22. Treatment of Seasonal Products

A. Problem of Seasonal Products

22.1 The existence of seasonal products poses some significant challenges for price statisticians. *Seasonal commodities* are products that are either (i) not available in the marketplace during certain seasons of the year or (ii) are available throughout the year but there are regular fluctuations in prices or quantities that are synchronized with the season or the time of the year.[1] A commodity that satisfies (i) is termed a *strongly seasonal commodity*, whereas a commodity that satisfies (ii) will be called a *weakly seasonal commodity*. It is strongly seasonal products that create the biggest problems for price statisticians in the context of producing a monthly or quarterly PPI. If a product price is available in only one of the two months (or quarters) being compared, then it is not possible to calculate a relative price for the product, and traditional bilateral index number theory breaks down. In other words, if a product is present in one month but not the next, how can the month-to-month amount of price change for that product be computed?[2] In this chapter, a solution to this problem will be presented that works even if the products produced are entirely different for each month of the year.[3]

[1]This classification of seasonal commodities corresponds to Balk's narrow and wide sense seasonal commodities; see Balk (1980a, p. 7; 1980b, p. 110; 1980c, p. 68). Diewert (1998b, p. 457) used the terms type 1 and type 2 seasonality.

[2]Zarnowitz (1961, p. 238) was perhaps the first to note the importance of this problem: "But the main problem introduced by the seasonal change is precisely that the market basket is different in the consecutive months (seasons), not only in weights but presumably often also in its very composition by commodities. This is a general and complex problem which will have to be dealt with separately at later stages of our analysis."

[3]However, the same products must reappear each year for each separate month!

22.2 There are two main sources of seasonal fluctuations in prices and quantities: (i) climate and (ii) custom.[4] In the first category, fluctuations in temperature, precipitation, and hours of daylight cause fluctuations in the demand or supply for many products; for example, think of summer versus winter clothing, the demand for light and heat, vacations, etc. With respect to custom and convention as a cause of seasonal fluctuations, consider the following quotation:

> Conventional seasons have many origins—ancient religious observances, folk customs, fashions, business practices, statute law…. Many of the conventional seasons have considerable effects on economic behaviour. We can count on active retail buying before Christmas, on the Thanksgiving demand for turkeys, on the first of July demand for fireworks, on the preparations for June weddings, on heavy dividend and interest payments at the beginning of each quarter, on an increase in bankruptcies in January, and so on. (Wesley C. Mitchell, 1927, p. 237)

22.3 Examples of important seasonal products are the following: many food items; alcoholic beverages; many clothing and footwear items; water, heating oil, electricity; flowers and garden supplies; vehicle purchases, vehicle operation; many entertainment and recreation expenditures; books; insurance expenditures; wedding expenditures; recreational equipment; toys and games; software; air travel; and tourism purchases. For a typical country, seasonal purchases will often amount to one-fifth to one-third of all consumer purchases.[5]

[4]This classification dates back to Mitchell (1927, p. 236) at least: "Two types of seasons produce annually recurring variations in economic activity—those which are due to climates and those which are due to conventions."

[5]Alterman, Diewert, and Feenstra (1999, p. 151) found that over the 40 months between September 1993 and December 1996, somewhere between 23 and 40 percent of U.S. imports and exports exhibited seasonal variations in (continued)

22.4 In the context of producing a monthly or quarterly PPI, it must be recognized that there is no completely satisfactory way of dealing with strongly seasonal products. If a product is present in one month but missing in the next, then none of the index number theories that were considered in Chapters 15–20 can be applied because all of these theories assumed that the dimensionality of the product space was constant for the two periods being compared. However, if seasonal products are present in the market during each season, then, in theory, traditional index number theory can be applied in order to construct month-to-month or quarter-to-quarter price indices. This traditional approach to the treatment of seasonal products will be followed in Sections H, I, and J of this chapter. The reason why this straightforward approach is deferred to the end of the chapter is twofold:

- The approach that restricts the index to products that are present in every period often does not work well in the sense that systematic *biases* can occur; and
- The approach is not fully *representative*; that is, it does not make use of information on products that are not present in every month or quarter.

22.5 In Section B, a modified version of Turvey's (1979) artificial data set is introduced. This data set will be used to numerically evaluate all of the index number formulas that are suggested in this chapter. It will be seen in Section G that large seasonal fluctuations in volumes combined with systematic seasonal changes in price can make month-to-month or quarter-to-quarter price indices behave rather poorly.

22.6 Even though existing index number theory cannot deal satisfactorily with seasonal products in the context of constructing month-to-month indices of producer prices, it can deal satisfactorily with seasonal products if the focus is changed from month-to-month PPIs to PPIs that compare the prices of one month with the prices of the *same* month in a previous year. Thus, in Section C, *year-over-year monthly* PPIs are studied. Turvey's seasonal data set is used to evaluate the performance of these indices, and they are found to perform quite well.

22.7 In Section D, the year-over-year monthly indices defined in Section C are aggregated into an *annual index* that compares all of the monthly prices in a given calendar year with the corresponding monthly prices in a base year. In Section E, this idea of comparing the prices of a current calendar year with the corresponding prices in a base year is extended to annual indices that compare the prices of the last 12 months with the corresponding prices in the 12 months of a base year. The resulting *rolling-year indices* can be regarded as seasonally adjusted price indices. The modified Turvey data set is used to test out these year-over-year indices, and they are found to work very well on this data set.

22.8 The rolling-year indices can provide an accurate gauge of the movement of prices in the current rolling year compared with the base year. However, this measure of price inflation can be regarded as a measure of inflation for a year that is centered around a month that is six months prior to the last month in the current rolling year. As a result, for some policy purposes, this type of index is not as useful as an index that compares the prices of the current month to the previous month, so that more up-to-date information on the movement of prices can be obtained. However, in Section F, it will be shown that under certain conditions, the current month year-over-year monthly index, along with last month's year-over-year monthly index, can successfully *predict* or *forecast* a rolling-year index that is centered around the current month.

22.9 The year-over-year indices defined in Section C and their annual averages studied in Sections D and E offer a theoretically satisfactory method for dealing with *strongly seasonal products*; that is, products that are available only during certain seasons of the year. However, these methods rely on the year-over-year comparison of prices; therefore, these methods cannot be used in the month-to-month or quarter-to-quarter type of index, which is typically the main focus of a producer price program. Thus, there is a need for another type of index, one that may not have strong theoretical foundations but can deal with seasonal products in the context of producing a *month-to-month index*. In Section G, such an index is introduced, and it is implemented using the artificial data set for the products that are available during

quantities, whereas only about 5 percent of U.S. export and import prices exhibited seasonal fluctuations.

Table 22.1. Artificial Seasonal Data Set: Prices

Year t	Month m	$p_1^{t,m}$	$p_2^{t,m}$	$p_3^{t,m}$	$p_4^{t,m}$	$p_5^{t,m}$
1970	1	1.14	0	2.48	0	1.30
	2	1.17	0	2.75	0	1.25
	3	1.17	0	5.07	0	1.21
	4	1.40	0	5.00	0	1.22
	5	1.64	0	4.98	5.13	1.28
	6	1.75	3.15	4.78	3.48	1.33
	7	1.83	2.53	3.48	3.27	1.45
	8	1.92	1.76	2.01	0	1.54
	9	1.38	1.73	1.42	0	1.57
	10	1.10	1.94	1.39	0	1.61
	11	1.09	0	1.75	0	1.59
	12	1.10	0	2.02	0	1.41
1971	1	1.25	0	2.15	0	1.45
	2	1.36	0	2.55	0	1.36
	3	1.38	0	4.22	0	1.37
	4	1.57	0	4.36	0	1.44
	5	1.77	0	4.18	5.68	1.51
	6	1.86	3.77	4.08	3.72	1.56
	7	1.94	2.85	2.61	3.78	1.66
	8	2.02	1.98	1.79	0	1.74
	9	1.55	1.80	1.28	0	1.76
	10	1.34	1.95	1.26	0	1.77
	11	1.33	0	1.62	0	1.76
	12	1.30	0	1.81	0	1.50
1972	1	1.43	0	1.89	0	1.56
	2	1.53	0	2.38	0	1.53
	3	1.59	0	3.59	0	1.55
	4	1.73	0	3.90	0	1.62
	5	1.89	0	3.56	6.21	1.70
	6	1.98	4.69	3.51	3.98	1.78
	7	2.07	3.32	2.73	4.30	1.89
	8	2.12	2.29	1.65	0	1.91
	9	1.73	1.90	1.15	0	1.92
	10	1.56	1.97	1.15	0	1.95
	11	1.56	0	1.46	0	1.94
	12	1.49	0	1.73	0	1.64
1973	1	1.68	0	1.62	0	1.69
	2	1.82	0	2.16	0	1.69
	3	1.89	0	3.02	0	1.74
	4	2.00	0	3.45	0	1.91
	5	2.14	0	3.08	7.17	2.03
	6	2.23	6.40	3.07	4.53	2.13
	7	2.35	4.31	2.41	5.19	2.22
	8	2.40	2.98	1.49	0	2.26
	9	2.09	2.21	1.08	0	2.22
	10	2.03	2.18	1.08	0	2.31
	11	2.05	0	1.36	0	2.34
	12	1.90	0	1.57	0	1.97

Table 22.2. Artificial Seasonal Data Set: Quantities

Year t	Month m	$q_1^{t,m}$	$q_2^{t,m}$	$q_3^{t,m}$	$q_4^{t,m}$	$q_5^{t,m}$
1970	1	3,086	0	82	0	10,266
	2	3,765	0	35	0	9,656
	3	4,363	0	98	0	7,940
	4	4,842	0	26	0	5,110
	5	4,439	0	75	700	4,089
	6	5,323	91	82	2,709	3,362
	7	4,165	498	96	1,970	3,396
	8	3,224	6,504	1,490	0	2,406
	9	4,025	4,923	2,937	0	2,486
	10	5,784	865	2,826	0	3,222
	11	6,949	0	1,290	0	6,958
	12	3,924	0	338	0	9,762
1971	1	3,415	0	119	0	10,888
	2	4,127	0	45	0	10,314
	3	4,771	0	14	0	8,797
	4	5,290	0	11	0	5,590
	5	4,986	0	74	806	4,377
	6	5,869	98	112	3,166	3,681
	7	4,671	548	132	2,153	3,748
	8	3,534	6,964	2,216	0	2,649
	9	4,509	5,370	4,229	0	2,726
	10	6,299	932	4,178	0	3,477
	11	7,753	0	1,831	0	8,548
	12	4,285	0	496	0	10,727
1972	1	3,742	0	172	0	11,569
	2	4,518	0	67	0	10,993
	3	5,134	0	22	0	9,621
	4	5,738	0	16	0	6,063
	5	5,498	0	137	931	4,625
	6	6,420	104	171	3,642	3,970
	7	5,157	604	202	2,533	4,078
	8	3,881	7,378	3,269	0	2,883
	9	4,917	5,839	6,111	0	2,957
	10	6,872	1,006	5,964	0	3,759
	11	8,490	0	2,824	0	8,238
	12	5,211	0	731	0	11,827
1973	1	4,051	0	250	0	12,206
	2	4,909	0	102	0	11,698
	3	5,567	0	30	0	10,438
	4	6,253	0	25	0	6,593
	5	6,101	0	220	1,033	4,926
	6	7,023	111	252	4,085	4,307
	7	5,671	653	266	2,877	4,418
	8	4,187	7,856	4,813	0	3,165
	9	5,446	6,291	8,803	0	3,211
	10	7,377	1,073	8,778	0	4,007
	11	9,283	0	4,517	0	8,833
	12	4,955	0	1,073	0	12,558

each month of the year. Unfortunately, due to the seasonality in both prices and quantities in the always available products, this type of index can be systematically biased. This bias is apparent in the modified Turvey data set.

22.10 Since many PPIs are month-to-month indices that use *annual basket quantity weights*, this type of index is studied in Section H. For months when the product is not available in the marketplace, the last available price is carried forward and used in the index. In Section I, an annual quantity basket is again used but instead of carrying forward the prices of seasonally unavailable items, an imputation method is used to fill in the missing prices. The annual basket-type indices defined in Sections H and I are implemented using the artificial data set. Unfortunately, the empirical results are not satisfactory because the indices show tremendous seasonal fluctuations in prices. This volatility makes them unsuitable for users who want up-to-date information on *trends* in general inflation.

22.11 In Section J, the artificial data set is used in order to evaluate another type of month-to-month index that is frequently suggested in the literature on how to deal with seasonal products: the *Bean and Stine Type C* (1924) or *Rothwell* (1958) index. Again, this index does not get rid of the tremendous seasonal fluctuations that are present in the modified Turvey data set.

22.12 Sections H and I show that the annual basket-type indices with carryforward of missing prices (Section H) or imputation of missing prices (Section I) do not get rid of seasonal fluctuations in prices. However, in Section K, it is shown how seasonally adjusted versions of these annual basket indices can be used to successfully *forecast* rolling-year indices that are centered in the current month. In addition, the results in Section K show how these annual basket-type indices can be seasonally adjusted (using information obtained from rolling-year indices from prior periods or by using traditional seasonal adjustment procedures). Hence, these seasonally adjusted annual basket indices could be used as successful indicators of general inflation on a timely basis.

22.13 Section L concludes with several suggestions for dealing with seasonal products.

B. A Seasonal Product Data Set

22.14 It will prove to be useful to illustrate the index number formulas that will be defined in subsequent sections by computing them for an actual data set. Turvey (1979) constructed an artificial data set for five seasonal products (apples, peaches, grapes, strawberries, and oranges) for four years by month, so that there are 5 times 4 times 12 observations, equal to 240 observations in all. At certain times of the year, peaches and strawberries (products 2 and 4) are unavailable, so in Tables 22.1 and 22.2, the prices and quantities for these products are entered as zeros.[6] The data in Tables 22.1 and 22.2 are essentially equal to that constructed by Turvey except that a number of adjustments were made to illustrate various points. The two most important adjustments were as follows:

- The data for product 3 (grapes) were adjusted, so that the annual Laspeyres and Paasche indices (which will be defined in Section D) would differ more than in the original data set;[7] and
- After the aforementioned adjustments were made, each price in the last year of data was escalated by the monthly inflation factor 1.008, so that month-to-month inflation for the last year of data would be at an approximate monthly rate of 1.6 percent per month, compared with about 0.8 percent per month for the first three years of data.[8]

[6]The corresponding prices are not zeros, but they are entered as zeros for convenience in programming the various indices.

[7]After the first year, the price data for grapes was adjusted downward by 30 percent each year and the corresponding volume was adjusted upward by 40 percent each year. In addition, the quantity of oranges (product 5) for November 1971 was changed from 3,548 to 8,548 so that the seasonal pattern of change for this product would be similar to that of other years. For similar reasons, the price of oranges in December 1970 was changed from 1.31 to 1.41 and in January 1971 from 1.35 to 1.45.

[8]Pierre Duguay of the Bank of Canada, while commenting on a preliminary version of this chapter, observed that rolling-year indices would not be able to detect the *magnitude* of systematic changes in the month-to-month inflation rate. The original Turvey data set was roughly consistent with a month-to-month inflation rate of 0.8 percent per month; that is, prices grew roughly at the rate 1.008 each month over the four years of data. Hence this second major

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22.15 Turvey sent his artificial data set to statistical agencies around the world, asking them to use their normal techniques to construct monthly and annual average price indices. About 20 countries replied; Turvey summarized the responses as follows:

> It will be seen that the monthly indices display very large differences, for example, a range of 129.12–169.50 in June, while the range of simple annual means is much smaller. It will also be seen that the indices vary as to the peak month or year. (Ralph Turvey, 1979, p. 13)

The (modified) data below will be used to test out various index number formulas in subsequent sections.

C. Year-over-Year Monthly Indices

22.16 It can be seen that the existence of seasonal products that are present in the marketplace in one month but absent the next causes the accuracy of a month-to-month index to fall.[9] A way of dealing with these strongly seasonal products is to change the focus from short-term month-to-month price indices to year-over-year price comparisons for each month of the year. In the latter type of comparison, there is a good chance that seasonal products that appear in February, for example, will also appear in subsequent Februarys, so that the overlap of products will be maximized in these year-over-year monthly indices.

22.17 For over a century, it has been recognized that making year-over-year comparisons[10] provides the simplest method for making comparisons that are free from the contaminating effects of seasonal fluctuations:

> In the daily market reports, and other statistical publications, we continually find comparisons between numbers referring to the week, month, or other parts of the year, and those for the corresponding parts of a previous year. The comparison is given in this way in order to avoid any variation due to the time of the year. And it is obvious to everyone that this precaution is necessary. Every branch of industry and commerce must be affected more or less by the revolution of the seasons, and we must allow for what is due to this cause before we can learn what is due to other causes. (W. Stanley Jevons, 1884, p. 3)

22.18 The economist Flux and the statistician Yule also endorsed the idea of making year-over-year comparisons to minimize the effects of seasonal fluctuations:

> Each month the average price change compared with the corresponding month of the previous year is to be computed. … The determination of the proper seasonal variations of weights, especially in view of the liability of seasons to vary from year to year, is a task from which, I imagine, most of us would be tempted to recoil. (A. W. Flux, 1921, pp. 184–85)

> My own inclination would be to form the index number for any month by taking ratios to the corresponding month of the year being used for reference, the year before presumably, as this would avoid any difficulties with seasonal commodities. I should then form the annual average by the geometric mean of the monthly figures. (G. Udny Yule, 1921, p. 199)

In more recent times, Zarnowitz also endorsed the use of year-over-year monthly indices:

> There is of course no difficulty in measuring the average price change between the same months of successive years, if a month is our unit “season”, and if a constant seasonal market basket can be used, for traditional methods of price index construction can be applied in such comparisons. (Victor Zarnowitz, 1961, p. 266)

22.19 In the remainder of this section, it is shown how year-over-year Fisher indices and ap-

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adjustment of the Turvey data was introduced to illustrate Duguay’s observation, which is quite correct: the centered rolling-year indices pick up the correct magnitude of the new inflation rate only after a lag of half a year or so. However, they do quickly pick up the direction of change in the inflation rate.

[9]In the limit, if each product appeared in only one month of the year, then a month-to-month index would break down completely.

[10]In the seasonal price index context, this type of index corresponds to Bean and Stine’s (1924, p. 31) Type D index.

proximations to them can be constructed.[11] For each month m = 1, 2,...,12, let $S(m)$ denote the set of products that are available for purchase in each year t = 0, 1,...,T. For t = 0, 1,...,T and m = 1, 2,...,12, let $p_n^{t,m}$ and $q_n^{t,m}$ denote the price and quantity of product n that is available in month m of year t for n belongs to $S(m)$. Let $p^{t,m}$ and $q^{t,m}$ denote the month m and year t price and quantity vectors, respectively. Then *the year-over-year monthly Laspeyres, Paasche, and Fisher indices* going from month m of year t to month m of year t + 1 can be defined as follows:

$$(22.1)\ P_L\left(p^{t,m}, p^{t+1,m}, q^{t,m}\right) = \frac{\sum_{n\in S(m)} p_n^{t+1,m} q_n^{t,m}}{\sum_{n\in S(m)} p_n^{t,m} q_n^{t,m}};$$

m = 1, 2,...12;

$$(22.2)\ P_P\left(p^{t,m}, p^{t+1,m}, q^{t+1,m}\right) = \frac{\sum_{n\in S(m)} p_n^{t+1,m} q_n^{t+1,m}}{\sum_{n\in S(m)} p_n^{t,m} q_n^{t+1,m}};$$

m = 1,,2,...12;

$$(22.3)\ P_F\left(p^{t,m}, p^{t+1,m}, q^{t,m}, q^{t+1,m}\right) \equiv \sqrt{P_L\left(p^{t,m}, p^{t+1,m}, q^{t,m}\right)}\sqrt{P_P\left(p^{t,m}, p^{t+1,m}, q^{t+1,m}\right)};$$

m = 1, 2,...,12.

