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A SOLUTION TO TWO PARADOXES OF INTERNATIONAL CAPITAL FLOWS

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Abstract

International capital flow from rich to poor countries can be regarded as either too low (the Lucas paradox in a one-sector model) or too high (when compared with the logic of factor price equalization in a two-sector model). To resolve the paradoxes, we propose a non-neo-classical theory that marries a model of financial contract between entrepreneurs and investors due to Holmstrom and Tirole (1998) and the Heckscher- Ohlin-Samuelson framework. Return to financial investment and marginal product of physical capital are naturally separate. The model generates a number of interesting predictions that seem to fit the data well. For example, between rich and poor countries, there can be massive, two-way gross capital flows but a small net flow. In fact, in the unique equilibrium in the world capital market, the relatively inefficient financial system is completely bypassed.

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1 Introduction

Lucas (1990) famously pointed out that relative to the implied difference in the marginal returns to capital between rich and poor countries in a one-sector model, it is a paradox that not more capital flows from rich to poor countries (the paradox of too little flow). The Lucas paradox could be turned on its head in a two-sector, two-factor, neoclassical trade model. A well known result in such a model is factor price equalization: with free trade in goods, returns to factors are equalized between countries even without factor mobility. Given this, any amount of observed capital flow is excessive (the paradox of too much capital flow).

A number of solutions to the Lucas paradox have been proposed in the literature: (a) thinking of a worker in a rich country as effectively equivalent to multiple workers in a poor country, (b) adding human capital as a new factor of production, (c) allowing for sovereign risk, and (d) adding costs of goods trade. We will argue in this paper that none of these explanations can escape from the tyranny of the factor price equalization.

We conclude that it is useful to think outside the neoclassical box, and propose a new micro-founded theory to understand goods trade and factor mobility. We introduce financial contract model of Holmstrom and Tirole (1998) into the Heckscher-Ohlin-Samuelson framework. A key feature of the new theory is that return to financial investment is generally not the same as return to physical investment. Financial investors (or savers) obtain only a slice of the return to physical capital, as they have to share the cake with entrepreneurs. The more developed a financial system is, the greater the slice to the investors. An important implication is that countries with low capital-labor ratio may experience large gross financial capital outflow, together with inward foreign direct investment, resulting either in a net capital outflow or a moderate capital inflow.

We assume firms' production function is constant return to scale. Entrepreneurs, however, are heterogeneous in their abilities to manage the capital. As a sector expands, more entrepreneurs enter the sector and the ability of the marginal entrepreneur is declining. Therefore, the return to external investment, which is determined by the marginal entrepreneur's ability to manage capital, is declining as the sector expands. Although free trade in goods equalizes product prices, factor returns, however, remain different across countries. Other things equal, the interest rate is lower and the wage rate is higher in the capital abundant country. In other words, our two-sector model restores these results from a typical one-sector model (but still predicts a small net capital flow between rich and poor countries).

Our model also differs from the literature in terms of welfare analysis. In most existing papers, removing barriers to capital flow improves welfare since it improves efficiency.[1] Such a view relies on the assumption that the return to investment equals the marginal product of physical capital. In our model, however, financial investors often gain at the expense of entrepreneurs. If the loss of the entrepreneurs is large enough, financial capital outflow can reduce the welfare.

This paper is related to the theoretical literature that investigates effects of financial market imperfection on capital flow. Gertler and Rogoff (1990) show that a moral hazard problem between foreign investors and domestic entrepreneurs may cause capital flow from poor to rich. Gordon and Bovenberg (1996) develop a model with asymmetric information between countries that explains possible differences in the real interest rates. Shleifer and Wolfenzon (2002) show that the country with better investor protection has a higher interest rate. Matsuyama (2005) studies the effect of credit market constraint on capital flows. Stulz (2005) develops a model of agency problems of government and entrepreneurs that limit the financial globalization. Caballero, Farhi, and Gourinchas (2005) show that lower capacity to generate financial assets reduces the interest rate. Our theory differs from these

[1] While the benefits of international capital flow are numerous in theory, all of which could raise the growth rate of developing countries, it is not straightforward to find strong and robust evidence according to Prasad, Rogoff, Wei and Kose (2003).

papers in three ways. First, all of the above papers use a one-sector model, whose prediction on capital flow does not generally survive an extension to a two-sector model. Second, our model endogenously generates two-way gross capital flows with a small net flow. Third, our model allows a formal welfare analysis that can highlight a possible welfare loss from financial capital outflow.[2]

Obstfeld and Rogoff (1997, pp 438) and Ventura (1997) have already pointed out that the sensitivity of interest rate to capital-labor ratio is a special feature of the one-sector model. They do not, however, develop a new two-sector model that breaks up the factor price equalization, and therefore, do not explain why some capital would flow internationally in a multi-sector model.

Our model features heterogeneous entrepreneurs, which is somewhat related to the models of heterogeneous firms in the international trade literature. Melitz (2003) develops a monopolistic competition model with heterogeneous firms. Bernard, Redding and Schott (2005) incorporates firm heterogeneity, product variety into HO framework and maintain the factor price equalization in their model. To the best of our knowledge, our model is the first that studies the effect of firm (entrepreneur) heterogeneity on international capital flow in a two-sector framework.

The rest of paper is organized as follows. Section 2 reviews the two paradoxes of capital flow in the neoclassical theory. Section 3 sets up our model. Sections 4 and 5 studies the aggregation and equilibrium conditions, and some key comparative statics, respectively. Section 6 analyses different forms of international capital flow under free trade in goods. Section 7 discusses the welfare impacts. Section 8 concludes. Two appendixes provide the formal proofs for the propositions in the text, and a table of the notations.

[2] Caballero, Farhi, and Gourinchas (2005) has as an extension of the model that includes a multiple sectors. Their purpose is to study the effect of exchange rate adjustment. Factor allocation across sectors and therefore possible factor price equalization across countries are not studied in their paper. While they also allow for two-way gross flows, its microfoundation, however, is not developed in the paper. Matsuyama (2004) discusses a possible welfare loss from financial globalization due to vicious circles of low-investment and low-wealth in unlucky countries.

2 Paradoxes of International Capital Flow

In this section we examine the return to capital in standard neoclassical models. The production functions are assumed to be constant return to scale and firms are perfect competitive. We will first discuss a one sector model and then a multiple-sector model.

2.1 The Lucas Paradox of Too Little Capital Flow

Using a one sector model, Lucas (1990) suggested that it was a paradox that more capital does not flow from rich to poor countries. His reasoning goes as follows. Let $y = f(L, K)$ be the production function where y is the output produced using labor L and capital K. Let p be the price of good, and w and r be returns to labor and capital, respectively. Firm's profit maximization problem gives

$$r = p\partial f(L, K)/\partial K = p\partial f(1, K/L)/\partial K \tag{1}$$

With free trade, the price of good is equalized across countries. The *Law of Diminishing Marginal Product* implies that r is higher in the country with lower per capita capital. As an illustration, Lucas calculated that the return to capital in India should be 58 times as high as that in U.S.. Facing a return differential of this magnitude, Lucas argued, we would observe a lot of capital to flow from rich to poor countries. This does not happen in the data has come to be known as the Lucas paradox. As we will see, this paradox is driven by one-sector model which ignores capital allocation across different sectors.

2.2 The Opposite Paradox of Too Much Capital Flow

The logic of the Lucas paradox can be turned on its head in a multi-sector model. Specifically, in a standard Heckscher-Ohlin-Samuelson 2 goods, 2 factors, and 2 countries model, firms earn zero profit. So we must have:

$$p_1 = c_1(w, r) \text{ and } p_2 = c_2(w, r) \tag{2}$$

where $c(.)$ is the unit cost function and subscripts represent sectors. Comparing to one-sector model, now

$$r = p_i \partial f_i(1, K_i/L_i)/\partial K = p_i \partial f_i(1, a_{iK}/a_{iL})/\partial K, \text{ for } i = 1, 2 \tag{3}$$

where $a_{iK}/a_{iL} = \frac{\partial c_i(w,r)/\partial r}{\partial c_i(w,r)/\partial w}$ is capital-labor ratio per unit of production. For given product prices, w and r, and therefore a_{iK}/a_{iL}, are determined and independent from factor endowments L and K–the well-known "factor price insensitivity" (Leamer 1995). Increases in K change the composition of outputs: more of capital intensive good and less of labor intensive good will be produced, but the marginal return to physical capital in each sector is maintained constant. Free trade equalizes product prices, and therefore equalizes the return to factors across countries, even in the absence of international factor movements. This result was first proved by Samuleson (1948 and 1949) and has come to be known as the "*Factor Price Equalization Theorem (FPE)*". Countries indirectly export their abundant factors through trade in goods. *The capital flow is completely substituted by goods trade.* There is no incentive for any amount of capital to flow across countries once there is free trade in goods.

One might think that the assumptions for the factor price equalization is surely too restrictive to be realistic and are not likely to hold once one goes beyond the $2 \times 2 \times 2$ model. Deardoff (1994) derives a necessary condition of the *FPE,* known as the "lens condition" in a n goods, m factors, and H countries model. The condition has been proved to be also sufficient in a model of n goods, 2 factors, and H countries by Xiang (2001).[3] We offer a new, but intuitive sufficient condition of *FPE* below. As we will see, such a condition is relatively weak in standard neoclassical model.

[3] For a recent discussion on the lens condition and additional literature review, readers are guided to Bernard, Robertson and Schott (2004).

