Managing Credit Booms and Busts: A Pigouvian Taxation Approach

Olivier Jeanne
Johns Hopkins University

Anton Korinek
University of Maryland

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Abstract

We study a dynamic model in which the interaction between debt accumulation and asset prices magnifies credit booms and busts. We show that these feedback effects create an externality since borrowers do not internalize their contribution to aggregate volatility and therefore take on excessive leverage. As a result the economy suffers from excessive volatility, i.e. large booms and busts in both credit flows and asset prices. We propose a Pigouvian tax on borrowing that induces agents to internalize their externalities. In a sample calibration, the optimal magnitude of this tax is around 2 to 3 percent. Our paper also develops a new numerical method of solving models with occasionally binding endogenous constraints.

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1 Introduction

The interaction between debt accumulation and asset prices contributes to magnify the impact of booms and busts. Increases in borrowing and in collateral prices feed each other during booms. In busts, the feedback turns negative, with credit constraints leading to fire sales of assets and further tightening of credit. It has been suggested that prudential policies could be used to mitigate the build-up in systemic vulnerability during the boom. However, there are few formal welfare analyses of the optimal policies to deal with booms and busts in credit and asset prices.

This paper makes a step toward filling this gap with a dynamic optimizing model of collateralized borrowing. We consider a group of individuals (the insiders) who enjoy a comparative advantage in holding an asset and who can use this asset as collateral on their borrowing from outsiders. The borrowing capacity of insiders is therefore increasing in the price of the asset. The price of the asset, in turn, is driven by the insiders' consumption and borrowing capacity. This introduces a mutual feedback loop between asset prices and credit flows: small financial shocks to insiders can lead to large simultaneous booms and busts in asset prices and credit flows.

The model attempts to capture, in a stylized way, a number of economic settings in which the systemic interaction between credit and asset prices may be important. The insiders could be interpreted as: a) a group of entrepreneurs who have more expertise than outsiders to operate a productive asset; b) households putting a premium on owning their homes; c) a group of investors who enjoy an informational advantage in dealing with a certain class of financial assets; or d) the residents of a small open economy borrowing from foreign lenders. One advantage of studying these situations with a common framework is to bring out the commonality of the problems and of the required policy responses—although, in the real world, those policies pertain to different areas such as financial regulation, individual and corporate taxation, or capital controls.

One of our main results is that the asset-debt loop entails systemic externalities that lead the borrowers to undervalue the benefits of conserving liquidity as a precaution against busts. A borrower who has one more dollar of liquid net worth when the economy experiences a bust not only relaxes his private borrowing constraint, but also relaxes the borrowing constraints of all other insiders. Not internalizing this spillover effect, the insider takes on too much debt during good times. In a benchmark calibration of our model, we find that it would be optimal to impose a 2 to 3 percent tax on borrowing by leveraged insiders to prevent them from taking on socially excessive debt.

Our model is related to the positive study of financial accelerator effects in closed and open economy macroeconomics. In closed-economy DSGE models, Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999) show that financial frictions amplify the response of an economy to fundamental shocks. However, models in this literature are traditionally solved by linearization, making them more appropriate to analyze regular business cycle fluctuations than systemic crises. In the open economy literature, Mendoza (2005) and Mendoza and Smith (2006), among others, have studied the nonlinear dynamics arising from financial accelerator effects during sudden stops in emerging market economies. These papers provide a positive analysis of how financial frictions can amplify shocks to the economy, but do not characterize welfare-maximizing policies. The central focus of our paper is to fill this gap.

This paper is also related to analyses of the ongoing world-wide credit crisis that emphasize the amplifying mechanisms involving asset price deflation and deleveraging in the financial sector (e.g., Adrian and Shin, 2009; Brunnermeier, 2009). Some earlier contributions have clarified the externalities involved in credit booms and busts and drawn some implications for policy in the context of stylized two- or three-period models (Korinek, 2009, 2010). By contrast, this paper gives a more realistic and quantitative flavor to the analysis, by considering an infinite-horizon model. This is particularly relevant for determining the optimal magnitude of regulatory measures in practice.

Recent papers have looked at the same kind of externalities as we do here in the context of a small open economy. Benigno et al (2009) and Bianchi (2009) also characterize welfare-maximizing policies in dynamic optimization models with collateralized debt for policy analysis. Their papers focus on the role of exchange rate depreciations in emerging market crises—the externality involves the real exchange rate rather than the price of a domestic asset. By contrast, our model does not have an exchange rate and attempts to capture the essence of the problem in a generic setting involving asset price deflation.

Finally, our paper develops a numerical solution method for DSGE models with occasionally binding endogenous constraints that extends the endogenous gridpoints method of Carroll (2006) to an environment with endogenous constraints. This endogenous gridpoints bifurcation method allows us to solve such models in a very efficient way and may enable researchers to analyze more complex models than what has been computationally feasible in the existing DSGE literature with endogenous constraints, ultimately producing policy guidance on richer and more realistic models of the economy.