22.20 The above formulas can be rewritten in price relative and monthly revenue share form as follows:

$$(22.4)\ P_L\left(p^{t,m}, p^{t+1,m}, s^{t,m}\right) = \sum_{n\in S(m)} s_n^{t,m}\left(p_n^{t+1,m}/p_n^{t,m}\right);$$

m = 1, 2,...12;

$$(22.5)\ P_P\left(p^{t,m}, p^{t+1,m}, s^{t+1,m}\right) = \left[\sum_{n\in S(m)} s_n^{t+1,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1};$$

m = 1, 2,...12;

$$(22.6)\ P_F\left(p^{t,m}, p^{t+1,m}, s^{t,m}, s^{t+1,m}\right) \equiv \sqrt{P_L\left(p^{t,m}, p^{t+1,m}, s^{t,m}\right)}\ \sqrt{P_P\left(p^{t,m}, p^{t+1,m}, s^{t+1,m}\right)};$$

m = 1, 2,...,12

$$= \sqrt{\sum_{n\in S(m)} s_n^{t,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)} \times \sqrt{\left[\sum_{n\in S(m)} s_n^{t,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1}};$$

where the monthly revenue share for product $n\in S(m)$ for month m in year t is defined as:

$$(22.7)\ s_n^{t,m} = \frac{p_n^{t,m} q_n^{t,m}}{\sum_{i\in S(m)} p_i^{t,m} q_i^{t,m}};\quad ; m = 1,2,...,12;$$

$n\in S(m)$; t = 0,1,...,T;

and $s^{t,m}$ denotes the vector of month m expenditure shares in year t, $[s_n^{t,m}]$ for $n\in S(m)$.

22.21 Current-period revenue shares $s_n^{t,m}$ are not likely to be available. As a consequence, it will be necessary to approximate these shares using the corresponding revenue shares from a base year 0.

22.22 Use the base-period monthly revenue share vectors $s^{0,m}$ in place of the vector of month m and year t expenditure shares $s^{t,m}$ in equation (22.4), and use the base-period monthly expenditure share vectors $s^{0,m}$ in place of the vector of month m and year t + 1 revenue shares $s^{t+1,m}$ in equation (22.5). Similarly, replace the share vectors $s^{t,m}$ and $s^{t+1,m}$ in equation (22.6) with the base-period expenditure share vector for month m, $s^{0,m}$. The resulting *approximate year-over-year monthly Laspeyres, Paasche, and Fisher indices* are defined by equations (22.8)–(22.10) below:[12]

[11]Diewert (1996b, pp. 17–19; 1999a, p. 50) noted various separability restrictions on purchaser preferences that would justify these year-over-year monthly indices from the viewpoint of the economic approach to index number theory.

[12]If the monthly revenue shares for the base year, $s_n^{0,m}$, are all equal, then the approximate Fisher index defined by equation (22.10) reduces to Fisher's (1922, p. 472) formula 101. Fisher (1922, p. 211) observed that this index was empirically very close to the unweighted geometric mean of the price relatives, while Dalén (1992a, p. 143) and Diewert (1995a, p. 29) showed analytically that these two indices approximated each other to the second order. The equally weighted version of equation (22.10) was recom-
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$$(22.8)\; P_{AL}\left(p^{t,m}, p^{t+1,m}, s^{0,m}\right) = \sum_{n\in S(m)} s_n^{0,m}\left(p_n^{t+1,m}/p_n^{t,m}\right);$$

$m = 1, 2,...12;$

$$(22.9)\; P_{AP}\left(p^{t,m}, p^{t+1,m}, s^{0,m}\right) = \left[\sum_{n\in S(m)} s_n^{0,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1};$$

$m = 1, 2,...12;$

$$(22.10)\; P_{AF}\left(p^{t,m}, p^{t+1,m}, s^{0,m}, s^{0,m}\right) \equiv \sqrt{P_{AL}\left(p^{t,m}, p^{t+1,m}, s^{0,m}\right)}\sqrt{P_{P}\left(p^{t,m}, p^{t+1,m}, s^{0,m}\right)};$$

$m = 1, 2,...,12$

$$= \sqrt{\sum_{n\in S(m)} s_n^{t,m}\left(p_n^{t+1,m}/p_n^{t,m}\right) \times \left[\sum_{n\in S(m)} s_n^{t,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1}}.$$

22.23 The approximate Fisher year-over-year monthly indices defined by equation (22.10) will provide adequate approximations to their true Fisher counterparts defined by equation (22.6) only if the monthly revenue shares for the base year 0 are not too different from their current-year t and $t + 1$ counterparts. Thus, it will be useful to construct the true Fisher indices on a delayed basis in order to check the adequacy of the approximate Fisher indices defined by equation (22.10).

22.24 The year-over-year monthly approximate Fisher indices defined by equation (22.10) will normally have a certain amount of upward bias, since these indices cannot reflect long-term substitution toward products that are becoming relatively cheaper over time. This reinforces the case for computing true year-over-year monthly Fisher indices defined by equation (22.6) on a delayed basis, so that this substitution bias can be estimated.

22.25 Note that the approximate year-over-year monthly Laspeyres and Paasche indices, P_{AL} and P_{AP} defined by equations (22.8) and (22.9), satisfy the following inequalities:

$$(22.11)\; P_{AL}\left(p^{t,m}, p^{t+1,m}, s^{0,m}\right) \times P_{AL}\left(p^{t+1,m}, p^{t,m}, s^{0,m}\right) \geq 1;$$

$m = 1, 2,...,12;$

$$(22.12)\; P_{AP}\left(p^{t,m}, p^{t+1,m}, s^{0,m}\right) \times P_{AP}\left(p^{t+1,m}, p^{t,m}, s^{0,m}\right) \leq 1;$$

$m = 1, 2,...,12;$

with strict inequalities if the monthly price vectors $p^{t,m}$ and $p^{t+1,m}$ are not proportional to each other.[13] Equation (22.11) says that the approximate year-over-year monthly Laspeyres index *fails the time reversal test* with an upward bias while equation (22.12) says that the approximate year-over-year monthly Paasche index *fails the time reversal test* with a downward bias. As a result, the fixed-weight approximate Laspeyres index P_{AL} has a built-in upward bias while the fixed-weights approximate Paasche index P_{AP} has a built-in downward bias. *Statistical agencies should avoid the use of these formulas*. However, they can be combined, as in the approximate Fisher formula in equation (22.10). The resulting index should be free from any systematic formula bias, although some substitution bias could still exist.

22.26 The year-over-year monthly indices defined in this section are illustrated using the artificial data set tabled in Section B. Although fixed-base indices were not formally defined in this section, these indices have similar formulas to the year-over-year indices that were defined, with the exception that the variable base year t is replaced by the fixed-base year 0. The resulting 12 year-over-year monthly fixed-base Laspeyres, Paasche, and Fisher indices are listed in Tables 22.3 to 22.5.

22.27 Comparing the entries in Tables 22.3 and 22.4, it can be seen that the year-over-year monthly fixed-base Laspeyres and Paasche price indices do not differ substantially for the early months of the year. There are, however, substantial differences between the indices for the last five months of the year by the time the year 1973 is reached. The largest percentage difference between the Laspeyres and Paasche indices is 12.5 percent for month 10 in 1973 (1.4060/1.2496 = 1.125).

mended as an elementary index by Carruthers, Sellwood, and Ward (1980, p. 25) and Dalén (1992a p. 140).

[13]See Hardy, Littlewood, and Polyá (1934, p. 26).

Table 22.3. Year-over-Year Monthly Fixed-Base Laspeyres Indices

	Month											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1085	1.1068	1.1476	1.1488	1.1159	1.0844	1.1103	1.0783	1.0492	1.0901	1.1284	1.0849
1972	1.2060	1.2442	1.3062	1.2783	1.2184	1.1734	1.2364	1.1827	1.1049	1.1809	1.2550	1.1960
1973	1.3281	1.4028	1.4968	1.4917	1.4105	1.3461	1.4559	1.4290	1.2636	1.4060	1.5449	1.4505

Table 22.4. Year-over-Year Monthly Fixed-Base Paasche Indices

	Month											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1074	1.1070	1.1471	1.1486	1.1115	1.0827	1.1075	1.0699	1.0414	1.0762	1.1218	1.0824
1972	1.2023	1.2436	1.3038	1.2773	1.2024	1.1657	1.2307	1.1455	1.0695	1.1274	1.2218	1.1901
1973	1.3190	1.4009	1.4912	1.4882	1.3715	1.3266	1.4433	1.3122	1.1664	1.2496	1.4296	1.4152

Table 22.5. Year-over-Year Monthly Fixed-Base Fisher Indices

	Month											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1080	1.1069	1.1474	1.1487	1.1137	1.0835	1.1089	1.0741	1.0453	1.0831	1.1251	1.0837
1972	1.2041	1.2439	1.3050	1.2778	1.2104	1.1695	1.2336	1.1640	1.0870	1.1538	1.2383	1.1930
1973	1.3235	1.4019	1.4940	1.4900	1.3909	1.3363	1.4496	1.3694	1.2140	1.3255	1.4861	1.4327

Table 22.6. Year-over-Year Approximate Monthly Fixed-Base Paasche Indices

	Month											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1077	1.1057	1.1468	1.1478	1.1135	1.0818	1.1062	1.0721	1.0426	1.0760	1.1209	1.0813
1972	1.2025	1.2421	1.3036	1.2757	1.2110	1.1640	1.2267	1.1567	1.0788	1.1309	1.2244	1.1862
1973	1.3165	1.3947	1.4880	1.4858	1.3926	1.3223	1.4297	1.3315	1.1920	1.2604	1.4461	1.4184

Table 22.7. Year-over-Year Approximate Monthly Fixed-Base Fisher Indices

	Month											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1081	1.1063	1.1472	1.1483	1.1147	1.0831	1.1082	1.0752	1.0459	1.0830	1.1247	1.0831
1972	1.2043	1.2432	1.3049	1.2770	1.2147	1.1687	1.2316	1.1696	1.0918	1.1557	1.2396	1.1911
1973	1.3223	1.3987	1.4924	1.4888	1.4015	1.3341	1.4428	1.3794	1.2273	1.3312	1.4947	1.4344

Table 22.8. Year-over-Year Monthly Chained Laspeyres Indices

	Month											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1085	1.1068	1.1476	1.1488	1.1159	1.0844	1.1103	1.0783	1.0492	1.0901	1.1284	1.0849
1972	1.2058	1.2440	1.3058	1.2782	1.2154	1.1720	1.2357	1.1753	1.0975	1.1690	1.2491	1.1943
1973	1.3274	1.4030	1.4951	1.4911	1.4002	1.3410	1.4522	1.3927	1.2347	1.3593	1.5177	1.4432

Table 22.9. Year-over-Year Monthly Chained Paasche Indices

	Month											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1074	1.1070	1.1471	1.1486	1.1115	1.0827	1.1075	1.0699	1.0414	1.0762	1.1218	1.0824
1972	1.2039	1.2437	1.3047	1.2777	1.2074	1.1682	1.2328	1.1569	1.0798	1.1421	1.2321	1.1908
1973	1.3243	1.4024	1.4934	1.4901	1.3872	1.3346	1.4478	1.3531	1.2018	1.3059	1.4781	1.4305

However, all of the year-over-year monthly series show a nice smooth year-over-year trend.

22.28 Approximate fixed-base year-over-year Laspeyres, Paasche, and Fisher indices can be constructed by replacing current-month revenue shares for the five products with the corresponding base-year monthly revenue shares for the same five products. The resulting approximate Laspeyres indices are equal to the original fixed-base Laspeyres indices, so there is no need to table the approximate Laspeyres indices. However, the approximate year-over-year Paasche and Fisher indices do differ from the fixed-base Paasche and Fisher indices found in Tables 22.4 and 22.5, so these new approximate indices are listed in Tables 22.6 (on preceding page) and 22.7.

22.29 Comparing the entries in Table 22.4 with the corresponding entries in Table 22.6, it can be seen that with few exceptions, the entries correspond fairly well. One of the bigger differences is the 1973 entry for the fixed-base Paasche index for month 9, which is 1.1664, while the corresponding entry for the approximate fixed-base Paasche index is 1.1920, for a 2.2 percent difference (1.1920/1.1664 = 1.022). In general, the approximate fixed-base Paasche indices are a bit bigger than the true fixed-base Paasche indices, as one might expect, because the approximate indices have some substitution bias built in. This is due to the fact that their revenue shares are held fixed at the 1970 levels.

22.30 Turning now to the chained year-over-year monthly indices using the artificial data set, the resultant 12 year-over-year monthly chained Laspeyres, Paasche, and Fisher indices, P_L, P_P, and P_F, where the month-to-month links are defined by equations (22.4)–(22.6), are listed in Tables 22.8 to 22.10.

22.31 Comparing the entries in Tables 22.8 and 22.9, it can be seen that the year-over-year monthly chained Laspeyres and Paasche price indices have smaller differences than the corresponding fixed-base Laspeyres and Paasche price indices in Tables 22.3 and 22.4. This is a typical pattern that was found in Chapter 19: *the use of chained indices tends to reduce the spread between Paasche and Laspeyres indices compared with their fixed-base counterparts*. The largest percentage difference between corresponding entries for the chained Laspeyres and Paasche indices in Tables 22.8 and 22.9 is 4.1 percent for month 10 in 1973 (1.3593/1.3059 = 1.041). Recall that the fixed-base Laspeyres and Paasche indices differed by 12.5 percent for the same month so that *chaining does tend to reduce the spread between these two equally plausible indices*.

22.32 The chained year-over-year Fisher indices listed in Table 22.10 are regarded as the best estimates of year-over-year inflation using the artificial data set.

Table 22.10. Year-over-Year Monthly Chained Fisher Indices

	Month											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1080	1.1069	1.1474	1.1487	1.1137	1.0835	1.1089	1.0741	1.0453	1.0831	1.1251	1.0837
1972	1.2048	1.2438	1.3052	1.2780	1.2114	1.1701	1.2343	1.1660	1.0886	1.1555	1.2405	1.1926
1973	1.3258	1.4027	1.4942	1.4906	1.3937	1.3378	1.4500	1.3728	1.2181	1.3323	1.4978	1.4368

Table 22.11. Year-over-Year Monthly Approximate Chained Laspeyres Indices

	Month											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1085	1.1068	1.1476	1.1488	1.1159	1.0844	1.1103	1.0783	1.0492	1.0901	1.1284	1.0849
1972	1.2056	1.2440	1.3057	1.2778	1.2168	1.1712	1.2346	1.1770	1.0989	1.1692	1.2482	1.1939
1973	1.3255	1.4007	1.4945	1.4902	1.4054	1.3390	1.4491	1.4021	1.2429	1.3611	1.5173	1.4417

Table 22.12. Year-over-Year Monthly Approximate Chained Paasche Indices

	Month											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1077	1.1057	1.1468	1.1478	1.1135	1.0818	1.1062	1.0721	1.0426	1.0760	1.1209	1.0813
1972	1.2033	1.2424	1.3043	1.2764	1.2130	1.1664	1.2287	1.1638	1.0858	1.1438	1.2328	1.1886
1973	1.3206	1.3971	1.4914	1.4880	1.3993	1.3309	1.4386	1.3674	1.2183	1.3111	1.4839	1.4300

Table 22.13. Year-over-Year Monthly Approximate Chained Fisher Indices

	Month											
Year	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1081	1.1063	1.1472	1.1483	1.1147	1.0831	1.1082	1.0752	1.0459	1.0830	1.1247	1.0831
1972	1.2044	1.2432	1.3050	1.2771	1.2149	1.1688	1.2317	1.1704	1.0923	1.1565	1.2405	1.1912
1973	1.3231	1.3989	1.4929	1.4891	1.4024	1.3349	1.4438	1.3847	1.2305	1.3358	1.5005	1.4358

22.33 The year-over-year chained Laspeyres, Paasche, and Fisher indices listed in Tables 22.8 to 22.10 can be approximated by replacing current-period product revenue shares for each month with the corresponding base-year monthly revenue shares. The resultant 12 year-over-year monthly approximate chained Laspeyres, Paasche, and Fisher indices (P_{AL}, P_{AP}, and P_{AF}), where the monthly links are defined by equations (22.8)–(22.10), are listed in Tables 22.11–22.13. (Tables 22.11 and 22.12 are on the preceding page.)

22.34 The year-over-year chained indices listed in Tables 22.11–22.13 approximate their true chained counterparts listed in Tables 22.8–22.10 closely. For 1973, the largest discrepancies are for the Paasche and Fisher indices for month 9: the chained Paasche is 1.2018, while the corresponding approximate chained Paasche is 1.2183, for a difference of 1.4 percent. The chained Fisher is 1.2181, while the corresponding approximate chained Fisher is 1.2305, for a difference of 1.0 percent. It can be seen that for the modified Turvey data set, the approximate year-over-year monthly Fisher indices listed in Table 22.13 approximate the theoretically preferred (but practically unfeasible) Fisher chained indices listed in Table 22.10 quite satisfactorily. Since the approximate Fisher indices are just as easy to compute as the approximate Laspeyres and Paasche indices, it may be useful to ask statistical agencies to make available to the public these approximate Fisher indices, along with the approximate Laspeyres and Paasche indices.