We assume $m \leq n$. Countries are ranked in the way that there are m products commonly produced by neighboring countries. For countries h and $h+1$, they both produce products $n_1^{hh+1}, n_2^{hh+1}, \cdots, n_m^{hh+1}$. Neighboring countries certainly may specialize in the rest of $n-m$ products. Note that we only require neighboring countries produce a common set of m products. They may even not trade directly with each other. Non-neighboring countries, may specialize in completely different set of products. Consider countries h and $h+1$. Zero profit conditions in sectors $n_1^{hh+1}, n_2^{hh+1}, \cdots, n_m^{hh+1}$ give

$$p_i^{hh+1} = c_i^{hh+1}(w_1^h, \cdots w_m^h) \text{ for } i = 1, \cdots, m \text{ and} \tag{4}$$

$$p_i^{hh+1} = c_i^{hh+1}(w_1^{h+1}, \cdots w_m^{h+1}) \text{ for } i = 1, \cdots, m \tag{5}$$

m equations in (4) determine m factor prices $w_1^h, \cdots w_m^h$ for country h, while m equations in (5) determine m factor prices $w_1^{h+1}, \cdots w_m^{h+1}$ for country $h+1$. Because technology is assumed to be identical across all countries and product prices are equalized due to free trade, factor prices in countries h and $h+1$ must be the same, which implies that factor prices are equalized across all countries. Consider the case of two factors, labor and capital. Both U.S. and Canada produce Car and Furniture, so the return to factors are equalized in U.S. and Canada; both Canada and Greece produce Furniture and TV, so the return to factors are equalized in Canada and Greece; both Greece and India produce Steel and Cloths, so the return to factors are equalized in Greece and India. As a result, the factor prices are equalized in U.S. and India, and across all countries in the world. We summarize our result by the following *chain rule of factor price equalization.*

Proposition 1 *Let number of factors be m in a standard neoclassical model. If any country in the world can find a "neighboring" country in the sense that they both produce a common set of m products. Then factor prices are equalized across all countries in a free trade world, even in the absence of international factor* *movements.*

Lucas (1990) himself provided three explanations for the puzzle of too little capital flow. The first is *effective labor* assumption: if each American worker was estimated to be productive equivalent of about five Indians, then the predicted return to capital in India became 5 rather than 58 times than the return to capital in U.S.. This result is again driven by one-sector assumption. Let production function be $y_i = f_i(EL_i, K_i)$ where E represents labor productivity. It can be easily shown that zero profit conditions in two goods, two factors model now become

$$p_1 = c_1(\frac{w}{E}, r) \text{ and } p_2 = c_2(\frac{w}{E}, r)$$

which gives unique solution $\left[\left(\frac{w}{E}\right), r\right]$. Note that $\left(\frac{w}{E}\right)$ and r are determined by $(p_{1,}p_2)$. For given product prices, the increase in labor productivity E will increase wage rate w proportionally to maintain $\left(\frac{w}{E}\right)$ as constant. The return to capital, r, is not affected by the increase in E. That is, using two-sector model, if American worker is 5 times more productive than Indian workers, then the wage rate in U.S. is exactly 5 times higher than that in India. The return to capital, however, is not affected.

Lucas's second explanation is *missing factor* assumption: if human capital was included as another factor, then the predicted return to capital in India was further reduced from 5 to 1.04 times than the return to capital in U.S.. This argument, once again, is derived from one-sector assumption. Using our *chain rule of factor price equalization*, if there are 3 factors, labor, capital and human capital, factor prices are equalized across all countries as long as at least 3 common products are produced by "neighboring" countries. Free goods trade substitute factor flows. The abundance of human capital in U.S. changes the composition of outputs, but has no effect on the return to capital. In multiple sectors model, human capital differentials *did not,* as Lucas argued, reduce the predicted return ratios between very rich and very poor countries from about 58 to unity. It is *free trade* in goods that equalize returns to capital across all countries.

Lucas's third explanation, downplayed by himself but emphasized by Reinhart and Rogoff (2004), is the *sovereign risk* assumption: the risk of sovereign default prevents the capital flow from rich to poor. This argument, and all other arguments without simultaneously considering capital flow together with free goods trade, run into the counter *factor price equalization* paradox: free trade in goods can completely substitute factor flows, so there is no incentive for capital flow at all.

A related explanation is costs of goods trade (see, for example, Obstfeld and Rogoff, 2000). Trading costs do break the *FPE*. However, as tariffs and transportation costs decline over the last four decades, goods prices should converge across countries. By the logic of *FPE*, factor returns should converge as well. So the logic of *FPE* would predict a decline in international capital flow, which obviously is contradicted by the data.

Another popular explanation for the Lucas paradox is cross-country differential in *total factor productivity (TFP),* of which difference in institutions is a special case. If *TFPs* are different, the returns to factors are, of course, different across two countries. The *TFP* explanation, besides that it is "too general" since anything can be counted as difference in *TFP,* may not predict the direction of capital flow. Let the *TFP* in foreign country be higher in the two goods, two factors and two countries model. That is,

$$p_1 = B_1 c_1(w^*, r^*) \text{ and } p_2 = B_2 c_2(w^*, r^*)$$

and $B_i < 1$.[4] Let sector 1 be labor intensive. Using the Stolper-Samuelson theorem, higher *TFP* ($B_1 < 1$) in sector 1 increases w^* but reduces r^*, while higher *TFP* in sector 2 increases r^* but reduces w^*. Unless we know exactly magnitudes of *TFP* in all sectors, the return to capital in more technologically advanced country can

[4] A superscript " $*$ " is used to denote variables in the foreign country.

be either higher or lower. Differences in institution may have asymmetric effects on productivities for different sectors. Unless a structural model of institution is developed, as we will do in next section, reduced form *TFP* may be too general to predict directions of capital flow.

It is useful to note that we are not claiming that factor price equalization is realistic. However, we point out that it is perhaps more difficult to escape from the tyranny of *FPE* than people thought. Both Lucas and *FPE* paradoxes rely on the assumption that marginal product of physical capital determines capital flow.[5] In fact, the return to investment, rather than marginal product of physical capital, determines financial capital flow. Those two differ since investors have to share the cake with entrepreneurs. Therefore, the financial contract between investors and entrepreneurs is crucial to determine the return to investment.

3 The Model

We introduce financial contracts between investors and entrepreneurs into an otherwise standard two-good, two-factor, and two-country HOS framework. Our model relies on the setup in Holmstrom and Tirole (1998), but extends it to a two-good and two-factor general equilibrium model and allows entrepreneurs to be heterogeneous.

3.1 Basic Setup

Focusing now on a single country, we assume that the production process takes two periods and the firm has a stochastic technology. The first period production function of industry i is $y_i^1 = G_i(L_i, K_i^1)$ $(i = 1, 2)$, where the superscript 1 denotes date 1. The initial labor-capital ratio, L_i/K_i^1, is assumed to be fixed and denoted as a_i. If the project succeeds, the gross return to capital is

[5] Such a view is popular in the literature of caital flow. If the risk premium is not considered, as Ventura (2003, pp. 488) states, the rule is: 'invest your wealth in domestic capital until its marginal product equals the world interest rate.'

$$p_i y_i^1 - wL_i = [p_i G_i(a_i, 1) - wa_i]\, K_i^1 = R_i K_i^1 \tag{6}$$

where $R_i = p_i G_i(a_i, 1) - wa_i$ represents the return to one unit of initial investment.

At the beginning of date 1, a financial contract is signed between the firm and investors and an initial investment K_i^1 is injected to the firm. Correspondingly, $a_i K_i^1$ of labor are hired. The gain to the capital is $R_i K_i^1$ if the project succeeds and zero if it fails. The labor is paid at w in the second period if the project succeeds and zero if it fails. At the beginning of date 2, a liquidity shock occurs. An additional and uncertain amount $\rho_i K_i^1 > 0$ of financing is needed to cover operating expenditures and other needs. ρ_i, the liquidity shock, is distributed according to the cumulative distribution function $F_i(\rho)$ with a density function $f_i(\rho)$. Correspondingly, $a_i \rho_i K_i^1$ of labor is hired for liquidity shock. If $\rho_i K_i^1$ is paid, the project continues and the output of $G_i(a_i, 1)K_i^1$ will be produced at the end of date 2. If $\rho_i K_i^1$ is not paid, the project terminates and produces nothing. Consumption takes place at the end of the second period.

Investment is subject to a moral hazard problem in the firm. Each entrepreneur is endowed with one unit of capital and heterogeneous in the cost of effort. The utility of managing one unit of capital in sector i for entrepreneur $n \geq 1$ is defined as

$$V_{ni}(e^j) = \lambda_i(e^j) R_{ni}^E - c_{ni}(e^j) \tag{7}$$

where $j = H$ and L, denoting high or low levels of effort, respectively, or "work" or "shirk" for short. R_{ni}^E is the amount that the entrepreneur receives from the revenue generated by one unit of initial investment if the project succeeds. If the entrepreneur works, the probability of success is $\lambda_i^H = \lambda_i(e^H)$; if the entrepreneur shirks, the probability of success is $\lambda_i^L = \lambda_i(e^L)$. In subsequent discussion, we define $\Delta\lambda_i = \lambda_i^H - \lambda_i^L > 0$. The probability of success is assumed to be identical across all entrepreneurs. However, the cost of effort, $c_{ni}(e^j)$, is heterogeneous across entrepreneurs. We normalize the cost of "shirk" to zero. The entrepreneur makes a decision on the effort level after the continuation decision is made in the second period.

At date 2, the first period investment K_i^1 is sunk. The net present value of the investment is maximized by continuing the project whenever the expected return from continuation, $\lambda_i^H R_i$, exceeds the cost ρ_i, that is, $\lambda^H R_i - \rho_i \geq 0$. Let

$$\rho_i^1 = \lambda_i^H R_i \tag{8}$$

be the *first-best cutoff value of* ρ_i. Following Holmstrom and Tirole (1998), we assume that the project's net present value is positive if the entrepreneur works but negative if she shirks. That is,

$$\begin{aligned} & \left(\frac{1}{1+r}\right) \int \max[\lambda_i^H R_i - c_{ni}(e^H) - \rho_i, 0] f_i(\rho) - 1 \\ > \quad & 0 > \left(\frac{1}{1+r}\right) \int \max[\lambda_i^L R_i - \rho_i, 0] f_i(\rho) - 1 \end{aligned} \tag{9}$$

Therefore, we only need to consider those contracts that implement a high level of effort or "work".