2 The model

We consider a group of identical atomistic individuals in infinite discrete time $t = 0, 1, 2, \ldots$. The individuals are indexed by $i \in [0, 1]$. The utility of individual i at time t is given by,

$$U_{i,t} = E_t \left(\sum_{s=t}^{+\infty} \beta^{s-t} u(c_{i,s}) \right), \tag{1}$$

where $u(\cdot)$ is strictly concave and satisfies the Inada conditions. We will generally assume that utility has constant relative risk aversion

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}.$$

These individuals (the insiders) receive two kinds of income, the payoff of an asset that can serve as collateral, and an endowment income. Insider i maximizes his utility under the budget constraint

$$c_{i,t} + d_{i,t} + \theta_{i,t+1} p_t = e_t + \theta_{i,t}(p_t + y_t) + \frac{d_{i,t+1}}{R}, \tag{2}$$

where $d_{i,t}$ the individual's debt, which we assume default-free; e_t is an endowment income; y_t is the payoff of an asset that can be pledged as collateral; $\theta_{i,t}$ is the insider's holdings of the asset and p_t its price. The debt is sold to outside investors (outsiders for short) who supply an indefinite supply of loans at interest rate $r = R - 1$. We assume that e_t and y_t follow a stationary Markov process (to be specified later).[1]

The collateral asset can be exchanged between insiders in a perfectly competitive market. However, the asset cannot be sold to outsiders: θ_t must be equal to 1 in equilibrium. We do not allow insiders to sell the asset to outsiders and rent it back because insiders derive important benefits from the control rights that ownership provides. This restriction can be relaxed to some extent (we will generalize it by assuming $1 \geq \theta_t \geq \underline{\theta}$ for some $\underline{\theta} > 0$), but we need a restriction of this form for insiders to issue collateralized debt.

Furthermore, we assume that the only financial instrument which can be traded between insiders and outsiders is uncontingent short-term debt. This assumption can be justified e.g. on the basis that shocks to the insider sector are not perfectly observable to outsiders and cannot be used to condition payments, or that short-term debt provides insiders with adequate incentives. This feature corresponds to common practice across a wide range of financial relationships.[2]

The value of the collateral asset determines how much debt insiders are able to roll over. We assume that outside lenders do not allow them roll over more than a fraction ϕ of the value of the collateral asset

$$\frac{d_{i,t+1}}{R} \leq \phi \theta_{i,t} p_t. \tag{3}$$

A micro-justification for this constraint– along the lines of Kiyotaki and Moore (1997) – would be that an indebted insider can walk away from the debt he just issued, in which case lenders can seize a fraction ϕ of his asset and sell it in the market. Equation (3) is then the incentive compatibility condition ensuring that debtors do not walk away from their debt. Subject to this constraint, debt issued in period t is repaid with certainty in period $t + 1$. [3]

It will be sometimes more convenient to write the model in terms of the insiders' net wealth $w_i = -d_i$, so that the collateral constraint becomes

$$\frac{w_{i,t+1}}{R} + \phi \theta_{i,t} p_t \geq 0.$$

3 Laissez-faire equilibrium

We characterize the laissez-faire equilibrium in several steps. First, we derive the equilibrium conditions for a symmetric equilibrium (in which all insiders behave

[1] We could introduce growth into the model. If the detrended payoff is Markov stationary and utility is CRRA, the model with growth, once detrended, is isomorphic to the model presented here.

[2] More generally, the findings of Korinek (2010) suggest that our results on excessive exposure to binding constraints would continue to hold when insiders have access to costly state-contingent financial contracts.

[3] We could introduce default risk, possibly linked to the next-period price of the asset. In addition, constraint (3) could involve the end-of-period holding of asset (θ_{t+1} instead of θ_t). Appendix A.4 derives the equilibrium conditions in this case. More generally, the right-hand side could involve a constant term or be a nonlinear increasing function of p_t. The only important assumption, to obtain the debt-asset deflation mechanism at the core of the model, is that the credit constraint depend on the asset price.

in the same way). We then present some considerations on equilibrium multiplicity and the possibility of self-fulfilling asset price and debt busts. Section 3.3 presents our numerical resolution method. Section 3.4 presents the results of a numerical simulation with booms and busts in the asset price and in credit flows. We assume hereafter that the non-pledgeable endowment is constant, $e_t = e$.

3.1 Equilibrium conditions

We derive in the appendix the first-order conditions for the optimization problem of an insider i. We then use the fact that in a symmetric equilibrium, all individuals are identical and hold one unit of collateral asset ($\forall i, t \quad \theta_{i,t} = 1$). Variables without the subscript i refer to the *representative* insider (or to aggregate levels, since the mass of insiders is normalized to 1). This gives the following two conditions

$$u'(c_t) = \lambda_t + \beta R E_t \left[u'(c_{t+1})\right], \tag{4}$$

$$p_t = \beta \frac{E_t \left[u'(c_{t+1})(y_{t+1} + p_{t+1}) + \phi \lambda_{t+1} p_{t+1}\right]}{u'(c_t)}, \tag{5}$$

where λ_t is the costate variable for the borrowing constraint. The first equation is the Euler condition and the second one is the pricing equation for the collateral asset. The asset-pricing equation has a term in $E_t \lambda_{t+1}$ in the numerator, reflecting the asset's expected utility as collateral in the next period.

