\eta-\int_{-\infty}^{0}h_t(0,\eta)d\eta\right|$
$\leq L|c^*|$ by assumption AN2(b)
$\leq Lc_T$
$=o(1)$

The result follows. Q.E.D.

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Table 1

Descriptive statistics of daily returns on stock market indices

This table reports the summary statistics of daily returns of the four country indices. Data are from MSCI and returns are continuously compounded. The significance level for excess skewness and excess kurtosis is based on test statistics developed by D'Agostino, Belanger and D'Agostino (1990). The Jarque-Bera (J-B) test for normality combines excess skewness and kurtosis, and is asymptotically distributed as χ^2_m with $m = 2$ degrees of freedom. * and ** denote 5% and 1% significance levels, respectively.

Panel A: Overall sample - December 31, 1987 – June 3, 2004

	Argentina	**Brazil**	**Chile**	**Mexico**
	Summary statistics			
Mean	0.29	0.49	0.08	0.11
Minimum	-20.40	-21.74	-6.05	-12.69
Maximum	39.04	24.66	8.60	12.14
Std. Dev.	3.35	2.68	1.14	1.61
Skewness	1.58**	0.25**	0.23**	0.07
Kurtosis	5.74**	11.98**	3.36**	4.86**
J–B	25055.82**	5354.92**	1894.82**	3881.02**
	Correlations and sample size			
Argentina	1.000	0.220	0.208	0.226
	3926	3682	3749	3718
Brazil		1.000	0.284	0.319
		3883	3721	3686
Chile			1.000	0.273
			3975	3741
Mexico				1.000
				3949

Table 1– continued

Panel B: Tranquil Days

	Argentina	**Brazil**	**Chile**	**Mexico**
Standard deviations, correlations and sample size				
Argentina	*3.372*	0.139	0.135	0.167
	3579	3350	3411	3396
Brazil		*2.532*	0.184	0.254
		3540	3396	3363
Chile			*1.072*	0.217
			3630	3417
Mexico				*1.522*
				3605

Panel C: Crisis Days

	Argentina	**Brazil**	**Chile**	**Mexico**
Standard deviations, correlations and sample size				
Argentina	*3.083*	0.812	0.724	0.673
	347	332	338	322
Brazil		*3.806*	0.704	0.602
		343	325	323
Chile			*1.693*	0.505
			345	324
Mexico				*2.321*
				344

Table 2

Test of difference in tail co-incidences between crisis and tranquil periods

This table reports the sum of $\hat{\hat{\alpha}}^2_{\theta_i}$ over θ, i.e. $\widehat{\delta}\left(\underline{\theta},\bar{\theta}\right) = \sum_{\theta\in[\underline{\theta},\bar{\theta}]} \hat{\hat{\alpha}}^2_{\theta_i}$, as well as the associated standard errors. The resulting t statistics provides a joint test for contagion which follows from Definition 1. Statistics indicated in bold are NOT significant at the 5% level.

Country pairs	Lower tail ($\theta \leqslant 0.5$)		Upper tail ($\theta > 0.5$)	
Panel A	$\widehat{\delta}(0, 0.5)$		$\widehat{\delta}(0.5, 1)$	
	Stat.	*s.e.*	Statistic	*s.e.*
Mex. – Bra.	7.34	*3.50*	**3.41**	*2.61*
Mex. – Arg.	9.67	*3.57*	7.27	*2.89*
Mex. – Chi.	**5.85**	*3.27*	**5.36**	*2.88*
Bra. – Arg.	13.36	*3.63*	12.28	*3.32*
Bra. – Chi.	9.26	*3.36*	8.86	*3.19*
Arg. – Chi.	10.35	*3.27*	10.51	*3.21*