3.2 Financial Contracts and Allocation of Capital within and across Sectors

Let each entrepreneur manage one project. Entrepreneur n invests her 1 unit of capital at the first period. The initial investment in the firm is the sum of internal capital and external capital, or $K_{ni}^1 = 1 + K_{ni}^{X1}$, where K_{ni}^{X1} is the funds that entrepreneur n raises from outside investors at date 1. Let $C_{ni} = \{K_{ni}^1, \mu_{ni}(\rho_i), R_{ni}^E(\rho_i)\}$ be the contract signed between entrepreneur n and outside investors, where $\mu_{ni}(\rho_i)$

is a state-contingent policy on project continuation (1 = *continue*, 0 = *stop*), and $R^E_{ni}(\rho_i)$ is the entrepreneur's portion of gross revenue from physical capital. Investors are left with $R_i - R^E_{ni}(\rho_i)$. If the project fails or is terminated, both sides are assumed to receive zero. An optimal contract can be found by choosing $\{K^1_{ni}, \mu_{ni}(\rho_i), R^E_{ni}(\rho_i)\}$ to solve the following entrepreneur's optimization problem (with moral hazard).

$$\max\ U_{ni} = \left(\frac{1}{1+r}\right) K^1_{ni} \int \lambda^H_i R^E_{ni}(\rho_i)\mu_{ni}(\rho_i) f_i(\rho_i) d\rho_i - 1 \tag{10}$$

subject to

$$\left(\frac{1}{1+r}\right) K^1_{ni} \int \{\lambda^H_i [R_i - R^E_{ni}(\rho_i)] - \rho_i\}\mu_{ni}(\rho_i) f_i(\rho_i) d\rho_i \geq K^{X1}_{ni} \tag{11}$$

and

$$\lambda^H_i R^E_{ni}(\rho_i) - c_{ni}(e^H) \geq \lambda^L_i R^E_{ni}(\rho_i) \tag{12}$$

Expression (10) is the present value of the firm's *net return to internal capital.* (11) is the participating constraint for outside investors, while (12) is the entrepreneur's incentive compatibility constraint. Solving the above problem, the optimal continuation policy $\mu_{ni}(\widehat{\rho}_{ni})$ takes the form of a cutoff rule so that the project continues, or $\mu_{ni}(\rho_i) = 1$ if $\rho_i \leq \widehat{\rho}_{ni}$ and the project terminates, or $\mu_{ni}(\rho_i) = 0$ if $\rho_i > \widehat{\rho}_{ni}$. Let $\rho^0_{ni} = \lambda^H_i [R_i - R^E_{ni}(\rho_i)]$, and rewrite expression (11) as $K^1_{ni} \leq \alpha_{ni}(\widehat{\rho}_{ni})$, where

$$\alpha_{ni}(\widehat{\rho}_{ni}) = \frac{1+r}{(1+r) - \int_0^{\widehat{\rho}_{ni}} \left(\rho^0_{ni} - \rho_i\right) f_i(\rho_i) d\rho_i} \tag{13}$$

represents an *equity multiplier* which is quasi-concave in $\widehat{\rho}_{ni}$ and maximized at $\widehat{\rho}_{ni} = \rho^0_{ni}$. If $\frac{1}{\alpha_{ni}} \leq 0$, the investors' expected marginal return to their investment at date 1 would be larger than the opportunity cost. The firm would be free to invest an arbitrarily large amount at date 1, making the moral hazard problem unconstrained.

Following Holmstrom and Tirole (1998), we assume

$$\int_0^{\widehat{\rho}_{ni}} \left(\rho_{ni}^0 - \rho_i\right) f_i(\rho_i) d\rho_i < 1 + r \tag{14}$$

to eliminate this case. The condition (14) is necessary and sufficient for the participating constraint (11) to be binding, which gives the firm's initial investment

$$K_{ni}^1 = \alpha_{ni}(\widehat{\rho}_{ni}) \tag{15}$$

Substituting binding constraint (11) into (10), the firm's objective function can be written as $U_{ni}(\widehat{\rho}_{ni}) = \beta_{ni}(\widehat{\rho}_{ni}) K_{ni}^1$, where

$$\beta_{ni}(\widehat{\rho}_{ni}) = \left(\frac{1}{1+r}\right) \int_0^{\widehat{\rho}_{ni}} \left(\rho_i^1 - \rho_i\right) f_i(\rho_i) d\rho_i - 1 \tag{16}$$

is the *marginal net social return* on investment. $\beta_{ni}(\widehat{\rho}_{ni})$ is quasi-concave and maximized at $\widehat{\rho}_{ni} = \rho_i^1$. Condition (9) ensures that $\beta_{ni}(\rho_i^1) > 0$. Note that $\widehat{\rho}_{ni} = \rho_i^1$ is feasible for the moral hazard problem. Thus, at the optimal contract $\widehat{\rho}_{ni} = \rho_{ni}^*$, we must have $U_{ni}(\rho_{ni}^*) = \beta_{ni}(\rho_{ni}^*) K_{ni}^1(\rho_{ni}^*) \geq \beta_{ni}(\rho_i^1) K_{ni}^1(\rho_i^1) > 0$. Given $\beta_{ni}(\rho_{ni}^*) > 0$, the firm will choose $R_{ni}^E(\rho_i)$ as small as possible to maximize $K_{ni}^1(\rho_{ni}^*)$. Thus, the incentive compatibility constraint (12) must be binding, which gives

$$R_{ni}^E = \frac{c_{ni}(e^H)}{\Delta\lambda_i} \text{ and } \rho_{ni}^0 = \lambda_i^H [R_i - \frac{c_{ni}(e^H)}{\Delta\lambda_i}] \tag{17}$$

Using (13), (15), and (16), the firm's *net return to internal capital* becomes

$$U_{ni}(\widehat{\rho}_{ni}) = \beta_{ni}(\widehat{\rho}_{ni}) \alpha_{ni}(\widehat{\rho}_{ni}) = \frac{\rho_i^1 - h(\widehat{\rho}_{ni})}{h(\widehat{\rho}_{ni}) - \rho_{ni}^0} \tag{18}$$

where

$$h_i(\widehat{\rho}_{ni}) = \frac{(1+r) + \int_0^{\widehat{\rho}_{ni}} \rho_i f_i(\rho_i) d\rho_i}{F_i(\widehat{\rho}_{ni})} \tag{19}$$

Because $\beta_{ni}(\widehat{\rho}_{ki})$ and $\alpha_{ni}(\widehat{\rho}_{ni})$ are both concave, we must have $\rho_{ni}^0 < \rho_{ni}^* < \rho_i^1$.

$h(\widehat{\rho}_{ni})$, in the terminology of Holmstrom and Tirole (1998), is called *expected unit cost of total investment*, which is the opportunity cost of initial investment at date 1, $(1+r)$, plus the expected financing for the liquidity shock at date 2, $\int_0^{\widehat{\rho}_{ni}} \rho_i f_i(\rho_i) d\rho_i$, under the condition that the project continues. Maximizing $U_{ni}(\widehat{\rho}_{ni})$ is equivalent to minimizing $h(\widehat{\rho}_{ni})$. The first order condition then gives

$$\int_0^{\rho_{ni}^*} F_i(\rho_i) d\rho_i = 1 + r \tag{20}$$

which implies that $h_i(\rho_{ni}^*) = \rho_{ni}^*$. Note that the equation (20) implies that ρ_{ni}^* is independent from n. Thus all entrepreneurs have the same optimal cutoff of the liquidity shock, $\rho_{ni}^* = \rho_i^*$. Equation (20) shows that optimal cutoff of the liquidity shock, ρ_i^*, increases as r increases. As the interest rate r becomes higher, the opportunity cost of the investment is higher. To attract investors into the project, the firm needs to promise higher probability that project continues under the liquidity shock, which implies higher optimal cutoff rate ρ_i^*.

If the financial system is underdeveloped, however, only can liquidity shocks $\widehat{\rho}_{ni} \leq \theta \rho_i^*$ be met by the financial system, where θ represents *the level of financial development* of the country. Higher θ represents a more developed financial system. θ can be interpreted as either each firm is financed up to the liquidity shock $\widehat{\rho}_{ni} = \theta \rho_i^*$, or θ portion of firms are financed up to ρ_i^* and $1-\theta$ portion of firms are not financed for any shock. The most that the firm (the entrepreneur) can promise to outside investors at data 2 is ρ_{ni}^0. If $\rho_i > \rho_{ni}^0$, even if the firm were allowed to issue senior securities at date 2, it could not raise enough cash. To cover liquidity shocks up to the second-best cutoff $\rho_i^* > \rho_{ni}^0$, the country must have a sufficiently developed financial system for outside investors to commit funds at date 1. That is, $\min\{\rho_{ni}^0/\rho_i^*$ for all n and $i\} \leq \theta \leq 1$.[6]

[6] Ju and Wei (2005) use the country's capacity of external capital to represent the level of financial development. Financial system consists of both financial market and public supply of liquidity. In an economy where both individual and aggregate uncertainties exist, Holmstrom and Tirole (1998) show that financial market alone can not provide enough funds to meet firms' liquidity

We will assume that $f_i(\rho_i)$ has a uniform distribution in $[0, \overline{\rho_i}]$ thereinafter. Then (20) gives the solution of ρ_i^* as

$$\rho_i^* = [2\,(1+r)\,\overline{\rho_i}]^{\frac{1}{2}} \tag{21}$$

Let $\widehat{\rho}_{ni} = \theta\rho_i^*$. Expression (19) now becomes

$$h_i(\widehat{\rho}_{ni}) = h(\theta\rho_i^*) = \left(\frac{1+\theta^2}{\sqrt{2}\theta}\right)[(1+r)\,\overline{\rho_i}]^{\frac{1}{2}} \tag{22}$$

Let there be a continuum of entrepreneurs (firms) in type n with unit mass. $F_i(\rho_i)$ denotes both the ex ante probability of a firm facing a liquidity shock below ρ_i, and the realized fraction of firms with liquidity shock below ρ_i in sector i. The total capital usage by type n entrepreneur is the sum of initial investment $K_{ni}^1(\rho_i^*)$ and expected liquidity shocks being paid. Denoting the total capital usage as K_{ni},

$$\begin{aligned} K_{ni} &= \left[1+\left(\frac{1}{1+r}\right)\int_0^{\theta\rho_i^*}\rho_i f_i(\rho_i)d\rho_i\right]K_{ni}^1(\theta\rho_i^*) \\ &= \left[1+\left(\frac{1}{1+r}\right)\int_0^{\theta\rho_i^*}\rho_i f_i(\rho_i)d\rho_i\right]\alpha_{ni}(\theta\rho_i^*) \\ &= \frac{(1+r)+\int_0^{\theta\rho_i^*}\rho_i f_i(\rho_i)d\rho_i}{(1+r)-\int_0^{\theta\rho_i^*}\left(\rho_{ni}^0-\rho_i\right)f_i(\rho_i)d\rho_i} \\ &= s_{ni}(\theta\rho_i^*) \end{aligned} \tag{23}$$

and the labor-capital ratio for firm n in the entire production process is

$$a_{ni}(\theta\rho_{ni}^*) = \frac{L_{ni}}{K_{ni}} = a_i \tag{24}$$

which is identical for all entrepreneurs in sector i.

There are two sectors in the economy. Sector 1 is assumed to be one in which entrepreneurs' cost of "work" differs. We rank entrepreneurs by their costs of "work"

demand at the optimal policy ρ_i^*.

from low to high, and index them by n directly. Entrepreneur n has lower cost of "work" than that of the entrepreneur n' if $n < n'$. In other words, the cost of "work" by entrepreneur n in sector 1, $c_{n1} = c_{n1}(e^H)$, is an increasing function in n. We will assume $c_{n1} = c_1 n$, being a linear function for simplicity. Expression (17) gives $\rho_{n1}^0 = \lambda_1^H[R_1 - (c_1 n)/\Delta\lambda_1]$, which is decreasing in n.

Expression (18) then implies that the *net return to internal capital* in sector 1 is decreasing in n, while Expression (23) implies that total capital managed by the entrepreneur is decreasing in n.

