A Macroeconomic Model with a Financial Sector.[†]

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PRELIMINARY AND INCOMPLETE

ABSTRACT. This paper studies a macroeconomic model in which financial experts borrow from less productive agents in order to invest in financial assets. We pursue three set of results: (i) Going beyond a steady state analysis, we show that adverse shocks cause amplifying price declines not only through the erosion of net worth of the financial sector, but also through increased price volatility, leading to precautionary hoarding and fire sales. (ii) Financial sector's leverage and maturity mismatch is excessive, since it does not internalize externalities it imposes on the labor sector and other financial experts due to a fire-sale externality. (iii) Securitization, which allows the financial sector to offload some risk, exacerbates the excessive risk-taking.

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1. Introduction

In many standard macroeconomic models, households directly invest without financial intermediaries. This approach can only yield realistic macroeconomic predictions if, in reality, there are no frictions in the financial sector. Yet, following the Great Depression, economists such as Fisher (1933), Keynes (1936) and Minsky (1986) have attributed the economic downturn to the failure of financial markets. The current financial crisis has underscored once again the importance of the financial sector for the business cycles.

We propose a model in which agents are heterogeneous and differ in their ability and/or willingness to invest in productive assets. Specifically, we assume that some agents, whom we call *experts*, are more productive - better at learning the creditworthiness of borrowers, better at making sure borrowers pay back, have special knowledge to do venture capital financing, better at picking stocks, etc. Alternatively, we could assume that experts are natural buyers because either they have a special skill that other agents do not have (as in Bernanke and Gertler (1989) and Kiyotaki (1998)) or they are more optimistic than everybody else (as in Geanakoplos (2003)). We interpret the experts as financial institutions - banks, hedge funds, private equity funds, insurance companies, etc - and study the role that the financial system plays in business cycles.

Because experts are more productive, they borrow from non-experts (households) to hold and manage assets. Leverage amplifies risks. In particular, negative macro shocks that hurt the collateral value of experts' assets can cause long lasting adverse feedback loops (Kiyotaki and Moore (1997)) and liquidity spirals as higher margins and haircuts force banks to delever (Brunnermeier and Pedersen (2009)). Deteriorating balance sheets lead to shifts of assets from experts to households, depressing prices and hurting further the experts' balance sheets. In our setting, borrowing is limited due to financial frictions in the form of moral hazard, and when experts lever up they are worried that they might be forced to fire-sell their assets in the future. This precautionary motive leads to hoarding of "dry powder", especially during crises periods when price volatility is high. In short, both price and volatility effects amplify initial macro shocks in our model.

We pursue three sets of results, and the first one concerns equilibrium dynamics. Traditionally, macro models have analyzed these amplifications primarily through steady-state analysis. In this paper, we derive full equilibrium dynamics, not just near the steady state, and argue that steady-state analysis misses important effects. Specifically, volatility effects and precautionary hoarding motives lead to a deleveraging phenomenon which makes the system significantly less stable and more volatile. As these effects significantly amplify initial shocks, a steady-state analysis severely underestimates the prominence of crisis episodes. In terms of asset pricing, we show in the time-series that asset prices are predictable, exhibit excess (and stochastic) volatility. In the cross-section, we show that asset price correlations increase in times of crisis. This property of the correlation of asset prices is important for risk models that are used by banks and for regulatory purposes.

Second, we study externalities, and find that generally experts lever up too much by taking on too much risk and by paying out funds too early. Experts impose an externality on the labor sector since when choosing their leverage they do not take fully into account the costs of adverse economic conditions that result in crises. Also, there are 'firesale' externalities within the financial sector when households can provide a limited liquidity cushion by absorbing some of the assets in times of crises. When levering up, experts do not take into account that they hurt other experts' ability to sell to households in times of crises. On top of it, low fire-sale prices also lower the fraction of outside equity financial experts can raise from households in times of crisis. Put together, this can also lead to overcapacity.

Third, we study the effects of securitization and financial innovation. Securitization of home loans into mortgage-backed securities allows institutions that originate loans to unload some of the risks to other institutions. More generally, institutions can share risks through contracts like credit-default swaps, through integration of commercial banks and investment banks, and through more complex intermediation chains (e.g. see Shin (2010)). To study the effects of these risk-sharing mechanisms on equilibrium, we add idiosyncratic shocks to our model. We find that when expert can hedge idiosyncratic shocks among each other, they become less financially constrained and take on more leverage, making the system less stable. Thus, while securitization is in principle a good thing - it reduces the costs of idiosyncratic shocks and thus interest rate spreads - it ends up amplifying systemic risks in equilibrium.

Literature review. Financial crises are common in history - having occurred at roughly 10-year intervals in Western Europe over the past four centuries, according Kindleberger (1993). Crises have become less frequent with the introduction of central banks and regulations that include deposit insurance and capital requirements (see Allen and Gale (2009) and Cooper (2008)). Yet, the stability of the financial system has been brought into the spotlight again by the events of the current crises, see Brunnermeier (2009).

The existence of the financial system is premised on the heterogeneity of agents in the economy – lenders and borrowers. In Bernanke and Gertler (1989), entrepreneurs have special skill and borrow to produce. In Kiyotaki (1998), more productive agents lever up by borrowing from the less productive ones, in Geanakoplos (2003) more optimistic and in Garleanu and Pedersen (2009) less risk-averse investors lever up. Intermediaries can facilitate lending – for example Diamond (1984) shows how intermediaries reduce the cost of borrowing. Holmström and Tirole (1997, 1998) also propose a model where both where both intermediaries and firms are financially constrained. Philippon (2008) looks at the financial system plays in helping young firms with low cash flows get funds to invest. In these models, financial intermediaries are also levered.

Leverage leads to amplification of shocks, and prices can play an important role in this process. Negative shocks erode borrowers' wealth, and impair their ability to perform their functions of production or intermediation. Literature presents different manifestations of how this happens. Shleifer and Vishny (1992) argue that when physical collateral is liquidated, its price is depressed since natural buyers, who are typically in the same industry, are likely to be also constrained. Brunnermeier and Pedersen (2009) study liquidity spirals, where shocks to institutions net worth lead to binding margin constraints and fire sales. The resulting increase in volatility brings about a spike in margins and haircuts forcing financial intermediaries to delever further. Maturity mismatch between the assets that borrowers hold and liabilities can lead to runs, such as the bank runs in Diamond and Dybvig (1983), or more general runs on non-financial firms in He and Xiong (2009). Allen and Gale (2000) and Adam Zawadowski (2009) look at network effects and contagion. In Shleifer and Vishny (2009) banks are unstable since they operate in a market influenced by investor sentiment.

These phenomena are important in a macroeconomic context – and many papers have studied the amplification of shocks through the financial sector near the steady state, using log-linearization. Prominent examples include Bernanke, Gertler and Gilchrist (1999), Carlstrom and Fuerst (1997) and Kiyotaki and Moore (1997) and (2007). More recently, Christiano, Eichenbaum and Evans (2005), Christiano, Motto and Rostagno (2005, 2007), Cordia and Woodford (2009), Gertler and Karadi (2009) and Gertler and Kiyotaki (2009) have studied related questions, including the impact of monetary policy on financial frictions.

We argue that the financial system exhibits the types of instabilities that cannot be adequately studied by steady-state analysis, and use the recursive approach to solve for full equilibrium dynamics. Our solution builds upon recursive macroeconomics, see Stokey and Lucas (1989) and Ljungqvist and Sargent (2004). We adapt this approach to study the financial system, and enhance tractability by using continuous-time methods, see Sannikov (2008) and DeMarzo and Sannikov (2006).

A few other papers that do not log-linearize include He and Krishnamurthy (2008 and 2009) and Mendoza (2010). Perhaps most closely related to our model, He and Krishnamurthy (2008) also model experts, but assume that only experts can hold risky assets. They derive many interesting asset pricing implications and link them to risk aversion. In contrast to He and Krishnamurthy (2008) we focus on the risk-neutral case and look at not only individual asset prices, but also in cross-section. We also study system dynamics through its stationary distribution, and analyze externalities and the effects of securitization.

Our result that pecuniary externalities lead to socially inefficient excessive borrowing, leverage and volatility can be related to Bhattacharya and Gale (1987) in which externalities arise in the interbank market and to Caballero and Krishnamurthy (2004) which study externalities an international open economy framework. On a more abstract level these effects can be traced back to inefficiency results within an incomplete markets general equilibrium setting, see e.g. Stiglitz (1982) and Geanakoplos and Polemarchakis (1986). In Lorenzoni (2007) and Jeanne and Korinek (2009) funding constraints depend on prices that each individual investor takes as given. Adrian and Brunnermeier (2008) provide a systemic risk measure and argue that financial regulation should focus on these externalities.

We build our model in steps, starting from a simple version and adding ingredients to illustrate externalities and the effects of securitization. We purposefully start in Section 2 building a basic model that has no externalities (for reasons that will become clear later), so that we can isolate precisely the ingredients that cause each externality in Sections 3 and 4. Nevertheless, the basic model exhibits amplifications and adverse feedback loops, and more importantly, illustrates how full equilibrium dynamics differs from steady-state dynamics.

2. The Model

The baseline model. We consider an economy populated by households and financial experts. Since, experts are better at managing capital, they find it profitable to invest in projects, such as productive firms, entrepreneurial ventures, home loans, etc.

When an expert holds capital k_t, he receives output at rate

$$y_t = a\, k_t.$$

where *a* is a parameter. New capital can be created through investment. At a cost of $\iota(g)$, k_t capital can be made to grow at an expected rate of g. We assume that

- the marginal cost of investment $\iota'(g)$ is increasing in g,
- without investment, capital depreciates at a rate δ, i.e. or $\iota(-\delta) = 0$
- experts can sell capital to households, who are less efficient in managing it and have a higher depreciation rate δ^*. This action is irreversible in the baseline model and so $\iota'(-\infty) = a/(r + \delta^*)$, where r is the risk-free rate. Later in the paper we allow households to provide a liquidity buffer by buying capital temporarily.
- $\iota'(r) = \infty$.

Holding capital is risky. The quantity of capital changes due to macro shocks, and evolves according to

$$dk_t = g\, k_t\, dt + \sigma\, k_t\, dZ_t,$$

where Z is a Brownian motion. Note that k_t reflects the "efficiency units" of capital, measured in output rather than in simple units of physical capital (number of machines). Hence, dZ_t also captures changes in expectations about the future productivity of capital.

