Linkages Across Sovereign Debt Markets

Cristina Arellano and Yan Bai

Federal Reserve Bank of Minneapolis and
University of Rochester

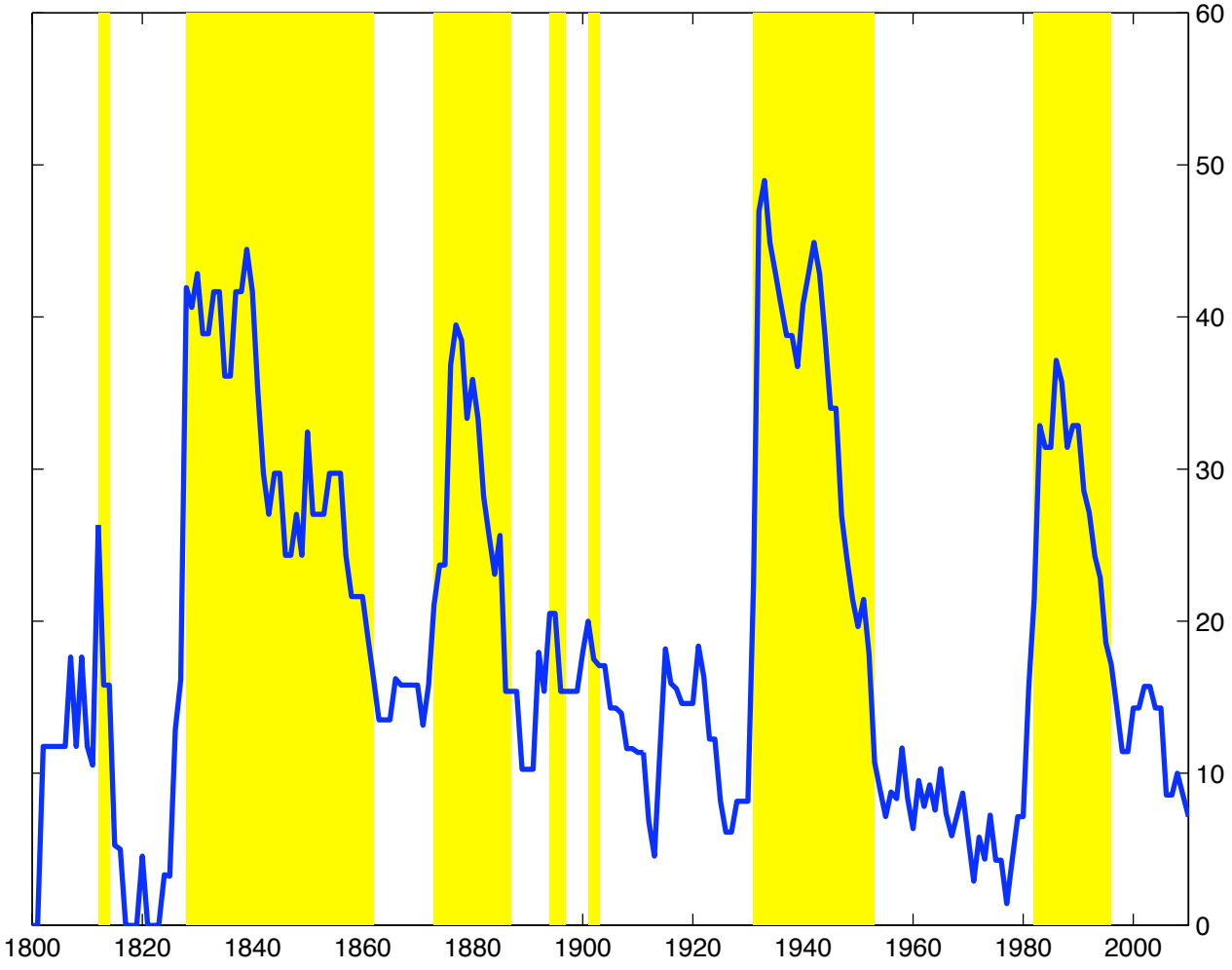
Jacques Polak Annual Research Conference

November 2014

Motivation

- Theory of sovereign default studies countries in isolation
- Sovereign debt crises happen in bunches