In Sector 2, all entrepreneurs are assumed to work at the same cost. That is, $c_{n2}(e^H) = c_2$. Expression (17) indicates that $\rho_{n2}^0 = \lambda_2^H[R_2 - c_2/\Delta\lambda_2]$, which is identical for all entrepreneurs. Thus, all entrepreneurs have the same *net return to internal capital,* $U_2(\theta\rho_2^*)$, in sector 2.

Let N_1 be the number of firms in Sector 1. Let N_1 solves for

$$U_{N_1 1}(\widehat{\rho}_{N_1 1}) = \frac{\rho_1^1 - h(\theta\rho_1^*)}{h(\theta\rho_1^*) - \lambda_1^H[R_1 - (c_1 N_1)/\Delta\lambda_1]} = U_2(\theta\rho_2^*) \tag{25}$$

Entrepreneurs in the interval of $[1, N_1]$ enter Sector 1 and earn the profit $U_{n1}(\widehat{\rho}_{n1}) \geq U_2(\theta\rho_2^*)$. Entrepreneurs of $n > N_1$ enter Sector 2 and earn the profit $U_2(\theta\rho_2^*)$. We summarize our results by the following proposition.

Proposition 2 *Productive entrepreneurs enter the heterogeneous sector, while less productive entrepreneurs enter the homogeneous sector. In the heterogeneous sector, more productive entrepreneurs manage more capital.*

4 Aggregation and Equilibrium Conditions

The first set of equilibrium conditions is free entry condition that equates the entrepreneur's *net return to internal capital* to her entry cost. We assume that a potential entrepreneur needs to pay an entry cost of f at the beginning of the first period to become an entrepreneur. The *net return to internal capital* in sector

2, $U_2(\theta\rho_2^*)$, should be equal to f. On the other hand, the marginal entrepreneur in sector 1, N_1, should have the same *net return to internal capital* as f, while all other entrepreneurs in sector 1 earn higher net returns. Using equation (25), the conditions can be stated as

$$\begin{aligned} U_{N_1 1}(\widehat{\rho}_{N_1 1}) &= \frac{\rho_1^1 - h(\theta\rho_1^*)}{h(\theta\rho_1^*) - \lambda_1^H[R_1 - (c_1 N_1)/\Delta\lambda_1]} = f \\ U_2(\theta\rho_2^*) &= \frac{\rho_2^1 - h(\theta\rho_2^*)}{h(\theta\rho_2^*) - \lambda_2^H[R_2 - c_2/\Delta\lambda_2]} = f \end{aligned} \tag{26}$$

Rewrite equations (26) as

$$\begin{aligned} \lambda_1^H R_1 &= h(\theta\rho_1^*) + \left(\frac{f}{1+f}\right)\left(\frac{\lambda_1^H c_1 N_1}{\Delta\lambda_1}\right) \\ \lambda_2^H R_2 &= h(\theta\rho_2^*) + \left(\frac{f}{1+f}\right)\left(\frac{\lambda_2^H c_2}{\Delta\lambda_2}\right) \end{aligned} \tag{27}$$

which we label as a set of *capital revenue sharing conditions (CRSC).* The left hand sides of equations (27) are expected marginal product of physical capital in sector i, which are shared between the expected unit cost of total investment, $h(\theta\rho_1^*)$, and the pays to entrepreneurs' efforts. Using (22), it is clear that R_i is uniquely determined by the interest rate r.

The second set of equilibrium conditions is full employment conditions. Each entrepreneur in sector 2 manages $s_2(\theta\rho_2^*)$ amount of capital. Expression (23) indicates that $s_2(\theta\rho_2^*)$ depends on prices, which we denote as $s_2(w,r)$ after suppressing notations of product prices. Entrepreneur n in sector 1 manages $s_{n1}(\theta\rho_1^*)$ amount of capital. $s_{n1}(\theta\rho_1^*)$ is a function of prices and n, which we denote as $s_1(w,r,n)$. (24) implies that labor-capital ratio is identical for all entrepreneurs within a sector. Let the number of entrepreneurs in sector 2 be N_2. Let L and K be the country's labor and capital endowments, respectively. The full employment conditions are

$$a_1 \int_1^{N_1} s_1(w,r,n)dn + a_2 s_2(w,r)N_2 = L \tag{28}$$

$$\int_1^{N_1} s_1(w,r,n)dn + s_2(w,r)N_2 = K \tag{29}$$

Substituting (17), (19), and (26) into (23), we obtain

$$s_1(w,r,n) = \frac{\Delta\lambda_1 h(\theta\rho_1^*)}{\lambda_1^H c_1 \left[n - (fN_1) / (1+f)\right]} \text{ and } s_2(w,r) = \frac{\Delta\lambda_2 h(\theta\rho_2^*)(1+f)}{\lambda_2^H c_2} \tag{30}$$

Applying expressions (30) and (22) to (28) and (29), we can rewrite the full employment conditions as follows:

$$a_{1L} \ln\left[\frac{N_1}{1+f-fN_1}\right] + a_{2L}N_2 = L \tag{31}$$

$$a_{1K} \ln\left[\frac{N_1}{1+f-fN_1}\right] + a_{2K}N_2 = K \tag{32}$$

where

$$\begin{aligned} a_{1L} &= \frac{a_1\Delta\lambda_1 (1+r)^{\frac{1}{2}} \overline{\rho_1}^{\frac{1}{2}} \left(1+\theta^2\right)}{\sqrt{2}\theta\lambda_1^H c_1}, a_{1K} = \frac{\Delta\lambda_1 (1+r)^{\frac{1}{2}} \overline{\rho_1}^{\frac{1}{2}} \left(1+\theta^2\right)}{\sqrt{2}\theta\lambda_1^H c_1} \\ a_{2L} &= \frac{a_2\Delta\lambda_2 (1+r)^{\frac{1}{2}} \overline{\rho_2}^{\frac{1}{2}} \left(1+\theta^2\right)(1+f)}{\sqrt{2}\theta\lambda_2^H c_2}, \text{ and} \\ a_{2K} &= \frac{\Delta\lambda_2 (1+r)^{\frac{1}{2}} \overline{\rho_2}^{\frac{1}{2}} (1+f)\left(1+\theta^2\right)}{\sqrt{2}\theta\lambda_2^H c_2} \end{aligned} \tag{33}$$

We close this section with the market clearing conditions in the product markets. The firms' expected output (or the realized industry output) in sector 1 is

$$\begin{aligned} y_1 &= F_1(\theta\rho_1^*)\lambda_1^H G_1(a_1^1, 1)\int_0^{N_1} \alpha_{ni}(\theta\rho_1^*)dn \\ &= \frac{G_1(a_1^1, 1)\Delta\lambda_1\,(1+r)}{c_1}\ln\left[\frac{N_1}{1+f-fN_1}\right] \end{aligned} \tag{34}$$

where we have used (13), (19) and (26) to derive the second equality. The expected output in sector 2 is

$$\begin{aligned} y_2 &= F_2(\theta\rho_2^*)\lambda_2^H G_2(a_2^1, 1)\alpha_2(\theta\rho_2^*)N_2 \\ &= \frac{G_2(a_2^1, 1)\Delta\lambda_2\,(1+r)\,(1+f)\,N_2}{c_2} \end{aligned} \tag{35}$$

We assume that the representative consumer's preference is homothetic. Thus, the ratio of the quantities consumed in the country depends only upon the relative goods price ratio, and can be represented by $D(\frac{p_1}{p_2})$. In equilibrium, the relative supply equals the relative demand. The condition is stated as

$$\frac{y_1}{y_2} = \left[\frac{G_1(a_1^1, 1)\Delta\lambda_1 c_2}{G_2(a_2^1, 1)\Delta\lambda_2\,(1+f)\,c_1}\right]\frac{\ln\left[\frac{N_1}{1+f-fN_1}\right]}{N_2} = D\,(p) \tag{36}$$

where $p = p_1/p_2$. Let good 2 be the numeraire good and we normalize p_2 as 1 in subsequent sections.

5 Comparative Statics

Substituting (8), (17), and (22) into (26), *CRSC* (27) can be written as

$$\lambda_1^H a_1 w + \frac{1+\theta^2}{2\theta}\left[2\,(1+r)\,\overline{\rho_1}\right]^{\frac{1}{2}} = \lambda_1^H\left[pG_1(a_1, 1) - \left(\frac{f}{1+f}\right)\frac{c_1 N_1}{\Delta\lambda_1}\right] \tag{37}$$

$$\lambda_2^H a_2 w + \frac{1+\theta^2}{2\theta}\left[2\,(1+r)\,\overline{\rho_2}\right]^{\frac{1}{2}} = \lambda_2^H\left[G_2(a_2, 1) - \left(\frac{f}{1+f}\right)\frac{c_2}{\Delta\lambda_2}\right] \tag{38}$$

The endogenous variables, w, r, p, N_1 and N_2 are determined by equations (31), (32), (36), (37), and (38). The outputs y_1 and y_2 are then derived from expressions (34) and (35). We will study effects of changes in endowments and the level of financial development on equilibrium prices and quantities.

5.1 Determination of Factor Prices

CRSC (37) and (38) are convex towards origin and downward sloping in (w, r) space. The slopes of the curves for given p, N_1 and θ are

$$\frac{dr}{dw} = -\frac{\lambda_i^H a_i 2^{\frac{3}{2}} (1+r)^{\frac{1}{2}} \theta}{\overline{\rho_i}^{\frac{1}{2}} \left(1+\theta^2\right)} = -\left[\frac{2^{\frac{3}{2}} (1+r)^{\frac{1}{2}} \theta}{\left(1+\theta^2\right)}\right] \widetilde{a}_i \text{ for } i = 1, 2 \tag{39}$$

where $\widetilde{a}_i = \left(\lambda_i^H a_i\right) / \overline{\rho_i}^{\frac{1}{2}}$, is defined as the *effective labor intensity.* Assume that $\widetilde{a}_1 < \widetilde{a}_2$, so sector 2 is effectively labor intensive than sector 1. As indicated in Figure 1, the *CRSC* of sector 2, S_2S_2, is steeper than that of sector 1, S_1S_1. Let the initial factor price equilibrium be given by point A. A decrease in the relative price of good 1, or an increase in N_1 will shift S_1S_1 inward as illustrated, and move the equilibrium to point B. It is clear that the wage has gone up, from w_0 to w_1, and the interest rate has declined, from r_0 to r_1. When θ is increased, both S_1S_1 and S_2S_2 shift out. As illustrated in Figure 1, the equilibrium moves from point A to point C which is vertically above A. The wage rate stays at exactly the same level, while the interest rate increases. A better financial system reduces the expected unit cost of total investment, $h(\theta\rho_i^*)$, and therefore increases the return to investment. The return to labor, however, is unaffected by the financial development due to the Leontif technology assumed in this paper. We summarize the above results as follows and relegate the formal proof to the Appendix.

