Using Common Features to Understand the Behavior of Metal-Commodity Prices and Forecast them at Different Horizons

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Common Features – Basic Ideas

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3. There exists a linear combination of them, $y_{1t} - \widetilde{\alpha} y_{2t}$, that does not have property A.
4. Cointegration is the most well-known example of common features.
5. Serial correlation-common features (SCCF) or *common cycles* are the also well-known: stationary series y_{1t} and y_{2t} both have serial correlation (are predictable), but there exists $y_{1t} - \widetilde{\alpha} y_{2t}$ which is white noise (unpredictable).

Common Features – Basic Ideas

Engle and Kozicki (1993) main example.

1. No cointegration for log-levels of GDP for the U.S. and Canada. Instantaneous growth rates of GDP for the U.S. and Canada have serial correlation and there is a linear combination of growth rates that is white noise. Cycles in U.S. and Canadian GDP growth are synchronized.

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2. This is our main finding between growth rates of industrial production and metal commodity prices.
3. Factor models and *latent* features:

$$
\begin{aligned}
\begin{pmatrix} \Delta \ln y_t^{US} \\ \Delta \ln y_t^{CAN} \end{pmatrix} &= \begin{pmatrix} \lambda \\ 1 \end{pmatrix} f_t + \begin{pmatrix} \varepsilon_t^{US} \\ \varepsilon_t^{CAN} \end{pmatrix}, \text{ or,} \\
\Delta \ln y_t^{US} - \lambda \Delta \ln y_t^{CAN} &= \varepsilon_t^{US} - \lambda \varepsilon_t^{CAN},
\end{aligned}
$$

$\begin{pmatrix} 1 & -\lambda \end{pmatrix}$ is the *cofeature* vector, eliminating the SCCF.

Common Features – Useful Dynamic Representations

Vahid and Engle (1993): *VAR* for y_t, an n-vector of $I(1)$ metal prices (or log metal prices):

$$y_t = \Gamma_1 y_{t-1} + \ldots + \Gamma_p y_{t-p} + \epsilon_t. \tag{1}$$

VECM:

$$\Delta y_t = \Gamma_1^* \Delta y_{t-1} + \ \ldots \ + \Gamma_{p-1}^* \Delta y_{t-p+1} + \gamma \alpha' y_{t-1} + \epsilon_t. \tag{2}$$

Normalized cofeature vectors:

$$\tilde{\alpha} = \begin{bmatrix} I_s \\ \tilde{\alpha}^*_{(n-s)\times s} \end{bmatrix}$$

Quasi-structural model (restricted VECM):

$$\begin{bmatrix} I_s & \tilde{\alpha}^{*\prime} \\ \underset{(n-s)\times s}{\mathbf{0}} & I_{n-s} \end{bmatrix} \Delta y_t = \begin{bmatrix} \underset{s\times(np+r)}{\mathbf{0}} \\ \Gamma_1^{**} \ \ldots \ \Gamma_{p-1}^{**} \ \gamma^* \end{bmatrix} \begin{bmatrix} \Delta y_{t-1} \\ \vdots \\ \Delta y_{t-p+1} \\ \alpha' y_{t-1} \end{bmatrix} + v_t. \tag{3}$$

Common Features: A Test for Common Cycles

GMM approach: exploits the following moment restriction and test H_0 :existence of s linearly independent SCCF:

$$0 = \mathbb{E}\left[\left(\begin{bmatrix} I_s & \tilde{\alpha}^{*\prime} \\ \underset{(n-s)\times s}{\mathbf{0}} & I_{n-s} \end{bmatrix} \Delta y_t - \begin{bmatrix} \underset{s\times(np+r)}{\mathbf{0}} \\ \Gamma_1^{**} \; \ldots \; \Gamma_{p-1}^{**} \; \gamma^* \end{bmatrix} \begin{bmatrix} \Delta y_{t-1} \\ \vdots \\ \Delta y_{t-p+1} \\ \alpha' y_{t-1} \end{bmatrix}\right) \otimes Z_{t-1}\right],$$

where the elements of Z_{t-1} are the instruments comprising past series: $\alpha' y_{t-1}, \Delta y_{t-1}, \Delta y_{t-2}, \cdots, \Delta y_{t-p+1}$. The test for common cycles is an over-identifying restriction test – the J test proposed by Hansen (1982). This test is robust to HSK of unknown form if it uses a White-correction in its several forms.