Percentage of Countries in Default

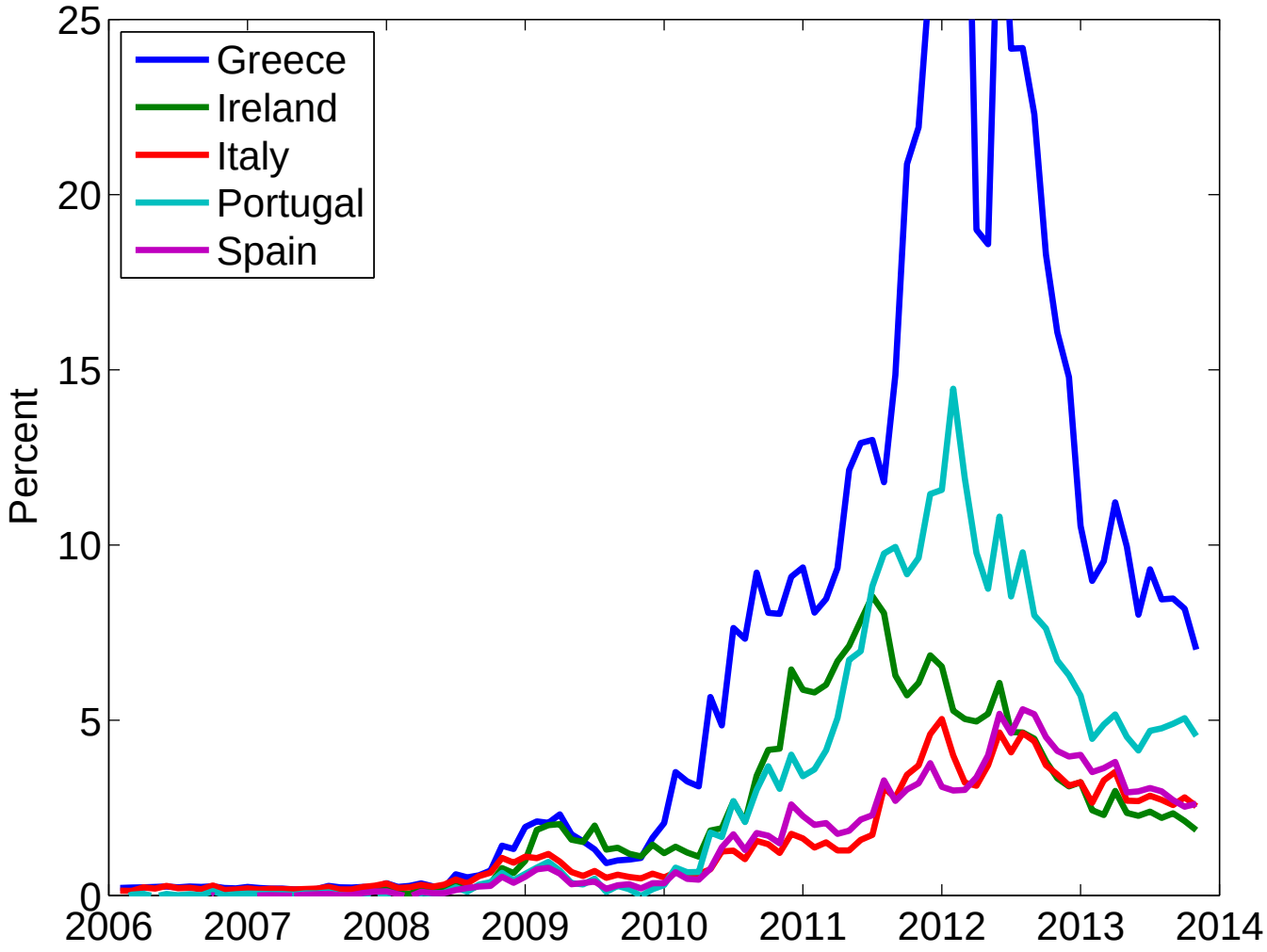


Source: Reinhart and Rogoff 2011

European Debt Crisis

- Default fears: Greece, Ireland, Italy, Portugal, Spain
  - Interest rate spreads co-move
- Lending banks at the center of crisis
  - Banks' exposure to GIIPS seen as a major concern
  - In German banks, loans to GIIPS are 130% of their capital

Spreads


Greece
Ireland
Italy
Portugal
Spain
Percent
25
20
15
10
5
0
2006
2007
2008
2009
2010
2011
2012
2013
2014

This Paper

- Dynamic multicountry model of sovereign debt linkages
  - Countries borrow, default, and renegotiate with common lenders
- Countries default together because
  - Renegotiating together lowers debt recovery
  - Rolling over debt is more expensive

This Paper

- Dynamic multicountry model of sovereign debt linkages
  - Countries borrow, default, and renegotiate with common lenders
- Countries default together because
  - Renegotiating together lowers debt recovery
  - Rolling over debt is more expensive
- Predictions consistent with historical cross-country data
  - Default probabilities higher when others default
  - Renegotiation probabilities higher when others renegotiate
  - Recovery rates lower when others renegotiate
- Model accounts for 50% of spread correlation across GIIPS

Literature

- Sovereign default and renegotiation in single countries: Eaton & Gersovitz (1981), Arellano (2008), Yue (2010), Benjamin & Wright (2011)

  Here multiple countries linked through debt markets

- Risk premia in sovereign bonds market: Borri and Verdelhan (2010), Presno and Pouzo (2012), Gilchrist, Yue, and Zakrajsek (2012)

  Here risk premium endogenous to countries' choices

- Contagion in capital flows through common lender: Calvo and Mendoza (2000), Kyle and Xiong (2001)

  Here contagion through default choices

- Default and contagion: Lizarazo (2010), Park (2012)

  Here strategic interactions among countries and renegotiation

Simple Model

- Two periods no-uncertainty
- Two borrowing countries and continuum of lenders
- Countries are strategic big players; lenders are competitive
- Countries differ in initial debt $b = \{b_1, b_2\}$
- Countries borrow, default, and renegotiate
  - Default entails costs: output and autarky

Borrowing Countries: Consumption

Period 1: Countries decide whether to repay or default

- If repay (d_i=0), borrow

$$c_i = y - b_i + \underbrace{q_i(b, d, b')}_{\text{bond price}} b_i'$$

- If default (d_i=1), output loss and not borrow

$$c_i = y^d$$

Borrowing Countries: Consumption

Period 1: Countries decide whether to repay or default

- If repay (d_i=0), borrow

$$c_i = y - b_i + \underbrace{q_i(b, d, b')}_{\text{bond price}} b_i'$$

- If default (d_i=1), output loss and not borrow

$$c_i = y^d$$

Period 2: Pay debt or recovery

- Non-defaulters pay debt: $c_i' = y' - b_i'$
- Defaulters renegotiate

$$c_i' = y' - \underbrace{\phi_i(d, b')}_{\text{recovery}}$$

Borrowing Countries: Consumption

Period 1: Countries decide whether to repay or default

- If repay (d_i=0), borrow

$$c_i = y - b_i + \underbrace{q_i(b, d, b')}_{\text{bond price}} b_i'$$

- If default (d_i=1), output loss and not borrow

$$c_i = y^d$$

Period 2: Pay debt or recovery

- Non-defaulters pay debt: $c_i' = y' - b_i'$
- Defaulters renegotiate

$$c_i' = y' - \underbrace{\phi_i(d, b')}_{\text{recovery}}$$

Countries are linked through bond price and recovery

Borrowing Countries: Default Decision

- Country i defaults if default value higher than repaying value

$$u(y - b_i + q_i(b, d_{-i})\bar{b}) + \beta u(y' - \bar{b}) < u(y^d) + \beta u(y' - \phi_i(d_{-i}))$$

