

# Macro-prudential Policy in a Fisherian Model of Financial Innovation

Javier Bianchi NYU and University of Wisconsin-Madison

> Emine Boz International Monetary Fund

Enrique Mendoza University of Maryland

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## Macro-prudential Policy in a Fisherian Model of Financial Innovation<sup>\*</sup>

Javier Bianchi Wisconsin and NYU Emine Boz IMF Enrique G. Mendoza University of Maryland and NBER

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#### Abstract

The interaction between credit frictions, financial innovation, and a switch from optimistic to pessimistic beliefs are thought to have played a central role in the 2008 financial crisis. This paper lays out a quantitative general equilibrium macro framework in which this interaction drives the financial amplification mechanism and studies its implications for the design of macroprudential policy. Financial innovation enhances the ability of agents to collateralize assets into debt, but the riskiness of this new regime can only be learned over time. Beliefs about the transition probabilities across states with high and low ability to borrow change as agents observe realizations of financial conditions, and in the long-run converge to the true probabilities. At the same time, the credit constraint that links debt to asset values introduces a pecuniary externality because agents fail to internalize the effect of their borrowing decisions on asset prices. The interaction between these forces strengthens the financial amplification mechanism, making macro-prudential policy more desirable. We show how the effectiveness of this policy depends on the government's information set; the policy is weaker when the government is as uninformed about the riskiness of the new financial regime as private agents.

Keywords: Financial crises, financial innovation, macro-prudential regulation, Bayesian learning JEL Codes: D62, D82, E32, E44, F32, F41

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## 1 Introduction

Policymakers have responded to the lapses in financial regulation in the years before the 2008 global financial crisis and the unprecedented systemic nature of the crisis itself with a strong push to revamp financial regulation following a "macro-prudential" approach. The aim is to focus on the macro (i.e. systemic) connections amongst credit market participants and to implement policies to influence behavior in credit markets in the "good times" in order to make financial crises less severe and less frequent. The design of macro-prudential policy requires, however, the development of models that are reasonably good at explaining the macro dynamics of financial crises and at capturing the connection between potential macro-prudential policy instruments and the actions of credit market participants.

The task of developing these models is particularly challenging because of the fast pace of financial development. Indeed, the decade before the 2008 crash was a period of significant financial innovation, which included both the introduction of a large set of complex financial instruments, such as collateralized debt obligations, mortgage backed securities and credit default swaps, and the enactment of major financial reforms of a magnitude and scope unseen since the end of the Great Depression. Thus, macro-prudential regulation has to take into account the changing nature of the financial environment, and hence deal with the fact that credit market participants, as well as policymakers, may be making decisions lacking perfect information about the true riskiness of a changing financial regime.

This paper proposes a dynamic stochastic general equilibrium model in which the interaction between financial innovation, credit frictions and imperfect information is at the core of the financial transmission mechanism, and uses it to so study its quantitative implications for the design and effectiveness of macro-prudential policy. In the model, a collateral constraint limits the agents ability to borrow to a fraction of the market value of the assets they can offer as collateral. Financial innovation enhances the ability of agents to securitize assets into debt, but also introduces risk because of the possibility of fluctuations in collateral coefficients or loan-to-value ratios.

We take literally the definition of financial innovation as a the introduction of a truly new financial regime. This forces us to walk away from the standard assumption that agents can formulate rational expectations with full information about the stochastic process driving fluctuations in credit conditions. In particular, agents learn (in Bayesian fashion) about the transition probabilities of financial regimes as they observe regimes with high and low ability to borrow over time. In the long run, and in the absence of new waves of financial innovation, they learn the true transition probabilities and form standard rational expectations, but in the short run agents' beliefs display waves of optimism and pessimism depending on their initial priors and on the market conditions they observe. These changing beliefs influence agents borrowing decisions and equilibrium asset prices, and together with the collateral constraint they form an amplification feedback loop: optimistic (pessimistic) expectations lead to over-borrowing (under-borrowing) and increased (reduced) asset prices, and as asset prices change the ability to borrow changes as well.

Our analysis focuses in particular on a learning scenario in which the arrival of financial innovation starts an "optimistic phase," in which a few observations of enhanced borrowing ability lead agents to believe that the financial environment is stable and risky assets are not "very risky." Hence, they borrow more and bid aggressively for risky assets pushing prices higher than in a full information rational expectations equilibrium. The higher value of assets in turn relaxes the credit constraint. Thus, the initial increase in debt due to optimism is amplified by the interaction with the collateral constraint via optimistic asset prices. Conversely, when the first realization of the low-borrowing-ability regime is observed, a "pessimistic phase" starts in which agents overstate the probability of continuing in poor financial regimes and overstate the riskiness of assets. This results in lower debt levels and lower asset prices, and the collateral constraint amplifies this downturn.

Macro-prudential policy action is desirable in this environment because the collateral constraint introduces a pecuniary externality in credit markets that leads to more debt and financial crises that are more severe and frequent than in the absence of this externality. The externality exists because individual agents fail to internalize the effect of their borrowing decisions on asset prices, particularly future asset prices in states of financial distress (in which the feedback loop via the collateral constraint triggers a financial crash). There are several studies in the growing literature on macroprudential regulation that have examined the implications of this externality, but typically under the assumption that agents form rational expectations with full information (e.g. Lorenzoni (2008), Stein (2011), Bianchi (2010), Bianchi and Mendoza (2010), Korinek (2010), Jeanne and Korinek (2010), Benigno, Chen, Otrok, Rebucci, and Young (2010)). In contrast, the novel contribution of this paper is in that we study the effects of macro-prudential policy in an environment in which the pecuniary externality is influenced by the interaction of the credit constraint with learning about the riskiness of a new financial regime. The findings of Boz and Mendoza (2010) suggest that taking this interaction into account can be very important, because the credit constraint in the learning setup produces significantly larger effects on debt and asset prices than in a full-information environment with the same credit constraint. Their study, however, focused only on the properties of decentralized competitive equilibria and abstracted from normative issues.

Our policy analysis considers a social planner under two different informational assumptions. First, a planner that is also subject to learning about the true riskiness of the new financial environment. We consider a baseline scenario in which private agents and the planner have the same initial priors and thus form the same sequence of beliefs. Second, a planner with full information, who therefore knows the true transition probabilities across financial regimes. Here we consider two alternatives, one in which the planner faces a set of feasible credit positions consistent with the collateral values (i.e. asset prices) of the full information, rational expectations competitive equilibrium, and one in which the planner's set of feasible credit positions is constrained to reflect the collateral values of the competitive equilibrium with learning. We compute the competitive equilibria of the model with imperfect information and contrast this case with the above social planner equilibria. We then compare the main features of these equilibria, in terms of the behavior of macroeconomic aggregates and asset pricing indicators, and examine the characteristics of macro-prudential policies that support the allocations of the planning problems as competitive equilibria.

Our analysis shows how the effectiveness of macro-prudential policy depends on the information set assumed for the social planner, and on whether the planner can enforce credit contracts that reflect full-information collateral prices or learning-distorted collateral prices. If the planner has the same information set as private agents and the same initial priors, it goes through the same learning process as the agents do. In this case, macro-prudential regulation, while still aiming to alter debt and asset pricing dynamics, is less effective at weakening the severity of financial crises than in the other scenarios. This is because, while the planner can still manage the pecuniary externality by affecting private borrowing plans, it can only do so around the asset prices that are already distorted by the mispricing of risk.

The quantitative analysis indicates that the interaction of the collateral constraint with optimistic beliefs can strengthen the case for introducing macro-prudential regulation compared with the full-information case. This is because the credit boom driven by this interaction generates larger amplification when the economy switches to the bad financial regime. On the other hand, the undervaluation of risk also weakens the incentives of a planner with subjective beliefs to build precautionary savings against states of nature with tight credit regimes over the long run, because this planner underestimates the probability of landing and remaining in those states. In contrast, if the government has full information to form rational expectations and prices risk correctly, optimal macro-prudential policy features a component that influences borrowing levels at given asset prices, and a component that influences portfolio choice of debt v. assets to address the effect of the agents' mispricing of risk on asset prices. Hence, by comparing the debt positions and asset prices that the different planners can support, our analysis emphasizes the potential limitations of macro-prudential policy in the presence of significant financial innovation (which makes the informational friction relevant), and highlights the relevance of taking into account informational frictions in evaluating the effectiveness of macro-prudential regulation.

Our model follows in a long and old tradition of models of financial crises in which credit frictions and imperfect information interact. This notion dates back to the classic work of Fisher (1933), in which he described his well-known debt-deflation financial amplification mechanism as the result of a feedback loop between agents' beliefs and credit frictions (particularly those that force fires sales of assets by distressed borrowers). Minsky (1992) work is along a similar vein. More recently, macroeconomic models of financial accelerators (e.g. Bernanke, Gertler, and Gilchrist (1999), Kiyotaki and Moore (1997), Aiyagari and Gertler (1999)) have focused on modeling financial amplification but focusing typically on models with rational expectations and full information about the stochastic processes of exogenous shocks.

The particular specification of imperfect information and learning that we use follows closely that of Boz and Mendoza (2010) and Cogley and Sargent (2008a), in which agents observe regime realizations of a Markov-switching process without noise but need to learn its transition probability matrix. The imperfect information assumption is based on the premise that the U.S. financial system went through significant changes beginning in the mid-90s as a result of financial innovation and deregulation that took place at a rapid pace. As in Boz and Mendoza, agents go through a learning process in order to "discover" the true riskiness of the new financial environment as they observe realizations of regimes with high or low borrowing ability.

Our quantitative analysis is related to Bianchi and Mendoza (2010)'s quantitative study of macro-prudential policy. They examined an asset pricing model with a similar collateral constraint and used comparisons of the competitive equilibria vis-a-vis a social planner to show that optimal macro-prudential policy curbs credit growth in good times and reduces the frequency and severity of financial crises. The government can accomplish this by using Pigouvian taxes on debt and dividends to induce agents to internalize the model's pecuniary externality. Bianchi and Mendoza's

framework does not capture, however, the role of informational frictions interacting with frictions in financial markets.

