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FOREWORD

Stress tests assess resilience of financial systems against various adverse shocks using quantitative tools. As such, they provide insight into the risks present in individual financial institutions and the financial system as a whole. The scenarios assessed under stress tests range from single-factor shocks to interest rates and liquidity, to complex macroeconomic scenarios linked to banks' internal credit risk models.

An increasing number of large banks and supervisory or financial stability authorities are employing stress tests as a part of their risk analysis. Indeed, stress tests are central to the Basel II supervisory framework, under which supervisors assess the quality of the banks' internal models and the stress tests they use. In addition, more and more financial stability reports worldwide include some form of stress tests in their analysis.

The recent global financial markets turmoil has highlighted the importance of stress testing. Even though no stress test had foreseen the depth and extent of the crisis, institutions that would regularly run a comprehensive set of stress tests arguably were better aware of the risks involved than others.
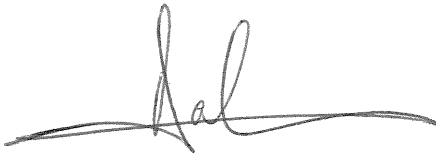
Stress testing methodologies remain under development. New techniques and data availability continue to improve modeling capacity in many countries and institutions. It is against this background that, together with a number of central banks, and with contributions from the private sector and academia, the Monetary and Capital Markets Department of the International Monetary Fund has been organizing a series of expert forums on advanced stress testing. The second of these expert forums was coorganized with and hosted by De Nederlandsche Bank in Amsterdam and took place over two days in October 2007.

The papers presented at this second expert forum covered several advanced technical topics. The first two sessions centered on the mapping from macroeconomic and financial risk factors to the banks' credit portfolios. The third session focused on the measurement of credit

and market risks. The second day of the forum highlighted cross-sector and cross-border risks, as well as asset concentration.

I join the conference participants in their keen interest in the further advancement of stress testing techniques and their applications for financial stability analysis. I would like to thank them very much for their contributions, and I hope that this collection of papers will help foster further progress in this important area of research and its applications.

Furthermore, it is my pleasure to inform you that the series of expert forums on stress testing will continue. The next expert forum will take place in May 2009 in Berlin, Germany, coorganized by the Deutsche Bundesbank and the IMF.

Jaime Caruana
Financial Counsellor
International Monetary Fund

Plausibility of Stress Scenarios[1]

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Introduction

This note briefly discusses the issue of the plausibility of stress scenarios for banks' risk management. The idea is mainly inspired by some concerns expressed by regulatory authorities as well as the banking industry on the possibility that the outcome of VaR and some types of stress testing might underestimate real amounts of risk faced by banks. The paper shows several approaches to the plausibility of stress scenarios, which could help banks' senior managers and other stakeholders including bank supervisors attain their risk management purposes through stress testing.

Various purposes of the stress testing often confuse banks when setting the plausibility of the stress scenarios. This paper first classifies different types of stress testing in terms of risk management purposes and then discusses the possible approaches to the plausibility of stress scenarios. In this process, the paper demonstrates some key issues to be considered, namely, the relationship between the degree of stresses and the confidence level used for VaR calculation, assumptions on the variability of external environments, and consistency of stresses between different risk categories. Finally, the paper suggests the next steps to be explored for further improvements of stress testing.

Some concerns of regulatory authorities and the banking industry

In the process of Basel II implementation, not a few regulatory authorities are showing some concerns on the credit risk amounts quantified by the banks under their supervision. Their risk asset amounts could sometimes be significantly lower than the amounts calculated under the Basel I. Their concerns often concentrate on the data used for risk parameter estimation. These are usually the data that have been collected during the last 5-7 years, a period during which some countries didn't experience a serious

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[1] This paper was prepared for the "2nd Expert Forum on Advanced Techniques on Stress Testing: Applications for Supervisors" hosted by the IMF and De Nederlandsche Bank on 23-24 October 2007. Views expressed in this paper are those of the author and do not necessarily reflect those of BOJ.

economic slump.

Even in Japan where banks were relieved from the massive NPL problem just 6-7 years ago, some regional banks have a concern that their estimation of credit VaR might underestimate their real credit risk amounts. Many senior managers, who experienced and managed out of the banking crisis, fret about the credit VaR outcome being significantly smaller than their intuitively alarming level.

Their concern might partly be evidenced by some simple comparisons of the estimated risk amounts with those calculated by other risk measurement methods. For example, the following is an image of risk amounts held by Japanese banks, which are estimated by the BOJ using basically the VaR method with 99% confidence level (*Bank of Japan, "Financial Stability Report" September, 2007*).

Major banks   Regional banks

☒ Credit risk [1]   ☐ Market risk associated with stockholdings [3]

☐ Interest rate risk [2]   ▨ Operational risk [4]
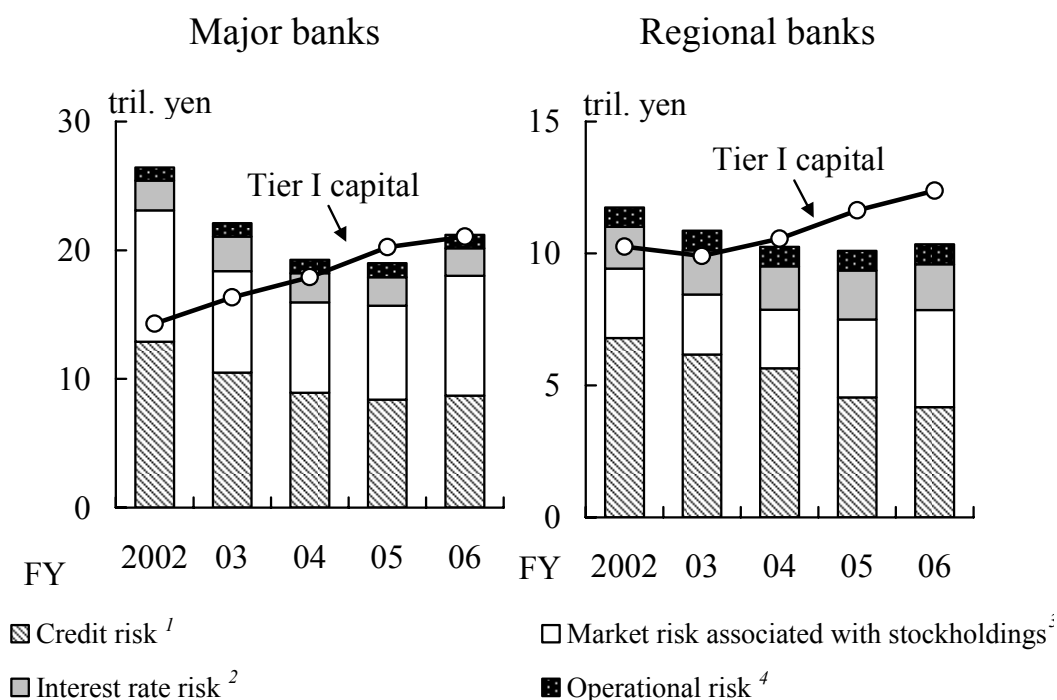
For more details, the following methods are used for measuring each broadly categorized risk.

  1: <u>Credit risk</u> is calculated by subtracting expected loss (EL) from the maximum loss (EL+UL) based on the Basel II risk weight formulas with a confidence level of 99

percent. In the estimation, borrowers classified as requiring "special attention" or below (in terms of credit quality) are considered. In FY 2006, credit risk accounts for around 41 % of Tier 1 capital for major banks.

2: Interest risk is limited to yen-denominated bond portfolio and estimated based on the assumption that market interest rates increase by 100 basis points on all maturities. In FY 2006, market risk accounts for around 10 % of Tier 1 capital for major banks.

3: Market risk associated with stockholdings is measured by 1-year, 99 percent VaR (using TOPIX as a risk factor). In FY 2006, market risk associated with stockholdings accounts for around 44 % of Tier 1 capital for major banks.

4: Operational risk is defined to be 15 percent of gross profits based on the Basel II Basic Indicator Approach (BIA). In FY 2006, operational risk accounts for around 5 % of Tier 1 capital for major banks.

As indicated by the chart, the aggregated risk amount of major banks is roughly equal to the level of their current Tier 1 capital. Although each bank actually uses more sophisticated techniques to measure their risks, the above chart shows a typical picture of the risk profiles which are generally shared by Japanese banks.

Another risk measurement method does not depend on distributional approaches but simply captures the worst loss cases experienced by major Japanese banking groups after the burst of bubble, i.e. during the last 20 or so years. The outcome is as follows.

1. Credit risk loss: The worst credit cost ratio is 4.7% (FY1998) → 55.6% of the current Tier1 capital based on the current loans outstanding.

2. Interest risk loss: The worst loss ratio is 1.3% (FY2005) → 4.3% of the current Tier1 capital based on the current securities outstanding.

3. Loss from market risk associated with stockholdings: The worst loss ratio is 61.4% (FY1991) → 70.8% of the current Tier1 capital based on the current equity outstanding.

4. Operational risk loss: Daiwa Bank NY Branch (around 100 billion yen, or 30% of the banking group's gross profit), Mizuho Security (around 40 billion yen, or 2% of the banking group's gross profit) → 0.7--9.6% of the current Tier1 capital based on the current level of gross profit .

Simple aggregation of all above risks amounts to 131.4%--140.3% of the current Tier1 capital.

We often refer to 99% confidence level of VaR as a metaphor of the worst event that could happen once every 100 years (or more strictly speaking, the second worst annual loss amount over the last 100 years). If this analogy can be applied to the latter estimates,, the worst loss amount over the last 20 years should correspond to roughly 95 % confidence level of VaR[2], which is significantly lower than the 99% confidence level used for the former estimates. Still, the total risk number of the second estimates is around 30--40 % larger than that of the first estimates. This may partly justify the concerns of Japanese regional banks.

Current state of stress testing and of stress assumptions

There are a number of different types of stress testing used for banks' risk management purpose. They usually share one aspect of the testing process, i.e. the use of "stress," but greatly differ over "what kind of stresses they use" and "for what kind of purposes they use these stresses. On the former, broadly speaking, there seems to be two types of stresses distinguished by the way how the variability of "external environments" is dealt with.

In the first type, a higher confidence level is used than the one used for usual VaR calculation.   In the world of VaR, plausibility of stresses is usually described by the frequency of stress events, and this frequency is determined by the confidence level (99% or 99.9% or 99.97%...). Thus, the "stress" with a higher confidence level means a lower frequency and consequent higher severity than that of the stresses with usual confidence level. This higher severity, however, follows the same external environment that is assumed by the VaR with the original confidence level. VaR often assumes some stability of external environments and accordingly uses the data over a limited period of time even if data over longer periods is available. This assumes that old data under different external environments would not help predict future loss. Thus, strictly speaking, 99% confidence level does not necessarily mean the worst event that could occur every 100 years because it is normally impossible to assume the stability of external environments over 100 years. This confidence level rather means the risk that could be faced by one among 100 institutions every year. As for credit VaR, for example,

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[2] If compared to the aggregation of individual banks' risk, which are measured by their worst annual loss amounts after the burst of bubble, the number here might underestimate the risk as I used the worst numbers of the "banking industry" (except operational risk), which reflect some offsetting effects of good and bad banks in the industry.

assumptions concerning the variability of external environments tend to cover only the business cycles of 4-5 years. This argument could also be true of macro stress testing using macroeconomic model, which usually does not assume any possible structural changes in macroeconomy in the future.

In some cases, banks might use risk amounts with a higher confidence level as a proxy of stress based on their historical experiences, which is a deviation from the original statistical meaning of stress. In the recent market turmoil, for example, many banks have seen extremely volatile movements in prices of credit products, which sometimes reached 10 sigma. Banks might use this newly observed number for stress testing not because they want to examine the impact of higher risk appetite (higher confidence level) but because they believe that this number might reflect some possible impacts of changes in external environments. This way of using confidence level falls under the second type, which will be explained below.

Second type of stress is defined in a more forward-looking way, often assuming the changes in external environments. This variability often depends on expert judgments and relatively long historical records. Use of historical records might reflect the belief that human beings repeat similar types of serious errors over relatively long time horizons, even though such errors are not exactly the same. This reminds us the famous "psychohistory," which was created by I. Asimov in his "*Foundation*." This science extracts some historical patterns of human actions from a myriad of observations in order to help predict the future history. Some Japanese regional bankers might have already studied this psychohistory and hence their concerns.

Which type of stress is used depends on the purpose of risk management. Generally speaking, risk management over relatively short time horizons or assuming the stability of external environments over long time horizons could be facilitated by the use of stress with distributional approach, such as VaR[3]. Otherwise, VaR outcome may not be enough for risk management. In particular, if structural changes of macroeconomy frequently (e.g. more than once every 20 years) occur in a society, exclusive use of stresses under stable external environments might be too optimistic for those who have strong interests in each institution's solvency or system-wide stability over long time

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[3] Here I assume the VaR that uses frequency distributions estimated only by actual observed data. In the operational risk management area, however, VaR also uses frequency distributions estimated by a combination of actual data and scenario data. For convenience, I classify this type of risk measurement into the second type of stress testing rather than the first type in the paper.

horizons.

On the issue of "for what kind of purpose banks use the stresses", again there seems to be two types of purposes; one is to confirm the capital adequacy by comparing capital with possible loss amounts caused by stresses, and another is to confirm the promptness and appropriateness of bank managers' reaction to possible events. The latter seems to be a simulation type of exercise and thus the plausibility of scenarios in a strict sense tends to be less important than in the former case.

Possible confusion between different types of stress testing

Owing to the same naming despite the variety of contents, the word "stress testing" sometimes causes unnecessary confusion within banks or between banks and supervisors. The following might be some representative cases.

1. A bank feels obliged to use the same stresses that are used to confirm the capital adequacy also for the purpose of confirming the promptness of mangers' reactions.

Needless to say, if the objective of stress testing is different, the plausibility of stresses could also be different. The plausibility of stresses, which is represented by VaR confidence level, could not be much useful for simulation exercises because they are too extreme on the one hand and assume too static external environments on the other.

2. A bank seeks credit risk scenarios which could occur once every 100 years, because this bank adopts 99% confidence level for credit VaR.

Again 99% confidence level corresponds to the event that could occur once every 100 years "only if" external environments assumed by VaR would be stable over 100 years. Otherwise, banks should not necessarily be constrained by the confidence level for VaR when setting up the frequency of scenarios.

3. A bank feels obliged to simply add up all stress testing outcome for different risk categories in order to confirm its capital adequacy, because this bank simply aggregates VaR numbers for each risk category in the integrated risk management framework.

For the purpose of integrating risk management, banks often use conservative assumptions on correlation between broad risk categories including positive correlation. As stress testing often assumes different plausibility for different scenarios, however, it

might be difficult to compare the risk amounts between different risk categories. Also, as stress testing often assumes a change in external environments, conservative positive correlation might be "too" conservative.

Some challenges in improving stress testing

Above arguments indicate that some steps might help banks and supervisors to have more fruitful dialogue on effective stress testing for banks' risk management.

As a first step, different types of stress testing should be clearly distinguished. In particular, we should better distinguish extreme events under stable external environments (ordinary environments), as in the case of VaR and extreme events under changing external environments (extra-ordinary environments) as in the case of typical stress testing.

The clear distinction could help highlight inconsistent treatments of different category of risks.. The following table shows how differently "low frequency but high severity" (LFHS) losses are dealt with in the management of different categories of risks.

	Market risk	Credit risk	Op risk
Number of observed loss data samples	Large	Not sufficiently but pretty large	Small
VaR based only on observed data	Good	Fair	Bad
Relative importance of LFHS cases	?	Big	Very big
Assumed frequency of stress scenarios	No consensus, often the worst event over the last 10-20 years, or higher confidence level of the VaR	No consensus, often the worst event over the last 10-20 years, or higher confidence level of the VaR	Often assuming 99.9% confidence level, or the frequency of every 1000 years
The way of using scenario outcome	Ad hoc, stress testing	Ad hoc, stress testing	Comprehensive scenario data is often used for VaR calculation

Given enough number of observed data samples, market and credit risks to some extent tend to use only those samples to estimate the VaR risk amounts. Mainly due to the fact that available data tend to be limited to the period of benign market condition, however,

not a few banks and supervisors feel a sense of underestimation in this risk outcome. Even in the case of stress testing using the worst numbers over the last 10-15 years, they tend to see many of their stress scenarios as being too weak to prepare themselves against a perfect storm. While supervisors often require banks to consider the outcome of stress testing over a whole business cycle in estimating risk parameters under the Pillar I and for other purposes under the Pillar II, the degree of stresses to be considered is not always clear.

Unlike the case of market and credit risks, operational risk tends to depend significantly on scenario data due to the lack of internally observed data. Banks often make a large number of scenario data, which is comprehensive enough to cover a fat tail part of loss distribution and then put them into the model for VaR calculation. Being unique to operational risk, scenarios are not limited to the events under the ordinary environments but also under the extra-ordinary environments, partly because the Basel II explicitly requires AMA (Advanced Measurement Approach) banks to consider so-called BEICF (Business Environments and Internal Control Factor) as one of the four minimum elements. Thus, some banks seek for events that could happen once every one thousand years, which is their interpretation of 99.9% confidence level required for AMA .

As a second step, some consensus on the degree of stress should be sought in order to avoid possible underestimation of risks as well as possible inconsistency between different risk categories in dealing with risk under extra-ordinary environments. We need some consensus not only in terms of the "horizontal frequency" under the ordinary environments (e.g. confidence level of VaR) but also in terms of the "historical frequency" under the variable environments. The former frequency can be represented by a metaphor such as an event faced by one among 1000 banks every year. The latter frequency can be represented by a metaphor such as an event faced by a bank once every 1000 years.

We have a certain consensus on horizontal frequency, which often falls under 99—99.97% ranges. However, we have only a rough idea on historical frequency, for which range of practice seems to be quite wide[4]. For example, some supervisors require banks to use a stress scenario that could occur once every 25 years for credit risk, and

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[4] In the credit VaR model, historical frequency might be implicitly expressed in the level of asset correlation with systemic factors being assumed in the model. I am not sure, however, if we can assume the same systematic factors over long time horizons.

others require 10 years for the same risk. I have no idea if any supervisor provides an indicator of historical frequency for the stress testing of market risk. As for operational risk, as stated above, some banks seek for a scenario that could occur once every 1000 years, while many others choose the maximum frequency of every 100 years, incapable of imaging less frequent scenarios.

The idea of minimum historical frequency would first come from the stakeholders that have strong interests in long-term financial system stability. Although this issue may depend on the tolerance level of the general public, our post-war experience of banking crises may provide us with some ideas. For example, if major post-war banking crises in the world can be categorized into two to three groups (e.g. intensified debt problem of Latin America and other developing countries during 1970-80s, banking crisis of many countries in the aftermath of financial liberalization during 1980-90s, massive market contagion cases including Asian and Russian crisis, LTCM and sub-prime loan? Shocks during 1990-2000s), the minimum historical frequency such as once every 20 or 25 years for credit and market risks could be used as an indicator of stress which helps to confirm the capital adequacy that ensures the post-war ordinary life in the coming half-century. Some types of operational risk, however, might need longer historical horizons, since longer stability of external environments could be assumed in terms of certain types of events, such as an earthquake.

The current practices of stress testing indicate that banks often use latest crisis experience as a benchmark. For example, Japanese banks tend to refer to their experiences of the recent banking crisis in a stress testing for confirming capital adequacy. Meanwhile, many Asian banks seem to use their experiences of Asian crisis as a benchmark of stresses. Many might agree that these crises would not repeat in the coming quarter-century and thus might satisfy the above conditions for the historical frequency of stresses.

The difficult cases are the stresses for banks in Europe, the US and Australia where the economy has never slumped for years and also where banking business model has significantly changed over the last 20 years. It is surely a huge challenge to extract common factors and possible size of impacts from the past crises, which can in turn be applied to possible future crises. Thus, the third step should be to restore "psychohistory" or a technique to extract historical lessons that can be applied to future events. Current discussions of scenario analysis in the context of AMA (Advanced

Measurement Approach <for operational risk>) implementation could have great potentialities to bring in more objectivity and comprehensiveness in scenario making process. We have no reason to restrict the use of this framework and technique to the area of operational risk.

A suite-of-models approach to stress-testing financial stability

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Norges Bank Financial Stability

Abstract

This paper presents a suite of models developed to stress-test financial stability. A macro model is linked to micro data-based models for households, firms and banks. The macro model includes credit- and consumer confidence-driven house prices and feed-back effects from credit and house prices to the real economy, i.e. a financial accelerator. The consumer confidence effect helps us mimic non-linearity in the housing market. We use the macro model to design stress scenarios, which are fed into the three micro models. The household and firm models enable us to analyse pockets of credit risk. The bank model sums it all up by providing estimates of bank profitability and capital adequacy.

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1   *Introduction*

In parallel with the strong growth in financial markets and more frequent instances of widespread financial distress during the last decades, financial stability has become an increasingly important objective in economic policymaking. In addition to a role in crisis resolution, many central banks have a clear mandate to promote financial stability.[1] The financial stability role involves analysis of potential threats to financial stability, assessment of the present situation and the outlook ahead, policy actions based on the risk assessment and external communication.

Financial stability is a complex concept and will in general depend on a wide range of risks and risk drivers. At present, neither academia nor central banks have reached a consensus definition of financial stability, a unified understanding of how to best model and analyse it, or concluded on how to promote financial stability most efficiently.[2] Probably spurred by the IMF's and the World Bank's Financial System Assessment Programme (FSAP), see IMF and World Bank (2003) and Hagen, Lund, Nordal and Steffensen (2005), many central banks have developed or are developing models for macro stress-testing. The purpose is to analyse the robustness of the financial system if severe negative events should occur. The methodology applied by central banks in this work differs, however.[3]

Norges Bank, as an inflation-targeting non-supervisory central bank, has adopted a macro-prudential approach to financial stability with strong focus on risks that originate and develop outside the financial system, i.e. external risks.[4] Previous crises in financial systems have often demonstrated a close linkage between financial stability and the health of the real economy, see, e.g., Crockett (1997).

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[1] See, e.g., Roger W. Ferguson Jr., Vice Chairman, Board of Governors of the Federal Reserve System, "Should Financial Stability Be An Explicit Central Bank Objective?", speech given at Challenges to Central Banking from Globalized Financial Systems, Conference at the IMF in Washington, D.C., September 16-17, 2002.

[2] For a discussion of alternative definitions, see Bank for International Settlements (1998), Schinasi (2004), Allen and Wood (2006) and Aspachs, Goodhart, Tsomocos and Zicchino (2007).

[3] For a review and discussion of alternative methodologies, see RTF Stress Testing Subgroup (2008), Sorge (2004), Sorge and Virolainen (2006) and ECB (2006). For the approaches chosen by different central banks, see, e.g., also Boss, Krenn, Puhr and Summers (2006) for Austria; Danmarks Nationalbank (2008, pp. 81-88) for Denmark; Bank of Finland (2007, pp. 33-34); Laviola, Marcucci and Quagliariello (2006) and Marcucci and Quagliariello (2005) for Italy; Bank of Japan (2008, pp. 60-70); De Nederlandsche Bank (2006, pp. 35-41), van den End, Hoeberichts and Tabbae (2006) and Lelyveld and Liedorp (2006) for the Netherlands; Jiménez and Mencía (2007) for Spain; Sveriges Riksbank (2006, pp. 75-88) and Åsberg and Shahnazarian (2008) for Sweden; Lehman and Manz (2006) for Switzerland; Haldane, Hall and Pezzini (2007) and references therein for UK.

[4] See keynote address by Governor Svein Gjedrem at the conference" Monetary Policy and Financial Stability", hosted by the Austrian National Bank in Vienna, May 2005, www.norges-bank.no/cgi-bin/pr.cgi.

Due to the complexity of financial stability and its dependence on a wide range of risks and risk drivers, one cannot expect one single model to include all important aspects or to be the preferred model in all analyses. Bårdsen, Lindquist and Tsomocos (2008) list ten desirable characteristics that the ideal financial stability model should possess. A financial stability model that encompasses all important issues would be very complicated, and Bårdsen et al. argue that a suite of models is probably needed. In addition, different datasets, such as aggregate macro data and micro data for different groups of agents, are likely to contain complementary information.

At Norges Bank, we have chosen to follow a suite-of-models approach, which enables us to take advantage of several data sets. The suite of models consists of a small macro model and micro data models for companies, households and banks. Much emphasis is put on linking the different models together as a system. This enables us to develop internally consistent scenarios on the different models. Alternatively, we may use the system to cross-check the output from the different models. All models are applied in the regular assessment and stress-testing of the financial system. Our priorities in the development of this system of models reflect, among other things, Norges Bank's definition of financial stability, as given in the bi-annual financial stability report, see Norges Bank (2007).

Financial stability means that the financial system is robust to disturbances to the economy and is able to channel funding, execute payments and redistribute risk in a satisfactory manner. Experience shows that the foundation for financial instability is laid during periods of strong growth in debt and asset prices. Banks play a central role in providing credit and executing payments and are therefore important to financial stability.

In accordance with this definition, we focus on banks and developments that can adversely affect banks, on credit growth and on asset prices. The emphasis is on external risks, as well as on feed-back effects from financial stability to the real economy.

Loans to domestic firms and households constitute about 70 per cent of the banks' total assets, while interbank and other fixed income instruments claims each constitute about 10 per cent of total assets. Only 1-2 per cent of assets are stocks. We therefore concentrate on credit risk, as driven by the development in debt holders' debt-servicing capability and collateral values.

Market risk, liquidity risk and operational risk can be evaluated in the bank model.[5] Our system for stress-testing does not include the endogenous risk created by self-enforcing processes between credit, market and liquidity risk that, it is often argued, are present. These processes would have increased the correlation between risks in stress scenarios. Neither does our system include contagion risk, i.e. the risk that difficulties in one financial institution may spread to other institutions and cause system-wide problems. Analyses on Norwegian data show, however, that contagion risk due to banks' credit risk exposures in the interbank market or to common third parties, is in general relatively small. The recent liquidity crisis in the international and domestic markets represented a form of contagion that is hard to model within our framework. See, however, Dungey, Fry, González-Hermosillo, Martin and Tang (2008) for an analysis of contagion in six recent financial crises.

Section 2 describes the suite of models developed at Norges Bank for stress-testing financial stability.[6] Section 3 presents stress-testing system simulations, and Section 4 summarises. In Appendix 1 and 2 respectively, we describe the small macro model and the bank model in more detail. Appendix 3 gives a detailed description of a bank model simulation.

2   A system for stress-testing

The models developed for stress-testing at Norges Bank, i.e. a small macro model and micro data-based models for the corporate, household and bank sector, can be simulated independently or as an integrated system. The structure of the system is recursive; with output from the macro model being used as input in the firm, household and bank models. We use the macro model to design alternative scenarios for the economy, primarily extreme stress scenarios, and follow the transmission of initial macro shocks through the set of models to get a more detailed picture of the consequences. Hence, we follow a top-down approach to study banks' credit risk. For a discussion of the pros and cons of this approach, see, e.g., Čihák (2007).[7] The relationship between the models is illustrated in Figure 1 below.

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[5] In addition to the bank model included in the stress-testing system, Norges Bank has developed a risk index for individual banks that predicts the probability of illiquidity or insolvency, see Andersen (2008).
[6] To allow for interaction with monetary policy (see Haugland and Vikøren, 2006), a financial stability satellite has been developed and linked to a New-Keynesian DSGE model used for inflation forecasting and policy analysis. For a short presentation of the satellite with an application, see Berge et al. (2007).
[7] Lessons learnt from simulating the micro data-based models may lead to a redesign of the stress scenario in the macro model.

The corporate and household sector models provide estimates of how individual agents or groups of agents are affected by alternative scenarios. These models are used to identify pockets of credit risks. Information on how debt, debt-servicing capability and debt at risk are distributed across firms and households can be important for the assessment of financial stability.[8] This information can be aggregated to produce estimates of the corporate sector's and household sector's debt at risk. These risk measures represent an upper limit to expected losses, since they do not take into account that loss-given-default (LGD) will normally be less than 100 per cent of debt at risk.

Figure 1. A system for stress-testing
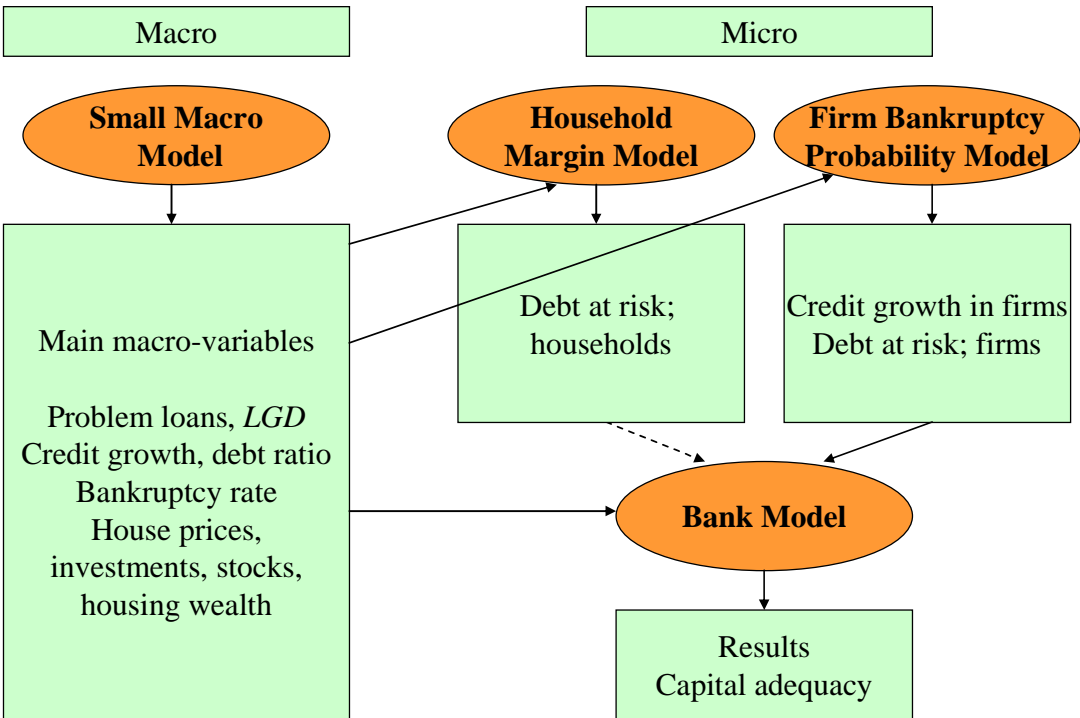
To calculate the impact of stress scenarios on the five largest banks' results and capital adequacy, output from both the macro model and the firm model are fed into the bank model. While growth in banks' losses on loans to firms is taken from the macro model, the distribution of losses across banks is done by matching information from the firm model and the bank model. We match information on how debt at risk is distributed across industries with information on banks' loans to different industries. Output from the household sector

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[8] Debt at risk is defined as bank debt multiplied by the bankruptcy probability in the corporate model and as the debt held by households with a negative margin in the household model. Household's margin is defined as income after tax minus standard living costs and interest payments.

model is used as additional information in an ad hoc way when we assess the strength of the banks, see ECB (2006, p. 149). This is illustrated by the dotted line in Figure 1. We will now present the different models in more detail.

2.1   The small macro model: SMM

Rather than developing a new macro model for the Norwegian economy, it was decided to build on an existing model. At Norges Bank Monetary Policy, a New-Keynesian DSGE type of model has been developed to support monetary policy decisions, see Brubakk, Husebø. Maih, Olsen and Østnor (2006). This model has forward-looking rational expectations. To extend this model with variables of interest to us and feed-back effects from financial variables to the real economy is complicated. We therefore decided to work with a model that is simpler in this respect. The chosen model is an estimated equilibrium-correction model, for a presentation of this model, see Bårdsen and Nymoen (2008) and Chapter 9 in Bårdsen, Eitrheim, Jansen and Nymoen (2005). This model is a macro model with, in general, backward-looking rather than forward-looking rational expectations. This simplifies the model and makes it fairly easy to extend and design the model to better fit our purpose.

Our extended version of the Bårdsen et al. model, which is called the small macro model (SMM), includes households' expectations about their own financial situation and the Norwegian economy, i.e. a consumer confidence indicator. These expectations need not be model-consistent, however, and the household sector may be overly optimistic or pessimistic.[9] At present, the extended model also includes estimated equations for household debt, house prices, housing investments, firms' bankruptcy rate, banks' problem loans to households and firms respectively and a GDP equation with feed-back effects from credit and house prices to real activity.[10] The interest rate works through three transmission channels; the exchange rate channel, the demand channel and the housing-credit market channel.

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[9] The consumer confidence indicator is based on a quarterly survey by TNS Gallupp. If more households are optimistic than pessimistic it takes on positive values, while the opposite is true if most households are pessimistic. It takes the value zero in the neutral case. In stress scenarios, the role of the consumer confidence indicator is to create a mismatch between house prices and the real economy, i.e. to create inconsistent price signals, bubbles and busts. This variable is exogenous in the macro model.
[10] Most of these equations are described in detail in Norges Bank's Economic Bulletin (and in Norges Bank's reports on Financial Stability), see Jacobsen and Naug (2004), Jacobsen and Naug (2005), Jacobsen and Kloster (2005), Jacobsen, Solberg-Johansen and Haugland (2006) and Berge and Boye (2007).

Problem loans consist of non-performing loans, i.e. defaulted loans, and performing loans with a high probability of becoming non-performing in the near future according to banks' financial statements. We use a loss-given-default (LGD) approach to predict banks' book losses, i.e., losses are calculated as a share of predicted problem loans. In simulations, we generally assume the loss-to-problem-loan ratio to be time-varying and reflect the development in collateral values, i.e. house prices, among other things.[11] Hence, in SMM, credit risk depends on the macroeconomic variables that determine problem loans and house prices. Internationally, there are a growing number of papers linking credit risk to macroeconomic variables using econometric models, see, e.g., Pesola (2005) or Čihák (2007) for a brief review. Appendix 1 gives a short description of the main equations in the present version of SMM.

Some properties of SMM are of particular interest in financial stability analysis.[12] The house price equation includes credit volume as well as the consumer confidence indicator described above. Hence, both an increase in available credit that gives rise to lending booms and overly optimistic households will boost house prices. Higher house prices increase collateral values, which in turn fuels credit growth. Lending booms typically coincide with highly optimistic agents. In a simple way, SMM internalises the co-movement, and also the procyclicality, of credit, asset prices and agents optimism discussed in the literature, see Borio and Lowe (2002), Allen (2005) and Goodhart and Hofmann (2007, particularly Chapters 1 and 6).

In SMM, house prices and credit volumes affect domestic activity, which is represented by a reduced-form aggregate demand equation. The house price effect includes a wealth effect in households' consumption, since house prices affect household wealth, and a positive effect from house prices to housing investments. The latter is consistent with our housing investment equation. While corporate credit affects GDP in the short run, household credit has

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[11] We plan to develop an alternative equation for banks' losses with households' and firms' debt at risk from the micro models and collateral values from the macro model as explanatory variables. The system will then give two alternative estimates on banks' losses. By comparing these two loss measures, we can evaluate how important is the information on heterogeneity and the distribution across industries of debt, income and other variables for financial stability analyses. At present, a cross check of the output from the macro and micro models is made on the basis of predictions on problem loans in the macro model and debt at risk in the micro models.

[12] SMM has proven useful also in other analyses than stress-testing. An early version of SMM has been used to analyse the consequences for inflation and financial stability of a house price shock and a credit shock, when the inflation-targeting central bank explicitly takes financial stability into account, see Akram, Bårdsen and Lindquist (2007).

long-run effects on GDP. The short-run effect is interpreted as reflecting frictions in the credit market, while the long-run effect points towards a form of rationing of the household sector.[13]

Through house prices and credit, SMM includes a financial accelerator with feed-back effects from financial markets to the real economy. A boom and subsequent bust in house prices, e.g. caused by changes in consumers' expectations as given by the consumer confidence indicator, will cause or amplify business cycles. Hence, in SMM, house prices have a role as both a source and transmitter of macroeconomic fluctuations. Furthermore, we can design scenarios in SMM with a credit-crunch were credit growth is severely cut back by a tightening of credit supply. For a discussion of the financial accelerator and the role of asset prices, see, e.g., Bjørnland and Jacobsen (2008), Bernanke, Gertler and Gilchrist (2000), Kiyotaki and Moore (1997) and Bernanke and Gertler (1989).

In stress-testing, low probability scenarios are designed, where the consequences of major adverse shocks to key financial stability variables are analysed. When stress-testing within our reduced-form and near-linear type of macro econometric model, we may suffer from the Lucas critique (Lucas, 1976). Agents' behaviour, and hence our reduced form equations, may be non-invariant to big stress events. A solution to this problem is not simple, however. First, even if we formulated a model with 'deep structural parameters', we would need to condition that on a specific representation of the utility function of agents. One can argue that in severe stress events, the utility function itself may shift, and the shift may depend on the specific stress scenario. Second, data from episodes with severe stress that could help us identify stress behaviour are rare, while the information needed to conduct different stress tests that are robust to the Lucas critique seems to be interminable. However, the estimation period of the core of SMM, i.e. the Bårdsen et al. model, includes the previous banking crisis in Norway. These equations pass standard stability tests, and we conclude that the core of SMM is invariant to this particular stress event. The added equations to this core model are in general estimated using a shorter sample, however. This is partly due to a lack of data and partly due to difficulties in finding overall stable equations.

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[13] For a review of the literature on credit market frictions on the firm side, see Hubbard (1998). For a discussion and analysis of household rationing, see Jappelli and Pagano (1989). Even if a high debt-to-income ratio in Norwegian households may suggest that rationing is not very important, the debt compared to their housing wealth, i.e. collateral value, gives some support to the opposite assumption.

Stress-testing is not forecasting, however. It is analysis of the robustness of the financial system to possible, but low-probability, events. The benefit from a stress test should not lie in the model being able to replicate the true consequence of the stress scenario, but rather in the model to help identify risks and how these risks may transmit through the economy and end up as negative events for banks.

Furthermore, SMM, as well as the other models in our stress-testing system, have proven to be helpful tools in our external communication. As a non-supervisory central bank, communication is an important instrument in promoting financial stability. For communication, we need a transparent model that is suitable for designing multivariate scenarios that illustrate major current or future risks to financial stability, see Drehmann (2008). In SMM, both the origin of risks, i.e. the triggers, and important (reduced-form) transmission channels, through which different shocks evolve, are represented. Furthermore, SMM includes variables measuring the fragility of both debt holders and collateral values, which are important for assessing the probability of a crisis and predicting the severity of a crisis if it occurs.

We continue to develop SMM to make it even more useful for designing and conducting stress tests. Much emphasis is put on improving the representation of feed-back effects from credit and housing markets to the real economy and endogenous risks drivers, i.e. second round effects.

2.2   The corporate sector model: SEBRA

SEBRA is a model designed to analyse the default and bankruptcy probabilities of all Norwegian limited liability companies. These probability estimates are used to assess the credit risk associated with bank loans to the corporate sector in more detail than in the macro model. Our data set consists of annual financial statements and bankruptcy information from 80 000-140 000 individual companies, starting in 1988. Bankruptcy probabilities are estimated as a generalised logistic function of accounting-data indicators representing earnings, liquidity, financial strength, industry, age and size of the company. Probabilities of default are estimated using the same variables in combination with a statistical model for

misclassification of the dependent variable.[14] The accuracy rate of the model is relatively high; the error I and error II probabilities are balanced at about 20 per cent of all actual bankruptcies and non-bankruptcies. Furthermore, averaged bankruptcy probabilities are very close to predicting the actual frequency of bankruptcies in any year and in different risk categories. The model is described in more detail by Bernhardsen (2001), Bernhardsen and Larsen (2007) and Bernhardsen and Syversten (2008).

The individual default probabilities are multiplied with the debt held by each company to produce the total *bank-debt at risk* held by companies. In simulations, this risk-measure is combined with a model for loss-given-default on corporate loans at the macro level. The latter is designed to fit our loss-predictions with banks' losses on corporate loans.[15]

Output from the macro model is used to project the financial statement of each firm using, to a large degree, estimated equations. The probability of default of each firm is then computed for the baseline scenario and stress scenario using the SEBRA model. By aggregation, debt at risk is derived for each industry. The method for projecting financial statements is described in detail in Bernhardsen and Syversten (2008), which also documents the results of a back-testing exercise of the method. This exercise, which applies the actual development in the macro variables, shows that projections starting in each year between 1988 and 2003, and reaching five years ahead, perform fairly well at the aggregate level.

The predictions in SEBRA on firms' debt growth and debt at risk at the industry level are used as input in the bank model. Hence, in the bank model, output from SEBRA supplement the predictions on macro variables in the macro model. In SEBRA, firms' debt growth is predicted using an estimated equation with the debt growth of a macro firm as the endogenous variable and GDP, inflation and the interest rate on bank loans as explanatory variables. The macro firm is defined by the value-weighted growth rate in moving balanced samples, i.e. by firms that are present at t and t-1. (See Bernhardsen and Syversten, 2008.)

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[14] See Hausman, Abrevaya and Scott-Morton (1998) for a discussion of the misclassification approach to binary dependent models.

[15] Loss given default (LGD) at the aggregate level is defined by the ratio of bank sector loan losses to potential losses. Although being a heuristic measure, it is model dependent so that an under prediction of potential losses will lead to an over prediction of LGD and vica versa. Thus misalignment of levels will cancel in projections of future loan losses as these are constructed by the product of projected LGD and potential loan losses.

2.3   The household model: Financial margins

The household sector model is designed to predict the probability of households defaulting on their bank loans. We do not observe default by individual households, however, and we instead proxy individual default probabilities by households' financial margins. Financial margin is defined as household income minus taxes, minus interest payments and repayment on debt and minus standard living costs. Repayment is calculated assuming a linear repayment profile over 20 years. The debt of households with a negative margin is termed *debt at risk*. For a discussion of households' margins, see Vatne (2006, 2007).

Our data set consists of annual household survey data from the Income and Property Statistics of Statistics Norway over 1986-2003. This survey includes 8 000-25 000 households per year. From 2004 on, we use tax return data from all Norwegian households. This gives us data for more than 2 millions households per year. Data on standard living costs are mainly from the National Institute for Consumer Research. These costs depend on key characteristics of the household. To these costs we add our own estimates of necessary housing maintenance costs and heating costs.

In forward projections of the household sectors' financial margins in different scenarios, the population is held fixed. Growth in income, debt and interest rates are taken from the macro model, and standard living costs are adjusted for consumer price inflation, which is also taken from the macro model. With respect to income, living expenses and interest rate, the same growth rate is applied for all households. Household debt growth is treated differently, however: If we assume that all households have the same debt growth, too many households with small margins at the outset may be pushed over the edge. Households with small margins are often recent home buyers that do not plan to increase their debt in near future. On the other hand, we do not want to restrict credit growth to households with a relatively large margin only. This problem of distributing debt growth on households is mitigated by dividing the households into 64 groups according to age, income and financial margin before and after new debt. The debt growth from the macro model is then distributed across these 64 groups according to the observed debt growth across the same groups from 2004 to 2005. This procedure gives us a projection of the financial margin of every individual household in the sample, and thus a distribution of households according to their financial position.

In addition, our data enables us to take into account that households' liquid financial wealth may serve as a buffer for households with a negative margin. Our data show, however, that households with a negative margin have relatively small financial buffers. Most of households' liquid financial wealth is held by households that do not experience a negative margin, not even in our stress scenarios. With respect to non-financial assets, i.e. real property, we only have tax-report valuations. These may deviate significantly from market values. At a later stage we expect to receive more reliable data on each household's real property wealth, starting with data from 2006.

2.4   The bank model: The five largest banks

The present bank model is a non-behaviour model. It includes disaggregated annual accounting information from the five largest Norwegian banks, i.e. DnB NOR Bank, SpareBank 1 SR-Bank, Sparebanken Vest, SpareBank 1 SMN and SpareBank 1 Nord-Norge. In 2007, these five banks had 45 per cent of total assets in the Norwegian banking industry. The market share of foreign branches and subsidiaries was 34 per cent.

Each bank is represented by a number of variables that are taken from their annual financial statement, end-year balance-sheet and capital adequacy reports (see Appendix 2 for a more detailed description of the bank model). The banks' accounts are projected forward by linking their main income and cost items to variables determined in the macro model. Banks' results affect their capital position, and the end-output of the bank model are banks' results and capital adequacy. See Appendix 3 for a more detailed description of the assumptions made in the bank model.

The present bank model does not enable us to evaluate how the macro scenarios affect individual bank behaviour. For this we would need a behavioural model of individual banks. For a more complete representation we would also need contagion between banks and feedback effects from bank behaviour to the real economy, see Goodhart, Sunirand and Tsomocos (2004, 2005, 2006a, 2006b).

3   Simulations on the stress-testing system

We will now demonstrate some of the properties of our stress-testing system. First we simulate the macro model assuming three different shocks, and then we put these three shocks together as a multivariate stress scenario. The output from this macro stress scenario is used as input in the firm, household and bank models. We start, however, by describing the background for this scenario.

For several years, the level of economic activity in Norway has been high, while core inflation and interest rates have been low. As a consequence, growth in credit and house prices has been high for a long period. At present, the unemployment rate is very low, around two per cent of the labour force, the debt-to-income ratio of households is very high, around 200 per cent of disposable income, and house prices are high according to most measures; see, e.g., Norges Bank (2007). Since summer 2005, the key policy interest rate has increased from 1¾ per cent to the present 5½ per cent. More recently, inflation in consumer prices has picked up, and growth in house prices and household debt has declined. According to Norges Bank's lending survey, there has been a tightening of banks' credit standards, see Norges Bank (2008a).

We design the stress scenario in the following way: Spurred by the increase in international prices on food and energy, domestic price and wage inflation increase. This sends price inflation above the policy target. In the model, this causes interest rates and unemployment to increase and growth in house prices to decline. Although the isolated macroeconomic implications of our price-wage shock are moderate, we assume that the rise in interest rates and the downward pressure in the housing market trigger a fall in consumer confidence. The fall in consumer confidence builds up to a severe confidence crisis as unemployment increases, and very much due to this, the economy enters a significant downturn. Finally, we assume that the turmoil in international credit markets and the fall in collateral values as house prices decline make banks adopt a much more restrictive lending policy. This generates a severe credit squeeze, as credit supply falls more than credit demand.

In the following we first present the wage and price shock, the shock to households' expectations and the fall in credit supply as three independent shocks. Then we present the multivariate stress scenario, which combines the three shocks. We simulate the model from

first quarter of 2007 to fourth quarter of 2011, denoted 2007Q1 – 2011Q4, and the shocks are introduced from 2008Q1 on. We compare the three shocks and the multivariate stress scenario with a common baseline scenario.

3.1   The wage and price shock

The wage and price shock is assumed to build up and fade out over the simulation period. We do this by adding a sequence of single-quarter shocks to the price and wage growth series. We add a maximum of 0.4 and 0.6 percentage points, respectively, to the four-quarter rise in prices and wages. Figure 2 shows the effect of the wage and price shock on selected variables measured as deviations from our baseline scenario in percentage points.

The four-quarter rise in consumer prices and wages is at most about 1½ percentage points higher than in the baseline scenario. The higher inflation rate causes the central bank to increase the interest rate. The higher interest rate causes the Norwegian krone to appreciate. The *exchange rate channel* of the interest rate dampens the initial price shock through reduced growth in import prices measured in kroner. Due to sticky prices, the real exchange rate also appreciates. As a result, the competitiveness of Norwegian industries deteriorates, output declines, unemployment increases and prices and wage growth decline. Hence, the exchange rate channel affects prices and wages indirectly through GDP and unemployment.

Furthermore, the interest rate affects the real economy through financial markets, where a higher money market interest rate is channelled into banks' lending rates. Higher lending rates affect GDP negatively. This is the *demand channel* found in main-stream monetary policy models, see, e.g., Ball (1999).

Figure 2. The effect of a shock to domestic wage and price inflation on selected variables. Deviations from the baseline scenario in percentage points. Quarterly data[1]
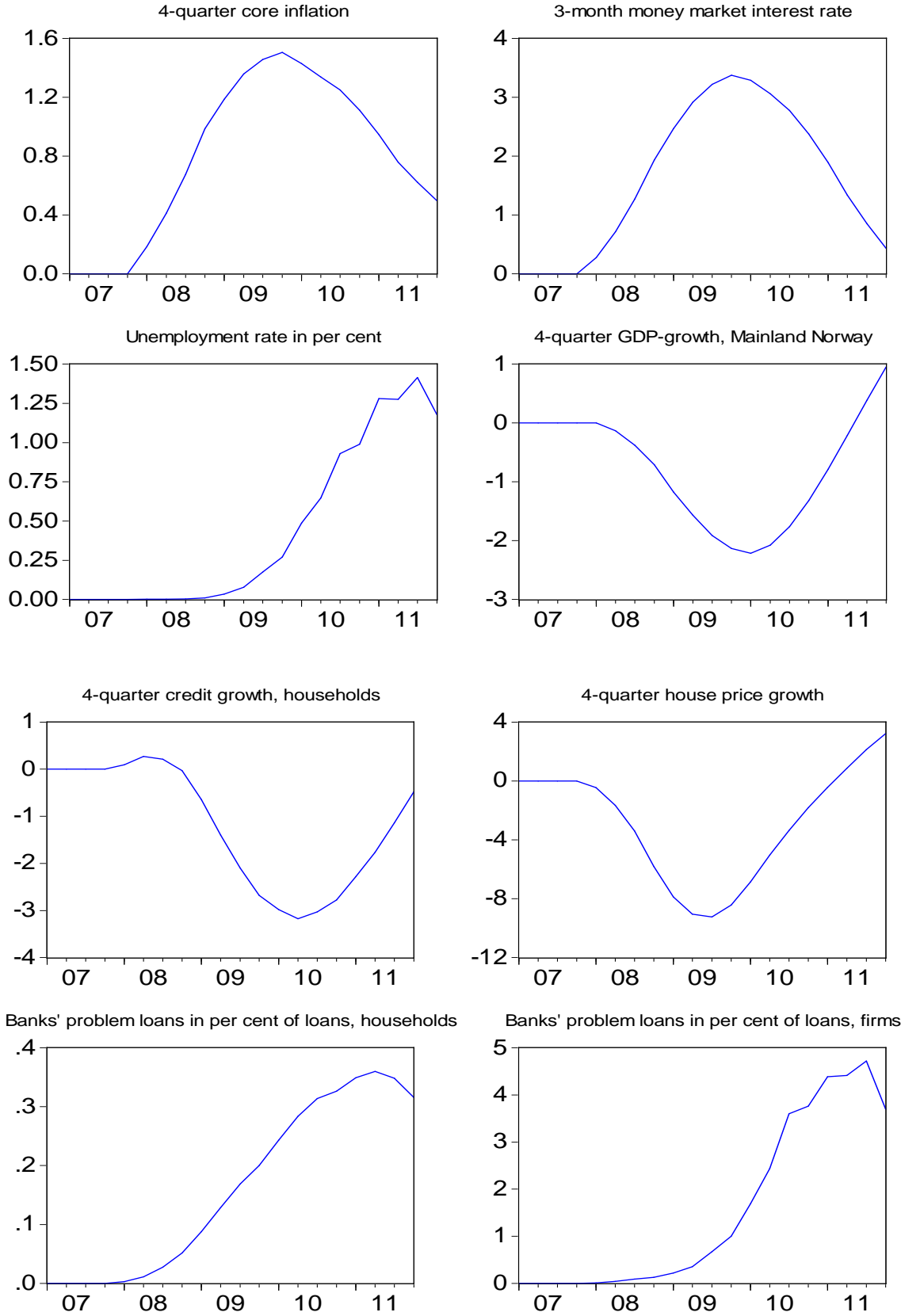
The model also includes a *housing-credit market channel* that is related to the financial accelerator, whereby interest rates affect output through house prices and credit. Higher interest rates increase the user cost of housing consumption, and as a result, housing demand and house prices decrease. Falling house prices reduces the collateral value of housing and affects consumption, housing investments and credit growth negatively. This drives down growth in GDP. A credit effect in the house price equation implies that interest rates also affect house prices indirectly through reduced credit growth. The interpretation of this effect is as follows: As interest rates increase, the debt-servicing capacity of home buyers falls and available credit in the housing market declines. This curbs the rise in house prices.

The volume of problem loans increases. Compared to the baseline scenario, the increase in the problem loans of households is very small, less than ½ percentage point at the most. Hence, despite an increase in the debt-servicing burden as interest rates increase by as much as 3½ percentage points, this does not cause large problems as long as the increase in unemployment stays modest. Firms' problem loans increase by more, and become close to 5 percentage points higher than in the baseline scenario. This increase reflects the higher interest rate, the stronger krone, which reduces domestic firms' competitiveness, and also reduced domestic demand due to higher unemployment. Hence, increased unemployment is likely to hit banks through the corporate sector rather than through the household sector of the economy. In Norway, about 80 per cent of total household debt is mortgages. Households that experience reduced financial margins and debt-servicing problems tend to cut back on consumption spending rather than default on their mortgage loans. The main effect of the deteriorated financial position of households is thus on firms' sales, income and debt-servicing capability.

3.2   A negative consumer confidence shock

In this simulation, we want to create a significant collapse in the housing market, and we do this by designing a drop in consumer confidence. (See footnote 8 for an explanation of the consumer confidence indicator and its role.) Our shock to consumer confidence starts in 2008Q1, builds up and fades out over three years. We calibrate the shock based on experiences from the spring of 2003, when house prices and consumer confidence both fell.

Figure 3. The effect of a shock to consumer confidence on selected variables. Deviations from the baseline scenario in percentage points. Quarterly data[1]

4-quarter core inflation

3-month money market interest rate

Unemployment rate in per cent

4-quarter GDP-growth, Mainland Norway

4-quarter credit growth, households

4-quarter house price growth

Banks' problem loans in per cent of loans, households

Banks' problem loans in per cent of loans, firms
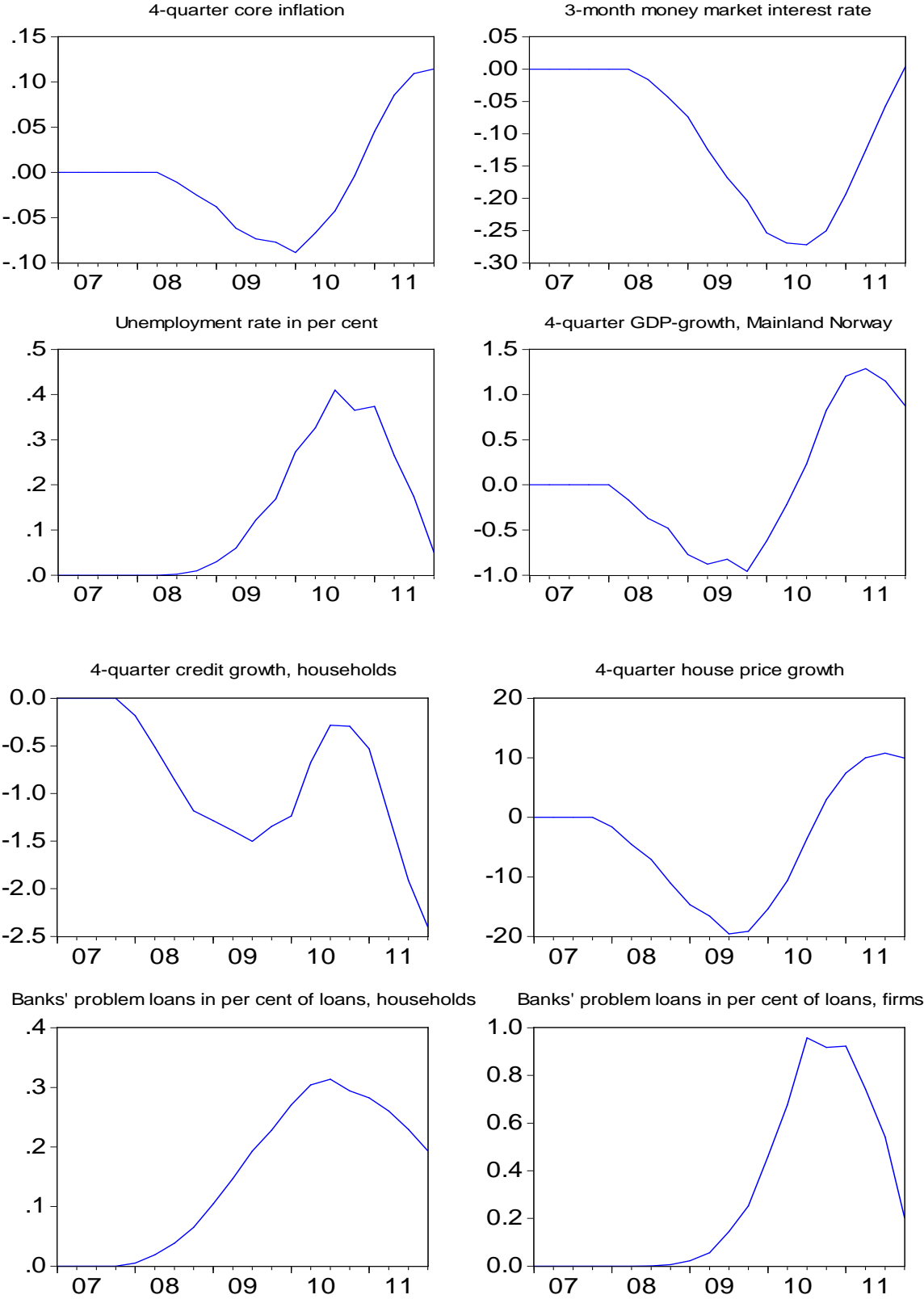
[1] Starting in 2008Q1, we add a sequence of single quarter shocks the consumer confidence indicator. The value of the indicator is: 2008: (0.5, 1.0, 1.0, 1.5); 2009: (2.0, 2.0, 2.5, 2.0); 2010: (1.5, 1.0, 0.5, 0.0); 2011: Zero, which is the neutral value of the indicator.

The negative shock to the consumer confidence indicator is about three times the amplitude of spring 2003. In addition, the indicator stays negative, indicating pessimistic households, for a longer period. Figure 3 shows the effect of the consumer confidence shock on selected variables, measured as percentage point deviations from our baseline scenario.

The macro model predicts that the fall in consumer confidence has a direct negative effect on growth in house prices, and compared to 2007, house prices are down by about 20 per cent in 2010. The fall in house prices affects growth in GDP negatively, and as a consequence, unemployment increases and domestic price and wage inflation falls. The decline in house prices also dampens households' credit growth. The central bank responds by lowering the interest rate, which stays below the rate in the baseline scenario until the very end of the simulation period. A lower interest rate helps the economy to recover, and growth in GDP, credit and house prices increases again. The development in these variables also reinforces each other, as explained in section 2.1.

Compared to the baseline scenario, this shock to consumer confidence increases households' and firms' problem loans at the most by only 0.3 and 1.0 percentage points respectively. The effect on households' debt-servicing capability and domestic demand is modest, since the increase in unemployment and fall in wage growth are relatively small, and since interest rates are reduced. Furthermore, firms are helped by a depreciation of the exchange rate that increases domestic firms' competitiveness relative to foreign firms.

3.3   A credit squeeze

We now look at the effects of a credit squeeze in our small macro model, i.e. a situation were credit supply to households and firms falls significantly. A more restrictive lending policy by banks can be motivated by the uncertainty from the continuous turmoil in international credit markets and from expected falls in collateral values as house prices decline.

Figure 4. The effect of a shock to credit supply on selected variables. Deviations from the baseline scenario in percentage points. Quarterly data[1]
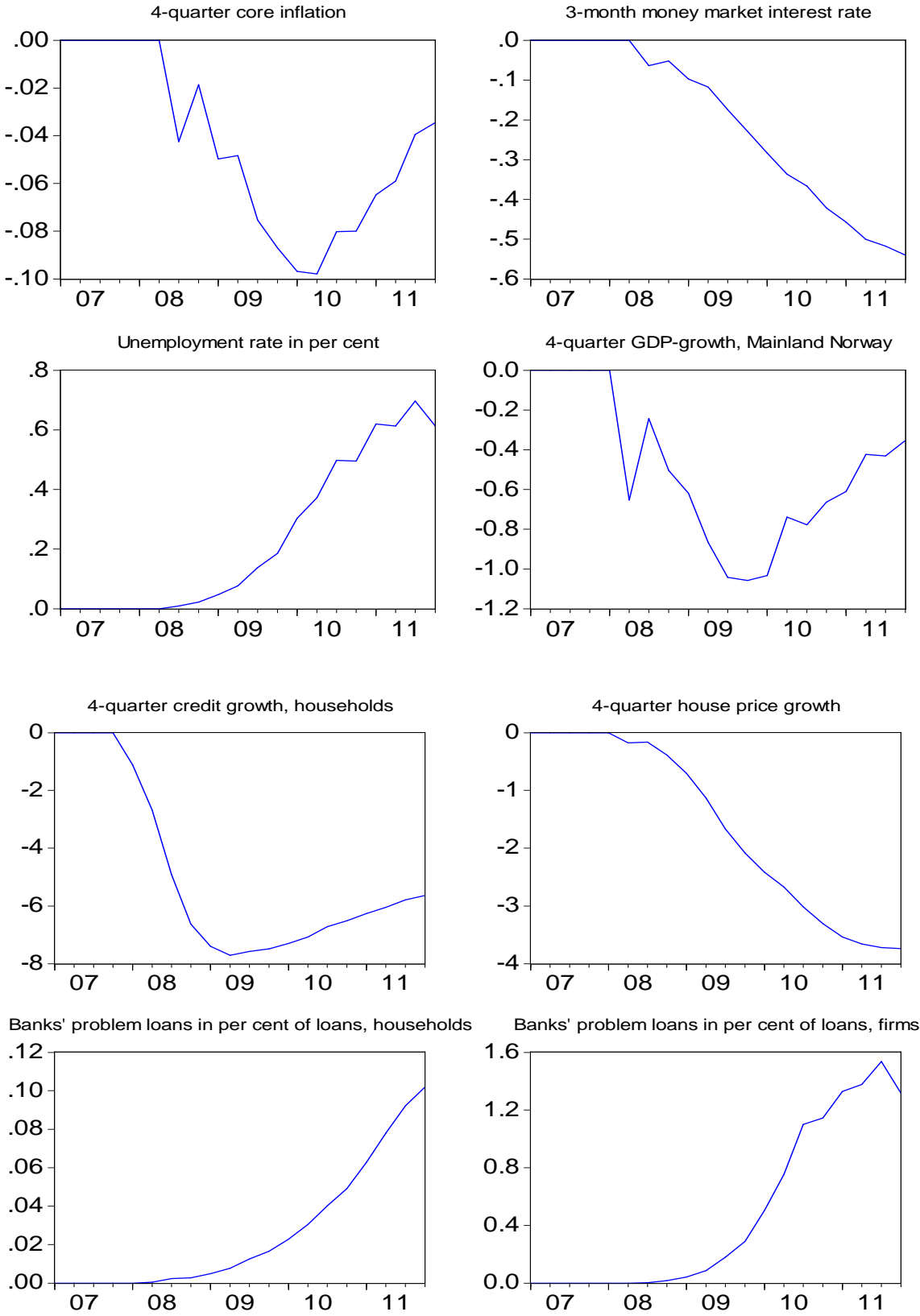
Figure 4 shows the results of the simulated fall in credit growth. The decline in credit growth has a direct negative effect on house prices and GDP, which cause inflation to decline, unemployment to increase, credit growth to fall even more, and the interest rate to be reduced. Monetary policy helps the economy improve, but banks' problem loans increase. Problem loans to households increase by only 0.1 percentage points compared to the baseline scenario, but problem loans to firms increase by more than 1½ percentage points compared to the baseline scenario. Firms' debt-servicing capability is hit by the fall in domestic demand caused by the reduction in available credit and increase in unemployment. This negative effect is partly counteracted, however, due to improved competitiveness as the real exchange rate depreciates when the interest rate falls.

3.4   A multivariate stress scenario

Finally we simulate a multivariate shock, where wage and price inflation increases, consumer confidence is eroded and banks' lending policy tightens to become a credit squeeze. This stress scenario combines the three shocks shown in Figures 2 - 4. The effects of this scenario on some selected variables are presented in Figure 5.

In this stress scenario, the positive impulse to monetary policy from the price and wage shock dominates the negative impulses from the fall in consumer confidence and credit growth. As a result, the three-month money market interest rate increases by close to 3 percentage points compared to the baseline scenario. This causes the exchange rate to appreciate, which erodes the competitiveness of domestic firms. As a result, GDP-growth declines even more and unemployment increases by almost 3 percentage points. This combined shock causes the housing market to collapse, and house prices fall by 35 per cent from 2007 to 2010. This is comparable to the experience from the 1988-1992 banking crisis in Norway, when house prices fell by about 30 per cent. The higher interest rate and negative demand shocks curb inflation, and the interest rate starts falling. This causes growth in GDP to pick up, unemployment to fall and the housing market to improve.

Figure 5. The effect on selected variables of a combined shock with high wage and price inflation, a fall in consumer confidence and a credit squeeze. Deviations from baseline scenario in percentage points. Quarterly data[1]

[1] Starting in 2008Q1, we add a sequence of single quarter shocks to price and wage inflation, to consumer fonfidence and to credit growth to households and firms. At the most, we add 0.4 and 0.6 percentage points to the 4-quarter rise in prices and wages respectively. The value of the consumer confidence indicator is: 2008: (0.5, 1.0, 1.0, 1.5); 2009: (2.0, 2.0, 2.5, 2.0); 2010: (1.5, 1.0, 0.5, 0.0); 2011: Zero, which is the neutral value of the indicator. We reduce the 4-quarter growth in household credit and firm credit by 2 (in general) and 20 (at most) percentage points respectively.
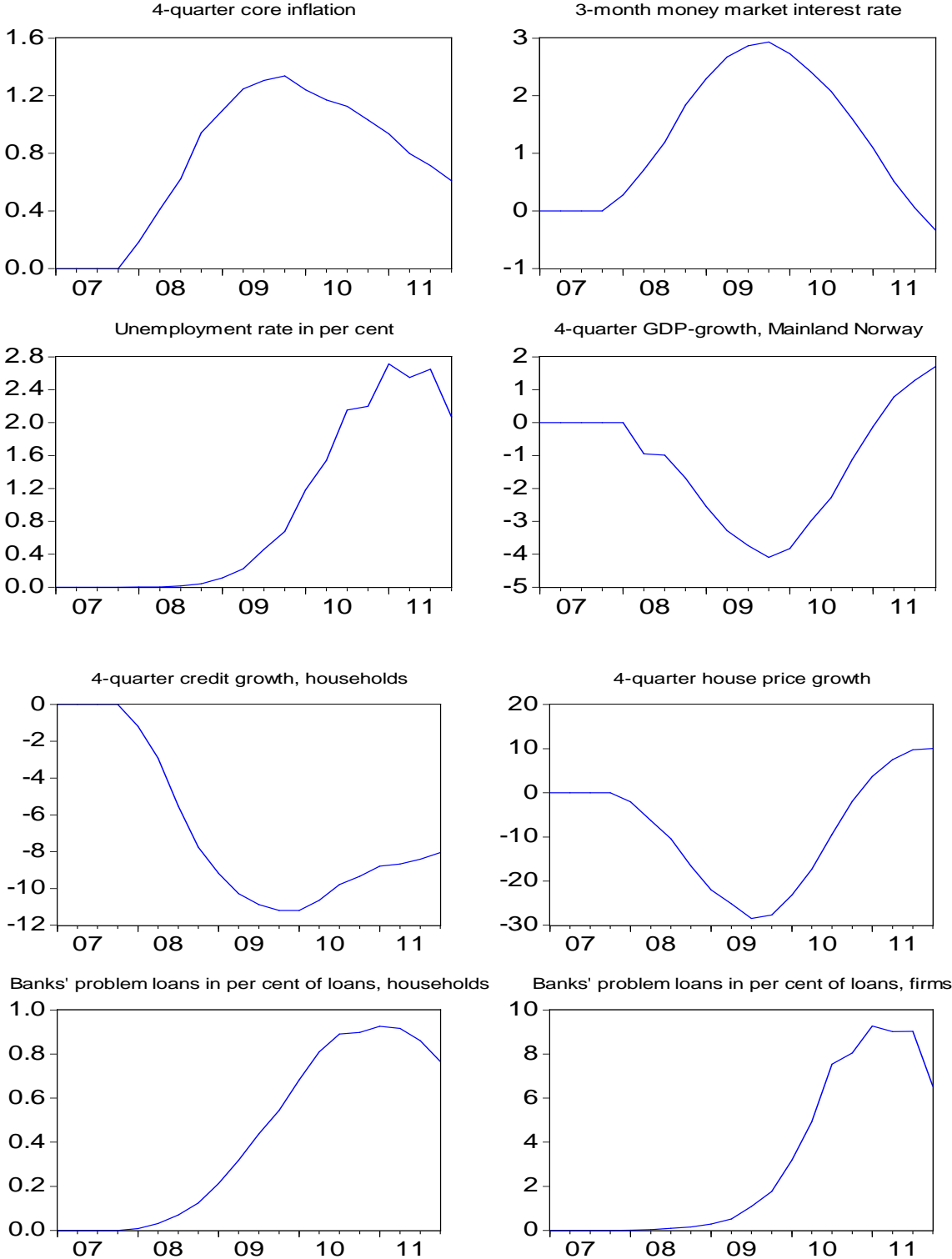
In this multivariate scenario, households' problem loans increase by about 0.9 percentage points compared to the baseline scenario. Households' capability to service their debt declines as both the interest rate and the level of unemployment increase significantly. The fall in house prices also contributes to the increase in problem loans. This effect may reflect that banks' credit supply declines as house prices fall, or that the willingness of households to service debt declines as the 'debt to value' ratio increases.

Firms' problem loans increase by close to 10 percentage points compared to the baseline scenario. This implies a default rate not far from the relatively high levels in the mid-nineties, i.e. just after the previous banking crisis in Norway. As with households, firms are hit by several factors that all contribute to reduce their ability to service their debt. The higher interest rate has a direct effect and also hits indirectly through the effect on the exchange rate and hence competitiveness. Higher unemployment has an additional strong effect.

3.5   Taking the multivariate stress scenario to the micro models

We now take the results from the macro model in the multivariate stress scenario to the micro models. This enables us to identify distributional effects and pockets of risk, and to evaluate the impact on the five largest Norwegian banks. We use the output from SMM as explanatory variables in the micro models. In the corporate sector model, i.e. SEBRA, we use the predictions on GDP (Mainland Norway), CPI inflation, wage growth, firm borrowing rate, the real exchange rate, and house prices as a proxy for commercial property prices. In the household-margin model, we use the CPI inflation, the wage growth, the interest rate charged on household loans and the household credit growth. In the bank model we use banks' loan losses, the three month money-market interest rate, the growth in credit to households, and the per hour wage growth. In addition, the bank model takes firms debt growth and the distribution of debt at risk from SEBRA as input.

Norges Bank's view on the economic development is published in the tertiary Monetary Policy Report, see, e.g., Norges Bank (2008b). Norges Bank publishes a baseline scenario based on models developed to support monetary policy and on judgement. In general, the baseline scenario that we produce in SMM may deviate from the official baseline scenario. When publishing results from our stress-testing exercise, we therefore adjust the scenarios to become consistent with the official baseline scenario in the latest available Monetary Policy

Report. This is relatively simple, since SMM is nearly linear. In the following, when results from the micro models are shown in level form, they have been adjusted to correctly represent deviations from the official baseline scenario in Norges Bank (2008b).

The corporate sector model; SEBRA

Figures 6 and 7 illustrate how SEBRA identifies pockets of risk in the corporate sector. These figures show that commercial-property firms are highly vulnerable to the shocks in our stress scenario. The increase in losses that banks suffer is very much a result of the fall in the debt-servicing capability in the real-estate sector. This sector is highly leveraged and thus heavily exposed to the increase in interest rates. Our assumption that commercial property-prices fall in line with house prices also contributes to the losses.

Figure 6. Banks' losses on loans to different industries as a share of total losses on loans to non-financial firms. Stress scenario, annual data
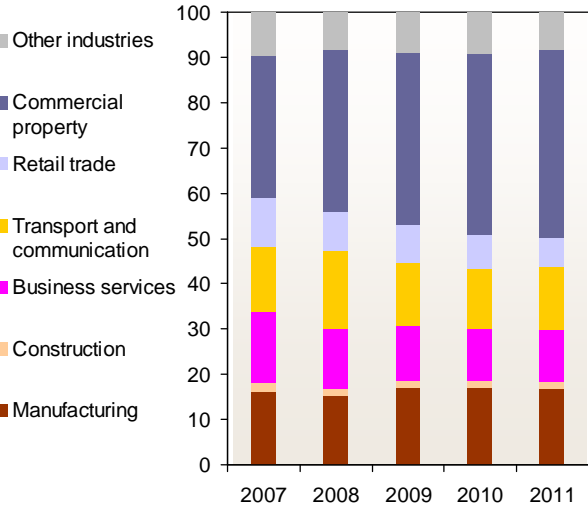
Figure 7. Debt-servicing capability of firms in the commercial property sector and other non-financial firms. Stress scenario, annual data
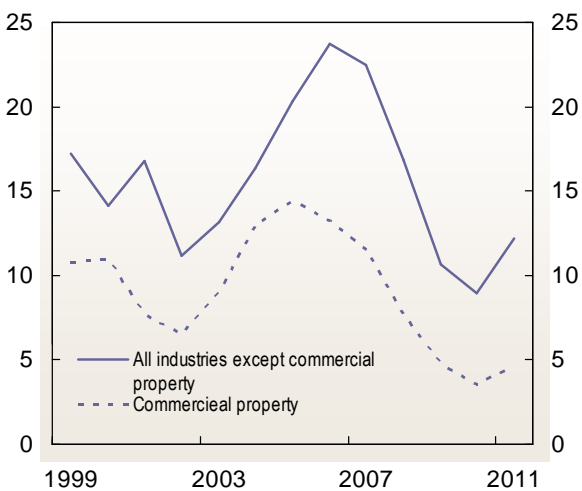
Source: Norges Bank

Source: Norges Bank

The household model

The household-margin model is used to identify households or groups of households that are likely to experience large increases in debt at risk in stress scenarios. We can split the households according to various characteristics that we find interesting.

In Figure 8 we show the share of households with a negative margin and their share of households' total debt. The three sets of bars to the right illustrate the results in 2010, since, in our scenario, households' situation improve somewhat again in 2011. 'Base' is our baseline scenario, 'stress' is our stress scenario, while 'stress + increased living expenses' is our stress scenario with the additional assumption that the annual rise in prices on basic consumption doubles compared to the stress scenario. According to the household model, close to 10 per cent of the households have a negative financial margin in our stress scenario, and they have 7½ per cent of total debt. In the 'stress + increased living expenses' case, 12 per cent of the households will have a negative margin. These households have 9 per cent of total debt. Hence, many households are vulnerable to the development in consumer prices, particularly if an increase in living expenses comes on top of an increase in interest rates and unemployment.

Figure 8. Percentage of households with negative margin and their debt in per cent of total debt. Annual data[1]

1) Margin = after tax income – standard cost of living – interest payments – calculated repayments (20 year)

Sources: Statistics Norway, National Institute for Consumer research and Norges Bank
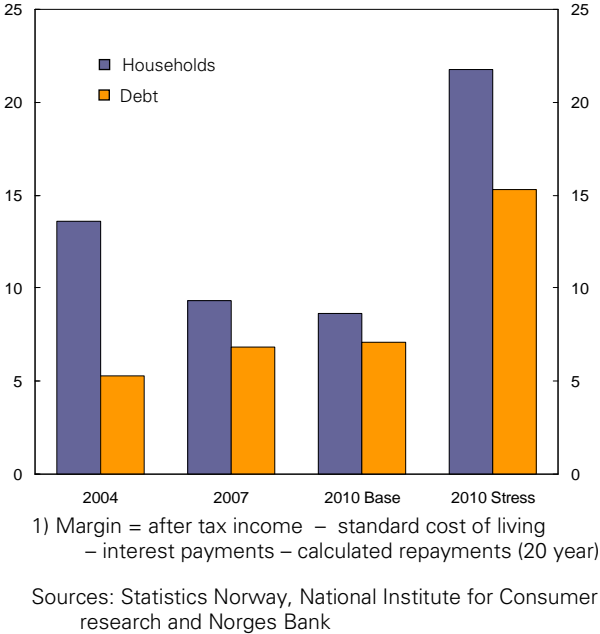
Figure 9. Debt in households with a negative margin in selected groups'. Percentage of group debt. In 2010 in stress scenario

[1] Debt to disposable income ratio

Sources: Statistics Norway and Norges Bank
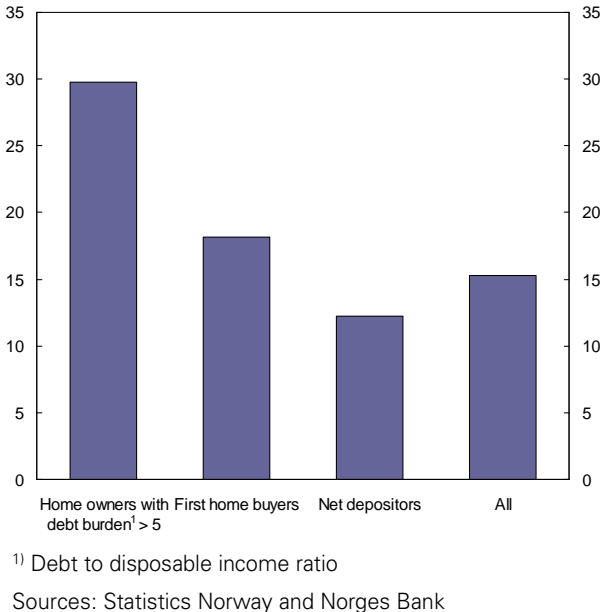
It is often argued that households with a high debt to income ratio and first-time home buyers are most vulnerable to negative events. At the same time, it is argued that many households with debt also have financial wealth that can help them out if negative events should occur. Figure 9 shows the situation in 2011 given our stress scenario. The first bar shows that about

13 per cent of the debt held by households with a debt-to-income ratio above 5 will be at risk, i.e. held by households with a negative margin. The second bar shows that about 11 per cent of total debt held by first-time home buyers will be at risk. The third bar shows that only 4-5 per cent of total debt in households with positive liquid net financial wealth will be at risk in our terminology. Liquid net financial wealth is defined as bank deposits minus debt. Hence, households with a buffer that can be drawn on in difficult times are less likely to run into a situation with a negative margin. Households with a high debt-to-income ratio and first-time home buyers are, as expected, vulnerable to negative events. In difficult times, liquid financial wealth is not mainly at the hands of those who may be needing it most.

The bank model

From the bank model we get the impact of the stress scenario on the five largest Norwegian banks' results and capital adequacy. The aggregate results are shown in Figure 10 and 11. Based on the baseline scenario for the Norwegian economy, banks' results after tax are expected to fall in 2008, and then remain at about 0.65 per cent of average total assets in the following years. Both in the baseline and the stress scenario, the banks' results after tax fall in 2008 due to a decline in other operating income. The main drivers behind the reduction are a decline in fee income and net losses on securities. In addition, DnB NOR had a 1.4 billion NOK gain on a property sale during the fourth quarter of 2007. As this is a one-time gain, other operating income is adjusted down by the same amount from 2007 to 2008.

In the stress scenario, bank's results after tax will fall substantially in 2009, and be negative as from 2010. The steep rise in loan losses is the main driver behind the negative results during the last two years of the stress scenario. Furthermore, the spread paid above the money market rate for market funding is assumed to be increasing in 2008 and again in 2009, and then falling somewhat in each of the years 2010 and 2011. This reduces the net interest income in the stress scenario.

Despite weaker results, capital adequacy ratios for the five banks as a group are not substantially weakened. This is due to the assumption that lending growth falls markedly, which reduces the capital adequacy requirements for these banks. One of the banks falls just below the minimum requirement of 8 per cent. However, a closer look at that bank indicates that its situation in the stress scenario is less critical than suggested by the model. At any rate, banks will not be passive bystanders to negative developments, as implicitly assumed in the

bank model. (From the macro model we have a fall in credit growth, however.) Banks can raise capital and subordinated debt in order to increase their capital adequacy. In addition, with loan losses of 1.9 per cent in 2010 and 2.3 per cent in 2011 banks may react by increasing their lending margins even more than what is assumed in the stress scenario.

Figure 10. Projections of post-tax profit as a percentage of average total assets in Norway's five largest banks.[1] Annual data
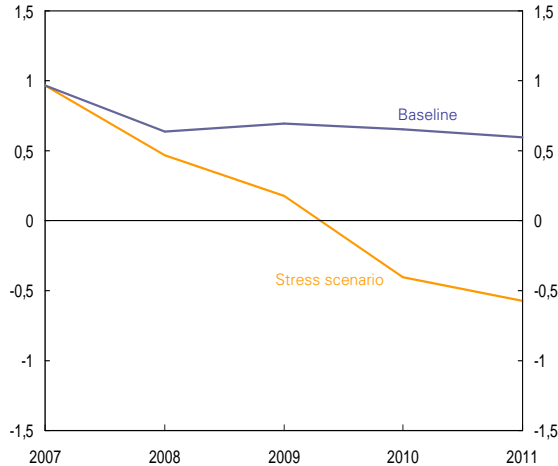
1) DnB NOR Bank (excl. branches abroad), SpareBank 1 SR Bank, Sparebanken Vest, SpareBank 1 Nord-Norge and SpareBank 1 SMN

Source: Norges Bank

Figure 11. Projections of capital adequacy in per cent in Norway's five largest banks. Annual data
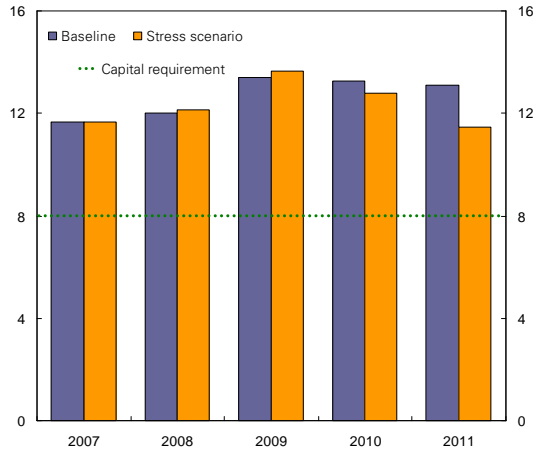
1) DnB NOR Bank (excl. branches abroad), SpareBank 1 SR-Bank, Sparebanken Vest, SpareBank 1 Nord-Norge and SpareBank 1 SMN

Source: Norges Bank

4   Summing up

This paper presents a system developed for stress-testing purposes, where an aggregative macro-model is linked to micro data-based models for households, firms and banks. The model structure is recursive; with output from the macro model being used as input into the micro data-based models. This enables us to follow the transmission of initial macro shocks through the set of models and to get a more detailed picture of the consequences. Information on how debt and probability of default are distributed across firms and households can be very important for the assessment of financial stability. The household and firm models are used to analyse pockets of risk. The bank model enables us to evaluate the consequence of different negative events on the five largest Norwegian banks' results and capital adequacy.

In addition to equations for the main macroeconomic variables, the macro model includes equations for household debt, house prices, housing investments, households' and firms' problem loans and firms' bankruptcy rate. The house price equation includes households' expectations about own financial situation and the Norwegian economy, i.e. a consumer confidence indicator. These expectations need not be model-consistent. While overly optimistic agents will fuel the rise in house prices, the opposite is true if agents are pessimistic. In addition to this consumer confidence effect on house prices, our macro model also includes other important properties from a financial stability assessment perspective. These are a credit driven house price effect, a long lasting effect of a rise in house prices on credit growth, and a feed-back effect from credit and house prices to the real economy. Hence, our macro model includes a financial accelerator.

Four simulation exercises on the macro model are presented; a wage and price shock, a shock to households' expectations, a credit crunch and a multivariate shock that combines the three shocks. As a consequence of the multivariable shock, households' problem loans increase, but by less than one percentage point compared to the baseline scenario. An increase in firms' problem loans by close to ten percentage points compared to the baseline scenario is rather dramatic, however. This implies a default rate on bank loans not far from the relatively high levels in the early nineties, i.e. at the end of the previous banking crisis in Norway. The multivariate shock is also fed into the firm, household margin and bank models. The predictions of the firm model are that the largest increase in debt at risk comes in the commercial real-estate sector. This result reflects that this sector is highly leveraged, and that commercial real-estate property prices are assumed to follow the fall in house prices. The household model predicts that the largest increase in debt at risk comes in households with a very high debt-to-income ratio and among first-time home buyers. Liquid financial wealth, i.e. bank deposits, is in general not at the hands of those households that will be mostly affected by our stress scenario.

The five largest banks' results deteriorate significantly in our stress scenario, very much due to the increase in losses. Despite weaker results, capital adequacy ratios for the five banks as a whole are not substantially weakened. This is due to the assumption that lending growth falls markedly, which reduces the capital adequacy requirement for these banks.

Although our model system has many favourable properties as a stress-testing tool as it stands today, it also has its weaknesses. We therefore continue to develop and improve the different models and the way they interact with each others. In the near future, the development of the bank model is a prioritised task. We would want to include more of the largest banks, to strengthen the relationship between the bank model and the household and corporate sector models and to include behavioural equations in the bank model.

References

Akram, Q.F., G. Bårdsen and K.-G. Lindquist (2007): Pursuing financial stability under an inflation-targeting regime, *Annals of Finance* 1, pp. 131-153.

Allen, F. (2005): Modelling financial instability, *National Institute Economic Review* No. 192, National Institute of Economic and Social Research.

Allen, W.A. and G. Wood (2006): Defining and achieving financial stability. *Journal of Financial Stability* 2, pp. 152-172.

Andersen, H. (2008): Failure prediction of Norwegian banks: A logit approach. Working Paper No. 2, Norges Bank.

Aspachs, O., C.A.E. Goodhart, D.P. Tsomocos and L. Zicchino (2007): Towards a measure of financial fragility. *Annals of Finance* 1, pp. 37-74.

Ball, L. (1999): Policy rules for open economics. In Taylor, J.B. (ed.), *Monetary Policy Raules*, A National Bureau of Economic Research Conference Report, 127-144. Chicago: University of Chicago Press.

Bank for International Settlements (1998): Implications of structural change for the nature of systemic risk. Basel, Switzerland.

Bank of Japan (2008): *Financial System Report*, March.
http://www.boj.or.jp/en/type/ronbun/fsr/data/fsr08a.pdf

Bank of Finland (2007): *Financial Stability*. Suomen Pankki.
http://www.bof.fi/NR/rdonlyres/845461E3-DE3E-4886-9452-755E0ECA2555/0/07Vakausraportti.pdf

Berge, T. and K.G. Boye (2007): Explaining the development in default by households and firms. *Economic Bulletin* 1, Norges Bank, forthcoming.

Berge, T., K.G. Boye, D.H. Jacobsen, K.-G. Lindquist and M. Skjæveland (2007): Measuring and predicting household housing wealth. *IFC Bulletin No. 25, pp. 252-263*, Bank for International Settlement.

Bernanke, B. and M. Gertler (1989): Agency Costs, Net Worth and Business Fluctuations. *American Economic Review*, 79, pp. 14-31.

Bernanke, B.S., M. Gertler and S. Gilchrist (2000): The Financial Accelerator in a Quantitative Business Cycle Framework, in John B. Taylor and Michael Woodford, eds., *Handbook of Macroeconomics*, Vol. 1C. New York: Elsevier Science, pp. 1341-1393.

Bernhardsen, E. (2001): A model of bankruptcy prediction. Working Paper No. 10, Norges Bank.

Bernhardsen, E. and K. Larsen (2007): Modelling credit risk in the enterprise sector - further development of the SEBRA model. *Economic Bulletin* 3, Norges Bank, pp. 102-108.

