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Macroeconomic Dimensions of Public-Private Partnerships

by Edward F. Buffie, Michele Andreolli, Bin Grace Li and Luis-Felipe Zanna

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Macroeconomic Dimensions of Public-Private Partnerships

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Abstract

The voluminous literature comparing public-private partnerships (P3s) and own-investment (OI) by the public sector is dominated by contributions from microeconomic theory. This paper gives macroeconomics a voice in the debate by investigating the repercussions of P3 vs. OI in a dynamic general equilibrium model featuring private capital accumulation and involuntary unemployment with efficiency wages. Typically P3s cost more but produce higher-quality infrastructure and boast a better on-time completion record than OI; consequently, they are comparatively more effective in reducing underinvestment in private capital, underinvestment in infrastructure, unemployment and poverty. The asymmetric impact on macro externalities raises the social return in the P3 2 - 9 percentage points relative to the social return to OI, depending on whether the externalities operate singly or in combination and on whether P3 enjoys an advantage in speed of construction.

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Table of Contents

Page

Tables

Figures

Appendix

1 Introduction

Public-private partnerships (P3s) to build and operate infrastructure assets are increasingly common in less-developed countries (LDCs). They are also highly controversial. Microeconomic theory and the evidence accumulated in case studies warn that P3s may be much more expensive than traditional procurement. The list of extra expenses is long and usually starts with the higher cost of private finance. P3s assign construction risk to the private partner to the tight complementarity between asset construction and quality of services.[1] But in a world with incomplete markets, the private sector cannot spread risk as widely as the public sector (Vickrey, 1964; Arrow, 1966; Arrow and Lind, 1970). The difference in the capacity to bear risk raises the return paid to the private partner above the interest rate on government debt.[2] In some LDCs, the return also includes compensation for country risk—for the perception that the government is an "unreliable business partner" (Estache et al., 2015).

There are numerous other costs to doing business with the private sector. At the procurement stage, the administrative costs of writing and tendering bids for complicated long-term contracts are non-trivial. High bid costs for private firms, in turn, limit ex ante competition (i.e., competition for the market). This, combined with the difficulty of designing auctions that prevent collusive behavior (Klemperer, 1999), limited information on firm costs, the urgent need to resolve infrastructure bottlenecks, and the illusory appeal of P3s as an off-budget operation, make it all too easy for inexperienced governments to overpay the private partner (IMF, 2007). Moreover, the operations phase is sure to bring additional costs. The complexity of the contracts, the impossibility of enumerating all contingencies in partnerships that last $20-30$ years, cumbersome legal systems that impede quick, efficient resolution of disputes, and the strong incentive for opportunistic behavior that arise after each party has made relationship-specific investments lead inevitably to frequent, costly renegotiations of the original contract. Last but not least, even if the government bargains exceptionally well and minimizes bid, tendering, and renegotiation costs, it cannot avoid the extra expense of monitoring compliance of the private partner; trust without verification is inadvisable in P3s and nuclear disarmament agreements.[3]

To date no study has compared the total cost of P3s and own investment by the public sector. We know from various fragmentary sources, however, that the extra costs associated with P3s add up to

[1] Shortcuts in construction reduce costs but compromise sevice quality. The problem disappears if asset quality is contractible—a condition unlikely to be satisfied.

[2] This is distinct from the effect of differences in default risk on borrowing costs. Private borrowers pay a premium over the risk-free rate to compensate lenders for the put option. Government borrowing does not involve a put option becuase the cost of failed projects falls on taxpayers.

[3] See de Bettignies and Ross (2004) and Dudkin and Valila (2005) for excellent, detailed discussion of these problems.

something substantial.

- Spackman (2002) and HM Treasury (2000) estimate that non-diversifiable risk pushes the interest rate on senior debt in P3s $2-3$ percentage points above the interest rate on government debt; junior debt and equity finance are presumably even more expensive.

- Measured as a percentage of capital costs, transaction costs average 10% for European governments (Dudkin and Valila, 2005) and $2-10\%$ for private firms in Great Britain (House of Commons, 2002-03).

- Monitoring costs in U.S. P3s magnify total costs $3-25\%$ (Torres and Pina, 2001).

- In Portugal, government officials acknowledge that a dearth of competitive bids and overly generous payments to the private partner have been persistent problems (Monteiro, 2005; Sarmento and Renneboog, 2014).

- While renegotiation costs have eluded quantification, the frequency of renegotiations is dismaying: the 35 Portuguese P3s studied by Sarmento and Renneboog (2014) gave birth to 254 contract renegotiations between 1995 and 2012; Chile's widely praised P3 has also suffered from constant renegotiation costs (IMF, 2004).

Certainly more hard data is needed to justify firm conclusions, especially in the case of LDCs. But the evidence that exists paints a disturbing picture. The verdict in Dudkin and Valila (2005) that "high transaction costs are perhaps the worst, and least studied, drawbacks of PPPs" is entirely fair.

Costs are only one-half of the equation. The other half contains everything the private partner brings to the table: superior technical expertise, greater implementation capacity, and fewer agency problems. These advantages translate into shorter construction periods—important in countries plagued by acute bottlenecks in transport, power, telecommunications, irrigation, etc.—and better, more productive infrastructure. The critical issue is whether the gains in speed and efficiency compensate for the higher cost. In the language favored by government bureaucrats, do P3s offer enough "value for money"? More precisely, do P3s offer better value for money than own investment by the public sector?

Compared to the microeconomic literature with its rich, expansive insights into how asymmetric information, moral hazard, uncertainty, risk allocation, imperfect monitoring, and incomplete contracts affect the pros and cons of P3s, the macroeconomic literature is almost mute. Its total contribution to the debate consists of the trivial recommendation that future payments to the private partner be explicitly incorporated into the government's fiscal accounts in order to highlight the point that P3s defer costs but create a greater overall (i.e., present value) fiscal burden than traditional procurement. Echoing the latter concern, the IMF (2007) has called for full integration of P3s into debt sustainability analysis.

This paper gives macroeconomics a greater voice in the conversation about how P3s compare with own investment (OI) by the public sector. We investigate the macroeconomic repercussions of P3 vs. OI in a dynamic general equilibrium macromodel featuring private capital accumulation and a flexible specification of the labor market that nests the cases of full employment with flexible wages and involuntary unemployment with efficiency wages.[4] The unifying theme in our welfare results is that general equilibrium effects shift the comparison in favor of P3. The direct return in the investment program equals the return on infrastructure minus either the return paid to the private partner (a foreign firm) or the interest rate paid on external debt. In partial equilibrium, the direct return picks the winner. In general equilibrium, the welfare calculation is more complicated. Because P3 builds better, higher quality infrastructure than OI, it crowds in private investment more and increases labor demand more. This helps P3 in the welfare comparison when (i) the private time preference rate exceeds the social time preference rate (implying that private investment is suboptimal at the initial equilibrium); (ii) real wage rigidity prevents full employment; or (iii) wage income carries a larger weight in the social welfare function than income of the representative agent. We show specifically that the asymmetric impact on macro externalities raises the social return in the P3 $2-9$ percentage points *relative to* the social return to OI, depending on whether the externalities operate singly or in combination and on whether P3 enjoys an advantage in speed of construction.[5] The ranking of direct returns is not therefore a reliable proxy for the welfare ranking: a P3 that pays a direct return of 2% may be preferable to OI that pays a direct return of 10%.

Macroeconomic effects also strongly condition the impact on the fiscal budget and the prospects for debt sustainability. P3s are considerably more expensive than OI. But their stronger positive effects on the supply of infrastructure services, private capital accumulation, and employment also generate more revenue from taxes and user fees. This can reduce the need for supporting fiscal adjustment to

[4] Formally, the model is a small open economy model where the interest rate on debt is exogenously given. Nevertheless we will still use the terms "general equilibrium" as is common practice in the macro literature.

[5] The output gain from reducing unemployment is clearly an externality. In the case where the social time preference rate is less than the private rate, induced increases in private investment raise social welfare. Since the private agent does not internalize the welfare gain, we call this an externality as well (recognizing that the terminology is slightly loose).

the point where debt sustainability is easier to achieve for P3 despite its higher cost. In some cases, P3 requires less fiscal support than OI at every point in time.

The rest of the paper is organized into six sections. Sections 2 and 3 present the model and solve for the transition path and the steady-state equilibrium. Following this, Sections 4 - 6 compare the effects of P3 and OI on welfare and fiscal sustainability. Section 7 concludes with a discussion of priorities for future research.

2 The Model

The model has three components: a government that builds new infrastructure on its own or by partnering with a foreign firm; the domestic private sector, run by a representative agent who pays efficiency wages and accumulates capital; and the national budget constraint, which requires that capital flows finance the current account deficit each period.

2.1 The Choice: P3 or OI?

Cobb-Douglas production functions convert infrastructure z, capital k, and labor L into output q:[6]

$$q = [a(z - z_o) + z_o]^{\psi} k^{\alpha} (\mathfrak{e}L)^{1-\alpha}, \tag{1}$$

with $a > 1$ for P3 and

$$q = z^{\psi} k^{\alpha} (\mathfrak{e}L)^{1-\alpha}, \tag{1'}$$

for OI. Infrastructure enters the production function as a shift factor, while effective labor input depends on the amount of effort $\mathfrak{e}$ that workers expend. The service flow from the infrastructure project is $a(z - z_o)$, with $a > 1$ measuring the gain in efficiency when z is built by the private partner instead of the government. z_o is the initial stock level of infrastructure

For simplicity, we assume the P3 contract is structured like an annuity. The foreign partner receives $(r_p + \delta)(z - z_o)$ in perpetuity, where $\delta(z - z_o)$ covers depreciation costs and r_p is the return earned on the investment.[7] The government levies a tax h on consumption and a user fee χ on infrastructure

[6] For simplicity in the exposition we omit the time subscript "t."

[7] The P3s in Mexico's PIDIREGAS scheme are structured as annuities (PIDIREGAS is the Spanish acronym for "long-term productive infrastructure projects with deferred impact on the recording of infrastructure.") Typically, however, the government buys out the private partner at some future date, say t_1. In this case, different dynamic systems operate during $(0, t_1)$ and (t_1, ∞). The economy follows a nonconvergent path until time t_1; it then moves on to the convergent saddle path that converges to the new steady state. (Boundary conditions at t_1 link the two systems.) This greatly

services. In the main variant of the model, lump-sum taxes T adjust to satisfy the government budget constraint:

$$T = r_p(z - z_o) + \delta z - \chi \underbrace{[a(z - z_o) + z_o]}_{\text{service flow}} - hc, \tag{2}$$

for P3.

The alternative to P3 is OI. In this case, the investment is financed by borrowing in the world capital market at the interest rate r. Analogous to the annuity-type P3 contract, the government rolls over the debt forever. The counterpart of (2) is[8]

$$T = r(z - z_o) + (\delta - \chi)z - hc, \tag{2'}$$

for OI.

Finally, P3 and OI differ in the speed of construction. The private partner not only builds a better road, it builds the road faster—a big selling point in countries suffering from large infrastructure deficits (Monteiro, 2005; Sarmento, 2010). To capture this, we specify different paths for investment i_z. The law of motion for the stock of infrastructure is

$$\dot{z} = i_z - \delta z, \tag{3}$$

where $\dot{z}$ represents the time derivative of z. In P3, i_z overshoots its new steady-state level $\bar{\imath}_z$ and z increases faster than under OI.[9] To capture this:[10]

$$di_z/dt = s(\bar{\imath}_z - i_z), \tag{4}$$

with $s > 0$ and $i_{z,o^+} > \bar{\imath}_z$, where i_{z,o^+} is the post-jump value of i_z. For OI, i_z is constant at its new steady-state level $\bar{\imath}_z$:

$$i_z = \bar{\imath}_z = \delta \bar{z}. \tag{4'}$$

complicates the algebra. The annuity specification conveys the key insights while keeping the model analytically tractable. We believe our (welfare) results will hold, to a great extent, under other specifications.

[8] The original budget constraint reads $T + \dot{d} = i_z + rd - \chi z - hc$. Since $\dot{d} = \dot{z} = i_z - \delta z$, this simplifies to $T = rd + \delta z - \chi z - hc = r(z - z_o) + (\delta - \chi)z - hc$, assuming $d_o = 0$.

[9] Faster growth of z may reflect either the superior implementation capacity of the private partner or expansion of the investment program when P3 supplements OI for a credit-constrained government.

[10] We represent the time derivative of investment in infrastructure by di_z/dt.

2.2 The Private Sector

The representative agent chooses consumption and effort to maximize

$$U = \int_{o}^{\infty} \left\{ \frac{c^{1-1/\tau}}{1-1/\tau} - [\mathfrak{e} - b_0 - b_1 \ln w - b_2 u]^2 \right\} \mathsf{e}^{-\rho t} dt \tag{5}$$

subject to

$$\dot{k} = q - T - (1+h)c - \chi[a(z - z_o) + z_o] - \delta k, \tag{6}$$

with $a \geq 1$ and where b_0, b_1, and b_2 are constants; labor supply is fixed at unity; $u = 1 - L$ is the unemployment rate; and w, ρ, and τ denote the real wage, the pure time preference rate, and the intertemporal elasticity of substitution. Following standard practice in the efficiency wage literature (e.g., Collard and de la Croix, 2000; Danthine and Kurmann, 2004, 2010), we assume that gratitude for having a job and the net utility loss from effort depends on how well the job pays and on whether jobs are easy or hard to come by (as determined by u).