Our paper is also in a similar vein as Gennaioli, Shleifer, and Vishny (2010), who study financial innovation in an environment in which "local thinking" leads agents to neglect low probability adverse events (see also Gennaioli and Shleifer (2010)). As in our model, this neglect distorts decision rules and equilibrium asset prices, but the modeling of the informational friction is different. In the model of Gennaioli et al. agents ignore part of the state space relevant for pricing risk by assumption, assigning zero probability to rare negative events. In contrast, in our learning framework agents always assign non-zero probability to all the regimes that are part of the realization vector of the Markov switching process.<sup>1</sup> However, agents do assign lower (higher) probability to bad states than they would under full information rational expectations when they are optimistic (pessimistic), and this lower probability is an outcome of a Bayesian learning process. In addition, the welfare analysis of Gennaioli, Shleifer, and Vishny (2010) focuses on the effect of financial innovation under local thinking, while we emphasize the interaction between a fire-sale externality and informational frictions.

Finally, our work is also related to Stein (2011) argument in favor of a cap and trade system to address a pecuniary externality that leads banks to issue excessive short-term debt in the presence of private information. Our analysis differs in that we study the implications of a form of model uncertainty (i.e. uncertainty about the transition probabilities across financial regimes) for macroprudential regulation, instead of private information, and we focus on Pigouvian taxes as a policy instrument to address the pecuniary externality.

The rest of the paper is organized as follows: Section 2 describes the model. Section 3 conducts the quantitative analysis comparing the decentralized competitive equilibrium with the various planning problems. Section 4 provides the main conclusions.

## 2 Model Economy

#### 2.1 Decentralized Competitive Equilibrium

Consider an economy inhabited by a continuum of identical agents who maximize a standard constant-relative-risk-aversion utility function. Agents choose consumption,  $c_t$ , holdings of a risky

<sup>&</sup>lt;sup>1</sup>Beliefs that assign zero probability to a state can only be achieved as a limiting case.

asset  $k_{t+1}$  (i.e. land), and holdings of a one-period discount bond,  $b_{t+1}$ , denominated in units of the consumption good. Land is a risky asset traded in a competitive market, where its price  $q_t$  is determined, and is in fixed unit supply. Individually, agents see themselves as able to buy or sell land at the market price, but since all agents are identical, at equilibrium the price clears the land market with all agents choosing the same land holdings. Bonds carry an exogenous price equal to 1/R, where R is an exogenous world-determined gross real interest rate.

The bond market is imperfect because agents face a collateral constraint that limits debt (a negative position in b) to a fraction  $\kappa$  of the market value of their individual land holdings. The collateral coefficient  $\kappa$  is stochastic and follows a Markov regime-switching process. Information is imperfect with respect to the true transition probability matrix governing the evolution of  $\kappa$ , and the agents learn about it by observing realizations of  $\kappa$  over time. We will model learning so that in the long-run the agents' beliefs converge to the true transition probability matrix, at which point the model yields the same competitive equilibrium as a standard rational-expectations asset pricing model with a credit constraint.

Agents operate a production technology  $\varepsilon_t Y(k_t)$  that uses land as the only input, and facing a productivity shock  $\varepsilon_t$ . This shock has compact support and follows a finite-state, stationary Markov process about which agents are perfectly informed.

The agents' preferences are given by:

$$E_0^s \left[ \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \right]. \tag{1}$$

 $E^s$  is the subjective conditional-expectations operator that is elaborated on further below,  $\beta$  is the subjective discount factor, and  $\sigma$  is the coefficient of relative risk aversion.

The budget constraint faced by the agents is:

$$q_t k_{t+1} + c_t + \frac{b_{t+1}}{R_t} = q_t k_t + b_t + \varepsilon_t Y(k_t)$$

$$\tag{2}$$

The agents' collateral constraint is:

$$-\frac{b_{t+1}}{R_t} \le \kappa_t q_t k_{t+1} \tag{3}$$

Using  $\mu_t$  for the Lagrange multiplier of (3), the first-order conditions of the agents optimization problem are given by:

$$u'(t) = \beta R E_t^s \left[ u'(t+1) \right] + \mu_t \tag{4}$$

$$q_t(u'(t) - \mu_t \kappa_t) = \beta E_t^s \left[ u'(t+1) \left( \varepsilon_{t+1} Y_k(k_{t+1}) + q_{t+1} \right) \right]$$
(5)

A decentralized competitive equilibrium (DE) is a sequence of allocations  $[c_t, k_{t+1}, b_{t+1}]_{t=0}^{\infty}$  and prices  $[q_t]_{t=0}^{\infty}$  that satisfy the above conditions together with the collateral constraint (3) and the market-clearing conditions for the markets of goods and assets:

$$c_t + \frac{b_{t+1}}{R_t} = b_t + \varepsilon_t Y(k_t)$$
$$k_t = 1$$

#### 2.2 Learning and the Interaction with the Credit Constraint

Expectations in the payoff function (1) are based on Bayesian beliefs agents form based on initial priors and information they observe over time. As mentioned in the Introduction, we model learning following closely Boz and Mendoza (2010) and Cogley and Sargent (2008a). Hence, we provide here only a short description and refer the interested reader to those other articles for further details.

The stochastic process of  $\kappa$  follows a classic two-point, Hamilton-style regime-switching Markov process. There are two realizations of  $\kappa$ , a regime with high ability to borrow  $\kappa^h$  and a regime with low ability to borrow  $\kappa^l$ . The "true" regime-switching Markov process has continuation transition probabilities defined by  $F_{hh}^a$  and  $F_{ll}^a$ , with the switching probabilities thus given by  $F_{hl}^a = 1 - F_{hh}^a$ and  $F_{lh}^a = 1 - F_{ll}^a$ . Hence, learning in this setup is about forming beliefs regarding the distributions of the transition probabilities  $F_{hh}^s$  and  $F_{ll}^s$  by combining initial priors with the observations of  $\kappa$ that arrive each period. After observing a sufficiently long and varied set of realizations of  $\kappa^h$  and  $\kappa^l$ , agents learn the true regime-switching probabilities of  $\kappa$ . Modeling of learning in this fashion is particularly useful for representing financial innovation as the introduction of a brand-new financial regime for which there is no data history agents could use to infer the true transition distribution of  $\kappa$ , while maintaining a long-run equilibrium that converges to a conventional rational expectations equilibrium.

Agents learn using a beta-binomial probability model starting with exogenous initial priors. Take as given a history of realizations of  $\kappa$  that agents will observe over T periods, denoted  $\kappa^T$ , and initial priors,  $F^s$ , of the distributions of  $F^s_{hh}$  and  $F^s_{ll}$  for date t = 0,  $p(F^s)$ . Bayesian learning with beta-binomial distributions will yield a sequence of posteriors  $\{f(F^s \mid \kappa^t)\}_{t=1}^T$ .

To understand how the sequence of posteriors is formed, consider first that at every date t, from 0 to T, the information set of the agent includes  $\kappa^t$  as well as the possible values that  $\kappa$  can take ( $\kappa^h$  and  $\kappa^l$ ). This means that agents also know the number of times a particular regime has persisted or switched to the other regime (i.e. agents know the set of counters  $[n_t^{hh}, n_t^{hl}, n_t^{lh}]_{t=0}^T$  where each  $n_t^{ij}$  denotes the number of transitions from state  $\kappa^i$  to  $\kappa^j$  that have been observed prior to date t).<sup>2</sup> These counters are important because together with the priors they form the arguments of the Beta-binomial distributions that characterize the learning process. For instance, the initial priors are given by  $p(F_{ii}^s) \propto (F_{ii}^s)^{n_0^{ij}-1}(1-F_{ii}^s)^{n_0^{ij}-1}$ . As in Cogley and Sargent (2008a), we assume that the initial priors are independent and determined by  $n_0^{ij}$  (i.e. the number of transitions assumed to have been observed prior to date t = 1).

The agents' posteriors about  $F_{hh}^s$  and  $F_{ll}^s$  have Beta distributions as well. The details of how they follow from the priors and the counters are provided in Cogley and Sargent (2008a) and in Boz and Mendoza (2010). Here we simply state that the posteriors are of the form  $F_{hh}^s \propto Beta(n_t^{hh}, n_t^{hl})$ and  $F_{ll}^s \propto Beta(n_t^{lh}, n_t^{ll})$ , and that the posterior means satisfy:

$$E_t[F_{hh}^s] = n_t^{hh} / (n_t^{hh} + n_t^{hl}), \quad E_t[F_{ll}^s] = n_t^{ll} / (n_t^{ll} + n_t^{lh})$$
(6)

This is a key result for the solution method we follow, as will be explained later in this Section. An important implication of this result is that posterior means change only when that same regime is observed at date t. Since in a two-point, regime-switching setup continuation probabilities also determine mean durations, it follows that both the persistence and the mean durations of the two financial regimes can be learned only as the economy actually experiences  $\kappa^l$  or  $\kappa^h$ .

The potential for financial innovation to lead to significant underestimation of risk can be inferred from the evolution of the posterior means implied by (6). Specifically, if we start from low values of  $n_0^{ij}$ , because the new financial regime is truly new, the initial sequence of realizations of  $\kappa^h$  observed until just before the first realization of  $\kappa^l$  generates substantial optimism (i.e. a sharp increase in  $E_t[F_{hh}^s]$  relative to  $F_{ll}^a$ ). Moreover, it also follows that the magnitude of the optimism that any subsequent sequence of realizations of  $\kappa^h$  generates will be smaller than in the initial optimistic phase. This is because it is only after observing the first switch to  $\kappa^l$  that agents rule

<sup>&</sup>lt;sup>2</sup>The number of transitions across regimes is updated as follows:  $n_{t+1}^{ij} = n_t^{ij} + 1$  if both  $\kappa_{t+1} = \kappa^j$  and  $\kappa_t = \kappa^i$ , and  $n_{t+1}^{ij} = n_t^{ij}$  otherwise.

out the possibility of  $\kappa^h$  being an absorbent state. Similarly, the first realizations of  $\kappa^l$  generate a pessimistic phase, in which  $E_t[F_{ll}^s]$  is significantly higher than  $F_{ll}^a$ , so the period of unduly optimistic expectations is followed by a period of pessimistic expectations.

Following Boz and Mendoza (2010), the effects of the interaction between the collateral constraint and learning can be explained intuitively by combining the Euler equations on land and bonds (Equations (4) and (5)) to obtain an expression for the model's land premium,  $E_t^s[R_{t+1+i}^q]$ , and then solving forward for the price of land in Equation (5). The expected land premium rises in every state in which the collateral constraint binds because of a combination of three effects: the increased shadow value of land as collateral (which is limited to the fraction  $(1 - \kappa_t)$  of  $\mu_t$  because only the fraction  $\kappa_t$  of land can be collateralized into debt), the lower covariance between marginal utility and land returns, and the increased expected marginal utility of future consumption. The latter two effects are caused by the fact that the binding credit constraint hampers the agents' ability to smooth consumption and tilts consumption towards the future.