Proposition 3 *Ceteris paribus, an improvement in the level of financial development increases the interest rate but has no effect on the wage rate. An increase in the number of entrepreneurs in the heterogeneous sector decreases the return to the* *factor used intensively (effectively) in that sector, and increase the return to the other factor. A decrease in the price of a good will decrease the return to the factor used intensively (effectively) in that good, and increase the return to the other factor.*

Where our model differs from the textbook version of the Heckscher-Ohlin model is that factor price equalization does not hold. As we will show next, more entrepreneurs enter the heterogeneous sector in the capital abundant country. Therefore, even if free trade in goods equalizes product prices across countries, larger N_1 results in lower interest rate r and higher wage rate w at home, which gives incentive for financial capital flow even with free goods trade.

5.2 Changes in Endowment and Financial Development

Let the equations (31) and (32) be denoted as LL curve and KK curve, respectively. The numbers of entrepreneurs (or amounts of internal capital) in equilibrium, $E = (N_1, N_2)$ are determined by the intersection of the LL and the KK curves, as indicated in Figure 2. KK is steeper than LL since sector 1 is capital intensive. Totally differentiating equations (31) and (32) and using the "Jones' algebra (Jones 1965)," we obtain

$$\begin{aligned} \xi_{1L}\widehat{N}_1 + \lambda_{2L}\widehat{N}_2 &= \widehat{L} - [\lambda_{1L}\widehat{a}_{1L} + \lambda_{2L}\widehat{a}_{2L}] \\ \xi_{1K}\widehat{N}_1 + \lambda_{2K}\widehat{N}_2 &= \widehat{K} - [\lambda_{1K}\widehat{a}_{1K} + \lambda_{2K}\widehat{a}_{2K}] \end{aligned} \tag{40}$$

We define $dN_1/N_1 = \widehat{N}_1$, and likewise for all other variables. In addition, we define the *fraction of labor used in industry i,*

$$\begin{aligned} \lambda_{1L} &= \frac{a_{1L} \ln\left[N_1/\left(1+f-fN_1\right)\right]}{L}, \lambda_{2L} = \frac{a_{2L}N_2}{L} \\ \text{and } \xi_{1L} &= \frac{a_{1L}\left(1+f\right)}{L\left(1+f-fN_1\right)} \end{aligned} \tag{41}$$

where $\lambda_{1L} + \lambda_{2L} = 1$. We define λ_{iK} and ξ_{1K} in an analogous manner.

Let the initial equilibrium output be point E. The effect of a change in the endowment is similar to the standard HOS model. $\widehat{L}$ and $\widehat{K}$ represent the *direct effect* of a change in endowment at given product prices, while the second terms in the right hand side of equations (40) represent the *feedback effect* of induced factor price changes on the factor usage per unit of production. For given factor prices, as depicted in Figure 2, the *direct effect* of an increase in the capital endowment shifts KK out to $K'K'$ and moves the equilibrium to point E'. It is clear that N_1 goes up, whereas N_2 declines. The increase in N_1 raises y_1, while the decrease in N_2 reduces y_2. Thus, the relative price of good 1, p, decreases. By Proposition 3, both the decrease in p and the increase in N_1 reduces r while increasing w. Using (33), we know that both labor and capital usages per unit of production decrease. Thus, the *feedback effect* shifts the $K'K'$ line out further to $K''K''$ and shifts the LL line out to $L''L''$, which moves the equilibrium from E' to E''. The shifting out of KK line further increases N_1 and reduces N_2, while the shifting out of LL line reduces N_1 and increases N_2. As we formally prove in the Appendix, if a *modified condition for non-reversal of factor intensity* is satisfied, the overall effect of an increase in K/L is to increase N_1. The overall effect on N_2 is ambiguous. However, the relative price p declines, and as a result, the relative output y_1 to y_2 increases.

We now discuss the effect of the change in θ. As Proposition 3 shows, the increase in θ raises the interest rate but has no effect on the wage w. That is, the impact of changing in θ is completely absorbed by the increase of r, while leaving w unaffected. Expression (6) and *CRSC* (27) then indicate that the change in θ must be offset by the change in r so that $h(\theta\rho_i^*)$ maintains constant. Comparing (22) with (33), we know that a_{ij} must maintain constant as θ changes. As the result, N_1, N_2, and p are not affected by the increase in θ.

Proposition 4 *Suppose a modified condition for non-reversal of factor intensity is satisfied, so that sector* 1 *is capital intensive globally. The increase in capital endowment will increase the number of entrepreneurs in sector* 1*, and decrease the* *relative price of good* 1. *The improvement in the level of financial development, however, has no effect on outputs and the product prices.*

If capital flows into the country, the above proposition indicates that N_1 will be larger and p smaller. Using Proposition 3, both effects reduce r, but increase w, which results in conflict interests towards to capital inflow.

6 Free Trade and Capital Flow

Using the comparative statics results derived above, we are now ready to describe patterns of goods trade and capital flow. Consider two countries with identical and homothetic tastes, identical technologies, identical liquidity shocks and managers' behavior, but different factor endowments and levels of financial development. Labor is immobile across countries. We will first study free trade in goods without international capital flow, and then move on to allow for just financial capital flow, just foreign direct investment, and both types of capital flow, respectively.

6.1 Pattern of Free Trade in Goods

Let the equilibrium autarky prices at home and abroad be p, and p^*, respectively. p^* differs from p since K^*/L^* and θ^* are different from corresponding domestic variables. Comparing p^* with p is equivalent to the exercise of comparative statics in the last section that changes K/L and θ to K^*/L^* and θ^*,respectively. Let $\widehat{p} = (p^* - p)/p$ be the percentage difference in the autarky prices. Ignoring a second order effect and using equation (65) in the Appendix., we have

$$A\widehat{p} = \widehat{L} - \widehat{K} \tag{42}$$

where $A = -|\lambda|\,\sigma_D/\sigma_N > 0$. $\widehat{L}$, $\widehat{K}$, and $\widehat{\theta}$ now are percentage differences in the labor and capital endowments, and financial development, respectively, between the two countries. Noting that $\widehat{\theta}$ has no effect on levels of output and product prices, our analysis of goods trade is essentially a generalized Heckscher-Ohlin model in an environment of imperfect capital market and heterogeneous entrepreneurs. The usual Heckscher-Ohlin result holds here: a labor-abundant country has a higher relative price of the capital-intensive good than the other country. Thus, it exports the labor-intensive product and imports the capital-intensive product.

Proposition 5 *In this model with financial market imperfection and heterogeneous entrepreneurs, the Heckscher-Ohlin result on trade patterns still holds: each country produces and exports the good that uses its relatively abundant factor intensively.*

6.2 Financial Capital Flow

In this model, there are two types of international capital flow that are associated with investors and entrepreneurs. International financial flow occurs when a financial investor takes her endowment out of the country and invests in a foreign financial system (or indirectly in a foreign entrepreneur's project). On the other hand, foreign direct investment (FDI) occurs when an entrepreneur takes her project to a foreign country (to use foreign labor but still her home country's financial system). Financial investors will invest in the country with a higher interest rate (return to financial investment), while entrepreneurs will locate their projects in the country with a lower production cost. In the rest of this sub-section, we discuss in which only financial capital flow is permitted (in addition to free trade in goods), but no FDI.

The direction of *financial flow* is determined by $\widehat{r} = (r^* - r)/r$. If $\widehat{r} > 0$, financial capital will flow from the home to the foreign country. Otherwise, it will flow in the reverse direction. As we discussed in the last section, if the country is either relatively abundant in labor, or more financially developed, its interest rate in the absence of international capital flow is higher.

In the equilibrium with free trade in goods, the endogenous variables in each country are still determined by equations (31), (32), (36), (37), and (38), or their foreign-country counterparts, except that the relative domestic demand in (36), $D(p)$, is replaced by the sum of domestic demand and the import demand from foreign country, $D(p) + M^*(p)$. Slightly abusing the notations, we still use σ_D to represent the elasticity of substitution between goods on the demand side in the free goods trade equilibrium. All proofs in Appendix A go through in the equilibrium with free trade in goods, though the percentage difference in product prices across countries $\widehat{p} = 0$. Using (55) in the appendix and ignoring the second order effect, we have

$$\widehat{r} = \frac{\pi_{2w}\pi_{1N}\widehat{N}_1}{|\pi|} + \frac{2(1+r)\left(1-\theta^2\right)\widehat{\theta}}{r\left(1+\theta^2\right)} \tag{43}$$

Substituting (64) into (43), we obtain

$$\widehat{r} = B_L\widehat{L} - B_K\widehat{K} + C\widehat{\theta} \tag{44}$$

where $B_L > 0$, $B_K > 0$, and $C > 0$. More capital or less labor endowment reduces the interest rate, while a higher level of financial development increases the interest rate. We can summarize the two polar cases with the following proposition.

Proposition 6 *Let there be free trade in goods, no barrier to international financial capital flow but no FDI is permitted. If the two countries have the same capital-labor ratio but different financial development, financial capital will flow out of the country with a less developed financial system, and into the one with more financial development. If the two countries are the same in financial development but different in endowment, then financial capital will flow out of the country which is relatively capital abundant, and into the one wich is relatively labor abundant.*

6.3 Foreign Direct Investment

We now allow projects and entrepreneurs to move freely across countries. Rewrite the expression of entrepreneur's *net return to internal capital* (18) as

$$U_{ni}(w^c) = \frac{\lambda_i^H \left[p_i^T G_i(a_i, 1) - w^c a_i^1\right] - h(\theta\rho_i^*)}{h(\theta\rho_i^*) - \lambda_i^H \left[p_i^T G_i(a_i, 1) - w^c a_i\right] + \lambda_i^H B_i(n)/\Delta\lambda_i} \tag{45}$$

where w^c is the wage rate in the host country for FDI. It is assumed that no fixed cost exists for locational choice and that domestic entrepreneurs utilize their domestic financial system even if they produce abroad. Therefore, domestic entrepreneurs will have an outbound *FDI* if and only if $w > w^*$. As we have shown in Proposition 3 and 4, w is not affected by θ. $w > w^*$ if and only if the home country is capital abundant. That is, entrepreneurs from a capital-abundant country will engage in outbound FDI to take advantage of a lower labor cost.[7]

Proposition 7 *With free trade in goods but prohibition of international financial capital flow, FDI will go from a capital-abundant country to a labor-abundant country.*

6.4 Capital Transfusion (or Bypass Circulation)

We now allow for both types of capital flows. For simplicity, we continue to assume no sovereign risk and no cost on capital movement. In addition, we assume that the countries are not fully specialized. For two countries with different capital-labor ratios and different levels of financial development, what is the equilibrium in the world capital market? The answer is somewhat surprising (to us at least): there is a unique equilibrium in which the less developed financial system is *completely bypassed.* All capital owned by financial investors in the country with a less developed financial system leaves the country in the form of a *financial capital outflow.* However,

[7] We abstract from expropriation risk in this model. A higher λ_i^H can be interpreted as reflecting a lower risk of expropriation. Ju and Wei (2006b) study the implications of expropriation risk, financial development and factor endowment for capital flows.

physical capital (and projects) reenters the country in the form of *FDI.* The less developed financial system receives no capital at all in the process.

