7 Seasonal Adjustment

The purpose of seasonal adjustment is to identify and estimate the different components of a time series, and thus provide a better understanding of the underlying trends, business cycle, and short-run movements in the series. Seasonal adjustment offers a complementary view on the current developments of macroeconomic series, allowing comparisons between quarters without the influence of seasonal and calendar effects. This chapter introduces the main principles of seasonal adjustment. Next, it outlines the main steps of the most commonly used seasonally adjustment procedures adopted by data producer agencies. Practical guidance is provided on how to evaluate and validate the quality of seasonally adjusted data. Finally, some specific issues arising from the application of seasonal adjustment in the framework of national accounts are considered, such as the direct versus indirect adjustment of quarterly national accounts (QNA) aggregates and the temporal consistency with annual benchmarks.

Introduction

7.1 Seasonal adjustment of the QNA allows a timely assessment of the current economic conditions and identification of turning points in key macroeconomic variables, such as quarterly gross domestic product (GDP). Economic variables are influenced by systematic and recurrent within-a-year patterns due to weather and social factors, commonly referred to as the seasonal pattern (or seasonality). When seasonal variations dominate period-to-period changes in the original series (or seasonally unadjusted series), it is difficult to identify nonseasonal effects, such as long-term movements, cyclical variations, or irregular factors, which carry the most important economic signals for QNA users.

7.2 Seasonal adjustment is the process of removing seasonal and calendar effects from a time series. This process is performed by means of analytical techniques that break down the series into components with different dynamic features. These components are unobserved and have to be identified from the observed data based on a priori assumptions on their expected behavior. In a broad sense, seasonal adjustment comprises the removal of both within-a-year seasonal movements and the influence of calendar effects (such as the different number of working days or moving holidays). By removing the repeated impact of these effects, seasonally adjusted data highlight the underlying long-term trend and short-run innovations in the series.

7.3 In trend-cycle estimates, the impact of irregular events in addition to seasonal variations is removed. Adjusting a series for seasonal variations removes the identifiable, regularly repeated influences on the series but not the impact of any irregular events. Consequently, if the impact of irregular events is strong, seasonally adjusted series may not represent a smooth, easily interpretable series. Standard seasonal adjustment packages provide an estimate of the trend-cycle component, representing a combined estimate of the underlying long-term trend and the business-cycle movements in the series. It should be noted, however, that the decomposition between the trend-cycle and the irregular components is subject to large uncertainty at the endpoint of the series, where it may be difficult to distinguish and allocate the effects from new observations.

7.4 A common solution to deal with seasonal patterns is to look at annual rates of change: that is, compare the current quarter to the same quarter of the previous year. Over-the-year comparisons present the disadvantage, however, of giving signals of outdated events.[1] Furthermore, these rates of change do not fully exclude all calendar-related effects (e.g., Easter

[1] Quenneville and Findley (2012) found analytically that year-on-year changes present 5.5 months of delay with respect to month-to-month growth rates. For quarterly series, this corresponds to a delay of almost two quarters.

may fall in the first or second quarter, and the number of working days of a quarter may differ between subsequent years). Finally, these year-on-year rates of change will be influenced by any eventual change in the seasonal pattern caused by institutional, climatic, or behavioral changes.

7.5 Several methods have been developed to remove seasonal patterns from a series.[2] Broadly speaking, they can be divided into two groups: moving average (MA) methods and model-based methods. Methods in the first group derive the seasonally adjusted data by applying a sequence of MA filters to the original series and its transformations. These methods are all variants of the X-11 method, originally developed by the U.S. Census Bureau (Shiskin and others, 1967).[3] The current version of the X-11 family is X-13ARIMA-SEATS (X-13A-S), which will often be referred to in this chapter. Model-based methods derive the unobserved components in accord with specific time series models, primarily autoregressive integrated moving average (ARIMA) models. The most popular model-based seasonal adjustment method is TRAMO-SEATS,[4] developed by the Bank of Spain (Gomez and Maravall, 1996). Box 7.1 illustrates the main characteristics of the X-13A-S and TRAMO-SEATS programs. Other available seasonal adjustment methods include, among others, BV4, SABLE, and STAMP.

7.6 Current seasonal adjustment packages offer built-in functionality to select between alternative modeling options in an automatic manner (e.g., ARIMA model, calendar effects, and additive versus multiplicative model). The selection process mostly relies on statistical tests or heuristic rules based on the seasonal adjustment results. These automatic features are very helpful when seasonal adjustment is to be applied to many time series at a time (hundreds, or even thousands), avoiding a series-by-series, time-consuming manual selection process. However, compilers should use these automatic features with care. Steps performed by the seasonal adjustment procedure used in the QNA should be assessed and comprehended, as with every other method applied in the national accounts. Seasonal adjustment options, at least for the most relevant QNA series, should always be tested for adequacy and monitored over time.

7.7 Seasonally adjusted data should not replace the original QNA data. Some users prefer to base their economic analysis on unadjusted data, as they treat seasonality as an integrated part of their modeling work. In this regard, seasonal adjustment adopted by statistical agencies is sometimes seen as a potentially dangerous procedure that may compromise the intrinsic properties of the original series. In fact, there is always some loss of information from seasonal adjustment, even when the seasonal adjustment process is properly conducted. For this reason, producers of seasonally adjusted data should employ sound and internationally accepted methodology for seasonal adjustment. More importantly, they should implement a transparent communication strategy, indicating the method in use and integrating seasonally adjusted figures with appropriate metadata that allow the results to be replicated and understood by the general public.

7.8 Countries that are yet to produce QNA in seasonally adjusted form may follow an evolutionary approach to seasonal adjustment. In a first stage, seasonal adjustment should be applied to the most important aggregates of the QNA (such as the GDP). For some time, these seasonally adjusted series may be used internally or published as experimental data. Next, seasonal adjustment could be expanded to the full set of QNA series once compilers gain more experience and confidence in the seasonal adjustment work. Albeit not published, seasonal adjustment of QNA data should at least be done internally; in effect, seasonally adjusted data often facilitate the identification of issues in the unadjusted data as seasonality may hide errors and inconsistencies in the original estimates.

7.9 This chapter is structured as follows. The next section illustrates the main principles of seasonal

[2] For a history of seasonal adjustment methods, see Ladiray and Quenneville (2001, chapter 1).

[3] The X-11 program was the first procedure designed for a large-scale application of seasonal adjustment. It emerged from decades of research originated in the early 1930s by researchers at the NBER. Subsequent improvements to the original X-11 program were implemented in the X-11-ARIMA program, developed by Statistics Canada (Dagum, 1980), and in the X-12-ARIMA program, developed by the U.S. Census Bureau (Findley and others, 1998). For further details on the history of X-11, see Box 7.1 in this chapter and Ghysels and Osborn (2001).

[4] TRAMO is the acronym for Time series Regression with Autoregressive integrated moving average (ARIMA) errors and Missing Observations. SEATS stands for Signal Extraction for ARIMA Time Series.

Box 7.1 Software for Seasonal Adjustment

Statistical agencies and international organizations have developed programs to facilitate the production process of seasonally adjusted and trend-cycle data. This box provides a brief overview of the three most used seasonal adjustment programs available at the time of writing:

- X-13-ARIMA-SEATS (U.S. Bureau of the Census),
- TRAMO-SEATS (Bank of Spain), and
- JDemetra+ (National Bank of Belgium, in cooperation with Deutsche Bundesbank and Eurostat).

X-13-ARIMA-SEATS (U.S. Bureau of the Census). The U.S. Bureau of the Census currently promotes and supports the X-13ARIMA-SEATS (X-13A-S) program. X-13A-S implements the two most widely used seasonal adjustment methods: the moving average X-11 method and the ARIMA model-based SEATS method. It also offers a module for selecting preadjustment effects using regARIMA models. X-13A-S produces basic and advanced diagnostics to evaluate the quality of the seasonal adjustment results. The program runs under DOS and Unix. A Windows interface, Win X-13, is also available. X-13A-S is the result of a series of developments of the original X-11 program developed by the U.S. Bureau of the Census in the late 1950s to early 1960s. Statistics Canada later developed X-11 ARIMA, which introduced ARIMA methods for backcasts and forecasts. The program, manual, and further reference can be found at https://www.census.gov/srd/www/x13as/.

TRAMO-SEATS (Bank of Spain). The TRAMO-SEATS program implements the ARIMA model-based seasonal adjustment method developed by Gomez and Maravall (1996). It is promoted and maintained by the Bank of Spain. The SEATS decomposition is based on the signal extraction method proposed by Burman (1980) and Hillmer and Tiao (1982). The preadjustment module (TRAMO) is similar to the one incorporated in the X-13A-S program. The software can be launched in both DOS and UNIX. A Windows interface, TRAMO-SEATS for Windows (TSW), is also available. The program, manual, and further reference can be found at http://www.bde.es/bde/en/secciones/servicios/Profesionales/Programas_estadi/Programas.html.

JDemetra+ (National Bank of Belgium, Deutsche Bundesbank, and Eurostat). JDemetra+ is a tool for seasonal adjustment developed by the National Bank of Belgium in cooperation with the Deutsche Bundesbank and Eurostat. It offers both X-11 and SEATS filters, and a common methodology for preadjustment. It allows to seasonally adjust large-scale sets of time series and provides user-friendly tools for checking the quality of the results. J-Demetra+ includes enhanced features and standardized outputs for comparing results from X-11 and SEATS. It also includes a plug-in for benchmarking (see Chapter 6).

The program, manual, and further reference can be found at https://ec.europa.eu/eurostat/cros/content/jdemetra-seasonal-adjustment-software_en.

adjustment. The third section "Seasonal Adjustment Procedure" outlines the two stages of seasonal adjustment procedures: preadjustment and time series decomposition. It also provides a brief illustration of the X-11 (MA) filter and SEATS (model-based) filter. The fourth section "Seasonal Adjustment and Revisions" stresses the importance of revisions in seasonal adjustment and how to handle and communicate them properly in a production context. Quality assessment tools for analyzing seasonal adjustment results are described in "Quality Assessment of Seasonal Adjustment" section. The sixth section "Particular Issues" addresses a set of critical issues on seasonal adjustment specifically related to QNA issues, such as preservation of accounting identities, seasonal adjustment of balancing items and aggregates, and the relationship between annual data and seasonally adjusted quarterly data. Finally, the last section "Status and Presentation of Seasonally Adjusted and Trend-Cycle QNA Estimates" discusses the presentation and status of seasonally adjusted and trend-cycle data.

Main Principles of Seasonal Adjustment

7.10 For seasonal adjustment purposes, a time series is generally assumed to be made up of four main components: (i) the trend-cycle component, (ii) the seasonal component, (iii) the calendar component, and (iv) the irregular component. These components are unobserved and have to be identified (and estimated) from the observed time series using a signal extraction technique.

7.11 The trend-cycle component (T_t) is the underlying path of the series. It includes both the long-term trend and the business-cycle movements in the data.

The long-term trend can be associated with structural changes in the economy, such as population growth and progress in technology and productivity. Business-cycle variations are related to the periodic oscillations of different phases of the economy (i.e., recession, recovery, growth, and decline), which generally repeat themselves with a period between two and eight years.

7.12 The seasonal component (S_t) includes those seasonal fluctuations that repeat themselves with similar annual timing, direction, and magnitude.[5] Possible causes of seasonal movements relate to climatic factors, administrative or legal rules, and social/cultural traditions and conventions—including calendar effects that are stable in annual timing (e.g., public holidays or other national festivities). Each of these causes (or a combination of them) can affect expectations in such a way that seasonality is indirectly induced. Similarly, changes in any of these causes may change the properties of the seasonal pattern.

7.13 The calendar component (C_t) comprises effects that are related to the different characteristics of the calendar from period to period. Calendar effects are both seasonal and nonseasonal. Only the "nonseasonal" part should be included in the calendar component and treated separately, as the "seasonal" one is already caught by the seasonal component.[6] The most used calendar effects include the following:

a. *Trading-Day or Working-Day Effect.* The trading-day effect detects the different number of each day of the week within a specific quarter relative to the standard weekday composition of a quarter. The working-day effect catches the difference between the number of working days (e.g., Monday through Friday) and the number of weekend days (e.g., Saturday and Sunday) in a quarter. The trading-day effect assumes an underlying pattern associated with each day of the week; the working-day effect postulates different behavior between the groups of weekdays and weekends.[7] Both the trading-day and working-day effects should incorporate the effects of national holidays (e.g., when Christmas falls on Monday, that Monday should not be counted as a trading/working day).

b. *Moving Holiday Effect*. A moving holiday is associated with events of religious or cultural significance within a country that change date from year to year (e.g., Easter or Ramadan).

c. *Leap Year Effect.* This effect is needed to account for the extra day in February of a leap year, which may generate a four-year cycle with a peak in the first quarter of leap years.

7.14 The irregular component (I_t) captures all the other fluctuations that are not part of the trend-cycle, seasonal, and calendar components. These effects are characterized by the fact that their timing, impact, and duration are unpredictable at the time of their occurrence. The irregular component includes the following effects:

a. *Outlier Effects*. These effects manifest themselves with abrupt changes in the series, sometimes related to unexpected weather or socioeconomic effects (such as natural disasters, strikes, or economic and financial crises). Such effects are not part of the underlying linear data generation process assumed for the original series. For these reasons, outlier effects are also called nonlinear effects. In the seasonal adjustment process, outliers should be removed by means of predefined intervention variables. Three main types of outliers are often used for economic time series:

   i. *additive outlier*, which relates to only one period;

   ii. *level shift*, which changes the level of a series permanently;[8] and

   iii. *transitory change*, whose effects on a series fade out over a number of periods.

   Other effects are seasonal outliers (which affect only certain quarters/months of the year), ramp outliers (which allow for a linear increase or decrease in the level of a series), or temporary level

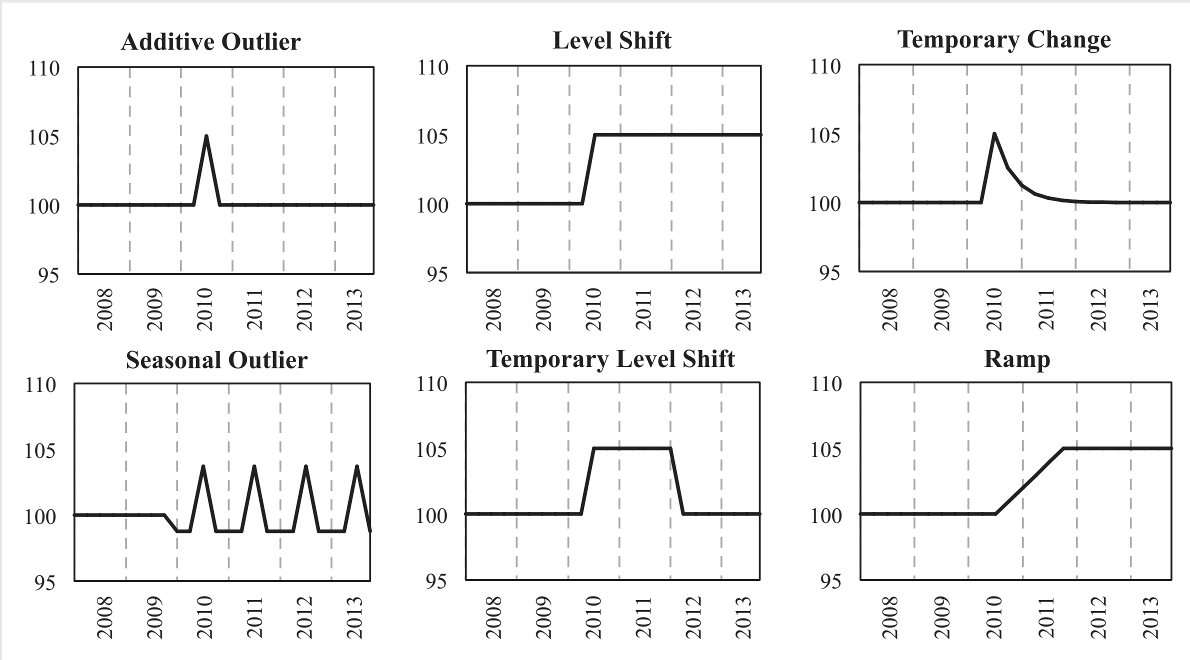
[5] Seasonality may be gradually changing over time. This phenomenon is called "moving seasonality."

[6] For example, the effect due to the different average number of days in each quarter is part of the seasonal effects.

[7] Trading-day effect is less important in quarterly data than in monthly data. Only quarters 3 and 4 contain different numbers of working days over time (excluding the leap year effect). The working-day effect is most commonly used for quarterly series.

[8] For seasonal adjustment purposes, outliers producing structural breaks in the series (such as a level shift or a seasonal outlier) may actually be allocated to the trend or seasonal components. Further details on the allocation of outliers is given in "Preadjustment" section.

Figure 7.1 Types of Outliers



shifts. These effects are modeled through specific intervention variables. The different types of outliers are shown in Figure 7.1. Further details on the treatment of outlier effects is given in "Preadjustment" section.

b. *White Noise Effects*. In the absence of outliers, the irregular component is assumed to be a random variable with normal distribution, uncorrelated at all times with constant variance. In statistical terms, such a variable is called a white noise process. Differently from outlier effects, a white noise process is assumed to be part of the underlying linear data generation process of the series.