Compare now what the land premium would look like in the learning economy v. a perfect information economy when the collateral constraint binds even at  $\kappa^h$ . If beliefs are optimistic (i.e.  $E_t[F_{hh}^s] > F_{hh}^a$ ), agents will assign lower probability to the risk of switching to  $\kappa^l$  (which has higher land returns because the constraint is more binding than with  $\kappa^h$ ) than they would under perfect information. This lowers the expected land premium because agents' beliefs put more weight on states with lower land returns.

To see how this would affect asset prices, consider the forward solution of  $q_t$ :

$$q_t = E_t^s \left[ \sum_{j=0}^\infty \left( \prod_{i=0}^j \left( \frac{1}{E_t^s [R_{t+1+i}^q]} \right) \right) \varepsilon_{t+1+j} Y_k(k_{t+1+j}) \right].$$
(7)

This expression shows that the lower land returns of the model with learning under optimistic beliefs, either at date t or expected along the equilibrium path for any future date, translate into higher land prices higher at t (and higher than under full information). But if the constraint was already binding with  $\kappa^h$  and  $\kappa^h$  is the current state, the value of collateral will rise and agents will borrow more. Hence, optimistic beliefs and the credit constraint interact to amplify the total effects on credit and prices.

When the financial regime does switch to  $\kappa^l$  after a spell of  $\kappa^{h's}$ , the opposite process is set in motion and it has an additional amplification mechanism via a Fisherian deflation. Now agents become pessimistic (i.e.  $E_t[F_{ll}^s] > F_{ll}^a$ ), so they assign excessive probability to staying in the poor credit regime. This increases the expected land premium because now agents' beliefs put more weight on states with higher land returns, and the higher expected premia lower asset prices relative to full information. As asset prices fall, and if  $\kappa^l$  is the current state, the collateral constraint becomes even more binding, which triggers a Fisherian deflation and fire sales of assets, which in turn put further upward pressure on land premia and downward on land prices, and agents continue to put higher probability in these states with even higher land returns and lower land prices. Unlike the case of the optimistic beliefs in the  $\kappa^h$  state, in which the price increase tends to lower the shadow value of the borrowing constraint, with pessimistic beliefs in the  $\kappa^l$  state the Fisherian deflation of asset prices increases this shadow value and feeds back into larger land premia and lower land prices, and thus larger contractions in credit.

#### 2.3 Recursive Anticipated Utility Competitive Equilibrium

The fact that this learning setup involves learning from and about an exogenous variable ( $\kappa$ ) allows us to separate the solution of the evolution of beliefs from the agents' dynamic optimization problem. Thus, we solve for the equilibrium dynamics following a two-stage solution method. In the first stage, we use the above Bayesian learning framework to generate the agents' sequence of posterior density functions  $\{f(F^s \mid \kappa^t)\}_{t=1}^T$ , from which we can compute the sequence of posterior means determined by (6). In the second stage, we characterize the agent's optimal plans as a recursive equilibrium by adopting Kreps's Anticipated Utility (AU) approach to approximate dynamic optimization with Bayesian learning. The AU approach focuses on combining the sequences of posterior means obtained in the first stage with chained solutions from a set of "conditional" AU optimization problems (AUOP).<sup>3</sup> Each of these problems solves what looks like a standard optimization problem with full information and rational expectations, but using the posterior means of each date t instead of the true transition probabilities (see Boz and Mendoza (2010) for further details).

We define the AU competitive equilibrium in recursive form. Consider the date-t AUOP. At this point agents have observed  $\kappa_t$ , and use it to update their beliefs so that (6) yields  $E_t[F_{hh}^s]$ and  $E_t[F_{ll}^s]$ . Using this posterior means, they construct the date-t beliefs about the transition

<sup>&</sup>lt;sup>3</sup>Cogley and Sargent (2008b) show that the AU approach is significantly more tractable than full Bayesian dynamic optimization and yet produces very similar quantitative results unless risk aversion coefficients are large. The full Bayesian optimization problem uses not just the posterior means but the entire likely evolution of posterior density functions to project the effects of future  $\kappa$  realizations on beliefs. This problem runs quickly into the curse of dimensionality because it requires carrying the counters  $[n_t^{hh}, n_t^{hl}, n_t^{ll}, n_t^{lh}]_{t=0}^T$  as additional state variables.

probability matrix across financial regimes  $E_t^s[\kappa'|\kappa] \equiv \begin{bmatrix} E_t[F_{hh}^s] & 1 - E_t[F_{hh}^s] \\ 1 - E_t[F_{ll}^s] & E_t[F_{ll}^s] \end{bmatrix}$ . The solution to the date-*t* AUOP is then given by policy functions  $(b'_t(b,\varepsilon,\kappa), c_t(b,\varepsilon,\kappa), \mu_t(b,\varepsilon,\kappa))$  and a pricing function  $q_t(b,\varepsilon,\kappa)$  that satisfy the following recursive equilibrium conditions:

$$u'(c_t(b,\varepsilon,\kappa)) = \beta R \left[ \sum_{\varepsilon' \in E_{\kappa'} \in \{\kappa^h,\kappa^l\}} E_t^s[\kappa'|\kappa] \pi(\varepsilon'|\varepsilon) u'(c_t(b',\varepsilon',\kappa')) \right] + \mu_t(b,\varepsilon,\kappa)$$
(8)

$$q_{t}(b,\varepsilon,\kappa) \left[ u'(c_{t}(b,\varepsilon,\kappa)) - \mu_{t}(b,\varepsilon,\kappa)\kappa \right] =$$

$$\beta \left[ \sum_{z'\in Z} \sum_{\kappa'\in\{\kappa^{h},\kappa^{l}\}} E_{t}^{s}[\kappa'|\kappa]\pi(\varepsilon'|\varepsilon)u'(c_{t}(b',\varepsilon',\kappa')) \left[\varepsilon'Y(1) + q_{t}(b',\varepsilon',\kappa')\right] \right] \\ c_{t}(b,\varepsilon,\kappa) + \frac{b'_{t}(b,\varepsilon,\kappa)}{R} = \varepsilon Y(1) + b$$

$$(10)$$

$$b(\varepsilon,\kappa) + \frac{c(\varepsilon,\kappa)}{R} = \varepsilon Y(1) + b \tag{10}$$
$$b_t'(b,\varepsilon,\kappa) > u_{\varepsilon}'(b,\varepsilon,\kappa) \tag{11}$$

$$\frac{b_t'(b,\varepsilon,\kappa)}{R} \ge -\kappa q_t(b,\varepsilon,\kappa) 1 \tag{11}$$

The time subscripts that index the policy and pricing functions indicate the date of the beliefs used to form the expectations (which is also the date of the most recent observation of  $\kappa$ , which is date t). Notice that these equilibrium conditions already incorporate the market clearing condition of the land market.

It is critical to note that solving for date-t policy and pricing functions means solving for a full set of optimal plans over the entire  $(b, \varepsilon, \kappa)$  domain of the state space and conditional on date-tbeliefs. Thus, we are solving for the optimal plans agents "conjecture" they would make over the infinite future acting under those beliefs. For characterizing the "actual" equilibrium dynamics to match against the data, however, the solution of the date-t AUOP determines optimal plans for date t only. This is crucial because beliefs change as time passes, and each subsequent  $\kappa_t$  is observed, which implies that the policy and pricing functions that solve each AUOP also change.

The model's recursive AU equilibrium is defined as follows:

**Definition** Given a *T*-period history of realizations  $\kappa^T = (\kappa_T, \kappa_{T-1}, ..., \kappa_1)$ , a recursive AU competitive equilibrium for the economy is given by a sequence of decision rules  $[b'_t(b, \varepsilon, \kappa), c_t(b, \varepsilon, \kappa), \mu_t(b, \varepsilon, \kappa)]_{t=1}^T$ and pricing functions  $[q_t(b, \varepsilon, \kappa)]_{t=1}^T$  such that: (a) the decision rules and pricing function for date tsolve the date-t AUOP conditional on  $E_t^s[\kappa'|\kappa]$ ; (b)  $E_t^s[\kappa'|\kappa]$  is the conjectured transition probability matrix of  $\kappa$  produced by the date-*t* posterior density of  $F^s$  determined by the Bayesian passive learning as defined in (6).

Intuitively, the complete solution of the recursive equilibrium is formed by chaining together the solutions for each date-t AUOP. For instance, the sequence of equilibrium bond holdings that the model predicts for dates t = 1, ..., T is obtained by chaining the relevant decision rules as follows:  $b_2 = b'_1(b, \varepsilon, \kappa), \ b_3 = b'_2(b, \varepsilon, \kappa), ..., \ b_{T+1} = b'_T(b, \varepsilon, \kappa).$ 

### 2.4 Constrained Planners' Problems

We examine macro-prudential policy by studying three versions of the optimal policy problem faced by a benevolent social planner who maximizes the agents' utility subject to the resource constraint and the collateral constraint. The key difference between the equilibria attained by these planners and the DE learning equilibrium is that the planners internalize the effects of their borrowing decisions on the market prices of assets. The three planner problems follow from different assumptions about the information set of the government and about the government's ability to alter the set of feasible credit positions faced by the private sector.

The first social planner (SP1) is subject to a similar learning problem as private agents. This planner observes the same history  $\kappa^T$  but starts learning off date-0 priors that may or may not be the same as those of the private sector. The second planner (SP2) is a fully informed planner (a planner who knows  $F_{hh}^a$  and  $F_{ll}^a$ ) but who is constrained to face the same set of feasible credit positions as private agents in the DE with learning. This implies imposing the equilibrium pricing functions of the DE with learning  $(q_t^{DEL}(b,\varepsilon,\kappa))$  to value collateral in the planner's collateral constraint. The third planner (SP3) is also a fully informed planner, but in addition this planner is able to implement the set of feasible credit positions of the full information DE, which implies imposing the time-invariant pricing function of the full information DE  $q^{DEF}(b,\varepsilon,\kappa)$  in the planner's collateral constraint.