We assume that experts and households are risk-neutral. Experts are financially constrained, and they borrow money from households at the risk-free rate r, which is lower than the experts' own discount rate ρ. We are imagining a story in which households hold money to ensure themselves against future shocks (large purchases, accidents, etc). Because of the option value of holding money, households are willing to lend it to experts (banks) at rate r, which is lower than their discount rate. Hence, the assumption $\rho > r$ is natural. For simplicity, we do not model money explicitly and assume that r is the households' discount rate.[1]

Experts' balance sheets. For most of the paper, we work with balance sheets that consist of assets, debt and equity as shown in Figure 1.

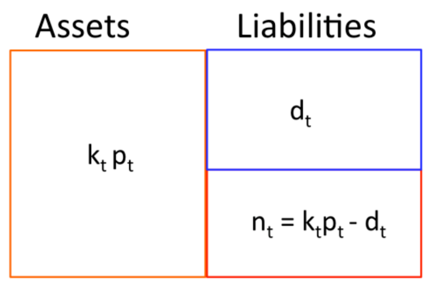

Figure 1. Expert balance sheet.

Debt is risk-free, and experts take on all of the risk of the assets they hold. In the next several paragraphs we justify balance sheets as an outcome of contracting, subject to informational problems, and get a slightly more general form of balance sheets (with inside and outside equity). However, exposition is much easier and all results hold with a simpler form of balance sheets, so we focus on them for the rest of the paper.[2]

Experts finance their capital holdings by borrowing from households, and also by selling to them a fraction of realized returns in the form of outside equity. Informational problems limit the fraction $1-\alpha_t$ of risk that the experts can offload. For convenience, we model informational asymmetry as moral hazard, and assume that if the agent does not put effort, then capital depreciates at a higher rate. The expert gets a benefit of b per unit of capital that disappears. The expert puts effort if he is liable for a fraction α_t of this loss such that

$$\alpha_t\, p_t \geq b,$$

where p_t is the equilibrium endogenous price of capital. This constraint is the one-shot deviation condition. Appendix A justifies this constraint formally using the theory of optimal dynamic contracting, in which the contracting variable is the market value of assets $k_t\, p_t$. By assuming that contracts depend on market value instead of k_t directly, we allow for an amplification channel in which market prices affect the expert's net worth. This assumption is consistent with what we see in the real world, and with the models of Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999). We are imagining that contracting on k_t directly is difficult because it is not something objective like the number of machines, but something much more forward looking, like the

[1] Of course, in a model with money rate r will depend on the banks' demand for deposits and the point in the economic cycle. We ignore these effects in our model.

[2] Balance sheets with inside and outside equity offer a new amplification channel, and, of course, the ratio of inside and outside equity would matter for calibration.

expected NPV of assets under a particular management strategy. Moreover, even though in our model there is a one-to-one correspondence between k_t and output, in a more general model this relationship could be different for different types of project, and private information of the expert. Moreover, output can be manipulated, e.g. by underinvestment. In extensions of our model, we relax the contracting assumption by allowing expert to hedge some of the risks of $k_t p_t$ (e.g. see the Section 4 on securitization).

With inside and outside equity and debt, the experts' balance sheets look as shown in Figure 2.

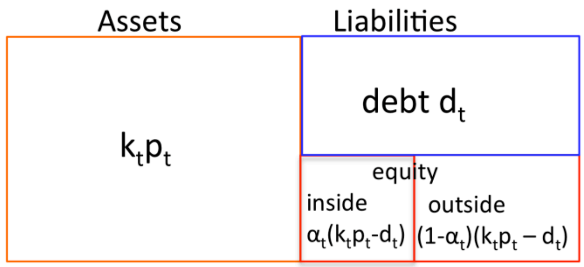

Figure 2. Expert balance sheet with inside and outside equity.

The contracting problem pins down the cash flows that go to inside equity n_t and implies a solvency constraint that the expert can operate only as long as $n_t \geq 0$. In contrast, cash flows to outside investors can be split arbitrarily between debt and equity-holders, by Modigliani and Miller (1959). We choose a particular capital structure that makes debt risk-free, because it simplifies exposition.

Note that the incentive constraint requires a higher α_t in downturns, when equilibrium prices p_t are depressed. This observation is consistent with higher informational asymmetry and lower liquidity in downturns.[3] This property of α_t also creates an additional reason why experts find it harder to hold assets in downturns - because they must retain a greater fraction of risk.[4]

However, aside from this amplification channel, almost all results in the paper hold in a simpler model where we assume that moral hazard effects are large such that $\alpha_t = 1$. We focus on this case for the rest of the paper.

Experts as dynamic optimizers and the evolution of balance sheets. The equilibrium

[3] See Leland and Pyle (1977) where managers must retain a greater fraction of equity when the informational asymmetry is greater, or DeMarzo and Duffie (1999) where informational sensitivity leads to lower liquidity.

[4] In a version of our model where $\alpha_t = b/p_t$ and households can provide liquidity support by buying assets temporarily in downturns (see Section 3), the equilibrium exhibits procyclical leverage in the region where households hold some of the assets. The reason is that α_t increases when p_t falls, making it harder for the financial sector to hold assets. Procyclical leverage is consistent with what we observe about financial firms in practice. In contrast, existing models, such as those of Bernanke, Gertler and Gilchrist (1999) and He and Krishnamurthy (2009), have countercyclical leverage.

in our economy is driven by exogenous shocks Z_t. These shocks affect the experts' balance sheets in the aggregate, and market prices of capital p_t are determined endogenously through supply and demand. That is, experts determine the sizes of their balance sheets by trading capital among each other at price p_t. They may also liquidate capital to households when $p_t = a/(r + \delta^*)$. Note that until Section 3.2 we assume that while households can buy assets from expert, they cannot sell them back. Hence, since capital in the hands of households depreciates at rate δ^*, they value a unit of capital at $a/(r + \delta^*)$ (by the Gordon growth formula). We denote the endogenous equilibrium law of motion of prices by

$$dp_t = \mu_t^p \, dt + \sigma_t^p \, dZ_t.$$

We assume that experts are small and act competitively, so individually they take prices p_t as given, but they affect prices in the aggregate through their leverage decisions.

Experts choose dynamic trading strategies to maximize their payoffs. They choose how much to lever up. There is a trade-off that greater leverage leads to both higher profit and greater risk. Greater risk means that experts will suffer greater losses exactly in the events when they value funds the most - after negative shocks when prices become depressed and profitable opportunities arise. We will see how this trade-off leads to an equilibrium choice of leverage.

Using Ito's lemma, without any sales or purchases of new capital the value of the assets on the balance sheet evolves according to

$$d(k_t p_t) = k_t (g \, p_t + \mu_t^p + \sigma\sigma_t^p) \, dt + k_t (\sigma p_t + \sigma_t^p) \, dZ_t,$$

where growth g is generated through internal investment. The term, $\sigma\sigma_t^p$, is due to Ito's lemma and reflects the positive covariance between the Z_t-shock to capital and price volatility.[5] . Debt evolves according to

$$dd_t = (r \, d_t - a \, k_t + \iota(g) \, k_t) \, dt - dc_t,$$

where $a \, k_t$ is output, $\iota(g) \, k_t$ is internal investment and dc_t are payouts (e.g. bonuses that experts pay themselves to consume). As a result, the expert's net worth $n_t = p_t k_t - d_t$ changes according to

$$dn_t = r \, n_t \, dt + k_t \, [(a - \iota(g) - (r - g)p_t + \mu_t^p + \sigma\sigma_t^p) \, dt + (\sigma p_t + \sigma_t^p) \, dZ_t] - dc_t.$$

Note that the optimal level of internal investment g is determined by the price through $\iota'(g) = p_t$.

Equilibrium. Our strategy for solving for the equilibrium is to combine the experts' dynamic optimization problems (expressed via Bellman equations) with the market

[5] The version of Ito's lemma we use is the product rule $d(X_t Y_t) = dX_t \, Y_t + X_t \, dY_t + \sigma_X \, \sigma_Y \, dt$.

clearing conditions.

In our economy, the key state variables are the aggregate expert net worth N_t and the aggregate amount of capital K_t. Because everything is proportionate with respect to K_t, we get scale invariance and the key state variable is the ratio

$$\eta_t = N_t/K_t.$$

Lemma 1. The equilibrium law of motion of η_t is

$$d\eta_t = (r - g + \sigma^2)\, \eta_t\, dt + (a - \iota(g) - (r - g + \sigma^2)p_t + \mu_t^p)\, dt + (\sigma p_t + \sigma_t^p - \sigma_t\, \eta_t)\, dZ_t - d\zeta_t,$$

where $d\zeta_t = dC_t/K_t$ and dC_t is aggregate consumption.

Proof. Aggregating over all experts, the law of motion of N_t is

$$dN_t = r\, N_t\, dt + K_t\, [(a - \iota(g) - (r - g)p_t + \mu_t^p + \sigma\sigma_t^p)\, dt + (\sigma p_t + \sigma_t^p)\, dZ_t] - dC_t,$$

where C_t is are aggregate payouts, and the law of motion of K_t is

$$dK_t = g\, K_t\, dt + \sigma\, K_t\, dZ_t.$$

Combining the two equations, and using Ito's lemma, we get a desired expression for η_t. QED

We look for an equilibrium that is Markov in η_t, and formally justify in Proposition 2 that this equilibrium is unique (among all equilibria, Markov or not). Denote by

$$p(\eta_t)$$

the market price of capital in equilibrium and by

$$f(\eta_t)\, n_t$$

the value function of an expert with net worth n_t. Note that the value function is proportional to net worth, because an expert with twice the net worth as another expert can replicate the strategy of the former, times two, and get twice the utility.