On an optimal path,

$$\dot{c} = \tau c(q_k - \rho - \delta) \tag{7}$$

and

$$\mathfrak{e} = b_0 + b_1 \ln w + b_2 u. \tag{8}$$

Equation (7) is the standard Euler equation for consumption, while equation (8) says that employees work harder when they are paid more or feel threatened by a high unemployment rate.

Firms recognize the connection between labor productivity (q_L) and the real wage (w). Hence they optimize over L and w. The profit-maximizing choices satisfy, as usual,

$$q_L = w \tag{9}$$

and

$$\frac{\partial \mathfrak{e}/\partial w}{\mathfrak{e}/w} = 1. \tag{10}$$

Equation (8) and the Solow condition in (10) imply, conveniently, that effort is constant in general equilibrium:[11]

$$\mathfrak{e} = b_1. \tag{11}$$

[11] Per the Solow condition, the optimal wage minimizes the cost of labor in efficiency units [i.e., $w = \arg\min w/\mathfrak{e}(w)$].

We set $b_1 = 1$ (without loss of generality), so the *wage curve* defined by (8) and (11) reads

$$\ln w = 1 - b_0 - b_2 \underbrace{(1 - L)}_{u}. \tag{12}$$

Unlike in Phillips Curve models, there is no natural rate of unemployment, just a *curve* relating the equilibrium wage to the unemployment rate.

2.3 The National Budget Constraint

The model is closed by the national budget constraint. Substituting for T in equation (6) produces[12]

$$\dot{k} = q - r_p(z - z_o) - c - \delta(z + k) \tag{13}$$

for P3 and

$$\dot{k} = q - r(z - z_o) - c - \delta(z + k) \tag{13'}$$

for OI.

2.4 The Case for Efficiency Wages

Efficiency wages are rarely seen in development macromodels. This needs to change. Over the past twenty years, empirical studies have amassed abundant, compelling evidence that efficiency wages operate throughout the non-agricultural sector in LDCs. Estimates of the impact of unemployment on real wages confirm the existence of wage curves in the formal and informal sectors in Argentina, Turkey, Colombia, Uruguay, Chile, S. Africa, Cote d'Ivoire, Mexico, and a host of other developing countries (Blanchflower and Oswald, 2005). There is also powerful, if indirect evidence supportive of efficiency wages in the stylized facts documented in microeconomic studies of LDC labor markets. Across the development spectrum, wage and employment data exhibit the same patterns: (i) firm-size wage premiums that start at very small establishment size (5+ employees) and are much larger than in developed countries; (ii) persistent, remarkably stable inter-industry wage differentials; (iii) high correlation of industry wage premiums across occupations; (iv) large wage premia for formal vs. informal sector employment and for informal non-agricultural employment vs. agricultural employ-

[12] Capital inflows and new infrastructure investment cancel out. In the case of OI, for example, $\dot{d} = q - c - i_z - \dot{k} - \delta k - rd$, which simplifies to (13′) after noting that $\dot{d} = i_z - \delta z$ and $d = z - z_o$.

ment; and (v) large cyclical flows into and out of unemployment in both the formal sector and the informal sector. At present, only efficiency wage models can explain all of these stylized facts. We do not have the space here to survey the literature in greater depth or to discuss myrial estimation issues. References and capsule summaries of the results for 47 studies are available, however, at http://mypage.iu.edu/~ebuffie/.

None of this denies the potential relevance of models that postulate full employment or alternative theories of wage rigidity. There is tremendous diversity in the structure of labor markets across and within LDCs. The right model depends therefore on the country and the type of infrastructure project under investigation. In South Africa, where the national unemployment rate (really) is 25%, trade unions are strong, and minimum wage laws are enforced in all regions, all industries, and at firms of all size, equation (12) with b_2 very small is probably a good fit.[13] In the most common case, high unemployment rates are confined to urban areas (see Table 1). The appropriate model then depends on the location of the project. If the project increases the supply of power to a major urban center, b_2 in equation (12) should take a value between 0.5 and 1.5 (the range in empirical estimates);[14] if it expands the irrigation network in an area populated by smallholder farms, full employment should be enforced by increasing b_2 to $100,000$.[15] The wage curve is simple but flexible; it can represent any degree of real wage rigidity.

3 Inputs

Before deriving any welfare results, we need to locate the steady-state equilibrium, delineate the transition paths for P3 and OI, and define the social welfare function. These tasks occupy the next three sections. To obtain exact closed-form solutions, we treat the increase in infrastructure investment as a small, differential change.

[13] But there is variation in wage-setting rules across the country. In the wage curves estimated by Kingdon and Knight (1999), $b_2 = 0$ for homeland regions and -.66 elsewhere (cluster means estimate evaluated at an unemployment rate of 25%).

[14] Estimates of b_2 cluster between 0.05 and 0.15 when the dependent variable is the logarithm of the unemployment rate. Evaluated at an unemployment rate of 10%, this corresponds to $b_2 = 0.5 - 1.5$ in equation (12).

[15] Or maybe not. In the great majority of LDCs, the gap between the marginal product of labor in non-agriculture and agriculture exceeds 70% (Gollin et al., 2013). The prevalence of surplus labor/underemployment in agriculture is a mystery. In some LICs, it appears to reflect insecure land titles (labor in agriculture receives its marginal product plus land rents).

3.1 The Long-Run Solution

Suppose policy makers opt for P3. At the new steady state,

$$q_k = \rho + \delta \tag{14}$$

and

$$c = q - r_p(z - z_o) - \delta(z + k). \tag{15}$$

Equations (9), (12), (14), and (15) can be solved for k, L, w, and c as a function of z. Straightforward algebra delivers

$$\begin{aligned} dc &= \rho dk + (R_p - r_p)dz + wdL, && (16)\\ dw &= b_2 wdL, && (17)\\ dL &= \frac{(R_p + \delta)(1-\alpha)}{wf_1}dz + \frac{(\rho+\delta)(1-\alpha)}{wf_1}dk, && (18) \end{aligned}$$

and

$$dk = \frac{R_p + \delta}{\rho + \delta}\left(\frac{\alpha}{1-\alpha}\right)dz + \frac{k}{L}dL, \tag{19}$$

where

$$f_1 \equiv \alpha + b_2 L = \alpha + b_2(1-u),$$

and $R_p = q_z - \delta$ is the return on infrastructure (net of depreciation).

Figure 1 depicts the solutions for the capital stock and employment in equations (18) and (19). Since k and L are gross complements in production, the **kk** and **LL** schedules slope upward: growth in employment increases the equilibrium capital stock and vice versa.[16]

An increase in the supply of infrastructure acts like a positive productivity shock, shifting **kk** vertically upward and **LL** horizontally to the right. In the long run,

$$dk = \frac{R_p + \delta}{\rho + \delta}\left(\frac{\alpha}{1-\alpha}\right)\underbrace{\frac{1 + b_2(1-u)}{b_2(1-u)}}_{\text{Multiplier}}dz, \tag{20}$$

[16] **LL** is steeper in slope than **kk**: $wf_1/(\rho+\delta)(1-\alpha) > k/L$ reduces to $f_1 > \alpha \Longrightarrow b_2(1-u) > 0$.

and

$$dL = \frac{R_p + \delta}{w f_1}(1-\alpha)\underbrace{\left[1 + \left(\frac{\alpha}{1-\alpha}\right)\frac{1 + b_2(1-u)}{b_2(1-u)}\right]}_{\text{Multiplier}} dz. \tag{21}$$

The corresponding solutions for OI substitute the return R for R_p. Thus

$$dk|_{\mathrm{P3}} - dk|_{\mathrm{OI}} = \frac{R_p - R}{\rho + \delta}\left(\frac{\alpha}{1-\alpha}\right)\frac{1 + b_2(1-u)}{b_2(1-u)} dz, \tag{22}$$

and

$$dL|_{\mathrm{P3}} - dL|_{\mathrm{OI}} = \frac{R_p - R}{w f_1}(1-\alpha)\left[1 + \left(\frac{\alpha}{1-\alpha}\right)\frac{1 + b_2(1-u)}{b_2(1-u)}\right] dz. \tag{23}$$

These solutions underpin many of the results in the paper. Two points merit emphasis. First, the positive direct effects on capital accumulation and employment depend on the return on infrastructure. Given the consensus view that R_p exceeds R, this strengthens the case for P3. Second, feedback effects between changes in the capital stock and employment *greatly* magnify the direct effects. Empirical estimates of b_2 cluster between 0.5 and 1.5. For values of b_2 in this range, the feedback multiplier is on the order of $1.5 - 3$ for k and $2 - 4$ for L. Point **D** in Figure 1 is far north of point **B** and far east of point **C**.

Return now to equation (16). Because the multiplier effects on capital accumulation and employment are so large, partial and general equilibrium analysis often reach different conclusions about whether P3 or OI pays the highest return. Substituting the solutions for k and L into (16) gives

$$\left.\frac{dc}{dz}\right|_{\mathrm{P3}} = R_p - r_p + (R_p + \delta)F, \tag{24}$$

where

$$F \equiv \left[\rho + \frac{(\rho+\delta)(1-\alpha)}{\alpha + b_2(1-u)}\right]\left[\frac{\alpha}{(\rho+\delta)(1-\alpha)}\right]\frac{1 + b_2(1-u)}{b_2(1-u)} + \frac{1-\alpha}{\alpha + b_2(1-u)} > 0.$$

The solution for OI replaces R_p and r_p with R and r:

$$\left.\frac{dc}{dz}\right|_{\mathrm{OI}} = R - r + (R + \delta)F. \tag{25}$$

Therefore

$$\left.\frac{dc}{dz}\right|_{\mathrm{P3}} - \left.\frac{dc}{dz}\right|_{\mathrm{OI}} = \underbrace{R_p - r_p - (R - r)}_{\text{Direct Return Gap}} + (R_p - R)F. \tag{26}$$

In the case of full employment ($b_2 \to \infty$), F becomes $\frac{\rho\alpha}{(\rho+\delta)(1-\alpha)}$ and consequently

$$\left.\frac{dc}{dz}\right|_{\mathrm{P3}} - \left.\frac{dc}{dz}\right|_{\mathrm{OI}} = \underbrace{R_p - r_p - (R-r)}_{\text{Direct Return Gap}} + (R_p - R)\frac{\rho\alpha}{(\rho+\delta)(1-\alpha)}. \tag{27}$$

Figure 2 plots the solutions in equations (26) and (27). The Direct Return Gap (DRG) is $R_p - r_p - (R-r)$.[17] The Total Return Gap (TRG) mirrors the full solution and includes the term $(R_p - R)F$. Everything depends on the size of this term relative to the DRG. To quantify the two competing effects, we computed solutions for

$$\alpha = .30 \text{-} .50,\ \delta = .05,\ r = .06,\ \rho = .10,\ R = .16,\ R_p = .16\ -\ .25,\ r_p = .15, \text{ and } \quad b_2 = .5 \text{-} 1.5, \infty.$$

The income share of capital, the depreciation rate, and the external borrowing rate take ordinary values.[18] For the private time preference rate, we chose the highish value 10% based on the estimates in Isham and Kaufmann (1999) and Dalgaard and Hansen (2005) of the return to private capital in LDCs. Estimates of the return on infrastructure in LDCs are all over the map, but the weight of the evidence in both micro and macro studies points to high average returns of 15 – 30%—see Buffie et al. (2012). Accordingly, we set the return at 16% for OI and 16 – 25% for P3. Finally, r_p is a pricey 15% because it reflects *all* of the costs associated with P3. In addition to the return paid to the private partner—many points above r for reasons discussed in the introduction—it includes annuitized payments for the very high transaction costs incurred in P3s.[19] Unfortunately, the paucity of data on transaction costs and actual payments to the private partner means there is a lot of uncertainty about the right value for r_p. Our choice of 15% is a crude, semi-educated guess. It is also, we admit, a debating tactic. Since R_p tops out at 25%, the DRG ranges from zero to -9%. The case for P3 rests entirely therefore with general equilibrium effects; by assumption, OI enjoys a big advantage in the comparison of direct returns.

Table 2 confirms that general equilibrium effects are potent, especially when increases in employment reinforce the gains from capital accumulation. In the column at the far right, where $R_p = .25$ and both direct returns equal 10%, the consumption return for P3 is 16 – 25% above the return for

[17] Note that what matters is the difference of net returns.

[18] Borrowing at 6% is in line with real interest rate paid by LICs in recent Eurobond issues and with the data presented in Gueye and Sy (2010).

[19] Some transaction costs do not result in higher payments to the private partner. The impact on the fiscal budget and national income is the same, however, if the transaction costs use up real resources.