- If repay borrow to limit: $\bar{b} = y' - y^d$
- States and choices of other country affects ϕ_i' and q_i
  - Low ϕ_i and low q_i increase default incentives
- Default is more likely when b_i is high
- Default cutoff $\hat{b}_i(b_{-i}, d_{-i})$

$$d_i = 1 \quad \text{if } b_i \geq \hat{b}_i(b_{-i}, d_{-i})$$

Recovery Functions

- Simultaneous renegotiation with generalized Nash Bargaining
- One country defaults and renegotiates

$$\max_{\phi_i} \left[u(y' - \phi_i) - u(y^d) \right]^{\theta} \left[u^L(y_L + \bar{b} + \phi_i) - u^L(y_L + \bar{b}) \right]^{1-\theta}$$

Recovery Functions

- Simultaneous renegotiation with generalized Nash Bargaining
- One country defaults and renegotiates

$$\max_{\phi_i} \left[u(y' - \phi_i) - u(y^d)\right]^{\theta} \left[u^L(y_L + \bar{b} + \phi_i) - u^L(y_L + \bar{b})\right]^{1-\theta}$$

- Two countries default and renegotiate

$$\max_{\phi_1,\phi_2} \left[u(y' - \phi_1) - u(y^d)\right]^{\theta} \left[u(y' - \phi_2) - u(y^d)\right]^{\theta} \left[u^L(y_L + \phi_1 + \phi_2) - u^L(y_L)\right]^{1-\theta}$$

Recovery Functions

- Simultaneous renegotiation with generalized Nash Bargaining
- One country defaults and renegotiates

$$\max_{\phi_i} \left[u(y' - \phi_i) - u(y^d)\right]^{\theta} \left[u^L(y_L + \bar{b} + \phi_i) - u^L(y_L + \bar{b})\right]^{1-\theta}$$

- Two countries default and renegotiate

$$\max_{\phi_1,\phi_2} \left[u(y' - \phi_1) - u(y^d)\right]^{\theta} \left[u(y' - \phi_2) - u(y^d)\right]^{\theta} \left[u^L(y_L + \phi_1 + \phi_2) - u^L(y_L)\right]^{1-\theta}$$

- Lenders' outside option lower when both countries renegotiate

Recovery Functions

- Simultaneous renegotiation with generalized Nash Bargaining
- One country defaults and renegotiates

$$\max_{\phi_i} \left[u(y' - \phi_i) - u(y^d)\right]^{\theta} \left[u^L(y_L + \bar{b} + \phi_i) - u^L(y_L + \bar{b})\right]^{1-\theta}$$

- Two countries default and renegotiate

$$\max_{\phi_1,\phi_2} \left[u(y' - \phi_1) - u(y^d)\right]^{\theta} \left[u(y' - \phi_2) - u(y^d)\right]^{\theta} \left[u^L(y_L + \phi_1 + \phi_2) - u^L(y_L)\right]^{1-\theta}$$

- Lenders' outside option lower when both countries renegotiate
- Joint renegotiation $\Rightarrow$ recovery lower $\Rightarrow$ defaults more, $\Rightarrow$ $\hat{b}_i$ tighter

Bond Price Functions

- Prices solve lenders' demand system
- Two countries repay

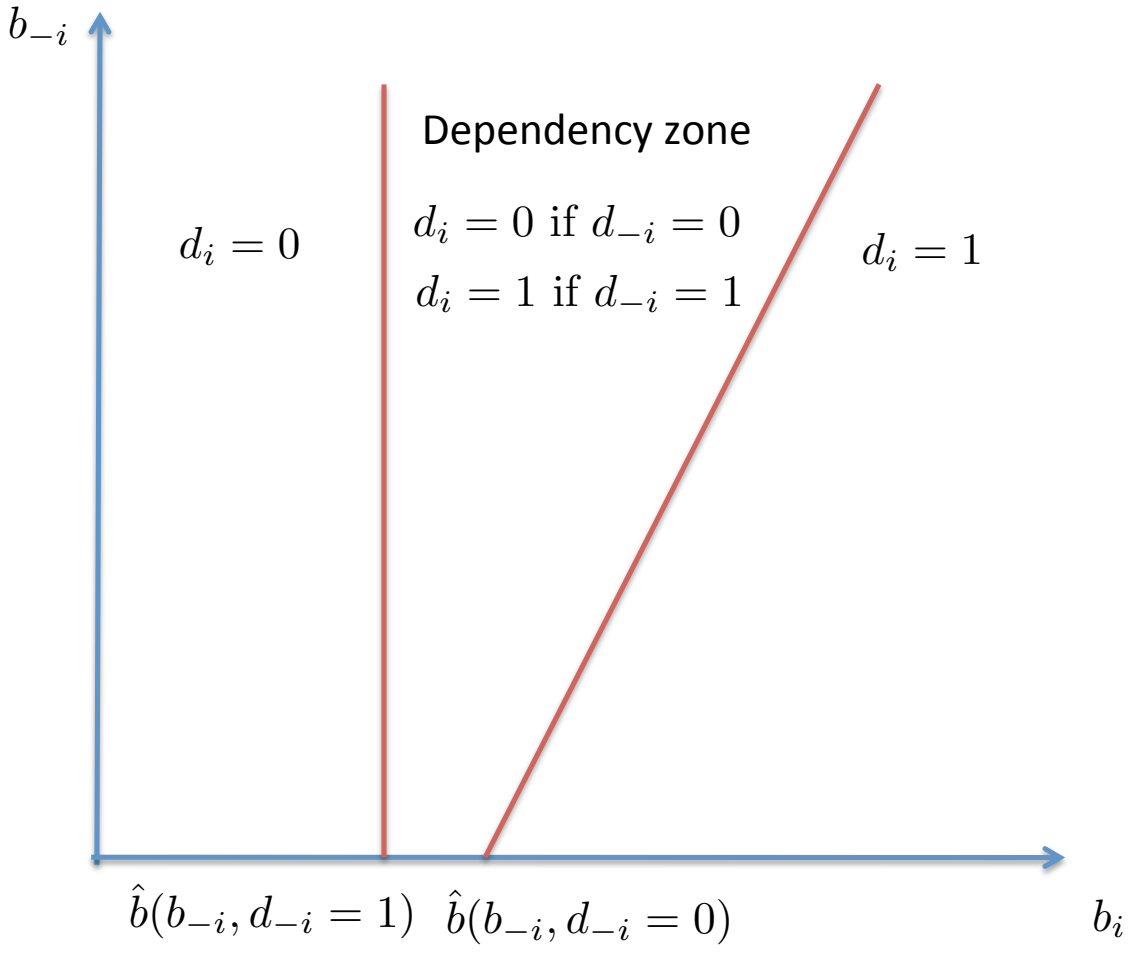
$$q_i = \delta \frac{u_L'(y_L + 2\bar{b})}{u_L'(y_L + b_i - q_i\bar{b} + b_{-i} - q_{-i}\bar{b})}$$