7.15 The purpose of seasonal adjustment is to identify and estimate the different components of a time series, and thus provide a better understanding of the underlying trends, business cycle, and short-run movements in the series. The target variable of a seasonal adjustment process is the series adjusted for seasonal and calendar effects (or seasonally and calendar adjusted series). As mentioned before, both seasonal and calendar effects should be removed from the original series to allow for a correct analysis of the current economic conditions.

7.16 A fundamental prerequisite for applying seasonal adjustment procedures is that the processed series should present clear and sufficiently stable seasonal effects. Series with no seasonal effects, or series with seasonal effects that are not easy to identify from the original series, should not be seasonally adjusted. As discussed in the next section, the original series should always be tested for the presence of identifiable seasonality. At the same time, the series should also be tested for the presence of calendar effects. Calendar effects are usually less visible than seasonal effects, therefore their identification relies on statistical tests that reveal when their contribution to the series is statistically different from zero.

7.17 Two observations on the limits of seasonal adjustment are worth noting here. First, seasonal adjustment is not meant for smoothing series. A seasonally adjusted series[9] is the sum of the trend-cycle component and the irregular component. As a consequence, when the irregular component is strong, the seasonally adjusted series may not present a smooth pattern

[9] Unless otherwise specified, we indicate hereafter with seasonally adjusted series a series adjusted for both seasonal and calendar effects (if they are present).

over time. To extract the trend-cycle component, the irregular component should be further removed from the seasonally adjusted series. Trend-cycle extraction is a difficult exercise and subject to greater uncertainty than seasonal adjustment, especially in the final period of a series.

7.18 Second, seasonal adjustment and trend-cycle estimation represent an analytical processing of the original data. As such, the seasonally adjusted data and the estimated trend-cycle component complement the original data, but they can never replace the original data for the following reasons:

a. Unadjusted data are useful in their own right. The nonseasonally adjusted data show the actual economic events that have occurred, while the seasonally adjusted data and the trend-cycle estimate represent an analytical elaboration of the data designed to show the underlying movements that may be hidden by the seasonal variations. Compilation of seasonally adjusted data, exclusively, represents a loss of information.

b. No unique solution exists on how to conduct seasonal adjustment.

c. Seasonally adjusted data are subject to revisions as future data become available, even when the original data are not revised.

d. When compiling QNA, balancing and reconciling the accounts are better done on the original unadjusted QNA estimates. While errors in the source data may be more easily detected from seasonally adjusted data, it may be easier to identify the source for the errors and correct the errors working with the unadjusted data.

7.19 Figure 7.2 shows a quarterly time series spanning 20 years of data. This series has been simulated using a well-known seasonal ARIMA model with calendar effects (see the figure for details). The series shows an evident upward trend, stable seasonal effects (high in quarters 3 and 4, low in quarters 1 and 2), plus other nonsystematic, random movements. This series will be used throughout this chapter to illustrate the different stages of a seasonal adjustment process, which are described in the following section. Seasonal adjustment results using X-11 of this series are shown in Example 7.1.

Seasonal Adjustment Procedure

7.20 A seasonal adjustment procedure follows a two-stage approach (see the diagram in Box 7.2). The first stage is called *preadjustment*. The objective of preadjustment is to select a regression model with ARIMA errors that best describes the characteristics of the original series. The chosen model is used to adjust the series for deterministic effects (from which the name “preadjustment” is taken) and to extend the series with backcasts and forecasts to be used in the time series decomposition process. The preadjustment stage comprises mainly the choice of (i) how the unobserved components are related to each other (additive,

Figure 7.2 A Simulated Series with Trend, Seasonal, Calendar, and Irregular Effects

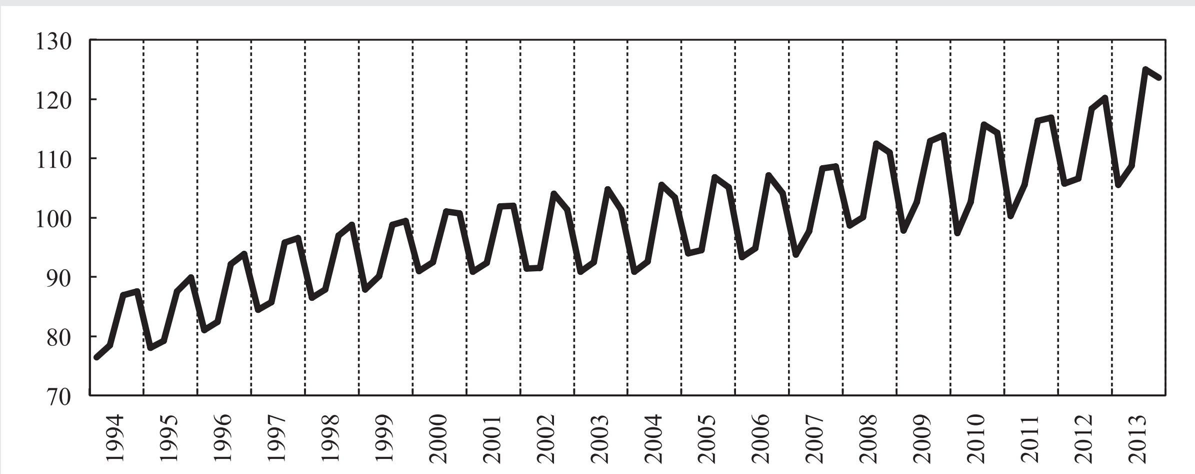


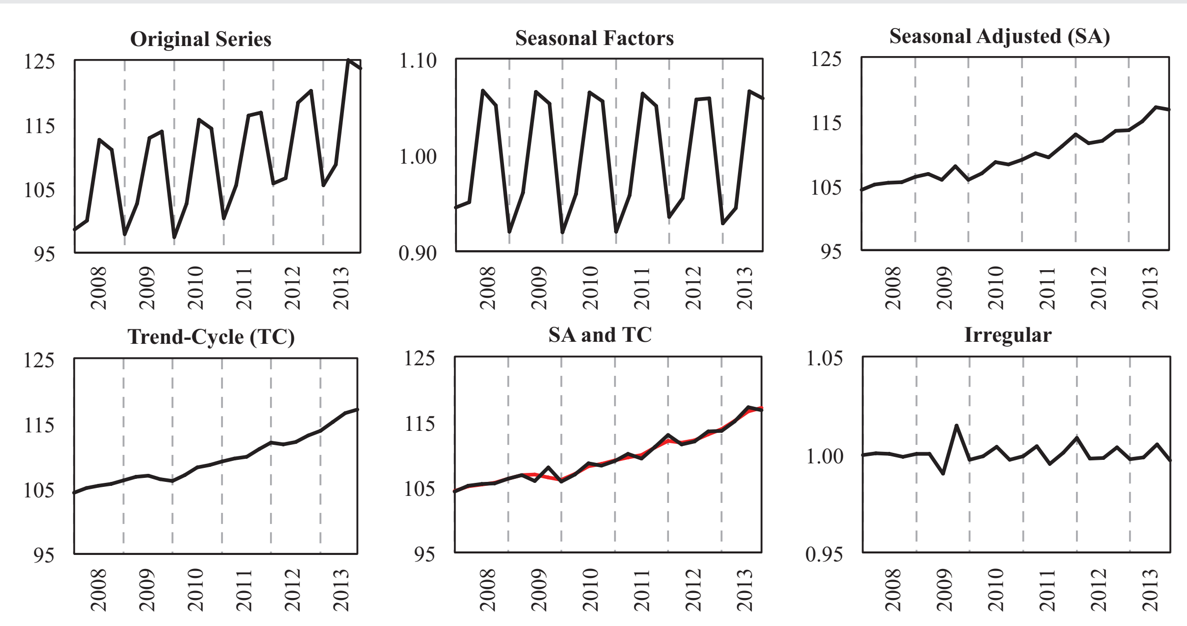
Figure 7.2 A Simulated Series with Trend, Seasonal, Calendar, and Irregular Effects (continued)

Quarter	Value	Quarter	Value	Quarter	Value	Quarter	Value	Quarter	Value
q1 1994	76.42	q1 1998	86.43	q1 2002	91.43	q1 2006	93.37	q1 2010	97.37
q2 1994	78.49	q2 1998	87.89	q2 2002	91.54	q2 2006	94.86	q2 2010	102.64
q3 1994	86.87	q3 1998	96.92	q3 2002	103.98	q3 2006	107.07	q3 2010	115.67
q4 1994	87.51	q4 1998	98.80	q4 2002	101.33	q4 2006	104.08	q4 2010	114.31
q1 1995	78.05	q1 1999	87.90	q1 2003	90.83	q1 2007	93.78	q1 2011	100.32
q2 1995	79.25	q2 1999	90.11	q2 2003	92.51	q2 2007	97.75	q2 2011	105.47
q3 1995	87.54	q3 1999	98.75	q3 2003	104.72	q3 2007	108.32	q3 2011	116.31
q4 1995	89.85	q4 1999	99.46	q4 2003	101.38	q4 2007	108.63	q4 2011	116.83
q1 1996	80.97	q1 2000	90.98	q1 2004	90.88	q1 2008	98.65	q1 2012	105.77
q2 1996	82.46	q2 2000	92.52	q2 2004	92.60	q2 2008	100.02	q2 2012	106.61
q3 1996	92.12	q3 2000	101.01	q3 2004	105.51	q3 2008	112.52	q3 2012	118.39
q4 1996	93.88	q4 2000	100.67	q4 2004	103.36	q4 2008	111.02	q4 2012	120.23
q1 1997	84.42	q1 2001	90.83	q1 2005	93.96	q1 2009	97.86	q1 2013	105.53
q2 1997	85.70	q2 2001	92.38	q2 2005	94.46	q2 2009	102.67	q2 2013	108.70
q3 1997	95.82	q3 2001	101.85	q3 2005	106.84	q3 2009	112.86	q3 2013	124.98
q4 1997	96.52	q4 2001	101.97	q4 1994	105.08	q4 2009	113.83	q4 2013	123.65

The series shown in Figure 7.2 was simulated using a seasonal ARIMA model $(0,1,1)(0,1,1)_4$. The series includes a composite deterministic effects proportional to the Easter period and the number of working days.

Example 7.1 Seasonally Adjusted Series, Seasonal, Irregular, and Trend-Cycle Components

Multiplicative decomposition approach using X-11

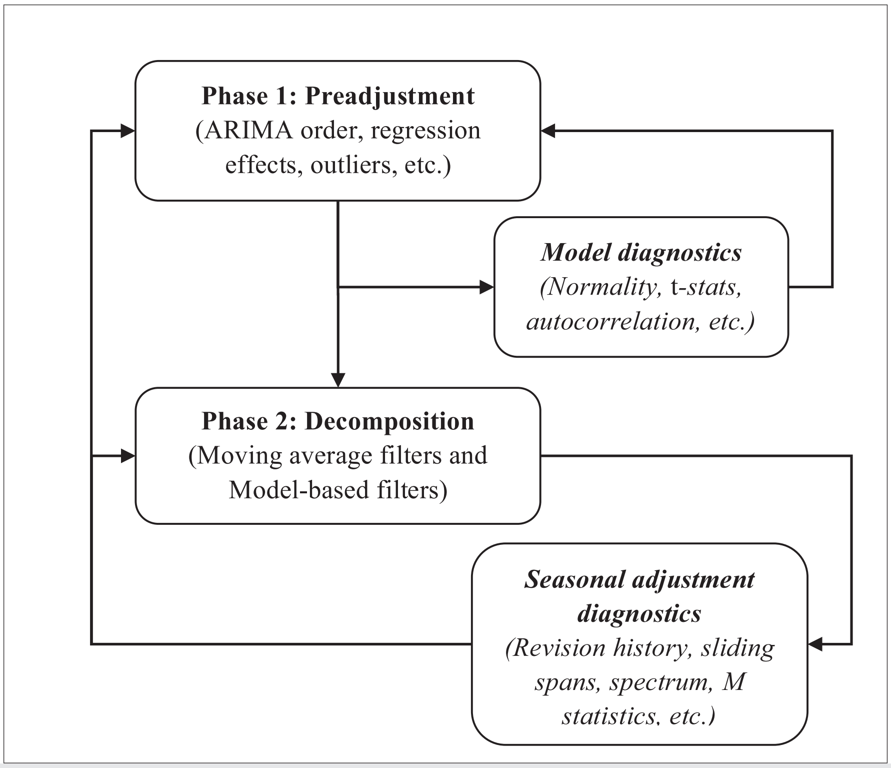


Example 7.1 Seasonally Adjusted Series, Seasonal, Irregular, and Trend-Cycle Components (continued)

	Original	Seasonal	Irregular	SA	TC
	(1)	(2)	(3)	(4)=(1) /(2)	(5)=(4) /(3)
q1 2009	97.9	0.920	1.000	106.3	106.3
q2 2009	102.7	0.961	1.000	106.9	106.8
q3 2009	112.9	1.066	0.990	105.9	106.9
q4 2009	113.8	1.054	1.015	108.0	106.5
q1 2010	97.4	0.920	0.997	105.9	106.1
q2 2010	102.6	0.960	0.999	106.9	107.0
q3 2010	115.7	1.065	1.004	108.6	108.2
q4 2010	114.3	1.056	0.997	108.3	108.6
q1 2011	100.3	0.920	0.999	109.0	109.1
q2 2011	105.5	0.958	1.004	110.1	109.6
q3 2011	116.3	1.063	0.995	109.4	109.9
q4 2011	116.8	1.051	1.001	111.2	111.1
q1 2012	105.8	0.936	1.008	113.0	112.1
q2 2012	106.6	0.955	0.998	111.6	111.8
q3 2012	118.4	1.057	0.998	112.0	112.2
q4 2012	120.2	1.059	1.004	113.5	113.1

The table and charts show the seasonal adjustment results for the simulated series shown in Figure 7.2. This decomposition has been obtained by the X-13A-S program, using the X-11 filter with default parameters and automatic identification of preadjustment effects.

Box 7.2 Main Elements of Seasonal Adjustment Procedures



[1] Diagram based on Findley and others (1998).

multiplicative, or other mixed forms), (ii) the order of the ARIMA model, (iii) calendar effects,[10] and (iv) outliers and other intervention variables.

7.21 The second stage performs a *decomposition* of the preadjusted series into unobserved components. The series adjusted for deterministic effects is decomposed into three unobserved components: trend-cycle, seasonal, and irregular. This section illustrates the two most applied decomposition methods for seasonal adjustment: the X-11 filter and the SEATS filter. After unobserved components are estimated, the adjustment factors identified in the first stage (calendar effects, outliers, etc.) are allocated to their respective component so to end up with a full decomposition of the original series into final trend-cycle, seasonal, calendar, and irregular components. The seasonally adjusted series is obtained as the series without seasonal and calendar effects.

7.22 Although the two steps are treated separately, they should be considered fully integrated in any seasonal adjustment procedure. Different choices in the preadjustment phase lead to different decomposition results. Also, results from the time series decomposition may point to changes in the preadjustment stage. A careful analysis of the diagnostics in the two stages (as discussed in "Quality Assessment of Seasonal Adjustment" section) is fundamental to determine whether the seasonal adjustment results are of acceptable quality.

7.23 The X-13A-S program implements this two-stage procedure. This software allows the user the choice between the X-11 and the SEATS filters within the same environment.[11] Because the same diagnostics are produced for the two filters, a comparative assessment between the two methods is now feasible for any series. Thanks to this flexibility, X-13A-S is (at the time of writing) the recommended seasonal adjustment procedure for producing seasonally adjusted QNA data.[12]

7.24 The rest of this chapter briefly presents the main elements of the preadjustment and decomposition steps.

Preadjustment

Model Selection

7.25 The first step in the preadjustment stage is to determine the decomposition model assumed for the series. For the X-11 decomposition, two main models are usually selected: the additive model and the multiplicative model.[13] In the additive model, the original series X_t can be thought of as the sum of unobserved components: that is,

$$X_t = T_t + S_t + C_t + I_t, \tag{1}$$

where

T_t is the trend-cycle component,

S_t is the seasonal component,

C_t is the calendar component, and

I_t is the irregular component.

The additive model assumes that the unobserved components are mutually independent from one another. The seasonal and calendar adjusted series for the additive model is derived by subtracting the seasonal and calendar components from the original series:

$$\begin{aligned} X_t^a &= X_t - (S_t + C_t) \\ &= T_t + I_t \, . \end{aligned} \tag{2}$$

7.26 In the multiplicative model, the series X_t is decomposed as the product of the unobserved components:

$$X_t = T_t \cdot S_t \cdot C_t \cdot I_t. \tag{3}$$

[10] The X-13A-S program offers an alternative method to estimate trading-day effects from the irregular component, inherited from the original X-11 method. However, the regression framework is the preferred approach for identifying and estimating calendar effects.

[11] X-13A-S implements a version of the SEATS procedure originally developed by Gomez and Maravall (1998). The X-13A-S manual cautions that "possible delays in the updating of versions may cause slight differences between the X-13A-S version of SEATS and the one at the Bank of Spain website." (U.S. Census Bureau, 2013).

[12] This chapter provides hints on how to set options in the X-13A-S input specification file. The input specification file contains a set of specifications (or "specs") that give information about the data and desired seasonal adjustment options. For more details on the theory and practice of seasonal adjustment, see the X-13A-S guide (U.S. Census Bureau, 2013) and the literature therein cited.