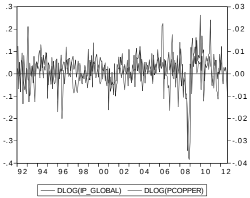
Common Cycles: Forecasting with Restricted VECM

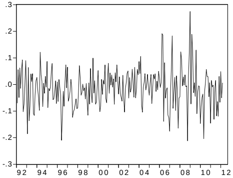
$$\Delta y_t = \begin{bmatrix} I_s & \tilde{\alpha}^{*\prime} \\ \underset{(n-s)\times s}{\mathbf{0}} & I_{n-s} \end{bmatrix}^{-1} \times \tag{4}$$

$$\begin{bmatrix} \underset{s\times(np+r)}{\mathbf{0}} \\ \Gamma_1^{**} \;\ldots\; \Gamma_{p-1}^{**} \;\gamma^* \end{bmatrix} \begin{bmatrix} \Delta y_{t-1} \\ \vdots \\ \Delta y_{t-p+1} \\ \alpha' y_{t-1} \end{bmatrix} + \epsilon_t .$$

- Forecasting gains with *real data*:
  - Issler and Vahid (2001) **find 25% reduction in** $|MSFE|$ for U.S. macroeconomic aggregates;
  - Vahid and Issler (2002) **find a reduction of 20%** for U.S. coincident series;
  - Athanasopoulos et al. (2011) **find a reduction of 47%** for Brazilian Inflation.

Common Cycles: Copper Prices and IP Global





- Deaton and Laroque (1996) stress the importance of demand factors for commodity prices.

Stylized Facts on Forecasts and Forecast Combinations

- Forecasting y_t, stationary and ergodic, using information up to h periods prior to t. Risk function is MSE. Optimal forecast is:

$$\mathbb{E}_{t-h}\left(y_t\right),$$

  - Bates and Granger (1969), Hendry and Clements (2002), *inter alia*: if $f_{i,t}^h$ is the h-step-ahead forecast of y_t using model (survey result) $i = 1, 2, \ldots, N$, $\frac{1}{N}\sum_{i=1}^{N} f_{i,t}^h$ performs better than $f_{i,t}^h$, so $f_{i,t}^h$ cannot approximate $\mathbb{E}_{t-h}\left(y_t\right)$, since $\mathbb{E}_{t-h}\left(y_t\right)$ is optimal.

Common-Feature Forecasts: Issler and Lima (JoE, 2009)

- Panel-data framework, with sequential asymptotics: first $T \longrightarrow \infty$ with N fixed. Then, $N \longrightarrow \infty$, written as $(T, N \longrightarrow \infty)_{\text{seq}}$.
- Propose the use of equal weights combination $(1/N)$ coupled with an average *bias correction term* (BCAF): $\frac{1}{N}\sum_{i=1}^{N} f_{i,t}^{h} - \widehat{B}$. Works even with *some* nested models.
- Propose a new test for the need to do *bias correction*: $H_0 : B = 0$.

Common-Feature Forecasts: Issler and Lima (JoE, 2009)

$$
\begin{aligned}
f_{i,t}^{h} &= \mathbb{E}_{t-h}(y_t) + k_i + \varepsilon_{i,t}, \qquad (5) \\
y_t &= \mathbb{E}_{t-h}(y_t) + \zeta_t, \text{ with } \mathbb{E}_{t-h}(\zeta_t) = 0, \text{ Then,} \\
f_{i,t}^{h} &= y_t - \zeta_t + k_i + \varepsilon_{i,t}, \text{ or,} \\
\text{Forecast error} &: \quad f_{i,t}^{h} - y_t = k_i + \eta_t + \varepsilon_{i,t}, \qquad i = 1, 2, \ldots, N, \\
\text{where } \eta_t &= -\zeta_t
\end{aligned}
$$

- The forecast error has a two-way decomposition (Wallace and Hussain (1969), Amemiya (1971), Fuller and Battese (1974)) with a long tradition in the econometrics literature.
- The goal is to estimate the unobserved common feature $\mathbb{E}_{t-h}(y_t)$ using (5), obtaining an optimal forecast.