Panel B	$\widehat{\delta}(0, 0.25)$		$\widehat{\delta}(0.25, 0.5)$		$\widehat{\delta}(0.5, 0.75)$		$\widehat{\delta}(0.75, 1)$	
	Stat.	*s.e.*	Stat.	*s.e.*	Stat.	*s.e.*	Stat.	*s.e.*
Mex. – Bra.	**3.74**	*2.28*	3.74	*1.73*	**3.23**	*1.65*	**0.33**	*1.42*
Mex. – Arg.	6.09	*2.29*	3.80	*1.76*	3.50	*1.68*	3.93	*1.68*
Mex. – Chi.	**3.03**	*2.16*	**2.87**	*1.63*	**2.85**	*1.59*	**2.59**	*1.79*
Bra. – Arg.	7.54	*2.28*	6.04	*1.82*	4.52	*1.71*	7.96	*2.02*
Bra. – Chi.	4.43	*2.13*	5.01	*1.74*	3.48	*1.61*	5.45	*2.00*
Arg. – Chi.	4.99	*2.08*	5.58	*1.71*	4.17	*1.64*	6.56	*2.01*

Panel C	$\widehat{\delta}(0, 0.1)$		$\widehat{\delta}(0.1, 0.5)$		$\widehat{\delta}(0.5, 0.9)$		$\widehat{\delta}(0.9, 1)$	
	Stat.	*s.e.*	Stat.	*s.e.*	Stat.	*s.e.*	Stat.	*s.e.*
Mex. – Bra.	**1.86**	*1.34*	5.61	*2.74*	**3.37**	*2.29*	**-0.02**	*0.74*
Mex. – Arg.	2.76	*1.29*	7.16	*2.83*	5.33	*2.44*	2.03	*0.87*
Mex. – Chi.	**2.13**	*1.33*	**3.77**	*2.47*	**3.63**	*2.29*	**1.77**	*1.09*
Bra. – Arg.	3.75	*1.23*	9.97	*2.92*	9.22	*2.79*	3.46	*1.01*
Bra. – Chi.	**2.15**	*1.26*	7.25	*2.70*	6.51	*2.62*	2.55	*1.10*
Arg. – Chi.	2.43	*1.23*	7.99	*2.56*	8.05	*2.62*	2.71	*1.15*

Figure 1
The Contagion Box

This figure plots the probability that a random variable y_t falls below its θ-quantile conditional on another random variable x_t being below its θ-quantile. The case of perfect positive correlation (co-monotonicity), independence, and perfect negative correlation (counter-monotonicity) are represented.

Theoretical co-incidences likelihood: co-monotonicity, independence, and counter-monotonicity

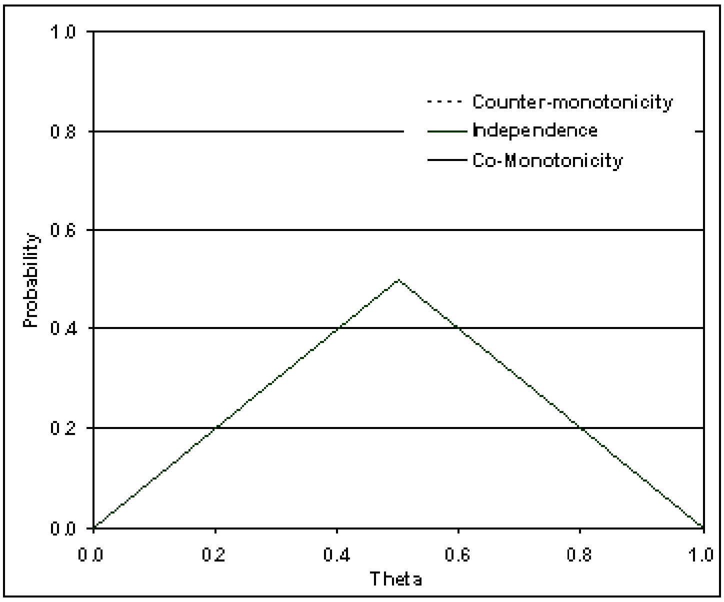
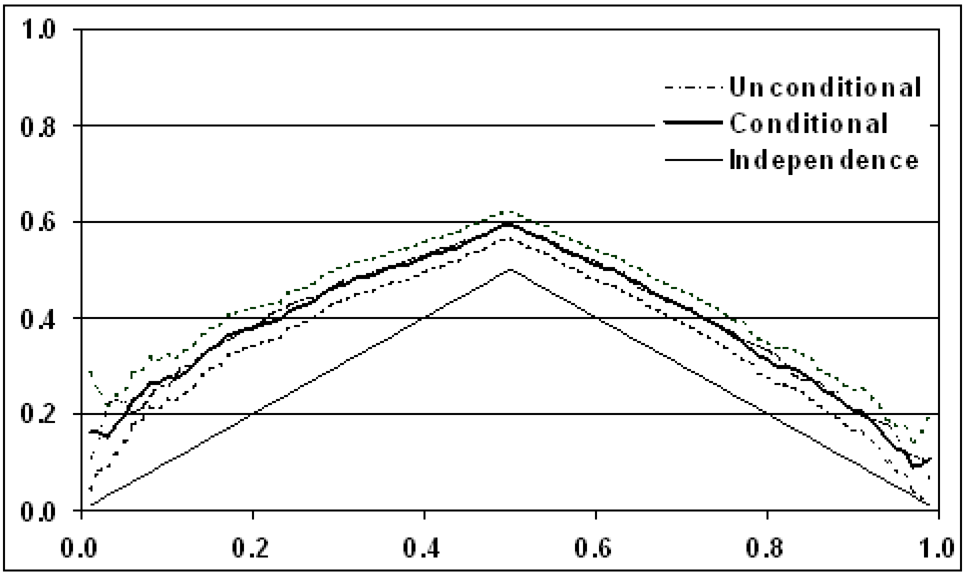