OI with full employment and $31-39\%$ higher when involuntary unemployment exists. Alternatively, consider the breakeven points in Figure 2. The schedule for full employment breaks into the first quadrant at $\bar{R}_p = .222$. This is neither exciting nor insignificant. But when infrastructure investment reduces unemployment, $\bar{R}_p$ drops to $.179-.196$. Strikingly, in five of nine runs where P3 pays a direct return of only 4%, it increases real income more than OI. Although these are not welfare results, they foreshadow the conclusion that P3 may dominate OI even when the DRG is very large.

3.2 The Transition Path

The transition path is governed by equations (3), (4), (7), and (13), along with equation (18), which links the path of employment to the paths of the capital stock and the stock of infrastructure. Linearizing (7) and (13) yields

$$\dot{c} = \tau c(q_{kk}dk + q_{kz}dz + q_{kL}dL) \tag{28}$$

and

$$\dot{k} = \rho dk + (R_p - r_p)dz + wdL - dc. \tag{29}$$

Substitute for dL from (18) and note that the second derivatives of the production function satisfy

$$q_{kk} = \frac{\alpha(\alpha-1)q}{k^2} = \frac{\alpha-1}{k}(\rho+\delta), \qquad q_{kz} = \frac{\alpha\psi aq}{kz} = \frac{\alpha}{k}(R_p+\delta), \qquad \text{and} \qquad q_{kL} = -q_{kk}\frac{k}{L}.$$

After collecting terms,

$$\dot{c} = \tau(c/k)(\rho+\delta)(\alpha-1)f_2(k-\bar{k}) + \tau(c/k)\alpha(R_p+\delta)f_3(z-\bar{z}) \tag{30}$$

and

$$\dot{k} = f_4(k-\bar{k}) + f_5(z-\bar{z}) - (c-\bar{c}), \tag{31}$$

where

$$f_2 \equiv 1 - \frac{\alpha}{f_1} > 0, \qquad f_3 \equiv 1 + \frac{1-\alpha}{f_1} > 0,$$

$$f_4 \equiv \rho + \frac{(\rho+\delta)(1-\alpha)}{f_1} > 0, \quad \text{and} \quad f_5 \equiv R_p - r_p + \frac{(R_p+\delta)(1-\alpha)}{f_1} > 0.$$

Until Section 4.3, we ignore differences in implementation and the speed of construction. Infrastructure investment is constant at $\bar{\imath}_z$ and

$$\dot{z} = -\delta(z - \bar{z}) \tag{32}$$

for both P3 and OI.

Equations (30)-(32) form a self-contained system of three differential equations in c, k, and z:

$$\begin{bmatrix} \dot{c} \\ \dot{k} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} 0 & \tau(c/k)(\rho+\delta)(\alpha-1)f_2 & \tau(c/k)\alpha(R_p+\delta)f_3 \\ -1 & f_4 & f_5 \\ 0 & 0 & -\delta \end{bmatrix} \begin{bmatrix} c-\bar{c} \\ k-\bar{k} \\ z-\bar{z} \end{bmatrix}. \tag{33}$$

The stationary equilibrium is a saddle point with two state variables, k and z, and two *negative* eigenvalues:

$$\lambda_1 = -\delta \qquad \text{and} \qquad \lambda_2 = \frac{f_4 - \sqrt{f_4^2 + 4\tau(1-\alpha)(\rho+\delta)(c/k)f_2}}{2}.$$

3.3 The Welfare Criterion

We measure social welfare as[20]

$$SW = \int_o^\infty \frac{c^{1-1/\tau}}{1-1/\tau}\mathrm{e}^{-\rho^* t}dt, \qquad \text{with} \qquad \rho^* \leq \rho. \tag{34}$$

SW allows the benevolent social planner to set ρ^* below the private time preference rate ρ. Should (s)he?

The position of policy makers is clear. In both developed and less developed countries, the social time preference rate used to calculate the cost-benefit ratio for public sector projects is usually much lower than the private rate. HM Treasury (2003) recommends, for example, $\rho^* = 2 - 3.5\%$.

Theory cannot tell us whether 3% is a sensible number for ρ^*, the view of HM Treasury notwithstanding. It does, however, provide cogent arguments for $\rho^* < \rho$. In Sen's (1967) isolation paradox, private saving is suboptimal because individuals would be willing to enter into a social contract that required everyone to save more. Feldstein (1964) and Baumol (1965) reach the same conclusion more quickly by appealing to the notion that economic development is partly a public good; if the premise is granted, then the social time preference rate "must be administratively determined as a matter of public policy [because] the market cannot express the 'collective' demand for investment to benefit

[20] The effort component in the private agent's utility function always equals zero in general equilibrium.

the future" (Feldstein, 1964, pp. 362, 365).

4 Welfare Comparisons

All of the machinery is now in place to rank the welfare gains for P3 vs. OI. We start with the simplest case.

4.1 Full Employment

We derive the solutions for c and SW in Appendix A. The welfare gain, measured in units of consumption, is

$$\left.\frac{SW - SW_o}{c^{-1/\tau}(\bar{z} - z_o)}\right|_{P3} = \int_o^{\infty} (c - c_o)\mathsf{e}^{-\rho^* t} dt = \left(\frac{R_p - r_p}{\rho + \delta}\right)\frac{\delta}{\rho^*}(1 + H_p), \tag{35}$$

for P3, where $\rho^* \leq \rho$ and

$$H_p \equiv \frac{\rho - \rho^*}{\rho^* + \delta}\Lambda + \frac{(R_p + \delta)\alpha(\rho - \rho^*)}{(\rho^* - \lambda_2)(1 - \alpha)(\rho + \delta)}\Gamma,$$

$$\Lambda \equiv 1 + \frac{\rho^*(\lambda_2 + \delta)}{J(\rho^* - \lambda_2)} \quad \text{with} \quad \Lambda > 0,$$

$$\Gamma \equiv \frac{\tau(c/k)(1 - \alpha)(\lambda_2 + \delta)}{J} - \lambda_2\frac{\rho^* + \delta}{\rho^*} \quad \text{with} \quad \Gamma > 0,$$

and

$$J \equiv \tau(c/k)(1 - \alpha) - \delta.$$

Similarly,

$$\left.\frac{SW - SW_o}{c^{-1/\tau}(z^* - z_o)}\right|_{OI} = \left(\frac{R - r}{\rho + \delta}\right)\frac{\delta}{\rho^*}(1 + H), \tag{35$'$}$$

with

$$H \equiv \frac{\rho - \rho^*}{\rho^* + \delta}\Lambda + \frac{(R + \delta)\alpha(\rho - \rho^*)}{(\rho^* - \lambda_2)(1 - \alpha)(\rho + \delta)}\Gamma$$

for OI. It can be shown with some effort that the terms involving $(\rho - \rho^*)$ in H_p and H are positive (see Appendix A).

The welfare solutions look forbidding, but they have the expected form and agree with intuition. When $\rho^* = \rho$, then $H_p = H = 0$. In this case, the impact on private capital accumulation is irrelevant.

The welfare gain is simply the capitalized value of the stream of returns generated by increases in the supply of infrastructure:[21]

$$\left.\frac{SW - SW_o}{c^{-1/\tau}}\right|_{P3,\,\rho^*=\rho} = \int_o^\infty (R_p - r_p)(z - z_o)\mathsf{e}^{-\rho t}dt = \left(\frac{R_p - r_p}{\rho + \delta}\right)\frac{\delta}{\rho}(\bar{z} - z_o) \tag{36}$$

and

$$\left.\frac{SW - SW_o}{c^{-1/\tau}}\right|_{OI,\,\rho^*=\rho} = \int_o^\infty (R - r)(z - z_o)\mathsf{e}^{-\rho t}dt = \left(\frac{R - r}{\rho + \delta}\right)\frac{\delta}{\rho}(\bar{z} - z_o). \tag{36$'$}$$

Discounting at $\rho^* < \rho$ changes the welfare calculation in two ways. First, the lower discount rate increases the present value welfare gain from increases in the stock of infrastructure. Second, since the private capital stock is below its socially optimal level, crowding in private investment generates an additional welfare gain. The impact of this gain on the welfare comparison depends on R_p vs. R as the increase in the service flow determines the stimulus to private investment. Thus P3 may increase welfare more than OI despite a lower direct return. From (35) and (35$'$),

$$SW|_{P3} > SW|_{OI}$$

if and only if

$$\underbrace{\frac{(R_p - R)\alpha(\rho - \rho^*)}{(\rho^* - \lambda_2)(1-\alpha)(\rho+\delta)G}\Gamma}_{\text{LHS}} > \underbrace{R - r - (R_p - r_p)}_{\text{Direct Return Gap (DRG)}}, \tag{37}$$

where

$$G \equiv \frac{\delta}{\rho^*(\rho+\delta)}\left(1 + \frac{\rho - \rho^*}{\rho^* + \delta}\Lambda\right) \quad \text{with} \quad G > 0 \quad \text{if } \rho^* < \rho.$$

The term on the left side (LHS) of (37) is small but significant. When $\rho^* = .02 - .06$, it lowers the breakeven value of R_p in Table 3 from .25 to $.228 - .239$. P3 wins close races where the DRG is $1 - 2$ points.

4.2 Unemployment

There is no need to repeat the algebraic derivations in the previous section. Obviously, the term multiplying $R_p - R$ in equation (37) is much larger and the case for P3 much stronger when infrastructure investment reduces unemployment. P3 produces an additional welfare gain of $27 - 34\%$ in the runs where direct returns are equal ($R_p = .25$ in Table 4), and the breakeven value of R_p

[21] Note here that $z - z_o = (1 - \mathsf{e}^{-\delta t})(\bar{z} - z_o)$.

decreases to $.17 - .22$ with ρ^* between .04 and .1 (Figure 3). Policy makers can rest assured that P3 is the right choice when the DRG is three points or less. In economies with highly rigid wages ($b_2 = .1 - .5$), it may be the right choice even when the direct return is $7 - 8$ points lower than for OI.

4.3 The Importance of Speed

When P3 builds infrastructure faster than OI, i_z enters the core dynamic system as a state variable. The new system is

$$\begin{bmatrix} \dot{c} \\ \dot{k} \\ \dot{z} \\ di_z/dt \end{bmatrix} = \begin{bmatrix} 0 & \tau(c/k)(\rho+\delta)(\alpha-1)f_2 & \tau(c/k)\alpha(R_p+\delta)f_3 & 0 \\ -1 & f_4 & f_5 & 0 \\ 0 & 0 & -\delta & 1 \\ 0 & 0 & 0 & -s \end{bmatrix} \begin{bmatrix} c-\bar{c} \\ k-\bar{k} \\ z-\bar{z} \\ i_z-\bar{\imath}_z \end{bmatrix}. \tag{38}$$

Let $n \equiv [i_{z,o^+} - i_{z,o}]/(\bar{\imath}_z - i_{z,o})$, where i_{z,o^+} is the post-jump value of i_z at $t = o$. n and s jointly determine the path of i_z and thus the path of z:[22]

$$z - z_o = \left[1 - \mathrm{e}^{-\delta t} + (n-1)\delta\frac{\mathrm{e}^{-st} - \mathrm{e}^{-\delta t}}{\delta - s}\right](\bar{z} - z_o).$$

When the government invests on its own, $n = 1$ and z traverses 50% of ground to its new steady-state level in $t_1 = -\ln(.50)/\delta$ years. In the comparison run for P3, we set $s = .20$ and chose n so that z reaches the 50% benchmark in 25% less time.[23] Figure 4 shows the two paths for z. In an infinite horizon model where the choice between P3 and OI is repeated in a sequence of projects, the gap between the paths widens for 15 years before starting to shrink.

Infrastructure deficits are large in low-income countries (LICs), middle-income countries (MICs) and most emerging economies (EMEs). Consequently, speed matters. It matters a lot in the full-employment economy, the case least favorable to P3. It matters even more in countries with unemployed workers because shorter construction periods for infrastructure increase labor demand directly ($q_{Lz} > 0$)and via faster growth in the private capital stock. Time is money, as they say in the business world.

The numbers in Tables 5 and 6 are surprising, nevertheless. Compare these tables with Tables 3 and 4. When direct returns are equal, the welfare-gain-advantage of P3 soars from $0 - 34\%$ in Tables

[22] The solution paths for c and k are presented in Appendix B.
[23] For $s = .20$, $n = 1.6$ solves $z(.75t_1) - z_o = .5(\bar{z} - z_o)$.