[13] In some cases, a mixed model may be chosen. In particular, X-13A-S includes a pseudo-additive model $X_t = T_t(S_t + I_t - 1)$ for series that show a multiplicative decomposition scheme but whose values are zero in some periods.

The multiplicative model assumes that the magnitude of the unobserved components is proportional to the level of the series. For the seasonal component, for example, a multiplicative model implies that seasonal peaks increase as the level of the series increases. Because the trend-cycle component determines the overall level of the series, the other unobserved components are expressed as percentages of T_t (usually called factors). The seasonal and calendar adjusted series for the multiplicative model is the ratio between the original series and the seasonal and calendar factors:

$$\begin{aligned} X_t^a &= X_t / (S_t \cdot C_t) \\ &= T_t \cdot I_t. \end{aligned} \tag{4}$$

7.27 With SEATS, the multiplicative model cannot be used directly because the model-based decomposition assumes that the unobserved components are additive. The multiplicative adjustment is approximated through the log-additive model:

$$\begin{aligned} \log(X_t) &= \log(T_t \cdot S_t \cdot C_t \cdot I_t) \\ &= \log(T_t) + \log(S_t) + \log(C_t) + \log(I_t). \end{aligned} \tag{5}$$

After an additive decomposition of the logged series is completed, the seasonally adjusted series is derived taking the exponential of the logged trend-cycle and irregular components:

$$\begin{aligned} X_t^a &= \exp[\log(T_t) + \log(I_t)] \\ &= T_t \cdot I_t. \end{aligned} \tag{6}$$

7.28 Sometimes, a graphical inspection of the series can give some clues about the best decomposition model for the series. If the variation of seasonal pattern increases with the level of the series, then the relationship between components is expected to be multiplicative and a multiplicative (or a log-additive) adjustment is recommended. Such a transformation allows stabilizing the evolution of the seasonal pattern and controlling for possible heteroscedasticity in the irregular component (and in the regression residuals). Alternatively, if the seasonal pattern appears to be stable over time and does not evolve in accordance with the movements of the trend, then no transformation is to be performed and the decomposition should follow an additive approach.

7.29 A visual inspection of the series may not be enough to determine the underlying relationship between components. In addition to the expert knowledge about the series, X-13A-S implements an automatic selection procedure to decide whether the series should be log transformed or not.[14] This automatic tool should be used when seasonal adjustment is applied for a large number of time series. However, the automatic choice from the X-13A-S should always be validated individually for important time series.

7.30 If the multiplicative approach is chosen, final components have the nature of multiplicative factors: that is, the seasonal and irregular components will be ratios centered around 1. On the other hand, if the additive approach is selected, seasonal and irregular components will have the form of addends and will be centered around 0 (additively neutral).

7.31 The next step in the preadjustment phase is to identify an ARIMA model for the series. The ARIMA selection process should be seen in conjunction with the choice of regression effects. In effect, using certain regression variables may change the order of the ARIMA model. An ARIMA with regression effects is called regARIMA model. For the sake of clarity, however, ARIMA model and regression effects are treated separately in this presentation.

7.32 Using the same notation of the X-13A-S manual, an ARIMA model for seasonal time series can be written as follows:

$$\phi(B)\Phi(B^s)(1-B)^d(1-B^s)^D Y_t = \theta(B)\Theta(B^s)\varepsilon_t, \tag{7}$$

where

Y_t is the original series (possibly preadjusted for deterministic effects);

B is the lag operator, which is defined by $Y_{t-1} = BY_t$;

s is the seasonal frequency, 4 for quarterly series and 12 for monthly series;

$\phi(B) = 1 - \phi_1 B - \ldots - \phi_p B^p$ is the regular autoregressive (AR) operator of order p;

$\Phi(B) = 1 - \Phi_1 B^4 - \ldots - \Phi_P B^{4P}$ is the seasonal AR operator of order P;

$\theta(B) = 1 - \theta_1 B - \ldots - \theta_q B^q$ is the regular MA operator of order q;

[14] The spec TRANSFORM enables the automatic transformation selection procedure. TRANSFORM can also be used to select automatically between the additive and multiplicative models for X-11.

$\Theta(B) = 1 - \Theta_1 B^Q - \ldots - \Theta_Q B^{4Q}$ is the seasonal MA operator of order Q; and

ε_t is a white noise process.

7.33 Identifying an ARIMA model consists in determining the orders of the AR operators (p for nonseasonal and P for seasonal), the MA operators (q and Q), as well as establishing the nonseasonal and seasonal integration orders (d and D). [15] A seasonal ARIMA model for a quarterly series is usually indicated as $(p,d,q)(P,D,Q)_4$. In X-13A-S, the following automatic selection procedure is implemented to identify the ARIMA order:[16]

- A default model is estimated. The default model for quarterly series is $(0,1,1)(0,1,1)_4$, also known as the "airline" model. This model is parsimonious (only two parameters are estimated) and usually fits very well-economic time series. Regression effects are also identified and removed using the default model.
- The differencing orders d and D are estimated by performing a series of unit root tests.
- The ARMA[17] order $(p,q)(P,Q)_4$ is selected by comparing values of a statistical information criterion[18] of a number of models, up to a maximum order for the regular and seasonal ARMA polynomial which can be specified by the user.
- Diagnostics on the residuals for the chosen ARIMA model are compared with those from the default model. Based on these tests, the final model is selected and validated.

7.34 Selecting the correct ARIMA model has important consequences in the seasonal adjustment process.

a. The chosen order of the ARIMA model may affect the automatic selection process of calendar effects and outliers.

b. The ARIMA model is used to produce forecasts and backcasts that are necessary to address the asymmetry of filters at the endpoints of the series.

c. The ARIMA order chosen for the original series is central to the SEATS decomposition, because the seasonal and trend filters are derived from the coefficients of the estimated ARIMA model (as discussed later in this section).

7.35 In general, users should consider the ARIMA order automatically selected by the X-13A-S program as the baseline model. Changes to the baseline model should be done when the model statistics are not satisfactory. For example, the model should be changed when diagnostics on the residuals signal misspecification of the automatically identified model. Parsimonious models should always be preferred to complex models. In most cases, the differencing orders d and D are 0 (if the component is stationary) or 1 (if the component is nonstationary). Double differencing may occur for some series with persistent movements (e.g., price data or deflators). Mixed ARMA models, which are models where AR and MA operators are both present in the seasonal or nonseasonal parts, should usually be avoided. For most series, the default model $(0,1,1)(0,1,1)_4$ often works very well with seasonal economic time series and should be considered when no other model provides satisfactory results.

Calendar Effects

7.36 Calendar effects should be removed from the series because they could affect negatively the quality of decomposition into unobserved components. For example, consider the effect from a different number of working days in two periods. When a month contains more working days than usual, series measuring economic activities may present a spike in that particular month due to the fact that there is more time for production. This effect cannot be captured by any linear representation of the series (like an ARIMA model), and will be allocated in the time series decomposition process mostly to the irregular component. As a result, the seasonally adjusted series will present an increase that is merely attributable to the different number of working days in the two periods compared. To avoid such distortions, calendar effects

[15] Integrated series refers to nonstationary underlying processes that must be differentiated in order to turn the series into a stationary process. The order of integration (i.e., number of unit roots in the autoregressive polynomial) reflects the need for differentiation of the time series to become stationary.

[16] The automatic ARIMA model identification procedure implemented in X-13A-S is based on the one available in the TRAMO program (Gomez and Maravall, 1996). In X-13A-S, the reference spec is AUTOMDL. For more details on the procedure, refer to the X-13A-S manual (U.S. Census Bureau, 2013).

[17] ARMA stands for autoregressive moving average. In contrast with ARIMA models, ARMA processes are stationary and do not require differentiation.

[18] X-13A-S uses the Bayesian information criterion (BIC) for selecting the ARMA order. The best model is the one with the minimum BIC value.

should be estimated and eliminated from the original series before the time series decomposition process.

7.37 All calendar effects are captured through specific deterministic effects that are meant to reproduce the changes in the calendar structure over time. These deterministic effects are called calendar regressors, as they are used as independent variables in the regARIMA model specified in the seasonal adjustment process. The most frequently used calendar regressors are formalized below.

7.38 The trading-days effect is defined by the following six regressors:[19]

$$\begin{aligned} td_t^1 &= (\#\,\text{Mondays} - \#\,\text{Sundays}) \\ td_t^2 &= (\#\,\text{Tuesdays} - \#\,\text{Sundays}) \\ &\dots \\ td_t^6 &= (\#\,\text{Saturdays} - \#\,\text{Sundays}) \end{aligned} \tag{8}$$

which calculate the difference between the number of each day of the week ($\#\,\text{Mondays}$, $\#\,\text{Tuesdays}$, ...) and the number of Sundays[20] ($\#\,\text{Sundays}$) in month t. The assumption is that each day of the week may influence the underlying phenomenon with different magnitude and direction.

7.39 The working-days effect is caught via a single regressor that compares the group of working days (e.g., Monday to Friday) with the group of weekend days (e.g., Saturday and Sunday) through the following equation:

$$wd_t = \left(\#\,\text{Weekdays} - \frac{5}{2}\#\,\text{Weekend days}\right). \tag{9}$$

The $5/2$ factor is needed to make the working-days regressor nil over a regular seven-day week composition. Any monthly deviation from the standard week will be reflected in the regressor (e.g., when wd_t is larger than zero, it means that month/quarter t has more working days than a standard week). This approach assumes that weekdays have similar effects (in sign and value) and are different from weekend-days effects.

7.40 The Easter date moves between March (q1) and April (q2).[21] The Easter regressor calculates the proportion of days before Easter falling in March (q1) and April (q2). After defining the length of the Easter effect, the regressor is calculated as follows:

$$e_t = \frac{W_t}{w} - \bar{W}, \tag{10}$$

where

W_t is the number of w days falling in month/quarter t and

$\bar{W}$ is the long-term proportion of days in month/quarter t.

Usually $\bar{W}$ can be approximated with 0.5 for both March (q1) and April (q2)[22]: that is, the number of days of the Easter effect is equally distributed between the two periods. In X-13A-S, the length w of the Easter effect can be provided by the user (from 1 to 25) or selected automatically by the program (lengths of 1, 8, and 15 are compared).

7.41 Finally, the leap year effect is captured as follows:

$$ly_t = \begin{cases} 0.75, & \text{if } t \text{ is February of a leap year} \\ -0.25, & \text{if } t \text{ is February of a non-leap year} \\ 0, & \text{otherwise.} \end{cases} \tag{11}$$

The regressor ly_t reproduces a deterministic four-year cycle with a peak in February of leap years; over a four-year period, the leap year effect is fully compensated by the negative effects in the subsequent non-leap years.

7.42 The adjustment for calendar effects should be performed only for those series for which there is both statistical evidence and economic interpretation of calendar effects. This assessment should be based on the statistical and economic significance of their regression coefficients. Statistically, a regression coefficient is said to be significantly different from zero

[19] The following presentation is based on the Monday-to-Friday workweek in place in most Western countries (the default one in X-13A-S). However, other countries have different workweeks (in particular, Sunday-to-Thursday in many Muslim countries). The groups of weekdays and weekend days should be defined according to the legal workweek in the country.

[20] National holidays should be considered non-working days. Therefore, the number of nonworking days should be increased by the number of national holidays and the number of working days should be decreased accordingly. The same holds true for the working-days regressor.

[21] Only the Catholic Easter is considered here. Orthodox Easter falls between April and May, thus it does not affect quarterly series.

[22] The Easter regressor may also be nonzero in February, but this happens very rarely.

Box 7.3 Test for Calendar Effects

A 1. Time Series Data (for the Span Analyzed)			
Regression Model			
Variable	Parameter Estimate	Standard Error	t-value
1-Coefficient trading day			
Weekday	0.0019	0.00065	2.97
**Saturday/Sunday (derived)	–0.0048	0.00162	–2.97
Leap year	0.0142	0.00384	3.71
Easter	0.0092	0.00250	3.66

Table A 1 from the X-13A-S output contains diagnostics on the estimated regression coefficients for the series presented in Figure 7.2. The first column shows the estimated values; the second column their standard errors; and the third column the *t*-statistics, which is the ratio between the parameter estimates and the standard errors. When the *t*-ratio (in absolute terms) is large enough (say, larger than 2), the regression effect is said to be significantly different from zero and should be kept in the model. In the above example, three calendar effects are estimated in the model: (i) the working-day regressor (called in X-13A-S "1-Coefficient trading day"), (ii) the leap year regressor, and (iii) the Easter regressor. All the effects are significantly different from zero (i.e., their *t*-values are larger than 2 in absolute terms). For calendar effects, it is also important to look at the sign of the estimated coefficients to validate them in economic terms. In most cases, calendar effects are expected to have a positive impact on national accounts transactions and should appear with positive sign; on the contrary, negative coefficients should be found when the effect is expected to reduce the activity (e.g., working days for tourism-related services).

when the associated *t*-statistic is higher (in absolute value) than a certain threshold (usually 2, but lower thresholds may be acceptable). Furthermore, the sign of the regression coefficient should be interpretable from an economic standpoint. For example, the leap year effect should always be positive, the working-days effect for economic activities where production is organized on a five-day week should be positive, the Easter effect should be positive for consumption of tourism-related services[23] and negative for other producing activities, etc. When the estimated coefficient for a calendar effect is not statistically significant (i.e., *t*-statistic lower than a chosen threshold) or is difficult to interpret in economic terms (i.e., implausible size or sign of the coefficient), the series should not be adjusted for that calendar effect. As an example, Box 7.3 shows the output returned by X-13A-S to evaluate the results on calendar effects.

7.43 Two compilation aspects concern the frequency of calculation of calendar effects. First, calendar effects are statistically more evident on monthly series than on quarterly series. The quarterly aggregation reduces (and sometimes eliminates) the variability of calendar regressors up to a level that makes them hardly detectable in the estimation process. For this reason, the adjustment for calendar effects should be preferably performed on monthly indicators and then the resulting effect aggregated at the quarterly level.

7.44 On the other hand, trading-days and working-days effects may also be relevant at the annual frequency. Adjacent years may contain up to three or four working days of difference, which may distort the comparison between annual observations. When such effects are significant on an annual basis, it may be necessary to calculate annual aggregates adjusted for calendar effects (mostly trading/working-days and leap year) and use these as annual benchmarks for the quarterly seasonally and calendar adjusted estimates.[24] When calendar effects are negligible on an annual basis, quarterly seasonally and calendar adjusted estimates can be benchmarked to the original annual national accounts (ANA) aggregates.

7.45 X-13A-S provides predefined calendar effects. Furthermore, the program allows user-defined regressors to be included in the regARIMA model. The user can prepare any specific calendar effect and test its economical and statistical significance from the results returned by the program. This functionality

[23] In some countries, Easter creates a peak in retail trade activity due to the increase in household spending; in other countries, however, most shops are closed during the Easter holiday.

[24] Annual data adjusted for calendar effects can be obtained by aggregating the quarterly calendar adjusted series returned by X-13A-S. When calendar adjustment is applied to monthly indicators, annual data of national accounts adjusted for calendar effects should be derived proportionally to the adjustment derived on the indicator or via a regression approach (Di Palma and Marini, 2004).

is important for adjusting QNA data for country-specific effects not included as a built-in option of X-13A-S (e.g., Chinese New Year, Ramadan, etc.). An automatic selection procedure of calendar effects (both built-in and user-defined) is available.[25] Similar to the ARIMA order, the automatic selection procedure should always be used when seasonal adjustment is applied to a large number of time series. However, the sign of each calendar effect accepted by X-13A-S should always be evaluated in economic terms. Moreover, regression coefficients associated with calendar effects should remain stable as new observations are incorporated in the series. Estimated calendar effects that are not supported by economic rationale should not be included in the adjustment.[26]

7.46 QNA series should not be adjusted for bridge days or extreme weather effects. Bridge days are working days that fall between a public holiday and the weekend. Because many employees take off bridge days for a long weekend, bridge days may result in lower output than on a normal working day. Extreme weather effects like heavy rain or snow days can affect the level of output in many industries, including construction and tourism activities. Nevertheless, their effects may be local rather than national. In some cases, the loss in production may be recovered in subsequent periods. Country experience shows that the estimation of bridge day and extreme weather effects is extremely uncertain.[27] Normal weather-related effects should be treated as part of the regular seasonal adjustment process, whereas extreme effects can be adjusted using outliers or ad hoc intervention variables.

Outliers and Intervention Variables

7.47 Unusual events cannot be predicted ex ante; but once they have manifested in the series, they should be understood and modeled in the seasonal adjustment process through specific regression variables. The reason is that leaving abnormal values in the series may lead to significant distortion in the decomposition of QNA series such as production, consumption, investment, etc. For instance, unexpected extreme weather conditions (droughts, floods, etc.) can seriously affect output of agricultural crops. The sudden fall in the agricultural activity should be allocated to the irregular component, without influencing the long-term trend or seasonality in agriculture. Other unusual events may be allocated to the trend (e.g., level shift) or seasonality (e.g., seasonal break). To achieve this, abnormal values (commonly known as outliers) should be taken out of the original series and reintroduced in the final components after the decomposition step has been applied to the series preadjusted for such events. Other known events that are supposed to have a significant impact on the series should be treated in the preadjustment step by means of intervention variables (e.g., strikes, temporal shutdowns, and quarantines).