Figure 2

Brazil–Argentina simulated and estimated tail co-dependence likelihood

These figures plot the estimated probability that the second country equity index returns falls below its θ-quantile conditional on the first country index returns being below its θ-quantile. The quantiles of each returns series are estimated using unconditional or conditional quantile regressions. The estimated co-incidence likelihood is compared to a benchmark of independence or to simulated tail co-incidence based on either a bivariate normal or a bivariate student$-t$ distribution with 5 degrees of freedom. The simulations are calibrated to match the sample volatilities and correlation of the returns series. Daily index returns are from MSCI and cover the period January 1, 1988 to June 4, 2004 ($n = 3682$).

Panel A: Unconditional and conditional quantile regressions



Panel B: Simulations and conditional quantile regression

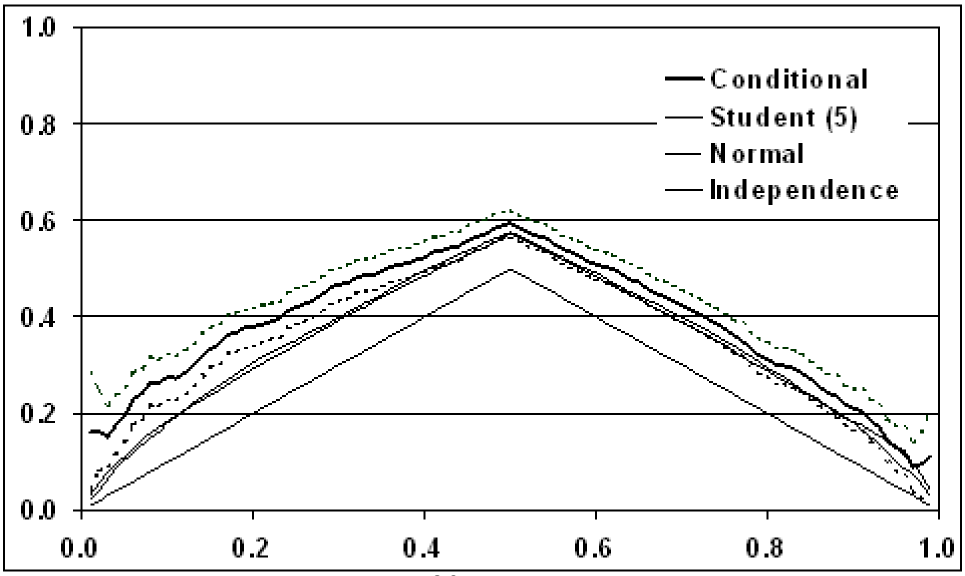
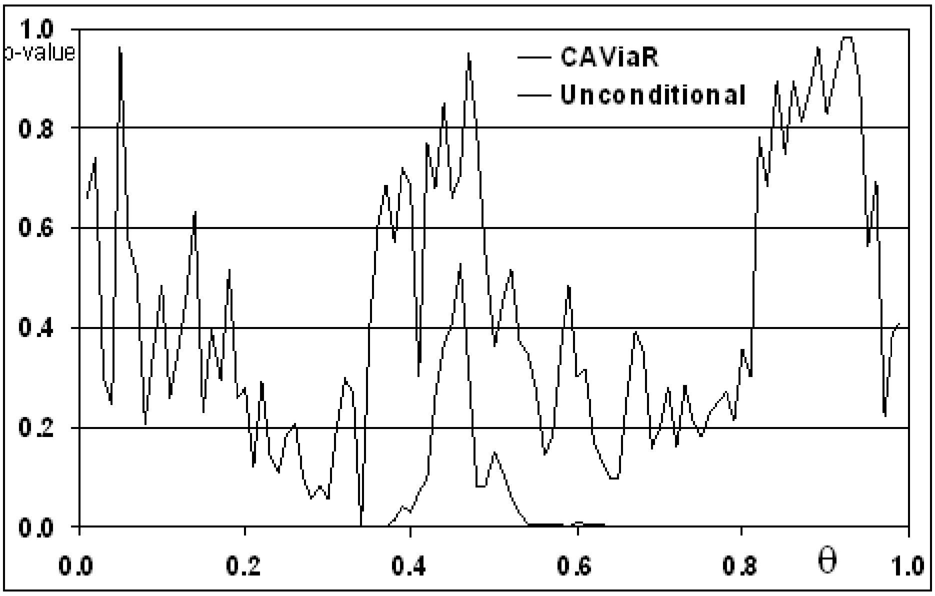


