When Grilli and Yang meet Prebisch and Singer: Piecewise linear trends in primary commodity prices

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March, 2013

Prebisch–Singer hypothesis

the secular decline of the terms of trade of primary commodity prices relative to those of manufactured goods.

Prebisch (1950): "the price relation turned steadily against primary production from the 1870's until the Second World War."

Singer (1950): "It is a matter of historical fact that ever since the [eighteen] seventies the trend of prices has been heavily against sellers of food and raw materials and in favor of the sellers of manufactured articles."

Prebisch–Singer hypothesis: evaluations

It is not easy to estimate the trend components of the primary commodity prices because the prices are volatile and the changes in their trends, if any, are relatively small; Cashin and McDermott (2002).

Deaton (1999): "commodity prices lack in trend."
"what commodity prices lack in trend, they make up for in variance."

Williamson (2012) declared that the Prebisch–Singer announcement of secular decline of relative primary commodity prices was premature.

Grilli–Yang data

Grilli and Yang (1988) built "a U.S. dollar index of prices of twenty–four internationally traded nonfuel commodities, beginning in 1900."

The original data series ended at 1986, but it is still regularly updated; see Pfaffenzeller, Newbold, and Rayner (2007). The current incarnation includes observations up to 2010.

very popular in economic development and international trade studies: see Cuddington, Ludema, and Shamila (2007) for a survey.

Our sample period: 1900–2010

Previous findings from the Grilli-Yang data

Kim, Pfaffenzeller, Rayner, and Newbold (2003) found strong evidence for negative trends for at most three commodities out of twenty–four: aluminum, hides, and rice for the sample ending at 1998. Moderately strong evidence was found for sugar and wheat. They showed that lamb and timber exhibited positive trends during the entire sample period.

Harvey, Kellard, Madsen, and Wohar (2010, 2012) found negative trends only for the following five commodities: aluminum, banana, rice, sugar, and tea. In contrast, tobacco had a significant positive trend for the full sample period. Their sample ended at 2003.

We ask a different question.

Does the Prebisch–Singer hypothesis hold *sometimes*?

We assume piecewise linear trends for primary commodity prices.

The number and location of change-points are not limited *a priori*.

Other stylized facts about commodity prices

they are often dominated by long periods of doldrums punctuated by sharp upward spikes; Deaton and Laroque (1992)

shocks to commodity prices tend to persist for several years at a time; Cashin, Liang, and McDermott (2000)

unrelated commodity prices move together; Pindyck and Rotemberg (1990)

Grilli–Yang's arguments for broken trends

- "The negative trend present in the relative prices of metals from 1900 to 1986 . . . is not uniform over time. A clear break in the price trend occurs in the early 1940s" (p. 18).

- "It was in the mid–1950s that petroleum–based synthetic products began to exercise strong downward pressure on natural rubber and natural fiber prices (cotton, jute, wool)" (p. 18).

multiple breaks

- "From 1900 to about 1941, the GYCPIM/MUV shows a strong negative trend (1.7 percent a year). Between 1942 and 1986 the trend turns positive (0.5 percent a year). The rising trend of the GYCPIM/MUV after 1941 was even stronger until the early 1970s" (p. 18).

- "An example of this can be found in the trend in the relative prices of metals in the 1900–86 period, which shows a clear primary tendency to fall, but a secondary tendency in the opposite direction" (p. 30).

- "Metals, conversely, though showing the strongest overall negative trend in their relative prices over the current century, did experience a precipitous fall until the early 1940s and a strong inversion of that tendency since then" (p. 34).

Unit root tests with breaks

Kellard and Wohar (2006) allowed (up to) two structural breaks. They found considerably less support for the hypothesis with the unit root test proposed by Lumsdaine and Papell (1997).

Zanias (2005) applied the same unit root test with two structural breaks.

León and Soto (1997) allowed up to one break and found evidence for negative trends in seventeen series out of twenty–four for all or most of their sample period of 1900–1992.

Ghoshray (2012) also applied other unit root tests, allowing for up to two structural breaks.

Balagtas and Holt (2009), Arezki, Hadri, Kurozumi, and Rao (2012), and Yang, Lin, and Kao (2012).

A different question

Does the Prebisch–Singer hypothesis hold *sometimes*?

Hodrick–Prescott filter

$$\sum_{t=1}^{T} (y_t - x_t)^2 + \lambda \sum_{t=3}^{T} \left(\Delta^2 x_t\right)^2, \tag{1}$$

or, in matrix notation,

$$\|\boldsymbol{y} - \boldsymbol{x}\|_2^2 + \lambda \|\boldsymbol{D}\boldsymbol{x}\|_2^2, \tag{2}$$

$$\boldsymbol{D} = \begin{pmatrix} 1 & -2 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & -2 & 1 \end{pmatrix} \in R^{(T-2)\times T}. \tag{3}$$