The equilibrium is characterized by a set of functions mapping the state of the economy into the endogenous variables. The state at period t can be characterized by two variables: the current return on the asset, y_t, and the beginning-of-period net wealth excluding the value of the collateral asset,

$$m_t \equiv e + y_t + w_t.$$

We do not include the asset in the definition of net wealth because its price, p_t, is an endogenous variable. The collateral constraint (3) can be written, in aggregate form,

$$c_t \leq m_t + \phi p_t. \tag{6}$$

The equilibrium, thus, is characterized by three non-negative functions, $c(m, y)$, $p(m, y)$ and $\lambda(m, y)$ such that

$$c(m, y) = \min \left\{ m + \phi p(m, y), \left[\beta R E \left(c(m', y')^{-\gamma}\right)\right]^{-1/\gamma} \right\}, \tag{7}$$

$$\lambda(m, y) = \left[c(m, y)^{-\gamma} - \beta R E \left(c(m', y')^{-\gamma}\right)\right]^{+}, \tag{8}$$

$$p(m, y) = \beta \frac{E \left[u'(c(m', y'))(y' + p(m', y')) + \phi \lambda(m', y') p(m', y')\right]}{u'(c(m, y))}, \tag{9}$$

where next-period values are denoted with primes and where $[\cdot]^+$ denotes $\max\{\cdot, 0\}$. The transition equation for net wealth is

$$m' = e + y' + R\,(m - c(m, y))\,. \tag{10}$$

The current asset return, y_t, provides information in addition to net wealth, m_t, only to the extent that it gives a signal about future returns. If y is i.i.d., the policy functions depend solely on m.

3.2 Multiple equilibria

Many papers, in the dynamic optimization literature on consumption and saving, compute the equilibrium policy functions by iterating on the first-order conditions, under the assumption that this method converges towards policy functions that exist and are unique.[4] However, we cannot make such an assumption here as our model generically gives rise to equilibrium multiplicity. We give in this section a heuristic account of the mechanism underlying multiplicity and of the conditions that tend to ensure uniqueness.[5]

The multiplicity comes from the self-reinforcing loop that links consumption to the price of the collateral. In the constrained regime, a fall in the price of the collateral asset decreases the insiders' level of consumption, which in turn tends to depress the price of the asset. This loop, which is essential for our results since it explains the financial magnification of real shocks, may also—if its effect is strong enough—lead to self-fulfilling crashes in the price of the asset.

More formally, the loop linking consumption to the asset price is captured by equations (5) and (6). Assuming that the policy functions $c(m, y)$, $p(m, y)$ and $\lambda(m, y)$ apply in the following period, equation (5) implicitly define the asset price as a function of the state and current consumption,

$$\widehat{p}(m, y, c) = \beta E\left[u'(c(m', y'))(y' + p(m', y')) + \phi\lambda(m', y')p(m', y')|m, y, c\right] c^{\gamma}, \tag{11}$$

where $m' = e + y' + R\,(m - c)$. The credit constraint (6) can then be written

$$c \leq m + \phi\widehat{p}(m, y, c). \tag{12}$$

The right-hand side of (12) is a priori increasing in c because the credit constraint on each individual is relaxed by a higher level of aggregate consumption that raises the price of the asset.[6] Multiplicity may arise if the left-hand side

[4] See Zeldes (1989) for an early example. Stokey et al (1989) present several fixed-point theorems guaranteeing that the equilibrium exists, is unique, and can be obtained by iterating on the problem's first-order conditions. However, the models considered in most of the literature (including Zeldes') do not satisfy the conditions under which those theorems are applicable—see the discussion in Carroll (2009).

[5] We are not aware of papers giving general conditions under which the equilibrium is unique in models of the type considered here (i.e., that extend the analysis of Carroll (2006) to the case with endogenous credit constaints). A rigorous treatment of this question is an interesting question for future research.

[6] This is captured by the factor c^{γ} on the r.h.s. of (11). However, because of the other terms in m', the sign of the variations of $\widehat{p}$ with c is a priori ambiguous.

and the right-hand side of (12) intersect for several levels of consumption. In this case, the consumption function is not uniquely defined at each step of the iteration on the first-order conditions.

We further explore the multiplicity of equilibria in the remainder of this section by considering a special case of the model that can be solved (almost completely) in closed form: the case where y is constant and $\beta R = 1$. One benefit of looking at this case is to suggest conditions under which the equilibrium is unique, which will be useful in the numerical simulations. We summarize the main results below (the details can be found in the appendix).

If y_t is constant, the state at time t is summarized by the level of net wealth m_t. We derive the equilibrium for $t = 1, 2...$ starting from an initial level of wealth m_1. It is easy to show (see the appendix) that the economy is in an unconstrained steady state from period 2 onwards, i.e., the economy may be constrained only in period 1. In the unconstrained steady state the price of the asset is given by

$$p^{\text{unc}} = \frac{y}{r}, \tag{13}$$

where $r = R - 1$ is the interest rate. The economy is constrained in period 1 if and only if the initial level of net wealth is lower than a threshold, and function $\widehat{p}(\cdot, \cdot)$ is given (in period 1) by

$$\widehat{p}(m_1, c_1) = p^{\text{unc}} \cdot \min\left[\left(\frac{c_1}{e + y + r(m_1 - c_1)}\right)^{\gamma}, 1\right], \tag{14}$$

(y is no longer an argument since it is constant). The price of the asset is convex and increasing in the level of aggregate consumption. If the left-hand side and right-hand side of constraint (12) intersect each other in several points, as shown in Figure 1, there are multiple equilibria. The unconstrained equilibrium (point A) coexists with a constrained equilibrium featuring lower levels of consumption and of the asset price (point C).[7] The multiplicity comes from the fact that the slope of the r.h.s. of (12) is larger than 1 over some range. Then a one-dollar fall in aggregate consumption tightens the credit constraint by more than one dollar for each individual, allowing a self-fulfilling downward spiral in consumption and the price of the asset.