Bernhardsen, E. and B.D. Syversten (2008): Stress testing of loans to the enterprise sector – a micro approach. Staff Memo. Norges Bank, forthcoming.

Bjørnland, H. and D.J. Jacobsen (2008): The role of house prices in the monetary policy transmission mechanism. Working Paper. Norges Bank, forthcoming.

Borio, C. and P. Lowe (2002): Asset prices, financial and monetary stability: exploring the nexus, BIS Working Paper no. 114 July.

Boss, M., G. Krenn, C. Puhr, M. Summer (2006): Systemic Risk Monitor: A modell for systemic risk analysis and stress testing of banking systems. In *Financial Stability Report* 11, Oesterreichische Nationalbank, pp. 83-95.

Brubakk, L., T.A. Husebø, J. Maih, K. Olsen and M. Østnor (2006): Finding NEMO: Documentation of the Norwegian economy model, Staff Memo No. 6, Norges Bank.

Bårdsen, G., Ø. Eitrheim, E.S. Jansen, and R. Nymoen (2005): *The econometrics of macroeconomic modelling*. Oxford: Oxford University Press.

Bårdsen, G., K.-G. Lindquist and D. Tsomocos (2008): Evaluation of macroeconomic models for financial stability analysis. *Journal of World Economic Review,* forthcoming. (Also available as Working paper No 1/2006, Norges Bank.)

Bårdsen, G., R. Nymoen (2008): Macroeconometric modeling for policy. Handbook of Econometrics Vol II, Applied Econometrics, Palgrave-MacMillan (forthcoming).

Castrén, Olli, S. Dées and F. Zaher (2008): Global macro-financial shocks and corporate sector expected default frequencies in the Euro area. Working Paper Series 875. European Central Bank.

Čihák, M. (2007): Introduction to applied stress testing. IMF Working Paper WP/07/59. International Monetary Fund.

Crockett, A. (1997): Why is financial stability a goal of public policy?, Federal Reserve Bank of Kansas City Symposium on "Maintaining Financial Stability in a Global Economy", August 28-30, Jackson Hole, Wyoming, pp. 7-36.

Danmarks Nationalbank (2008): *Financial Stability*.
http://www.nationalbanken.dk/C1256BE9004F6416/side/DF4F96A8070AA8C2C125744A002B6506/$file/FinancialStability_2008.pdf

De Netherlandsche Bank (2006): Financial stability: is the Dutch financial sector stress-resistant?. DNB Quarterly Bulletin, December, pp. 35-41.
http://www.dnb.nl/dnb/home/file/Quarterly_Bull_Dec06_tcm47-147090.pdf

Drehmann, M. (2008): Stress tests: Objectives, challenges and modelling choices. Manuscript March 2008. Bank of International Settlement.

Dungey, Mardi, R. Fry, B. Gonzáles-Hermosillo, V.L. Martin and C. Tang (2008): Are financial crises alike?. Unpublished paper dated April 2008. Presented at Norges Bank May 2008.

ECB (2006): Financial Stability Review, June, pp. 147-154. European Central Bank.

Goodhart, C.A.E. and B. Hofmann (2007): *House prices and the macroeconomy*, New York: Oxford University Press.

Goodhart, C.A.E., P. Sunirand and D.P. Tsomocos (2004): A model to analyse financial fragility: Applications. *Journal of Financial Stability* 1, pp. 1-30.

Goodhart, C.A.E., P. Sunirand and D.P. Tsomocos (2005): A risk assessment model for banks. *Annals of Finance* 1, pp. 197-224.

Goodhart, C.A.E., P. Sunirand and D.P. Tsomocos (2006a): A model to analyse financial fragility. *Economic Theory* 27, pp. 107-142.

Goodhart, C.A.E., P. Sunirand and D.P. Tsomocos (2006b): A time series analysis of financial fragility in the UK banking system. *Annals of Finance* 2, pp. 1-21.

Haldane, A., S. Hall and S. Pezzini (2007): A new approach to assessing risks to financial stability, *Financial Stability Paper* No. 2, Bank of England.

Hagen, J., A. Lund, K.B. Nordal and E. Steffensen (2005): The IMF's stress testing of the Norwegian financial sector. *Economic Bulletin* 4, Norges Bank, pp. 202-211.

Haugland, K. and B. Vikøren (2006): Financial stability and monetary policy – theory and practice. *Economic Bulletin* 1, Norges Bank, pp. 24-31.

Hausman, J. A., J. Abrevaya and F. M. Scott-Morton (1998): Misclassification of the Dependent Variable in a Discrete-Response Setting. *Journal of Econometrics*, 87, pp. 239-269.

Hubbard, G. (1998): Capital-market imperfections and investment. *Journal of Economic Literature* 36, pp. 193-225.

IMF and World Bank (2003): Analytical tools of the FSAP.
http://www.imf.org/external/np/fsap/2003/022403a.pdf

Jacobsen, D.H. and T.B. Kloster (2005): What influences the number of bankruptcies? *Economic Bulletin* 4, Norges Bank, pp. 191-201.

Jacobsen, D.H. and B.E. Naug (2004): What influences the growth of household debt? *Economic Bulletin* 3, Norges Bank, pp. 103-111.

Jacobsen, D.H. and B.E. Naug (2005): What drives house prices? *Economic Bulletin* 1, Norges Bank, pp. 29-41.

Jacobsen, D.H., K. Solberg-Johansen and K. Haugland (2007): Housing investments and house prices. *Economic Bulletin* 1. Norges Bank, pp. 33-46.

Jappelli, T. and M. Pagano (1989): Consumption and capital market imperfections: An international comparison. *American Economic Review* 79, pp. 1088-1105.

Jiménez, G. and J. Mencía (2007): Modelling the distribution of credit losses with observable and latent factors. Documentos de Trabajo N. 0709, Banco de España.

Kiyotaki, N. and J. Moore (1997): Credit cycles. *Journal of Political Economy* 105, pp. 211-248.

Laviola, S., J. Marcucci and M. Quagliariello (2006): Stress testing credit risk: Experience from the Italian FSAP. *Banca Nazionale del Lavoro Quarterly Review,* vol. LIX, no. 238.

Lehman, H. and M. Manz (2006): The exposure of Swiss banks to macroeconomic shocks – an empirical investigation. Working Paper no. 4, Swiss National Bank.

Lucas, R.E. (1976): Econometric policy evaluation: A critique. Carnegie-Rochester Conference Series on Public Policy 1, pp. 19-46.

Marcucci, J. and M. Quagliaiello (2005): Is bank portfolio riskiness procyclical? Evidence from Italy using a vector autoregression. *Journal of International Financial Markets, Institutions & Money* 18, pp. 46-83.

Norges Bank (2007): *Report on Financial Stability*, No. 2.
http://www.norges-bank.no/Upload/64823/Financial_Stability_2_07.pdf

Norges Bank (2008a): Survey of Bank Lending 2008 Q1.
http://www.norges-bank.no/Pages/Article_____69133.aspx.

Norges Bank (2008b): *Monetary Policy Report*, No. 1.
http://www.norges-bank.no/Upload/66309/Monetary_Policy_Report_0108.pdf

Pesola, Jarmo (2005): Banking fragility and distress: An econometric study of macroeconomic determinants. Bank of Finland Research Discussion Papers no 13, Bank of Finland.

RTF Stress Testing Subgroup (2008): Stress testing credit risk: A survey of authorities' approaches. RTF/07/16, RTFST/07/13, April 2008. Basel Committee on Banking Supervision, Bank for International Settlement. Restricted availability, contact editor A. Foglia, Banca d'Italia, Banking and Financial Supervision.

Schinasi, G.J. (2004): Defining financial stability, IMF Working Paper 04/187. Washington: International Monetary Fund.

Sorge, M. (2004): Stress-testing financial systems: An overview of current methodologies, BIS Working Paper No. 165. Bank for International Settlements.

Sorge, M. and K. Virolainen (2006): A comparative analysis of macro stress-testing methodologies with application to Finland. *Journal of Financial Stability* 2, pp. 113-151.

Sveriges Riksbank (2006): *Financial Stability Report* 1/2006.
http://www.riksbank.com/pagefolders/26147/2006_1_eng.pdf

van den End, J.W., M. Hoeberichts and M. Tabbae (2006): Modelling Scenario Analysis and Macro Stress-testing. Working Paper No. 119, De Nederlandsche Bank.

van Lelyveld, I. and F. Liedorp (2006): Interbank contagion in the Dutch banking sector. *International Journal of Central Banking* 2, pp. 99-133.

Vatne, B.H. (2007): Financial margins in Norwegian households - An analysis of micro data for the period 1987-2003. *IFC Bulletin* No. 26, pp. 40-51, Bank for International Settlements.

Vatne, B.H. (2006): How large are the financial margins of Norwegian households? An analysis of micro data for the period 1987–2004. *Economic Bulletin* 4, Norges Bank, pp. 173-180.

Åsberg, P. and H. Shahnazarian (2008): Macroeconomic impact on expected default frequency. Working Paper 219, Sveriges Riksbank.

Appendix 1: The main equations of the small macro-econometric model

The small macro model is an extension of the model reported in Bårdsen and Nymoen (2008) and Bårdsen et al. (2005).[16] It is a macro-econometric model estimated on quarterly data. The model explicitly takes into account several channels of interplay between output, inflation and financial stability. The equations are in equilibrium-correction form, with backward-looking expectations formation.

We present a stylized version of the model in Equations (1)-(13). Small letters denote natural logarithms of the variable, Δ denotes the first difference operator, Δ_j denotes the j-period difference operator, and foreign variables are denoted with starred superscripts. In general, intercept terms and seasonal effects have been omitted from the equations for ease of exposition. The identities that complete the model are not reported.

Aggregate demand

$$
\begin{aligned}
\Delta y_t = & -0.6\Delta y_{t-1} + 0.7\Delta g_t + 0.4\Delta g_{t-1} \\
& + 0.1\Delta(ph - p)_{t-1} + 0.1\Delta(cr^e - p)_{t-1} + 0.2\Delta(cr^h - p)_{t-3} \\
& -0.3[(y_{t-2} - 0.8g_{t-2} - 0.1(v + p^* - p)_{t-1} - 0.1(cr^h - p)_{t-4} + 0.01(RL - \pi)_{t-1}]
\end{aligned}
\tag{1}
$$

Estimation period 1991Q1-2006Q4

Exchange rate

$$
\begin{aligned}
\Delta v_t = & \varphi(-0.04\Delta R_t + 0.05\Delta R^*_t - 0.1\Delta po_t - 0.07u_{t-1}) \\
& -0.1[(v + p^* - p)_{t-1} + 0.03((R - \pi)_{t-1} - (R^* - \pi^*)_{t-1}) + 0.1(po + usd - p)_{t-1} - \mu_v]
\end{aligned}
\tag{2}
$$

Estimation period 1994Q2-2007Q2

Import prices

$$
\begin{aligned}
\Delta pi_t = & 0.4\Delta v_t + 1.3\Delta pi^*_t \\
& -0.4[(pi - pi^* - v)_{t-1} - 0.6(p - p^* - v)_{t-1}]
\end{aligned}
\tag{3}
$$

Estimation period 1990Q1-2007Q2

---

[16] The presentation of the core part of the macro model is based on Bårdsen and Nymoen (2008).

Unemployment

$$\Delta u_t = 0.4 u_{t-1} - 1.6(\Delta_4 \frac{1}{2}\sum_{j=1}^{2} y_{t-j} - mean(\Delta_4 \frac{1}{2}\sum_{j=1}^{2} y_{t-j}))$$
$$-0.03[u_{t-2} - 11.1\Delta(w-p)_t] \tag{4}$$

Estimation period 1979Q3-2007Q4

Wages

$$\Delta w_t = \Delta z_t - 0.5\Delta(w_{t-1} - z_{t-1})$$
$$-0.1[w_{t-2} - p_{t-1} - z_{t-2} + 0.001 u_{t-1} - \mu_w] \tag{5}$$

Estimation period 1978Q4-2007Q4

Consumer prices

$$\Delta p_t = 0.3\Delta p_{t-2} + 0.1\Delta y_{t-1} + 0.1\Delta(w_{t-1} - z_{t-1}) + 0.1\Delta pe_t$$
$$-0.06[p_{t-3} - 0.65(w_{t-3} - z_{t-2}) - 0.35 pi_{t-1} - \mu_p] \tag{6}$$

Estimation period 1978Q4-2007Q4

Money market interest-rate

$$\Delta R_t = 1.5(\pi^c_t - 2.5) - 0.6(R_{t-1} - R^*_{t-1} - 1) + 0.4\Delta R^*_t - 0.5(\frac{1}{4}\sum_{j=1}^{4} u_{t-j} - 2) \tag{7}$$

Estimation period 1991Q1-2007Q2

Banks' lending rate

$$\Delta RL_t = 0.8\Delta R_t + 0.2\Delta R_{t-1} - 0.35[RL_{t-1} - (R_{t-1} + RLM)] \tag{8}$$

Household debt

$$\Delta(cr^h - p_t) = -0.01(\Delta RL_{t-2} + \Delta RL_{t-3}) + 0.3\Delta(inc - p)_{t-1}$$
$$+0.1(\Delta(ph-p)_t - \Delta(ph-p)_{t-3})$$
$$-0.04[(cr^h - p)_{t-1} - 0.7(ph-p)_{t-4} + 0.04 RL_{t-4} - 1.2(inc-p)_{t-2}] \tag{9}$$

Estimation period 1991Q1-2007Q2

House prices

$$\Delta ph_t = 0.2\Delta inc_t - 0.03\Delta RL_t - 0.02\Delta RL_{t-1} + 0.03 H^e_t$$
$$-0.1[ph_{t-1} + 0.05 RL_{t-1} + 0.5 u_t - 1.3(inc-hs)_{t-1} - 0.3 cr^h_{t-1}] \tag{10}$$

Estimation period 1990Q2-2006Q4

Housing investments

$$\Delta j_t = -0.04\Delta_4(RL - \frac{1}{3}\sum_{j=-1}^{1}\pi^c_{t-j})_t - 0.01(RL - \frac{1}{3}\sum_{j=-1}^{1}\pi^c_{t-j})_{t-4}$$
$$-0.1[(j_{t-1} - hs_{t-10}) - (ph - p)_{t-4} - (inc - p)_{t-1} - (pj - p)_{t-4}] \tag{11}$$

Estimation period 1991Q1-2007Q4

Household default rate

$$\Delta(d^h - cr^h) = -0.2\Delta_3(d^h - cr^h)_{t-1} + 0.02\Delta_2(RL - \pi)_t$$
$$+0.02\Delta_2(RL - \pi)_{t-2} - 0.5\Delta_4(ph - p)_t$$
$$-0.2[(d^h - cr^h)_{t-4} - 0.4u_{t-3} - 0.08(RL - \pi)_{t-4} \tag{12}$$
$$+1.2(inc - p)_{t-1} + 1.2(ph - p)_{t-4}]$$

Estimation period 1993Q1-2005Q4

Firm default

$$\Delta(d^e - p)_t = -0.3\Delta_2(d^e - p)_{t-1} + 0.02\Delta_2(RL - \pi)_t + 0.9\Delta u_t + 0.7\Delta u_{t-1}$$
$$+1.5\Delta(cr^e - p)_{t-3} - 0.4\Delta(po + usd - p)_t - 0.5[(d^e - p)_{t-3}$$
$$-(cr^e - p)_{t-4} - 0.05(RL - \pi)_{t-3} \tag{13}$$
$$-1.7u_{t-2} + 0.7(v + p^* - p)_{t-3} + 0.5(po + usd - p)_t]$$

Estimation period 1992Q1-2005Q4

where $\pi = 100\frac{\Delta_4 P_t}{P_{t-4}}$ is the inflation rate ; $\pi^u = 100\frac{\Delta_4 P^u_t}{P^u_{t-4}}$ is the core inflation rate, i.e.

inflation adjusted for changes in energy prices and taxes; $\pi^* = 100\frac{\Delta_4 P^*_t}{P^*_{t-4}}$ is the foreign

inflation rate.

Growth in real aggregate demand (Δy) is modelled in Equation (1). Aggregate demand is affected by the real interest rate ($RL - \pi$), real government expenditure (g) and the real exchange rate ($v + p^* - p$). Thus, a change in the nominal exchange rate would directly affect aggregate demand. Aggregate demand is also affected by house prices and credit. Changes in real house prices ($ph - p$) have short run effects on aggregate demand through a wealth effect on consumption and through housing investments not captured by the real interest rate.

Real corporate credit $(cr^e - p)$ affects GDP in the short run, while real household credit $(cr^h - p)$ has long-run effects on GDP. The short-run effect is interpreted as reflecting frictions in the credit market, while the long-run effect points towards a form of rationing of the household sector.

The exchange rate (in logs denoted v) expresses the number of domestic currency units per unit of foreign currency. The equation of growth of the nominal effective exchange rate (Δv) in Equation (2) reacts to deviations from PPP ($v + p^* - p$) and hence contributes to stabilizing the real exchange rate. φ is a dummy for inflation targeting, and takes the value 0 up until 2001Q1 and the value 1 from 2001Q2. In the long run, the nominal exchange rate reflects the difference between domestic and foreign prices and the difference between domestic and foreign real interest rates ($R - \pi$) – ($R^* - \pi^*$). Accordingly, domestic inflation becomes fully reflected in the nominal exchange rate in the long run.

Import prices measured in domestic currency (pi) are a homogenous function of the nominal exchange rate (v) and foreign producer prices measured in foreign currency (pi^*). On the other hand, import prices increase if the real exchange rate (in terms of consumer prices) appreciates. This is due to pricing-to-markets in import price setting.

The unemployment rate (u) follows output growth (Δy) in the short run as an Okun's law relationship, see Equation (4). In addition, it exhibits slow reversion towards its equilibrium rate; an intercept term has been omitted.

There is a pass-through of consumer price inflation (Δp) to nominal wage growth (Δw) in the short run; see Equation (5). In each period, nominal wages adjust towards their long-run relationship where there is a full pass-through of consumer prices and productivity (z). However, the mark-up of wages on prices and productivity is inversely related to the unemployment rate (u).[17]

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[17] The constant mark-up term is suppressed. In the full econometric model, productivity (z) is an endogenous variable that depends on real wages ($w - p$), unemployment (u) and a deterministic trend.

In the short run, consumer price inflation varies with changes in aggregate demand (Δy) and to some extent nominal wage growth (Δw); see Equation (6). In addition, it adjusts to deviation from the long-run relationship for consumer prices. In the long run, consumer prices (p) reflect a weighted average of domestic and imported costs, represented by unit labour costs ($w - z$) and import prices ($v + p^*$). It follows that the initial effect of a change in nominal exchange rate on aggregate demand would become modified over time due to the echange rate pass-through to inflation, which would have an effect opposite that of the nominal exchange rate on the real exchange rate. The model also includes an equation for the underlying, i.e. core, inflation rate (p^c), which is linked to consumer price inflation.

The three-month money market interest rate (R) follows an estimated Taylor-type rule in Equation (7). Since March 2001, Norwegian monetary policy is aimed at targeting the annual core inflation rate (π^c) at 2.5 per cent. Despite the fact that Norwegian monetary policy has changed over time, see, e.g., Akram (2004)[18], the estimated equation is stable over the estimation period 1991-2006. The interest rate responds to deviation from target in domestic core inflation and to deviation in unemployment from 2 per cent. This unemployment gap represents the output gap. If the interest rate deviates from the foreign interest rate inclusive a premium of 1 percentage point, this also affects the interest rate.

Banks' lending rate (RL) is defined to follow the money market rate. A lending margin (RLM), i.e. the margin between the lending rate and the money market rate, is an exogenous variable in the model. The coefficients of this equation are calibrated and not estimated.

The relationship explaining movements in household debt in Equation (9) builds on the work presented in Jacobsen and Naug (2004). Growth in household debt (Δcr^h) reacts positively to growth in income (Δinc) and housing prices (Δph), and decreases with higher interest rate on loans (RL) see Jacobsen and Naug (2004) for further details.

The model of house prices (ph) in Equation (10) is based on Jacobsen and Naug (2005). The growth rate of nominal house prices (Δph) is explained by growth in nominal income (inc)

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[18] At the very beginning of the sample, NOK was pegged to the ECU, but went floating in December 1992. Although inflation targeting was formally introduced in March 2001, it is a common view that this regime was gradually introduced from early 1999 on.

and household expectations about their own financial situation and the Norwegian economy (H^e), i.e. a survey based consumer confidence indicator, as well as interest rate changes (ΔRL) and deviations from steady state. In steady state, house prices (ph) are mainly determined by income (inc) and housing capital (hc) in addition to the interest rate (RL), the unemployment rate (u), and household debt (cr^h).

The equation for gross fixed housing investments (j) is based on Jacobsen, Solberg-Johansen and Haugland (2007), see Equation (11). Growth in gross fixed housing investments (Δj) depends on the change in the real lending rate $\Delta_4(RL - \frac{1}{3}\sum_{j=-1}^{1}\pi^c_{t-j})_t$. In steady state, gross fixed investments depend on the level of housing capital (hs) due to replacement investments, real house prices ($ph - p$), real investment price ($pj - p$), households' real wage income ($inc - p$) as a proxy for land costs, and the real lending rate $(RL - \frac{1}{3}\sum_{j=-1}^{1}\pi^c_{t-j})_{t-4}$.

The equations of default[19] by households and firms in (12) and (13) respectively are based on Berge and Boye (2007). Households' default rate ($d^h - cr^h$), i.e., default as a share of total household bank debt, depends on households' real income ($inc - p$), unemployment (u), the real interest rate ($RL - \pi$) and real house prices ($ph - p$). With respect to firms' default, there is not homogeneity between default and debt in the short run, only in the long run. Firms' default, measured in real terms ($d^e - p$), depends on the level of debt ($cr^e - p$), the real interest rate ($RL - \pi$), domestic demand proxied by the unemployment rate (u), the real exchange rate ($v + p^* - p$) as a measure of competitiveness and the real oil price ($po + usd - p$). The latter variable captures that the level of activity and investments in the oil sector affect other industries.

In addition, SMM includes estimated equations for bankruptcies in firms adapted from Jacobsen and Kloster (2005), productivity (z), and bond rates (RB).

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[19] Our data on problem loans include both default and loans with a high probability of default as reported by the banks.

Appendix 2: The bank model

The bank model is a static non-behaviour model consisting of three main components, namely a profit and loss account, a balance sheet and a capital adequacy calculation.

1. The profit and loss account

The profit and loss account includes the following items:

- Net interest income
- Other operating income
- Other operating costs
- Loan losses

The profit before taxes and dividends is given by:

Profit before taxes and dividends = Net interest income + Other operating income – Other operating costs – Loan losses

Net interest income has become less important since the mid 1990's, but still remained the dominant component with 67 per cent of banks' operating income in 2007. This makes it particularly important to make as good predictions of net interest income as possible. Thus, the bank model includes a detailed net interest income calculation, based on projections of lending and deposit interest rates and interest rates on other interest bearing assets and liabilities. The growth rates of loans, deposits and other interest bearing assets and liabilities also affect the calculated net interest income. The net interest income is computed as:

$Net\ interest\ income_t = ((Loans_{t-1}+Loans_t)/2)*Average\ lending\ rate_t$
$+ ((Other\ interest\ bearing\ assets_{t-1}+Other\ interest\ bearing\ assets_t)/2)$
$*Averate\ interest\ rate\ on\ other\ interest\ bearing\ assets_t$
$- ((Deposits_{t-1}+Deposits_t)/2)*Average\ deposit\ rate_t$
$- ((Other\ interest\ bearing\ liabilities_{t-1}+Other\ interest\ bearing\ liabilities_t)/2)$
$*Average\ interest\ rate\ on\ other\ interest\ bearing\ liabilities_t$

Subscript t denotes the year of the predicted result. Notice that 'Other interest bearing liabilities' include both market funding, subordinated loans and other debt. It follows from the equation that a rise in the interest rates on loans and other interest bearing assets increases the net interest income, while an increase in the interest rates on deposits and other interest bearing liabilities pulls the net interest income in the opposite direction. In addition to this

price effect, a positive volume growth in the assets and liabilities boosts the net interest income, given that the marginal interest rates on interest bearing assets are higher than the marginal interest rates on interest bearing liabilities.

Other operating income consists of fee income and capital market income, i.e. net gains and dividends on securities, currency trade and derivatives. Other operating income is given by:

Other operating income = Fee income + Net gains and dividends on securities + Net gains on currency trade + Net gains on derivatives + Other gains and income

Fee income has in recent years accounted for about 20 per cent of total bank income.

Apart from the funding costs included in the net interest income calculation, *Other operating costs* are the dominant cost component in the profit and loss account. 55 per cent of Other operating costs were labour costs in 2007. *Loan losses* have been close to zero in recent years. However, banks losses may increase substantially. During the Norwegian banking crisis of 1988-93 bank losses were by far the major cost component.

2. The balance sheet

The asset side of the balance sheet includes the following items:

- Loans to households and enterprises
- Securities and deposits
- Other assets

The liability side of the balance sheet includes the following items:

- Deposits
- Market funding
- Other debt
- Subordinated debt
- Equity

While loans are the dominant component on the asset side (67 per cent of total assets in 2007), deposits is the dominant component on the liability side (62 per cent of total liabilities in 2007). Market funding includes bonds, short-term paper and loans from financial institutions.

Figure A1
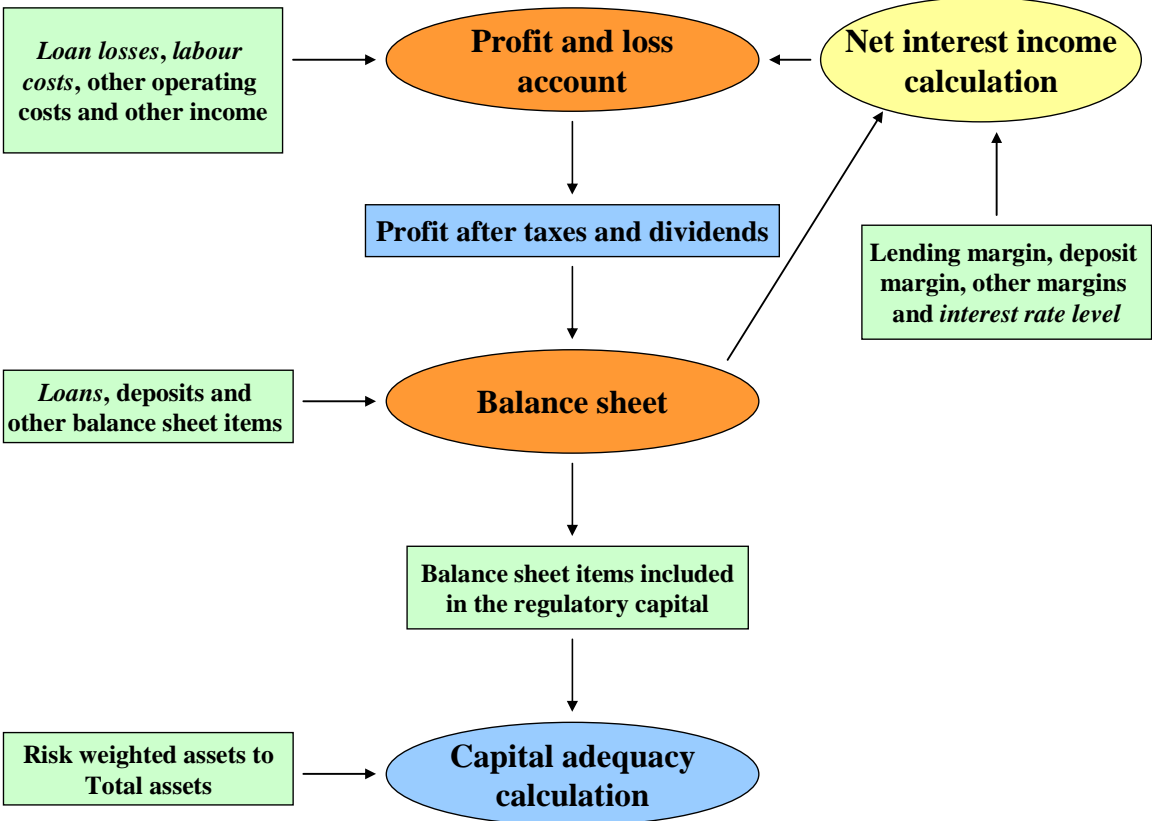
Banks' results after taxes and dividends affect their capital, and the balance sheet growth affects the risk weighted assets, confer figure A1 above. The end-output of the bank model are banks' results and capital adequacy.

3. The capital adequacy calculation

The future capital adequacy ratio is calculated based on projections of the regulatory capital and the risk-weighted assets. The regulatory capital is approximated based on balance sheet items. However, it is not possible to identify every single regulatory capital component in the balance sheet. Thus, a residual, i.e. the difference between the last reported regulatory capital and the sum of the regulatory capital components identified in the last reported balance sheet, is being predicted as well.

The risk-weighted assets are approximated based on the assumption that the ratio of risk-weighted assets to total assets remains constant during the simulation period. Thus, it is assumed that the risk parameters and composition of the banks' assets remains the same

during the prediction period. This runs contrary to the hypothesis that the risk parameters are responsive to the business cycle. Studies simulating the internal rating based approach of Basle II find significant cyclicality in the capital requirements caused by internally estimated risk parameters. Thus, a natural extension of the bank model would be to calculate risk-weighted assets based on risk parameters from the enterprise sector model which are responsive to the development in bankruptcy probabilities.

Appendix 3: Simulations on the bank model

In simulations, the bank model builds on projections of money market interest rates, loan losses, labour cost growth and loan growth to households from the macro model. The loan growth to the enterprise sector and the distribution of loan losses from different industries is predicted by the SEBRA enterprise sector model, which is a satellite to the macro model. We apply predictions of fee income from a separate error correction model estimated on macro variables (the GDP level, the GDP growth and the difference between the five year and the three month real yield on Treasuries)[20]. Projections of the remaining variables are based on analysis undertaken in Norges Bank.

1. The balance sheet

The bank model builds on projections of *loan growth to households* from the macro model for both the baseline and the stress scenario, see table 1. The *loan growth to the enterprise sector* is in both scenarios predicted by the enterprise sector model. The macro model predicts a steep increase in loan losses from the enterprise sector in the stress scenario. Due to these predicted problems in the enterprise sector, both the loan supply from the banks and the loan demand from the enterprises may fall substantially. Therefore, the loan growth to the enterprise sector is adjusted down in the enterprise sector model from 8.1 to 3.0 per cent in 2010 and from 16.6 to 5.0 per cent in 2011 in order to be in line with the predicted steep increase in loan losses from the enterprise sector. This is more in line with the experiences from the Norwegian bank crisis of 1988-93 when the loan growth to the enterprise sector remain below 5 per cent until 1996.

Securities and other assets are assumed to be growing at the same rate as the loan growth. This assumption keeps the composition of the banks' assets unchanged and is, in turn, consistent with the assumption that the ratio of risk-weighted assets to total assets remains constant during the simulation period.

As a simplification, *deposit growth* is assumed to mirror the wage growth from the macro model. Finally, the growth of *other interest bearing liabilities* (bonds, short-term paper, loans from financial institutions, subordinated debt and other debt) is set as a residual in order to

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[20] $\Delta lnFee\ income_t = -5.000 - 0.380lnFee\ income_{t-1} + 0.616lnGDP_{t-1} + 1.721(Five\ year\ real\ yield\ -\ Three\ month\ real\ yield)_{t-1} + 0,847\Delta lnGDP_t + 0.032Second\ quarter + 0.024Third\ quarter + 0.030Fourth\ quarter$

make the total liabilities equal to the total assets. The growth of *equity capital* is endogenously determined by the profit after taxes and dividends.

While the total asset growth is higher than the deposit growth (and the labour costs growth) in the baseline scenario, the opposite is true in the stress scenario. Thus, the assumptions above make the growth rate of market funding higher than the deposit growth in the baseline scenario. This is in line with the fact that the Norwegian banks' use of market funding has grown faster than their deposits during the last decade. However, the banks' need for market funding is substantially lower in the stress scenario due to the low loan growth. Thus, the above assumptions make the growth rate of market funding lower than the deposit growth in the stress scenario.

2. The profit and loss account

The *net interest income* is calculated based on projections of lending and deposit interest rates and interest rates on other interest bearing assets and liabilities. The growth rates of loans, deposits and other interest bearing assets and liabilities also affect the calculated net interest income. Projections of the balance sheet variables included in the net interest income calculation are described in chapter 2.1.

For both the baseline and the stress scenario, lending and deposit interest rates and interest rates on other interest bearing assets are assumed to change in line with the lending rate predicted by the macro model. This can be justified by the predominance of floating rate lending in Norwegian banking, which may have enabled the banks to eliminate most maturity mismatches. As banks largely extend long-term loans at floating rates, they also prefer floating rates on long-term borrowing. When banks issue bonds at fixed rates, they convert their interest payments to floating money market rates by means of interest rate swap agreements. This means that higher money market rates make both short-term and long-term funding more expensive.

However, during financial turbulence, the spread between fixed swap rates and fixed rates on long term borrowing may increase substantially. When converting their interest payments to floating money market rates, the banks have to pay this spread above the floating money market rates. Thus, in the stress scenario, the additional spread paid above the money market rate for market funding is assumed to increase by 20 basis points in 2008 and again in 2009,

and then falling by 10 basis points in each of the years 2010 and 2011. Thus, the spread in 2011 is 20 basis points higher than the initial spread in 2007. The spread increases gradually, because it takes time before the whole balance of market funding has been refinanced.

We compare the calculated net interest income to projections of net interest income from a separate error correction model estimated on macro variables (the GDP level and the three month real yield on Treasuries)[21]. The comparison is done to make sure that the calculated net interest income is in line with the scenarios for the Norwegian economy. Thus, the projections from the error correction model are only used as a cross-check. The comparison unveils that the calculated net interest income represents a plausible development given the macro economic scenario. Thus, the projections of the input variables in the net interest income calculation are left unchanged.

Predictions of *other operating income* are a function of several predicted components. The bank model applies predictions of fee income from the separate error correction model estimated on macro variables. Dividends received on securities are in 2008 assumed to be the same amount as in 2007, then 20 per cent lower in 2009, 2010 and 2011. The net losses on securities are in 2008 set equal to the net losses from the first quarter of 2008. For the remaining prediction period zero gains/losses are assumed. The net gains on currency trade and derivatives are not assumed to be cyclically sensitive. Thus, the amounts of net gains on currency trade and derivatives are assumed to be the same as in 2007 during the whole prediction period. During the fourth quarter of 2007, DnB NOR had a 1.4 billion NOK gain on a property sale. As this is a one-time gain, other operating income falls by almost the same amount from 2007 to 2008. For the remaining period other operating income (i.e. exclusive of net interest and fee income) grows at the same rate as the inflation target, i.e. 2.5 per cent per year.

The banks' *labour costs* are assumed to be growing at the same rate as the labour costs (including both employment and salary changes) predicted by the macro model for both the baseline and the stress scenario. The year-on-year rise in *other operating costs* of Norwegian banks has only been around 0.5 per cent during the last five years. However, the potential for

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[21] *ΔlnNet interest income $_t$ = -0.674 – 0.448lnNet interest income $_{t-1}$ + 0.36lnGDP $_{t-1}$ + 1.168Three month real yield $_t$ – 0,02ΔMarket share of foreign branches $_t$ + 0.035Second quarter + 0.039Third quarter + 0.004Fourth quarter*

further cost reduction may be limited. Thus, non-labour operating costs are assumed to be growing at the same rate as the inflation target in both scenarios. Finally, the bank model builds on projections of *loan losses* from the macro model for both the baseline and the stress scenario. The distribution of loan losses from different industries is predicted by the enterprise sector model.

The banks are assumed to distribute dividends of 50 per cent when the profit after taxes is positive and 0 per cent when the profit after taxes is negative.

Systemic Risk Monitor: A Model for Systemic Risk Analysis and Stress Testing of Banking Systems[*]

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Abstract

In 2002 the Oesterreichische Nationalbank (OeNB) launched in parallel several projects to develop modern tools for systemic financial stability analysis, off-site banking supervision and supervisory data analysis. In these projects the OeNB's expertise in financial analysis and research was combined with expertise from the Austrian Financial Market Authority (FMA) and from academia. Systemic Risk Monitor (SRM) is part of this effort. SRM is a model to analyze banking supervision data and data from the Major Loans Register collected at the OeNB in an integrated quantitative risk management framework to assess systemic risk in the Austrian banking system at a quarterly frequency. SRM is also used to perform regular stress testing exercises. This paper gives an overview of the general ideas used by SRM and shows some of its applications to a recent Austrian dataset.

1 Introduction

The primary mandate of central banks is to achieve and maintain price stability. Safeguarding and maintaining financial stability has always been regarded as a necessary prerequisite for this task. Institutionally, this combination of tasks was until very recently achieved by putting the central bank in charge of the oversight of individual financial institutions. Following the lead of the U.K., many countries, including Austria, have transferred responsibility for the oversight of individual financial institutions to newly established financial supervisory authorities, while the central banks kept the mandate to safeguard and maintain systemic financial stability. These institutional developments have forced central banks to arrive at answers to the new question what it means to maintain systemic financial stability without having ultimate responsibility for the oversight of individual financial institutions.

In 2002 the Oesterreichische Nationalbank (OeNB) launched in parallel several projects that aim to develop modern tools for systemic financial stability analysis and off-site banking supervision. In these projects the OeNB's expertise in financial analysis and research was combined with expertise from the University of Vienna, the University of Applied Sciences Vorarlberg, the Vienna University of Technology and the Austrian Financial Market Authority (FMA; see OeNB and FMA, 2005).

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Systemic Risk Monitor (SRM) is part of this effort. SRM is a model to analyze banking supervision data and data from the Major Loans Register collected at the OeNB in an integrated quantitative risk management framework. The purpose of SRM is to assess systemic risk in the Austrian banking system at a quarterly frequency. SRM is also used to perform regular stress testing exercises.

1.1 An overview of the model

The basic idea of the SRM model is to combine standard techniques from modern quantitative market and credit risk management with a network model of the banking system. In contrast to standard risk management models, SRM makes the step from the individual institution perspective to the system level. This step is the major challenge to be met by any systemic risk model. Only at the system level the two major reasons for simultaneous problems become visible: correlated exposures and financial interlinkages. The risk of simultaneous difficulties of institutions and the financial losses incurred in such events is the key focus of systemic financial stability analysis.

The model intentionally does not rely on a sophisticated theory of economic behavior. The consequences from a given liability and asset structure being exposed to realistic shock scenarios are uncovered in terms of problems of institutions. The model is designed to exploit existing data sources. Although these sources are not ideal, our approach shows that with the available data we can start to consider financial stability at the system level and provide quantitative judgements of systemic financial stability and systemic risk.

1.2 Related research

SRM can draw on a rich modern literature dealing with risk management and risk monitoring problems for banks or insurance companies (see McNeil et al. (2005) for an overview). The change of perspective from the individual institution level to the system level is the main methodological innovation of SRM. It is this system perspective, where SRM had to explore new territory. SRM mainly builds on research by Elsinger et al. (2006b) and Boss (2002). This paper gives an overview of the general ideas used by SRM and shows some of its applications to a recent Austrian dataset. Readers interested in technical details are referred to the model documentation, which can be received from the authors upon request (see Boss et al., 2006).

2 The SRM Model

The basic structure of the SRM model can be best described at an intuitive level by a simple picture showing the individual model components as well as their interrelation. Chart 1 displays the modular construction of SRM.

Chart 1: Basic Structure of SRM[1]
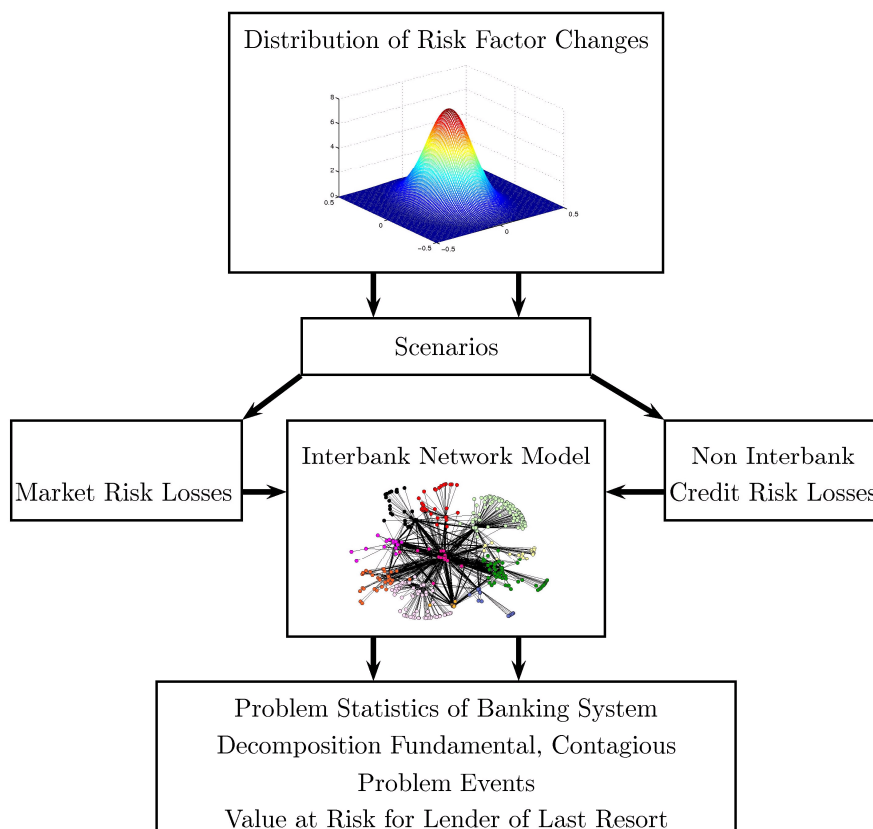
[1] Chart 1 shows the basic structure of the SRM model. Banks' noninterbank portfolios are exposed to shocks from a risk factor change distribution of market and credit risk factors. The value of interbank positions is determined endogenously by the network model and a clearing mechanism that makes all financial claims consistent ex post after shocks have been realized. The clearing of the interbank market determines the solvency of other banks and defines endogenous probabilities of problem events as well as the respective recovery rates. The output consists of statistics on problem events, a decomposition into fundamental and contagious problem events and an estimate of the amounts of liquidity a lender of last resort has to stand ready to inject into the system.

As a starting point it is perhaps best to begin with the middle layer of Chart 1, showing three boxes: *Market risk losses*, *Noninterbank credit risk losses* and *Interbank network model*.

SRM describes the Austrian banking system at the end of each quarter as a system of portfolios. Each portfolio in the system belongs to one bank and typically consists of collections of securities such as stocks and bonds across domestic and foreign markets (the *Market risk losses* box), a collection of corporate loans and loans to households (the *Noninterbank credit risk losses* box) as well as interbank positions (the *Interbank network model* box).

The value of each portfolio is observed from the data at the end of each quarter. The future portfolio values one quarter later (approximately 60 trading days) are random variables. Thus the difference between the portfolio values at the observation date and the portfolio values a quarter from the observation date, i.e. the gains and losses in the banking system, is subject to uncertainty. It is the distribution of these gains and losses we are interested in.

We adopt the usual risk management practice of thinking of future portfolio values as a function of time as well as of risk factors. Risk factors are market prices that determine portfolio values, such as stock market indices, interest rates and foreign exchange rates, as well as macroeconomic variables that have an impact on the quality of loan portfolios. To analyze the distribution of portfolio gains and losses in the banking system, we have to specify the distribution of *risk factor changes*. All individual modeling steps as well as the practical challenges that arise in SRM have to do with the details of how we describe the functional relation between risk factor changes and portfolio losses.

The top box of Chart 1 symbolizes a multivariate risk factor change distribution. In SRM such a distribution is estimated every quarter based on past observations of market price changes and changes of macroeconomic variables that have an impact on problem event probabilities.

The modeling strategy treats the marginal risk factor distributions and the dependency structure separately. While marginal distributions are chosen according to statistical tests that select for each risk factor a model which gives the best out-of-sample density forecast of changes in each risk factor over a three-month horizon, dependency is modeled by fitting a grouped t-copula to the data. Together, the marginal distributions and the copula characterize the multivariate risk factor change distribution.

For the simulation of scenarios, vectors of risk factor changes are drawn at random from this distribution. Each drawing of risk factor changes from the multivariate distribution characterizes a scenario, symbolized by the box *Scenarios*. Scenarios are then translated into profits and losses at the system level in two steps. In a first step each scenario is analyzed with respect to its impact on the value of market and noninterbank credit positions.

In a second step, these positions are combined with the network model. The network model basically checks whether given the gains and losses from the portfolio positions and given the capital of the banks, they are able to fulfill the financial obligations resulting from their interbank relations. Thus the network model combines all financial positions and bank capital in an overall system of bank net values. The network model does this by applying a clearing procedure that provides the final system of bank net values for each scenario. Simulating many scenarios, we get a distribution of problem events and gains and losses that allows us to make probability assignments for problem events over a three-month horizon.

The market risk losses and the losses from noninterbank credit risk are generated by two submodels that translate scenarios of risk factor changes into the respective scenario losses: a market and a credit risk model.

For marketable securities the situation is fairly simple. Supervisory data allow us a fairly coarse reconstruction of positions of securities at market values that are held on the bank balance sheet. The picture is coarse because individual stocks are lumped into Austrian and foreign, and interest rate- and currency-sensitive instruments are mapped into broad maturity and currency buckets. Consider, for instance, a simple stock portfolio consisting of Austrian and foreign stocks. Risk factor changes are then the logarithmic changes in the Austrian and a foreign stock price index. To calculate gains or losses from the stock portfolios, we can use a linearized approximation of the loss function. This amounts to simply multiplying the position values with the risk factor changes to get the portfolio gains and losses. For interest rate- and

currency-sensitive positions, we can equally arrive at gains and losses by using linearized losses and the relevant risk factor changes, which are changes in different exchange rates or interest rate changes for different maturities and different currencies.

For loans to nonbanks the situation is more complicated because the dependence between loan losses and risk factors is more indirect. We do not have a simple analogue to market returns. Defaults of loans in certain industry sectors – the units into which we break down loans in SRM – depend mainly on risk factors describing the aggregate state of the economy. Due to the discrete nature of the default events (either an obligor defaults or not), linearized losses are of little importance for the analysis of credit risk. Therefore SRM uses a credit risk model to calculate losses from loan portfolios. Our credit risk model is based on Credit Risk+ (see Credit Suisse, 1997) and has been adapted to explicitly take into account the dependency of default rates on the state of the macroeconomy. The basic idea is that the default probability of a loan in a particular industry sector, for instance construction, depends on a set of macroeconomic variables according to a function the parameters of which are statistically estimated from historical data. Given a realization of macroeconomic variables and the implied probability of default for different industry sectors, loan defaults are assumed to be conditionally independent. Under this assumption a loan loss distribution can be derived for each bank for each value of macroeconomic risk factor changes. Loan losses are then calculated by independent draws from these loan loss distributions.

From this discussion we see a fundamental modeling choice taken in SRM: Following the literature on risk management of individual institutions, the analysis is undertaken for a given set of portfolios observed at the observation time. The value of the portfolio is assumed to be completely determined by the risk factors and no behavioral considerations are taken into account. The longer the time horizon under consideration, the more problematic is such an assumption. In particular, in our framework, where we aim at an integrated analysis of portfolio positions which can be easily changed with other positions that are much more difficult to change, even at a 60-trading day horizon, this assumption is debatable for some of the portfolio positions. We ask the following question: given the portfolio positions we observe today in the system and given the future realizations of risk factors, how would these changes influence portfolio values ceteris paribus? This allows a statement about the risk inherent in the current banking system.

2.1 Using SRM for Financial Stability Analysis

We use four main risk concepts to look at the simulation output:

1) analysis of fundamental and contagious problem events;

2) analysis of probability distribution of problem events according to rating classes;

3) analysis of aggregate loss distributions;

4) quantification of resources that might have to be mobilized by a lender of last resort.

Since the risk of bank problems is a major concern for a central bank, we put a particular focus on probabilities of problem events. The network model allows us to distinguish problem events that result directly from changes in risk factors from events that result indirectly from

contagion through interbank relations. We call problem events fundamental if they result directly from risk factor movements and we call them contagious if they are a consequence of interbank relations. Apart from analyzing the number of fundamental and contagious problem events, we look at the probability distribution of problem events according to the OeNB's rating classes. We look at the aggregate loss distribution both for all risk categories taken together and for certain subcomponents such as market risk, credit risk and contagion risk. Finally we make an attempt to quantify the resources a lender of last resort might have to mobilize to prevent problems in the banking system.

2.2 Using SRM for Stress Testing

One advantage of a quantitative model is that it allows the consideration of hypothetical situations. In the context of systemic risk assessment, one kind of thought experiment is of particular importance. Usually it is of interest to know how the risk measures for the banking system will behave when there are extreme risk factor changes. Such thought experiments are known as stress tests. Systemic risk monitor provides a coherent framework to consistently conduct such stress testing exercises.

In a stress test, one or more risk factors of interest are constrained to take extreme values, like a certain drop in GDP or a hike in interest rates. Since we have a complete model of the multivariate risk factor distribution we can then perform a model simulation on the constraint that certain risk factors are at their stressed values. The risk measures of the model can then be studied relative to the baseline simulation based on the unconditional risk factor change distribution calibrated to historical data. The main advantage of this approach is its consistency with the dependency structure of the risk factors and therefore its consistency with the quantitative framework. Such an approach is advocated by Elsinger, Lehar and Summer (2006a) or by Bonti, Kalkbrener, Lotz and Stahl (2005).

3 Data

The main sources of data used by SRM are bank balance sheet and supervisory data from the monthly reports to the OeNB (known by their German acronym MAUS) and the OeNB's Major Loans Register (Großkreditevidenz, GKE). In addition we use default frequency data in certain industry groups from the Austrian business information provider and debt collector Kreditschutzverband (KSV), financial market price data from Bloomberg and Datastream and macroeconomic time series from the OeNB, the OECD and the IMF International Financial Statistics.

Banks in Austria file monthly reports on their business activities to the central bank. In addition to balance sheet data, the so-called MAUS reports contain a fairly extensive assortment of other data that are required for supervisory purposes. They include figures on capital adequacy, interest rate sensitivity of loans and deposits with respect to various maturity buckets and currencies, and foreign exchange exposures with respect to different currencies.

To estimate shocks on bank capital stemming from market risk, we include positions in foreign currency, equity, and interest rate-sensitive instruments from MAUS. For each bank, we collect foreign exchange exposures in USD, JPY, GBP and CHF only, as no bank in our sample

reports had open positions of more than 1% of total assets in any other currency at the observation date. We collect exposures to foreign and domestic stocks, which are equal to the market value of the net position held in these categories. For the exposure to interest rate risk, we use the interest rate risk statistics, which provide exposures of all interest-sensitive on- and off balance sheet assets and liabilities with respect to 13 maturity buckets for EUR, USD, JPY, GBP and CHF as well as a residual representing all other currencies. On the basis of this information we calculate the net positions in the available currencies – neglecting the residual – with respect to four different maturity buckets: up to 6 months, 6 months to 3 years, 3 to 7 years, more than 7 years. For the valuation of net positions in these maturity buckets, we use the 3-month, 1-year, 5-year and 10-year interest rates in the respective currencies.

To analyze credit risk we use, in addition to the data provided by MAUS, the Major Loans Register, which provides us with detailed information on banks' loan portfolios to nonbanks. This database contains all loans exceeding a volume of EUR 350,000 on an obligor-by-obligor basis.

We assign the domestic loans to nonbanks to 13 industry sectors (basic industries, production, energy, construction, trading, tourism, transport, financial services, public services, other services, health, households, and a residual sector) based on the NACE classification of the debtors. Furthermore we add regional sectors (Western Europe, Central and Eastern Europe, North America, Latin America and the Caribbean, Middle East, Asia and Far East, Pacific, Africa, and a residual sector) for both foreign banks and nonbanks, which leaves us with a total of 18 nondomestic sectors. Since only loans above a threshold volume are reported to the GKE we assign domestic loans below this threshold to the domestic residual sector. This is done on the basis of a report that is part of MAUS and provides the number of loans to domestic nonbanks with respect to different volume buckets. No comparable statistics are available for nondomestic loans. However, one can assume that the largest part of cross-border lending exceeds the threshold of EUR 350,000 and hence we do not lose much information on smaller cross-border exposures.

The riskiness of an individual loan to domestic customers is assumed to be characterized by two components: the rating which is assigned by the bank to the respective customer and the default frequency of the industry sector the customer belongs to. The bank's rating is reported to the GKE and is mapped at the OeNB onto a master scale, which allows assigning a probability of default to each loan. The default frequency data are from the Austrian business information provider and debt collector Kreditschutzverband (KSV). The KSV database provides us with time series of insolvencies and the total number of firms in most NACE branches at a quarterly frequency starting in 1969. This allows us to calculate a time series of historically observed default frequencies for our 13 industry sectors by dividing the number of insolvencies by the number of total firms for each industry sector and quarter. The time series of default frequencies is explained by macroeconomic risk factor changes, for which we use an econometric model. This estimated equation enables us to translate macroeconomic risk factor changes into probabilities of default for each industry branch. These default probabilities serve as input to the credit risk model. To construct insolvency statistics for the private and the residual sectors, where no reliable information on the number of insolvencies and sample sizes is available, we take averages from the data that are available. Default probabilities for the

nondomestic sectors are calculated as averages of the default probabilities according to the ratings that are assigned by all banks to all customers within a given foreign sector.

4 Applications

The OeNB uses the SRM model mainly for two applications: systemic risk assessment and stress testing. Systemic risk assessment involves a simulation at the end of each quarter as soon as all new data are available. The output of this simulation is a risk report with a detailed account of our four risk measures. In the stress tests one or more risk factors of interest are deliberately set to an extreme value and the simulation is performed conditional on the assumption that these risk factors are at their hypothetical extreme realizations. The output of this simulation can then be compared with the baseline simulation.

To make SRM operational, it is implemented such that it can be accessed via an interface called from the analyst's desk. The interface is a Java client application which gives users the possibility to run certain predefined simulations (including a variety of regular stress tests) as well as to parameterize individual simulations. The level of parameterization covers the point in time for which the simulation is run, data included in the model, various alternative model components as well as their parameters. Additionally, stress tests can be defined for market and credit risk factors. The parameters chosen are stored at database level and written to configuration files, which are read by the application at runtime. The models themselves are implemented in Matlab script language, version 14.3, a programming language for technical computing, which provides object-oriented means to include various model components and store complex data sets. Although SRM functionality can be accessed through Matlab's standard user interface, in its end-user implementation the source code of SRM is compiled as C Code and called via the SRM interface. In either case output is written to Microsoft Excel files for further analysis, which are sent as an e-mail attachment to the analyst's desk by SRM after a simulation request has been finished. A screenshot of the interface is shown in Chart 2.

Chart 2: Screenshot of the SRM Interface
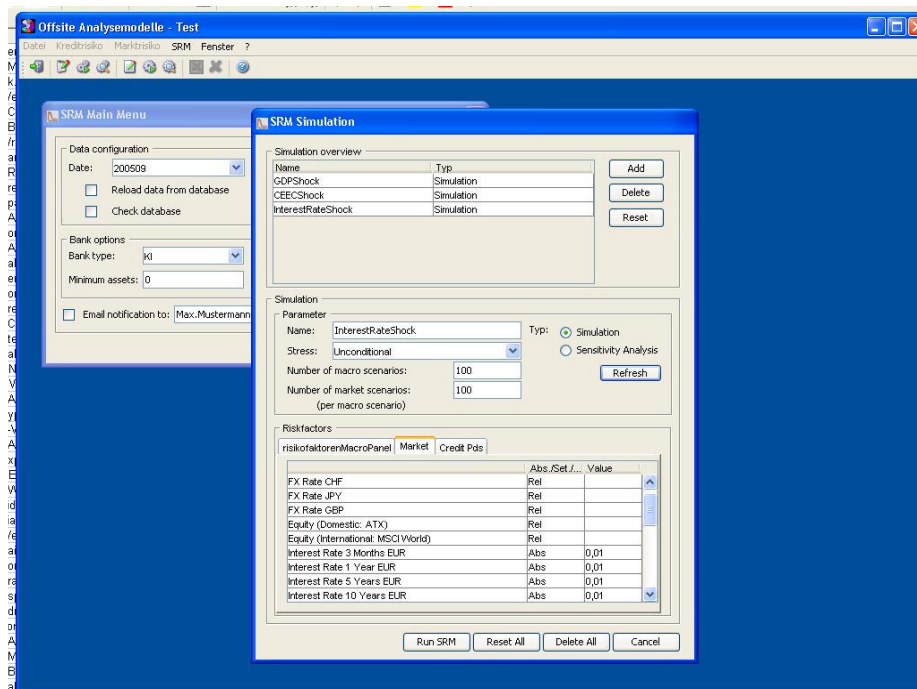
Source: OeNB

4.1 Regular Supervisory Data Analysis and Stress Tests

Systemic Risk Monitor will be used to perform regular analyses of supervisory data with respect to systemic risk problems. It will also be used as a stress testing tool. We will now illustrate output generated by SRM by looking at some examples based on a recent simulation for the last quarter of 2005. We present our results always for a regular simulation of the current economic situation together with two stress tests: Stress test number one simulates an unexpected drop in GDP. Stress test number two assumes a parallel upward shift in the euro yield curve.

4.2 Fundamental and Contagious Problem Events

The network model generates a multivariate distribution of bank's problem events across scenarios. We interpret the relative frequency of problem events as a probability.

Our method allows a decomposition of problem events into events resulting directly from shocks to the risk factors and those that are consequences of a domino effect. Bank problems may be driven by losses from market and credit risks (fundamental problem events). Bank problems may, however, also be initiated by contagion: as a consequence of other bank problems in the system (contagious problem events).

We can quantify these different cases and are able to give a decomposition into fundamental and contagious problem events. Table 1 summarizes the according probabilities both in the current situation as well as under both stress scenarios. These probabilities are grouped by the number of fundamental problem events. The column "fundamental" shows the percentage of scenarios where we encounter such events. The number of scenarios where in addition contagion occurs is reported in the "contagious" column.

Table 1: Probabilities of Fundamental and Contagious Problem Events[1]

	Current situation		GDP stress		Interest rate stress	
	Fundamental	Contagious	Fundamental	Contagious	Fundamental	Contagious
0	74.49%	0.00%	68.53%	0.00%	60.27%	0.00%
1 to 5	25.51%	0.00%	31.27%	0.00%	39.73%	0.00%
6 to 10	0.00%	0.00%	0.13%	0.00%	0.00%	0.00%
11 to 20	0.00%	0.00%	0.05%	0.00%	0.00%	0.00%
21 to 50	0.00%	0.00%	0.02%	0.02%	0.00%	0.00%
More than 51	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Total	100.00%	0.00%	100.00%	0.02%	100.00%	0.00%

Source: OeNB.

[1] A fundamental problem event is due to the losses arising from exposures to market risk and nonbank credit risk, while a contagion is triggered by problems of another bank that cannot fulfill its promises in the interbank market. The probability of occurrence of fundamental problem events alone and concurrently with contagious problem events is observed. The time horizon is one quarter. The column *Current situation* shows the result for a simulation without stress. The Column *GDP stress* shows the case of a stress test with an unexpected drop in GDP. The column *Interest rate stress* shows the stress test with a parallel upward shift in the euro yield curve. Data are from December 2005.

Table 1 shows that in the base case simulation of the current situation we have no scenario with more than 5 fundamental problem events . None of the scenarios including up to 5 fundamental problem events shows contagion. This result is consistent with the findings in Elsinger, Lehar and Summer (2006a), who show that contagion is a rare event given a risk factor change distribution calibrated to historical data. In situations of stress, the picture changes: When we have a drop in GDP, up to 50 fundamental problem events can occur, and there can also be some contagion once we have 21 to 50 fundamental problem events. The stress test for an interest rate hike looks less spectacular. The simulations show no contagion effects but the number of scenarios where at least one and up to at most five problem events are expected to occur increases. The analyst using SRM has the opportunity to look deeper into the microstructure of these results and find out details about the institutions that are most severely hit under the stress scenario.

4.3 Probability Distribution of Problem Events According to the OeNB Master Scale

To get a more precise idea about the distribution of risk within the banking system, we map the probabilities of problem events into the OeNB master scale. This distribution of ratings, which is implied by our simulation, is shown in table 2.

Table 2: Probability Distribution of Problem Events According to the OeNB Master Scale[1]

		Current situation		GDP stress		Interest rate stress	
OeNB MS	S&P	abs.	rel.	abs.	rel.	abs.	rel.
1 to 2	AAA to AA	800	94.67%	779	92.19%	791	93.61%
3 to 4	A to BBB	23	2.73%	35	4.14%	22	2.61%
5 to 7	BB to CCC	22	5.22%	31	7.46%	31	6.05%

Source: OeNB.

[1] Share of Banks in OeNB rating classes. Data are from December 2005.

Table 2 shows that in the base case simulation, about 95% of banks are expected to be in a triple or double A rating at the end of the first quarter of 2006. Under the assumptions of our two stress scenarios, the number of top-rated institutions decreases slightly. The biggest increase under stress can be observed in the lower rating classes.

4.4 Aggregate Loss Distributions

Turning from problem events to the distribution of losses over the next quarter, we can draw pictures of the losses due to credit risk, market risk and contagion risk as well as due to the combination of all of these risks. Contrary to familiar pictures from the practice of risk management, these distributions are derived from an integrated analysis of all portfolio positions and their change in value due to the entire distribution of risk factor changes. Thus rather than analyzing credit and market risk in isolation, these graphs give us the results of an integrated analysis.

Chart 3: Loss Distributions: Total, Market, Credit and Contagion Risk[1]
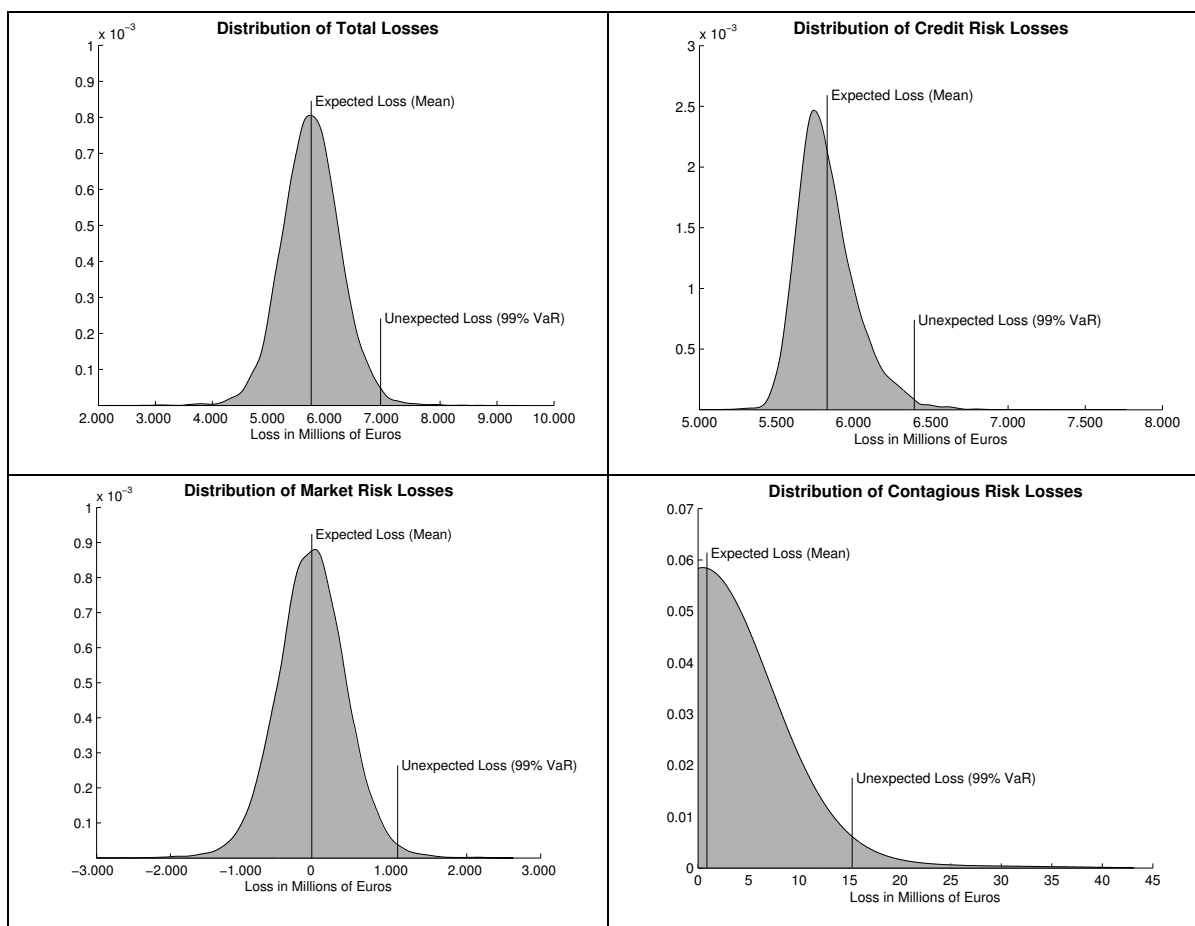
Source: OeNB.

[1] Densities of loss distribution for the entire banking system. The densities are shown for the entire portfolio and separately for market and credit risk as well as for the losses due to contagion. Data are from December 2005.

Chart 3 shows four loss distributions. From the figures we can see – as in standard quantitative risk management – whether or not the system has enough capital to absorb extreme losses.

Therefore loss distribution figures give a first overview of the shock absorption capacity of the system.

4.5 Changes in System-Wide VaR under Stress

We analyze the distribution of losses relative to regulatory capital, that is, we look at the distribution of losses as a percentage of regulatory capital and determine certain quantiles of this distribution. In our case we analyze the mean and the 99% quantile (or the 99% value at risk). We look at these measures for the different subcategories, total losses, market losses, credit losses and contagion losses. The results for the base case as well as for the stress scenarios are reported in table 3.

Table 3: Mean and 99% Quantile of Loss Distribution Relative to Regulatory Capital[1]

	Total[2]		Market		Credit (*)		Contagion	
Rel. VaR	Mean	99%	Mean	99%	Mean	99%	Mean	99%
Current situation	1,56%	4,04%	-0.18%	2.11%	1,74%	2,82%	0.00%	0.03%
GDP stress	1,68%	7,42%	-0.15%	5.68%	1,82%	2,99%	0.01%	0.05%
Interest rate stress	3,87%	6,23%	2.11%	4.34%	1,75%	2,87%	0.01%	0.04%

Source: OeNB.

[1] Mean and 99% quantile of the distribution of losses relative to regulatory capital for total losses, losses from market risk, losses from credit risk and losses from contagion risk. This relative VaR is shown for the baseline simulation, for the case of a GDP stress test and for the case of the euro yield curve stress test. Data are from December 2005.

[2] In order to reflect the risk-bearing capacity with respect to different risk categories, the volume of specific and general provisions for credit risk losses as of end-2005 was substracted from the mean and the 99% quantile of the distribution of credit losses and total losses, respectively, before the respective numbers were divided by regulatory capital.

Table 3 shows that the Austrian banking system is very well capitalized. Even under the stress scenarios capital is sufficient to absorb potential losses that result from risk factor movements.

4.6 Value at Risk for the Lender of Last Resort

A relevant aspect of our model for the regulator is that it can be used to estimate the cost of crisis intervention. We estimate the funds that would have to be available to avoid contagion or even fundamental problem events for different confidence levels. A lender of last resort's cost of preventing problems in the banking system is calculated as the amount required to prevent problem events. A lender of last resort's cost of preventing contagion is calculated as the amount required to prevent all but fundamental problem events. Hence, interbank liabilities are not fully insured but just sufficiently to prevent contagion.

Table 4: Costs of Avoiding Problem Events[1]

	Current situation		GDP stress		Interest rate stress	
Quantiles	95%	99%	95%	99%	95%	99%
Resources	29.16	31.58	29.16	44.71	1.24	21.4

Source: OeNB

[1] In the first bottom row we give estimates for the 95% and 99% percentiles of the avoidance cost distribution across scenarios. Amounts are in EUR million. Data are from December 2005. Source: OeNB.

Since problem events occur rarely in the base scenario the amounts that must be available to prevent these events are low. The analysis shows that for the quarter ending in December 2005 a lender of last resort can expect that even if crisis scenarios simulated by the model do actually occur, the amounts to be mobilized for crisis intervention will be small.

5 Conclusions

Systemic Risk Monitor implements a new framework for banking system risk assessment. The innovation is that SRM analyzes risk at the level of the entire banking system rather than at the level of an individual institution.

Conceptually, it is possible to take this perspective by carrying out a systematic analysis of the impact of a set of market and macroeconomic risk factors on banks in combination with a network model of mutual credit relations.

Whereas the modelling of noninterbank market and credit losses is rooted in standard quantitative risk management techniques, the combination with an interbank network model to arrive at total gains and losses in the banking system in SRM is new. Both the generalizations of standard individual risk management techniques and the simultaneous consideration of portfolio values across the system for given risk factor changes as well as the resolution of bilateral claims via a network clearing model focus on the main issues for an institution in charge of monitoring systemic financial stability: the probability of joint problems of institutions and their financial consequences. The system perspective uncovers exposures to aggregate risk that remain invisible for banking supervision that relies on the assessment of single institutions only. We distinguish problems caused directly by a macroeconomic shock from those triggered by problems of other banks in the interbank market.

We hope that SRM will prove useful as a tool of macro-prudential risk analysis and that the framework will be of interest to other institutions with a mandate to safeguard and maintain systemic financial stability.

References

Bonti, Gabriel, Michael Kalkbrener, Christopher Lotz and Gerhard Stahl. 2005. Credit Risk Concentration under Stress. Contribution to the conference 'Concentration Risk in Credit Portfolios', Bundesbank. Journal of Credit Risk and Basel Committee.

Boss, Michael. 2002. A Macroeconomic Credit Risk Model for Stress Testing the Austrian Credit Portfolio. OeNB Financial Stability Report 4. 64–82.

Boss, Michael, Thomas Breuer, Helmut Elsinger, Martin Jandacka, Gerald Krenn, Alfred Lehar, Claus Puhr and Martin Summer. 2006. Systemic Risk Monitor: A Model for Systemic Risk Analysis and Stress Testing of Banking Systems. Technical Report. Oesterreichische Nationalbank.

Credit Suisse. 1997. Credit Risk+, A Credit Risk Management Framework. Credit Suisse Financial Products.

Elsinger, Helmut, Alfred Lehar and Martin Summer. 2006a. Risk Assessment for Banking Systems, Management Science. Forthcoming.

Elsinger, Helmut, Alfred Lehar and Martin Summer. 2006b. Using Market Information for Banking System Risk Assessment. International Journal of Central Banking 2(1). 137–165.

McNeil, Alexander, Rüdiger Frey and Paul Embrechts. 2005. Quantitative Risk Management: Concepts, Techniques and Tools. Princeton University Press.

OeNB and FMA (Financial Market Authority). 2005. Off-Site Analysis Framework of Austrian Banking Supervision. Austrian Banking Business Analysis. Technical Report. Oesterreichische Nationalbank.

Modelling the distribution of credit losses with observable and latent factors [1]

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Abstract

This paper proposes a dynamic model to estimate the credit loss distribution of the aggregate portfolio of loans granted in a banking system. We consider a sectorial approach distinguishing between corporates and individuals. The evolution of their default frequencies and the size of the loans portfolio are expressed as functions of macroeconomic conditions as well as unobservable credit risk factors, which capture contagion effects between sectors. In addition, we model the distributions of the Exposures at Default and the Losses Given Default. We apply our framework to the Spanish banking system, where we find that sectorial default frequencies are not only affected by economic cycles but also by a persistent latent factor. Finally, we identify the riskier sectors and perform stress tests.

1   Introduction

During the last years, a more volatile and dynamic financial environment has caused an increasing concern about the stability of banking systems. In this sense, it is widely agreed that credit risk is one of the variables that are more directly related to financial stability. Indeed, the Basel II framework has put forward the need of measuring this type of risk accurately. As a consequence, there has been a number of papers that estimate the credit loss distributions of the loans portfolios of different countries.[1]

These papers generally follow a top-down approach by analysing the banking sector as a whole. Most of them also emphasise the need of assessing the variability of credit risk across different sectors. In addition, since the early works of Wilson (1997a,b), most subsequent studies relate changes in the probabilities of default to changes in macroeconomic conditions (see also Demchuk and Gibson, 2006). Specifically, it is usually assumed that, conditional on the macroeconomic explanatory variables, defaults are independent across sectors. However, this assumption might yield strongly biased results if a relevant factor is omitted. What is more important, on top of macroeconomic variables, there might exist some credit risk factors that induce contagion across sectors, but which we cannot directly observe. This issue has already been a cause of concern in the literature. Unfortunately, most of the empirical research has generally focused on either large corporates or publicly traded instruments, such as bonds or stock returns. For instance, Schuermann and Stiroh (2006) have found an important presence of "hidden risk factors" in U.S. banks stock returns, while Duffie, Eckner, Horel, and Saita (2006) have noticed that the effects of these factors on the correlation of defaults might be larger if they are persistent. However, much less is known about the presence of latent factors in the credit loss distribution of loans.

This paper proposes a credit risk model that allows for the presence of persistent latent factors. We express loans losses in terms of four stochastic components: default frequencies, the size of the loans portfolio, the exposures at default and the losses given

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[1]To cite a few examples, Boss (2002) has developed a credit risk model for Austria, Virolainen (2004) has considered the case of Finland, Misina, Tessier, and Dey (2006) have analysed the Canadian loans portfolio, Drehmann (2005) and Drehmann, Patton, and Sorensen (2006) have studied the credit loss distribution in the U.K., while Pesaran, Schuermann, Treutler, and Weiner (2006) have considered an international credit risk model.

default. The importance of modelling the size of the loans portfolio has been traditionally neglected. However, it is necessary to take into account this variable if we want to study the total losses of a banking system, and not just those due to a fixed number of loans. For each of the economic sectors in which we arrange the loans, we assume that changes in the default frequencies and the total number of loans are a function of past observations of the dependent variables, a set of observable characteristics, some potentially persistent common latent factors and one idiosyncratic component. The effect of observable factors is to introduce correlation between different loans due to clearly identifiable shocks, such as a fall in GDP growth. In contrast, the latent components will generate contagion effects that are orthogonal to the observable events. Conditional on default, the loss given default and the exposure at default are initially assumed to be independent of default rates and the size of the credit market, although they are allowed to have a different distributional shape for each sector. With the exception of Madan and Unal (2006) in the context of deposit insurance, the literature has paid little attention to the distribution of exposures at default. However, we believe that it is necessary to account for the variability of exposures within each sector in order to correctly describe the heterogeneity of loans. Specifically, we employ either the Inverse Gaussian or the Gamma distribution. Both are flexible distributions whose statistical properties can be exploited to reduce by a considerable amount the computational demands of our model. Additionally, we propose a generalisation in which these distributions can change as a function of the observable macroeconomic factors. Finally, we consider the usual Beta distribution to describe the loss given default (see e.g. Gupton and Stein, 2002).

We use our model to estimate the credit loss distribution of the Spanish banking system. We have quarterly loan data from 1984.Q4 to 2006.Q4, obtained from the Spanish Credit Register. This database contains information on every loan granted in Spain with an exposure above €6,000. Since this threshold is very low, we can safely assume that we have data on virtually every loan granted in Spain. Hence, we use high quality loan data at a frequency at which it is not usually available. In this sense, it is worth remarking that we are able to obtain actual default rates from our database. In contrast, most of the literature usually relies on bankruptcy rates, which are imperfect proxies of

defaults.[2] We consider 10 corporate sectors plus one group for mortgages and another one for consumption loans. We first estimate a simple model with changes in GDP growth and three-month interest rates as our macroeconomic factors. Then, we obtain the credit loss distribution by simulating losses from our model under the current economic conditions and under some stressed scenarios. Interestingly, we are able to identify a persistent unobservable factor that generates dependence between sectorial default frequencies, and an analogous effect on the growth of the number of loans. These factors remain significant when we reestimate our model with an augmented set of macroeconomic characteristics. We also determine which sectors are riskier, and compare our model with simpler versions that have been previously implemented. In this sense, we show that latent factors are crucial to capture the empirical correlations between sectorial default frequencies. In addition, we assess the out-of-sample stability of our model. Finally, we explore the relationship between exposures at default and macroeconomic conditions, where we find that they tend to be higher on average during recessions than during expansions. This result is consistent with the findings of Jiménez, López, and Saurina (2007), who find, also for the Spanish loan market, that a higher usage rate of credit lines during recessions induces higher exposures at default in these periods.

In summary, we believe that our paper provides some important contributions to the literature. Firstly, this paper introduces unobservable common shocks in a credit risk model of loans losses. Secondly, the paper takes advantage of the use of a very rich dataset which contains precise information about almost all the loans granted in the Spanish economy. In particular, we are able to model the distribution of exposures at default, as well as the loan market dynamics. In addition, we consider an extensive sectorial structure that includes mortgages and consumption loans. Thirdly, our results show that value at risk can be significantly underestimated if contagion effects between sectors are not allowed. Finally, we dramatically reduce the computational demands of our model by exploiting its statistical properties.

The rest of the paper is organised as follows. We describe our model in the next section, and discuss the estimation of its parameters in Section 3. In Section 4, we consider an empirical application to Spanish loan data. Finally, concluding remarks and directions

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[2]See the discussion by Duffie, Eckner, Horel, and Saita (2006)

for future research are suggested in Section 5.

2   The credit risk model

We are interested in modelling credit risk in an economy with K sectors. We will consider a sample of T periods of data. In this context, the losses due to a loan i from sector k can be decomposed at any time period t as

$$L_{i,k,t} = D_{i,k,t} LGD_{k,t} EAD_{i,k,t},$$

where $D_{i,k,t}$ is a binary variable that equals 1 in case of default and 0 otherwise, while $LGD_{k,t} \in (0,1)$ and $EAD_{i,k,t} > 0$ are, respectively, the loss given default and the exposure at default. We will denote the proportion of non-performing loans in sector k at time t as p_{kt}, i.e. the ratio of the number of loans in default to the total number of loans in each sector. This variable is usually known as default frequency. Hence, the losses from sector k at time t can be expressed as

$$L_{k,t} = \sum_{i=1}^{n_{k,t}} L_{i,k,t} = LGD_{k,t} S_k(p_{kt} n_{k,t}), \tag{1}$$

where $n_{k,t}$ is the total number of loans in sector k and

$$S_{kt} = \sum_{i=1}^{\lfloor p_{kt} n_{k,t} \rceil} EAD_{i,k,t}. \tag{2}$$

where $\lfloor p_{kt} n_{k,t} \rceil$ rounds $p_{kt} n_{kt}$ to the nearest integer. Without loss of generality, we have assumed that the first loans in the sum (1) are those that default. We have also supposed that the losses given default are homogeneous in each sector because this type of information is rarely available for loans at a more disaggregated level. If we assume that the probability of default is constant in each sector, p_{kt} will converge to the probability of default of sector k as n_{kt} grows to infinity. However, for small n_{kt}, they will not necessarily coincide.

The main dynamic features of our model are introduced with a joint model for p_{kt} and n_{kt}. In order to work with variables with support on the whole real line, we transform the default frequencies by means of the probit functional form $y_{kt} = \Phi^{-1}(p_{kt})$, where $\Phi^{-1}(\cdot)$ is the inverse of the standard normal cumulative distribution function. Alternatively, a logit

model could also be adopted. For every sector, we define the growth of the number of loans as $\Delta n_{kt} = \log(n_{kt}) - \log(n_{kt-1})$, while the changes in the transformed default frequencies are defined as $\Delta y_{kt} = y_{kt} - y_{kt-1}$.[3] We propose the following vector autoregression for these variables:

$$\Delta n_{kt} = \alpha_{1,k} + \sum_{j=1}^{q} \rho_{1,j} \Delta n_{kt-j} + \sum_{j=1}^{r} \boldsymbol{\gamma}'_{1,j} \mathbf{x}_{t-j} + \boldsymbol{\beta}_{1,k} f_{1,t} + u_{1,kt}, \qquad (3)$$

$$\Delta y_{kt} = \alpha_{2,k} + \sum_{j=1}^{q} \rho_{2,j} \Delta y_{kt-j} + \sum_{j=1}^{r} \boldsymbol{\gamma}'_{2,j} \mathbf{x}_{t-j} + \boldsymbol{\beta}_{2,k} f_{2,t} + u_{2,kt}. \qquad (4)$$

In consequence, the evolution of Δn_{kt} and Δy_{kt} depends on their previous history, a set of m observable characteristics $\mathbf{x}_t$, two unobservable common factors, $f_{1,t}$ and $f_{2,t}$, and the idiosyncratic shocks $u_{1,kt} \sim N(0, \sigma_{1k}^2)$ and $u_{2,jt} \sim N(0, \sigma_{2k}^2)$, for $j, k = 1, \cdots, K$. These idiosyncratic terms are assumed to be *iid* jointly Gaussian and independent from the common shocks. In addition, we only allow for correlation between the two idiosyncratic terms from the same sector, i.e. $\text{cov}(u_{1,kt}, u_{2,jt}) = 0$ for $k \neq j$.

We consider the following vector autoregressive structure for the observable factors:

$$\mathbf{x}_t = \delta_0 + \sum_{j=1}^{s} \mathbf{A}_j \mathbf{x}_{t-j} + \mathbf{v}_t, \qquad (5)$$

where $\mathbf{v}_t \sim N(\mathbf{0}, \boldsymbol{\Omega})$. To ensure the identification of the model, we assume that f_{1t} only affects (3), whereas f_{2t} can only influence default frequencies. However, we allow for correlation between these factors. In particular, if we define the vector $\mathbf{f}_t = (f_{1t}, f_{2t})'$, the dynamics of $\mathbf{f}_t$ can be expressed in terms of the following VAR(1) model:

$$\mathbf{f}_t = \mathbf{R} \mathbf{f}_{t-1} + \mathbf{w}_t. \qquad (6)$$

where

$$\mathbf{R} = \begin{bmatrix} \phi_1 & 0 \\ 0 & \phi_2 \end{bmatrix}.$$

and $\mathbf{w}_t$ is Gaussian with zero mean and

$$V(\mathbf{w}_t) = \begin{bmatrix} 1 - \phi_1^2 & \rho\sqrt{(1-\phi_1^2)(1-\phi_2^2)} \\ \rho\sqrt{(1-\phi_1^2)(1-\phi_2^2)} & 1 - \phi_2^2 \end{bmatrix}. \qquad (7)$$

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[3]We specify our model in first differences because the levels are usually nonstationary in this type of applications (see e.g. Boss, 2002, and our empirical application). However, it will be straightforward to rewrite our model in levels if necessary.

Hence, ϕ_i is the first order autocorrelation of $f_{i,t}$, for $i = 1, 2$, and ρ is the conditional correlation between $f_{1,t}$ and $f_{2,t}$. Since $\mathbf{f}_t$ is unobservable, we have to fix its scale to ensure the identification of the model. This is why we have parametrised (7) so that the latent factors have unit unconditional variances. In addition, we assume that $\mathrm{cov}(\mathbf{v}_t, \mathbf{w}_t) = \mathbf{0}$, which implies that the latent factors are orthogonal to the observable characteristics. Hence, these unobservable components introduce a source of contagion between sectors that cannot be attributable to the observable shocks. Giesecke and Weber (2004) show that these effects may be caused by the interaction of firms with their business partners, while Kiyotaki and Moore (1997) argue that the relationship between credit limits and asset prices can create a transmission mechanism by which shocks will persist and spill over to other sectors. Nevertheless, our approach is focused on empirically assessing the existence of latent factors, without precluding or favouring any of these explanations.

Finally, we will suppose that, conditional on default and the current macroeconomic conditions, $LGD_{k,t}$ are random Beta variates, while $EAD_{i,k,t}$ are independent Inverse Gaussian or Gamma variates.[4] We will first suppose that the parameters of these distributions are constant over time but possibly different for each sector. This implies that their distributions do not depend on the cycle. Later on, we will extend this model by allowing the mean of $EAD_{i,k,t}$ to depend on the macroeconomic factors. Specifically, if we denote the mean of the exposures at default in sector k and period t as μ_{kt}, we propose the following parametrisation:

$$\mu_{kt} = \mu_{kt-1} \exp\left[\eta_k + \boldsymbol{\varphi}_k' \mathbf{v}_{t-1} - \frac{1}{2} \boldsymbol{\varphi}_k' \boldsymbol{\Omega} \boldsymbol{\varphi}_k \right] \tag{8}$$

where η_k captures a time trend, $\mathbf{v}_{t-1}$ is the lagged vector of innovations in equation (5) and $\boldsymbol{\Omega}$ is its covariance matrix. Thus, we allow μ_{kt} to be influenced by the same shocks that affect $\mathbf{x}_t$. Of course, if $\boldsymbol{\varphi}_k = \mathbf{0}$ we are back in the static setting. The time trend component turns out to be important for estimation purposes. For example, in a context of historically decreasing exposures, this component will be negative. However, when we compute the credit loss distribution, we will assume no particular trend by setting this parameter to zero. In consequence, it is important to include the term $\boldsymbol{\varphi}_k' \boldsymbol{\Omega} \boldsymbol{\varphi}_k / 2$ in (8)

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[4]We have compared the empirical performance of these two distributions with other potential candidates. Our results show that the Gamma and the Weibull yield a similar empirical fit, while the shapes generated by the IG are similar to those of the log-normal. These results are available on request. However, we will not consider the Weibull nor the Log-normal because they are not closed under aggregation.

to ensure that

$$E\left[\exp\left[\boldsymbol{\varphi}_k'\mathbf{v}_{t-1} - \frac{1}{2}\boldsymbol{\varphi}_k'\boldsymbol{\Omega}\boldsymbol{\varphi}_k\right]\right] = 1.$$

This result, which is a consequence of the normality of $\mathbf{v}_t$, ensures the constancy of the unconditional mean of (8) when η_k is set to zero. It is also possible to consider a dynamic parametrisation of the distribution of the loss given default (see Bruche and González-Aguado, 2006). However, due to lack of data in our application, we will not be able to explore this extension.

3   Estimation and simulation of the model

To estimate the parameters in (3) and (4), we need to use the Kalman filter to deal with the unobserved factors. The intuition of this procedure is as follows. To evaluate the likelihood at each period t, we first compute the expected value of the factors given the information available up to time $t-1$:

$$\mathbf{f}_{t|t-1} = E(\mathbf{f}_t|\{\Delta\mathbf{n}_s, \Delta\mathbf{y}_s, \mathbf{x}_s\}_{1 \leq s \leq t-1}),$$

where $\Delta\mathbf{n}_s = (\Delta n_{1,s}, \cdots, \Delta n_{K,s})'$ and $\Delta\mathbf{y}_s = (\Delta y_{1,s}, \cdots, \Delta y_{K,s})'$. In addition, since $\mathbf{f}_{t|t-1}$ is a noisy estimate of the true realisation $\mathbf{f}_t$, we also need to measure the uncertainty of this estimate:

$$\mathbf{P}_{t|t-1} = V\left[\mathbf{f}_t|\{\Delta\mathbf{n}_s, \Delta\mathbf{y}_s, \mathbf{x}_s\}_{1 \leq s \leq t-1}\right].$$

Finally, the estimation procedure consists basically in treating (3) and (4) as a pure vector autoregressive model, by using the series of $\mathbf{f}_{t|t-1}$ as if they were actually observed. However, we must adjust the variance of the model with $\mathbf{P}_{t|t-1}$ to account for the fact that $\mathbf{f}_{t|t-1}$ is not equivalent to the true realisation $\mathbf{f}_t$ (see e.g. Hamilton, 1994, for a formal discussion).