D. Year-over-Year Annual Indices

22.35 Assuming that each product in each season of the year is a separate annual product is the simplest and theoretically most satisfactory method for dealing with seasonal products when the goal is to construct annual price and quantity indices. This idea can be traced back to Mudgett in the consumer price context and to Stone in the producer price context:

> The basic index is a yearly index and as a price or quantity index is of the same sort as those about which books and pamphlets have been written in quantity over the years. (Bruce D. Mudgett, 1955, p. 97)

> The existence of a regular seasonal pattern in prices which more or less repeats itself year after year suggests very strongly that the varieties of a commodity available at different seasons cannot be transformed into one another without cost and that, accordingly, in all cases where seasonal variations in price are significant, the varieties available at different times of the year should be treated, in principle, as separate commodities. (Richard Stone, 1956, pp. 74–75)

22.36 Using the notation introduced in the previous section, the *Laspeyres, Paasche, and Fisher annual (chain link) indices* comparing the prices of year t with those of year $t+1$ can be defined as follows:

$$(22.13)\quad P_L\left(p^{t,1},\ldots,p^{t,12};p^{t+1,1},\ldots,p^{t+1,12};q^{t,1},\ldots,q^{t,12}\right) \equiv \frac{\sum_{m=1}^{12}\sum_{n\in S(m)} p_n^{t+1,m}q_n^{t,m}}{\sum_{m=1}^{12}\sum_{n\in S(m)} p_n^{t,m}q_n^{t,m}};$$

$$(22.14)\quad P_P(p^{t,1},\ldots,p^{t,12};p^{t+1,1},\ldots,p^{t+1,12}; q^{t+1,1},\ldots,q^{t+1,12})$$

$$\equiv \frac{\sum_{m=1}^{12} \sum_{n \in S(m)} p_n^{t+1,m} q_n^{t+1,m}}{\sum_{m=1}^{12} \sum_{n \in S(m)} p_n^{t,m} q_n^{t+1,m}};$$

$$(22.15)\ P_F(p^{t,1},\ldots,p^{t,12};p^{t+1,1},\ldots,p^{t+1,12}; q^{t,1},\ldots,q^{t,12};q^{t+1,1},\ldots,q^{t+1,12})$$
$$\equiv \sqrt{P_L\left(p^{t,1},\ldots,p^{t,12};p^{t+1,1},\ldots,p^{t+1,12};q^{t,1},\ldots,q^{t,12}\right)}$$
$$\times \sqrt{P_P\left(p^{t,1},\ldots,p^{t,12};p^{t+1,1},\ldots,p^{t+1,12};q^{t+1,1},\ldots,q^{t+1,12}\right)}.$$

22.37 The above formulas can be rewritten in price relative and monthly revenue share form as follows:

$$(22.16)\ P_L(p^{t,1},\ldots,p^{t,12};p^{t+1,1},\ldots,p^{t+1,12}; \sigma_1^t s^{t,1},\ldots,\sigma_{12}^t s^{t,12})$$
$$\equiv \sum_{m=1}^{12} \sum_{n \in S(m)} \sigma_m^t s_n^{t,m}\left(p_n^{t+1,m}/p_n^{t,m}\right);$$

$$(22.17)\ P_P(p^{t,1},\ldots,p^{t,12};p^{t+1,1},\ldots,p^{t+1,12};\sigma_1^{t+1}s^{t+1,1}, \sigma_1^{t+1}s^{t+1,1},\ldots,\sigma_{12}^{t+1}s^{t+1,12})$$
$$\equiv \sum_{m=1}^{12} \sum_{n \in S(m)} \sigma_m^t s_n^{t,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)$$
$$\equiv \left[\sum_{m=1}^{12} \sum_{n \in S(m)} \sigma_m^{t+1} s_n^{t+1,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1}$$
$$= \left[\sum_{m=1}^{12} \sigma_m^{t+1} \sum_{n \in S(m)} s_n^{t+1,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1}$$
$$= \left[\sum_{m=1}^{12} \sigma_m^{t+1}\left[P_P\left(p^{t,m},p^{t+1,m},s^{t+1,m}\right)\right]^{-1}\right]^{-1};$$

$$(22.18)\ P_F(p^{t,1},\ldots,p^{t,12};p^{t+1,1},\ldots,p^{t+1,12}; \sigma_1^t s^{t,1},\ldots,\sigma_{12}^t s^{t,12};\sigma_1^{t+1}s^{t+1,1},\ldots,\sigma_{12}^{t+1}s^{t+1,12})$$
$$\equiv \sqrt{\sum_{m=1}^{12} \sum_{n \in S(m)} \sigma_m^t s_n^{t,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)}$$
$$\times \sqrt{\left[\sum_{m=1}^{12} n \sum_{\in S(m)} \sigma_m^{t+1} s_n^{t+1,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1}}$$
$$= \sqrt{\sum_{m=1}^{12} \sigma_m^t\left[P_L\left(p^{t,m},p^{t+1,m},s^{t,m}\right)\right]}$$
$$\times \sqrt{\left[\sum_{m=1}^{12} \sigma_m^{t+1}\left[P_P\left(p^{t,m},p^{t+1,m},s^{t+1,m}\right)\right]^{-1}\right]^{-1}},$$

where the *revenue share* for month m in year t is defined as

$$(22.19)\ \sigma_n^t \equiv \frac{\sum_{n \in S(m)} p_n^{t,m} q_n^{t,m}}{\sum_{i=1}^{12} \sum_{j \in S(i)} p_j^{t,i} q_j^{t,i}};\ m = 1,2,\ldots,12;$$

$t = 0,1,\ldots,T$;

and the year-over-year monthly Laspeyres and Paasche (chained-linked) price indices $P_L(p^{t,m},p^{t+1,m},s^{t,m})$ and $P_P(p^{t,m},p^{t+1,m},s^{t+1,m})$ were defined in the previous section by equations (22.4) and (22.5), respectively. As usual, the annual chain-linked Fisher index P_F defined by equation (22.18), which compares the prices in every month of year t with the corresponding prices in year t + 1, is the geometric mean of the annual chain-linked Laspeyres and Paasche indices, P_L and P_P, defined by equations (22.16) and (22.17). The last equation in equations (22.16), (22.17), and (22.18) shows that these annual indices can be defined as (monthly) share weighted averages of the year-over-year monthly chain-linked Laspeyres and Paasche indices, $P_L(p^{t,m},p^{t+1,m},s^{t,m})$ and $P_P(p^{t,m},p^{t+1,m},s^{t+1,m})$, defined earlier by equations (22.4) and (22.5). Hence, once the year-over-year monthly indices defined in the previous section have been numerically calculated, it is easy to calculate the corresponding annual indices.

22.38 Fixed-base counterparts to the formulas defined by equations (22.16)–(22.18) can readily be defined: simply replace the data pertaining to period t with the corresponding data pertaining to the base period 0.

22.39 Using the data from the artificial data set in Table 22.1 of Section B, the annual fixed-base Laspeyres, Paasche, and Fisher indices are listed in Table 22.14. Viewing Table 22.14, it can be seen that by 1973, the annual fixed-base Laspeyres index exceeds its Paasche counterpart by 4.5 percent. Note that each series increases steadily.

22.40 The annual fixed-base Laspeyres, Paasche, and Fisher indices can be approximated by replacing any current shares with the corresponding base-year shares. The resulting annual approximate fixed-base Laspeyres, Paasche, and Fisher indices are listed in Table 22.15. Also listed in the last column of Table 22.15 is the fixed-base geometric Laspeyres annual index, P_{GL}. It is the weighted geometric mean counterpart to the fixed-base Laspeyres index, which is equal to a base-period weighted arithmetic average of the long-term price relative (see Chapter 19). It can be shown that P_{GL} approximates the approximate fixed-base Fisher index P_{AF} to the second order around a point where all of the long-term price relatives are equal to unity.[14] It is evident that the entries for the Laspeyres price indices are exactly the same in Tables 22.14 and 22.15. This is as it should be because the fixed-base Laspeyres price index uses only revenue shares from the base year 1970; consequently, the approximate fixed-base Laspeyres index is equal to the true fixed-base Laspeyres index. Comparing the columns labeled P_P and P_F in Table 22.14 and P_{AP} and P_{AF} in Table 22.15 shows that the approximate Paasche and approximate Fisher indices are quite close to the corresponding annual Paasche and Fisher indices. Thus, for the artificial data set, *the true annual fixed-base Fisher can be closely approximated by the corresponding approximate Fisher index P_{AF}* (or the geometric Laspeyres index P_{GL}), *which can be computed using the same information set that is normally available to statistical agencies*.

Table 22.14. Annual Fixed-Base Laspeyres, Paasche, and Fisher Price Indices

Year	P_L	P_P	P_F
1970	1.0000	1.0000	1.0000
1971	1.1008	1.0961	1.0984
1972	1.2091	1.1884	1.1987
1973	1.4144	1.3536	1.3837

Table 22.15. Annual Approximate Fixed-Base Laspeyres, Paasche, Fisher, and Geometric Laspeyres Indices

Year	P_{AL}	P_{AP}	P_{AF}	P_{GL}
1970	1.0000	1.0000	1.0000	1.0000
1971	1.1008	1.0956	1.0982	1.0983
1972	1.2091	1.1903	1.1996	1.2003
1973	1.4144	1.3596	1.3867	1.3898

22.41 Using the data from the artificial data set in Table 22.1 of Section B, the annual chained Laspeyres, Paasche, and Fisher indices can readily be calculated using the equations (22.16)–(22.18) for the chain links. The resulting indices are listed in Table 22.16. Viewing Table 22.16, it can be seen that the use of chained indices has substantially narrowed the gap between the Paasche and Laspeyres indices. The difference between the chained annual Laspeyres and Paasche indices in 1973 is only 1.5 percent (1.3994 versus 1.3791), whereas from Table 22.14, the difference between the fixed-base annual Laspeyres and Paasche indices in 1973 is 4.5 percent (1.4144 versus 1.3536). *Thus, the use of chained annual indices has substantially reduced the substitution (or representativity) bias of the Laspeyres and Paasche indices*. Comparing Tables 22.14 and 22.16, it can be seen that for this particular artificial data set, the annual fixed-base Fisher indices P_F are close to their annual chained Fisher counterparts P_{AF}. However, the annual chained Fisher indices should normally be regarded as the more desirable target index to approximate, since this index will normally give better results if prices and revenue shares are changing substantially over time.[15]

22.42 The current-year weights, $s_n^{t,m}$ and $\sigma_m^{\ t}$ and $s_n^{t+1,m}$ and $\sigma_m^{\ t+1}$, which appear in the chain-linked equations (22.16)–(22.18), can be approximated by the corresponding base-year weights, $s_n^{0,m}$ and $\sigma_m^{\ 0}$. This leads to the annual approximate chained

[14]See footnote 12.

[15]"Better" in the sense that the gap between the Laspeyres and Paasche indices will normally be reduced using chained indices under these circumstances. Of course, if there are no substantial trends in prices so that prices are just randomly changing, then it will generally be preferable to use the fixed-base Fisher index.

Table 22.16. Annual Chained Laspeyres, Paasche, and Fisher Price Indices

Year	P_L	P_P	P_F
1970	1.0000	1.0000	1.0000
1971	1.1008	1.0961	1.0984
1972	1.2052	1.1949	1.2001
1973	1.3994	1.3791	1.3892

Table 22.17. Annual Approximate Chained Laspeyres, Paasche, and Fisher Price Indices

Year	P_{AL}	P_{AP}	P_{AF}
1970	1.0000	1.0000	1.0000
1971	1.1008	1.0956	1.0982
1972	1.2051	1.1952	1.2002
1973	1.3995	1.3794	1.3894

Laspeyres, Paasche, and Fisher indices listed in Table 22.17.

22.43 Comparing the entries in Tables 22.16 and 22.17 shows that the approximate chained annual Laspeyres, Paasche, and Fisher indices are extremely close to the corresponding true chained annual Laspeyres, Paasche, and Fisher indices. Therefore, for the artificial data set, the true annual chained Fisher can be closely approximated by the corresponding approximate Fisher index, which can be computed using the same information set that is normally available to statistical agencies.

22.44 The approach to computing annual indices outlined in this section, which essentially involves taking monthly expenditure share-weighted averages of the 12 year-over-year monthly indices, should be contrasted with the approach that simply takes the arithmetic mean of the 12 monthly indices. The problem with the latter approach is that months where revenues are below the average (for example, February) are given the same weight in the unweighted annual average as months where revenues are above the average (for example, December).

E. Rolling-Year Annual Indices

22.45 In the previous section, the price and quantity data pertaining to the 12 months of a calendar year were compared to the 12 months of a base calendar year. However, there is no need to restrict attention to calendar year comparisons; any 12 consecutive months of price and quantity data could be compared to the price and quantity data of the base year, provided that the January data in the noncalendar year is compared to the January data of the base year, the February data of the noncalendar year is compared to the February data of the base year, and so on.[16] Alterman, Diewert, and Feenstra (1999, p. 70) called the resulting indices *rolling-year* or *moving-year* indices.[17]

22.46 In order to theoretically justify the rolling-year indices from the viewpoint of the economic approach to index number theory, some restrictions on preferences are required. The details of these assumptions can be found in Diewert (1996b, pp. 32–34; 1999a, pp. 56–61).

22.47 The problems involved in constructing rolling-year indices for the artificial data set that was introduced in Section B are now considered. For both fixed-base and chained rolling-year indices, the first 13 index number calculations are the same. For the year that ends with the data for December of 1970, the index is set equal to 1 for the Laspeyres, Paasche, and Fisher moving-year indices. The base-year data are the 44 nonzero price and quantity observations for the calendar year 1970. When the data for January of 1971 become available, the three nonzero price and quantity entries for January of calendar year 1970 are dropped and replaced with the corresponding entries for January of 1971. The data for the remaining months of the comparison year remain the same; that is, for February through December of the comparison year, the data for the rolling year are set equal to the corresponding entries for February through December of 1970. Thus, the Laspeyres, Paasche, or Fisher rolling-year index value for

[16]Diewert (1983b) suggested this type of comparison and termed the resulting index a *split year* comparison.

[17]Crump (1924, p. 185) and Mendershausen (1937, p. 245), respectively, used these terms in the context of various seasonal adjustment procedures. The term *rolling year* seems to be well established in the business literature in the United Kingdom.

January of 1971 compares the prices and quantities of January 1971 with the corresponding prices and quantities of January 1970, and for the remaining months of this first moving year, the prices and quantities of February through December of 1970 are simply compared with the exact same prices and quantities of February through December of 1970. When the data for February of 1971 become available, the three nonzero price and quantity entries for February for the last rolling year (which are equal to the three nonzero price and quantity entries for February of 1970) are dropped and replaced with the corresponding entries for February of 1971. The resulting data become the price and quantity data for the second rolling year. The Laspeyres, Paasche, or Fisher rolling-year index value for February of 1971 compares the prices and quantities of January and February of 1971 with the corresponding prices and quantities of January and February of 1970. For the remaining months of this first moving year, the prices and quantities of March through December of 1971 are compared with the exact same prices and quantities of March through December of 1970. This process of exchanging the price and quantity data of the current month in 1971 with the corresponding data of the same month in the base year 1970 in order to form the price and quantity data for the latest rolling year continues until December of 1971 is reached, when the current rolling year becomes the calendar year 1971. Thus, the Laspeyres, Paasche, and Fisher rolling-year indices for December of 1971 are equal to the corresponding fixed-base (or chained) annual Laspeyres, Paasche, and Fisher indices for 1971 listed in Tables 22.14 or 22.16.

22.48 Once the first 13 entries for the rolling-year indices have been defined as indicated, the remaining fixed-base rolling year Laspeyres, Paasche, and Fisher indices are constructed by taking the price and quantity data of the last 12 months and rearranging them so that the January data in the rolling year is compared with the January data in the base year, the February data in the rolling year is compared with the February data in the base year, and so on. The resulting fixed-base rolling-year Laspeyres, Paasche, and Fisher indices for the artificial data set are listed in Table 22.18.

22.49 Once the first 13 entries for the fixed-base rolling-year indices have been defined as indicated, the remaining *chained* rolling-year Laspeyres, Paasche, and Fisher indices are constructed by taking the price and quantity data of the last 12 months and comparing them with the corresponding data of the rolling year of the 12 months preceding the current rolling year. The resulting chained rolling-year Laspeyres, Paasche, and Fisher indices for the artificial data set are listed in the last three columns of Table 22.18. Note that the first 13 entries of the fixed-base Laspeyres, Paasche, and Fisher indices are equal to the corresponding entries for the chained Laspeyres, Paasche, and Fisher indices. Also, the entries for December (month 12) of 1970, 1971, 1972, and 1973 for the fixed-base rolling-year Laspeyres, Paasche, and Fisher indices are equal to the corresponding fixed-base annual Laspeyres, Paasche, and Fisher indices listed in Table 22.14. Similarly, the entries in Table 22.18 for December (month 12) of 1970, 1971, 1972, and 1973 for the chained rolling-year Laspeyres, Paasche, and Fisher indices are equal to the corresponding chained annual Laspeyres, Paasche, and Fisher indices listed in Table 22.16.

22.50 In Table 22.18, the rolling-year indices are smooth and free from seasonal fluctuations. For the fixed-base indices, each entry can be viewed as a *seasonally adjusted annual PPI* that compares the data of the 12 consecutive months that end with the year and month indicated with the corresponding price and quantity data of the 12 months in the base year, 1970. Thus, rolling-year indices offer statistical agencies an *objective* and *reproducible* method of seasonal adjustment that can compete with existing time series methods of seasonal adjustment.[18]

22.51 The use of chained indices substantially narrows the gap between the fixed-base moving-year Paasche and Laspeyres indices as shown in

[18]For discussions on the merits of econometric or time-series methods versus index number methods of seasonal adjustment, see Diewert (1999a, pp. 61–68) and Alterman, Diewert, and Feenstra (1999, pp. 78–110). The basic problem with time-series methods of seasonal adjustment is that the target seasonally adjusted index is difficult to specify in an unambiguous way; that is, there are an infinite number of possible target indices. For example, it is impossible to identify a temporary increase in inflation within a year from a changing seasonal factor. Thus, different econometricians will tend to generate different seasonally adjusted series, leading to a lack of reproducibility.

Table 22.18. Rolling-Year Laspeyres, Paasche, and Fisher Price Indices

Year	Month	P_L (fixed)	P_P (fixed)	P_F (fixed)	P_L (chain)	P_P (chain)	P_F (chain)
1970	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1	1.0082	1.0087	1.0085	1.0082	1.0087	1.0085
	2	1.0161	1.0170	1.0165	1.0161	1.0170	1.0165
	3	1.0257	1.0274	1.0265	1.0257	1.0274	1.0265
	4	1.0344	1.0364	1.0354	1.0344	1.0364	1.0354
	5	1.0427	1.0448	1.0438	1.0427	1.0448	1.0438
	6	1.0516	1.0537	1.0527	1.0516	1.0537	1.0527
	7	1.0617	1.0635	1.0626	1.0617	1.0635	1.0626
	8	1.0701	1.0706	1.0704	1.0701	1.0706	1.0704
	9	1.0750	1.0740	1.0745	1.0750	1.0740	1.0745
	10	1.0818	1.0792	1.0805	1.0818	1.0792	1.0805
	11	1.0937	1.0901	1.0919	1.0937	1.0901	1.0919
	12	1.1008	1.0961	1.0984	1.1008	1.0961	1.0984
1972	1	1.1082	1.1035	1.1058	1.1081	1.1040	1.1061
	2	1.1183	1.1137	1.1160	1.1183	1.1147	1.1165
	3	1.1287	1.1246	1.1266	1.1290	1.1260	1.1275
	4	1.1362	1.1324	1.1343	1.1366	1.1342	1.1354
	5	1.1436	1.1393	1.1414	1.1437	1.1415	1.1426
	6	1.1530	1.1481	1.1505	1.1528	1.1505	1.1517
	7	1.1645	1.1595	1.1620	1.1644	1.1622	1.1633
	8	1.1757	1.1670	1.1713	1.1747	1.1709	1.1728
	9	1.1812	1.1680	1.1746	1.1787	1.1730	1.1758
	10	1.1881	1.1712	1.1796	1.1845	1.1771	1.1808
	11	1.1999	1.1805	1.1901	1.1962	1.1869	1.1915
	12	1.2091	1.1884	1.1987	1.2052	1.1949	1.2001
1973	1	1.2184	1.1971	1.2077	1.2143	1.2047	1.2095
	2	1.2300	1.2086	1.2193	1.2263	1.2172	1.2218
	3	1.2425	1.2216	1.2320	1.2393	1.2310	1.2352
	4	1.2549	1.2341	1.2444	1.2520	1.2442	1.2481
	5	1.2687	1.2469	1.2578	1.2656	1.2579	1.2617
	6	1.2870	1.2643	1.2756	1.2835	1.2758	1.2797
	7	1.3070	1.2843	1.2956	1.3038	1.2961	1.3000
	8	1.3336	1.3020	1.3177	1.3273	1.3169	1.3221
	9	1.3492	1.3089	1.3289	1.3395	1.3268	1.3331
	10	1.3663	1.3172	1.3415	1.3537	1.3384	1.3460
	11	1.3932	1.3366	1.3646	1.3793	1.3609	1.3700
	12	1.4144	1.3536	1.3837	1.3994	1.3791	1.3892

Table 22.18. The difference between the rolling-year chained Laspeyres and Paasche indices in December of 1973 is only 1.5 percent (1.3994 versus 1.3791), whereas the difference between the rolling-year fixed-base Laspeyres and Paasche indices in December of 1973 is 4.5 percent (1.4144 versus 1.3536). *Thus, the use of chained indices has substantially reduced the substitution (or representativity) bias of the Laspeyres and Paasche indices*. As in the previous section, the chained Fisher rolling-year index is regarded as the *target seasonally adjusted annual index* when seasonal products are in the scope of the CPI. This type of index is also a suitable index for central banks to

use for inflation targeting purposes.[19] The six series in Table 22.18 are charted in Figure 22.1. The fixed-base Laspeyres index is the highest one, followed by the chained Laspeyres, the two Fisher indices (which are virtually indistinguishable), the chained Paasche, and, finally, the fixed-base Paasche. An increase in the slope of each graph can clearly be seen for the last 8 months, reflecting the increase in the month-to-month inflation rates that was built into the last 12 months of the data set.[20]

22.52 As in the previous section, the current-year weights, $s_n^{t,m}$ and $\sigma_m^{\ t}$ and $s_n^{t+1,m}$ and $\sigma_m^{\ t+1}$, which appear in the chain link equations (22.16)–(22.18) or in the corresponding fixed-base formulas, can be approximated by the corresponding base-year weights, $s_n^{0,m}$ and $\sigma_m^{\ 0}$. This leads to the annual approximate fixed-base and chained rolling-year Laspeyres, Paasche, and Fisher indices listed in Table 22.19.