7.48 X-13A-S contains a procedure for automatic identification of additive outliers, temporary change outliers, and level shifts (see Figure 7.1).[28] This procedure consists of including dummy-type variables in the regression model for all possible periods within a specified time span. The program calculates regression coefficients for each type of outlier specified and adds to the model all outliers with absolute *t*-statistics exceeding a critical value.[29] Furthermore, X-13A-S allows the use of predefined intervention variables.[30] As mentioned in "Main Principles of Seasonal Adjustment" section, three common intervention variables are temporary level shifts, seasonal outliers, and ramps (see Figure 7.1). Other intervention variables can be created from the user externally and given as input to the program.

7.49 Seasonal adjustment results are severely affected by outliers and intervention variables. A different combination of regression effects can produce significant changes in the estimation of trend and seasonal components. As for any other regression effects, outliers should be evaluated based on the statistical significance of their regression coefficients (through

[25] In X-13A-S, the spec name for the automatic identification of calendar effects is REGRESSION. The test is based on the Akaike Information Criterion.

[26] X-13A-S also provides another option for estimating trading-day and Easter effects based on ordinary least square (OLS) regression analysis from the final irregular component. The spec name for this option is X11REGRESSION. However, regARIMA models for estimating calendar effects are preferred because they usually provide better results than OLS regressions.

[27] See Deutsche Bundesbank (2012).

[28] In X-13A-S, the spec name for the automatic identification of outliers is OUTLIER.

[29] Default critical values for the outlier *t*-statistics depend on the series length.

[30] In X-13A-S, the spec name for including intervention variables in the regARIMA model is REGRESSION.

t-statistics) and their robustness. Outliers are particularly difficult to detect and interpret in real time, especially during periods of strong economic changes such as recessions.[31] When abnormal values first arise in a series, they should either be adjusted as additive outliers or left unadjusted. Level shifts or other transitory effects involving more than one period should be taken into consideration only when future observations of the series make clear the nature of the event.

Time Series Decomposition Methods

7.50 For seasonal adjustment of QNA data, a choice should be made between two alternative methods: the X-11 filter and the SEATS filter. These methods are well documented and have become standard methods for seasonal adjustment of official statistics. Furthermore, their use increases comparability of seasonally adjusted time series across countries.

7.51 Both methods give satisfactory results for most time series and are equally recommendable. Countries should choose their preferred method according to statistical and practical considerations. The fact that X-13A-S offers both filters in the same program allows an easy comparison on series with different characteristics using a common set of diagnostics.[32] However, the choice may also be grounded on past experience, internal expertise, and subjective judgment. Once the choice is made, the same method should be used to seasonally adjust all the QNA series (indicators or final results) and clearly communicated to the public. Mixing different seasonal adjustment methods in the same statistical domain may reduce the level of comparability of seasonally adjusted series and cause confusion in the users.

7.52 Both X-11 and SEATS apply symmetric filters to the preadjusted series to derive estimates of trend-cycle, seasonal, and irregular components. However, the nature of such filters differs significantly from one another. The following provides a brief description and highlights the main differences between the two methods.

The X-11 Filter

7.53 The X-11 filter is derived as an iterative process, which consists in applying a sequence of predefined MA filters. After the series is preadjusted and extended with backcasts and forecasts, it goes through three rounds of filtering and extreme value adjustments called "B, C, and D iterations."

7.54 The MA filtering procedure implicitly assumes that the irregular effect is approximately symmetrically distributed around their expected value (1 for a multiplicative model and 0 for an additive model) and thus can be fully eliminated by using a symmetric MA filter. Therefore, seasonality and trend-cycle components are isolated from the irregular components by means of a successive application of ad hoc MA filters.

7.55 The main steps of the X-11[33] multiplicative adjustment[34] for quarterly data in the B, C, and D iterations are reproduced as follows:

Iteration B. Initial Estimates

a. *Initial Trend Cycle* (T_t^1). The original series Y_t is filtered using a weighted 5-term (2×4)[35] centered MA, which extracts an initial trend component from the series.

b. *Initial Seasonal–Irregular (SI) Ratios* (SI_t^1). The original series is divided by T_t^1 to give an initial (joint) estimate of the seasonal and irregular components SI_t^1.

c. *Initial Preliminary Seasonal Factors*. Irregular effects from the initial SI ratios are removed by applying a weighted 5-term (3×3) centered seasonal MA so as to derive an initial preliminary estimate of the seasonal factors.

d. *Initial Seasonal Factors* (S_t^1). Preliminary seasonal factors are then normalized to ensure that the annual average of the initial seasonal factors is close to 1.

[31] For a discussion on how to handle recession effects on seasonal adjustment, see the experiments in Ciammola and others (2010) and in Lytras and Bell (2013).

[32] The original version of SEATS, developed by Gomez and Maravall (1996), is implemented in the program TRAMO-SEATS (available at the Bank of Spain Web site). X-13A-S provides a close approximation of the SEATS decomposition results.

[33] The procedure described below is a summary of the detailed steps performed by the X-11 filter. For a more precise description of the X-11 filter, see Ladiray and Quenneville (2001, chapter 4).

[34] Multiplicative decomposition is the default X-11 method in X-13A-S. However, the decomposition method should be consistent with the type chosen in the preadjustment stage (automatically or manually). In the case of additive decomposition, subtractions are used instead of divisions. The spec to modify options of the standard X-11 filter is X11.

[35] An *N*×*M* centered moving average is obtained by applying simple moving averages of length *N* and *M* in succession. See Ladiray and Quenneville (2001) for a discussion of the properties of moving averages used in X-11.

e. *Initial Seasonally Adjusted Series* (A_t^1). The initial estimate of the seasonally adjusted series A_t^1 is derived by dividing the original series by the initial seasonal factors S_t^1: that is,

$$A_t^1 = \frac{Y_t^1}{S_t^1} = T_t^1 I_t^1 .$$

Iteration C. Final Seasonal–Irregular Ratios

a. *Intermediate Trend Cycle* (T_t^2). A revised estimate of the trend cycle is derived by applying a Henderson filter[36] to the initial seasonally adjusted series A_t^1. The Henderson filter is a (2*h*+1)-term symmetric filter whose values are designed to extract a trend component from the input series. For quarterly series, X-13A-S selects automatically a 5- or a 7-term Henderson MA based on the statistical characteristics of the data.

b. *Revised SI Ratios* (SI_t^2). Revised SI ratios are derived by dividing the original series Y_t by the intermediate trend cycle T_t^2.

c. *Revised Preliminary Seasonal Factors.* Revised preliminary seasonal factors are derived by applying a (3×5)-centered seasonal MA to the revised SI ratios SI_t^2.

Iteration D. Final Components

a. *Final Seasonal Factors* (S_t^2). As in stage B, preliminary seasonal factors are normalized to produce final seasonal factors.

b. *Final Seasonally Adjusted Series* (A_t^2). The original series is divided by the revised seasonal factors S_t^2 to derive the final seasonally adjusted series.

c. *Final Trend Cycle* (T_t^3). A final estimate of the trend-cycle component is derived by applying a Henderson MA to the final seasonally adjusted series A_t^2.

d. *Final Irregular* (I_t^3). A final estimate of the irregular component is derived by dividing the final seasonally adjusted series A_t^2 by the final trend cycle T_t^3.

7.56 In addition to the three-stage procedure described above, the X-11 filter implements an algorithm to reduce the impact of extreme values in the adjustment process. Based on a statistical analysis of SI ratios, extreme values are identified and temporarily replaced with average values in stages B and C, so as to eliminate their effects from seasonal factors.

7.57 One quick way to analyze the X-11 results is to look at the final SI ratios. X-13A-S produces a chart comparing the final seasonal factors with the SI ratios (see the example in Figure 7.3). Seasonality is expected to be stable over time. If SI ratios are too volatile relative to the seasonal factors, this indicates that the series contains a strong irregular component and the seasonal effects may absorb too much volatility. Shorter filters for extracting seasonal effects from SI ratios are warranted when the irregular component is large relative to the seasonal effects; longer filters are better to extract stable seasonal factors.[37]

The SEATS Filter

7.58 The SEATS filter is based on the ARIMA model-based (AMB) approach for seasonal adjustment. This approach consists of estimating an ARIMA model for the original (possibly preadjusted) series, deriving consistent ARIMA models for the unobserved components (trend-cycle, seasonality, and irregular), and estimating the components using an optimal signal extraction technique. An important property of the AMB approach is that the seasonal adjustment filter adapts itself to the particular structure of the series. Conversely, X-11 is an ad hoc seasonal adjustment filter that applies to every single series in the same manner regardless of the structure of the seasonal and nonseasonal components (although the filter length may be changed to better suit different characteristics).

7.59 The AMB approach implemented by SEATS is briefly illustrated below.[38] The following presentation of SEATS is informal, as a comprehensive illustration of the AMB approach for seasonal adjustment requires the use of advanced concepts of time-series analysis (such as spectral analysis and signal extraction theory) that go beyond the scope of this manual. The advantage of using seasonal adjustment programs such as X-13A-S[39] or TRAMO-SEATS is that they have been designed and equipped with automatic features that facilitate

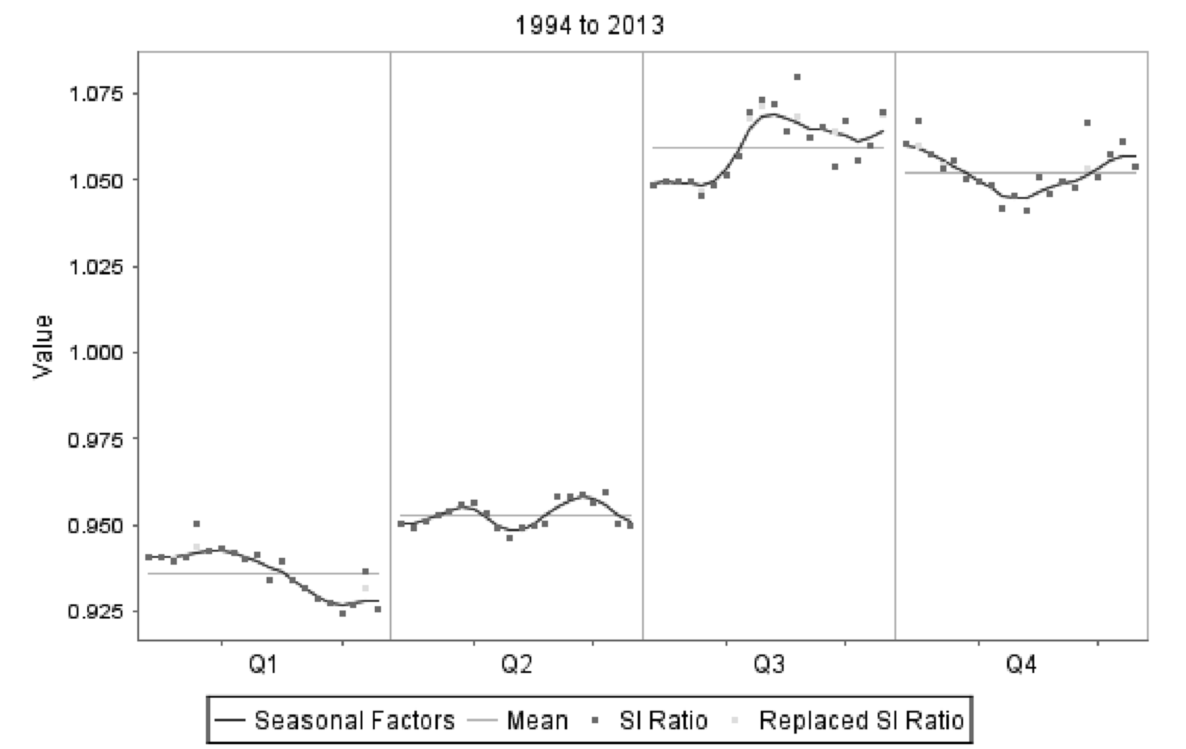
[36] A Henderson filter is a centered moving average whose weights are designed to extract a smooth trend cycle from series with noise.

[37] In X-13A-S, the spec name for modifying the length of the standard X-11 filter is X11.

[38] For an introduction to the AMB decomposition of time series and further reference, see Kaiser and Maravall (2000).

[39] In X-13A-S, the spec name for running SEATS is SEATS.

Figure 7.3 Seasonal Factors and Seasonal-to-Irregular Ratios



This chart is taken from the output of Win X-13, the windows interface of X-13A-S. The chart shows the seasonal factors by quarter, their average, and the seasonal-to-irregular (SI) ratios for the series presented in Figure 7.2.

the selection of seasonal adjustment options, making this task easy even for less experienced seasonal adjustment users. However, compilers who are interested in applying SEATS in the QNA should acquire a sound knowledge of the method, which will be necessary to help them evaluate and validate the results and be able to handle the adjustment of problematic series.

7.60 The ARIMA model (7) (see paragraphs 7.31–35), identified and estimated on the input series, is decomposed into ARIMA models for the trend-cycle, seasonal, and irregular components. A number of assumptions are made to derive an optimal decomposition (among infinite ones) of the estimated ARIMA model. First, components are assumed to be mutually independent (components are said to be orthogonal). This is not a harmless assumption, as it requires, for example, that the trend-cycle and seasonal components are independent from one another. However, this assumption is traditionally accepted in model-based seasonal adjustment methods. Second, the seasonal component captures all the seasonal movements in the series.[40] Finally, the variance of the irregular component is maximized over the variance of the other components.[41] This assumption implies that the trend-cycle and seasonal components estimated by SEATS tend to be stable, as most of the volatility is assigned to the irregular component.

7.61 The optimal estimator for a component[42] is derived by applying a specific symmetric filter to the input series:

$$\begin{aligned} A_t &= \ldots + v_2 Y_{t-2} + v_1 Y_{t-1} + v_0 Y_t + v_1 Y_{t+1} + v_2 Y_{t+2} + \ldots \\ &= v_0 Y_t + \sum_{j=1}^{\infty} v_j (B^j + F^j) Y_t = v(B,F) \end{aligned} \tag{12}$$

[40] In the frequency domain, this means that peaks at the seasonal frequencies in the spectrum are allocated to the ARIMA model of the seasonal component.
[41] A decomposition where the innovation of the irregular is maximized is called canonical decomposition.
[42] The estimator is optimal because it is the minimum mean square error (MMSE) estimator of the components.

where B is the backward operator and F is the forward operator, such that $Y_{t-j} = B^j Y_t$ and $Y_{t+j} = F^j Y_t$. The filter $\nu(B,F)$ is called the Wiener–Kolmogorov (WK) filter. Its weights depend on the ARIMA models derived for the unobserved components and vary with the characteristics of the series. Likewise the X-11 filter, the WK filter uses backcasts and forecasts at both ends of the series calculated from the ARIMA model estimated for the series. Because the WK filter is a convergent filter, it can be truncated after a relatively large number of periods to yield a finite-sample approximation of the infinite filter.

7.62 Several empirical studies have been conducted to compare the seasonal adjustment results from the X-11 and SEATS filters.[43] In general, both methods give satisfactory results and the choice should be made based on practical considerations and subjective judgment. However, a few general points for guidance can be given. Because of the canonical decomposition, SEATS tends to produce more stable seasonal components than X-11. Consequently, seasonally adjusted series with SEATS are expected to be more volatile than seasonally adjusted series with X-11. Furthermore, the quality of the SEATS decomposition mainly depends on the quality of the estimated ARIMA model, as a poorly fit model is likely to lead to a bad-quality (or even inadmissible) decomposition. Finally, the SEATS results are more subject to the effect of parameter uncertainty than for X-11. A greater instability in the estimation process of the regARIMA model may be expected for short series (5–6 or less years), as parameters are estimated on a small number of observations, and long series (20 or more years), which may present evolving patterns.

Seasonal Adjustment and Revisions[44]

7.63 Seasonal effects may change over time. The seasonal pattern may gradually evolve as economic behavior, economic structures, and institutional and social arrangements change. The seasonal pattern may also change abruptly because of sudden institutional changes. Seasonal filters estimated using centered MAs (like the X-11 and SEATS filters) allow the seasonal pattern of the series to change over time and allow for a gradual update of the seasonal pattern. This results in a more correct identification of the seasonal effects influencing different parts of the series.

7.64 On the other hand, centered MA seasonal filters also imply that the final seasonally adjusted values depend on both past and future values of the series. Thus, to be able to seasonally adjust the earliest and latest observations of the series, either asymmetric filters have to be used for the earliest and latest observations of the series or the series has to be extended by use of backcasts and forecasts based on the pattern of the time series. While the original X-11 program used asymmetric filters at the beginning and end of the series, X-13A-S (and its predecessors X-11-ARIMA and X-12-ARIMA programs) use ARIMA modeling techniques to extend the series so that less asymmetric filters can be used at both ends of the series.