- Price q_i increases with repayment of large b_{-i}
- Foreign defaults

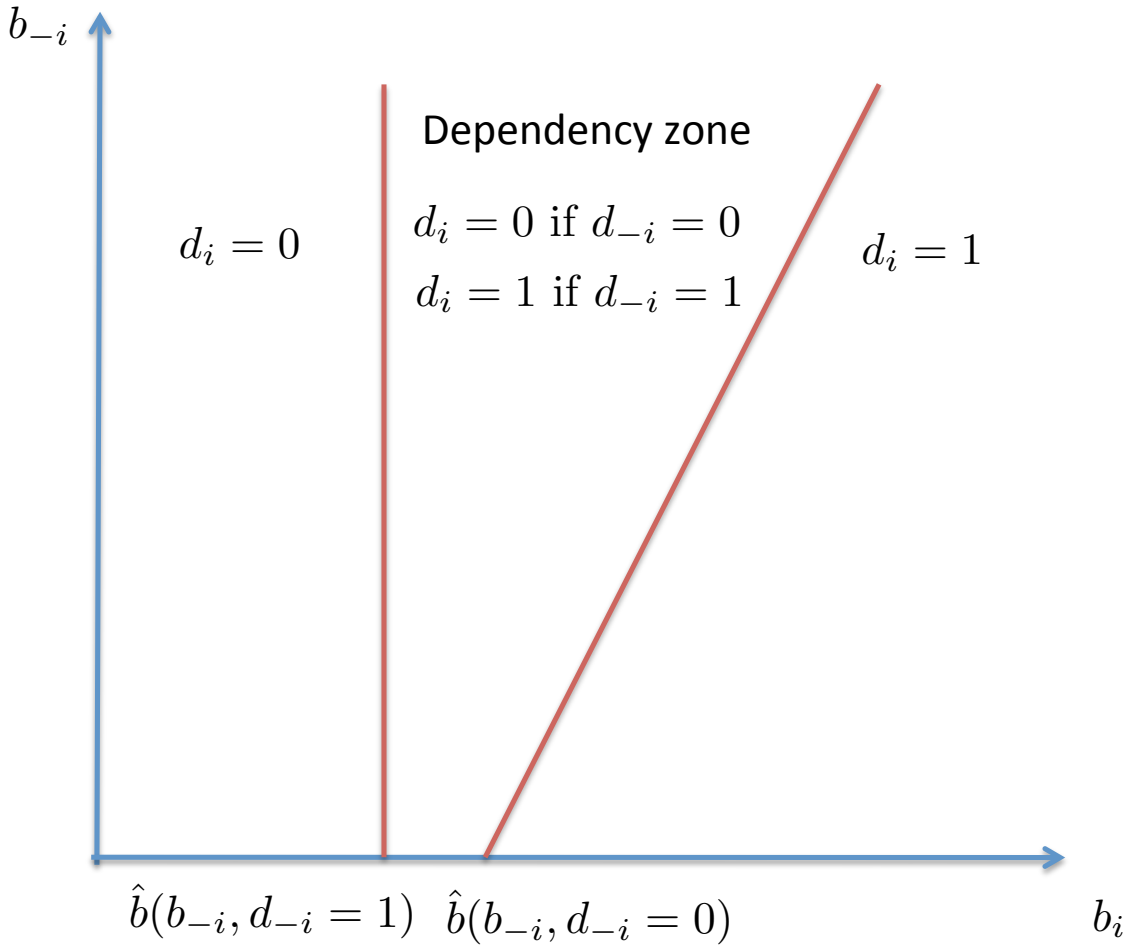
$$q_i = \delta \frac{u_L'(y_L + 2\bar{b})}{u_L'(y_L + b_i - q_i\bar{b})}$$