22.53 Comparing the indices in Tables 22.18 and 22.19, it can be seen that the approximate rolling-year fixed-base and chained Laspeyres, Paasche, and Fisher indices listed in Table 22.19 are close to their true rolling-year counterparts listed in Table 22.18. In particular, the approximate chain rolling-year Fisher index (which can be computed using just base-year expenditure share information along with current information on prices) is close to the preferred target index, the rolling-year chained Fisher index. In December of 1973, these two indices differ by only 0.014 percent (1.3894/1.3892 = 1.00014). The indices in Table 22.19 are charted in Figure 22.2. Figures 22.1 and 22.2 are similar; in particular, the Fisher fixed-base and chained indices are virtually identical in both figures.

[19]See Diewert (2002c) for a discussion of the measurement issues involved in choosing an index for inflation targeting purposes.

[20]The arithmetic average of the 36 month-over-month inflation rates for the rolling-year fixed-base Fisher indices is 1.0091; the average of these rates for the first 24 months is 1.0076; for the last 12 months it is 1.0120; and for the last 2 months it is 1.0156. Thus, the increased month-to-month inflation rates for the last year are not *fully* reflected in the rolling-year indices until a full 12 months have passed. However, the fact that inflation has *increased* for the last 12 months of data compared to the earlier months is picked up almost immediately.

22.54 These tables demonstrate that year-over-year monthly indices and their generalizations to rolling-year indices perform very well using the modified Turvey data set; that is, like is compared to like and the existence of seasonal products does *not* lead to erratic fluctuations in the indices. The only drawback to the use of these indices is that it seems that they cannot give any information on *short-term month-to-month fluctuations in prices. This is most evident if seasonal baskets are completely different for each month, since in this case there is no possibility of comparing prices on a month-to-month basis*. However, in the following section, we learn that a current-period year-over-year monthly index *can* be used to predict a rolling-year index that is centered at the current month.

F. Predicting Rolling-Year Index Using Current-Period Year-over-Year Monthly Index

22.55 In a regime where the long-run trend in prices is smooth, changes in the year-over-year inflation rate for this month compared with last month theoretically could give valuable information about the long-run trend in price inflation. For the modified Turvey data set, this conjecture turns out to be true, as will be seen below.

22.56 The basic idea will be illustrated using the fixed-base Laspeyres rolling-year indices that are listed in Table 22.18 and the year-over-year monthly fixed-base Laspeyres indices listed in Table 22.3. In Table 22.18, the fixed-base Laspeyres rolling-year entry for December of 1971 compares the 12 months of price and quantity data pertaining to 1971 with the corresponding prices and quantities pertaining to 1970. This index number is the first entry in the first column of Table 22.20 and is labeled as P_L. Thus, in the first column of Table 22.20, the fixed-base rolling-year Laspeyres index, P_{LRY} taken from Table 22.18, is tabled starting at December of 1971 and carrying through to December of 1973, 24 observations in all. The first entry of this column shows that the index is a weighted average of year-over-year price relatives over all 12 months in 1970 and 1971. Thus, this index is an average of year-over-year monthly price changes, centered between June and July of the two years whose prices are being compared. As a result, an *approximation* to this annual index

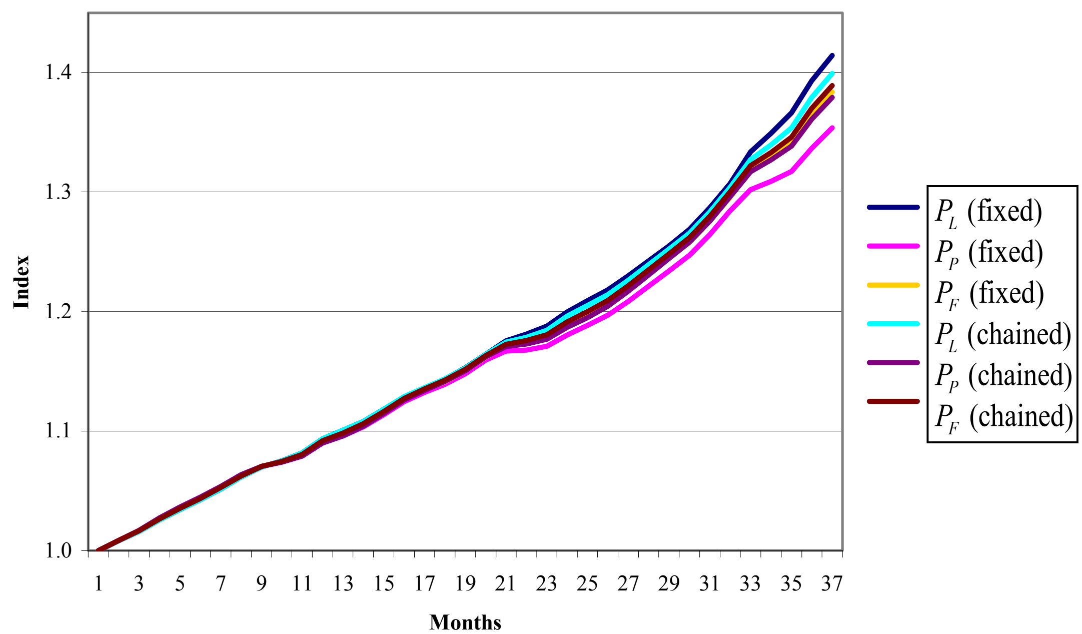
Table 22.19. Rolling-Year Approximate Laspeyres, Paasche, and Fisher Price Indices

Year	Month	P_{AL} (fixed)	P_{AP} (fixed)	P_{AF} (fixed)	P_{AL} (chain)	P_{AP} (chain)	P_{AF} (chain)
1970	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1	1.0082	1.0074	1.0078	1.0082	1.0074	1.0078
	2	1.0161	1.0146	1.0153	1.0161	1.0146	1.0153
	3	1.0257	1.0233	1.0245	1.0257	1.0233	1.0245
	4	1.0344	1.0312	1.0328	1.0344	1.0312	1.0328
	5	1.0427	1.0390	1.0409	1.0427	1.0390	1.0409
	6	1.0516	1.0478	1.0497	1.0516	1.0478	1.0497
	7	1.0617	1.0574	1.0596	1.0617	1.0574	1.0596
	8	1.0701	1.0656	1.0679	1.0701	1.0656	1.0679
	9	1.0750	1.0702	1.0726	1.0750	1.0702	1.0726
	10	1.0818	1.0764	1.0791	1.0818	1.0764	1.0791
	11	1.0937	1.0881	1.0909	1.0937	1.0881	1.0909
	12	1.1008	1.0956	1.0982	1.1008	1.0956	1.0982
1972	1	1.1082	1.1021	1.1051	1.1083	1.1021	1.1052
	2	1.1183	1.1110	1.1147	1.1182	1.1112	1.1147
	3	1.1287	1.1196	1.1241	1.1281	1.1202	1.1241
	4	1.1362	1.1260	1.1310	1.1354	1.1268	1.1311
	5	1.1436	1.1326	1.1381	1.1427	1.1336	1.1381
	6	1.1530	1.1415	1.1472	1.1520	1.1427	1.1473
	7	1.1645	1.1522	1.1583	1.1632	1.1537	1.1584
	8	1.1757	1.1620	1.1689	1.1739	1.1642	1.1691
	9	1.1812	1.1663	1.1737	1.1791	1.1691	1.1741
	10	1.1881	1.1710	1.1795	1.1851	1.1747	1.1799
	11	1.1999	1.1807	1.1902	1.1959	1.1855	1.1907
	12	1.2091	1.1903	1.1996	1.2051	1.1952	1.2002
1973	1	1.2184	1.1980	1.2082	1.2142	1.2033	1.2087
	2	1.2300	1.2074	1.2187	1.2253	1.2133	1.2193
	3	1.2425	1.2165	1.2295	1.2367	1.2235	1.2301
	4	1.2549	1.2261	1.2404	1.2482	1.2340	1.2411
	5	1.2687	1.2379	1.2532	1.2615	1.2464	1.2540
	6	1.2870	1.2548	1.2708	1.2795	1.2640	1.2717
	7	1.3070	1.2716	1.2892	1.2985	1.2821	1.2903
	8	1.3336	1.2918	1.3125	1.3232	1.3048	1.3139
	9	1.3492	1.3063	1.3276	1.3386	1.3203	1.3294
	10	1.3663	1.3182	1.3421	1.3538	1.3345	1.3441
	11	1.3932	1.3387	1.3657	1.3782	1.3579	1.3680
	12	1.4144	1.3596	1.3867	1.3995	1.3794	1.3894

could be obtained by taking the arithmetic average of the June and July year-over-year monthly indices pertaining to the years 1970 and 1971 (see the entries for months 6 and 7 for the year 1971 in Table 22.3, 1.0844 and 1.1103).[21] For the next rolling-year fixed-base Laspeyres index corresponding to the January 1972 entry in Table 22.18, *an approximation to this rolling-year index*, P_{ARY}, could be derived by taking the arithmetic average of the July and August year-over-year monthly

[21]If an average of the year-over-year monthly indices for May, June, July, and August were taken, a better approximation to the annual index could be obtained, and if an average of the year-over-year monthly indices for April, May, (continued) June, July, August, and September were taken, an even better approximation could be obtained to the annual index, and so on.

Figure 22.1. Rolling-Year Fixed-Base and Chained Laspeyres, Paasche, and Fisher Indices



indices pertaining to the years 1970 and 1971 (see the entries for months 7 and 8 for 1971 in Table 22.3, 1.1103 and 1.0783, respectively). These arithmetic averages of the two year-over-year monthly indices that are in the middle of the corresponding rolling-year are listed in the third column of Table 22.20. Table 22.20 shows that column 3, P_{ARY}, does not approximate column 1 particularly well, since the approximate indices in column 3 have some pronounced seasonal fluctuations, whereas the rolling-year indices in column 1, P_{LRY}, are free from seasonal fluctuations.

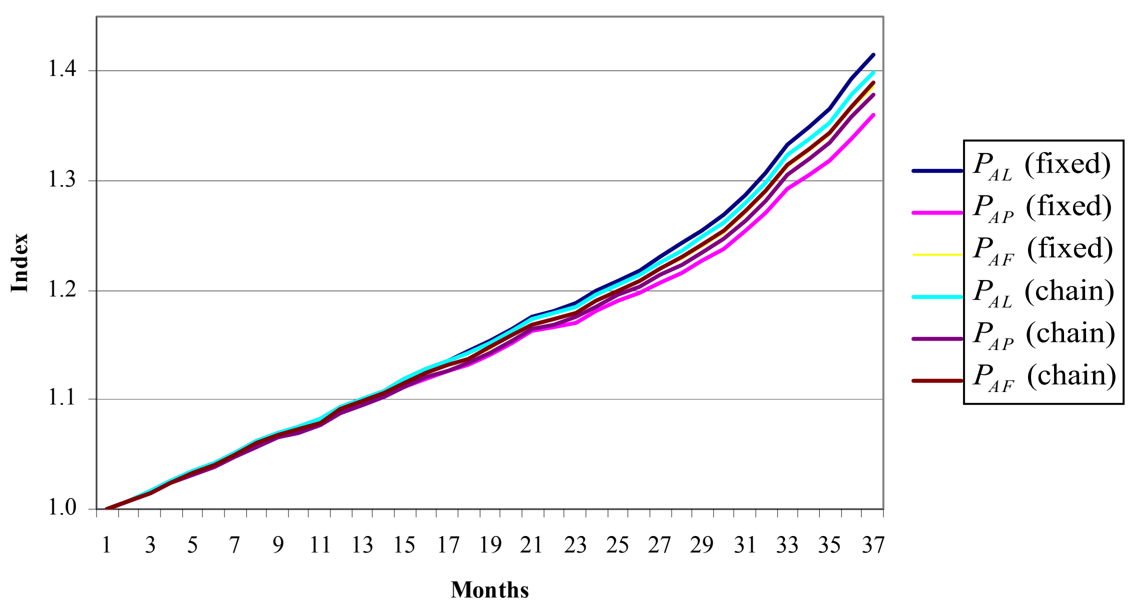
22.57 In the fourth column of Table 22.20, some *seasonal adjustment factors* are listed. For the first 12 observations, the entries in column 4 are simply the ratios of the entries in column 1 divided by the corresponding entries in column 3; that is, for the first 12 observations, the seasonal adjustment factors, *SAF*, are simply the ratio of the rolling-year indices starting at December of 1971 divided by the arithmetic average of the two year-over-year monthly indices that are in the middle of the corresponding rolling year.[22] The initial 12 seasonal adjustment factors are then just repeated for the remaining entries for column 4.

22.58 Once the seasonal adjustment factors have been defined, the approximate rolling-year index P_{ARY} can be multiplied by the corresponding seasonal adjustment factor, *SAF*, to form a *seasonally adjusted approximate rolling-year index*, P_{SAARY}, which is listed in column 2 of Table 22.20.

22.59 Compare columns 1 and 2 in Table 22.20: the rolling-year fixed-base Laspeyres index P_{LRY} and the seasonally adjusted approximate rolling-year index P_{SAARY} are identical for the first 12 observations, which follows by construction since P_{SAARY} equals the approximate rolling-year index

[22]Thus, if *SAF* is *greater than* 1, this means that the two months in the middle of the corresponding rolling year have year-over-year rates of price increase that average out to a number *below* the overall average of the year-over-year rates of price increase for the entire rolling year. The opposite is true if *SAF* is less than 1.

Figure 22.2. Rolling-Year Approximate Laspeyres, Paasche, and Fisher Price Indices



P_{ARY} multiplied by the seasonal adjustment factor *SAF*, which in turn is equal to the rolling-year Laspeyres index P_{LRY} divided by P_{ARY}. However, starting at December of 1972, the rolling-year index P_{LRY} differs from the corresponding seasonally adjusted approximate rolling-year index P_{SAARY}. It is apparent that for these last 13 months, P_{SAARY} is surprisingly close to P_{LRY}.[23] P_{LRY}, P_{SAARY}, and P_{ARY} are graphed in Figure 22.3. Due to the acceleration in the monthly inflation rate for the last year of data, it can be seen that the seasonally adjusted approximate rolling-year series, P_{SAARY}, does not pick up this accelerated inflation rate for the first few months of the last year (it lies well below P_{LRY} for February and March of 1973), but in general, it predicts the corresponding centered year quite well.

22.60 The above results for the modified Turvey data set are quite encouraging. If these results can be replicated for other data sets, *statistical agencies will be able to use the latest information on year-over-year monthly inflation to predict reasonably well the (seasonally adjusted) rolling-year inflation rate for a rolling year that is centered around the last two months*. Thus, policymakers and other interested users of the PPI could obtain a reasonably accurate forecast of trend inflation (centered around the current month) some six months in advance of the final estimates.

22.61 The method of seasonal adjustment used in this section is rather crude compared with some of the sophisticated econometric or statistical methods that are available. These more sophisticated methods could be used to improve the forecasts of trend inflation. However, it should be noted that if improved forecasting methods are used, it will be useful to use the rolling-year indices as *targets* for the forecasts rather than using a statistical package that simultaneously seasonally adjusts current data and calculates a trend rate of

[23]The means for the last 13 observations in columns 1 and 2 of Table 22.20 are 1.2980 and 1.2930. A regression of P_L on P_{SAARY} leads to an R^2 of 0.9662 with an estimated variance of the residual of .000214.

Table 22.20. Rolling-Year Fixed-Base Laspeyres and Seasonally Adjusted Approximate Rolling-Year Price Indices

Year	Month	P_{LRY}	P_{SAARY}	P_{ARY}	SAF
1971	12	1.1008	1.1008	1.0973	1.0032
1972	1	1.1082	1.1082	1.0943	1.0127
	2	1.1183	1.1183	1.0638	1.0512
	3	1.1287	1.1287	1.0696	1.0552
	4	1.1362	1.1362	1.1092	1.0243
	5	1.1436	1.1436	1.1066	1.0334
	6	1.1530	1.1530	1.1454	1.0066
	7	1.1645	1.1645	1.2251	0.9505
	8	1.1757	1.1757	1.2752	0.9220
	9	1.1812	1.1812	1.2923	0.9141
	10	1.1881	1.1881	1.2484	0.9517
	11	1.1999	1.1999	1.1959	1.0033
	12	1.2091	1.2087	1.2049	1.0032
1973	1	1.2184	1.2249	1.2096	1.0127
	2	1.2300	1.2024	1.1438	1.0512
	3	1.2425	1.2060	1.1429	1.0552
	4	1.2549	1.2475	1.2179	1.0243
	5	1.2687	1.2664	1.2255	1.0334
	6	1.2870	1.2704	1.2620	1.0066
	7	1.3070	1.2979	1.3655	0.9505
	8	1.3336	1.3367	1.4498	0.9220
	9	1.3492	1.3658	1.4943	0.9141
	10	1.3663	1.3811	1.4511	0.9517
	11	1.3932	1.3827	1.3783	1.0032
	12	1.4144	1.4188	1.4010	1.0127

inflation. What is being suggested here is that the rolling-year concept can be used to make reproducible the estimates of trend inflation that existing statistical methods of seasonal adjustment generate.[24]

22.62 In this section and the previous sections, all of the suggested indices have been based on year-over-year monthly indices and their averages. In the subsequent sections of this chapter, attention will be turned to more traditional price indices that attempt to compare the prices in the current month with the prices in a previous month.