<sup>&</sup>lt;sup>4</sup>These pricing functions are time invariant because they correspond to the solutions of a standard recursive rational expectations equilibrium. The resulting planning problem is analogous to the one solved in Bianchi and Mendoza (2010).

The three planners' optimization problems in standard intertemporal form can be summarized as follows:

$$E_0^i \left[ \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \right] \qquad \text{for} \quad i = SP1, SP2, SP3 \tag{12}$$

s.t. 
$$c_t + \frac{b_{t+1}}{R_t} = b_t + \varepsilon_t Y(1)$$
 (13)

$$\frac{b_{t+1}}{R_t} \leq \kappa_t q_t^i \tag{14}$$

with  $q_t^i = q_t^{DEL}$  for i = SP1, SP2 and  $q_t^{SP3} = q_t^{DEF}$ . Note that in SP1, the planner solves a similar Bayesian learning problem as private agents observing the same history of credit regimes  $\kappa^T$ . This planner's initial priors are denoted  $p_0^{ij}$  for i, j = h, l. If  $p_0^{ij} = n_0^{ij}$ , which will be our baseline scenario, both SP1 and private agents have identical beliefs at all times. In SP2 and SP3 the planners use the true transition probabilities  $F_{hh}^a$  and  $F_{ll}^a$ .

We solve the problem of each planner in recursive form, and to simplify the exposition we represent all three as AU recursive equilibria. For each planner i = SP1, SP2, SP3 the solution to the date-t AUOP is given by policy functions  $(b'_t(b,\varepsilon,\kappa), c_t(b,\varepsilon,\kappa), \mu_t(b,\varepsilon,\kappa))$  that satisfy the following recursive equilibrium conditions:

$$u'(c_t(b,\varepsilon,\kappa)) - \mu_t(b,\varepsilon,\kappa) =$$
(15)

$$\beta R \left[ \sum_{\varepsilon' \in E} \sum_{\kappa' \in \{\kappa^h, \kappa^l\}} E_t^i[\kappa'|\kappa] \pi(\varepsilon'|\varepsilon) \left[ u'(c_t(b', \varepsilon', \kappa')) + \kappa' \mu_t(b', \varepsilon', \kappa') \frac{\partial q_t^i(b', \varepsilon', \kappa')}{\partial b'} \right] \right]$$

$$c_t(b,\varepsilon,\kappa) + \frac{b'_t(b,\varepsilon,\kappa)}{R} = \varepsilon Y(1) + b \tag{16}$$

$$\frac{b_t'(b,\varepsilon,\kappa)}{R} \ge -\kappa q_t^i(b,\varepsilon,\kappa)$$
(17)

where the pricing functions for each planner are  $q_t^i(b,\varepsilon,\kappa) = q_t^{DEL}(b,\varepsilon,\kappa)$  for i = SP1, SP2and  $q_t^{SP3}(b,\varepsilon,\kappa) = q^{DEF}(b,\varepsilon,\kappa)$  Moreover, expectations in each planner's date-t AUOP are taken using  $E_t^{SP1}[\kappa'|\kappa] \equiv \begin{bmatrix} E_t[F_{hh}^g] & 1 - E_t[F_{hh}^g] \\ 1 - E_t[F_{ll}^g] & E_t[F_{ll}^g] \end{bmatrix}$  and  $E_t^i[\kappa'|\kappa] \equiv \begin{bmatrix} F_{hh}^a & 1 - F_{hh}^a \\ 1 - F_{ll}^a & F_{ll}^a \end{bmatrix}$  for i = SP2, SP3.<sup>5</sup> Note also that in these problems the time subindexes of expectations operators, deci-

sion rules and pricing functions represent the date of the AUOP to which they pertain, and not the

<sup>&</sup>lt;sup>5</sup> By analogy with the results in (6), the posterior means of the government's learning dynamics satisfy:  $E_t[F_{hh}^g] = p_t^{hh}/(p_t^{hh} + p_t^{hl})$ ,  $E_t[F_{ll}^g] = p_t^{ll}/(p_t^{ll} + p_t^{lh})$ . Note that, since both the private sector and the government observe the same  $\kappa$  sequence, these counters can differ from those of private agents only because of differences in date-0 priors.

indexing of time within each AUOP. That is, in the date-t AUOP the planner creates expectations of the prices and allocations of future periods using the date-t recursive decision rules and pricing functions (e.g. in the date-t AUOP, consumption projected for t + 1 is given by the expectation of  $c_t(b', \varepsilon', \kappa')$ ). Moreover, for SP3, since the planner has full information and can implement the credit feasibility set of the full information DE, the decision rules are actually time-invariant at equilibrium (all date-t AUOP's for SP3 are identical because they use the true Markov process of  $\kappa$  and the time-invariant pricing functions of the full information DE).

We can now define the three recursive social planner equilibria for a given *T*-period history of realizations  $\kappa^T = (\kappa_T, \kappa_{T-1}, ..., \kappa_1)$ :

**SP1 Equilibrium** Given the time-varying pricing functions from the DE with learning  $q_t^{DEL}(b,\varepsilon,\kappa)$ , a recursive AU equilibrium for a planner with imperfect information is given by a sequence of decision rules  $[b'_t(b,\varepsilon,\kappa), c_t(b,\varepsilon,\kappa), \mu_t(b,\varepsilon,\kappa)]_{t=1}^T$  such that: (a) the decision rules for date t solve SP1's date-t AUOP conditional on  $E_t^g[\kappa'|\kappa]$ ; and (b) the elements of  $E_t^g[\kappa'|\kappa]$  are the posterior means produced by the date-t posterior densities of  $F_{hh}^g$  and  $F_{ll}^g$  determined by the Bayesian passive learning process.

**SP2 Equilibrium** Given the time-varying pricing functions from the decentralized equilibrium with learning  $q_t^{DEL}(b,\varepsilon,\kappa)$ , a recursive AU equilibrium for a planner with perfect information who faces the set of feasible credit positions of the DE with learning is given by a sequence of decision rules  $[b'_t(b,\varepsilon,\kappa), c_t(b,\varepsilon,\kappa), \mu_t(b,\varepsilon,\kappa)]_{t=1}^T$  such that the decision rules for date t solve SP2's date-t AUOP conditional on  $E^a[\kappa'|\kappa]$ .

**SP3 Equilibrium** Given the time-invariant pricing functions from the decentralized equilibrium with full information  $q^{DEF}(b,\varepsilon,\kappa)$ , a recursive AU equilibrium for a planner with perfect information who faces the set of feasible credit positions of the DE with full information is given by time-invariant decision rules  $[b'(b,\varepsilon,\kappa), c(b,\varepsilon,\kappa), \mu(b,\varepsilon,\kappa)]$  such that the decision rules solve SP3's date-t AUOP conditional on  $E^a[\kappa'|\kappa]$  for all t.

#### 2.5 Pecuniary Externality and Decentralization of Planners' Allocations

The key difference between the first-order conditions of the social planners and those obtained in the private agent's DE is the pecuniary externality reflected in the right-hand-side of the Euler equation for bonds: The planners' internalize how, in states in which the collateral constraint is expected to bind next period (i.e.  $\mu_t(b', \varepsilon', \kappa') > 0$  for at least some states), the choice of debt made in the current period, b', will alter prices in the next period  $\left(\frac{\partial q_t^i(b', \varepsilon', \kappa')}{\partial b'}\right)$ . This derivative represents the response of the land price tomorrow to changes in the debt chosen today, which can be a particularly steep function when the collateral constraint binds because of the Fisherian deflation mechanism.

While the three planning problems internalize the above price derivative, they differ sharply in how they do it. Consider for example a period of optimism produced by the effect of a spell of  $\kappa^h$  realizations on the private agents' beliefs. The planner SP1 (assuming  $p_0^{ij} = n_0^{ij}$  so its beliefs are identical to those of private agents) will share in the agent's optimism both in terms of beliefs about transition probabilities of  $\kappa$  and in terms of facing the feasible set of credit positions implied by optimistic asset prices in the DE with learning. This planner will still want to use macroprudential policy to dampen credit growth because it internalizes the slope of the asset pricing function when the collateral constraint on debt is expected to bind, but this planner's expectations are as optimistic as the private agents' and hence it assigns very low probability to a financial crash (i.e. a transition from  $\kappa^h$  to  $\kappa^l$ ), and it internalizes a pricing function inflated by optimism. As we explain in the Section on Quantitative Findings, depending on priors and initial debt, these limitations can result in SP1 attaining equilibrium debt and land prices close to those of the DE, particularly if during the optimistic phase the collateral constraint binds and beliefs are such that  $E_t[F_{hh}^s]$  is close to 1.

SP2 and SP3 differ sharply because they do not share in the private agents' optimistic beliefs and thus assign higher probability to the likelihood of observing a  $\kappa^{h}$ -to- $\kappa^{l}$  transition, which will therefore strengthen their incentive to build precautionary savings and borrow less. But SP2 and SP3 do not adopt identical policies. SP2 faces the pricing functions (or credit feasibility sets) of the DE with learning, which again are influenced by the agents optimism. This planner is more cautious than SP1 because it assigns higher probability to transitions from states with optimistic prices to those with pessimistic crash prices, but the levels and slopes of the pricing functions it faces are the same as for SP1. In contrast, SP3 faces the time-invariant pricing function (or credit feasibility set) faced by private agents who are not affected by the learning friction, which "endogenously" constraints the set of feasible debt positions relative to those SP2 deals with. Hence, SP3 assigns higher probability to switches from  $\kappa^{h}$  to  $\kappa^{l}$  than private agents just like SP2, but facing pricing functions across those two states that display sharply smaller collapses than those captured in the pricing functions of the DE with learning (which are the ones SP2 uses). Again depending on whether the constraint binds and how optimistic are beliefs, at equilibrium both SP2 and SP3 acquire less debt and experience lower land price booms than both SP1 and the DE with learning during the optimistic phase, but for the same reason their use of macro prudential policy is more intensive.

Given the model's pecuniary externality, the most natural choice to analyze particular macroprudential policies are Pigouvian taxes. In particular, using taxes on debt  $(\tau_{b,t}^i)$  and land dividends  $(\tau_{l,t}^i)$  we can fully implement each of the constrained planner problems' allocations (for i = SP1, SP2, SP3). With these taxes, the budget constraint of private agents becomes:

$$q_t k_{t+1} + c_t + \frac{b_{t+1}}{R_t (1 + \tau_{b,t}^i)} = q_t k_t + b_t + \varepsilon_t Y(k_t) (1 - \tau_{l,t}^i) + T_t^i.$$
(18)

 $T_t^i$  represents lump-sum transfers by which the government rebates all its tax revenue (or a lumpsum tax in case the tax rates are negative, which is not ruled out).