When choosing asset holdings k_t experts affect only the law of motion of their own net worth, and take the law of motion of η_t as given. Their value functions satisfy the Bellman equation

$$\rho f(\eta_t) n_t\, dt = \max_{k,dc} E[dc + d(f(\eta_t) n_t)] = dc_t + \mu_t^f\, n_t\, dt +$$

$$f(\eta_t)\, (r\, n_t\, dt + k_t\, (a - \iota(g) - (r - g)p_t + \mu_t^p + \sigma\sigma_t^p)\, dt - dc_t) + \sigma_t^f\, k_t\, (\sigma p_t + \sigma_t^p)\, dt.$$

The first-order condition with respect to k_t is

$$a - \iota(g) - (r - g)p_t + \mu_t^p + \sigma\sigma_t^p + \sigma_t^f/f(\eta_t)\ (\sigma p_t + \sigma_t^p) = 0,$$

and the expert consumes only when $f(\eta_t) = 1$ (when $f(\eta_t) > 1$, then choosing $dc_t = 0$ is optimal). The first-order condition with respect to g is an equation that we already saw,

$$\iota'(g) = p_t.$$

Together, the Bellman equation and the two first-order conditions are sufficient to find three functions of η_t that characterize the equilibrium, $f(\eta_t)$, $p(\eta_t)$ and $g(\eta_t)$. Proposition 1 expresses our characterization in terms of differential equations, using Ito's lemma, and provides appropriate boundary conditions.

Proposition 1. Functions $f(\eta_t)$ and $p(\eta_t)$ solve the differential equations

$$(\rho - r)\ f(\eta_t) = f'(\eta_t)\ \mu_t^\eta + \tfrac{1}{2}\ f''(\eta_t)\ (\sigma_t^\eta)^2 \qquad \text{and}$$

$$a - \iota(g) - (r - g)p_t + p'(\eta_t)\ \mu_t^\eta + \tfrac{1}{2}\ p''(\eta_t)(\sigma_t^\eta)^2 + \sigma p'(\eta_t)\ \sigma_t^\eta + f'(\eta_t)/f(\eta_t)\ \sigma_t^\eta\ (\sigma p_t + \sigma_t^p) = 0,$$

where $\iota'(g) = p_t$, and μ_t^η, σ_t^η are given by Lemma 1. In equilibrium η_t evolves over the range $[0, \eta^*]$, where η^* is a reflecting point where the experts consume and 0 (if ever reached) is an absorbing point. Experts do not consume when $\eta_t < \eta^*$.

The boundary conditions are

$$p(0) = a/(r + \delta^*),\ \ p'(\eta^*) = 0,\ f(\eta^*) = 1 \qquad \text{and} \qquad f'(\eta^*) = 0.$$

Note that when experts' net wealth reaches the point η^*, the marginal value of an extra dollar for them is simply one dollar and hence they start consuming. Since experts consume any extra wealth at $\eta_t = \eta^*$, payouts fully adjust for the shocks to balance sheets, and so the price volatility is 0, i.e. $p'(\eta^*) = 0$. The boundary condition, $f'(\eta^*) = 0$ states that there are no kinks in experts' value function. Finally, the first boundary conditions follows simple from the fact that households are willing to absorb all assets at a price of $a/(r + \delta^*)$, given their discount rate of r and the fact that assets in their hands depreciate at the rate of δ^*.

Figure 3 shows an example, in which we computed functions $f(\eta)$ and $p(\eta)$ numerically. As expected, asset prices $p(\eta_t)$ increase when experts have more net worth. At the same time, experts get more value per dollar of net worth when prices are depressed and they can buy assets cheaply, so function $f(\eta_t)$ is decreasing.

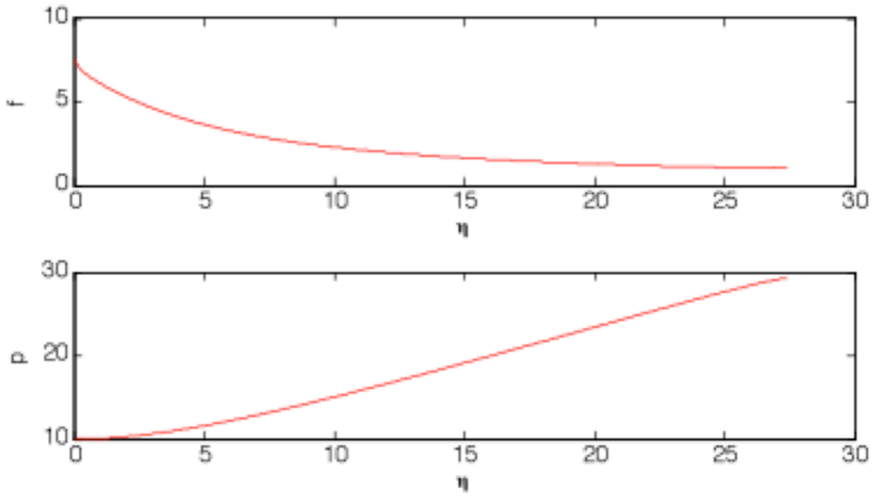

Figure 3. The marginal component of experts' value function and the price of capital as functions of η.

For completeness, we show that the equilibrium characterized in Proposition 1 is unique not only among equilibria that are Markov in η_t but among all competitive rational expectations equilibria.

Proposition 2. Our economy has a unique equilibrium, which is described by Proposition 1.

We defer the proof until Section 3 - this proposition is a corollary of Proposition 4.

Unstable dynamics. Let us compare full system dynamics in our model to log-linearized steady-state dynamics in traditional models like Kiyotaki and Moore (1997) or Bernanke, Gertler and Gilchrist (1999). Standard steady-state dynamics involves a mean-reverting process, which pushes the state variable towards the steady state with a drift that is proportional to the distance away from steady state. Shocks are small, and volatility is constant in the neighborhood near the steady state. The stationary distribution is normal around the steady state, as illustrated on the left panel of Figure 4. The right panel stylistically illustrates impulse response functions that illustrate how various components of the system return to the steady state following a negative macro shock.

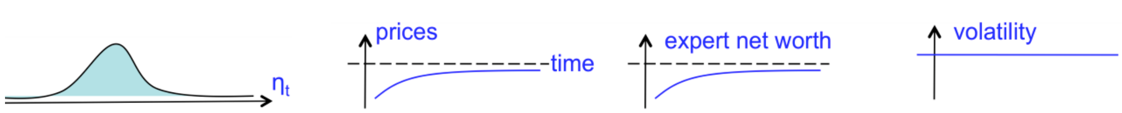

Figure 4. Steady-state dynamics around steady state.

In our model, the steady state is at $\eta_t = \eta^*$. The state variable is pushed towards the steady state from the left by positive drift, and reflected from the right. It is somewhat non-standard because of a reflecting boundary. However, a modification of our model with an exogenous exit rate of experts (as in Bernanke, Gertler and Gilchrist (1999)) would reproduce a more typical steady state.[6]

The system is very stable near the steady state in our model, where the price volatility $\sigma_t^p = p'(\eta_t)\, \sigma_t^\eta$ is close to 0. Recall that $p'(\eta^*) = 0$ is one of the boundary conditions in Proposition 1, which is related to the reflecting nature of the boundary. However, below the steady state prices fall and $p'(\eta)$ becomes larger. This leads to adverse feedback loops, in which a negative shock to k_t erodes expert capital η_t, leading to a drop in prices, further erosion of expert capital and so on. As a result, shocks to k_t are amplified and the resulting volatility of η_t satisfies

$$\sigma_t^\eta = \frac{\sigma(p_t - \eta_t)}{1 - p'(\eta_t)},$$

since by Lemma 1, $\sigma_t^\eta = \sigma(p_t - \eta_t) + \sigma_t^p = \sigma(p_t - \eta_t) + p'(\eta_t)\, \sigma_t^\eta$. Note that as $p'(\eta_t) \to 1$, the adverse feedback loop becomes completely unstable and never converges, leading to infinite volatility. Of course, in equilibrium we always have $p'(\eta_t) < 1$. Figure 5 illustrates the drift and volatility of the full system dynamics of η_t:

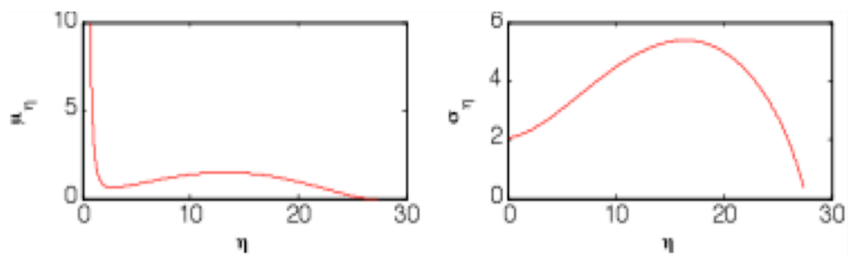

Figure 5. The drift and volatility of η in equilibrium.

Since the volatility is greatest below the steady state, in the middle range of η, it means that the system moves through that region very fast and does not spend much time there. At the same time, it also means that the system is likely to end up in the range of very low η_t occasionally, despite profit-making and the positive drift of η_t. This high positive drift for low η_t values is consistent with the sizable profits banks made in the spring and summer 2009.

Figure 6 shows the stationary distribution of η_t. As we expected, it is thin in the middle range of η_t and has two peaks. There is a large mass near the steady state, and also a

[6] By exogenous exit, we mean that with Poisson intensity γ experts are hit by a shock that makes them lose their expertise. In that case, they unwind their positions to other experts, retire, and just consume their net worth. Exogenous exit adds a negative term to the drift of η_t, which produces an internal point where the drift of η_t is 0 (i.e. steady state). If the sole purpose of our paper were to compare steady-state and full dynamics, then we would write our model in this way, but because our aim is much broader (externalities, securitization, regulations) we assume instead that $\rho > r$. We find this assumption much more natural.

smaller mass in the range of very low η_t. It is typical for the system to dip below the steady state and enter volatile destructive eposides, which occasionally lead to very large downturns.

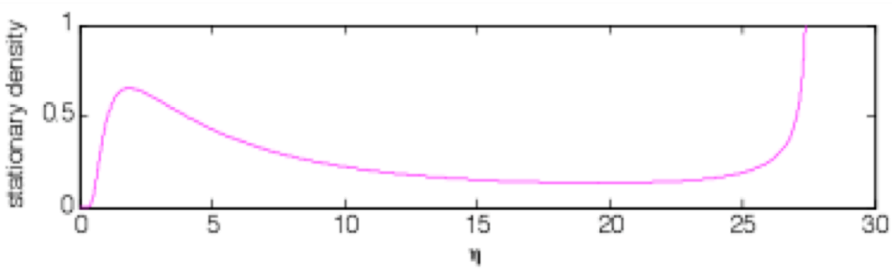

Figure 6. The stationary distribution of η_t.