G. Maximum Overlap Month-to-Month Price Indices

22.63 A reasonable method for dealing with seasonal products in the context of picking a target index for a month-to-month PPI is the following:[25]

- Identify products that are produced in both months; and
- For this maximum overlap set of products, calculate one of the three indices recommended in previous chapters; that is, the Fisher, Walsh, or Törnqvist-Theil index.[26]

Thus, the bilateral index number formula is applied only to the subset of products that are present in both periods.[27]

22.64 The question now arises: should the comparison month and the base month be adjacent months (thus leading to chained indices), or should the base month be fixed (leading to fixed-base indices)? It seems reasonable to prefer chained indices over fixed-base indices for two reasons:

- The set of seasonal products that overlaps during two consecutive months is likely to be much larger than the set obtained by comparing the prices of any given month with a fixed-base month (such as January of a base year). The comparisons made using chained indices, therefore, will be more comprehensive and accurate than those made using a fixed-base; and

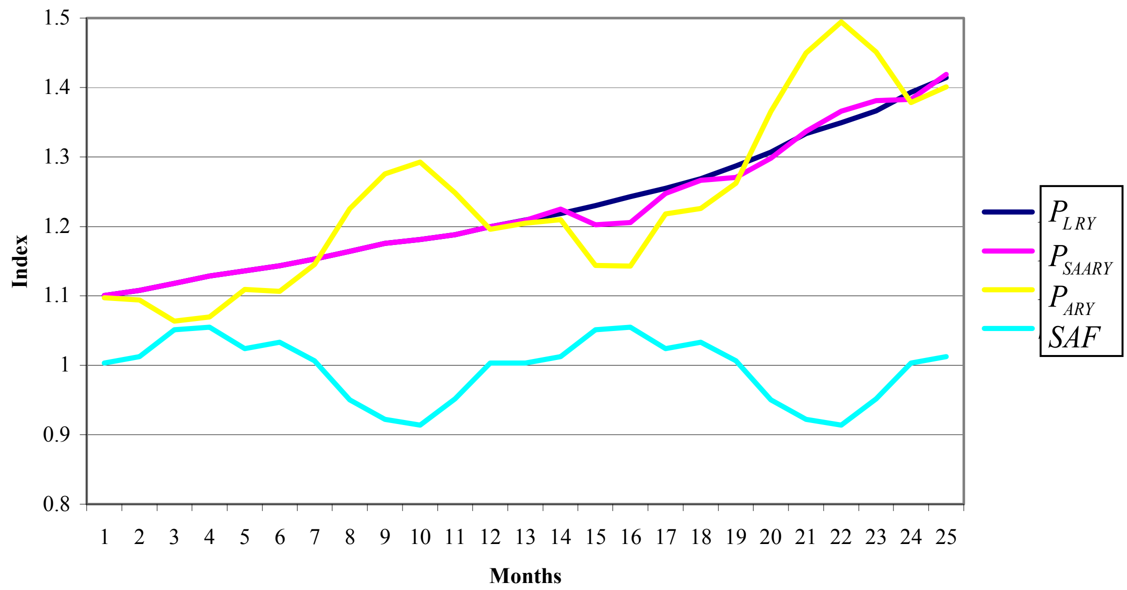
[24]The operator of a statistical seasonal adjustment package has to make somewhat arbitrary decisions on many factors; for example, are the seasonal factors additive or multiplicative? How long should the moving average be and what type? Thus, different operators of the seasonal adjustment package will tend to produce different estimates of the trend and the seasonal factors.

[25]For more on the economic approach and the assumptions on consumer preferences that can justify month-to-month maximum overlap indices, see Diewert (1999a, pp. 51–56).

[26]In order to reduce the number of equations, definitions, and tables, only the Fisher index will be considered in detail in this chapter.

[27]Keynes (1930, p. 95) called this the highest common factor method for making bilateral index number comparisons. This target index drops those strongly seasonal products that are not present in the marketplace during one of the two months being compared. Thus, the index number comparison is not completely comprehensive. Mudgett (1951, p. 46) called the error in an index number comparison that is introduced by the highest common factor method (or maximum overlap method) the homogeneity error.

Figure 22.3. Rolling-Year Fixed-Base Laspeyres and Seasonally Adjusted Approximate Rolling-Year Price Indices



- In many economies, on average 2 or 3 percent of price quotes disappear each month due to the introduction of new products and the disappearance of older ones. This rapid sample attrition means that fixed-base indices rapidly become unrepresentative; as a consequence, it seems preferable to use chained indices, which can more closely follow market developments.[28]

22.65 It will be useful to review the notation at this point and define some new notation. Let there be N products that are available in some month of some year and let $p_n^{t,m}$ and $q_n^{t,m}$ denote the price and quantity of product n that is in the marketplace[29] in month m of year t (if the product is unavailable, define $p_n^{t,m}$ and $q_n^{t,m}$ to be 0). Let $p^{t,m} \equiv [p_1^{t,m}, p_2^{t,m}, \ldots, p_N^{t,m}]$ and $q^{t,m} \equiv [q_1^{t,m}, q_2^{t,m}, \ldots, q_N^{t,m}]$ be the month m and year t price and quantity vectors, respectively. Let $S(t,m)$ be the set of products that is present in month m of year t *and the following month*. Then the maximum overlap Laspeyres, Paasche, and Fisher indices going from month m of year t to the following month can be defined as follows:[30]

$$(22.20)\ P_L\left(p^{t,m}, p^{t,m+1}, q^{t,m}, S(t,m)\right) = \frac{\sum_{n\in S(t,m)} p_n^{t,m+1} q_n^{t,m}}{\sum_{n\in S(t,m)} p_n^{t,m} q_n^{t,m}};$$

$m = 1, 2, \ldots 11;$

[28]This rapid sample degradation essentially forces some form of chaining at the elementary level in any case.

[29]As was seen in Chapter 20, it is necessary to have a target concept for the individual prices and quantities $p_n^{t,m}$ and $q_n^{t,m}$ at the finest level of aggregation. In most circumstances, these target concepts can be taken to be unit values for prices and total revenues for the quantities purchased.

[30]The equations are slightly different for the indices that go from December to January of the following year. In order to simplify the exposition, these equations are left for the reader.

$$(22.21)\ P_P\left(p^{t,m}, p^{t,m+1}, q^{t,m+1}, S(t,m)\right) = \frac{\sum_{n\in S(t,m)} p_n^{t,m+1} q_n^{t,m+1}}{\sum_{n\in S(t,m)} p_n^{t,m} q_n^{t,m+1}}\ ;\ m = 1, 2,...11;$$

$$(22.22)\ P_F\left(p^{t,m}, p^{t,m+1}, q^{t,m}, q^{t,m+1}, S(t,m)\right) \equiv \sqrt{P_L\left(p^{t,m}, p^{t,m+1}, q^{t,m}, S(t,m)\right)} \times \sqrt{P_P\left(p^{t,m}, p^{t,m+1}, q^{t,m+1}, S(t,m)\right)}\ ;$$

$m = 1, 2,...,11.$

Note that P_L, P_P, and P_F depend on the two (complete) price and quantity vectors pertaining to months m and $m + 1$ of year t, $p^{t,m}$,$p^{t,m+1}$,$q^{t,m}$,$q^{t,m+1}$, but they also depend on the set $S(t,m)$, which is the set of products that are present in both months. Thus, the product indices n that are in the summations on the right-hand sides of equations (22.20)–(22.22) include indices n that correspond to products that are present in *both* months, which is the meaning of $n\in S(t,m)$; that is, n belongs to the set $S(t,m)$.

22.66 To rewrite equations (22.20)–(22.22) in revenue share and price relative form, some additional notation is required. Define the revenue shares of product n in month m and $m + 1$ of year t, using the set of products that are present in month m of year t and the subsequent month, as follows:

$$(22.23)\ s_n^{t,m}(t,m) = \frac{p_n^{t,m} q_n^{t,m}}{\sum_{i\in S(t,m)} p_i^{t,m} q_i^{t,m}}\ ;\ n\in S(t,m)\ ;$$

$m = 1, 2,...,11;$

$$(22.24)\ s_n^{t,m+1}(t,m) = \frac{p_n^{t,m+1} q_n^{t,m+1}}{\sum_{i\in S(t,m)} p_i^{t,m+1} q_i^{t,m+1}}\ ;\ n\in S(t,m)\ ;$$

$m = 1, 2,...,11.$

The notation in equations (22.23) and (22.24) is rather messy because $s_n^{t,m+1}(t,m)$ has to be distinguished from $s_n^{t,m+1}(t,m+1)$. The revenue share $s_n^{t,m+1}(t,m)$ is the share of product n in month $m + 1$ of year t but where n is restricted to the set of products that are present in month m of year t and the subsequent month, whereas $s_n^{t,m+1}(t,m+1)$ is the share of product n in month $m + 1$ of year t but where n is restricted to the set of products that are present in month $m + 1$ of year t and the subsequent month. Thus, the set of superscripts, $t,m+1$ in $s_n^{t,m+1}(t,m)$, indicates that the revenue share is calculated using the price and quantity data of month $m + 1$ of year t and (t,m) indicates that the set of admissible products is restricted to the set of products that are present in both month m and the subsequent month.

22.67 Now define vectors of revenue shares. If product n is present in month m of year t and the following month, define $s_n^{t,m}(t,m)$ using equation (22.23); if this is not the case, define $s_n^{t,m}(t,m) = 0$. Similarly, if product n is present in month m of year t and the following month, define $s_n^{t,m+1}(t,m)$ using equation (22.24); if this is not the case, define $s_n^{t,m+1}(t,m) = 0$. Now define the N dimensional vectors:

$$s^{t,m}(t,m) \equiv [s_1^{t,m}(t,m), s_2^{t,m}(t,m),..., s_N^{t,m}(t,m)] \text{ and}$$
$$s^{t,m+1}(t,m) \equiv [s_1^{t,m+1}(t,m), s_2^{t,m+1}(t,m),..., s_N^{t,m+1}(t,m)]\ .$$

Using these share definitions, the month-to-month Laspeyres, Paasche, and Fisher equations (22.20)–(22.22) can also be rewritten in revenue share and price form as follows:

$$(22.25)\ P_L\left(p^{t,m}, p^{t,m+1}, s^{t,m}(t,m)\right) \equiv \sum_{n\in S(t,m)} s_n^{t,m}(t,m)\ \left(p_n^{t,m+1}/p_n^{t,m}\right);$$

$m = 1, 2,...11;$

$$(22.26)\ P_P\left(p^{t,m}, p^{t,m+1}, s^{t,m+1}(t,m)\right) \equiv \left[\sum_{n\in S(t,m)} s_n^{t,m+1}(t,m)\ \left(p_n^{t,m+1}/p_n^{t,m}\right)^{-1}\right]^{-1};$$

$m = 1, 2,...11;$

$$(22.27)\ P_F\left(p^{t,m}, p^{t,m+1}, s^{t,m}(t,m), s^{t,m+1}(t,m)\right) \equiv \sqrt{\sum_{n\in S(t,m)} s_n^{t,m}(t,m)\ \left(p_n^{t,m+1}/p_n^{t,m}\right)} \times \sqrt{\left[\sum_{n\in S(t,m)} s_n^{t,m+1}(t,m)\ \left(p_n^{t,m+1}/p_n^{t,m}\right)^{-1}\right]^{-1}}\ ;$$

$m = 1, 2,...,11.$

22.68 It is important to recognize that the revenue shares $s_n^{t,m}(t,m)$ that appear in the maximum overlap month-to-month Laspeyres index defined by equation (22.25) are *not* the revenue shares that could be taken from an establishment production survey for month m of year t; instead, they are the shares that result from revenues on seasonal products that are present in month m of year t but are not present in the following month. Similarly, the revenue shares $s_n^{t,m+1}(t,m)$ that appear in the maximum overlap month-to-month Paasche index defined by equation (22.26) are *not* the expenditure shares that could be taken from an establishment production survey for month $m + 1$ of year t; instead, they are the shares that result from revenues on seasonal products that are present in month $m + 1$ of year t but are not present in the preceding month.[31] The maximum overlap month-to-month Fisher index defined by equation (22.27) is the geometric mean of the Laspeyres and Paasche indices defined by equations (22.25) and (22.26).

22.69 Table 22.21 lists the maximum overlap chained month-to-month Laspeyres, Paasche, and Fisher price indices for the data listed in Section B. These indices are defined by equations (22.25), (22.26), and (22.27).

22.70 The chained maximum overlap Laspeyres, Paasche, and Fisher indices for December of 1973 are 1.0504, 0.1204, and 0.3556, respectively. Comparing these results to the year-over-year results listed in Tables 22.3, 22.4, and 22.5 indicate that the results in Table 22.21 are not at all realistic! These hugely different direct indices compared with the last row of Table 22.21 indicate that *the maximum overlap indices suffer from a significant downward bias for the artificial data set.*

22.71 What are the factors that can explain this downward bias? It is evident that part of the problem has to do with the seasonal pattern of prices for peaches and strawberries (products 2 and 4). These products are not present in the market for each month of the year. For the first month of the year when they become available, they have relatively high prices; in subsequent months, their prices drop substantially. The effects of these initially high prices (compared with the relatively low prices that prevailed in the last month that the

[31]It is important that the revenue shares that are used in an index number formula add up to unity. The use of unadjusted expenditure shares from an establishment survey would lead to a systematic bias in the index number formula.

Table 22.21. Month-to-Month Maximum Overlap Chained Laspeyres, Paasche, and Fisher Price Indices

Year	Month	P_L	P_P	P_F
1970	1	1.0000	1.0000	1.0000
	2	0.9766	0.9787	0.9777
	3	0.9587	0.9594	0.9590
	4	1.0290	1.0534	1.0411
	5	1.1447	1.1752	1.1598
	6	1.1118	1.0146	1.0621
	7	1.1167	1.0102	1.0621
	8	1.1307	0.7924	0.9465
	9	1.0033	0.6717	0.8209
	10	0.9996	0.6212	0.7880
	11	1.0574	0.6289	0.8155
	12	1.0151	0.5787	0.7665
1971	1	1.0705	0.6075	0.8064
	2	1.0412	0.5938	0.7863
	3	1.0549	0.6005	0.7959
	4	1.1409	0.6564	0.8654
	5	1.2416	0.7150	0.9422
	6	1.1854	0.6006	0.8438
	7	1.2167	0.6049	0.8579
	8	1.2230	0.4838	0.7692
	9	1.0575	0.4055	0.6548
	10	1.0497	0.3837	0.6346
	11	1.1240	0.3905	0.6626
	12	1.0404	0.3471	0.6009
1972	1	1.0976	0.3655	0.6334
	2	1.1027	0.3679	0.6369
	3	1.1291	0.3765	0.6520
	4	1.1974	0.4014	0.6933
	5	1.2818	0.4290	0.7415
	6	1.2182	0.3553	0.6579
	7	1.2838	0.3637	0.6833
	8	1.2531	0.2794	0.5916
	9	1.0445	0.2283	0.4883
	10	1.0335	0.2203	0.4771
	11	1.1087	0.2256	0.5001
	12	1.0321	0.1995	0.4538
1973	1	1.0866	0.2097	0.4774
	2	1.1140	0.2152	0.4897
	3	1.1532	0.2225	0.5065
	4	1.2493	0.2398	0.5474
	5	1.3315	0.2544	0.5821
	6	1.2594	0.2085	0.5124
	7	1.3585	0.2160	0.5416
	8	1.3251	0.1656	0.4684
	9	1.0632	0.1330	0.3760
	10	1.0574	0.1326	0.3744
	11	1.1429	0.1377	0.3967
	12	1.0504	0.1204	0.3556

products were available in the previous year) are not captured by the maximum overlap month-to-month indices, so the resulting indices build up a tremendous downward bias. The downward bias is most pronounced in the Paasche indices, which use the quantities or volumes of the current month. These volumes are relatively large compared to those of the initial month when the products become available, reflecting the effects of lower prices as the quantity made available in the market increases.

22.72 Table 22.22 lists the results using chained Laspeyres, Paasche, and Fisher indices for the artificial data set where the strongly seasonal products 2 and 4 are dropped from each comparison of prices. Thus, the indices in Table 22.22 are the usual chained Laspeyres, Paasche, and Fisher indices restricted to products 1, 3, and 5, which are available in each season. These indices are labeled as $P_L(3)$, $P_P(3)$, and $P_F(3)$.

22.73 The chained Laspeyres, Paasche, and Fisher indices (using only the three year-round products) for January of 1973 are 1.2038, 0.5424, and 0.8081, respectively. From Tables 22.8, 22.9, and 22.10, the chained year-over-year Laspeyres, Paasche, and Fisher indices for January of 1973 are 1.3274, 1.3243, and 1.3258, respectively. Thus, the chained indices using the year-round products, which are listed in Table 22.22, evidently *suffer from substantial downward biases*.

22.74 The data in Tables 22.1 and 22.2 demonstrate that the quantity of grapes (product 3) available in the market varies tremendously over the course of a year, with substantial increases in price for the months when grapes are almost out of season. Thus, the price of grapes decreases substantially as the quantity increases during the last half of each year, but the annual substantial increase in the price of grapes takes place in the first half of the year, when quantities in the market are small. This pattern of seasonal price and quantity changes will cause the overall index to take on a downward bias.[32] To verify that this conjecture is true, see the last three columns of Table 22.22, where chained Laspeyres, Paasche, and Fisher indices are calculated using only products 1 and 5. These indices are labeled $P_L(2)$, $P_P(2)$, and $P_F(2)$, respectively, and for January of 1973, they are equal to 1.0033, 0.9408, and 0.9715, respectively. These estimates based on two year-round products are much closer to the chained year-over-year Laspeyres, Paasche, and Fisher indices for January of 1973, which were 1.3274, 1.3243, and 1.3258, respectively, than the estimates based on the three year-round products. However, it is clear that *the chained Laspeyres, Paasche, and Fisher indices restricted to products 1 and 5 still have substantial downward biases for the artificial data set.* Basically, the problems are caused by the high volumes associated with low or declining prices and the low volumes caused by high or rising prices. These weight effects make the seasonal price declines bigger than the seasonal price increases using month-to-month index number formulas with variable weights.[33]

22.75 In addition to the downward biases that show up in Tables 22.21 and 22.22, all of these month-to-month chained indices show substantial seasonal fluctuations in prices over the course of a year. Therefore, these month-to-month indices are of little use to policymakers who are interested in short-term inflationary trends. *If the purpose of the month-to-month PPI is to indicate changes in general inflation, then statistical agencies should be cautious about including products that show strong seasonal fluctuations in prices in the*

[32]Baldwin (1990) used the Turvey data to illustrate various treatments of seasonal products. He has a good discussion of what causes various month-to-month indices to behave badly. "It is a sad fact that for some seasonal product groups, monthly price changes are not meaningful, whatever the choice of formula" (Andrew Baldwin, 1990, p. 264).

[33]This remark has an application to Chapter 20 on elementary indices where irregular sales during the course of a year could induce a similar downward bias in a month-to-month index that used monthly weights. Another problem with month-to-month chained indices is that purchases and sales of individual products can become irregular as the time period becomes shorter and shorter and the problem of zero purchases and sales becomes more pronounced. Feenstra and Shapiro (2003, p. 125) find an *upward* bias for their chained *weekly* indices for canned tuna compared to a fixed-base index; their bias was caused by variable weight effects due to the timing of advertising expenditures. In general, these drift effects of chained indices can be reduced by lengthening the time period, so that the *trends* in the data become more prominent than the *high-frequency fluctuations*.