Let $\widehat{L} = (L^* - L)/L = 0$. The patterns of capital flows as a function of relative factor endowment and relative financial development are depicted in Figure 3. The home country locates at origin where $\widehat{K} = 0$ and $\widehat{\theta} = 0$. The RR' curve represents the equilibrium condition of *financial capital flow,* $\widehat{r} = 0$, where $C\widehat{\theta} = B_K\widehat{K}$.[8] $\widehat{r} < 0$ on the right side of RR' curve; and $\widehat{r} > 0$ on the left side of RR' curve.

The foreign country locates at point F where $\widehat{K} = (K^* - K)/K > 0$ and $\widehat{\theta} = (\theta^* - \theta)/\theta > 0$. The vertical line, WW', represents the equilibrium condition of *FDI,* $\widehat{w} = 0$. Recall that $w^* = w$ is observed in equilibrium only if the same amount of capital is used in both countries. Starting from point F, both *financial capital* and *FDI* flow from foreign country to home country. After the RR' curve is reached, it then comes to an area which we label *capital bypass circulation,* WHR. The foreign country is still more capital abundant than the home in the area WHR so that *FDI* continues to flow from the foreign to the home country, but *financial capital* now flows from the home to abroad since $r^* > r$. In the equilibrium all home capital must flow out through *financial capital flow.* If it were not, then the equilibrium would have been at WW'. Any capital served by the home financial system would earn a lower return since $\theta^* > \theta$ and the same amount of capital were used in both countries now. Those home capital would continue to flow out. At equilibrium E, all home capital flows out and reenters back home country through *FDI*; all world capitals are served by foreign financial system; wages and interest rates are equalized between two countries.

If the foreign country locates at point F' where $\theta^* < \theta$, the analysis is similar. In a second area of *capital bypass circulation,* $W'HR'$, foreign financial capital flows into the home country, and reenters the foreign country in the form of *FDI* until all foreign financial capital flows out and equilibrium E' is reached. The analysis can

[8] The notations are abused here slightly. $\widehat{K}$ represents the percentage difference in capital endowments between the two countries. It changes as capital crosses the border.

be easily extended to the case where $L \neq L^*$. Summarizing the above analysis, we have:

Proposition 8 *Suppose the two countries (with the same population) differ in their levels of financial development. In the unique equilibrium in the world capital market, the less developed financial system is completely bypassed. All capital owned by financial investors in the country with a less developed financial system will leave the country in the form of financial capital flow, but reenter the country in the form of FDI.*

Figure 3 indicates that even in a frictionless world, different components of gross capital flows can move in either the same or the opposite directions. A rich country may exhibit both financial capital outflow and outflow of FDI at point F; a combination of an inflow of financial capital but an outflow of FDI inside the area WHR; or a combinatioin of outflow of financial capital flow but inflow of *FDI* inside the area $W'HR'$. Thus, patterns of *gross capital flow* are diverse, and different gross flow patterns can be consistent with a given pattern of *net flow.*

7 Welfare Impacts

We now investigate the impact of goods trade and capital flow on social welfare in this model. Let the representative consumer's utility function be $W(c_1, c_2)$ where $c_i (i = 1, 2)$ is the aggregate consumption. Let κ be the amount of capital flow into the home country: $\kappa > 0$ represents capital inflow, while $\kappa < 0$ represents capital outflow. The GDP function for domestic economy is defined as

$$\begin{aligned} R(p, L, K, \kappa) &= \max_{y_1, y_2} py_1 + y_2 \\ &\quad \text{s.t. equations (31), (32), (34),} \\ &\quad \text{(35), (37), and (38).} \end{aligned} \tag{46}$$

Note that the capital endowment on the right hand side of equation (32), K, should be replaced by $K + \kappa$ now.

The GNP function is then defined as

$$\widetilde{R}(p, L, K, \kappa) = R(p, L, K, \kappa) + \phi(.)\kappa \tag{47}$$

where $\phi(.)$ is the return to capital flow. In the case of only financial capital flow $\phi(.) = -(1+r)$ if $\kappa > 0$ and $(1+r^{*})$ if $\kappa < 0$. The value of $\phi(.)$ will be discussed later in the case of only *FDI*. The representative consumer's indirect utility function becomes

$$V(p, L, K, \kappa) = \max_{c_1, c_2}\{W(c_1, c_2) : pc_1 + c_2 \leq \widetilde{R}(p, L, K, \kappa)\} \tag{48}$$

The welfare impact of free goods trade when $\kappa = 0$ follows the standard analysis. Let p^T be the equilibrium price under free trade. Assuming the home exports good 1, so we have $p^T > p$ and $y_1 > c_1$. The effect of goods trade is equivalent to the effect of increasing p on $V(p, L, K, \kappa)$. Differentiating $V(p, L, K, \kappa)$ with respect to p and using the envelope theorem, we obtain

$$\frac{\partial V(p, L, K, \kappa)}{\partial p} = \lambda \left(\frac{\partial R(p, L, K, \kappa)}{\partial p} - c_1 \right) = \lambda\,(y_1 - c_1) > 0$$

where λ is the marginal utility of income, which proves the gain from free trade in goods. We now turn to the effects of capital flows.

7.1 The Case of Financial Capital Flow

We first consider the effect of *financial capital outflow*, assuming *FDI* is prohibited. This is the case when $r < r^{*}$ and $\kappa < 0$. Differentiating $V(p, L, K, \kappa)$ with respect to κ gives

$$\frac{1}{\lambda}\frac{dV(p^T, L, K, \kappa)}{d\kappa} = (y_1 - c_1)\,\frac{\partial p^T}{\partial \kappa} + \frac{\partial R(p^T, L, K, \kappa)}{\partial \kappa} + (1 + r^{*}) \tag{49}$$

The first term on the right hand side represents a terms of trade effect. The capital outflow reduces K at home and increases K^* abroad by the same amount. The former decreases y_1 and increases y_2, while the later increases y_1^* and decreases y_2^*. The two effects approximately cancel each other out since both countries have the same Leontif production function. We assume that the world relative supply, $\frac{y_1+y_1^*}{y_2+y_2^*}$, is not affected by the capital flow. Thus, $\frac{\partial p^T}{\partial \kappa} = 0$.[9] The third term represents the return to financial investors who invest in the foreign country.

The second term is the marginal contribution of physical capital to GDP. As shown in the appendix, it is the sum of the return to financial investors and the expected payment to entrepreneurs' efforts in sector i if the capital was invested in sector i before it flows out.[10] Hence,

$$\frac{\partial R(p^T, L, K, \kappa)}{\partial \kappa} = -\frac{F_i(\theta\rho_i^*)\lambda_i^H R_i}{1+\theta^2} = -(1+r) - E_i \tag{50}$$

where

$$E_i = \frac{F_i(\theta\rho_i^*)\lambda_i^H f B_i(N_i)}{\left(1+\theta^2\right)(1+f)\,\Delta\lambda_i} = \frac{\theta\left[2(1+r)\right]^{\frac{1}{2}} \lambda_i^H f B_i(N_i)}{\overline{\rho_i}^{\frac{1}{2}}\left(1+\theta^2\right)(1+f)\,\Delta\lambda_i} \tag{51}$$

represents the expected pay to entrepreneurs' efforts. Note that we have used (22) and (27) to derive the second equality in (50).

Expression (50) highlights a tradeoff in the welfare implication of *financial capital flow.* There is a wedge E_i between the marginal contribution of physical capital to GDP and the return to financial investors. Although financial capital outflow brings a higher return to financial investors, represented by $r^* - r > 0$, but it causes a loss in the expected pay to entrepreneurs' efforts at home by an amount equal to E_i. The overall welfare effect is determined by the trade off between the financial investors' gain, $r^* - r$, and the entrepreneurs' loss, E_i. Substituting (50) and (44) into (49),

[9] The small country assumption would be a special case of this.

[10] Since pays to entrepreneurs' efforts differ across sectors, the marginal contribution of physical capital to GDP in general differs across sectors.

we obtain:

$$\frac{1}{\lambda}\frac{dV}{d\kappa} = r^* - r - E_i = r\left(-B_K\widehat{K} + C\widehat{\theta} - \frac{E_i}{r}\right)$$

Let VV' curve represent $C\widehat{\theta} - B_K\widehat{K} - E_i/r = 0$, which shifts in RR' curve by E_i/r and is drawn in Figure 3. $\frac{dV}{d\kappa} > 0$ on the left side of VV' curve and $\frac{dV}{d\kappa} < 0$ on the right side of VV' curve. If a foreign country locates in the area between RR' and VV' curves, *financial capital outflow* reduces welfare at home.

We now turn to the effect of *financial capital inflow* where $r > r^*$ and $\kappa > 0$. Differentiating $V(p, L, K, \kappa)$ with respect to κ gives

$$\frac{1}{\lambda}\frac{dV(p^T, L, K, \kappa)}{d\kappa} = (y_1 - c_1)\frac{\partial p^T}{\partial \kappa} + \frac{\partial R(p^T, L, K, \kappa)}{\partial \kappa} - (1 + r) \qquad (52)$$

Note that $\frac{\partial R(p^T, L, K, \kappa)}{\partial \kappa} = (1 + r) + E_i$ in this case. Financial capital inflow brings additional expected pay to entrepreneurs' efforts. Thus, the economy-wide welfare must be improved at home. Summarizing the results above, we have:

Proposition 9 *Suppose there is no* FDI. *Financial capital inflow unambiguously improves the welfare at home. On the other hand, the welfare effect of financial capital outflow is ambiguous, and determined by the trade-off between the financial investors' gain and the entrepreneurs' loss. If the later dominates the former, welfare is reduced at home.*

7.2 The Case of FDI

We now investigate the welfare effect of *FDI.* We continue to assume that entrepreneurs use only domestic financial services when they produce abroad. The effect of *FDI* outflow can be examined by

$$\frac{1}{\lambda}\frac{dV(p^T, L, K, \kappa)}{d\kappa} = \frac{\partial R(p^T, L, K, \kappa)}{\partial \kappa} + \phi(.) = -\frac{F_i(\theta\rho_i^*)\lambda_i^H R_i}{1 + \theta^2} + \frac{F_i(\theta\rho_i^*)\lambda_i^H R_i^*}{1 + \theta^2} > 0$$

since $R_i^* > R$. Entrepreneurs produce abroad only if it is more profitable to do so, and they bring all returns back home. Thus *FDI* improves welfare at home unambiguously. The effect of *FDI* inflow is

$$\frac{1}{\lambda}\frac{dV(p^T, L, K, \kappa)}{d\kappa} = \frac{\partial R(p^T, L, K, \kappa)}{\partial \kappa} - \phi(.) = \frac{F_i(\theta^* \rho_i^*)\lambda_i^H R_i}{1+\theta^{*2}} - \frac{F_i(\theta^* \rho_i^*)\lambda_i^H R_i}{1+\theta^{*2}} = 0$$

Now the foreign financial system θ^* is used by foreign entrepreneurs. They take all their returns out so that the home's welfare is not affected in the equilibrium by FDI inflows.