ℓ_1 trend filtering

$$\sum_{t=1}^{T} (y_t - x_t)^2 + \lambda \sum_{t=3}^{T} |\Delta^2 x_t| \tag{4}$$

$$\|\boldsymbol{y} - \boldsymbol{x}\|_2^2 + \lambda \|\boldsymbol{D}\boldsymbol{x}\|_1, \tag{5}$$

only a numerical solution available: $\boldsymbol{x}^{\mathtt{lt}}$

Kim, Koh, Boyd, and Gorinevsky (2009, SIAM Review)

Lasso representation of ℓ_1 trend filtering

(5) is equivalent to

$$\|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{\theta}\|_2^2 + \lambda \sum_{t=3}^{T} |\theta_t|, \tag{6}$$

where $\boldsymbol{\theta} = (\theta_1 \quad \dots \quad \theta_T)' \in R^T$ and $\boldsymbol{A}$ is a lower triangular matrix,

$$\boldsymbol{A} = \begin{pmatrix} 1 & & & & & \\ 1 & 1 & & & & \\ 1 & 2 & 1 & & & \\ 1 & 3 & 2 & \ddots & & \\ \vdots & \vdots & \vdots & \ddots & 1 & \\ 1 & T-1 & T-2 & \cdots & 2 & 1 \end{pmatrix} \in R^{T \times T}.$$

(6) is the Lasso (least absolute shrinkage and selection operator, Tibshirani, 1996) representation of ℓ_1 trend filtering.

We can now easily explain why $\boldsymbol{x}^{\text{lt}}$ converges to the best affine fit as $\lambda \to \infty$. When $\lambda \to \infty$, $\lambda \sum_{t=3}^{T} |\theta_t| \to \infty$ unless $\boldsymbol{\theta}_{3:T} = \mathbf{0}_{(T-2)\times 1}$. Therefore, $\boldsymbol{\theta}_{3:T} = \mathbf{0}$ is required to minimize (6). With this requirement, the first component in (6) is

$$\sum_{t=1}^{T} \left(y_t - \theta_1 - \theta_2(t-1)\right)^2 . \tag{7}$$

This is a simple trend regression, and it explains why we should have an affine fit when $\lambda \to \infty$.

HP filter as a ridge regression

$$\|\boldsymbol{y} - \boldsymbol{A}\boldsymbol{\theta}\|_2^2 + \lambda \sum_{t=3}^{T} \theta_t^2. \tag{8}$$

This is the ridge regression representation of the HP filter.

Generalizations of ℓ_1 filtering

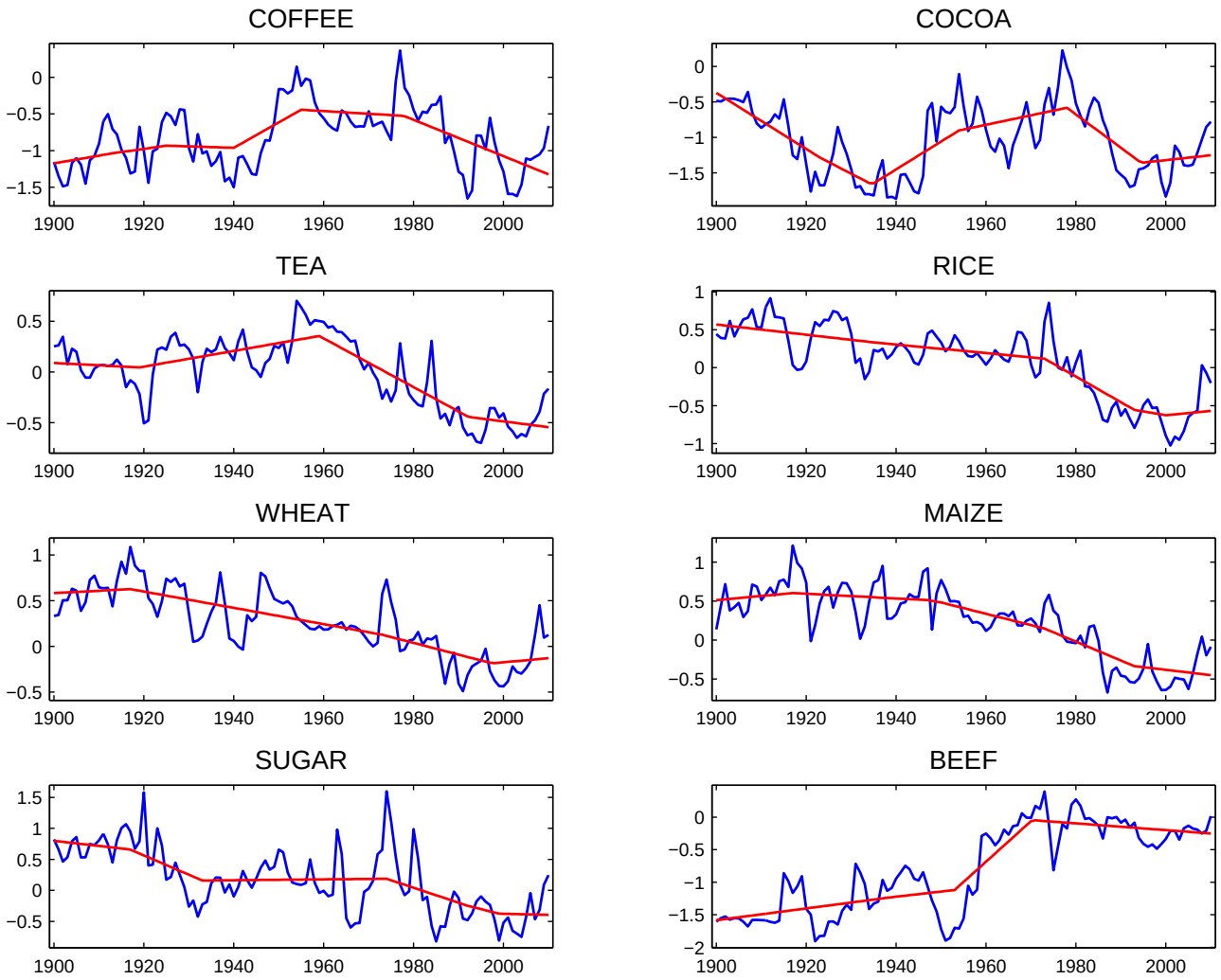
$$\sum_{t=1}^{T} (y_t - x_t)^2 + \lambda \sum_{t=k+1}^{T} |\Delta^k x_t| \tag{9}$$