3 and 4 to $35 - 147\%$ in Tables 5 and 6. The breakeven value of R_p decreases to $.186 - .212$ in the full-employment case and to $.119 - .180$ in the runs with efficiency wages and $b_2 = .5 - 1.5$. It should be emphasized in connection with this last result that the distinction between perfect and extremely high wage flexibility is important. A value of 1.5 for b_2 is at the upper end of empirical estimates. Yet, in runs where $b_2 = 1.5 - 4$, $\rho^* \geq .04$, and the DRG is $7 - 8$ percentage points ($R_p = .17 - .18$), P3 beats OI $62 - 88\%$ of the time. This simplifies life for some policy makers. Arguably, there is no need for a public sector comparator or complex general equilibrium analysis in countries where involuntary unemployment exists: if P3 passes the benefit-cost test by a small margin, based on its direct return only, then it is virtually certain to dominate OI.[24]

5 The Fiscal Challenge

Lump-sum taxes adjust to pay the private partner in P3 and foreign bond holders in OI. Since $r_p >> r$, it seems obvious that the fiscal challenge is greater for P3 than for OI. This is not necessarily the case, however. P3s are much more expensive than OI, but they also increase the supply of infrastructure services and real output more than OI, generating more revenue at existing tax rates. Let Y denote net national income—e.g., $Y = q - r_p(z - z_o) - \delta(k + z)$ for P3—and define $\bar{\chi} \equiv \chi/(R+\delta)$, the ratio of the user fee to the monetary value of infrastructure services produced by OI. Solving the government budget constraints (2) and (2′) for $v \equiv T/Y$ then gives

$$(v - v_o)|_{P3} = [r_p + \delta - (R_p + \delta)\Omega - v(R_p - r_p)]\,\frac{z - z_o}{Y} - h\frac{c - c_o}{Y} - v\Phi\frac{k - k_o}{Y} \tag{39}$$

and

$$(v - v_o)|_{OI} = [r + \delta - (R + \delta)\Omega - v(R - r)]\,\frac{z - z_o}{Y} - \frac{h}{Y}(c - c_o) - v\Phi\frac{k - k_o}{Y}, \tag{39′}$$

where

$$\Omega \equiv \bar{\chi} + v(1 - \alpha)/f_1 > 0 \qquad \text{and} \qquad \Phi \equiv \rho + (\rho + \delta)(1 - \alpha)/f_1 > 0.$$

[24] A reminder, however: the social time preference rate cannot be too low. The gains from faster construction decrease rapidly with ρ^*. In the limiting case $\rho^* = 0$, speed counts for nothing; we return to Table 2 to pick the winner.

When the speed of construction is the same for P3 and OI,

$$
\begin{aligned}
(v - v_o)|_{P3} - (v - v_o)|_{OI} \quad = \quad & (r_p - r)(1+v)\frac{z - z_o}{Y} - (R_p - R)(\Omega + v)\frac{z - z_o}{Y} \\
& -h\left(\left.\frac{c - c_o}{Y}\right|_{P3} - \left.\frac{c - c_o}{Y}\right|_{OI} \right) - v\Phi \left(\left.\frac{k - k_o}{Y}\right|_{P3} - \left.\frac{k - k_o}{Y}\right|_{OI} \right).
\end{aligned} \tag{40}
$$

The existing literature worries, rightly, that the first term is positive and large. But the difference in endogenous revenue—the sum of the last three terms—carries the opposite sign and is also potentially large.

We track $CFS \equiv \frac{(v - v_o)}{(\bar{\imath}_z - i_{z,o})/Y}$, the ratio of the increase in v to to the increase in infrastructure investment measured as a percentage of National Net Income (NNI). Translating into plain English, CFS is the "coefficient of fiscal stress." $CFS(t_1) = .75$ means, for example, that the government has to collect an additional .75% of NNI in lump-sum taxes at time t_1 to finance a permanent increase in infrastructure investment equal to 1% of NNI.[25]

Figure 5 shows the path of v for a variety of cases in which $r_p = .10 - .15$ and P3 ties or wins the welfare comparison.[26] The diversity of outcomes is extreme.[27] Sometimes the P3 line is always above the OI line, sometimes it is always below; sometimes the two lines cross; sometimes the lines stay in the first quadrant, sometimes they end up in the fourth quadrant. Clearly, there is no general presumption that P3 creates more fiscal stress than OI. Much depends on the productivity differential $(R_p - R)$, the structure of taxes, and the labor market:

- When $R_p = .20$, $r_p = .10$, (i.e., the $DRG = 0$) and user fees cover only depreciation costs ($\bar{\chi} = .25$), the gap between the P3 and OI lines is positive and large for several decades (Figures 5a and 5e). But the gap shrinks considerably when $R_p = .25$, and in the model with unemployment the sign changes to the advantage of P3 at year 13 (Figures 5b and 5f).

- Because P3 builds better, more productive infrastructure than OI, modest user fees that cover O+M costs plus $30 - 50\%$ of interest/return payments make a big difference to the fiscal outcome. In the runs where $R_p = .25$, $r_p = .10$, and $\bar{\chi} = .40$, P3 pays for itself immediately or within a few

[25] In the more common arrangement where the government eventually takes ownership of the infrastructure asset, net revenue gains must be invested in anticipation of the future buyout of the private partner. The present-value impact on the fiscal budget is the same as with the annuity contract, but if the government lacks discipline the large outlay at the buyout date may provoke a fiscal crisis.

[26] We are interested in whether the fiscal challenge is greater for P3 when it maximizes welfare. In a wide search of the relevant parameter space, we did not come across any cases where OI maximized welfare but P3 minimized fiscal stress.

[27] v decreases a small amount at $t = 0$ because consumption rises in anticipation of an increase in future income.

years (Figures 5d and 5g) and delivers a much larger fiscal dividend than OI. The comparison still favors OI when $R_p = .20$ (Figures 5c and 5h), but the choice in Figure 5h is between good and better—fiscal adjustment is easy and stress-free for both P3 and OI.

The import of these results is that policy makers should not pre-judge how fiscal concerns affect the case for P3 vs. OI. P3 wins the welfare comparison decisively when $R_p = .25$ or when $R_p = .20$ and involuntary unemployment exists. In half of the runs, it is also the choice that minimizes fiscal stress, *provided* a little help is available from user fees. Indeed, Figure 5 suggests that welfare-maximizing P3s may *ease* fiscal constraints and *improve* the prospects for debt sustainability. The Ministry of Finance and the Ministry of Public Works should agree as often as not.

6 Extensions

The benchmark model assumes that lump-sum taxes adjust in the government budget constraint and that social welfare depends solely on consumption of the representative agent. We relax both assumptions below. The first proves innocuous, but the second is potentially important.

6.1 Distortionary Taxes Adjust

Distortionary taxes may bear part or all of the cost of financing public investment. To investigate this scenario, treat c as a CES aggregate of two consumer goods and assume that only c_2 is subject to the tax h. The exact consumer price index is

$$P = [\kappa(P_1^*)^{1-\epsilon} + (1-\kappa)P_2^{1-\epsilon}]^{1/(1-\epsilon)}, \tag{41}$$

where

$$P_2 = P_2^*(1+h), \tag{42}$$

P_i^* is the world market price of good i; κ is a CES distribution parameter; and ϵ is the elasticity of substitution between c_1 and c_2. We choose units so that $P_1^* = 1$ and $P_2^* = 1/(1+h_o)$. Thus $P_1 = P_2 = P = 1$ at the initial equilibrium.

The public, private, and national budget constraints are

$$hP_2^*c_2 = r_p(z - z_o) + \delta z - \chi[a(z - z_o) + z_o] - T, \tag{43}$$

$$\dot{k} = q - T - Pc - \delta k - \chi[a(z - z_o) + z_o], \tag{44}$$

and

$$\dot{k} = q - r_p(z - z_o) - Pc - \delta(z + k) + hP_2^* c_2. \tag{45}$$

Demand for good 2 is

$$c_2 = (1 - \kappa) \left(\frac{P_2}{P} \right)^{-\epsilon} c \tag{46}$$

and h adjusts endogenously in (43) to satisfy the government budget constraint.

Efficiency gains/losses from the tax distortion show up in the term $hP_2^* c_2$ in the national budget constraint. Digging into the details, linearize equation (45) and solve (43) and (46) for c_2 and h. This yields

$$\dot{k} = f_4 dk + f_5 dz - dc + hP_2^* dc_2, \tag{47}$$

$$\frac{dc_2}{c_2} = \frac{dc}{c} - \epsilon(1 - \gamma)\frac{dh}{1 + h}, \tag{48}$$

and

$$\frac{dh}{1 + h} = \frac{r_p + \delta - \bar{\chi}(R_p + \delta)}{g_0} dz - \frac{h}{cg_0} dc, \tag{49}$$

where

$$g_0 \equiv 1 + h - \epsilon h(1 - \gamma),$$

differentials refer to deviations from the steady state and $\gamma \equiv P_2 c_2 / Pc$, the consumption share of good 2.

Consistent with the results in Section 5, the impact on the fiscal budget and the consumption tax can go either way. Regardless of whether h rises or falls, however, the welfare effects are merely "little triangles." Consider the solution for steady-state consumption. From (47)-(49), and the solution for dk in (20),

$$\begin{aligned} \left.\frac{dc}{dz}\right|_{P3} &= (1 - h\gamma/g_0)^{-1} \left\{ \left[\rho + \frac{(\rho + \delta)(1 - \alpha)}{f_1} \right] a_2 + R_p - r_p \right. \\ &\quad \left. + \frac{(R_p + \delta)(1 - \alpha)}{f_1} - \frac{h\epsilon(1 - \gamma)}{g_0} [r_p + \delta - \bar{\chi}(R_p + \delta)] \right\}. \end{aligned} \tag{50}$$

where

$$a_2 \equiv \frac{R_p+\delta}{\rho+\delta}\left(\frac{\alpha}{1-\alpha}\right)\frac{1+b_2(1-u)}{b_2(1-u)}.$$

The term $r_p + \delta - \bar{\chi}(R_p+\delta)$ picks up the direct effect on the budget—see (49). It is negative for plausible values of r_p, R_p, and $\bar{\chi}$. But since ϵ is small at high levels of aggregation—a universal finding in estimates of demand systems with 5-10 goods—the coefficient on the direct effect, $h\epsilon(1-\gamma)/g_0$, is *extremely* small. Moreover, the small negative term in the numerator is counterbalanced by a multiplier slightly larger than unity (reflecting the gain from increases in c that translate into increases in c_2). Consumption may increase more or less than in the case where lump-sum taxes adjust, but the difference is pocket change. And the impact on the P3-OI ranking, which depends on the size difference in two little triangles, is even smaller. The solution for OI substitutes R and r for R_p and r_p in (50). Hence

$$\left.\frac{dc}{dz}\right|_{P3} - \left.\frac{dc}{dz}\right|_{OI} > 0$$

if and only if

$$\underbrace{R_p - r_p - (R-r) + (R_p - R)F}_{\text{Solution in the Lump-Sum Tax Case in (26)}} + \frac{h\epsilon(1-\gamma)}{g_0}[\bar{\chi}(R_p-R) - (r_p - r)] > 0. \qquad (51)$$

Barring an exceptionally large productivity gap, $(R_p - R)/(r_p - r) < 1/\bar{\chi}$ and the comparison of consumption gains is more favorable to OI than before. To repeat, however, the little triangles are truly little. The term involving h functions essentially as a tiebreaker: for $R_p = .23$, $R = .16$, $r_p = .15$, $r = .06$, $\delta = .05$, $\epsilon = \gamma = .5$, $b_2 = 1$, $\alpha = .40$, $\bar{\chi} = .25$, and $h = .15$, it equals all of $-.00261$ vs. 123 for the component that mirrors lump-sum tax solution.

At the risk of beating a dead horse, we went to the trouble of deriving the transition path (see Appendix C) and computing exact welfare comparisons for lump-sum vs. distortionary tax adjustment. Unsurprisingly, the numbers in Table 7 are always very close. In many cases, the difference amounts to rounding error.

6.2 Incorporating Distributional Concerns

Policy makers obsess about creating more and better-paying jobs. Short of building a model with heterogeneous agents, the most natural way to incorporate the gains from more/better jobs is to enter real wage income in the social welfare function. In this section,

$$\frac{SW - SW_o}{c^{-1/\tau}} = \int_o^\infty [c - c_o + \mathfrak{f}(wL - w_o L_o)]\mathsf{e}^{-\rho^* t} dt. \tag{52}$$

In principle, $\mathfrak{f}$ is the answer to the problem: when real wage income decreases by one dollar, social welfare is unchanged if aggregate consumption increases by ____ dollars. Although policy makers will not reveal their answer, Benthamite utilitarian calculations shed some light on reasonable values for $\mathfrak{f}$. In out model, a utilitarian welfare function implies $\mathfrak{f} = .47 - 2.05$ when $\tau = .35 - 1$ and other parameters take their usual values.[28]

Our earlier results predict the qualitative and quantitative changes in the welfare arithmetic reported in Tables 8 and 9. Naturally, since P3 increases labor demand real wages more than OI, it benefits much more from the distributional externality. For $\rho^* = .06$ and values of $\mathfrak{f}$ that we consider (too) conservative (.5) or reasonable $(1, 1.5)$, the breakeven value of R_p drops from .202 to $.169 - .185$ in the runs with efficiency wages and from .239 to $.189 - .214$ for projects in regions with full employment. Distributional concerns increase the social return for both P3 and OI, but the gain for P3 is much larger, especially in the full-employment case.