Proposition 10 *FDI outflow improves welfare at home unambiguously, but FDI inflow has no effect on marginal welfare at home.*

We highlight entrepreneur's locational choice based on the labor cost. Entrepreneurs collect the capital at home, produce at the country where the labor cost is the lowest, and bring all returns to capital back home. Therefore, FDI is always beneficial. We note that a more complete analysis of FDI needs to introduce a fixed cost to location choice, the risk of investment abroad, and difference in business environment into the model, which is left for future research.

8 Conclusion

In this paper, we aim to provide a solution to two prominent paradoxes on international capital flow: the paradox of too little flow in a one-sector model and the paradox of too much flow in a multi-sector trade model. Our model uses entrepreneur heterogeneity to partially restore the intuition of one-sector models in a two-sector setting that the interest rate is lower in a capital-abundant county. A revenue sharing rule between financial investors and entrepreneurs, together with marginal product of capital, determine the interest rate. In addition, quality of financial system plays a crucial role in the model. The interest rate is higher in the country with a better financial system. Financial capital flow and *FDI* can move in either the same or the opposite directions, and therefore form rich patterns of *gross capital flows*. The equilibrium in a frictionless world capital market, however, is unique and labeled as *capital bypass circulation:* less developed financial system of the two is completely bypassed.

The model also highlights a number of conflict of interest. While financial capital outflows tend to benefit domestic financial investors, they also tend to hurt domestic entrepreneurs. This tension adds a new dimension to the usual conflict of interest between capital and labor. Financial capital outflows increase r and decrease w at home. Thus, they may be appreciated by financial investors, but opposed by unskilled labor. Conversely, financial capital inflows reduce r but increase w at home. So they may be welcomed by labor but resisted by domestic investors.

Our focus in this paper is to provide a simple framework to solve the two paradoxes of capital flow. The current model is static; extending it to dynamic analysis will be a fruitful direction for future research.

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9 Appendix A

1. Proof of Proposition 3

Totally differentiating equations (37) and (38), we obtain

$$\begin{aligned} \pi_{1w}\widehat{w} + \pi_{1r}\widehat{r} &= \pi_{1\theta}\widehat{\theta} + \pi_{1p}\widehat{p} - \pi_{1N}\widehat{N}_1 \\ \pi_{2w}\widehat{w} + \pi_{2r}\widehat{r} &= \pi_{2\theta}\widehat{\theta} \end{aligned} \tag{53}$$

where $\widehat{w} = dw/w$ denotes the percentage change in wage rate and likewise for other variables. We define $\pi_{iw} = \lambda_i^H a_i w$, $\pi_{ir} = \left[\overline{\rho_i}^{\frac{1}{2}} r\left(1+\theta^2\right)\right] / \left[2^{\frac{3}{2}}\theta(1+r)^{\frac{1}{2}}\right]$, and $\pi_{i\theta} = \left[\overline{\rho_i}^{\frac{1}{2}}\left(1+r\right)^{\frac{1}{2}}\left(1-\theta^2\right)\right] / \left(2^{\frac{1}{2}}\theta\right)$, while $\pi_{1p} = p\lambda_1^H G_1(a_1, 1)$ and $\pi_{1N} = \lambda_1^H f B_1 N_1 / \left[(1+f)\,\Delta\lambda_1\right]$. We can solve for the change in factor prices from equations (53) as

$$\widehat{w} = \frac{\pi_{2r}\left(\pi_{1p}\widehat{p} - \pi_{1N}\widehat{N}_1\right)}{|\pi|} \text{ and} \tag{54}$$

$$\widehat{r} = \frac{\pi_{2w}\left(\pi_{1N}\widehat{N}_1 - \pi_{1p}\widehat{p}\right)}{|\pi|} + \frac{2\left(1+r\right)\left(1-\theta^2\right)\widehat{\theta}}{r\left(1+\theta^2\right)} \tag{55}$$

where $|\pi| = \pi_{1w}\pi_{2r} - \pi_{1r}\pi_{2w} < 0$ if sector 1 is effectively capital intensive than sector 2. Then results in Proposition 3 are immediately seen from expressions (54) and (55).

2. Proof of Proposition 4

Using (33), we have

$$\widehat{a}_{iL} = \frac{r}{2(1+r)}\widehat{r} - \left(\frac{1-\theta^2}{1+\theta^2}\right)\widehat{\theta} \text{ and } \widehat{a}_{iK} = \frac{r}{2(1+r)}\widehat{r} - \left(\frac{1-\theta^2}{1+\theta^2}\right)\widehat{\theta} \tag{56}$$

These solutions for $\widehat{a}_{ij}(j = L, K)$ can then be substituted into equation (40) to obtain

$$\xi_{1L}\widehat{N}_1 + \lambda_{2L}\widehat{N}_2 = \widehat{L} - \frac{r}{2(1+r)}\widehat{r} + \left(\frac{1-\theta^2}{1+\theta^2}\right)\widehat{\theta} \tag{57}$$

$$\xi_{1K}\widehat{N}_1 + \lambda_{2K}\widehat{N}_2 = \widehat{K} - \frac{r}{2(1+r)}\widehat{r} + \left(\frac{1-\theta^2}{1+\theta^2}\right)\widehat{\theta} \tag{58}$$

Let $|\lambda|$ denote the determinant of the 2×2 matrix on the left hand side of the above system. It is immediately seen that $|\lambda| < 0$ if and only if $a_1 < a_2$.

Totally differentiating equation (36), we obtain

$$\sigma_N \widehat{N}_1 - \widehat{N}_2 = -\sigma_D \widehat{p} \tag{59}$$

where $\sigma_D > 0$ is the elasticity of substitution between goods on the demand side, and

$$\sigma_N = \frac{1+f}{(1+f-fN_1)\ln\left[N_1/\left(1+f-fN_1\right)\right]} \tag{60}$$

Now substituting (59) into (55), we have

$$\widehat{r} = \frac{\pi_{2w}}{|\pi|\,\sigma_D}\left[\left(\pi_{1N}\sigma_D + \pi_{1p}\sigma_N\right)\widehat{N}_1 - \pi_{1p}\widehat{N}_2\right] + \frac{2\left(1+r\right)\left(1-\theta^2\right)\widehat{\theta}}{r\left(1+\theta^2\right)} \tag{61}$$

Then substituting the above expression into equations (57) and (58), we obtain

$$
\begin{aligned}
(\xi_{1L}-\eta_1)\,\widehat{N}_1+(\lambda_{2L}+\eta_2)\,\widehat{N}_2 &= \widehat{L} \\
(\xi_{1K}-\eta_1)\,\widehat{N}_1+(\lambda_{2K}+\eta_2)\,\widehat{N}_2 &= \widehat{K}
\end{aligned} \tag{62}
$$

where

$$
\eta_1=-\frac{r\pi_{2w}\left(\pi_{1N}\sigma_D+\pi_{1p}\sigma_N\right)}{2(1+r)\,|\pi|\,\sigma_D},\eta_2=-\frac{r\pi_{2w}\pi_{1p}}{2(1+r)\,|\pi|\,\sigma_D} \tag{63}
$$

Both η_1 and η_2 are positive. Let $|\Lambda|$ denote the determinant of the 2×2 matrix on the left hand side of (62). We assume a *modified condition for non-reversal of factor intensity* that $|\lambda|$ and $|\Lambda|$ have the same sign, which implies that $|\Lambda|<0$. The condition ensures that sector 1 is capital intensive both before and after changes in factor endowments and the level of financial development. Solving for $\widehat{N}_1$ gives

$$
\widehat{N}_1=\frac{(\lambda_{2K}+\eta_2)\,\widehat{L}-(\lambda_{2L}+\eta_2)\,\widehat{K}}{|\Lambda|},\widehat{N}_2=\frac{(\xi_{1L}-\eta_1)\,\widehat{K}-(\xi_{1K}-\eta_1)\,\widehat{L}}{|\Lambda|} \tag{64}
$$

$\widehat{N}_1>0$ when $\widehat{K}>0$, $\widehat{L}=0$.

Using the fact that $\xi_{1L}-\xi_{1K}=\sigma_N\left(\lambda_{2K}-\lambda_{2L}\right)$, we have $|\lambda|=\sigma_N\left(\lambda_{2K}-\lambda_{2L}\right)$. Subtracting (57) from (58), and using (59), we obtain

$$
\left[-\frac{|\lambda|\,\sigma_D}{\sigma_N}\right]\widehat{p}=\widehat{L}-\widehat{K} \tag{65}
$$

$\widehat{p}<0$ when $\widehat{K}-\widehat{L}>0$. Note that $\widehat{\theta}$ has no effect on $\widehat{N}_1$, $\widehat{N}_2$ and $\widehat{p}$. Thus, we have proved Proposition 4.

3. The Marginal Contribution of physical Capital to GDP

Rewriting (6) as

$$
F_i(\theta\rho_i^*)\lambda_i^H p_i G_i(a_i,1)K_i^1=F_i(\theta\rho_i^*)\lambda_i^H wL_i+F_i(\theta\rho_i^*)\lambda_i^H R_i K_i^1 \tag{66}
$$

and then using expressions of outputs (34) and (35), we obtain:

$$R(p, L, K, \kappa) = \sum_{i=1}^{2} \left[F_i(\theta\rho_i^*)\lambda_i^H w L_i + F_i(\theta\rho_i^*)\lambda_i^H R_i K_i^1 \right] \tag{67}$$

Recall that K_i^1 is the investment in the first period. Substituting (23) into the above expression, we write the GDP function (46) as its dual problem

$$\begin{aligned} R(p, L_i, K_i) \quad &= \quad \min_{w,r} \sum_{i=1}^{2} \left[F_i(\theta\rho_i^*)\lambda_i^H w L_i + F_i(\theta\rho_i^*)\lambda_i^H R_i \left(\frac{K_i}{1+\theta^2} \right) \right] \\ &\quad \quad \text{s.t. equations (37) and (38)} \end{aligned} \tag{68}$$

Note that we need to replace N_1 by corresponding K_1 in equation (37). Using envelope theorem, we have

$$\frac{\partial R(p, L_i, K_i)}{\partial K_i} = \frac{F_i(\theta\rho_i^*)\lambda_i^H R_i}{1+\theta^2} \tag{69}$$

Thus, the effect of capital outflow from sector i is the negative value of the right hand side of the above expression.