The Euler equations of the competitive equilibrium with the macro-prudential policy in place are:

$$u'(t) = \beta R(1 + \tau_{b,t}^i) E_t^s \left[ u'(t+1) \right] + \mu_t$$
(19)

$$q_t(u'(t) - \mu_t \kappa) = \beta E_t^s \left[ u'(t+1) \left( \varepsilon_{t+1} Y_k(k_{t+1}) (1 - \tau_{l,t}^i) + q_{t+1} \right) \right].$$
(20)

We compute the state-contingent, time-varying schedules of these taxes by replacing each planner's allocations in these optimality conditions and then solving for the corresponding tax rates, so that the DE with learning and macro-prudential policy supports the same allocations of each planner's problem (along with the corresponding pricing functions used in each planner's collateral constraint). The tax schedules in recursive form are denoted  $\tau_{b,t}^i(b,\varepsilon,\kappa)$  and  $\tau_{l,t}^i(b,\varepsilon,\kappa)$ . There will be one of these schedules for each date-t AUOP solved by private agents in the DE with learning.

In addition to solving for the tax schedules, the debt tax can be decomposed into three terms that are useful for interpreting how macro-prudential policy responds to the effects of imperfect information, the pecuniary externality and the interaction of these two. In particular, combining Equations (19) and (15) and rearranging terms, the debt tax for each planner can be expressed as follows:

$$\tau_{b,t}^{i} = \underbrace{\frac{E_{t}^{i}[u'(t+1)]}{E_{t}^{s}[u'(t+1)]} - 1}_{\text{information}} + \underbrace{\frac{E_{t}^{i}\left[\kappa'\mu(b',\varepsilon',\kappa')\frac{\partial q_{t}^{i}(.)}{\partial b'}\right] - E_{t}^{s}\left[\kappa'\mu(b',\varepsilon',\kappa')\frac{\partial q_{t}^{i}(.)}{\partial b'}\right]}_{\text{interaction}} + \underbrace{\frac{E_{t}^{s}\left[\kappa'\mu(b',\varepsilon',\kappa')\frac{\partial q_{t}^{i}(.)}{\partial b'}\right]}{E_{t}^{s}[u'(t+1)]}}_{\text{externality}}$$
(21)

where  $q_t^i(.) = q_t^i(b', \varepsilon', \kappa')$  and  $E^i$  are calculated using the information set of planner *i*. The first term in the right-hand-side of this expression is labeled "information" because it reflects the contribution to the debt tax that arises from differences in the information sets of the private agents and planner *i* that lead to different one-period-ahead expected marginal utilities. If the two information sets are identical, as they are for the case of SP1 in our assumed baseline, this term vanishes, but for SP2 and SP3 it does not vanish. The second term, labeled "interaction", reflects differences in the expected value of the externality when evaluated using the beliefs of each planner v. the private agents' beliefs. This term is zero when either the information sets are the same or the DE with learning is far from the region where the constraint binds, and hence the externality term is zero for all possible states in t + 1. Thus, the label "interaction" is inteded to capture that fact that both the informational difference and the externality need to be present for this term to be nonzero. Finally, the third labeled "externality" is simply the value of the externality evaluated using the beliefs of private agents.

## 3 Quantitative Analysis

This Section explores the quantitative implications of the model. We discuss first the baseline calibration, and then compare the equilibria produced by the decentralized economy with those of the three planning problems. We also quantify the macro-prudential tax schedules that decentralize the planners' allocations and decompose them into their three components.

#### 3.1 Baseline Calibration

The model is calibrated to U.S. quarterly data at annualized rates and assuming a learning period of length T in which  $\kappa = \kappa^h$  from t = 1, ..., J (the optimistic phase) and  $\kappa = \kappa^l$  from J + 1 to T (the pessimistic phase). The calibration is the same as in Boz and Mendoza (2010), so we keep the description here short. The parameter values are listed in Table 1.

$\beta$	Discount factor (annualized)	0.91
$\sigma$	Risk aversion coefficient	2.0
c	Consumption-GDP ratio	0.670
A	Lump-sum absorption	0.321
r	Interest rate (annualized)	2.660
ho	Persistence of endowment shocks	0.869
$\sigma_e$	Standard deviation of TFP shocks	0.008
$\alpha$	Factor share of land in production	0.025
$\kappa^h$	Value of $\kappa$ in the high securitization regime	0.926
$\kappa^l$	Value of $\kappa$ in the low securitization regime	0.642
$F^a_{hh}$	True persistence of $\kappa^h$	0.964
$F^a_{ll}$	True persistence of $\kappa^l$	0.964
$n_0^{hh}, n_0^{hl}$	Priors	0.0205

As in Boz and Mendoza (2010), we set the start of the learning dynamics and the dates T and J by following observations from a timeline of the financial innovation process and events leading to the U.S. financial crisis. Financial innovation is defined as a structural change from a regime with only a time-invariant collateral coefficient  $\kappa^l$  to one where the can be switches between  $\kappa^h$  and  $\kappa^l$ . We set the date of this structural change in 1997Q1 to be consistent with two important facts. First, 1997 was the year of the first publicly-available securitization of mortgages under the New Community Reinvestment Act and the first issuance of corporate CDS's by JPMorgan. Second, this was also the year in which the U.S. households' net credit assets-GDP ratio started on a declining trend that lasted until the end of 2008, while prior to 1997 this ratio was quite stable at about -30 percent. We date the start of the financial crisis at 2007Q1, consistent with the initial nation-wide decline in home prices and the early signs of difficulties in the subprime mortgage market. The experiment ends two years later. These assumptions imply setting T = 48 and J = 40 (i.e. 40 consecutive quarters of  $\kappa^h$  realizations followed by 8 consecutive quarters of  $\kappa^l$ ).

The model's parameters are calibrated as follows: First, the values of  $(\sigma, R, \rho, \sigma_e, \kappa^l, \kappa^h, F_{hh}^a, F_{ll}^a)$  are calculated directly from the data or set to standard values from the quantitative DSGE literature. Second, the values of  $(\alpha, \beta)$  are calibrated such that the model's pre-financial innovation stochastic stationary state is consistent with various averages from U.S. data from the pre-financialinnovation period (i.e. pre-1997), assuming that in that period the financial constraint with  $\kappa^l$  was binding on average. Finally, the initial priors are calibrated assuming that they are symmetric, with the common value of  $n_0$  for all transitions targeted to match an estimate of observed excess land returns, as described later in this Section.

We set the real interest rate to the average ex-post real interest rate on U.S. three-month T-bills during the period 1980Q1-1996Q4, which is 2.66 percent annually. The utility function is of the constant-relative-risk aversion (CRRA) type  $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$ . The value of  $\sigma$  is set to  $\sigma = 2.0$ , the standard value in DSGE models of the U.S. economy.

To pin down  $\kappa^l$  and  $\kappa^h$ , we use the data on net credit market assets of U.S. households and non-profit organizations from the *Flow of Funds* as a proxy for *b* in the model. The proxy for *ql* is obtained from the estimates of the value of residential land provided by Davis and Heathcote (2007). On average over the 1980Q1-1996Q4 period, the ratios of the value of residential land and net credit market assets relative to GDP were stable around 0.477 and -0.313, respectively. Next, we construct a macro estimate of the household leverage ratio, or the loan-to-value ratio, by dividing net credit market assets by the value of residential land. We set the value of  $\kappa^l$  by combining the 1980Q1-1996Q4 average of this ratio with the calibrated value of *R* which yields  $\kappa^l = 0.659/1.0266 = 0.642$ . Following a similar idea, we set  $\kappa^h$  to the 2006Q4 value of the estimated leverage ratio, hence  $\kappa^h = 0.926$ .

We calibrate  $F_{hh}^a$  based on Mendoza and Terrones (2008)'s finding that the mean duration of credit booms in industrial economies is 7 years. To match this mean duration, we set  $F_{hh}^a = 0.964$ . We assume a symmetric process by setting  $F_{ll}^a = 0.964$ . Notice that the true transition probability matrix is not needed to solve the model with learning, but  $F_{hh}^a$  and  $F_{ll}^a$  necessary for solving the planner problems with full information and the pricing function of the DE with full information.

We assume a standard Cobb-Douglas production function:  $Y(k_t) = k_t^{\alpha}$ . Using the 1980Q1-1996Q4 average of the value of residential land to GDP, the value of R, and the condition that arbitrages the returns on land and bonds, which follows from the optimality conditions (4)-(5), the implied value for  $\alpha$  is  $\alpha = 0.0251^{-6}$ .

The stochastic process for  $\varepsilon$  is set to approximate an AR(1)  $(\ln(\varepsilon_t) = \rho \ln(\varepsilon_{t-1}) + e_t)$  fitted to HP-filtered real U.S. GDP per capita using data for the period 1965Q1-1996Q4. We estimate  $\rho = 0.869$  and  $\sigma_e = 0.00833$ , which imply a standard deviation of TFP of  $\sigma_{\varepsilon} = 1.68$  percent.

<sup>&</sup>lt;sup>6</sup>Since the model with a single financial regime set at  $\kappa^l$  (i.e., the pre-financial-innovation regime) yields a collateral constraint that is almost always binding and a negligible excess return on land, we use the approximation  $E[R^q] \approx R$ , and then conditions (4) and (5) imply:  $\alpha = (ql/l^{\alpha})[R - 1 + \beta^{-1}(1 - \beta R)(1 - \kappa^l)]$ 

The value of  $\beta$  is set so that in the pre-financial-innovation stochastic steady state the model matches the observed standard deviation of consumption relative to output over the 1980Q1-1996Q4 period, which is 0.8. This yields  $\beta = 0.91$ .