- $k = 1$: ℓ_1 mean filtering–piecewise constant trend functions
- $k = 2$: ℓ_1 trend filtering–piecewise linear trend functions
- $k = 3$: ℓ_1 quadratic filtering–piecewise quadratic trend functions
- $k = 4$: ℓ_1 cubic filtering–piecewise cubic trend functions

also, applicable to time series with seasonality, cyclic components, outliers, or multivariate time series data.

Empirical results for twenty–four primary commodity prices in the Grilli–Yang data set

Figures 1 and 2, and Tables 1 and 2


COFFEE
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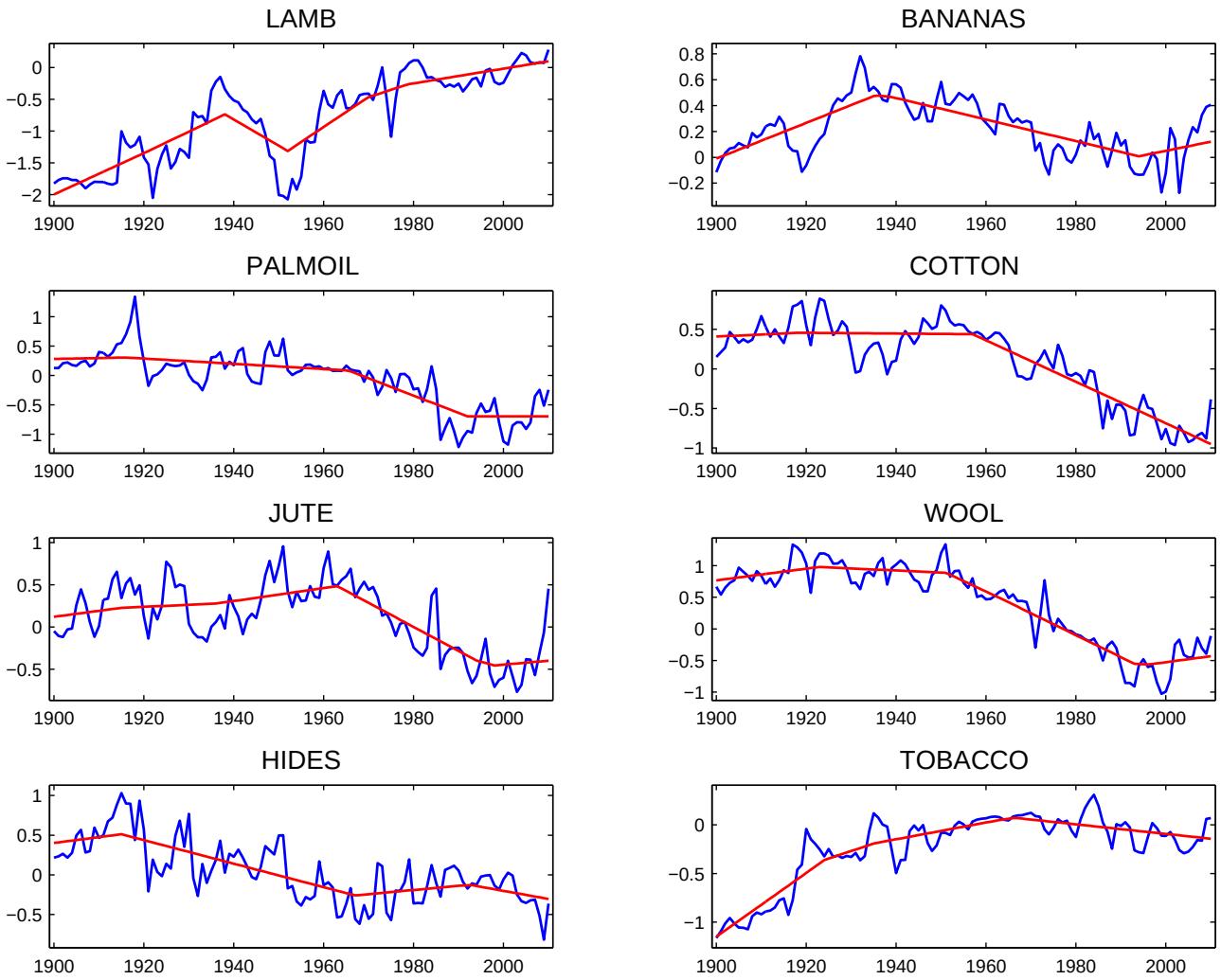
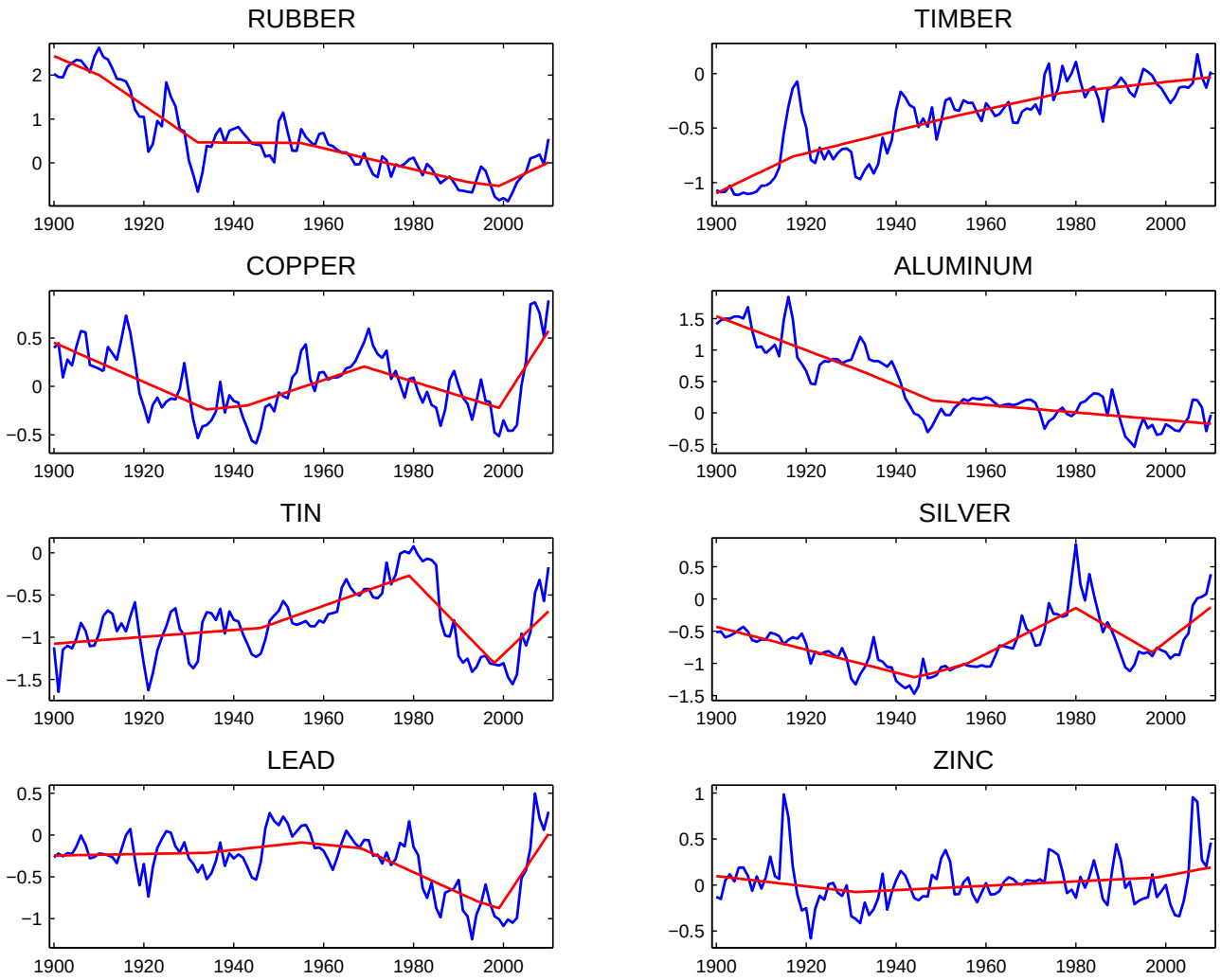
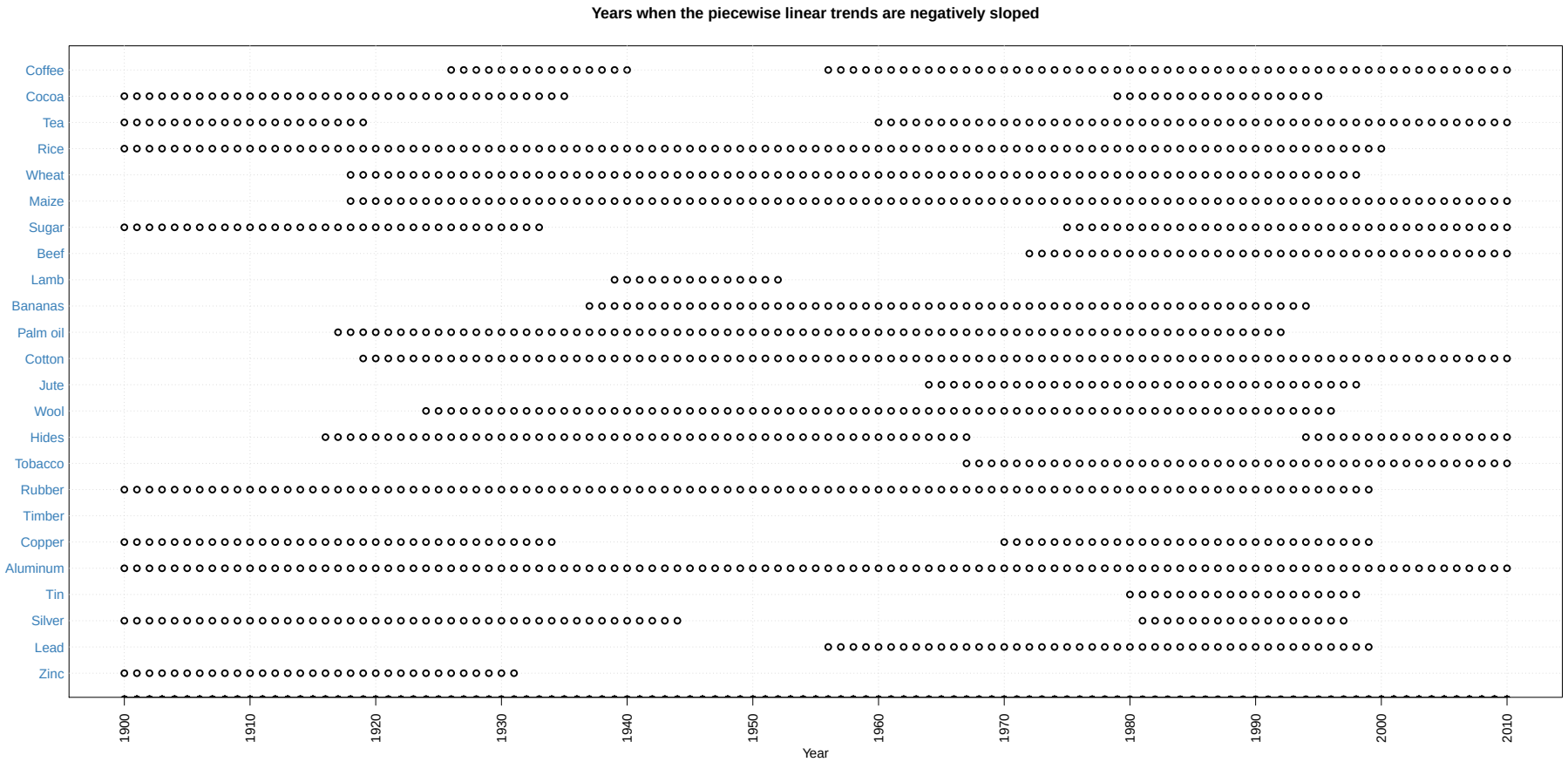
TEA
RICE
WHEAT
MAIZE
SUGAR
BEEF
1900
1920
1940
1960
1980
2000