Interestingly, as new data arrives, we can update our previous estimates of the realisations of the factors, and obtain more accurate ones. For example, given the whole sample of data, we can estimate the evolution of the latent factors as:

$$\mathbf{f}_{t|T} = E(\mathbf{f}_t|\{\Delta\mathbf{n}_s, \Delta\mathbf{y}_s, \mathbf{x}_s\}_{1 \leq s \leq T}).$$

To identify the factors, we need at least two sectors. In fact, the more sectors we have, the more precise our estimates of $\mathbf{f}_t$ will be. Hence, latent factors are particularly

valuable in models with many sectors, since they allow for rich dynamics and correlation structures without requiring too many parameters.

As we have remarked, we consider two possible distributions for $EAD_{i,k,t}$: the Inverse Gaussian (IG) and the Gamma distribution. For each sector, we choose the one that best fits the data from the sector. Their parameters are estimated by maximum likelihood, where their density functions can be expressed as:

$$f_{IG}\left(EAD_{i,k,t} = x; \mu_k, \lambda_k\right) = \left(\frac{\lambda_k}{2\pi x^3}\right)^{1/2} \exp\left[-\frac{\lambda_k}{2\mu_k^2 x}\left(x - \mu_k\right)^2\right] \qquad (9)$$

$$f_{Gamma}(EAD_{i,k,t} = x; \nu_k, \tau_k) = \frac{(x/\tau_k)^{\nu_k/2-1}}{2^{\nu_k/2}\Gamma(\nu_k/2)\tau_k} \exp\left(\frac{-x}{2\tau_k}\right) \qquad (10)$$

We will denote these distributions as $IG(\mu_k, \lambda_k)$ and $Gamma(\nu_k, \tau_k)$, respectively. In the IG case μ_k is the mean, and μ_k^3/λ_k is the variance, whereas for the Gamma distribution the mean is $\nu_k\tau_k$ and the variance $\nu_k\tau_k^2$. The subindices indicate that these parameters are sector specific. As we show in the empirical application, both distributions provide a good fit of the data, although the IG generally outperforms the Gamma. In addition, it can be shown that sums of *iid* IG or Gamma variates remain within the same family (see Johnson, Kotz, and Balakrishnan, 1994). Due to this property, we can express the distribution of S_{kt} in closed form for a given number of defaults $\lfloor p_{kt}n_{kt}\rfloor$. Specifically, it can be shown that the distribution of S_{kt} conditional on the number of defaults at t is a $IG[\lfloor p_{kt}n_{kt}\rfloor\mu_k, \lfloor p_{kt}n_{kt}\rfloor^2\lambda_k]$ in the IG case, while it is a $Gamma(\lfloor p_{kt}n_{kt}\rfloor\nu_k, \tau_k)$ in the Gamma case. From this result, we can express the distribution of the sum of EAD's given only the information known at $t - s$ by means of the following sum:

$$f\left(S_{kt} \mid I_{t-s}\right) = \sum_{i=0}^{\infty} g\left(S_{kt} \mid p_{kt}n_{kt} = i, I_{t-s}\right) \Pr\left(\lfloor p_{kt}n_{kt}\rfloor = i \mid I_{t-s}\right) \qquad (11)$$

where $g\left(S_{kt} \mid \lfloor p_{kt}n_{kt}\rfloor = i, I_{t-s}\right)$ is the conditional density function of S_{kt} given i defaults occurring at t, while I_{t-s} denotes the information known at $t-s$. Finally, $\Pr(\lfloor p_{kt}n_{kt}\rfloor = i \mid I_{t-s})$ is the probability of i defaults occurring at t given I_{t-s}.

Unfortunately, we cannot compute (11) in closed form because it is extremely difficult to obtain the exact values of $\Pr(\lfloor p_{kt}n_{kt}\rfloor = i \mid I_{t-s})$ due to the dynamic features of the model followed by p_{kt} and n_{kt}. Moreover, when we consider the dynamic parametrisation (8) for the means of exposures at default, we will only be able to express $g(S_{kt} \mid p_{kt}n_{kt} = i, I_{t-s})$ in closed form for $s = 1$. Due to this complexity, we will have to compute the

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credit loss distribution by simulation. However, the IG and the Gamma distributions offer important computational advantages. In particular, thanks to their properties, we do not need to simulate individual exposures at default, but just their sum S_{kt}, which will severely speed up the computation of the credit loss distribution.

4 Empirical application

We use loan data from the Credit Register of the Bank of Spain (CIR). This database records monthly information about all the loans granted by credit institutions in Spain (commercial banks, savings banks, credit cooperatives and credit finance establishments) for a value above €6,000. Although the database offers a wider amount of information, we will focus on the particular details directly related to our application (see Jiménez and Saurina, 2004, and Jiménez, Salas, and Saurina, 2006, for a thorough description). In particular, the database reports the amount drawn and available for each loan, and whether its borrower is an individual or a company. In the latter case, the specific economic sector to which the borrower belongs is reported as well. There is also information available about the state of the loans. Every new loan is assigned a code which only changes if its situation deteriorates or if it matures. A loan that is expected to fail in the near future is classified as "doubtful". If the loan eventually defaults, every month the database reports the time elapsed since its default. In particular, we will know whether it has been in default from 3 to 6, 6 to 12, 12 to 18, 18 to 21, or more than 21 months.

From the CIR, we have obtained quarterly series from 1984.Q4 to 2006.Q4 of sectorial default frequencies (p_{kt}), the total number of loans per sector (n_{kt}) and the exposures of the defaulting loans. Most papers usually focus on corporate loans. Typically, this is due to lack of available data on loans to individuals. However, we believe that loans to individuals, and specially mortgages, play an important role in the credit loss distribution of banks. In consequence, we consider 2 sectors for individuals and 10 corporate sectors. For individuals, we consider one group of mortgages and another one for consumption loans. For corporate loans, we define the following economic sectors: (1) Agriculture, livestock and fishing; (2) Mining; (3) Manufacture; (4) Utilities; (5) Construction and real estate; (6) Commerce; (7) Hotels and restaurants; (8) Transport, storage and communications; (9) Renting, computer science and R&D. Finally, those companies that cannot be classi-

fied in any of the previous sectors are gathered in an additional group denoted as Other Corporates (10). However, we remove from the database all the companies from the financial sector, because of their particular characteristics.

In each quarter, we compute the default rates as the ratio of the number of loans that have been in default from 3 to 6 months to the total number of loans in each sector. This definition is consistent with the Basel II framework. Those loans that have been in default for more than 6 months are left out because they were already considered in one of the previous quarters. Thus, only newly defaulted loans are considered at each period. Additionally, we have also obtained the individual exposures of the non-performing loans for every quarter.

Figure 1 (a) shows the historical evolution of default frequencies. For the sake of comparability, we represent in Figures 1 (c) and 1 (d) the quarterly series of the Spanish GDP annual growth and the 3-month real interest rates, respectively.[5] We can observe an increasing trend of default frequencies in all sectors from the end of the 1980s until almost the mid 1990s. This period coincides with a strong recession in the Spanish economy which had its trough in 1993, as we can check in Figure 1 (c). In addition, interest rates also increased from 4% in 1988 to values above 8% in the first half of the 1990's. Loans to construction companies and hotels were more affected than the rest in this recession, with default frequencies peaking at 4%. In contrast, the default frequencies of mortgages reached 1.5% at the worst moment of the recession. From 1995 to the present, economic conditions have steadily improved, except for a brief period from 2000 to 2001. Interest rates have experienced a sharp decline in the last decade due to the convergence and integration in the European Monetary Union, and GDP growth has remained positive and less volatile than in the past (see Martín, Salas, and Saurina, 2005, for a more detailed analysis). As a consequence, during this expansionary period default frequencies have dropped to the lowest historical values in the sample. Under the current conditions, hotels and communications are the two sectors with higher default frequencies. In comparison, defaults in the construction sector are remarkably low at the moment.

Figure 1 (b) shows the quarterly series of the total number of loans in each sector. The loan market size has steadily grown in all sectors during the sample period under

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[5]Following the methodology of Davidson and MacKinnon (1985), we have obtained real interest rates from the nominal rates and inflation.

analysis. From this impressive growth it is not difficult to conclude that assuming a constant number of loans could yield inaccurate results. In addition, if we take a closer look at this figure, we can see that the rate of growth decreased for almost all sectors in the first half of the previous decade, that is, during the last recession. In consequence, the evolution of these variables seems to be correlated with the economic cycle. However, this conjecture will have to be confirmed with more formal results.

4.1   A simple model with two macroeconomic factors

We will start with a simple model that only considers two macroeconomic factors: the quarterly change in real GDP growth and the variation of three-month real interest rates.[6] We employ these two factors because they are generally regarded in the literature as the most important macroeconomic determinants of credit risk fluctuations. In addition, in this first set of estimations, we will assume that the parameters of the distribution of the exposures are constant over time.

Default frequency and market size growth.   Let us consider the estimation of (3) and (4).[7] We will introduce the lags $2, 3$ and 4 of our two macroeconomic variables. To save parameters, we do not include the first lag, because we obtain insignificant estimates for this lag once the subsequent 3 lags are considered. The intuition of this result relies in the definition of default: not meeting the scheduled payments for at least one quarter. In consequence, the default frequencies of period t are related to borrowers who originally became insolvent in period $t - 2$. In this sense, it seems reasonable that we do not obtain significant sensitivities with respect to the first lag of the observable factors. As for the autoregressive structure, we consider the effect of the first lag of the dependent variables, as well as a seasonal effect by means of the fourth lag. Finally, we consider three dummies whose values are 1 in 1988.Q1, 1988.Q4 and 1996.Q2, respectively, and zero otherwise.[8] These dummies are intended to capture the effects of historical exogenous changes in the

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[6]A similar analysis has been conducted with nominal interest rates yielding similar results, which are available on request.

[7]Prior to estimation, we have conducted a series of unit root tests on the data (see Breitung and Pesaran, 2005, for a review of this literature). Our results have shown us that we need to model default rates and the total number of loans in first differences to ensure their stationarity.

[8]The first dummy only affect mortgages, the second dummy affects mortgages and consumption loans, whereas the third dummy affects all sectors.

database (see Delgado and Saurina, 2004, for a formal justification).

The estimates of the default frequency model are shown in Table 1, whereas analogous results for the evolution of the size of the credit portfolio can be found in Table 2. Intuitively, an increase in GDP growth tends to reduce default frequencies and induce an expansion of the loan market. This is why we observe that GDP growth generally has a negative impact on the variation of default frequencies and a positive effect on the growth of the credit market. As Table 1(a) shows, the effect of GDP on default frequencies seems to be more important for most sectors, with the first two lags being highly significant in many of them. Nevertheless, mining and utilities react less to the cycle, while some sectors seem to respond more slowly to aggregate shocks. For instance, we only observe a significant effect on R&D and mortgages two quarters after a shock to GDP has occurred. In Table 2(a), we can observe that the effect of GDP on the size of the credit market is smaller, although it is still significant for manufacture, construction, commerce, and R&D.

As for interest rates, higher values generally tend to increase default frequencies, with significant coefficients for agriculture, hotels and communications. However, the overall effect of higher interest rates on the size of the loan industry is less clear. In some cases, they may even strengthen its growth. Nevertheless, from a theoretical point of view, it is unclear how interest rates should affect the growth of the number of loans. On the one hand, higher interest rates will reduce the demand of loans. On the other hand, on the supply side banks will have incentives to grant more loans if interest rates rise. Nevertheless, the effect of interest rates seems to be less important than the impact of GDP. This may well be due to the fact that, until very recently, most Spanish borrowers, either corporates or individuals, preferred fixed to variable interest rates. For instance, in 1992 only 26.11% of the credit granted in Spain was linked to variable interest rates. This proportion has steadily increased in subsequent years, reaching 55.02% in 2000, and 74.47% in 2005. However, the predominant fixed interest rates for most of our sampling period have surely weakened the impact of interest rates variations in our model.

The last column of Tables 1(a) and 2(a) report the loadings of the unobservable factors. Although we consider two latent factors, we have explained in Section 2 that f_{2t} only affects default frequencies, whereas f_{1t} exclusively alters the size of the credit portfolio. As

we can see, we obtain significant estimates for both factors in all sectors. In addition, we find a significant correlation of -0.473 between f_{1t} and f_{2t} (see Table 3). In consequence, a high value of f_{2t} in a given quarter will induce an increase in default frequencies in all sectors. Moreover, through the negative correlation with f_{1t}, it will tend to cause a reduction in the growth of the loan market. Likewise, a low (negative) value of f_{1t} would produce a similar effect. Hence, f_{1t} and f_{2t} are able to capture a presence of contagion between sectors that the observable factors cannot account for.[9] Furthermore, the time series structure of these factors also deserves some attention. Table 3 shows the autoregressive structure of the observable and unobservable factors. As we can observe, f_{2t} has a significant first order autocorrelation of 0.198. Hence, since shocks to f_{2t} tend to persist through time, their effect on default frequencies will die away slowly. In contrast, f_{1t} has a significant negative autocorrelation of -0.193. In consequence, the effect of a shock to f_{2t} will tend to be reverted in the following periods. For the observable factors, we find a positive (first order) autocorrelation for interest rates, and a negative autocorrelation for GDP growth.

We report the remaining parameters of the model in the lower panels of Tables 1 and 2. The first column of Table 2 (b) shows the positive and highly significant intercept terms that we obtain for the market size growth, which are consistent with the expansion of the loan market already documented in Figure 1 (b). These intercepts are negative but statistically insignificant for default frequencies, as Table 1 (b) shows. The second column of Table 1 (b) shows that the marginal effect of lagged default frequencies from the previous quarter is negative, whereas the seasonal effect (third column) is positive when it is significant. In contrast, both terms are generally positive in the market size equation. Finally, we can observe in the last columns of both tables that the correlation between the idiosyncratic terms from the same sector are generally negative in the significant cases. Hence, shocks that increase the growth of the number of loans in a particular sector tend to be correlated with declines in the rate of defaults from the same sector.

These results can be compared with the estimates reported in Tables 4 and 5, which correspond to a restricted version of our model, where no latent factors are considered. GDP and interest rates have a qualitatively similar impact in this model. However,

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[9]Notice that the latent factors are independent from the observable factors by construction.

the absence of latent factors causes an increase in the absolute correlations between the idiosyncratic terms of default frequencies and loan market growth in each sector (see the last column of Tables 4 (b) and 5 (b)).

Exposure at default. For each sector, we estimate the parameters of the static specifications of the IG and the Gamma distributions by maximum likelihood. Since we assume that these parameters remain constant over time, we focus on the current situation. Hence, we only use the exposures of the loans that defaulted in 2006 to fit the parameters of these distributions. Prior to estimation, we have adjusted the data for inflationary effects. In Figures 2 and 3 we compare for each sector the empirical fit at the right tail of the IG and the Gamma with a Kernel estimate of the empirical density. Except for mortgages, the IG distribution provides a better fit in all sectors. In consequence, we will model the exposures of non-performing mortgages with the Gamma distribution and employ the Inverse Gaussian in the remaining cases.

Loss given default. Unfortunately, we do not have data on the loss given default of the loans in our database. However, Spanish banks have reported the historical average loss given default for corporate, consumption and mortgage loans to the QIS5.[10] Using this data, we choose the parameters of the Beta distribution so that the mean loss given default is 35% for corporates, 25% for consumption loans and 15% for mortgages. Finally, we choose 20% as the standard deviation in the three cases, which is close to the values reported by Altman, Resti, and Sironi (2004).

Credit loss distribution. We estimate the credit loss distribution by simulating losses from our model. For each quarter of the horizon that we consider, we first obtain draws of the total number of loans and the default rates per sector. In particular, we use (3) and (4), where we sample the idiosyncratic terms from their joint Gaussian distribution, and generate the draws of the observable and latent common factors by means of (5) and (6), respectively. In these simulations, we set to zero the unconditional means of the changes of default frequencies, since a positive (negative) intercept would imply that default frequencies would tend to 1(0) in the long run. Thus, our restriction rules out

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[10]Fifth Quantitative Impact Study of the Basel Committee on Banking Supervision.

these extreme cases. Finally, given the total number of defaults, we can generate random replications of (2) and the loss given default from their respective distributions. To ensure the stability of our results, we obtain one million simulated losses from our model.

We report descriptive statistics of the credit loss distribution in Table 6 for the model with latent factors. Specifically, we focus on the expected loss, the Value at Risk (VaR) at the 99.9% level and the unexpected loss, defined as the difference between the first two measures. We consider three different time horizons: 1, 3 and 5 years.[11] We can see that, due to higher uncertainty, the three measures increase more than proportionately as the horizon increases. In terms of expected losses, consumption loans is the riskiest group for short horizons, followed by construction and manufacture. However, for longer horizons mortgages and specially construction also have high expected losses. These three sectors are also the riskiest ones in terms of unexpected losses, specially for long horizons. Again, the VaR of the construction sector seems to grow relatively more with the horizon than in the other cases. This is due to the strong dependence of this sector on cyclical effects, as we already observed in Tables 1 and 2.

Table 7 reports analogous results for the model without latent factors. The differences between sectors are qualitatively similar in this model. For instance, construction and consumption loans are still the riskiest categories. In addition, if we view each sector individually, there are not large quantitative discrepancies between the two models. If anything, it seems that the model without latent factors yields higher sectorial losses. However, as the last row of the table shows, total unexpected losses are much lower in this model, specially for longer horizons. This is due to the fact that we are underestimating contagion effects across sectors when we do not consider the unobservable factors. For example, the unexpected loss at a three year horizon is about 15% larger in the model with latent factors than in the model with only observable explanatory variables. Graphically, we perform a similar comparison in Figure 4, where we plot the total credit loss densities for the two models. Again, we can observe that the model that allows for unobservable factors has fatter tails.

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[11]These horizons start at the end of December 2006, because we are conditioning on the final date of our sample. For instance, three-year horizon losses add all losses that occur up to three years after the start date.

4.2 Extensions and robustness checks

To begin with, we will determine whether we are still able to identify contagion through latent factors when we consider a richer set of observable explanatory variables. Specifically, we will consider, as an additional common factor, the spread between three-month and six-year interest rates. This variable, related to the slope of the term structure of interest rates, will affect all sectors. Moreover, we consider six additional variables that will only have an impact on those sectors that are more related to these characteristics. In particular, we allow the change in the unemployment rate to affect consumption loans and mortgages; gross value added of market services will affect communications, hotels and commerce; gross value added of industry will affect manufacture and mining; and the gross value added series of agriculture, energy and construction will affect agriculture, utilities and construction, respectively. The coefficients obtained with this specification are displayed in Tables 8 and 9. We can observe some significant values for the impact of the spread variable, specially in the evolution of the growth of the number of loans. Specifically, a steepening of the term structure seems to induce an expansion of the number of loans in some sectors. Unfortunately, at least in terms of statistical significance, most of the sectorial factors yield somewhat unsatisfactory results. Nevertheless, in spite of the additional factors, we still obtain highly significant factor loadings for the unobservable effects.

We will now compare the ability of the three different specifications of the VAR model to fit the empirical correlations between default frequencies.[12] To do so, we compute the fitted residuals of the default frequencies in (4) for the three cases. That is, we compute $\varepsilon_{kt}(\hat{\boldsymbol{\theta}}_T) = \Delta y_{kt} - E(\Delta y_{kt-1}|I_{t-1}; \hat{\boldsymbol{\theta}}_T)$ for $k = 1, \cdots, K$, where the expectation is based on the information known at time $t-1$ and the maximum likelihood estimates of the parameters, denoted by the vector $\hat{\boldsymbol{\theta}}_T$. The specification that does not include latent factors assumes that these fitted residuals are uncorrelated because in this case intersectorial correlations are only captured by the observable common characteristics, which are part of the information set I_{t-1}. In contrast, the model with latent factors introduces a factorial structure for these correlations: $\text{cov}(\varepsilon_{it}(\hat{\boldsymbol{\theta}}_T), \varepsilon_{jt}(\hat{\boldsymbol{\theta}}_T)) = \beta_{2,i}\beta_{2,j}$. We test in Table 10

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[12] For the sake of brevity, we focus only on default frequencies. However, we have obtained similar results with the residuals of the equation for the number of loans, which are available upon request.

whether the empirical correlations of the fitted residuals are equal to those hypothesised by each of these specifications. As we can observe in Panel (a), most correlations are not adequately captured when latent factors are neglected. In contrast, Panels (b) and (c) show that these unobservable effects are able to yield a very accurate fit of the empirical residual correlations. Although these results show the good in-sample performance of our model, we are also interested in assessing its out of sample reliability. We will consider the period from 2004.Q1 to 2006.Q4 for this analysis. Hence, we need to reestimate the three specifications of our VAR model using only data up to 2003.Q4. With these estimates, we again compute the fitted residuals of (4), but in this case we will also consider those of (3). We could use these residuals to compute tests analogous to those of Table 10. However, since we only have 12 periods, these tests will have low power. Thus, we prefer to follow a different approach in this case. In particular, we standardise the residuals with the inverse of the Cholesky factorisation of their hypothesised covariance matrices under each specification. The resulting values should be *iid* standard normal under the correct specification. We check this hypothesis in Table 11 by means of a Kolmogorov test. This table shows that the null can be easily rejected when we do not consider latent factors, but it can no longer be rejected once these factors are included. Hence, this result confirms the out-of-sample stability of our model.

Finally, we will explore the linkages between aggregate macroeconomic shocks and the distribution of exposures at default. We have estimated by maximum likelihood the parameters of the IG distribution, substituting (8) for μ_k in (9). Although we have also estimated an analogous model with the Gamma distribution, we do not report the results for this model due to its poorer empirical fit. For the sake of parsimony, we will only consider the effect of the innovations to GDP growth and real interest rate variations. The results are displayed in Table 12. As expected, the estimated means at the end of our sample period, displayed in the first column of Table 12, reflect the differences between the loan sizes across sectors. Specifically, loans to individuals, either mortgages or consumption loans, are characterised by small mean exposures when compared to the much larger sizes of loans to corporates. As for corporates, the more capital intensive sectors have larger mean exposures. For instance, utilities is a sector with relatively few but very large loans. We can also observe in the second column that the time trend

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coefficients are generally negative though small in magnitude. Imposing $\eta_k = 0$ in these estimations would have yielded unstable estimates of the factor loadings. Specifically, the interest rates would then be forced to capture the time effects, because of their decreasing historical trend (see Figure 1d). In the third column, we can observe that GDP generally has a negative and significant effect. In consequence, higher GDP growth will tend to reduce the magnitude of exposures at default on average. Conversely, these exposures will be higher during economic downturns. As for interest rates, we generally obtain positive coefficients. Hence, higher interest rates tend to increase the means of the exposures. These results are consistent with the use of credit lines as a liquidity management tool by firms, as Jiménez, López, and Saurina (2007) show. Moreover, the observed dependence of EAD on the business cycle can reinforce the pro-cyclicality of the Basel II framework. The impact of Basel II on pro-cyclicality has been extensively debated in the literature.[13]The main conclusion is that the minimum capital requirements computed under the Internal Ratings Based (IRB) approach will be more risk-sensitive under Basel II, increasing during recessions and falling as the economy enters expansions. Thus, this will make the lending decisions of banks more pro-cyclical, which, in turn, will amplify the economic cycle. In this sense, our results support the concerns of this literature about the strong relationship between economic cycles and credit risk. However, the global impact of Basel II on the financial stability of the banking system is an issue beyond the scope of this paper.

4.3 Stress tests

We will end this empirical study by assessing the consequences of a strong shock to either GDP or interest rates. We follow the standard practice in stress testing exercises and introduce artificial shocks in the vector of innovations of the factors (see (5)). In particular, we stress our model with a 3-standard deviation shock that occurs in the first quarter of the period under study. We consider separate shocks to each of the two macroeconomic factors that we stress. The GDP shock will be negative, whereas the interest rate shock will be positive. Thus, these tests are designed to induce a recession in both cases.

As in the previous sections, we will start with our baseline model, in which GDP

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[13]See for instance Goodhart (2005), Goodhart and Taylor (2005), Gordy and Howels (2006), Kashyap and Stein (2004) and Ayuso, Pérez, and Saurina (2004)

and interest rates are the only observable characteristics. We report in Table 13 the percentage change in the expected loss and the VaR caused by these shocks. The effect of the GDP shock is similar for most sectors, although it is relatively larger for manufacture, construction and mortgages, and smaller for utilities. In contrast, due to its poorer explanatory power, the interest rate shock causes more heterogeneous responses. In Table 14, we compare these results with the ones obtained from our two extensions. In the first extension we assess the effect of including the augmented set of macroeconomic factors, while in the second one we analyse the impact of modelling the dynamics of the mean of the exposures at default. In both cases, we allow for the presence of latent factors, although in the latter extension we only consider our specification with two observable factors. In addition, we assume that the unconditional means of the exposures at default will remain constant over time.[14] The two models that use a static distribution for exposures at default yield fairly close results. Indeed, both seem to respond more to a GDP shock than to an interest rate shock. For example, at a three-year horizon, the expected loss and the value at risk increase by 17% under the GDP shock, but only by 5-7% under the interest rate shock. This result is a direct consequence of the much higher explanatory power of GDP in the VAR models of Tables 1, 2 and 8.

In contrast, we find larger effects when we allow for time varying means of exposures at default. Although the expected loss and the VaR under normal conditions are similar for short horizons, we now obtain fatter tails at the five-year horizon, where VaR reaches €50 billion. We also find a higher sensitivity to the GDP and interest rate shocks. These larger losses are mainly due to two sources. Firstly, exposures at default deteriorate as the economy worsens, whereas in the previous models they remained unaltered. Secondly, we have introduced correlation between default frequencies and exposures at default, since both of them are influenced by the same macroeconomic factors. For instance, increments in default frequencies due to a lower GDP growth are reinforced with higher exposures at default. In consequence, the overall effect is fatter tails and larger responses to stress tests of the same magnitude.

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[14]Hence, we directly simulate from (8), by imposing $\eta_k = 0$, because we do not expect that the downward trend documented in Table 12 will persist in the future.

5 Conclusions

We develop a flexible model to estimate the credit loss distribution of the loans port-folio in a national banking system. We classify the loans in sectors, and model default frequencies, individual exposures at default, losses given default and the total number of loans in each sector. This latter variable has not been previously considered in the liter-ature. However, we believe that the growth of the credit industry may have important effects on total credit losses, specially for medium and long term horizons. We propose a dynamic model for default frequencies and the growth of the credit industry, using as explanatory variables a set of macroeconomic factors. As a distinguishing feature of our approach, we also allow for the presence of unobservable common factors. These fac-tors are able to capture contagion effects between sectors, which are orthogonal to the observable macroeconomic conditions. Both observable and unobservable variables are modelled with a vector autoregressive structure. In addition, we model the loss given de-fault with a Beta distribution. Finally, we fit the distributions of the exposures at default with the Gamma and the Inverse Gaussian distributions, where we propose a dynamic parametrisation that relates their expected values to macroeconomic shocks.

In the second part of the paper we apply our model to analyse the loss distribution of the total credit portfolio of Spanish banks. We use quarterly loan data from the Spanish Credit Register. Our database starts in 1984.Q4 and ends in 2006.Q4. It contains infor-mation on every loan granted in Spain with an exposure above €6,000. Hence, we are able to analyse the whole Spanish loan market. We consider 10 corporate sectors. Further-more, we also investigate the role of consumption loans and mortgages in the credit loss distribution by including an additional group for each of these categories. We first study a simple model that uses the quarterly changes in GDP growth and the variation in three-month real interest rates as the only macroeconomic explanatory variables. Exposures are modelled in a static setting for each sector with the Inverse Gaussian distribution, except for mortgages, where we employ the Gamma because of its better fit. We estimate the parameters by maximum likelihood and obtain the credit loss distribution for the 1, 3 and 5 year horizons by simulation. Despite the analytical complexity of our model, we show that we can generate extremely fast simulations by exploiting the statistical properties of

the Gamma and the Inverse Gaussian distributions. In particular, we compute for each sector the expected loss, the unexpected loss and the value at risk of credit losses. We also estimate the density function of losses. Our results show that credit losses in the Spanish economy are mainly due to the manufacture, construction, consumption loans and mortgages. The result for the latter two sectors should be interpreted in absolute terms. Despite the typically low losses given default and exposures at default in loans to individuals, there is such a large number of loans in these groups that they are one of the main sources of credit risk in Spain. At the other extreme, mining and utilities are the sectors with lower absolute risk in Spain. We compare our results with the losses generated by a simpler model that does not take into account the presence of "hidden" factors. Although the two models provide similar results for sectorial losses viewed separately, aggregate or total losses are larger in the more general setting, due to the higher correlation between sectors introduced by the latent factors. In this sense, we show by means of in and out-of-sample specification tests that latent factors capture the intersectoral correlations very accurately, whereas a model with only observable explanatory variables misses important contagion effects. Furthermore, we are also able to find a significant impact of macroeconomic cycles on the distribution of exposures at default.

Finally, we perform two stress tests to assess the sensitivity of credit losses to macro shocks. In particular, we assess the separate effects of a sudden drop in GDP growth and a sharp increase in interest rates. Both shocks occur in just one quarter, and they have a magnitude of three standard deviations. Overall, stressed GDP has a stronger effect than the interest rate shock. However, we obtain a higher sensitivity once we account for the dependence of exposures at default on the cycle.

A fruitful avenue for future research would be to integrate this credit risk model with market risk and operational risk models, as Rosenberg and Schuermann (2006) propose. It would also be interesting to combine our model with one for the interbank market, such as those developed by Goodhart (2005) and Elsinger, Lehar, and Summer (2006). These types of general models could be extremely helpful in providing analytical systemic risk measures.

References

Altman, E., A. Resti, and A. Sironi (2004). Default recovery rates in credit risk modelling: a review of the literature and empirical evidence. *Economic Notes 33*, 183–208.

Ayuso, J., D. Pérez, and J. Saurina (2004). Are capital buffers procyclical? Evidence from Spanish panel data. *Journal of Financial Intermediation 13*, 249–264.

Boss, M. (2002). A macroeconomic credit risk model for stress testing the Austrian credit portfolio. *Financial Stability Report of the Austrian National Bank 4*, 64–82.

Breitung, J. and M. H. Pesaran (2005). Unit roots and cointegration in panels. Forthcoming in L. Matyas and P. Sevestre "The Econometrics of Panel Data (Third Edition)", Kluwer Academic Publishers.

Bruche, M. and C. González-Aguado (2006). Recovery rates, default probabilities and the credit cycle. CEMFI Working Paper No. 0612.

Davidson, R. and J. G. MacKinnon (1985). Testing linear and loglinear regressions against Box-Cox alternatives. *Canadian Journal of Economics 18*, 499–517.

Delgado, J. and J. Saurina (2004). Riesgo de crédito y dotaciones a insolvencias. Un análisis con variables macroeconómicas. *Moneda y Crédito 219*, 11–41. Also available in English by request to the authors.

Demchuk, A. and R. Gibson (2006). Stock market performance and the term structure of credit spreads. *Journal of Financial and Quantitative Analysis 41*, 863–887.

Drehmann, M. (2005). A market based macro stress test for the corporate credit exposures of UK banks. Bank of England Working Paper.

Drehmann, M., A. J. Patton, and S. Sorensen (2006). Corporate defaults and macroeconomic shocks: non-linearities and uncertainty. Bank of England Working Paper.

Duffie, D., A. Eckner, G. Horel, and L. Saita (2006). Frailty correlated default. Working paper.

Elsinger, H., A. Lehar, and M. Summer (2006). Risk assessment for banking systems. *Management Science 52*, 1301–1315.

Giesecke, K. and S. Weber (2004). Cyclical correlations, credit contagion and portfolio losses. *Journal of Banking and Finance 28*, 3009–3036.

Goodhart, C. A. E. (2005). Financial regulation, credit risk and financial stability. *National Institute Economic Review 192*, 118–127.

Goodhart, C. A. E. and A. Taylor (2005). Procyclicality and volatility in the financial system: The implementation of Basel II and IAS 39. In S. Gerlach and P. Gruenwald (Eds.), *Procyclicality of Financial Systems in Asia*. Palgrave Macmillan.

Gordy, M. B. and B. Howels (2006). Procyclicality in Basel II: Can we treat the disease without killing the patient? *Journal of Financial Intermediation 15*, 395–417.

Gupton, G. M. and R. M. Stein (2002). Losscalc^{TM}: Model for predicting Loss Given Default (LGD). Moody's KMV, New York.

Hamilton, J. D. (1994). *Time series analysis*. Princeton: Princeton University Press.

Jiménez, G., J. A. López, and J. Saurina (2007). Empirical analysis of corporate credit lines. Unpublished manuscript.

Jiménez, G., V. Salas, and J. Saurina (2006). Determinants of collateral. *Journal of Financial Economics 81*, 255–281.

Jiménez, G. and J. Saurina (2004). Collateral, type of lender and relationship banking as determinants of credit risk. *Journal of Banking and Finance 28*, 2191–2212.

Johnson, N. L., S. Kotz, and N. Balakrishnan (1994). *Continuous univariate distributions*. New York: John Wiley and Sons.

Kashyap, A. K. and J. C. Stein (2004). Cyclical implications of the Basel II capital standards. *Economic Perspectives 1Q/2004*, 18–31.

Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of Political Economy 105*, 211–248.

Madan, D. B. and H. Unal (2006). Designing countercyclical and risk based aggregate deposit insurance premia. CFR Seminar Series Library, FDIC.

Martín, V. Salas, and J. Saurina (2005). A test of the law of one price in retail banking. Bank of Spain Working Paper No. 0530.

Misina, M., D. Tessier, and S. Dey (2006). Stress testing the corporate loans portfolio of the Canadian banking sector. Bank of Canada Working Paper No. 2006-47.

Pesaran, M. H., T. Schuermann, B. J. Treutler, and S. M. Weiner (2006). Macroeconomic dynamics and credit risk: a global perspective. *Journal of Money, Credit and Banking 38*, 1211–1261.

Rosenberg, J. V. and T. Schuermann (2006). A general approach to integrated risk management with, skewed, fat-tailed risks. *Journal of Financial Economics 79*, 569–614.

Schuermann, T. and K. J. Stiroh (2006). Visible and hidden risk factors for banks. Financial Institutions Center, Wharton School of Business. Working paper No. 06-10.

Virolainen, K. (2004). Macro stress testing with a macroeconomic credit risk model for Finland. Bank of Finland discussion paper No. 18.

Wilson, T. C. (1997a). Portfolio credit risk (I). *Risk 10*, 56–61.

Wilson, T. C. (1997b). Portfolio credit risk (II). *Risk 10*, 111–117.

Table 1

Model for default frequencies with GDP, interest rates and latent factors

(a) Explanatory variables

	GDP_{t-2}	GDP_{t-3}	GDP_{t-4}	INT_{t-2}	INT_{t-3}	INT_{t-4}	f_{2t}
Agriculture	-1.133**	-1.129**	-0.432	-0.281	1.453**	-0.336	3.335**
Mining	-1.162	-1.248	0.122	0.291	0.316	-1.094	5.791**
Manufacture	-1.515**	-1.740**	-0.862*	0.383	0.668	-0.469	4.447**
Utilities	-0.097	0.087	-0.494	0.073	0.647	-0.847	5.129**
Construction	-0.958**	-0.988*	-0.875**	0.702	0.093	0.259	3.411**
Commerce	-1.267**	-1.213**	-0.606	-0.198	0.712	-0.119	4.038**
Hotels	-1.304**	-0.826	-0.141	-0.101	1.849**	-0.348	4.038**
Communications	-0.953**	-1.053**	-0.857*	0.138	1.125**	-0.435	3.673**
R&D	-0.403	-1.421**	-1.486**	0.156	-0.187	-0.096	3.697**
Other Corp.	-0.331	-0.888*	-0.256	0.644	0.881*	-0.242	3.191**
Cons. loans	-0.840**	-1.026**	-0.526	0.020	0.604	0.219	3.261**
Mortgages	-0.805	-1.608**	-1.329**	0.364	0.022	0.029	1.668**

(b) Dynamics

	α	$\Delta y_{k,t-1}$	$\Delta y_{k,t-4}$	$corr(u_{1k,t}, u_{2k,t})$
Agriculture	-0.605	-0.362**	0.215**	0.429**
Mining	-1.080	-0.327**	-0.074	0.017
Manufacture	-0.554	-0.329**	-0.013	0.084
Utilities	-1.122	-0.377**	-0.135	0.058
Construction	-0.368	-0.079	0.176**	-0.354**
Commerce	-0.459	-0.237**	0.038	0.052
Hotels	-0.395	-0.340**	-0.003	0.145
Communications	-0.420	-0.317**	0.120*	0.319**
R&D	-0.494	-0.160**	0.070	-0.116
Other Corp.	-0.625	-0.219**	0.141*	-0.322**
Cons. loans	-0.594	-0.277**	-0.030	-0.304**
Mortgages	-0.520	0.049	0.058	-0.162

Notes: Two asterisks indicate significance at the 5% level, while one asterisk denotes significance at the 10% level. Prior to estimation, the dependent and the explanatory variables have been multiplied by 100. GDP_{t-i} and INT_{t-i} for $i = 2, 3, 4$ denote, respectively, the effect of lagged observations of changes of GDP growth and three-month real interest rates on the dependent variables. α is the intercept of the VAR model, and the columns labelled $\Delta y_{k,t-1}$ and $\Delta y_{k,t-4}$ denote the effect of lagged observations of the dependent variables. "$corr(u_{1k,t}, u_{2k,t})$" refers to the correlation between the two idiosyncratic residuals that affect the same sector.

Table 2

Model for the growth of the number of loans with GDP, interest rates and latent factors

(a) Explanatory variables

	GDP$_{t-2}$	GDP$_{t-3}$	GDP$_{t-4}$	INT$_{t-2}$	INT$_{t-3}$	INT$_{t-4}$	f_{1t}
Agriculture	0.250	0.171	0.189	-0.200	0.059	-0.078	1.258**
Mining	0.197	-0.249	0.038	-0.056	-0.064	0.226	1.375**
Manufacture	0.383**	0.062	0.120	-0.072	-0.074	0.090	1.600**
Utilities	0.246	-0.110	-0.097	-0.863**	0.562	-0.499	1.211**
Construction	0.321*	0.086	0.137	-0.240	0.068	-0.126	1.470**
Commerce	0.463**	0.127	0.072	0.086	-0.201	0.158	1.793**
Hotels	0.210	-0.070	0.063	0.023	0.027	-0.242	1.991**
Communications	0.126	0.537	0.424	0.621	-0.113	0.141	2.069**
R&D	0.623**	0.225	-0.059	-0.055	-0.096	-0.201	1.591**
Other Corp.	-0.902**	-0.805*	0.205	0.359	-0.261	0.544	1.019**
Cons. loans	0.029	0.058	0.522*	0.514	0.311	0.042	0.781**
Mortgages	0.155	0.038	0.116	0.756**	-0.516	-0.118	0.589*

(b) Dynamics

	α	$\Delta n_{k,t-1}$	$\Delta n_{k,t-4}$	corr($u_{1k,t}, u_{2k,t}$)
Agriculture	1.309**	0.308**	0.130	0.429**
Mining	0.917**	0.293**	0.081	0.017
Manufacture	0.659**	0.374**	0.186**	0.084
Utilities	1.199**	0.194*	-0.191*	0.058
Construction	1.002**	0.575**	0.249**	-0.354**
Commerce	0.846**	0.447**	0.289**	0.052
Hotels	1.303**	0.286**	0.488**	0.145
Communications	0.908**	0.514**	0.252**	0.319**
R&D	1.579**	0.314**	0.416**	-0.116
Other Corp.	1.649**	0.477**	0.094	-0.322**
Cons. loans	2.465**	0.094*	0.033	-0.304**
Mortgages	2.681**	-0.023	0.235**	-0.162

Notes: Two asterisks indicate significance at the 5% level, while one asterisk denotes significance at the 10% level. Prior to estimation, the dependent and the explanatory variables have been multiplied by 100. GDP$_{t-i}$ and INT$_{t-i}$ for $i = 2, 3, 4$ denote, respectively, the effect of lagged observations of changes of GDP growth and three-month real interest rates on the dependent variables. α is the intercept of the VAR model, and the columns labelled $y_{k,t-1}$ and $y_{k,t-4}$ denote the effect of lagged observations of the dependent variables. "corr($u_{1k,t}, u_{2k,t}$)" refers to the correlation between the two idiosyncratic residuals that affect the same sector.

Table 3

Dynamics of the factors

	Intercept	First lag	Second lag	Conditional covariance matrix			
				GDP	INT	f_{1t}	f_{2t}
GDP	0.035	-0.425**	-0.056	1.259**			
INT	-0.094	0.549**	-0.511**	-0.117	0.933**		
f_{1t}	0	-0.193*	0	0	0	1	
f_{2t}	0	0.198*	0	0	0	-0.473**	1

Notes: Two asterisks indicate significance at the 5% level, while one asterisk denotes significance at the 10% level. Prior to estimation, the dependent and the explanatory variables have been multiplied by 100. GDP and INT denote, respectively, the changes of GDP growth and three-month real interest rates.

Table 4

Model for default frequencies with GDP and interest rates

(a) Explanatory variables

	GDP_{t-2}	GDP_{t-3}	GDP_{t-4}	INT_{t-2}	INT_{t-3}	INT_{t-4}	f_{1t}
Agriculture	-1.058**	-1.105**	-0.326	-0.096	1.349**	-0.067	0.000
Mining	-0.984	-1.171	0.205	0.685	0.251	-0.949	0.000
Manufacture	-1.509**	-1.613**	-0.686	0.646	0.681	-0.430	0.000
Utilities	-0.076	0.071	-0.394	0.451	0.390	-0.491	0.000
Construction	-0.783*	-0.712	-0.770*	1.190**	-0.308	0.593	0.000
Commerce	-1.203**	-1.029**	-0.431	0.069	0.702	-0.073	0.000
Hotels	-1.273**	-0.688	-0.017	0.155	1.714**	-0.156	0.000
Communications	-0.745*	-0.800	-0.652	0.567	0.999*	-0.218	0.000
R&D	-0.207	-1.364**	-1.454**	0.412	-0.428	0.178	0.000
Other Corp.	-0.290	-0.840*	-0.192	0.736	0.766	-0.013	0.000
Cons. loans	-0.650*	-0.893**	-0.418	0.308	0.472	0.452	0.000
Mortgages	-0.825	-1.654**	-1.440**	0.530	-0.224	0.103	0.000

(b) Dynamics

	α	$\Delta y_{k,t-1}$	$\Delta y_{k,t-4}$	$\text{corr}(u_{1k,t}, u_{2k,t})$
Agriculture	-0.311	-0.329**	0.467**	0.061
Mining	-0.985	-0.338**	-0.002	-0.360**
Manufacture	-0.375	-0.237**	0.146	-0.458**
Utilities	-1.010	-0.357**	-0.053	-0.103
Construction	-0.156	0.047	0.393**	-0.256**
Commerce	-0.278	-0.131	0.253**	-0.431**
Hotels	-0.287	-0.301**	0.118	-0.227**
Communications	-0.254	-0.244**	0.382**	0.083
R&D	-0.352	-0.125	0.264**	-0.103
Other Corp.	-0.450	-0.203*	0.306**	-0.242**
Cons. loans	-0.405	-0.239**	0.174	-0.025
Mortgages	-0.553	0.034	0.105	-0.141

Notes: Two asterisks indicate significance at the 5% level, while one asterisk denotes significance at the 10% level. Prior to estimation, the dependent and the explanatory variables have been multiplied by 100. GDP_{t-i} and INT_{t-i} for $i = 2, 3, 4$ denote, respectively, the effect of lagged observations of changes of GDP growth and three-month real interest rates on the dependent variables. α is the intercept of the VAR model, and the columns labelled $\Delta y_{k,t-1}$ and $\Delta y_{k,t-4}$ denote the effect of lagged observations of the dependent variables. "$\text{corr}(u_{1k,t}, u_{2k,t})$" refers to the correlation between the two idiosyncratic residuals that affect the same sector.

Table 5

Model for the growth of the number of loans with GDP and interest rates

(a) Explanatory variables

	GDP$_{t-2}$	GDP$_{t-3}$	GDP$_{t-4}$	INT$_{t-2}$	INT$_{t-3}$	INT$_{t-4}$	f_{2t}
Agriculture	0.282	0.174	0.146	-0.223	-0.008	-0.114	0.000
Mining	0.198	-0.212	-0.044	-0.086	-0.111	0.201	0.000
Manufacture	0.455**	0.166	0.095	-0.155	-0.111	-0.016	0.000
Utilities	0.242	-0.085	-0.112	-0.832**	0.486	-0.471	0.000
Construction	0.392**	0.124	0.122	-0.299	0.017	-0.243	0.000
Commerce	0.514**	0.208	0.022	0.011	-0.232	0.019	0.000
Hotels	0.211	-0.088	-0.023	-0.018	0.004	-0.347	0.000
Communications	0.220	0.712*	0.465	0.787*	-0.109	0.050	0.000
R&D	0.794**	0.460*	-0.052	-0.152	-0.045	-0.415	0.000
Other Corp.	-0.913**	-0.843*	0.152	0.328	-0.265	0.538	0.000
Cons. loans	0.012	0.021	0.531*	0.505	0.312	-0.023	0.000
Mortgages	0.162	0.041	0.121	0.730**	-0.463	-0.153	0.000

(b) Dynamics

	α	$\Delta n_{k,t-1}$	$\Delta n_{k,t-4}$	corr($u_{1k,t}, u_{2k,t}$)
Agriculture	1.197**	0.208*	0.293**	0.061
Mining	1.103**	0.063	0.173*	-0.360**
Manufacture	0.622**	0.159	0.413**	-0.458**
Utilities	1.332**	0.112	-0.191	-0.103
Construction	0.791**	0.461**	0.522**	-0.256**
Commerce	0.688**	0.261**	0.547**	-0.431**
Hotels	1.010**	0.171*	0.643**	-0.227**
Communications	0.813*	0.446**	0.410**	0.083
R&D	1.085**	0.115	0.685**	-0.103
Other Corp.	1.782**	0.443**	0.088	-0.242**
Cons. loans	2.383**	0.071	0.084	-0.025
Mortgages	2.648**	-0.033	0.251**	-0.141

Notes: Two asterisks indicate significance at the 5% level, while one asterisk denotes significance at the 10% level. Prior to estimation, the dependent and the explanatory variables have been multiplied by 100. GDP$_{t-i}$ and INT$_{t-i}$ for $i = 2, 3, 4$ denote, respectively, the effect of lagged observations of changes of GDP growth and three-month real interest rates on the dependent variables. α is the intercept of the VAR model, and the columns labelled $y_{k,t-1}$ and $y_{k,t-4}$ denote the effect of lagged observations of the dependent variables. "corr($u_{1k,t}, u_{2k,t}$)" refers to the correlation between the two idiosyncratic residuals that affect the same sector.

Table 6

Descriptive statistics of the credit loss distribution.
Model with GDP, interest rates and latent factors

	Expected loss			VaR(99.9%)			Unexpected loss		
	1 year	3 years	5 years	1 year	3 years	5 years	1 year	3 years	5 years
Agriculture	38.32	128.39	244.97	124.45	534.22	1281.41	86.13	405.83	1036.43
Mining	5.65	19.41	37.26	26.04	116.48	284.98	20.39	97.07	247.72
Manufacture	285.39	929.76	1697.45	948.88	3863.20	8379.99	663.49	2933.44	6682.54
Utilities	4.09	14.13	27.09	26.43	91.85	217.93	22.34	77.71	190.84
Construction	318.00	1153.53	2344.96	1051.21	5113.96	13162.55	733.21	3960.43	10817.59
Commerce	189.87	638.15	1203.09	609.49	2522.91	5661.09	419.62	1884.76	4458.00
Hotels	32.10	114.78	232.83	110.14	497.27	1232.32	78.03	382.49	999.49
Communications	36.88	126.52	246.08	122.23	560.86	1399.85	85.35	434.33	1153.78
R&D	68.39	252.79	532.46	235.98	1167.62	3140.44	167.59	914.82	2607.99
Other Corp.	33.97	121.02	245.51	111.12	536.74	1411.08	77.14	415.72	1165.57
Cons. loans	408.74	1412.72	2719.85	1692.62	6870.98	15186.85	1283.88	5458.26	12466.99
Mortgages	257.88	975.90	2116.68	1568.66	8599.30	25027.81	1310.78	7623.41	22911.12
Total	1679.29	5887.11	11648.23	3889.04	17443.22	43715.87	2209.75	11556.11	32067.64

Notes: results in millions of euros. The unexpected loss is defined as the difference between the VaR(99.9%) and the expected loss. Statistics obtained from 1 million simulations of the credit risk model.

Table 7

Descriptive statistics of the credit loss distribution.
Model with GDP and interest rates

	Expected loss			VaR(99.9%)			Unexpected loss		
	1 year	3 years	5 years	1 year	3 years	5 years	1 year	3 years	5 years
Agriculture	39.11	131.01	254.02	124.82	566.39	1505.67	85.71	435.38	1251.65
Mining	5.81	19.59	37.21	25.68	110.27	263.80	19.87	90.68	226.59
Manufacture	293.13	952.26	1742.00	948.98	3944.37	8769.62	655.85	2992.11	7027.62
Utilities	4.16	14.24	27.26	26.55	92.13	215.66	22.39	77.89	188.40
Construction	335.68	1276.94	2720.49	1092.19	6099.12	18175.05	756.50	4822.18	15454.56
Commerce	194.19	649.47	1226.39	610.49	2585.41	6025.71	416.31	1935.93	4799.32
Hotels	32.65	116.01	233.56	110.08	500.78	1253.15	77.43	384.77	1019.59
Communications	36.49	123.76	246.24	120.14	604.35	1750.34	83.65	480.59	1504.09
R&D	69.25	255.10	543.98	234.30	1219.38	3532.81	165.05	964.28	2988.83
Other Corp.	34.49	122.97	251.52	111.14	547.53	1507.93	76.66	424.56	1256.41
Cons. loans	423.79	1487.22	2890.92	1719.56	7269.91	16734.72	1295.77	5782.69	13843.80
Mortgages	259.55	983.23	2144.15	1555.19	8743.74	25720.14	1295.64	7760.51	23575.99
Total	1728.28	6131.82	12317.74	3661.07	16058.23	41024.64	1932.79	9926.42	28706.91

Notes: results in millions of euros. The unexpected loss is defined as the difference between the VaR(99.9%) and the expected loss. Statistics obtained from 1 million simulations of the credit risk model.

Table 8

Model with latent factors, GDP, interest rates, spread and six sectorial effects

(a) Default frequencies

	GDP$_{t-2}$	GDP$_{t-3}$	GDP$_{t-4}$	INT$_{t-2}$	INT$_{t-3}$	INT$_{t-4}$	SPR$_{t-2}$	SPR$_{t-3}$	SPR$_{t-4}$	SEC$_{t-2}$	SEC$_{t-3}$	SEC$_{t-4}$	f_{2t}
Agriculture	-0.927**	-1.098**	-0.473	0.597	0.316	0.421	0.672	-1.025	0.731	0.038	0.035	-0.036	3.320**
Mining	-0.882	-0.843	0.647	0.835	-0.969	-1.407	0.935	-2.019	-0.436	0.004	-0.551	-0.586*	5.121**
Manufacture	-1.353**	-1.458**	-0.593	0.652	0.008	-0.931	0.267	-1.169	-0.932	-0.125	-0.208*	-0.185*	4.029**
Utilities	-0.408	0.406	-0.712	-1.536	2.411	-3.211**	-1.191	1.235	-2.566**	-0.142	0.191	-0.313	4.918**
Construction	-0.794*	-0.533	-0.852*	0.760	0.351	-0.345	0.262	0.159	-0.778	-0.192*	-0.003	-0.075	3.160**
Commerce	-1.161**	-1.199**	-0.904**	0.315	-0.032	-0.155	0.656	-0.854	-0.165	-0.089	0.445*	0.073	3.856**
Hotels	-1.208**	-0.506	-0.331	0.233	1.824*	-0.063	0.054	0.047	0.374	-0.694	0.247	0.393	4.122**
Communications	-0.824*	-1.132**	-1.130**	1.183	-0.156	0.168	1.049	-1.114	0.569	-0.023	0.335	0.275	3.665**
R&D	-0.460	-1.289**	-1.329**	-0.818	0.685	-0.666	-1.248	0.630	-0.791	-	-	-	3.632**
Other Corp.	-0.317	-0.877*	-0.181	0.029	1.134	0.140	-1.277*	0.120	0.265	-	-	-	3.270**
Cons. loans	-0.857**	-1.001**	-0.573	0.172	0.756	-0.292	0.472	0.180	-0.572	0.021	0.216	-0.580	3.193**
Mortgages	-0.882	-1.753**	-1.506**	1.781*	0.094	-0.694	2.734**	0.143	-0.509	-0.911	-0.347	0.166	1.860**

(b) Growth of the number of loans

	GDP$_{t-2}$	GDP$_{t-3}$	GDP$_{t-4}$	INT$_{t-2}$	INT$_{t-3}$	INT$_{t-4}$	SPR$_{t-2}$	SPR$_{t-3}$	SPR$_{t-4}$	SEC$_{t-2}$	SEC$_{t-3}$	SEC$_{t-4}$	f_{1t}
Agriculture	0.187	0.175	0.199	-0.194	0.390	-0.336	0.191	0.369	-0.197	0.028	0.023	0.017	1.246**
Mining	0.118	-0.303	0.023	-0.041	0.305	0.000	0.238	0.422	-0.028	0.072	-0.051	-0.036	1.302**
Manufacture	0.284*	0.032	0.106	-0.108	0.396	-0.299	0.246	0.510**	-0.231	0.055*	-0.038	-0.013	1.516**
Utilities	0.279	-0.103	-0.211	0.159	0.133	-0.862*	1.588***	-0.353	-0.427	0.076	0.034	-0.011	1.095***
Construction	0.214	0.029	0.111	-0.253	0.386	-0.483*	0.187	0.352	-0.299	0.012	0.000	0.070**	1.364**
Commerce	0.351**	0.174	0.177	-0.073	0.455	-0.346	0.056	0.714**	-0.441**	0.003	-0.162**	0.052	1.711**
Hotels	0.063	0.098	-0.081	-0.293	1.134**	-1.399**	0.226	1.073***	-0.992**	-0.285	0.212	0.063	1.774**
Communications	0.024	0.595	0.298	1.112*	-0.434	-0.311	0.918	-0.307	-0.574	-0.156	0.125	0.523*	1.950**
R&D	0.585**	0.238	-0.085	0.062	0.359	-0.426	0.345	0.632*	0.018	-	-	-	1.494**
Other Corp.	-0.840**	-0.854*	0.078	1.334**	-0.975	0.626	1.407**	-0.525	0.143	-	-	-	1.056**
Cons. loans	-0.010	0.067	0.503*	0.551	0.527	-0.191	0.221	0.257	-0.186	0.010	0.009	-0.041	0.720**
Mortgages	0.177	0.024	0.064	0.851*	-1.343**	0.260	-0.251	-0.797*	0.196	-0.220	0.078	0.208	0.779**

Notes: Two asterisks indicate significance at the 5% level, while one asterisk denotes significance at the 10% level. Prior to estimation, the dependent and the explanatory variables have been multiplied by 100. GDP$_{t-i}$, INT$_{t-i}$, SPR$_{t-i}$ for $i = 2, 3, 4$ denote, respectively, the effect of lagged observations of GDP growth, the variation of three-month real interest rates, and the spread between six-year and three-month interest rates on the dependent variables. Except for R&D and Other Corp., each sector is additionally allowed to depend on an additional sectorial variable, whose effects are reported in the columns SEC$_{t-i}$. SEC denotes gross value added by sector for corporates and the unemployment rate for consumption loans and mortgages.

Table 9

Dynamics of the factors

Model with latent factors, GDP, interest rates, spread and six sectorial effects

	Intercept	First lag	Second lag
GDP	0.029	-0.430**	0.017
INT	-0.096	0.534**	-0.559**
SPR	0.018	-0.068	-0.165*
GVA$_{Agriculture}$	0.290	0.161*	0.057
GVA$_{Industry}$	0.094	-0.214**	-0.030
GVA$_{Energy}$	0.077	0.491**	0.145
GVA$_{Construction}$	0.306	0.154*	-0.076
GVA$_{Services}$	0.058	-0.281**	-0.052
Unemployment	-0.016	0.255**	0.124
f_{1t}	0.000	-0.291**	0.000
f_{2t}	0.000	0.136	0.000

Conditional covariance matrix

	GDP	INT	SPR	GVA$_{Agr.}$	GVA$_{Ind.}$	GVA$_{Ene.}$	GVA$_{Con.}$	GVA$_{Ser.}$	UNP	f_{1t}	f_{2t}
GDP	1.268**										
INT	-0.105	0.935**									
SPR	0.262*	0.135	1.367**								
GVA$_{Agriculture}$	1.038	0.430	0.217	32.089**							
GVA$_{Industry}$	0.502	-0.283	0.629*	-9.211**	8.087**						
GVA$_{Energy}$	-0.376	-0.675*	1.327**	-0.167	2.436**	14.685**					
GVA$_{Construction}$	0.196	-0.471	0.638	-4.605**	3.708**	5.285**	12.588**				
GVA$_{Services}$	0.103	0.009	0.325**	0.128	0.897**	1.465**	1.782**	1.617**			
Unemployment	-0.095	0.088	-0.048	0.323	-0.246	0.033	-0.373	-0.058	0.361		
f_{1t}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	
f_{2t}	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	-0.468**	1.000

Notes: Two asterisks indicate significance at the 5% level, while one asterisk denotes significance at the 10% level. GDP, INT, SPR and GVA denote, respectively, GDP growth, the variation of three-month real interest rates, the spread between six-year and three-month interest rates, and gross value added.

Table 10

P-values of specification tests of the correlation matrix of default frequencies

(a) Model with GDP and Interest rates

		1	2	3	4	5	6	7	8	9	10	11
Agriculture	1											
Mining	2	**0.00**										
Manufacture	3	**0.00**	**0.00**									
Utilities	4	**0.00**	**0.00**	**0.00**								
Construction	5	**0.00**	**0.00**	**0.00**	**0.00**							
Commerce	6	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**						
Hotels	7	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**					
Communications	8	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**				
R&D	9	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**			
Other Corp.	10	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**		
Cons. loans	11	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	
Mortgages	12	**0.03**	0.17	0.06	0.07	**0.01**	0.11	0.28	0.11	**0.01**	**0.00**	**0.00**

(b) Model with GDP, Interest rates and latent factors

		1	2	3	4	5	6	7	8	9	10	11
Agriculture	1											
Mining	2	0.30										
Manufacture	3	0.67	**0.03**									
Utilities	4	0.85	0.74	0.89								
Construction	5	0.76	0.24	0.57	0.14							
Commerce	6	0.67	0.95	0.69	0.59	0.36						
Hotels	7	0.43	0.27	0.50	0.38	0.72	0.99					
Communications	8	0.67	0.52	0.88	0.93	0.44	0.99	0.72				
R&D	9	0.44	0.15	0.15	0.09	0.40	0.35	0.94	0.51			
Other Corp.	10	0.76	0.71	0.57	0.13	0.27	0.41	0.77	0.35	**0.00**		
Individuals	11	0.39	0.34	0.20	0.92	0.52	0.28	0.64	0.25	0.24	0.78	
Mortgages	12	0.73	0.72	0.60	0.51	0.43	0.51	0.58	0.62	0.39	0.40	0.18

(c) Model with GDP, Interest rates, spread, six sectorial effects and latent factors

		1	2	3	4	5	6	7	8	9	10	11
Agriculture	1											
Mining	2	0.33										
Manufacture	3	0.88	0.06									
Utilities	4	0.62	0.85	0.94								
Construction	5	0.75	0.16	0.44	0.29							
Commerce	6	0.91	0.94	0.60	0.98	0.71						
Hotels	7	0.73	0.41	0.55	0.65	0.83	0.90					
Communications	8	0.74	0.57	0.87	0.87	0.53	0.94	0.82				
R&D	9	0.39	0.40	0.22	0.34	0.59	0.37	0.84	0.73			
Other Corp.	10	0.65	0.93	0.69	0.55	0.53	0.44	0.53	0.46	0.10		
Individuals	11	0.26	0.41	0.22	0.53	0.36	0.32	0.63	0.33	0.45	0.92	
Mortgages	12	0.67	0.68	0.80	0.19	0.34	0.43	0.63	0.41	0.11	0.15	0.24

Notes: in each cell the null hypothesis is that the empirical correlation between the corresponding sectorial default frequencies equals the one hypothesised by the model. The p-values below 5% are expressed in bold.

Table 11

Kolmogorov specification tests of the out-of-sample distribution of the standardised fitted residuals of the model of default frequencies and number of loans

Factors	Kolmogorov test	P-value
GDP, INT	0.103	0.004
GDP, INT, f_t	0.051	0.446
GDP, INT, SPR, SEC, f_t	0.046	0.573

Notes: The model has been estimated with data from 1984.Q4 to 2003.Q4. The test studies whether the orthogonalised residuals from 2004.Q1 to 2006Q4, a total number of 288 values, are independent standard normal. INT, SPR and SEC denote, respectively, real interest rates, interest rate effects and sectorial factors.

Table 12

Effect of macroeconomic factors on the expected exposures at default

	Mean in 2006.Q4	η_k	GDP_{t-1}	INT_{t-1}
Agriculture	0.107	-0.002	-0.054**	0.131**
Mining	0.089	-0.018**	-0.011	0.059*
Manufacture	0.096	-0.010**	-0.029**	0.041**
Utilities	0.178	0.028	-0.150**	-0.218**
Construction	0.092	-0.021**	-0.076**	0.051**
Commerce	0.090	-0.007**	-0.043**	0.024**
Hotels	0.062	-0.023**	-0.115**	-0.026*
Communications	0.054	-0.018**	-0.061**	-0.021**
R&D	0.057	-0.014**	-0.111**	0.002
Other Corp.	0.094	-0.015**	-0.029**	-0.002
Cons. loans	0.016	-0.018**	0.017**	0.018**
Mortgages	0.062	0.004**	-0.042**	0.022**

Notes: Two asterisks indicate significance at the 5% level. Means in millions of euros. GDP and INT denote, respectively, GDP growth and the variation of three-month real interest rates. Data sample for the estimation: 1989.Q4 - 2006.Q4.

Table 13

Changes in the credit loss distribution caused by macroeconomic stress tests (3 standard-deviation shocks)

Model with GDP, interest rates and latent factors

| | GDP shock | | | | | | Interest rate shock | | | | | |
| | Expected loss | | | VaR(99.9%) | | | Expected loss | | | VaR(99.9%) | | |
	1 year	3 years	5 years	1 year	3 years	5 years	1 year	3 years	5 years	1 year	3 years	5 years
Agriculture	7	13	15	7	12	13	2	5	5	2	5	4
Mining	8	10	10	7	10	10	2	-2	-2	2	-1	-2
Manufacture	10	17	18	10	15	17	3	3	3	3	4	4
Utilities	0	2	2	0	2	2	1	-2	-3	1	-1	-2
Construction	6	16	17	6	14	16	4	7	7	4	6	7
Commerce	7	13	14	8	13	13	1	3	3	1	3	2
Hotels	7	10	10	7	10	10	4	7	7	5	6	6
Communications	6	10	11	6	10	10	5	10	11	5	9	12
R&D	4	14	16	4	14	13	0	-2	-3	0	-3	-3
Other Corp.	7	16	18	8	16	18	6	13	15	7	13	15
Individuals	6	11	11	6	10	10	3	8	8	3	8	8
Mortgages	10	29	32	9	28	30	4	5	5	3	4	5
Total	7	16	18	7	18	21	3	5	6	3	5	5

Notes: percentage changes with respect to the normal scenario. The unexpected loss is defined as the difference between the VaR(99.9%) and the expected loss. Statistics obtained from 1 million simulations of the credit risk model. GDP is stressed with a negative 3 standard deviation shock, whereas interest rates are stressed with a positive shock of the same magnitude.

Table 14

Comparison of credit loss distributions

	(1)	(2)	(3)
Characteristics			
Included Factors			
-GDP, Interest rates	✓	✓	✓
-Spread, GVA's, Unemployment		✓	
Model of the distribution of exposures	Static	Static	Dynamic
Normal Scenario			
Expected loss			
1 year	1679	1671	1486
3 years	5887	5769	5288
5 years	11648	11335	10647
VaR (99.9%)			
1 year	3889	3821	3501
3 years	17443	16693	17811
5 years	43716	40708	50076
Change due to -3 s.d. GDP shock (%)			
Expected loss			
1 year	7	6	20
3 years	16	16	32
5 years	18	18	35
VaR (99.9%)			
1 year	7	7	17
3 years	18	17	33
5 years	21	20	37
Change due to +3 s.d. Interest rate shock (%)			
Expected loss			
1 year	3	6	10
3 years	5	6	14
5 years	6	6	15
VaR (99.9%)			
1 year	3	7	10
3 years	5	7	14
5 years	5	7	15

Notes: results in millions of euros. "Spread" denotes the difference between six-year and three-month interest rates. "GVA's" denotes gross value added factors, namely: agriculture, industry, energy, construction and market services. Statistics obtained from 1 million simulations of the credit risk model. All models include latent factors.
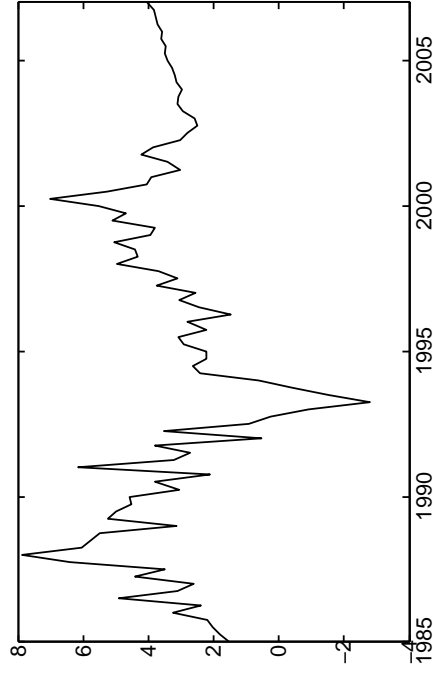
Figure 1:

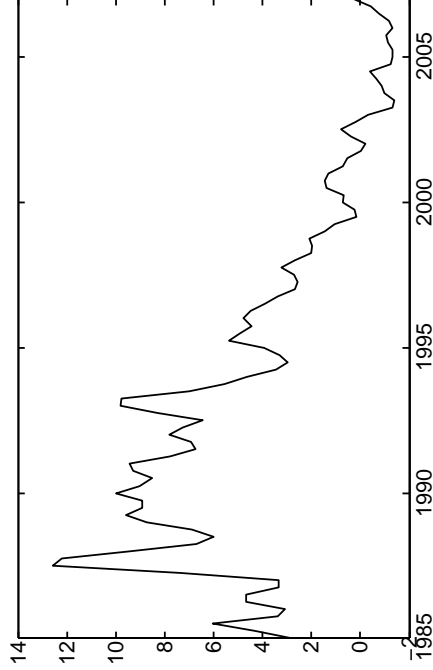(a) Historical default frequencies in the Spanish Economy (%)
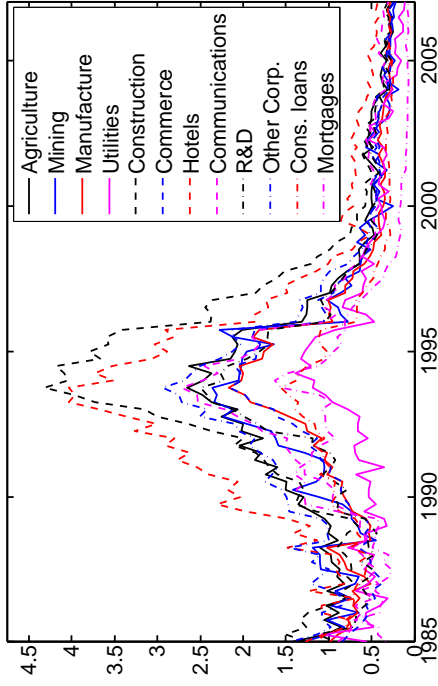
(b) Historical evolution of the total number of loans in the Spanish loan market
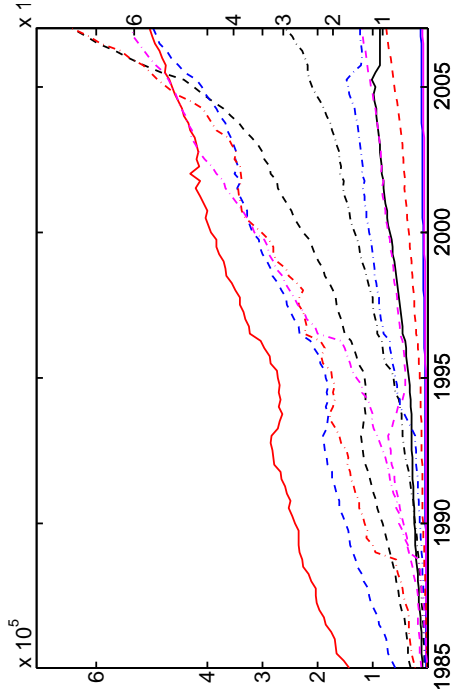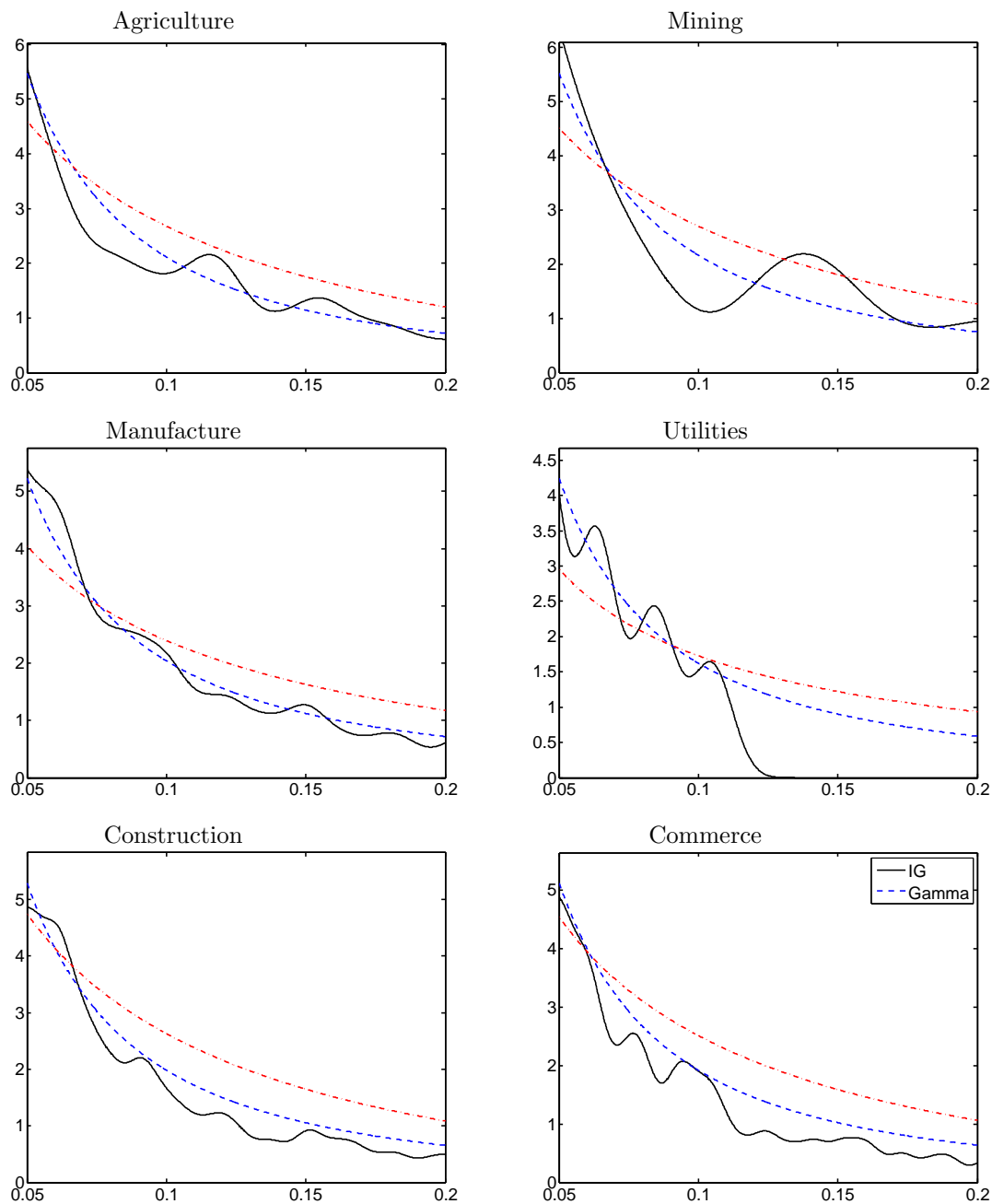
(c) Annual Spanish GDP growth (%)

(d) Three-month real interest rates in Spain (%)

Notes: (a) and (b) share the same legends. The right scale on the y axis of figure (b) correspond to consumption loans and mortgages, whereas the left axis corresponds to the remaining cases.

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Figure 2:

Kernel estimate and fitted densities of the right tail of the distribution of exposures at default
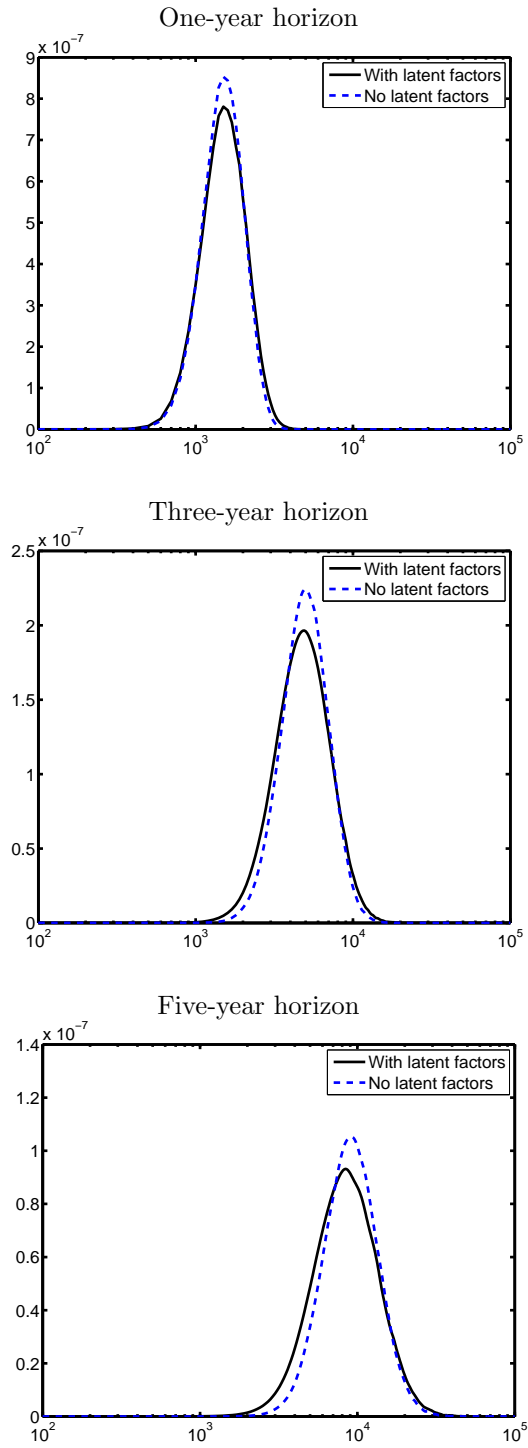

Notes: the x-axis is expressed in millions of euros. Both the kernel and the fitted densities are based on exposure data from 2001 to 2006.

Figure 3:

Kernel estimate and fitted densities of the right tail of the distribution of exposures at default
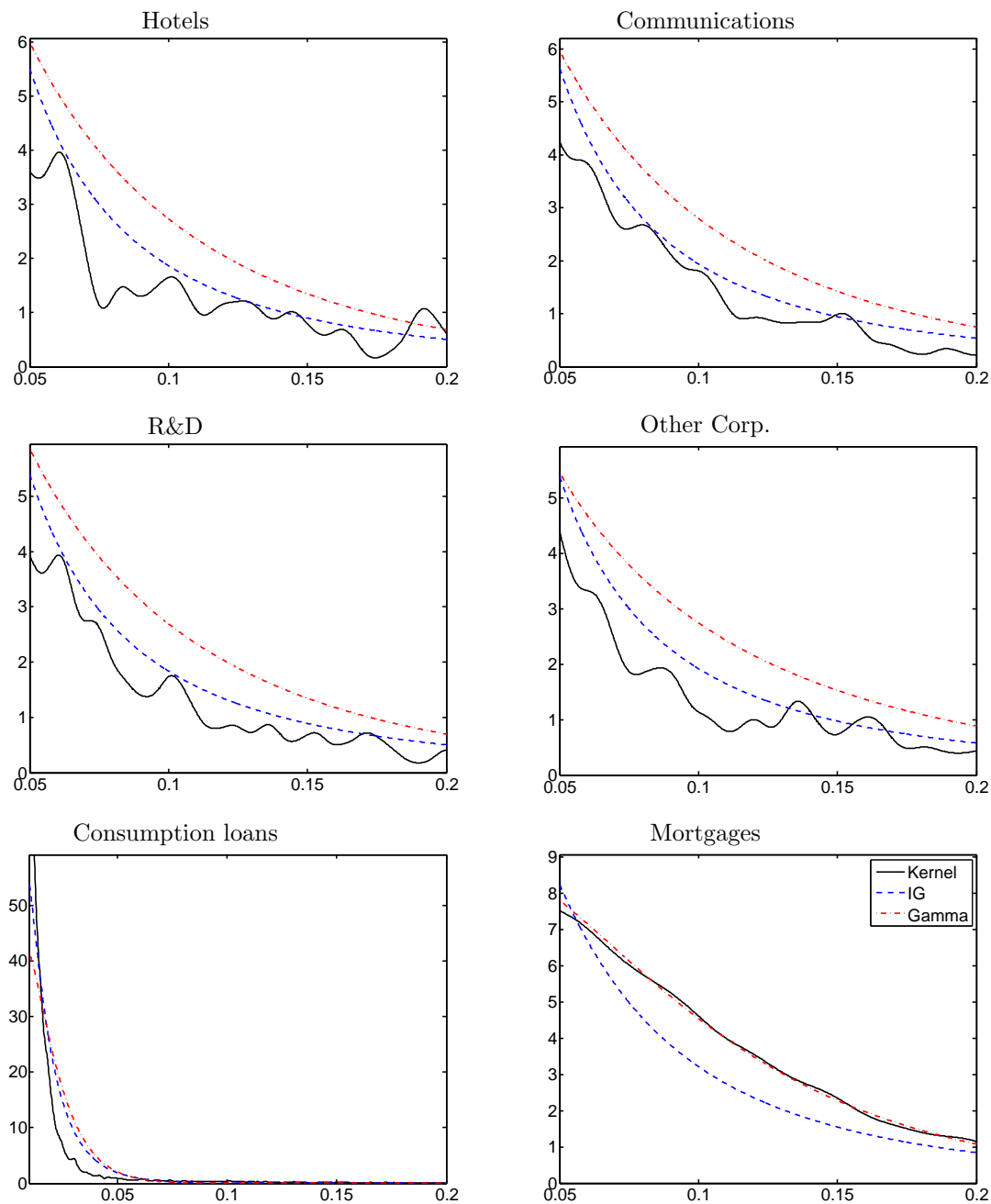
Notes: the x-axis is expressed in millions of euros. Both the kernel and the fitted densities are based on exposure data from 2001 to 2006.

Figure 4:

Kernel estimates of the total credit loss distribution

One-year horizon

Three-year horizon

Five-year horizon

Note: the x-axis is expressed in millions of euros, where a log-scale is employed. Estimates based on 100,000 simulations.

Estimating macroeconomic credit risk and sectoral default rate correlations for the Italian economy

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ABSTRACT

This paper studies how sectoral default rates are influenced by macroeconomic variables. The data on business default risk are taken from the Central Credit Register's archives of default rates by branch of economic activity that are in turn aggregated into six homogeneous sectors (clusters) in terms of credit risk. A system of equation is then estimated to relate the default rates of the six clusters to the main macroeconomic variables to identify the impact of economic performance on the riskness of the various sectors and quantify the component of credit risk attributable to common factors (systematic risk) and the corresponding inter-sectoral default correlations

The results of the econometric analysis show that there is only a partial influence of common factors (the macroeconomic variables) on business default risk. Secondly, the presence of a residual correlation between the error terms of the model's equations after the estimation suggests the existence of sectoral interdependence that might give rise to contagion following idiosyncratic shocks within the sectors.

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1.   Introduction

The measurement of credit risk has become a leading field of research in finance in recent years. The necessity of measuring default losses with appropriate methodologies was validated by the Basel Committee, which in June 2004, after five years of work, modified the criteria for determining banks' minimum capital requirement in respect of credit risk.

In credit risk analysis, the most difficult aspect to evaluate is the probability of joint default by borrowers. Measuring the correlations between default events lies at the basis of portfolio-risk models developed by the industry and in the academic literature.

Despite the consensus that the state of the economy influences the profitability and financial conditions of firms, it was not until recently that a series of works explicitly studied this issue within these models. The basic hypothesis of these studies is that the occurrence of defaults and their correlation differ according to the growth opportunities of the sector of economic activity to which firms belong, the sector's degree of internationalization and its dependence on other sectors. These sectoral characteristics impinge in turn on the financial situation of firms.

This paper will study if and how far Italian firms' default risk is influenced by the performance of macroeconomic variables and by interdependence between different sectors of economic activity. The introduction of macroeconomic variables makes it easy to interpret the effects of the economic cycle on the credit risk of firms, allowing the impact of cyclical fluctuations to be distinguished from that of firm- or sector-specific conditions.

The data on business default risk are taken from the Central Credit Register's archives of default rates by branch of economic activity. These data are in turn aggregated into six homogeneous groups of economic activity in terms of risk on the basis of a statistical analysis of indicators of economic growth and financial fragility. The groups (clusters) are: agriculture; cyclical consumer goods, including typical Italian export products; engineering and construction; trade, transport and communications; mining and quarrying and energy products; and other market services. The default rates of the six clusters are then set in relation with the main macroeconomic variables.

The results of the econometric analysis show that there is only a partial influence of macroeconomic variables on business default risk. Secondly, the presence of a residual correlation between the error terms of the model's equations after the estimation suggests the existence of sectoral interdependence that might give rise to contagion following idiosyncratic shocks within the sectors.

The paper is organized as follows. Section 2 reviews the main contributions of the academic and professional literature. Section 3 analyzes the time series of default rates used and the construction of the clusters, Sections 4 and 5 describe the estimation model and the treatment of the macroeconomic variables, and Section 6 presents the results. Section 7 summarizes the main conclusions and suggests areas for further study.

2.   Credit risk and the economic cycle

In credit risk analysis, the most difficult aspect to evaluate is the probability of joint default by borrowers. Measurement of the correlations between default events lies at the basis of the estimation of the loss distribution on a portfolio of loans over a specific time horizon due to default by the borrowers[1].

Two statistics summarize that distribution: the expected loss, i.e. the monetary value that is expected to be lost on average from the occurrence of defaults, and the unexpected loss, i.e. the uncertainty (volatility) around the level of the expected loss. The unexpected loss, which represents the financial risk of the portfolio, is usually divided into an idiosyncratic and a systematic component. Idiosyncratic risk is the component linked to specific characteristics of each debtor and is generally diversifiable. Systematic risk by contrast is non-diversifiable, as it represents the effect of common factors that affect all debtors, generating correlations between default events. Once systematic risk is taken into account, default events are assumed to be independent of each other (conditional independence).

Multifactor models presume the existence of different systematic risk factors connected with specific industries, geographical areas or markets. A low degree of correlation between the risk factors and/or a difference in debtors' sensitivity to those factors means that an appropriate composition of the loan portfolio, for example by economic sector and geographical area, can reduce the portfolio's credit risk. Hanson, Pesaran and Schuermann (2005) find a significant reduction in the risk of a portfolio of Japanese and American firms as a result of geographical and sectoral diversification.

It was not until recently that a series of works explicitly analyzed the effects of the performance of macroeconomic variables within portfolio models. The introduction of directly observable macroeconomic variables makes the effects of the economic cycle on borrowers' default risk immediately interpretable, allowing them also to be distinguished from those due to the specific situations of the units analyzed (specific risk of firms or sectors)[2].

Pesaran, Schuermann, Treutler and Weiner (2004) estimate a structural model in which the equity returns of 119 companies from 26 countries — grouped in turn into 11 macro-regions — are set in relation with the changes in the macroeconomic variables of their respective regions (GDP, inflation, share market index, exchange rate and interest rate), with the same variables for the other regions (external variables) and, to capture the performance of the world economy, with the price of

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[1] In general, models of this type distinguish between losses due to non-performance or default (a change in the status of the borrower from "performing" to defaulting) and value gains or losses due to a change in credit rating (upgrade or downgrade event). In this paper we focus on losses caused by default events.
[2] Portfolio models most widely used by banks are based on the so-called structural models, which adopt the Merton's concept that a firm defaults when the market value of its assets falls below that of its liabilities. The correlation between the default events of individual firms stems from the common sensitivity of the market value of their respective assets to the systematic factors. Application of this model include CreditMetrics and PortfolioManager of the consultancy Moody's-KMV, in which the change in the value of firms' assets is proxied by the equity returns of listed companies with similar characteristics. In the second model, in particular, the systematic component of each company's equity return is estimated by decomposing the share market index into non-observable, orthogonal factors, generally attributable to regional and industry-wide factors. In this formulation, therefore, the influence of macroeconomic conditions on firms' probability of default is mediated by their impact on the equity indices.

oil[3]. The results show that the changes in the macroeconomic factors explain between 11 per cent (for Latin America) and 41 per cent (for Europe) of the total variance in equity returns and that the changes in the prices of the domestic and foreign equity markets are the most significant macroeconomic factors[4].

Drehman (2005) too estimates a multifactor model to identify the macroeconomic and market factors that determine the systematic risk on the equity return of 556 companies listed in the United Kingdom in the period 1980-2003, grouped into six sectors of economic activity. For each sector the model considers the macroeconomic variables (GDP, short and long-term interest rates, inflation rate, effective exchange rate and the price of oil) and financial market variables (indices of volatility and price/earnings ratios). The results of the estimation show a very weak correlation of equity returns with systematic factors for all sectors, with not more than 20 per cent of variance explained.

Pain and Vesala (2004) use principal-components analysis to estimate the incidence of common factors on the expected default frequency (EDF)[5] of 1,118 European listed firms in the period 1992-2003. They find that for around two thirds of the firms common factors explain less than 40 per cent of the variability of the EDF. They too conclude that the most significant determinants of corporate risk are connected not with systematic risk but with firm-specific features (idiosyncratic risk).

A series of recent works use information on the credit situation of the customers of some Swedish banks in the period 1994-2000 to study the factors determining their probability of failure or survival (Carling. Jacobson, Lindé and Roszbach, 2004; Jacobson, Lindé and Roszbach, 2005). These studies demonstrate that macroeconomic variables increase the explanatory power of models based only on firms' financial statement information. However, the authors suggest that the macroeconomic variables' strong explanatory power might not only reflect their direct impact but also incorporate a sectoral effect, which is absent in their estimations. Carling, Ronnegard and Roszbach (2004) use Carling, Jacobson, Lindé and Roszbach's (2004) model supplemented by the hypothesis that inter-firm default risk correlation is determined not only by common sensitivity to macroeconomic factors but also by direct links due to firms' belonging to the same economic sector. The results of an estimation of the model for seven macro-sectors show high intra-sectoral interdependence which, if ignored, would result in a substantial underestimation of the risk and of the economic capital needed to face it.