Proposition A1 in the appendix provides equations that characterize this stationary distribution.

Asset-pricing implications. Our equilibrium analysis implies interesting results for asset pricing - predictability, excess volatility, etc. More importantly, however, by reinterpreting our model to allow multiple assets we get a conclusion that cross-sectionally, prices of different assets become more correlated in crises. This phenomenon is important in practice, and it has been pointed out that risk models used by many financial institutions have failed in recent crisis because they did not take these correlation effects into account.[7] These important effects are also absent from asset pricing model - to our knowledge we are the first to offer a fully dynamic model that exhibits increased correlation of asset prices in crises.[8]

Regarding asset pricing, the Bellman equation and the first-order condition with respect to k imply that the value n_t of any portfolio of risky capital and cash satisfies

$$\rho f(\eta_t) n_t \, dt = E[d(f(\eta_t) n_t)]$$

when internal investment is done optimally, according to $\iota'(g) = p_t$. It follows that any portfolio held by an expert can be priced using the stochastic discount factor

$$e^{-\rho t} f(\eta_t)/f(\eta_0).$$

In contrast, households have a different stochastic discount factor in our model, e^{-rt}, because they get a different return from holding risky capital.

[7] See "Efficiency and Beyond" in *The Economist*, July 16, 2009.
[8] For example see Erb, Harvey and Viskanta (1994).

Our model predicts *excess volatility*. The volatility of $p_t k_t$ is $\sigma + \sigma_t^p/p_t$, where σ is the volatility of earnings (per dollar invested). Our model also implies that asset returns are *predictable*. From the first-order condition, the expected return from investing a dollar into the risky asset is

$$(a - \iota(g) + gp_t + \mu_t^p + \sigma\sigma_t^p)/p_t = r - \sigma_t^f/f(\eta_t)\ \ (\sigma + \sigma_t^p/p_t),$$

where $-\sigma_t^f/f(\eta_t)(\sigma + \sigma_t^p/p_t)$ is the risk premium, which is time-varying. The risk premium is zero at η^*, since $\sigma_t^f = f'(\eta_t)\ \sigma_t^\eta$ and $f'(\eta^*) = 0$. Below η^*, the risk premium is positive.

To look at asset prices in cross section, we reinterpret the model to allow for multiple assets. Suppose that there are many types of capital, and each is hit by aggregate and type-specific shocks. Specifically, capital of type *j* evolves according to

$$dk_t^j = g\ k_t^j\ dt + \sigma\ k_t^j\ dZ_t + \sigma'\ dZ_t^j,$$

where dZ_t^j is type-specific Brownian shock uncorrelated with the aggregate shock dZ_t.

In aggregate, idiosyncratic shocks cancel out and the total amount of capital in the economy still evolves according to

$$dK_t = g\ K_t\ dt + \sigma\ K_t\ dZ_t.$$

Then, in equilibrium experts hold fully diversified portfolio and experience only aggregate shocks. The equilibrium looks identical to one in the single-asset model, with price of capital of any kind given by p_t per unit of capital.

Then

$$d\ (p_t k_t^j) = \text{drift} + (\sigma\ p_t k_t^j + \sigma_t^p\ k_t^j)\ dZ_t + \sigma'\ p_t k_t^j\ dZ_t^j.$$

The correlation between assets i and j is

$$Cov(p_t k_t^i,\ p_t k_t^j)/(Var(p_t k_t^i)Var(p_t k_t^j)) = (\sigma + \sigma_t^p/p_t)^2/(((\sigma + \sigma_t^p/p_t)^2 + (\sigma')^2).$$

Near the steady state $\eta_t = \eta^*$, there is only as much correlation between the prices of assets i and j as there is correlation between shocks. Specifically, $\sigma_t^p = 0$ near the steady state, and so the correlation is

$$\sigma^2/(\sigma^2 + (\sigma')^2).$$

Away from η^*, correlation increases as σ_t^p increases. Asset prices become most correlated in prices when σ_t^p is the largest, and as $\sigma_t^p \rightarrow \infty$, the correlation coefficient tends to 1.

3. Externalities

So far, we set up our baseline model intentionally in a way that has no externalities. In this section we add ingredients to our model to isolate externalities. We show that there can be externalities both between the financial sector and households, and within the financial sector. To illustrate the former, we add a labor market, in which households' labor income depends on the amount of capital in the economy. When levering up and choosing bonus payouts, experts do not take internalize the damages that crises bring onto the labor market. This externality does not depend on competition within the financial sector - a fact that we illustrate by a class of model in which the competitive equilibrium is identical to the optimal policy with a monopolist financial institution.

Within the financial sector, we identify a *firesale externality*, which is an inefficient pecuniary externality in an incomplete markets setting. This externality does not exist in our baseline setting because experts disinvest internally in the event of crises. The firesale externality appears when in the event of crises (1) experts are able to sell assets to another sector (e.g. vulture investors or the government) and (2) the new asset buyers provide a downward-sloping demand function.[9] In this case, when levering up in good times financial institutions do not take into account that in the event of crises, its own fire sales will depress prices that other institutions are able to sell at. This effect leads to excess leverage due to competition among the financial institution - a monopolist expert would lever up less.

No externalities in the baseline model. We show that the competitive rational expectations equilibrium in our baseline model coincides with a policy that a monopolist expert would choose.

Consider a monopolist with discount rate ρ, who can borrow from households at rate r. His debt and the total amount of capital in the economy evolve according to

$$dD_t = (r\, D_t - a\, K_t + \iota(g)\, K_t)\, dt - dC_t \quad \text{and} \quad dK_t = gK_t\, dt + \sigma K_t\, dZ_t,$$

where dC_t is the monopolist's consumption. It is convenient to express the monopolist's value function as $h(\omega_t)K_t$, where $\omega_t = -D_t/K_t$.[10] The value function is homogenous in D_t and K_t of degree 1 because of scale invariance. From the liquidation value of assets, the monopolist's debt capacity is $D_t \leq aK_t/(r+\delta^*)$, and so $\omega_t \geq -a/(r+\delta^*)$.

Using Ito's lemma

$$d\omega_t = ((r - g + \sigma^2)\, \omega_t + a - \iota(g))\, dt - \sigma\omega_t\, dZ_t - dC_t/K_t.$$

[9] He, Khang and Krishnamurthy (2009) document that government support for commercial banks allowed them to buy mortgage and other asset-backed securities during the great liquidity and credit crunch of 2007-09. Warren Buffet also provided additional funds to Goldman Sachs and Wells Fargo.

[10] It convenient to analyze the monopolist's behavior using ω_t, instead of the more economically meaningful variable $\eta_t = N_t/K_t$, because η_t depends on market prices, which are endogenous in equilibrium. Proposition 4 provides a one-to-one map between variables ω_t and η_t in equilibrium.

The following proposition summarizes the Bellman equation and the optimal policy of the monopolist.

Proposition 3. The monopolist's value function solves equation

$$(\rho - g)\, h(\omega) = h'(\omega)\, [(r - g)\, \omega + a - \iota(g)] + ½\, h''(\omega)(\sigma\omega)^2$$

with boundary conditions $h(-a/(r+\delta^*)) = 0$, $h'(\omega^*) = 1$ and $h''(\omega^*) = 0$. The optimal policy has investment with $\iota(g)$ with $(\omega + \iota'(g))\, h'(\omega) = h(\omega)$. Payouts occur exactly when ω_t reaches ω^*, and prevent ω_t from exceeding ω^*. Thus, technically, ω^* is the reflecting boundary for the process ω_t.[11]

Proof. The value function must satisfy the Bellman equation

$$\rho h(\omega)K\, dt = \max_{g,dC}\; dC + E[d(h(\omega)K)] =$$

$$dC + h'(\omega)\, ((r - g + \sigma^2)\, \omega + a - \iota(g) - dC)\, K + ½\, h''(\omega)(\sigma\omega)^2\, K + h(\omega)\, gK - h'(\omega)\, \sigma^2\omega\, K.$$

When $h'(\omega) > 1$, then $dC = 0$ is optimal and the equation reduces to (*). The optimal choice of g is determined by $(\omega + \iota'(g))\, h'(\omega) = h(\omega)$.

To justify the boundary conditions, we extend function $h(\omega)$ that satisfies them beyond ω^* according to $h(\omega) = h(\omega^*) + \omega - \omega^*$, and show that the Bellman equation holds on the entire domain $[-a/(r+\delta^*), \infty)$. For $\omega < \omega^*$, it holds because $h'(\omega) > 1$ and so $dC = 0$ is the optimal choice. The value function for $\omega \geq \omega^*$ can be attained by making a one-time payment of $dC/K = \omega - \omega^*$, and moreover, $dC = 0$ is suboptimal since

$$h(\omega^*) = 1,\ h''(\omega^*) = 0 \Rightarrow (\rho - g)\, h(\omega^*) = (r - g)\, \omega^* + a - \iota(g)$$

$$\Rightarrow (\rho - g)\, h(\omega) < (r - g)\, \omega + a - \iota(g) \quad \text{for all } \omega > \omega^*,$$

since $\rho > r$. QED

Figure 7 illustrates the monopolist's value function.

[11] Our analysis here can be related to Bolton, Chen and Wang (2009). They study optimal investment and payouts of a single firm, which faces output shocks (rather than capital shocks, as in our setting).

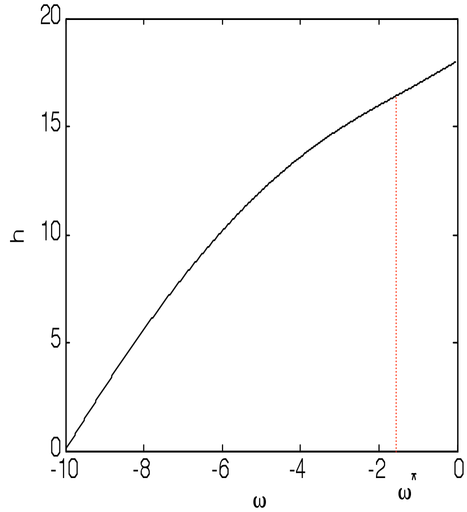

Figure 7. The value function of a monopolist expert.