7.65 Studies have shown that using ARIMA models to extend the series before filtering generally significantly reduces the size of these revisions compared with using asymmetric filters.[45] These studies have shown that, typically, revisions to the level of the series as well as to the period-to-period rate of change are reduced. Use of regARIMA models, as offered by X-13A-S, may make the backcasts and forecasts more robust and thus further reduces the size of these revisions compared with using pure ARIMA models. The reason for this is that regARIMA models allow calendar effects and other effects captured by the regressors to be taken into account in the forecasts in a consistent way. Availability of longer time series should result in a more precise identification of the regular pattern of the series (the seasonal pattern and the ARIMA model) and, in general, also reduce the size of the revisions.

7.66 Consequently, new observations generally result in changes in the estimated seasonal pattern for the latest part of the series and subject seasonally adjusted data to more frequent revisions than the original nonseasonally adjusted series. This is illustrated in Example 7.2. Furthermore, revisions to one observation in the original series may lead to changes in some of the estimated parameters, which in turn results in revisions to more than one period in the seasonally

[43] For example, see Hood and Findley (1999) and Scott, Tiller, and Chow (2007).

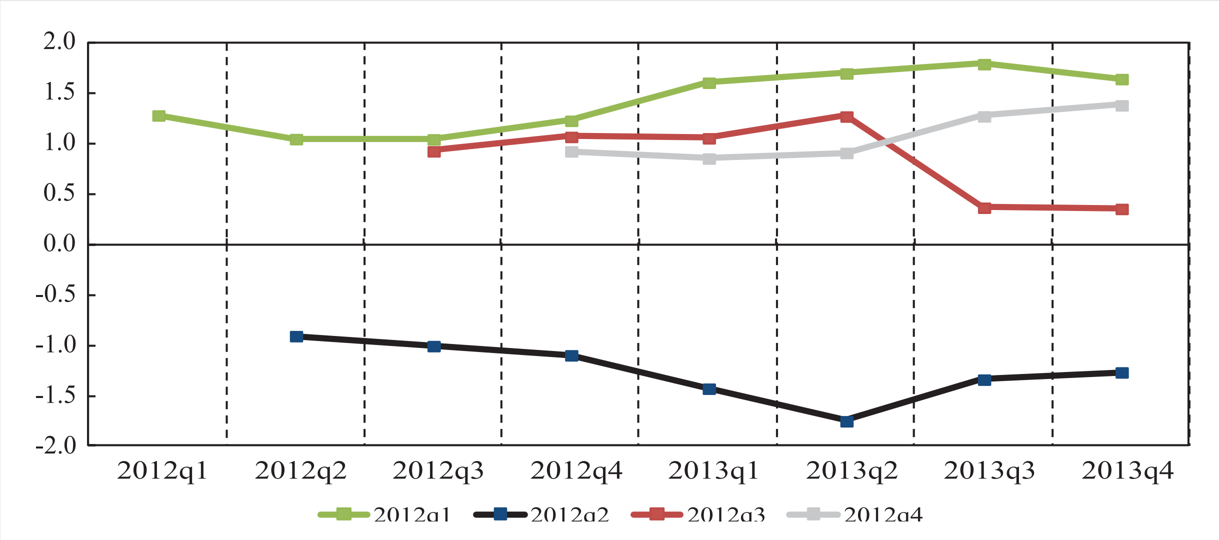
[44] This section focuses on revisions generated by seasonal adjustment. For more general considerations on the revision policy of QNA, see Chapter 12.

[45] See among others Bobbitt and Otto (1990), Dagum (1987), Dagum and Morry (1984), and Huyot and others (1986).

Example 7.2 Revisions to the Seasonally Adjusted Series

Revisions to the seasonally adjusted estimates by adding new observations

(Original unadjusted data in Figure 7.2)



	Data Until															
	q1 2012		q2 2012		q3 2012		q4 2012		q1 2013		q2 2013		q3 2013		q4 2013	
Quarter	Index	Rate of Change (%)	Index	Rate of Change (%)	Index	Rate of Change (%)	Index	Rate of Change (%)	Index	Rate of Change (%)	Index	Rate of Change (%)	Index	Rate of Change (%)	Index	Rate of Change (%)
q1 2010	105.5	–2.4	105.6	–2.4	105.6	–2.3	105.7	–2.2	105.8	–2.1	105.9	–2.1	105.9	–2.0	105.9	–2.0
q2 2010	106.8	1.2	106.9	1.3	106.9	1.2	106.9	1.1	106.9	1.0	106.8	0.9	106.9	1.0	106.9	1.0
q3 2010	108.8	1.9	108.9	1.8	108.9	1.8	108.8	1.8	108.7	1.7	108.7	1.8	108.7	1.6	108.6	1.6
q4 2010	108.6	–0.2	108.5	–0.3	108.5	–0.3	108.4	–0.4	108.3	–0.4	108.4	–0.3	108.3	–0.4	108.3	–0.3
q1 2011	108.7	0.1	108.6	0.1	108.6	0.1	108.7	0.3	108.9	0.5	109.0	0.6	109.1	0.8	109.0	0.7
q2 2011	109.7	0.9	110.0	1.3	109.9	1.2	109.9	1.1	109.9	0.9	109.8	0.7	110.1	0.9	110.1	0.9
q3 2011	109.7	0.0	109.7	–0.3	109.7	–0.2	109.8	–0.2	109.7	–0.2	109.6	–0.1	109.4	–0.6	109.4	–0.6
q4 2011	111.5	1.6	111.3	1.5	111.4	1.5	111.2	1.4	111.1	1.3	111.2	1.4	111.1	1.6	111.2	1.7
q1 2012	112.9	1.3	112.5	1.0	112.5	1.0	112.6	1.2	112.9	1.6	113.0	1.7	113.1	1.8	113.0	1.6
q2 2012			111.5	–0.9	111.4	–1.0	111.4	–1.1	111.3	–1.4	111.1	–1.8	111.6	–1.3	111.6	–1.3
q3 2012					112.4	0.9	112.6	1.1	112.5	1.1	112.5	1.3	112.0	0.4	112.0	0.4
q4 2012							113.6	0.9	113.4	0.9	113.5	0.9	113.4	1.3	113.5	1.4
q1 2013									113.3	–0.2	113.5	0.0	113.7	0.2	113.6	0.1
q2 2013											114.3	0.7	115.1	1.2	115.0	1.3
q3 2013													117.2	1.9	117.2	1.9
q4 2013															116.8	–0.3

Note how the seasonally adjusted data (like the trend-cycle data presented in Example 7.3 but less so) for a particular period are revised as later data become available, even when the *unadjusted data for that period are not revised*. The chart shows how the seasonally adjusted estimates of the 2012 quarters that are revised as new observations are added in the adjustment. For example, adding q2 2012 results in a downward adjustment of the q1 2012 rate of change in the seasonally adjusted series (1.0% from 1.3%) and considering the data up to q3 2013 brings the rate of change to its highest level (1.8%).

adjusted series. Seasonal adjustment is a major source of revisions of quarterly seasonally adjusted data, as discussed in Chapter 12.

7.67 In particular, estimates of the underlying trend-cycle component for the most recent parts of the time series may be subject to relatively large revisions at the first updates.[46] However, theoretical and empirical studies indicate that the trend cycle converges much faster to its final value than the seasonally adjusted series. In contrast, the seasonally adjusted series may be subject to lower revisions at the first updates but not negligible revisions even after one to two years. There are two main reasons for slower convergence of the seasonal estimates. First, the seasonal MA filters are significantly longer than the trend-cycle filters.[47] Second, revisions to the estimated regression parameters for deterministic effects may affect the entire time series.

7.68 Trend-cycle estimates for the most recent parts of the series must be interpreted with care, because they may be subject to strong revisions. Outliers may be one of the causes of significant revisions to the trend-cycle endpoint estimates, since it is usually not possible to distinguish an outlier from a change in the underlying trend cycle from a single observation. In general, several observations are needed to verify if changes are due to changes in the cycle or should be part of the irregular. Second, trend filters used at the end of the series will implicitly be applied on the most recent observed series as well as on the forecasts (which in turn depend on the observed data). Consequently, when a turning point appears at the current end of the series, it is not possible to discern whether it is a change in the trend, so its initial estimate will most likely make it persist in function of the previous trend. It is only after a lag of several observations that the change in the trend comes to light. While the trend-cycle component may be subject to large revisions at the first updates, however, it typically converges relatively fast to its final value. An illustration of this can be found by comparing the data presented in Example 7.2 (seasonally adjusted estimates) with those in Example 7.3 (trend-cycle estimates).

7.69 Producing seasonally adjusted (and trend-cycle) data on a continuous basis makes necessary to develop a well-defined and coherent revision policy. A revision policy should aim at minimizing both (i) the size and (ii) the frequency of revisions of seasonally adjusted data. Moreover, it should avoid the publication of unnecessary revisions that may be reverted when new observations are added to the series, since this confuses the users by generating uncertainty in the seasonally adjusted estimates.

7.70 A revision policy comprises at least two elements: the update strategy and the revision period. The update strategy defines the way in which options and models for seasonally adjustment are modified as new observations become available (or past observations are revised). This strategy plays an important role in the calculation of seasonally adjusted data, while the revision period has to do with the dissemination stage as it establishes the number of periods to be revised and released to the public each time new QNA results are published. These two components of revision policy are discussed below.

Update Strategies

7.71 Seasonal adjustment can be carried out using different update strategies. Basically, these strategies differ in how often models and options[48] for seasonal adjustment are reidentified as new or revised observations are available. Two strategies with opposite characteristics are usually compared: the concurrent adjustment strategy and the current adjustment strategy. Broadly speaking, they can be described as follows:

- In the concurrent adjustment, models, options, and parameters of seasonal adjustment are identified and estimated every time new or revised observations are made available. A concurrent strategy generates the most accurate seasonally adjusted data as they incorporate all the

[46] As illustrated in Example 7.3.

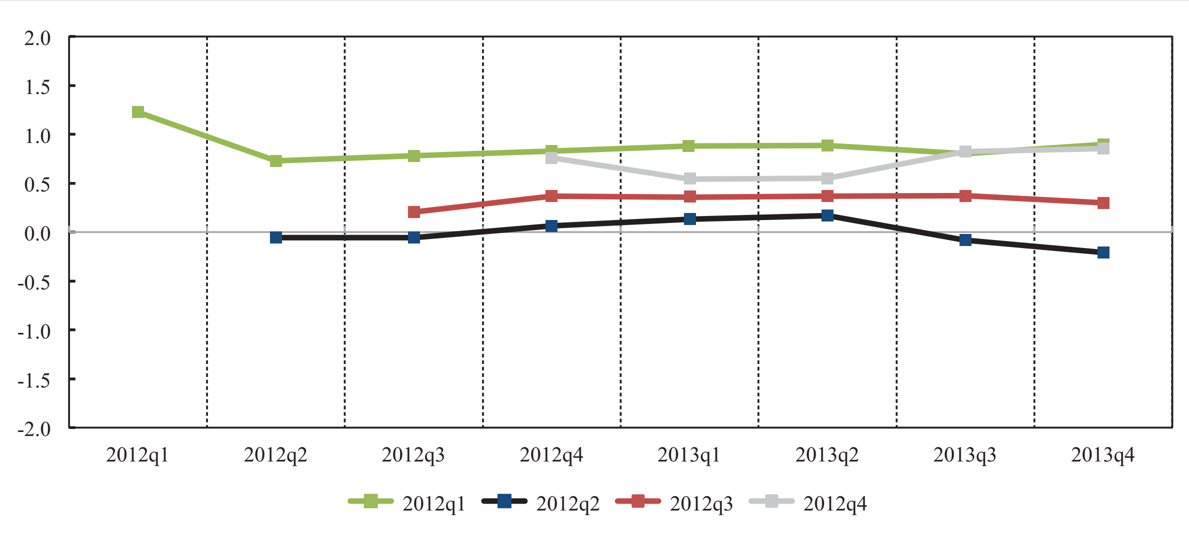
[47] For instance, the seasonal factors will be final after two years with the default 5-term (3×3) moving average seasonal filter (as long as any adjustments for calendar effects and outliers are not revised). In contrast, the trend-cycle estimates will be final after two quarters with the 5-term Henderson moving average trend-cycle filter (as long as the underlying seasonally adjusted series is not revised).

[48] Models and options correspond to the set of choices to be made in the preadjustment (i.e., calendar effects, outliers, etc.) and decomposition stages (i.e., filter length, allocation of autoregressive roots, etc.).

Example 7.3 Revisions to the Trend-Cycle Component

Revisions to trend-cycle estimates by adding new observations

(Original unadjusted data in Figure 7.2)



	Data Until															
	q1 2012		q2 2012		q3 2012		q4 2012		q1 2013		q2 2013		q3 2013		q4 2013	
Quarter	Index	Rate of Change (%)	Index	Rate of Change (%)	Index	Rate of Change (%)	Index	Rate of Change (%)	Index	Rate of Change (%)	Index	Rate of Change (%)	Index	Rate of Change (%)	Index	Rate of Change (%)
q1 2010	106.5	–0.1	106.4	–0.1	106.3	–0.1	106.2	–0.2	106.1	–0.3	106.1	–0.3	106.1	–0.2	106.1	–0.3
q2 2010	107.2	0.6	107.2	0.8	107.2	0.8	107.1	0.9	107	0.9	107.0	0.9	107.1	0.9	107.0	0.9
q3 2010	108.3	1.1	108.4	1.1	108.4	1.1	108.4	1.2	108.3	1.2	108.2	1.2	108.2	1.1	108.2	1.1
q4 2010	108.7	0.4	108.6	0.2	108.6	0.2	108.6	0.2	108.6	0.3	108.7	0.4	108.6	0.3	108.6	0.3
q1 2011	108.9	0.1	108.9	0.2	108.9	0.2	108.9	0.2	109	0.3	109.0	0.3	109.1	0.5	109.1	0.5
q2 2011	109.3	0.4	109.5	0.6	109.5	0.6	109.5	0.6	109.6	0.5	109.5	0.4	109.6	0.5	109.6	0.4
q3 2011	110.0	0.6	110.1	0.6	110.1	0.6	110.1	0.5	110	0.4	110.0	0.4	110.0	0.3	109.9	0.3
q4 2011	111.4	1.2	111.3	1.0	111.3	1.0	111.2	1.0	111.1	0.9	111.0	1.0	110.9	0.9	111.1	1.1
q1 2012	112.7	1.2	112.1	0.7	112.2	0.8	112.1	0.8	112	0.9	112.0	0.9	111.8	0.8	112.1	0.9
q2 2012			112.0	–0.1	112.1	–0.1	112.2	0.1	112.2	0.1	112.2	0.2	111.7	–0.1	111.8	–0.2
q3 2012					112.3	0.2	112.6	0.4	112.6	0.4	112.6	0.4	112.2	0.4	112.2	0.3
q4 2012							113.5	0.8	113.2	0.5	113.2	0.6	113.1	0.8	113.1	0.9
q1 2013									113.4	0.2	113.7	0.4	113.9	0.7	113.9	0.6
q2 2013											114.3	0.5	115.3	1.2	115.2	1.2
q3 2013													116.8	1.3	116.6	1.2
q4 2013															117.2	0.5

Large revisions may be expected to the initial estimates of the trend-cycle component. In this example, the q1 2012 rate of change is revised downward to 0.7 percent from 1.2 percent when the q2 2012 observation is added. However, the concurrent estimates of the trend cycle tend to converge faster to the final ones than the seasonally adjusted estimates. This appears evident in this example by noting the stability of the trend-cycle estimates in this chart in comparison with the revision pattern shown by the corresponding seasonally adjusted estimates in Example 7.2.

revisions to seasonal factors from current and updated observations. However, it may lead to more frequent revisions generated by (possible) changes in models and options.

- In the current adjustment, models, options, and parameters of seasonal adjustment are identified and estimated during specific review periods conducted, at a minimum, every year or every time a major revision in the original data occurs. Models, options, and parameters are kept fixed between two review periods. In-between review periods, seasonally adjusted data are obtained by dividing the original series by extrapolated seasonal and calendar factors (which implicitly means that models and options, including estimated parameters, are the same of the last review period). This strategy concentrates revisions to seasonally adjusted data during review periods, while revisions are not presented during non-review periods (unless past observations in the original data are revised). On the opposite, seasonally adjusted data during non-review periods may be less accurate as they do not incorporate all the up-to-date information in the calculation of seasonal and calendar factors.

7.72 From a purely theoretical point of view, and excluding the effects of outliers and revisions to the original unadjusted data, concurrent adjustment is preferable. New data carry new information about changes in the seasonal pattern, which should preferably be incorporated into the estimates as early as possible. Consequently, use of one-year-ahead forecasts of seasonal factors results in loss of information and, as empirical studies[49] have shown and as illustrated in Example 7.4, often in larger, albeit less frequent, revisions to the levels as well as the period-to-period rates of change in the seasonally adjusted data. Theoretical studies[50] support this finding.

7.73 Potential gains from concurrent adjustment can be significant but are not always. In general, potential gains depend on, among other things, the following factors:

- The stability of the seasonal component. A high degree of stability in the seasonal factors implies that the information gain from concurrent adjustment is limited and makes it easier to forecast the seasonal factors. On the contrary, rapidly moving seasonality implies that the information gain can be significant.
- The size of the irregular component. A high irregular component may reduce the gain from concurrent adjustment because there is a higher likelihood for the signals from the new observations about changes in the seasonal pattern to be false, reflecting an irregular effect and not a change in the seasonal pattern.
- The size of revisions to the original unadjusted data. Large revisions to the unadjusted data may reduce the gain from concurrent adjustment because there is a higher likelihood for the signals from the new observations about changes in the seasonal pattern to be false.