7 Concluding Remarks

There is a clean division of labor between microeconomic and macroeconomic analysis of P3s. The job of microeconomics is to secure the best possible deal for the LDC by structuring the P3 contract so as to maximize the incentives for delivery of high-quality services while minimizing transaction costs

[28] Suppose there are $1-n$ capitalists and n workers. The Benthamite social welfare function is $U = (1-n)y_k^{1-1/\tau}/(1-1/\tau) + n y_w^{1-1/\tau}/(1-1/\tau)$, where y_k and y_w denote income of the represenative capitalist and income of the representative worker. Then

$$\begin{aligned} \frac{du}{y_k^{-1/\tau}} &= (1-n)dy_k + \left(\frac{y_k}{y_w}\right)^{1/\tau} n dy_w, \\ \Longrightarrow \frac{du}{y_k^{-1/\tau}} &= dy + \left[\left(\frac{y_k}{y_w}\right)^{1/\tau} - 1\right] dwL, \end{aligned}$$

as $y = (1-n)y_k + n y_w$ and $n dy_w = dwL$. Assume taxes claim the same share v of capitalists' and workers' income. The private budget constraints then imply $dc = (1-v)dy$. Thus

$$(1-v)\frac{du}{y_k^{-1/\tau}} = dy + \left[\left(\frac{y_k}{y_w}\right)^{1/\tau} - 1\right](1-v)dwL,$$

where v is set by the government budget constraint. $[v(q - \delta k) = i_z \Longrightarrow v = (i_z/q)/[1 - \delta\alpha/(\rho+\delta)].]$ $\mathfrak{f}$ is the coefficient $[(y_k/y_w)^{1/\tau} - 1](1-v)$. For $y_k/y_w = 1.5$, $\delta = i_z/q = .05$, $\alpha = .40$, $\rho = .10$, and $\tau = .35 -\ 1$, $\mathfrak{f}$ ranges from .47 to 2.05.

for the government and rents paid to the private partner. In the existing literature, the story ends here. The final step in the evaluation process compares the net return in the optimal P3 contract, calculated as simply the return on infrastructure net of total costs for the government, to the net return on OI by the public sector.

But the partial equilibrium comparison of net returns should not be the final step. In a world that is irredeemably second-best, general equilibrium macroeconomic analysis is required for an accurate comparison of social returns to P3 and OI. Typically P3s cost more but produce higher-quality infrastructure and boast a better on-time completion record than OI. Because they score better on efficiency and speed, P3s are comparatively more effective in reducing underinvestment in private capital, underinvestment in infrastructure, unemployment and poverty. How much should policy makers be willing to pay for these benefits? Our two-word answer is "a lot." The welfare gains from ameliorating major macroeconomic externalities raise the social return in P3 by $5-8$ percentage points *relative to* the return on OI.

The current paper is part of a larger project the IMF has initiated to improve the analysis of P3s in LDCs. Future research in the project will focus on construction of policy tools that are theory-based but flexible and user-friendly. We plan specifically to integrate P3s into the DIG (Debt, Investment, and Growth) model developed by Buffie et al. (2012). The P3-DIG model promises to cover all the bases: policy makers will be able to specify the exact terms in annuity or DBOT (design, build, operate, and transfer) contracts, to rank the welfare gains for P3 vs. OI, and to track the paths of key macroeconomic variables (sectoral wages, employment, and output, private investment, the real exchange rate, external debt and the current account deficit, etc.).

Table 1: Unemployment rates in LDCs

Country	Year	National	Rural	Urban
Algeria	2013	9.8	8.1	10.6
Bahamas	2013	15.8	-	15.8[a]
Bangladesh	2010	4.5	4.0	6.5
Belize	2013	11.7[d]	11.8	11.5
Botswana	2010	17.9	-	9.8[b]
Brazil	2009	8.4[d]	3.4	9.3
Cameroon	2010	3.8	1.4	8.1
Colombia	2014	9.1	5.7	10.0
Costa Rica	2014Q2	8.6[d]	8.8	8.5
Cyprus	2014	16.1	17.1	15.7
Cote d'Ivoire	2015	6.9[e]	3.0	13.4, 7.7[c]
Dominican Rep.	2013	15.0[d]	14.2	15.4
Egypt	2013	13.2	10.7	16.5
Gabon	2010	20.4	19.0	21.0
Gambia	2012	29.8	31.1	28.4
Guyana	2002	-	-	11.8[b]
India	2014	4.9[d]	5.5	4.7
Indonesia	2012	6.1[f]	4.7	7.7
Iran	2013	10.4	7.0	11.8
Jamaica	2013	15.2	-	-
Jordon	2012	12.2	14.2	11.8
Kyrgyzstan	2012	8.4[f]	7.9	9.5
Malawi	2013	6.6	6.0	11.5
Malaysia	2014	2.9	3.2	3.1
Mauritania	2012	10.1	-	-
Mauritius	2014	7.8	-	-
Mexico	2014	4.8	2.8	5.3
Morocco	2014	9.7	4.2	15.0
Mozambique	2012	22.6[d]	-	-
Namibia	2014	28.1[e]	30.2	26.2
Nigeria	2015Q3	9.9[d,f]	9.0	12.1
Philippines	2014	6.8	-	9.3[b]
Senegal	2015	13.4[g]	10.1	17.2
South Africa	2012	24.7	28.0	24.3
Tajikistan	2009	11.5	9.6	16.8
Thailand	2015	.9	1.0	.9
Tunisia	2012Q4	16.7[f]	-	20.4[b]
Turkey	2013	9.7	6.1	11.5
Uruguay	2014	6.6	4.7	6.7, 7.0[c]
Zambia	2012	7.8	3.3	14.2

Sources: International Labor Office, World Economic Outlook, World Bank, CIA World Factbook, and National Statistical Offices (Labor Force Surveys, Censuses, Poverty Surveys, AIDS Surveys, and Household Surveys). Unless otherwise noted, figures for the national unemployment rate are based on two sources that agree to within one percentage point.

[a] Unemployment rate for New Providence Island (where the capital city is located).

[b] Unemployment rate for the capital city.

[c] Unemployment rates in the capital city and other urban centers, respectively.

[d] Disagreement between two or more sources.

[e] Only one source (Labor Force Survey for Cote d'Ivoire and Namibia; Population Census for Guyana).

[f] Data cross-checked for a different year.

[g] Unemployment rate changed more than 10% across two years in the Labor Force Survey and the Census.

Table 2: Percentage difference in the increase in steady-state consumption for P3 vs. OI.[1]

α = .50										
b_2 \ R_p	.16	.17	.18	.19	.20	.21	.22	.23	.24	.25
.5	-8.8	-3.6	1.7	7.0	12.3	17.5	22.8	28.1	33.4	38.6
1	-14.3	-8.7	-3.1	2.5	8.1	13.7	19.3	24.9	30.5	36.0
1.5	-18.0	-12.2	-6.4	-.6	5.2	11.0	16.8	22.7	28.5	34.3
FE[2]	-37.5	-30.6	-23.6	-23.6	-9.7	-2.8	4.2	11.1	18.1	25.0

α = .40										
b_2 \ R_p	.16	.17	.18	.19	.20	.21	.22	.23	.24	.25
.5	-10.4	-5.0	.4	5.7	11.1	16.5	21.8	27.2	32.6	37.9
1	-17.0	-11.2	-5.5	.3	6.0	11.8	17.5	23.3	29.0	34.8
1.5	-21.5	-15.5	-9.5	-3.5	2.5	8.5	14.6	20.6	26.6	32.6
FE	-46.6	-39.1	-31.6	-24.1	-16.7	-9.2	-1.7	5.7	13.2	20.7

α = .30										
b_2 \ R_p	.16	.17	.18	.19	.20	.21	.22	.23	.24	.25
.5	-11.8	-6.4	-.9	4.5	10.0	15.4	20.9	26.3	31.8	37.2
1	-19.6	-13.7	-7.8	-1.9	4.0	9.9	15.8	21.7	27.6	33.5
1.5	-25.0	-18.8	-12.6	-6.3	-.1	6.1	12.3	18.5	24..7	31.0
FE	-56.2	-48.2	-40.2	-32.1	-24.1	-.16.1	-8.0	0	8.0	16.1

[1] R = .16, r = .06, r_p = .15, δ = .05, and ρ = .10. The direct return in the P3 equals the direct return on own investment when R_P = .25.

[2] Solution with full employment.

Table 3: Ratio of welfare gain in P3 vs. OI in the model with full employment.[1]

R_P	ρ^*				
	.02	.04	.06	.08	.10
.16	.47	.39	.30	.20	.10
.17	.55	.47	.39	.30	.20
.18	.63	.56	.48	.39	.30
.19	.70	.64	.57	.49	.40
.20	.78	.72	.65	.58	.50
.21	.86	.81	.74	.67	.60
.22	.94	.89	.83	.77	.70
.23	1.02	.97	.92	.86	.80
.24	1.10	1.06	1.01	.96	.90
.25	1.18	1.14	1.10	1.05	1.0
$\overline{R_P}$	.228	.233	.239	.245	.250

[1] R = .16, r = .06, r_p = .15, α = .40, τ = .50, δ = .05, and ρ = .10. The direct return in the P3 equals the direct return on own investment when R_P = .25.

Table 4: Ratio of welfare gain in P3 vs. OI in the model with open employment and $b_2 = 1$.[1]

R_P	ρ^*				
	.02	.04	.06	.08	.10
.16	.81	.77	.74	.70	.66
.17	.86	.83	.80	.77	.73
.18	.92	.90	.86	.83	.80
.19	.98	.96	.93	.90	.87
.20	1.04	1.02	.99	.96	.93
.21	1.10	1.08	1.05	1.03	1.0
.22	1.16	1.14	1.12	1.09	1.07
.23	1.22	1.20	1.18	1.16	1.13
.24	1.28	1.26	1.24	1.22	1.20
.25	1.34	1.32	1.30	1.29	1.27
$\overline{R_P}$	.193	.197	.202	.206	.210

[1] $R = .16$, $r^* = .06$, $r_p = .15$, $\alpha = .40$, $\tau = .50$, $\delta = .05$, $u = .10$, and $\rho = .10$. The direct return in the P3 equals the direct return on own investment when $R_P = .25$.

Table 5: Ratio of welfare gain in P3 vs. OI and the breakeven value of R_P when there is full employment and P3 builds infrastructure faster than OI.[1]

R_P	ρ^*				
	.02	.04	.06	.08	.10
.16	.53	.53	.52	.51	.51
.17	.62	.64	.66	.67	.69
.18	.71	.75	.79	.83	.88
.19	.80	.86	.92	.99	1.07
.20	.90	.97	1.06	1.15	1.26
.21	.99	1.08	1.19	1.31	1.45
.22	1.08	1.19	1.32	1.47	1.63
.23	1.17	1.30	1.46	1.63	1.82
.24	1.26	1.41	1.59	1.79	2.01
.25	1.35	1.52	1.72	1.95	2.20
$\overline{R_P}$	.212	.203	.196	.190	.186
DRG[2]	.038	.047	.054	.06	.064

[1] $R = .16$, $r = .06$, $r_p = .15$, $\alpha = .40$, $\tau = .50$, $\delta = .05$, $\rho = .10$, $s=.20$, and $n=1.6$. The direct return in the P3 equals the direct return on own investment when $R_P = .25$.

[2] Direct return gap for $R_p = \overline{R_P}$.

Table 6: Ratio of welfare gain in P3 vs. OI and the breakeven value of R_P when $b_2 = 1$ in the model with open employment and P3 builds infrastructure faster than OI.[1]

R_P	ρ*				
	.02	.04	.06	.08	.10
.16	.91	1	1.10	1.20	1.30
.17	.98	1.08	1.20	1.32	1.43
.18	1.05	1.17	1.29	1.43	1.56
.19	1.12	1.25	1.39	1.54	1.69
.20	1.18	1.33	1.49	1.66	1.82
.21	1.25	1.41	1.59	1.77	1.95
.22	1.32	1.49	1.68	1.88	2.08
.23	1.39	1.57	1.78	2.0	2.21
.24	1.46	1.66	1.88	2.11	2.34
.25	1.53	1.74	1.97	2.22	2.47

	$\overline{R_P}$ (DRG)[2]				
b_2 \ ρ*	.02	.04	.06	.08	.10
.5	.160 (.09)	.145 (.105)	.133 (.117)	.126 (.124)	.119 (.131)
1	.173 (.077)	.160 (.09)	.150 (.10)	.142 (.108)	.137 (.113)
1.5	.180 (.07)	.168 (.082)	.159 (.091)	.152 (.098)	.147 (.103)
2	.186 (.064)	.174 (.076)	.165 (.085)	.158 (.092)	.153 (.097)
3	.192 (.058)	.181 (.069)	.173 (.077)	.166 (.084)	.161 (.089)
4	.196 (.054)	.185 (.065)	.177 (.073)	.171 (.079)	.166 (.084)

[1] R = .16, $r^* = .06$, $r_p = .15$, $\alpha = .40$, $\tau = .50$, $\delta = .05$, u = .10, $\rho = .10$, s=.20, and n=1.6. The direct return in the P3 equals the direct return on own investment when $R_P = .25$.