Issler and Lima (JoE, 2009) Main Results

Under some assumptions, the following are consistent estimators of k_i, B, η_t, and $\varepsilon_{i,t}$, respectively:

$$\widehat{k}_i = \frac{1}{R}\sum_{t=T_1+1}^{T_2} f_{i,t}^h - \frac{1}{R}\sum_{t=T_1+1}^{T_2} y_t, \qquad \operatorname*{plim}_{T\to\infty}\left(\widehat{k}_i - k_i\right) = 0,$$

$$\widehat{B} = \frac{1}{N}\sum_{i=1}^{N}\widehat{k}_i, \qquad \operatorname*{plim}_{(T,N\to\infty)_{\text{seq}}}\left(\widehat{B} - B\right) = 0,$$

$$\widehat{\eta}_t = \frac{1}{N}\sum_{i=1}^{N} f_{i,t}^h - \widehat{B} - y_t, \qquad \operatorname*{plim}_{(T,N\to\infty)_{\text{seq}}}\left(\widehat{\eta}_t - \eta_t\right) = 0,$$

$$\widehat{\varepsilon}_{i,t} = f_{i,t}^h - y_t - \widehat{k}_i - \widehat{\eta}_t, \qquad \operatorname*{plim}_{(T,N\to\infty)_{\text{seq}}}\left(\widehat{\varepsilon}_{i,t} - \varepsilon_{i,t}\right) = 0.$$

Issler and Lima (JoE, 2009) Main Results

Under some assumptions, the feasible bias-corrected average forecast $\frac{1}{N}\sum_{i=1}^{N} f_{i,t}^{h} - \widehat{B}$ obeys:

$$\underset{(T,N\to\infty)_{\text{seq}}}{\text{plim}} \left(\frac{1}{N}\sum_{i=1}^{N} f_{i,t}^{h} - \widehat{B} \right) = y_t + \eta_t = \mathbb{E}_{t-h}(y_t),$$

and has a mean-squared error as follows:

$$\mathbb{E}\left[\underset{(T,N\to\infty)_{\text{seq}}}{\text{plim}} \left(\frac{1}{N}\sum_{i=1}^{N} f_{i,t}^{h} - \widehat{B} \right) - y_t \right]^2 = \sigma_\eta^2.$$

Therefore it is an optimal forecasting device.

Issler and Lima (JoE, 2009) Main Results

Consider the sequence of deterministic weights $\{\omega_i\}_{i=1}^{N}$, such that $|\omega_i| \neq 0$, $\omega_i = \mathbf{O}\left(N^{-1}\right)$ uniformly, with $\sum_{i=1}^{N} \omega_i = 1$ and $\lim_{N\to\infty} \sum_{i=1}^{N} \omega_i = 1$. Then,

$$\mathbb{E}\left[\underset{(T,N\to\infty)_{\text{seq}}}{\text{plim}}\left(\sum_{i=1}^{N} \omega_i f_{i,t}^{h} - \sum_{i=1}^{N} \omega_i \widehat{k}_i\right) - y_t\right]^2 = \sigma_\eta^2,$$

i.e., weighted forecasts are optimal as well.

Issler and Lima (JoE, 2009) Main Results

Under the null hypothesis $H_0 : B = 0$, the test statistic:

$$\widehat{t} = \frac{\widehat{B}}{\sqrt{\widehat{V}}} \quad \xrightarrow[(T,N\to\infty)_{\text{seq}}]{d} \mathcal{N}\left(0,1\right),$$

where $\widehat{V}$ is a consistent estimator of the asymptotic variance of $\overline{B} = \frac{1}{N}\sum_{i=1}^{N} k_i$.

- $\widehat{V}$ is estimated using a cross-section analog of the Newey-West estimator due to Conley (1999), where a natural order in the cross-sectional dimension requires matching spatial dependence to a metric of economic distance.
- If $B = 0$, the average forecast $\frac{1}{N}\sum_{i=1}^{N} f_{i,t}^{h}$ is an optimal forecasting device.