The importance of direct contagion between firms is also highlighted by Giesecke and Weber (2003). They argue that the effect of variations in the macroeconomic variables on firms'

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[3] The external variables are constructed as weighted averages of the variables for the different regions, with different weights depending on the firm's country/region. The weights are constructed using the shares of exports and imports between the firm's country and the other 10 regions.

[4] The authors' rationale for their study is the necessity, with increasing economic and market globalization, of taking into account not only domestic economic conditions but also those of the countries that directly or indirectly influence the distribution of banks' loan losses, particularly as regards the major international banks. They therefore estimate a global macroeconomic model in order to take interdependence between national and international factors explicitly into account. The global macroeconomic model is used to estimate, by applying Montecarlo simulations, the distribution of the portfolio losses and the shocks on the macroeconomic variables selected.

[5] This is the measure of the probability of default supplied by the Moody's-KMV model.

risk can be greatly amplified by direct connections between firms due to reciprocal debit-credit relationships, legal ties (such as membership of the same group) or supply relationships.

While the papers discussed above analyze the structure of correlations of the probability of default between pairs of individual firms, a different strand of research has empirically examined the relation between the macroeconomic variables and the time series of default frequencies observed for the firms belonging to homogeneous groups (sectors of economic activity).

Wilson (1997) uses CreditPortfolioView, a model developed for McKinsey, in which the default rate of a homogeneous group of debtors depends on several macroeconomic factors, summarized in an index, and factors specific to each group. The macroeconomic variables are modeled as ARIMA processes.

Similar analyses on sectoral data for the Finnish and Italian economies are performed respectively in Virolainen (2004) and Botticini, Marchesi and Toffano (2000). Virolainen estimates an econometric model in which the default rates of four sectors are set in relation with two macroeconomic variables (GDP and the interest rate) and one sectoral variable (the sectoral ratio of debt to value added). The results show that there is a significant correlation of the sectoral default rates with GDP and sectoral debt, but not with the interest rate. Botticini, Marchesi and Toffano describe a model developed by Prometeia, using the sectoral time series of the default rates of Italian firms; according to their estimates, the portion of the variance of the default rates explained by macroeconomic factors for the different sectors ranges between 30 and 40 per cent.

Lastly, Alves (2004) analyzes sectoral data of European firms. The firms are grouped into seven macro-sectors; the median value of the Moody's-KMV expected default frequencies of the firms belonging to each sector is used as a summary index of sectoral risk. The correlation between the sectoral risk indices is modeled not only through common sensitivity to some macroeconomic variables (growth rate of industrial production, price of oil, three-month Euribor and an index of share market volatility) but also hypothesizing relations of sectoral interdependence. The results of the estimation of a system of equations by means of a VAR model show that the macroeconomic variables do not have a significant impact on sectoral risk, and that the variability of the sectoral risk index is largely explained by sectoral interdependence. In particular, the performance of the cyclical consumer goods sector would appear to determine the degree of risk of the other sectors.

The present paper is part of the strand of research on the link between sectoral default frequencies and macroeconomic variables. As in the works by Wilson and Violainen, the performance of the macroeconomic variables is represented by a set of autoregressive processes. The expected changes in the macroeconomic variables are subsequently inserted into a system of equations in order to explain the risk observed in the different economic sectors.

However, our analysis differs from the preceding studies in some respects. Firms are classified into six homogeneous groups in terms of risk by means of a statistical analysis based on predictive variables of default (sectoral value added, debt level, leverage, the ratio of net interest expense to gross operating profit and bank debt as a percentage of value added). In contrast with Virolainen, therefore, the sector-specific variables have been used to identify the homogeneous groups of firms and are not included into the model, since they are strongly correlated with the

macroeconomic variables. The set of macroeconomic variables considered is broader too. A factor analysis allowed us to identify a limited number of factors and select the most significant variables for each factor. The results of the econometric model were used to estimate the impact of extreme variations in the macroeconomic variables observed between 1990 and 2004, the period covered by our analysis.

3.   Sectoral default rates

The definition of default used to estimate the model is based on the concept of adjusted bad debts used in the supervision analysis[6]. According to this definition, a firm that is the client of a bank or a financial company is taken to have defaulted when it is reported for the first time to the Central Credit Register for adjusted bad debts. The series of default rates considered is quarterly and is constructed as the ratio of the number of new defaults to the number of performing borrowers at the beginning of the reference period, from 1990 to 2004. The positions refer to non-financial companies and producer households, divided into the 23 branches of economic activity used by the Central Credit Register.

In the period considered the Italian economy, after a long expansion beginning in the early 1980s, experienced a major crisis in the second half of 1993 and a subsequent slowdown in the last part of 1995 and most of 1996. The default rates of non-financial companies and producer households were affected by these cyclical fluctuations, with a marked deterioration especially in conjunction with the first crisis of the 1990s. The peak was reached in December 1993, when the default rate rose to 2.9 per cent. The rates gradually declined during the expansionary phase that began towards the end of 1996 and fell to around 1 per cent, which was lower than at the beginning of the period.

With a view to improving the interpretability of the results and making the estimation model more compact, the default rates for the 23 branches were aggregated in homogeneous risk classes using variables serving to predict defaults: the growth rate of value added, the ratio of bank debt to value added, the degree of utilization of current account credit facilities, the coverage of financial costs by gross operating profit and leverage[7]. The dynamics of value added can be considered an indicator of the growth of the sector, while, read together, the financial indicators permit an assessment of firms' health in terms of capital solidity, liquidity and debt sustainability. To this end use was made of a cluster analysis algorithm[8].

The results of the statistical analysis led to the division of the branches into six homogeneous groups (clusters): agriculture; the consumer goods industry, including traditional

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[6] Adjusted bad debts is the total loans from the financial system outstanding when a borrower is reported to the Central Credit Register: a) as a bad debt by the only intermediary that disbursed credit; b) as a bad debt by one intermediary and as having an overshoot by the only other intermediary exposed; c) as a bad debt by one intermediary and the amount of the bad debt is at least 70% of its exposure towards the financial system or as having overshoots of at least 10% of its total loans outstanding; d) as a bad debt by at least two intermediaries for at least 10% of its total loans outstanding.

[7] The data on firms' financial conditions were obtained from the Company Accounts Data Service.

[8] Cluster analysis consists of a series of multivariate statistical analysis techniques used to classify statistical units into a small number of homogeneous groups. We used two clustering methods (the Wald method and the K-means method).

Italian export products; mechanical engineering and construction; wholesale and retail trade, transport and communications; mining and quarrying and energy products; and other market services. Table 1 shows the branches of economic activity falling within each cluster, together with some statistics on the default rates of the various clusters.

Two points are worth noting. The first is that the branches of manufacturing industry, which are normally considered together, fall into two clusters. One of these mainly comprises the consumer goods industry and traditional Italian export products (textiles and clothing, tanning and leather products, paper and paper products, timber and wood products); the second cluster sees mechanical engineering (base metals, machinery and means of transport) combined with construction in a group that can be loosely defined as the investment goods group. The second point is that the service sector is also divided into two clusters: one containing tourism, which includes hotel and restaurant services, together with transport and communication services and wholesale and retail trade, and the other containing other market services, including real-estate and business activities. Graphical analysis of the default rates by cluster suggests there is a link with the economic cycle common to all the sectors of activity (Chart 1).

Table 2 reports the statistics of the variables used in the cluster analysis. In the period considered the firms that recorded higher average rates of growth of value added are those operating in the wholesale and retail trade sectors and those providing transport and communication services (cluster 4, 1.8 per cent) and those providing other market services to households and businesses (cluster 6, 1.6 per cent). Firms in these clusters are also marked by less use of financial leverage and hence by a lower ratio of financial costs to earnings.

The agricultural sector (cluster 1) and the energy and mineral mining sectors (cluster 5) are marked by less growth in value added (respectively 0.6 and -0.3 per cent) and a higher-than-average volatility. The average number of insolvencies for the two sectors nonetheless remained below the average for the whole economy. The mechanical engineering industry (cluster 3) and the sector producing consumer goods and traditional Italian export products (cluster 2) show a higher degree of financial leverage and a higher ratio of bank debt to value added. Cluster 4 (firms operating in the wholesale and retail trade sectors and those providing transport and communication services) and clusters 2 and 3 are marked by above-average riskiness.

4.   The model

Like Wilson (1997), in order to estimate the model we have transformed the default rates of the six clusters into indices of economic soundness by the following formula:

$$(1) \qquad y_{j,t} = \ln\left(\frac{1 - p_{j,t}}{p_{j,t}}\right)$$

in which $y_{j,t}$ denotes the index of soundness of cluster j at time t and $p_{j,t}$ is its default rate. The functional form adopted for the transformation ensures that the simulated value of the default rate always falls in the interval between 0 and 1. Since the soundness index falls as the default rate

rises, the variables that are positively correlated with the latter are negatively correlated with the former.

The sectoral soundness index depends on a number of macroeconomic variables:

(2) $\qquad y_{j,t} = \beta_{j,0} + \beta_{j,1}x_{1,t} + \beta_{j,2}x_{2,t} + ... + \beta_{j,n}x_{n,t} + \mu_{j,t}$

in which β_j is a set of regression coefficients to be estimated for the jth cluster (j=1,…6), x_i are the n independent macroeconomic variables and $\mu_{j,t}$ is a random error term.

Equations (1) and (2) constitute a multifactor model in which the variability of the sectoral soundness index due to the systematic components is captured by the influence of the x macroeconomic variables and that due to the idiosyncratic component is captured by the error term μ.

Each macroeconomic factor in turn has a dynamic that is explained by a stochastic autoregressive moving average process (ARMA) of order (p_i, q_i).

(3) $\qquad x_{i,t} = k_{i,0} + k_{i,1}x_{i,t-1} + ... + k_{i,p}x_{i,t-p} + \varepsilon_{i,0} + \theta_{i,1}\varepsilon_{i,t-1} + ... + \theta_{i,q}\varepsilon_{i,t-q}$

in which k_i and θ_i are a set of regression coefficients to be estimated.

Identifying the process that governs the evolution of each time series enables us to separate the predictable from the unexpected component. The expected variation in macroeconomic factors is substituted into (2)[9].

Equations (1)-(2)-(3) for the six clusters define a system of equations describing the joint trend in the default rates (transformed into soundness indices) of the various clusters on the basis of the trend in the economy. The estimated coefficients thus enable us to measure the impact of adverse variation in the macroeconomic variables on the default rates of the single clusters.

As in Virolainen (2004), the system of equations was estimated by the SUR (Seemingly Unrelated Regression) method, which unlike OLS uses an estimate of simultaneous correlations of errors between the different equations to improve the efficiency of the estimates of the coefficients[10].

Assuming the model captures the whole systematic component, the idiosyncratic sectoral component should be uncorrelated. The existence of a correlation between the residuals would thus be of great interest, in that it would give an indication of the extent to which the hypothesis of conditional independence in multifactor models is violated[11]. In other words, a correlation between the residuals of the estimates would indicate that the correlation between the sectoral soundness indices is not due solely to the macroeconomic variables common to various equations (and/or to

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[9] The unexpected component of the macro variables will be used in a later phase to generate the scenarios for simulating the distribution of banks' portfolio losses around the expected value.

[10] The presence in the system's equations of correlated dependent variables induces a simultaneous correlation of error terms. The SUR method (or JGLS, Joint Generalized Least Squares) consists in generalizing the OLS method for multi-equation systems and increases efficiency if the equations have different regressors. In the limiting case in which the same regressors appear in each equation, SUR gives the same results as estimating each equation singly by OLS.

[11] This is discussed at length in Hanson, Pesaran, and Schermann (2005) and in Carling, Ronnegard and Roszbach (2004).

the correlations between macroeconomic variables), as in multifactor models, but also to direct interconnection between firms in contiguous sectors.

For the econometric estimate, the time series of financial soundness indices for the six clusters have been seasonally adjusted using ARIMA X11. The seasonally adjusted series were then subjected to unit root tests to check the stationarity; appropriate transformations, if needed, are performed to make them stationary. The results of the test on the soundness indices are given in Table 3. They suggest using the first difference of the dependent variable in every cluster.

5.   The macroeconomic variables

The time series of the variables considered in the econometric analysis are also quarterly and run from 1990 through 2004.

The first stage of the analysis covered a large number of variables (Table 4) that could affect the economic and financial condition of firms, such as GDP, inflation, interest rates, share market prices and exchange rates. These variables were also used, in various combinations, in the empirical works described above.

To identify the main factors driving the movements of default rates and detect collinearity between variables, a preliminary factor analysis to check correlations between the macroeconomic variables was run to identify a small set of unobservable common factors summarizing the information contained in the original set of variables. On this basis we identified and grouped the macrovariables that weigh most heavily in the variability of each factor.

The results of the analysis are given in Tables 5 and 6. There are five macro-factors, grouping indicators for the following: i) the business cycle, ii) external competitiveness, iii) debt cost, iv) world economy, and v) price stability.

The first factor consists of variables identifying the business cycle, such as: real GDP, output gap, industrial production index, index of forecast orders, business confidence index, fixed capital formation over real GDP. In a cyclical downturn firms' profitability tends to decline, adversely affecting their ability to meet their obligations. The cyclical variables should thus be correlated positively with the financial soundness index and negatively with the default rate. The ratio of fixed capital formation to GDP is expected to show a positive sign, in that investment implies greater potential for expansion and growth and thus, if productive, lower probability of default. However, a high incidence of invested capital can produce greater leverage and a higher incidence of depreciation on operating profits, resulting in lower profitability, so that in some cases the sign of this coefficient could actually be negative.

The second factor consists of variables summarizing the competitiveness of the Italian economy on the world scene (the effective real exchange rate, the prices of imports and exports). A rise in the effective exchange rate has adverse effects on externally-oriented sectors while favoring those with foreign debts. The expected sign of the correlation is therefore not unequivocal.

The third factor is the cost of debt (money market rate and rate on bank loans to firms). The short-term interest rate reflects monetary policy action as regards the outlook for overall economic growth and affects the evolution of the rate on bank loans to firms, which represents the cost of debt and is thus inversely related to the default rate.

The fourth factor reflects world economic performance (the S&P 500 index and the price of oil). A rise in US stock markets proxies an uptrend in the world economy. The price of oil also reflects the state of the global economic cycle. A significant rise in oil prices increases the cost of inputs in all sectors of the economy, the most severe consequences coming in industries like basic metals, whose output prices are fixed in the short term. The rise in energy costs also affects both firms and the disposable income of households, making default more likely. Obviously, the sign of the correlation is inverse for firms in the mining and energy sectors, for which the price of oil is the benchmark for their own output.

The fifth factor is price stability. The consumer price index affects domestic consumption, and thereby above all the demand for domestic consumer goods and durables.

For each of these latent factors (business cycle, external competitiveness, cost of debt, world economy, price stability) we have selected one macroeconomic variable to relate with the soundness index. All the macroeconomic variables, seasonally adjusted, are transformed to make the series stationary. The explanatory variable used in the econometric estimate is the expected component of the relevant economic variable, if present, or the variable itself if the time series is *white noise*.

The main descriptive statistics of the original time series, the details on the transformations (log difference or first difference), the expected signs of the impact on the soundness index and the results of the ARMA estimation are given in Table 4. The regression also includes two dummies, one for the second quarter of 1991 and one for the fourth quarter of 1997, to account for changes in the reporting threshold of the Central Credit Register[12].

6. The estimates

Tables 7a and 7b show the normal and standardized coefficients for the six-equation model estimated by SUR. They were tested for robustness and stability by performing univariate regressions for each cluster, with errors adjusted for heteroskedasticity and autocorrelation of residuals[13]. The procedure moved from general to particular, at each step eliminating the variables that proved not to be significant, to streamline the estimation and forecasting model.

To take account of the autocorrelation of residuals, the specification of each equation included among the regressors the dependent variable with a one- or two-quarter lag[14]. The soundness index shows an autoregressive pattern in five of the six clusters, indicating that the default rate has a certain time persistence and that in the presence of a shock to the macroeconomic

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[12] In 1991 the reporting threshold was raised to 150 million lire; in 1997 coverage was extended to financial companies.
[13] The variance and covariance matrix was corrected for heteroskedasticity and autocorrelation by the Newey-West method.

variables the readjustment to equilibrium takes place over several successive periods (generally, one or two quarters)[15].

The macroeconomic variables that influence the largest number of clusters are real GDP, the real effective exchange rate, and the nominal rate of interest on loans to firms. The latter incorporates information on price trends as well, as is shown by the fact that including the inflation rate among the explanatory variables, an independent factor according to the results of the principal-components analysis, gives the regressions no additional explanatory power.

The correlation with GDP is positive. In cyclically weak phases firms' earnings tend to decline and the financial soundness index consequently diminishes. The clusters most heavily affected by GDP trends are energy and mining (cluster 5), "other" services (cluster 6) and agriculture (cluster 1). In all except cluster 3 (engineering and construction), GDP shocks are transmitted to the soundness index with a one-quarter lag, indicating that it takes some time before the business cycle impacts on the default rate.

The coefficient of the effective exchange rate is significant in all sectors but the fifth. The sign is positive.

The interest rate on loans to businesses has a significant effect on three clusters: agriculture, consumer and typical export goods, and engineering and construction. The sign of the coefficient is negative, in that as interest rates rise so does the cost of debt, which results in a deterioration of the soundness index in the two or three quarters following the interest-rate rise. Clusters 2 and 3 show a higher ratio of credit used to value added than the other clusters.

Some clusters respond more specifically to other macroeconomic variables. The trade, transport and communications cluster is sensitive to changes in the price of oil, i.e. energy input costs. The correlation is negative: as the price of oil rises, the soundness index falls and the default rate increases. Cluster 5 (mining and energy products) is also sensitive to the price of oil, but in this case the correlation is positive: the price of oil is their output price, and as it rises their earnings increase and their soundness index improves.

Finally, fixed capital formation as a ratio to GDP, which in our factor analysis was one of the variables defining the business cycle (first factor), retains independent explanatory power for cluster 3, that of investment goods and construction, and cluster 5, mining and energy. The sign of the coefficient is negative. This could be due to lower profitability for firms with a heavy incidence of fixed capital, because they presumably have greater leverage and a higher incidence of depreciation on operating income.

In macroprudential terms, the key point is the effect of macroeconomic variables on bank portfolio risk. What matters for the overall stability of the banking system is shocks that can damage a number of portfolios at the same time and that originate in the real economy and the

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[14] The SUR method assumes that all the explanatory variables, including endogenous ones with various lags, are exogenous or predetermined variables; on this assumption, it is not necessary to use instrumental variables.
[15] In all except cluster 3 (engineering and construction) the coefficient of the lagged dependent variable was negative and significant. The estimate is also robust to univariate specification with Newey-West errors.

financial markets. Our econometric analysis (χ^2 test) shows that overall the macroeconomic variables have a significant effect on sectoral soundness indices.

Nevertheless, only in two clusters does the systematic component explain more than half the variation in the sectoral indices. The percentage of the variation explained by macroeconomic factors ranges from a low of 31 per cent (agriculture) to a high of 56 per cent (mining and energy) of the overall change in soundness indices. In four of the six clusters most company risk depends not on systematic factors but on sector-specific factors, a result that is in line with the empirical literature cited earlier for both Italy and Europe (in particular, Botticini, Marchesi and Toffano, 2000, and Chionsini, Foglia and Marullo Reedtz, 2004, who also use default rates taken from the Central Credit Register).

The energy sector is the one most sensitive to systematic risk, the agriculture cluster the least sensitive. Consequently, the largest benefits from diversification would be obtained when lending to the agricultural sector, while the greatest risks of concentration are in the energy sector.

A second interesting point is the correlation between error terms in the equations (Table 8). The Bresch-Pagan test rejects the hypothesis of independence of residuals, suggesting the existence of sectoral interdependence originating in the idiosyncratic component of each sector. This interdependence is especially strong for clusters 2, 3 and 4. Presumably there are specific business relations that result in direct contagion between firms, even when shocks originate in a single sector. The only sector relatively independent of the others is agriculture, for which the correlation of residuals is significant only with the sixth cluster.

The extent of business relations between firms, which is generally cited at the intrasectoral level, thus appears to be significant at the intersectoral level as well (Alves, 2004). Neglecting this effect in portfolio models would result in an underestimation of risk[16].

7.   Conclusions

This paper analyzes the mechanisms linking the riskiness of firms to the performance of macroeconomic variables when interrelations exist between various sectors.

Multivariate statistical analysis has been used to group the default rates of 23 branches of economic activity recorded by the Central Credit Register into six clusters with homogeneous risk.

The default rates of the six clusters were then related with the main macroeconomic variables to identify the impact of economic performance on the risk of the various sectors and quantify the component of credit risk attributable to common factors (the systematic risk).

One aspect of interest for macro-prudential analysis is the influence of systemic factors on bank portfolios' risk. The results show that, overall, the macroeconomic variables have a significant impact on the indices of sector soundness; however, they mainly depend on factors specific to each

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[16] Giesecke and Weber (2003) observe that the intensity of this effect depends on the complexity of the economic environment, as gauged by the number of counterparties of each firm. As complexity increases, the risk of contagion diminishes, so there is a lower probability of significant unexpected losses.

sector. This result is in line with the findings of other related studies based on data for both European and Italian firms.

In particular, the energy sector would appear to be the most vulnerable to systematic risk, while agriculture is the least "cyclical" cluster.

A significant correlation between the residuals of the sectoral regressions shows, moreover, that although macroeconomic factors do have a common influence on most of the clusters, this does not fully explain the correlation between the risk of the various economic sectors, which is largely due to direct contagion between firms in different sectors. Neglecting this component when estimating the distribution of losses on a portfolio would lead to undervaluation of the risk.

These results are consistent with Italy's productive structure, with its multitude of small businesses, often organized into chains, districts or business groups, for which specific risks predominate and which have mainly direct business relations with few counterparts.

As robustness checks, we plan to: (i) use a different cluster specification, based on NACE industry classification; (ii) estimate the credit risk model in terms of unobservable latent factors.

The model is also suitable for stress test analysis. Following Wilson (1997), it is possible to utilise the parameter estimates and the error terms together with the system of equations to simulate future paths of joint default rates across all industries over some desired time horizon and to determine a credit loss distribution conditional on the simulated macro scenarios. The simulation takes into account correlations between the macroeconomic factors as well as any industry-specific shocks.

For stress testing purposes, Sorge and Virolainen (2005) introduce an artificial shock in the vector of errors and in the first step of each simulation round; the shock impacts the other macro-factors through the variance-covariance matrix; loss distribution conditional on the assumed stressed scenario can then be calculated. This stress test method is also applied in the stress test software developed at the Austrian central bank for financial stability purposes (ONB, 2006).

REFERENCES

Alves, Ivan (2004), "Sectoral fragility: factors and dynamics", paper prepared for the ECB Workshop on Financial Stability, 16-17 June, mimeo.

Botticini, Angela, M. Marchesi and Carlo Toffano (2000), "Un modello di stima delle correlazioni tra le probabilità di insolvenza per il portafoglio di una banca commerciale", in "La gestione del rischio di credito", edited by P. Savona and A. Sironi, EDIBANK.

Carling, Kenneth, T. Jacobson, J. Lindé and Kasper Roszbach, (2004), "Corporate Credit Risk Modelling and the Macroeconomy", mimeo.

Carling, Kenneth. L. Ronnegard and Kasper Roszbach (2004), "Is Firm Interdependence within Industries Important for Portfolio Credit Risk?", Sveriges Riksbank Working Paper, No. 168.

Chionsini, Gaetano, A. Foglia and Paolo Marullo Reedtz, (2004), "Bank Mergers, Diversification and Risk", paper prepared for the ECB Workshop on Financial Stability, 16-17 June, mimeo.

Drehmann, Mathias, (2005) "A Market Based Macro Stress Test for the Corporate Credit Exposures of UK Banks", mimeo.

Giesecke, Kay, and Stefan Weber, (2003), "Cyclical correlations, credit contagion, and portfolio losses", forthcoming, Journal of Banking and Finance.

Hanson, Samuel G., M.H. Pesaran and Til Schuermann , (2005), "Firm Heterogeneity and Credit Risk Diversification", Wharton Financial Institutions Center Working Paper No. 05-05.

Jacobson, Tor, J. Lindé and Kasper Roszbach, (2005), "Exploring Interactions between Real Activity and the Financial Stance", mimeo.

ONB (2006), Systemic Risk Monitor, Model documentation.

Pain, Darren and Jukka Vesala, (2004), "Driving Factors of Credit Risk in Europe", paper prepared for the ECB Workshop on Financial Stability, 16-17 June, mimeo.

Pesaran, M. Hashem, T. Schuermann, B. Treutler and Scott M. Weiner, (2004), "Macroeconomics and Credit Risk: A Global Perspective", forthcoming, Journal of Money, Credit and Banking, available as Wharton Financial Institutions Center Working Paper No. 03-13B.

Sorge, Marco and Kimmo Virolainen, (2005), "A comparative analysis of macro-stress testing with application to Finland", Journal of Financial Stability, 2, pp. 113-151.

Virolainen, Kimmo, (2004), "Macro stress testing with a macroeconomic credit risk model for Finland", paper prepared for the ECB Workshop on Financial Stability, 16-17 June, mimeo.

Wilson, Thomas (1997) "Portfolio credit risk (I)", Risk 10 (9), 111-116.

Chart1. Annual default rates by cluster of economic activity (1990-2004)

Legend:
- ◆ AGRICOLTURE
- ■ CONSUMER GOODS AND TYPICAL ITALIAN PRODUCTS
- ▲ ENGINEERING AND CONSTRUCTIONS
- ✕ TRADE, TRANSPORT AND COMMUNICATIONS
- ✳ MINING AND QUARRYING AND ENERGY PRODUCTS
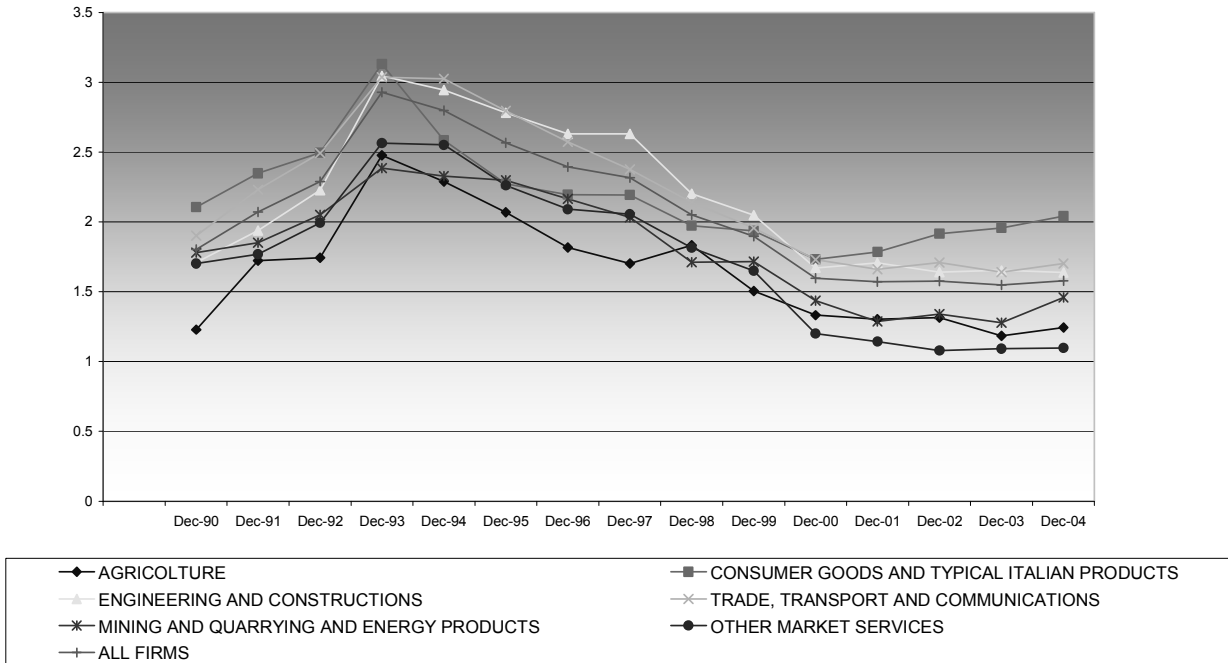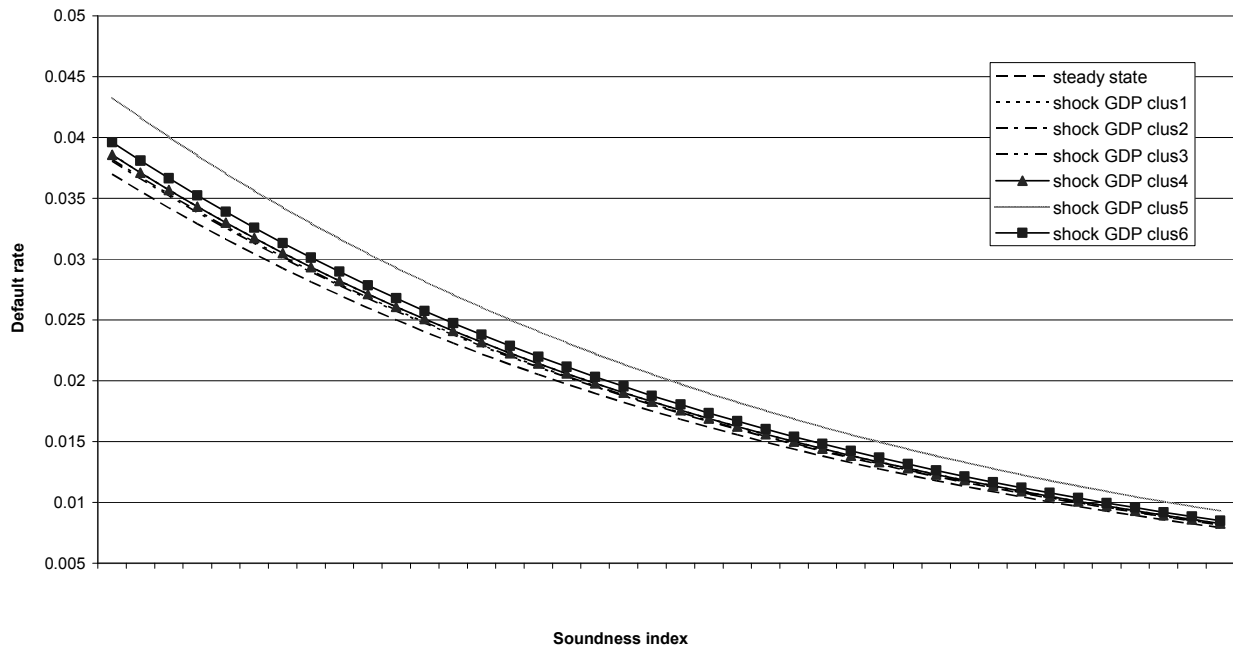- ● OTHER MARKET SERVICES
- ┼ ALL FIRMS

Chart 2. Impact on the annual default rate of each cluster of a GDP shock equal to twice the standard deviations, at the various levels of the soundness index

Legend:
- steady state
- shock GDP clus1
- shock GDP clus2
- shock GDP clus3
- shock GDP clus4
- shock GDP clus5
- shock GDP clus6

Default rate

Soundness index

16

CLUSTER	NAME	BRANCHES OF ECONOMIC ACTIVITY	Average annual default rate (%)	Relative volatility (§)	Max (Year)	Min (Year)
		Table 1: Statistics for annual default rates by clusters of economic activity				
		(1990 - 2004; firms and producer households).				
TOTAL			2.07	1.00	2.93 (1993)	1.55 (2003)
1	*Agriculture*	Agriculture, forestry and fishing	1.65	0.86	2.48 (1993)	1.18 (2003)
2	*Consumer goods and typical Italian products*	Manufacture of food products, beverages and tobacco; Manufacture of textiles and clothing;Manufacture of leather and leather products; Manufacture of paper and paper products; Publishing and printing; Manufacture of rubber and plastic products; Manufacture of w ood and w ood products; Other manufacturing; Manufacture of chemicals, chemical products and man-made fibres.	2.18	0.77	3.13 (1993)	1.73 (2000)
3	*Engineering and building industry*	Manufacture of basic metals and fabricated metal products; Manufacture of machinery and equipment including repair and maintenance; Manufacture of electrical and optical equipment; Manufacture of transport equipment;Building industry	2.16	1.11	3.05 (1993)	1.64 (2003)
4	*Trade, transport and communications*	Wholesale and retail trade and repairs; Hotels, restaurants and bars; Transport and communications	2.20	1.06	3.04 (1993)	1.64 (2003)
5	*Mining and quarrying and energy products*	Mining and quarrying of energy producing materials; Mining and quarrying except energy producing materials;Manufacture of coke, refined petroleum products and nuclear fuel	1.81	0.84	2.39 (1993)	1.28 (2003)
6	*Other market services*	Real estate and business activities; Other service activities	1.74	1.12	2.56 (1993)	1.08 (2002)

(*) Source: Banl of Italy's Central Credit Register; (§) Volatility with respect to the cluster average

Table 2. Statistics for clusters

Agriculture

Value added (*)	31-Dec-03	25,452
Credit used as a % of total	31-Dec-04	3.6

Variables	Average (1990-2004)	Volatility with respect to system average
No. of firms	45,377	0.11
VA growth rate (%)	0.64	5.40
Credit used as % of VA	1.92	0.65
Credit used/credit granted	0.80	1.16
Leverage	0.93	1.01
Interest coverage ratio (%)	14.00	2.00

Manufacture of consumer goods and typical Italian products

Value added (*)	31-Dec-03	106,474
Creidt used as % of total	31-Dec-04	16.0

Variables	Average (1990-2004)	Volatility with respect to system average
No. of firms	95,158	0.10
VA growth rate (%)	0.96	2.10
Credit used as % of VA	2.69	0.67
Credit used/credit granted	0.57	0.93
Leverage	0.97	0.40
Interest coverage ratio (%)	8.2	1.22

Engineering and construction

Value added (*)	31-Dec-03	134,177
Credit used as a % of total	31-Dec-04	24.4

Variables	Average (1990-2004)	Volatility with respect to system average
No. of firms	154,670	0.22
VA growth rate (%)	0.65	2.43
Credit used as % of VA	3.23	0.71
Credit used/credit granted	0.61	1.31
Leverage	0.94	1.05
Interest coverage ratio (%)	7.93	1.11

Trade, transport and communications

Value added (*)	31-Dec-03	241,550
Credit used as % of total	31-Dec-04	25.0

Statistics	Average (1990-2004)	Volatility with respect to system average
No. of firms	221,468	0.36
VA growth rate (%)	1.82	1.31
Credit used as % of VA	1.69	0.40
Credit used/credit granted	0.64	0.96
Leverage	0.52	1.29
Interest coverage ratio (%)	4.38	1.04

Mining and quarrying and energy products

Value added (*)	31-Dec-03	74,801
Credit used as % of total	31-Dec-04	6.3

Variables	Average (1990-2004)	Volatility with respect to system average
No. of firms	13,683	0.02
VA growth rate (%)	-0.30	4.12
Creidt used as % of VA	1.41	0.73
Credit used/credit granted	0.56	2.14
Leverage	0.99	3.10
Interest coverage ratio (%)	2.13	0.73

Other market services

Value added (*)	31-Dec-03	448,541
Credit used as % of total	31-Dec-04	24.7

Variables	Average (1990-2004)	Volatility with respect to system average
No. of firms	97,196	0.29
VA growth rate (%)	1.60	0.70
Credit used as % of VA	0.57	0.41
Credit used/credit granted	0.72	1.95
Leverage	0.40	1.33
Interest coverage ratio (%)	0.02	0.73

(*) 2003 value added measured in millions of eurolire at constant 2005 prices. Source: Bank of Italy, Economic Research Dept., Economic Analysis Office.

TABLE 3

UNIT ROOT TEST

Augmented Dickey-Fuller tests (see Said and Dickey, 1984)
The lagged differences in the models are included to obtain white noise residuals. The maximum lag parameter p* is computed using information criteria (Schwarz, Hannan and Quinn, Final Prediction Error) and miss-specification tests.

Series	p*	Model B $\Delta y_t = \mu + \rho\, y_{t-1} + \sum_{j=1}^{p^*-1} \rho_j \Delta y_{t-1} + \varepsilon_t$			Model A $\Delta y_t = \rho\, y_{t-1} + \sum_{j=1}^{p^*-1} \rho_j \Delta y_{t-1} + \varepsilon_t$
		$\rho=0$ τ_μ	$\mu=0$ $\tau_{\alpha\mu}$	$\rho=\mu=0$ Φ_1	$\rho=0$ τ
Default rates 1990:1-2004:4					
Cluster1		-1.18	1.05	0.80	-2.39
Cluster2		-1.70	1.58	1.64	-2.32
Cluster3		-1.13	1.00	0.78	-2.07
Cluster4		-0.89	0.70	0.74	-1.96
Cluster5		-0.81	0.67	0.54	-2.13
Cluster6		-0.71	0.47	0.56	-2.26
Seasonally adjusted soundness indices 1990:1-2004:4					
Cluster1		-0.71	0.74	0.45	-2.55
Cluster2		-1.31	1.33	1.17	-2.02
Cluster3		-0.51	0.54	0.41	-1.88
Cluster4		-0.28	0.32	0.68	-1.98
Cluster5		-0.78	0.81	0.61	-1.98
Cluster6		0.04	0.04	0.89	-2.10
Critical values	5%	-2.93	2.56	4.86	-3.50
	1%	-3.58	3.28	7.06	-4.15

Table 4. Main statistics for macroeconomic variables (original series)

Time series	Variables	Average	Standard deviation	Max	Min	Expected sign (*)	Trans-formation	Arma estimation
Business cycle indicators								
Output gap - % deviation from an estimated figure	OUTQ	- 0.3	3.6	9.0	- 7.5	+	Xt -Xt-1	AR(1)
GDP at constant prices - million eurolire	PILQKDSP	238,681.8	16,617.2	264,040.0	215,301.0	+	ln(Xt) -ln(Xt-1)	White Noise
Index of industrial production - Series 66..c - Seasonally adjusted series, 1995 price index	INDPSA	92.6	5.6	101.7	82.1	+	ln(Xt) -ln(Xt-1)	AR(1)
IT COMPOSITE LEADING INDICATOR: PRODN. - FUTURE TENDENCY SADJ	LAEPROD	0.1	8.3	17.2	- 21.8	+	Xt	AR(1)
IT COMPOSITE LEADING INDICATOR: ORDERBOOKS OR DEMAND(FUT.TE SADJ)	ITOL0633	- 0.0	9.0	16.3	- 28.1	+	Xt	White Noise
IT BUSINESS INDUSTRIAL CONFIDENCE INDICATOR(DISC.) SADJ	INDCONF	92.5	7.8	107.4	75.3	+	Xt	White Noise
IT GROSS CAPITAL FORMATION (% CONSTANT GDP) SADJ	FIXCAP	20.3	1.2	22.5	17.2	+ / -	Xt -Xt-1	AR(3)
Indicators of external competitiveness								
Real effective exchange rate - Average for the period	REU	107.3	9.8	128.2	88.4	+ / -	ln(Xt) -ln(Xt-1)	White Noise
Export prices - Series 74..d	EXPPRI	108.4	11.5	148.3	91.2	+	ln(Xt) -ln(Xt-1)	White Noise
Import prices - Series 75..d	IMPPRI	107.7	10.5	140.2	92.2	-	ln(Xt) -ln(Xt-1)	White Noise
Cost of debt								
Money market rate Series Flinp 60b	MMR	7.3	4.0	16.4	2.1	-	Xt -Xt-1	White Noise
Interest rate on business loans	CORY	- 0.16	0.62	- 2.14	2.49	-	Xt -Xt-1	AR(1)
World economic indicators								
S&P 500 COMPOSITE DS CALCULATED - PRICE INDEX	SPCOMZ	773.6	406.3	1,515.3	249.2	+ / -	ln(Xt) -ln(Xt-1)	White Noise
Brent Crude - Current month, fob U$/BBL	OILBREN	21.8	6.8	47.0	10.5	-	ln(Xt) -ln(Xt-1)	AR(2)
Price stability indicators								
Consumer price index - Series 64	CPI	91.7	12.7	111.1	66.6	-	ln(Xt) -ln(Xt-1)	D=1 AR(4)

(*) The expected sign is of the index of financial soundness, which is inversely related to the default rate.

Table 5: Results of factor analysis

Factor	Eigenvalue	Difference	Proportion	Cumulative
1	**4.37318**	**1.33091**	**0.2915**	**0.2915**
2	**3.04227**	**1.2468**	**0.2028**	**0.4944**
3	**1.79547**	**0.5705**	**0.1197**	**0.6141**
4	**1.22497**	**0.18161**	**0.0817**	**0.6957**
5	**1.04337**	**0.15146**	**0.0696**	**0.7653**
6	0.8919	0.1194	0.0595	0.8247
7	0.77251	0.20506	0.0515	0.8762
8	0.56745	0.14657	0.0378	0.9141
9	0.42087	0.1604	0.0281	0.9421
10	0.26047	0.0554	0.0174	0.9595
11	0.20507	0.01834	0.0137	0.9732
12	0.18674	0.06291	0.0124	0.9856
13	0.12383	0.06873	0.0083	0.9939
14	0.05509	0.01828	0.0037	0.9975
15	0.03681	.	0.0025	1

Table 6 : Factor loadings

Variable	1	2	3	4	5
outq	0.76578	-0.32157	-0.05939	-0.12402	0.08871
pilqkdsp	0.71112	-0.37182	-0.19287	0.15725	0.11607
indpsa	0.79382	-0.32496	-0.03777	-0.12482	0.05591
laeprod	0.89073	-0.03316	0.17872	-0.07124	0.11568
itol0633	0.87042	-0.02799	0.12385	-0.06218	0.00994
indconf	0.8106	0.09263	0.01879	-0.12227	-0.26447
reu	0.02154	0.51876	-0.64764	0.21644	0.10488
mmr	0.36007	0.64304	0.43574	0.3154	-0.16251
exppri	0.19788	0.80033	-0.40473	0.05004	0.2627
imppri	0.27846	0.82267	-0.21863	-0.19405	0.25648
spcomz	0.08079	-0.36994	-0.13857	0.48776	0.25783
oilbren	0.04246	0.55587	0.31346	-0.56559	-0.02392
cpi	-0.01216	0.02216	0.52161	0.18905	0.72598
fixcap	0.41132	0.14785	-0.3624	0.35332	-0.33588
cory	0.13544	0.46521	0.60324	0.49044	-0.2374

Table 7a: Results of SUR model

Independent variables	Variables	Cluster 1			Cluster 2			Cluster 3			Cluster 4			Cluster 5			Cluster 6		
		Lag	Coeff	P-value	Lag	Coeff	P-value	Lag	Coeff	P-value	Lag	Coeff	P-value	Lag	Coeff	P-value	Lag	Coeff	P-value
Constant			-0.117	0.355		-0.009	0.305		-0.010	0.242		0.001	0.913		-0.447	0.023		0.000	0.971
Dependent variable t-1			-0.385	0.000		-0.267	0.003					-0.271	0.000		-0.598	0.000		-0.237	0.001
Dependent variable t-2															-0.339	0.000			
GDP at constant prices	PILQKDSP	1	3.457	0.048	1	2.329	0.071	0	2.628	0.003	1	3.471	0.002	1	12.896	0.000	1	5.643	0.000
Real effective exchange rate	REU	2	1.040	0.006	1	0.502	0.053	1	0.850	0.000	2	1.006	0.000				2	1.057	0.001
Interest rate on business loans	CORY	1	-0.060	0.057	1	-0.051	0.026	2	0.036	0.004									
Brent crude - Current month, fob U$/BBL	OILBREN										1	-0.138	0.098	1	0.564	0.020			
Gross fixed investment (% GDP at constant prices)	FIXCAP							0	-0.100	0.000				4	-0.110	0.041			
Dummy 9106	dd9106					0.118	0.035		0.200	0.000									
Dummy 9712	dd9712																	-0.205	0.000
"R^2"		30.8%			38.7%			52.3%			32.9%			56.1%			34.7%		
Chi2 (pvalue)		24.46	(0.000)		41.78	(0.000)		102.05	(0.000)		45.52	(0.000)		86.24	(0.000)		61.18	(0.000)	

The dependent variable for each cluster is the "health index", calculated as $y_{jt} = \ln((1-p_{jt})/p_{jt})$.

Table 7b: Results of SUR model - Standard coefficients

Independent variables	Variables	Cluster 1			Cluster 2			Cluster 3			Cluster 4			Cluster 5			Cluster 6		
		Lag	Coeff	P-value	Lag	Coeff	P-value	Lag	Coeff	P-value	Lag	Coeff	P-value	Lag	Coeff	P-value	Lag	Coeff	P-value
Constant			0.961	0.353		0.053	0.597		0.464	0.599		0.115	0.22		0.002	0.979		0.068	0.536
Dependent variable t-1			-0.385	0.000		-0.267	0.003					-0.271	0.000		-0.598	0.000		-0.237	0.001
Dependent variable t-2															-0.339	0.000			
GDP at constant prices	PILQKDSP	1	0.217	0.048	1	0.181	0.071	0	0.192	0.003	1	0.286	0.002	1	0.408	0.000	1	0.354	0.000
Real effective exchange rate	REU	2	0.283	0.006	1	0.169	0.053	1	0.269	0.000	2	0.360	0.000				2	0.287	0.001
Interest rate on business loans	CORY	1	-0.185	0.055	1	-0.193	0.026	2	-0.128	0.039									
Brent crude - Current Month,fob U$/BBL	OILBREN										1	-0.116	0.098	1	0.182	0.020			
Gross fixed investment (% GDP at constant prices)	FIXCAP							0	-0.302	0.000				4	-0.143	0.041			
Dummy 9106	dd9106					1.470	0.035		2.338	0.000									
Dummy 9712	dd9712																	-2.055	0.000
"R^2"		30.8%			38.7%			52.3%			32.9%			56.1%			34.7%		
Chi2 (pvalue)		24.46	(0.000)		41.78	(0.000)		102.05	(0.000)		45.52	(0.000)		86.24	(0.000)		61.18	(0.000)	

The dependent variable for each cluster is the "health index", calculated as yjt= ln ((1-pjt) / pjt).

Table 8: Regression residuals correlation matrix

	clus1	clus2	clus3	clus4	clus5	clus6
clus1	1					
clus2	-0.06	1				
	0.68					
clus3	-0.02	0.34	1.00			
	0.91	*0.01*				
clus4	0.18	0.50	0.43	1.00		
	0.18	*0.00*	*0.00*			
clus5	-0.05	-0.01	0.40	0.32	1.00	
	0.70	*0.97*	*0.00*	*0.02*		
clus6	0.29	0.33	0.66	0.61	0.08	1
	0.03	*0.01*	*0.00*	*0.00*	*0.57*	

Breusch-Pagan independence test - Chi2(15): 102.003 (p: 0.000)
(p-value in Italics)

A FRAMEWORK FOR STRESS TESTING BANKS' CREDIT RISK[1]

Key Points:

- *This paper develops a framework for stress testing the credit exposures of Hong Kong's retail banks to macroeconomic shocks. It involves the construction of macroeconomic credit risk models, each consisting of a multiple regression model explaining the default rate of banks, and a set of autoregressive models explaining the macroeconomic environment estimated by the method of seemingly unrelated regression.*

- *Specifically, two macroeconomic credit risk models are built. One model is specified for the overall loan portfolios of banks and, to illustrate how the same framework can be applied for stress testing loans to different economic sectors, the other model is specified for the banks' mortgage exposures only.*

- *The empirical results suggest a significant relationship between the default rates of bank loans and key macroeconomic factors including Hong Kong's real GDP, real interest rates, real property prices and Mainland China's real GDP.*

- *Macro stress testing is then performed to assess the vulnerability and risk exposures of banks' overall loan portfolios and mortgage exposures. By using the framework, a Monte Carlo method is applied to estimate the distribution of possible credit losses conditional on an artificially introduced shock. Different shocks are individually introduced into the framework for the stress tests. The magnitudes of the shocks are specified according to those occurred during the Asian financial crisis.*

- *The result shows that even for the Value-at-Risk (VaR) at the confidence level of 90%, banks would continue to make a profit in most stressed scenarios, suggesting that the current credit risk of the banking sector is moderate. However, under the extreme case for the VaR at the confidence level of 99%, banks' credit loss would range from a maximum of 3.22% to a maximum of 5.56% of the portfolios, and if a confidence level of 99.9% is taken, it could range from a maximum of 6.08% to a maximum of 8.95%. These estimated maximum losses are very similar to what the market experienced one year after the Asian financial crisis shock. However, the probability of such losses and beyond is very low.*

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I.   INTRODUCTION

Macro stress testing refers to a range of techniques used to assess the vulnerability of a financial system to "exceptional but plausible" macroeconomic shocks.[2] Increasingly, macro stress testing plays an important role in the macro-prudential analysis of public authorities. The main objective is to identify structural vulnerability and overall risk exposures in a financial system that could lead to systemic problems. In conjunction with stress testing to assess the vulnerability of the portfolios of individual institutions, macro stress testing forms the main part of system-wide analysis, which measures the risk exposure of a group of financial institutions to a specific stress scenario. It can also serve as a tool for cross-checking results obtained by financial institutions' internal models.

In this paper, a macro stress testing framework is developed for testing the loan portfolios of retail banks in Hong Kong. It involves the construction of macroeconomic credit risk models, each consisting of a multiple regression model and a set of autoregressive models (for examining the relationship between the default rate of bank loans and different macroeconomic values based on historical data) estimated by the method of seemingly unrelated regression. Two macroeconomic credit risk models are built. One model is specified for the overall loan portfolios of banks and, to illustrate how the same framework can be applied for stress testing loans to different economic sectors, the other model is for the banks' mortgage exposures only.

Macro stress testing is then performed to assess the vulnerability and risk exposures of banks' overall loan portfolios and mortgage exposures. Adverse macroeconomic scenarios are taken and, using the framework, the possible combinations of stressed macroeconomic values are obtained from a Monte Carlo simulation. Based on this, distributions of possible default rates of bank loans under a specific shock can be generated. Value-at-Risk (VaR) is computed to evaluate how the stressed macroeconomic environment may affect the default probability of banks' loan portfolios.[3]

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[2]   This follows the IMF definition. See Blaschke et al. (2001) and Sorge (2004).
[3]   VaR refers to the maximum amount of money that may be lost over a certain period at a specific confidence level.

II.   ELEMENTS OF STRESS TEST AND THE COMMON METHODOLOGY

Macroeconomic stress tests involve two major elements. First, scenarios of extreme but plausible adverse macroeconomic conditions need to be devised.[4] Secondly, the adverse macroeconomic scenarios need to be mapped onto the impact on banks' balance sheets. Through this, the robustness of banks can be evaluated.

For the first element, given that since an adverse macroeconomic scenario refers to a combination of adverse developments in several macroeconomic variables, it is important to ensure its internal consistency and that the specified values of the macroeconomic variables constitute a realistic mix. The conventional approach, as adopted by Froyland and Larsen (2002), Hoggarth and Whitley (2003), Mawdsley et al. (2004) and Bunn et al. (2005), is to devise scenarios that imitate historical episodes of tail events or to generate scenarios with the aid of a macro-econometric model.

After devising the scenarios, the impact on banks will be estimated. This usually requires first estimating an empirical model that relates a certain financial soundness indicator y to a number of macroeconomic variables $x_1,\ldots, x_M$ that the scenarios encompass:

$$y = f(x_1,\ldots,x_M) + \varepsilon,$$

where ε is an error term capturing determinants of the indicator other than $x_1,\ldots, x_M$. The values of $x_1,\ldots, x_M$ given by the scenarios will then be substituted into the estimated equation and the predicted values of y are computed under the assumption that $\varepsilon = 0$. These predicted values are (point) estimates of the expected values of y conditional on the occurrence of the scenarios. Changes in the predicted values of y as a result of the imposition of the scenarios are usually regarded as the estimated impacts. This approach suffers from two problems: first, once a scenario is chosen, how likely it is to occur is no longer an issue in the stress test;[5] secondly, even if the predicted value of the soundness indicator is not significantly affected by the realisation of the adverse scenario, it is hard to conclude that the risk is low because a large deviation from the average may occur with a "tangible" probability.

By taking into account the possibility that ε is non-zero in the y equation and there is randomness in the behaviour of the macroeconomic variables with the various

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[4]   The importance of the first element lies in the fact that relying on an improperly specified scenario would render the stress test useless as a way to uncover systemic risk. For example, if the specified scenarios have a negligible probability of occurring, the exercise will be irrelevant. On the other hand, if they are too mild to pose a challenge, the exercise will be unable to reveal the downside risk that the financial system is exposed to.

[5]   This treatment is criticised by Berkowitz (1999).

stochastic components being correlated, Wilson (1997a, 1997b) and Boss (2002) developed a stress-testing framework that examines default risk and the development of macroeconomic conditions. Their framework has several advantages over the conventional approach since it takes into account the probabilistic elements and explicitly considers the variation of ε and its correlation with the macroeconomic variables $x_1, \cdots, x_M$. Boss (2002) and Virolainen (2004) applied this framework to conduct credit-risk stress tests for the corporate loan portfolio of Austrian and Finnish banks respectively.

III. THE FRAMEWORK

A framework for stress testing the credit exposure of Hong Kong's retail banks to macroeconomic shocks is developed based on Wilson (1997a, 1997b), Boss (2002), and Virolainen (2004). In essence, our framework comprises:

(i)   an empirical model with a system of equations on credit risk and macroeconomic dynamics, and

(ii)   a Monte Carlo simulation for generating distribution of possible default rates (or credit losses).

3.1   The system of empirical equations

Suppose there are J economic sectors to which banks lend.[6] Let $p_{j,t}$ be the average default rate in sector j observed in period t, where $j = 1, \ldots, J$. As $p_{j,t}$ is bound between zero and one, we use its logit-transformed value $y_{j,t}$ as the regressand. That is,

$$y_{j,t} = \ln\left(\frac{1 - p_{j,t}}{p_{j,t}}\right)$$

is applied to transform $p_{j,t}$ to $y_{j,t}$, hence $-\infty < y_{j,t} < +\infty$.[7] Obviously, $p_{j,t}$ and $y_{j,t}$ are negatively related; a higher $y_{j,t}$ is associated with a better credit-risk status.

Let $\mathbf{y}_t = (y_{1,t}, \ldots, y_{J,t})'$. We model it as depending linearly on its lags and on the current and lagged values of M macroeconomic variables:

$$\mathbf{y}_t = \mathbf{m} + \mathbf{A}_1\mathbf{x}_t + \cdots + \mathbf{A}_{1+s}\mathbf{x}_{t-s} + \mathbf{\Phi}_1\mathbf{y}_{t-1} + \cdots + \mathbf{\Phi}_k\mathbf{y}_{t-k} + \mathbf{v}_t, \tag{1}$$

---

[6]   Boss (2002) and Virolainen (2004) analyse loans to different sub-sectors of the corporate sector. However, there is no impediment in the framework to covering loans to the household sector as well.

[7]   This treatment represents a common practice (see, for example, Pain (2003), Boss (2002) and Virolainen (2004)). Alternative ways of transformation, such as the probit, have also been attempted, and similar results are obtained.

where $\mathbf{x}_t$ is an $M \times 1$ vector of macroeconomic variables; $\mathbf{m}$ is a $J \times 1$ vector of intercepts; $\mathbf{A}_1,\ldots, \mathbf{A}_{1+s}$ are $J \times M$ and $\mathbf{\Phi}_1,\ldots, \mathbf{\Phi}_k$ are $J \times J$ coefficient matrices; and $\mathbf{v}_t$ is a $J \times 1$ vector of disturbances. The characterisation of equation (1) explicitly links the default behaviours in the J economic sectors to the macroeconomic conditions. In Wilson (1997a, 1997b), $\mathbf{y}_t$ is assumed to depend only on $\mathbf{x}_t$. Similar to Virolainen (2004), our specification is more general, allowing the impact of a macroeconomic shock to be prolonged and defaults in different economic sectors to be correlated.[8]

Another part of the equation system in Wilson's framework is on the dynamics of the M macroeconomic variables. In his original specification, each of them follows an autoregressive (AR) process. We generalise it by adopting the following specification:

$$\mathbf{x}_t = \mathbf{n} + \mathbf{B}_1\mathbf{x}_{t-1} + \cdots + \mathbf{B}_p\mathbf{x}_{t-p} + \mathbf{\Theta}_1\mathbf{y}_{t-1} + \cdots + \mathbf{\Theta}_q\mathbf{y}_{t-q} + \mathbf{\varepsilon}_t , \qquad (2)$$

where $\mathbf{n}$ is an $M \times 1$ vector of intercepts; $\mathbf{B}_1,\ldots, \mathbf{B}_p$ are $M \times M$ and $\mathbf{\Theta}_1,\ldots, \mathbf{\Theta}_q$ are $M \times J$ coefficient matrices; and $\mathbf{\varepsilon}_t$ is an $M \times 1$ vector of disturbances. Our specification is similar to Virolainen (2004) and has two advantages over Wilson's. First, equation (2) embodies a more realistic dynamic process in which the macroeconomic variables are mutually dependent. Secondly, equation (2) explicitly models the feedback effects of bank performances on the economy by letting $\mathbf{x}_t$ depend on $\mathbf{y}_{t-1},\ldots, \mathbf{y}_{t-q}$.[9] Equations (1) and (2) together define a system of equations governing the joint evolution of the economic performance, the associated default rates, and their error terms.

In this system, we assume that $\mathbf{v}_t$ and $\mathbf{\varepsilon}_t$ are serially uncorrelated and normally distributed with variance-covariance matrices $\mathbf{\Sigma}_{\mathbf{v}}$ and $\mathbf{\Sigma}_{\mathbf{\varepsilon}}$ respectively; $\mathbf{v}_t$ and $\mathbf{\varepsilon}_t$ are correlated, with variance-covariance matrix $\mathbf{\Sigma}_{\mathbf{v},\mathbf{\varepsilon}}$. In sum, the structure of the disturbances is as follows:

$$\mathbf{e}_t = \begin{pmatrix} \mathbf{v}_t \\ \mathbf{\varepsilon}_t \end{pmatrix} \sim N(\mathbf{0}, \mathbf{\Sigma}), \quad \mathbf{\Sigma} = \begin{pmatrix} \mathbf{\Sigma}_{\mathbf{v}} & \mathbf{\Sigma}_{\mathbf{v},\mathbf{\varepsilon}} \\ \mathbf{\Sigma}_{\mathbf{v},\mathbf{\varepsilon}} & \mathbf{\Sigma}_{\mathbf{\varepsilon}} \end{pmatrix}. \qquad (3)$$

Allowing the off-diagonal elements of $\mathbf{\Sigma}_{\mathbf{v}}$, $\mathbf{\Sigma}_{\mathbf{\varepsilon}}$ and $\mathbf{\Sigma}_{\mathbf{v},\mathbf{\varepsilon}}$ to be non-zero is desirable. First, influences stemming from factors affecting the dependent variables but not explicitly incorporated in equations (1) and (2) will not be omitted altogether. Secondly, the contemporaneous correlation between the two disturbances in Equations (1) and (2) can be

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[8] As pointed out by Sorge (2004), the impact of a macroeconomic shock may persist for a number of years. Therefore, a dynamic specification like equation (1) is more desirable.

[9] See Hoggarth et al. (2005).

captured and the feedback effects of bank performances on the economy can be more accurately assessed.[10]

3.2   Monte Carlo simulations and stress tests

In our framework, stress tests are conducted by comparing the estimated frequency or probability distribution of credit losses of the stressed scenario, where an artificial adverse macroeconomic development is introduced, with that of the baseline scenario, where no artificial adverse shock takes place. Estimated frequency distributions of the horizon-end default rates for each sector corresponding to stressed and baseline scenarios are obtained separately from simulating a large number of future joint sector-specific default rates by applying a Monte Carlo method.

Let us first discuss the estimation of the baseline distribution. To simulate a vector of one-period-ahead values of joint sector-specific default rates, we first draw a vector of random variables $\mathbf{r}$ from the multivariate normal distribution with mean being zero and variance-covariance matrix being the estimated Σ. The vector so drawn represents a realisation of the vector of disturbances $\mathbf{e}$. Given the current and past values of the M macroeconomic variables, the J default rates and the realisation $\mathbf{r}$, the associated one-period-ahead values $y_{j,t+1}$ and $x_{i,t+1}$ can be calculated based on the estimated equations (1) and (2). Similarly, the two-period-ahead values can be calculated with another independently drawn $\mathbf{r}$ and the one-period-ahead values previously obtained. Repeating the same procedure yields a future path of the joint sector-specific default rates, given the time horizon. By simulating a large number of such paths, a frequency distribution of the horizon-end default rates (of the baseline scenario) for each of the J sectors can be constructed. These paths stem from different future evolutions of the macroeconomic environment and the innovations $\mathbf{v}_t$ in equation (1). With specific assumptions or actual data on the loss given default (LGD), the associated distribution of possible credit losses can be estimated.

In constructing the distribution of possible credit losses for a stressed scenario, we introduce an artificial adverse macroeconomic development subject to which another set of paths of future joint sector-specific default rates is simulated. Consider first the simulation of the one-period-ahead default rates of a particular path. We introduce in the vector of innovations an artificial shock over a macroeconomic variable through replacing the corresponding element in $\mathbf{r}$ by the assumed shock (normalised by the respective standard deviation), so $\mathbf{r}$ becomes pseudo random. Nevertheless, the other macroeconomic variables would be accordingly affected, since the off-diagonal elements of Σ_{ε} need not be zero. In other words, the artificially shocked macroeconomic variable

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[10]   In other words, the two disturbances in Equations (1) and (2) of the same time period are allowed to correlate.

would not be the only variable that is affected adversely in the simulation because the shock would be transmitted to other variables through the impact on its disturbance to other disturbances.

The simulation of the two-period-ahead default rates requires drawing another **r**. If the adverse development of the previously shocked macroeconomic variable is assumed to last, we continue to make the **r** pseudo random. Otherwise, we simply let the **r** to be random, as in the baseline simulations. The farther-ahead default rates can be simulated in the same manner. Based on this procedure, a path of future joint sector-specific default rates can be constructed, given the duration of the artificial shock. With a sufficiently large number of simulated paths, the distribution of credit losses for a stressed scenario can be estimated. Note that our equation system stated in Section 3.1 characterises both the dynamics of sector-specific default rates and the macroeconomic variables. In both baseline and stressed scenarios, a simulated future path of joint sector-specific default rates is partly governed by the simulated future paths of the macroeconomic variables. The reasonableness of the simulated mixes of macroeconomic variables is supported by the estimated relationships based on historical data.

Intuitively, the baseline simulations produce an estimated unconditional probability distribution of possible credit losses, without the information about the occurrence of a particular shock. In some simulations, a serious credit loss occurs because there can be adverse macroeconomic developments in the baseline simulations due to randomness. On the other hand, in stressed simulations, as the different future evolutions of the macroeconomic environment and the innovations $\mathbf{v}_t$ that the simulated paths involved share the same artificial economic shocks, the estimated distribution is conditional on the occurrence of such shocks.[11] Hence, comparing the conditional loss distribution of the stressed scenario with the unconditional distribution of the baseline scenario provides information on the possible impact of adverse macroeconomic conditions triggered by the shock that we introduce.

A better understanding on the adopted stress-testing approach can be gained by comparing it to the conventional approach as described in Section 2. Consider for simplicity the aggregate case where borrowers of different sectors are not distinguished. Given a particular pre-selected macroeconomic scenario, in the conventional stress-testing approach, the impact is mapped out by substituting into equation (1) the values of the M macroeconomic variables given by the scenario. In constructing the scenario, with the aid of a macro-econometric model, a shock can be artificially introduced over a particular macroeconomic variable, which is the stress origin, and the responses of the other variables in the model can be computed assuming that all disturbances are zero -- the scenario is the combination of the obtained numerical values of the macroeconomic

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[11] As mentioned earlier, the artificial shocks can be specified to last for one period or longer.

variables. Conceptually, the values so computed are, on average, what the responses would be. However, in our approach the responses of the other macroeconomic variables to the shock of the stress origin are probabilistic because the disturbances are not assumed to be zero in our Monte Carlo exercise. The effect of this probabilistic treatment is represented in the numerous simulated paths which associate with different **r**s.[12] With this, the framework allows us to assess banks' vulnerability through the use of VaR statistics.

The above illustrates an essential feature of our stress-testing approach: the probabilistic components of the default rates and the macroeconomic variables are not ignored, but are used to produce information on responses that deviate from the average. This feature is important because in stress testing public authorities are concerned with "exceptional but plausible" shocks, which are usually accompanied by rather abnormal behaviour of the macroeconomic variables.

IV.   THE MODEL AND ESTIMATION RESULTS

The equation system on default probability and macroeconomic dynamics is estimated by using retail banks' data covering the period from 1994 Q4 to 2006 Q1. The default rates for the overall loan portfolios and mortgage exposures of banks are chosen in this study.[13,14] In particular, the default rate is specified to depend on the following macroeconomic variables:

(i)   real GDP growth of Hong Kong (g^{HK})

(ii)   real GDP growth of Mainland China (g^{CN})

(iii)   real interest rates in Hong Kong (r)[15]

(iv)   real property prices in Hong Kong (*prop*).[16]

● The default rate is measured as a ratio of the amount of loans which have been overdue for more than three months to the total amount of loans. The data series on default rate is transformed by the logit formula to produce the y_t series. Results obtained from an augmented Dickey-Fuller test suggest that y_t is an I(1) process. Thus, we opt to model its first difference Δy_t.

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[12] This means that, even in the baseline credit loss distribution, there could be certain simulated paths of default rates that accompany extreme movements in the macroeconomic variables.

[13] The framework can also be applied for stress testing loans to other economic sectors.

[14] The time series of classified loans of retail banks can be an alternative measure of default rates. However, such data only became available from 1997 Q1, which is too short for the estimation.

[15] Real interest rates are calculated as $[(1+r^n_t)/(1+\pi_{t+1})]-1$, where r^n_t and π_{t+1} are the nominal interest rate in period t and the inflation rate in period $t+1$ respectively. We use the seasonally adjusted CPI to calculate the inflation rate.

[16] The real rate of property prices is calculated as $[(1+prop^n_t)/(1+\pi_t)]-1$, where $prop^n_t$ is the change of nominal property prices in period t.

- GDP governs the ability of agents in the economy to service their debt. For loans used to finance economic activities in the domestic market, the GDP of Hong Kong should be an important factor influencing the ability to repay.

- We also incorporate the GDP of Mainland China because the Hong Kong and Chinese economies are closely integrated.

- The reason for incorporating interest rates as an explanatory variable is obvious: they directly affect the burden of the debt. We use the three-month HIBOR to represent nominal interest rates.

- We consider property prices relevant because real estate is the major item of collateral. If the collateral value declines, the incentive to continue servicing the debt will weaken. The property price index compiled by the Rating and Valuation Department is used to calculate the variations in property prices in Hong Kong.

The equation system, which consists of equations (1) to (3), is estimated by the seemingly unrelated regression (SUR) method. The four macroeconomic series stated above are I(0), as suggested by the results of an augmented Dickey-Fuller test, so we do not use their first differences in the regression. The SUR estimation results are presented in Table 1. For the Δy_t equation, the results shown in the table are obtained by removing the insignificant variables from the more general specification in which g_t^{HK}, g_{t-1}^{HK}, g_t^{CN}, g_{t-1}^{CN}, Δr_t, Δr_{t-1}, $\Delta prop_t$, $\Delta prop_{t-1}$, Δy_{t-1} and Δy_{t-2} are incorporated as explanatory variables. Similarly, the results from the equations of the macroeconomic variables are also obtained by removing the insignificant variables from a more general specification.

As shown in Table 1, the signs of the coefficients of the macroeconomic variables in the Δy_t equation are all as expected. The results suggest that the default rate would become higher if real GDP growth in Hong Kong and the Mainland deteriorated, property prices in Hong Kong declined, and interest rates rose, and vice versa. The coefficient of the lagged default rate Δy_{t-2} is positive and significant, so there is positive autocorrelation in default rates, suggesting that a macroeconomic shock can produce a prolonged impact on the default rate. This leads us to analyse the development of the default rate over a time horizon that is longer than the duration of the artificial shock in order to reflect the long-term impact of the stress.

Table 1: SUR estimates for the equation system (sample period: 1994 Q4 to 2006 Q1)

	Dependent Variable				
Variable	Δy_t	g_t^{HK}	g_t^{CN}	Δr_t	$\Delta prop_t$
Intercept	-0.087**	0.510**	1.858***	-0.051	0.180
	(0.037)	(0.220)	(0.267)	(0.080)	(0.731)
g_{t-1}^{HK}	0.034***	0.475***			
	(0.008)	(0.117)			
g_t^{CN}	0.032**				
	(0.016)				
g_{t-1}^{CN}			0.198*		
			(0.108)		
Δr_t	-0.024**				
	(0.011)				
Δr_{t-1}				-0.173**	
				(0.071)	
$\Delta prop_{t-1}$	0.005**				0.629***
	(0.002)				(0.104)
Δy_{t-2}	0.512***				
	(0.089)				
Adj. R^2	0.631	0.191	0.113	0.682	0.336
DW statistic	1.756	1.94	2.129	1.689	1.978
No. of obs.	43	64	64	56	51

Notes:
1. In the estimation, dummy variables are added respectively in the g^{HK}, g^{CN} and r equations to control for the effects of structural breaks.
2. Standard errors are in parentheses.
3. *, ** and *** indicate significance at the 10%, 5% and 1% levels respectively.

Data Sources: CEIC, Census & Statistics Department of Hong Kong, HKMA.

V. THE SIMULATION OF FUTURE CREDIT LOSSES AND STRESS-TESTING

We now proceed to simulate paths of future default rates based on the SUR estimates and to construct the accompanying distributions of credit losses.[17] The time horizon of a path is one year. As most of the shocks last four quarters, taking the macroeconomic conditions in 2006 Q1 as the current environment, a simulated future path has the eight time points covering a two-year period from 2006 Q2 to 2008 Q1.

As mentioned earlier, in constructing the loss distribution for the baseline scenario, no artificial adverse shock is introduced. For the four stressed scenarios, different shocks arising from four different stress origins are considered:

(i)   reductions in Hong Kong's real GDP by 1.7%, 3.9%, 0.8% and 1.1% respectively in each of the four consecutive quarters starting from 2006 Q2;

(ii)   a fall in Mainland China's real GDP by 3% in only the first quarter (i.e. 2006 Q2);

(iii)   a rise of real interest rates by 300 basis points in the first quarter, followed by no change in the second and third quarters and another rise of 300 basis points in the fourth quarter; and

(iv)   reductions in real property prices by 4.4%, 14.5%, 10.8% and 16.9% respectively in each of the four consecutive quarters starting from 2006 Q2.

These are quarter-to-quarter changes and are supposed to change separately from 2006 Q2 to 2007 Q1. Their magnitudes are in general similar to those during the Asian financial crisis.[18] No further artificial shock is introduced for the subsequent quarters. For each of the baseline scenario and stressed scenarios, we simulate 10,000 future paths and use the simulated 10,000 default rates in 2008 Q1 to construct a frequency distribution of credit loss percentages.[19]

If no formal statistics are available for the loss given default (*LGD)*, some studies assign a rough constant ratio based on market information to obtain the estimated credit loss. If no market information is available, a ratio of 0.5 may be assumed for the calculation of loss figures. In this paper, we assume the *LGD* will vary with property prices as properties are by far the most important collateral for lending. Property prices should therefore have an impact on how much banks can recover from their losses. For

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[17]   A random vector of multivariate normal distribution can be obtained by first computing the Cholesky decomposition $\mathbf{C}$ of the variance-covariance matrix $\mathbf{\Sigma}$, where $\mathbf{C}$ is defined by $\mathbf{\Sigma} = \mathbf{CC'}$. Pre-multiplying a random vector $\mathbf{z}$ whose entries are independently drawn from the standardised normal distribution $N(0,1)$ by $\mathbf{C'}$ gives $\mathbf{r}$.

[18]   Note that during the Asian financial crisis (from 1997 Q4 to 1998 Q3), real interest rates rose by 306 bps in the first quarter (i.e. 1997 Q4), but dropped by 90 bps and 86 bps in the second and third quarters respectively before rising again by 314 bps in the final quarter. Also, China's GDP in all quarters recorded positive growth. Our assumed shocks are therefore more severe than the actual situation.

[19]   The percentage of credit loss is simply the product of the default rate and the LGD.

simplicity, we assume the *LGD* in 2006 Q1 to be 0.5 and the *LGD* in 2008 Q1 to be inversely proportional to the percentage change in the property price index (*PI*) around the initial level 0.5, as follows[20]:

$$LGD_{2008Q1} = 0.5 - 0.5 \times \frac{PI_{2008Q1} - PI_{2006Q1}}{PI_{2006Q1}}.$$

The simulated frequency distributions of the baseline and stressed scenarios are depicted in Chart 1. Introducing a shock shifts the loss distribution to the right, representing an increase in the frequency of the higher credit loss percentages at the expense of the lower ones.

Chart 1a: A GDP shock: simulated frequency distributions of credit loss under baseline and stressed scenarios
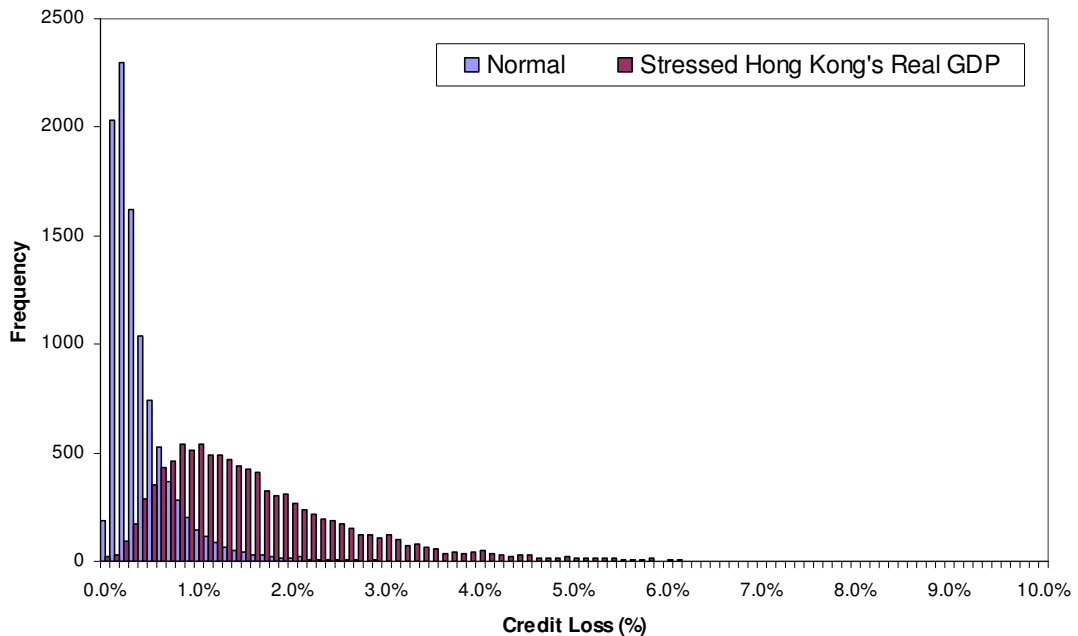
Note: Each distribution is constructed with 10,000 simulated future paths of default rates.

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[20] This is indeed a very crude assumption.

Chart 1b: A China-GDP shock: simulated frequency distributions of credit loss under baseline and stressed scenarios
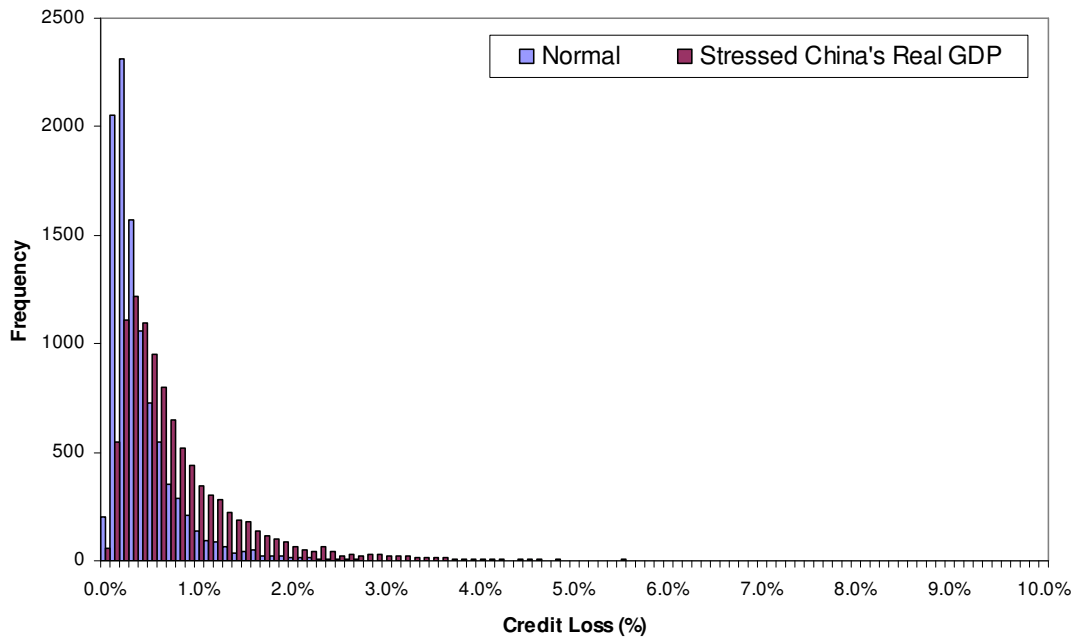
Note: Each distribution is constructed with 10,000 simulated future paths of default rates.

Chart 1c: An interest-rate shock: simulated frequency distributions of credit loss under baseline and stressed scenarios
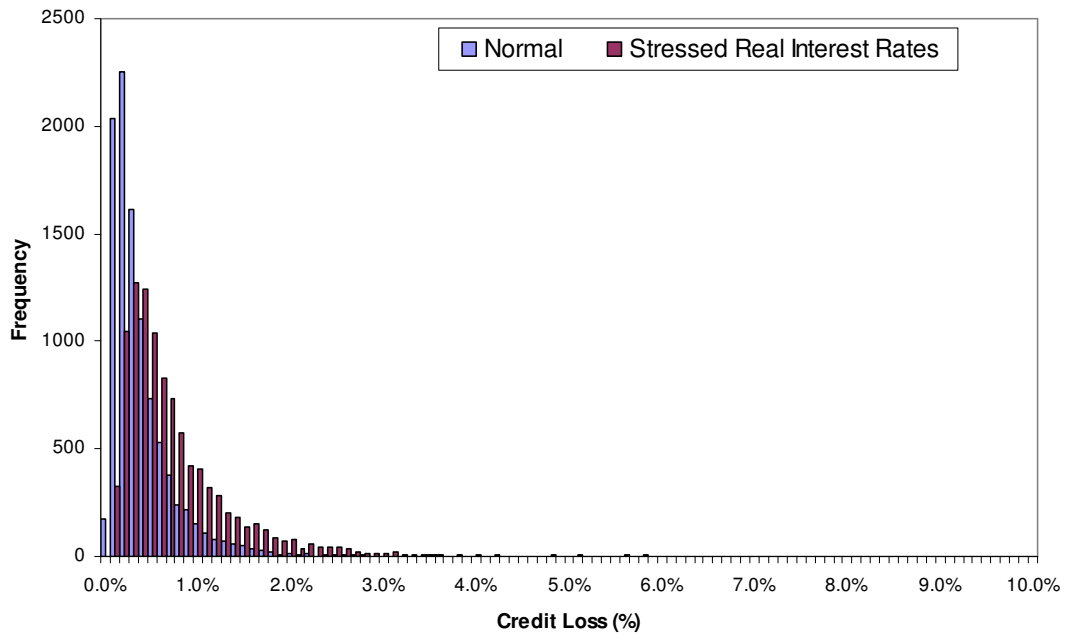
Note: Each distribution is constructed with 10,000 simulated future paths of default rates.

Chart 1d: A property-price shock: simulated frequency distributions of credit loss under baseline and stressed scenarios
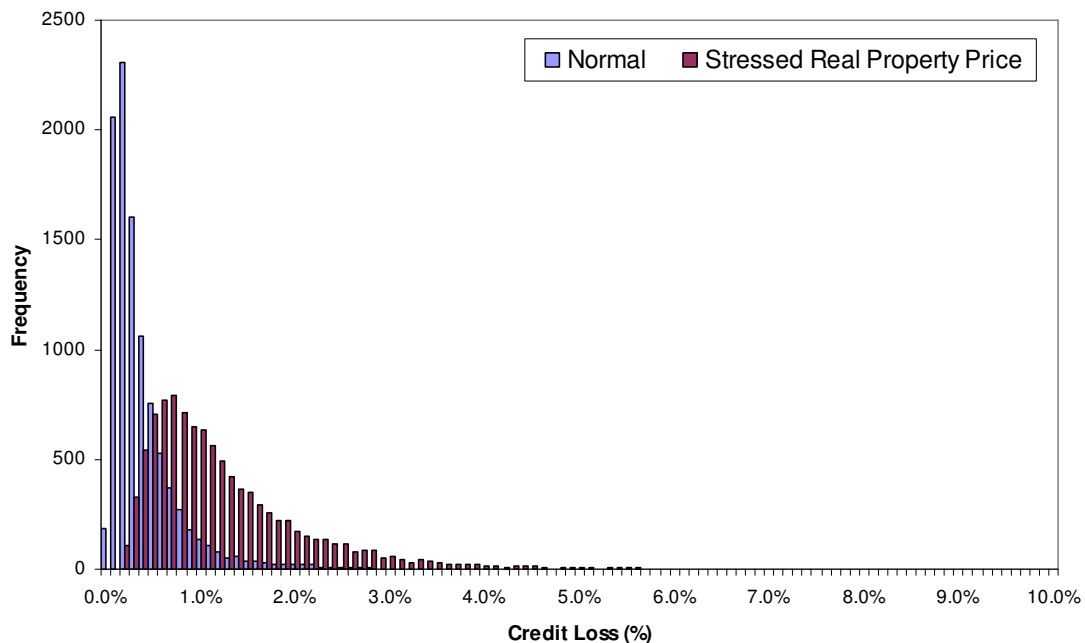
Note: Each distribution is constructed with 10,000 simulated future paths of default rates.

Salient statistics are presented in Table 2 to provide highlights of the distributions of credit losses for the baseline scenario and for the four stressed scenarios with different macroeconomic variables as the stress origin. In the baseline scenario, the percentage of credit loss that is expected to prevail in 2008 Q1 (or the mean of the credit loss distribution) is 0.34%. Introducing the artificial shocks substantially increases the expected percentage of credit loss. For example, it becomes 1.59% in the stressed scenario where Hong Kong's real GDP growth rate is shocked from 2006 Q2 to 2007 Q1.

However, our focus is on the more-than-average adverse responses of the other macroeconomic variables and the default behaviour. In particular, we are more interested in the tails of the credit loss distributions. Table 2 shows that even for the VaR at the confidence level of 90%, banks would continue to make a profit in most of the stressed scenarios, suggesting that the current credit risk of the banking sector is moderate. However, under the extreme case for the VaR at the confidence level of 99%, banks' credit loss with shocks from different origins would range from a maximum of 3.22% to a maximum of 5.56% of the portfolios, and if a confidence level of 99.9% is taken, it could range from a maximum of 6.08% to a maximum of 8.95%. The estimated maximum losses are very similar to what the market experienced one year after the Asian financial shock.[21] However, the probability of such losses and beyond is very low.

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[21] In the event, the credit loss of banks is estimated to have risen from 1.4% before the Asian financial crisis to 6.0% one year after the shock. These rough estimates are based on an assumed LGD of 70%, and the actual default rates of overall loans at 2.01% in 1997 Q3 and 8.58% in 1998 Q4.

We can also map out the impact of credit losses on banks' profitability. For a given bank or the entire banking sector, the amount of credit losses is simply the product of the percentage of credit loss and the amount of loans and advances. Suppose the future level of operating profit before provisions was the same as the current level, if no default were to take place. After realising defaults, the level of operating profit before provisions falls by the amount of credit losses. Table 3 shows post-default levels of operating profit before provisions of a hypothetical bank corresponding to the credit loss percentages given in Table 2. The operating profit before provisions and the amount of loans and advances of the hypothetical bank are assumed to be HK$3 billion and HK$130 billion respectively. We can see that for the more extreme situations, the bank may incur a loss as a result of the materialisation of credit risk alone. Under the VaR at the 90% confidence level with the GDP shock, banks could incur a loss of HK$882 million. The bank may also suffer a loss under shocks from other origins under the VaR at the 99% confidence level. However, the occurrence of such extreme scenarios resulting in the estimated maximum loss and beyond would have a very small probability of only 1%.

Table 2: The mean and VaR statistics of simulated credit loss distributions

Credit loss (%)	Baseline scenario	Stressed scenarios			
		GDP shock[a]	Property price shock[b]	Interest rate shock[c]	Mainland China GDP shock[d]
Mean	0.34	1.59	1.21	0.71	0.73
VaR at 90% CL[e]	0.76	2.99	2.30	1.48	1.56
VaR at 95% CL	1.05	3.77	2.88	1.94	2.12
VaR at 99% CL	1.91	5.56	4.54	3.22	3.73
VaR at 99.9% CL	3.13	8.95	8.29	6.08	6.66
VaR at 99.99% CL	4.38	12.56	10.92	9.15	9.38

Notes: a) Reductions in Hong Kong's real GDP by 1.7%, 3.9%, 0.8% and 1.1% respectively in each of the four consecutive quarters starting from 2006 Q2.
b) Reductions in real property prices by 4.4%, 14.5%, 10.8% and 16.9% respectively in each of the four consecutive quarters starting from 2006 Q2.
c) A rise of real interest rates by 300bps in the first quarter, followed by no change in the second and third quarters and another rise of 300 bps in the fourth quarter.
d) A fall in Mainland China's real GDP by 3.0% in only the first quarter (i.e. 2006 Q2).
e) CL denotes the confidence level

Table 3: Post-default operating profit of a hypothetical local bank [1,2] (in HK$m)

Profit (HK$m)	Baseline scenario	Stressed scenarios			
		GDP shock[a]	Property price shock[b]	Interest rate shock[c]	Mainland China GDP shock[d]
Mean	2,554	927	1,427	2,078	2,051
VaR at 90% CL[e]	2,013	-882	5	1,075	970
VaR at 95% CL	1,636	-1,900	-746	477	242
VaR at 99% CL	517	-4,226	-2,903	-1,182	-1,844
VaR at 99.9% CL	-1,066	-8,629	-7,774	-4,905	-5,661
VaR at 99.99% CL	-2,690	-13,332	-11,193	-8,900	-9,195

Notes: 1) The operating profit before provisions and the amount of loans and advances are assumed to be HK$3 billion and HK$130 billion respectively.
2) A positive figure indicates a profit while a negative figure indicates a loss.
3) For (a) to (e), see Table 2.

VI.   A STRESS TEST FOR BANKS' MORTGAGE PORTFOLIO

The same framework can be applied for stress testing loans to different economic sectors.   In this section, we apply the framework to analyse the default behaviour of residential mortgage loans (RMLs).   This is of particular interest because banks in Hong Kong generally have a substantial exposure to this type of loan.   For this exercise, the first difference of the logit-transformed default rate for RMLs Δy_t^{RML} is modelled as dependent on five macroeconomic variables: real GDP growth of Hong Kong (g^{HK}), the best lending rate in real terms (*BLR*), real property prices in Hong Kong (*prop*), real GDP growth of Mainland China and Hong Kong's unemployment rate.

Table 4 presents the SUR estimates for the equation system for RMLs. Similar to the treatment in Table 1, results in Table 4 are derived by removing the insignificant variables (including Mainland China's real GDP growth and Hong Kong's unemployment rate) from a more general specification.   As expected, the performance of the RMLs depends negatively on the *BLR* and positively on Hong Kong's real GDP growth rate and changes in real property prices.[22]   Similar to the model for overall loans, the coefficient of the lagged dependent variable in the Δy_t^{RML} equation is positive and significant, so the impact of an economic shock on the credit risk associated with RMLs is likely to be prolonged.