Figure 3

P-values of the Dynamic Quantile test

These figures plot the p-values of the in-sample DQ test statistic of Engle and Manganelli (2004). The DQ statistic tests the null hypothesis of no autocorrelation in the exceedances of the quantiles, as correct specification would require.

Panel A: Argentina



Panel B: Brazil

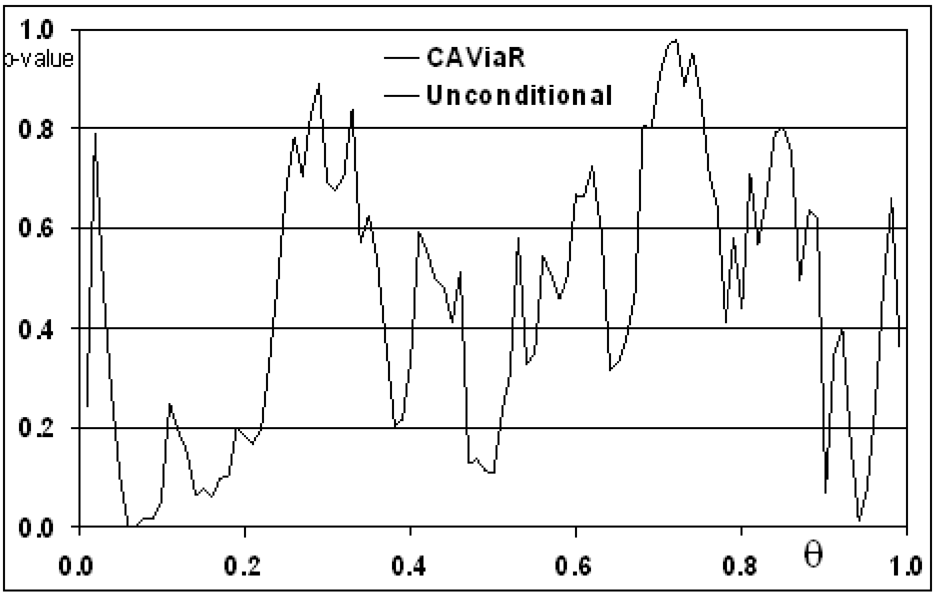


Figure 4

Brazil–Argentina estimated probabilities in crisis vs. tranquil periods

This figure plots the estimated probability that the second country equity index returns falls below its θ-quantile conditional on the first country index returns being below its θ-quantile in crisis and in tranquil periods. The quantiles of each returns series are estimated using conditional quantile regressions. The two standard error bands around the estimated co-incidence likelihood in crisis periods are plotted as dashed line. Daily index returns are from MSCI and cover the period January 1, 1988 to May 31, 2004 ($n = 3682$). The crisis sample includes 332 observations and cover the sub-periods November 1, 1994 to March 31 1995 (Tequila crisis), June 2, 1997 to December 31, 1997 (Asian crisis), and August 3, 1998 to December 31, 1998 (Russian and LTCM crisis).