LAMB
BANANAS
PALMOIL
COTTON
JUTE
WOOL
HIDES
TOBACCO
1900
1920
1940
1960
1980
2000


RUBBER
TIMBER
COPPER
ALUMINUM
TIN
SILVER
LEAD
ZINC
1900
1920
1940
1960
1980
2000


Years when the piecewise linear trends are negatively sloped
Coffee
Cocoa
Tea
Rice
Wheat
Maize
Sugar
Beef
Lamb
Bananas
Palm oil
Cotton
Jute
Wool
Hides
Tobacco
Rubber
Timber
Copper
Aluminum
Tin
Silver
Lead
Zinc
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
Year

Findings

- Various numbers of structural breaks are found: two for aluminum and zinc, and seven for cocoa, lamb, jute, wool, and silver, for instance.
- Continuous change-points should be regarded as a transition period to a new regime. See, for instance, the result for coffee.
- Many of the slope estimates are very small. The two steepest ones are for lead and copper after 2000: 8.08% and 7.24% per annum, respectively.
- For aluminum, the slopes of the estimated ℓ_1 piecewise linear trend are all negative: -2.70% (1900–1933), -2.98% (1934–1948), and -0.60% (1949–2010). In contrast, they are all positive for timber.

- For lamb, tin, and silver, while there are some periods in which the slopes of their trends are negative, the slopes have been positive for most of the sample period.

- The slopes of their trends were negative for most of the sample periods: rice, wheat, maize, bananas, palm oil, cotton, wool, and rubber. Interestingly, they are all food and nonfood primary commodities. Except for maize and cotton, however, the slopes of their piecewise linear trends became positive recently.

- Therefore, even though we find strong evidence that the Prebisch–Singer hypothesis sometimes holds for these primary commodities, the strength of the hypothesis becomes somewhat weaker during the last decade or so.