We introduce an exogenous, time-invariant amount of autonomous spending in order to make the model's average consumption-output ratio and average resource constraint consistent with the data. As noted earlier, the *Flow of Funds* data show that the observed average ratio of net credit assets to GDP in the 1980Q1-1996Q4 period was very stable at  $\bar{b} = -0.313$ . In the case of the consumption-GDP ratio, the data show a slight trend, so we use the last observation of the prefinancial-innovation regime (1996Q4), which implies  $\bar{c} = 0.670$ .<sup>7</sup> To make these ratios consistent with the model's resource constraint in the average of the stochastic stationary state for that same financial regime, we introduce autonomous spending by the a share A of GDP, so that the long-run average of the resource constraint is given by  $1 = \bar{c} + A - \bar{b}(R-1)/R$ . Given the values for  $\bar{b}$ ,  $\bar{c}$ and R, A is calculated as a residual  $A = 1 - \bar{c} + \bar{b}(R-1)/R = 0.321$ .<sup>8</sup> This adjustment represents the averages of investment and government expenditures, which are not explicitly modeled.

The only remaining parameters are the counters of the beta-binomial distribution that determine the initial priors. As noted earlier, the assumption of symmetric priors implies  $n_0 = n_0^{hl} = n_0^{hh} =$  $n_0^{ll} = n_0^{lh}$ , so that there is only one parameter to calibrate. We set  $n_0$  so that the implied expected excess return one period ahead at t = 0 from the DE matches the annualized 1997Q2 spread on the Fannie Mae residential MBS with 30-year maturity over the T-bill rate. This excess return was equal to 47.6 basis points and the model matches it with  $n_0 = 0.0205$ .

#### 3.2 Quantitative Findings

The main quantitative experiment compares the time-series dynamics triggered by financial innovation over the learning period (t = 0, ..., 48) in the DE with those of the three planning problems. These dynamics are computed by solving the sequence of AUOPs of each date that define each equilibrium, and constructing forecast functions that chain together the decision rules of each datet AUOP as described in Boz and Mendoza (2010).<sup>9</sup> The forecast functions keep TFP unchanged at its mean value ( $\varepsilon = 1$ ) and start from the initial condition  $b_0 = -0.345$ , which corresponds to

<sup>&</sup>lt;sup>7</sup>Consumption and GDP data were obtained from the *International Financial Statistics* of the IMF.

<sup>&</sup>lt;sup>8</sup>Note that, since land is in fixed unit supply and the unconditional mean of  $\varepsilon$  equal to 1, the mean of output in the model is also 1.

<sup>&</sup>lt;sup>9</sup>Recall that, as explained in Section 2, the decision rules of DE and SP1 change every period as their beliefs evolve, and hence the dynamics shown for these scenarios result from chaining together the corresponding period's bond decision rules and equilibrium prices. The decision rules of SP2 also change every period because this planner internalizes the time-varying DE asset pricing function driven by the private agent's learning.

the net credit market assets-GDP ratio of U.S. households observed in the data in 1996Q4. Figure 1 plots the forecast functions for bonds and land prices (Panels (a) and (b)), for the shadow value of collateral  $\mu_t$  (Panel (c)) and the evolution of beliefs (Panel (d)).

Consider first the evolution of beliefs. As shown in Panel (d),  $E_t[F_{hh}^s]$  rises gradually from 0.980 to 0.999 from t = 1 to t = 40, as agents observe the long spell of  $\kappa^h$ s. Since there are no observations of  $\kappa^l$ , the beliefs about  $\kappa^l$  do not change during this time (recall Equation (6). At date 41, when the economy switches to  $\kappa^l$  for the first time,  $E_{41}[F_{hh}^s]$  falls to 0.975, and more importantly  $E_{41}[F_{ll}^s]$  rises sharply from 0.5 to 0.98. Hence, beliefs turn pessimistic very quickly after the first realization of  $\kappa^l$ . Panel (d) also shows the time-invariant true values of  $F_{hh}^a$  and  $F_{ll}^a$ , which are the same because we assumed a symmetric process for  $\kappa$ . The excesses of  $E_t[F_{hh}^s]$  over  $F_{hh}^a$  and  $F_{ll}^a$  over  $E_t[F_{ll}^s]$ , for t = 1, ..., 40, measure the degree of optimism built during the optimistic phase.

The increase in  $E_t[F_{hh}^s]$  from date 1 to 40 may appear small (from 0.98 to 0.999) and the difference relative to  $F_{hh}^a$  (which is set at 0.964) may also seem small. However, even these small differences have important implications for the perception of riskiness of the financial environment, particularly for the expected mean duration of the  $\kappa^h$  regime and the "perceived" variability of the  $\kappa$  process. The expected mean duration of  $\kappa^h$  rises from 50 quarters with  $E_1[F_{hh}^s] = 0.98$  to 1,000 quarters with  $E_{40}[F_{hh}^s] = 0.999$  at the peak of the optimistic phase, and the coefficient of variation of  $\kappa$  based on date-40 beliefs is about 1/4 of that based on date-1 beliefs. Thus, agents' expectations of the riskiness of the new financial environment drop dramatically as the optimistic phase progresses. This is also true relative to  $F_{hh}^a = 0.964$ , which implies that in the true regimeswitching Markov process the  $\kappa^h$  regime has a significantly shorter mean duration of 28 periods. This is about half of what the agents that are learning perceive already at t = 1 of the optimistic phase, and a negligible fraction of the mean duration they expect by t = 40.

The difference between  $E_t[F_{ll}^s]$  as learning progresses and  $F_{ll}^a$  has a similar implication. During the optimistic phase, in which  $E_t[F_{ll}^s]$  remains constant at 1/2, and since  $F_{ll}^a = 0.964$ , agents' beliefs imply a projected mean duration for the  $\kappa^l$  regime of only 2 periods, whereas the true mean duration is 28 periods.

These sharp differences in projected mean durations of both  $\kappa$  regimes play a key role in driving the much stronger incentives for precautionary savings of SP2 and SP3. These two planners anticipate that  $\kappa^h$  ( $\kappa^l$ ) will arrive less (more) often and that sequences of  $\kappa^h$  ( $\kappa^l$ ) are likely to be of much shorter (longer) duration than what DE and SP1 believe. Thus, DE and SP1 perceive much less riskiness in the new financial environment than SP2 and SP3. In line with the above description of the evolution of beliefs, Panel (a) of Figure 1 shows that in DE there is a large and sustained increase in debt for the first 40 periods and a very sharp correction at date 41. This increase in debt accounts for about 2/3rds of that observed in the U.S. household data. Panel (b) shows that the surge in debt in the DE is accompanied by a sharp increase in the price of the risky asset, which is about 44 percent of what was observed in U.S. residential land prices. These two results are reassuring, because they show that the model's baseline DE is a reasonable laboratory in which to conduct our macro-prudential policy experiments. That is, in the baseline DE with learning the interaction of the financial friction and financial innovation produce sizable, sustained booms in debt and land prices. Moreover, as Boz and Mendoza (2010) showed, these booms are twice as large as what the model would predict by either removing the debt-deflation amplification mechanism or the informational friction.

Panel (a) of Figure 1 also shows that all three social planners choose lower debt positions than the DE during the optimistic phase, but the size of the adjustment differs across the three planners. SP1 chooses only slightly smaller debt (higher bonds) than DE, while both SP2 and SP3 choose similar debt levels that are much smaller than those of SP1 and DE. To understand these differences, recall that there are two key factors driving the three planners' actions: differences in beliefs, which are used to project the future evolution of the  $\kappa^h$  and  $\kappa^l$  regimes, and differences in the pricing functions that support feasible credit positions, which are used to evaluate the derivatives that reflect the planners' efforts to internalize how projected prices respond to debt choices when the collateral constraint binds.

In this experiment, SP1 borrows less than DE bond holdings in the early periods but then bond holdings and asset prices become identical at about t = 6. The reason why bond holdings become identical is that both SP1 and DE act with the same optimistic perceived riskiness of the new financial environment. In particular, households willingness to borrow induce them to face a high shadow value from relaxing the collateral constraint. Since the planner also considers the high value from current consumption attributed by households, it also decides to borrow up to the limit. Notice that although borrowing decisions coincide, the fact that the collateral constraint might become binding next periods still generates an externality, but this is not strong enough to offset the high value from borrowing which pushes both private agents and the planner to borrow up to the limit.

Moreover, as panel (e) of Figure 1 shows, this externality becomes weaker over time. This is illustrated in Figure 2, which plots the bond decision rules and pricing functions for t = 40 in the

two  $\kappa$  regimes (for  $\varepsilon = 1$ ). As Panel (b) of the Figure shows, the pricing function  $q_t^{DEL}(b, 1, \kappa^h)$  is relatively flat, which means that land prices do not differ much for different choices of b'. The pricing function  $q_t^{DEL}(b, 1, \kappa^l)$  is very steep (see Panel (d)), but this plays a decreasing role over time because optimistic beliefs imply that, conditional on having observed  $\kappa_t = \kappa^h$  at each date of the optimistic pase, SP1's perceived probability of a switch to the low credit regime is very low (i.e.  $E_t[F_{hh}^s]$  is close to zero). Hence, as time goes by, SP1 evaluates the externality assigning a large weight to the one-period-ahead state with the small derivative  $\frac{\partial q_t^{DEL}(b', 1, \kappa^h)}{\partial b'}$  and nearly zero weight to the state with the large derivative  $\frac{\partial q_t^{DEL}(b', 1, \kappa^h)}{\partial b'}$ . Notice that there is another important effect that goes in the opposite direction. At higher levels of debt along the transition path, the slope of the pricing function becomes higher, which implies a higher tax on debt to correct the externality. While this would make optimism generate a larger case for macro-prudential regulation, this is dominated by the previous effects.

We would like to point out that the small differences between SP1 and DE in this simulation is not a general result. In particular, as we show in the sensitivity analysis, it hinges on two features of the baseline calibration: A pre-financial-innovation regime setting initial conditions in which the collateral constraint was binding at  $\kappa^l$  and initial priors that set  $E_1[F_{hh}^s]$  close to 1 and  $E_1[F_{ll}^s]$ relatively far from 1.

Using the true regime-switching transition probabilities across the  $\kappa$  regimes, SP2 and SP3 perceive higher risk in the new financial environment (both in terms of the likelihood of switching to  $\kappa^l$  one period ahead of each date t = 1, ..., 40 and in terms of the long-run perceived mean duration of the  $\kappa^h$  regime and the volatility of the  $\kappa$  process). Thus, they have significantly stronger precautionary savings motives, and choose much lower debt levels than SP1 and DE during the optimistic phase. The asset prices each equilibrium supports are, however, quite different. SP2 supports prices very similar to DE and SP1 while SP3 yields significantly lower prices.