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[22] The estimated coefficient for real GDP growth of Mainland China is insignificantly different from zero. This may reflect that, unlike the part of business credit of overall loan exposures, mortgage loans are more affected by domestic factors and are less directly affected by the China factor.

Table 4: SUR estimates for the equation system for RMLs
(sample period: 1998 Q2 to 2006 Q1)

Variable	Dependent variable			
	Δy_t^{RML}	g_t^{HK}	ΔBLR_t	$\Delta prop_t$
Intercept	0.014	0.530**	-0.025	0.127
	(0.014)	(0.228)	(0.065)	(0.728)
g_t^{HK}	0.011*			
	(0.006)			
g_{t-1}^{HK}		0.452***		
		(0.124)		
ΔBLR_{t-1}	-0.029**		0.189*	
	(0.014)		(0.104)	
$\Delta prop_{t-1}$	0.008***			0.541***
	(0.008)			(0.106)
Δy_{t-1}^{RML}	0.562***			
	(0.072)			
Adj. R^2	0.842	0.190	0.363	0.336
DW statistic	2.191	1.892	2.051	1.819
No. of obs.	30	64	56	51

Notes:
1. In the estimation, dummy variables are added respectively in the g^{HK} and r equations to control for the effects of structural breaks.
2. Standard errors are in parentheses.
3. *, ** and *** indicate significance at the 10%, 5% and 1% levels respectively.

Data Sources: CEIC, Census & Statistics Department of Hong Kong, HKMA.

The credit loss is simulated over a one-year horizon after the three different shocks, originating separately from (1) real Hong Kong GDP, (2) real property prices, and (3) real interest rates, the magnitudes of which are similar to those during the Asian financial crisis. As in Chart 1, Chart 2 shows that the distribution of losses of the stressed scenarios shifts towards the right compared with the baseline scenarios, suggesting that the shocks have resulted in increases in the expected percentage of credit losses.

Chart 2a: A GDP shock: simulated frequency distributions of credit loss for RMLs under baseline and stressed scenarios
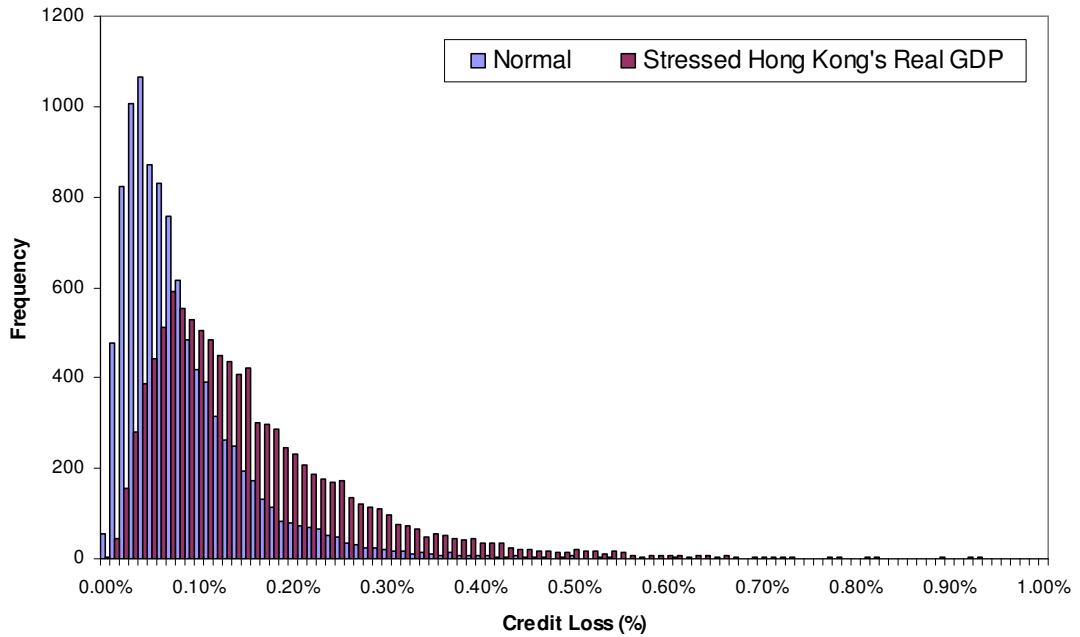
Note: Each distribution is constructed with 10,000 simulated future paths of default rates.

Chart 2b: A property-price shock: simulated frequency distributions of credit loss for RMLs under baseline and stressed scenarios
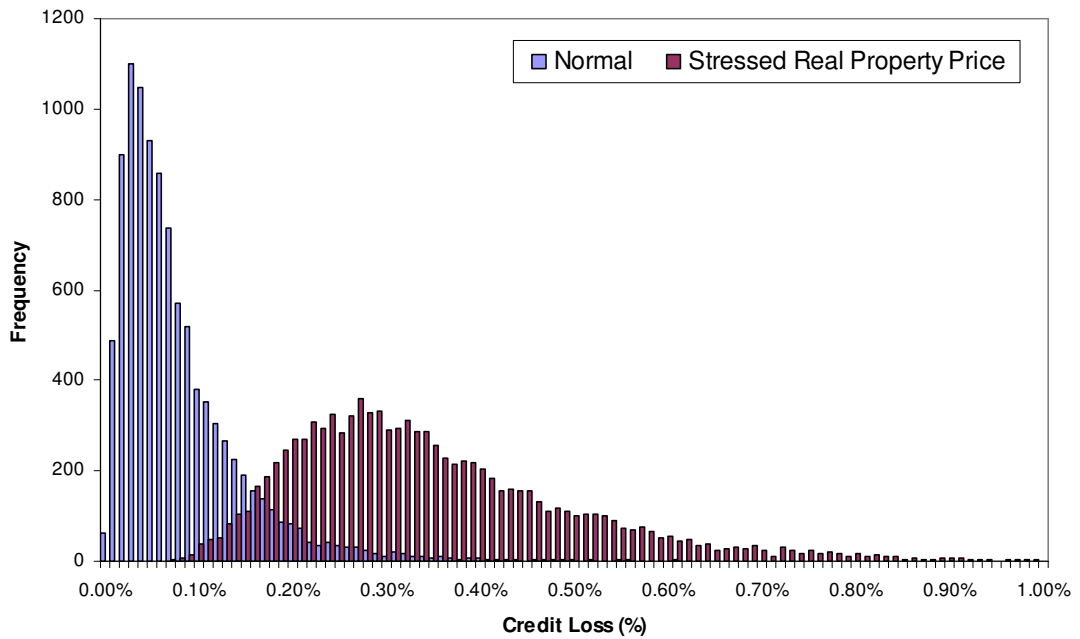
Note: Each distribution is constructed with 10,000 simulated future paths of default rates.

Chart 2c: An interest-rate shock: simulated frequency distributions of credit loss for RMLs under baseline and stressed scenarios
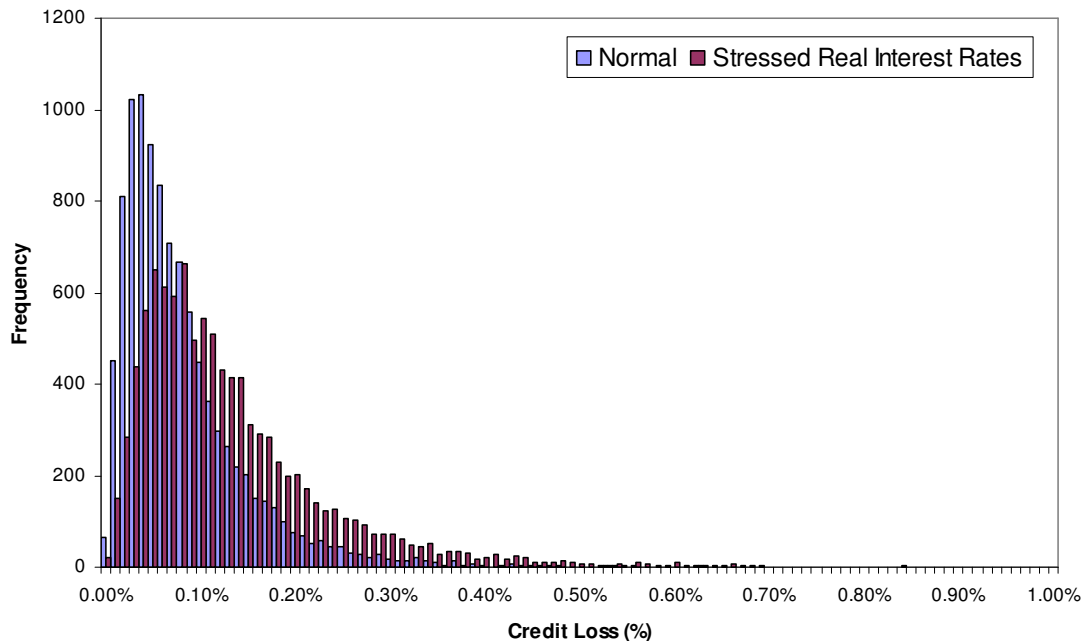
Note: Each distribution is constructed with 10,000 simulated future paths of default rates.

The simulation results also show that the impact on banks' profit would be moderate. For all the three shocks of different origins, even with a high confidence level of VaR measure, banks would continue to make a profit. As shown in Table 5, the expected credit losses (under the mean credit losses) for the given severe shock are moderate, ranging from 0.08% to 0.34% of the bank's total RMLs. Such credit loss may rise to a maximum of 1.12% at the 99.9% confidence level, which suggests that there is a probability of 0.1% for banks to suffer from a credit loss of 1.12% or more. Assuming that the hypothetical bank's outstanding loans for RMLs in 2006 Q1 is HK$ 39 billion, the cut in profit is found to be at most HK$436.8 million at the 99.9% confidence level, which amounts to 14.6% of total operating profit before provisions (see Table 6).[23] However, the occurrence of such adverse market conditions has a very low probability.

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[23] It is assumed that the share of mortgage loans to the bank's total loans for use in Hong Kong is 30%, which is about the industry average. Note that this loss figure arises from only the bank's RML portfolio.

Table 5: The mean and VaR statistics of the simulated credit loss distributions for RMLs

Credit loss (%)	Baseline scenario	Stressed scenarios		
		GDP shock[a]	Property price shock[b]	Interest rate shock[c]
Mean	0.08	0.15	0.34	0.13
VaR at 90% CL[d]	0.16	0.29	0.55	0.25
VaR at 95% CL	0.21	0.38	0.64	0.32
VaR at 99% CL	0.35	0.58	0.83	0.50
VaR at 99.9% CL	0.53	0.91	1.12	0.84
VaR at 99.99% CL	0.69	1.07	1.41	1.33

Notes: a) Reductions in Hong Kong's real GDP by 1.7%, 3.9%, 0.8% and 1.1% respectively in each of the four consecutive quarters starting from 2006 Q2.
b) Reductions in real property prices by 4.4%, 14.5%, 10.8% and 16.9% respectively in each of the four consecutive quarters starting from 2006 Q2.
c) A rise of real interest rates by 300bps in the first quarter, followed by no change in the second and third quarters and another rise of 300 bps in the fourth quarter.
d) CL denotes the confidence level

Table 6: Post-default operating profit of a hypothetical local bank for RML [1,2] (in HK$m)

Profit (HK$m)	Baseline scenario	Stressed scenarios		
		GDP shock[a]	Property price shock[b]	Interest rate shock[c]
Mean	2,970	2,941	2,866	2,951
VaR at 90% CL[d]	2,937	2,885	2,787	2,901
VaR at 95% CL	2,916	2,853	2,751	2,875
VaR at 99% CL	2,865	2,774	2,677	2,803
VaR at 99.9% CL	2,793	2,644	2,564	2,674
VaR at 99.99% CL	2,731	2,582	2,449	2,481

Notes: 1) The operating profit before provisions and the amount of loans for RML are assumed to be HK$3 billion and HK$39 billion respectively.
2) A positive figure indicates a profit while a negative figure indicates a loss.
3) For (a) to (d), see Table 5.

VII.   CONCLUSIONS

This paper studied a macro stress testing framework for loan portfolios of banks in Hong Kong. Two macroeconomic credit risk models, each comprising a multiple regression model explaining the default probability and a set of autoregressive models describing the macroeconomic environment, were constructed for the overall loan portfolios and mortgage exposures of banks respectively. The analysis suggests a significant relationship between the default rates of bank loans and key macroeconomic factors, including Hong Kong's real GDP, real interest rates, real property prices and Mainland China's real GDP.

Macro stress testing is then performed to assess the vulnerability and risk exposures of banks' overall loan portfolios and mortgage exposures. By using the framework, a Monte Carlo method is applied to estimate the distribution of possible credit losses conditional on an artificially introduced shock. Different shocks, the magnitude of which are specified according to those occurring during the Asian financial crisis, are individually introduced into the framework for the stress tests. The results show that even for the VaR at the confidence level of 90%, banks would continue to make a profit in most of the stressed scenarios, suggesting that the current credit risk of the banking sector is moderate. Under extreme cases for the VaR at the confidence level of 99%, banks could incur material losses. However, the probability of the occurrence of such events is extremely low.

Using a hypothetical bank as an example, this paper illustrates how estimates obtained from aggregate default-rate data can be applied to stress test individual banks. The framework can also be applied in a more comprehensive manner to assess the vulnerability of individual (or groups of) banks by using bank level (or group level) data to obtain bank-specific (or group-specific) estimates for the macro credit risk model and VaR statistics.

References

Berkowitz, J. (1999). "A Coherent Framework for Stress-testing," *Finance and Economics Discussion Series,* Federal Reserve Board.

Blaschke, W, M T Jones, G Majnoni and S M Peria (2001). "Stress Testing of Financial Systems: An Overview of Issues, Methodologies, and FSAP Experiences", International Monetary Fund.

Boss, M. (2002). "A Macroeconomic Credit Risk Model for Stress Testing the Austrian Credit Portfolio," *Financial Stability Report* 4, Oesterreichische Nationalbank.

Bunn, P. Cunningham, A. and Drehmann, M. (2005). "Stress Testing As a Tool for Assessing Systemic Risks," *Financial Stability Review*, June 2005, Bank of England.

Froyland E. and Larsen K. (2002). "How Vulnerable are Financial Institutions to Macroeconomic Changes? An Analysis Based on Stress Testing," *Economic Bulletin*. October 2002, Norges Bank.

Hoggarth, G. and Whitley, J. (2003). "Assessing the Strength of UK Banks through Macroeconomic Stress Tests," *Financial Stability Review*, June 2003, Bank of England.

Hoggarth, G., Sorenen, S. and Zicchino, L. (2005). "Stress Tests of UK Banks Using a VAR Approach," Bank of England Working Paper, no. 282.

Mawdsley, A., McGuire, M. and O'Donnell, N. (2004). "The Stress Testing of Irish Credit Institutions," *Financial Stability Report*, 2004, Central Bank and Financial Services Authority of Ireland.

Pain, D. (2003). "The Provisioning Behaviour Experience of the Major UK Banks: A Small Panel Investigation," Bank of England Working Paper, no. 117.

Sorge, M. (2004). "Stress-testing Financial Systems: An Overview of Current Methodologies," BIS Working Papers, no. 165.

Virolainen, K. (2004). "Macro Stress-testing with a Macroeconomic Credit Risk Model for Finland," Bank of Finland Discussion Paper, no. 18/2004.

Wilson, T. C. (1997a). "Portfolio Credit Risk (I)," *Risk*, vol. 10, issue 9, pp. 111-17.

Wilson, T. C. (1997b). "Portfolio Credit Risk (II)," *Risk*, vol. 10, issue 10, pp. 56-61.

Stress Testing SME Portfolios Using Loan-Level Data: An Integrated Approach

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Abstract: In this paper, we develop an integrated stress testing model for SME (Small and Medium size Enterprise) portfolio that allows us to evaluate the impact of stress scenarios on expected and unexpected losses. We use loan-level data to construct default, prepayment, and loss models that are conditional on loan-specific, market and macroeconomic conditions generally used to evaluate the resilience of a small-business portfolio to various stress scenarios. These loan-specific default and prepayment rates makes it possible to obtain more accurate credit loss estimates than those obtained with more aggregated models typically employed. Using these time dependent parameter estimates, we examine the impact of a credit-risk shock and find that not only is the magnitude of the initial shock (first-order effects) important, but so is the time path of the adjustment (second-order effects) as the shock resonates through the portfolio over time.

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The views expressed here are those of the authors and do not necessarily reflect the views of the Office of the Comptroller of the Currency, the U.S. Treasury Department, or International Monetary Fund. Glennon is the corresponding author.

I. Introduction

Stress-testing is a risk-management tool that quantifies a bank's potential exposure to extreme, but plausible, shocks or stress-based changes in financial markets. Banks, supervisory agencies, and rating agencies typically stress test portfolios or lines-of-businesses that are highly sensitive to external shocks to assess their impact on the institution's balance sheet. A well-developed bottom-up stress-testing framework helps develop a set of actions or strategies that can limit a bank's exposure to extreme shocks. For example, stress testing may place limits on the amount of losses, the level of provisions, the percentage of criticized loans or the re-allocation of resources across lines-of-business to better reflect the bank's exposure to risk of loss. These figures are periodically evaluated to make certain the bank's exposure does not exceed the limit.

Until recently, stress testing models have primarily examined a bank's exposure to market riskin the trading book. Thus, the stress-testing techniques developed for this purpose reflects the required mark-to-market valuation of assets over a relatively short (instantaneous) performance horizon that characterizes the bank's exposure to market risk. The development of a stress-testing framework for credit-risk exposures is inadequate, especially when compared to the market risk models (CGFS, 2005; Hagan et al., 2005). As a result, the methods used to stress test a portfolio for credit risk mimic models developed for market-risk. The near instantaneous adjustment to an extreme shock implicit in stress-testing a bank's exposure to market risk, however, clearly misrepresents the extended (i.e., several quarter/years) adjustment process related to credit risk. The impact of a credit-risk shock depends not only on the magnitude of the initial shock (first-order effects), but the time path of the adjustment (second-order effects) as the shock resonates through the portfolio over time. This extended time horizon framework for stress-testing credit risk is especially important due to the interaction of behavior across competing risks (Cihak, 2004) and it differentiates our approach from others methods proposed in the literature.

In this paper, we outline a modeling framework that captures the dynamic adjustments in a bank's credit-risk exposure to time-dependent external shocks. We build our stress-testing framework around a dynamic net cash-flow design that captures the relationship between default risk, prepayment risk, loss severity, and exposure (i.e., factoring in the effects of both amortizard and prepayment) conditional on economic and credit-cycle factors. This approach has several advantages over the static modeling design of a typical P&L-based approach used in practice, especially with respect to a scenario-based stress-testing analysis. Under a dynamic model design, a base-case/benchmark scenario is generated using average values for the risk drivers derived from a development sample; the impact of a historical or plausible extreme shock that plays out over several quarters or years can be tracked over an extended time horizon. More importantly, the model could eventually form the basis of a Monte Carlo study of the bank's exposure to losses from the evolution of extreme events over time.[1]

Our objective is to integrate the effect of both credit risk and interest rate risk (i.e., market risk) on the value of the banking book. Jarrow and Turnbull (2000) argue that market and credit risk are intrinsically related but not separable and use a reduced form modeling approach to integrate these risks. Drehmann, Sorensen and Stringa (2006) also measure the impact of correlated interest rate and credit risk jointly on the whole portfolio of banks to assess their impact on the bank's economic value. Neither of these papers, however, model the time path of credit risk and interest rate risk as we propose in this paper.

The remainder of the paper is organized as follows. In Section II, we present a conceptual overview of our model design. We apply our model to a portfolio of SME loans underwritten through the government-sponsored Small Business Administration (SBA). In Section III, we summarize our data and formalize our method of estimating the prepayment and default hazard models that form the core of our stress-testing model. We report our results in Section IV and illustrate the impact of a relatively large

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[1] We recognize that a potential disadvantage to our approach is the introduction of prediction errors due to mis-specifying the dynamic models. However, this criticism is applicable to all model-based methods and can be addressed as part of the model development process.

shock on our portfolio in Section V. Finally, Section VI provides conclusions and discusses areas for further research.

II. Background and relevant literature

The stress-testing of a bank's loan portfolio requires the ability to adequately track the impact of a shock on a borrowers' payment behavior both over time and across multiple risks (e.g., defaults and prepayments). In practice, however, conventional methods developed to assess the impact of *credit-risk* related stress scenarios on performance do not adequately reflect this underlying dynamic process. Credit-risk models are often constructed using a static model design that implicitly assume that default behavior is a time-insensitive process. As a result, these models lack a cohesive framework for linking the interactions between changes in market conditions, payment and loss behavior, and the magnitude of the stress. In the absence of a coherent modeling framework that captures the dynamic process implicit in the concept of a stress-based scenario analysis, the results and the strategies derived are, at best, without merit or, at worse, misleading.

To capture the dynamic process over an extended time horizon, we construct our loan-level stress model around a simple life-of-loan net cash flow process. The net cash flow design captures the time-path of revenues net of costs (including defaults) conditional on the macroeconomic and industry (i.e., systemic) factors over the expected life of the loan. More specifically, we define the bank's net revenues ($\Re_t$) as the net cash flow, $CF_{i,t}$, in time t = 1, 2, …, T, summed over all individual loans i = 1, 2, …, N:

$$\Re_t = \sum_{i}^{N} [E[R_{i,t} \mid t < T, r, X_t] - E[L_{i,t} \mid t < T, r, Y_t] - K_{i,t}] \tag{1}$$

where

$E[R_{i,t} \mid t < T, r, X]$ the expected (gross) revenue R from loan i, in time t, given that the loan survives to time t-1, conditional on the contract interest rate, r, and macroeconomic and industry-specific factors, X;

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$E[L_{,i,t} \mid t < T, \mathbf{Y}]$ the expected loss due to default L from loan i, in time t, given that the loan

survives to time t-1, conditional on macroeconomic and industry-specific

factors, $\mathbf{Y}$; and

$K_{i,t}$ (fixed) costs associated with underwriting and managing the loan portfolio

net of fee income attributed to each loan i, in time t.

To allow more flexibility in modeling the effects of (stress-based) changes in macroeconomic and

industry conditions on net revenues, we model the expected (gross) revenue independently of expected

loss and costs.[2] Each component of the net revenue equation including revenue, (dollar) loss, and costs is

modeled separately as a time-dependent process. Although net revenue could be estimated directly as a

function of systemic risk factors, we argue that the loan-level model design better captures the time-

dependent process underlying the development of a coherent stress-testing framework.[3]

Gross revenue from the ith loan, in time t, is defined as the product of the contract rate times the

remaining outstanding balance as of time t.[4] For a portfolio of fixed maturity, amortizing loans, the

expected conditional gross revenue for each loan depends on the contract rate, r_t, the amortization rule,

$a(bal_{t=0})$, and economic and loan-specific factors (both time-varying, $\mathbf{X_t}$, and time-invariant, $\mathbf{W_i}$) that

affect the survival (i.e., prepayment) of a loan and the exposure-at-risk at each point in time over the life

of the loan. As a result, the expected gross revenue is defined as:

$$E_i[R_{i,t} \mid r_t, X_t, W_i] = \sum_{t}^{T} r_t \, f(a(bal_{t=0}), X_t, W_i) \tag{2}$$

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[2] We note that $\mathbf{X}, \mathbf{Y} \in \Phi$, where Φ is a complete set of macro economic and industry variable such as: employment, output/production, and interest rates.

[3] Under the restricted conditions that (i) $\mathbf{X}=\mathbf{Y}=\mathbf{Q}$ and (ii) the weights assigned to each set of conditions variables are the same, then $E[R_{t,i} \mid t < T, r, \mathbf{X}] - E[L_{t,i} \mid t < T, \mathbf{Y}] = E[R_{t,i} - L_{t,i} \mid t < T, \mathbf{Q}]$. In this case, net revenue $(R_{t,i}-L_{t,i})$ could be estimate directly using a single equation model – a special case of the modeling approach outlined below.

[4] Fee income is assumed to be independent of the outstanding balances.

We assume that the amortization rule, $a(bal_{t=0})$, is a function of the original loan amount (i.e., $bal_{t=0}$). For illustrative purposes, we assume a straight-line amortization rule such that for any given loan to borrower i ($i=1,...,N$) at any given time $t=\tau$ (for $t=1,...,T$), the remaining outstanding balances ($bal_{i,t=\tau}$) is:

$$bal_{i,\,t=\tau} = f_\tau(bal_{i,t=0})$$
$$= bal_{i,t=0} - \sum_{t=1}^{\tau-1}\left[\frac{(bal_{i,t=0})}{T}\right]_t \tag{3}$$

where τ represents a specific point in time since origination.[5] We define the contract interest rate as:

$$r_t = \begin{cases} r & \textit{fixed rate} \\ \rho_t + \textit{sprd} & \textit{variable rate} \end{cases}$$

where ρ_t represents the time-varying index rate and *sprd* the contractual spread over the index set at time of origination. Under this rule, the interest rate for variable-rate loans adjusts to maintain a constant spread over an index rate.

The outstanding balances at time t, however, are also conditional on the likelihood the loan survives through $t-1$ periods. It is not unusual for a large percentage of amortizing loans to prepay-in-full before their contractual maturity date, which can materially affect the amount of interest income collected over the life of the loan. For that reason, we adjust the aggregate dollar exposure in each time period $t=\tau$ (i.e., $f(bal_{i,t=\tau})$) to reflect the likelihood that the loan prepays. More specifically, we weight the remaining balances in time t by the likelihood that the loans survived through time $t-1$ – a survival probability that reflects the accumulative likelihood a loan does not prepay, π, through the initial $t-1$ periods. We define the survival probability as a conditional probability derived from the likelihood a borrower prepays, $pr(\pi_t)$, in period t given borrower, lender, and loan characteristics, $\mathbf{W}_i$, and time-dependent macro-

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[5] Alternative amortization rule, such as the following non-linear rule:

$$bal_{i,\,t=\tau} = f_\tau(bal_{i,t=0})$$
$$= bal_{i,t=0}\left[\frac{1 - e^{(-r(T-t))}}{1 - e^{-rT}}\right]$$

generates similar results as those derived from a simple straight-line amortization rule.

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/regional economic variables, $\mathbf{X}_t$. That is, we define the survival probability in terms of the prepayment function [1-pr*(prepay)*$_\tau$] in which:

$$\text{pr}(prepay)_\tau = \text{pr}(\pi_t \mid \mathbf{X}_t, \mathbf{W}_i); \tag{4}$$

is the instantaneous probability of prepayment (π) in time $t = \tau$ conditioned on $\mathbf{X}_t$ and $\mathbf{W}_i$. Using equations (3) and (4), we can more formally write the expected gross revenue equation, in time t, as:

$$E[R_{t,i}|t < T, r, X]_t = \sum_{i=1}^{N} \left(r_t \left[bal_{i,t=0} - \sum_{t=1}^{\tau-1} \left(\frac{bal_{i,t=0}}{T} \right)_t \right] \left(1 - pr(\pi_t | X_t, W_i)\right) \right) \tag{5}$$

Equation 5, however, does not yet take into default and the associated portfolio losses. As outlined above, the exposure at any time t over the life of the loan depends on the amortization rule and the likelihood of prepayment. The exposure in time t is converted to a measure of the outstanding balances "at risk" by multiplying by the probability of default, d, in time t. The probability of default is conditional on loan-, borrower-, lender-specific characteristics denotedn by $\mathbf{W}_i$, as well as time-varying, macro/regional economic and industry conditions, denoted by $\mathbf{Y}_t$. Specifying the default probability, as a time-dependent process, captures the effect of loan seasoning on expected performance. Thus, the probability of default can be written as the conditional probability:

$$\text{pr}(default)_t = \text{pr}(d_t \mid \mathbf{W}_i, \mathbf{Y}_t). \tag{6}$$

and the "at risk" balances as:

$$l_{t,i} = \sum_{t}^{T} f(bal_t) pr(d_t \mid W_i, Y_t). \tag{7}$$

Expected loss, however, is derived from the amount of exposure "at risk" after adjusting for expected recoveries. For secured loans, the value of the collateral protects the lender's interest and provides for at least a partial recovery of the balance outstanding at time of default. The recovery rate, η, is presumed to be time-dependent (e.g., the value of the collateral as a percentage of outstanding balances

is likely to increase as the loan seasons) and conditional on borrower and loan characteristics.[6] We define the likelihood of recovery in time t as:

$$\text{pr}(recovery)_\tau = \text{pr}(\eta_t \mid \mathbf{W_i}, \mathbf{Z_t}). \tag{8}$$

where $\mathbf{Z}_t$ represents time-dependent conditioning variables. Loss severity is the estimated non-recoverable portion of the gross exposure equal to $[1 - \text{pr}(\eta_t \mid \mathbf{W_i}, \mathbf{Z_t})]$.

Substituting equation (5) into equation (7) and adjusting for prepayment (equation (4)) and loss severity (equation (8)), we can re-write the expected loss equation, in time t, as:

$$E[L_{t,i} \mid t < T, Y]_t = \sum_{i=1}^{N} \left([1 - pr(\eta \mid W_i, Z_t)] \left\{ \left[bal_{i,t=0} - \sum_{t=1}^{\tau-1} \left(\frac{bal_{i,t=0}}{T} \right)_t \right] (1 - pr(\pi_t \mid X_t, W_i)) \right\} pr(d_t \mid W_i, Y_t) \right) \tag{9}$$

Substituting equations (5) and (9) into equation (1) and rearranging terms, the expected net cash flow for a portfolio of (at least partially) secured amortizing loans, in time t, is:

$$\Re_t = \sum_{i}^{N} (E[R_{t,i} \mid t < T, r, X_t] - E[L_{t,i} \mid t < T, r, Y_t] - K_{t,i})$$

$$\Re_t = \sum_{i}^{N} \left\{ \left(r_t \left[bal_{i,t=0} - \sum_{t=1}^{\tau-1} \left(\frac{bal_{i,t=0}}{T} \right)_t \right] (1 - pr(\pi_t \mid X_t, W_i)) \right) \right.$$
$$\left. - \left([1 - pr(\eta_t \mid Z_{i,t})] \left\{ \left[bal_{i,t=0} - \sum_{t=1}^{\tau-1} \left(\frac{bal_{i,t=0}}{T} \right)_t \right] (1 - pr(\pi_t \mid X_t, W_i)) \right\} pr(d_t \mid W_i, Y_t) \right) - K_{t,i} \right\}$$

$$\Re_t = \sum_{i}^{N} \left[\left(r_t - pr(d_t \mid W_i, Y_t)[1 - pr(\eta_t \mid Z_{i,t})] \right) \left(bal_{i,t=0} - \sum_{t=1}^{\tau-1} \left(\frac{bal_{i,t=0}}{T} \right)_t \right) (1 - pr(\pi_t \mid X_t, W_i\}) - K_{t,i} \right] \tag{10}$$

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[6] Frye (2000) argues that recovery rates may also hinge on macroeconomic conditions. Thus, a downturn in economic conditions will not only lead to higher default rates, but lower recoveries due to the fire-sale of assets. We leave this as a potential area for further exploration.

Under this design, for every loan in the portfolio we estimate the expected exposure at each point in time over its life as outlined in equation (10). The advantage of this portfolio level design is reflected in the stylized example illustrated in Figure 1. The instantaneous default probabilities are represented by three identically shaped individual humped shaped hazard curves that all reflect the impact of seasoning on the likelihood of default. The three loans, however, were originated at different points in time 1997Q1, 1998Q1, and 1999Q3. For stress testing purposes, however, we are interested in evaluating the impact of a shock that takes place over a specific observation period: 2000Q2 through 2001Q2. As a result, the exposure to loss varies significantly across the three loans. The time-path of default for the least seasoned loan (bottom of the panel) is expected to increase over the observation period, while that for the most seasoned loan (top of the panel) it is expected to fall. Similarly, the time path for prepayment will also varies by loan seasoning. This example illustrates the importance of capturing the time path of both the default and prepayment rates over the observation period, as well as the age distribution of loans in the portfolio.

Using equation (10) we estimate the impact of a shock at a specific point in time, t, over the full loan portfolio (i.e., $i = 1, 2, \ldots, N$). The individual estimates, at each point in time over the observation period, can be used to identify the maximum loss associated with a specific adverse shock – a value of interest if the objective is to assess a bank's capital adequacy. Alternatively, we can use this approach to value the portfolio by extending the model to incorporate expected principal payments – including the quarterly pay down required under the amortization rule and the expected payoff in the event of prepayment – into the quarterly cash flows, and discounting the expected cash flows back to the observation period. More specifically, the expected value of the portfolio in time t is:

$$V_t = \sum_i^N \sum_t^T \Phi_t [\ (r'_t E_t S_t) + (pr(\pi_t \mid X_t, W) E_t S_t) + ((1 - pr(\pi_t \mid X_t, W_i)) \ (E_t - E_{t-1}) \\ - ([1 - pr(\eta_t \mid Z_{i,t})] \ E_t \ pr(d_t \mid W_i, Y_t)) - K_{t,i} \}]$$

(11)

where

       r'_t contract interest rate (net of cost-of-funds);

E_t exposure at time t: $bal_{i,t=0} - \sum_{t=1}^{\tau-1}\left(\dfrac{bal_{i,t=0}}{T}\right)_t$

S_t survival probability at time t: $\left(1 - pr(\pi_t \mid X_t, W_i)\right)\left(1 - pr(d_t \mid W_i, Y_t)\right)$

Φ_t discount factor: $\dfrac{1}{(1+r_t)^t}$

Equation (11) can be used to evaluate the impact of various stress scenarios on the value of the portfolio.

Using equations (10) and (11), we identify five key time-varying components of our model. They are: (i) the contractual interest rate (r_t), (ii) the pay down rate based on the loan amortization rule ($f(bal_t)$), (iii) the probability of default and the effect of seasoning ($pr(d_t)$), (iv) the probability of prepayment ($pr(\pi_t)$), and (v) the probability of recovery ($pr(\eta_t)$). We link each of these key components of the expected net revenue equation to time-varying macroeconomic and market conditions through the conditioning factors $\mathbf{Y_t}$, $\mathbf{X_t}$, and $\mathbf{Z}_t$.

We develop a two-step process to introduce and track the expected impact of an exogenous shock on the portfolio. The first step involves modeling the time path of changes (or shocks) in economic or market variables (i.e., $\mathbf{Y_t}$, $\mathbf{X_t}$, and $\mathbf{Z}_t$). The economic variables are then used as inputs into the expected net revenue equation via their impact on default, prepayment, and recovery behavior. To illustrate the development of this process, we use a sample of small business loans underwritten through the U.S. Small Business Administrations (SBA) 7(a) loan program. Our sample is illustrative of a typical medium- to large-size commercial bank SME portfolio. It includes both borrower- and loan-specific information that we use to evaluate the feasibility of developing a loan-level (and bottom-up) method of tracking the impact of various stress scenarios on a bank's net revenues.

III. A Stylized Small Business Portfolio, Data and Estimation Approach

SBA guaranteed loans are underwritten by a large number of financial institutions including banks, thrifts, and credit unions typically using underwriting guidelines set by the SBA. The guarantee

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represents a *pro rata* sharing of loss given default – a SBA guarantee is not a first-loss position. Under the *pro rata* sharing of losses, the SBA guarantee is not much different than any other risk sharing arrangement a bank could use to manage its exposure to losses (DeYoung, Glennon, and Nigro, 2007). Because the lenders use a common set of underwriting standard imposed by the conditions of the 7(a) loan guarantee program, we can treat the sample of SBA loans *as if* the loans were underwritten by a single lending institution with a decentralized staff of underwriters.

The data set represents a 20 percent random sample of medium-maturity (i.e., 7-years) loans disbursed under the SBA 7(a) loan guarantee program from 1985 through 1998 with loan performance infmorations through 2002:3.[7] The data set includes information on: (i) loan-specific characteristics such as the guarantee percentage, loan amount, initial interest rate, interest rate type (i.e., fixed or variable), and a low-documentation indicator; (ii) lender characteristics such as SBA lender type (Preferred Lender Program (PLP), Certified Lender Program (CLP), and the Regular Lender Program) and loan originator/servicer status; and (iii) borrower characteristics such as corporate structure (i.e., corporation, partnership and sole proprietor), SIC division (industry classification), number of employees, and new/existing-firm status. Finally, to control for changing economic conditions, we augment the data set to include several region and economic variables.[8]

Panel A of Table 1 provides summary statistics on defaults and prepayments for the full SBA sample. The data set includes 19,063 individual loans in which 4,109 defaulted (21.6 percent) and 10,033 prepaid (52.6 percent) over the sample period. Only a small number (4,921) or 25.8 percent of the portfolio were "right" censored (i.e., did not default or prepay by the end of the observation period) or paid at maturity?. Panel B of Table 1 reports more detailed summary statistics for a selected set of key borrower-, loan- and lender-specific variables in Table 1. Columns B and C of Table 1 show that borrower default behavior is sensitive to such factors as loan amount, SBA guarantee percentage, loan

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[7] Because the term to maturity of SBA loans is closely identified with loan purpose (e.g., working capital, long-term capital improvement, etc.), we intentionally limited the scope of our analysis to loans with the same term to maturity. We use only seven-year maturity loans as they represent the largest portion of loans in the SBA portfolio.
[8] See Glennon and Nigro (20005a, 2005b) for a more detailed discussion of the data.

amount, new business status, corporate structure, and lending program. For example, new firms are significantly more likely to default (38.7 percent) relative to their older counterparts (33.8 percent). More generally, defaulted loans tend to be, on average, underwritten at higher interest rates for lower loan amounts, with low documentation and a higher SBA guarantee, through the regular loan program, to businesses structured as a sole proprietorship relative to non-defaulted loans – factors that tend to reflect lower credit-quality borrowers. Similarly, Columns D and E of Tables 1 shows that prepaying loans tend to be, on average, to established firms with a corporate structure, for larger amounts, through the certified lender program, low documentation and higher SBA guarantee. These results are interesting insofar as they are generally consistent with a priori expectations. These univariate results, although limited in usefulness, form the basis of our specification of the hazard models.

In Table 2 we report the survivor status by disbursement date (i.e., loan cohort year). We include the total number of loans, as well as the cumulative and average-annual default and prepayment rates in of each cohort, as well its contribution to the overall sample . To capture some of the changes to the SBA program over our sample period, we also include the average dollar loan amounts and mean SBA guarantee percentages by loan cohort. Table 2 shows that our sample is heavily weighted toward loans disbursed in the 1990's, reflecting the recent growth in the SBA 7(a) program. There was an upward trend in the average nominal loan amount until 1994 at which time the SBA introduced their Low-Doc program in which the maximum loan amount was limited to $100,000. In addition to the low-doc program, the SBA has taken steps to lower their exposure to loss that is reflected in the decline in the maximum allowable guarantee rate in recent years. Although the average guarantee percentage of the SBA loans in our sample is roughly 84 percent, the mean guarantee percentage declined to roughly 78 percent for those loans originated since 1996.

In Table 2, also shows that the cohort cumulative default rate varies from a high of 29.1 percent for the 1986 cohort to a low of 14.5 percent for the 1993 cohort. The default and prepayment figures for the latter cohorts (1996-98), however, are right censored and biased downward to varying degrees since

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the last performance period observed is 2002:3. The average annual default rates vary from a low of 2.1 percent to a high of 5.2 percent over the full sample (i.e., 1985 through 1998) although the 1998 rate is likely over-estimated due to censoring. The cumulative prepayment rate for cohorts originated in the 1980's averaged just over 45 percent increasing significantly (to nearly 60 percent) for cohort groups originated in the early 1990's. The cumulative prepayment rate of the more recent cohort groups appears to be declining suggesting that the prepayment behavior was sensitive to the rising interest rates in the later half of the 1980's and the falling rates in the early 1990's.

We argue above that a coherent, loan-level stress-testing model (i.e., equations 10 and (11)) must reflect the dynamic nature of the prepayment and default behavior over the life of the loan. For that reason, we use a hazard model approach to capture the changes in the (instantaneous) prepayment and default probabilities in each quarter over the life of the loan. More specifically, we use a discrete-time hazard model to estimate the prepayment and default models.[9]

The discrete-time hazard framework is an empirical analog to the semi-parametric Cox proportional hazard model (Allison 1990; Shumway 2001; Brown and Goetzmann 1995; Deng 1995). Consistent with all empirical approaches based on hazard functions, we measure the likelihood that loan i ($i = 1,2,\ldots,N$) originated at time $t = 0$ will run off during some time period $t > 0$ ($t= 1,2,\ldots T$), given that it has survived up until that time. More specifically, the discrete-time hazard approach requires us to report our data in an 'event history' format: a series of binary variables $d_i(1),\ldots d_i(T)$, where $d_i(t)=1$ if loan i either defaults or prepays during time period t, and $d_i(t)=0$ otherwise.[10] These N separate event histories for each loan i are 'stacked' one on top of the other, resulting in a column of zeros and ones having

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[9] As noted above, the pattern in the cumulative (or, average annual) default and prepayment rates by cohort, however, may be misleading due to censoring of the performance horizon for the 1996-98 cohorts. Because the hazard model approach measures the instantaneous event probabilities, our estimates of the prepayment and default probabilities will not be affected by the censoring of the data.

[10] Measuring time in quarters, the event history $d_i(1),\ldots d_i(t),\ldots d_i(T)$ for a 7-year loan that defaults is the sixth quarter will be five zeros followed by a one (0,0,0,0,0,1). Alternatively, a 7 year that does not default and survives 28 quarters will be represented by a string of twenty-eight zeros. Loans that are prepaid prior to their contractual maturity, or right-censored loans (still performing but not yet mature at the end of our sample period), are also represented by strings of zeros until their censoring time when estimating the default parameters.

$\sum_{i=1}^{N} T_i$ rows. This event-history data design permits a hazard model to be estimated using qualitative

dependent variable (e.g., logit or probit) techniques. For example, if we define d^*_{it} as a latent index value

that represents the unobserved propensity of loan i to default during time period t, conditional on

covariates X and W, then we can represent the default behavior as:

$$d^*_{it} = \mathbf{X_t}\boldsymbol{\beta} + \mathbf{W_i}\boldsymbol{\gamma} + \varepsilon_{it} \tag{12}$$

$$= \Delta\boldsymbol{\phi} + \varepsilon_{it}$$

where $\mathbf{X_t}$ is a vector of time-varying covariates, $\mathbf{W_i}$ is a vector of time-invariant covariates, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ are the

corresponding vectors of parameters to be estimated, and ε is an error term assumed to be distributed as

standard logistic. We write (12) more compactly using $\Delta = [\mathbf{X},\mathbf{W}]$ and $\boldsymbol{\phi} = \begin{bmatrix} \beta \\ \gamma \end{bmatrix}$ to represent the full set

of time-invariant and time-varying covariates and parameters, respectively. We further define:

$$d_{it} = 0 \text{ if } d^*_{it} \leq 0$$

$$d_{it} = 1 \text{ if } d^*_{it} > 0$$

so that the probability that $d_{it} = 1$ (i.e., the probability that loan i defaults during period t conditional on

having survived until period t-1, or the hazard rate) is given by:

$$pr(d^*_{it} > 0 \mid \Delta) = prob(\Delta\boldsymbol{\phi} + \varepsilon > 0)$$

$$pr(d^*_{it} > 0 \mid \Delta) = prob(\varepsilon > -\Delta\boldsymbol{\phi})$$

$$pr(d_{it} = 1 \mid \Delta) = \Lambda(\Delta\boldsymbol{\phi}) \tag{13}$$

where $\Lambda(\cdot)$ is the logistic cumulative distribution function.[11] Using equation (13), we can estimate the

conditional default probabilities for each i in time t, $pr(d_{it}|\Delta)$ – i.e., the conditional hazard rate (Jenkins,

2003). The event-history sample design can also be used to estimate the prepayment hazard rates.[12]

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[11] We estimate equation (12) as a standard binomial logit model. A logit model estimated using an event-history sample design is generally referred to as a 'stacked-logit' model. The stacked-logit is a very flexible approach compared to most other multivariate hazard function models: in addition to allowing for time-varying covariates on

Under the event-history sample design, our model development data is expanded to include an observation in each quarter for each loan over the life of the loan. As a result, the 19,063 loans generate a total of 336,822 event-time observations.

IV. Summary of Results: Default and Prepayment Hazard Models

We report our preliminary estimation results for the default and prepayment hazard models in Table 3.[13] We capture the effects of loan seasoning using a sixth-order polynomial of time-since-origination (i.e., age) to capture the underlying shape of the hazard function.[14] Although preliminary, the estimated results are generally consistent with the previous literature and our expectations. Loans underwritten through the SBA's low doc program, to new firm, and to firms in the retail industry have an increased likelihood to default in each period over the life of the loan. In contrast, loans underwritten by experienced SBA lenders (i.e., preferred and certified lenders), or to firms in the service industry are less likely to default in each period.

More importantly for our purposes, the default and prepayment models are sensitive to time-varying systemic factors. Our results show that the effects of changing regional economic conditions are felt over several quarters for both defaults and prepayments, as reflected in the lagged structure of the

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the right-hand-side of the logit model, this approach does not require us to impose any parametric restrictions (e.g., a Weibull distribution) on the loan default distribution (the hazard function).

[12] Implicit in our model design is the assumption that the default and prepayment probabilities are independent of after conditioning on the macroeconomic, industry, and borrower/loan-specific characteristics (i.e., X, Y, and W). We also estimated the default and prepayment models simultaneously, however, using a competing risk model design (McDonald and ven de Gultch, 1999) with very similar estimates. Because the discrete-time hazard approach is easier to incorporate into the loan-level stress-testing modeling framework, we use the discrete-time hazard models at this time. A more complex competing risk approach will be addressed in future work.

[13] At this time, our data set does not include recovery/loss information. As a result, we are not able to empirically estimate loan-specific recovery probabilities. We assume that in the event of default the lender will recover on average 55 percent of the exposure at time of default. We apply this rate uniformly in the simulations below.

[14] We tested several transformations including a piecewise transformation using annual and quarterly dummies, a quadratic function of "time since origination" (i.e., age), and several higher-order polynomials of the age variable. The results were consistent with the hypothesis that loan default is a time-dependent process and that a sixth-order polynomial fit the data best based on a comparison of log-likelihood ratios.

state unemployment rate. Moreover, SBA defaults and prepayments are influenced by the general state of the macro economy, with defaults increasing as general business conditions decline, e.g., business bankruptcies increase or the industrial price and industrial production indices decrease; and, prepayments increasing as the conditions improve, e.g., leading economic index increases and inflation declines. We assume borrowers with fixed rate loans will react differently to a change in interest rates spread than those with variable rates. Our results suggests that borrowers with fixed rate loans react more strongly to a widening of the gap between the contract rate and the seven-year government bond rate than those with variable rates: increasing their likelihood of prepaying presumably to take advantage of current lower rates. Borrowers with variable rates are also more likely to prepay, although at a lower rate, reflecting possibly institutional factors that limit the speed at which the variable contract rate adjusts to changing market conditions. Indicator variables were added to reflect structural changes in the SBA 7(a) program in the early 1990s and seasonal dummies that reflect a tendency for prepayment to surge in the third quarter and decline in the fourth quarter relative to first quarter behavior, all else equal.

Overall, the models perform well at predicting both borrower default and prepayment behavior. We use a Hosmer-Lemeshow goodness-of-fit test to evaluate the models accuracy. For both the default and prepayment hazard models, we fail to reject the hypotheses of no difference between the actual and predicted performance distributions (i.e., HL_d=0.6657 and HL_p=0.4424).

As an initial test of the model, we use historical data from 1985.1 through 1998.4 to generate in-sample estimates of expected revenue, losses, and market value of the portfolio. We report the results in Table 4. Using a fixed recovery rate of 55 percent and contract interest rate adjusted for costs (i.e., estimated cost-of-funds, fee, etc.), we underestimate the actual losses and revenues by 1.0 percent and 4.7 percent, respectively; estimate that are reasonable given the limitations of our data. Using equation (11), we estimate that the market value, at time of origination over the full time horizon would have exceeded

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the actual loan amount by 3.5 percent.[15] We use these estimates as a baseline for evaluating alternative

stress scenarios in the analysis below.

V. Stress Testing Analysis Using a Dynamic Modeling Approach

As the basis of our stress test, we use a set of macroeconomic variables (or state variables) that

are typically used in practice to represent changing market conditions. These include: GDP, interest rates,

unemployment, input prices (e.g., oil prices), and production indices.[16] We use the state variables to

introduce specific exogenous shocks – e.g., a one-time increase in oil prices, or a sudden and sustained

increase in the unemployment rate – and track the impact of these shocks through a set of "auxiliary"

equations that link the state variables to the conditioning variables (i.e., $\mathbf{X_t}$ and $\mathbf{Y_t}$) used in the

specification of the prepayment and default (hazard) equations.[17] The impact and speed of the adjustment

to a specific stress scenario depends on (i) the size and duration of the initial shock,[18] (ii) the lag structure

of the auxiliary equations, and (iii) the time-dependent process underlying the default and prepayment

behavior.

For example, suppose we are interested in evaluating the impact of a sudden large decline in the

growth in the national economy (i.e., GDP) – a shock due possibly to a drop in housing sales – over the

next eight to twelve quarters. In our hazard models specified in Table 3, neither the default nor

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[15] Our results are illustrative only and are not necessarily representative of actual SBA loan performance or portfolio value. Limitations include using a sample of SBA loans, imposing a recovery rate that is not empirically verified for the SBA and not including all costs (i.e., $K_t=0$). We also assume a cost-of-funds that is the interest rate on 7-year government bonds, net fees (i.e., fee revenue less administrative costs) of 50 bps and, for variable rate loans, the contract rate changes by a fixed spread over the 7-year government bond rate,.

[16] We limit our initial analysis to a small set of macroeconomic variables typically used by banks in practice for illustrative purpose only. We believe the scenarios chosen are realistic and potentially valuable to risk management objectives. Our modeling approach, however, is flexible enough to allow one to test a large array of stress-related variables limited only by the availability of data and scope of the analysis. For example, one could evaluate the impact of a region-specific (e.g., Northeast), industry-specific (e.g., retail or construction), or program-specific (e.g., increase in low doc loans) shocks by stressing the default and prepayment factors directly.

[17] This macroeconomic stress tests are especially important for bank's SME portfolio's since in most larger bank's employ scoring models which are typically void of macroeconomic conditions.

[18] The time-path adjustment of the state variables to an initial shock can be modeled in several ways: (i) historical trends, (ii) imposing a deterministic mean-reverting process, or (iii) simulate a time-path using a Monte Carlo based process.

prepayment models are directly affected by a change in national economic conditions. We can, however,

capture the effects of changes in national economic conditions on the systemic factors in the hazard

equations using auxiliary equations that link the macroeconomic variables directly to regional economic

variables. For example, in Table 5 we link the state unemployment rates to GDP (not seasonally

adjusted), oil prices, the national unemployment rate, and the interest rate on 7-year maturity government

bonds.[19] The state unemployment rate is linked directly to a change in GDP using a simple lagged

dependent variable model. Under this design, the length of the lag is used to capture the speed of

adjustment to the initial macroeconomic shock over the observation period (e.g., eight to twelve quarters).

We use these models to simulate the impact of a change in GDP, unemployment, and oil prices

over a 12 month performance window. We simulate the impact on losses, revenues, and market value for

the portfolio of loans existing as of first quarter 1994 and track their behavior through fourth quarter

1996.[20] As a result, our data reflects the structure presented in Figure 1 (i.e., loans of varying levels of

seasoning). We evaluate the impact of two moderate to severe shocks: (i) a sharp decline in GDP over

the first year of the simulation, followed by a gradual increase in GDP by the end of the third year; and

(ii) the same change in GDP combined with a rise in oil prices to a high of $60 dollars a barrel – a tripling

of the price at that time. Table 6, we report the results of our simulation relative to the baseline estimates

derived from the models in Section IV.

The decline in GDP has a direct affect on the unemployment rate in each state, although the

impact on the state unemployment rate is lagged two periods. The decline in GDP, however, is also likely

to affect the national unemployment rate. We know from Okum's Law that there is roughly a 2 percent

decrease in output for every one percent increase in unemployment. We use this "law" to construct a

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[19] In Table 5, we report the regression results for the unemployment rate for the state of California. At this point in
time, we used a simple fixed-effect model design to estimate the state unemployment rate as a function of
macroeconomic conditions.

[20] Because our data set includes no new loans after 1998, and we wanted to simulate our model over a period in
which we have both entry and exit, we begin our stress scenarios in 1994. This will allow us to simulate changes
over periods up to 5 years. At this time, however, we use only a three-year performance horizon.

time-path for the simultaneous adjustment in the national unemployment rate that is consistent with the exogenous shock to GDP.

Under Scenario I, we impose a sharp 4.75 percent decline in GDP over the first year, with an initial 1 percent decline in the first quarter; follow by a 1.5 percent decline in the second quarter. The quarterly rate of decline in GDP is allowed to fall after the second quarter resulting in a 1.1 percent annual decline in the second year; and an actual rise in GDP of 0.8 percent in the third year. By Okum's Law, the national unemployment rate follows the same (although inverted) time path. The impact of this shock increases losses over the three-year time horizon by 25 percent (i.e., $32.3 million to $40.4 million) relative to the baseline forecasts; and revenues fall by 1 percent to $180.0 million. There is a 6.7 percent decrease in profits, and a 5.1 percent decline in the value of the portfolio relative to the baseline forecast.

Under Scenario II, we allowed oil prices to rise quickly from roughly $20 a barrel during this time period, to $50 by the end of the first year, and $60 after a year-and-half. Oil prices are then allowed to fall back to $35 a barrel by the end of the third year. GDP and the national unemployment rate follow the same time path as outlined under Scenario I. Losses increase an additional 10.6 percent to $44.7 million and revenues fall an additional 0.6 percent to $179.0 million. The impact of the oil price shock on profits and the market value of the portfolio are small after accounting for the impact of a change in GDP and the national unemployment rate. Profits fall an additional 3.7 percent (to $134.3 million) and the market value falls an additional 0.7 percent resulting in an overall decline of -5.8 percent in the value of the portfolio relative to the baseline forecast.

VI.   Conclusions

Stress testing models for credit risk are still in their infancy, relying instead mostly on more developed stress testing models for market risk.   These standard market risk models, however, suffer from several limitations that are especially important when stress testing credit portflios. The paper has several distinguishing characteristics. First, we argue that credit risk often involves opaque assets that are

cannot be easily marked to market and are also typically amortizing in nature. Second, the performance horizon for business loans is often much longer than the performance horizon evaluating the bank's exposure to market risk. To deal with these concerns, we model the expected gross revenues of the bank that captures both default and prepayment likelihoods, while incorporating the time-dependent dynamic nature of borrower behavior through the use of hazard functions.

Using this unique approach, we argue that the impact of a credit-risk shock depends not only on the magnitude of the initial shock (first-order effects), but the time path of the adjustment (second-order effects) as the shock resonates through the portfolio over time. This extended time horizon framework for stress-testing credit risk is especially important due to the interaction of behavior across competing risks (default and prepayment) and it differentiates our approach from others methods proposed in the literature. After estimating default and prepayment hazards, we generate in-sample estimates of expected revenue, losses and the market value of the portfolio that are used as baseline estimates for evaluating various stress scenarios.

We focus our stress on a set of macroeconomic variables (GDP, interest rates, unemployment and input process) typically used in practice to represent changing market conditions. We believe the scenarios chosen are realistic and potentially valuable to risk management objectives. We evaluate the impact of these shocks in a modeling framework that incorporates competing risks, the time dependence of prepayment and default and the amortizing nature of small business loans. Future research will incorporate realized loss given default estimates.

References

Allison, Paul D. (1995). *Survival Analysis Using the SAS System: a Practical Guide*. Cary, North Carolina: SAS Institute.

Brown, Stephen, and William Goetzmann (1995). "Performance Persistence." *Journal of Finance* 50, 679–698.

Cihak, M. (2004). "**Stress Testing: A Review of key Concepts**," Research and Policy Notes 2004/02, Czech National Bank, Research Department.

Committee on the Global Financial System. 2005. "Stress Testing at Major Financial Institutions: Survey Results and Practice." Working group report, Bank for International Settlements. *http://www.bis.org/publ/cgfspubl.htm*

Deng, Yongheng (1997). "Mortgage Termination: an Empirical Hazard Model with Stochastic Term Structure." *Journal of Real Estate Finance and Economics* 14, 309–331.

DeYoung, R, D. Glennon and P. Nigro (2007) "Borrower-Lender Distance, Credit Scoring and the Performance of Small Business Loans," forthcoming in the *Journal of Financial Intermediation*.

Glennon, D. and P. Nigro (2005a). "The Default History of Small Business Loans: A Survival Analysis Approach," *Journal of Money, Credit and Banking (2005), 37 (5), pp.923-946*

Glennon, D. and P. Nigro (2005a).. "An Analysis of SBA Small Business Loan Defaults by Maturity Structure," *Journal of Financial Services Research* (2005), 28 (1/2), pp. 77-111

*Hagan, *** (2005)*

Jarrow, R.A, Turnbull, S.M. (2000) The intersection of market and credit risk. Journal of Banking & Finance 24, 271–299.

Jenkins (2003)

McDonald, Cynthia. G., and Linda M. Van de Gucht (1999). "High-yield Bond Default and Call Risks." *The Review of Economics and Statistics* 81, 409–419.

Shumway, Tyler (2001). "Forecasting Bankruptcy More Accurately: a Simple Hazard Approach." *Journal of Business* 74, 101–124.

Table 1: Mean Values by Survivor Category[1]

	Column A. Total	Colum B. Non-Defaults	Column C. Defaults	Column D. Non-Prepaid	Column E. Prepaid
Panel A					
Surivor Status Full Sample	19,063	14,954	4,109	9,030	10,033
Number of Observations					
Panel B. Survivor Status by Selected Variables					
Employees	13.891	13.276	16.132	14.304	13.520
SBA Guarantee Percentage	0.834	0.833	0.838 *	0.831	0.836 *
Loan Amount	131,222	132,604	126,191 *	128,531	133,644 **
Loan Interest Rate	0.102	0.101	0.104 *	0.102	0.102 *
New Business	0.349	0.338	0.387 *	0.363	0.336 *
Sole Proprietor	0.354	0.350	0.371 **	0.373	0.337 *
Corporation	0.578	0.581	0.564 **	0.560	0.593 *
Partnership	0.068	0.069	0.065	0.067	0.069
Preferred Lender	0.113	0.118	0.096 *	0.119	0.108 **
Certified Lender	0.185	0.191	0.161 *	0.171	0.196 *
Regular Lender	0.702	0.691	0.743 *	0.710	0.696 **
Low Documentation[2]	0.334	0.325	0.367 *	0.351	0.319 *
Fixed Interest Rate Loan	0.157	0.157	0.157	0.156	0.158

1. A simple t-test is used to evaluate the difference between the mean value for survivors relative to that of defaulted loans (i.e., columns B and C); and the mean values for non-prepaid and prepaid loans (i.e., columns D and E).   * (**) indicates the difference is significant at the .01 (.05) level.
2. Low Documentation loans were first issued in 1994 and, therefore, few low doc loans have matured beyond 14 quarters.

Table 2: Descriptive Statistics for SBA 7(a) Loans by Loan Cohort[1]

Year Loan Disbursed (Cohort)[2]	Total		Cumulative Default Rate				Cumulative Prepayment Rate				Averages	
	Number of Loans in Each Cohort	% of Total in Each Cohort	Number of Loans in Default, by Cohort	Cumulative Default Rate, by Cohort	Cohort % of Total Defaults	Average Annual Default Rate	Number of Loans Pre-paid, by Cohort	Cumulative Prepayment Rate, by Cohort	Cohort % of Total Prepaid	Average Annual Prepayment Rate	Average Loan Amount by Cohort	Average Guarantee Percentage by Cohort
1985	537	2.8	151	28.1	3.7	4.0	273	50.8	2.7	7.3	$139,637	87.7
1986	721	3.8	210	29.1	5.1	4.2	329	45.6	3.3	6.5	$139,474	85.1
1987	711	3.7	193	27.1	4.7	3.9	326	45.9	3.2	6.6	$135,469	84.5
1988	613	3.2	151	24.6	3.7	3.5	297	48.5	3.0	6.9	$158,852	84.3
1989	727	3.8	161	22.1	3.9	3.2	449	61.8	4.5	8.8	$144,583	84.8
1990	815	4.3	180	22.1	4.4	3.2	468	57.4	4.7	8.2	$154,041	84.9
1991	776	4.1	157	20.2	3.8	2.9	474	61.1	4.7	8.7	$150,169	85.1
1992	1124	5.9	189	16.8	4.6	2.4	669	59.5	6.7	8.5	$165,624	85.1
1993	1203	6.3	174	14.5	4.2	2.1	731	60.8	7.3	8.7	$178,449	85.1
1994	1953	10.2	370	18.9	9.0	2.7	1083	55.5	10.8	7.9	$138,636	85.3
1995	3642	19.1	802	22.0	19.5	3.1	2071	56.9	20.6	8.1	$103,350	87.2
1996	2530	13.3	577	22.8	14.0	3.8	1335	52.8	13.3	8.8	$104,795	79.4
1997	2714	14.2	587	21.6	14.3	4.3	1182	43.6	11.8	8.7	$125,459	78.0
1998	997	5.2	207	20.8	5.0	5.2	346	34.7	3.4	8.7	$131,833	77.9
	19,063	100	4,109	21.6	100.0		10,033	52.6	100.0			

1. Based on a 20 percent sample of seven-year maturity SBA 7(a) loans originated between 1985.3 and 1998.3 and tracked through 2002.3.
2. Loans originated after 1995 are right censored. The annual averages values for the censored data were adjusted to reflect the shorter performance period.

Table 3: Hazard Models

Default Model	β_i	p-value	Prepayment Model	β_i	p-value
Intercept	-9.6770	0.0001	Intercept	-10.2491	0.0001
Age	3.1813	0.0001	Age	2.6548	0.0001
Age^2	-58.5184	0.0001	Age^2	-53.4916	0.0001
Age^3	541.60	0.0001	Age^3	571.40	0.0001
Age^4	-2685.00	0.0001	Age^4	-3253.10	0.0001
Age^5	6782.10	0.0001	Age^5	9310.60	0.0001
Age^6	-6848.70	0.0001	Age^6	-10493.3	0.0001
Time-invariant factors (W_i)			*Time-invariant factors (W_i)*		
Low documentation loan	0.1902	0.0001	Seasonal dummy (2 qtr)	-0.0154	0.6108
New business (fewer than 3 yrs old)	0.1911	0.0001	Seasonal dummy (3 qtr)	0.1703	0.0001
Preferred lender program	-0.1160	0.0546	Seasonal dummy (4 qtr)	-0.1162	0.0010
Certified lender program	-0.1715	0.0002	Dummy for 1991 – 2002	-0.2132	0.0001
Borrower in service industry	-0.1663	0.0001	Dummy for 1994 – 2002	0.2147	0.0001
Borrower in retail industry	0.1276	0.0005	Low documentation loan	-0.0833	0.0012
Time-varying factors (X_t)			*Time-varying factors (Y_t)*		
Business bankruptcies (national)	0.0001	0.0007	Composite leading index lag 1 qtr	0.0084	0.0011
State unemployment rate	0.0444	0.0846	State unemployment rate	-0.0711	0.0001
State unemployment rate lag 1 qtr	0.0609	0.0172	State unemployment rate lag 2 qtr	0.1154	0.0001
PPI: finished goods (national)	-0.0271	0.0001	State unemployment rate lag 3 qtr	-0.1030	0.0001
Industrial production index	-0.0985	0.0001	Spread over 7yr gov't bond (fixed-rate loan)	0.0285	0.0016
Industrial prod index lag 1 qtr	0.1167	0.0001	Spread over 7yr gov't bond (variable-rate loan)	0.0217	0.0023
			CPI - inflation lag 1 qtr	-0.2383	0.0001
			CPI - inflation lag 2 qtr	0.2307	0.0001
Number of Events	336822		Number of Events	336822	
Defaults	4109		Prepayment	10033	
-2 Log-likelihood	43077.4		-2 Log-likelihood	83920.4	
Likelihood Ratio	1301.8		Likelihood Ratio	6349.1	
H-L	0.6657		H-L	0.4424	

Table 4: Valuation Results: Historical Data (1985.1 – 1998.4)

	Predicted	Actual
Loss (Eq. 9)	$107,540,180	$108,673,546
Revenue (Eq. 5)	$537,934,991	$564,919,135
Profit	$430,394,811	$456,245,589
Exposure	$2,456,816,337	$2,456,816,337
Valuation (Eq. 11)	$2,543,090,671	
Percentage (+/-)	3.51%	

Table 5: State Unemployment Rate (California)

Variables (Limited Set)	β_i	p-value
Intercept (California)	-0.78259	0.0157
State unemployment rate lag 1 qtr	0.61960	0.0001
State unemployment rate lag 2 qtr	0.10024	0.0001
GDP (not seasonally adj) lag 2 qtr	0.00017	0.0378
Oil Price – Average Crude Price ($/barrel)	0.01649	0.0001
National Unemployed Rate	0.35350	0.0001
7-yr govt bond rate	-0.08003	0.0032
7-yr govt bond rate lag 1 qtr	0.08039	0.0046
Number of Observations	3409	
Adjusted R^2 (full model)[1]	98.7	

1. The full model includes a dummy variable for each state. We suppress all but the dummy for California (intercept) in this table

Table 6: Valuation Results: Stress-Test (1994.1 – 1996.4)

	Baseline	Scenario I	Scenario II
Loss (Eq. 9)	$32,331,748	$40,439,795	$44,711,885
Revenue (Eq. 5)	$181,943,503	$180,003,681	$179,039,102
Profit	$149,611,755	$139,563,886	$134,327,217
Exposure	$1,329,053,680		
Valuation (Eq. 11)	$1,279,731,654	$1,261,598,920	1,252,247,680
Percentage (+/-)	-3.7%	-5.1%	-5.8%

Figure 1: Default Hazard Curves
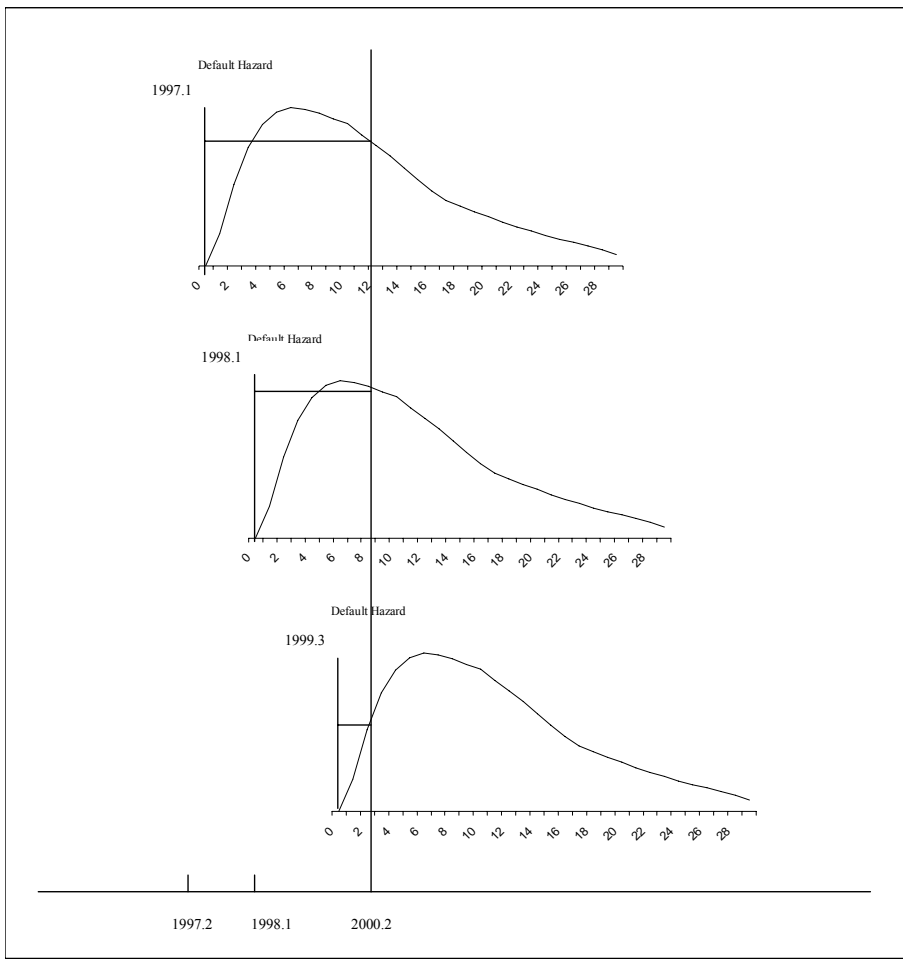
Non – Parametric Estimation of Conditional and Unconditional Loan Portfolio Loss Distributions with Public Credit Registry Data

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Abstract

Employing a resampling-based Monte Carlo simulation developed in Carey (2000, 1998) and Majnoni, Miller and Powell (2004), in this paper we estimate conditional and unconditional loss distributions for loan portfolios of argentine banks in the period 1999-2004, controlling by type of borrower and type of bank. The exercise, performed with data contained in the public credit registry of the Central Bank of Argentina, yields economic estimates of expected and unexpected losses useful in bank supervision and in the prudential regulation of credit risk.

I. Introduction

In the last decade, attempts to model portfolio credit losses have proliferated, the most known among them being CreditRisk[+] (Credit Suisse Financial Products (1997)), CreditMetrics[TM] (J.P. Morgan (1997)), KMV's Portfolio Manager (O. A. Vasicek (1984)), McKinsey's CreditPortfolio View (Wilson (1987, 1998)) and recently, the Asymptotic Single Risk Factor Model (Gordy (2002)), featured in Basel II's

Internal Ratings Based approach. While on the one hand these model-based approaches yield similar and plausible results, on the other they rely on parametric assumptions to assess the likelihood of losses in the loan portfolios, therefore being subject to model risk, i.e., the risk of obtaining misleading results as a consequence of mistaken assumptions regarding the structure of the model (such as number of systematic factors or the nature of assets' correlations) or the behaviour of random variables (such as the distribution of the systematic factor, for example gaussian in the IRB approach). In addition to this, the loss distributions are obtained using individual loans' estimated default probabilities (PDs) as an input. This introduces another source of risk, as a result of the simplifying assumptions embedded in the probit models or logistic regressions used to estimate those PDs.

Following the approach proposed in Carey (2000, 1998), we use a resampling-based Monte Carlo simulation to estimate conditional and unconditional distributions for the losses observed in loan portfolios, using the data contained in the public credit registry of the Central Bank of Argentina, the *Central de Deudores del Sistema Financiero* (CENDEU). The use of resampling-based procedures in statistics gained prominence in the last decades, in particular as from the mid 70's with the introduction of Efron's bootstrapping procedure (Efron (1979)). Efron's non-parametric bootstrap is also a resampling technique, useful to infer the distribution of test statistics. The bootstrap procedure estimates a distribution resampling repeatedly from one sample, and computing the value of the desired statistic after each iteration.

Conditional distributions are computed for each of the five years comprised between 1999 and 2004, while the estimation of unconditional distributions covers the whole period altogether. To control for differences in credit risk management

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policies and other factors that may influence the shape of the distribution, separate estimations are carried out for different types of banks and borrowers. The estimated distributions allow the computation of expected losses and measures of unexpected losses at various confidence levels. These economic measures of risk are useful to detect discrepancies with their regulatory counterpart, namely provisioning and capital requirements for credit risk. In addition to this, the results can be used to evaluate the extent to which an IRB approach is suitable to specific portfolios in an emerging economy, and in particular if its adoption would deliver the desired level of risk coverage. Adapting an exercise performed in Majnoni, Miller and Powell (2004), with the expected losses associated to the unconditional distributions and using their corresponding loss rate as a proxy of the average PD in the portfolio, we solve for an average LGD consistent with that expected loss. Having obtained these risk dimensions, we compute the capital requirement that would result from the IRB approach and we compare the results with the Monte Carlo simulated unexpected loss at the 99.9% confidence level. The paper is organized as follows: section II describes the data used in the estimations, while section III introduces the methodology: the resampling-based Monte Carlo simulation. Section IV comments the results and compares the capital requirements that would result from this methodology with those obtained with the IRB approach. Finally, section V presents the conclusions.

II. Description of the Data

The sample used in the estimation of the loan loss distributions was constructed with information obtained from the public credit registry of the Central Bank of Argentina (BCRA), the *Central de Deudores del Sistema Financiero*

(CENDEU). Data of December of each of the years in the period 1999 to 2003 was included in the sample: identification of the borrower, identification of the creditor (bank and non-bank financial institutions), type of borrower (commercial, SME or retail), business sector, total outstanding debt with the creditor, amount collateralised (with eligible financial or real assets) and risk classification one year ahead.

Following detailed guidelines set by the BCRA, risk classifications are assigned to borrowers (not to their credits) by each of their creditors (individuals with operations with many banks receive one risk classification by each creditor) and range between 1 and 5[2] depending on the perceived risk of each borrower. In the case of retail borrowers, the risk classification depends on their payment behaviour, in particular of the days past due, with borrowers having less than 90 days past due being classified 1 or 2. On the other hand, for commercial borrowers the relationship between days in arrears and the risk classification is less direct, and there are more criteria other than payment behaviour to decide how the firm will be classified, such as the projected cash-flow, business sector, etc.

Tables I and II depict the characteristics of the information contained in CENDEU, which registers every outstanding debt above AR$50 (US$16).

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[2] There is a sixth category which is assigned to borrowers in unusual situations, such as non-performing borrowers of liquidated institutions. However, not all of them are riskier than those in situations 4 and 5, or even non-performing. Therefore, to ease computations they have been removed from the sample.

Table I. Distribution of Borrowers by Risk Classification

– Non Financial Private Sector –

	1999	2000	2001	2002	2003
Fraction of Borrowers per Risk Classification					
1	80%	78%	74%	61%	66%
2	5%	5%	5%	5%	2%
3	3%	3%	3%	3%	1%
4	4%	4%	4%	6%	2%
5	8%	10%	13%	25%	27%
6	0%	0%	1%	1%	1%
Total	**7.711.858**	**7.945.971**	**8.265.319**	**6.321.842**	**6.034.802**

Source: Superintendencia de Entidades Financieras y Cambiarias, BCRA. Figures are year-end.

Table II. Outstanding Debt by Risk Classification (AR$ millions)

– Non Financial Private Sector –

	1999	2000	2001	2002	2003
Fraction of Debt per Risk Classification					
1	77%	75%	69%	43%	47%
2	5%	4%	5%	10%	8%
3	2%	3%	3%	10%	5%
4	5%	6%	6%	11%	8%
5	10%	11%	16%	25%	30%
6	1%	1%	1%	1%	2%
Total	**79.291**	**75.345**	**67.329**	**55.535**	**49.589**

Source: Superintendencia de Entidades Financieras y Cambiarias, BCRA. Figures are year-end.

After experiencing years of growth, the argentine economy entered a recession in 1999, which among other consequences affected banks' loan portfolios with a reduction of the share of performing borrowers (i.e., borrowers classified 1 or 2). While on December 1999 performing borrowers and their corresponding obligations represented respectively 85% and 82% of the total, these shares where 79% and 74% in 2001. After three years of stagnation, though, the crisis unfolded in 2002, triggered by a deposit freeze, the devaluation of the argentine peso and the default of the public debt, dragging the economy into a more severe recession with

real GDP shrinking 11% that year. The crisis reinforced the worsening of banks' loan portfolios, increasing the fraction of non-performing borrowers and debt, and reducing the depth of the financial system. Bank credit to the non-financial private sector fell from 23.3% of GDP in December 1999, to 19.2% in December 2001 and 7.5% in December 2003. Besides, by the end of 2003 nearly 50% of the outstanding bank credit to the non-financial private sector was in default.

III. Methodology

Following the approach employed in Carey (2000, 1998) we use a resampling-based Monte Carlo simulation to estimate conditional and unconditional distributions of the annual losses observed in banks' loan portfolios, using the data contained in the public credit registry of the Central Bank of Argentina, *Central de Deudores del Sistema Financiero* (CENDEU). The computations are performed controlling by type of obligor or portfolio (corporate, SME and retail) and by type of financial institution (bank and non-bank, public, foreign owned, cooperative, etc.). Therefore, for each year and each type of bank three conditional distributions are obtained, as well as one unconditional distribution for each combination of type of bank and portfolio. By this token, should differences exist in the credit policies followed by different types of institutions (i.e. private banks vs. public banks, banks vs. financial companies) these are likely to be captured by the shape of their respective distributions.

As explained in the introduction, the objective of the paper is to obtain conditional distributions for each of the five years comprised in the period 1999-2004: 1999-2000, 2000-2001, 2001-2002, 2002-2003 and 2003-2004. These estimates are deemed as conditional since, for sufficiently diversified or fine grained portfolios, their shape will generally depend on the realization of the systematic factor(s) and on

obligors' asset or default correlation. In this paper, we assume that there is only one systematic factor affecting obligors' credit stance, which is the state of the economy and is proxied by the observed behaviour of the GDP.

For each portfolio and type of bank an unconditional distribution is also computed. In this case, for each combination of portfolio and type of bank the behaviour of the borrowers in the period 1999-2004 is taken altogether in the simulation, therefore allowing for the coexistence of different patterns of credit risk in response to different realizations of the systematic factor.

Before estimating a conditional distribution a sub-set of the obligors' population is assembled; this sub-set will later be used to perform the resampling. First, from the total population of obligors belonging to the non-financial private sector only those with a positive amount of outstanding debt at the outset of the chosen period are retained. Second, given that the conditional distribution is computed for one particular combination of type of bank and portfolio, we choose those borrowers that meet this criteria. Third, borrowers that are already in default at the outset of each period are removed from the sample. Besides, some obligors that exist at the outset of a period disappear from the CENDEU during the following 12 months. This is because they may either have defaulted, been written-off and removed from the bank's balance sheet and from CENDEU, or they may have cancelled their debts and also been deleted from the CENDEU. In both cases they are removed from the sample as well; the empirical evidence found in Balzarotti, Gutiérrez Girault and Vallés (2006) shows that the potential bias introduced by removing these borrowers is negligible. For the remaining borrowers, their initial total indebtedness and eligible collateral with the bank are computed, and their risk classification in that bank one year ahead, be it indicative of default or not, is attached.

The sample constructed in this way enables the computation of an observed default rate and, together with assumptions regarding recovery rates, of a loss rate. The aforementioned procedure, while informative as to the loss experienced in the chosen portfolio, is a snapshot which yields no additional information such as what other values the loss rate may have taken and with what probability, what is the average loss rate or, perhaps more importantly, what are the worse loss rates that the portfolio may suffer, no matter how unlikely they are. Namely, we are interested in knowing the range of possible values that loan portfolios' losses may take with their associated probability, which is the output of our resampling-based Monte Carlo simulation.

To perform the Monte Carlo simulation we construct many simulated portfolios by drawing borrowers randomly and with replacement from the corresponding sub-set for which the distribution is to be computed. When simulating the portfolios we tried to mimic as far as possible the actual characteristics of the segment under study. Therefore, besides limiting the data to those borrowers that met the characteristics of the portfolios to be modelled (type of borrower and of bank), the size of the simulated portfolios (measured by the number of obligors in them) was set to equal the average number of obligors in the portfolio under study, with a cap of 500 obligors for corporates and 1,000 for SMEs and retail. For example, when simulating the distribution of corporate clients of foreign banks, the simulated portfolios were constructed drawing randomly from a pool of corporate borrowers of foreign banks, with the restriction that the size of each portfolio matched the average size of this sort of portfolio, subject to the mentioned cap. In addition to this, the resampling introduces a source of randomness, and of error, in the results, which shrinks with the number of portfolios simulated. Our results didn't show a clear

pattern of change when increasing the number of resamples from 5,000 to 20,000. Therefore, to ease the speed of computation but keeping the error as low as possible we limited the number of iterations to 10,000. Consequently, the results that follow in the paper were obtained resampling 10,000 portfolios according to the already explained data generation process. Having simulated 10,000 portfolios of the desired group of borrowers, the loss rate is estimated for each portfolio. The resulting set of 10,000 loss rates, which can be displayed diagrammatically in a histogram, constitutes our estimated loan loss distribution.

To illustrate the procedure with an example, assume we want to understand the behaviour of the loss rate of loans granted by foreign banks to corporate borrowers in a specific period, such as December 2002 – December 2003. After removing the borrowers already in default in December 2002, as well as those that disappeared during the course of the year, we attach to the remaining ones their risk classification in December 2003. Subsequently we simulate 10,000 portfolios drawing randomly from the sub-set of borrowers with the restriction that the number of obligors is consistent with the observed size of the portfolio being analysed, and for each simulated portfolio we compute the loss rate. Finally, with the 10,000 loss rates we compute the average (expected) loss and different percentiles that will provide us with measures of unexpected losses, at various confidence levels.

Conditional distributions summarize the potential credit losses that banks may experience as a result of credit events in one particular year and thus, for one particular realization of the systematic factor (the behaviour of the GDP). Conditioning in the realization of the systematic factor, the variability of the portfolio losses displayed in the distribution results from the randomness introduced by the resampling procedure coupled with the observed default rate in the assembled sub-

set, the heterogeneity of the loans in the portfolio and the existence of collaterals. However, when comparing observed loss rates in different periods of time, their difference may result not only from the abovementioned factors but also from the state of the economy. The unconditional distribution may also be understood as being a weighted average of the distributions observed in different realizations of the systematic factor, as a result of which the dynamic of the borrowers switches from one of low risk to a dynamic of high risk. Thus, the unconditional distribution is the mixture of conditional distributions that switch between regimes of high or low risk according to the observed realizations of the systematic factor. Figure I shows an example of the interpretation of unconditional distributions as the summation of densities corresponding to different regimes, weighted by the likelihood of occurrence of each regime[3].

Figure I. Unconditional distributions as mixture-distributions
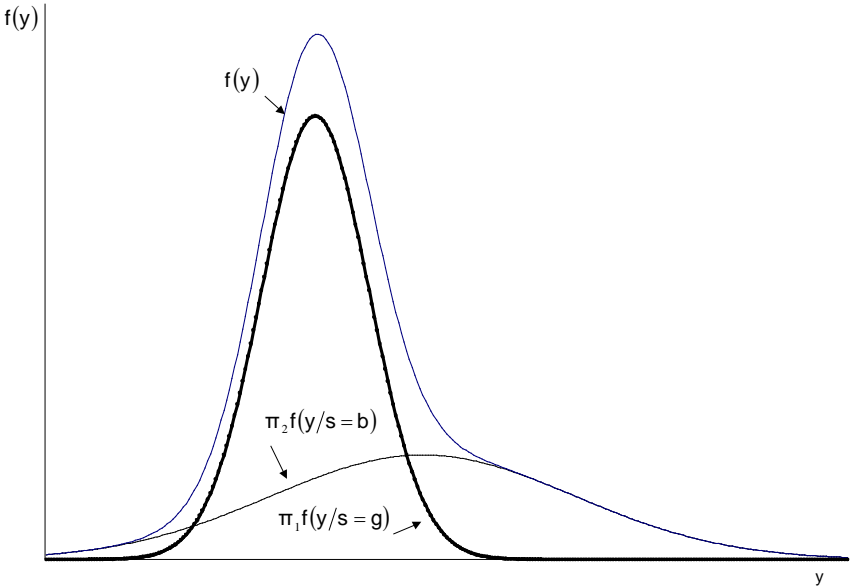
In Figure I f(y/s=b) represents the distribution of y_t/s_t=b, which is assumed to be normal with mean 2 and variance 8, and that may represent the behaviour of losses in bad realizations of the systematic factor (s=b) (i.e., y_t/s_t = b ~ N(2,8)). On

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[3] For a thorough explanation of mixture densities, see Hamilton (1994).

the other hand, representing the behaviour of losses in good realizations of the systematic factor the graph shows $y_t/s_t = g \sim N(0,1)$. The unconditional distribution is obtained as the vertical summation of densities for each level of loss, weighted by the probability of occurrence of each state of the economy. The difference between the two conditional densities is reflecting that during economic downturns credit losses are higher on average and more volatile.

IV. Empirical Results

The principal results of the simulations are summarized in tables III and IV. In Table III we assume that in each defaulted loan the loss equals 50% of the uncovered tranche of the exposure. Results in Table IV reflect a much conservative stance and assume the loss amounts to 100% of the uncovered tranche plus 50% of the collateral. Therefore the difference in the expected and unexpected losses for the same portfolio (i.e., type of borrower and of bank) in both tables is the assumption regarding the recoveries or the effective Loss Given Default (LGD), since in both cases the underlying loss rate is the same. In what follows, the discussion will be centred on the results displayed in the first table. Nevertheless, and taking into consideration that during economic downturns LGDs are likely to be larger than in normal times, since the market value of collaterals may decline, the results shown in Table IV are more suitable to assess the behaviour of credit losses during deep recessions, such as the 2001-2002 period.

Table III shows, for each type of bank and borrower, the resampled conditional expected and unexpected losses. In each case the simulations were computed for each of the abovementioned 12-month periods, while on the other hand the unconditional estimates correspond to the whole 1999-2004 period. Unexpected

losses are those that exceed the expected ones, and that usually correspond to the 90th, 95th, 99th and 99.9[th] percentiles. The latter, however, are of particular relevance since most model-based portfolio models yield estimates of the unexpected loss at this confidence level, such as Basel II's IRB. Therefore, to facilitate the comparability of results with the model-based alternatives only the unexpected losses at the 99.9% confidence are shown.