[2] Direct return gap for $R_p = \overline{R_P}$.

Table 7: Ratio of welfare gain in P3 vs. OI for distortionary and lump-sum taxation when $b_2 = 1$ in the model with open unemployment.[1]

	$\bar{\chi} = 0$					
	$r_p = .10$			$r_p = .15$		
R_P	$\rho^* = .02$	$\rho^* = .06$	$\rho^* = .10$	$\rho^* = .02$	$\rho^* = .06$	$\rho^* = .10$
.20	1.14 (1.15)	1.12 (1.14)	1.10 (1.12)	1.02 (1.04)	.94 (.99)	.89 (.93)
.21	1.20 (1.21)	1.19 (1.20)	1.18 (1.19)	1.08 (1.10)	1.01 (1.05)	.95 (1.0)
.22	1.26 (1.27)	1.25 (1.26)	1.25 (1.25)	1.14 (1.16)	1.07 (1.62)	1.02 (1.07)
.23	1.32 (1.33)	1.32 (1.32)	1.32 (1.32)	1.20 (1.22)	1.14 (1.18)	1.09 (1.13)
.24	1.39 (1.39)	1.39 (1.39)	1.39 (1.39)	1.26 (1.28)	1.21 (1.24)	1.16 (1.20)
.25	1.45 (1.44)	1.45 (1.45)	1.46 (1.46)	1.32 (1.34)	1.27 (1.30)	1.24 (1.27)

	$\bar{\chi} = .25$					
	$r_p = .10$			$r_p = .15$		
R_P	$\rho^* = .02$	$\rho^* = .06$	$\rho^* = .10$	$\rho^* = .02$	$\rho^* = .06$	$\rho^* = .10$
.20	1.14 (1.15)	1.12 (1.14)	1.11 (1.12)	1.02 (1.04)	.95 (.99)	.89 (.93)
.21	1.20 (1.21)	1.19 (1.20)	1.18 (1.19)	1.08 (1.10)	1.01 (1.05)	.96 (1.0)
.22	1.26 (1.27)	1.25 (1.26)	1.25 (1.25)	1.14 (1.16)	1.08 (1.62)	1.03 (1.07)
.23	1.32 (1.33)	1.32 (1.32)	1.32 (1.32)	1.20 (1.22)	1.14 (1.18)	1.10 (1.13)
.24	1.39 (1.39)	1.39 (1.39)	1.39 (1.39)	1.26 (1.28)	1.21 (1.24)	1.17 (1.20)
.25	1.45 (1.44)	1.45 (1.45)	1.46 (1.46)	1.32 (1.34)	1.28 (1.30)	1.24 (1.27)

	$\bar{\chi} = .40$					
	$r_p = .10$			$r_p = .15$		
R_P	$\rho^* = .02$	$\rho^* = .06$	$\rho^* = .10$	$\rho^* = .02$	$\rho^* = .06$	$\rho^* = .10$
.20	1.14 (1.15)	1.12 (1.14)	1.11 (1.12)	1.02 (1.04)	.94 (.99)	.89 (.93)
.21	1.20 (1.21)	1.19 (1.20)	1.18 (1.19)	1.08 (1.10)	1.02 (1.05)	.96 (1.0)
.22	1.26 (1.27)	1.26 (1.26)	1.25 (1.25)	1.14 (1.16)	1.08 (1.62)	1.03 (1.07)
.23	1.32 (1.33)	1.32 (1.32)	1.32 (1.32)	1.20 (1.22)	1.15 (1.18)	1.10 (1.13)
.24	1.39 (1.39)	1.39 (1.39)	1.39 (1.39)	1.26 (1.28)	1.21 (1.24)	1.17 (1.20)
.25	1.45 (1.44)	1.45 (1.45)	1.46 (1.46)	1.32 (1.34)	1.28 (1.30)	1.24 (1.27)

[1] $R = .16$, $r^* = .06$, $r_p = .15$, $\alpha = .40$, $\tau = .50$, $\delta = .05$, $u = .10$, and $\rho = .10$. The solution with lump-sum taxation is shown in parentheses.

Table 8: Ratio of welfare gain in P3 vs. OI when real wage income enters the social welfare function in the model with full employment.[1]

R_P	ʄ = 0			ʄ = .5		
	ρ* = .02	ρ* = .06	ρ* = .10	ρ* = .02	ρ* = .06	ρ* = .10
.16	.47	.30	.10	.68	.63	.55
.17	.55	.39	.20	.75	.70	.63
.18	.63	.48	.30	.82	.77	.70
.19	.70	.57	.40	.88	.84	.77
.20	.78	.65	.50	.95	.91	.85
.21	.86	.74	.60	1.01	.98	.92
.22	.94	.83	.70	1.08	1.04	.99
.23	1.02	.92	.80	1.15	1.11	1.07
.24	1.10	1.01	.90	1.21	1.18	1.14
.25	1.18	1.10	1.0	1.28	1.25	1.21
$\overline{R_P}$	.228	.239	.250	.208	.214	.221

R_P	ʄ = 1			ʄ = 1.5		
	ρ* = .02	ρ* = .06	ρ* = .10	ρ* = .02	ρ* = .06	ρ* = .10
.16	.78	.76	.73	.84	.84	.82
.17	.84	.83	.79	.89	.90	.88
.18	.90	.89	.85	.95	.95	.94
.19	.96	.95	.92	1.01	1.0	1.0
.20	1.02	1.01	.98	1.06	1.07	1.05
.21	1.08	1.07	1.04	1.12	1.12	1.11
.22	1.14	1.13	1.11	1.18	1.18	1.17
.23	1.20	1.19	1.17	1.24	1.24	1.23
.24	1.26	1.26	1.24	1.29	1.29	1.29
.25	1.32	1.32	1.30	1.35	1.35	1.34
$\overline{R_P}$	.196	.198	.203	.189	.189	.191

[1] R = .16, r^* = .06, r_p = .15, α= .40, τ = .50, δ = .05, u = .10, and ρ = .10. The solution for ʄ = 0 reproduces the solution in the benchmark model where social welfare depends only on consumption of the representative agent.

Table 9: Ratio of welfare gain in P3 vs. OI when real wage income enters the social welfare function in the model with open employment.[1]

	ƒ = 0			ƒ = .5		
R_P	$\rho^* = .02$	$\rho^* = .06$	$\rho^* = .10$	$\rho^* = .02$	$\rho^* = .06$	$\rho^* = .10$
.16	.81	.74	.66	.88	.86	.83
.17	.86	.80	.73	.93	.91	.88
.18	.92	.86	.80	.99	.97	.94
.19	.98	.93	.87	1.04	1.03	1.0
.20	1.04	.99	.93	1.10	1.08	1.06
.21	1.10	1.05	1.0	1.15	1.14	1.11
.22	1.16	1.12	1.07	1.21	1.19	1.17
.23	1.22	1.18	1.13	1.26	1.25	1.23
.24	1.28	1.24	1.20	1.32	1.31	1.29
.25	1.34	1.30	1.27	1.37	1.36	1.35
$\overline{R_P}$	.193	.202	.210	.182	.185	.190

	ƒ = 1			ƒ = 1.5		
R_P	$\rho^* = .02$	$\rho^* = .06$	$\rho^* = .10$	$\rho^* = .02$	$\rho^* = .06$	$\rho^* = .10$
.16	.92	.92	.90	.94	.95	.95
.17	.97	.97	.96	.99	1	1
.18	1.02	1.02	1.01	1.04	1.05	1.05
.19	1.07	1.08	1.06	1.09	1.11	1.10
.20	1.13	1.13	1.12	1.14	1.16	1.15
.21	1.18	1.18	1.17	1.20	1.21	1.20
.22	1.23	1.23	1.22	1.25	1.26	1.25
.23	1.28	1.29	1.28	1.30	1.31	1.30
.24	1.34	1.34	1.33	1.35	1.36	1.36
.25	1.39	1.39	1.38	1.40	1.41	1.41
$\overline{R_P}$	.176	.176	.178	.172	.169	.170

[1] $R = .16$, $r^* = .06$, $r_p = .15$, $\alpha = .40$, $\tau = .50$, $\delta = .05$, $u = .10$, b_2, and $\rho = .10$. The solution for ƒ = 0 reproduces the solution in the benchmark model where social welfare depends only on consumption of the representative agent.

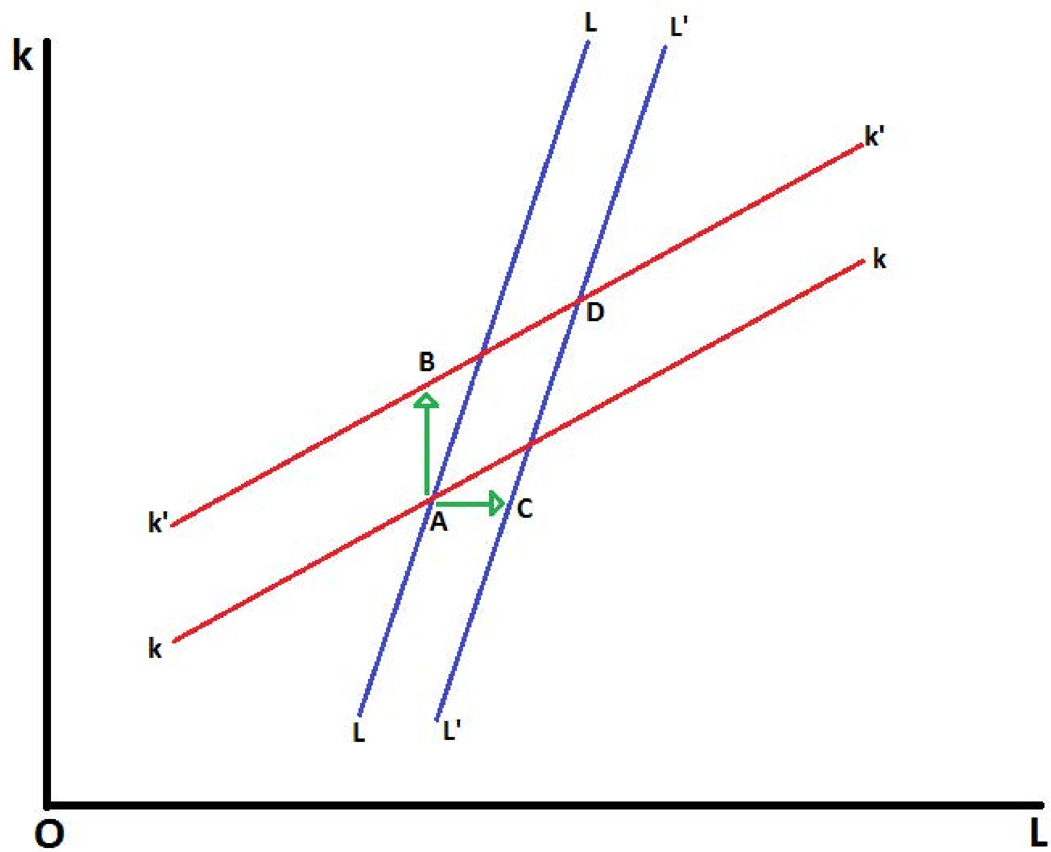


Figure 1: Interdependence of Capital Accumulation and Employment

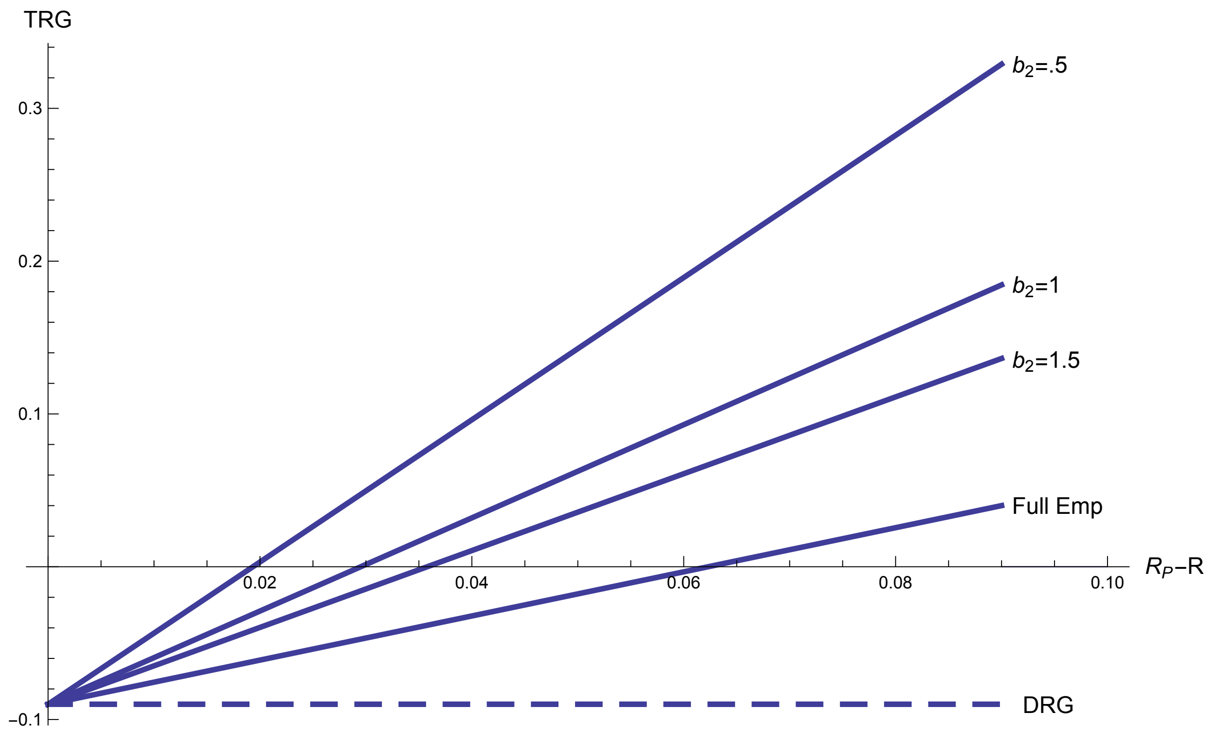


Figure 2: Total Return Gap (TRG) vs. Direct Return Gap (DRG)