Estimated probabilities from conditional quantile regression in tranquil and crisis times

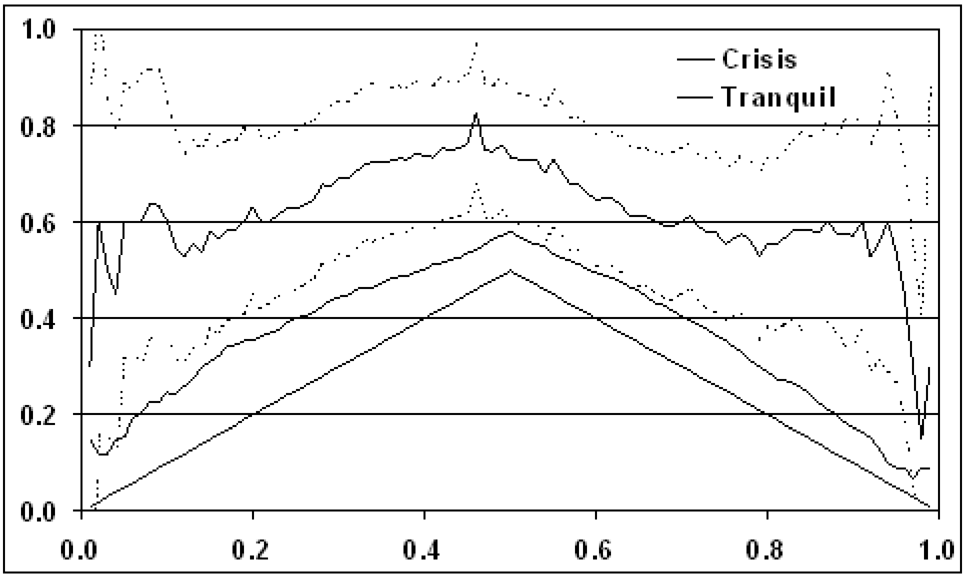
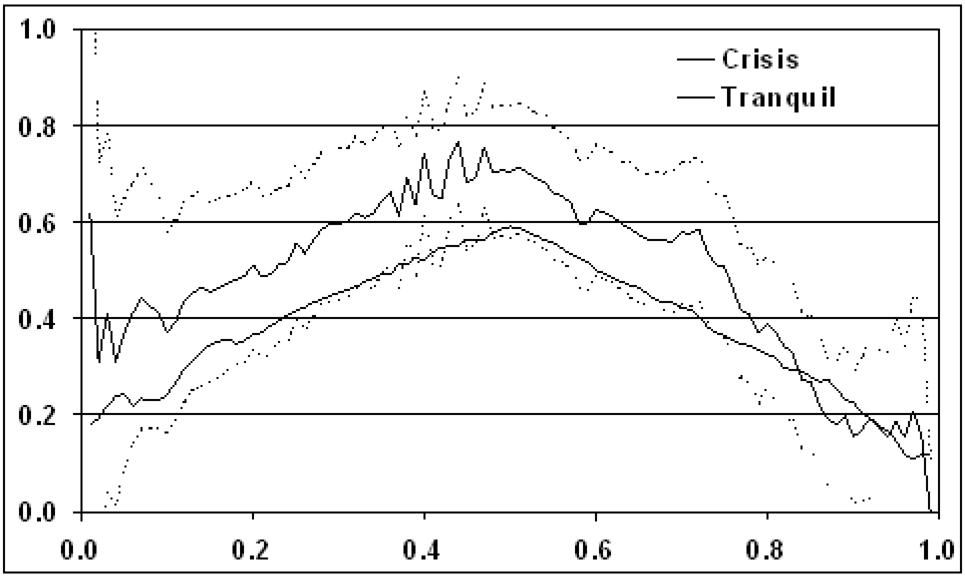


Figure 5

Estimated tail co-dependence likelihood in crisis vs. tranquil periods

These figures plot the estimated probability that the second country equity index returns falls below its θ-quantile conditional on the first country index returns being below its θ-quantile in crisis and in tranquil periods. The quantiles of each returns series are estimated using conditional quantile regressions. The two standard error bands around the estimated co-incidence likelihood in crisis periods are plotted as dashed line. Daily index returns are from MSCI and cover the period January 1, 1988 to May 31, 2004 (n_{Max}=3,749, Chile-Argentina, n_{Min}=3,682, Brazil-Argentina). The crisis sample includes a maximum of 338 (Min: 322) observations and cover the sub periods November 1, 1994 to March 31 1995 (Tequila crisis), June 2, 1997 to December 31, 1997 (Asian crisis), and August 3, 1998 to December 31, 1998 (Russian crisis).