Common-Feature Tests (Monthly)

Table: Global Industrial Production

$\Delta y_{1,t}$ (1,	$\Delta y_{2,t}$ $\tilde{\alpha}^*$)	$\tilde{\alpha}^*$	J-statistic
Aluminum	Global Industrial Production	-5.316***	0.0427
		(0.969)	[0.036]
Lead	Global Industrial Production	-4.052*	0.0331
		(2.101)	[0.047]
Copper	Global Industrial Production	-7.523***	0.0310
		(1.504)	[0.189]
Tin	Global Industrial Production	-5.23***	0.0096
		(1.603)	[0.512]
Nickel	Global Industrial Production	-6.034***	0.0292
		(1.728)	[0.219]
Zinc	Global Industrial Production	-5.827***	0.0337
		(1.601)	[0.329]

Common-Feature Tests (Monthly)

Table: US Industrial Production

$\Delta y_{1,t}$ (1,	$\Delta y_{2,t}$ $\tilde{\alpha}^*$)	$\tilde{\alpha}^*$	J-statistic
Aluminum	US Industrial Production	-2.683***	0.0558
		(0.897)	[0.434]
Lead	US Industrial Production	0.839	0.0577
		(1.799)	[0.056]
Copper	US Industrial Production	-3.033	0.0513
		(2.018)	[0.094]
Nickel	US Industrial Production	-2.622	0.0650
		(1.683)	[0.03]
Tin	US Industrial Production	-2.524*	0.0429
		(1.301)	[0.176]
Zinc	US Industrial Production	-1.923	0.0619
		(1.357)	[0.484]

Forecast Results (Monthly)

Table: Forecast Root-Mean-Squarred-Error (Monthly)

Aluminum	BCAF	Weighted Average (MSE)	Average Forecast	Median	Best Model (BIC)	5 Best Models	10 Best Models
1 step-ahead	0.619	0.605	0.678	0.663	0.732	0.688	0.684
2 step-ahead	0.926	0.888	1.049	1.005	1.158	1.096	1.057
3 step-ahead	1.402	1.327	1.635	1.570	1.914	1.804	1.699
4 step-ahead	1.674	1.548	1.981	1.911	2.461	2.315	2.129
5 step-ahead	1.833	1.647	2.197	2.161	2.866	2.722	2.436
6 step-ahead	2.125	1.861	2.564	2.530	3.459	3.330	2.912
R^2 with drift	−0.123	−0.095	−0.227	−0.199	−0.208	−0.202	−0.208
R^2 without drift	−0.124	−0.097	−0.229	−0.2	−35.847	−35.841	−35.175
Lead	BCAF	Weighted Average (MSE)	Average Forecast	Median	Best Model (BIC)	5 Best Models	10 Best Models
1 step-ahead	0.074	0.074	0.073	0.072	0.076	0.073	0.074
2 step-ahead	0.143	0.145	0.144	0.143	0.150	0.145	0.146
3 step-ahead	0.2	0.206	0.206	0.205	0.214	0.209	0.209
4 step-ahead	0.247	0.256	0.258	0.257	0.272	0.267	0.264
5 step-ahead	0.281	0.293	0.293	0.291	0.310	0.307	0.303
6 step-ahead	0.329	0.341	0.343	0.342	0.367	0.367	0.359
R^2 with drift	−0.123	−0.126	−0.099	−0.095	−0.136	−0.142	−0.139
R^2 without drift	−0.122	−0.125	−0.098	−0.094	−11.703	−12.063	−12.047

Forecast Results (Monthly)

Table: Forecast Root-Mean-Squarred-Error (Monthly)

Copper	BCAF	Weighted Average (MSE)	Average Forecast	Median	Best Model (BIC)	5 Best Models	10 Best Models
1 step-ahead	0.647	0.575	0.571	0.574	0.55	0.566	0.569
2 step-ahead	1.999	1.706	1.716	1.717	1.631	1.695	1.707
3 step-ahead	3.378	2.779	2.817	2.804	2.698	2.795	2.815
4 step-ahead	4.471	3.557	3.587	3.579	3.485	3.604	3.634
5 step-ahead	5.219	4.123	4.091	4.060	4.04	4.157	4.193
6 step-ahead	5.718	4.619	4.592	4.55	4.566	4.672	4.711
R^2 with drift	0.039	0.141	0.135	0.135	0.137	0.117	0.124
R^2 without drift	0.04	0.141	0.136	0.135	-6.901	-7.037	-7.317