- Foreign repays small $b_{-i} \Rightarrow \hat{b}_i$ tighter

Best Responses

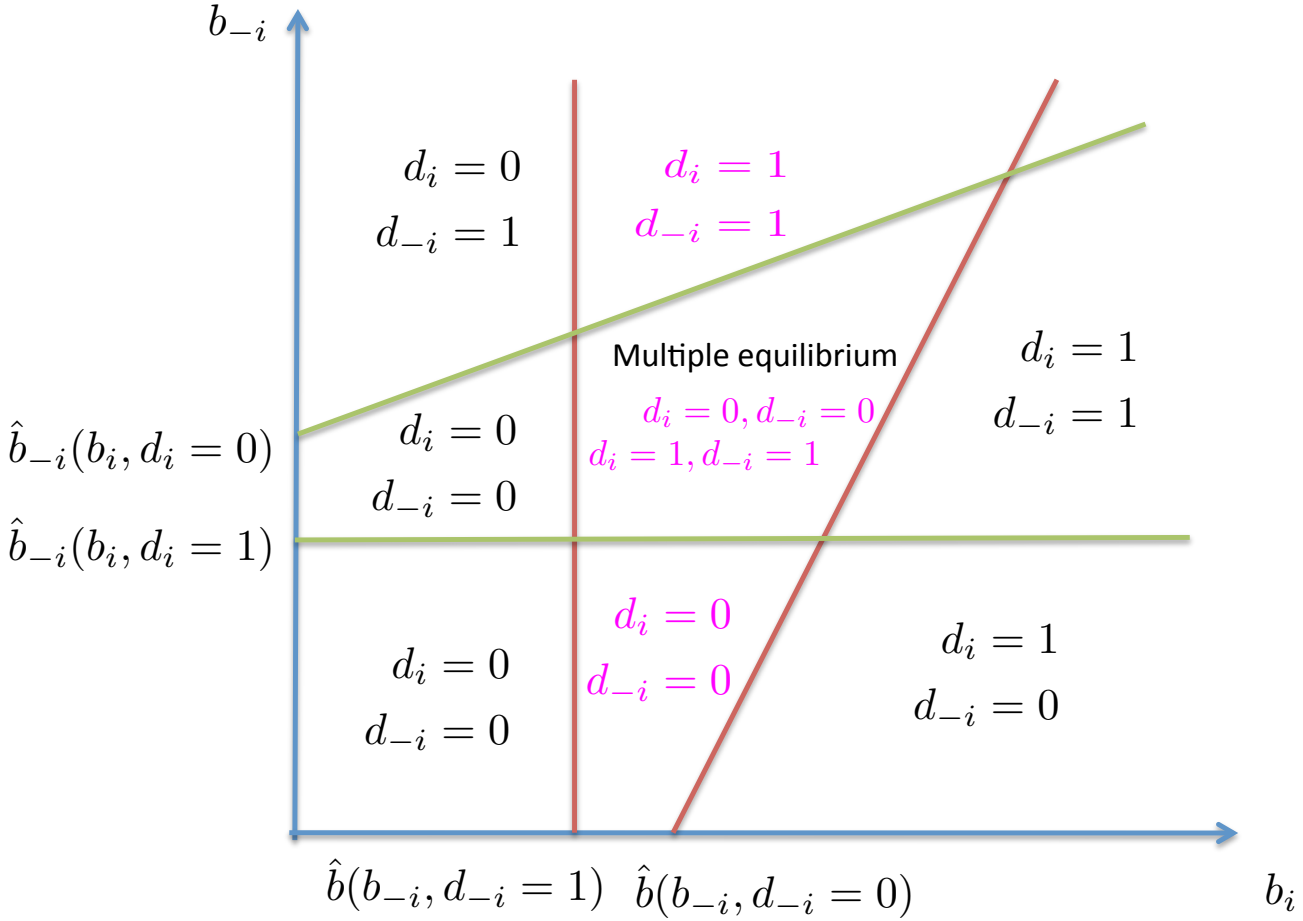


Best Responses

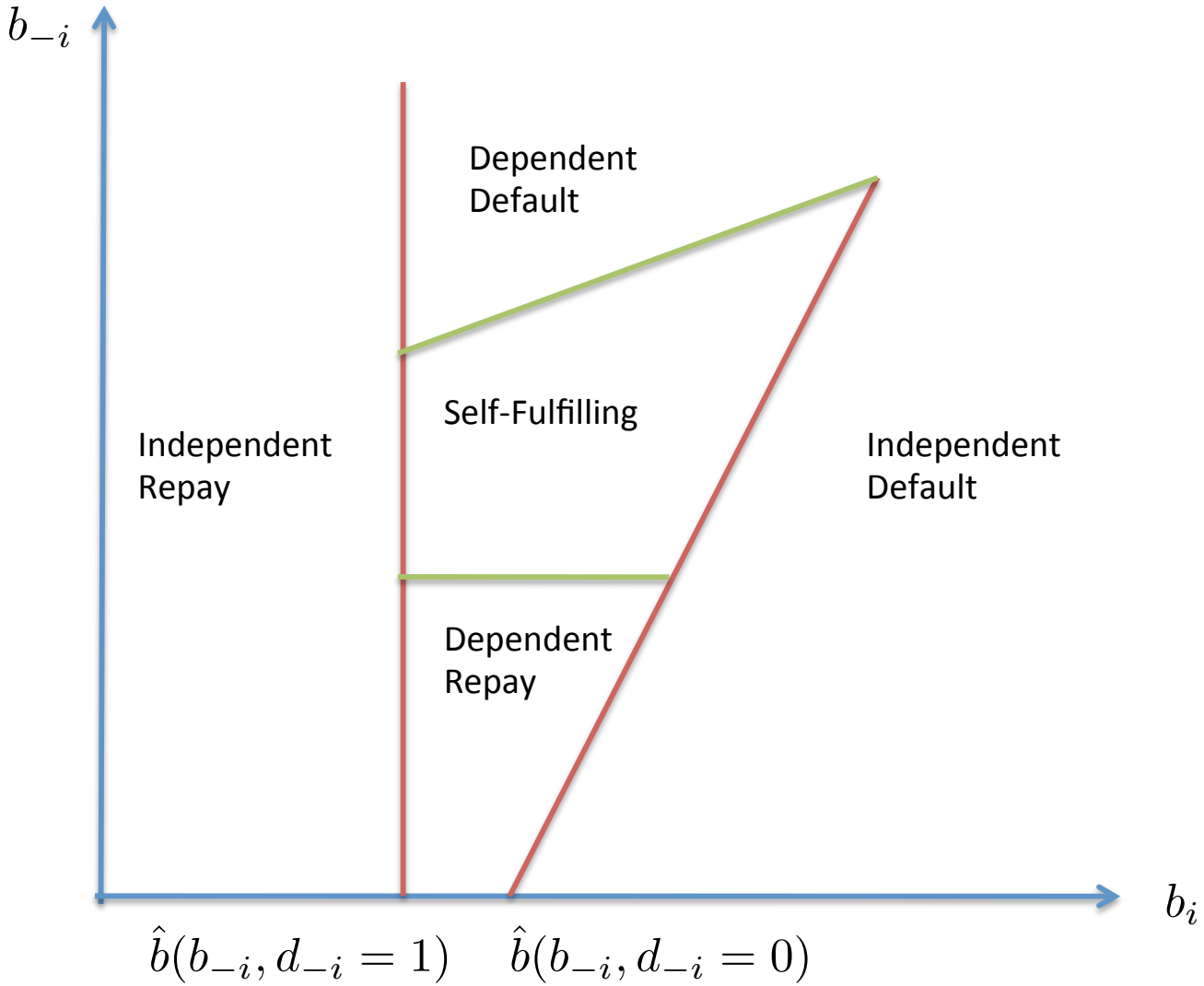


In dependency zone, foreign default leads to home default

Equilibrium



Equilibrium: Zones



Main Predictions Simple Model

- Default more when others default
- Renegotiate more when others renegotiate
- Recovery lower with joint renegotiation

Cross Country Data

	Default	Renegotiation	Recovery
Fraction in Default$_{it}$	1.36***	-0.88***	0.92***
Fraction Renegotiating$_{it}$	-2.13*	4.60**	-7.39***
Debt/GDP$_{it}$	0.11**	-0.03*	-0.21***
Country fixed effects	Yes	Yes	No
Adjusted R^2	0.28	0.06	0.34
Observations	2682	552	139

Dataset of 77 countries 1970-2011 from S&P and Cruces and Trebesch (2013)

- Theory predictions are consistent with historical data

Quantitative Model

- Infinite horizon and stochastic income
- Time varying debt and default choices
- Spreads compensates for expected default loss
- Same strategic interactions for default and renegotiation
- Markov equilbirum:
  - Optimal choices for default, renegotiation, and debt are Nash
  - Bond price and recovery functions are consistent with default and renegotiation decisions
- Equilibrium selection: Outcome that maximizes country values

Quantitative analysis

- Use data from Greece, Italy, Spain, and historical debt recoveries to parameterize model
- Model can account for half of the correlation in spreads and debt exposures across countries
- Strategic interactions and joint renegotiations are most important forces

Model and Data

	Data	Model
Calibrated moments (%)		
Mean risk free rate	4.0	4.2
Mean spread	1.4	1.6
Volatility risk free rate	1.4	1.6
Volatility spread	2.6	1.8
Volatility exposure	15	16
Mean recovery	60	66
Δ in recovery with multiple rene.	-16	-13
Other moments		
Correlation of spread	0.97	0.43