Conversely, as shown in the appendix, a necessary and sufficient condition for the equilibrium to be unique is that the slope of the right-hand side of equation (12) be smaller than 1 on the l.h.s. of the kink. This is true if and only if ϕ is small enough,

$$\phi \leq \frac{1 + e/y}{1 + \gamma(1 + 1/r)}. \tag{15}$$

[7] The figure was constructed for $r = 0.04$, $\gamma = 2$, $e = 0.9$, $y = 0.1$ and $\phi = 0.3$ and $m_1 = 0.3$. There is one more intersection (point B) between A and C. However, point B corresponds to an unstable equilibrium in the sense that a one dollar change in aggregate consumption changes the maximum level of consumption by more than one dollar for each individual, so that the economy would tip toward points A or C following a small perturbation in consumption.

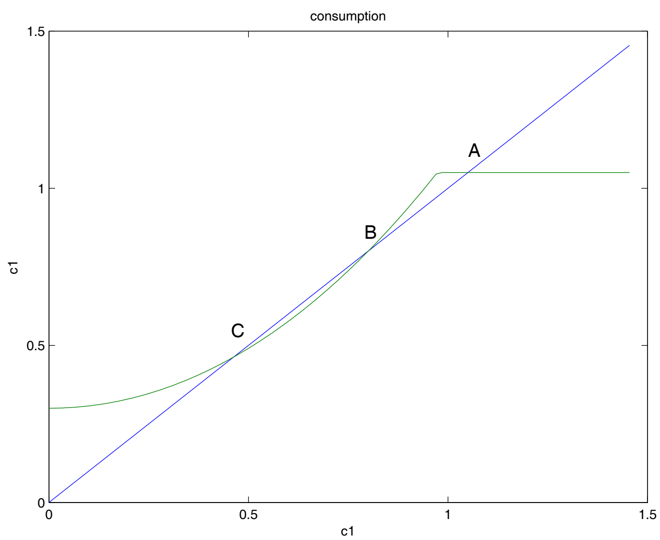

Figure 1: Equilibrium multiplicity (y constant and $\beta R = 1$)

For the parameter values used to construct Figure 1, for example, equilibrium uniqueness is ensured by taking $\phi \leq 0.189$. A small ϕ contains the strength of the externality below the level where it leads to multiple equilibria.

Although the multiplicity of equilibria is interesting, it is not essential for the main insights of our analysis. We will thus ensure (for now) that the equilibrium is unique by assuming sufficiently low values of ϕ.[8]

3.3 Numerical resolution

We now go back to the general case with a stochastic y. In order to generate a persistent motive for borrowing, we need to assume that insiders are impatient relative to outsiders, i.e.,[9]

$$\beta R < 1.$$

We may make conjectures about the form of the solution by analogy with the deterministic case studied in the previous section. In order to rule out multiple equilibria, the attention will be restricted to equilibria in which the consumption function $m \mapsto c(m, y)$ is a continuously increasing function of wealth for any y. Let us denote by $\underline{m}(y)$ the level of wealth for which consumption is equal to zero,

$$c(\underline{m}(y), y) = 0.$$

[8] We found that condition (15) was generally sufficient to ensure convergence in the stochastic version of the model. For high levels of ϕ the numerical method presented in the following section typically leads to a cycle in policy functions and does not converge.

[9] With trend growth at a growth factor G, we could allow $\beta R \geq 1$ as long as $\beta R G^{1-\gamma} < 1$.

By analogy with the deterministic case, we would expect the insiders to be credit-constrained in a wealth interval $m \in [\underline{m}(y), \overline{m}(y)]$, and to be unconstrained for $m \geq \overline{m}(y)$. The thresholds $\underline{m}(y)$ and $\overline{m}(y)$ are key endogenous variables to determine in deriving the equilibrium.

It is not difficult to see that the lower threshold must be equal to zero,

$$\forall y, \ \underline{m}(y) = 0.$$

This results from the facts that $c(m, y) \leq m + \phi p$, and that p converges to zero as c goes to zero (by equation (5)). Since $m = e + y + w$ must always be positive and the level of debt is set before the realization of y, we must have

$$w + e + \min y \geq 0,$$

(assuming that $\min y$ is always a possible realization in the following period). As for the higher threshold, $\overline{m}(y)$, it must be determined numerically.

The numerical resolution method that we develop here is an extension of the endogenous grid points method of Carroll (2006) to the case where the credit constraint is endogenous. The basic idea is to perform backwards time iteration on the agent's optimality conditions, i.e., to define a grid $\mathbf{w}^g$ for next period wealth levels w' and combine the next period policy functions with agent's optimality conditions to obtain current period policy functions until the resulting functions converge. The only difference with Carroll (2006) is that the minimum level of wealth is itself a function of the state, which is obtained by iterating on the asset pricing equation (9). The details of the numerical resolution method can be found in the appendix.