The prices of SP1 and SP2 are similar despite SP2 choosing significantly less debt because both of SP1 and SP2 face the same set of feasible credit positions, which means the same asset pricing functions  $q_t^{DEL}(b, 1, \kappa)$ , and because these pricing functions feature a weak externality in the  $\kappa^h$ state. In particular, since  $\frac{\partial q_t^{DEL}(b', 1, \kappa^h)}{\partial b'}$  is small for SP1 and SP2 for t = 1, ..., 40, their different choice of bond positions translates into small differences in land prices. If the pricing functions were steeper at the optimal debt choices, the equilibrium dynamics of land prices would be very different even though both SP1 and SP2 use the same pricing functions, because the lower debt levels chosen by SP2 would imply different date-t prices picked from the same pricing function (i.e. the same  $q_t^{DEL}(b', 1, \kappa^h)$  would return different prices for each planner because of the different choices of b'). Hence, the weak externality in the  $\kappa^h$  state plays a key role in producing the result that DE and SP1 support nearly identical debt and land prices and in the result that SP1 and SP2 support similar land prices.

The equilibrium prices for SP3 are lower than the rest, and particularly SP2 (who uses the same beliefs based on the true transition probabilities) because SP3 carries lower debt levels and faces the full-information pricing functions,  $q_t^{DEF}(b', 1, \kappa^h)$ . These pricing functions are also relatively flat, but in addition they return uniformly lower prices than the optimism-driven prices of the DE with learning (see Panel (b) of Figure 2). Hence, SP3 chooses lower debt levels because of precautionary reasons, and these credit positions support lower land prices because this planner can attain credit positions that undo the effect of optimistic expectations on prices.

The dynamics of consumption are easy to infer from the debt and price dynamics. During the early periods of the optimistic phase, consumption in DE and SP1 exceeds that of SP2 and SP3, in line with the faster debt buildup in those equilibria. Consumption for SP3 lies slightly above the other equilibria after about period 25 due to the lower level of debt this economy is converging to before the switch of financial regime, which thus entails low debt service payments.

Consider now the outcomes predicted by the DE and the three planners when the first switch to  $\kappa^l$  arrives at t = 41, which we define as a "crisis episode." To illustrate more clearly the dynamics around this crisis episode, Figure 4 shows event windows for seven quarters before and after the crisis. As shown in panel (a) of this figure, SP3, who chose the lowest levels of debt in the optimistic phase, experiences the smallest correction in debt. This is consistent with the macro-prudential behavior that led SP3 to take precautionary action and choose lower debt levels, because SP3 can correct the optimism of private beliefs and their effect on the set of feasible credit positions (i.e. it can support collateral values consistent with those of the full information DE). With both sources of overborrowing shut down, the smaller correction in debt at t = 41 is in response to the exogenous tightening of the constraint due to a lower realization of  $\kappa$ . This exogenous debt correction cannot be avoided because even with full information about the transition probability matrix,  $\kappa$  remains as a stochastic process.

The realization of  $\kappa^l$  in period 41 leads to a change in the beliefs of SP1 about the persistence of the  $\kappa^h$  regime making the debt correction for SP1 more pronounced. Since this social planner could not undo the informational source of overborrowing, this planner cannot avoid arriving at date 40 with debt levels that leave the economy vulnerable to large adjustments in case of a transition to  $\kappa^l$ . As a result, the change in debt that takes place in date 41 is more than twice as large as SP2 or SP3.

The ranking of the price decline in SP2, SP1 and DE (with SP2 smaller and DE and SP1 the largest) follows the ranking of the debt correction. Consistent with the sharp change in beliefs at date 41, having built a larger debt than the other SPs and facing the same set of feasible credit positions as DE, SP1 cannot avoid falling on the relatively steep portion of the pricing function, as plotted in panel (d) of Figure 3. In the region where SP1 choosed debt levels, prices vary significantly across debt positions, leading to a large in decline in the asset price for SP1, as shown in panel (b) of Figure 4. Notice also how the differences across the different equilibria shrink for all the macroeconomic series plotted in this figure towards the end of the time series experiment as the beliefs get closer to rational expectations.

Figure 5 shows the taxes on debt,  $\tau_b$ , and dividends,  $\tau_l$ , necessary to support each planners' allocations as competitive equilibria as characterized in Equations 19 and 20. The taxes on debt required to achieve constrained efficient allocations are in the range of 1-2 percent for SP1 during the optimistic phase. Consistent with the findings reported in Figure 1 where SP1 bond holdings were close to DE, SP1 taxes on debt are smaller than those for SP2 and SP3 which hover around 8 percent.

For SP2 and SP3, there is a significant increase in the taxes as optimism builds and Figure 6 sheds light to these dynamics. Consistent with Equation 21, we decompose the taxes on debt into the parts that arise from information, externality and the interaction of the two. Since SP1 goes through the same learning process as the private agents, the information as well as the interaction parts of the taxes for this planner are zero. Both information and interaction parts increase in periods 1 through 40 as agents turn more optimistic and SP2 and SP3 need to levy higher taxes to correct for the distortions induced by distorted beliefs.

Interestingly, the interaction part which arises from different expectations of the one period ahead externality between SPs and private agents, is larger than the other two parts. This is true for both SP2 and SP3 for which the information and interaction parts are non-zero revealing that the smaller borrowing observed in panel (a) of Figure 1 for these two planners compared to SP1 is largely related to these different valuations of the expected externality. Remember that the information part captures the correction by the social planners of the distorted evaluations of the one period ahead consumption. The interaction part being larger than the information part may be due to the externality term not being bounded from above while consumption can not fall below zero and cannot exceed the sum of output and the maximum that can be borrowed in this class of models. In other words, the externality term and the shadow value can have a much larger variance than consumption and a difference in the probabilities assigned to the future states can make a larger difference in the expected shadow value than does in expected consumption.

The externality part of taxes is the largest for SP1. Despite SP1 being fairly close to DE in panel (a) of Figure 1, the taxes required to correct for the externality is larger than the other two planners. As explained above, the other two planners borrow less with the goal of correcting for both distortions and their required correction for only the externality part is smaller than information and interaction parts, and it is also smaller than the externality correction of SP1.

#### 3.3 Sensitivity Analysis

The result that equilibrium debt and prices of SP1 and DE are similar, and hence that macroprudential policy makes little difference during the optimistic phase in this comparison, should not be taken as a general result. In particular, it depends heavily on assumptions about the parameterization of the informational frictions. To illustrate the point, in this subsection we show how the comparisons across DE and the planners change under a different parameterization, and in particular how the equilibrium debt and prices of DE and SP1 can differ.

Consider a parameterization under which we relax the assumption of symmetric initial priors, and instead assume that initial beliefs are set at the true transition probabilities. In particular, we assume  $n_0^{hl} = 0.4$ ,  $n_0^{hh} = 12.1$ ,  $n_0^{ll} = 0.18$ ,  $n_0^{hl} = 0.02$ , which gives to less optimistic beliefs than the baseline. We also assume a slightly asymmetric process for  $\kappa$ . As can be seen from 7, in this scenario there is a sharp differences between DE and SP1. In particular, SP1 accumulates significantly less debt during the transition phase which leads to a smaller crash at t = 41.

The key reason for this difference between the baseline and this scenario is that in the baseline, the level of optimism and impatience of households lead them to borrow up to the limit and have a high shadow value from relaxing the collateral constraint. Since the planner also considers the high value from current consumption attributed by households, it also decides to borrow up to the limit. In fact, as 7 shows the differences in bond positions between DE and SP1 is reduced, as the shadow value of relaxing the collateral constraint increases over time. This occurs at approximately t=30.

In general, we find that for moderate degrees of optimism, the planner with full information reduces significantly the build-up of risk. The behavior for the planner that shares the beliefs of private agents depend on how binding is the collateral constraint on private agents in the transition phase. As long as the collateral constraint for private agents remain slack or only marginally binding, there remains a strong case for macro-prudential regulation, even if the planner shares the optimism of private agents.

## 4 Conclusion

This paper provides a quantitative dynamic stochastic general equilibrium framework for studying macro-prudential policy that incorporates two key elements of the financial amplification mechanism: Imperfect information about the true riskiness of new financial regimes and a credit constraint that limits the debt of agents to a fraction of the market value of their assets. The fraction of the value of assets that can be pledged as collateral increases with financial innovation, but risk also increases because this collateral coefficient also becomes stochastic, and the persistence of regimes with high and low ability to borrow needs to be learned over time. As learning progresses, agents go through waves of optimism and pessimism which distort their debt decisions and hence equilibrium asset prices. In addition, the credit constraint introduces a pecuniary externality whereby individual agents do not internalize the effect of their borrowing decisions on equilibrium prices. The interaction of waves of optimistic and pessimistic beliefs with this pecuniary externality produces a powerful amplification mechanism that can yield large increases in debt and asset prices in a decentralized competitive equilibrium.

We study the effects of macroprudential policies in the form of Pigouvian taxes on debt and dividends in this environment, considering three different social planners that face different information sets and feasible credit positions. The first planner faces a similar learning problem as private agents and faces the set of feasible credit positions of the DE with learning. The second planner has full information about transition probabilities across financial regimes but faces the same set of feasible credit positions as the first planner. The third planner has full information and in addition it faces the set of feasible credit positions of the full-information DE. The different sets of credit positions that these planners face reflect different assumptions about the pricing functions the planners can support for the valuation of collateral. The first and second planners are constrained to value collateral using the same asset pricing function of the DE with learning, while the third planner uses the pricing function of the full-information DE.

In the baseline calibration to U.S. data, the third social planner supports debt positions and land prices that are much lower than those in the DE with learning, and hence faces smaller corrections in debt, consumption and land prices when financial crises hit. The second planner supports debt allocations only slightly larger than the third, but land prices are as inflated as in the DE with learning. Finally, the first planner supports allocations and prices that deviate only slightly from those of the DE with learning. Thus, in the baseline parameterization, macro-prudential policy is significantly more effective when the planner has full information and can support collateral values free from the effect of optimistic beliefs, and has negligible effects when the planner is subject to the same subjective beliefs and collateral pricing conditions of the DE with learning. Sensitivity analysis shows, however, that by varying the structure of the learning setup, particularly the initial priors, and the tigthness of the borrowing constraint during the optimistic phase of a credit boom, it is possible even for the first planner to use macro-prudential policy to attain different equilibrium debt and prices than the DE with learning.