Correlation of exposure	0.56	0.30

Debt Linkages

	Overall	Foreign Good Credit		Foreign Bad Credit	
Home	Mean	Repay	Default	Rene.	Nonrene.
Default prob.	4.5	2.9	37.3	0.03	100
Rene. prob.	98	100	1	100	–
Recovery	66	71	90	58	–
Spread	1.6	1.6	1.9	1.1	–

- Default more when others default or do not renegotiate
- Renegotiate more when others repay or renegotiate
- Recovery is reduced when others repay or renegotiate

What Drives Results?

	Benchmark	Decomposing Mechanism			Correlated Shocks
		Linear	Low IES	Small	
Correlations					
Spreads	0.42	0.28	0.52	0.17	0.67
Exposure	0.30	0.34	0.51	0.07	0.74
Default	0.34	0.45	0.32	0.11	0.59
Volatility					
Risk free rate	1.6	0.0	4.0	1.6	1.6
Dependent events					
Default	25	35	31	–	41
Repay	27	27	22	–	22

What Drives Results?

	Benchmark	Decomposing Mechanism			Correlated Shocks
		Linear	Low IES	Small	
Correlations					
Spreads	0.42	0.28	0.52	0.17	0.67
Exposure	0.30	0.34	0.51	0.07	0.74
Default	0.34	0.45	0.32	0.11	0.59
Volatility					
Risk free rate	1.6	0.0	4.0	1.6	1.6
Dependent events					
Default	25	35	31	–	41
Repay	27	27	22	–	22

Strategic interactions are most important force

Conclusion

- Developed a model of sovereign debt linkages
- Defaults and renegotiations are correlated
  - Strategic interactions of countries trading with common lenders

Parameters

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \qquad g(c_L) = \frac{c_L^{1-\alpha}}{1-\alpha}$$

Uncorrelated AR(1) shock process from Greek GDP $\rho = 0.88$, $\eta = 0.03$
Borrowers' risk aversion $\sigma = 2$

	Value	Target
Lender's income	$y_L = 1.4$	German yield: mean and vol.
Lender's risk aversion	$\alpha = 0.65$	
Lender's discount factor	$\delta = 0.96$	Greek spread: mean and vol.
Output cost after default	$\lambda = 0.016$	
Borrowers' discount factor	$\beta = 0.82$	Recovery rate mean and cond.
Borrower's bargaining power	$\theta = 0.38$	
		Exposure vol.