Nickel	BCAF	Weighted Average (MSE)	Average Forecast	Median	Best Model (BIC)	5 Best Models	10 Best Models
1 step-ahead	17.916	17.670	15.688	15.817	16.221	17.095	16.231
2 step-ahead	48.508	47.784	48.316	49.425	50.291	51.964	49.464
3 step-ahead	76.682	75.53	83.621	86.079	88.583	91.630	86.640
4 step-ahead	92.064	91	106.000	109.447	112.618	117.515	110.329
5 step-ahead	100.430	99.401	119.853	123.473	124.583	131.369	122.996
6 step-ahead	105.895	104.082	126.449	131.179	127.222	136.305	126.641
R^2 with drift	0.035	0.05	0.159^{**}	0.162^{**}	0.143^{**}	0.149^{**}	0.139^{**}
R^2 without drift	0.031	0.045	0.155^{**}	0.158^{**}	-5.547	-5.541	-9.213

Forecast Results (Monthly)

Table: Forecast Root-Mean-Squarred-Error (Monthly)

Tin	BCAF	Weighted Average (MSE)	Average Forecast	Median	Best Model (BIC)	5 Best Models	10 Best Models
1 step-ahead	3.497	3.374	2.614	2.535	2.479	2.497	2.502
2 step-ahead	11.160	9.501	7.130	7.002	6.853	6.778	6.757
3 step-ahead	23.028	18.181	13.257	13.039	13.128	12.894	12.853
4 step-ahead	37.227	27.824	20.298	19.901	20.564	20.184	20.094
5 step-ahead	51.951	36.476	26.372	25.575	26.988	26.381	26.214
6 step-ahead	67.825	44.722	31.727	30.62	32.705	31.877	31.612
R^2 with drift	−0.273	−0.227	0.049	0.079	0.103	0.099	0.097
R^2 without drift	−0.269	−0.223	0.052	0.081	−5.641	−5.485	−5.511

Zinc	BCAF	Weighted Average (MSE)	Average Forecast	Median	Best Model (BIC)	5 Best Models	10 Best Models
1 step-ahead	-	-	-	-	-	-	-
2 step-ahead	-	-	-	-	-	-	-
3 step-ahead	-	-	-	-	-	-	-
4 step-ahead	-	-	-	-	-	-	-
5 step-ahead	-	-	-	-	-	-	-
6 step-ahead	-	-	-	-	-	-	-
R^2 with drift	-	-	-	-	-	-	-
R^2 without drift	-	-	-	-	-	-	-

Forecast Results (Annual)

Table: Forecast Root-Mean-Squarred-Error (Annual)

(RMSE are divided by 10^4)

Aluminum	BCAF	Weighted Average (MSE)	Average Forecast	Median	Best Model (BIC)	5 Best Models	10 Best Models
1 step-ahead	37.206	40.657	38.358	40.861	37.900	36.715	36.116
2 step-ahead	59.374	70.052	63.762	66.324	67.485	65.130	63.431
3 step-ahead	65.961	82.476	70.684	75.429	77.922	74.876	72.269
4 step-ahead	69.016	93.859	75.082	81.393	83.052	79.635	76.129
5 step-ahead	69.13	99.694	75.681	84.285	87.665	83.348	79.164
R^2 with drift	0.086^{**}	0.001^{**}	0.058^{**}	-0.004^{**}	0.069^{**}	0.098^{**}	0.113^{**}
R^2 without drift	0.032^{**}	-0.058^{**}	0.002^{**}	-0.063^{**}	-4.852	-7.53	-10.269

Lead	BCAF	Weighted Average (MSE)	Average Forecast	Median	Best Model (BIC)	5 Best Models	10 Best Models
1 step-ahead	87.215	88.310	95.419	102.963	98.490	96.390	96.919
2 step-ahead	164.499	161.849	182.150	199.458	188.621	180.429	180.717
3 step-ahead	194.459	199.206	231.145	247.535	225.020	218.144	219.826
4 step-ahead	197.945	217.125	261.908	280.107	231.141	228.625	232.594
5 step-ahead	204.004	223.123	274.640	288.786	226.445	230.735	237.098
R^2 with drift	0.159^{**}	0.148^{**}	0.08^{**}	0.007^{**}	0.05^{**}	0.07^{**}	0.065^{**}
R^2 without drift	0.136^{**}	0.125^{**}	0.055^{**}	-0.02^{**}	-1.847	-3.365	-3.222