7.74 Furthermore, a concurrent adjustment strategy may not be ideal from the users' perspective. It is generally observed that most users of QNA data prefer a strategy where seasonally adjusted data are stable and not subject to frequent revisions. With a pure concurrent seasonal adjustment strategy, the risk of generating excessive noise in the revision process is very high. This is particularly true when options for seasonal adjustment are chosen on the basis of automatic selection procedures, which could modify previously selected choices on the basis of new or revised data. It is also not ideal from the producers' perspective. A concurrent adjustment requires human intervention to control and validate the results of seasonal adjustment, and this would have to be done during peak production times of QNA.

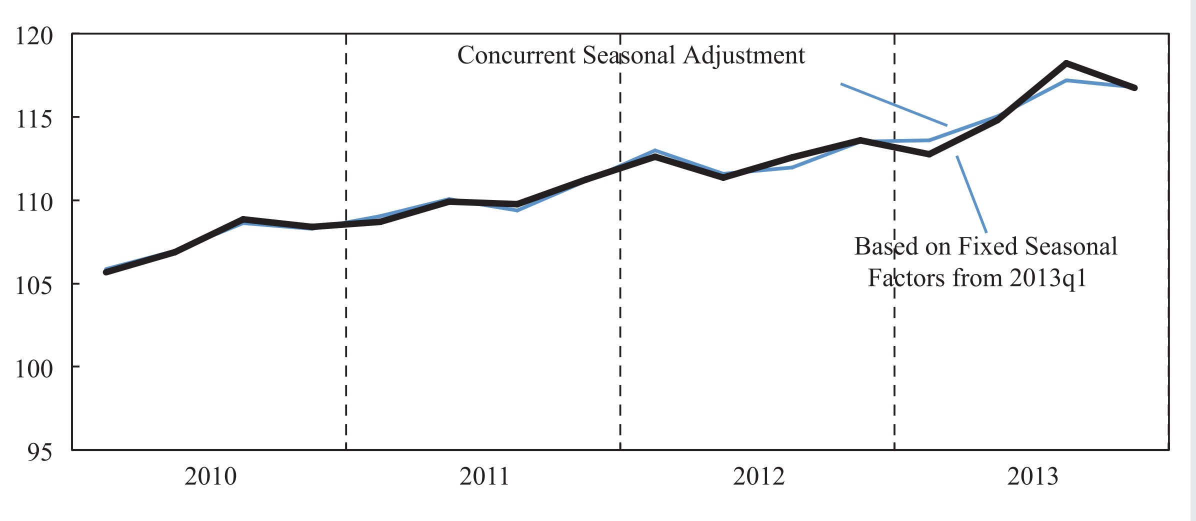
7.75 A more balanced alternative to the current and concurrent strategies is the so-called partial concurrent adjustment. Models and options are identified in every review period (once a year or anytime a major revision occurs) and are maintained up to the next review period. However, parameters are reestimated every time new observations are added to the series (i.e., parameters are concurrently estimated every time new data are available). Between two review periods, models and options should be checked for adequacy. Changes between review periods should be made only when exceptional events

[49] See among others Dagum and Morry (1984), Huyot and others (1986), Kenny and Durbin (1982), and McKenzie (1984).
[50] See among others Dagum (1982) and Wallis (1982).

Example 7.4 Concurrent Adjustment versus Current Adjustment

Concurrent adjustment versus current adjustment (one-year-ahead forecast of seasonal factors)

(Original unadjusted data in Figure 7.2)



Quarter	Concurrent Seasonal Adjustment (up to 2013q4)	Rate of Change (%)	Fixed Seasonal Factors from 2013q1	Rate of Change (%)
q1 2010	105.9	–2.0	105.7	–2.2
q2 2010	106.9	1.0	106.9	1.1
q3 2010	108.6	1.6	108.8	1.8
q4 2010	108.3	–0.3	108.4	–0.4
q1 2011	109.0	0.7	108.7	0.3
q2 2011	110.1	0.9	109.9	1.1
q3 2011	109.4	–0.6	109.8	–0.2
q4 2011	111.2	1.7	111.2	1.4
q1 2012	113.0	1.6	112.6	1.2
q2 2012	111.6	–1.3	111.4	–1.1
q3 2012	112.0	0.4	112.6	1.1
q4 2012	113.5	1.4	113.6	0.9
q1 2013	**113.6**	**0.1**	**112.8**	**–0.8**
q2 2013	**115.0**	**1.3**	**114.8**	**1.8**
q3 2013	**117.2**	**1.9**	**118.2**	**3.0**
q4 2013	**116.8**	**–0.3**	**116.7**	**–1.3**

The chart and table show the differences between concurrent adjustment (i.e., seasonal adjustment of data up to q4 2013) and current adjustment (i.e., fixed seasonal factors extrapolated from seasonal adjustment up to q4 2012). The latter series is taken from column "q4 2012" in Example 7.2 and extrapolated with the "Final adjustment ratio forecasts" provided by X-13A-S in table E 18.A. In this example, using one-year-ahead seasonal factors gives a decline of 1.3 percent in the seasonally adjusted series for q4 2013; instead, the concurrent adjustment, which incorporates the full sample of observations available, shows a much smaller reduction (–0.3%). However, the use of concurrent adjustment may produce significant revisions in the series. In this example, the seasonally adjusted rate of change of q3 2012 goes down to 0.4 percent from 1.1 percent.

occur and require special treatment in the adjustment. Otherwise, revisions to seasonally adjusted data are determined only by changes in the estimated parameters.

7.76 As an example of partial concurrent adjustment, consider the case of a review period scheduled for March of year T (when q4 of year $T-1$ is first released). ARIMA models, regression effects, outliers, and other intervention variables are identified including observations up to q4 of year $T-1$ (possibly using automatic selection features of seasonal adjustment programs). In the next estimation period (say, June), all the options selected in March are not changed (unless extraordinary revisions are done to the original series). Diagnostics on the residuals should be checked to evaluate if the new observation (i.e., q1 of year T) is an outlying observation. In that case, an additive outlier may be included in the model and tested for adequacy. The same approach should be taken for the following quarters until the next review period arrives (March of year $T+1$), where all models and options are reidentified and tested for adequacy.[51] This cycle then repeats every year.

7.77 A partial concurrent adjustment strategy represents the best compromise in the trade-off between preserving the accuracy of seasonally adjusted data and minimizing the size and frequency of revisions. An uncontrolled concurrent strategy should not be used in a production context, as changes in the seasonal adjustment options (especially if they rely on automatic procedures) may introduce large and unnecessary revisions from one quarter to the next. A current adjustment strategy could be an acceptable one for series with a stable seasonal component and low-variance irregular component.

Revision Period

7.78 The other element of revisions policy is to establish the revision period of QNA publications: that is, the number of previously published quarterly observations subject to revisions. In a concurrent approach (partial or full), the seasonally adjusted series changes in its entirety every time a new observation is added to the series (or an old observation is revised). The same happens in the review period when a current adjustment approach is adopted. Revisions may be sizeable up to four to five years before the last revised observation in the original series; for more distant observations, revisions tend to be rather small. This happens because seasonal adjustment filters assign larger weights to nearby observations than to distant ones.[52] However, reidentification of regression effects (e.g., outliers) or changes in the estimated regression coefficients may generate significant revisions in the whole seasonally adjusted series.

7.79 In a review period (i.e., when seasonal adjustment options are reidentified and models are reestimated), the best approach is to revise the entire seasonally adjusted series. At a minimum, revisions to seasonally adjusted data should be made for four or five completed years before the revision period of the original data. The revision period may be shortened when the reidentified models and options do not lead to long significant revisions to previously published seasonally adjusted data.

7.80 In a non-review period, the revision period should be selected on the basis of the update strategy:

a. In a partial concurrent adjustment strategy, seasonally adjusted series should be revised a minimum of two complete years before the revision period of the original data. Such a window permits the incorporation of the effects from reestimated regression coefficients and newly identified outliers in the seasonally adjusted data for the most recent periods. A minimum of two completed years is required to calculate quarter-to-quarter rates of change for the current year and the previous one using seasonally adjusted data coming from the same adjustment process. Previously published seasonally adjusted data before the two-year (or longer) revision period could be frozen provided there are no artificial breaks induced in the series. As an alternative, the entire seasonally adjusted series

[51] To preserve stability of seasonally adjusted series, previously identified models and options should be maintained as much as possible. Changes should be done only when they are supported by better statistical tests and diagnostics.

[52] For example, in the standard X-11 filter for quarterly series, zero weights are assigned to observations that are distant five or more years.

could be published when the size of historical revisions is within acceptable limits.

b. In a current adjustment strategy, the revision period of seasonally adjusted data should at least cover the revision period of the original data. When the original series is not revisable, this practice implies that the seasonally adjusted data for each current quarter (derived with extrapolated seasonal factors) is added to the previously published seasonally adjusted series until the next review period.

Quality Assessment of Seasonal Adjustment

7.81 The validation of seasonally adjusted results is an integral part of any seasonal adjustment procedure. Seasonal adjustment programs may return "seasonally adjusted" data even when the input data does not contain seasonal effects. On the other hand, they may provide seasonally adjusted series that still contain residual seasonal effects. Both situations can be avoided by looking at the output of seasonal adjustment programs.

7.82 Seasonally adjusted results should be evaluated and assessed on the basis of specific diagnostics on the preadjustment and decomposition results. "True" seasonally adjusted data do not exist, as the components are unobserved and can only be estimated from the original series. As a consequence, the quality of seasonally adjusted data should be judged upon the quality of the estimation process that has generated them and the dynamic features of the estimated components. The main diagnostics on the seasonal adjustment process are presented in this section. Furthermore, seasonally adjusted data in the QNA should also be viewed in the general framework of national accounts statistics. These other aspects of quality are discussed in "Particular Issues" section.

7.83 A key prerequisite for seasonal adjustment is that the original data should present clear and stable characteristic patterns. Without good-quality original data, there is no chance to get good-quality seasonally adjusted data. In particular, it is required that seasonal effects repeat themselves with similar pattern and intensity over time. Unstable seasonal effects increase the level of uncertainty in the decomposition, as it becomes more difficult to distinguish seasonal movements from other signals when they are not regular. The situation gets worse when the irregular component dominates the series' variability.

7.84 This section introduces both basic and advanced diagnostics on seasonal adjustment; all of them are produced by the X-13A-S program.[53] Basic diagnostics should include at a minimum tests for presence of identifiable seasonality in the original series, tests for residual seasonality in the seasonally adjusted series, significance tests of calendar effects and other regression effects identified in the preadjustment stage, and diagnostics on residuals from the estimated regARIMA model.

7.85 Advanced diagnostics of seasonal adjustment include sliding spans and revision history. Both of them look at the stability of the seasonal adjustment results as more observations are included in the estimation process. Because they require more time to be implemented and monitored than basic diagnostics, these tools should only be considered during review periods for the most relevant series in the QNA (or for series showing problematic issues).[54]

Basic Diagnostics

7.86 A visual inspection of the series to be processed is the first step when conducting seasonal adjustment. For most series, a simple visualization of observations against time highlights the most visible features of the series, such as an upward/downward trend, cyclical patterns, seasonal effects, outliers, and volatility. A seasonal plot[55] can also be used for a better understanding of the seasonal component. When the quarters move around different levels, this is a clear signal of seasonal effects in the series.

7.87 Seasonal adjustment should not be applied to series that do not present seasonal movements, or present seasonal movements that are hardly identifiable. X-13A-S calculates a combined test to check for

[53] X-13A-S provides a wide range of diagnostics to assess the seasonal adjustment results, both for the preadjustment and decomposition stages. Most of them apply equally to the X-11 and SEATS filters; therefore with X-13A-S, it is possible to compare alternative adjustments with a common set of quality measures.

[54] Both sliding spans and revisions history require a minimum length of the time series which depend on the length of the filters used in seasonal adjustment.

[55] A seasonal plot for quarterly series splits the series into four subseries by quarter and plots each quarter against the years. The average value for each quarter is usually shown in each subplot. Chart 7.3 is an example of seasonal plot of seasonal–irregular ratios.

Box 7.4 Test for the Presence of Seasonality in the Original Series

D 8. Final Unmodified SI Ratios

D 8.A. F-Tests for Seasonality

Test for the Presence of Seasonality Assuming Stability

	Sum of Squares	Degrees of Freedom	Mean Square	*F*-Value
Between quarters	2,509.7	3.0	836.6	1505.733[a]
Residual	42.2	76.0	0.6	
Total	2,551.9	79.0		

[a]*Seasonality present at the 0.1 percent level.*

Nonparametric Test for the Presence of Seasonality Assuming Stability

Kruskal–Wallis Statistic	Degrees of Freedom	Probability Level
70.2823	3	0.000%

Seasonality present at the 1 percent level.

Moving Seasonality Test

	Sum of Squares	Degrees of Freedom	Mean Square	*F*-Value
Between years	9.3038	20	0.465189	0.802
Error	34.8074	60	0.580123	

No evidence of moving seasonality at the 5 percent level.

Combined test for the presence of identifiable seasonality

Identifiable Seasonality Present

Table D 8.A informs the user whether stable seasonality is identified in the series. Three outcomes are possible: (i) identifiable seasonality present, (ii) possible identifiable seasonality present, and (iii) identifiable seasonality not present. The decision is made on the basis of the values of three tests: an *F*-test for the presence of seasonality assuming stability, the Kruskall and Wallis test (a nonparametric test for stable seasonality), and an *F*-test for moving seasonality (an *F*-test is a statistical test in which the test statistic has an *F*-distribution under the null hypothesis). Identifiable seasonality is shown when the first two tests signal the presence of stable seasonality, whereas the third test shows no evidence of moving seasonality. Table D 8.A should always be checked when a series is seasonally adjusted for the first time. When the program returns "identifiable seasonality not present," the series should not be seasonally adjusted.

the presence of identifiable seasonality (see Box 7.4). The decision is based on statistical tests that identify whether seasonality is present and, when present, look at whether the seasonal effects are sufficiently stable over the years. These tests are calculated on the preliminary SI ratios. Based on this combined test, X-13A-S returns in output one of the following outcomes: (a) identifiable seasonality is present, (b) identifiable seasonality is probably not present, or (c) identifiable seasonality is not present. In general, seasonal adjustment should not be performed when identifiable seasonality is not present (case c).[56]

7.88 After seasonal adjustment, an immediate check on results should be to verify that the seasonally adjusted series is free from seasonal effects. No residual seasonality or calendar effects should appear in the seasonally adjusted series. A statistical test is used in X-13A-S to check that there is no residual seasonality in the seasonally adjusted series (see Box 7.5). The test is similar to the one used to verify identifiable seasonality in the original series. A good seasonal adjustment process requires the test to be rejected, as this indicates that seasonal effects are not present in the seasonally adjusted series.[57]

7.89 The regARIMA model specified in the pre-adjustment phase should be assessed using standard

[56] The M7 statistic can also be helpful to determine whether identifiable seasonality is present, as explained later.

[57] Absence of seasonal and calendar effects can also be verified by looking at the spectrum diagnostics available in X-13A-S. For more details, see section 6.1 "Spectral Plots" of the X-13A-S reference manual (U.S. Census Bureau, 2013).

Box 7.5 Test for the Presence of Seasonality in the Seasonally Adjusted Series

D 11. Final Seasonally Adjusted Data

Test for the presence of residual seasonality

No evidence of residual seasonality in the entire series at the 1 percent level: $F = 0.02$

No evidence of residual seasonality in the last 3 years at the 1 percent level: $F = 0.14$

Table D 11 provides the results of the test for the presence of residual seasonality in the seasonally adjusted series. Residual seasonality in the seasonally adjusted series is a sign of misspecification of the seasonal adjustment model, and consequently a warning for users to modify the chosen specification. An *F*-test is calculated, similar to the one used in table D 8.A, to determine whether stable seasonality is present in the series. Differently from table D 8.A, a positive outcome is when the null of presence of seasonality is rejected. For the series of Figure 7.2, the program shows no evidence of seasonality in the entire series and in the last three years. The latter test is useful to identify a possible deterioration of the seasonal adjustment quality for the most recent periods.

regression diagnostics. Failures in the model specification may lead to incorrect seasonal adjustment results. Estimated residuals should be normally distributed and uncorrelated. X-13A-S provides normality tests and sample autocorrelation tests (the Ljung–Box Q tests) on the estimated residuals. If these tests indicate non-normality or autocorrelation in the residuals, actions should be taken in order to improve the fit of the regARIMA model. Non-normality may derive from large residuals not properly accounted for in the estimation process, which may be fixed using outliers or intervention variables. The presence of autocorrelation in the residuals may derive, in turn, from model misspecification.

7.90 Regression effects such as calendar effects, outliers, and any additional intervention variables should be retained in the model only when they are both statistically significant and economically meaningful. Standard *t*-statistics are used to assess the statistical significance of individual regressors; combined tests (such as *F*-tests) are used to assess the significance of a group of regressors (like the six-regressor trading-days effect). Particular care should be given to outliers, as different selections of outliers may generate large differences in the results. A regression effect is economically meaningful when the magnitude and sign of the estimated regression coefficient is in line with economic rationale. Box 7.3 provides an example on how to assess statistical significance and economic meaningfulness of calendar effects using the X-13A-S output.