Table 22.22. Month-to-Month Chained Laspeyres, Paasche, and Fisher Price Indices

Year	Month	$P_L(3)$	$P_P(3)$	$P_F(3)$	$P_L(2)$	$P_P(2)$	$P_F(2)$
1970	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	2	0.9766	0.9787	0.9777	0.9751	0.9780	0.9765
	3	0.9587	0.9594	0.9590	0.9522	0.9574	0.9548
	4	1.0290	1.0534	1.0411	1.0223	1.0515	1.0368
	5	1.1447	1.1752	1.1598	1.1377	1.1745	1.1559
	6	1.2070	1.2399	1.2233	1.2006	1.2424	1.2214
	7	1.2694	1.3044	1.2868	1.2729	1.3204	1.2964
	8	1.3248	1.1537	1.2363	1.3419	1.3916	1.3665
	9	1.0630	0.9005	0.9784	1.1156	1.1389	1.1272
	10	0.9759	0.8173	0.8931	0.9944	1.0087	1.0015
	11	1.0324	0.8274	0.9242	0.9839	0.9975	0.9907
	12	0.9911	0.7614	0.8687	0.9214	0.9110	0.9162
1971	1	1.0452	0.7993	0.9140	0.9713	0.9562	0.9637
	2	1.0165	0.7813	0.8912	0.9420	0.9336	0.9378
	3	1.0300	0.7900	0.9020	0.9509	0.9429	0.9469
	4	1.1139	0.8636	0.9808	1.0286	1.0309	1.0298
	5	1.2122	0.9407	1.0679	1.1198	1.1260	1.1229
	6	1.2631	0.9809	1.1131	1.1682	1.1763	1.1723
	7	1.3127	1.0170	1.1554	1.2269	1.2369	1.2319
	8	1.3602	0.9380	1.1296	1.2810	1.2913	1.2861
	9	1.1232	0.7532	0.9198	1.1057	1.0988	1.1022
	10	1.0576	0.7045	0.8632	1.0194	1.0097	1.0145
	11	1.1325	0.7171	0.9012	1.0126	1.0032	1.0079
	12	1.0482	0.6373	0.8174	0.9145	0.8841	0.8992
1972	1	1.1059	0.6711	0.8615	0.9652	0.9311	0.9480
	2	1.1111	0.6755	0.8663	0.9664	0.9359	0.9510
	3	1.1377	0.6912	0.8868	0.9863	0.9567	0.9714
	4	1.2064	0.7371	0.9430	1.0459	1.0201	1.0329
	5	1.2915	0.7876	1.0086	1.1202	1.0951	1.1075
	6	1.3507	0.8235	1.0546	1.1732	1.1470	1.1600
	7	1.4091	0.8577	1.0993	1.2334	1.2069	1.2201
	8	1.4181	0.7322	1.0190	1.2562	1.2294	1.2427
	9	1.1868	0.5938	0.8395	1.1204	1.0850	1.1026
	10	1.1450	0.5696	0.8076	1.0614	1.0251	1.0431
	11	1.2283	0.5835	0.8466	1.0592	1.0222	1.0405
	12	1.1435	0.5161	0.7682	0.9480	0.8935	0.9204
1973	1	1.2038	0.5424	0.8081	1.0033	0.9408	0.9715
	2	1.2342	0.5567	0.8289	1.0240	0.9639	0.9935
	3	1.2776	0.5755	0.8574	1.0571	0.9955	1.0259
	4	1.3841	0.6203	0.9266	1.1451	1.0728	1.1084
	5	1.4752	0.6581	0.9853	1.2211	1.1446	1.1822
	6	1.5398	0.6865	1.0281	1.2763	1.1957	1.2354
	7	1.6038	0.7136	1.0698	1.3395	1.2542	1.2962
	8	1.6183	0.6110	0.9944	1.3662	1.2792	1.3220
	9	1.3927	0.5119	0.8443	1.2530	1.1649	1.2081
	10	1.3908	0.5106	0.8427	1.2505	1.1609	1.2049
	11	1.5033	0.5305	0.8930	1.2643	1.1743	1.2184
	12	1.3816	0.4637	0.8004	1.1159	1.0142	1.0638

month-to-month index.[34] If seasonal products are included in a month-to-month index that is meant to indicate general inflation, then a seasonal adjustment procedure should be used to remove these strong seasonal fluctuations. Some simple types of seasonal adjustment procedures will be considered in Section K.

22.76 The rather poor performance of the month-to-month indices listed in the last two tables does not always occur in the context of seasonal products. In the context of calculating import and export price indices using quarterly data for the United States, Alterman, Diewert, and Feenstra (1999) found that maximum overlap month-to-month indices worked reasonably well.[35] However, statistical agencies should check that their month-to-month indices are at least approximately consistent with the corresponding year-over-year indices.

22.77 The various Paasche and Fisher indices computed in this section could be approximated by indices that replaced all current-period revenue shares with the corresponding revenue shares from the base year. These approximate Paasche and Fisher indices will not be reproduced here because they resemble their real counterparts and are themselves subject to tremendous downward bias.

H. Annual Basket Indices with Carryforward of Unavailable Prices

22.78 Recall that the Lowe (1823) index defined in earlier chapters had two reference periods:[36]

- The vector of quantity weights; and
- The base-period prices.

The *Lowe index* for month m was defined by the following equation:

$$(22.28)\quad P_{LO}\left(p^0, p^m, q\right) = \frac{\sum_{n=1}^{N} p_n^m q_n}{\sum_{n=1}^{N} p_n^0 q_n},$$

where $p^0 \equiv [p_1^0,\ldots,p_N^0]$ is the price reference period price vector, $p^m \equiv [p_1^m,\ldots,p_N^m]$ is the current month m price vector, and $q \equiv [q_1,\ldots,q_N]$ is the weight reference year quantity vector. For the purposes of this section, where the modified Turvey data set is used to numerically illustrate the index, the weight reference year will be 1970, and the resulting reference year quantity vector turns out to be:

$$(22.29)\quad q \equiv [q_1,\ldots,q_5] = [53889, 12881, 9198, 5379, 68653].$$

The price reference period for the prices will be December of 1970. For prices that are not available in the current month, the last available price is carried forward. The resulting Lowe index with carryforward of missing prices using the modified Turvey data set can be found in column 1 of Table 22.23.

22.79 Baldwin's comments on this type of annual basket (AB) index are worth quoting at length:

> For seasonal goods, the AB index is best considered an index partially adjusted for seasonal variation. It is based on annual quantities, which do not reflect the seasonal fluctuations in the volume of purchases, and on raw monthly prices, which do incorporate seasonal price fluctuations. Zarnowitz (1961, pp. 256–257) calls it an index of "a hybrid sort." Being neither of sea nor land, it does not provide an appropriate measure either of monthly or 12 month price change. The question that an AB index answers with respect to price change from January to February say, or January of one year to January of the next, is "What would have the change in consumer prices have been if there were no seasonality in purchases in the months in question, but prices

[34]However, if the purpose of the index is to compare the prices that producers *actually receive* in two consecutive months, ignoring the possibility that the purchasers may regard a seasonal good as being qualitatively different in the two months, then the production of a month-to-month PPI that has large seasonal fluctuations can be justified.

[35]They checked the validity of their month-to-month indices by cumulating them for four quarters and comparing them to the corresponding year-over-year indices. They found only relatively small differences. However, note that irregular high-frequency fluctuations will tend to be smaller for quarters than for months. For this reason, chained quarterly indices can be expected to perform better than chained monthly or weekly indices.

[36]In the context of seasonal price indices, this type of index corresponds to Bean and Stine's (1924, p. 31) Type A index.

> nonetheless retained their own seasonal behaviour?" It is hard to believe that this is a question that anyone would be interested in asking. On the other hand, the 12 month ratio of an AB index based on seasonally adjusted prices would be conceptually valid, if one were interested in eliminating seasonal influences (Andrew Baldwin, 1990, p. 258).

In spite of Baldwin's somewhat negative comments on the Lowe index, it is the index that is preferred by many statistical agencies, so it is necessary to study its properties in the context of strongly seasonal data.

22.80 Recall that the *Young* (1812) *index* was defined in Chapters 1 and 15 as follows:

$$(22.30)\quad P_Y\left(p^0, p^m, s\right) = \sum_{n=1}^{N} s_n\left(p_n^m / p_n^0\right),$$

where $s \equiv [s_1,\ldots,s_N]$ is the weight reference year vector of revenue shares. For the purposes of this section, where the modified Turvey data set is used to numerically illustrate the index, the weight reference year will be 1970 and the resulting revenue share vector turns out to be

$$(22.31)\quad s \equiv [s_1,\ldots,s_5] = [0.3284, 0.1029, 0.0674, 0.0863, 0.4149].$$

Again, the base period for the prices will be December 1970. For prices that are not available in the current month, the last available price is carried forward. The resulting Young index with carry-forward of missing prices using the modified Turvey data set can be found in column 2 of Table 22.23.

22.81 The *geometric Laspeyres index* was defined in Chapter 19 as follows:

$$(22.32)\quad P_{GL}(p^0, p^m, s) \equiv \prod_{n=1}^{N} (p_n^m / p_n^0)^{s_n}.$$

Thus, the geometric Laspeyres index makes use of the same information as the Young index, except that a geometric average of the price relatives is taken instead of an arithmetic one. Again, the weight reference year is 1970, the price reference-period is December 1970, and the index is illustrated using the modified Turvey data set with carryforward of missing prices. See column 3 of Table 22.23.

Table 22.23. Lowe, Young, Geometric Laspeyres, and Centered Rolling-Year Indices with Carry-forward Prices

Year	Month	P_{LO}	P_Y	P_{GL}	P_{CRY}
1970	12	1.0000	1.0000	1.0000	1.0000
1971	1	1.0554	1.0609	1.0595	1.0091
	2	1.0711	1.0806	1.0730	1.0179
	3	1.1500	1.1452	1.1187	1.0242
	4	1.2251	1.2273	1.1942	1.0298
	5	1.3489	1.3652	1.3249	1.0388
	6	1.4428	1.4487	1.4068	1.0478
	7	1.3789	1.4058	1.3819	1.0547
	8	1.3378	1.3797	1.3409	1.0631
	9	1.1952	1.2187	1.1956	1.0729
	10	1.1543	1.1662	1.1507	1.0814
	11	1.1639	1.1723	1.1648	1.0885
	12	1.0824	1.0932	1.0900	1.0965
1972	1	1.1370	1.1523	1.1465	1.1065
	2	1.1731	1.1897	1.1810	1.1174
	3	1.2455	1.2539	1.2363	1.1254
	4	1.3155	1.3266	1.3018	1.1313
	5	1.4262	1.4508	1.4183	1.1402
	6	1.5790	1.5860	1.5446	1.1502
	7	1.5297	1.5550	1.5349	1.1591
	8	1.4416	1.4851	1.4456	1.1690
	9	1.3038	1.3342	1.2974	1.1806
	10	1.2752	1.2960	1.2668	1.1924
	11	1.2852	1.3034	1.2846	1.2049
	12	1.1844	1.2032	1.1938	1.2203
1973	1	1.2427	1.2710	1.2518	1.2386
	2	1.3003	1.3308	1.3103	1.2608
	3	1.3699	1.3951	1.3735	1.2809
	4	1.4691	1.4924	1.4675	1.2966
	5	1.5972	1.6329	1.5962	1.3176
	6	1.8480	1.8541	1.7904	1.3406
	7	1.7706	1.8010	1.7711	0.0000
	8	1.6779	1.7265	1.6745	0.0000
	9	1.5253	1.5676	1.5072	0.0000
	10	1.5371	1.5746	1.5155	0.0000
	11	1.5634	1.5987	1.5525	0.0000
	12	1.4181	1.4521	1.4236	0.0000

22.82 It is interesting to compare the above three indices that use annual baskets to the fixed-base Laspeyres rolling-year indices computed earlier.

However, the rolling-year index that ends in the current month is centered five and a half months backward. Thus, the above three annual basket-type indices will be compared with an arithmetic average of two rolling-year indices that have their last month five and six months forward. This latter centered rolling-year index is labeled P_{CRY} and is listed in the last column of Table 22.23.[37] Note that zeros are entered for the last six rows of this column, since the data set does not extend six months into 1974. As a result, the centered rolling-year indices cannot be calculated for these last six months.

22.83 It can be seen that the Lowe, Young, and geometric Laspeyres indices have a considerable amount of seasonality in them and do not at all approximate their rolling-year counterparts listed in the last column of Table 22.23.[38] Therefore, without seasonal adjustment, the Lowe, Young, and geometric Laspeyres indices are not suitable predictors for their seasonally adjusted rolling-year counterparts.[39] The four series, P_{LO}, P_Y, $P_{GL,}$ and P_{CRY}, listed in Table 22.23 are also plotted in Figure 22.4. The Young price index is generally the highest, followed by the Lowe index, and then the geometric Laspeyres. The centered rolling-year Laspeyres counterpart index, P_{CRY}, is generally below the other three indices (and does not have the strong seasonal movements of the other three series), but it moves in a roughly parallel fashion to the other three indices.[40] Note that the seasonal movements of P_{LO}, P_Y, and P_{GL} are quite regular. This regularity will be exploited in Section K in order to use these month-to-month indices to predict their rolling-year counterparts.

22.84 Part of the problem may be the fact that the prices of strongly seasonal goods have been carried forward for the months when the products are not available. This will tend to add to the amount of seasonal movements in the indices, particularly when there is high general inflation. For this reason, the Lowe, Young, and geometric Laspeyres indices will be recomputed in the following section, using an imputation method for the missing prices rather than simply carrying forward the last available price.

I. Annual Basket Indices with Imputation of Unavailable Prices

22.85 Instead of simply carrying forward the last available price of a seasonal product that is not sold during a particular month, it is possible to use an *imputation method* to fill in the missing prices. Alternative imputation methods are discussed by Armknecht and Maitland-Smith (1999) and Feenstra and Diewert (2001), but the basic idea is to take the last available price and *impute* prices for the missing periods that trend with another index. This other index could be an index of available prices for the general category of product or higher-level components of the PPI. For the purposes of this section, the imputation index is taken to be a price index that grows at the multiplicative rate of 1.008, since the fixed-base rolling-year Laspeyres indices for the modified Turvey data set grow at approximately 0.8 percent per month.[41] Using this imputation method to fill in the missing prices, the Lowe, Young, and geometric Laspeyres indices defined in the previous section can be recomputed. The resulting indices are listed in Table 22.24, along with the centered rolling-year index P_{CRY} for comparison purposes.

22.86 As could be expected, the Lowe, Young, and geometric Laspeyres indices that used imputed prices are on average a bit *higher* than their counterparts that used carryforward prices, but the variability of the imputed indices is generally a bit

[37]This series was normalized to equal 1 in December 1970 so that it would be comparable to the other month-to-month indices.

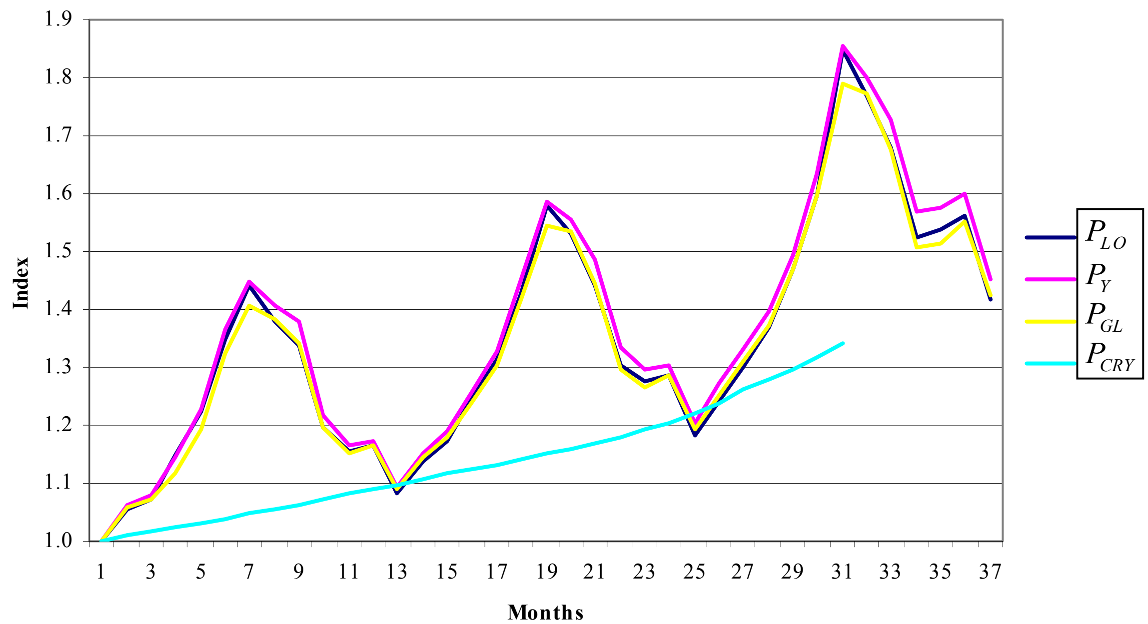
[38]The sample means of the four indices are 1.2935 (Lowe), 1.3110 (Young), 1.2877 (geometric Laspeyres) and 1.1282 (rolling-year). The geometric Laspeyres indices will always be equal to or less than their Young counterparts, since a weighted geometric mean is always equal to or less than the corresponding weighted arithmetic mean.

[39]In Section K, the Lowe, Young, and Geometric Laspeyres indices will be seasonally adjusted.

[40]In Figure 22.4, P_{CRY} stops at the June 1973 value for the index, which is the last month that the centered index can be constructed from the available data.

[41]For the last year of data, the imputation index is escalated by an additional monthly growth rate of 1.008.

Figure 22.4. Lowe, Young, Geometric Laspeyres, and Centered Rolling-Year Indices with Carry-forward Prices



lower.[42] The series that are listed in Table 22.24 are also plotted in Figure 22.5. It is apparent that the Lowe, Young, and geometric Laspeyres indices that use imputed prices still have a huge amount of seasonality in them and do not closely approximate their rolling-year counterparts listed in the last column of Table 22.24.[43] Consequently, without seasonal adjustment, the Lowe, Young, and geometric Laspeyres indices using imputed prices are not suitable predictors for their seasonally adjusted rolling-year counterparts.[44] As these indices stand, they are not suitable as measures of general inflation going from month to month.

J. Bean and Stine Type C or Rothwell Indices

22.87 The final month-to-month index[45] that will be considered in this chapter is the *Bean and Stine Type C* (1924, p. 31) or *Rothwell* (1958, p. 72) index.[46] This index makes use of *seasonal baskets* in the base year, denoted as the vectors $q^{0,m}$ for the months $m = 1, 2,\ldots,12$. The index also makes use

[42]For the Lowe indices, the mean for the first 31 observations increases (with imputed prices) from 1.3009 to 1.3047, but the standard deviation decreases from 0.18356 to 0.18319; for the Young indices, the mean for the first 31 observations increases from 1.3186 to 1.3224, but the standard deviation decreases from 0.18781 to 0.18730; and for the geometric Laspeyres indices, the mean for the first 31 observations increases from 1.2949 to 1.2994, and the standard deviation also increases slightly from 0.17582 to 0.17599. The imputed indices are preferred to the carryforward indices on general methodological grounds: in high inflation environments, the carryforward indices will be subject to sudden jumps when previously unavailable products become available.

[43]Note also that Figures 22.4 and 22.5 are similar.

[44]In Section K, the Lowe, Young, and geometric Laspeyres indices using imputed prices will be seasonally adjusted.

[45]For other suggested month-to-month indices in the seasonal context, see Balk (1980a, 1980b, 1980c, 1981).

[46]This is the index favored by Baldwin (1990, p. 271) and many other price statisticians in the context of seasonal products.