For a monopolist expert, the optimal payout point ω^* is determined by the trade-off between the benefits of being able to borrow at rate r, which is less than his discount rate, to consume, and the liquidation costs that are incurred when ω_t gets close to $-a/(r+\delta^*)$. It is optimal to pay out when there is a sufficient amount of financial slack ω^*, which determined by Proposition 3.

Proposition 4 shows that in our baseline model, the outcome with a monopolist investor is identical to that under competition. The intuition is that even though in a competitive equilibrium experts do affect prices in the aggregate by their choices of compensation and investment, they are isolated from market prices because they do not trade in equilibrium (due to symmetry).[12]

Proposition 4. The competitive equilibrium in our baseline economy is equivalent to the outcome with a monopolist. The following equations summarize the map between the two:

$$\eta_t = h(\omega_t)/h'(\omega_t), \qquad p_t = h(\omega_t)/h'(\omega_t) - \omega_t, \qquad \text{and} \qquad f_t = h'(\omega_t).$$

Proof. First, since the monopolist chooses g and dC_t to maximize his payoff, the sum of all experts' utilities in the competitive equilibrium cannot be greater than that of a monopolist. On the other hand, each expert can guarantee his fraction of the monopolist's utility (weighted by his net worth) by trading to a fraction of the aggregate portfolio at time 0, and by copying the monopolist's policy in isolation thereafter. Thus, the sum of all experts' utilities in the competitive equilibrium must equal the monopolist's payoff.

[12] The argument of Proposition 4 can be easily generalized to show that in the baseline model, the equilibrium is the same under oligopolistic competition as well.

It follows that the aggregate behavior in the competitive equilibrium is equivalent to the monopolist's optimal policy. In particular, since growth chosen by the monopolist satisfies $(\omega + \iota'(g))\, h'(\omega) = h(\omega)$, the competitive equilibrium has prices

$$p_t = h(\omega_t)/h'(\omega_t) - \omega_t.$$

Under these prices, $\eta_t = N_t/K_t = (p_tK_t - D_t)/K_t = p_t - \omega_t = h(\omega_t)/h'(\omega_t)$. Finally, the sum of the experts' utilities is $f_t\, N_t = h(\omega_t)\, K_t \Rightarrow f_t = h(\omega_t)/\eta_t = h'(\omega_t)$. QED

As a corollary of Proposition 4, we conclude that the competitive equilibrium in our baseline model is unique.

Corollary. In our baseline model, equilibrium prices, expert value function $f_t\, n_t$, and the law of motion of η_t are uniquely determined.

Proof. Note that the proof of Proposition 4 does not assume any properties of the competitive equilibrium (such as that it is Markov in η_t). Uniqueness follows from the uniqueness of the monopolist's optimal policy. QED

Proposition 4 provides an alternative convenient way to compute equilibria in our baseline setting, by solving a singe equation for $h(\omega)$ instead of a system of equations for $p(\eta)$ and $f(\eta)$.

In our baseline setting, there are no externalities only because we do not model households, and their welfare, explicitly. In the following section we introduce a labor market with wages that depend on the aggregate amount of capital in the economy, and illustrate externalities between the financial sector and households.

Externalities between the financial sector and labor sector. We can use the baseline model directly to illustrate externalities, by modeling a labor market in a way that does not directly interfere with the equilibrium among financial intermediaries.

As Bernanke and Gertler (1989), suppose that households in the economy supply a fixed and inelastic amount of labor L. The production function is Cobb-Douglas in labor and capital, and it depends on the aggregate amount of capital in the economy,

$$y_t = A\, l_t^{\alpha}\, k_t^{1-\alpha}\, K_t^{\alpha}.$$

The total amount of capital K_t in the production function reflects the idea from endogenous growth literature that technological progress increases productivity of everyone in the economy (e.g. see Romer (1986)). Recall that we do not measure capital k_t as the number of machines, but rather k_t is the cash-flow generating potential of capital under appropriate management. That is why it is difficult quantify k_t and contract on it directly - the quantification of k_t involves something intangible. Therefore, a part of K_t is the level of knowledge and technological progress of the economy as a whole, and that part enters the production function of everyone.

In equilibrium capital and labor is used for production proportionately, with $l_t = k_t\,(L/K_t)$. Wages per unit of labor and in the aggregate are given by

$$w_t = \alpha\, A/L^{1-\alpha}\, K_t \qquad \text{and} \qquad W_t = \alpha\, A\, L^{\alpha}\, K_t.$$

Capital owners receive output net of wages, which is

$$a\, k_t = (1 - \alpha)\, A\, L^{\alpha}\, k_t.$$

We see immediately that there are externalities between households, who supply labor, and the financial sector. Financial experts receive only a fraction $1-\alpha$ of total output. Therefore, when they take actions that increase the likelihood or a downturn, such as taking on too much risk for the sake of short-term profits or paying out bonuses, they do not take into account the full extent of the damage of these downturns to the labor market.

To illustrate this point most clearly, we assume a constant marginal cost of capital production of $\varphi'(g) = a/(r + \delta^*)$ for $g \le g^*$, take $\varphi(g) = \infty$ for $g > g^*$, and normalize $\varphi(g^*) = 0$. That is, without investment capital grows according to

$$dk_t = g^*\, k_t\, dt + \sigma\, k_t\, dZ_t,$$

it cannot be made to grow any faster, but it can be liquidated in any amount at a constant price of $a/(r + \delta^*)$ per unit of capital. Under these assumptions, the experts' investment decisions are totally passive - and capital grows at rate g^* - whenever $p_t > a/(r + \delta^*)$. The only active decision involves bonus payouts. We call it the *passive investment economy*. The following proposition characterizes the equilibrium, which is the same with competitive investors and with a monopolist.

Proposition 5. In the passive investment economy, the equilibrium law of motion of $\omega_t = -D_t/K_t$ is given by

$$d\omega_t = ((r - g^* + \sigma^2)\,\omega_t + a)\, dt - \sigma\omega_t\, dZ_t - dC_t/K_t$$

on the interval $[-a/(r+\delta^*), \omega^*]$, with a reflecting boundary at ω^* at which bonuses are paid out. The aggregate expert payoff function $h(\omega_t)K_t$ and point ω^* can be found from the equation

$$(\rho - g)\, h(\omega) = ((r - g^*)\,\omega + a)\, h'(\omega) + \tfrac{1}{2}\,(\sigma\omega)^2\, h''(\omega)$$

with boundary conditions $h(-a/(r+\delta^*)) = 0$, $h'(\omega^*) = 1$ and $h''(\omega^*) = 0$.

Proof. The desired conclusions follow directly from Proposition 3, which characterizes the optimal policy of a monopolist, and Proposition 4, which shows that the monopolist solution coincides with the competitive equilibrium. QED

We would like to argue that a regulator can improve social welfare by a policy that limits bonus payouts within the financial sector. Specifically, suppose that experts are not allowed to pay themselves as long as financial experts are not sufficiently capitalized. Formally, as long as ω_t reaches some level $\omega^{**} > \omega^*$. This type of a regulation keeps capital within the financial system longer, and makes it more stable. The following proposition characterizes the equilibrium with such a regulatory policy, and the value functions of the experts and the households.

Proposition 6. If experts are not allowed to pay out bonuses until ω_t reaches $\omega^{**} = \omega^*$, they will pay at ω^{**}. The process ω_t follows the same equation, but with a reflecting boundary at ω^{**}. Expert value function is given by

$$\tilde{h}(\omega) = h(\omega)/h'(\omega^{**}),$$

where $h(\omega)$ is as in Proposition 5. Household value function is $H(\omega_t)K_t$, where $H(\omega)$ solves equation

$$(r - g)\, H(\omega) = \alpha A L^{\alpha} + ((r - g)\, \omega + a)\, H'(\omega) + \tfrac{1}{2}\, (\sigma\omega)^2\, H''(\omega), \qquad (**)$$

with boundary conditions $H(-a/(r+\delta^*)) = AL^{\alpha}/(r+\delta^*)$ and $H'(\omega^{**}) = -1$.

Proof. Then the household value function $H(\omega_t)K$ satisfies

$$r\, H(\omega_t)\, K_t = (a + b)\, K + ((r - g + \sigma^2)\, \omega_t + a)\, H'(\omega)\, K + \tfrac{1}{2}\, (\sigma\omega_t)^2\, H''(\omega_t)\, K + H(\omega)\, g\, K - H'(\omega)\, \sigma^2\, \omega \Rightarrow$$

To be completed.

How does such a regulatory policy affect welfare? For experts, note that $h'(\omega^{**}) > 1$ for $\omega^{**} > \omega^*$. Therefore, for a fixed level of ω_t, a restriction on compensation practices reduces expert welfare. However, since $h''(\omega^*) = 0$, $h'(\omega^{**})$ increases very little with ω^{**} near ω^*, and the effect on expert welfare is second-order.

For households, for welfare analysis it is convenient to write $H(\omega)$ as a linear combination of the solutions of the homogeneous equation

$$(r - g)\, h_i(\omega_t) = ((r - g)\, \omega + a)\, h_i'(\omega) + \tfrac{1}{2}\, (\sigma\omega)^2\, h_i''(\omega).$$

Denote by h_1 and h_2 the functions that solve it with boundary conditions

$$h_1(-a/(r+\delta^*)) = 0,\ \ h_1'(-a/(r+\delta^*)) = 1,\ \ h_2(-L) = AL^{\alpha}/(r + \delta^*) - \alpha AL^{\alpha}/(r - g)\ \text{ and }\ h_2(0) = 0.$$

Functions h_1 and h_2 are illustrated in Figure 8.