3.4 Booms and busts

Booms and busts in credit can be modeled by assuming a simple two-state Markov process for y. Assume that the return on the collateral can be high, $y = y_H$, or low, $y = y_L$. The resulting debt and consumption dynamics are shown in Figure [.] for the following calibration of the model:

β	R	γ	e	y_L	y_H	P	ϕ
.94	1.04	2	.9	.08	.12	$\begin{pmatrix} .9 & .1 \\ .5 & .5 \end{pmatrix}$	.20

Table 1: Parameter values for stochastic simulation with $\beta R < 1$

If y is high ($y = y_H$), the price of collateral is also high, allowing the insiders to bear high levels of debt. Debt converges to a level that keeps insiders unconstrained (they maintain a precautionary margin of safety if the risk of a bust is not too small).

By contrast, if $y = y_L$, the price of the asset is low and so is the level of debt allowed by the collateral constraint. Because they are impatient ($\beta R < 1$), insiders increase their level of debt until the collateral constraint binds. We observe that conditional on the low state, the economy converges to a deterministic cycle with oscillations between the constrained state and the unconstrained state (endogenous cycle). This is because if the constraint is binding in one period, saving is high, so that the constraint is not binding the following period.[10]

State-switching thus generates booms and busts in credit and asset prices. When y increases from y_L to y_H, insiders can suddenly borrow more and increase their leverage. If the boom is long enough, the level of debt exceeds the threshold that makes the economy vulnerable to a credit crunch if y falls back to y_L. When y falls, there is a bust with a sharp contraction of credit and downward overshooting in the price of the asset.

4 Social planner

4.1 The social planner solution

We assume that the social planner of the economy determines the amount of insiders' borrowing, but does not directly interfere in asset markets—that is, the social planner takes as given that insiders trade the collateralizable asset at a price that is determined by their private optimality condition (5). The social optimum differs from the laissez-faire equilibrium because the social planner internalizes that future asset prices and insiders' borrowing capacity depend on the aggregate level of debt accumulated by insiders. A possible motivation for this setup is that decentralized agents are better than the planner at observing the fundamental payoffs of financial assets, while only the social planner has the capacity of internalizing the costs of debt deflation dynamics that may arise from high levels of debt.

In period t, the social planner chooses the debt level of the representative insider, d_{t+1}, before the asset market opens at time t. The asset market remains perfectly competitive, i.e., individual market participants optimize on θ_{t+1} subject to (2), yielding the optimality condition (5). We look for time-consistent equilibria in which the social planner optimizes on d_{t+1} taking the future policy functions $c(m, y)$ and $p(m, y)$ as given. (Although we do not change the notation, those policy functions are not the same as in the laissez-faire equilibrium.)

Through savings, the social planner determines the price of the asset, which at time t is given by

$$p_t = \widehat{p}(m_t, y_t, m_t + d_{t+1}/R)$$

where function $\widehat{p}(\cdot, \cdot, \cdot)$ was defined by (11). The social planner internalizes that he can affect the price of the asset through his decision on the current level of aggregate debt. Increasing d_{t+1} lowers the marginal utility of consumption, which tends to increase the price of the asset.

[10] If ϕ is smaller, there are no oscillations. The conditions under which there is a cycle in the deterministic model are derived in the appendix.

Since insiders can still not borrow more than a fraction ϕ of the value of their asset holdings, the social planner sets d' subject to the constraint

$$\frac{d'}{R} \leq \phi \widehat{p}\left(m, y, m + d'/R\right). \tag{16}$$

If ϕ is small enough to avoid multiple equilibria (as we have assumed), the right-hand side increases less with d' than the left-hand side, so that this inequality determines an *upper bound* on aggregate debt. Then the social planner's credit constraint can be rewritten in reduced form,

$$\frac{d'}{R} \leq \phi \overline{p}(m, y), \tag{17}$$

where $\overline{p}(m, y)$ is the level of $p\left(m, y, m + d'/R\right)$ such that (16) is an equality. The maximum price level $\overline{p}(m, y)$ is increasing in m because $p\left(m, y, m + d'/R\right)$ is increasing in m for any y and d' (as can be readily seen from (11)). Note that per the definition of function $\overline{p}(\cdot, \cdot)$, we have $\overline{p}(m, y) = p(m, y)$ for all the states (m, y) in which the social planner's constraint is binding.

The social planner solves the same optimization problem as decentralized agents, except that he takes $\theta_t = 1$ as given in the aggregate budget constraint, and that his credit constraint is given by (17). As shown in the appendix, the social planner's Euler equation is,

$$u'(c_t) = \lambda_t + \beta R E_t \left(u'(c_{t+1}) + \lambda_{t+1} \phi \frac{\partial \overline{p}_{t+1}}{\partial m_{t+1}} \right). \tag{18}$$

The derivative of the next-period asset price with respect to aggregate wealth, $\partial \overline{p}_{t+1} / \partial m_{t+1}$, is positive. Comparing (4) and (18), this implies that the social planner raises saving above (lowers consumption and debt below) the laissez-faire level. The saving wedge is proportional to the expected product of the shadow cost of the credit constraint times the derivative of the debt ceiling with respect to wealth. This reflects that the social planner internalizes the endogeneity of next period's asset price and credit constraint to this period's aggregate saving.