Borrowers Countries: Payoffs

Consumption and expected value for arbitrary strategy (d, b')

$$w_i(s, d, b') = u(c_i(s, d, b')) + \beta \sum \pi(y', y) v_i(s'(d, b'))$$

- v_i: future value given by the Markov allocations
- w_i: payoff for arbitrary strategies (d, b')
- Future state $s' = (b', h', y')$ induced by strategies (d, b')

Borrowing Countries: Stages of Game

Each period has two sequential stages:

- Default-renegotiation stage

  Countries decide on d_i: repay/default or renegotiate/not

- Borrowing stage: given states (s, d)

  Non-defaulting countries decide on borrowing b'

  Cournot competition

We consider Markov equilibrium

Borrowers Countries: Borrowing Stage

- Borrowing determined by Cournot competition
  - Borrowing best response of country i

$$x_i^b(b'_{-i}, s, d) = \{b'_i : \max_{b'_i} \; w_i(s, d, b')\}$$

  - Optimal borrowing $(B_1(s,d), B_2(s,d))$ is Nash

$$B_i(s,d) = x_i^b(B_{-i}(s,d), s, d) \quad \text{for all } i$$

Borrowers Countries: Default-Renegotiation Stage

- Optimal default and renegotiation is Nash
  - Default-renegotiation best response of country i

$$x_i^d(d_{-i}, s) = \{d_i : \max_{d_i} w_i(s, d, B(s, d))\}$$

  - Optimal default-renegotiation strategy $(D_1(s), D_2(s))$

$$D_i(s) = x_i^d(D_{-i}(s), s)$$

- Markov equilibrium given price and recovery functions

$$v_i(s) = w_i(s, D(s), B(s, D(s)))$$

Lenders

- Competitive with preferences: $E\sum_{t=0}^{\infty}\delta^t g(c_{Lt})$
- More patient than borrowers $\delta > \beta$, dislike volatility $g''(.) < 0$
- Dividends depend on credit phases and countries' choices

$$c_L = y_L + \sum_{i=1}^{2} \underbrace{[1 - D_i(s)]\left[(1-h_i)\left(b_i - Q_i(s)b_i'\right) + h_i\Phi_i(s)\right]}_{\text{net repayment from country } i}$$