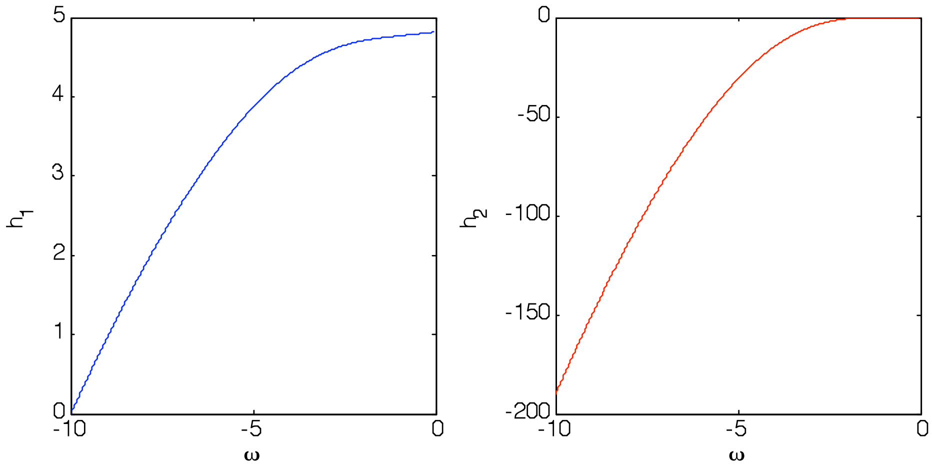

Figure 8: Solutions to the homogenous version of the household Bellman equation.

Lemma 2. Household welfare function under the policy that limits compensation for $\omega_t < \omega^{**}$ is given by

$$H(\omega) = \alpha AL^{\alpha}/(r - g) + q\, h_1(\omega) + h_2(\omega),$$

with $q = -(h_2'(\omega^{**}) + 1)/h_1'(\omega^{**})$. As ω^{**} increases, q increases.

Proof. It is easy to see that any function of the form $\alpha AL^{\alpha}/(r - g) + q_1\, h_1(\omega) + q_2\, h_2(\omega)$ satisfies the non-homogenous equation (**). Coefficient $q_2 = 1$ follows from the boundary condition $H(-a/(r+\delta^*)) = AL^{\alpha}/(r+\delta^*)$, since $h_1(-a/(r+\delta^*)) = 0$. Coefficient q_1 can be found from the boundary condition $H'(\omega^{**}) = -1$.

Since h_2 and h_1 are concave functions and $h_2'(\omega) < 1$ for $\omega > -a/(r+\delta^*)$, … *to be completed.* QED

Because q is increasing in ω^{**}, the effect of ω^{**} on household welfare is first-order. Figure 9 shows the experts' and households' value functions for various choices of ω^{**} by the social planner.

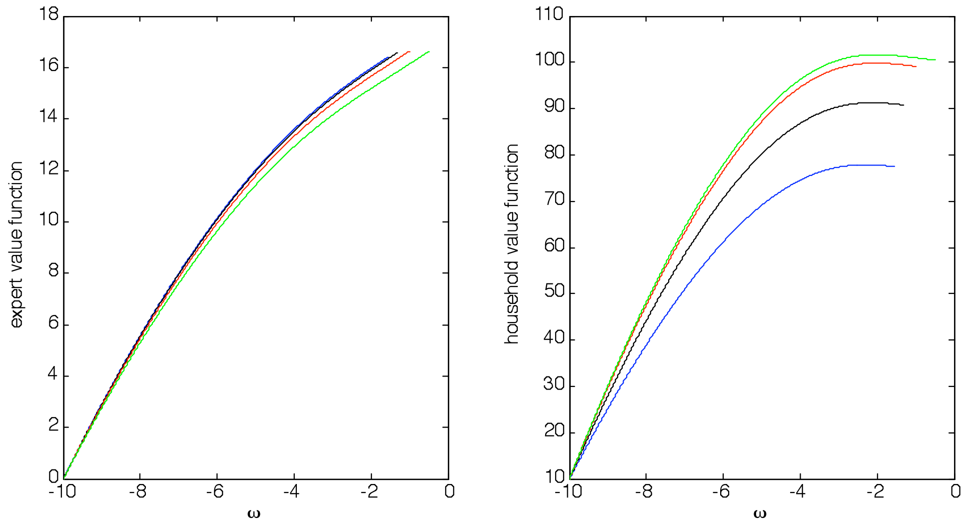

Figure 9: Value functions experts and households for different regulatory policies; the blue functions corresponds to $\omega^{**} = \omega^{*}$.

We see that the central planner can improve efficiency by setting $\omega^{**} > \omega^{*}$. When ω^{**} is close to ω^{*}, the effect of policy on expert welfare is second-order, but the effect on households is first-order. Relative to the equilibrium without regulation, a social planner can implement a Pareto improvement by a policy that combines a transfer from households to the financial sector together with a regulation that limits bonus payouts. When ω_t is small, such a transfer can be interpreted as a bailout.

Without an accompanying transfer, regulation always hurts the financial experts in our baseline model. However, next we modify our baseline model to highlight possible externalities within the financial sector. In such a context, regulation can be welfare-improving even without accompanying transfers.

Pecuniary externalities within the financial sector. Externalities within the financial sector are pecuniary externalities in an incomplete market setting.[13] They arise whenever experts' welfare depends directly on market prices, which are affected by the actions of other experts. In our baseline model there are no pecuniary externalities because in equilibrium experts do not trade with each other at market prices, and prices do not enter the experts' payoffs or action sets through contracts. However, there are many natural extensions that give rise to externalities. There are externalities when experts trade, e.g. if they invest not internally but by buying capital from capital producers, and if they liquidate assets by selling them to households gradually. Externalities may exist even without trade when the experts' contracts depend on prices, such as in the following examples:

- when experts can unload a fraction $1\text{-}\alpha_t$ or risk to outside investors, there are

[13] Bhattacharya and Gale (1987) were among the first to highlight the inefficiency of a pecuniary externality. A recent application of this inefficiency within a finance context, see Lorenzoni (2007).

externalities because $\alpha_t = b/p_t$ depends on prices

- the terms of borrowing - the spread between the interest rate experts need to pay and the risk-free rate - may depend on prices. For example, there are externalities in the setting of Section 4, where experts face idiosyncratic shocks.
- experts may be bound by margin requirements, which may depend both on price level and price volatility
- in asset management, the willingness of investors to keep money in the fund depends on short-term returns, and thus market prices

Overall, it may be hard to quantify the effects of many of these externalities directly, because each action has rippling effects through future histories, and there can be a mix of good and bad effects. To see how this can happen, let us explore how increased internal investment by one expert affects future values of α_t for everybody. Since volatility increases with higher leverage, investment leads to higher values of p_t and lower values of α_t in good states and vice versa in bad states. Given the mix of effects, it is best to study the overall significance of various externalities, as well as the welfare effects of possible regulatory policies, numerically on a calibrated model.

However, one type of an externality seems very prominent - the firesale externality. This externality arises when households offer a downward-sloping demand function for asset purchases from the financial sector in the event of crises. Firesale externality arises because, when levering up, experts do not take into account that their fire sale will depress prices at which other experts sell. We extend our model to illustrate the effects of fire sales in the remainder of this section.

Equilibrium when households provide liquidity support. So far, we assumed that the sale of assets from experts to households in the event of crises is irreversible. However, in practice the economy has resources to pick up some of the functions of the traditional financial sector in times of crises. In the spring of 2009, the Fed introduced the Term Asset-Backed Securities Loan Facility in order to entice hedge funds to buy some of the asset-backed securities. Investors like Warren Buffet has helped institutions like Goldman Sachs and Wells Fargo with capital infusions. More generally, governments have played a huge role in providing capital to financial institutions in various ways.

We extend the model to allow household sector to provide some liquidity support to the financial sector, by buying assets at depressed prices and holding them until the economy comes back. This model can have many uses, such as studying shifts in asset holdings in times of crises (see He, Khang and Krishnamurthy (2009)). In this section specifically, we use the model to study fire sale externalities. These stem from the fact that the economy has a bounded capacity to absorb assets when the financial sector fails. When investing and levering up in good times, experts do not take into account that in the event of a crisis their fire-sales would depress prices at which other institutions are able to sell assets.

Specifically, suppose that some households are sophisticated enough so they can buy assets from experts, and sell them back. Sophisticated households have discount rate r.

Capital held by households depreciates at a higher rate $\delta^* > \delta$. For simplicity, assume that households cannot invest internally. Then the law of motion for capital held by households is

$$dk_t = -\delta^* k_t\, dt + \sigma\, k_t\, dZ_t.$$

If the value function of a sophisticated household with net worth n_t' is given by $f_t'n_t'$, with

$$df_t' = \mu_t^{f'}\, dt + \sigma_t^{f'}\, dZ_t,$$

then the first-order condition for the optimal investment strategy of sophisticated households is

$$a - (r + \delta^*)\, p_t + \mu_t^p + \sigma\sigma_t^p + \sigma_t^{f'}/f_t'\, (\sigma p_t + \sigma_t^p) = 0. \qquad (***)$$

Denote by N_t' the total amount of sophisticated household capital in the economy. Then equilibrium dynamics is characterized by two state variables, $\eta_t = N_t/K_t$ and $\eta_t' = N_t'/K_t$. The following proposition extends our equilibrium characterization to a model with sophisticated households.

Proposition 7. If households can provide liquidity support, then aggregate capital in the economy follows

$$dK_t = (\psi_t\, g_t - (1 - \psi_t)\, \delta^*)\, K_t\, dt + \sigma\, K_t\, dZ_t,$$

where ψ_t is the fraction of capital held by households. The state variables η_t and η_t' follow

$$d\eta_t = (r - \psi_t\, g_t + (1 - \psi_t)\, \delta^* + \sigma^2)\, \eta_t\, dt + \psi_t\, (a - \varphi(g) - (r - g + \sigma^2)p_t + \mu_t^p)\, dt + (\psi_t(\sigma p_t + \sigma_t^p) - \sigma_t\, \eta_t)\, dZ_t - d\zeta_t \quad \text{and}$$

$$d\eta_t' = (r - \psi_t\, g_t + (1 - \psi_t)\, \delta^* + \sigma^2)\, \eta_t\, dt + (1 - \psi_t)\, (a - (r + \delta^* + \sigma^2)p_t + \mu_t^p)\, dt + ((1 - \psi_t)(\sigma p_t + \sigma_t^p) - \sigma_t\, \eta_t')\, dZ_t.$$

The equilibrium is characterized by four functions of (η_t, η_t'), p_t, f_t, f_t' and ψ_t, which are determined by the equations (***)

$$a - (r + \delta^*)\, p_t + \mu_t^p + \sigma\sigma_t^p + \sigma_t^{f'}/f_t'\, (\sigma p_t + \sigma_t^p) = 0, \qquad (\rho - r)f_t = \mu_t^f \qquad \text{and} \qquad \mu_t^{f'} = 0.$$

Proof. To be completed.