Table III. Expected and Unexpected Losses (99.9% confidence level)

- Scenario I: loss equals 50% of uncovered exposure -

	1999 - 2000		2000 - 2001		2001 - 2002		2002 - 2003		2003 - 2004		Unconditional	
	EL	UL	EL	UL	EL	UL	EL	UL	EL	UL	EL	UL
Real GDP Growth												
	-0,8%		-4,4%		-10,9%		8,8%		9%			
National State Banks												
Corporates	2,0%	4,9%	7,2%	14,9%	4,3%	7,4%	2,8%	8,9%	1,2%	6,9%	3,6%	11,4%
SMEs	-	-	-	-	-	-	4,3%	6,2%	0,3%	1,2%	1,7%	3,6%
Retail	1,2%	1,3%	1,4%	2,2%	3,3%	2,9%	1,4%	2,2%	2,6%	10,0%	1,4%	1,3%
Non-Bank Financial Institutions												
Corporates	5,1%	15,4%	7,9%	21,2%	12,7%	21,8%	0,8%	12,7%	0,3%	6,5%	6,8%	20,4%
SMEs	-	-	9,7%	14,0%	12,3%	13,0%	0,6%	4,4%	1,0%	8,2%	8,0%	20,4%
Retail	4,9%	4,7%	4,8%	4,0%	8,7%	4,2%	2,1%	1,5%	3,0%	2,2%	5,4%	4,7%
Wholesale and Investment Banks												
Corporates	0,0%	0,0%	0,2%	2,1%	5,4%	21,1%	0,0%	0,9%	0,0%	0,0%	1,2%	19,5%
Retail	1,6%	13,8%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,7%	12,1%
Large Retail Banks												
Corporates	1,6%	4,9%	2,7%	7,8%	11,4%	14,6%	1,4%	3,3%	0,7%	1,4%	3,7%	7,4%
SMEs	-	-	3,1%	2,5%	8,7%	4,1%	0,9%	1,8%	1,3%	2,5%	4,1%	3,2%
Retail	2,4%	3,4%	2,2%	2,4%	4,2%	2,8%	0,5%	1,3%	0,3%	5,3%	2,2%	3,2%
Medium-Sized Retail Banks												
Corporates	1,5%	4,2%	2,2%	9,2%	11,2%	15,8%	1,6%	8,2%	0,2%	1,0%	3,6%	11,1%
SMEs	-	-	3,1%	2,2%	9,1%	3,5%	1,6%	3,4%	1,1%	2,3%	4,0%	2,7%
Retail	2,9%	3,8%	2,9%	6,9%	5,7%	4,0%	1,0%	3,5%	0,7%	2,9%	3,6%	6,7%
Small Retail Banks												
Corporates	3,8%	12,8%	4,1%	11,4%	10,0%	22,0%	0,7%	12,1%	0,4%	2,8%	3,9%	19,5%
SMEs	-	-	4,0%	9,6%	8,9%	6,5%	3,5%	13,1%	4,6%	8,7%	7,1%	7,6%
Retail	5,9%	8,0%	5,5%	9,7%	8,1%	6,3%	2,1%	4,0%	2,1%	5,1%	5,1%	9,2%
Other Wholesale and Investment Banks												
Corporates	2,5%	9,0%	2,2%	9,6%	8,3%	20,9%	4,5%	28,9%	0,5%	6,0%	3,4%	17,5%
SMEs	-	-	7,9%	8,8%	11,5%	13,7%	3,0%	10,5%	4,6%	7,9%	7,5%	13,8%
Retail	7,1%	9,9%	8,1%	9,5%	10,1%	9,5%	4,4%	6,2%	0,4%	4,4%	6,4%	9,1%
Provincial and Municipal Banks												
Corporates	3,0%	11,2%	5,5%	26,1%	5,9%	11,8%	4,8%	32,7%	2,2%	12,6%	4,5%	28,1%
SMEs	-	-	5,9%	2,9%	12,0%	3,4%	1,9%	2,2%	5,9%	3,5%	7,4%	3,2%
Retail	3,2%	3,2%	2,8%	3,0%	4,2%	3,1%	0,7%	1,8%	1,3%	2,4%	3,7%	16,6%

Table IV. Expected and Unexpected Losses (99.9% confidence level)

- Scenario II: loss equals uncovered exposure plus 50% of collateral -

	1999 - 2000		2000 - 2001		2001 - 2002		2002 - 2003		2003 - 2004		Unconditional	
	EL	UL	EL	UL	EL	UL	EL	UL	EL	UL	EL	UL
Real GDP Growth												
	-0,8%		-4,4%		-10,9%		8,8%		9%			
National State Banks												
Corporates	9,5%	18,4%	19,0%	28,9%	13,3%	16,7%	9,2%	20,1%	2,7%	14,4%	11,6%	22,8%
SMEs							10,5%	12,6%	0,9%	2,9%	4,2%	7,9%
Retail	7,1%	3,9%	6,7%	6,5%	13,4%	6,9%	5,7%	6,7%	9,0%	21,5%	8,2%	4,4%
Non-Bank Financial Institutions												
Corporates	11,6%	33,8%	16,8%	44,9%	27,0%	44,9%	1,9%	27,6%	0,6%	13,1%	14,8%	42,4%
SMEs			20,5%	27,8%	25,9%	26,4%	1,3%	9,1%	2,5%	17,3%	16,9%	41,8%
Retail	13,3%	9,7%	12,0%	8,1%	20,3%	8,5%	4,6%	4,5%	6,3%	4,8%	13,4%	9,2%
Wholesale and Investment Banks												
Corporates	0,0%	0,0%	0,4%	4,3%	10,7%	42,3%	0,1%	1,7%	0,0%	0,0%	2,4%	39,0%
Retail	3,2%	27,5%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	1,3%	24,2%
Large Retail Banks												
Corporates	3,8%	10,1%	6,5%	15,7%	24,7%	28,8%	3,3%	6,7%	1,6%	2,7%	8,5%	14,7%
SMEs			7,4%	5,6%	19,7%	8,8%	2,3%	4,2%	3,2%	5,3%	9,7%	6,4%
Retail	5,9%	7,1%	5,5%	6,0%	10,7%	6,7%	1,3%	3,3%	1,1%	10,8%	5,7%	7,4%
Medium-Sized Retail Banks												
Corporates	3,8%	8,7%	5,3%	18,7%	24,7%	31,4%	3,6%	16,5%	0,5%	2,4%	8,1%	22,0%
SMEs			9,0%	5,3%	25,2%	6,9%	4,2%	7,0%	3,0%	4,7%	11,2%	5,7%
Retail	7,8%	8,5%	8,1%	13,3%	15,4%	8,8%	2,4%	7,0%	2,2%	7,4%	8,1%	13,6%
Small Retail Banks												
Corporates	9,0%	25,9%	9,5%	24,4%	22,5%	43,6%	1,6%	24,4%	1,1%	6,5%	9,0%	39,3%
SMEs			9,3%	19,5%	22,3%	12,4%	7,9%	27,6%	11,4%	17,2%	17,2%	16,0%
Retail	12,8%	16,9%	11,7%	20,2%	17,3%	12,6%	4,3%	8,0%	4,4%	10,3%	11,0%	18,2%
Other Wholesale and Investment Banks												
Corporates	5,1%	18,0%	4,7%	20,8%	17,6%	43,5%	9,3%	59,1%	1,2%	11,9%	7,2%	34,7%
SMEs			18,2%	17,3%	25,3%	27,2%	7,2%	21,8%	9,2%	15,9%	16,9%	27,5%
Retail	14,5%	20,9%	16,7%	19,3%	21,0%	18,8%	8,8%	12,4%	0,9%	8,8%	13,1%	18,4%
Provincial and Municipal Banks												
Corporates	9,8%	25,4%	15,4%	51,0%	18,4%	26,0%	11,7%	64,6%	6,6%	36,2%	13,0%	55,8%
SMEs			14,0%	5,9%	29,0%	7,2%	4,8%	5,5%	15,6%	7,6%	18,0%	7,0%
Retail	8,6%	7,6%	7,6%	6,9%	13,4%	7,4%	2,6%	4,2%	4,4%	5,4%	9,8%	34,4%

Conditional Distributions

The results of the simulated conditional distributions show that, across the economic cycle, the expected losses corresponding to the different portfolios are quite correlated, although their behaviour presents differences. Figures II, III and IV show the conditional expected losses for corporates, SMEs and the retail portfolio, by type of financial institution. As expected, conditional expected losses are cyclical: around 2% in years of high economic growth (such as 2003 and 2004), increasing in 2001 up to 8% in the case of the retail portfolio and corporates and 10% for SMEs. It

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is worth mentioning that by December 2001 the argentine economy had been in recession for three years, with real GDP falling 3.4% in 1999, 0.8% in 2000 and 4.4% in 2001. At the outset of the year 2002, the devaluation of the argentine Peso and the default of the public debt transformed the recession into a major crisis, with real GDP falling 11% that year. As a result of this, the expected loss rates conditional on the events of 2002 soared to 12% in the case of corporates and SMEs, and to 10% for the retail portfolio.

Figure II. Conditional Expected Losses: Retail Portfolio
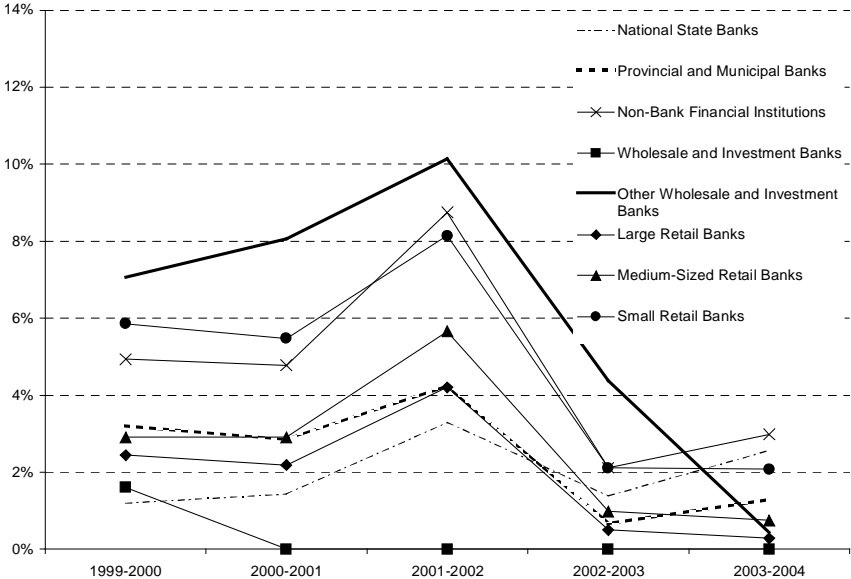
Figure III. Conditional Expected Losses: SME Portfolio
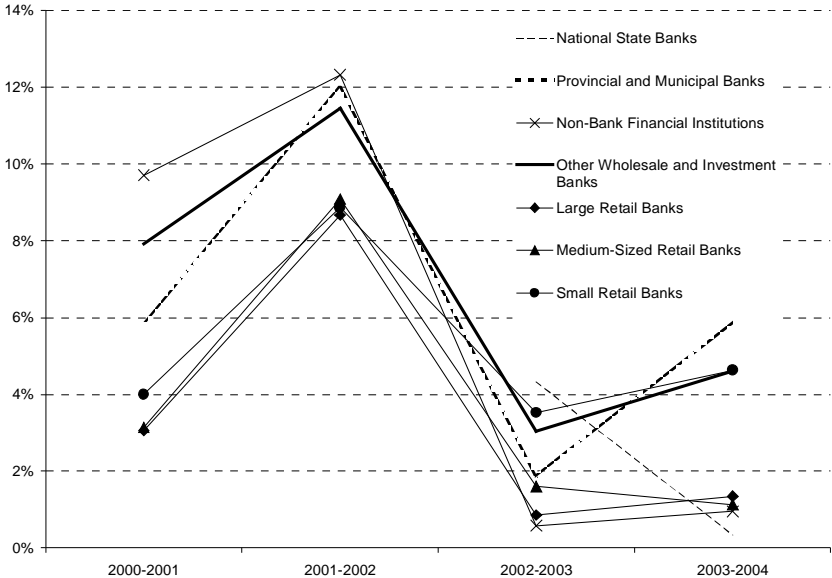
Figure IV. Conditional Expected Losses: Corporate Portfolio
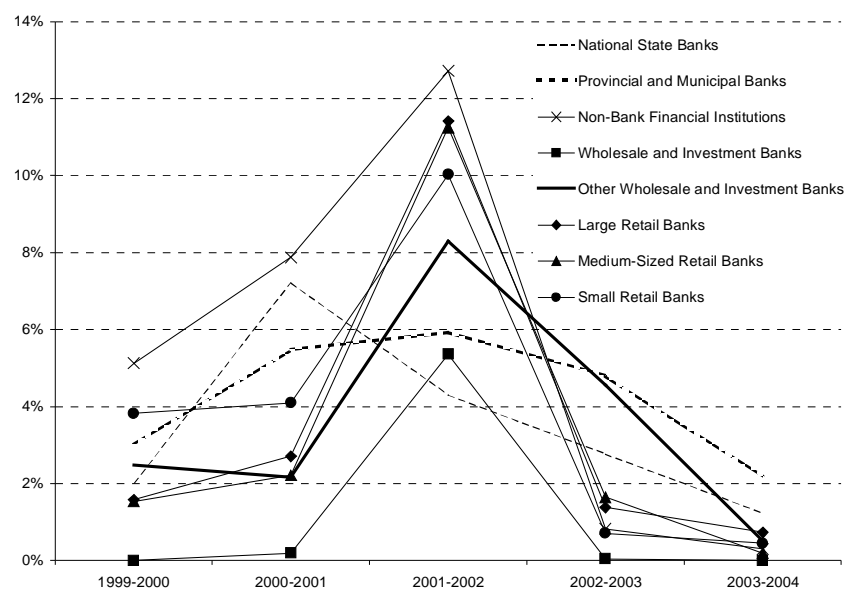
Figures II - IV above also show that the cyclical pattern of expected losses for each type of borrower is very similar across all the institutions, but it shows differences between types of borrowers. On the other hand, figures V, VI and VII below depict the behaviour of the conditional unexpected losses. In the case of the retail and SME portfolio, our results show that although the estimates react to the business cycle, they are less sensible to the state of the economy than the expected losses. With the exception of wholesale and investment banks, unexpected losses of the retail portfolio range between 0% and 10% during the three years comprised between 1999 and 2002, peaking slightly during 2001, and reduced subsequently to a range below 5% in years of high economic growth. The unexpected losses of SMEs present a similar pattern, although they take values up to 15% (on top of the expected losses) and the effect of the state of the economy on them seems to be even milder. Finally, corporate borrowers are much more responsive to the realizations of the systematic factor. With the exception of state-owned banks, the unexpected losses of this portfolio increased significantly during 2002 in response to the economic crisis, with unexpected losses in some cases above 20% of the

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portfolio and on top of the expected losses. These findings regarding the higher sensitivity of corporate obligors to the realizations of the systematic factor and, conversely, the fact that defaults of retail and SME obligors are more idiosyncratic and less dependent on the economic cycle, are reflected in the calibration of the IRB approach, as explained in BCBS (2004).

Figure V. Conditional Unexpected Losses: Retail Portfolio
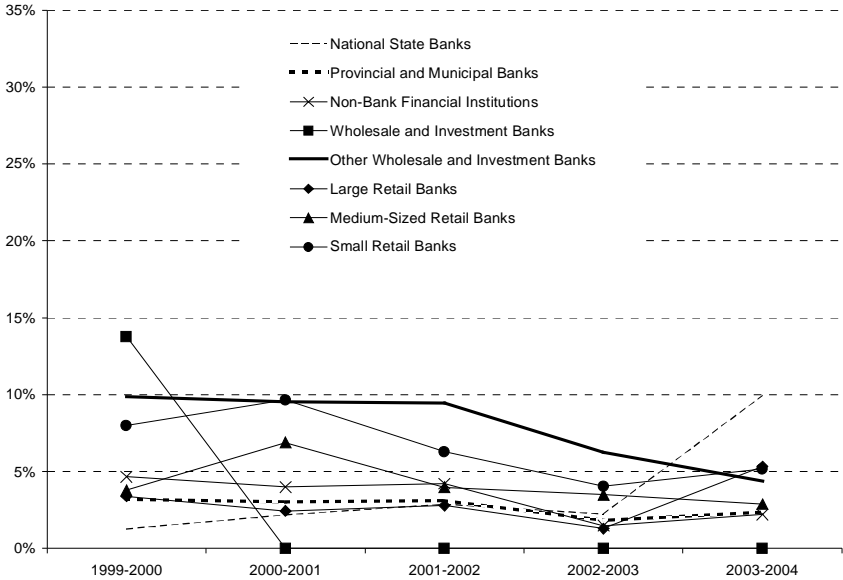
Figure VI. Conditional Unexpected Losses: SME Portfolio
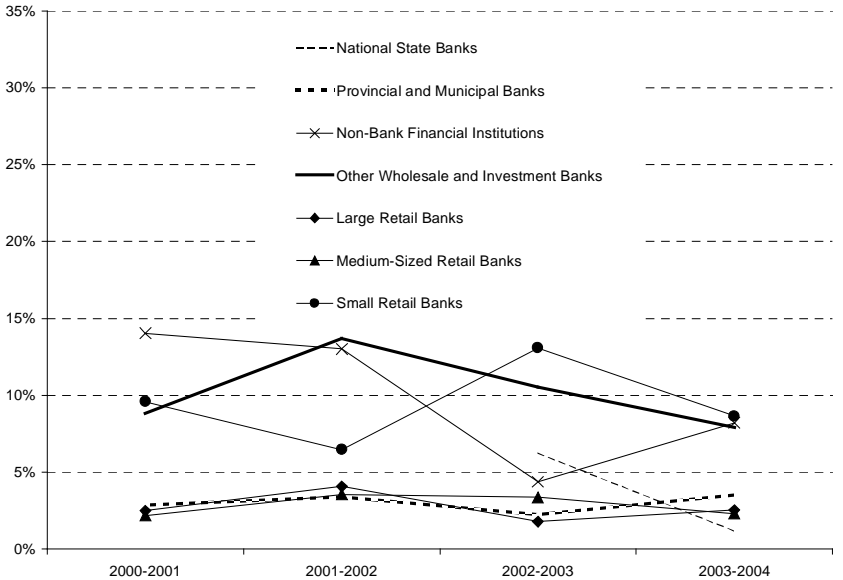
Figure VII. Conditional Unexpected Losses: Corporate Portfolio
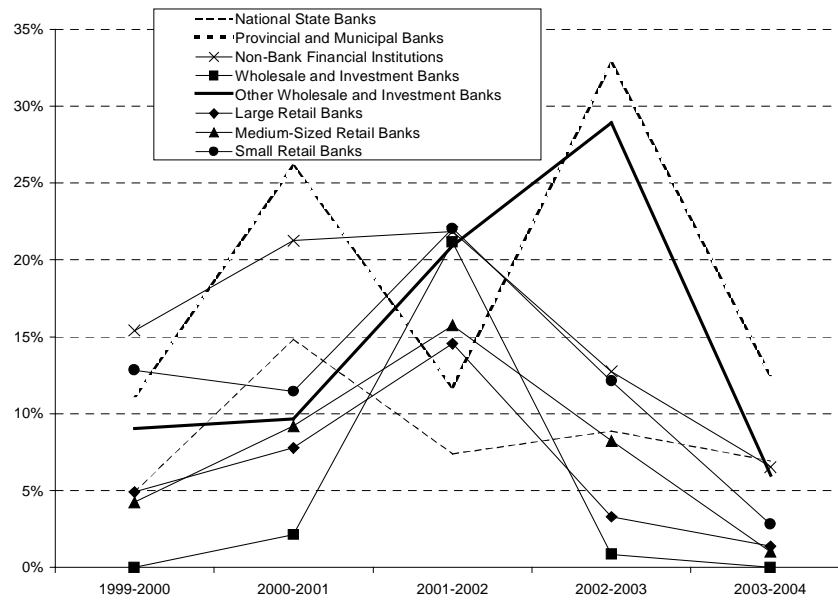
The findings regarding the conditional expected and unexpected losses shown thus far reflect the shifting of the loss distributions as a consequence of the realizations of the systematic factor. Those findings, also, reflect the higher loss volatility observed in bad years (recessions), and the lower volatility observed in good years (expansions of the economy). Figures VIII, IX and X show the impact of the systematic factor on (conditional) loss volatilities.

Figure VIII. Conditional Loss Volatilities: Retail Portfolio
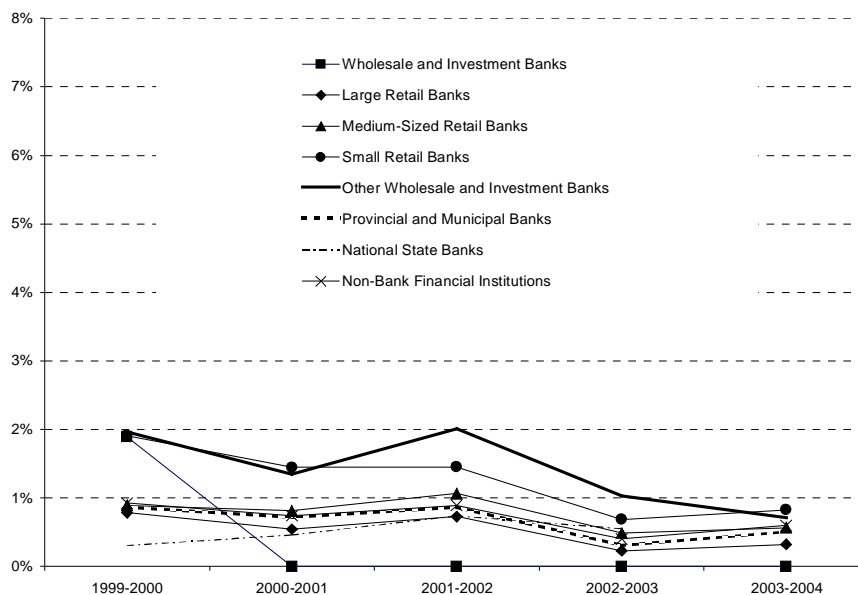
Figure IX. Conditional Loss Volatilities: SME Portfolio
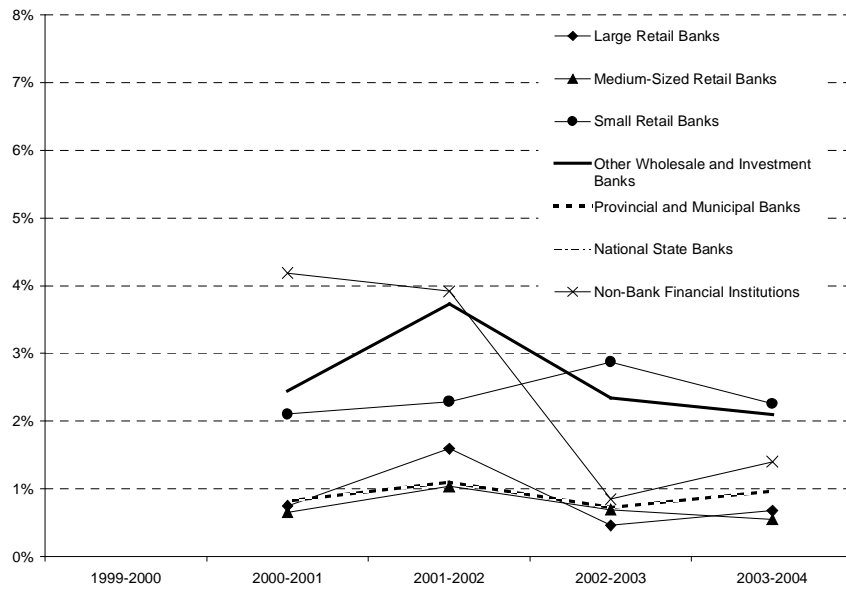
Figure X. Conditional Loss Volatilities: Corporate Portfolio
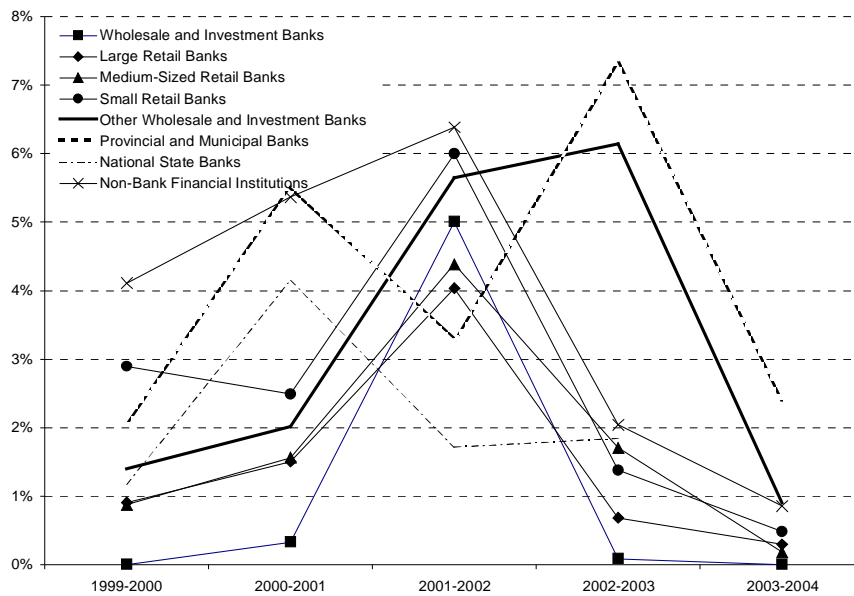
While the three figures reflect the behaviour of unexpected losses through-the-cycle, in all three cases our simulated loss volatilities show the expected behaviour, in the sense that in years of bad realizations of the systematic factor the loss volatility is higher, and lower in good years.

Unconditional Distributions

In this sub-section we discuss the results obtained when computing the unconditional distributions, applying the resampling-based simulation to a chosen sub-set of borrowers but for the period 1999-2004 altogether. The resulting distribution can be understood as an average of the conditional distributions that correspond to different realizations of the systematic factor, weighted by the likelihood of occurrence of that particular realization.

Figure XI shows an example of the unconditional distribution of retail obligors of big retail banks. In the graph it can be seen how the conditional distributions shift according to the realizations of the systematic factor, with bad realizations shifting the conditional distributions to the right, increasing their mean (expected loss) and standard deviation. In the figure below the unconditional distribution is indicated with a grey area.

Figure XI. Unconditional Distribution as Mixture of Conditional Distributions:

Retail Portfolio of Big Retail Banks
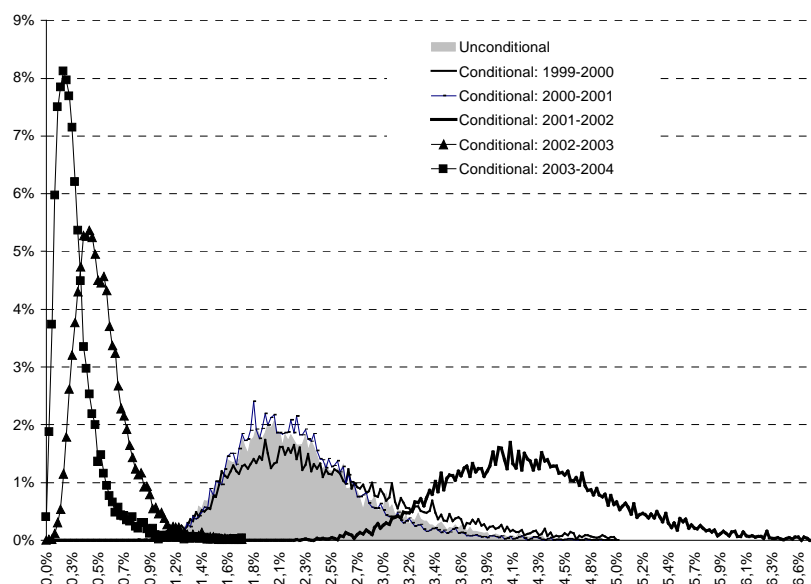
The resulting estimations of unconditional expected and unexpected losses (at the 99.9% confidence level in the last case) are shown by type of borrower and bank in figures XII through XIV.

Figure XII. Expected and Unexpected Unconditional Losses: Retail Portfolio
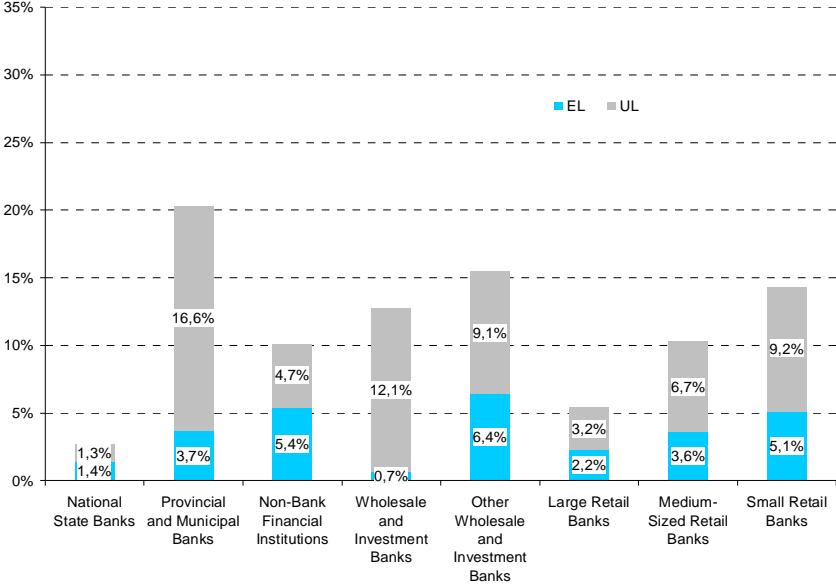
Figure XIII. Expected and Unexpected Unconditional Losses: SME Portfolio
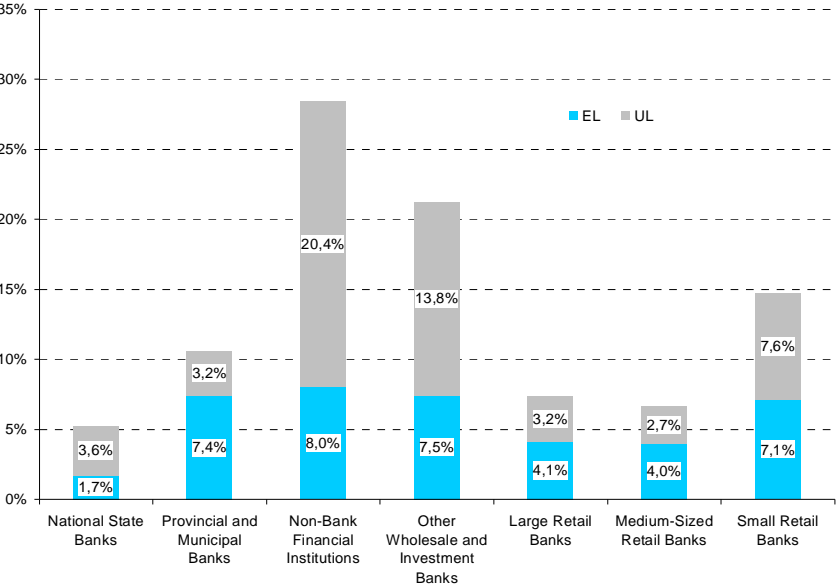
Figure XIV. Expected and Unexpected Unconditional Losses: Corporate Portfolio
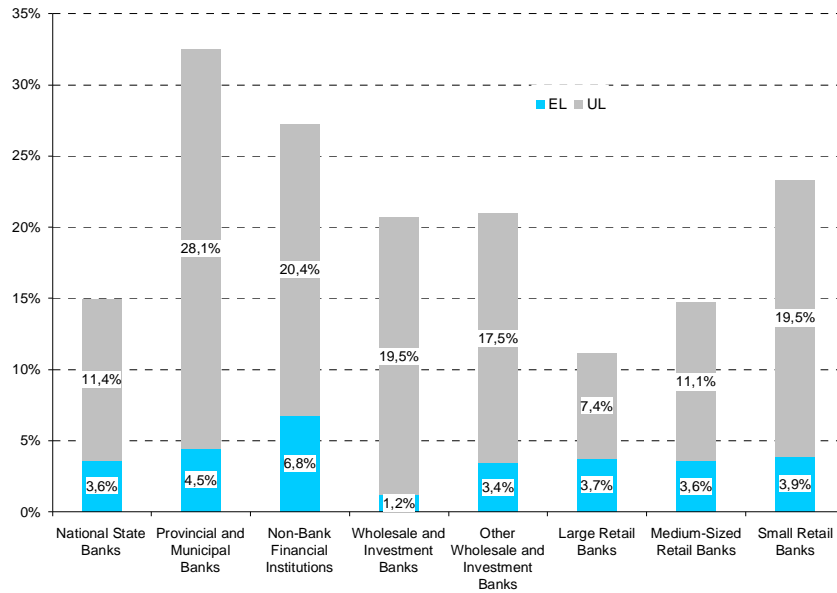
The results reflect our findings regarding the behaviour of corporate obligors' losses through-the-cycle: their unconditional unexpected losses are particularly larger than those of SMEs and retail obligors, therefore meriting larger capital requirements as reflected in the design of the IRB approach. Notwithstanding the type of borrower involved, there are also clear differences in unconditional distributions across banks, with large and medium-sized retail banks and national state banks showing the lowest risk, measured by the loss rate at the 99.9% confidence level (i.e., the 99.9[th] percentile of the unconditional loss distribution). Within each asset class, banks' risk profile varies considerably by type of bank. In the case of the retail portfolio, provincial and municipal banks have the largest loss at the 99.9[th] percentile, 20% of its retail portfolio, followed by wholesale and investment banks. Regarding SMEs, non-bank financial institutions are those with the riskiest portfolio, with a loss rate at the 99.9[th] percentile of more than 28% of the portfolio. On the other hand, national state banks and large retail banks have the lowest loss rates at that percentile of the

tail distribution. As to the corporates, provincial and municipal banks have loss rates higher than 30% at a 99.9% Value-at-Risk, followed by the non-bank financial institutions and small retail banks. In general, the abovementioned differences in risk profiles may be attributed to differences in the granularity of the corresponding portfolios, in the respective obligors sensitivity to the systematic factor and in their risk management policies and tools (i.e., application and behavioural scorings). In this last case, it is worth mentioning that among the financial institutions with the highest risk profiles are some which may not seem proficient enough or with the necessary expertise with respect to the corresponding borrowers, such as wholesale and investment banks in the retail portfolio, non-bank financial institutions with SMEs and corporates and small retail banks with corporates.

Comparison with a model-based approach: the advanced IRB

Among other possible uses, the results obtained with this methodology can be compared with Basel II's IRB approach. In what follows, we perform an exercise adapted from Majnoni, Miller and Powell (2004) in which we compare the capital requirements needed to cover unexpected losses at the 99.9% confidence level of our unconditional distributions, with those resulting from the IRB approach. Taking the estimated unconditional expected loss of any portfolio, assuming its corresponding default rate is a good proxy of the average PD of the obligors and that the exposure at default equals their outstanding debt, we find an implicit LGD. To perform this computations we use unconditional estimates since they incorporate the loss experience in adverse scenarios. With these risk dimensions we compute the (advanced) IRB capital requirements. The results, expressed as the ratio of the IRB

capital requirements to the Monte Carlo estimated capital requirements, are shown in Table V.

Table V. IRB capital requirements vs. Non-parametric Monte Carlo based

	Corporates	SMEs	Retail
National State Banks	0.8	3.4	2.5
Non-Bank Financial Institutions	0.9	0.9	1.8
Wholesale and Investment Banks	0.6	-	0.9
Large Retail Banks	1.8	3.6	1.6
Medium-Sized Retail Banks	1.0	4.1	1.2
Small Retail Banks	0.6	2.0	1.2
Provincial and Municipal Banks	0.5	5.0	0.5

The results show that, on average, the IRB yields capital requirements which would be insufficient to cover unexpected losses at the 99.9% confidence level for corporate obligors. This effect is particularly important for wholesale and investment, small retail and provincial and municipal banks. Besides suggesting a possible miscalibration of the IRB model, these results may reflect the fact that these banks' portfolios are not sufficiently fine-grained. Conversely, for the retail and SME portfolios we find that the coverage produced by and IRB approach would be overly conservative, yielding capital requirements more than enough to cover unexpected losses at the 99.9% VaR. For example, in the case of large retail banks IRB capital requirements for SME obligors would be 260% larger than the unexpected losses, and 60% larger in the case of retail borrowers. Our results for corporates reinforce those obtained in Majnoni, Miller and Powell (2004) who also found that the IRB approach yielded insufficient coverage for corporate obligors. However, their resampled distributions included corporate and some SME obligors, did not control by type of bank (was performed for the whole financial system as a whole) and corresponded to the period 2000-2001 only.

V. Conclusions

In this paper we used data of the public credit registry of the Central Bank of Argentina to implement a non-parametric method to estimate loan portfolio loss distributions. The method, which is a resampling-based Monte Carlo simulation, enabled us to obtain conditional distributions for the five 12-month periods comprised between 1999 and 2004, and an unconditional distribution for the whole period. In both cases, separate computations where performed by type of borrower and bank. In all cases the estimated distributions allow the computation of economic (risk-based) measures of expected and unexpected losses for credit risk, to be covered with provisions and capital requirements. However, whether the supervisor must use conditional or unconditional measures to set the prudential regulation depends, among other factors, on the degree of risk sensitivity the regulation is expected or desired to show, and on the national supervisor's leeway to deal with the prociclicality that conditional measures exacerbate.

As it was explained during the paper, unconditional distributions can be interpreted as an average of the conditional distributions. Therefore, had the exercise in this paper included information of the years 2005 and 2006, in which the economy grew at 9.2% and 8.5% and with obligors' average default rates at 3.2% and 3.6% respectively, the estimated unconditional expected and unexpected losses would have been lower than those here obtained (shown in tables III and IV). According to the information of the BCRA, while at the end of 2003 only 66% of the obligors of the financial system were risk classified as 1 (see Table I), by the end of 2006 that fraction had risen to 86%. Therefore, for this methodology to be useful in bank regulation and supervision it is of paramount importance that the model is computed

with a sample that covers a sufficiently long time period, and that its estimates are updated on a regular basis.

The comparison between our resampled unconditional unexpected losses and the IRB capital requirements for the same portfolios allows to detect discrepancies between risk and coverage. These may be caused by less than sufficient granularity in banks' portfolios or by problems in the calibration of the IRB. Our study shows there is a tendency of IRB capital requirements to exceed unexpected losses for SMEs and the retail portfolio, while they fall short in the case of corporate obligors. In this case, our findings support similar results obtained for corporate obligors of argentine banks in Majnoni, Miller and Powell (2004).

References

Balzarotti, V., M. A. Gutiérrez Girault y V. A. Vallés (2006). "Modelos de Scoring Crediticio con Muestras Truncadas y su Validación", Banco Central de la República Argentina.

Basel Committee on Banking Supervision (2004). "An Explanatory Note on the Basel II IRB Risk Weight Functions", Bank for International Settlements.

Carey, M. (1998). "Credit Risk in Private Debt Portfolios", The Journal of Finance, Vol. LIII, N° 4, pp.1363-1387.

Carey, M. (2000). "Dimensions of Credit Risk and their Relationship to Economic Capital Requirements", National Bureau of Economic Research, WP 7629.

CreditMetrics (1997). "Technical Document", J.P. Morgan.

Credit Suisse Financial Products (1997). "CreditRisk+: A Credit Risk Management Framework", Credit Suisse First Boston.

Efron, B. (1979). "Bootstrap Methods: Another Look at the Jacknife", The Annals of Statistics, Vol. 7, N° 1, pp. 1-26.

Gordy, M. (2002). "A Risk-Factor Model Foundation for Ratings-Based Bank Capital Rules", Board of Governors of the Federal Reserve System.

Hamilton, J. D. (1994). Time Series Analysis, Princeton University Press, Princeton, New Jersey

Majnoni, G., M. Miller and A. Powell (2004). "Bank Capital and Loan Loss Reserves under Basel II: Implications for Latin America and Caribbean Countries", The World Bank and Universidad Torcuato Di Tella.

Vasicek, O. A. (1984). "Credit Valuation", K.M.V.

Wilson, T. (1997). "Portfolio Credit Risk", Risk Magazine, pp. 111–117.

Wilson, T. (1997). "Portfolio Credit Risk", Risk Magazine, pp. 56–61.

Systemic Risk: Stress Testing the Banking System[*]

Javier Márquez Diez Canedo, Serafín Martínez Jaramillo

Abstract

Although there are many definitions of Systemic Risk, most coincide that it manifests itself by an initial shock that results in the failure of one or more banks, and then spreads out to the entire system by a contagion mechanism which can result in the failure of more banks in the system. Assuming that bank failures in the initial shock are random depending on the failure probabilities of the individual banks, and that the ensuing contagion process is deterministic, depending on interbank exposures, in this paper we propose a network model to analyze systemic risk in the banking system, which in contrast to other proposed models, seeks to obtain the probability distribution of losses for the financial system resulting from the shock/contagion process.

Thus, assuming that individual bank default probabilities are independent[1] and provided exogenously, and that the matrix of bilateral interbank exposures is known, we represent systemic risk in the financial system by means of a graph and use discrete modeling techniques to characterize the dynamics of contagion and corresponding losses within the network. The probability distribution of losses, *risk profile for the financial system* is obtained through an efficient, complete enumeration procedure of all possible bank default events in the system. This in turn allows the use of the wide variety of well established risk measures to describe the fragility of the financial system. Additionally, the model allows us to perform stress tests along both the bank default probabilities and the interbank exposures.

1 Introduction

Systemic risk is a a subject of paramount importance for regulators responsible for financial stability, but its measurement poses a formidable technical problem. Part of the difficulty is that the initial shock which causes the failure of

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[1]We are currently working to relax this assumption.

one or more banks, and then spreads out to the entire system, can arise from a wide variety of sources e.g. default in large payment systems or as counter party of a contingent claim of a derivatives contract in the interbank lending market. Another important difficulty is how to associate a risk measure to the contagion process itself; i.e. once the initial event occurs, what is the impact on the financial system provoked by the ensuing contagion process, due to banks exposures to each other. Whereas measures such as value at risk (VaR), Tail-VaR and stress tests have been developed for market and credit risk, no comparable measures have been developed for systemic risk. This makes it difficult for financial authorities to design regulation that specifically addresses systemic risk related issues in an efficient way. A case in point is deposit insurance, the cost of which must not only contemplate the individual probabilities of bank failures, but also the contagion capability that particular banks have on the entire system. Thus, financial contagion is an integral part of systemic risk and cannot be disassociated from it.

In our study we employ a network model to study systemic risk and capture both the initial random shock and the ensuing contagion process. The *Systemic Risk Network Model* permits the estimation of the distribution of losses for the financial system due to the initial shock and the contagion process, to perform some stress tests and develop a measure of financial fragility.

The paper is organized as follows: In Section 2, we begin by reviewing some of the literature we consider relevant to our work; both on systemic risk and financial contagion. We provide a brief summary of the main approaches proposed to study the phenomenon in the following section. In this section, after a brief mention of some applications of graphs and networks to economic and financial problems, we discuss what in our opinion is the most relevant work on financial contagion using graph theoretical and network models, as they relate to our particular approach.

Section 3 deals with the details of our network model to study systemic risk. We explain how the proposed model captures the relationship between banks through interbank loans and how the dynamics of the contagion mechanism is characterized using discrete modeling techniques. By incorporating the individual failure probabilities of the banks in the system (assuming independence), we show how to obtain the distribution of losses in the financial system due to initial shock contagion, through an efficient, complete enumeration procedure.

Section 4 presents an initial proposal of a measure of financial fragility for the banking system. In Section 5, we provide the full details of the enumeration procedure. We then go on to show how the model can be used for stress testing the financial system, and explain the experiments performed, the data used and the results obtained. Finally, in Section 6, we summarize our findings and propose possible lines of further research.

2   Systemic Risk and Financial Contagion

The importance of systemic risk is its link with financial stability. In fact, one implies the other to the point that in order to ensure financial stability, it is necessary to measure, and "manage things" so that the risk of occurrence of events that could lead to a systemic crises can be avoided or mitigated. Although much has been written on systemic risk [BH00][2] and although there is a good idea of what systemic risk is about, there is no precise, widely accepted definition, nor is there such a thing as an accepted analytical framework. The dominant idea in any definition is that systemic risk has to do with "the risk of experiencing an event that will affect the well-functioning of the entire financial system"[3]. It is interesting to note however, that alternative definitions also refer to the nature of the event and the mechanism of propagation that could affect the financial system. For example, contagion could occur through failures in payment systems, counter party defaults in derivatives contracts, defaults in interbank loans or a combination of these. As to the nature of the event that could cause a widespread failure, two are readily apparent; namely: a shock that causes a severe dysfunctionality in a group of financial institutions or, the failure of a certain number of financial institutions, either of which is transmitted to the entire financial system through one of the mechanisms previously mentioned.

Particularly noteworthy, is the definition given by the Bank for International Settlements in its annual report of 1993-1994 [fIS94]: "Systemic risk is the risk that the failure of a participant to meet its contractual obligations may in turn cause other participants to default, with the chain reaction leading to broader financial difficulties". This definition of systemic risk highlights the role of financial contagion in a systemic crisis. From this discussion one can infer that systemic risk has two components; namely: An event that causes the failure or dysfunctionality of a critical number of market participants, and a contagion mechanism which propagates the failure and/or dysfunctionality to a broader number of participants or the entire system.

2.1   The role of financial contagion in systemic risk.

That financial contagion is a real threat is evidenced by financial crisis of varying degrees of severity and detonated from different sources, experienced in several countries in the last two decades. The savings and loans crisis in the U.S. in the late eighties and early nineties, the Mexican crisis ("Tequila Effect") of 1994-96, the Russian ("Long term Capital") and Asian crisis at the end of the century are among the most notable and are still fresh in our memories.

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[2]De Bandt et. al. provide a useful survey of published research on systemic risk up to the year 2000.

[3]For example, Kaufman defines it as "the probability that cumulative losses will accrue from an event that sets in motion a series of successive losses along a chain of institutions or markets comprising a system. . . . That is, systemic risk is the risk of a chain reaction of falling interconnected dominos' (Kaufman 1995a, 47). According to De Bandt and Hartmann "Systemic risk (in the narrow and broad sense) can then be defined as the risk of experiencing systemic events in the strong sense". [BH00]

Furthermore, the globalization of the financial system resulted in cross border spill over effects, that only a few years earlier would have been inconceivable and highly unlikely. It is thus important to understand the causes, the mechanics and the consequences of financial contagion, which is not an easy task. The complex way in which today's financial institutions are related to each other make it difficult to understand conceptually and verify empirically, the different sources and nature of possible destabilizing events with the ensuing contagion process and its consequences. Although the types of shocks to which financial systems are particularly sensitive are fairly evident, it is not clear what causes contagion between financial institutions. From herding behavior to sun spots, all sorts of explanations have been provided, none of which is either totally inclusive or conclusive.

Although much has been written on systemic risk and contagion, in what follows, we only mention those references that provided the conceptual framework for the study of systemic risk due to financial contagion as they relate to our approach. Rochet et. al.[RT96] do not deal specifically with financial contagion, but the authors provide a theoretical framework to investigate interbank lending and systemic risk and arrive at the important conclusion that, in an environment of market discipline, interbank lending could be beneficial for prudential control. Of the empirical studies on financial contagion, the most cited is Furfine [Fur99a] and [Fur99b] . In his study of contagion in US banks, bank failures of "significant" banks are simulated and the effect on the remaining banks is measured by estimating the expected loss in each case. Furfine acknowledges that he underestimates the size of the interbank market as he only uses interbank federal funds exposures for his study. There are a number of papers that analyze contagion in different countries along the same lines e.g. [SM98] and [Mul06] for Switzerland, [Wel02] for the United Kingdom, [BN02] for Sweden, [UW04] for Germany, [DN04] for Belgium, [GGLG05] for Mexico and [BEST04a] and [ELS06] for Austria. In later research, there is a conscious effort to compensate for the underestimation of losses by considering all interbank exposures. The difficulty here is in the data which is vague on how these are distributed. In order to deal with this problem and with the exception of Graf [GGLG05], [Mul06], the lack of counter party information is dealt with by assuming that the distribution of interbank exposures is uniform. Another interesting approach is to use market data on the movements of stock prices, interest rates and exchange rates to infer statistically whether or not contagion occurred. In [GDV06], the authors analyze contagion using distance to default measures for European banks and find evidence of cross-border contagion in Europe.

An interesting paper by Upper [Upp07], besides summarizing the previously mentioned group of papers related to simulations of financial contagion, goes on to evaluate the assumptions made by other authors and discusses their use for the analysis of financial stability. In his paper, the author clearly states: "Going forward, more work is needed on how to attach probabilities to the individual scenarios and on the micro foundations of the models." In the approach followed in this paper, we show how the network model permits the association bank

default probabilities to the initial shock scenarios of bank failures followed by failures due to contagion, which permits the estimation of the distribution of losses for the financial system, which can in turn be used to obtain a measure of fragility.

When applicable, graph and network models possess many advantages. Besides the fact that a vast knowledge base and analytical tools are available in this field, network formulations are highly visual and dynamic, and it is possible to gain much insight and understanding on a problem by simply examining its graphical representation. Graph theory can be traced as far back as 1763 with the paper by Euler on the solution of the "Königsberg Bridge problem". Euler "invented" graph theory in order to solve this puzzle. In 1758 Quesnay represented the financial funds' flow in an economy as a network and it can be considered the first financial network model. In the twentieth century, first Pigou(1920) and later Kantorovich, Hitchcock and Koopmans used a graph representation for the minimum cost transportation problem. The final breakthrough occurred in the late fifties and early sixties, with the work by Dantzig, Ford and Fulkerson, which paved the way for the development of a host of efficient algorithms to solve network flow problems. Applications of graph and network models in economics and finance range from currency translation to the portfolio optimization problem. Nagurney [Nag03] provides a comprehensive survey of the literature on networks in finance and economics.

It is very natural to use network and graph models to study financial contagion since banks can be represented by vertices or nodes and the bilateral exposures as edges or arcs in a graph. Thus, it is not by chance that many people have chosen this path for modeling contagion. Building on previous research ([AG98] and [DD83]), Allen and Gale [AG00] provide the microeconomic foundations to study financial contagion on two different structures: the complete graph and the cycle, which they called a complete and an incomplete market structure respectively. Allen and Gale concluded that the complete structure is more resilient to liquidity shocks than the cycle. Despite their undeniable contribution, the drawback is that real financial networks differ significantly from those two extreme cases, as illustrated by Boss [BEST04a] who gives a glimpse of what a real interbank market looks like.

Based on certain characteristics of the model by Eboli [Ebo04], Nier et. al. [NYYA06], propose a model that captures a more general structure of the financial system. In order to gain insight on financial contagion, the authors randomly generate graphs to simulate interbank markets and then explore the impact of variations in different parameters (e.g. the bank's capitalization) on the possibility of occurrence of bank failures due to contagion. In [IdMP+05], Iori et. al. analyze the network of the Italian overnight market and provide some useful measures to characterize the network at different points in time. Additionally, in [VL04], the author studies network structures that would enable banks to improve depositors utility by means of small-world networks. Small world networks are networks which have a small clustering coefficient and average shortest path length [WS98]. Such networks have been found to exist in a wide number of social and natural phenomena like the Internet, genetics,
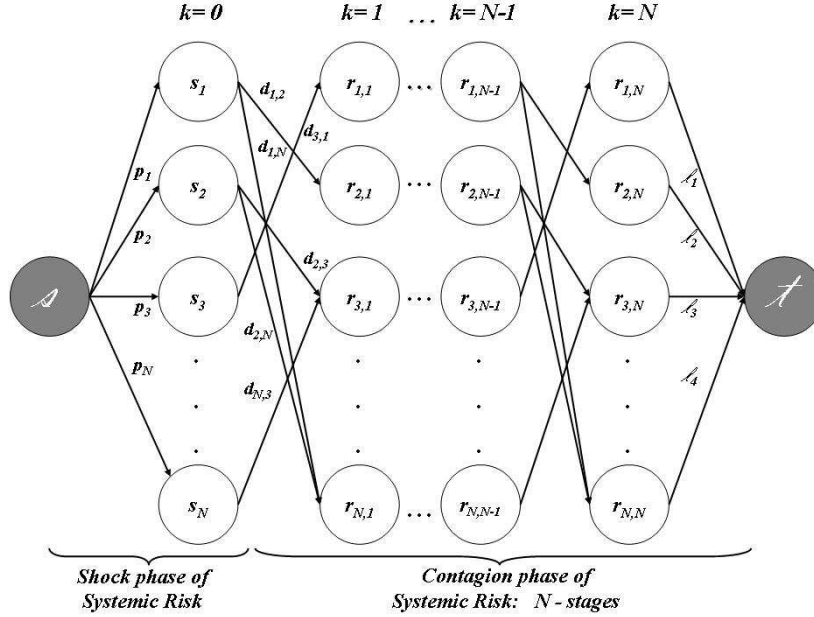
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Figure 1: Systemic Risk Network Model.

scientific collaboration, etc. The reader is also encouraged to read the articles by Boss [BEST04b] and Muller [Mul06] who also study financial contagion by means of network or graph models.

Finally, a word is in order on the measurement of contagion, and as pointed out by Rigobon [Rig01], this is not easy to do. There are however some interesting proposals in which contagion is measured through equity prices as opposed to banks' balance sheets[GM04].

3   A Network Model of Systemic Risk

The following systemic risk model traces losses to the system due to bank failures, whether they are due to the initial stochastic shock or determined by contagion, on a network $G[N, A]$. The nodes N of the graph are partitioned as $N = \{s, S, R, t\}$, where s is the node that represents the initial shock to the system; S is the set of nodes that represents the banks which are the "sources" of contagion into the system given the initial shock; R is the set of "relay" nodes which are banks in the possible contagion tree at the different "stages" of contagion and t represents the sink node where all systemic losses concur. The network is represented schematically in Figure 1.

From Figure 1 it is seen how systemic risk is divided into its two phases; namely, the shock phase and the contagion phase. It is also seen that depending

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on the phase, the arcs in the network are labeled with different attributes. Thus, the arcs that go from the shock node s to the source nodes are labeled with the individual bank failure probabilities p_i. The labels on the arcs during the contagion phase are the exposures d_{ij} that banks have to each other; which in this model are assumed constant through all possible contagion stages N. Finally, the arcs that go from the terminal relay nodes r_{in} to the sink node t are labeled with the loss to the system l_i given failure of bank i. It should be noticed that in this simple model, only the initial shock is a random event, and the ensuing contagion process is deterministic.

3.1   The loss distribution

Let F denote the set of failed banks in the initial shock, and $L\,(F)$ denote the losses to the system if *"scenario F occurs"*. Note that L considers both the losses of the banks that fail in the initial shock, and the losses due to the contagion generated by these banks. Furthermore, since contagion is a deterministic process, the banks that fail due to contagion of initially failed banks in F is unique; so let $C(F)$ denote the set of banks that fail due to contagion whose source is F. Then, the loss to the system given scenario F is simply:

$$L(F) = \sum_{i \in F} l_i + \sum_{i \in C(F)} l_i \tag{1}$$

Furthermore, assuming that during the initial shock, the failure probabilities of banks are independent, this loss has probability of occurrence

$$P(F) = \prod_{i \in F} p_i \prod_{i \in F^\sim} (1 - p_i);$$

where $F^\sim$ is the complement of F.

Thus, doing this for all possible F, the distribution of losses in the system is obtained.

For the sake of clarity, in this simple model it is assumed that bank failures are independent of each other. Although this is a rather strong assumption it facilitates understanding how the full distribution of losses can be obtained. However, the more realistic case of dependence can be addressed in several ways:

- One can assume that there exist "implicit correlations" in the default probabilities; i. e., default probabilities are correlated to the extent that they respond to common risk factors in some degree.

- Although complex, it is also possible to derive a formula that contemplates the "explicit" correlations.

- Finally, it is possible to deal with dependence of joint failures by using a copulas based approach.

3.2 Discrete Modeling of contagion in the Network

In order to model contagion, assume that at every stage of contagion and for each bank i in the system, there is a certain *"threshold"* u_i^k such that if the banks exposure to previously defaulted banks exceeds the threshold, the bank will also fail. Formally, let D^k be the set of all banks that have failed by stage k. Then bank i will fail at stage $k+1$ if,

$$\sum_{j \in D^{k-1}} d_{ji} \leq u_i^k \text{ and } \sum_{j \in D^k} d_{ji} > u_i^{k+1}$$

We define a state variable to indicate whether a bank is failed or not at stage k of the contagion process as:

$$\theta_i^k = \left\{ \begin{array}{ll} 1 & \text{if } \sum_{j \in D^k} d_{ji} > u_i^k \\ 0 & \text{otherwise} \end{array} \right.$$

From here, the modeling of contagion is straightforward:

a) $\sum_i \theta_i^k \, d_{ij} =$ Sum of defaulted exposures to bank j at stage k.

b) $u_j^{k+1} = \max \left\{ u_j^k - \sum_i \theta_i^{k-1} \, d_{ij} \; ; \; 0 \right\}$

c) $\theta_j^k \geq \frac{\sum_i \theta_i^{k-1} d_{ij} - u_j^k}{1 + \sum_i \theta_i^{k-1} d_{ij}}$;

d) $\theta_j^k < \frac{\sum_i \theta_i^{k-1} d_{ij} + \varepsilon}{u_j^k + \varepsilon}$; $\varepsilon > 0$;

e) $\theta_j^{k+1} \geq \theta_j^k$ and $\theta_j^k \in \{0, 1\} \;\; \forall j, k$.

Now, to verify that the above logic will give the state of any bank at every stage , first assume that $\sum_i \theta_i^{k-1} \, d_{ij} < u_j^k$ so that bank j *does not fail at stage k*. From (c) and (d) we have that:

$$\alpha \leq \theta_j^k < 1; \; where \; \alpha = \frac{\sum_i \theta_i^{k-1} \, d_{ij} - u_j^k}{1 + \sum_i \theta_i^{k-1} \, d_{ij}} < 0$$

In other words, θ_j^k must be strictly less than one and greater than some negative number. Then $\theta_j^k = 0$ since θ_j^k can only be zero or one.

Similarly, assume $\sum_i \theta_i^{k-1} \, d_{ij} > u_j^k$ so that bank j fails at stage k. Again, from (c) and (d):

$$0 < \frac{\sum_i \theta_i^{k-1} \, d_{ij} - u_j^k}{1 + \sum_i \theta_i^{k-1} \, d_{ij}} \leq \theta_j^k < \beta; \; \beta = \frac{\sum_i \theta_i^{k-1} \, d_{ij} + \varepsilon}{u_j^k + \varepsilon} > 1$$

This means that in this case θ_j^k must be strictly greater than zero and less than some number which is greater than one. Thus $\theta_j^k = 1$, since $\theta_j^k \in \{0, 1\}$ ∎

Additionally, we now define a very important concept on the contagion phase: *overexposure*. We say that a bank i is *overexposed* if:

$$\sum_{j \in N^-(i)} d_{ji} > u_i^0. \tag{2}$$

where $N^-(i)$ is the set of inner neighbors of the bank i.

We can infer from the above definitions that the contagion phase only depends on the set of overexposed nodes and the set of their inner neighbors. This means that in order to study contagion on a specific network of interbank lending, it is only necessary to focus on the sub-network that consists of overexposed nodes, their inner neighbors and the respective links. This simplification of the network considerably reduces the computational effort.

Finally, the losses in the system are computed as:

$$L = \sum_i \theta_i^N l_i. \tag{3}$$

3.3 A toy example

For illustrative purposes, in this section we give a toy example of systemic risk measurement in a system with only four banks (A, B, C, D) and assume that their probability of default, thresholds and losses given default are as shown in Table 1.

Bank	A	B	C	D
Probability (%)	1	4	2	1
Threshold	11	5	7	7
Loss	16	20	12	8

Table 1: Probability of default, threshold and loss for banks A, B, C, and D.

Assume their exposures on the interbank market are as shown in Table 2. From Table 2 we know, for example, that bank A owes 10 units to bank D and the total exposure of bank C is 14 units. Clearly, if none of the banks fail (no shock occurs), for every i we have θ_i^0 is equal to zero. This means that $D^0 = \emptyset$ therefore $\sum_{j \in D^0} d_{ji} \leq u_i^0$ for all banks as can be seen in Table 1 and Table 2. Note that in this system, banks B, C, and D are overexposed.

Debt	A	B	C	D	Total debt
A	0	6	0	10	16
B	0	0	4	8	12
C	2	0	0	0	2
D	0	0	10	0	10
Total Exposures	2	6	14	18	40

Table 2: Debts of each bank

Now assume that bank failure probabilities p_i are independent and as shown in Table 1. Examine the case where only bank A fails and is unable to honor

its commitments. From Table 2 it is seen that only banks B and D are exposed to bank A and that

$$d_{AB} = 6 > u_B = 5$$

and

$$d_{AD} = 10 > u_D = 7$$

so that in the first stage of contagion both banks B and D will fail once A has failed. In the next stage, bank C is exposed to banks B and D so that,

$$d_{BC} + d_{DC} = 4 + 10 > u_C = 7$$

Thus, bank C fails in the second stage, so that if bank "A" fails in the initial shock, the whole system will fail due to contagion, for a total loss of:

$$L = \sum_i \theta_i^N l_i = 56$$

Assuming independence, the probability of this happening is:

$$P = p_A(1 - p_B)(1 - p_C)(1 - p_D) = 0.93\%$$

If one repeats the procedure assuming bank B fails in the initial shock, then bank D will fail in the first stage of contagion and bank C will fail on the second. The total loss in this case is $L = 36$ and the probability of this scenario is 3.48%.

Thus, by repeating the procedure for all possible combinations of bank failures during the initial shock (i.e.that 1, 2, 3 or all four banks fail in the initial shock) and computing the corresponding losses and probabilities, it is possible to obtain the complete loss distribution. For this simple example, the results are summarized in Table 3 .

Loss	Probability (%)
0	92.12
12	1.88
20	1.88
36	3.96
56	1.00

Table 3: Loss Distribution

4 A measure of system fragility

As previously mentioned, it is difficult to measure systemic risk and to assign a risk measure to the contagion process and how fragile a banking system is. Moreover, as in the case of Systemic Risk, there is little consensus of what *financial fragility* actually is. For example, in [Tso03] Tsomocos provides the following definition of financial fragility:

When substantial default of a 'number' of households and banks (ie a liquidity 'crisis'), without necessarily becoming bankrupt, occurs and the aggregate profitability of the banking sector decreases significantly (ie a banking 'crisis').

Some insight can be obtained from our model. After having experimented with the model, and as will be illustrated in the next section, it appears that considering the topological aspects of the interbank exposures network and the probability distribution of the initial shock, a financial system becomes more fragile when:

- There are more overexposed banks.

- There are more paths going through overexposed banks.

- The probability distribution over the shock scenarios weighs more heavily on banks that trigger contagion.

In summary, from our experience with the model we infer that system fragility is characterized by high default probabilities (initial shock), the associated losses and the propensity to contagion (overexposed nodes). The loss distribution combines all these elements and can be used to derive a fragility measure. For example, the expected loss could be used as a fragility measure, but disregarding the variance of the distribution could be a very misleading appreciation of the actual risk. This observation immediately suggests that a better measure would be related to some quantile of the distribution or to use VaR directly to derive a measure of fragility for a financial system; for example:

$$\mu = \frac{VaR(\alpha)}{L} \tag{4}$$

where L represents the total losses for the financial system as it was previously defined in Equation 3.

5 Testing the fragility of the Mexican banking system.

In this section we report the results of applying our model 3 to analyze the robustness/fragility of the Mexican banking system, under normal and stressed conditions. We ran the systemic risk model with information corresponding to the interbank loans reported at the end of 2006 by all the banks on the Mexican financial system. Additionally, since the model is parameterized it is possible to consider different percentages and types of losses; this is a topic in itself. Namely, when a bank fails, different actors lose different things; most notably: Shareholders lose the capital invested, creditors lose what is their due, depositors will lose anything over what is ensured, and ultimately taxpayers will have to pay for the cost of the resolution process. In the literature, the most

explored cases are asset losses and the losses due to interbank defaulted loans. Although, there are different percentages of losses reported on the literature, we will only examine the case when the failure of a bank causes a loss of the bank's total assets, this being the most pessimistic case. The data used in our tests is described in Subsection 5.1.

5.1   Data

The central bank has daily data that can be used to calculate the matrix of interbank exposures of the Mexican financial system, from January 2004. The period of time contemplated on this study goes from this date to December 2006. Although there are 31 banks in the system, the exercises performed only included 25, since the remaining six are relatively new charters for which information is scarce and inconsistent. The interbank exposures considered comprise all the possible deposits, credits and loans including credit lines as part of the interbank market. For a correct analysis, it is important to know what the real network of exposures looks like and and how it changes over time. As Graf [GGLG05] points out, the assumption of maximum entropy on the distribution of the interbank exposures is not realistic, at least not in the Mexican case. Bank failure probabilities for banks in the Mexican Financial System are those calculated by the Central Bank.

5.2   Computational aspects

At first sight to compute all the possible shock scenarios and contagion paths appears to be a formidable task. Since there are 25 banks included in the model, there are 2^{25} different combinations of failures due to the initial shock, to which one must add the computation of all the ensuing contagion trees. In the case of the Mexican banking system however, it is a relatively easy task. Since the only relevant banks in the contagion process are those that are overexposed, and resorting to some of the techniques commonly used on the Constraint Satisfaction field and implicit enumeration, it is possible to program the algorithm to run in a few hours[4]. In the context of our problem and referring to our toy example, note that the failure of bank A leads to the complete breakdown of the system, therefore, any combination that includes the failure of bank A is going to cause the system's collapse. Thus, it is not necessary to explicitly enumerate all the combinations since the outcome is known beforehand. Similarly, since no bank that is underexposed can fail due to contagion, eliminating them from the contagion network also reduces the search space; the more of these there are, the less cases that have to be explicitly enumerated.

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[4]Constraint satisfaction techniques (e.g. constraint propagation, domain reduction and learning no goods) are used to reduce the solution search space. Thus a large number of cases are enumerated implicitly and not explicitly greatly reducing the necessary computation.

5.3 Reference case and stress test results

In order to illustrate the use of the model we now present the results obtained in four different cases. The first case can be considered as the reference case, and the other three are stressed scenarios. In posing stress scenarios for the financial system it must be recalled that system fragility has to do with bank failure probabilities, the number and importance of overexposed banks, and the number of paths that go through them. Thus, a stress situation has two distinct elements; namely: The bank failure probabilities in the initial shock phase, and the interbank exposures in the contagion phase. The reference case is the analysis of the Mexican Banking system under current "normal" conditions. In stress case 1, the interbank exposures are stressed while maintaining bank failure probabilities as in the reference case. In stress case 2, interbank exposures are as in the reference case and bank failure probabilities are stressed. Finally in stress case 3, both failure probabilities and interbank exposures are stressed. Specifically:

- **Reference Case.** For this case the interbank exposures are taken as those observed at the end of December 2006. The failure threshold values are taken as tier 1 capital at the end of December 2006. Banks' failure probabilities are estimated from market and credit risk data over the period 2001-2006. These default probabilities can be considered as the probabilities under "normal" conditions since the 2001-2006 horizon does not include periods of crises. Finally, the losses are taken as a percentage of the banks' total assets in December 2006.

- **Stress Case 1.** For this case the interbank exposures are taken as the maximum registered historic values. The rationale being that we wanted to investigate what would happen in a network that possesses a large number of links and over exposed banks. The other parameters are the same as for the reference case.

- **Stress Case 2**. For this case the interbank exposures and thresholds are the same as for the reference case, while banks' of failure probabilities are estimated considering the period 1994-2001, where the Mexican banking system went through several critical periods. The stressed probabilities were calculated so as to characterize a period of extreme financial distress for the banking system, such as the Mexican 1994 crisis. As in the previous, losses are taken as a percentage of the banks' total assets in December 2006.

- **Stress Case 3.** For this case the interbank exposures are as in stress case 1 and failure probabilities are as in stress case 2. As usual, the losses are taken as a percentage of the banks' total assets in December 2006. This case is obviously the most dramatic one as the network contains a larger number of links and overexposed banks and the failure probabilities are those of the stress period.

Figure 2 shows the two different graphs representing interbank exposures. First of the reference case which is the state of the interbank market at the end of December 2006 (Figure 2(a)). Note that there is only one overexposed bank (represented by a red circle). The stressed graph shows the maximum historic exposures between banks (Figure 2(b)), where almost every bank is overexposed.

Table 4 shows the two different sets of bank failure probabilities: The reference case and the stressed case where one can see that the latter are much larger than the former.

	Normal probabilities	Stressed probabilities
Average	0.18%	9.31%
Std. Dev.	0.40%	11.5%
Maximum	1.36%	35.84%
Minimum	0.01%	0.01%

Table 4: Statistics for the normal and stressed probabilities.

Figure 3 shows the cumulative distribution of losses for the reference case. It should be observed that there is a very high probability of loosing nothing and there are big jumps for small losses. As we move along the x axis, it is seen that the probabilities of large losses are very small. We can say that this is a typical appearance of a distribution of losses for a financial system as the probability of individual failures of banks are generally small and the probabilities of joint failures (the ones that would carry lager losses) are even smaller. This is further enforced by the fact that there is practically no possibility of contagion.

In Figure 4, we can observe the distribution of losses for the Stress Case 1 which does not change very much with respect to the Reference Case. This means that despite the fact that the network topology is dramatically different, the shape of the distribution does not change much. In fact, as we will see later, the reported VaR is the same for both cases.

In Figure 5, we can observe the distribution of losses for the Stress Case 2. In this figure we can observe that the distributions of losses change dramatically in relation to the Reference Case and the Stress Case 1. In fact, the shape of the distribution is totally different to the two previous cases. Although we acknowledge that we are using "extreme" probabilities, it is remarkable that a drastic change in topology of the interbank exposure network has such a small effect on the distribution of losses whereas the opposite is true when we change the failure probabilities.

In Figure 6, we can observe the distribution of losses for Stress Case 3. In it we observe that the distribution of losses does change dramatically in relation to the Reference Case and Stress Case 1 but less in relation to the Stress Case 2. This reinforces the fact that system fragility is more sensitive to failure probabilities than it is to interbank exposures.

In Table 5 we provide a statistical summary for the distributions of losses for the four cases presented; namely: mean, variance, skewness, kurtosis, and $VaR(99)$. The mean divided by the total losses L and the $VaR(99)$ divided
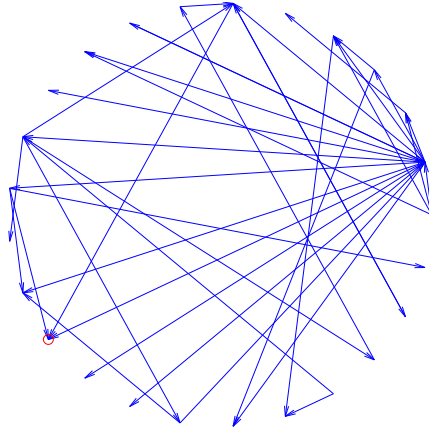
(a)
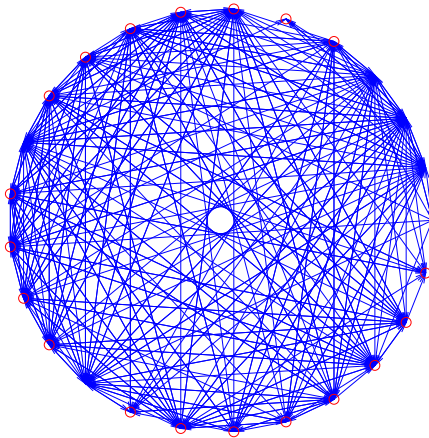
(b)

Figure 2: Two different exposures networks: (a) December 2006 and (b) maximum historic loans.
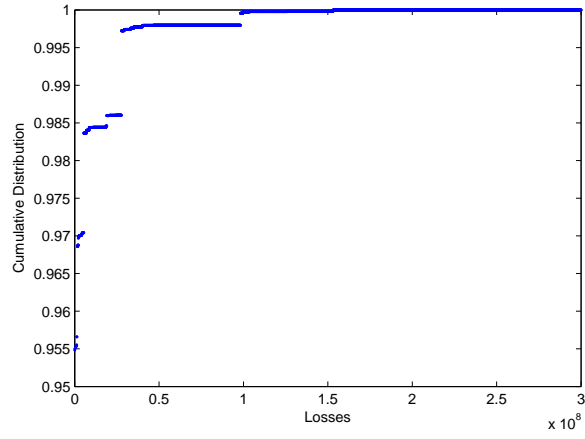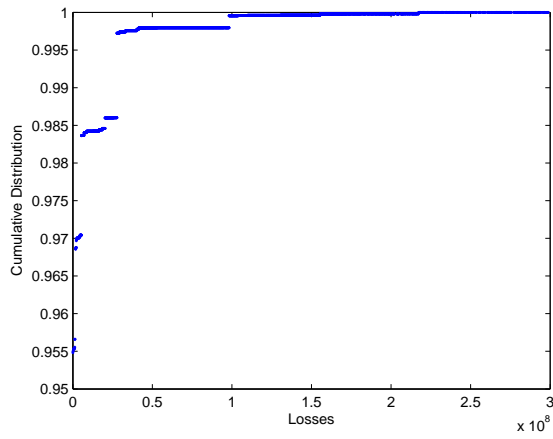
Figure 3: Distribution of losses for reference case.
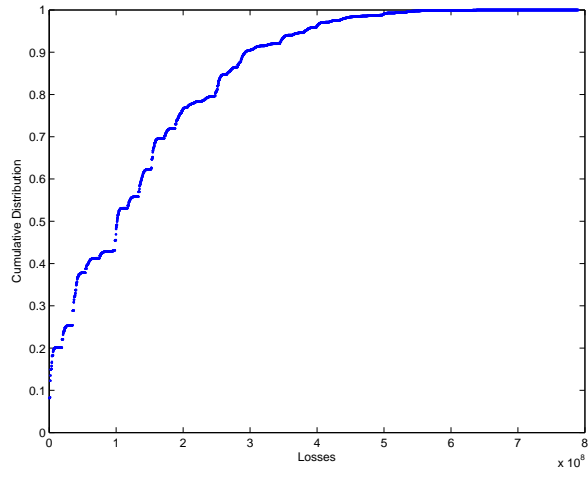

Figure 4: Distribution of losses for the Stress Case 1.

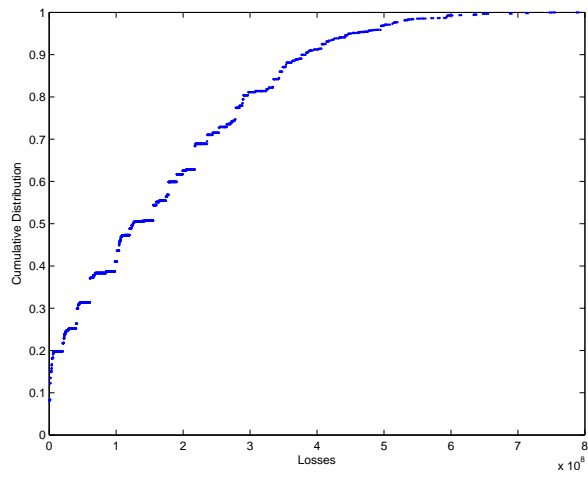16

Figure 5: Distribution of losses for the Stress Case 2.

Figure 6: Distribution of losses for the Stress Case 3.

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by the total losses L. In the table we can observe that although the Reference case and Stress Case 1 have the same $VaR(99)$ value, the other statistics of the distributions are significantly different, which means that the shapes of the distributions do matter. Obviously, the Reference Case and Stress Case 1 are very different from the remaining two cases, being much less critical. The most important inference that we can make based on the previous results is that apparently the loss distribution is much more sensitive to failure probabilities than to the interbank exposure network. It is also interesting to see that the *VaR/Total loss* measure behaves well as a measure of system fragility.

	Reference Case	Stress Case 1	Stress Case 2	Stress Case 3
Mean	683,431	710,966	129,744,000	167,476,000
Std. Dev.	5,753,762	6,371,700	122,875,000	153,211,000
Skewness	14.88	18.05	1.03	0.85
Kurtosis	295.71	457.42	3.60	3.15
VaR	27,718,638	27,718,638	499,317,801	594,429,329
Mean/L	0.09%	0.09%	16.42%	21.19%
VaR/L	3.50%	3.50%	63.19%	75.22%

Table 5: Summary statistics for the loss distributions.

6   Conclusions

The most important conclusions that we can extract from our work are: First, although the proposed network model to study systemic risk is very simple, it captures the essential elements to analyze systemic risk in its two main components; namely the initial shock and the ensuing financial contagion. Next, for banking systems with relatively few banks such as the case of Mexico, it is possible to estimate the distribution of losses for the financial by total enumeration using efficient computational tools. In larger banking systems one would probably have to resort to Monte Carlo simulation with reasonable accuracy. The model allows the researcher to investigate different aspects of systemic distress. We illustrated the model's flexibility by computing the distribution of losses in four cases of varying conditions of stress. Since the model is totally parameterized, it is a simple matter to study the system changing any of the parameters. For example a stress test could be performed by varying the banks failure thresholds as well as the other characteristics. It can also be used to determine the losses to different actors; i.e. banks creditors, depositors and the taxpayers as previously pointed out. In fact all of these types of losses can be estimated simultaneously. Another important finding is that the banking system's fragility is determined by bank default probabilities and the overexposed banks in the network. And apparently, the banking system is more sensitive to default probabilities than to Network structure (overexposed banks).

We are currently working on the relaxation of the independence of bank failures assumption, by including explicit failure correlations in the estimation

of the loss distribution. Since contagion comes in many forms, we believe that more research needs to be done in order to model losses due to contagion in a more realistic way .

References

[AG98] Franklin Allen and Douglas Gale. Optimal financial crises. *Journal of Finance*, 53:1245–1284, 1998.

[AG00] Franklin Allen and Douglas Gale. Financial Contagion. *Journal of Political Economy*, 108:1–33, 2000.

[BEST04a] M. Boss, H. Elsinger, M. Summer, and S. Thurner. An empirical analysis of the network structure of the austrian interbank market. *Oesterreichische Nationalbank Financial Stability Report*, pages 77–87, June 2004.

[BEST04b] M. Boss, H. Elsinger, M. Summer, and S. Thurner. The network topology of the interbank market. *Quantitative Finance*, 4:677–684, December 2004.

[BH00] Olivier De Bandt and Philipp Hartmann. Sistemic Risk: A Survey. Working Paper 35, European Central Bank, November 2000.

[BN02] M. Blavarg and B. Nimander. Interbank Exposures and Systemic Risk. *Sveriges Riskbank Economic Review*, 2:19–45, 2002.

[DD83] Douglas W. Diamond and Philip H. Dybvig. Bank runs, deposit insurance, and liquidity. *The Journal of Political Economy*, 91:401–419, June 1983.

[DN04] Degryse and Nguyen. Interbank exposures: An empirical examination of systemic risk in the belgian banking system. Discussion Paper 2004-04, Tilburg University, 2004.

[Ebo04] M. Eboli. Systemic risk in financial networks: A graph theoretic approach. Mimeo, Universita di Chieti Pescara, 2004.

[ELS06] Helmut Elsinger, Alfred Lehar, and Martin Summer. Risk assessment for banking systems. *Management Science*, 52:1301–1314, September 2006.

[fIS94] Bank for International Settlements. Annual report 1993-1994. Annual report, BIS, Basle, Switzerland, June 1994.

[Fur99a] Craig Furfine. The pricing of bank lending and borrowing: evidence from the federal funds market. BIS Working Papers 62, Bank for International Settlements, 1999.

[Fur99b] Craig H. Furfine. Interbank Exposures: Quantifying the Risk of Contagion. BIS Working Papers 70, Bank for International Settlements, June 1999.

[GDV06] Reint Gropp, Marco Lo Duca, and Jukka Vesala. Cross-Border Bank Contagion In Europe. Working Paper 662, European Central Bank, July 2006.

[GGLG05] Juan Pablo Graf, Sergio Guerrero, and Fabrizio Lopez-Gallo. Interbank exposures and contagion: An empirical analysis for the mexican banking sector. Mimeo, Banco de México, May 2005.

[GM04] Reint Gropp and Gerard Moerman. Measurement of contagion in bank's equity prices. *Journal of International Money and Finance*, 23:405–459, 2004.

[IdMP+05] Giulia Iori, Giulia de Masi, Ovidiu V. Precup, Giampaolo Gabbi, and Guido Caldarelli. A network analysis of the italian overnight money market. Discussion Paper Series 05/05, City University London, July 2005.

[Mul06] Jeannette Muller. Interbank credit lines as a channel of contagion. *Journal of Financial Services Research*, 29:37–60, February 2006.

[Nag03] Anna Nagurney. *Innovations in Financial and Economic Networks*, chapter Chapter 1: Financial and Economic Networks: An Overview, pages 1–25. Edward Elgar Publishing, 2003.

[NYYA06] Erlend Nier, Jing Yang, Tanju Yorulmazer, and Amadeo Alentorn. Network models and financial stability. *Journal of Economic, Dynamics & Control*, 2006.

[Rig01] Roberto Rigobon. Contagion: How to measure it? NBER Working Paper 8118, National Bureau of Economic Research, February 2001.

[RT96] Jean-Charles Rochet and Jean Tirole. Interbank Lending and Systemic Risk. *Journal of Money, Credit and Banking*, 28:733–762, November 1996.

[SM98] G. Sheldon and M. Maurer. Interbank Lending and Systemic Risk: An Empirical Analysis of Switzerland. *Swiss Journal of Economics and Statistics*, 134:685–704, 1998.

[Tso03] Dimitrios P Tsomocos. Equilibrium analysis, banking, contagion and financial fragility. Working Paper 175, Bank of England, 2003.

[Upp07] Christian Upper. Using counterfactual simulations to assess the danger of contagion in interbank markets. BIS Working Papers 234, Monetary and Economic Department of the Bank for International Settlements, CH-4002 Basel, Switzerland, August 2007.

[UW04] Christian Upper and A. Worms. Estimating Bilateral Exposures in the German Interbank Market: Is there a Danger of Contagion? *European Economic Review*, 48:827–849, 2004.

[VL04] Sbastien Vivier-Lirimont. Interbanking networks : towards a small financial world ? Cahiers de la Maison des Sciences Economiques v04046, Universit Panthon-Sorbonne (Paris 1), May 2004.

[Wel02] Simon Wells. UK Interbank Exposures: Systemic Risk Implications. *Financial Stability Review 2002*, pages 175–182, December 2002.

[WS98] Duncan J. Watts and Steven H. Strogatz. Collective dynamics of 'small-world' networks. *Nature*, 393:440–442, June 1998.

A   Algorithms

In Figure 7 we can observe the algorithm that we implemented to perform an exhaustive check of all the possible combinations of bank failures and the loss incurred by each of such cases.

```
1    Initialize()
2    powerSet = CreatePowerSetList()
3    noGoods = ∅
4    FOR EACH s ∈ powerSet DO
5       ResetThetas()
6       FOR EACH i ∈ s DO
7          θ_i = 1
8          L_i = 0
9          failure = TRUE
10         count = 1
11         exit = FALSE
12         IF s ∈ noGoods
13            exit = TRUE
14         END
15         WHILE (NOT(EXIT) AND (count < (‖V‖ − ‖S‖)) AND (failure)) DO
16            UpdateExposures()
17            FOR j ∈ V − s DO
18               IF (PII_j ≥ u_j) THEN
19                  θ_j = 1
20                   failure = TRUE
21               ELSE
22                  θ_j = 0
23               END
24            END
25            count + +
26            exit = CheckDefaults()
27         END
28         L_i = CalculateSystemicLost()
29         IF (exit) THEN
30            noGoods = noGoods ⋃ {i}
31         END
32      END
33   END
```

Figure 7: Financial Contagion Algorithm

Cross-sectoral stress-testing

Paper prepared for the IMF Expert Forum on Advanced Techniques on Stress-Testing,

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Abstract

The cross-sector dimension is increasingly important for macro stress-testing since interlinkages between financial sectors and the sensitivity to common risk factors have increased. This paper describes principles and tools for jointly stress-testing the banking, insurance and pension sectors through both bottom-up and top-down approaches. A distinction is made between methods to assess first and second round effects of shocks. Second round effects owing to indirect contagion between sectors are simulated by using the combined outcomes from multiple models. Direct cross-sectoral contagion is assessed by bottom-up tests for intra-group contagion. These simulate the effects of rising correlations and intra-group spill-overs within financial conglomerates. Cross-sectoral aspects of credit risk transfer are also stress-tested and the results are presented in this paper. The various stress-testing methods are applied to the Dutch financial sector.

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Introduction

The cross-sectoral dimension is increasingly important for stress-testing. Due to the rise of large and complex financial institutions, traditionally different sectors are increasingly being driven by *common risk factors*. Advances in risk management techniques, the use of complex financial instruments and increased reliance on financial markets have added to the commonalities of risk profiles of financial firms. This concerns banks and investment funds, but also insurance companies and pension funds. Moreover, product innovations have *blurred the distinctions* between financial sectors and hybrid financial products have induced a broad array of financial firms to compete with each other. Furthermore, *cross-sectoral interlinkages* have increased through financial innovation, for instance through risk transfers between sectors. These could be the transfer of credit risk from banks to other sectors, but also hedging of market risk by insurers and pension funds, with banks acting as counterparties.

DNB's institutional set-up has main advantages to address risks across financial sectors. In the Netherlands, prudential supervision is entrusted to the central bank; DNB is central bank and integrated prudential supervisor on banks, insurance companies, pension funds and investment firms.[1] This integrated set-up improves the cross-sectoral risk orientation, both on the micro as well as on the macro level. On a micro-level, intra-group risks of financial conglomerates can be assessed more closely, by combining the expertise from various supervisory angles. The integrated approach is also important for macro-prudential reasons, since the combined central bank and cross-sectoral prudential supervisor oversees the entire financial system (including the payment and settlement infrastructure) and has multiple instruments available to safeguard its stability. Macro stress-testing is one of these instruments.

Given the importance of the cross-sector dimension and DNB's institutional set-up, its macro stress-testing approach is cross-sectoral by nature. Section 1 illustrates this by describing the sector-wide stress-testing framework of DNB. Section 2 deals with the specific risks and stress-tests of financial conglomerates. Section 3 describes the cross-sectoral aspects of credit risk transfer and shows the results of a stress-test for these exposures.

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[1] The Netherlands Authority for the Financial Markets (AFM) is responsible for regulating behaviour on the financial markets.

1. Sector-wide macro stress-test

DNB applies both top-down (centralised by in-house models) and bottom-up stress-tests (decentralised by involving the institutions) across all sectors. Both methods are complementary and comparing their outcomes can provide both additional insight as well as a cross-check.

1.1 DNB's macro stress-testing framework

Concerning the *top-down* method, DNB uses simple reduced form models for the banking sector (Van den End et al, 2006). These explain credit risk and interest rate risk out of some key macro economic variables. The models are used to quantify the (first round) impact of the macro scenarios on the financial sector, through deterministic and stochastic simulations. The latter produce distributions of losses and income projections for banks. To simulate (second round) contagion effects in the banking sector DNB has developed an interbank contagion model (Van Lelyveld and Liedorp, 2006) and has applied the Bank of Finland Payment and Settlement System Simulator (Ledrut, 2007). For the pension sector DNB uses the Pension Asset and Liability Model (PALMNET, Van Rooij et al, 2004). This model can test the sensitivity of pension funds' solvency to shocks in equity returns, interest rates, asset mix, etc. For the insurance sector, DNB is currently developing a reinsurance contagion model (Van Lelyveld, Liedorp and Kampman, forthcoming). Hence, DNB uses a suite of models to analyse the risks to the financial system top-down. These are partial approaches and developed for a single sector. Ideally the models should be integrated to detect the interlinkages between financial sectors.

The *bottom-up* approach of DNB is more integrated across sectors. In this approach, DNB regularly requests the main Dutch banks, pension funds and insurance companies to run the same set of scenarios with their internal models. DNB prepares the scenarios with a view to obtaining an overall picture and aggregates the outcomes to the level of the financial system as a whole.

Both the top-down and the bottom-up methods have their merits. The top-down stress-tests provide for a greater comparability of the individual firm outcomes and are flexible to run different scenarios. Bottom-up stress-tests have the advantage that the involvement of financial firms raises their risk awareness and gives the supervisor deeper insight into their risk management. Bottom-up stress-tests are also closely aligned to the risk profile and management of the individual firms. Hence, they produce distributions of outcomes which reflect differences in risk sensitivities of firms and sectors (besides differences in assumptions or interpretations by the firms when performing the tests). Both methods however have their limitations with respect to simulating second round effects. In bottom-up stress-tests the firms involved usually have no models in place which take into account economic or

financial market effects that may reverberate on them. With regard to top-down methods, usually there is a set of models needed to assess potential second round effects (Haldane et al, 2007).

Comparing the top-down and bottom-up outcomes of the Dutch banking sector reveals that the direction of the impact of the scenarios is similar for both approaches (Van den End et al, 2006). However, the level of losses turns out to be higher in the bottom-up stress-tests. One reason for this is that the results are driven by extreme outliers, which are not produced by the top-down model, since this assumes the same marginal effects for each bank. Other reasons are that the bottom-up tests include dynamic effects in the banks' balance sheets (i.e. respect to new production, re-investments etc.) which are hard to capture in top-down models, while the latter are also based on annual data, which do not capture intra-year effects.

1.2 Principles for sector-wide tests

DNB applies various principles to enhance a consistent approach of macro stress-testing across financial sectors.

° Design scenarios that are relevant for different financial sectors. Scenarios should be directed to the main risks in each sector. In case of banks these are for instance upward interest rate shocks, owing to their net long positions in their loan books. Pension funds and insurers are sensitive to declining rates due to their negative net duration. Hence, to stress-test different sectors DNB has developed scenarios with both downward and upward shifts of interest rates.