Table 1: Slopes of the estimated ℓ_1 piecewise linear trends and structural break date

Food	1	2	3	4	5	6	7	8
Coffee	1.05	0.88	−0.20	3.46	−0.36	−0.72	−2.48	
		[1914]	[1926]	[1941]	[1956]	[1978]	[1979]	
Cocoa	−3.95	−3.28	−0.35	3.92	1.33	−4.69	−2.11	0.68
		[1924]	[1935]	[1936]	[1955]	[1979]	[1995]	[1996]
Tea	−0.23	0.77	−2.40	−1.03	−0.57			
		[1920]	[1960]	[1993]	[1994]			
Rice	−0.67	−0.56	−3.38	−1.01	0.58			
		[1936]	[1974]	[1994]	[2001]			
Wheat	0.26	−0.89	−1.28	−1.22	−0.62	0.47		
		[1918]	[1974]	[1995]	[1998]	[1999]		
Maize	0.53	−0.30	−1.01	−1.46	−2.41	−0.68		
		[1918]	[1948]	[1951]	[1974]	[1994]		
Sugar	−0.81	−1.05	−3.12	0.07	−2.41	−1.84	−0.17	
		[1917]	[1918]	[1934]	[1975]	[1993]	[2000]	
Beef	0.91	0.79	6.23	1.19	−0.52			
		[1940]	[1954]	[1971]	[1972]			
Lamb	3.25	3.41	−4.12	4.72	4.29	2.32	1.88	1.16
		[1923]	[1939]	[1953]	[1970]	[1971]	[1979]	[1980]
Bananas	1.38	0.42	−0.38	−0.82	−0.84	0.71		
		[1936]	[1937]	[1940]	[1952]	[1995]		
Palm oil	0.15	−0.45	−1.59	−2.95	0.01			
		[1917]	[1966]	[1967]	[1993]			

Nonfood	1	2	3	4	5	6	7	8
Cotton	0.27	−0.05	−2.60	−2.62				
		[1919]	[1958]	[1959]				
Jute	0.71	0.24	0.76	−2.72	−2.85	−1.45	0.46	
		[1916]	[1937]	[1964]	[1965]	[1995]	[1999]	
Wool	0.92	−0.33	−3.11	−3.35	−3.47	−0.36	0.62	0.96
		[1924]	[1952]	[1954]	[1965]	[1994]	[1997]	[1998]
Hides	0.73	−1.48	0.53	0.15	−1.02			
		[1916]	[1968]	[1992]	[1994]			
Tobacco	3.28	1.56	0.86	0.69	−0.16	−0.49		
		[1925]	[1936]	[1964]	[1967]	[1968]		
Rubber	−4.30	−6.99	−0.07	−2.41	−1.27	4.90		
		[1911]	[1933]	[1956]	[1993]	[2000]		
Timber	1.98	1.03	0.90	0.43				
		[1918]	[1954]	[1978]				

Metals	1	2	3	4	5	6	7	8
Copper	−2.02	0.45	1.54	−1.42	7.24			
		[1935]	[1944]	[1970]	[2000]			
Aluminum	−2.70	−2.98	−0.60					
		[1934]	[1949]					
Tin	0.41	1.88	−5.46	−4.71	5.07			
		[1947]	[1980]	[1998]	[1999]			
Silver	−1.78	1.49	1.96	2.07	3.56	−4.05	−3.60	5.35
		[1945]	[1949]	[1955]	[1957]	[1981]	[1997]	[1998]
Lead	0.10	0.59	−0.52	−1.85	−2.46	−1.74	8.08	
		[1935]	[1956]	[1969]	[1970]	[1995]	[2000]	
Zinc	−0.56	0.23	0.92					
		[1932]	[1999]					

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Wool	0.92	−0.33	−3.11	−3.35	−3.47	−0.36	0.62	0.9
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		[1911]	[1933]	[1956]	[1993]	[2000]		
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		[1934]	[1949]					
Tin	0.41	1.88	-5.46	-4.71	5.07			
		[1947]	[1980]	[1998]	[1999]			
Silver	-1.78	1.49	1.96	2.07	3.56	-4.05	-3.60	5
		[1945]	[1949]	[1955]	[1957]	[1981]	[1997]	[19
Lead	0.10	0.59	-0.52	-1.85	-2.46	-1.74	8.08	
		[1935]	[1956]	[1969]	[1970]	[1995]	[2000]	
Zinc	-0.56	0.23	0.92					
		[1932]	[1999]					