Appendix B. Table of Notations (in the order of appearance in the text)

Notations	**Definitions**
p_i	The price of good i
w	Wage rate
r	Interest rate
K_i	Amount of capital used in sector i
L_i	Amount of labor used in sector i
K	Total capital endowment of the country
L	Total labor endowment of the country
y_i^1	The output of a project in sector i if the project succeeds
K_i^1	The investment in sector i at period 1
$G_i(L_i, K_i^1)$	Production function in sector i at period 1
R_i	The return to a unit of period 1 investment if the project succeeds
a_i	Labor-capital ratio in sector i
ρ_i	Liquidity shock in sector i
$F_i(\rho)$	Distribution function of liquidity shock in sector i
$f_i(\rho)$	Density function of liquidity shock in sector i
λ_i^H	The probability of success if entrepreneur works
λ_i^L	The probability of success if entrepreneur shirks
R_{ni}^E	The entrepreneur's share in capital revenue if the project succeeds
$c_{ni}(.)$	The entrepreneur n's cost of effort.
ρ_i^1	$= \lambda_i^H R_i$
K_{ni}^1	The amount of first period investment managed by entrepreneur n
K_{ni}^{X1}	The external capital that entrepreneur raises at date 1
$\mu_{ni}(\rho_i)$	The state-contingent continuation policy (1 = *continue*, 0 = *stop*)
ρ_{ni}^0	$= \lambda_i^H \left(R_i - R_{ni}^E\right)$

Notations	**Definitions**
$\alpha_{ni}(.)$	Equity multiplier, and equals K_{ni}^1 in the equilibrium
$\beta_{ni}(.)$	The marginal net social return on investment
$h(\widehat{\rho}_{ni})$	Expected unit cost of total investment
$U_{ni}(.)$	Firm's net return to internal capital (profit)
ρ_i^*	Optimal cutoff of the liquidity shock
θ	The level of financial development
K_{ni}	$= s_{ni}(\theta\rho_i^*)$, is the total amount of capital managed by entrepreneur n
N_i	Number of entrepreneurs in sector i
f	Fixed cost to become an entrepreneur
y_i	The expected (or the realized) output in sector i
$D(.)$	The relative demand
$\widetilde{a}_i$	The effective labor intensity
$\widehat{N}_i$	$= dN_i/N_i$, and likewise for all other variables
λ_{ij}	The fraction of factor j used in sector i
$M^*(p)$	The import demand from foreign country
$W(c_1, c_2)$	The representative consumer's utility function
κ	The amount of capital flow
$R(p, L, K, \kappa)$	The GDP function
$\widetilde{R}(p, L, K, \kappa)$	The GNP function
p^T	The equilibrium price under free trade
$\phi(.)$	The return to capital flow
$V(p, L, K, \kappa)$	The representative consumer's indirect utility function
E_i	The expected pays to entrepreneurs' efforts

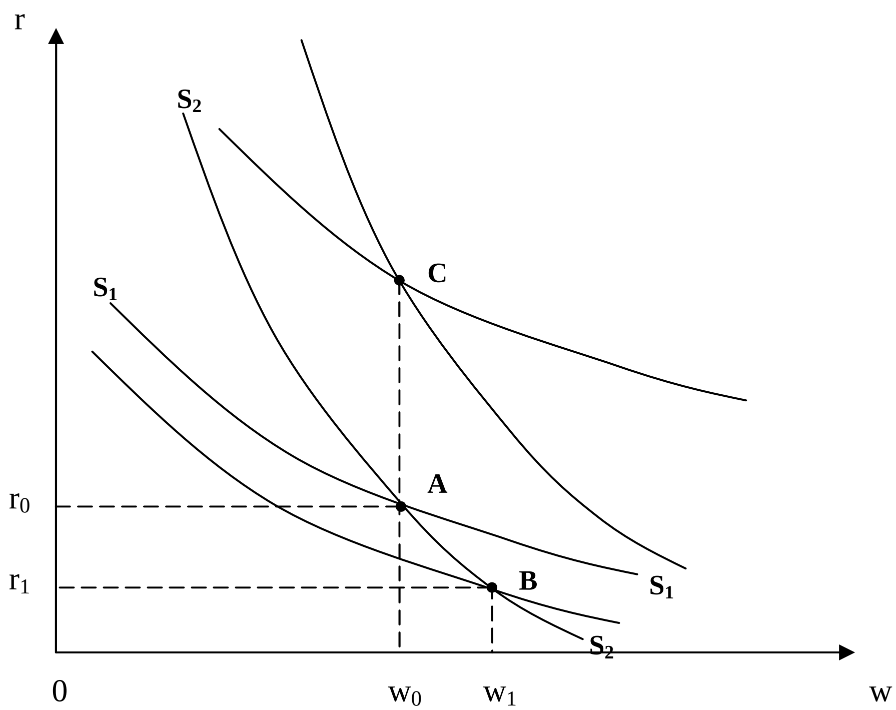

Figure 1

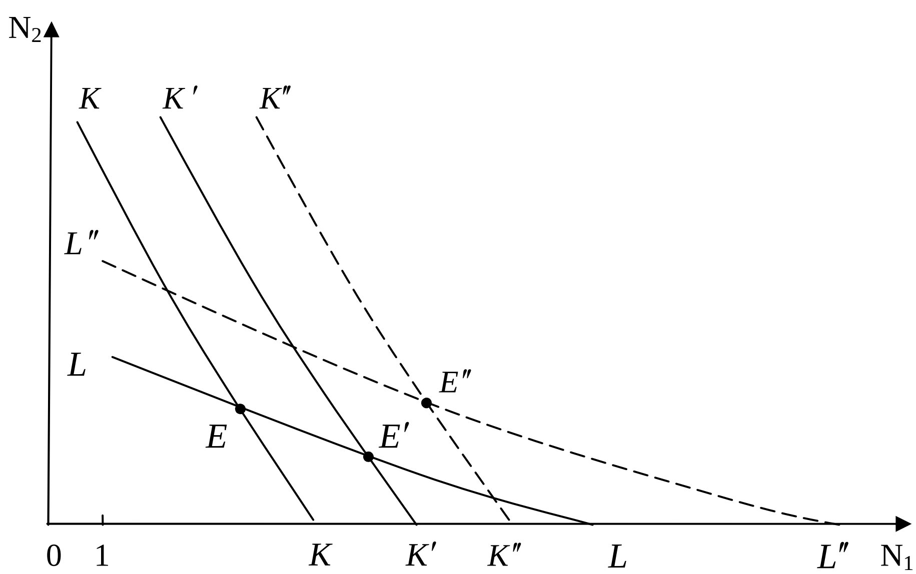

Figure 2

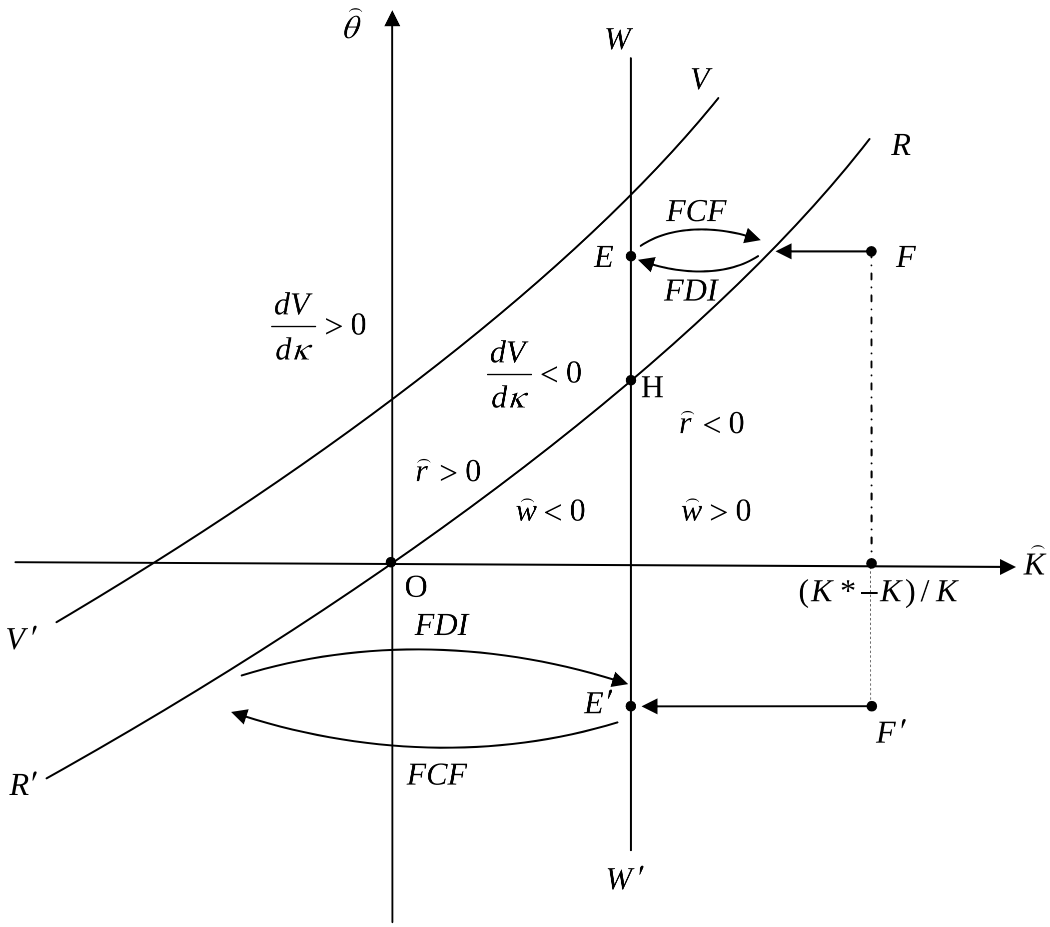
$\hat{\theta}$
W
V
R
FCF
E
F
FDI
$\frac{dV}{d\kappa} > 0$
$\frac{dV}{d\kappa} < 0$
H
$\hat{r} < 0$
$\hat{r} > 0$
$\hat{w} < 0$
$\hat{w} > 0$
$\hat{K}$
O
$(K^* - K)/K$
V'
FDI
E'
F'
R'
FCF
W'

Figure 3