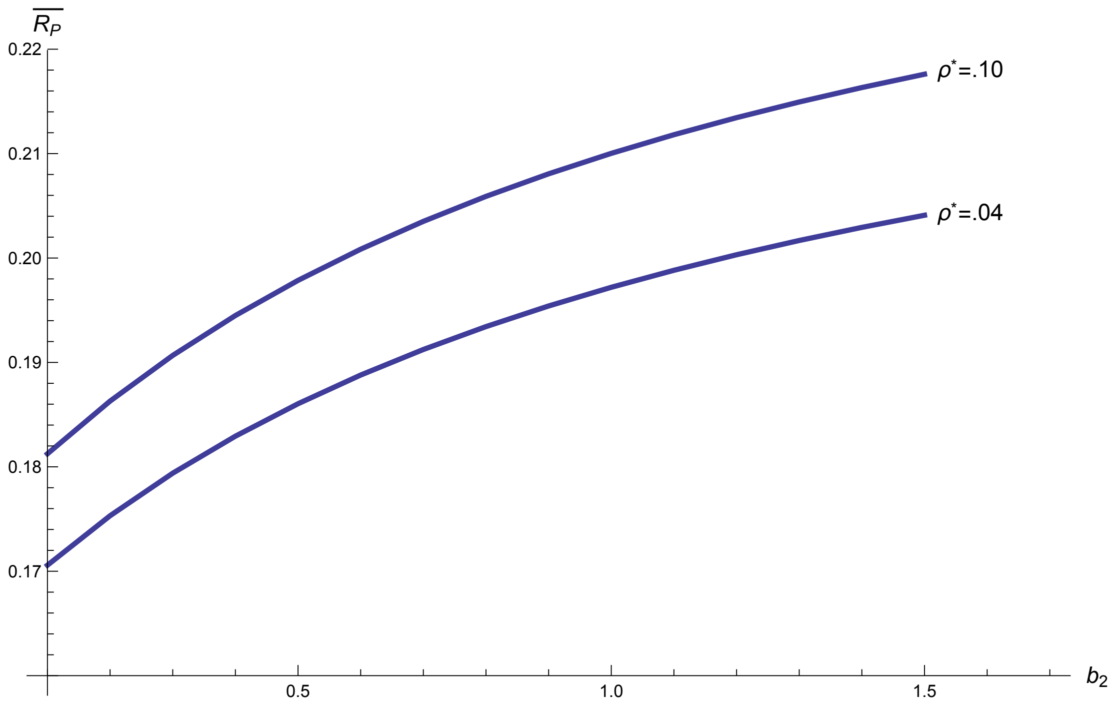


Figure 3: Breakeven value of R_P in the model with efficiency wages.

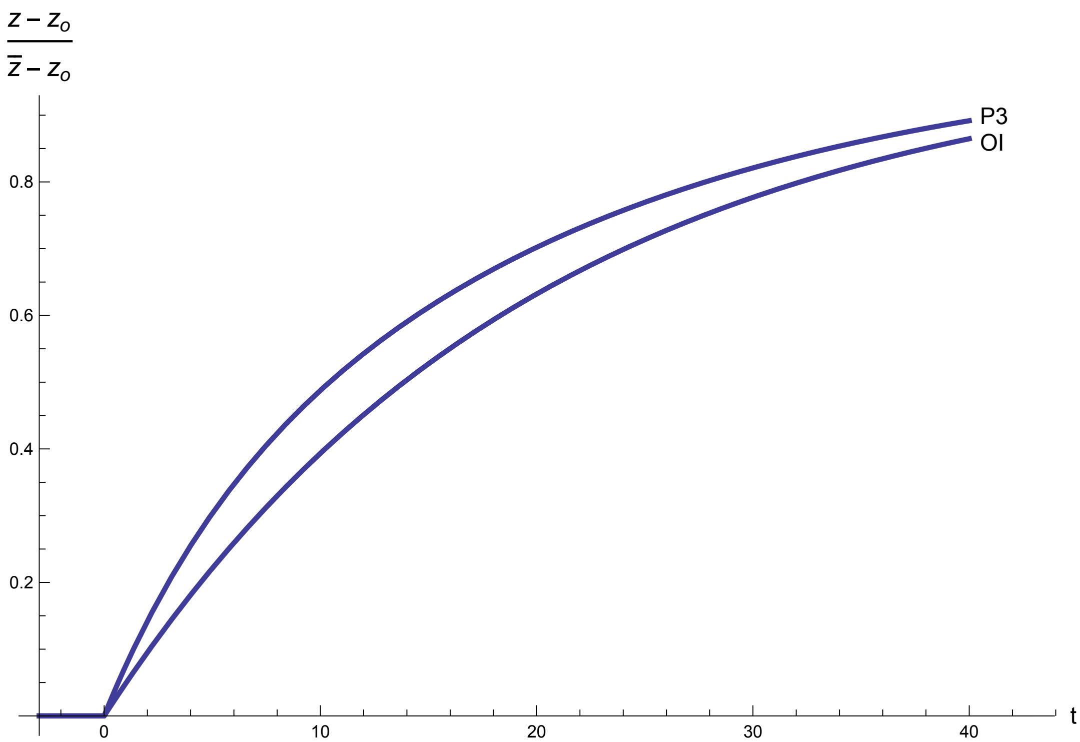
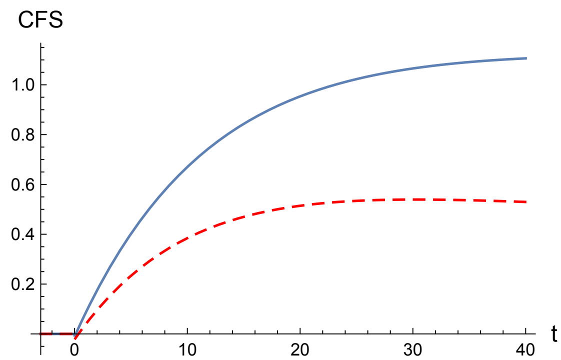
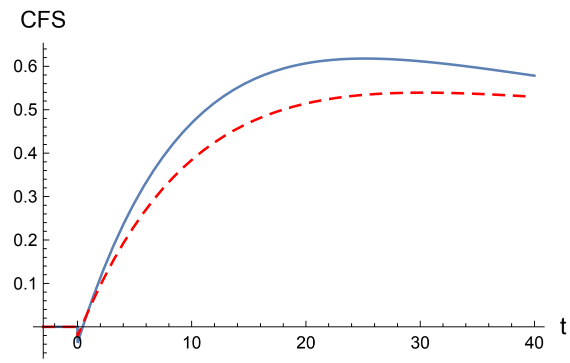


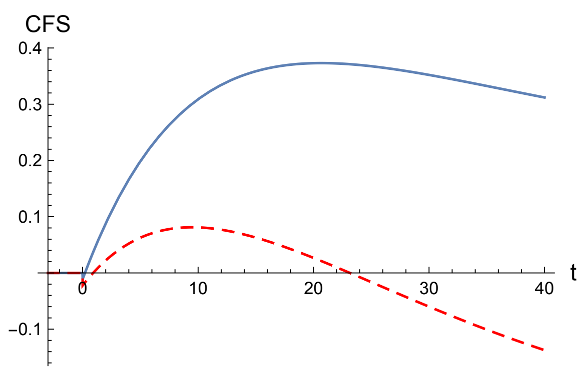
Figure 4: P3 builds infrastructure faster than OI.



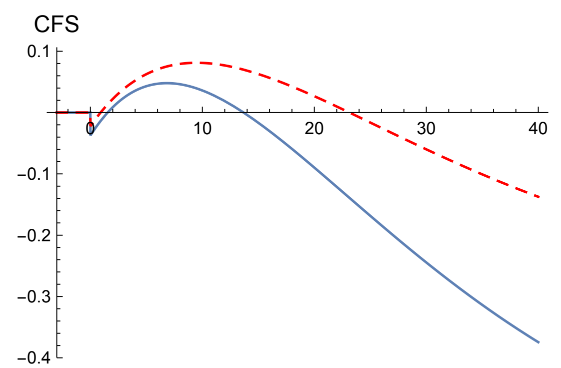
5a. Full employment: R_P = .20 and $\overline{\chi}$ = .25.



5b. Full employment: R_P = .25 and $\overline{\chi}$ = .25.

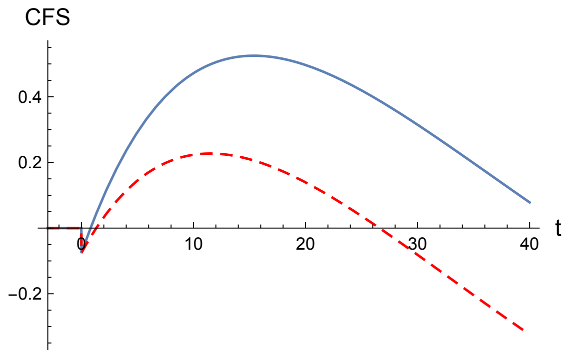


5c. Full employment: R_P = .20 and $\overline{\chi}$ = .40.

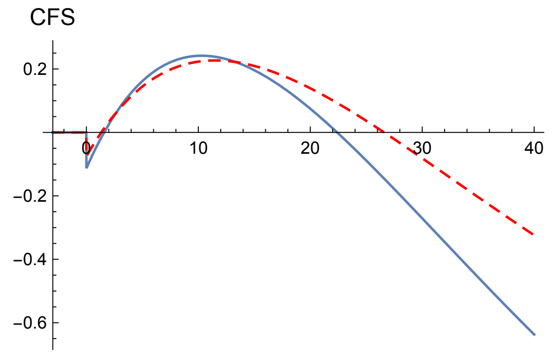


5d. Full employment: R_P = .25 and $\overline{\chi}$ = .40.

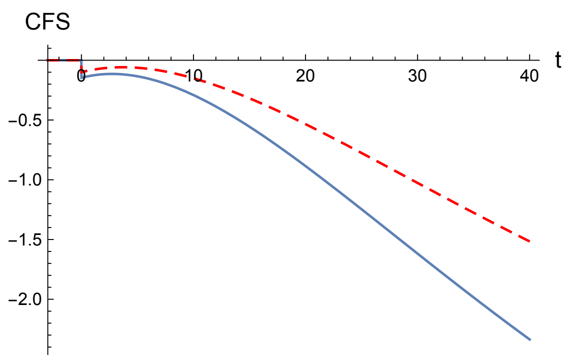
Figure 5: Path for the coefficient of fiscal stress (solid blue for P3, red dashed for OI).
r_p = .10 in Figures 5a-5h.



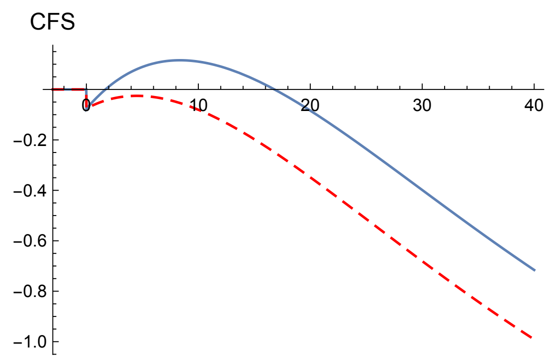
5e. R_P = .20, b_2 = 1, and $\overline{\chi}$ = .25.



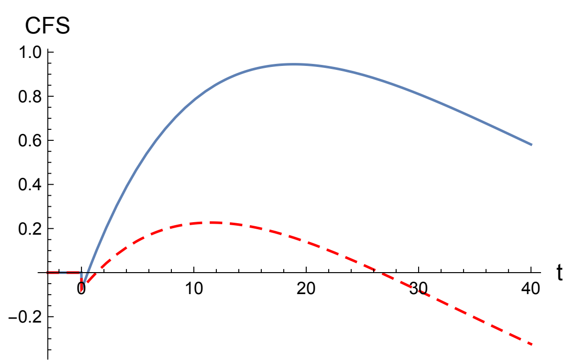
5f. R_P = .25, b_2 = 1, and $\overline{\chi}$ = .25.