7.91 The ARIMA order should be validated carefully, especially when it is automatically identified by the program. The ARIMA order is particularly relevant for SEATS adjustment, as the AMB decomposition implemented by SEATS completely relies on the specified ARIMA model; but it is also important for X-11 as the ARIMA model is used to calculate backcasts and forecasts needed to extend the series at both ends. In general, parsimonious models should be preferred as they are more likely to lead to admissible decompositions than models with many parameters. In this regard, the airline model $(0,1,1)(0,1,1)_4$ is a particularly suitable model because it only has two parameters to be estimated (the regular and the seasonal MA coefficients) and provides an admissible decomposition for a large region of the parameter space.[58]

7.92 Other useful diagnostics on seasonal adjustment are the 11 M diagnostics calculated by X-13A-S. All the M diagnostics (and the Q aggregate measure) take values between 0 and 3. Values higher than 1 indicate potential issues in the adjustment, while values between 0 and 1 are acceptable. The most important M diagnostics are the following:

- M7 measures the relation between moving and stable seasonality. High values of M7 may indicate an excessive amount of moving seasonality in relation to the stable seasonality. The M7 diagnostic may also be used as a test for existence of seasonality in the original series.
- M1 and M2 show how large is the irregular component in the series. M1 assesses the contribution of variance of the irregular to the original series in terms of lag 3 differences; M2 compares the irregular with the original series made stationary. High values of M1 and M2 may signal highly irregular series, which are more difficult to adjust.
- M6 compares the (annual) stability of seasonality with respect to changes in the irregular component. This diagnostic may suggest the use of filters with different lengths in order to split evolving seasonal patterns from irregular movements.

[58] The airline model may not provide an admissible decomposition when the seasonal MA parameter is large and positive.

- M8 and M9 deal with the stability of the seasonal component. High values of M8 and M9 may indicate high fluctuations in the seasonal pattern, which may reveal the existence of abrupt seasonal breaks. M10 and M11 are the same M8 and M9 diagnostics calculated using the last three years of data. They may be used to help identify problems at the end of the series.
- M3 and M5 calculate the significance of the irregular component in relation to the trend cycle. High values of M3 and M5 may indicate difficulty in extracting the trend-cycle component from the seasonally adjusted series. These diagnostics are relevant for trend-cycle estimation and interpretation of the results.

7.93 None of the M diagnostics can be used individually to assess the overall quality of seasonal adjustment, because each of them focuses on particular aspects of the results. Specific problems can be detected when monitoring these measures, which should be addressed as much as possible, but the quality of the whole process could still be deemed appropriate even when some of the M diagnostics are larger than one. Naturally, the adjustment should be considered unacceptable when all diagnostics fail. In order to give an overall assessment of the adjustment, the M diagnostics are aggregated in a single quality control indicator called Q (see Box 7.6).

Box 7.6 The M Diagnostics

	F 3. Monitoring and Quality Assessment Statistics	
No.	**Statistics**	**Value**
1.	The relative contribution of the irregular over one quarter span (from table F 2.B)	M1 = 0.020
2.	The relative contribution of the irregular component to the stationary portion of the variance (from table F 2.F)	M2 = 0.013
3.	The amount of quarter-to-quarter change in the irregular component as compared to the amount of quarter-to-quarter change in the trend cycle (from table F2.H)	M3 = 0.200
4.	The amount of autocorrelation in the irregular as described by the average duration of run (from table F 2.D)	M4 = 0.535
5.	The number of quarters it takes the change in the trend cycle to surpass the amount of change in the irregular (from table F 2.E)	M5 = 0.200
6.	The amount of year-to-year change in the irregular as compared to the amount of year-to-year change in the seasonal (from table F 2.H)	M6 = 0.482
7.	The amount of moving seasonality present relative to the amount of stable seasonality (from table F 2.I)	M7 = 0.056
8.	The size of the fluctuations in the seasonal component throughout the whole series	M8 = 0.242
9.	The average linear movement in the seasonal component throughout the whole series	M9 = 0.074
10.	Same as 8, calculated for recent years only	M10 = 0.233
11.	Same as 9, calculated for recent years only	M11 = 0.188
	ACCEPTED at the level 0.15	
	Q (without M2) = 0.17 ACCEPTED	

Table F 3 reports the M diagnostics and the aggregate Q measure. All the M diagnostics are defined in the range from 0 to 3, with an acceptance region from 0 to 1. The Q measure is a weighted average of the M diagnostics. In the above example (for the series from Figure 7.2), all the Ms are within the acceptance region.

Advanced Diagnostics

7.94 X-13A-S offers two advanced diagnostic tools for evaluating the reliability of seasonal adjustment results. The first tool is the sliding spans diagnostic. It measures how stable the seasonal adjustment estimates are when different spans of data in the original series are considered in the estimation process. When the sliding spans statistics signal instability of the seasonally adjusted data, this may indicate the presence of breaks in the series or moving seasonality. The second tool is the revisions history diagnostic. It looks at the revisions of seasonally adjusted data for the most recent quarters when new data points are introduced. Both tools are very useful for comparing alternative options for the same seasonal adjustment filter (either X-11 or SEATS) or for comparing the same options using two different filters (X-11 vs. SEATS).

7.95 When the sliding spans diagnostic is enabled,[59] the program selects four spans of data from the series. The span length is automatically chosen between 6 and 11 years, depending on the seasonal filter selected, the length of the series, and its frequency (monthly or quarterly). In the final span, the final observation is the last available period of the series. The other spans progressively exclude one year from the end and include one year at the beginning.[60] Seasonal adjustment is then performed on each span separately and the seasonally adjusted data are compared for the overlapping periods. Summary statistics are calculated to measure the stability of the estimates in the different spans. In particular, the sliding spans evaluation is carried out on the estimated seasonal factors and on quarter-to-quarter changes in the seasonally adjusted series. The program alerts the user when there is too much variation in the estimates for the same quarter and when the number of unstable seasonal factors or changes in the seasonally adjusted series exceeds recommended limits. Box 7.7 illustrates the sliding spans statistics.

7.96 The revision history diagnostic measures how much the seasonally adjusted figures change as new observations are introduced. A start date for the revisions history analysis is automatically selected by the program (or specified by the user). The program adjusts the series up to the start period of the revisions analysis; next, it adjusts the series including the next quarter; and so forth. The process is repeated until the whole series is seasonally adjusted. By default, the program calculates the differences between the concurrent estimates (first seasonal adjustment of a data point) and the final estimates (seasonal adjustment of the whole series) in the revision period. Other history analysis can be specified by the user. Summary statistics on the revisions to the seasonal adjusted and trend-cycle values (both in the levels and in percent changes) are calculated. This tool is particularly helpful when comparing different seasonal adjustment methods, with the method presenting the smaller statistics of revisions generally being preferable. Furthermore, revisions history can be useful when comparing direct and indirect seasonal adjustment of aggregates: the approach with the smallest amount of revisions should be preferred.[61] Conversely, it is less helpful when assessing the quality of adjustment of a single method as it is difficult to decide an acceptable level of revisions in absolute terms. Box 7.8 provides an example of revisions history.

Particular Issues

7.97 This section addresses a series of more QNA-specific issues related to seasonal adjustment. A first group of issues is related to how seasonal adjustment should be applied to maintain consistency in the framework of national accounts. Ideally, seasonally adjusted QNA variables should preserve the same accounting relationships existing between unadjusted variables. However, seasonal adjustment procedures may generate inconsistencies across variables and across frequencies because of existing nonlinearities in the estimation process. The issues considered here are the direct versus indirect calculation of seasonally adjusted aggregates; the relationship between price, volume, and value indices for seasonally adjusted series; and the temporal consistency between quarterly seasonally adjusted data with the annual benchmarks.

7.98 Further practical issues need consideration when producing seasonally adjusted QNA data. When the original series is too short (or too long),

[59] In X-13A-S, the spec name for the sliding spans diagnostic is SLIDINGSPANS; the spec name for revisions history is HISTORY.

[60] For the series used in this chapter, the span length selected is eight years and the four spans are q1 2004–q4 2010, q1 2005–q4 2011, q1 2006–q4 2012, and q1 2007–q4 2013.

[61] For a discussion on the direct versus indirect adjustment in the QNA, see paragraph 134.

Box 7.7 Sliding Spans Tables

S 1. Quarterly Means of Seasonal Factors
(movements within a quarter should be small)

	Span 1		Span 2		Span 3		Span 4		Max Difference (%)	All Spans	
First	93.04	min	92.96	min	93.07	min	92.77	min	0.33	92.96	min
Second	95.48		95.70		95.74		95.61		0.27	95.64	
Third	106.62	max	106.25	max	106.17	max	106.32	max	0.43	106.33	max
Fourth	104.94		105.15		105.07		105.35		0.39	105.14	

S 2. Percentage of Quarters Flagged as Unstable				
Seasonal factors	0 out of 32	(0.0%)		
Quarter-to-quarter changes in seasonally adjusted series	0 out of 31	(0.0%)		
Recommended Limits for Percentages				
Seasonal factors	15% is too high.			
	25% is much too high.			
Quarter-to-quarter changes in seasonally adjusted series	35% is too high.			
	40% is much too high.			
Threshold Values Used for Maximum Percent Differences to Flag Quarters as Unstable				
Seasonal factors	Threshold = 3.0%			
Quarter-to-quarter changes in seasonally adjusted series	Threshold = 3.0%			

Table S 1 presents average seasonal factors for each quarter calculated from four spans of data. For stable seasonal adjustment, the seasonal factors should be similar in the different spans.

Table S 2 calculates statistics on the stability of seasonal factors and quarter-to-quarter changes in the seasonally adjusted series. A quarter is flagged “unstable” when the seasonal factor (or the quarter-to-quarter change in the seasonally adjusted series) deviates more than 3.0 percent (a default threshold) from the average. Recommended limits are given by the program. For stable seasonal adjustment, the number of unstable seasonal factors and quarter-to-quarter changes should not exceed 15 percent.

additional efforts are required to produce seasonally adjusted data with an acceptable level of quality. A decision should also be made as to whether to apply seasonal adjustment to indicators (monthly or quarterly) or to QNA series, considering the pros and cons of both solutions. Finally, some suggestions are given on how the responsibility for producing seasonally adjusted QNA data should be organized in the QNA.

Direct versus Indirect Seasonal Adjustment of Aggregates

7.99 Seasonally adjusted series of aggregates can be derived (i) directly by adjusting the aggregates or (ii) indirectly by aggregating seasonally adjusted data of the component series. A typical example in the QNA is a seasonally adjusted estimate for GDP derived either by seasonally adjusting GDP directly or as the sum of seasonally adjusted data of value added by economic activity (plus net taxes on products). The two approaches are also alternatives for deriving balancing items; value added, for instance, can be derived either by seasonally adjusting value added directly or as the difference between independently derived seasonally adjusted data of output and intermediate consumption. Generally, the results will differ, sometimes significantly.

7.100 Conceptually, neither the direct approach nor the indirect approach is optimal. There are arguments in favor of both approaches. It is convenient, and for some uses crucial, that accounting and aggregation relationships are preserved.[62] Studies,[63] however, have

[62] However, for chain-linked series, these accounting relationships are already broken (see Chapter 8 on nonadditivity of chain-linked measures in monetary terms).
[63] See, among others, Dagum and Morry (1984).

Box 7.8 Revisions History Tables

R 1 Percent revisions of the concurrent seasonal adjustments		R 2 Percent revisions of the quarter-to-quarter percent change of the adjustments		R 1.S Summary statistics : average absolute percent revisions of the seasonal adjustments		R 2.S Summary statistics : average absolute revisions of the quarter-to-quarter percent change of the adjustments	
Date	Concurrent - Final	Date	Concurrent - Final	Date	Concurrent - Final	Date	Concurrent - Final
2009		2009		Quarters		Quarters	
1st	0.50	1st	0.69	1st	0.20	1st	0.30
2nd	0.01	2nd	-0.38	2nd	0.22	2nd	0.37
3rd	-0.29	3rd	-0.57	3rd	0.24	3rd	0.37
4th	-0.13	4th	0.26	4th	0.11	4th	0.38
2010		2010					
1st	0.03	1st	-0.11	Years:		Years:	
2nd	0.24	2nd	0.21	2009	0.23	2009	0.47
3rd	0.32	3rd	0.42	2010	0.16	2010	0.30
4th	-0.04	4th	-0.45	2011	0.14	2011	0.27
2011		2011		2012	0.16	2012	0.44
1st	-0.10	1st	0.13	2013	0.32	2013	0.25
2nd	0.16	2nd	0.36				
3rd	0.14	3rd	-0.25	Total:	0.20	Total:	0.35
4th	0.18	4th	0.35				
2012		2012		Hinge Values:		Hinge Values:	
1st	0.09	1st	0.36	Min	0.01	Min	0.01
2nd	0.09	2nd	-0.36	25%	0.08	25%	0.23
3rd	-0.40	3rd	-0.57	Med	0.14	Med	0.36
4th	-0.08	4th	0.46	75%	0.29	75%	0.45
2013		2013		Max	0.61	Max	0.69
1st	0.29	1st	0.21				
2nd	0.61	2nd	0.54				
3rd	-0.04	3rd	-0.01				

Tables R 1 and R 2 show the differences between the final seasonally adjusted estimates and the concurrent seasonally adjusted estimates (i.e., the first seasonal adjustment of a data point) for the last five years of data (in levels and period-to-period changes). In other terms, these differences are the revisions to the concurrent estimates when the full sample of observations is considered. Revisions history tables are particularly helpful when comparing alternative seasonal adjustment models: the one with the smallest amount of revisions should be preferred.

shown that the quality of the seasonally adjusted series, and especially estimates of the trend-cycle component, may be improved, sometimes significantly, by seasonally adjusting aggregates directly or at least at a more aggregated level. Practice has shown that seasonally adjusting the data at a detailed level may leave residual seasonality in the aggregates, result in less smooth seasonally adjusted series, and provide series more subject to revisions. Which compilation level for seasonal adjustment gives the best results varies from case to case and depends on the properties of the particular series.

7.101 For aggregates, the direct approach may give the best results if the component series show similar seasonal patterns and the trend cycles are highly correlated. In such cases, aggregation often reduces the amplitude of the irregular in the component series, which at the most detailed level may be too dominant for proper seasonal adjustment. This effect may be particularly important for small economies where irregular events have a stronger impact on the data. Similarly, when trend cycles are highly correlated, aggregation reduces the impact of both the seasonal and irregular components of the component series.

7.102 Conversely, the indirect approach may give the best results when the component series show very different seasonal patterns. Aggregation may cause large, highly volatile seasonality overshadow stable seasonal effects, making it difficult or impossible to identify seasonality in the aggregate series. Moreover, it may be easier to identify breaks, outliers, calendar effects, and so on in detailed series than directly from the aggregates, because at the detailed level, these effects may display a simpler pattern and be more interpretable in economic terms.

7.103 For balancing items (such as value added), the indirect approach may give better results than the direct approach. Balancing items are often derived as the difference between two correlated component series (e.g., gross output and intermediate consumption of the same industry). Irregular effects estimated from two (or more) correlated series are also likely to be correlated. Correlated movements in the component series, when subtracted, will cancel each other out in the balancing item, resulting in a more regular seasonally adjusted series. For value added, however, the estimate of intermediate consumption at the quarterly level may be absent or subject to high uncertainty; in that case, a direct seasonal adjustment of value added should be followed.

7.104 In practice, the choice between direct and indirect seasonal adjustment may be guided by the expected uses of seasonally adjusted data. For some uses, preserving accounting and aggregation relationships in the data may be crucial, and the smoothness and stability of the derived series secondary. For other uses, the time-series properties of the derived estimates may be crucial, while accounting and aggregation relationships may be less important. If the differences are insignificant, accounting and aggregation relationships in the seasonally adjusted data should be guaranteed. When the indirect approach is preferred, seasonally adjusted aggregates should be checked to exclude the presence of residual seasonality using the *F*-test available in X-13A-S (see Box 7.4).

7.105 Countries' practices vary with respect to the choice between direct and indirect seasonal adjustment. Many countries obtain the seasonally adjusted QNA aggregates as the sum of adjusted components, while others prefer to adjust the totals independently, showing discrepancies between the seasonally adjusted total and the sum of the seasonally adjusted component series. Allocating discrepancies on components to achieve consistency should be avoided.

7.106 X-13A-S offers a diagnostic tool to evaluate the direct and indirect adjustment of aggregates.[64] The program calculates seasonally adjusted aggregates using the direct and indirect approaches and provides in output a set of statistics to compare the results (M diagnostics, measures of smoothness, frequency spectrum diagnostics, etc.). Furthermore, sliding spans and revision history diagnostics can be requested to assess which of the two approaches provides more stable and reliable seasonally adjusted results.

Relationship among Price, Volume, and Value

7.107 As for balancing items and aggregates, seasonally adjusted estimates for national accounts price indices, volume measures, and current price data can be derived either by seasonally adjusting the three series independently or by seasonally adjusting two of them and deriving the third as a residual, if all three show seasonal variations.[65] Again, because of nonlinearities in the seasonal adjustment procedures, the alternative methods will give different results; however, the differences may be minor. Preserving the relationship among the price indices, volume measures, and the current price data is convenient for users.[66] Thus, it seems reasonable to seasonally adjust two of them and derive the seasonally adjusted estimate of the third residually.