Decentralized agents are aware of the risk of credit crunch and maintain a certain amount of precautionary saving (they issue less debt than if this risk were absent), but they do not internalize the contribution of their precautionary savings to reducing *systemic* risk. With the social planner, precautionary savings is augmented by a systemic component (i.e., the social planner implements a policy of *systemic precautionary saving*).

4.1.1 Pigouvian taxation

The social planner's Euler equation also provides guidance for how the socially optimal equilibrium can be implemented via taxes on external borrowing. Decentralized agents undervalue the social cost of debt by the term $\phi E_t \left[\lambda_{t+1} \frac{\partial p_{t+1}}{\partial m_{t+1}} \right]$

on the right-hand side of the social planner's Euler equation (18), which depends on the state of the economy (m_t, y_t). The planner's equilibrium can be implemented by a Pigouvian tax $\tau_t = \tau(m_t, y_t)$ on borrowing that introduces a wedge in insiders' Euler equation and that is rebated as a lump sum transfer $T_t = \tau_t w_{t+1}/R$:

$$c_t = e_t + y_t + w_t - \frac{w_{t+1}}{R}(1+\tau_t) + T_t$$

This modifies insiders' Euler equation to

$$u'(c_t) = \lambda_t + (1+\tau_t)\,\beta R E_t\left[u'(c_{t+1})\right]$$

The tax replicates the constrained social optimum as chosen by the constrained planner if it is chosen such that

$$\begin{aligned}
(1+\tau_t)\,E_t\left[u'(c_{t+1})\right] &= E_t\left[u'(c_{t+1}) + \lambda_{t+1}\phi\frac{\partial p_{t+1}}{\partial m_{t+1}}\right] \\
\text{or}\quad \tau(m_t, y_t) &= \frac{\lambda_{t+1}\phi\frac{\partial p_{t+1}}{\partial m_{t+1}}}{E_t\left[u'(c_{t+1})\right]}
\end{aligned}$$

where all variables are evaluated at the social optimum.

The tax would be levied at time t when the borrowing decision w_{t+1} for next period is made; therefore such a measure avoids any commitment problems. A Ramsey-equivalent approach would be to impose a tax whenever borrowing constraints are binding and the externality materializes. However, this would potentially face two important political economy constraints: First, it would require that higher taxes are imposed in the midst of large downturns – precisely when consumption among insiders falls sharply. Secondly, it would create a commitment problem for the planner – the measure is only effective if insiders in period t when borrowing choices are made believe that the tax will indeed be imposed in period $t+1$.

In our simulations, we find that the optimal magnitude of this tax is on average 2.41%.

5 Extensions

5.1 Interest Rates and Financial Fragility

5.2 Debt moratorium and bailouts

5.3 FDI Liberalization

6 Conclusion

This paper has developed a simple model to study the optimal policy response to booms and busts in credit and asset prices. We found that decentralized agents do not internalize that their borrowing choices in boom times render the economy more vulnerable to credit and asset price busts involving debt deflation in bust times. Therefore their borrowing imposes an externality on the economy.

In our baseline calibration, a social planner would impose an ex-ante tax of 2.41% per dollar on insider borrowing so as to reduce the debt burden of insiders and mitigate the decline in consumption in case of crisis.

7 References

Adrian, Tobias and Hyun Song Shin, 2009, "Liquidity and Leverage," manuscript, Department of Economics, Princeton University.

Aguiar, Mark and Gita Gopinath, 2007, "Emerging Market Business Cycles: The Cycle is the Trend," *Journal of Political Economy* 115(1), 69-102.

Benigno, Gianluca, Huigang Chen, Christopher Otrok, Alessandro Rebucci and Eric R. Young, 2009, "Optimal Policy with Occasionally Binding Credit Constraints," manuscript, IADB.

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist, 1999, "The Financial Accelerator in a Quantitative Business Cycle Framework," *Handbook of Macroeconomics*, Vol 1C, edited by John B. Taylor and Michael Woodford, Elsevier.

Bianchi, Javier, 2009, "Overborrowing and Systemic Externalities in the Business Cycle," manuscript, Department of Economics, University of Maryland.

Brunnermeier, Markus, 2009, "Deciphering the Liquidity and Credit crunch," *Journal of Economic Perspectives* 23(1), 77-100.

Carlstrom, Charles T. and Timothy S. Fuerst, 1997, "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis," *American Economic Review* 87, 893-910.

Carroll, Christopher D., 2006, "The Method of Endogenous Gridpoints for Solving Dynamic Stochastic Optimization Problems,' *Economics Letters*, 312–320.

Kaldor, Nicholas, 1934, "A Classificatory Note on the Determination of Equilibrium," *Review of Economic Studies* 1, 122-136.

Kiyotaki, Nobuhiro and John H. Moore, 1997, "Credit Cycles," *Journal of Political Economy* 105(2), 211-248.

Korinek, Anton, 2009, "Regulating Capital Flows to Emerging Markets: An Externality View," manuscript, Department of Economics, University of Maryland.

Korinek, Anton, 2010, "Systemic Risk-Taking: Amplification Effects, Externalities, and Regulatory Responses," manuscript, Department of Economics, University of Maryland.

Lorenzoni, Guido, 2008, "Inefficient Credit Booms," *Review of Economic Studies* 75(3), 809-833.