° Apply identical scenarios across sectors. Although the scenarios are designed to capture sector specific risk profiles, they should be uniformly applied to all sectors. This provides a picture of the system's resilience to particular scenarios and identifies weak spots. However, applying the multi-year and multi-factor scenarios uniformly across sectors is not always consistent with market practices. For instance, to stress test their trading books, banks use sensitivity tests with a 10 days horizon, while institutional investors usually apply a horizon of one year for their market risks. These differences could be addressed by complementing the uniform scenarios with sector-specific sensitivity tests.

° Instruct the firms carefully in bottom-up tests to enhance a common understanding and application of assumptions. In particular the assumptions used by firms to quantify interest rate risk (model risk) may impinge upon the comparability of the outcomes. Insight in the assumptions is important to interpret the stress test results across firms and sectors. Therefore, DNB asks the firms to report their assumptions for prepayment behaviour, dynamics of balance sheet items, hedging policy etc., in addition to the quantitative results.

° Harmonize the output indicators of the stress-test as much as possible. This could be done by requiring the different sectors to report the impact of the scenario on core indicators that are common across sectors, such as capital and assets. The comparability of indicators is further

improved when valuation techniques are harmonised, for instance by asking firms across different sectors to report changes in market values. Where possible, bottom-up stress-test results should be reported in common templates.

° Aggregate the results for each sector to enable a cross-sectoral comparison. This might reveal vulnerable spots in the system under uniform scenarios and indicate potential cross-sector contagion. This could be the case if a sector that is closely linked to another sector shows large losses.

1.3 First round effects

Stress-testing the first round effects of shocks usually refers to estimating the impact of macro-financial scenarios on the financial sector. For assessing these effects, DNB applies a sector-wide approach in its bottom-up stress-tests. This can be illustrated by a recent stress-test (2006) in which five large Dutch banks, five large pension funds and three large (predominantly life) insurers calculated the impact of uniform scenarios prescribed by DNB. These involved sensitivity tests for market risk, in which the most extreme case assumes full correlation of risk factors. This resembles a scenario in which risk aversion in financial markets rises sharply and asset prices fall in tandem. In addition, two multi-year, multi-factor scenarios were developed, representing downside deviations from the base line. In these hypothetical scenarios an extreme but plausible concurrence of shocks was assumed. The 'malaise-scenario' implied a sharp fall in long-term interest rates and a flattening yield curve. In the hypothetical 'global correction scenario', the dollar, equity prices and house prices fell worldwide, while bond yields rose strongly (DNB, 2007).

An overview of the effects on the overall Dutch financial system was obtained by aggregating and comparing the outcomes of the three sectors (in terms of first-round effects). The outcomes reveal differences in risk profiles and risk sensitivities. In case of the extreme sensitivity test with full correlation between risk factors, the pension sector appeared to be most vulnerable (Chart 1). On average, pension funds would lose over 100% of their surplus capital. This reflects their relatively large exposures to market risk, through the large interest rate mismatch and equity holdings. The insurance sector appeared to be less sensitive than pension funds, due to their smaller interest rate mismatch and more conservative asset mix. The banks turned out to be least vulnerable to shocks in market risk factors; the full correlation scenario would cost less than 5% of their own funds. The main reason for this is that Dutch banks' trading book exposures are relatively small, while these risks are mostly hedged. All in all the results indicate that supervisory action in case of serious market corrections will be most needed in the pension sector.

Chart 1, Sensitivity to risk-aversion
Decline of own funds (%) due to correlated shocks in various risk factors
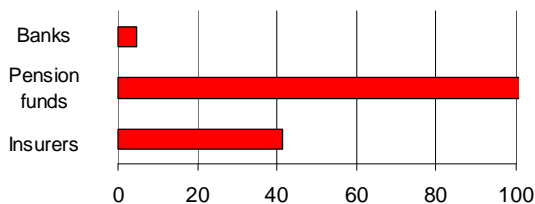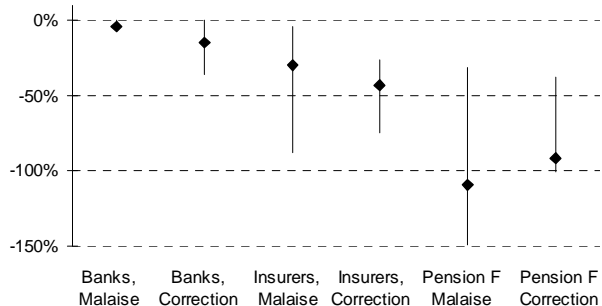
Chart 2, Scenarios: impact on capital
Percentage of 2005 own funds, 3 years horizon (range of outcomes across firms, diamond shows the aggregate weighted impact)

The sectoral differences also appeared from the multi-year hypothetical scenarios (Chart 2). The outcomes suggest that Dutch banks have sufficient capital buffers to withstand both a scenario with a strong rise of interest rates ('global correction scenario') and a fall of rates ('malaise-scenario'). Even the capital of the bank that was hardest hit would decline by less than 40%. The insurance and pension sectors would be more severely affected, although differently in each scenario. While pension funds are worst hit by the malaise scenario with steep declines in bond yields, insurers are more sensitive to a scenario with falling equity and real estate prices, as in the global correction scenario. This illustrates that for the insurance sector the impact of equity risk is stronger than interest rate risk and vice versa for the pension sector. Overall, insurers are less vulnerable to the stress scenarios than pension funds, because their balance sheet mismatches are smaller and they more actively hedge their investment risks.

1.4 Second round effects

Next to identifying the first round effects of shocks (originating from the economy or financial markets) on financial sectors, potential second round effects due to contagion between financial sectors are relevant as well (Chart 3). Firstly, these effects could result from *direct* cross-sectoral contagion through exposures or other financial linkages. Secondly, behavioural reactions of firms in one sector could affect the economy or the financial markets with subsequent *indirect* feedback-effects on other sectors.

6

Chart 3, Cross sector contagion effects

Indirect contagion   *Direct contagion*
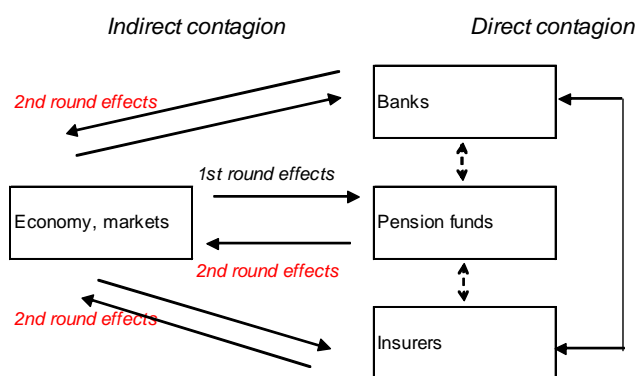
Direct contagion between banks and insurers is possible through counterparty exposures, ownership links and risk transfers. There is empirical evidence for cross-sector contagion between the two sectors. In the UK, the most important channel for spill-overs appeared to be banks' ownership of life insurers (Stringa and Monks, 2007). The same study concludes that *indirect* contagion through links in capital markets and confidence were not found to be significant. A previous study by DNB shows that co-movements of equity returns of insurance companies explain co-movements of bank equity returns.[2] This interdependence was found to be highest in times of stress: the joint occurrences of extreme returns is stronger than of average returns. Moreover, when extreme co-movements of market prices become more systemic within one sector, the probability of systemic co-movements in the other sector increases (Chart 4).

Chart 4, Co-exceedance response curves, bank & insurance sectors
P (x-axis) is the number of negative (co-)exceedances of insurers affecting
the probability of negative (co-)exceedances of banks (y-axis)

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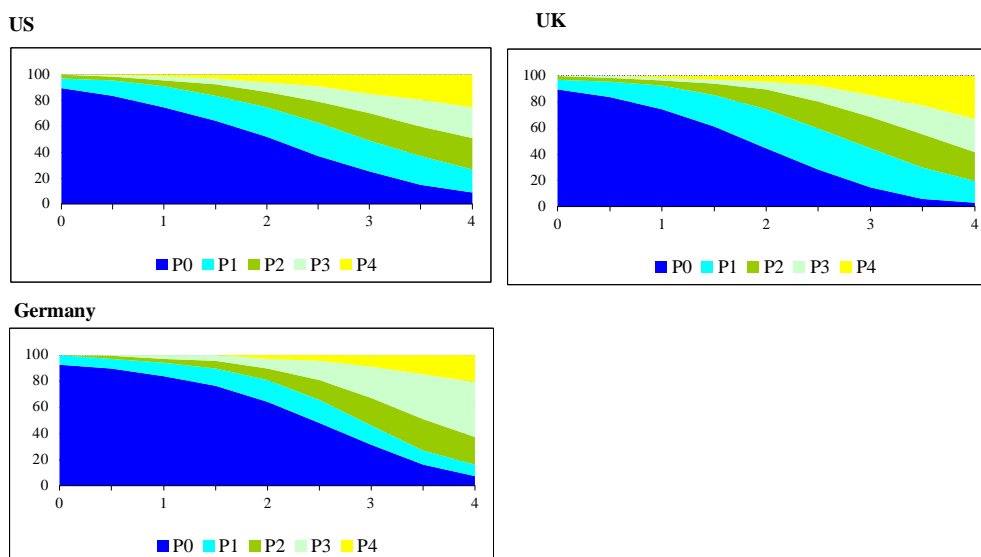
[2] Co-movements are meant to be the number of days at which firms show extreme returns at the same time (Minderhoud, 2003).

Stress-tests for direct contagion between banks and insurers are described in sections 2 and 3. With regard to *indirect* contagion, DNB does not use a model approach to simulate behavioural reactions by banks and insurers in response to shocks, but, as part of the bottom-up stress-tests, these institutions are surveyed about their likely responses to a scenario. This delivers qualitative information which helps to assess the risks of indirect contagion. For instance, if firms collectively would respond with similar measures this could engender second round effects on the economy and financial markets which could feed back into the financial sector (Chart 5).

Chart 5, Stress-testing cross sector contagion effects
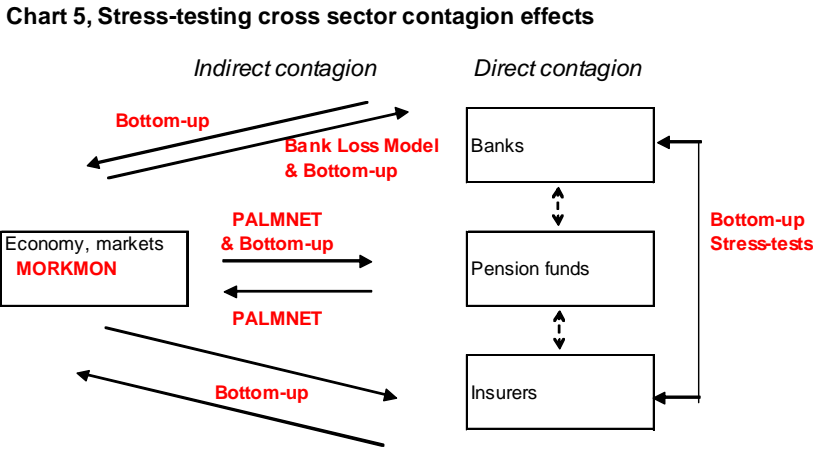
Pension funds have little *direct links* with other financial sectors, beyond banks and insurers being sponsor of their own pension fund and counterparty exposures in the credit risk transfer market (section 3). Hence, identifying the pension funds as the most vulnerable sector in the stress-tests is comforting from the perspective of cross-sector contagion. Potential second round effects from behavioural responses of pension funds on the economy or the financial markets (*indirect* contagion) could be material though. By raising pension contributions or limiting the indexation of benefits, the responses of pension funds can affect disposable income and hence economic growth. Besides, financial markets could be influenced by changes in pension funds' investment strategies.

Subsequent risks for *indirect contagion* from the pension sector to other sectors can be assessed by combining the outcomes of different models (Chart 5). PALMNET produces first round effects of macro shocks on pension funds' solvency position and simulates possible policy reactions by pension funds. These policy measures can be used as input in MORKMON (DNB's macro-econometric model) to simulate their possible feedback effects on the economy and markets (see Box). The impact of economic effects on other sectors (indirect cross-sector contagion) can be simulated by using other models. In case of banks, the indirect impact on credit risk can be estimated by plugging a GDP-shock in DNB's bank loss model. This is an illustration of how DNB uses multiple models to simulate

(indirect) second round effects on sectors (Van den End et al, 2006). Disadvantages of not having one single model could be that the ultimate estimations are inefficient and probably not very precise.


Simulating second round effects from the pension sector

Simulations by PALMNET indicate that a decrease of equity risk premium or short term interest rates by 1 percent lead to a decline of the nominal funding ratio of the average Dutch pension fund by around 10, respectively, 30 percent points in the long run (Kakes and Broeders, 2006). In response of these first round effects, pension funds may adjust their premium, indexation or investment policies. According to PALMNET this leads to a long run increase of pension premiums in the Netherlands by 5 percent points. The impact of this shock on the Dutch economy (the second round effect) can be simulated by MORKMON. This model estimates that a 5 percent point rise of pension premiums would have a negative effect on GDP volume of 1.5% in the long run. By plugging this GDP shock in DNB's model for bank losses, it turns out that credit losses would erase around 1% of the capital of Dutch banks.

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2. Intra-group contagion

A main channel for direct contagion between banks and insurers is contagion within financial conglomerates (FC), which combine both activities. Within Europe there are around 85 systemically relevant FCs[3]. On the one hand, diversification of bank and insurance activities leads to risk reduction. This is found in many studies (for example in Boyd et al, 1993, Lown et al, 2000, Bikker and Van Lelyveld, 2003). Risk reduction follows from the imperfect correlation of risk drivers and off-setting exposures, for instance opposite interest rate positions of banks and insurers. Simulations show that FCs are substantially less sensitive to interest rate shocks than individual banks and insurers (Chart 6). However, in times of stress the correlations between risk factors may rise which would result in an underestimation of the default risk of an FC. This can be called a *weak form of conglomerate risk*, which will be further explained in section 2.1. A FC may also be less stable than its constituting parts. This can be the case if the group is threatened with a loss due to problems in one of its constituent parts. Such potential intra-group contagion can be called a *strong form of conglomerate risk*, which will be further explained in section 2.2. Both weak and strong conglomerate risks can be stress-tested.

Chart 6, Diversification effects fin. conglomerates
Change profits fico as percentage of change profits bank

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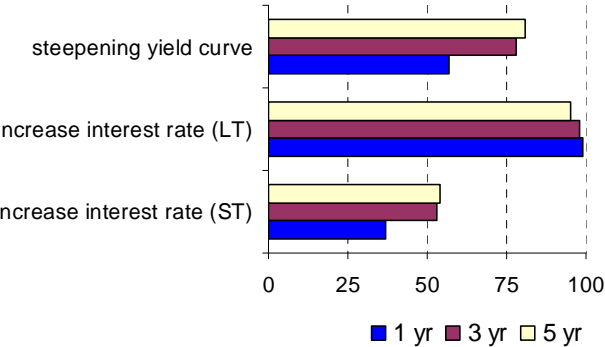
[3] As identified on the basis of the EU Financial Conglomerates Directive and total assets.

2.1 Correlation in times of stress

Correlation between risk divers can be measured by economic capital models in two ways (Van Lelyveld (ed.) 2006).

°   <u>Statistical approach</u> in which diversification effects are estimated by using a correlation matrix. A primary challenge is to determine the correlation values used. A recent survey among large FCs indicates that they account for diversification effects in their economic capital ranging from 30 to 60%[4]. However, in times of stress it is more appropriate to use tail or stress correlations than average correlations. Averages measure linear dependency which does not include extreme cases.

°   <u>Scenario analysis</u> which determines diversification benefits by a set of scenarios that provide the combined effect of different risk drivers. The challenge here is to select shocks that result in a set of worst case scenarios that match a certain probability.

For macro stress-testing DNB has applied a statistical approach (which individual firms applied in their economic capital models). The stress-test was conducted bottom-up, by asking three large Dutch FCs to calculate their economic capital on a group level, based on the stress assumption of full correlation between all risk factors. By comparing these outcomes with economic capital based on average correlation in normal times, the impact of rising correlation during stress was measured. Chart 7 shows that the unexpected loss would rise substantially in times of stress. Economic capital increases by 10 to 40% if correlations increase to 1. These percentages come close to the diversification effects as estimated by Oliver, Wyman & Co.[5]

Chart 7, Diversification effects

EUR bn, Economic capital

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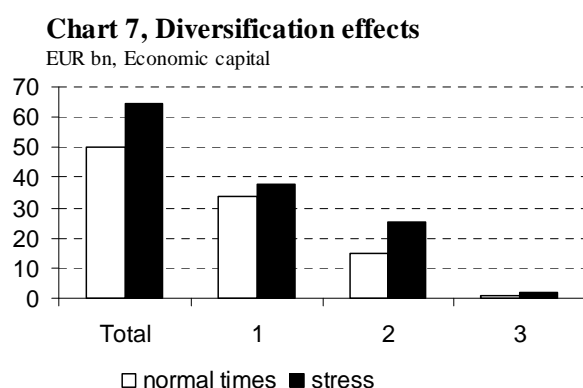
[4] Chief Risk Officer Forum, June 2005. These are diversification effects resulting from moving from a simple sum of the parts to a fully diversified capital requirement.
[5] Level II diversification i.e. between risk factors for banks 15% and insurers 25-35%; level III diversification i.e. between business units 5-10% (OWC, 2001).

These stress-test outcomes support the approach of supervisors not to allow for diversification benefits in the capital requirements for FCs. Besides supervisory concern about the plausibility of the parameter values (e.g. correlation) under stressed conditions, risk models may also be incomplete, e.g. not all risk factors are included. For instance the risk of intra-group contagion.
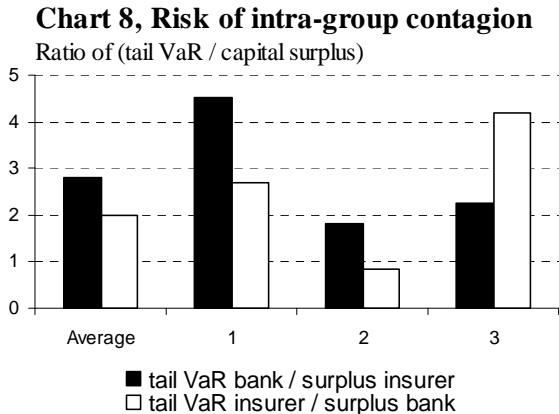
2.2 Intra-group contagion

FC risks are contained by legal restrictions, prudential limits for intra-group exposures and internal firewalls. These procedures within a FC should prevent contagion across entities (Freshfields, 2004). However, internal firewalls could crack or be ignored by the public, when financial difficulties in one of the entities have consequences on another entity. Intra-group risks may arise through various direct and indirect contagion channels, such as:

° Accounting losses, if losses in one entity burden the profitability of other entities or the group. The 2002 stock market crisis illustrated this risk, as write-downs of the insurers' share holdings showed up in P&L accounts of bank-insurance companies.

° Intra-group support, through loans, guarantees or intra-group hedging of investment risks. Entities can also be supported by the transfer of surplus capital from the (bank)holding or via transfers of profits through dividend payments. This might be accompanied by moral hazard risks, if non-bank entities count upon support by the bank in case of problems, or by the deposit insurance scheme implicitly supporting the whole conglomerate (Freixas et al, 2005).

° Reputation risk, through negative developments in one entity affecting other entities or the whole group through loss of goodwill or deteriorating funding conditions. This again can be illustrated by the 2002 stock market crisis when the weakened financial position of the insurance entity of some FCs tarnished the reputation of the banking entity, which faced a higher credit spread as a consequence. Loss of reputation presents a risk to a bank in particular, since it has a more vulnerable financing profile than an insurer, owing to its dependency on short-term funding (which in case of deposits are of a first come, first serve nature).

° Legal risk, if the parent (bank) is held responsible for obligations of her entities this could 'pierce the corporate veil' (Freshfields, 2004). The court could enforce this if the conglomerate structure has been misused, for instance in case of fraud or if a branch acts as an agent of the parent (e.g. in case an entity uses its parent's guarantee in marketing).

DNB has designed a stress-test for intra-group contagion in FCs (i.e. the strong form of conglomerate risk). The underlying scenario did not describe the type of intra-group contagion, but is a what/if-type sensitivity test. It was conducted by asking three large Dutch FCs (as part of the bottom-up macro stress-test) to estimate the tail-VaR for the bank and the insurance entity, e.g. the expected size of a shortfall below the minimum capital requirement, assuming that such a shortfall occurs. From comparing the tail-VaRs with the actual capital surplus within the entities, an impression was gained

of the absorption possibilities within the group, should the bank or the insurer be unable to meet the minimum requirement at some point in time. The underlying assumption that one entity incurs an extreme loss, while the other does not, may be seen as a mechanical (probably not very realistic) first-round effect of a component-specific shock. Second-round effects could of course also threaten the stability of both components of a financial conglomerate.

The outcomes of this stress-test show that none of the FCs could compensate an extreme loss in its banking entity with the capital buffer of the insurer (loss bank / capital surplus insurer is larger than 1 in all cases, Chart 8). This means that extreme shocks at the banking side in all instances lead to problems for the group as a whole. With two institutions the possibility that the banking entity supports the insurance part is larger (white bar is smaller than black bar in Chart 8). One firm could compensate the total loss at the insurance side with the capital buffer of the bank (loss insurer / capital buffer of the bank is smaller than 1). For one FC the possibility that the insurance entity can support the bank is lager than vice versa (white bar is larger than black bar in Chart 8).

Chart 8, Risk of intra-group contagion

Ratio of (tail VaR / capital surplus)

■ tail VaR bank / surplus insurer
□ tail VaR insurer / surplus bank

3. Cross-sector contagion, the case of CRT

Contagion between financial sectors has potentially increased with the rapid growth of credit risk transfers (CRT). Credit risk is increasingly being transferred across sectors, by the use of credit derivatives and structured credits. According to the British Bankers' Association the importance in these markets of non-bank counterparties has risen over the last years (Chart 9). Of the non-bank sectors, insurers and pension funds are the second and the third largest sellers of credit protection, after hedge funds. Studies suggest that CRT affects the stability of banks and insurers and that these transactions are a potential source of spill-over effects between both sectors (Chan-Lau and Ong, 2006, Bernoth and Pick, 2007). This is indicated by the recently increased dependence of distance to default measures of banks and insurers (Table).

Chart 9, Sellers of credit protection

Percentage (share of CDS market)
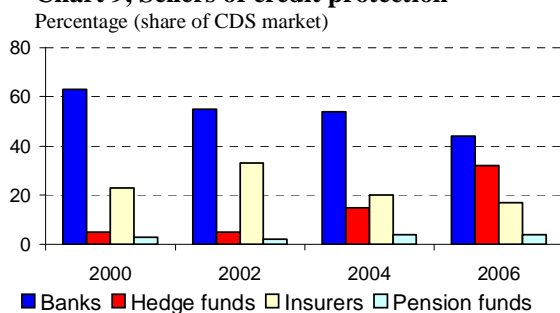
Source: British Bankers' Association, 2006

Cross-section dependence Banks & Insurers		
correlation coefficient of distance to default		
	1992-05	2002-05
All	0,27	0,36
North America	0,47	0,57
EU-15	0,28	0,46
Japan/Korea	0,36	0,47
Source: Bernoth and Pick, 2007		

3.1 Stress-test scenario

To assess the risks related to CRT exposures across different sectors, DNB performed a specific macro stress-test in 2007. Herein, the Dutch banks, insurers and pension most active in the CRT market ran a uniform scenario, embodying both elements of a market crisis and risks at individual firms. It has parallels with the recent crisis in credit markets (although it has been developed earlier this year), of which it could be seen as a hypothetical extreme outcome, affecting Dutch financial firms. By this, the stress-test is an attempt to simulate a tail event, which structural models can not capture since in extreme situations existing relationships may break down or become non-linear.

The scenario assumed that a large investment bank active in the credit derivatives market defaults and contests its obligation to make payments under credit default swap (CDS) contracts. This leads to general, market-wide uncertainty about the legal status of CDS and securitisation contracts. In this climate, market liquidity dries up, CDS spreads soar, with default correlations between different credit tranches rising to 1. Dutch firms are being hit by assuming that the investment bank concerned is their largest counterparty in the CDS market. As a consequence they partly lose their credit risk protection. Besides, the market turbulence impacts them along various channels; it is assumed that liquidity facilities dry up and the value of their credit exposures and collateral fall. Furthermore, it is assumed that liquidity facilities of existing conduits supplied by the institutions are fully drawn upon. Due to market-wide doubts about the quality of assets in SPVs, it is further assumed that the Dutch firms are forced to unwind their latest securitisation transaction and take back the related assets.

The scenario has been applied by the participating firms on their credit exposures that would be affected. For the banks these are exposures in the trading and banking books (among which loans in the banking book that are eligible for trading according to the originate-to-distribute model). The pension funds and insurers included the credit exposures in their investment portfolios. Their main investments in credit risk relate to structured credits like MBS, CDOs and CLOs. These exposures

indicate that institutional investors have taken over credit risks from the banks (Chart 10). Credit derivatives are another instrument for a cross-sectoral transfer of risk. However, the CDS-positions of the firms participating in the stress-test do not indicate a clear distribution of credit risk from the banks to the pension funds and insurers, since positions bought and sold are nearly balanced (Charts 11, 12). This would appear to limit the risk of direct contagion between sectors through counterparty exposure.

Chart 10, Credit exposures Dutch firms
Percentag of total assets (weighted average per sector)

Chart 11, CDS positions banks
CDS notional outstandings as percentage of total assets (weighted average)
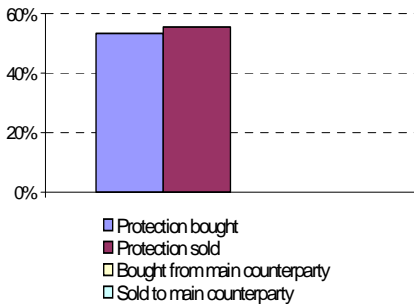
Chart 12, CDS positions IC & PF
CDS notional outstandings as percentage of total assets average)

□ Bonds* ■ Structured credits □ Loans
* Corporate and emerging market bonds

□ Protection bought
■ Protection sold
□ Bought from main counterparty
□ Sold to main counterparty

□ Protection bought
■ Protection sold
□ Bought from main counterparty
□ Sold to main counterparty
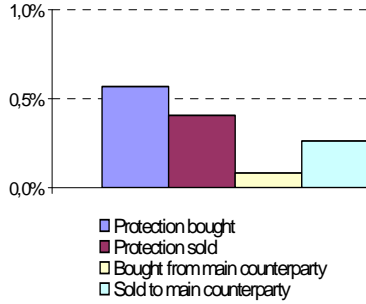
3.2 Outcomes

The stress-test appeared to have the most impact on the banks, due to their relatively large credit and liquidity exposures to the CRT market compared to the pension funds and insurers (Chart 13). On average, the banks appear to have sufficient capital buffers to withstand the scenario, but there is a wide dispersion of the impact among individual banks. These differences are explained by banks' relative open credit exposures and their treatment of loans in the banking book (for banks that do not earmark their loans for trading and do not apply fair value valuation, the impact is partly hidden). The extent to which firms use instruments for credit protection influences the outcomes as well. In case of the banks the average credit risk hedge declines by one fifth due to the reduced effectiveness of credit derivatives and securitisation in the scenario. On average, the reduced credit protection has a larger impact on the capital ratio of the banks than the increase of credit spreads (Chart 14). In case of the pension funds and insurers, more than half of the negative impact is caused by the revaluation of structured credit portfolios (Chart 15). Since these were mostly originated by banks, this adverse impact indicates the potential losses relating to the transfer of credit risks between sectors.

Chart 13, Impact scenario on sectoral balance sheets
Scenario effect* relatve to own funds

Chart 14, Decomposition impact op BIS ratio
Contribution to change of BIS ratio, weighted average
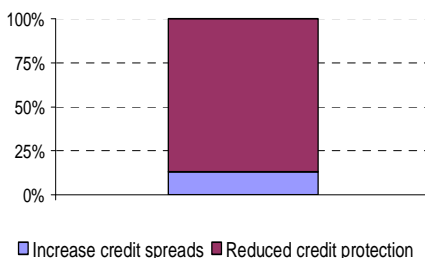
Chart 15, Impact on credit exposures IC & PF
Share in total loss (weighted average)

* Impact on P&L and revaluations of available for sale instruments and investments
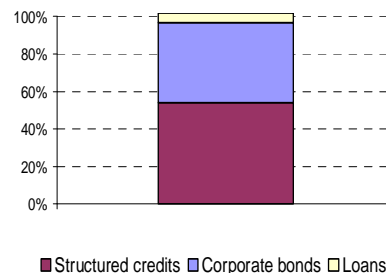
The liquidity of financial firms is also affected by the scenario, which assumed that funding and market liquidity dry up. Although this reduces the liquidity ratio of the banks, they appeared to have sufficient liquidity buffers to meet the additional liquidity needs. For the pension funds and insurers, (funding) liquidity risk was not an issue since they barely had short term liabilities while most of their assets are highly liquid. This illustrates that although credit risks have been spread in the financial system, banks remain vulnerable to liquidity risks stemming from the CRT-market. This owes to the banks' dependency on market liquidity to trade credit and to their role as liquidity provider, for instance to structured credit vehicles (which has come to the fore in the recent crisis on the credit markets).

3.3 Second round effects

Most firms participating in the stress-test indicated that they would not respond to the scenario by behavioural actions. Main arguments for this were the limited initial impact on them and the expectation that they would be able to enter the market again to recapitalise after the scenario horizon. As a consequence, DNB was not able to quantify any potential second round effects of the scenario related to behavioural reactions. However, the assumptions of participating firms could have major flaws. The initial impact on individual firms could for instance lead to rating downgrades which will worsen funding terms more that was assumed. Moreover, the scenario effects on the financial system could be reinforced by collective reactions of market participants. For instance by banks which hoard liquidity or increase their demand for liquidity in the interbank market. This would raise interbank rates and could lead to a drying up of the interbank market (as witnessed recently). Such second round effects could undermine the effectiveness of the risk management of financial firms and might force them to additional responses. This has been an important element in DNB's feedback to the financial institutions with the purpose of improving their risk awareness and risk management capacities.

4. Conclusion

First round effects of shocks on different financial sectors can be measured by well-established methods for bottom-up and top-down stress-testing. To apply these consistently across sectors, we defined some principles in this paper. Tools are also available to stress-test the second round effects arising from *direct* contagion between sectors, e.g. through intra-group linkages and the credit risk transfer market. Challenges remain for stress-testing *indirect* contagion effects between financial sectors, resulting from behavioural reactions of one sector feeding back on other sectors, through disturbances in the economy or financial markets. There is not a single top-down model to stress-test these risks and using multiple models in stead could lead to inefficient estimates. The bottom-up approach also has its shortcomings, since firms usually can not foresee their actions in stress situations. Nevertheless, understanding the vulnerabilities of firms to market disruptions, and reduced market liquidity in particular, has become more important, since financial firms increasingly rely on markets for their funding and portfolio management.

References

Bikker and Van Lelyveld (2003), 'Economic versus regulatory capital for financial conglomerates', in: Kuppens, Prast en Wesseling (eds.), Supervision on a cross-road, Edward Elgar, 169-216 (plus Reply, 222-223).

Bernoth and Pick (2007), 'Linkages between the banking and insurance sector', DNB Working Paper Series X.

Chan-Lau and Ong (2006), 'The Credit Risk Transfer Market and Stability Implications for UK Financial Institutions', IMF Working Paper, no. 139.

DNB (2007), Overview of Financial Stability in the Netherlands, March 2007.

Freixas, Loranth and Morrison (2005), 'Regulating Financial Conglomerates', Oxford Financial Research Centre Working Paper No. 2005-FE-03.

Haldane, Hall and Pezzini (2007), 'A new approach to assessing risks to financial stability', Financial Stability Papers no. 2.

Kakes and Broeders (2006), 'The sustainability of the Dutch pension system', DNB Occasional Studies No. 6.

Minderhoud (2003), 'Extreme co-movements of financial institutions: contagion or interdependence', DNB Meb series No. 2003-16.

Ledrut (2007), 'Simulating retaliation in payment systems. Can banks control their exposure to a failing participant?', DNB Working Paper Series 133.

OWC (2001), 'Study on the risk profile and capital adequacy of financial conglomerates', London: Oliver, Wyman & Company.

Stringa and Monks (2007), 'Inter-industry contagion between UK life insurers and UK banks: an event study', Bank of England Working Paper no. 325.

Van den End, Hoeberichts and Tabbae (2006), 'Modelling scenario analysis and macro stress-testing', DNB Working Paper Series 119.

Van Lelyveld and Liedorp (2006), "Interbank Contagion in the Dutch Banking Sector: A Sensitivity Analysis", International Journal of Central Banking, vol. 2(2).

Van Lelyveld (ed.) (2006), 'Economic Capital Modelling: Concepts, Measurement and Implementation, Risk'

Van Lelyveld, Liedorp and Kampman, forthcoming

Van Rooij, Siegmann and Vlaar (2004), "A Pension Asset and Liability Model for the Netherlands", DNB Research Memorandum Wo 760.

A New Risk Indicator and Stress Testing Tool:
A Multifactor Nth-to-Default CDS Basket

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Abstract

This paper presents a market-based indicator for financial sector surveillance using a multifactor latent structure in the determination of the default probabilities of an nth-to-default credit default swap (CDS) basket of large complex financial institutions (LCFIs). To estimate the multifactor latent structure, we link the market risk (the covariance of the LCFIs' equity) to credit risk (the default probability of the CDS basket) in a coherent manner. In addition, to analyze the response of the probabilities of default to changing macroeconomic conditions, we run a stress test by generating shocks to the latent multifactor structure. The results unveil a rich set of default probability dynamics and help in identifying the most relevant sources of risk. We anticipate that this approach could be of value to financial supervisors and risk managers alike.

I. INTRODUCTION

The costly financial crises of the 1990s sparked the interest of supervisory agencies and central banks in developing a broader understanding of financial markets and institutions through macro-prudential analysis. Such analysis is intended to complement the micro analysis of individual institutions, as it aims at unveiling aggregate risks emerging from common shocks and risk correlations across institutions (Crocket, 2000). Large complex financial institutions (LCFIs) play a key role in the stability of global financial markets, and, as such, their surveillance has features of both micro- and macro-prudential analysis. The health of LCFIs can be analyzed by looking at levels and trends in financial soundness indicators, also referred to as macro-prudential indicators.

Financial soundness indicators are based on balance sheet information usually published quarterly, semi-annually, or annually.[2] A problem with these indicators is their use of lagged, historical information based on balance sheet items, which represent a decreasing proportion of LCFIs activities. To analyze forward-looking information, prices of debt, equity, and derivatives have been proposed in devising early warning indicators of bank performance.

One of the most widely used market-based indicators is distance-to-default, which is based on Merton's seminal contribution (Merton, 1974). Other market measures are based on spreads on (primary and secondary market) subordinated debt issued by banks.[3] A problem with the distance-to-default indicators is that they also need some information on balance sheet items and, therefore, they only partially reflect current market information. Additionally, the information content of secondary markets on senior- and subordinated-debt spreads is also hampered by insufficient liquidity in bank bond markets. For these reasons, such indicators have a limited role as timely early warning measures of risks and vulnerabilities emerging in financial institutions.

This paper develops a market-based indicator for financial sector surveillance using a basket of credit default swaps (CDSs). It generalizes the approach taken in Avesani (2005) in two main directions. First, it determines and analyzes the multivariate latent factor structure which underpins the LCFIs' correlation dynamics. By doing so, we move from a framework in which the risk factor sensitivities are the same across institutions and regions, as in the capital asset pricing model (CAPM), to one in which the multifactor risk sensitivities are institution-specific, as in the arbitrage pricing theory

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[2] The International Monetary Fund (2003) has developed a core set of financial soundness indicators covering the financial sector.

[3] See, among others, Flannery and Sorescu (1996) and, more recently, Gropp, Vesala, and Vulpes (2002).

(APT).[4] Second, it uses the identified latent factor structure for conducting stress tests in a coherent fashion. Specifically, the risk profile of each institution and of the entire group of LCFIs is stressed through shocks applied to the default correlations and to the values of the identified factors.

The paper is organized as follows. The next section provides a description of the market-based indicator. Section III briefly describes the methodology for computing the probabilities of default of a CDS basket in a multifactor framework. This section also presents the econometric estimation of the latent factor structure through factor analysis. Section IV describes the CDS spread data and Section V shows the results of the factor analysis estimation. Section VI shows the results of the computation of the default probabilities for different horizons. Sections VII and VIII contain the sensitivity analysis and stress testing of the probabilities of default to shocks in the correlation and factor structures. Section IX concludes.

II. DESCRIPTION OF THE INDICATOR

Many studies on macro-prudential analysis have been based on the lessons learned from the banking crises of the 1980s and 1990s. In this paper we take a more financial-oriented approach by focusing on the information content, relevant for financial stability analysis, of an nth-to-default CDS basket.[5] An nth-to-default CDS basket is the simplest example of a collateralized debt obligation (CDO). A CDO is the securitization of a pool of debt obligations, generally corporate debt, into classes (i.e., the "tranches") of securities with various levels of exposure to the underlying credit risk. The CDO exposure to the underlying pool of debt securities can be direct, that is, a cash transaction where the CDO owns the actual debt securities (cash CDO), or indirect, that is, a synthetic transaction where the CDO writes CDSs on a pool of corporate names or asset backed securities (synthetic CDO).

In a synthetic CDO, the reference portfolio is made up of credit default swaps (CDSs). Much of the risk transfer that occurs in the credit derivatives market is in the form of synthetic CDOs. Understanding the risk characteristics of synthetic CDOs is important for understanding the nature and magnitude of credit risk transfer. In this paper, the synthetic CDO is composed by the actively traded CDSs of 15 large complex financial institutions (LCFIs).

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[4] See Berndt and others (2005) for an alternative approach to model firm-specific sensitivities to risk factors.

[5] CDSs are financial contracts where the financial guarantor agrees to make a payment, sometimes subject to a loss threshold, contingent on a credit event concerning the reference asset in exchange for a periodic fee.

The main differences between an nth-to-default basket of CDSs and a CDO are the leverage and exposure, which the CDO provides and the CDSs don't. The buyer of a CDO (investor) is exposed to the risk caused by the different credit events, which affect the tranches at different points in time. The seller of protection (investor) on an nth-to-default basket of CDSs is exposed only until the specific event on which protection has been sold takes place. For example, in a first-to-default basket, protection on the basket is provided and paid for only until the first name in the basket is subject to a credit event. After that, the financial instrument ceases to exist. In this sense, a first-to-default basket is similar to the equity tranche of a CDO.

Default correlations are the main driver of the CDS basket's value. Let us suppose that we have a basket of five credits where each CDS pays a spread of 100 basis points (bps). In the case of zero correlation, the first-to-default swap would have a spread of 500 bps, i.e., the simple sum of the individual credit spreads. If, instead, the correlation is one, the spread for the basket would be 100 bps, i.e., the maximum of the individual swap spread. Given the relevance of default correlations, this paper concentrates on the determinants of the default correlation structure underlying an nth-to-default basket of CDSs.

III. MODEL DESCRIPTION

Our modeling strategy is based on two key elements. First, following Hull and White (2005), Gibson (2004), and Andersen, Sidenius, and Basu (2003), we compute the probabilities of default conditional on a multifactor structure. Second, the multifactor structure is estimated by factor analysis and will allow us to express the LCFI's correlation structure in terms of a set of common factors related to the macroeconomic conditions in which financial institutions operate. As a by-product, the multifactor structure also serves as a platform to conduct stress testing of the default probabilities to shocks in all or some of these factors.

The Pricing Model

This section presents a schematic representation of the multifactor pricing model. In pricing a CDO or a CDS basket, it is assumed, following Vasicek (1987), that the asset value of each institution in the portfolio is influenced by a common set of factors and an idiosyncratic factor. The lower the value the common factors and/or the idiosyncratic factor, the earlier a default is likely to occur. Thus, the asset value of financial institution i can be expressed as a random variable x_i, $i = 1, ..., N$,

$$x_i = a_{i1} M_1 + a_{i2} M_2 + ... + a_{im} M_m + Z_i \sqrt{1 - a_{i1}^2 - a_{i2}^2 - ... - a_{im}^2} \tag{1}$$

where the common factors $M = (M_1, ..., M_m)$ and the idiosyncratic factor Z_i have independent zero-mean and unit-variance distributions. The factor loadings a_{ij} are such

that $a_{ij} \in \{-1,1\}$ and $a_{i1}^2 + a_{i2}^2 + ... + a_{im}^2 \leq 1$. The correlation matrix among the N institutions, Σ, is such that the pair-wise correlation between asset i and j can be expressed as $a_{i1}a_{j1} + a_{i2}a_{j2} + ... + a_{im}a_{jm}$.

Let H be the cumulative distribution of the Z_i. Following Merton (1974), the default probability of x_i, i.e., the probability of x_i falling below a threshold $\bar{x}_i$, is characterized as:

$$p(x_i < \bar{x}_i \mid M) = Q_i(t \mid M) = H\left[\frac{\bar{x}_i - (a_{i1}M_1 + ... + a_{im}M_m)}{\sqrt{1 - a_{i1}^2 - ... - a_{im}^2}}\right] \qquad (2)$$

Let $p(l,t \mid M)$, $l = 0,...,N$, denote the probability that exactly l defaults occur by time t, conditional on the common factors M, in a reference portfolio of N financial institutions. Let F_i be the cumulative distribution of x_i. The copula model maps x_i to t_i using a percentile-to-percentile transformation. The percentile point in the probability distribution for x_i is transformed to the same percentile point in the probability distribution of t_i. Defining $Q_i(t)$ as the cumulative risk-neutral probability that institution i will default before time t, the point $x_i = \bar{x}_i$ is mapped to $t_i = t$ where $t = Q_i^{-1}[F_i(\bar{x}_i)]$. If the default probability for each entity i is characterized by a (forward) default hazard rate $\lambda_i(t)$, then $Q_i(t)$ can be expresssed as:

$$Q_i(t_i \leq t) = 1 - e^{-\int_0^t \lambda_i(u)du}. \qquad (3)$$

The functions $Q_i(t)$, $i = 1,...,N$, can be bootstrapped from the quoted CDS spreads and are assumed known for all t.

The distribution of the number of defaults conditional on the common factors M can be computed through recursion.[6] Once we have the conditional default distribution, the unconditional default distribution $p(l,t)$ can be solved as

$$p(l,t) = \int_{\mathbb{R}^m} p(l,t \mid M)g(M)dM. \qquad (4)$$

The joint density distribution of M $g(M)$, is the product of m standard (independent) Gaussian densities. As we can see, the probability of default is conditional on the factor structure which approximates the correlation among the 15 financial institutions. We describe the estimation of such factor structure in the next section.

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[6] For details, see Andersen, Sidenius and Basu (2003) and Gibson (2004).

Factorization of the Correlation Matrix: "Factor Analysis"

We approximate the copula default correlation matrix between the N institutions is by their equity return correlation matrix. The factorization of the equity return correlation matrix is accomplished through "factor analysis," whereby the 15-dimensional matrix of observed LCFIs' equity returns can be expressed as the sum of an unobserved systematic part and an unobserved error part:

$$X = \mu + AF + U \tag{5}$$

The vector of observed equity returns (X), the error term or idiosyncratic variable (U), and the constant vector of means (μ) are column vectors of N components (i.e., 15 LCFIs). The common factors (F) is a column vector of m factors, with $m \leq N$. The factor loadings matrix (A) is a $N \times m$ matrix (where a_{ij} in equation 1 is the generic element of A). The N components of F are assumed to be independent standard Gaussian variables. U is assumed to be independently distributed of F with zero mean and covariance matrix Ψ.[7] Under these assumptions, the maximum likelihood (ML) estimator of A and Ψ are determined by the following two conditions:

$$A(A'\Psi^{-1}A + I) = C\Psi^{-1} \qquad \text{and} \qquad \text{diag}(AA' + \Psi) = \text{diag}(C) \tag{6}$$

where C is defined as $(1/T)\sum_{t=1}^{T}(x_t - \bar{x})(x_t - \bar{x})'$ and T is the number of observations (for details, see Anderson, 2003).[8]

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[7] For identification purposes, we need to add the restriction that $A'\Psi^{-1}A$ is diagonal. If the diagonal elements of $A'\Psi^{-1}A$ are ordered and different, then A is uniquely determined (Anderson, 2003).

[8] It is important to note that the factor model specification is consistent with the multifactor pricing model, where the factors are independent i.i.d. random variables. Although a dynamic factor model with multivariate GARCH effects (or a model with explicit macroeconomic factors such as Pesaran et al., 2005), is likely to capture better the stochastic properties of the equity return series, it would not be consistent with the pricing model.

IV. DATA DESCRIPTION

This paper focuses on the group of LCFIs as defined by the Bank of England (2004).[9] The financial institutions are ABN Amro (ABN), Bank of America (BoA), Barclays (BARC), BNP Paribas (BNP), Citigroup (CITI), Credit Suisse (CS), Deutsche Bank (DB), Goldman Sachs (GS), HSBC Holdings (HSBC), JP Morgan Chase Chase (JPM), Lehman Brothers (LEH), Merrill Lynch (ML), Morgan Stanley (MS), Société Génerale (SG), and UBS. We used the daily quotation of five years CDS spreads, the most liquid contract (computed as the end of day average bid-ask spread), from 2003 to 2005.

Figure 1 shows the CDS spreads of the 15 LCFIs in the basket. The last three years are characterized by an overall improvement (shrinking) in the credit spreads, with a few exceptions for some specific periods, such as Spring 2005. At that time the spreads experienced a temporary increase following the downgrading of General Motors. It is also interesting to note that since the second half of 2004, the market has identified three main groups of financial institutions and ranked them according to their perceived relative riskiness.

The first group with the largest credit spreads corresponds to the financial institutions more active in investment banking (i.e. Lehman Bothers, Morgan Stanley, Goldman Sachs, JP Morgan Chase and Merrill Lynch). The second group includes institutions with more diversified activities, such as the largest U.S. and European banks (e.g. Citigroup, Bank of America, Deutsche Bank and Credit Suisse). The third group, with the lowest spreads, corresponds to banks that are seen by the market as well diversified and with a very good quality credit portfolio (e.g. HSBC, UBS, Société Génerale, BNP Paribas, ABN Amro, and Barclays). Overall, the very benign market conditions keep the spreads in a very narrow band that ranges from 7–8 to 24–25 bps.

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[9] The financial institutions selected are ranked in the top ten in at least two of the following six categories: (i) equity book runners, (ii) bond book runners, (iii) syndicated loans book runners, (iv) interest rate derivatives outstanding, (v) foreign exchange revenues, and (vi) holders of custody assets.

Figure 1. Large Complex Financial Institutions: Credit Spreads
(in basis points)
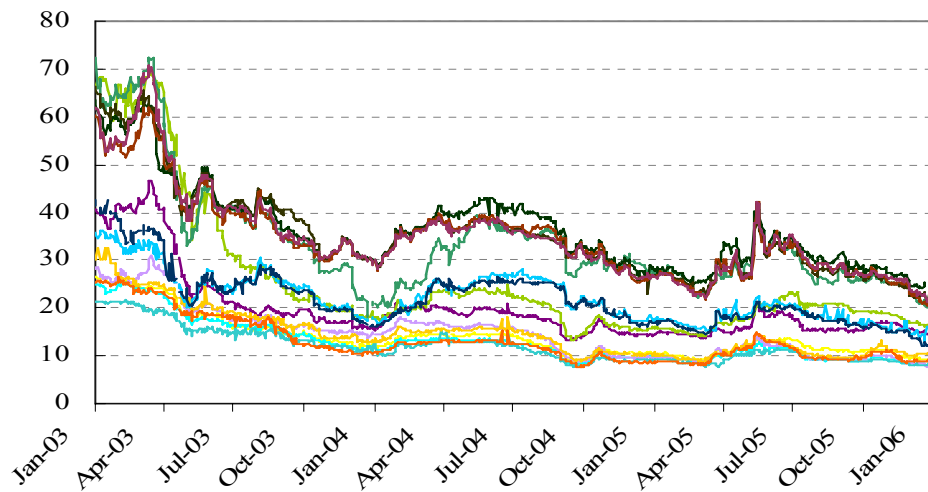
Source: Bloomberg.

Another important feature of LCFIs is their high degree of cross correlation. Following Hull and White (2005), we use stock returns of the reference entities to estimate their correlation structure. The average estimated correlation for the period (2003–05) was about 40 percent, well above the correlations observed for non-financial companies (Table 1).[10] The correlations seem to have a marked geographical pattern, with the correlations observed among the European-based institutions and among the U.S.-based institutions being higher than cross-continent correlations.

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[10] As pointed out in FitchRatings (2005), correlation estimates based on equity-price movements may tend to overestimate actual correlations on average by 10-15 percent.

Table 1. Large Complex Financial Institutions: Estimated Correlations

	SG	BNP	DB	ABN	HSBC	BARC	UBS	CS	BoA	CITI	JPM	LEH	ML	GS	MS
SG	1.00	0.87	0.70	0.80	0.65	0.66	0.73	0.67	0.01	0.08	0.06	0.04	0.06	0.05	0.08
BNP		1.00	0.69	0.73	0.62	0.67	0.67	0.59	0.04	0.11	0.08	0.04	0.09	0.04	0.13
DB			1.00	0.67	0.55	0.61	0.65	0.64	0.27	0.24	0.29	0.21	0.36	0.30	0.33
ABN				1.00	0.68	0.68	0.75	0.63	0.05	0.11	0.13	0.06	0.08	0.01	0.04
HSBC					1.00	0.78	0.68	0.63	-0.06	0.16	0.06	0.06	0.15	0.02	0.16
BARC						1.00	0.73	0.62	0.04	0.16	0.09	0.15	0.15	0.07	0.16
UBS							1.00	0.81	-0.01	0.04	0.08	0.05	0.09	0.03	0.00
CS								1.00	0.09	0.13	0.05	0.07	0.10	0.13	0.14
BoA									1.00	0.76	0.75	0.61	0.71	0.59	0.62
CITI										1.00	0.66	0.51	0.67	0.50	0.67
JPM											1.00	0.53	0.68	0.53	0.46
LEH												1.00	0.71	0.82	0.64
ML													1.00	0.67	0.75
GS														1.00	0.64
MS															1.00

Note: Calculations are based on 2005:QIV equity returns.

There are also important variations in the cross-correlations of the financial institutions as different macroeconomic and financial shocks affect their correlation structure. The common factors underlying the variation in the correlation matrix are analyzed next.

V. FACTOR ANALYSIS: ESTIMATION RESULTS

One of the key inputs in the computation of the probability of default and the pricing of the CDS basket are the factor loadings in the asset valuation equation. Following Hull and White (2005), the factor loadings in (1) are estimated so that they "best" approximate the correlation structure observed in the asset returns series of the LCFIs. To this end, the latent factor model in (5) is fit the asset return data, where the number of underlying common factors is an important choice variable. In fact, we want to determine the smallest number of factors that make the factor model consistent with the observed data.

We start by testing that the number of common factors is m_0 (e.g., $m_0 = 1$). If this hypothesis is rejected, we proceed to test for m_0+1 and continue iteratively until the null hypothesis is accepted or until $((N - m)((N - m) - 1))/2 \leq 0$.[11] The likelihood ratio test for the null hypothesis is distributed asymptotically as a chi-square with $((N - m)((N - m) - 1))/2$ degrees of freedom (Amemiya and Anderson, 1990). The results of the chi-square goodness-of-fit test indicate that 5 common factors fit best the LCFI's

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[11] Although this iterative procedure is typically followed in empirical applications, the probabilities of errors under this procedure are unknown, even asymptotically (Anderson, 2003).

asset-return data.[12] The 5 common factors explain 78 percent of the variance of the asset returns, the remaining 22 percent is the institution-specific or idiosyncratic variance.

In order to provide an interpretation of the sizes and signs of the estimates of the factor loadings, we undertook an exploratory, principal-component analysis (PCA) of the asset return data for the 15 LCFIs. [13] This analysis revealed that the first 5 principal components could be interpreted as: (i) a factor common to all financial institutions; (ii) a factor mainly related to European institutions; (iii) a factor mainly related to U.S. institutions; and two other factors that could be related to institutions mainly (iv) in commercial banking and (v) in investment banking (similar results have been reported by Hawkesby, Marsh, and Stevens, 2005).

Following the PCA results, we rotated the ML estimates of the factor loadings matrix A in order to facilitate the interpretation of the factors—i.e., to make the factors look "similar" to the 5 components described above. Each row of A, i.e., the factor loading vector for each financial institution, can be interpreted as coordinates of a point in our m-dimensional space. Thus each factor corresponds to a coordinate axis, and factor rotation is equivalent to rotating those axes and computing new loadings in the rotated coordinate system. Consequently, the factor rotation leaves the statistical properties of our ML estimates unchanged, including the common factors' variance and the residuals' variance.

Table 2 shows the rotated, ML estimates of A. The results show clear patterns related to "geography" and "line of business," in particular:

- The estimates of the first common factor (i.e., "financial institution" factor) in the first column of Table 2 show that all institutions are positively affected by the "financial-institution" factor, with values ranging from 0.64 (Société Génerale) to 0.23 (HSBC).

- The second factor is related to a regional European effect whereby all European institutions are positively affected by it. Factor loadings range from 0.84 (HSBC) to 0.34 (Deutsche Bank). The U.S. banks are also affected by the European factor, but its effect is negative. The negative effect appears to be significant for Bank of America, JP Morgan Chase, and Citigroup, and close to zero for the rest.

- The third factor is related to a regional U.S. effect whereby all U.S. institutions are positively affected by it. Factor loadings range from 0.67 (Citigroup) to 0.42 (Goldman Sachs). The factor loadings for the European banks are all negative—with

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[12] The chi-square statistic with 40 degrees of freedom has a p-value of 8.4 percent. Therefore, we fail to reject the null hypothesis of 5 factors at the 95 percent level. The ML estimates are based on the most recent CDS data—daily CDS spread data, computed as the bid/ask average, corresponding to the last quarter of 2005.

[13] The results are available from the authors upon request.

the exception of the two U.K. banks in the sample (HSBC and Barclays)—and are generally much smaller than those of the European institutions.

- The fourth factor is related to commercial banking business. All but four banks have a positive factor loading. Merrill Lynch, Morgan Stanley, Lehman Brothers, and Goldman Sachs, i.e., the more investment bank oriented, have a factor loading close to zero or negative.

- The fifth factor is related to investment banking business. Goldman Sachs and Lehman Brothers, among the U.S. banks, and UBS and Credit Suisse among the European are the banks with the highest factor loading. The U.S. institutions have higher loadings than the European ones. Finally, a group of banks have factor loadings that are very small or negative, including Société Génerale, BNP Paribas, HSBC, and ABN Amro.

Table 2. Rotated Factor Loadings, Maximum Likelihood Estimates

	1st Factor	2nd Factor	3rd Factor	4th Factor	5th Factor	Residual Variance
SG	0.645	0.456	-0.285	0.462	-0.076	0.076
BNP	0.615	0.435	-0.204	0.454	-0.117	0.172
DB	0.572	0.342	-0.024	0.420	0.188	0.344
ABN	0.437	0.497	-0.178	0.516	0.032	0.264
HSBC	0.227	0.836	0.148	0.346	-0.025	0.108
BARC	0.307	0.686	0.025	0.387	0.109	0.274
UBS	0.231	0.601	-0.331	0.538	0.417	0.014
CS	0.303	0.515	-0.181	0.450	0.298	0.319
BoA	0.359	-0.398	0.572	0.338	0.370	0.134
CITI	0.296	-0.150	0.671	0.355	0.229	0.262
JPM	0.299	-0.247	0.506	0.330	0.363	0.352
LEH	0.494	-0.046	0.464	-0.194	0.541	0.208
ML	0.394	-0.072	0.618	0.112	0.441	0.250
GS	0.552	-0.089	0.417	-0.238	0.542	0.163
MS	0.444	-0.028	0.621	0.018	0.274	0.341

Note: Calculations are based on 2005:QIV equity returns.

Table 3 shows the contributions to the asset-return variance of each common factor. The variance contribution is the squared value of the estimated factor loadings, so that the sum of each row gives the proportion of the variance explained by the common factors. By institution, the factor model seems to provide the best fit for UBS—with 98.6 of the variance explained by the common factors (the European factor and the commercial banking factor are the main contributors, explaining 64 percent of the variance). In contrast, the worst fit is for JP Morgan Chase—with only 64.8 percent of the variance accounted for by the common factors (the U.S. and investment banking factors are the main contributors, explaining 39 percent of the variance).

Table 3. Variance Contribution

	1st Factor	2nd Factor	3rd Factor	4th Factor	5th Factor	Idiosyncratic Variance
SG	0.416	0.208	0.082	0.214	0.006	0.076
BNP	0.378	0.189	0.042	0.206	0.014	0.172
DB	0.327	0.117	0.001	0.176	0.035	0.344
ABN	0.191	0.247	0.032	0.266	0.001	0.264
HSBC	0.052	0.698	0.022	0.119	0.001	0.108
BARC	0.094	0.470	0.001	0.149	0.012	0.274
UBS	0.054	0.361	0.110	0.289	0.174	0.014
CS	0.092	0.265	0.033	0.202	0.089	0.319
BoA	0.129	0.158	0.328	0.114	0.137	0.134
CITI	0.088	0.022	0.450	0.126	0.053	0.262
JPM	0.090	0.061	0.256	0.109	0.132	0.352
LEH	0.244	0.002	0.215	0.038	0.293	0.208
ML	0.155	0.005	0.382	0.013	0.194	0.250
GS	0.305	0.008	0.174	0.057	0.294	0.163
MS	0.197	0.001	0.386	0.000	0.075	0.341
Average	0.187	0.187	0.167	0.139	0.101	0.219

Note: Calculations are based on 2005:QIV equity returns.

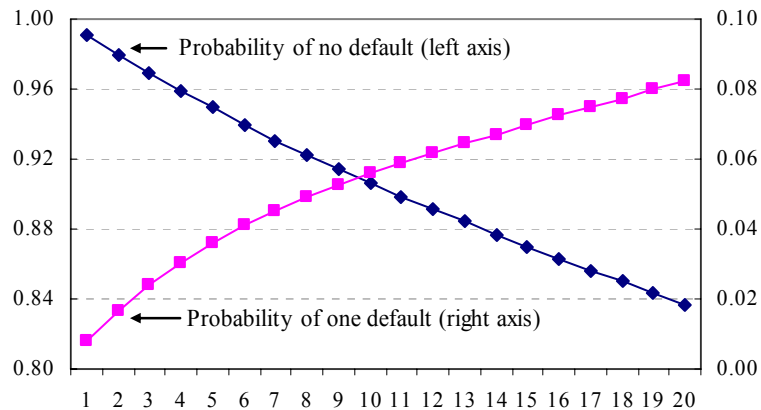## VI. COMPUTATION OF THE PROBABILITIES OF DEFAULT

With the estimated matrix of factor loadings A, we are now in a position to compute the implied probabilities of default. It has to be noted that the probabilities of default computed here are forward probabilities, e.g., the current market expectations of future default probabilities. Moreover, these probabilities are risk-neutral, i.e., they are obtained under the assumption that the underlying asset value growth is in line with the risk-free rate, and not with its own actual (e.g., historically observed) rate of growth.[14] Further, we estimate the default hazard rate as the ratio of the CDS spread to the loss-given-default (LGD), which is assumed to remain constant. The calculation of the probability of default is done for every period in which there is a payment of the CDS basket. For example, in the case of a basket of 5-year CDSs there would be 20 payment dates.

- Figure 2 shows the probability of 0 and 1 defaults over a 5 year horizon based on daily CDS data for end-2005. Several features of the probability of defaults can be highlighted. First, the one quarter forward probability of no defaults is very high (0.99 percent). This is typical of CDS baskets of highly rated financial institutions,

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[14] It is known that risk-neutral probabilities are in general higher than actual default probabilities. Recent estimates for nonfinancial-sector firms (Berndt et al., 2005) suggest that the ratio of risk neutral to actual default probabilities is in the range of 1.73 to 1.79 for rating levels that are comparable with those of the LCFIs (i.e., A-AAA).

such as the LCFIs. Second, the probability of no default falls systematically from one quarter ahead to 20 quarters ahead (83.7 percent), logically implying that the market sees the likelihood of defaults increasing as time passes. Third, the other side of the coin is that the probability of one default over the next quarter is very small (0.8 percent) and it increases over the 5 year horizon up to 8.2 percent. Although not shown in the figure, a similar pattern can be observed for the probability of 2 defaults, 3 defaults, etc.

Figure 2. Probability of Default: One- to Twenty-Quarter Ahead

Note: Calculations are based on end-2005 CDS spreads, correlations, and factor structure.

The implied probabilities of default provide a good indication of the market's views on the underlying credit quality of the financial institutions in the basket. To illustrate how this indicator captures changes in market perceptions, Figure 3 shows daily estimates for 2005 of the probability of no default over a 2-year horizon. To simplify the computation, the correlation and factor structure are held constant over the sample. They are set equal to the correlation and common factor estimates based on the end-2005 data. We can see how the credit events related to the downgrading of GM are captured by a significant decrease in the probability of observing zero default in May 2005 (e.g., this corresponds to an increase in the probability of observing some defaults). In the second half of 2005, the probability of zero default climbs back to pre-shock levels.

Figure 3. Two-Year-Ahead Probability of No Defaults
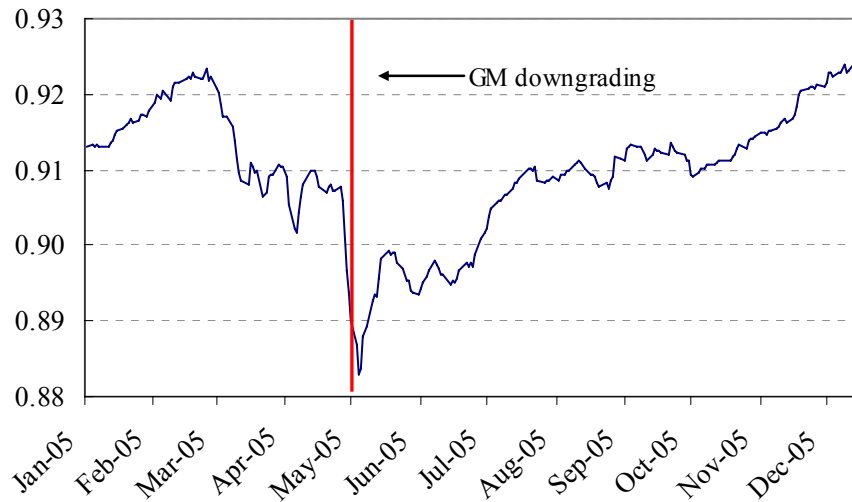
Note: Calculations are based on CDS spreads for 2005 (constant correlation and factor structure).

While a constant correlation and factor structure may be a reasonable approximation over a short period of time, macroeconomic shocks, financial shocks, and, generally, changes in the business cycle do affect both the correlation and the underlying common-factor structure which drives the LCFIs' dynamics.

Figure 4 shows the estimates of the average cross-correlation for the asset returns of the LCFIs in the period 2001–05.[15] The estimates are based on a one quarter window (75 days) to be consistent with market practice. The first salient feature of the estimates is that the average correlation ranges between approximately 25 and 65 percent (obviously with much higher pair-wise correlations, especially within the European and U.S. groups). The second main feature is that the optimal common factor approximation to the estimated correlation matrix varies between 2 and 5 factors.

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[15] Note that for the estimation of the correlation and factor structure based on daily equity returns, we added 2001–02 to the sample in order to incorporate information from the stock-market crisis (the pre-2003 CDS prices were discarded because the market lacked sufficient liquidity).

Figure 4. Estimated Correlation and Factor Structure (2001–05)
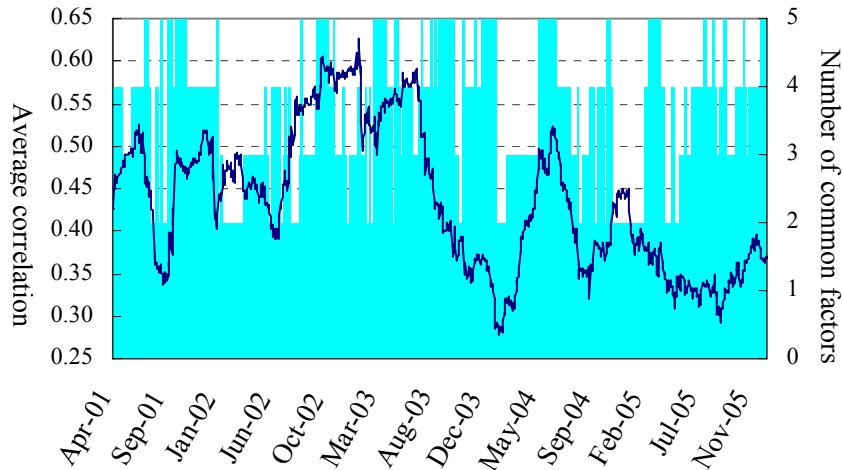
Note: Estimates are based on daily equity-return data for the 15 LCFIs for 2001−05.

In sum, the rich dynamics in the correlation and common factor structure of the equity returns suggest that a multifactor approach is better suited than a single factor model to compute the probability of default and the pricing of the CDS basket. How sensitive is the implied probability of default to a time-varying correlation and a multifactor representation? Can the multifactor approximation to the correlation matrix serve as a platform to conduct stress test analysis of the LCFIs to macroeconomic and financial shocks? These and other issues involving the sensitivity analysis are examined in the next section.

VII. SENSITIVITY ANALYSIS

To assess the impact on the implied probability of default of different correlations and their multifactor representation, we selected some extreme correlation values from their observed historical distribution. In particular, we selected the highest and lowest average correlations among those with a 5-factor representation, which correspond to the 99.2 percentile and the 1.3 percentile of the historical distribution of average correlation, respectively. They correspond to the 3-month period ending in January 14, 2004 and October 21, 2002, respectively. Figure 5a shows how in the high-correlation scenario the implied probability of no defaults—over one-quarter to a five-year horizon—is much higher than that in the low-correlation scenario. The computation of the probability of default is again based on end-2005 data on CDS spreads.

Figure 5. Default Probabilities under High and Low Correlation Scenarios

| Figure 5a	Figure 5b
Probability of no default	Probability of one default
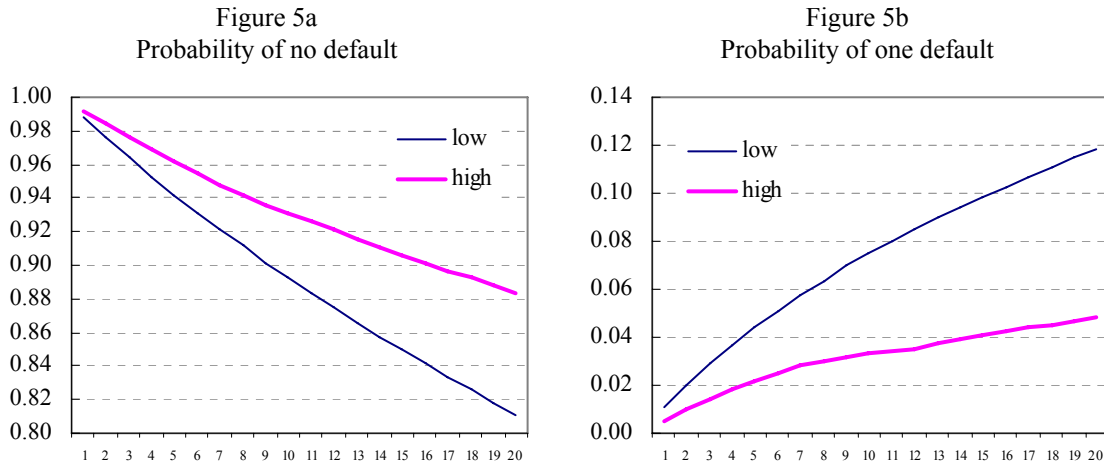
Note: Calculations are based on CDS spreads for end-2005.

In contrast, Figure 5b shows how in the high correlation state the implied probability of one default is below the implied probability in the low correlation state. These results are consistent with what one would expect: as correlation increases, the financial institutions behave more similarly, therefore the likelihood of observing no defaults tends to be high (i.e., all institutions being in a similar state of credit strength), and similarly, the likelihood of observing only one default falls.

To complement this analysis, we computed the probabilities of no default and one default over the entire historical distribution of correlations and factor structures (ranging from 2 to 5 factors) using the whole sample period (2001–05, about 1250 observations). Figure 6 presents the results in a three-dimensional picture: (i) the vertical axis represents probabilities, (ii) the left axis represents the approximately 1250 observations for the ordered, average correlations (from low correlation, about 25 percent, to high correlation, about 65 percent), and (iii) the right axis represents the number of quarters forward for the estimation of the probabilities of default (from 1 to 20 quarters ahead). The estimates confirm the results obtained earlier, i.e., high correlation gives a higher value for the probability of no default. Additionally, different factor structures also result in varying default probabilities.

Figure 6. Probability of default over range of correlation and factor structures

Probability of no default   Probability of one default
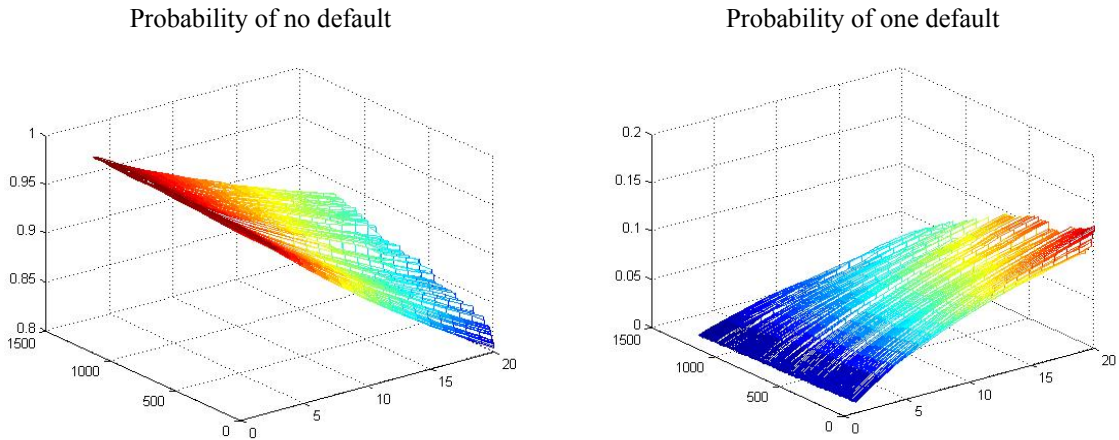
Note: Calculations are based on CDS spreads for end 2005.

Finally, we re-estimated all the probabilities of default for the entire sample of CDS spreads (2003–05). Figure 7 shows the 2-year-ahead probability of zero and one defaults. First, we kept the correlation and factor structure constant and equal to the median average correlation estimated over the period 2001–05 (represented by the thin line in the graph). Second, we let the correlation and factor structure be re-estimated over time as new information becomes available using a 3-month estimation window. This results in significant changes in the estimates of probabilities of default (thick line), indicating that updating the correlation structure, as well as its multifactor representation, is critical in the computation of the probabilities of default.

Figure 7. Two-year Ahead Probability of Default: Variable vs. Fixed Correlation and Factors

Probability of no default   Probability of one default
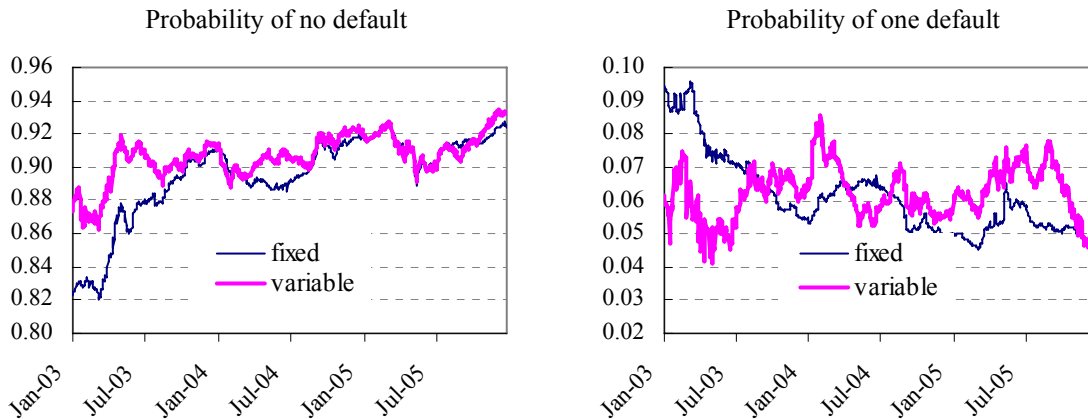
Note: "Variable" shows the two-year-ahead probability of default using the rolling estimation of the correlation and factor structure using a 75 day window. "Fixed" corresponds to estimates of the probability of default with a constant correlation structure set equal to the median correlation from its historical distribution (based on 2001–05).

So far we have assumed that the common factors ($M_1,\ldots, M_m$) and the idiosyncratic factor Z_i follow standard normal distributions, i.e. a Gaussian copula model. Since the choice of the copula determines the default dependence, different distributional assumptions, such as a copula where the common factors have heavy tails, will change the dependency dynamics and the estimates of the probabilities of default. Table 4 shows the one-period ahead PDs for different mixtures of normal and t-distributions for the common and idiosyncratic factors. To focus on the effect of the distributional assumptions, we assumed an equal factor loading structure corresponding to an average correlation of 40 percent (the average of the period). We also assumed two different scenarios, namely a single common factor and a multifactor scenario with 5 common factors.

For the single common factor scenario the following results emerge. First, under the assumption of a t-distribution for the idiosyncratic factor and a Gaussian distribution for the common factor, the probability of observing a single name default increases relative to the all Gaussian case. Second, under the assumption of a t-distribution for the common factors and a Gaussian distribution for the idiosyncratic factor, the probability of observing several joint default events increases relative to the all Gaussian case. As pointed out by Hull and White (2005), as the distribution of the common factors accumulates more mass in the tail, the PDs behave in a similar fashion to an increase in the equity return correlation. Third, when both the idiosyncratic and common factors have t-distributions, we observe how both the probability of one default as well as the probability of joint defaults increase relative to the all Gaussian case. These effects are more pronounced as the number of degrees of freedom of the t-distribution decreases, i.e. under heavier tails. For the multifactor case, the effects are by in large qualitatively similar to the single factor case. However, there is a "diversification" effect relative to the single factor case that leads to relatively lower PDs. This effect can be more clearly observed in the comparatively lower probability of joint defaults relative to the single common factor case. Yet, as the degrees of freedom of the t-distribution decrease, there is an increase in the probability of joint default relative to the Gaussian case (as in the single common factor case).

Table 4. Effect on the probabilities of default of different distributions for the idiosyncratic factor and the common factors

	m=1; correl=40%; T-student d.o.f.=6				m=5; correl=40%; T-student d.o.f.=6			
	Idiosyncratic Factor / Common Factor				Idiosyncratic Factor / Common Factor			
	Gauss / Gauss	T-student / T-student	T-student / Gauss	Gauss / T-student	Gauss / Gauss	T-student / T-student	T-student / Gauss	Gauss / T-student
PD(0)	0.98616	0.96048	0.96158	0.98403	0.98611	0.96245	0.96174	0.98645
PD(1)	0.01209	0.03465	0.03586	0.01150	0.01275	0.03547	0.03611	0.01245
PD(2)	0.00135	0.00283	0.00221	0.00215	0.00104	0.00195	0.00200	0.00102
PD(3)	0.00030	0.00069	0.00029	0.00081	8.9E-05	0.00013	0.00014	7.7E-05
PD(4)	8.0E-05	0.00021	5.4E-05	0.00032	6.3E-06	7.3E-06	9.1E-06	4.4E-06
PD(5)	1.9E-05	7.2E-05	1.0E-05	0.00012	3.7E-07	3.2E-07	5.0E-07	1.9E-07
PD(6)	3.9E-06	6.0E-05	1.7E-06	0.00009	1.7E-08	1.1E-08	2.2E-08	6.2E-09
PD(7)	6.2E-07	0.00011	2.2E-07	0.00013	6.6E-10	3.1E-10	8.1E-10	1.5E-10
PD(8)	7.6E-08	0.00017	2.2E-08	0.00019	2.0E-11	6.6E-12	2.4E-11	3.0E-12
PD(9)	7.3E-09	0.00022	1.7E-09	0.00022	4.8E-13	1.1E-13	5.5E-13	4.5E-14
PD(10)	5.3E-10	0.00022	1.0E-10	0.00020	8.9E-15	1.4E-15	1.0E-14	5.1E-16
PD(11)	2.9E-11	0.00016	4.8E-12	0.00014	1.2E-16	1.4E-17	1.4E-16	4.4E-18
PD(12)	1.2E-12	8.6E-05	1.6E-13	7.1E-05	1.3E-18	9.8E-20	1.5E-18	2.8E-20
PD(13)	3.2E-14	3.2E-05	3.6E-15	2.5E-05	9.0E-21	4.8E-22	1.1E-20	1.2E-22
PD(14)	5.5E-16	7.3E-06	5.1E-17	5.3E-06	3.9E-23	1.5E-24	4.7E-23	3.2E-25
PD(15)	4.3E-18	7.7E-07	3.3E-19	5.3E-07	7.8E-26	2.1E-27	9.7E-26	3.9E-28
	m=1; correl=40%; T-student d.o.f.=3				m=5; correl=40%; T-student d.o.f.=3			
	Idiosyncratic Factor / Common Factor				Idiosyncratic Factor / Common Factor			
	Gauss / Gauss	T-student / T-student	T-student / Gauss	Gauss / T-student	Gauss / Gauss	T-student / T-student	T-student / Gauss	Gauss / T-student
PD(0)	0.98616	0.84808	0.85952	0.96954	0.98611	0.84548	0.85965	0.96643
PD(1)	0.01209	0.12433	0.12599	0.01545	0.01275	0.13006	0.12616	0.02385
PD(2)	0.00135	0.01544	0.01289	0.00421	0.00104	0.01919	0.01284	0.00694
PD(3)	0.00030	0.00354	0.00138	0.00222	0.00009	0.00408	0.00123	0.00211
PD(4)	0.00008	0.00161	0.00018	0.00144	6.3E-06	0.00095	0.00011	0.00054
PD(5)	1.9E-05	0.00095	2.8E-05	0.00098	3.7E-07	0.00019	9.1E-06	0.00011
PD(6)	3.9E-06	0.00065	4.1E-06	0.00070	1.7E-08	3.3E-05	6.2E-07	1.8E-05
PD(7)	6.2E-07	0.00052	5.1E-07	0.00055	6.6E-10	4.5E-06	3.5E-08	2.3E-06
PD(8)	7.6E-08	0.00044	5.2E-08	0.00045	2.0E-11	4.8E-07	1.6E-09	2.3E-07
PD(9)	7.3E-09	0.00035	4.1E-09	0.00036	4.8E-13	4.2E-08	5.9E-11	1.8E-08
PD(10)	5.3E-10	0.00032	2.6E-10	0.00033	8.9E-15	2.8E-09	1.7E-12	1.1E-09
PD(11)	2.9E-11	0.00043	1.2E-11	0.00045	1.2E-16	1.4E-10	4.0E-14	5.1E-11
PD(12)	1.2E-12	0.00065	4.2E-13	0.00067	1.3E-18	5.3E-12	6.7E-16	1.7E-12
PD(13)	3.2E-14	0.00079	9.9E-15	0.00078	9.0E-21	1.4E-13	8.0E-18	3.9E-14
PD(14)	5.5E-16	0.00065	1.5E-16	0.00058	3.9E-23	2.2E-15	6.0E-20	5.5E-16
PD(15)	4.3E-18	0.00124	1.0E-18	0.00127	7.8E-26	1.6E-17	2.1E-22	3.6E-18

Note: "m" is the number of common factors; "correl" is the average correlation coefficient among the equity return series in the CDS basket; and "T-student d.o.f." is the number of degrees of freedom of the T-student distribution . We also conducted other sensitivity analysis for different values of the correlation coefficient and different number of degrees of freedom of the T-distribution.

VIII. STRESS TESTING

In the previous section we have shown how the probabilities of default are sensitive to changes in the correlation, the factor structure, and distributional assumptions. The multifactor structure allows us to analyze also the response of the probabilities of default to shocks in the factors themselves. As we have seen, the estimated factors are in fact related to the state of the financial and macroeconomic conditions in which the LCFIs operate. For example, if a global recession hits both the European and U.S. financial institutions, the probabilities of default are likely to increase. It then becomes important to understand the relative significance of the different channels (i.e., factors) through which this scenario affects the default probabilities of the institutions.

Specifically, to implement a stress test, we can estimate the probabilities of default (0 defaults, 1 default, 2 defaults, etc.) conditional on a certain value of the common factor. For example, to examine the effect of a "recession" ("boom") on a given factor, we can integrate over the set the values of the factor in the left (right) tail of the factor's distribution (Gibson, 2004).

Figure 8 shows such a calculation. We first computed the implied probability of default over the next 5 years (i.e., 20 quarters). Figure 8a shows the probability of zero default (left axis) and one, two, and three defaults (right axis) in the baseline scenario. The baseline-scenario probabilities are computed based on the end-2005 correlation and factor structure (i.e., the 5 factor structure described earlier). In general, given the high quality of the LCFIs the probabilities of two and three defaults are well below 5 percent even at the 5-year horizon.

In contrast to the baseline, when all factors enter simultaneously into a generalized recession, the probabilities of default change substantially. As Figure 8b shows, the probability of zero defaults falls from around 90 percent (one quarter ahead), to around 30 percent (two years ahead), and below ten percent (5 years ahead). The flip side of the coin is that the probability of one default over a two-year horizon jumps significantly up to about 40 percent. The pattern of the different default probability dynamics in a recession is in fact intuitively very appealing. At longer time horizons, there is a progressive worsening of the credit conditions. This shows up as an increase in the probability of observing a larger number of defaults. [16]

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[16] Indeed, the estimates show that the probability of observing only one default falls after two years and, after three years, the probability of two defaults is even higher than the probability of one default. Between 3 to 5 years ahead, a similar pattern emerges, namely, the probability of observing three defaults increases, rising above the probability of observing two and one defaults, which then start to decline.

Figure 8. Stress Testing: Probabilities of Default Under an "All Factor" Recession Scenario

Figure 8a
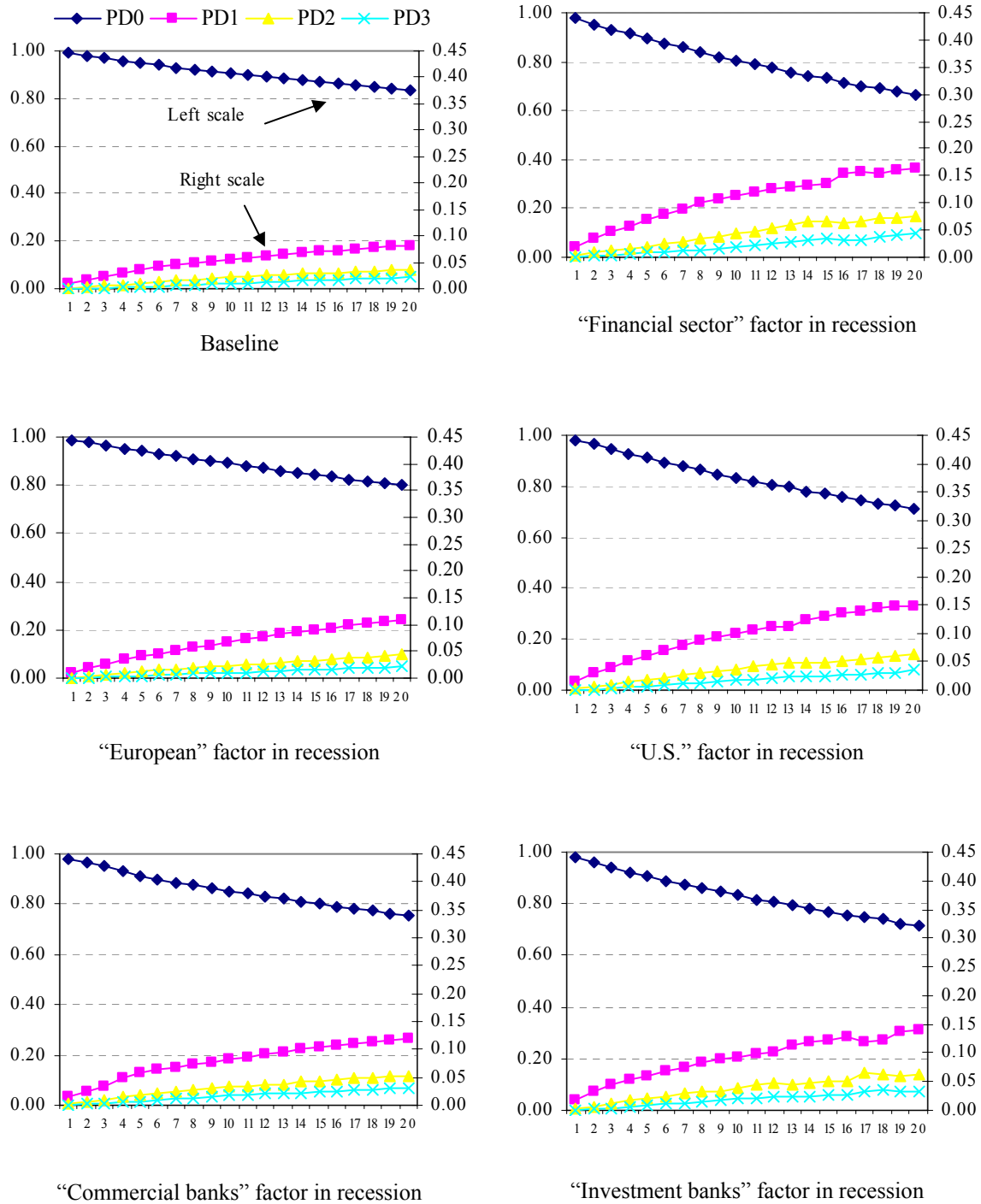
Figure 8b

Baseline

All factors in recession
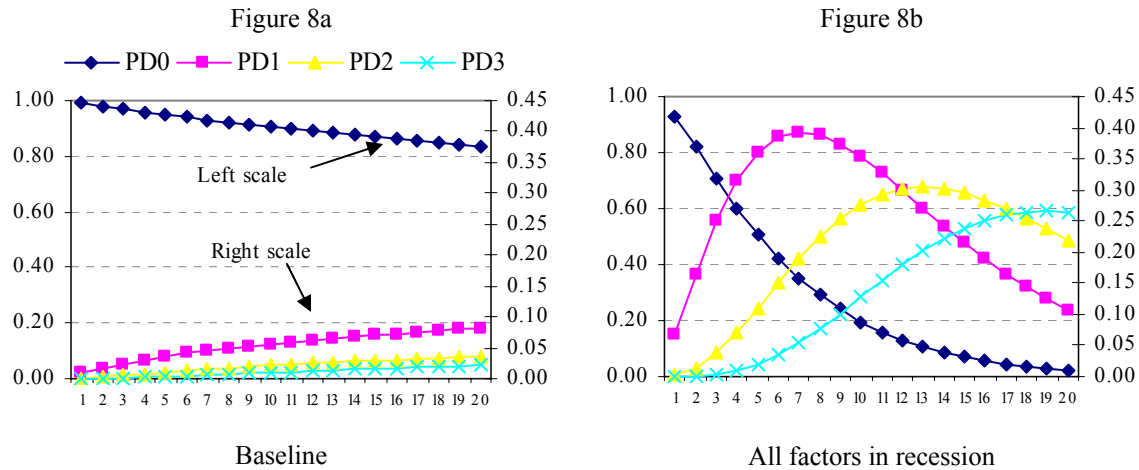
Note: Calculations are based on CDS spreads for end 2005.