Table 22.24. Lowe, Young, Geometric Laspeyres, and Centered Rolling-Year Indices with Imputed Prices

Year	Month	P_{LOI}	P_{YI}	P_{GLI}	P_{CRY}
1970	12	1.0000	1.0000	1.0000	1.0000
1971	1	1.0568	1.0624	1.0611	1.0091
	2	1.0742	1.0836	1.0762	1.0179
	3	1.1545	1.1498	1.1238	1.0242
	4	1.2312	1.2334	1.2014	1.0298
	5	1.3524	1.3682	1.3295	1.0388
	6	1.4405	1.4464	1.4047	1.0478
	7	1.3768	1.4038	1.3798	1.0547
	8	1.3364	1.3789	1.3398	1.0631
	9	1.1949	1.2187	1.1955	1.0729
	10	1.1548	1.1670	1.1514	1.0814
	11	1.1661	1.1747	1.1672	1.0885
	12	1.0863	1.0972	1.0939	1.0965
1972	1	1.1426	1.1580	1.1523	1.1065
	2	1.1803	1.1971	1.1888	1.1174
	3	1.2544	1.2630	1.2463	1.1254
	4	1.3260	1.3374	1.3143	1.1313
	5	1.4306	1.4545	1.4244	1.1402
	6	1.5765	1.5831	1.5423	1.1502
	7	1.5273	1.5527	1.5326	1.1591
	8	1.4402	1.4841	1.4444	1.1690
	9	1.3034	1.3343	1.2972	1.1806
	10	1.2758	1.2970	1.2675	1.1924
	11	1.2875	1.3062	1.2873	1.2049
	12	1.1888	1.2078	1.1981	1.2203
1973	1	1.2506	1.2791	1.2601	1.2386
	2	1.3119	1.3426	1.3230	1.2608
	3	1.3852	1.4106	1.3909	1.2809
	4	1.4881	1.5115	1.4907	1.2966
	5	1.6064	1.6410	1.6095	1.3176
	6	1.8451	1.8505	1.7877	1.3406
	7	1.7679	1.7981	1.7684	0.0000
	8	1.6773	1.7263	1.6743	0.0000
	9	1.5271	1.5700	1.5090	0.0000
	10	1.5410	1.5792	1.5195	0.0000
	11	1.5715	1.6075	1.5613	0.0000
	12	1.4307	1.4651	1.4359	0.0000

of a vector of *base-year unit-value prices*, $p^0 \equiv [p_1^0,\ldots,p_5^0]$, where the *n*th price in this vector is defined as

$$(22.33)\quad p_n^0 \equiv \frac{\sum_{m=1}^{12} p_n^{0,m} q_n^{0,m}}{\sum_{m=1}^{12} q_n^{0,m}}\,;\; n = 1,\ldots,5.$$

The *Rothwell price index for month* m *in year* t can now be defined as follows:

$$(22.34)\quad P_R\left(p^0, p^{t,m}, q^{0.m}\right) \equiv \frac{\sum_{n=1}^{5} p_n^{t,m} q_n^{0,m}}{\sum_{n=1}^{5} p_n^0 q_n^{0.m}}\,;$$

$m = 1,\ldots,12.$

Thus, as the month changes, the quantity weights for the index change. The month-to-month movements in this index, therefore, are a mixture of price and quantity changes.[47]

22.88 Using the modified Turvey data set, the base year is chosen to be 1970 as usual, and the index is started off at December of 1970. The Rothwell index P_R is compared to the Lowe index with carryforward of missing prices P_{LO} in Table 22.25. To make the series a bit more comparable, the *normalized Rothwell index* P_{NR} is also listed in Table 22.25; this index is simply equal to the original Rothwell index divided by its first observation.

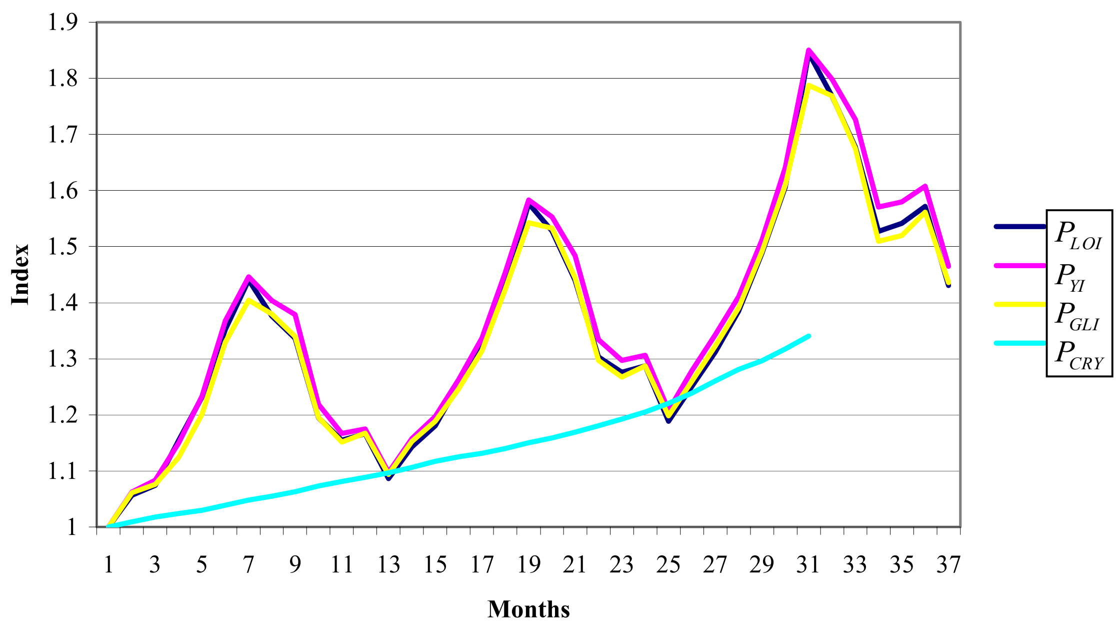
22.89 Viewing Figure 22.6, which plots the Lowe index with the carryforward of the last price and the normalized Rothwell index, it is clear that the Rothwell index has smaller seasonal movements than the Lowe index and is less volatile in general.[48] However, it is evident that there still are large seasonal movements in the Rothwell index, and it may not be a suitable index, for measuring general inflation without some sort of seasonal adjustment.

22.90 In the following section, the annual basket-type indices (with and without imputation) defined earlier in Sections H and I will be season-

[47] Rothwell (1958, p. 72) showed that the month-to-month movements in the index have the form of an expenditure ratio divided by a quantity index.

[48] For all 37 observations in Table 22.25, the Lowe index has a mean of 1.3465 and a standard deviation of 0.20313, while the normalized Rothwell has a mean of 1.2677 and a standard deviation of 0.18271.

Figure 22.5. Lowe, Young, Geometric Laspeyres, and Centered Rolling-Year Indices with Imputed Prices



ally adjusted using essentially the same method that was used in Section F and compared with a standard seasonal adjustment using X-11.

K. Forecasting Rolling-Year Indices Using Month-to-Month Annual Basket Indices

22.91 Recall that Table 22.23 in Section H presented the Lowe, Young, geometric Laspeyres (using carryforward prices), and centered rolling-year indices for the 37 observations running from December 1970 to December 1973 (P_{LO}, P_Y, P_{GL}, and P_{CRY}, respectively). For each of the first three series, define a seasonal adjustment factor, *SAF*, as the centered rolling-year index P_{CRY} divided by P_{LO}, P_Y, and P_{GL} , respectively, for the first 12 observations. Now for each of the three series, repeat these 12 seasonal adjustment factors for observations 13–24 and then repeat them for the remaining observations. These operations will create three *SAF* series for all 37 observations (label them SAF_{LO}, SAF_Y, and SAF_{GL}, respectively), but only the first 12 observations in the P_{LO}, P_Y, P_{GL}, and P_{CRY} series are used to create the three *SAF* series. Finally, define *seasonally adjusted Lowe, Young, and geometric Laspeyres indices* by multiplying each unadjusted index by the appropriate seasonal adjustment factor:

$$(22.35)\quad P_{LOSA} \equiv P_{LO}\, SAF_{LO}\,;\ P_{YSA} \equiv P_Y\, SAF_Y\,;\ P_{GLSA} \equiv P_{GL}\, SAF_{GL}\,.$$

Figure 22.6. Lowe and Normalized Rothwell Indices

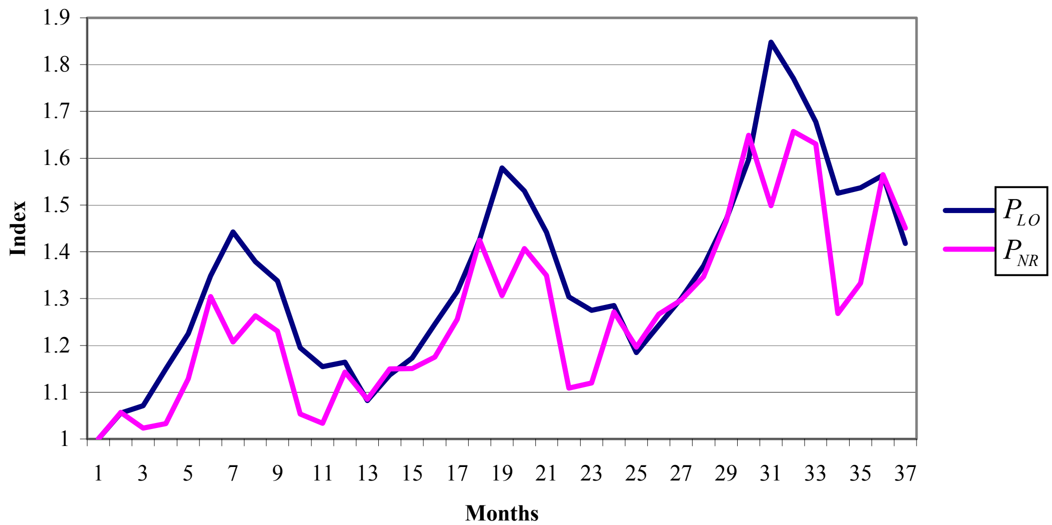


Figure 22.7a. Seasonally Adjusted Lowe, Young, Geometric Laspeyres, and Centered Rolling-Year Indices

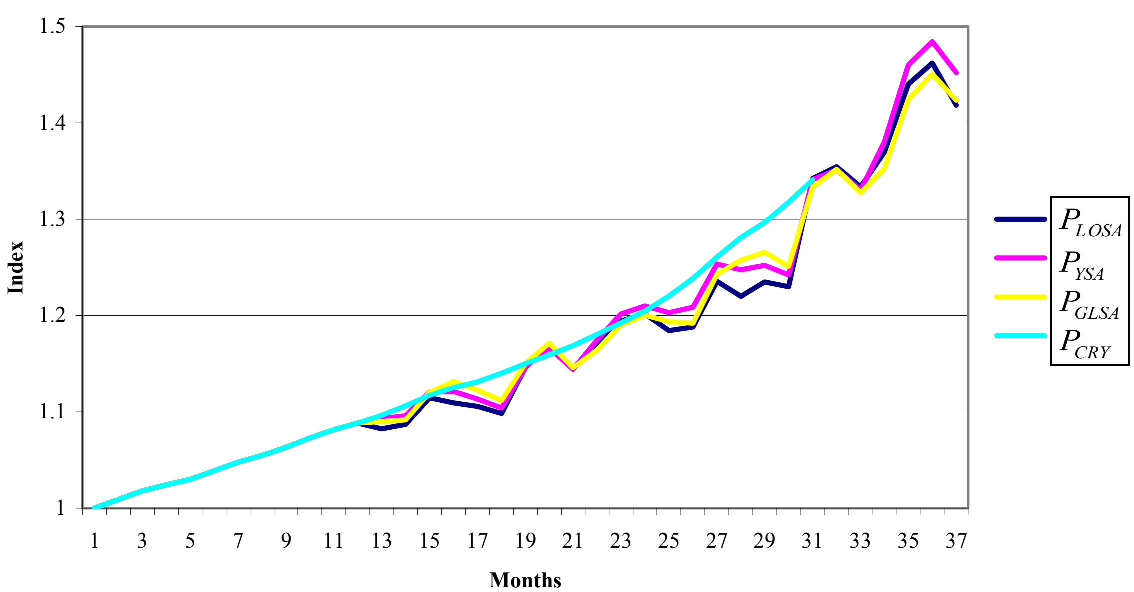


Table 22.25. Lowe with Carryforward Prices, Normalized Rothwell, and Rothwell Indices

Year	Month	P_{LO}	P_{NR}	P_R
1970	12	1.0000	1.0000	0.9750
1971	1	1.0554	1.0571	1.0306
	2	1.0711	1.0234	0.9978
	3	1.1500	1.0326	1.0068
	4	1.2251	1.1288	1.1006
	5	1.3489	1.3046	1.2720
	6	1.4428	1.2073	1.1771
	7	1.3789	1.2635	1.2319
	8	1.3378	1.2305	1.1997
	9	1.1952	1.0531	1.0268
	10	1.1543	1.0335	1.0077
	11	1.1639	1.1432	1.1146
	12	1.0824	1.0849	1.0577
1972	1	1.1370	1.1500	1.1212
	2	1.1731	1.1504	1.1216
	3	1.2455	1.1752	1.1459
	4	1.3155	1.2561	1.2247
	5	1.4262	1.4245	1.3889
	6	1.5790	1.3064	1.2737
	7	1.5297	1.4071	1.3719
	8	1.4416	1.3495	1.3158
	9	1.3038	1.1090	1.0813
	10	1.2752	1.1197	1.0917
	11	1.2852	1.2714	1.2396
	12	1.1844	1.1960	1.1661
1973	1	1.2427	1.2664	1.2348
	2	1.3003	1.2971	1.2647
	3	1.3699	1.3467	1.3130
	4	1.4691	1.4658	1.4292
	5	1.5972	1.6491	1.6078
	6	1.8480	1.4987	1.4612
	7	1.7706	1.6569	1.6155
	8	1.6779	1.6306	1.5898
	9	1.5253	1.2683	1.2366
	10	1.5371	1.3331	1.2998
	11	1.5634	1.5652	1.5261
	12	1.4181	1.4505	1.4143

These three seasonally adjusted annual basket-type indices are listed in Table 22.26, along with the target index, the centered rolling-year index, P_{CRY}. In addition, one could seasonally adjust the original Lowe, Young, and geometric Laspeyres indices using a standard seasonal adjustment procedure such as X-11. Table 22.26 also contains Lowe, Young, and geometric Laspeyres series that have been seasonally adjusted using the X-11 multiplicative model with default settings.[49] The series have been normalized to set December 1970 = 1.0. They are labeled P_{LOx11}, P_{Yx11}, and P_{GLx11}, respectively.

22.92 The first four series in Table 22.26 coincide for their first 12 observations, which follows from the way the seasonally adjusted series were defined. Also, the last six observations are missing for the centered rolling-year series, P_{CRY}, because data for the first six months of 1974 would be required to calculate all of these index values. Note that from December 1971 to December 1973, the three seasonally adjusted annual basket-type indices (P_{LOSA}, P_{YSA}, and P_{GLSA}) can be used to *predict* the corresponding centered rolling-year entries; see Figure 22.7a for plots of these predictions. What is remarkable in Table 22.26 and Figure 22.7a is that *the predicted values of these seasonally adjusted series are fairly close to the corresponding target index values*.[50] This result is somewhat unexpected since the annual basket indices use price information for only two consecutive months, whereas the corresponding centered rolling-year index uses price information for some 25 months![51] It should also be noted that the seasonally adjusted geometric Laspeyres index is generally the best predictor of the corresponding rolling-year index for this data set. In viewing Figure 22.7a, for the first few

[49]Many statistical offices have access to moving average seasonal adjustment programs such as the X-11 system developed by the U.S. Census Bureau and Statistics Canada. The seasonal adjustment performed here ran the data through the multiplicative version of X-11.

[50]For observations 13 through 31, one can regress the seasonally adjusted series on the centered rolling-year series. For the seasonally adjusted Lowe index, an R^2 of 0.8816 is obtained; for the seasonally adjusted Young index, an R^2 of 0.9212 is derived; and for the seasonally adjusted geometric Laspeyres index, an R^2 of 0.9423 is derived. These fits are not as good as the fit obtained in Section F above where the seasonally adjusted approximate rolling-year index was used to predict the fixed-base Laspeyres rolling-year index. This R^2 was 0.9662; recall the discussion around Table 22.20.

[51]However, for seasonal data sets that are not as regular as the modified Turvey data set, the predictive power of the seasonally adjusted annual basket-type indices may be considerably less; that is, if there are abrupt changes in the seasonal pattern of prices, one could not expect these month-to-month indices to accurately predict a rolling-year index.

Table 22.26. Seasonally Adjusted Lowe, Young, and Geometric Laspeyres Indices with Carryforward Prices and Centered Rolling-Year Index

Year	Month	P_{LOSA}	P_{YSA}	P_{GLSA}	P_{CRY}	P_{LOX11}	P_{YX11}	P_{GLX11}
1970	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1	1.0091	1.0091	1.0091	1.0091	1.0077	1.0088	1.0088
	2	1.0179	1.0179	1.0179	1.0179	1.0009	1.0044	0.9986
	3	1.0242	1.0242	1.0242	1.0242	1.0208	1.0205	1.0029
	4	1.0298	1.0298	1.0298	1.0298	1.0314	1.0364	1.0157
	5	1.0388	1.0388	1.0388	1.0388	1.0604	1.0666	1.0490
	6	1.0478	1.0478	1.0478	1.0478	1.0302	1.0402	1.0258
	7	1.0547	1.0547	1.0547	1.0547	1.0237	1.0409	1.0213
	8	1.0631	1.0631	1.0631	1.0631	1.0572	1.0758	1.0561
	9	1.0729	1.0729	1.0729	1.0729	1.0558	1.0665	1.0626
	10	1.0814	1.0814	1.0814	1.0814	1.0500	1.0598	1.0573
	11	1.0885	1.0885	1.0885	1.0885	1.0598	1.0714	1.0666
	12	1.0824	1.0932	1.0900	1.0965	1.0828	1.0931	1.0901
1972	1	1.0871	1.0960	1.0919	1.1065	1.0856	1.0957	1.0916
	2	1.1148	1.1207	1.1204	1.1174	1.0963	1.1059	1.0992
	3	1.1093	1.1214	1.1318	1.1254	1.1056	1.1173	1.1083
	4	1.1057	1.1132	1.1226	1.1313	1.1076	1.1203	1.1072
	5	1.0983	1.1039	1.1120	1.1402	1.1211	1.1334	1.1229
	6	1.1467	1.1471	1.1505	1.1502	1.1276	1.1387	1.1264
	7	1.1701	1.1667	1.1715	1.1591	1.1361	1.1514	1.1343
	8	1.1456	1.1443	1.1461	1.1690	1.1393	1.1580	1.1385
	9	1.1703	1.1746	1.1642	1.1806	1.1517	1.1676	1.1531
	10	1.1946	1.2017	1.1905	1.1924	1.1599	1.1777	1.1640
	11	1.2019	1.2102	1.2005	1.2049	1.1703	1.1912	1.1762
	12	1.1844	1.2032	1.1938	1.2203	1.1848	1.2031	1.1938
1973	1	1.1882	1.2089	1.1922	1.2386	1.1940	1.2163	1.1998
	2	1.2357	1.2536	1.2431	1.2608	1.2260	1.2480	1.2314
	3	1.2201	1.2477	1.2575	1.2809	1.2296	1.2569	1.2469
	4	1.2349	1.2523	1.2656	1.2966	1.2529	1.2764	1.2678
	5	1.2299	1.2425	1.2514	1.3176	1.2628	1.2820	1.2743
	6	1.3421	1.3410	1.3335	1.3406	1.3175	1.3285	1.3035
	7	1.3543	1.3512	1.3518	0.0000	1.3123	1.3313	1.3069
	8	1.3334	1.3302	1.3276	0.0000	1.3254	1.3460	1.3186
	9	1.3692	1.3800	1.3524	0.0000	1.3489	1.3739	1.3411
	10	1.4400	1.4601	1.4242	0.0000	1.4016	1.4351	1.3962
	11	1.4621	1.4844	1.4508	0.0000	1.4308	1.4691	1.4296
	12	1.4181	1.4521	1.4236	0.0000	1.4332	1.4668	1.4374

months of 1973, the three month-to-month indices underestimate the centered rolling-year inflation rate, but by the middle of 1973, the month-to-month indices are right on target.[52]

22.93 The last three series in Table 22.26 reflect the seasonal adjustment of the Lowe, Young, and geometric Laspeyres using the X-11 program. The seasonally adjusted series (P_{LOx11}, P_{Yx11}, and P_{GLx11})

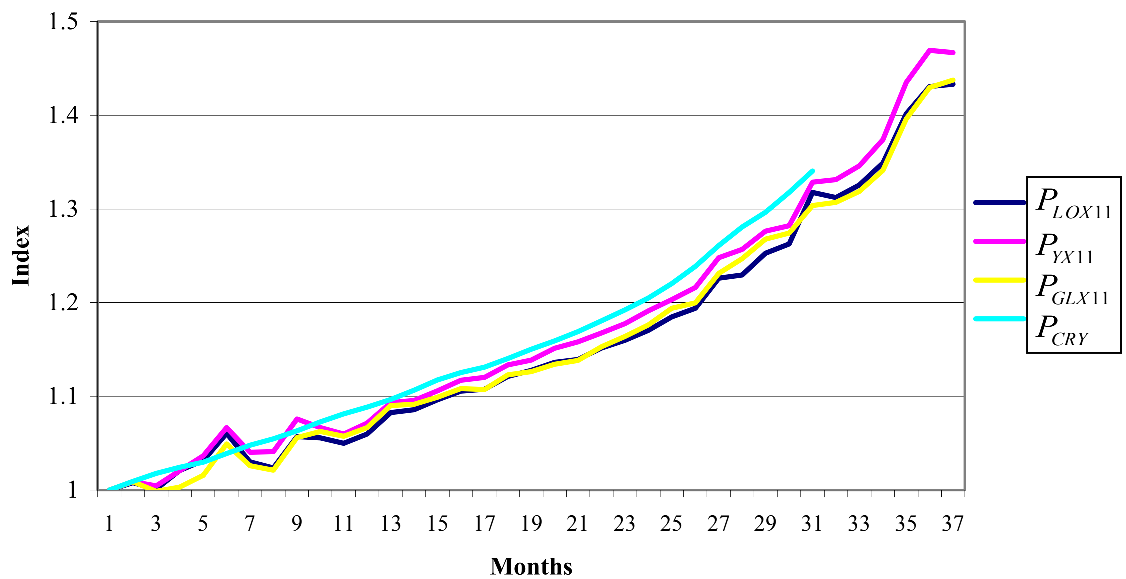
[52]Recall that the last six months of P_{CRY} are missing; six months of data for 1974 would be required to evaluate these (continued) centered rolling-year index values, and these data are not available.