7.108 Choosing which series to derive residually should be determined on a case-by-case basis,

[64] In X-13A-S, the spec name for comparing direct and indirect adjustment is COMPOSITE.
[65] Experience has shown that the price data may not always show identifiable seasonal variations.
[66] Note that chain-linking preserves this relationship (V = P • Q).

depending on which alternative seems to produce the most reasonable result. Theoretically, seasonality in current price data is generated by seasonality from price and volume effects. Therefore, the best approach is to apply seasonal adjustment to price and volume series and derive seasonally adjusted data in current prices indirectly. However, seasonal adjustment of data in current prices should be preferred when the main data source is available in nominal terms. In any case, the residual item should always be tested because indirect adjustment may induce residual seasonality.

Temporal Consistency with the Annual Accounts

7.109 Annual totals based on the seasonally adjusted data will not automatically—and often should not conceptually—be equal to the corresponding annual totals based on the original unadjusted data. The number of working days, the impact of moving holidays, and other calendar effects vary from year to year. Similarly, moving seasonality implies that the impact of seasonal effects will vary from year to year. Thus, conceptually, for series with significant calendar effects or moving seasonality effects, the annual totals of a seasonally adjusted series should differ from the unadjusted series.

7.110 Seasonal adjustment based on the additive model (1) without calendar effects or moving seasonality will produce seasonally adjusted data that add up to the corresponding unadjusted annual totals. In the case of multiplicative seasonal adjustment with no significant calendar or moving seasonality effects, the difference between the annual totals of the adjusted and unadjusted series will depend on the amplitude of the seasonal variation, the volatility of the seasonally adjusted series, and the pace of the change in the underlying trend cycle. The difference will be small, and often insignificant, for series with moderate to low seasonal amplitudes and for series with little volatility and trend-cycle change.

7.111 In the QNA, it is generally considered acceptable to force seasonally adjusted series to annual benchmarks of the national accounts. From a user's point of view, consistent quarterly and annual estimates are generally preferred. However, there are no reasons for forcing seasonally adjusted series when there are significant calendar effects or evolving seasonal patterns.[67] In fact, consistency with the annual series would be achieved at the expense of the quality of the seasonal adjustment.[68]

7.112 When a series is adjusted for calendar effects and these effects lead to significant changes in the annual rates, the seasonally adjusted data should be benchmarked to the annual data adjusted for calendar effects (see also paragraph 7.44). The annual data adjusted for calendar effects should be derived as the sum of quarterly data adjusted for calendar effects. For practical considerations, however, countries may choose to benchmark seasonally adjusted data to the original annual aggregates of national accounts. Maintaining two systems of annual data (unadjusted and adjusted for calendar effects) can be a challenging task for national accounts compilers. Users may also be puzzled when the results are different and the differences are not sufficiently explained by metadata.

7.113 X-13A-S provides an option for benchmarking the seasonally adjusted data to the annual original (or calendar adjusted) totals.[69] When this option is not chosen (i.e., when seasonally adjusted data are not benchmarked to the annual unadjusted data), the differences between the annual unadjusted data and the annual aggregation of seasonally (and calendar) adjusted data should be checked for plausibility. For example, different rates of change between unadjusted data and adjusted data for working days should be coherent with movements in the number of working days. When the number of working days in a year is larger (smaller) than the number of working days in the previous one, the rate of change in the annual adjusted series should be lower (higher) than the rate of change in the unadjusted series.

Length of the Series for Seasonal Adjustment

7.114 Seasonal adjustment requires sufficiently long time series for delivering results with acceptable

[67] The X-13A-S manual does not recommend the forcing option when trading-day (or working-day) adjustment is performed and if the seasonal pattern is undergoing changes.

[68] The effects of forcing the seasonally adjusted data to the annual totals (table D11.A) on the quarterly growth rates can be checked from table E4 in the X-13A-S output file.

[69] In X-13A-S, the spec name for benchmarking seasonally adjusted data to annual totals is FORCE. The default benchmarking method is the Cholette–Dagum method with autoregressive (AR) error (see Chapter 6), with the AR parameter sets to 0.9 for monthly series and 0.9^3 for quarterly series.

quality. When a series is too short, it may be difficult to identify a stable seasonal pattern and significant calendar effects from a small number of observations. Moreover, the estimated coefficients of the regARIMA model would be characterized by large uncertainty. This may have consequences in the reliability of the seasonally adjusted series, with the potential risk of large revisions when new data points are added to the series. Such risks are higher with model-based methods than with MA methods, as model-based methods (like SEATS) rely heavily on the results of the estimation process.

7.115 For QNA variables, it is recommended that at least five years of data (20 quarters) be used for seasonal adjustment. Time series with less than five years of data[70] may be seasonally adjusted for internal use, but not published until five complete years are available and the stability of results seems acceptable. Better results are to be expected when the data span more than five years. When starting a new QNA system, unadjusted data should be reconstructed as far back as possible before applying seasonal adjustment procedures.

7.116 Seasonal adjustment may also return questionable results for very long series. A long series may be affected by discontinuities and structural breaks in the seasonal pattern due to the existence of different economic conditions over a long period of time. Some breaks can be accounted for in the regARIMA model by means of outliers or specific intervention variables in the preadjustment stage, but it may be more difficult to model an evolving seasonal pattern. Furthermore, the assumption of fixed calendar effects may not be tenable for long time spans. For example, the average impact of one working day on production activities today is likely to be different from the impact of one working day 20–30 years ago due to productivity changes, different working regulations, or other structural factors.

7.117 When seasonal adjustment results are unsatisfactory for long series, it may be worth dividing the series in two (or more) contiguous periods characterized by relative stability and applying seasonal adjustment to each subperiod separately. The resulting seasonally adjusted series should be linked together to create a consistent, long time series. For calendar effects, stability of parameter estimates should be evaluated over time. When the impact of calendar effects significantly changes over time, it is advisable to estimate these effects from the most recent span in order to increase the precision of the latest seasonally adjusted data.

Seasonally Adjusting Indicators or QNA Series?

7.118 Seasonal adjustment can be applied either to monthly or quarterly indicators, or to unadjusted QNA series (i.e., to quarterly series benchmarked to ANA levels and consistent with other QNA variables). When seasonal adjustment is applied to indicators, the seasonally adjusted indicator is used to derive QNA data in seasonally adjusted form. When seasonal adjustment is applied to unadjusted QNA series, the seasonally adjusted QNA series is obtained as a result from the chosen seasonal adjustment method. Both approaches are equally acceptable. An advantage of applying seasonal adjustment directly to indicators is that seasonal effects originate from actual data sources; instead, unadjusted QNA series may contain artificial seasonality introduced by QNA techniques (e.g., benchmarking or chain-linking methods). On the other hand, unadjusted QNA series have the advantage of being consistent with other variables in the QNA. When seasonal adjustment is applied to consistent QNA series, one may expect to see a high degree of consistency in the seasonality of production, expenditure, and income components of the GDP.

7.119 This choice should consider the effects of temporal aggregation on seasonal adjustment, in particular whether the seasonal and calendar adjustment should be done at the monthly or quarterly frequency. As explained in "Preadjustment" section, calendar effects are better identified and estimated on monthly series than on quarterly series. Calendar adjustment on quarterly data should only be considered when indicators are not available on a monthly basis. Because QNA series are not available at the monthly frequency, the best approach is to identify and estimate calendar effects on monthly indicators. For seasonal adjustment, the choice between monthly or quarterly adjustment is less obvious. Studies have been con-

[70] At a minimum, X-13A-S requires three years of data (12 quarters) to produce seasonally adjusted data.

ducted on this issue but conclusions are still unclear.[71] In general, it is preferred to apply seasonal adjustment on quarterly series when temporal aggregation reduces the variance of the irregular component in the monthly series.

Organizing Seasonal Adjustment in the QNA

7.120 Many series are well behaved and easy to seasonally adjust, requiring little intervention from the user. Seasonal adjustment programs (such as X-13A-S or TRAMO-SEATS) offer automatic selection procedures that give satisfactory results for the majority of time series. Thus, lack of experience in seasonal adjustment or lack of staff with particular expertise in seasonal adjustment should not preclude one from starting to compile and publish seasonally adjusted estimates. Before compiling seasonally adjusted estimates, however, the main focus of compilation and presentation should be on the unadjusted QNA data.

7.121 For problematic series, substantial experience and expertise may be required to determine whether the seasonal adjustment is done properly or to fine-tune the seasonal adjustment options. In particularly unstable series with a strong irregular component (e.g., outliers and other special events, seasonal breaks, or level shifts), it may be difficult to derive satisfactory results without sufficient experience. In the medium term, the team responsible for seasonal adjustment should build skills and knowledge (both theoretical and practical) to be able to handle the adjustment of problematic series.

7.122 It is generally recommended that the statisticians who compile the statistics should also be responsible—either solely or together with seasonal adjustment specialists—for seasonally adjusting the statistics. This arrangement should give them greater insight into the data, make their job more interesting, help them understand the nature of the data better, and lead to improved quality of both the original unadjusted data and the seasonally adjusted data. However, it is advisable in addition to set up a small central group of seasonal adjustment experts, because the in-depth seasonal adjustment expertise required to handle ill-behaved series can only be acquired by hands-on experience with seasonal adjustment of many different types of series.

[71] For example, see Di Palma and Savio (2001), Burgess (2007), Zhang and Apted (2008), and Ciammola, Cicconi, and Di Palma (2013).

Status and Presentation of Seasonally Adjusted and Trend-Cycle QNA Estimates

7.123 The status and presentation of seasonally adjusted and trend-cycle QNA estimates vary. Some countries publish seasonally adjusted estimates for only a few main aggregates and present them as additional (sometimes unofficial) analytical elaborations of the official data. Other countries focus on the seasonally adjusted and trend-cycle estimates and publish an almost complete set of seasonally adjusted and trend-cycle QNA estimates in a reconciled accounting format. Data adjusted for calendar effects may also be published separately. This presentation makes the impact of calendar effects on the QNA aggregates visible for users.

7.124 The mode of presentation also varies substantially. Seasonally adjusted and trend-cycle data can be presented as charts; as tables with the actual data, either in money values or as index series; and as tables with derived measures of quarter-to-quarter rates of change. Calendar adjusted data should be presented in the same manner as the original unadjusted data (generally in levels and year-on-year changes). Quarter-to-quarter rates of change are not appropriate for calendar adjusted data, because these data still contain seasonal effects which may dominate quarterly movements.

7.125 Quarter-to-quarter rates of change should be presented as actual rates of change between one quarter and the previous one. Growth rates are sometimes annualized to make it easier for the user to interpret the data. Most users have a feel for the size of annual rates of change but not for monthly or quarterly ones. Annualizing growth rates, however, also means that the irregular effects are compounded. Irrespective of whether the actual or annualized quarterly rates of change are presented, it is important to clearly indicate what the data represent.

7.126 Growth rates representing different measures of change can easily be confused unless it is clearly indicated what the data represent. For instance, terms like "annual percentage change" or "annual rate of growth" can mean (a) the rate of change from one quarter to the next annualized (at annual rate); (b) the change from the same period of the previous year; (c) the change from one year to the next in annual data or, equivalently, the change from the average of one year to the average of the next year; or (d) the change from the end of one year to the end of the next year.

7.127 Some countries also present the level of quarterly data at annualized levels by multiplying the actual data by four. This presentation seems artificial, does not make the data easier to interpret, and may be confusing because annual flow data in monetary terms no longer can be derived as the sum of the quarters. Users not familiar with the practice of annualizing levels of current price data and volume data by multiplying the actual data by four may confuse annualized levels with forecast annual data. For these reasons, this practice is not recommended.

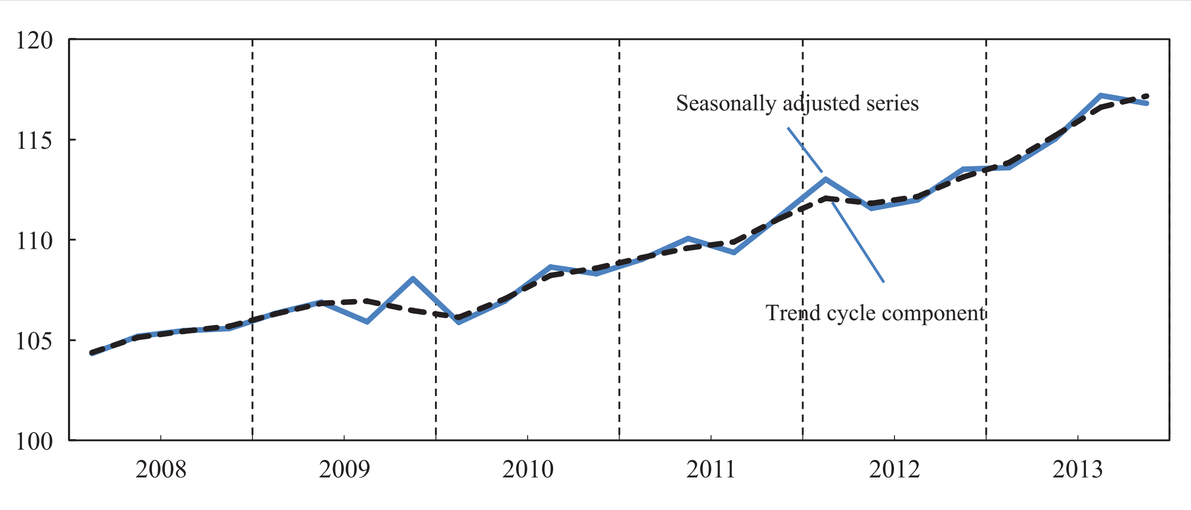
7.128 Finally, whether to present seasonally adjusted data or estimates of the trend-cycle component is still the subject of debate between experts in this area. In this manual, it is recommended to present both, preferably in the form of graphs incorporated into the same chart, as illustrated in Figure 7.4.

7.129 An integrated graphical presentation highlights the overall development in the two series over time, including the uncertainties represented by the irregular component. In contrast, measures of quarter-to-quarter rates of change (in particular, annualized rates) may result in an overemphasis on the short-term movements in the latest and most uncertain observations at the expense of the general trend in the series. The underlying data and derived measures of quarter-to-quarter rates of change, however, should be provided as supplementary information.

7.130 The presentation should highlight the lower reliability, particularly for the trend-cycle component, of the estimates for the latest observations as discussed in this section. Means of highlighting the lower quality of the endpoint estimates include noting past revisions to these estimates or showing the confidence interval of trend-cycle estimates in graphical and tabular presentations. When the irregular is particularly strong, trend-cycle estimates for the latest observations (up to two quarters) could be removed from graphical presentations.

Figure 7.4 Presentation of the Seasonally Adjusted Series and Trend-Cycle Component

(Based on data from Example 7.1)



Presenting the seasonally adjusted series and the trend-cycle component in the same chart highlights the overall development in the two series over time, including the uncertainty of the irregular component. Users should be informed that trend-cycle estimates of the latest observations are subject to high uncertainty and should be taken with caution.

Summary of Key Recommendations

- *Seasonally adjusted data in the QNA should be calculated to facilitate the analysis of current economic developments without the influence of seasonal and calendar effects. However, seasonally adjusted data should not replace the unadjusted QNA data.*
- *A series should be seasonally adjusted only when there is evidence of identifiable seasonality. Series with no seasonality or too unstable seasonality should not be seasonally adjusted.*
- *QNA series should also be adjusted for calendar effects. However, the adjustment should be done only for those series for which there is statistical evidence and economical interpretation of calendar effects.*
- *In the preadjustment stage, deterministic effects should be identified and removed from the series using regression models and diagnostics.*
- *Decomposition of the (preadjusted) series should be conducted using either the moving average X-11 method or the model-based SEATS method. The X-13A-S program, which implements both X-11 and SEATS, is the recommended procedure for seasonal adjustment in the QNA.*
- *Seasonal adjustment results should be evaluated using basic and advanced diagnostics. Seasonally adjusted series with residual seasonality should not be accepted.*
- *Seasonally adjusted data should be updated using a partial concurrent strategy. In a partial concurrent strategy, models and options for seasonal adjustment are selected at established review periods (usually once a year). In non-review periods, seasonal adjustment models and options are kept fixed but parameters are reestimated each time a new observation is added.*
- *The full seasonally adjusted series should be revised anytime the seasonal adjustment model is changed or updated. In non-review periods, seasonally adjusted data should at least cover the revision period of the unadjusted data.*
- *Revisions studies of seasonally adjusted QNA data should be conducted on a regular basis to identify where revisions are large and systematic.*
- *Due to moving seasonality and calendar effects, seasonally adjusted data may not be consistent with corresponding annual data. However, seasonally adjusted data could be benchmarked to annual benchmarks of national accounts for consistency reasons. When the series is adjusted for calendar effects, the seasonally adjusted data should be benchmarked to annual benchmarks adjusted for calendar effects.*
- *A minimum of five years is required to seasonally adjust QNA series. Shorter series may be adjusted for internal use but not disseminated.*
- *Seasonally adjusted data of the main QNA aggregates should be released to the public. Trend-cycle component and calendar adjusted data could also be disseminated. Metadata on seasonal adjustment models and revision policy should be made available for transparency.*

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