These results highlight the importance of considering the information set of policymakers in the design of macro-prudential policies. If regulators operate with the same incomplete information set as the private agents, the effects of these policies are more limited and can even be negligible. This is particularly important in a boom-bust cycle in credit largely driven by financial innovation, about which the regulators are likely to be just as uninformed as the private agents. If on the other hand, in a credit boom episode where the private agents operate under incomplete or misleading information while the regulators can acquire better information, say by looking at similar previous episodes in the history of the country or other countries in similar situations, then the macro-prudential policy has good potential to contain the amplitude of the boom-bust cycle.

An interesting extension of our framework would be to model agents with heterogenous beliefs as in Cao (2011) who shows that heterogeneous beliefs under incomplete markets can have important implications for asset prices and investment variability. Financial innovation in such a framework may generate asset price and debt dynamics that are closer to data and potentially shed further light on the effectiveness of macro-prudential policy.

## 5 Appendix: Recursive Optimization Problems

We assume that agents make decisions according to the anticipated utility approach. Accordingly, the recursive optimization problem can be written as

$$V_{t}(b,k,B,\varepsilon) = \max_{b',k',c} \frac{c^{1-\sigma}}{1-\sigma} + \beta E_{t}^{s} \left[ V_{t+1}(b',k',B',\varepsilon') \right]$$
(22)  
s.t.  $q(B,\varepsilon)k' + c + \frac{b'}{R} = q(B,\varepsilon)k + b + \varepsilon F(k)$   
 $B' = \Gamma(B,\varepsilon)$   
 $\frac{b'}{R} \le \kappa q(B,\varepsilon)k'$ 

Notice that the value function is indexed by t because beliefs are changing over time. In rational expectations, instead, the value function would be a time-invariant function of the individual and aggregate state variables.

#### (Recursive Competitive Equilibrium)

The (AU) recursive competitive equilibrium under is defined by a subjective conditional-expectation operator  $E_t^s$ , an asset pricing function  $q_t(B,\varepsilon)$ , a perceived law of motion for aggregate bond holdings  $\Gamma_t(B,\varepsilon)$ , and a set of decision rules  $\left\{ \hat{b}'_t(b,k,B,\varepsilon), \hat{k}'_t(b,k,B,\varepsilon), \hat{c}_t(b,k,B,\varepsilon) \right\}$  with associated value function  $V_t(b,k,B,\varepsilon)$  such that:

- **Definition** 1.  $\left\{ \hat{b}'_t(b,k,B,\varepsilon), \hat{k}'_t(b,k,B,\varepsilon), \hat{c}_t(b,k,B,\varepsilon) \right\}$  and  $V_t(b,k,B,\varepsilon)$  solve (22), taking as given  $q_t(B,\varepsilon), \Gamma_t(B,\varepsilon)$  and the evolution of .
  - 2. The perceived law of motion for aggregate bonds is consistent with the actual law of motion:  $\Gamma_t(B,\varepsilon) = \hat{b}'_t(B,\bar{K},B,\varepsilon).$
  - 3. Land prices satisfy  $q(B,\varepsilon) = E_{\varepsilon'|\varepsilon} \left\{ \frac{\beta u'(\hat{c}(\Gamma(B,\varepsilon),\bar{K},\Gamma(B,\varepsilon),\varepsilon')) \left[\varepsilon'F_k(\bar{K},\varepsilon') + q(\Gamma_t(B,\varepsilon),\varepsilon')\right]}{u'(\hat{c}(B,\bar{K},B,\varepsilon)) \kappa \max[0,u'(\hat{c}(B,\bar{K},B,\varepsilon)) \beta RE_{\varepsilon'|\varepsilon}u'(\hat{c}(\Gamma(B,\varepsilon),\bar{K},\Gamma(B,\varepsilon),\varepsilon')]} \right\}$
  - 4. Goods and asset markets clear:  $\frac{\hat{b}'(B,\bar{K},B,\varepsilon)}{R} + c(B,\bar{K},B,\varepsilon) = \varepsilon f(\bar{K}) + B_t$  and  $\hat{k}(B,\bar{K},B,\varepsilon) = \bar{K}$

## References

- AIYAGARI, R., AND M. GERTLER (1999): "Overreaction of Asset Prices in General Equilibrium," *Review of Economic Dynamics*, 2(1), 3–35.
- BENIGNO, G., H. CHEN, C. OTROK, A. REBUCCI, AND E. YOUNG (2010): "Financial Crises and Macro-Prudential Policy," Mimeo, University of Virginia.
- BERNANKE, B., M. GERTLER, AND S. GILCHRIST (1999): "The financial accelerator in a quantitative business cycle model," in *Handbook of Macroeconomics*, ed. by J. Taylor, and M. Woodford, vol. 1C. by North-Holland.
- BIANCHI, J. (2010): "Overborrowing and Systemic Externalities in the Business Cycle," American Economic Review, Forthcoming.
- BIANCHI, J., AND E. G. MENDOZA (2010): "Overborrowing, Financial Crises and 'Macroprudential' Policy," NBER Working Paper No. 16091.
- BOZ, E., AND E. G. MENDOZA (2010): "Financial Innovation, the Discovery of Risk, and the U.S. Credit Crisis," NBER Working Paper 16020.
- CAO, D. (2011): "Collateral Shortages, Asset Price and Investment Volatility with Heterogeneous Beliefs," mimeo, Georgetown University.
- COGLEY, T., AND T. SARGENT (2008a): "The market price of risk and the equity premium: A legacy of the Great Depression?," *Journal of Monetary Economics*, 55(3), 454–476.
- COGLEY, T., AND T. J. SARGENT (2008b): "Anticipated Utility and Rational Expectations as Approximations of Bayesian Decision Making," *International Economic Review*, 49, 185–221.
- DAVIS, M., AND J. HEATHCOTE (2007): "The price and quantity of residential land in the United States," *Journal of Monetary Economics*, 54(8), 2595–2620.
- FISHER, I. (1933): "The debt-deflation theory of great depressions," *Econometrica*, 1(4), 337–357.
- GENNAIOLI, N., AND A. SHLEIFER (2010): "What comes to mind," The Quarterly Journal of Economics, 125(4), 1399.
- GENNAIOLI, N., A. SHLEIFER, AND R. VISHNY (2010): "Neglected risks, financial innovation, and financial fragility," Journal of Financial Economics, forthcoming.

- JEANNE, O., AND A. KORINEK (2010): "Managing Credit Booms and Busts: A Pigouvian Taxation Approach," Discussion paper, NBER Working Paper.
- KIYOTAKI, N., AND J. MOORE (1997): "Credit Cycles," Journal of Political Economy, 105, 211–248.
- KORINEK, A. (2010): "Systemic Risk-Taking: Accelerator Effects, Externalities, and Regulatory," Mimeo, University of Maryland.
- LORENZONI, G. (2008): "Inefficient Credit Booms," Review of Economic Studies, 75, 809-833.
- MENDOZA, E. G., AND M. E. TERRONES (2008): "An Anatomy Of Credit Booms: Evidence From Macro Aggregates And Micro Data," NBER Working Paper 14049.
- MINSKY, H. (1992): "The financial instability hypothesis," The Jerome Levy Economics Institute Working Paper No. 74.
- STEIN, J. (2011): "Monetary policy as financial-stability regulation," Discussion paper, NBER Working Paper.



Figure 1: Time Series Simulations in the Baseline Calibration

Notes: DE: Imperfect information decentralized equilibrium, SP1: Social planner with imperfect information implementing the set of feasible credit positions of imperfect information DE, SP2: Social planner with full information implementing the set of feasible credit positions of imperfect information DE, SP3: Social planner with full information implementing the set of feasible credit positions of full information DE.



Figure 2: Period 40 Bond Holdings and Asset Prices

Notes: SP1: Social planner with imperfect information implementing the set of feasible credit positions of imperfect information decentralized equilibrium, SP2: Social planner with full information implementing the set of feasible credit positions of imperfect information decentralized equilibrium, SP3: Social planner with full information implementing the set of feasible credit positions of full information decentralized equilibrium.



Figure 3: Period 41 Bond Holdings and Asset Prices

Notes: SP1: Social planner with imperfect information implementing the set of feasible credit positions of imperfect information decentralized equilibrium, SP2: Social planner with full information implementing the set of feasible credit positions of imperfect information decentralized equilibrium, SP3: Social planner with full information implementing the set of feasible credit positions of full information decentralized equilibrium.



Figure 4: Time Series Simulations: Crisis Episode

Notes: This figure plots the time series dynamics in periods  $41 \pm 7$ . DE: Imperfect information decentralized equilibrium, SP1: Social planner with imperfect information implementing the set of feasible credit positions of imperfect information DE, SP2: Social planner with full information implementing the set of feasible credit positions of imperfect information DE, SP3: Social planner with full information implementing the set of feasible credit positions of full information DE.



Figure 5: Time Series Simulations: Taxes on Debt and Land Dividends
(a) Taxes on Debt

Notes: This figure plots the taxes on debt and on land dividends that support the corresponding planners allocations as competitive equilibrium. SP1: Social planner with imperfect information implementing the set of feasible credit positions of imperfect information DE, SP2: Social planner with full information implementing the set of feasible credit positions of imperfect information DE, SP3: Social planner with full information implementing the set of feasible credit positions of full information DE.



Figure 6: Time Series Simulations: Decomposition of Taxes on Debt

Notes: This figure plots the decomposition of taxes on debt to three distinct parts: 'information' arises due to the differences in the expectation of one period ahead consumption between private agents and the social planner, 'externality' captures the pecuniary externality, 'interaction' is due to the differences in the expectation of the one period ahead externality between private agents and the social planner. SP1: Social planner with imperfect information implementing the set of feasible credit positions of imperfect information DE, SP2: Social planner with full information implementing the set of feasible credit positions of imperfect information DE, SP3: Social planner with full information implementing the set of feasible credit positions of full information DE.



Figure 7: Time Series Simulations with Alternative Learning Parameters

Notes: DE: Imperfect information decentralized equilibrium, SP1: Social planner with imperfect information implementing the set of feasible credit positions of imperfect information DE, SP2: Social planner with full information implementing the set of feasible credit positions of imperfect information DE, SP3: Social planner with full information implementing the set of feasible credit positions of full information DE.