Table 22.27. Seasonally Adjusted Lowe, Young, and Geometric Laspeyres Indices with Imputed Prices, Seasonally Adjusted Rothwell, and Centered Rolling-Year Indices

Year	Month	P_{LOSA}	P_{YSA}	P_{GLSA}	P_{ROTHSA}	P_{CRY}	P_{LOX11}	P_{YX11}	P_{GLX11}
1970	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1	1.0091	1.0091	1.0091	1.0091	1.0091	1.0125	1.0131	1.0133
	2	1.0179	1.0179	1.0179	1.0179	1.0179	1.0083	1.0109	1.0057
	3	1.0242	1.0242	1.0242	1.0242	1.0242	1.0300	1.0288	1.0121
	4	1.0298	1.0298	1.0298	1.0298	1.0298	1.0418	1.0460	1.0267
	5	1.0388	1.0388	1.0388	1.0388	1.0388	1.0680	1.0753	1.0574
	6	1.0478	1.0478	1.0478	1.0478	1.0478	1.0367	1.0485	1.0362
	7	1.0547	1.0547	1.0547	1.0547	1.0547	1.0300	1.0450	1.0251
	8	1.0631	1.0631	1.0631	1.0631	1.0631	1.0637	1.0807	1.0615
	9	1.0729	1.0729	1.0729	1.0729	1.0729	1.0607	1.0713	1.0685
	10	1.0814	1.0814	1.0814	1.0814	1.0814	1.0536	1.0634	1.0615
	11	1.0885	1.0885	1.0885	1.0885	1.0885	1.0631	1.0741	1.0704
	12	1.0863	1.0972	1.0939	1.0849	1.0965	1.0867	1.0973	1.0940
1972	1	1.0909	1.0999	1.0958	1.0978	1.1065	1.0948	1.1043	1.1004
	2	1.1185	1.1245	1.1244	1.1442	1.1174	1.1079	1.1168	1.1109
	3	1.1129	1.1250	1.1359	1.1657	1.1254	1.1191	1.1300	1.1224
	4	1.1091	1.1167	1.1266	1.1460	1.1313	1.1220	1.1341	1.1233
	5	1.0988	1.1043	1.1129	1.1342	1.1402	1.1298	1.1431	1.1328
	6	1.1467	1.1469	1.1505	1.1339	1.1502	1.1345	1.1476	1.1377
	7	1.1701	1.1666	1.1715	1.1746	1.1591	1.1427	1.1559	1.1386
	8	1.1457	1.1442	1.1461	1.1659	1.1690	1.1464	1.1632	1.1444
	9	1.1703	1.1746	1.1642	1.1298	1.1806	1.1570	1.1729	1.1594
	10	1.1947	1.2019	1.1905	1.1715	1.1924	1.1639	1.1818	1.1685
	11	1.2019	1.2103	1.2005	1.2106	1.2049	1.1737	1.1943	1.1805
	12	1.1888	1.2078	1.1981	1.1960	1.2203	1.1892	1.2079	1.1983
1973	1	1.1941	1.2149	1.1983	1.2089	1.2386	1.1906	1.2118	1.1954
	2	1.2431	1.2611	1.2513	1.2901	1.2608	1.2205	1.2415	1.2244
	3	1.2289	1.2565	1.2677	1.3358	1.2809	1.2221	1.2483	1.2370
	4	1.2447	1.2621	1.2778	1.3373	1.2966	1.2431	1.2656	1.2542
	5	1.2338	1.2459	1.2576	1.3131	1.3176	1.2613	1.2833	1.2694
	6	1.3421	1.3406	1.3335	1.3007	1.3406	1.3298	1.3440	1.3208
	7	1.3543	1.3510	1.3518	1.3831	0.0000	1.3246	1.3407	1.3158
	8	1.3343	1.3309	1.3285	1.4087	0.0000	1.3355	1.3531	1.3266
	9	1.3712	1.3821	1.3543	1.2921	0.0000	1.3539	1.3780	1.3470
	10	1.4430	1.4634	1.4271	1.3949	0.0000	1.4023	1.4346	1.3971
	11	1.4669	1.4895	1.4560	1.4903	0.0000	1.4252	1.4617	1.4237
	12	1.4307	1.4651	1.4359	1.4505	0.0000	1.4205	1.4540	1.4250

are normalized to December 1970, so that they may easily be compared with the centered rolling-year index, P_{CRY}. Again, these seasonally adjusted series compare rather well with the trend of P_{CRY} and appear to predict the corresponding target values. Figure 22.7b shows a graph of these series, and the X-11 seasonal adjustment appears to provide a somewhat smoother series than those for the first three series in Table 22.26. This occurs because the X-11 program estimates seasonal factors over the whole data series but requires a minimum of three years of monthly data. The seasonal factors (*SAF*) for the first three series are based on the 12 estimated monthly factors for 1971 that are simply re-

Figure 22.7b. Lowe, Young, Geometric Laspeyres, and Centered Rolling Indices Using X-11 Seasonal Adjustment



peated for subsequent years.[53] Although the trends of X-11 series and the target index (P_{CRY}) are similar, the X-11 series are consistently lower than the target series due to the normalization of the X-11 series. December is a month that has a larger seasonal component in the X-11 adjustment than that for the series using the rolling average. Normalizing the X-11 adjusted series for December results in the first few months of the series showing relatively little growth.

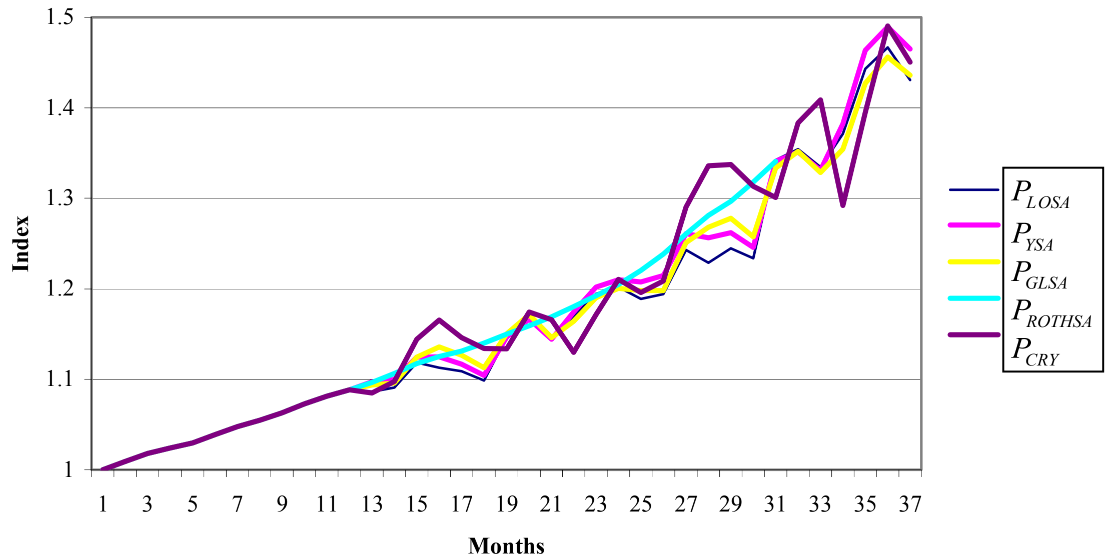
22.94 The manipulations above can be repeated, replacing the *carryforward* annual basket indices with their *imputed* counterparts; that is, using the information in Table 22.24 (instead of Table 22.23) and Table 22.27 replacing Table 22.26. A seasonally adjusted version of the Rothwell index presented in the previous section may also be found in Table 22.27.[54] The eight series in Table 22.27 are also graphed in Figures 22.8a and 22.8b.

22.95 Again, the seasonally adjusted annual basket-type indices listed in the first three data columns of Table 22.27 (using imputations for the missing prices) are reasonably close to the corresponding centered rolling-year index listed in the

[53]Again, for observations 13 through 31, one can regress the seasonally adjusted series on the centered rolling-year series. For the X-11 seasonally adjusted Lowe index, an R^2 of 0.9873 is derived; for the X-11 seasonally adjusted Young index, an R^2 of 0.9947 is derived; and for the X-11 seasonally adjusted geometric Laspeyres index, an R^2 of 0.9952 is derived. These fits are better than those obtained above and in Section F. However, the X-11 seasonal adjustment procedure uses the entire data set to do the adjusting, whereas the index number methods of seasonal adjustment used only the first 12 months of data.

[54]The same seasonal adjustment technique that was defined by equation (22.35) was used.

Figure 22.8a. Seasonally Adjusted Lowe, Young, and Geometric Laspeyres Indices with Imputed Prices; Seasonally Adjusted Rothwell and Centered Rolling-Year Indices

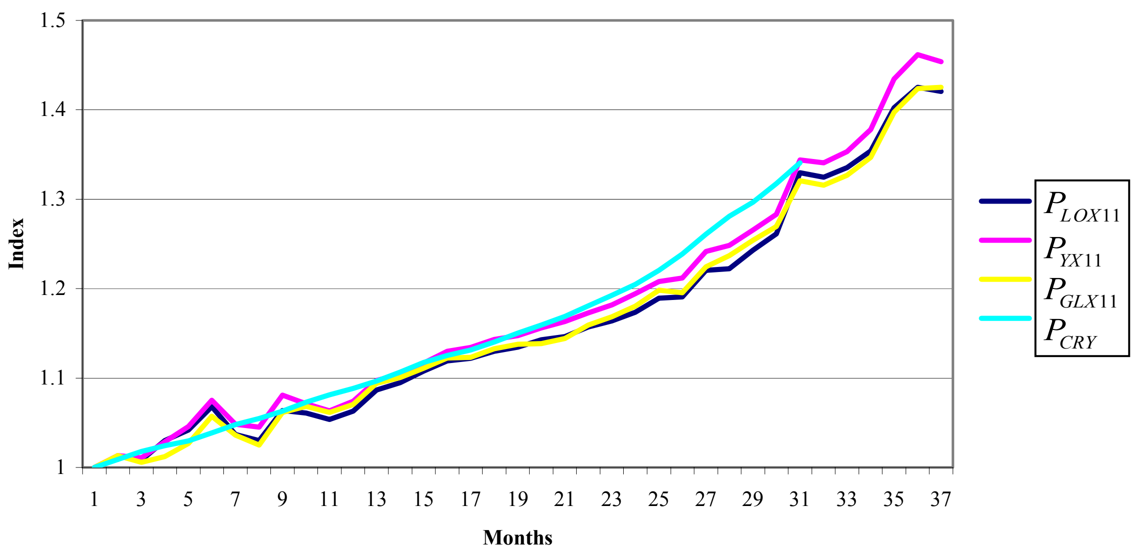


fifth data column of Table 22.27.[55] The seasonally adjusted geometric Laspeyres index is the closest to the centered rolling-year index, and the seasonally adjusted Rothwell index is the furthest away. The three seasonally adjusted month-to-month indices that use annual weights—P_{LOSA}, P_{YSA}, and P_{GLSA}—dip below the corresponding centered rolling-year index, P_{CRY}, for the first few months of 1973 when the rate of month-to-month inflation suddenly increases. But by the middle of 1973, all four indices are fairly close to each other. The seasonally adjusted Rothwell does not do a very good job of approximating P_{CRY} for this particular data set, although this could be a function of the rather simple method of seasonal adjustment that was used. The series adjusted using X-11 again are smoother than the other series and show very similar trends to the target index.

22.96 In comparing the results in Tables 22.26 and 22.7, one can see that it did not make a great deal of difference for the modified Turvey data set whether missing prices are carried forward or imputed; the seasonal adjustment factors picked up the lumpiness in the unadjusted indices that happens when the carryforward method is used. However, the three month-to-month indices that used annual weights and imputed prices did predict the corresponding centered rolling-year indices somewhat better than the three indices that used carry forward prices. Therefore, the use of imputed prices over carryforward prices is recommended.

[55]Again, for observations 13 through 31, one can regress the seasonally adjusted series on the centered rolling-year series. For the seasonally adjusted Lowe index, an R^2 of 0.8994 is derived; for the seasonally adjusted Young index, an R^2 of 0.9294 is derived; and for the seasonally adjusted geometric Laspeyres index, an R^2 of 0.9495 is derived. For the seasonally adjusted Rothwell index, an R^2 of 0.8704 is derived, which is lower than the other three fits. For the X-11 seasonally adjusted series, the R^2 values are 0.9644 for the Lowe, 0.9801 for the Young, and 0.9829 for the geometric Laspeyres. All of the Lowe, Young, and geometric Laspeyres indices, using imputed prices, have higher R^2 values than those obtained using carryforward prices.

Figure 22.8b. Lowe, Young, and Geometric Laspeyres Indices Using X-11 Seasonal Adjustment with Imputed Prices and Centered Rolling-Year Indices



22.97 The conclusions that emerge from this section are rather encouraging for statistical agencies that wish to use an annual basket-type index as their flagship index.[56] It appears that for product groups that have strong seasonality, an annual basket-type index can be seasonally adjusted,[57] and the resultant seasonally adjusted index value can be used as a price relative for the group at higher stages of aggregation. The preferred type of annual basket-type index appears to be the geometric Laspeyres index, rather than the Lowe index, but the differences between the two were not large for this data set.

[56]Using the results of previous chapters, the use of the annual basket Young index is not encouraged because of its failure of the time reversal test and the resultant upward bias.

[57]It is not necessary to use rolling-year indices in the seasonal adjustment process, but the use of rolling-year indices is recommended because they will increase the objectivity and reproducibility of the seasonally adjusted indices.

L. Conclusions

22.98 A number of tentative conclusions can be drawn from the results of the sections in this chapter:

- The inclusion of seasonal products in maximum overlap month-to-month indices will frequently lead to substantial biases. Therefore, unless the maximum overlap month-to-month indices using seasonal products cumulated for a year are close to their year-over-year counterparts, the seasonal products should be excluded from the month-to-month index or the seasonal adjustment procedures suggested in Section K should be used;
- Year-over-year monthly indices can always be constructed even if there are strongly seasonal products.[58] Many users will be interested in

[58]There can be problems with the year-over-year indices if shifting holidays or abnormal weather changes normal seasonal patterns. In general, choosing a longer time period will mitigate these types of problems; that is, quarterly sea- (continued)

these indices; moreover, these indices are the building blocks for annual indices and for rolling-year indices. As a result, statistical agencies should compute these indices. They can be labeled analytic series in order to prevent user confusion with the primary month-to-month PPI;

- Rolling-year indices should also be made available as analytic series. These indices will give the most reliable indicator of annual inflation at a monthly frequency. This type of index can be regarded as a seasonally adjusted PPI. It is the most natural index to use as a central bank inflation target. It has the disadvantage of measuring year-over-year inflation with a lag of six months; thus, it cannot be used as a short-run indicator of month-to-month inflation. However, the techniques suggested in Sections F and K could be used so that timely forecasts of these rolling-year indices can be made using current-price information;
- Annual basket indices can also be successfully used in the context of seasonal commodities. However, many users of the PPI will want to use seasonally adjusted versions of these annual basket-type indices. The seasonal adjustment can be done using the index number methods explained in Section K or traditional statistical agency seasonal adjustment procedures;[59]
- From an a priori point of view, when making a price comparison between any two periods, the Paasche and Laspeyres indices are of equal importance. Under normal circumstances, the spread between the Laspeyres and Paasche indices will be reduced by using chained indices rather than fixed-base indices. As a result, when constructing year-over-year monthly or annual indices, choose the chained Fisher index (or the chained Törnqvist-Theil index, which closely approximates the chained Fisher) as the target index that a statistical agency should aim to approximate. However, when constructing month-to-month indices, chained indices should always be compared with their year-over-year counterparts to check for chain drift. If substantial drift is found, the chained month-to-month indices must be replaced with fixed-base indices or seasonally adjusted annual basket-type indices;[60]
- If current-period revenue shares are not all that different from base-year revenue shares, approximate chained Fisher indices will normally provide a close practical approximation to the chained Fisher target indices. Approximate Laspeyres, Paasche, and Fisher indices use base-period expenditure shares whenever they occur in the index number formula in place of current-period (or lagged current-period) revenue shares. Approximate Laspeyres, Paasche, and Fisher indices can be computed by statistical agencies using their normal information sets; and
- The geometric Laspeyres index is an alternative to the approximate Fisher index that uses the same information. It will normally be close to the approximate Fisher index.

It is evident that more research needs to be done on the problems associated with the index number treatment of seasonal products. A consensus on what is best practice in this area has not yet formed.

sonal patterns will be more stable than monthly patterns, which in turn will be more stable than weekly patterns.

[59]However, there is a problem with using traditional X-11-type seasonal adjustment procedures for adjusting the PPI because final seasonal adjustment factors are generally not available until an additional two or three years' data have been collected. If the PPI cannot be revised, this may preclude using X-11-type seasonal adjustment procedures. Note that the index number method of seasonal adjustment explained in this chapter does not suffer from this problem. It does, however, require the use of seasonal factors derived from a single year of data, so that the year used should reflect a normal seasonal pattern. If the seasonal patterns are irregular, it may be necessary to use the average of two or more years of past adjustment factors. If the seasonal patterns are regular but slowly changing, then it may be preferable to update the index number seasonal adjustment factors on a regular basis.

[60]Alternatively, some sort of multilateral index number formula could be used; for example, see Caves, Christensen, and Diewert (1982a) or Feenstra and Shapiro (2003).