We illustrate the equilibrium for the case when sophisticated households are not financially constrained, so $f_t' = 1$. Figure 10 illustrates functions f_t, p_t and asset holdings ψ_t by the financial sector.

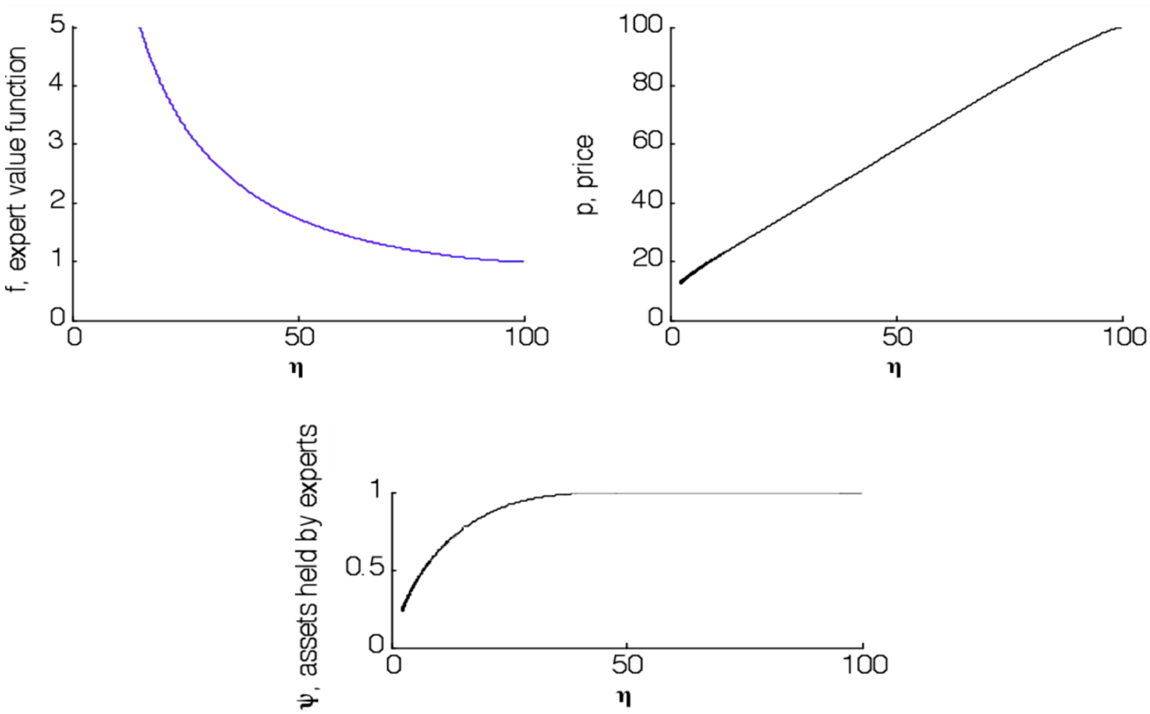

Figure 10: Equilibrium when households provide liquidity support.

We see that when η_t becomes small, then experts sell assets to households at market prices, and $\psi_t < 1$. Unlike in our baseline setting, experts do not just hold assets, and so market prices directly enter the experts' welfare. There are pecuniary externalities within the financial sector. However, when households are not financially constrained, the effect of these externalities is unclear: when the economy and the financial sector expand, the households' willingness to pick up assets also expands. When K_t grows, in equilibrium households start absorbing assets at a larger value of N_t, and their capacity expands proportionately to the size of the economy. On the other hand, if households are financially constrained, then firesale externalities become prominent.

Firesale externalities and overcapacity. Financial expansion is connected with the debate on trade-offs between growth and volatility. In our model, experts have superior ability to make assets more productive, but their ability to function and absorb shocks depends on price levels. When prices become depressed more easily then shocks are amplified and the financial system becomes less stable. On the other hand, when households can add liquidity to the financial markets picking up risky assets off of the financial institutions' balance sheets in times when the institutions become constrained, then the financial system is more stable.

We want to argue that when households have a limited capacity to provide liquidity, then when levering up, financial institutions impose externalities on one another. They do not take into account that their fire-sales in the event of a crisis would depress prices at which other institutions would be able to sell. We call this phenomenon a *fire-sale externality.* A monopolist financial institution would take the households' limited capacity to provide liquidity into account, and expand more slowly. In contrast, competition leads to overexpansion and instabilities.

4. Idiosyncratic Shocks and Securitization

By securitization/hedging, we refer to various mechanisms by which financial institutions can share risks among each other. These mechanisms include pooling and tranching, by which the issuer can diversify and slice risks. Credit default swaps, and various options and futures contracts allow financial institutions to hedge specific risks. Furthermore, more efficient risk-sharing can be attained by longer intermediation chains between households and borrowers (e.g. see Shin (2010)).

A model with idiosyncratic shocks. A natural simple way to capture these phenomena, is to augment our baseline model to allow idiosyncratic shocks, which may be hedged within the financial sector, in the same spirit as BGG. Specifically assume that capital k_t managed by expert i evolves according to

$$dk_t = g\, k_t\, dt + \sigma\, k_t\, dZ_t + k_t\, dJ_t^i,$$

where dJ_t^i is an idiosyncratic Poisson loss process. As BGG we make the simplifying assumption that when experts get bigger, their idiosyncratic shocks are amplified proportionately, that is, there is no diversification of idiosyncratic shocks within any expert.

Losses after an idiosyncratic jump are characterized by the distribution function $F : [0, 1] \to [0, 1]$, which describes the percentage of capital that is recovered in the event of a loss. We can capture additional volatility effects by allowing the intensity of losses $\lambda(\sigma_t^p)$ to depend on the volatility asset prices p_t. This assumption is consistent with the general idea that interest rate spreads and margins are set by debt holders who worry about potential losses (which depend on volatility). It can be justified through an informal story that idiosyncratic shocks have to do with liquidity (such as the difficulty to find an acceptable buyer and having to sell assets at fire-sale prices). Note that this assumption would be vacuous under steady-state analysis, since price volatility is constant near the steady state.

To extend our agency model to idiosyncratic losses, we assume that an expert may generate losses for benefit extraction, getting b units of private financial benefit from a single unit of lost physical capital. While the expert's stake α_t in the assets prevents losses of size n_t/b or less, we assume that costly state verification is possible to prevent larger losses.[14] We assume that if verification is immediate when an expert simulates a loss in order to steal money, the fraud is revealed and the expert cannot get any private benefit. As in BGG, we assume that the verification cost is a fraction $c \in (0,1)$ of the

[14] It is optimal to trigger verification if and only if k_t^i drops below n_t/b because, as we will see later, in equilibrium the expert is risk-neutral towards idiosyncratic risk that does not lead to default.

amount of capital recovered.[15]

Default and costly state verification occur when the value of the assets $v = p_t k_t$ falls below the value of debt $d = v - n_t/\alpha_t$ i.e. k_t falls by more than $n_t/(\alpha_t p_t) = n_t/\alpha_t$. Note that the expected loss in the event of default is

$$vL(d/v) = v\int_0^{\frac{d}{v}} (d/v - x)\; dF(x)$$

where x is the fraction of assets left after loss. Default occurs in the event that $x < d/v$. The expected verification cost is

$$vC(d/v) = v\int_0^{\frac{d}{v}} cx\; dF(x).$$

To break even, households who lend money to the expert must get not only the interest rate ρ, but also compensation for the possible losses and verification costs at a total rate of $\lambda(\sigma_t^p)$ V (L(d/v) + C(d/v)). This quantity defines the spread that the expert needs to pay to borrow from households.

As in the remainder of the paper, we now narrow down analysis to the case when $\alpha_t = 1$, which occurs when the agent is able to fully extract the value of lost capital from the losses he generates.

As before, the equilibrium is characterized by the state variable η_t, and prices $p_t = p(\eta_t)$ and the expert's value function $f_t = f(\eta_t)$ are functions of η_t. The net worth of an individual expert evolves according to

$$dn_t = r\, n_t\, dt +$$

$$k_t\, [(a - \iota(g) - (r + \lambda(\sigma_t^p)\, (L(\xi_t) + C(\xi_t)) - g + dJ_t^i)\, p_t + \mu_t^p + \sigma\sigma_t^p)\, dt + (\sigma p_t + \sigma_t^p)\, dZ_t] - dc_t,$$

where $\xi_t = 1 - n_t/(p_t k_t)$ is the expert's leverage ratio.

In the aggregate, idiosyncratic losses cancel out and total expert capital evolves according to

[15] The basic costly state verification framework, developed by Townsend (1979) and adopted by Bernanke-Gertler and Gilchrist (1999) is a two-period contracting framework. At date 0, the agent requires investment I from the principal, and at date 1 he receives random output y distributed on the interval $[0, \bar{y}]$. The agent privately observes output y, but the principal can verify it at a cost. The optimal contract under commitment is a standard debt contract. If the agent receives $y \geq D$, the face value of debt, then he pays the principal D and there is no verification. If $y < D$, the agent cannot pay D and costly state verification (bankruptcy) is triggered, and debtholders receive all of output.

$$dN_t = rN_t\, dt + K_t\, [(a - \iota(g) - (r + \lambda(\sigma_t^p)C(\xi_t) - g)p_t + \mu_t^p + \sigma\sigma_t^p)\, dt + (\sigma p_t + \sigma_t^p)\, dZ_t] - dC_t,$$

where the term $L(\xi_t)$ disappears because of limited liability. The modified law of motion of $\eta_t = N_t/K_t$ is

$$d\eta_t = (r - g + \sigma^2)\, \eta_t\, dt + (a - \iota(g) - (r + \lambda(\sigma_t^p)C(\xi_t) - g + \sigma^2)p_t + \mu_t^p)\, dt$$

$$+ (\sigma p_t + \sigma_t^p - \sigma_t\, \eta_t)\, dZ_t - d\zeta_t.$$

The Bellman equation and the first-order condition with respect to k_t are now

$$(\rho - r)\, f(\eta_t)\eta_t = \mu_t^f\, \eta_t + f(\eta_t)\, (a - \iota(g) - (r + \lambda(\sigma_t^p)C(\xi_t) - g)p_t + \mu_t^p + \sigma\sigma_t^p)) + \sigma_t^f\, (\sigma p_t + \sigma_t^p)$$

$$\text{and} \quad a - \iota(g) - (r + \lambda(\sigma_t^p)(C(\xi_t) + (1-\xi_t)C'(\xi_t)) - g)p_t + \mu_t^p + \sigma\sigma_t^p + \sigma_t^f/f(\eta_t)\ (\sigma p_t + \sigma_t^p) = 0.$$

As before, in equilibrium η_t evolves on the range $[0, \eta^*]$, with a different boundary η^*. Experts pay themselves bonuses only when η_t is at η^*.