Mendoza, Enrique G., 2005, "Real Exchange Rate Volatility and the Price of Nontradables in Sudden-Stop-Prone Economies," *NBER Working Paper* 11691.

Mendoza, Enrique G. and Katherine Smith, 2006, "Quantitative Implications of a Debt-Deflation Theory of Sudden Stops and Asset Prices," *Journal of International Economics* 70, 82-114.

A Solution of Benchmark Model

A.1 Laissez-faire

Decentralized agents solve the Lagrangian

$$\mathcal{L}_t = E_t \sum_{s=t}^{+\infty} \beta^{t-s} \left\{ u\left(e_s + \theta_{i,s}(y_{i,s} + p_s) + \frac{d_{i,s+1}}{R} - d_{i,s} - \theta_{i,s+1} p_s \right) + \lambda_{i,s} \left[\phi \theta_{i,s} p_s - \frac{d_{i,s+1}}{R} \right] \right\},$$

Given CRRA utility, this implies the first-order conditions

$$\begin{aligned} FOC\,(d_{i,s+1}) \quad &: \quad c_{i,s}^{-\gamma} = \beta R E_s \left[c_{i,s+1}^{-\gamma} \right] + \lambda_{i,s}, \\ FOC\,(\theta_{i,s+1}) \quad &: \quad p_s c_{i,s}^{-\gamma} = \beta E_s \left[c_{i,s+1}^{-\gamma} (y_{s+1} + p_{s+1}) + \phi \lambda_{i,s+1} p_{s+1} \right]. \end{aligned}$$

In a symmetric equilibrium with a representative agent, this gives (4) and (5).

A.2 Deterministic case with $\beta R = 1$

In a steady state, equation (5) implies that the price of the asset is given by (13). The collateral constraint is satisfied if and only if $w/R + \phi p^{\mathrm{unc}} \geq 0$, that is if wealth is larger than a threshold

$$m \geq \overline{m} \equiv e + y \left(1 - \frac{\phi}{1-\beta} \right).$$

If $m_1 \geq \overline{m}$, the economy settles in the steady-state equilibrium in period 1. In this equilibrium, consumption is constant and given by

$$c^{\mathrm{unc}}(m_1) = e + y + (1-\beta) w_1 = \beta(e+y) + (1-\beta) m_1.$$

By contrast, if $m_1 < \overline{m}$, the economy is constrained in period 1 and $w_2/R \geq -\phi p^{\mathrm{unc}}$, which implies $m_2 = e + y + w_2 \geq \overline{m}$ so that the economy is in the unconstrained steady state from period 2 onwards. This implies $c_2 = e + y + r(m_1 - c_1)$ and $\lambda_2 = 0$, so that the period-1 price is given by (14), which can also be written

$$\widehat{p}(m_1, c_1) = p^{\mathrm{unc}} \min \left[r^{-\gamma} \left(\frac{e + y + r m_1}{e + y + r(m_1 - c_1)} - 1 \right)^{\gamma}, 1 \right]. \tag{19}$$

One can show that this is a strictly convex function of c_1 for $c_1 \leq c^{\mathrm{unc}}(m_1)$ (to the left of the kink). Hence the slope is everywhere lower than the slope on the l.h.s. of the kink, which is given by,

$$\left. \frac{\partial \widehat{p}}{\partial c_1} \right|_{c_1 = c^{\mathrm{unc}}(m_1)} = \left(1 + \frac{1}{r} \right) \frac{\gamma y}{c^{\mathrm{unc}}(m_1)}. \tag{20}$$

This slope is decreasing in m_1. To avoid multiplicity, it is necessary and sufficient to have

$$\phi \left. \frac{\partial \widehat{p}}{\partial c_1} \right|_{c_1 = c^{\text{unc}}(\overline{m})} < 1.$$

If this condition is satisfied, then the slope of the r.h.s. of (12) is lower than 1 for any $m_1 \geq \overline{m}$, so that the unconstrained equilibrium is unique. Conversely, if this condition is not satisfied, then there is multiplicity for m_1 slightly below $\overline{m}$. Using (20), the condition can be rewritten as (15).

A.3 Social planner

The social planner maximizes the utility of the representative insider subject to the budget constraint (2) taking $\theta_t = 1$ as given, and to the credit constraint (17). The Lagrangian of the social planner is

$$\mathcal{L}_t^{SP} = E_t \sum_{s=t}^{+\infty} \beta^{t-s} \left\{ u\left(e + y_s + \frac{d_{s+1}}{R} - d_s\right) + \lambda_s \left[\phi \overline{p}(e + y_s - d_s, y_s) - \frac{d_{s+1}}{R}\right] \right\},$$

FOC d_{t+1}

$$u'(c_t) = \lambda_t + \beta R E_t \left[u'(c_{t+1}) + \phi \lambda_{t+1} \frac{\partial \overline{p}(m_{t+1}, y_{t+1})}{\partial m} \right].$$

Using the fact that $\overline{p}(\cdot, \cdot) = p(\cdot, \cdot)$ in the constrained states, we have

$$\lambda_{t+1} \frac{\partial \overline{p}(m_{t+1}, y_{t+1})}{\partial m} = \lambda_{t+1} \frac{\partial p(m_{t+1}, y_{t+1})}{\partial m},$$

so that the Euler condition can be written like (18).