Panel A: Mexico – Brazil



Panel B: Mexico – Argentina

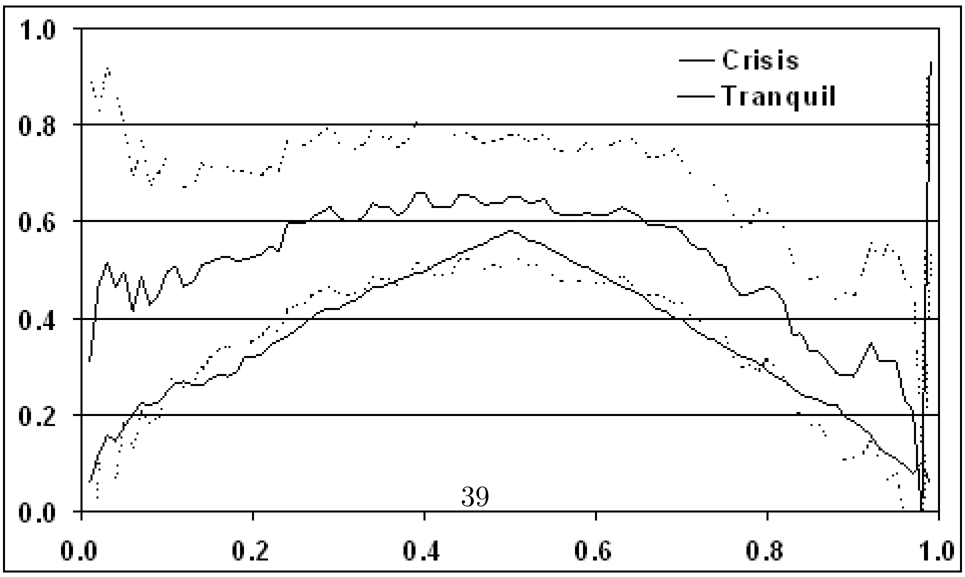
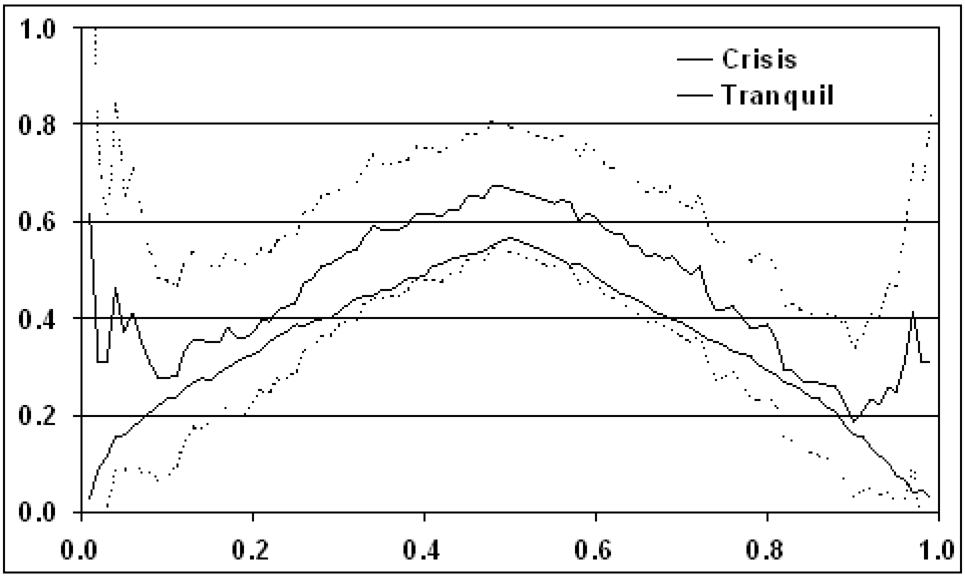