In equilibrium experts borrow at a rate higher than r due to verification costs - they pay the rate $r + \lambda(\sigma_t^p)\ (L(\xi_t) + C(\xi_t))/\xi_t$. This is the promised interest rate - due to limited liability in the event of default the actual cost of borrowing is only $r + \lambda(\sigma_t^p)/\xi_t$. Higher cost of borrowing makes equilibrium leverage relative to our baseline model without idiosyncratic shocks.

Securitization. We model securitization as risk-sharing within the financial sector. Specifically, assume that all shocks, both idiosyncratic J_t^i and aggregate Z_t, are observable and contractible among the experts, but not between experts and households.

Denote by ω_t the risk-premium on aggregate risk and by ω_t^i the risk premium on idiosyncratic risk. A hedging contract for aggregate risk adds

$$\theta_t\, (\omega_t\, dt + dZ_t)$$

to the law of motion of expert *i*'s wealth, where θ_t is the overall risk exposure. A contract on expert *j*'s idiosyncratic risk adds

$$\theta_t^j\, (\omega_t^j\, dt + dJ_t^j)$$

to the law of motion of expert *i*'s wealth, and may affect the verification region and verification costs. The following proposition characterizes the equilibrium when hedging within the financial sector is possible.

Proposition. *If hedging within the financial sector is possible, then in equilibrium experts will fully hedge idiosyncratic risk, which carries the risk premium of $\omega_t^i = 0$. Nobody hedges aggregate risk, which carries the risk premium of $\omega_t = -\sigma_t^f/f_t > 0$.*

Since idiosyncratic shocks are fully hedged, the equilibrium is identical to one in a setting without those shocks.

Proof. It is easy to see that the idiosyncratic risks are fully hedged and that the risk premia are zero, since market clears when each expert optimally chooses to offload his own idiosyncratic risk, and take on a little bit of everybody's risks (which cancel out). Once idiosyncratic risks are removed, the law of motion of individual expert's capital is

$$dn_t = \rho\, n_t\, dt + k_t\, ((a - (\rho - g)\, p_t + \mu_t^p + \sigma\, \sigma_t^p)\, dt + \alpha_t\, (\sigma p_t + \sigma_t^p)\, dZ_t) - \theta_t\, (\omega_t\, dt + dZ_t),$$

where the optimal choice of θ_t must be zero in order for hedging markets to clear. The appropriate risk premium for aggregate risk can be found from the Bellman equation

$$\rho f_t n_t = \max_{k,\theta}\, \mu_t^f\, n_t + f_t\, (\rho\, n_t + k\, (a - (\rho - g)\, p_t + \mu_t^p + \sigma\sigma_t^p) + \theta\omega_t) + \sigma_t^f\, (k\alpha_t(\sigma p_t + \sigma_t^p) + \theta).$$

In order for $\theta = 0$ to be optimal, we need $\omega_t = -\sigma_t^f/f_t$. QED

Experts fully hedge out idiosyncratic shocks when securitization is allowed, they face the cost of borrowing of only r, instead of $r + \lambda(\sigma_t^p)/\xi_t$. Lower cost of borrowing leads to higher leverage quicker payouts. As a result, the financial system becomes less stable. Thus, even though in principle securitization is a good thing, as it allows financial institutions to share idiosyncratic risks better, it leads to greater leverage and the amplification of systemic risks.

Remark. By varying the verification costs and the loss distribution, our framework can capture several other models. Kiyotaki-Moore assume that financial experts can borrow only up to fraction θ of the market value of assets. Thus, someone with net worth n_t can hold at most $1/(1-\theta)n_t$ worth of assets, by financing $\theta/(1-\theta)n_t$ of the assets with debt and the rest, n_t, with personal wealth. This is captured in our framework by setting the verification costs to zero up to a certain level and infinity afterwards. Alternatively, one can assume that margins are set equal to the value-at-risk (VaR) as in Shin (2010). In Brunnermeier and Pedersen (2009), margins increase with endogenous price volatility. These effects are captured in our model through the dependence of potential losses on price volatility. The framework of BGG, who use the costly state verification model of Townsend (1979), corresponds to the assumptions that $\alpha_t = 1$ and $\lambda(\sigma_t^p)$ is a constant.

5. Conclusions and Regulatory Implications

Events during the great liquidity and credit crunch in 2007-09 have highlighted the importance of financing frictions for macroeconomics. Unlike many existing papers in macroeconomics, our analysis is not restricted to local effects around the steady state. Importantly, we show that non-linear effects in form of adverse feedback loops and liquidity spirals are significantly larger further away from the steady state. Especially volatility effects and behavior due to precautionary motives cause these large effects. Second, we identify and isolate several externalities both within the financial sector and also from the financial sector to the real sector of the economy. Due to these externalities, financial experts leverage and maturity mismatch is excessive. We argue that financial regulation should aim to internalize these externalities. For this purpose co-risk measures have to be developed.

Appendix A: Our contracting space.

Consider a principal-agent environment, in which the agent generates cash flows

$$dX_t = \pi(e_t, k_t, \eta_t)\, dt + \sigma(k_t, \eta_t)\, dZ_t,$$

where $e_t \in \{0, -E\}$ is the agent's effort, k_t is the scale of production and η_t is the state of the economy that evolves according to

$$d\eta_t = \mu_t^\eta\, dt + \sigma_t^\eta\, dZ_t',$$

and cannot be controlled by the agent or the principal. Brownian motions Z and Z′ may be correlated.

In our model

$$\pi(e_t, \eta_t, k_t) = k_t\,(a - \iota(g) - (r - g)p_t + \mu_t^p + \sigma\sigma_t^p + p_t\, e_t) \qquad \text{and} \quad \sigma(\eta_t, k_t) = k_t\,(\sigma p_t + \sigma_t^p).$$

Both the principal and the agent are risk-neutral, and the agent's discount rate ρ is greater than the risk-free rate r. The agent's payoff flow is $dc_t - h(e_t, k_t, \eta_t)\, dt$, where $h(e, k, \eta) = b\, f(\eta)ke$ in our setting. We assume that $b < a/(r + \delta^*)$, so it is never optimal to let the agent not put effort (because it is always better to liquidate capital).

There is a standard way to solve these problems using the agent's continuation value as a state variable, but those solutions may be challenging to relate to real world. Much more intuitive is the approach of Fudenberg, Holmström and Milgrom (1990), who propose an implementation of the optimal contract with the agent's wealth as a state variable, in which the principal breaks even at any moment of time. Such an implementation exists whenever the principal's profit function $F(w_t, \eta_t)$ is decreasing in the agent's continuation value W_t, and the agent's continuation payoff as a function of wealth n_t and the state of the economy η_t is

$$w_t = H(n_t, \eta_t), \qquad \text{such that } F(w_t, \eta_t) = -n_t.$$

Agents enter short-term contracts with principals, characterized by variables β_t, k_t and β_t'. Under this contract the agent collects a fraction of output β_t, the principal collects $1 - \beta_t$

and pays the agent the fee $(1 - \beta_t)\,\pi(e_t, k_t, \eta_t)$ (so the principal breaks even) and the agent hedges β_t' of aggregate risk, so that

$$dn_t = \beta_t\, dX_t + (1 - \beta_t)\,\pi(e_t, k_t, \eta_t)\, dt + \beta_t'\, dZ_t' - dc_t$$

Optimal short-term contracts (β_t, k_t, β_t') can be found from the Bellman equation

$$\rho H(n_t, \eta_t)dt = \max_{\beta,\, k,\, \beta',c,e} dc_t - h(e, k, \eta_t) + E\,[\, dH(n_t, \eta_t)\,]$$

subject to the incentive-compatibility constraint that e maximizes

$$\beta\, H_1(n,\eta)\,\pi(e, k, \eta) - h(e, k, \eta).$$

We do not allow the full contracting space in our paper, but limit the hedging of aggregate risk by forcing β' to be 0. With limited instruments, optimization leads to the value functions

$$H(n_t,\eta_t) = f(\eta_t)\, n_t \qquad \text{and} \qquad F(w_t,\eta_t) = -w_t/f(\eta_t),$$

with $F(w_t,\eta_t)$ decreasing in w_t as required by Fudenberg, Holmström and Milgrom (1990). Thus, contracts in our paper are optimal dynamic contracts from a smaller contracting space, so we have contract incompleteness.

In the section on securitization, we allow hedging of aggregate risk through a market mechanism within the financial sector. This leads to a risk premium λ_t on aggregate risk, so the agent's wealth evolve according to

$$dn_t = \beta_t\, dX_t + (1 - \beta_t)\,\pi(e_t, k_t, \eta_t)\, dt + \beta_t'\,(dZ_t' + \lambda_t\, dt) - dc_t.$$

Appendix B.

Proposition A1. There is a stationary distribution of ω_t only if the system never becomes absorbed at $-a/(r+\delta^*)$ because assets are liquidated sufficiently fast when ω_t approaches $-a/(r+\delta^*)$. In that case, the stationary density must satisfy the standard equation

$$\tfrac{1}{2}\, d^2/d\omega^2\, (\sigma^\omega(\omega)^2\, d(\omega)) + d/d\omega\, (\mu^\omega(\omega)\, d(\omega)) = 0,$$

where $\mu^\omega = ((r - g + \sigma^2)\,\omega + a - \varphi(g)$ and $\sigma^\omega = -\sigma\omega$. The relevant boundary conditions are $d'(\omega^*) = 0$ (because it is a reflecting boundary) and

$$\int_{-L}^{\omega^*} d(\omega)d\omega = 1.$$

Proof. To be completed.

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