Forecast Results (Annual)

Table: Forecast Root-Mean-Squarred-Error (Annual)

(RMSE are divided by 10^4)

Copper	BCAF	Weighted Average (MSE)	Average Forecast	Median	Best Model (BIC)	5 Best Models	10 Best Models
1 step-ahead	17.665	17.128	13.821	12.165	12.414	15.894	15.021
2 step-ahead	44.467	46.675	35.102	34.609	31.649	32.732	32.260
3 step-ahead	75.580	83.605	59.676	62.879	55.337	55.212	55.486
4 step-ahead	106.519	121.296	100.515	100.425	81.601	82.790	84.118
5 step-ahead	128.250	150.484	128.840	128.84	100.425	102.647	105.021
R^2 with drift	-0.259	-0.221	0.015^{**}	0.133^{**}	0.115^{**}	-0.133^{**}	-0.07^{**}
R^2 without drift	-0.182	-0.146	0.076^{**}	0.186^{**}	-6.441	-5.7	-6.981

Nickel	BCAF	Weighted Average (MSE)	Average Forecast	Median	Best Model (BIC)	5 Best Models	10 Best Models
1 step-ahead	780.106	791.635	784.654	760.113	728.444	739.280	747.302
2 step-ahead	1727.239	1840.379	1825.805	1898.553	1475.888	1421.853	1441.278
3 step-ahead	1967.854	2148.107	2060.348	2160.290	1696.273	1587.752	1607.805
4 step-ahead	2263.931	2663.476	2317.876	2342.069	2065.623	1971.324	2000.096
5 step-ahead	2678.195	3306.581	2872.925	2839.961	2335.586	2306.738	2352.873
R^2 with drift	0.071^{**}	0.057^{**}	0.066^{**}	0.095^{**}	0.133^{**}	0.12^{**}	0.11^{**}
R^2 without drift	0.068^{**}	0.055^{**}	0.063^{**}	0.092^{**}	-8.365	-17.765	-23.419

Forecast Results (Annual)

Table: Forecast Root-Mean-Squared-Error (Annual)

(RMSE are divided by 10^4)

Tin	BCAF	Weighted Average (MSE)	Average Forecast	Median	Best Model (BIC)	5 Best Models	10 Best Models
1 step-ahead	64.457	72.979	64.111	67.698	74.204	72.228	72.420
2 step-ahead	159.763	168.255	141.708	147.906	161.558	148.089	148.736
3 step-ahead	221.401	216.056	176.578	183.942	192.313	179.855	179.121
4 step-ahead	203.960	258.841	204.587	210.945	201.923	199.095	197.25
5 step-ahead	201.165	313.839	226.033	229.935	204.341	214.522	211.547
R^2 with drift	0.085^{**}	-0.036^{**}	0.09^{**}	0.039^{**}	-0.054^{**}	-0.025^{**}	-0.028^{**}
R^2 without drift	0.074^{**}	-0.048^{**}	0.079^{**}	0.028^{**}	-0.835^{**}	-2.004	-1.773

Zinc	BCAF	Weighted Average (MSE)	Average Forecast	Median	Best Model (BIC)	5 Best Models	10 Best Models
1 step-ahead	141.114	143.832	195.797	161.538	150.502	149.399	153.070
2 step-ahead	244.172	251.799	479.166	292.524	254.273	255.094	267.004
3 step-ahead	245.351	266.658	875.957	291.309	247.958	248.965	260.206
4 step-ahead	242.105	279.292	250.617	253.082	233.096	236.309	241.427
5 step-ahead	250.851	301.163	264.693	269.355	235.248	242.127	248.668
R^2 with drift	0.238^{**}	0.223^{**}	-0.057^{**}	0.128^{**}	0.187^{**}	0.193^{**}	0.174^{**}
R^2 without drift	0.231^{**}	0.216^{**}	-0.067^{**}	0.12^{**}	-12.232	-16.386	-22.693