A.4 Alternative Specification of Constraint

If the collateral constraint in subsection **??** was written in terms of future asset holdings

$$\frac{d_{s+1}}{R} \leq \phi \theta_{s+1} p_s,$$

the second first-order condition would read as

$$p_s (c_s^{-\gamma} - \phi \lambda_s) = \beta E_s \left[c_{s+1}^{-\gamma} (y_{s+1} + p_{s+1}) \right].$$

In this case, we would have

$$p_s = \frac{E_s \left[c_{s+1}^{-\gamma} (y_{s+1} + p_{s+1}) \right]}{c_s^{-\gamma} - \phi \lambda_s},$$

As long as $\phi < 1$, there is again a positive feedback effect from c_s to p_s, giving rise to debt deflation dynamics that are equivalent to our benchmark specification.

B Numerical resolution method

In order to solve the system numerically using this method, we define a grid $\mathbf{y}^g$ containing the possible realizations of the output shock and a grid $\mathbf{w}^g$ for wealth. In iteration step k, we start with a triplet of functions $c_k(m,y)$, $p_k(m,y)$ and $\lambda_k(m,y)$ where $c_k(m,y)$ and $p_k(m,y)$ are weakly increasing in m and $\lambda_k(m,y)$ is weakly decreasing in m for a given y. For each $w' \in \mathbf{w}^g$ and $y \in \mathbf{y}^g$ we solve the system of optimality conditions from section 3.1 under the assumption that the borrowing constraint is loose, noting that $m' = e + y' + w'$:

$$\begin{aligned}
c^{\text{unc}}(w',y) &= \beta RE\left\{c_k(m',y')^{-\gamma}|y\right\}^{-\frac{1}{\gamma}}, \\
p^{\text{unc}}(w',y) &= \frac{\beta E\left\{c_k(m',y')^{-\gamma}\cdot[y' + p_k(m',y')] + \phi\lambda_k(m',y')\,p_k(m',y')\,|y\right\}}{c^{\text{unc}}(w',y)^{-\gamma}}, \\
\lambda^{\text{unc}}(w',y) &= 0, \\
m^{\text{unc}}(w',y) &= c^{\text{unc}}(w',y) + \frac{w'}{R}.
\end{aligned}$$

By the same token, we can solve for the constrained branch of the system for each non-negative $w' \in \mathbf{w}^g$ s.t. $w' \leq 0$ and $y \in \mathbf{y}^g$ under the assumption that the borrowing constraint is binding in the current period as

$$\begin{aligned}
p^{\text{con}}(w',y) &= \frac{-w'}{\phi R}, \\
c^{\text{con}}(w',y) &= \left[\frac{\beta E\left\{c_k(m',y')^{-\gamma}\cdot[y' + p_k(m',y')] + \phi\lambda_k(m',y')\,p_k(m',y')\,|y\right\}}{p^{\text{con}}(w',y)}\right]^{-\frac{1}{\gamma}}, \\
\lambda^{\text{con}}(w',y) &= c^{\text{con}}(w',y)^{-\gamma} - \beta RE\left[c_k(m',y')^{-\gamma}|y\right], \\
m^{\text{con}}(w',y) &= c^{\text{con}}(w',y) + \frac{w'}{R}.
\end{aligned}$$

We determine for each level of $y \in \mathbf{y}^g$ the next period wealth threshold $\overline{w}(y)$ s.t. the borrowing constraint in the unconstrained system being just marginally binding, i.e., such that

$$\frac{-w'}{R} = \phi p^{\text{unc}}(w',y).$$

This is the lowest possible wealth level that the economy can support for a given level of y. By construction of this threshold, $c^{\text{unc}}(\overline{w}(y),y) = c^{\text{con}}(\overline{w}(y),y)$ for consumption as well as for the other policy variables. This threshold debt level corresponds to a beginning-of-period liquid net wealth $\overline{m}(y) = m^{\text{unc}}(\overline{w}(y),y) = m^{\text{con}}(\overline{w}(y),y)$. The lowest possible level of m is $\underline{m}(y) = m^{\text{con}}(0,y)$. We can construct for each y the step $k+1$ policy function $c_{k+1}(m,y)$ for the interval $\underline{m}(y) \leq m < \overline{m}(y)$ by interpolating on the pairs $\{(c^{\text{con}}(w',y), m^{\text{con}}(w',y))\}_{y\in\mathbf{y}^g, w'\in\mathbf{d}^g}$ where $0 \leq w' < \overline{w}(y)$, and then for the interval $m \geq \overline{m}(y)$ by interpolating on the pairs $\{(c^{\text{unc}}(w',y), m^{\text{unc}}(w',y))\}_{y\in\mathbf{y}^g, w'\in\mathbf{w}^g, w'\leq\overline{w}(y)}$ for $w' < \overline{w}(y)$. The resulting consumption function $c_{k+1}(m,y)$ is again monotonically increasing in

m. We proceed in the same manner for the policy functions $p_{k+1}(m, y)$ and $\lambda_{k+1}(m, y)$, which are, respectively, monotonically increasing and decreasing in m for a given y. The iteration process is continued until the distance between two successive functions $c_k(m, y)$ and $c_{k+1}(m, y)$ (or other policy functions) is sufficiently small.