- $Q_i(s)$ is prices of bonds; $\Phi_i(s)$ is the recovery

Lenders FOC

- Lenders' kernel

$$m(s', s) = \frac{\delta \pi(y', y) g'(c_L(s'))}{g'(c_L(s))}$$

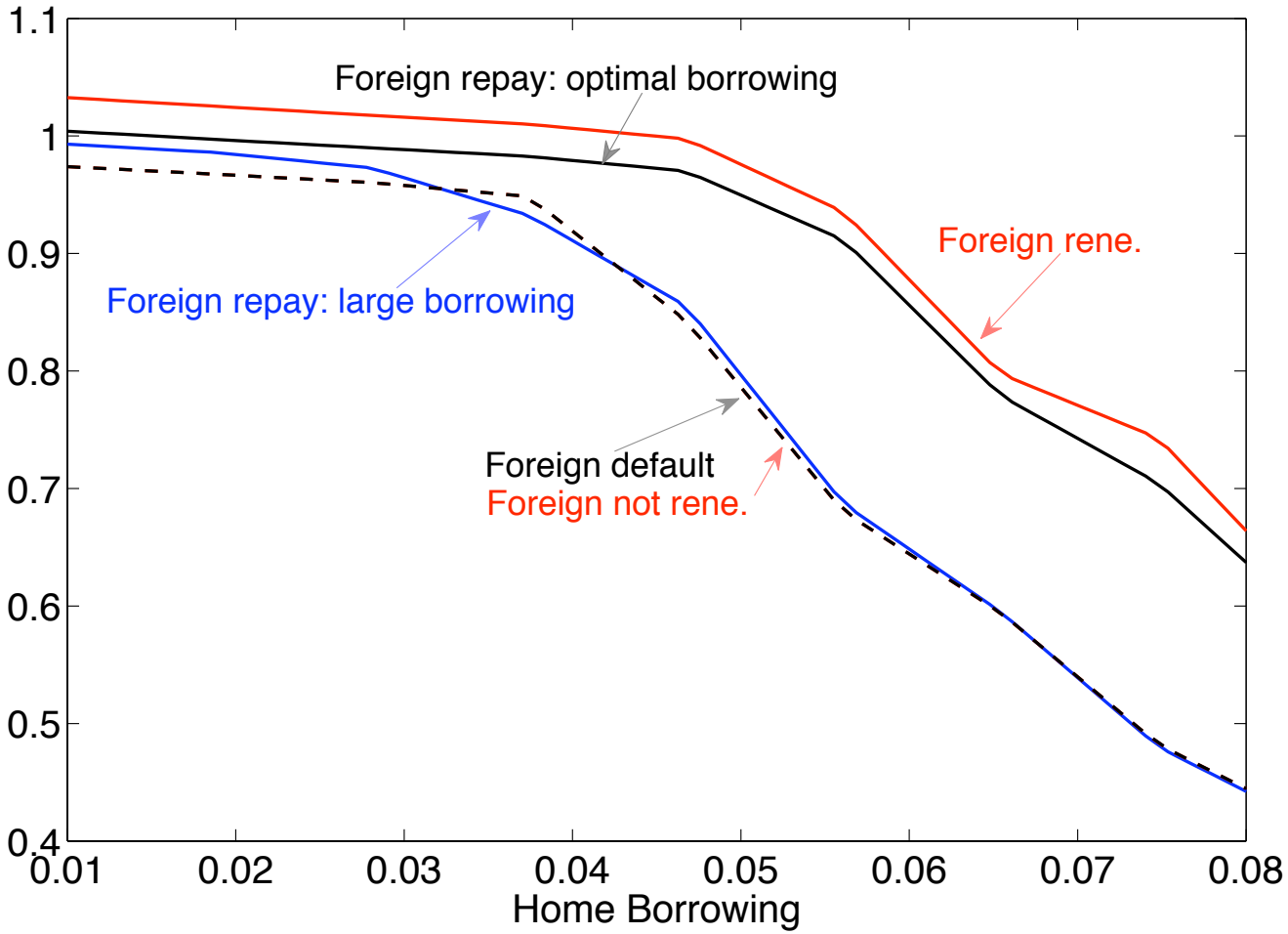
- Price of bonds

$$Q(s) = E\ m(s', s) \left[(1 - D(s')) + D(s') \zeta(s') \right]$$

- Risk adjusted present value of recovery rate

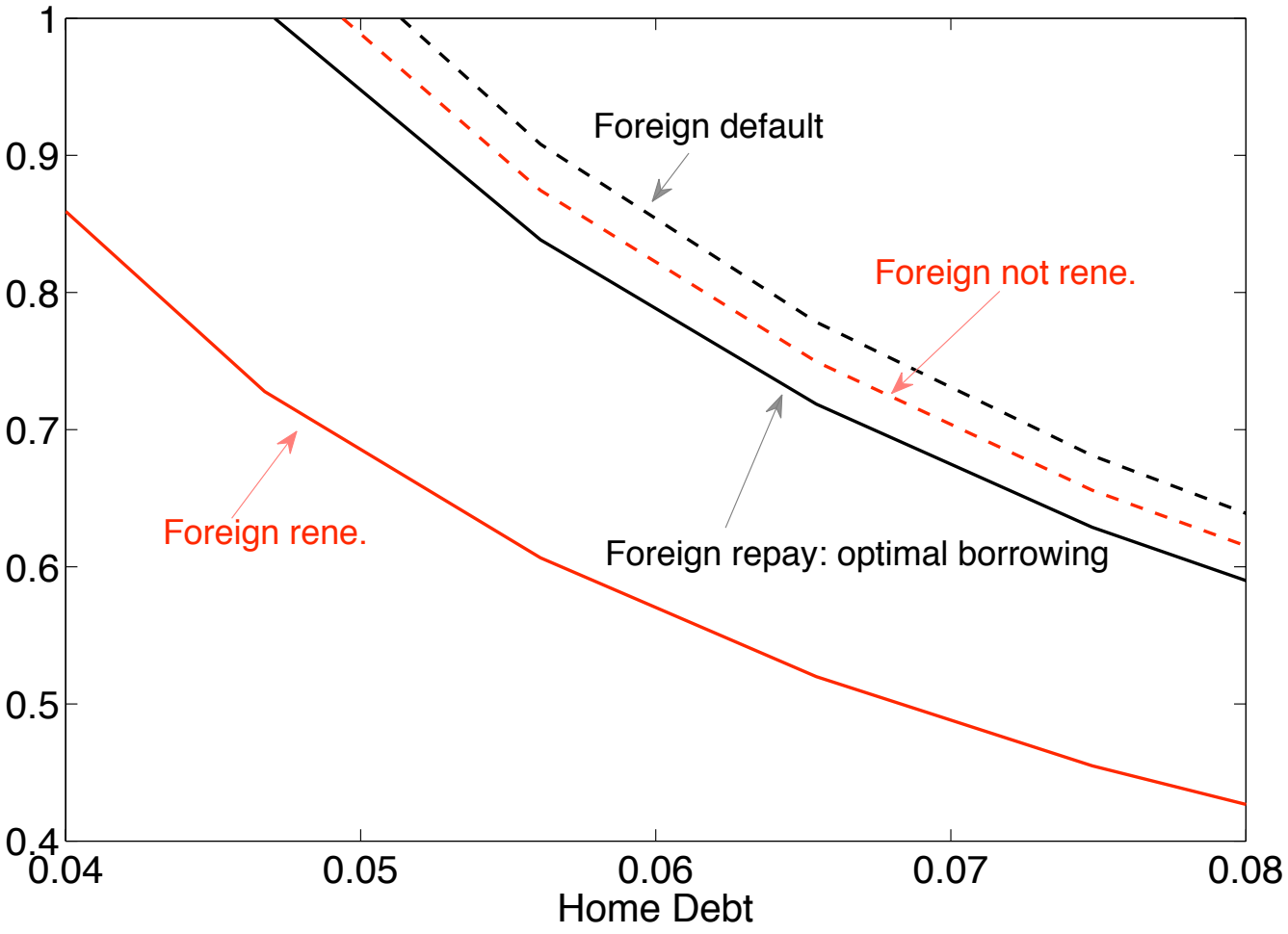
$$\zeta(s) = E\ \left[m(s', s)(1 - D(s')) \frac{\Phi(s')}{b'} + D(s') \zeta(s') \right]$$

Bond price schedule



- Price tight with foreign default, high borrowing, and not renegotiate

Recovery schedule



- Recovery low in joint renegotiations