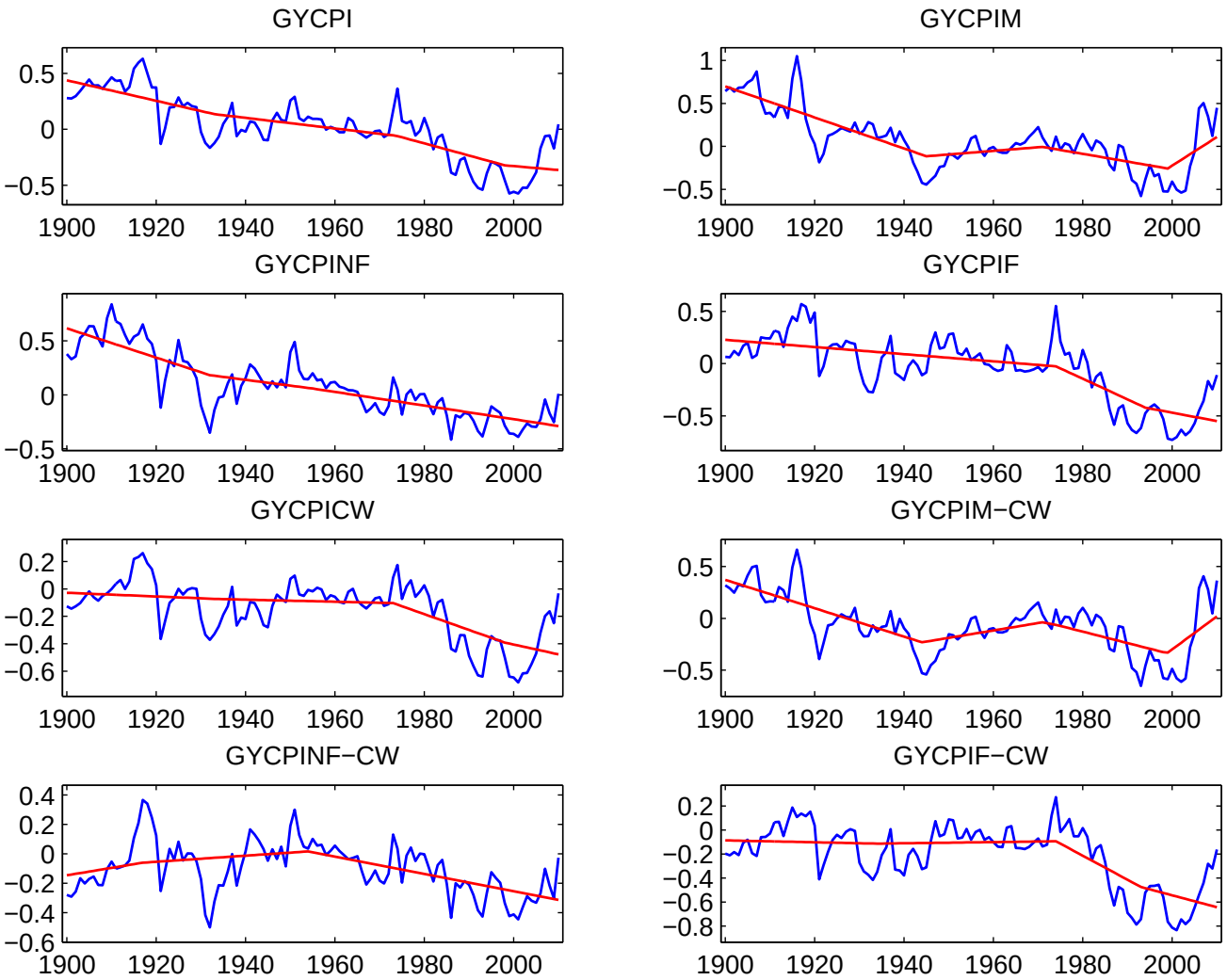
Table 2: The strength of the Prebisch–Singer hypothesis in each decade

decade	number
1900s	9
1910s	9
1920s	14
1930s	13
1940s	12
1950s	10
1960s	14
1970s	17
1980s	20
1990s	20
2000s	9

- The evidence for the Prebisch–Singer hypothesis is quite strong during the 1970s and 1980s.
- However, many series, twelve in total, have changed the signs of their trend functions from negative to positive since 1992.
- Rice and rubber, which had negatively sloped piecewise linear trends for about 100 years, now have positively sloped trends.
- Only nine prices are found to have negatively sloped trends since 2000, and evidence for the Prebisch–Singer hypothesis has recently become much weaker.