Figure 5 - continued

Estimated tail co-dependences in crisis vs. tranquil periods

Panel C: Mexico – Chile



Panel D: Argentina – Chile

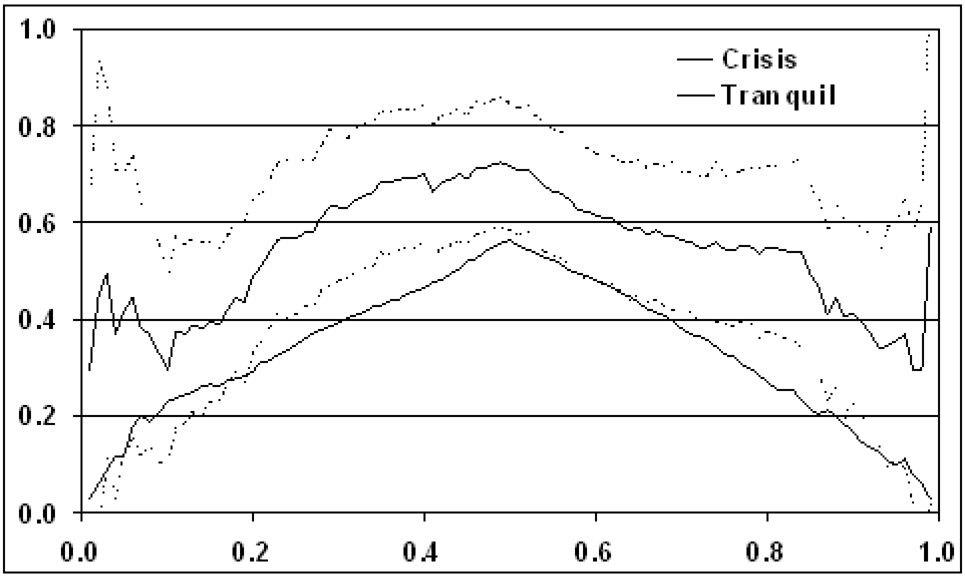


Figure 5 - continued

Estimated tail co-dependences in crisis vs. tranquil periods

Panel E: Brazil – Chile

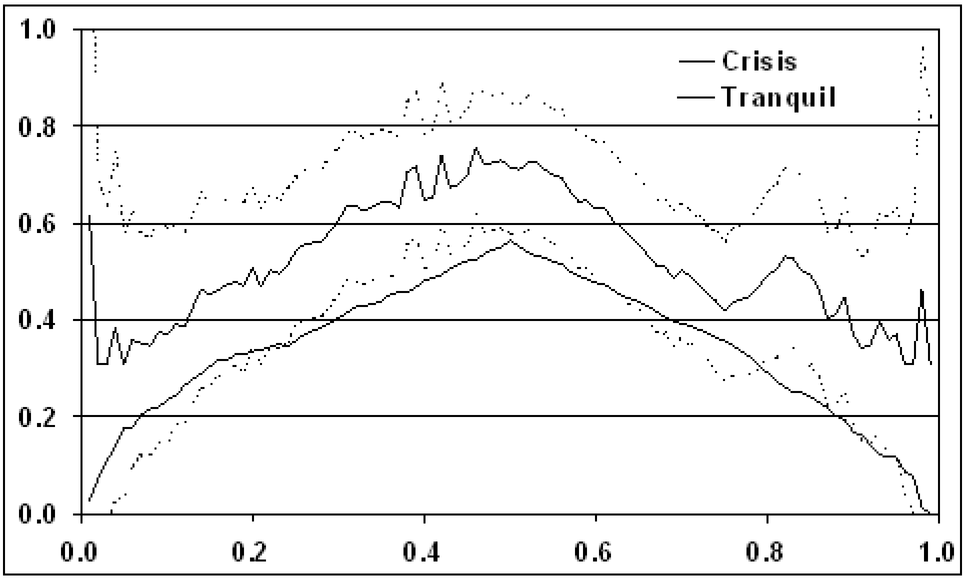


Figure 6

Volatility Crisis for Brazil and Argentina

This figure plots the probability of co-movements between Argentina and Brazil in high and low volatility periods.