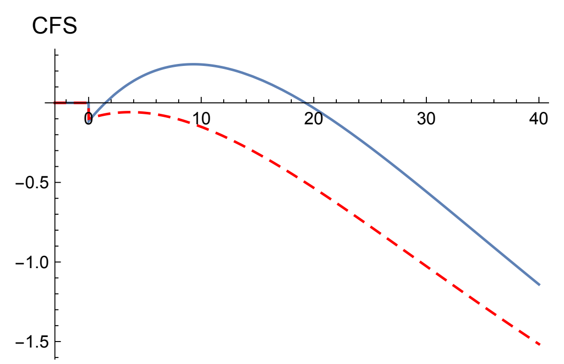
5g. R_P = .25, b_2 = .5, and $\overline{\chi}$ = 40.



5h. R_P = .20, b_2 = 1, and $\overline{\chi}$ = .40.



5i. R_P = .25, r_p = .15, b_2 = 1, $\overline{\chi}$ = .25.



5j. R_P = .25, r_p = .15, b_2 = .5, $\overline{\chi}$ = .40.

Figure 5 (Continued): Path for the coefficient of fiscal stress (solid blue for P3, red dashed for OI). r_p = .10 in Figures 5a-5h.

A Appendix A

The core dynamic system in the benchmark model is given in equation (33). On the convergent saddle path,

$$c - c_o = \bar{c} - c_o + [(f_4 - \lambda_2)(X_{21} - a_2)\mathrm{e}^{\lambda_2 t} - X_{11}\mathrm{e}^{-\delta t}](\bar{z} - z_o), \tag{A1}$$

and

$$k - k_o = [a_2(1 - \mathrm{e}^{\lambda_2 t}) + X_{21}(\mathrm{e}^{\lambda_2 t} - \mathrm{e}^{-\delta t})](\bar{z} - z_o), \tag{A2}$$

where

$$\begin{aligned}
X_{11} &= \frac{\tau(c/k)[(1-\alpha)f_5 f_2 + \alpha(R_p + \delta)f_3^2]}{V}, \\
X_{21} &= \frac{(R_p + \delta)[\tau\alpha(c/k)f_3 + \delta(1-\alpha)/f_1] + \delta(R_p - r_p)}{(\rho + \delta)V}, \\
V &\equiv \tau(c/k)(1-\alpha)f_2 - \delta f_3, \\
a_2 &\equiv \frac{\bar{k} - k_o}{\bar{z} - z_o} = \frac{R_p + \delta}{\rho + \delta}\left(\frac{\alpha}{1-\alpha}\right)\frac{1 + b_2(1-u)}{b_2(1-u)}.
\end{aligned}$$

The Welfare Solution Under Full Employment

In the full-employment case, $f_4 \equiv \rho$ and $a_2 \equiv (R_p + \delta)\alpha/(1-\alpha)(\rho + \delta)$. Recall that $\rho^* \leq \rho$. Thus

$$\begin{aligned}
\frac{SW - SW_o}{c^{-1/\tau}} &= \int_o^\infty (c - c_o)\mathrm{e}^{-\rho^* t}dt, \\
\Longrightarrow \frac{SW - SW_o}{c^{-1/\tau}(\bar{z} - z_o)} &= \int_o^\infty \left(\rho\frac{R_p + \delta}{\rho + \delta}\frac{\alpha}{1-\alpha} + R_p - r_p\right)\mathrm{e}^{-\rho^* t}dt \\
&\quad - \int_o^\infty X_{11}\mathrm{e}^{-(\rho^* + \delta)t}dt + \int_0^\infty (\rho - \lambda_2)\left(X_{21} - \frac{R_p + \delta}{\rho + \delta}\frac{\alpha}{1-\alpha}\right)\mathrm{e}^{(\lambda_2 - \rho^*)t}dt, \\
\Longrightarrow \frac{SW - SW_o}{c^{-1/\tau}(\bar{z} - z_o)} &= \frac{\lambda_2(\rho^* - \rho)}{\rho^*(\rho^* - \lambda_2)}\frac{R_p + \delta}{\rho + \delta}\frac{\alpha}{1-\alpha} + \frac{R_p - r_p}{\rho^*} + \frac{\rho - \lambda_2}{\rho^* - \lambda_2}X_{21} - \frac{X_{11}}{\rho^* + \delta}.
\end{aligned}$$

Substitute for X_{11} and X_{21}. After rearranging terms, the solution may be written as

$$\left.\frac{SW - SW_o}{c^{-1/\tau}(\bar{z} - z_o)}\right|_{P3} = \int_o^\infty (c - c_o)\mathrm{e}^{-\rho^* t}dt = \left(\frac{R_p - r_p}{\rho + \delta}\right)\frac{\delta}{\rho^*}(1 + H_p), \tag{A3}$$

for P3, where

$$H_p \equiv \frac{\rho - \rho^*}{\rho^* + \delta}\Lambda + \frac{(R_p + \delta)\alpha(\rho - \rho^*)}{(\rho^* - \lambda_2)(1-\alpha)(\rho + \delta)}\Gamma,$$

$$\Lambda \equiv 1 + \frac{\rho^*(\lambda_2 + \delta)}{J(\rho^* - \lambda_2)} \quad \text{with} \quad \Lambda > 0,$$

$$\Gamma \equiv \frac{\tau(c/k)(1-\alpha)(\lambda_2 + \delta)}{J} - \lambda_2 \frac{\rho^* + \delta}{\rho^*} \quad \text{with} \quad \Gamma > 0,$$

and

$$J \equiv \tau(c/k)(1-\alpha) - \delta.$$

To show that $\Lambda > 0$ and $\Gamma > 0$, note first that $\lambda_2 + \delta$ and J are opposite in sign. Hence

$$\frac{\lambda_2 + \delta}{J} > -1. \tag{A4}$$

is sufficient for $\Lambda \equiv 1 + \frac{\rho^*(\lambda_2+\delta)}{J(\rho^*-\lambda_2)}$ to be positive. When $J > 0$, the condition holds if

$$\begin{aligned} \lambda_2 + \delta + J &> 0, \\ \Longrightarrow \lambda_2 + N &> 0, \end{aligned}$$

where $N \equiv \tau(c/k)(1-\alpha)$. Substituting for λ_2 produces

$$\begin{aligned} \frac{\rho - \sqrt{\rho^2 + 4(\rho+\delta)N}}{2} + N &> 0, \\ \Longrightarrow \rho + 2N &> \sqrt{\rho^2 + 4(\rho+\delta)N}, \\ \Longrightarrow NJ &> 0, \end{aligned}$$

which holds as $J > 0$ in the case under examination.

Suppose next that $J < 0$. In this case, we require

$$\lambda_2 + \delta + J < 0.$$

The same algebra delivers

$$NJ < 0,$$

which also holds.

Turn now Γ which affects H_p in (A3). Since $J = N - \delta$, the bracketed term multiplying $\rho - \rho^*$ is positive if

$$\frac{N(\lambda_2 + \delta)}{N - \delta} - \lambda_2 > 0,$$

or, equivalently,

$$\lambda_2 + N \gtrless 0 \quad \text{as} \quad J = N - \delta \gtrless 0. \tag{A5}$$

But we already know that $\lambda_2 + N$ and J take the same sign. Thus the condition in (A5) holds.

The solution for OI takes the same form as (A3), with R and r replacing R_p and r_p.

B Appendix B

When P3 builds infrastructure faster than OI, the core dynamic system is

$$\begin{bmatrix} \dot{c} \\ \dot{k} \\ \dot{z} \\ di_z/dt \end{bmatrix} = \begin{bmatrix} 0 & \tau(c/k)(\rho+\delta)(\alpha-1)f_2 & \tau(c/k)\alpha(R_p+\delta)f_3 & 0 \\ -1 & f_4 & f_5 & 0 \\ 0 & 0 & -\delta & 1 \\ 0 & 0 & 0 & -s \end{bmatrix} \begin{bmatrix} c-\bar{c} \\ k-\bar{k} \\ z-\bar{z} \\ i_z-\bar{\imath}_z \end{bmatrix}. \tag{B1}$$

The saddle point solutions for c, k, and z are

$$\begin{bmatrix} c-c_o \\ k-k_o \\ z-z_o \end{bmatrix} = \begin{bmatrix} \bar{c}-c_o \\ \bar{k}-k_o \\ \bar{z}-z_o \end{bmatrix} + \begin{bmatrix} X_{11} & f_4-\lambda_2 & X_{13} \\ X_{21} & 1 & X_{23} \\ 1 & 0 & 1/(\delta-s) \end{bmatrix} \begin{bmatrix} h_1\mathsf{e}^{-\delta t} \\ h_2\mathsf{e}^{\lambda_2 t} \\ h_3\mathsf{e}^{-st} \end{bmatrix}, \tag{B2}$$

where

$$\begin{aligned} X_{13} &= \frac{\tau(c/k)}{(\delta-s)V_1}\left[(1-\alpha)f_2f_5+\alpha(R_p+\delta)f_3\frac{f_4+s}{\rho+\delta}\right], \\ X_{23} &= \frac{\tau(c/k)\alpha(R_p+\delta)f_3+sf_5}{(\delta-s)(\rho+\delta)V_1}, \\ V_1 &\equiv \tau(c/k)(1-\alpha)f_2-s\frac{f_4+s}{\rho+\delta}, \end{aligned}$$

and h_1 - h_3 are determined by the initial jump in i_z and the initial values of k and z:

$$\begin{aligned} h_1 &= [(1-n)\delta/(\delta-s)-1](\bar{z}-z_o), \\ h_2 &= \{X_{21}-a_2+(n-1)\delta[X_{21}/(\delta-s)+X_{23}](\bar{z}-z_o), \\ h_3 &= (n-1)\delta(\bar{z}-z_o), \\ n &= \frac{i_{z,o^+}-i_{z,o}}{\bar{\imath}_z-i_{z,o}}. \end{aligned}$$

C Appendix C

In the model where the distortionary consumption tax adjusts to satisfy the government budget constraint,

$$\dot{c} = \tau c(q_k-\rho-\delta)-\tau c\gamma\frac{\dot{h}}{1+h} \tag{C1}$$

replaces equation (7). Interpret the differentials in (49) as time derivatives and substitute for $\dot{h}$. After collecting terms, we have

$$(1 - \tau h\gamma/g_0)\dot{c} = \tau c(q_k - \rho - \delta) - \tau c\gamma g_1(i_z - \delta z), \tag{C2}$$

where

$$g_0 \equiv 1 + h - \epsilon h(1-\gamma) \quad \text{and} \quad g_1 \equiv \frac{r_p + \delta - \bar{\chi}(R_p + \delta)}{P_2^* c_2 g_0}.$$

The linearized version of (C2) is

$$\dot{c} = \frac{\tau(c/k)(\alpha-1)(\rho+\delta)f_2}{g_3}(k-\bar{k}) + \frac{\tau(c/k)(\alpha-1)(R_p+\delta)f_3 - g_4}{g_3}(z-\bar{z}), \tag{C3}$$

where

$$g_3 \equiv 1 - \frac{\tau h\gamma}{g_0} \quad \text{and} \quad g_4 \equiv \frac{\tau\delta(1+h)}{g_0}[r_p + \delta - \bar{\chi}(R_p+\delta)].$$

Turn next to equation (47). Using equations (48) and (49) to solve for c_2 and substituting it into equation (47) leads to

$$\dot{k} = f_4(k-\bar{k}) + f_6(z-\bar{z}) - f_7(c-\bar{c}), \tag{C4}$$

where

$$f_6 = f_5 - \frac{h\epsilon(1-\gamma)[r_p + \delta - \bar{\chi}(R_p+\delta)]}{g_0} \quad \text{and} \quad f_7 = 1 - \frac{h\gamma}{g_0}.$$

Equations (C3), (C4), and

$$\dot{z} = -\delta(z - \bar{z}) \tag{C5}$$

comprise the core dynamic system. The solution for the path of consumption is

$$c - c_o = \bar{c} - c_o + \left[\frac{f_4 - \lambda_2}{f_7}(X_{21} - a_2)\mathsf{e}^{\lambda_2 t} - X_{11}\mathsf{e}^{-\delta t}\right](\bar{z} - z_o), \tag{C6}$$

but now

$$\begin{aligned}
\lambda_2 &= \frac{f_4 - \sqrt{f_4^2 + 4\tau(1-\alpha)(\rho+\delta)(c/k)f_2 f_7/g_3}}{2} \\
X_{11} &= \frac{f_3}{g_3 V}[\tau(c/k)(1-\alpha)f_6 f_2/f3 + \tau(c/k)\alpha(R_p+\delta)f_3 + g_4], \\
X_{21} &= \frac{\delta f_6 + [\tau\alpha(c/k)(R_p+\delta)f_3 + g_4]f_7/g_3}{(\rho+\delta)V}, \\
V &\equiv \tau(c/k)(1-\alpha)f_2 f_7/g_3 - \delta f_3.
\end{aligned}$$

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