Empirical results for eight aggregate price indexes in the Grilli–Yang data set

See Figures 3 and 4, and Tables 3 and 4


GYCPI
GYCPIM
GYCPINF
GYCPIF
GYCPICW
GYCPIM−CW
GYCPINF−CW
GYCPIF−CW
1900
1920
1940
1960
1980
2000

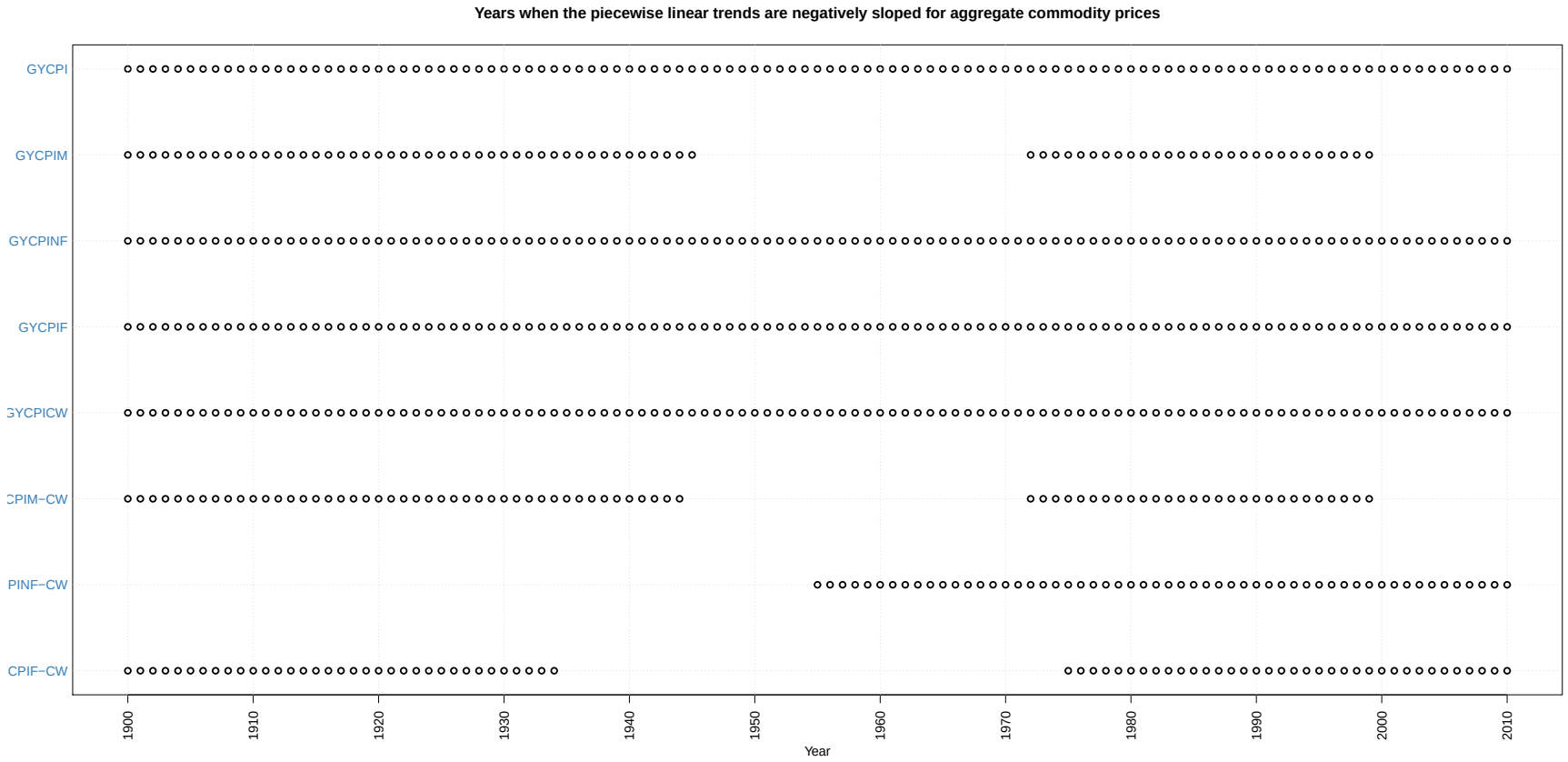

Years when the piecewise linear trends are negatively sloped for aggregate commodity prices
GYCPI
GYCPIM
GYCPINF
GYCPIF
GYCPICW
CPIM−CW
PINF−CW
CPIF−CW
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
Year

Table 4: Slopes of the estimated ℓ_1 piecewise linear trends and structural break dates of aggregate commodity prices

data	1	2	3	4	5	6
GYCPI	−0.92	−0.47	−0.63	−1.09	−0.35	
		[1934]	[1974]	[1975]	[1999]	
GYCPIM	−1.81	0.42	−0.90	3.33		
		[1946]	[1972]	[2000]		
GYCPINF	−1.36	−0.54	−0.63			
		[1933]	[1956]			
GYCPIF	−0.34	−1.97	−0.80			
		[1975]	[1995]			
GYCPICW	−0.13	−0.08	−1.15	−0.72		
		[1935]	[1974]	[1999]		
GYCPIM–CW	−1.37	0.71	−0.80	−1.10	−0.49	3.17
		[1945]	[1972]	[1975]	[1999]	[2000]
GYCPINF–CW	0.50	0.21	−0.59			
		[1918]	[1955]			
GYCPIF–CW	−0.08	0.05	−2.00	−0.99		
		[1935]	[1975]	[1994]		

- Most aggregate commodity prices exhibited negatively sloped trends for all or most of the sample periods, especially those constructed with arithmetic means.
- They also became negative or more negative after around the mid–1970s.
- The recent movement of metal prices, GYCPIM and GYCPIM–CW, is noteworthy in that they have increased sharply since 2000.
- The slopes of their trend functions are 3.33% and 3.17% per annum, respectively. The increase in metal prices should be attributed to the rapid industrialization and higher demand for metals in China.

Two problems of ℓ_1 filtering

- how to choose optimally λ, the tuning parameter in (4)
- inference about the solution $x^{\mathtt{lt}}$ of (4), such as confidence intervals