The multifactor framework also allows for an analysis of shocks to each of the factors individually. Figure 9 shows the probability of default under a negative shock (a recession) to each of the 5 factors individually. Under a negative shock to the first factor or, in other words, when the financial-sector factor enters into a recession, the probability of observing zero defaults falls relative to the baseline scenario. This is a result of all the loadings for the first factor being large and positive. Consistently, the probability of observing 1, 2, and 3 defaults rises above the baseline-scenario probabilities. When shocks affect the other 4 factors, similar patterns emerge. However, the overall impact in terms of the probability of default has a comparatively smaller effect than for the first-factor shock, since some of the loadings are small and/or negative.

We also conducted a stress test analysis for a positive shock ("boom") to all factors as well as one factor at a time.[17] When all factors are jointly in a boom scenario, most of the probability mass concentrates on zero defaults. In the case of a boom for each factor at a time, the probability of zero defaults also has higher values than in the baseline; however, the probability of observing just one default increases over time relative to the baseline. And, consistently, the probability of observing more than one default falls relative to the baseline.

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[17] The results of the boom scenarios are available from the authors upon request.

Figure 9. Stress Testing: Probabilities of Default under Alternative Recession Scenarios

Baseline

"Financial sector" factor in recession

"European" factor in recession

"U.S." factor in recession

"Commercial banks" factor in recession

"Investment banks" factor in recession

Note: Calculations are based on CDS spreads for end 2005

IX. CONCLUDING REMARKS

This paper develops a market-based indicator for financial sector surveillance. Building on Hull and White (2005) and Gibson (2004), our approach generalizes Avesani (2005) by adopting a multifactor latent structure in the determination of the default probabilities of a credit default swap basket of large complex financial institutions. Factor analysis shows that the correlation among the financial institutions requires a multifactor representation, which is critical for the computation of the default correlations and, therefore, for the accuracy of this indicator.

The identification and estimation of the factors, which drive the covariance-matrix dynamics, offer an opportunity to bring macroeconomic-related factors to bear in a purely financial model. By doing so, we are proposing a new angle from which to approach stress testing. In fact, the impact of changing macroeconomic conditions (e.g., a recession) is directly modeled through shocks to the multifactor structure that is generated within the financial model.

Our empirical results, based on end-2005 credit default swap spreads and stress testing analysis, provide the following insights. First, the two-year forward probability of no default (92 percent) has increased markedly compared to the one observed during the May 2005 credit events related to the downgrading of General Motors (88 percent). Second, the stress-testing results for a scenario where all common factors enter a recession simultaneously show that the two-year forward probability of no default would fall to around 30 percent. And, third, a recession in the U.S. factor, that is, a more similar shock to the May 2005 credit events, which affected mostly U.S. institutions, would result in two-year forward probability of default of about 80 percent. Overall, the results obtained from the application of these shocks unveil a rich set of default probability dynamics and help in identifying the most relevant sources of risk.

References

Amemiya, Yasuo, and T.W. Anderson, 1990, "Asymptotic Chi-Square Tests for a Large Class of Factor Analysis Models", The Annals of Statistics, Vol. 18, No. 3, pp. 1453—1463.

Andersen, Leif, Jakob Sidenius and Susanta Basu, 2003, "All Your Hedges in One Basket," *Risk*, November, pp. 67–72.

Anderson, T.W., 2003, *An Introduction to Multivariate Analysis*, 3rd Edition John Wiley & Sons (New York).

Avesani, Renzo G., 2005, "FIRST: A Market-Based Approach to Evaluate Financial System Risk and Stability," Working Paper No. 05/232 (Washington: International Monetary Fund).

Bank of England, 2004, *Financial Stability Review*, (London: Bank of England), December.

Berndt, A., R. Douglas, D. Duffie, M. Ferguson, and D. Schranz, 2005, "Measuring Default Risk Premia from Default Swap Rates and EDFs," (unpublished; Palo Alto: Stanford University).

Black, F., and M. Scholes, 1973, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy,* Vol. 81, No. 3 (May/June), pp. 637–54.

Crockett, Andrew, 2000, "Marrying the Micro- and Macro-Prudential Dimensions of Financial Stability," Eleventh International Conference of Banking Supervisors, Basel, September 2000.

Elsinger, Helmut, Alfred Lehar, Martin Summer, and Simon Wells, 2004, "Using Market Information for Banking System Risk Assessment," (unpublished; London: Bank of England).

FitchRatings, 2005, "A Comparative Empirical Study of Asset Correlations," *Structured Finance*, July.

Flannery, M.J., and S.M. Sorescu, 1996, "Evidence of Bank Market Discipline in Subordinated Debenture Yields: 1983–1991," *Journal of Finance* 51, (September), pp. 1347–77.

Gibson, Michael, 2004, "Understanding the Risk of Synthetic CDOs" (unpublished Washington: Federal Reserve Board).

Gropp, R., J. Vesala, and G. Vulpes, 2002, "Equity and Bond Market Signals As Leading Indicators of Bank Fragility," Working Paper No. 150 (Frankfurt: European Central Bank).

Hawkesby, Christian, Ian Marsh, and Ibrahim Stevens, 2005, "Comovements in the Price of Securities Issued by Large Complex Financial Institutions," Working Paper No. 256, (London: Bank of England).

Hull, John, and Alan White, 2005, "Valuation of a CDO and an Nth -to-default CDS Without Monte Carlo Simulation," *Journal of Derivatives*, Vol. 12, No. 2, pp. 8–23.

International Monetary Fund, 2003, "Compilation Guide on Financial Soundness Indicators," March draft (Washington: International Monetary Fund). Available on the Internet at http://www.imf.org/external/np/sta/fsi/eng/guide/index.htm.

Kevin J. Stiroh, 2005, "Bank Risk and Revenue Diversification: An Assessment of Using Equity Returns" (unpublished; New York: Federal Reserve Bank System).

Merton, R.C., 1974, "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance,* Vol. 29, pp. 449–70.

Pesaran, M.H., T. Schuermann, B.-J. Treutler, and S. M. Weiner, 2005, "Macroeconomic Dynamics and Credit Risk: A Global Perspective," *Journal of Money Credit and Banking*, Vol. 38 No. 5, pp. 1211-1262.

Vasicek, O., 1987, "Probability of Loss on Loan Portfolio," KMV Working Paper (San Francisco: KMV Corporation).

Granularity Adjustment for Basel II[*]

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Abstract

The credit value-at-risk model underpinning the Basel II Internal Ratings-Based approach assumes that idiosyncratic risk has been diversified away fully in the portfolio, so that economic capital depends only on systematic risk contributions. We develop a simple methodology for approximating the effect of undiversified idiosyncratic risk on VaR. The supervisory review process (Pillar 2) of the new Basel framework offers a potential venue for application of the proposed granularity adjustment (GA).

Our GA is a revision and extension of the methodology proposed in the Basel II Second Consultative Paper. The revision incorporates some technical advances as well as modifications to the Basel II rules since the Second Consultative Paper of 2001. Most importantly, we introduce an "upper bound" methodology under which banks would be required to aggregate multiple exposures to the same underlying obligor only for a subset of their obligors. This addresses what appears to be the most significant operational burden associated with any rigorous assessment of residual idiosyncratic risk in the portfolio. For many banks, this approach would permit dramatic reductions in data requirements relative to the full GA.

Key words: Basel II, granularity adjustment, value-at-risk, idiosyncratic risk

JEL Codes: G31, G28

1. Introduction

In the portfolio risk-factor frameworks that underpin both industry models of credit VaR and the Internal Ratings-Based (IRB) risk weights of Basel II, credit risk in a portfolio arises from two sources, systematic and idiosyncratic. Systematic risk represents the effect of unexpected changes in macroeconomic and financial market conditions on the performance of borrowers. Borrowers may differ in their degree of sensitivity to systematic risk, but few firms are completely indifferent to the wider economic conditions in which they operate. Therefore, the systematic component of portfolio risk is unavoidable and only partly diversifiable. Idiosyncratic risk represents the effects of risks that are particular to individual borrowers. As a portfolio becomes more fine-grained, in the sense that the largest individual exposures account for a smaller share of total portfolio exposure, idiosyncratic risk is diversified away at the portfolio level.

Under the Asymptotic Single Risk Factor (ASRF) framework that underpins the IRB approach, it is assumed that bank portfolios are *perfectly* fine-grained, that is, that idiosyncratic risk has been fully diversified away, so that economic capital depends only on systematic risk. Real-world portfolios are not, of course, perfectly fine-grained. The asymptotic assumption might be approximately valid for some of the largest bank portfolios, but clearly would be much less satisfactory for portfolios of smaller or more specialized institutions. When there are material name concentrations of exposure, there will be a residual of undiversified idiosyncratic risk in the portfolio. The IRB formula omits the contribution of this residual to required economic capital.

The impact of undiversified idiosyncratic risk on portfolio VaR can be assessed via a methodology known as *granularity adjustment*. The basic concepts and approximate form for the granularity adjustment were first introduced by Gordy in 2000 for application in Basel II (see Gordy, 2003). It was then substantially refined and put on a more rigorous foundation by Wilde (2001b) and Martin and Wilde (2003).[1] In this paper, we propose and evaluate a granularity adjustment (GA) suitable for application under Pillar 2 of Basel II (Basel Committee on Bank Supervision, 2006).

Our proposed methodology is similar in form and spirit to the granularity adjustment that was included in the Second Consultative Paper (CP2) of Basel II (Basel Committee on Bank Supervision, 2001). Like the CP2 version, the revised GA is derived as a first-order asymptotic approximation for the effect of diversification in large portfolios within the CreditRisk$^+$ model of portfolio credit risk. Also in keeping with the CP2 version, the data inputs to the revised GA are drawn from

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[1]The results of Martin and Wilde (2003) can be viewed as an application of theoretical work by Gouriéroux, Laurent, and Scaillet (2000). Other early contributions to the GA literature include Wilde (2001a) and Pykhtin and Dev (2002). Gordy (2004) presents a survey of these developments and a primer on the mathematical derivation.

quantities already required for the calculation of IRB capital charges and reserve requirements.

In practical application, it is the data inputs (and not the formulae applied to those inputs) that can pose the most serious obstacles to cost-effective implementation. For this reason, we should elaborate here on an important caveat to our claim that all GA inputs are made available in the course of calculating IRB capital and reserve requirements. When a bank has multiple exposures to the same underlying obligor, it is required that these multiple exposures be aggregated into a single exposure for the purpose of calculating GA inputs. For the purpose of calculating IRB capital requirements, by contrast, the identity of the obligor is immaterial, as capital charges depend only on characteristics of the loan and obligor (e.g., type of loan, default probability, maturity) and not on the name of the borrower per se. This is a great convenience when data on different sorts of exposures are held on different computer systems, as the job of calculating capital may be delegated to those individual systems and reported back as subportfolio aggregates which can then be added up in a straightforward fashion to arrive at the bank-level capital and reserve requirements. When we measure granularity, we cannot ignore borrower identity. From the perspective of single name concentration, ten loans of 1 million Euros each to ten distinct borrowers jointly carry much less idiosyncratic risk than the same ten loans made to a single borrower. It is understood that the need to aggregate information across computer systems on multiple exposures to a single borrower is the most significant challenge for banks in implementing a granularity adjustment. In defense of this aggregation requirement, we note that such aggregation would be necessary in *any* effective measure of granularity, and so is not a drawback peculiar to the GA we propose in this paper. Furthermore, one might ask how a bank can effectively manage its name concentrations without the ability to aggregate exposures across the different activities of the bank.

To reduce the burden associated with exposure aggregation, the revised GA provides for the possibility that banks be allowed to calculate the GA on the basis of the largest exposures in the portfolio, and thereby be spared the need to aggregate data on each and every obligor. To permit such an option, regulators must be able to calculate the largest possible GA that is consistent with the incomplete data provided by the bank. Our approach, therefore, is based on an upper bound formula for the GA as a function of data on the m largest capital contributions out of a portfolio of n loans (with $m \leq n$). As m grows towards n (i.e., as the bank provides data on a larger share of its portfolio), the upper bound formula converges to the "full portfolio" GA. The advantage to this approach is that the bank can be permitted to choose m in accordance with its own trade-off between higher capital charges (for m small) and higher data collection effort (for m large).

Our revised methodology takes advantage of theoretical advances that have been made since the time of CP2. In particular, the GA of CP2 required a first-stage

calculation in which the portfolio would be mapped to a homogeneous portfolio of similar characteristics. In the revision GA, the heterogeneous portfolio is used directly in the formula. The resulting algorithm is both simpler and more accurate than the one of CP2.

Last, our revised methodology is adapted to the changes in the definition of regulatory capital. At the time of CP2, capital requirements were expressed in terms of expected loss (EL) plus unexpected loss (UL), whereas the finalized Basel II distinguishes UL capital requirements from EL reserve requirements. The GA is invariant to EL so is unaffected by this definitional issue. However, the *inputs* to the GA do depend on the distinction between EL and UL, and so the formulae have been modified accordingly.

The methodology for the GA is set out in Section 2. In Section 3, we show how to construct an upper bound based on partial information for the portfolio. Section 4 describes the dataset that we have used for our numerical studies. The performance of the GA is assessed in various ways in Section 5. We conclude with some thoughts on the role of model choice in crafting a granularity adjustment and with a list of some tasks left for future work.

2. Methodology

In principle, the granularity adjustment can be applied to *any* risk-factor model of portfolio credit risk, and so we begin with a very general framework. We mainly follow the treatment of Martin and Wilde (2003) in the mathematical presentation, though our parameterization of the GA formula will differ. Let X denote the systematic risk factor. For consistency with the ASRF framework of Basel II and for ease of presentation, we assume that X is unidimensional (i.e., that there is only a single systematic factor). Let n be the number of positions in the portfolio, and assume that exposures have been aggregated so that there is a unique obligor for each position. Let U_i denote the loss rate on position i, let A_i denote its exposure at default (EAD$_i$), and let L_n be the loss rate on the portfolio of the first n positions, i.e.,

$$L_n = \sum_{i=1}^{n} s_i \cdot U_i, \tag{1}$$

where s_i denotes the portfolio share of each instrument $s_i = A_i / \sum_{j=1}^{n} A_j$.

Let $\alpha_q(Y)$ denote the qth percentile of the distribution of some random variable Y. When economic capital is measured as value-at-risk at the qth percentile, we wish to estimate $\alpha_q(L_n)$. The IRB formula, however, delivers the qth percentile of the conditional expected loss $\alpha_q(E[L_n|X])$. The difference $\alpha_q(L_n) - \alpha_q(E[L_n|X])$

is the "exact" adjustment for the effect of undiversified idiosyncratic risk in the portfolio. Such an exact adjustment cannot be obtained in analytical form, but we can construct a Taylor series approximation in orders of $1/n$. Define the functions $\mu(X) = E[L_n|X]$ and $\sigma^2(X) = V[L_n|X]$ as the conditional mean and variance of the portfolio loss respectively, and let h be the probability density function of X. Wilde (2001b) shows that the first-order granularity adjustment is given by

$$GA = \frac{-1}{2h(\alpha_q(X))} \frac{d}{dx} \left(\frac{\sigma^2(x)h(x)}{\mu'(x)} \right) \bigg|_{x=\alpha_q(X)} \tag{2}$$

This general framework can accommodate any definition of "loss." That is, we can measure the U_i on a mark-to-market basis or an actuarial basis, and either inclusive or exclusive of expected loss. The latter point is important in light of the separation of "total capital" (the concept used in CP2) into its EL and UL components in the final Basel II document. Say we measure the U_i and L_n inclusive of expected loss, but wish to define capital on a UL basis. Let UL_n be the "true" UL for the portfolio, and let UL_n^{asympt} be its asymptotic approximation which assumes that the idiosyncratic risk is diversified away. Then

$$\alpha_q(L_n) - \alpha_q(E[L_n|X]) = (UL_n + EL_n) - \left(UL_n^{asympt} + E[E[L_n|X]]\right) = UL_n - UL_n^{asympt}$$

because the unconditional expected loss ($EL_n = E[L_n]$) is equal to the expectation of the conditional loss ($E[E[L_n|X]]$). Put more simply, expected loss "washes out" of the granularity adjustment.

In the GA formula, the expressions for $\mu(x)$, $\sigma^2(x)$ and $h(x)$ are model-dependent. For application of the GA in a supervisory setting, it would be desirable to base the GA on the same model as that which underpins the IRB capital formula. Unfortunately, this is not feasible for two reasons: First, the IRB formula is derived within a single-factor mark-to-market Vasicek model closest in spirit to KMV Portfolio Manager. The expressions for $\mu(x)$ and $\sigma^2(x)$ in such a model would be formidably complex. The effect of granularity on capital is sensitive to maturity, so simplification of the model to its default-mode counterpart (closest in spirit to a two-state CreditMetrics) would entail a substantive loss of fidelity. Furthermore, even with that simplification, the resulting expressions for $\mu(x)$ and $\sigma^2(x)$ remain somewhat more complex than desirable for supervisory application. The second barrier to using this model is that the IRB formula is not fit to the model directly, but rather is linearized with respect to maturity. The "true" term-structure of capital charges in mark-to-market models tends to be strongly concave, so this linearization is not at all a minor adjustment. It is not at all clear how one would alter $\mu(x)$ and $\sigma^2(x)$ to make the GA consistent with the linearized IRB formula.

As fidelity to the IRB model cannot be imposed in a direct manner, we adopt an indirect strategy. We base the GA on a model chosen for the tractability of the

resulting expressions, and then reparameterize the inputs in a way that restores consistency as much as possible. Our chosen model is an extended version of the single factor CreditRisk$^+$ model that allows for idiosyncratic recovery risk.[2] As CreditRisk$^+$ is an actuarial model of loss, we define the loss rate as $U_i = \text{LGD}_i \cdot D_i$, where D_i is a default indicator equal to 1 if the obligor defaults, 0 otherwise. The systematic factor X generates correlation across obligor defaults by shifting the default probabilities. Conditional on $X = x$, the probability of default is

$$\text{PD}_i(x) = \text{PD}_i \cdot (1 - w_i + w_i \cdot x).$$

where PD_i is the unconditional probability of default. The factor loading w_i controls the sensitivity of obligor i to the systematic risk factor. We assume that X is gamma-distributed with mean 1 and variance $1/\xi$ for some positive ξ.[3] Finally, to obtain an analytical solution to the model, in CreditRisk$^+$ one approximates the distribution of the default indicator variable as a Poisson distribution.

In the standard version of CreditRisk$^+$, the recovery rate is assumed to be known with certainty. Our extended model allows LGD_i to be a random loss-given-default with expected value ELGD_i and variance VLGD_i^2. The LGD uncertainty is assumed to be entirely idiosyncratic, and therefore independent of X.

We next obtain the $\mu(x)$ and $\sigma^2(x)$ functions for this model. Let us define at the instrument level the functions $\mu_i(x) = E[U_i|x]$ and $\sigma_i^2(x) = V[U_i|x]$. By the conditional independence assumption, we have

$$
\begin{aligned}
\mu(x) &= E[L_n|x] = \sum_{i=1}^{n} s_i \mu_i(x) \\
\sigma^2(x) &= V[L_n|x] = \sum_{i=1}^{n} s_i^2 \sigma_i^2(x).
\end{aligned}
$$

In CreditRisk$^+$, the $\mu_i(x)$ function is simply

$$\mu_i(x) = \text{ELGD}_i \cdot \text{PD}_i(x) = \text{ELGD}_i \cdot \text{PD}_i \cdot (1 - w_i + w_i \cdot x).$$

For the conditional variance, we have

$$\sigma_i^2(x) = E[\text{LGD}_i^2 \cdot D_i^2|x] - \text{ELGD}_i^2 \cdot \text{PD}_i(x)^2 = E[\text{LGD}_i^2] \cdot E[D_i^2|x] - \mu_i(x)^2. \quad (3)$$

As D_i given X is assumed to be Poisson distributed, we have $E[D_i|X] = V[D_i|X] = \text{PD}_i(X)$, which implies

$$E[D_i^2|X] = \text{PD}_i(X) + \text{PD}_i(X)^2.$$

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[2]CreditRisk$^+$ is a widely-used industry model for portfolio credit risk that was proposed by Credit Suisse Financial Products (1997).

[3]Note that we must have $E[X] = 1$ in order that $E[\text{PD}_i(X)] = \text{PD}_i$.

For the term $E[\text{LGD}_i^2]$ in the conditional variance, we can substitute

$$E[\text{LGD}_i^2] = V[\text{LGD}_i] + E[\text{LGD}_i]^2 = \text{VLGD}_i^2 + \text{ELGD}_i^2$$

This leads us to

$$
\begin{aligned}
\sigma_i^2(x) &= \left(\text{VLGD}_i^2 + \text{ELGD}_i^2\right) \cdot \left(\text{PD}_i(X) + \text{PD}_i(X)^2\right) - \mu_i(x)^2 \\
&= \mathcal{C}_i \mu_i(x_q) + \mu_i(x_q)^2 \cdot \frac{\text{VLGD}_i^2}{\text{ELGD}_i^2}
\end{aligned}
$$

where $\mathcal{C}_i$ is defined as

$$\mathcal{C}_i \equiv \frac{\text{ELGD}_i^2 + \text{VLGD}_i^2}{\text{ELGD}_i}. \tag{4}$$

We substitute the gamma pdf $h(x)$ and the expressions for $\mu(x)$ and $\sigma^2(x)$ into equation (2), and then evaluate the derivative in that equation at $x = \alpha_q(X)$. The resulting formula depends on the instrument-level parameters PD_i, w_i, ELGD_i and VLGD_i.

We now reparameterize the inputs. Let $\mathcal{R}_i$ be the EL reserve requirement as a share of EAD for instrument i. In the default-mode setting of CreditRisk$^+$, this is simply

$$\mathcal{R}_i = \text{ELGD}_i \cdot \text{PD}_i.$$

Let $\mathcal{K}_i$ be the UL capital requirement as a share of EAD. In CreditRisk$^+$, this is

$$\mathcal{K}_i = E[U_i | X = \alpha_q(X)] = \text{ELGD}_i \cdot \text{PD}_i \cdot w_i \cdot (\alpha_q(X) - 1) \tag{5}$$

When we substitute $\mathcal{R}_i$ and $\mathcal{K}_i$ into the CreditRisk$^+$ GA, we find that the PD_i and w_i inputs can be eliminated. We arrive at the formula

$$
\begin{aligned}
GA_n &= \frac{1}{2\mathcal{K}^*} \sum_{i=1}^n s_i^2 \Big[\left(\delta \mathcal{C}_i (\mathcal{K}_i + \mathcal{R}_i) + \delta (\mathcal{K}_i + \mathcal{R}_i)^2 \cdot \frac{\text{VLGD}_i^2}{\text{ELGD}_i^2} \right) \\
&\qquad\qquad - \mathcal{K}_i \left(\mathcal{C}_i + 2(\mathcal{K}_i + \mathcal{R}_i) \cdot \frac{\text{VLGD}_i^2}{\text{ELGD}_i^2} \right) \Big],
\end{aligned} \tag{6}
$$

where $\mathcal{K}^* = \sum_{i=1}^n s_i \mathcal{K}_i$ is the required capital per unit exposure for the portfolio as a whole and where

$$\delta \equiv (\alpha_q(X) - 1) \cdot \left(\xi + \frac{1 - \xi}{\alpha_q(X)} \right).$$

Note that the expression for δ depends only on model parameters, not data inputs, so δ is a regulatory parameter. It is through δ that the variance parameter ξ influences the GA. In the CP2 version, we set $\xi = 0.25$. Assuming that the target solvency probability is $q = 0.999$, this setting implies $\delta = 4.83$. This is the value used in the numerical exercises of Section 5, but we also examine the sensitivity of the GA to the

choice of ξ. Alternative calibrations of ξ are explored in the Appendix. For policy purposes, it is worthwhile to note that setting $\xi = 0.31$ would be well within any reasonable empirical bounds on this parameter, and would yield the parsimonious integer value $\delta = 5$.

The volatility of LGD (VLGD) neither is an input to the IRB formula, nor is it restricted in any way within the IRB model. Banks could, in principle, be permitted or required to supply this parameter for each loan. Given the scant data currently available on recoveries, it seems preferable to impose a regulatory assumption on VLGD in order to avoid the burden of a new data requirement. We impose the relationship as found in the CP2 version of the GA:

$$\text{VLGD}_i^2 = \gamma \, \text{ELGD}_i (1 - \text{ELGD}_i) \tag{7}$$

where the regulatory parameter γ is between 0 and 1. When this specification is used in industry models such as CreditMetrics and KMV Portfolio Manager, a typical setting is $\gamma = 0.25$.

The GA formula can be simplified somewhat. The quantities $\mathcal{R}_i$ and $\mathcal{K}_i$ are typically small, and so terms that are products of these quantities can be expected to contribute little to the GA. If these second-order terms are dropped, we arrive at the simplified formula:

$$\widetilde{GA}_n = \frac{1}{2\mathcal{K}^*} \sum_{i=1}^{n} s_i^2 \mathcal{C}_i \left(\delta (\mathcal{K}_i + \mathcal{R}_i) - \mathcal{K}_i \right). \tag{8}$$

Here and henceforth, we use the tilde to indicate this simplified GA formula. The accuracy of this approximation to equation (6) is evaluated in Section 5.

Before proceeding, we pause to mention some alternative methodologies. Perhaps the very simplest approach would be based on a Herfindahl-Hirschman Index (HHI), which is defined as the sum of the squares of the portfolio shares of the individual exposures. Holding all else equal, the closer the HHI of a portfolio is to 1 the more concentrated the portfolio is, so the higher the appropriate granularity add-on charge. As with any ad hoc approach, it is difficult to say what the "appropriate" add-on for a given HHI should be. Furthermore, as we will see in Section 5, the effect of granularity on economic capital is quite sensitive to the credit quality of the portfolio, so the HHI approach would need to somehow take this into account. One suspects that an appropriately modified HHI-based approach would be no less complex than a model-based approach and certainly would be less robust. Finally, an HHI-based approach does not avoid in any way the operational burden associated with aggregation of multiple exposures to a single exposure per obligor.

Another approach, due to Vasicek (2002), lies somewhere between ad hoc and model-based. In this method, one augments systematic risk (by increasing the factor

loading) in order to compensate for ignoring the idiosyncratic risk. The trouble is that systematic and idiosyncratic risk have very different distribution shapes. This method is known to perform quite poorly in practice.

Much closer to our proposal in spirit and methodology is the approach of Emmer and Tasche (2005). Emmer and Tasche (2005) offer a granularity adjustment based on a one-factor default-mode CreditMetrics model, which has the advantage of relative proximity of the model underpinning the IRB formula. As discussed earlier, however, we believe this advantage to be more in appearance than in substance because of the importance of maturity considerations in the IRB model. As a mark-to-market extension of the Emmer and Tasche (2005) GA appears to be intractable, maturity considerations would need to be introduced indirectly (as in our proposal) through the inputs. Reparameterization along these lines is feasible in principle, but would lead to a rather more complicated formula with more inputs than our CreditRisk$^+$-based GA.

Finally, an alternative that has not been much studied is the saddlepoint based method of Martin and Wilde (2003). Results in that paper suggest that it would be quite similar to the GA in performance and pose a similar tradeoff between fidelity to the IRB model and analytical tractability. Indeed, it is not at all likely that the saddlepoint GA would yield a closed-form solution for any industry credit risk model other than CreditRisk$^+$.

3. An upper bound based on incomplete data

As discussed in the introduction, aggregation of multiple exposures into a single exposure per obligor is very likely to be the only substantive challenge in implementing the granularity adjustment. To reduce this burden on the banks, we propose that banks be permitted to calculate the GA based on a *subset* consisting of the largest exposures. An upper bound can be calculated for the influence of exposures that are left out of the computation. This approach is conservative from a supervisory point of view because the upper bound is always at least as large as the "true" GA. The bank can therefore be given the flexibility to find the best trade-off between the cost of data collection and the cost of the additional capital associated with the upper bound.

In order to convey most clearly the intuition behind our approach, we first present the upper bound in the special case of a portfolio that is homogeneous in PD and ELGD. We then present the upper bound for the more realistic case of a heterogeneous portfolio.

3.1. Homogeneous case

The simplest upper bound is for the case in which exposures are homogeneous in PD and ELGD, but heterogeneous in exposure size. Assume that the bank has determined the m largest aggregate exposures in the portfolio of n obligors ($m \leq n$), and that we have sorted these aggregated EAD values as $A_1 \geq A_2 \geq \ldots \geq A_m$. The shares $s_1 \geq s_2 \geq \ldots \geq s_m$ are, as in Section 2, calculated with respect to the *total* portfolio EAD in the denominator. This latter quantity certainly will be available in the bank's balance sheet.

When PD and ELGD are homogeneous, we have $\mathcal{K}_i = \mathcal{K}^* = \mathcal{K}$ and $\mathcal{R}_i = \mathcal{R}$ for all i, and similarly $\mathcal{C}_i = \mathcal{C}$ is also independent of i. Hence the simplified GA reads

$$\widetilde{GA}_n = \frac{1}{2\mathcal{K}}\mathcal{C}(\delta(\mathcal{K} + \mathcal{R}) - \mathcal{K}) \cdot HHI,$$

where HHI is the Herfindahl-Hirschman Index

$$HHI = \sum_{i=1}^{n} s_i^2.$$

Using only the first $m \leq n$ exposures, and defining S_m as the cumulative share of these exposures, $S_m = \sum_{i=1}^{m} s_i$, we know that HHI is bounded by

$$HHI = \sum_{i=1}^{m} s_i^2 + \sum_{i=m+1}^{n} s_i^2 \leq \sum_{i=1}^{m} s_i^2 + s_m \cdot \sum_{i=m+1}^{n} s_i = \sum_{i=1}^{m} s_i^2 + s_m \cdot (1 - S_m).$$

This leads to the following upper bound for the simplified granularity adjustment

$$\widetilde{GA}_n^{upper} = \frac{1}{2\mathcal{K}}\mathcal{C}(\delta(\mathcal{K} + \mathcal{R}) - \mathcal{K}) \cdot \left(\sum_{i=1}^{m} s_i^2 + s_m \cdot (1 - S_m) \right). \tag{9}$$

3.2. Heterogeneous case

In the general case of a heterogeneous portfolio, the upper bound becomes more complicated because the meaning of "largest exposures" is no longer unambiguous. Do we mean largest by EAD, by capital contribution, or by some other measure? It turns out that we require information on both the distribution of aggregated positions by EAD and by capital contribution. Specifically, we assume:

1. The bank has identified the m obligors to whom it has the largest aggregated exposures measured in capital contribution, i.e., $A_i \cdot \mathcal{K}_i$. Denote this set of obligors as Ω. For each obligor $i \in \Omega$, the bank knows $(s_i, \mathcal{K}_i, \mathcal{R}_i)$.

2. For the $n - m$ exposures that are unreported (that is, exposures for which the obligor is not in Ω), the bank determines an upper bound on share (denoted $\bar{s}$) such that $s_i \le \bar{s}$ for all i in the unreported set.

3. The bank knows $\mathcal{K}^*$ and $\mathcal{R}^*$ for the portfolio as a whole.

The first assumption is straightforward and unavoidable, as this is where the need arises to aggregate multiple exposures for a subset of obligors in the portfolio. Internal risk management reporting typically includes a list of the "tallest trees" in capital usage by customer, and therefore it is reasonable to assume that aggregated capital contribution data for the largest customers are internally available. If such data are unavailable, we might question whether the bank is making any substantive business use of its internal economic capital models.

The second assumption is perhaps more difficult, but is necessary in order to obtain a bound on unreported exposure shares. A bank can easily identify $\bar{s}$ if, for example, internal risk management systems report on the obligors to which the bank has the greatest exposure in EAD.[4] Denote this set by Λ, and let λ be the smallest s_i in this set. Then $\bar{s}$ is either the largest of the s_i which is in Λ but not in Ω or (if this set is empty) simply λ, i.e.,

$$\bar{s} = \max\{s_i : s_i \in \Lambda \backslash \Omega \cup \{\lambda\}\}$$

The third assumption hardly needs justification, as these porfolio-level quantities are calculated in the course of determining IRB capital requirements. In particular, $\mathcal{K}^*$ and $\mathcal{R}^*$ can be obtained in the usual manner without aggregation of exposures by obligor.

We generalize the notation $\mathcal{K}^*$ and $\mathcal{R}^*$ so that

$$\mathcal{K}_k^* = \sum_{i=1}^{k} s_i \mathcal{K}_i \qquad \text{and} \qquad \mathcal{R}_k^* = \sum_{i=1}^{k} s_i \mathcal{R}_i,$$

i.e., $\mathcal{K}_k^*$ and $\mathcal{R}_k^*$ are partial weighted sums of the $\mathcal{K}_i$ and $\mathcal{R}_i$ sequences, respectively. Finally, for notational convenience define

$$Q_i \equiv \delta(\mathcal{K}_i + \mathcal{R}_i) - \mathcal{K}_i.$$

Using the above notation, the GA can be reformulated as follows

$$
\begin{aligned}
\widetilde{GA}_n &= \frac{1}{2\mathcal{K}^*} \sum_{i=1}^{n} s_i^2 \mathcal{C}_i (\delta(\mathcal{K}_i + \mathcal{R}_i) - \mathcal{K}_i) \\
&= \frac{1}{2\mathcal{K}^*} \cdot \left(\sum_{i=1}^{m} s_i^2 Q_i \mathcal{C}_i + \sum_{i=m+1}^{n} s_i^2 Q_i \mathcal{C}_i \right).
\end{aligned}
\tag{10}
$$

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[4]For example, there may be a lending rule that requires the director of the bank to sign off on all loans above a certain threshold.

The summation over 1 to m is known by Assumption 1. By Assumption 2, we know that $\bar{s} \geq s_i$ for $i = m+1, \ldots, n$. Our assumption on VLGD in equation (7) is sufficient to guarantee that $\mathcal{C}_i \leq 1$. Therefore,

$$\sum_{i=m+1}^{n} s_i^2 Q_i \mathcal{C}_i \leq \bar{s} \sum_{i=m+1}^{n} s_i Q_i = \bar{s} \left(\delta \sum_{i=m+1}^{n} s_i(\mathcal{K}_i + \mathcal{R}_i) - \sum_{i=m+1}^{n} s_i \mathcal{K}_i \right).$$

Next observe that

$$\begin{aligned}
\sum_{i=m+1}^{n} s_i \mathcal{K}_i &= \mathcal{K}^* - \mathcal{K}_m^* \\
\sum_{i=m+1}^{n} s_i \mathcal{R}_i &= \mathcal{R}^* - \mathcal{R}_m^*.
\end{aligned}$$

Assumption 1 implies that $\mathcal{K}_m^*$ and $\mathcal{R}_m^*$ are known to the bank. Thus we arrive at

$$\sum_{i=m+1}^{n} s_i^2 Q_i \mathcal{C}_i \leq \bar{s}((\delta-1)(\mathcal{K}^* - \mathcal{K}_m^*) + \delta(\mathcal{R}^* - \mathcal{R}_m^*)). \tag{11}$$

Finally we obtain the following upper bound for the heterogeneous case

$$\widetilde{GA}_m^{upper} = \frac{1}{2\mathcal{K}^*} \left(\sum_{i=1}^{m} s_i^2 Q_i \mathcal{C}_i + \bar{s}\left((\delta-1)(\mathcal{K}^* - \mathcal{K}_m^*) + \delta(\mathcal{R}^* - \mathcal{R}_m^*)\right) \right). \tag{12}$$

4. Data on German bank portfolios

To show the impact of the granularity adjustment on economic capital we need to apply the GA to realistic bank portfolios. We use data from the German credit register, which includes all bank loans greater or equal to 1.5 Million Euro. This data set has been matched to the firms' balance sheet data to obtain obligor specific PDs. More specifically, a logistic regression model based on balance sheet data between 12 and 24 months before default classified as default balance sheets has been used.[5] The resulting portfolios are much smaller than the portfolios reported in the German credit register, however, there are still a number of banks with more than 300 exposures in this matched data set which we consider as an appropriate size for calculating the GA. We grouped the banks in large, medium, small and very small banks where large refers to a bank with more than 4000 exposures, medium refers to one with $1000 - 4000$ exposures, small refers to a bank with $600 - 1000$ exposures and very small to a bank with $300 - 600$ exposures.

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[5]The model has been found to provide a high accuracy in terms of an area under the ROC curve of more than 0.8. See Gerke et al. (2006).

To accommodate privacy restrictions on these data, we aggregate portfolios for three different banks into a single data set. We then sort the loans by exposure size and remove every third exposure. The resulting portfolio of 5289 obligors is still realistic in terms of exposure and PD distribution and is similar in size to some of the larger portfolios in the matched data set of the German credit register and the firm's balance sheet data. The mean of the loan size distribution is 3973 thousand Euros and the standard deviation is 9435 thousand Euros. Quantiles are reported in Table 1. Henceforth, we refer to this portfolio as "portfolio A."

Table 1
Exposure distribution in Portfolio A

| Level	Quantile
5%	50.92
25%	828.80
50%	1811.75
75%	3705.50
95%	13637.36

Figure 1 shows the PD distribution for the aggregated portfolio A for different PD categories which we denote here by S&P's common rating grades. The PD ranges for the different rating grades are listed in Table 2 below.

Table 2
PD ranges associated with rating buckets

Rating Grade	PD Ranges in %
AAA	$PD \leq 0.02$
AA	$0.02 \leq PD \leq 0.06$
A	$0.06 \leq PD \leq 0.18$
BBB	$0.18 \leq PD \leq 1.06$
BB	$1.06 \leq PD \leq 4.94$
B	$4.94 \leq PD \leq 19.14$
C	$19.14 \leq PD$

The average PD of the data set is 0.43% and hence lower than the average PD of a portfolio of a smaller or medium sized bank in Germany, which is approximately 0.8% (Kocagil et al., 2001, p. 8). Moody's, for example, understates average net loan provisions of 0.77% for German banks during the period $1989 - 1999$ (Kocagil

Figure 1. Borrower Distribution by Rating Grade
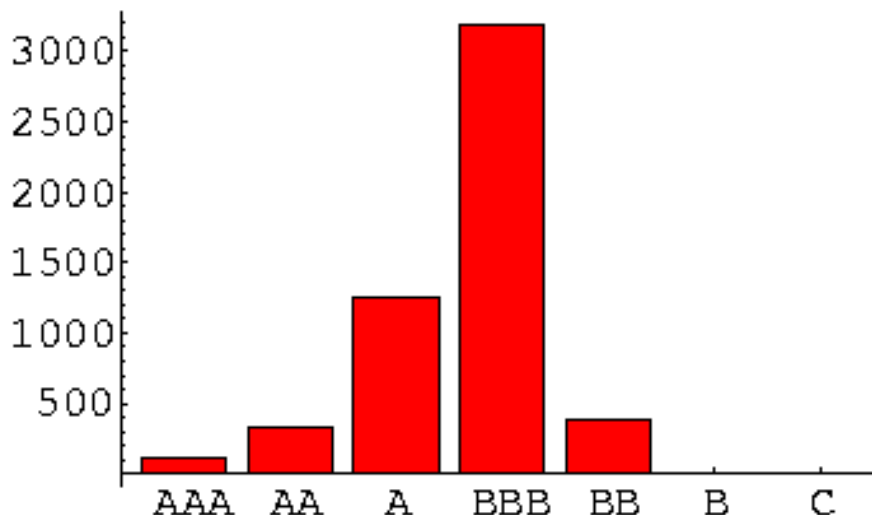
et al., 2001, p. 7), which is more than two times the average loss of the firms in our sample during the same period. Approximately 70% of the portfolio in our data set belongs to the investment grade domain (i.e., rated BBB or better) and the remaining 30% to the subinvestment grade. In smaller or medium sized banks in Germany the percentage of investment grade exposures in a portfolio is approximately 37% (Taistra et al., 2001, p. 2). As a consequence the value of the GA in our aggregated portfolio A will be smaller than the GA in a true bank portfolio of similar exposure distribution.

The data set does not contain information on LGD, so we impose the Foundation IRB assumption of ELGD = 0.45.

5. Numerical results

In Table 3, we present granularity adjustments calculated on real bank portfolios varying in size and degree of heterogeneity. As we would expect, the GA is invariably small (12 to 14 basis points) for the largest portfolios, but can be substantial (up to 161 basis points) for the smallest. The table demonstrates the strong correlation between Herfindahl index and GA across these portfolios, though of course the correspondence is not exact as the GA is sensitive to credit quality as well. As a reference portfolio, we included a portfolio with 6000 loans each of PD = 0.01 and

ELGD = 0.45 and of homogeneous EAD. The GA for the largest real portfolio is roughly six times as large as the GA for the homogeneous reference portfolio, which demonstrates the importance of portfolio heterogeneity in credit concentrations.

Table 3
Granularity Adjustment for real bank portfolios

Portfolio	Number of Exposures	HHI	GA (in %)
Reference	6000	0.00017	0.018
Large	> 4000	< 0.001	$0.12 - 0.14$
Medium	$1000 - 4000$	$0.001 - 0.004$	$0.14 - 0.36$
Small	$600 - 1000$	$0.004 - 0.011$	$0.37 - 1.17$
Very Small	$250 - 600$	$0.005 - 0.015$	$0.49 - 1.61$

We have also computed the VaR in the CreditRisk$^+$ model and the relative add-on for the GA on the VaR. For a large portfolio this add-on is 3% to 4% of VaR. For a medium sized bank the add-on lies between 5% and 8% of VaR. In a study based on applying a default-mode multi-factor CreditMetrics model to US portfolio data, Heitfield et al. (2006) find that name concentration accounts for between 1% and 8% of VaR depending on the portfolio size. These results are quite close to our own for the GA, despite the difference in model and data.

Table 4 shows the relative add-on for the granularity adjustment on the Risk Weighted Assets (RWA) of Basel II for small, medium and large portfolios as well as for the reference portfolio with 6000 exposures of unit size. The reference portfolio is used to point out the influence of the GA even for large portfolios that would be seen as very fine-grained. For the reference portfolio of 6000 exposures of unit size with homogeneous PD = 1% and ELGD = 45% the GA is approximately 0.018% and the IRB capital charge is 5.86%. Thus the add-on due to granularity is approximately 0.3% and the economic capital to capture both systematic risk and risk from single name concentration is 5.878% of the total portfolio exposure. For the real bank portfolios of our data set the add-on for the GA is higher than for the reference portfolio, although it is still quite small for large and even for some of the medium sized bank portfolios. For smaller portfolios with 300 to 1000 exposures the add-on for the GA is more significant.

Figure 2 shows the dependence of the simplified GA on the default probability. Each point on the curve represents a homogeneous portfolio of $n = 100$ borrowers of the given PD. Dependence on portfolio quality is non-negligible, particularly for lower-quality portfolios. Such dependence cannot be accommodated naturally and accurately in ad hoc methods of granularity adjustment based on exposure HHI.

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Table 4
GA as percentage add-on to RWA

Portfolio	Number of Exposures	Relative Add-On for RWA
Reference	6000	0.003
Large	> 4000	0.04
Medium	$1000 - 4000$	$0.04 - 0.10$
Small	$300 - 1000$	$0.17 - 0.32$

Figure 2. Effect of Credit Quality on Simplified GA
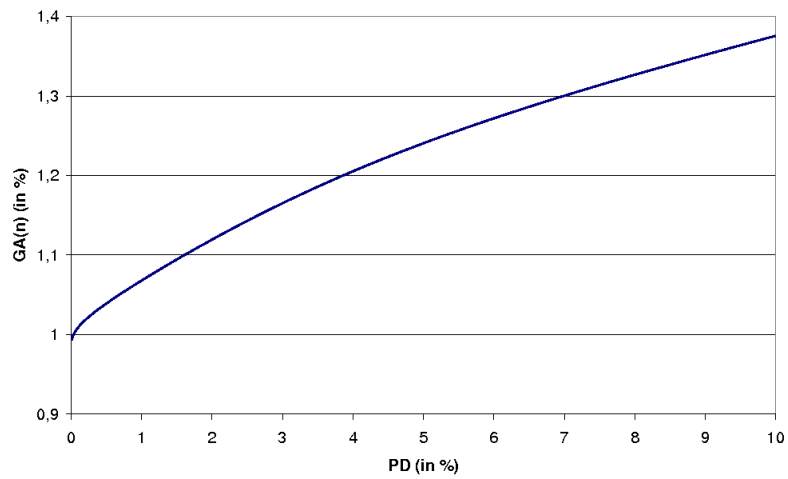
The sensitivity of the GA to the variance parameter ξ of the systematic factor X is explored in Figure 3. We see that the granularity adjustment is strictly increasing in ξ, and that the degree of sensitivity is not negligible. Increasing ξ from 0.2 to 0.3 causes a 10% increase in the GA for Portfolio A. Uncertainty in dependence parameters of this sort is a perennial challenge in portfolio credit risk modeling. A guiding principle in the design of Basel II has been to impose regulatory values on parameters (e.g., the asset correlation parameter ρ) that cannot be estimated to reasonable precision with extant data. Similar judgmental treatment is required here. While the absolute magnitude of the GA is sensitive to ξ, its relative magnitude across bank portfolios is much less so. In this sense, the proper functioning of the GA as a supervisory tool does not materially depend on the precision with which ξ is calibrated.

Figure 3. Effect of the Variance of the Systematic Factor on Simplifed GA
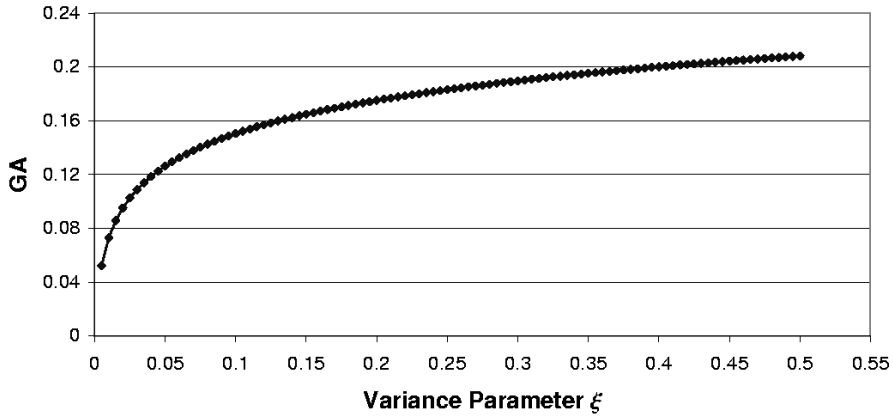
Our next task is to verify the accuracy of the simplified granularity adjustment $\widetilde{\text{GA}}$ as an approximation to the "full" GA of equation (6). We construct six stylized portfolios of different degrees of exposure concentrations. Each portfolio consists of $n = 1000$ exposures and has constant PD and ELGD fixed at 45%. Portfolio P0 is completely homogeneous whereas portfolio P50 is highly concentrated since the largest exposure $A_{1000} = 1000^{50}$ accounts for 5% of the total exposure of the portfolio. The values for both the simplified $\widetilde{GA}_n$ and the full GA for each of these portfolios are listed in Table 5. We see that the approximation error increases with concentration and with PD. For realistic portfolios, the error is trivial. Even for the case of portfolio P10 and PD = 4%, the error is only 3 basis points. The error grows to 12 basis points in the extreme example of P50 and PD = 4%, but even this remains small relative to the size of the GA.

Finally, we use Portfolio A to demonstrate the effectiveness of the upper bound provided in Section 3. In Figure 4, we show how the gap between the upper bound

Table 5
Approximation error of the simplified $\widetilde{GA}_n$

Portfolio	P0	P1	P2	P10	P50
PD = 1%					
Exposure A_i	1	i	i^2	i^{10}	i^{50}
$\widetilde{GA}$ in %	0.107	0.142	0.192	0.615	2.749
GA in %	0.109	0.146	0.197	0.630	2.814
PD = 4%					
Exposure A_i	1	i	i^2	i^{10}	i^{50}
$\widetilde{GA}$ in %	0.121	0.161	0.217	0.694	3.102
GA in %	0.126	0.168	0.227	0.726	3.243

and the "whole portfolio" GA shrinks as m (the number of positions included in the calculation) increases. With only 150 exposures included out of 5289 in the whole portfolio, this gap is only 10 basis points. With 300 exposures included, the gap shrinks to 5 basis points. The tightness of the upper bound is undoubtedly somewhat sensitive to the characteristics of the portfolio, but from these results we can tentatively conclude that the upper bound approach performs quite well.

Figure 4. Tightness of the Upper Bound
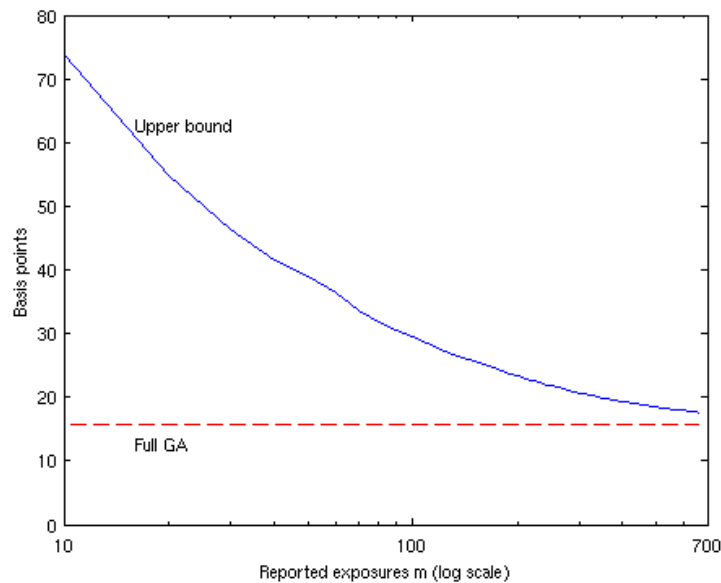
6. Discussion

This paper sets forth a granularity adjustment for portfolio credit VaR that accounts for a risk that is not captured by the Pillar 1 capital requirement of the Basel II IRB approach. Our GA is a revision and extension of the methodology first introduced in the Basel II Second Consultative Paper. The revision incorporates some technical advances as well as modifications to the Basel II rules since CP2. Most importantly, we introduce an "upper bound" methodology that addresses the most significant source of operational burden associated with the assessment of residual idiosyncratic risk in the portfolio (whether through the proposed GA or by any other rigorous methodology). For many banks, this approach would permit dramatic reductions in data requirements at modest cost in additional capital requirement.

We have examined the numerical behavior of the GA across a range of portfolio types and studied its robustness to model parameters. Two further potential sources of inaccuracy should be considered. First, the GA formula is itself an asymptotic approximation, and so might not work well on very small portfolios. We do not see this issue as a material concern. In general, the GA errs on the conservative (i.e., it overstates the effect of granularity), but is quite accurate for modest-sized portfolios of as few as 200 obligors (for a low-quality portfolio) or 500 obligors (for an investment-grade portfolio). Second, the IRB formulae are based on a rather different model of credit risk, so we have a form of "basis risk" (or "model mismatch"). This is potentially a more serious issue. However, the great advantage to the particular model we use to underpin the GA is its analytical tractability. This tractability permits us to reparameterize the GA formula in terms of the IRB reserve requirement and capital charge, the latter of which includes a maturity adjustment. In effect, we obtain an indirect form of maturity adjustment in the GA through maturity-adjustment of the inputs, rather than maturity adjustment in the formula itself. Furthermore, without the analytical tractability of our approach, it would not have been possible to derive a useful upper bound methodology.

For application in practice, a more important limitation of our methodology is that we assume each position is an unhedged loan to a single borrower. How should we incorporate credit default swaps (CDS) and loan guarantees in a granularity adjustment? Credit risk mitigation activities can decrease name concentration (say, through purchase of CDS on the largest exposures in the portfolio) or actually indirectly give rise to name concentration in exposure to providers of credit protection. We will address this problem in future work.

Appendix: Calibration of variance parameter ξ

In models such as CreditMetrics that assume Gaussian systematic factors, the shape of the distribution for X does not depend on the variance. For this reason, one can normalize the variance to one without any loss of generality. By contrast, when X is gamma-distributed as in CreditRisk$^+$, skewness and kurtosis and other shape measures for X are not invariant to the variance, and so this parameter must be calibrated. In principle, the parameter ξ presents an extra degree of freedom for better fitting the model to data, and so is welcome. In practice, however, extremely long time-series would be required to get a reasonably precise fit. One sees users impose a fairly wide range of values for ξ, say between 0.2 and 2. Lower values of ξ imply greater systematic risk, which generally leads to higher economic capital requires, but which minimizes the GA as a share of economic capital.

Recall that ξ influences the GA through the δ parameter. In Table 6, we report δ for representative values of ξ (holding fixed $q = 0.999$). From this, we conclude that a range of values $4.5 < \delta < 6.5$ would not be out of line with common practice.

Table 6
δ as a function of ξ ($q = 0.999$)

ξ	0.20	0.25	0.35	0.50	0.75	1.00	1.50	2.00
δ	4.66	4.83	5.09	5.37	5.68	5.91	6.23	6.45

Another way to calibrate ξ is to match the variance of the default probability when portfolio maturity is one year. When $M = 1$, the IRB model collapses to the default-mode CreditMetrics model, and this variance has tractable form Gordy (2000)

$$V_i^{\text{CM}} = Var[PD_i(X)] = \Phi_2(\Phi^{-1}(\text{PD}_i), \Phi^{-1}(\text{PD}_i), \rho_i) - PD_i^2. \tag{13}$$

where ρ_i is the Basel II asset correlation parameter and Φ_2 denotes the bivariate normal cdf. The corresponding variance for CreditRisk$^+$ is

$$V_i^{CR+} = Var[PD_i(X)] = (PD_i \cdot w_i)^2/\xi. \tag{14}$$

Equating the two variance expressions gives

$$\xi = \frac{\Phi(\Phi^{-1}(\text{PD}_i), \Phi^{-1}(\text{PD}_i), \rho_i) - PD_i^2}{PD_i^2 \cdot w_i^2}. \tag{15}$$

Next, we obtain an expression for the factor loading w_i by matching asymptotic UL capital charges across the same two models:

$$\mathcal{K}_i^{CR+} = \text{ELGD}_i \cdot \text{PD}_i \cdot w_i \cdot (\alpha_q(X) - 1)$$

$$\mathcal{K}_i^{CM} = \Phi\left(\sqrt{\frac{1}{1 - \rho_i}}\Phi^{-1}(\text{PD}_i) + \Phi^{-1}(q)\sqrt{\frac{\rho_i}{1 - \rho_i}}\right)$$

and so

$$w_i = \frac{\Phi\left(\sqrt{\frac{1}{1-\rho}}\Phi^{-1}(\text{PD}_i) + \Phi^{-1}(q)\sqrt{\frac{\rho}{1-\rho}}\right) - \text{PD}_i}{\text{PD}_i \cdot (\alpha_q(X) - 1)}. \tag{16}$$

We substitute this expression for w_i into equation (15) to get an implicit formula for ξ that depends only on PD, the corresponding ρ in the IRB formula, and $\alpha_q(X)$. This last quantity depends on ξ, so we must solve using a nonlinear root-finding algorithm.

An obvious drawback to this method is that the estimated value of ξ depends on the chosen PD, whereas ξ ought to be independent of portfolio characteristics. When PD is set to 1%, we obtain the value $\xi = 0.206$, which is roughly consistent with the our baseline parameterization of $\xi = 0.25$.

References

Basel Committee on Bank Supervision. The New Basel Capital Accord. Second Consultative Paper, Bank for International Settlements, January 2001.

Basel Committee on Bank Supervision. Basel II: International convergence of capital measurement and capital standards: A revised framework. Publication No. 128, Bank for International Settlements, June 2006.

Credit Suisse Financial Products. *CreditRisk+: A Credit Risk Management Framework*. London, 1997.

S. Emmer and D. Tasche. Calculating credit risk capital charges with the one-factor model. *Journal of Risk*, 7:85–101, 2005.

W. Gerke, F. Mager, T. Reinschmidt, and C. Schmieder. Empirical risk analysis of pension insurance – the case of Germany. Discussion Paper, Series 2: Banking and Financial Studies 07/2006, Deutsche Bundesbank, 2006.

M. Gordy. A comparative anatomy of credit risk models. *Journal of Banking and Finance*, 24:119–149, 2000.

M. Gordy. A risk-factor model foundation for ratings-based bank capital rules. *Journal of Financial Intermediation*, 12:199–232, 2003.

M. Gordy. Granularity adjustment in portfolio credit risk measurement. In Giorgio P. Szegö, editor, *Risk Measures for the 21st Century*. Wiley, 2004.

C. Gouriéroux, J.P. Laurent, and O. Scaillet. Sensitivity analysis of Values at Risk. *Journal of Empirical Finance*, 7:225–245, 2000.

Erik Heitfield, Steve Burton, and Souphala Chomsisengphet. Systematic and idiosyncratic risk in syndicated loan portfolios. *Journal of Credit Risk*, 2(2):3–31, Fall 2006.

A.E. Kocagil, F. Glormann, and P. Escott. Moody's RiskCalc for private companies: The German Model. Moody's Investors Service. www.moodyskmv.com., 2001.

R. Martin and T. Wilde. Unsystematic credit risk. *Risk Magazine*, 15:123–128, 2003.

M. Pykhtin and A. Dev. Analytical approach to credit risk modelling. *Risk*, 15(3): 26–32, 2002.

G. Taistra, C. Tiskens, and D. Glüder. Basel II – Auswirkungen auf typische Mittelstandsportfolien. KfW, Abteilung Volkswirtschaft, 2001.

O. Vasicek. Loan portfolio value. *Risk Magazine*, pages 160–162, 2002.

T. Wilde. IRB approach explained. *Risk Magazine*, 14(5):87–90, 2001a.

T. Wilde. Probing granularity. *Risk Magazine*, 14(8):103–106, 2001b.

Stress Testing Banking Book Positions under Basel II
By

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Preliminary Draft

The Basel II Advanced Internal Ratings Based (AIRB) approach, an approach derived from the Vasicek single factor credit loss model, sets minimum regulatory capital requirements using an estimate of the 99.9 percentile of the loss distribution of an asymptotic credit portfolio. This exceptionally high solvency standard begs the question as to why stress testing is necessary given that losses beyond the AIRB capital threshold should be extremely rare. In practice, the highly restrictive assumptions underlying the AIRB framework are unlikely to be satisfied; consequently actual solvency standards may be below the 99.9 percent regulatory target. This paper analyzes conditions under which the AIRB is likely to understate credit losses. It formally models correlation risks that may be generated by stochastic exposure at default (EAD), loss given default (LGD), probability of default (PD) and default correlation (ρ). Analysis of the generalized model indicates that these additional sources of systematic risk magnify unexpected portfolio credit losses. By providing a clear understanding of the limitations of the AIRB framework and developing methods to quantify loss rates under alternative assumptions, the analysis provides a framework that can be used to design and calibrate stress loss scenarios that usefully supplement Basel II AIRB minimum capital requirements.

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Stress Testing Banking Book Positions under Basel II

1. Introduction

  The Basel II Advanced Internal Ratings Based (AIRB) approach is a detailed set of rules and guidelines that prescribe bank minimum capital requirements for credit risk. In addition to rule-based minimum capital requirements, Basel II requires banks to proactively manage their asset composition and risk exposures to ensure that their overall capitalization remains adequate under conditions that may arise outside of those implicitly considered when setting AIRB minimum capital requirements. This requirement is clearly articulated in Pillar II of the Comprehensive Framework,[1]

Banks must be able to demonstrate that chosen internal capital targets are well founded and that these targets are consistent with their overall risk profile and current operating environment. … Rigorous, forward-looking stress testing that identifies possible events or changes in market conditions that could adversely impact the bank should be performed. Bank management clearly bears primary responsibility for ensuring that the bank has adequate capital to support its risks. (p. 205, paragraph 726).

  Supplemental stress testing, also a qualitative feature of the Pillar I minimum capital rules, is required to supplement the capital adequacy standards set by the AIRB capital rules:

An IRB bank must have in place sound stress testing processes for use in the assessment of capital adequacy. Stress testing must involve identifying possible events or future changes in economic conditions that could have unfavourable effects on a bank's credit exposures and assessment of the bank's ability to withstand such changes (p. 96, paragraph 434).

  While the Basel II framework is rarely explicit as to how stress tests interact with minimum required capital levels, the comprehensive framework language suggests that, in some circumstances, stress test results may indicate the need for banks to maintain minimum capital levels in excess of AIRB minimums:

Where stress tests reveal particular vulnerability to a given set of circumstances, prompt steps should be taken to manage those risks appropriately (e.g. by hedging against that outcome or reducing the size of the bank's exposures, or increasing capital) (page 192 , paragraph 718).

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[1] Basel Committee on Banking Supervision (2006)

The prior discussion highlights the importance of stress testing in the Basel II framework. The AIRB minimum regulatory capital framework is rich in detail regarding quantitative and qualitative requirements that must be satisfied when calculating minimum regulatory capital. In contrast, while there are numerous references to supplemental stress testing requirements in Basel II, the 2006 Comprehensive Framework offers little guidance on the nature of the supplemental stress tests that are required:

Examples of scenarios that could be used are (i) economic or industry downturns; (ii) market-risk events; and (iii) liquidity conditions...In addition to the more general tests described above, the bank must perform a credit risk stress test to assess the effect of certain specific conditions on its IRB regulatory capital requirements. The test to be employed would be one chosen by the bank, subject to supervisory review. The test to be employed must be meaningful and reasonably conservative. Individual banks may develop different approaches to undertaking this stress test requirement, depending on their circumstances. For this purpose, the objective is not to require banks to consider worst-case scenarios... one example might be to use two consecutive quarters of zero growth to assess the effect on the bank's PDs, LGDs and EADs (page 96, paragraphs 434-435).

This paper considers the design of stress tests that are used to validate capital adequacy or, in some cases, augment basic minimum capital requirements set under the Basel II AIRB approach. The AIRB approach specifies a minimum capital adequacy standard that is adequate for absorbing 99.9 percent of the one-year unexpected credit loss distribution on a well-diversified credit portfolio. This nominal loss coverage rate suggests that only a very rare set of potential events might generate credit losses that exceed AIRB minimum capital requirements. If this interpretation is true, then stress tests need only consider very extreme events or events that are not captured by the AIRB modeling framework.

In reality, Basel II AIRB minimum capital requirements are derived from a highly stylized model of portfolio credit losses. The AIRB credit loss distribution formally incorporates only a single source of unexpected credit loss while large complex banks' true unexpected credit loss distributions are attributable to multiple sources of credit loss uncertainty. Recognition of additional sources of loss uncertainty will often result in unexpected loss estimates that exceed AIRB minimum capital values. When designing a supplemental system of stress tests to be used in conjunction with an AIRB framework, it is important to understand the limitations of the AIRB model and how

stress tests might be designed to quantify the importance of these limitations and the magnitude of the capital shortfalls they might generate.

This paper revisits the AIRB framework and discusses the restrictive assumptions that are used to derive AIRB minimum capital requirements. We then consider tractable generalizations of the AIRB framework that introduce additional sources of systematic credit risk. The results show that, in many cases, these additional sources of systematic risk generate larger unexpected credit loss exposures and an additional need for capital. The generalizations considered include stochastic probability of default (PD), loss given default (LGD), exposure at default (EAD), and stochastic default correlation (ρ). Each of the stochastic generalizations may introduce additional sources of systematic risk that alter the shape of the portfolio credit loss distribution.

An outline of the paper follows. Section 2 discusses the Vasicek single factor asymptotic portfolio model of credit losses, the model that underlies the Basel II AIRB framework. Section 3 reviews the Basel II AIRB minimum capital calculations. Section 4 generalizes the Vasicek model to include random exposures at default (EAD), and random loss given default (LGD), where EAD and LGD include a common systematic risk factor. Section 5 generalizes the model to include a random probability of default, where the individual credits in a portfolio share an exposure to a common factor that in part determines their *ex ante* probability of default. Section 6 models an asymptotic portfolio's credit loss distribution when default correlations are stochastic. In Section 7, generalized unexpected credit loss measures are compared to Basel II AIRB required minimum capital levels. The comparisons identify credit portfolio features that may lead to inadequate capital under AIRB capital rules. This information can be used to design supplemental stress tests to identify and quantify additional capital needs. A final section concludes the paper.

2. *The Vasicek Portfolio Credit Loss Distribution Model*

The Vasicek single common factor model of portfolio credit losses assumes that uncertainty on credit i is driven by a latent unobserved factor, $\widetilde{V}_i$, with the following properties:

$$\tilde{V}_i = \sqrt{\rho_V}\, \tilde{e}_M + \sqrt{1-\rho_V}\, \tilde{e}_{id}$$
$$\tilde{e}_M \sim \phi(e_M)$$
$$e_{id} \sim \phi(e_{id}),$$
$$E(\tilde{e}_{id}\, \tilde{e}_{jd}) = E(\tilde{e}_M\, \tilde{e}_{jd}) = 0, \ \ \forall i, j. \tag{1}$$

where $\phi(\cdot)$ represents the standard normal density function. $\tilde{V}_i$ is distributed standard normal, $E(\tilde{V}_i) = 0$, and $\sigma^2(\tilde{V}_i) = E(\tilde{V}_i^2) - E(\tilde{V}_i)^2 = 1$. $\tilde{V}_i$ is often interpreted as a proxy for the market value of the firm that issued credit i. The common factor, $\tilde{e}_M$, induces correlation between individual credits' latent factors, $\rho_V = \dfrac{Cov(\tilde{V}_i, \tilde{V}_j)}{\sigma(\tilde{V}_i)\sigma(\tilde{V}_j)}$.

Credit i is assumed to default when its latent factor takes on a value less than a credit-specific threshold, $\tilde{V}_i < D_i$. The unconditional probability that credit i defaults is $PD = \Phi(D_i)$, where $\Phi(\cdot)$ represents the cumulative standard normal density function. Time is not an independent factor in this model, but is implicitly recognized through the calibration of input values for PD.

Consider a portfolio composed of N credits with identical initial market values, promised future values, correlations, ρ, and default thresholds, $D_i = D$. It is useful to define a default indicator function for each credit,

$$\tilde{I}_i = \begin{cases} 1 & if \ \tilde{V}_i < D \\ 0 & otherwise \end{cases} \tag{2}$$

$\tilde{I}_i$ has a binomial distribution with an expected value of $\Phi(D)$. Define $\tilde{X}$ to be the proportion of credits in the portfolio that default, $\tilde{X} = \dfrac{\sum_{i=1}^{n} \tilde{I}_i}{n}$.

In an asymptotic portfolio, the number of individual credits is assumed to increase without bound, $n \to \infty$. In the limit as $n \to \infty$, idiosyncratic risks are completely diversified within the portfolio and portfolio default rate uncertainty is driven by the common market

factor alone. The unconditional distribution function of $\widetilde{X}$, the asymptotic portfolio's default rate, is given by,

$$\Pr\left[\widetilde{X} \le x\right] = \Phi\left(\frac{\sqrt{1-\rho}\ \Phi^{-1}(x) - \Phi^{-1}(PD)}{\sqrt{\rho}}\right), \quad x \in [0,1] \tag{3}$$

In the Vasicek model, individual credit *LGD*s and *EAD*s are assumed to be known constants. The uncertainty in portfolio credit losses is driven entirely by the portfolio default rate and so the critical values of the loss rate distribution are determined by the critical values of the default rate distribution. Let $x_{.999}$ represent the 99.9 percent critical value of the portfolio default rate distribution. The 99.9 percent critical value of the default rate distribution is given by,

$$\begin{aligned}
0.999 &= \Phi\left(\frac{\sqrt{1-\rho}\ \Phi^{-1}(x_{.999}) - \Phi^{-1}(PD)}{\sqrt{\rho}}\right) \\
&\Rightarrow \Phi^{-1}(.999) = \frac{\sqrt{1-\rho}\ \Phi^{-1}(x_{.999}) - \Phi^{-1}(PD)}{\sqrt{\rho}} \\
&\Rightarrow x_{.999} = \Phi\left(\frac{\Phi^{-1}(.999)\sqrt{\rho} + \Phi^{-1}(PD)}{\sqrt{1-\rho}}\right)
\end{aligned} \tag{4}$$

Using expression (4), the 99.9 percent critical value of the portfolio loss rate distribution is,

$$\Phi\left(\frac{\Phi^{-1}(.999)\sqrt{\rho} + \Phi^{-1}(PD)}{\sqrt{1-\rho}}\right) \times LGD \times EAD \tag{5}$$

Expected portfolio losses are $PD \times LGD \times EAD$. As a consequence, the 99.9 percent unexpected portfolio loss rate is,

$$\Phi\left(\frac{\Phi^{-1}(.999)\sqrt{\rho} + \Phi^{-1}(PD)}{\sqrt{1-\rho}}\right) \times LGD \times EAD - (PD \times LGD \times EAD) \tag{6}$$

3. The Basel II AIRB Model

The Basel II AIRB approach for setting minimum capital requirements is derived from expression (6). Under the AIRB capital rule, minimum capital (K) for corporate, bank and sovereign credits is,[2]

$$K = EAD \cdot \left[LGD \times \Phi \left[\frac{1}{\sqrt{1-R}} \Phi^{-1}(PD) + \sqrt{\frac{R}{1-R}} \Phi^{-1}(.999) \right] - PD \times LGD \right] \left(\frac{1 + (M - 2.5)b}{1 - 1.5b} \right) \quad (7)$$

where,

$$R = 0.12 \left(\frac{1 - e^{-50PD}}{1 - e^{-50}} \right) + 0.24 \left(1 - \frac{1 - e^{-50PD}}{1 - e^{-50}} \right), \quad b = \left(0.11852 - .05478 \, Ln(PD) \right)^2 .$$

In expression (7), EAD is exposure at default, PD is a credit's probability of default expressed as a percentage, LGD is a credit's expected loss given default expressed as a percentage, M is the credit's maturity in measured in years, and K represents the dollar capital requirement. The R function is a regulatory rule that links a portfolio's asset correlation to the PD of its individual credits–low PD credits are specified to have higher asset correlation values. The final term in expression (7), $\left(\frac{1 + (M - 2.5)b}{1 - 1.5b} \right)$, is a maturity adjustment factor. When $M = 1$, the adjustment factor equals 1. The R function and maturity adjustment factor are *ad hoc* functions that were introduced by the BCBS as a means for "tuning" the capital calibration.

4. Stochastic LGD and EAD

4.1 Background

The Vasicek and Basel II AIRB models assume EAD and LGD are fixed parameters. These assumptions preclude these models from capturing two important sources of systematic credit risk that are present in historical loss rate data. In many cases, LGD and EAD are themselves random variables. Available evidence suggests that

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[2] The minimum capital formula depends on the type of credit categories (e.g. mortgage, retail, SMEs, specialized lending categories, etc.). For other credit categories, capital is set using a modified version of expression (7). These modifications include alternative regulatory correlation functions to reflect different assumptions about portfolio default correlations and in some cases adjustments to reflect portfolio income, collateral, or third party credit guarantees.

both components may introduce additional sources of systematic risk into portfolio credit losses. The assumptions underlying the Basel II AIRB model are particularly limiting in the case of *EAD*, as the AIRB model is used to set minimum capital requirements on revolving lines of credit without incorporating any structure to account for a stochastic facility draw rates.

Model recognition of the stochastic nature of *EAD* and *LGD* may lead to significantly different minimum capital requirements if there is systematic time variability among portfolio *LGD* and *EAD* realizations. Available evidence suggests that *LGD*s increase in periods when default rates are elevated.[3] A positive correlation between *LGD*s and portfolio default rates suggests that *LGD*s values are in part driven by systematic factors. This additional source of systemic risk will magnify unexpected credit losses relative to AIRB model estimates unless a bank inputs a "downturn LGD" estimate that is very large relative to its historical average LGD experience.

The AIRB approach does not include formal modeling for the *LGD* parameter. The Basel Committee did however recognize the potential importance of correlations among portfolio *LGD*s and attempted to account for the potential capital effects of LGD correlation by introducing a requirement for that banks use so-called "downturn *LGD*" to calculate capital when warranted:

Paragraph 468 of the Framework Document requires that the LGD parameters used in Pillar1 capital calculations must "reflect economic downturn conditions where necessary to capture the relevant risks." The purpose of this requirement is to ensure that LGD parameters will embed forward-looking forecasts of recovery rates on exposures that default during conditions where credit losses are expected to be substantially higher than average. Under such conditions default rates are expected to be high so that if recovery rates are negatively related to default rates, LGD parameters must embed forecasts of future recovery rates that are lower than those expected during more neutral conditions. In those cases where future recovery rates are expected to be independent of future default rates there is no supervisory expectation that the forward-looking forecasts of recovery rates embedded in LGD parameters will differ from those expected during more neutral conditions.(BCBs (2005). p. 2).

The AIRB downturn *LGD* requirement directs banks to adjust historical *LGD*s to reflect heavier than historical average losses should a class of credits show loss rates that vary with economic conditions. Downturn *LGD* is not formally modeled in the AIRB or

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[3] See for example, Frye (2000b), Hu and Perraudin (2002), Schuermann (2004), Araten, Jacobs, and Varshney (2004), Altman, Brady, Resti, and Sironi (2004), Hamilton, Varma, Ou, and Cantor (2004), Carey and Gordy (2004), or Emery, Cantor, and Arnet (2004).

related Basel Committee guidance and so procedures for estimating downturn *LGD* are left to bank judgment and the adequacy of the resulting estimates are, for all practical purposes, a Pillar 2 issue.

In contrast to the literature on *LGD*, published research on *EAD* behavior is more limited. The available evidence suggests that obligors draw on committed lines of credit as their credit quality deteriorates.[4] In many cases, creditors' draw rates are positively correlated with default rates. This relationship suggests that there are commons factor that simultaneously determine portfolio *EAD* and default rate realizations. Again, this additional source of systemic risk will increase unexpected loss estimates relative to those calculated under the AIRB framework. Unlike *LGD*, the AIRB has not introduced an explicit requirement for using "stress" or "downturn" in the capital calculations.

4.2 A Model of Stochastic EAD

Account-level *EAD* is modeled as an initial outstanding exposure and a random draw rate, $\tilde{\delta}_i$, on an accounts remaining line of credit. Assume that an individual account, account, i, begins the period with a drawn exposure and has a maximum line of credit, M_i, upon which it may draw. The account utilization rate $\tilde{X}_i \in [0,1]$ is a random variable that determines the end-of-period account exposure, $\tilde{X}_i M_i$.

Let the initial account exposure be represented by, $d_{i0} M_i$, where d_{i0} is the initial share of the account line of credit that is used. The line of credit that can be drawn by the creditor over the subsequent period is $(1 - d_{i0})M_i$. Let $\tilde{\delta}_i \in [0,1]$ represent the share of the remaining line of credit that is borrowed over the period, and let $\Omega(\tilde{\delta}_i)$ represent the cumulative density function for $\tilde{\delta}_i$. The model can be generalized to recognize creditors' ability to reduce or eliminate their outstanding balances by setting $d_{i0} = 0$ and directly modeling an account's end-of-period utilization rate $\tilde{X}_i \in [0,1]$ instead of modeling an account's draw rate $\tilde{\delta}_i$. Under the draw rate specification, the account's end-of-period exposure is,

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[4] The available evidence includes studies by Allen and Saunders (2003), Asarnow and Marker (1995), Araten and Jacobs (2001), and Jiménez, Lopez, and Saurina (2006).

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$$M_i \widetilde{X}_i = M_i \left(d_{i0} + (1 - d_{i0}) \widetilde{\delta}_i \right), \quad \widetilde{d}_i \sim \Omega(\delta_i), \quad \delta_i \in [0,1]. \tag{8}$$

Systematic dependence among individual accounts' draw rates is incorporated by assuming that account draw rates are driven by a latent Gaussian factor, $\widetilde{Z}_i$, with the following properties:

$$
\begin{aligned}
\widetilde{Z}_i &= \sqrt{\rho_Z}\, \widetilde{e}_M + \sqrt{1 - \rho_Z}\, \widetilde{e}_{iZ} \\
\widetilde{e}_M &\sim \phi(e_M) \\
e_{iZ} &\sim \phi(e_{iZ}), \\
E(\widetilde{e}_{iZ}\, \widetilde{e}_{jZ}) &= E(\widetilde{e}_M\, \widetilde{e}_{jZ}) = E(\widetilde{e}_{iZ}\, \widetilde{e}_{jd}) = 0 \; \forall\, i, j.
\end{aligned}
\tag{9}
$$

The correlation between the latent variables that determines each account's draw rate is $\rho_Z = \dfrac{Cov(\widetilde{Z}_i, \widetilde{Z}_j)}{\sigma(\widetilde{Z}_i)\sigma(\widetilde{Z}_j)}$, and the correlation between the latent factors that drive account

exposures and defaults is, $\sqrt{\rho_Z \rho_V} = \dfrac{Cov(\widetilde{V}_i, \widetilde{Z}_j)}{\sigma(\widetilde{V}_i)\sigma(\widetilde{Z}_j)}$. To induce a positive correlation between

a portfolio's default rate and its draw rate, we adopt the normalization convention that higher account draw rates are associated with smaller realizations of the latent variable, $\widetilde{Z}_i$.

For any random variable $\widetilde{s}$ with continuous density function, $f(\widetilde{s})$, the probability integral transformation requires that the random variable $\widetilde{S}$ be distributed uniformly over the interval $[0,1]$, when the random variable $\widetilde{S}$ is defined by the integral

transformation, $S_i = \displaystyle\int_{-\infty}^{s_i} f(s)\, ds$. Using this transformation, we introduce correlation

structure into the realizations of the draw rate process by equating the probability integral transformations for the physical draw rate $\widetilde{\delta}_i$ and the latent variable, $\widetilde{Z}_i$, $\Omega(\delta_i) = 1 - \Phi(Z_i)$. The probability integral transformation implies a one-to-one mapping between $\widetilde{z}_i$ and $\widetilde{\delta}_i$,

$$\widetilde{\delta}_i = \Omega^{-1}\left(1 - \Phi(\widetilde{Z}_i)\right). \tag{10}$$

4.3 A Model of Stochastic LGD

Let $\tilde{\lambda}_i \in [0,1]$ represent the loss rate that that will be experienced on credit i's outstanding balance should the borrower default. Let $\Theta(\tilde{\lambda}_i)$ represent the cumulative density function for $\tilde{\lambda}_i$. Systematic dependence among individual credits' loss rates is introduced by assuming that $\tilde{\lambda}_i$ is driven by a latent Gaussian factor, $\tilde{Y}_i$, with the following properties:

$$
\begin{aligned}
&\tilde{Y}_i = \sqrt{\rho_Y}\, \tilde{e}_M + \sqrt{1-\rho_Y}\, \tilde{e}_{iY} \\
&\tilde{e}_M \sim \phi(e_M) \\
&e_{iY} \sim \phi(e_{iY}), \\
&E(\tilde{e}_{iY}\, \tilde{e}_{jY}) = E(\tilde{e}_M\, \tilde{e}_{jY}) = E(\tilde{e}_{iY}\, \tilde{e}_{jZ}) = E(\tilde{e}_{iY}\, \tilde{e}_{id}) = 0 \quad \forall\, i,j.
\end{aligned}
\tag{11}
$$

To induce positive correlation between a portfolio's default rate and its loss rate given default, we adopt the normalization convention that higher account draw rates are associated with smaller realizations of the latent variable, $\tilde{Z}_i$. The correlation between the latent factors that determine default and loss given default is $\sqrt{\rho_V \rho_Y} > 0$, and the correlation between the Gaussian drivers of default and exposure at default is $\sqrt{\rho_V \rho_Z} > 0$. Using the inverse integral transformation to introduce a correlation structure, the mapping between $\tilde{\lambda}_i$ and $\tilde{Y}_i$ is given by

$$
\tilde{\lambda}_i = \Theta^{-1}\!\left(1 - \Phi(\tilde{Y}_i)\right).
\tag{12}
$$

4.4 Credit Loss Distribution for an Asymptotic Portfolio

Consider a portfolio with N accounts that have identical credit limits, $M_i = M$, identical initial drawn balances, $d_{i0} M_i = d_0 M$, identical latent factor correlations, $\{\rho_V, \rho_X, \rho_Y\}$, and identical default thresholds, $D_i = D$. Assume that all credits' end-of-period draw rates, $\tilde{\delta}_i$, and loss rates given default, $\tilde{\lambda}_i$, are, respectively, taken from unconditional distributions that are identical across credits (the distributions for $\tilde{\delta}_i$ and $\tilde{\lambda}_i$ generally differ). Let $\tilde{\Lambda}_p$ represent the loss rate on the portfolio of N credits from this

homeogeneous class. It can be shown[5] that, as $n \to \infty$, the asymptotic portfolio credit loss distribution, $\widetilde{\Lambda}_P$, has a probability density that can be written as an implicit function of the common factor e_M and its density $\phi(e_M)$,

$$\widetilde{\Lambda}_p \sim \left\{ \Phi\left(\frac{D - \sqrt{\rho_V}\, e_M}{\sqrt{1 - \rho_V}} \right) \cdot \left[d_0 + (1 - d_0)\, E\!\left(\Omega^{-1}\!\left(1 - \Phi\!\left(\widetilde{Z}_i \mid e_M \right) \right) \right) \right] E\!\left(\Theta^{-1}\!\left(1 - \Phi\!\left(\widetilde{Y}_i \mid e_M \right) \right) \right), \phi(e_M) \right\},$$

$$\text{for } e_M \in (-\infty, \infty). \tag{13}$$

The portfolio loss rate consistent with a cumulative probability of α is

$$\Phi\left(\frac{\Phi^{-1}(PD) + \sqrt{\rho_V}\, \Phi^{-1}(\alpha)}{\sqrt{1 - \rho_V}} \right) \times \left[d_0 + (1 - d_0)\, E\!\left(\Omega^{-1}\!\left(1 - \Phi\!\left(\widetilde{Z}_i \mid e_M = -\Phi^{-1}(\alpha) \right) \right) \right) \right]$$

$$\times E\!\left(\Theta^{-1}\!\left(1 - \Phi\!\left(\widetilde{Y}_i \mid e_M = -\Phi^{-1}(\alpha) \right) \right) \right), \qquad \text{for } \alpha \in [0,1]. \tag{14}$$

Expression (14) is used to define the portfolio minimum capital requirement.

The first term in expression (14), $\Phi\left(\dfrac{\Phi^{-1}(PD) + \sqrt{\rho_V}\, \Phi^{-1}(\alpha)}{\sqrt{1 - \rho_V}} \right)$, is the inverse of an asymptotic portfolio's cumulative default rate distribution evaluated at a probability of α. The remaining terms in expression (14) are the α-level critical values for the asymptotic portfolio's *EAD* distribution, $d_0 + (1 - d_0)\, E\!\left(\Omega^{-1}\!\left(1 - \Phi\!\left(\widetilde{Z}_i \mid e_M = -\Phi^{-1}(\alpha) \right) \right) \right)$, and the asymptotic portfolio's *LGD* distribution, $E\!\left(\Theta^{-1}\!\left(1 - \Phi\!\left(\widetilde{Y}_i \mid e_M = -\Phi^{-1}(\alpha) \right) \right) \right)$. In general, the critical values of the asymptotic portfolio *EAD* and *LGD* distributions must be calculated using numerical techniques.

An asymptotic portfolio's *EAD* and *LGD* distribution have at least two properties that hold regardless of the individual credit's univariate *LGD* and *EAD* distributional characteristics. First, it can be shown that as the correlations in their latent factors converge to 0, the asymptotic portfolio draw rate and *LGD* distributions converge to a point distribution located at their unconditional expected values:

$$\lim_{\rho_Y \to 0} E\!\left(\Theta^{-1}\!\left(1 - \Phi\!\left(\widetilde{Y}_i \mid e_M = -\Phi^{-1}(\alpha) \right) \right) \right) = E(\widetilde{\lambda}) \quad \forall \alpha \in [0,1] \tag{15}$$

$$\lim_{\rho_Y \to 0} E\!\left(\Omega^{-1}\!\left(1 - \Phi\!\left(\widetilde{Z}_i \mid e_M = -\Phi^{-1}(\alpha) \right) \right) \right) = E(\widetilde{\delta}) \quad \forall \alpha \in [0,1]. \tag{16}$$

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[5] See Kupiec (2007a) for the derivation of expression (13).

In this case, idiosyncratic LGD and EAD are completely diversified within an asymptotic portfolio.

A second important property is the characteristics of an asymptotic portfolio's *EAD* and *LGD* distributions as the correlations in individual credits' *LGD* and *EAD* latent factors approach 1. In this case, it can be shown that the distributions of the portfolio *EAD* and LGD distributions converge to distributions that characterize the loss or exposure behavior of a single credit (i.e., there is no diversification in the portfolio-level distributions):

$$\lim_{\rho_Y \to 1} E\left(\Theta^{-1}\left(1 - \Phi\left(\widetilde{Y}_i \mid e_M = -\Phi^{-1}(\alpha)\right)\right)\right) = \Theta^{-1}(\alpha) \quad \forall \alpha \in [0,1] \tag{17}$$

$$\lim_{\rho_Z \to 1} E\left(\Omega^{-1}\left(1 - \Phi\left(\widetilde{Z}_i \mid e_M = -\Phi^{-1}(\alpha)\right)\right)\right) = \Omega^{-1}(\alpha) \quad \forall \alpha \in [0,1]. \tag{18}$$

For correlations between 0 and 1, partial diversification benefits provide for some reduction in portfolio unexpected credit losses relative to the case of perfect positive correlation.

5. Stochastic Probability of Default
5.1 Model of an Individual Account Default

All of the credit loss distribution models discussed thus far assume that the probability that an individual credit defaults is a known constant value. In practice, the probability of an individual credit assigned to a given class by a rating or underwriting system (these terms will be used interchangeably) are random variables with distributions that have substantial variability relative to the mean outcome. In this section we model the implication of stochastic probability of default for an asymptotic portfolio's credit loss distribution.

Let $\widetilde{P}_i \in [0,1]$ represent the stochastic default rate on credit i. Let $\Xi(\widetilde{P}_i)$ represent the cumulative density function for $\widetilde{P}_i$. Systematic dependence among individual credits'

loss rates is introduced by assuming that $\widetilde{P}_i$ is driven by a latent Gaussian factor, $\widetilde{Y}_i$, with the following properties:

$$\begin{aligned}
\widetilde{Q}_i &= \sqrt{\rho_Q}\,\widetilde{e}_M + \sqrt{1-\rho_Q}\,\widetilde{e}_{iQ} \\
\widetilde{e}_M &\sim \phi(e_M) \\
e_{iY} &\sim \phi(e_{iQ}), \\
E(\widetilde{e}_{iQ}\,\widetilde{e}_{jQ}) &= E(\widetilde{e}_M\,\widetilde{e}_{jQ}) = E(\widetilde{e}_{iQ}\,\widetilde{e}_{jd}) = 0 \quad \forall\, i, j.
\end{aligned} \tag{19}$$

If we adopt the normalization convention that an account has a higher default rate for smaller realizations of its latent variable $\widetilde{Q}_i$, then there will be positive correlation between an individual credit's probability of default and the default rate in the macro economy. Using the inverse integral transformation to introduce a correlation structure, the mapping between $\widetilde{P}_i$ and $\widetilde{Q}_i$ is given by

$$\widetilde{P}_i = \Xi^{-1}\big(1 - \Phi\big(\widetilde{Q}_i\big)\big). \tag{20}$$

In this Gaussian latent factor framework, each credit (credit i) has an associated latent unobserved factor, $\widetilde{V}_i$ with properties given in expression (1). Credit i is now assumed to default when its associated latent factor takes on a value less than a credit-specific stochastic threshold, $\widetilde{V}_i < \widetilde{D}_i$. The stochastic threshold is determined by equating the alternative expressions for the probability of default,

$$\widetilde{P}_i = \Xi^{-1}\big(1 - \Phi\big(\widetilde{Q}_i\big)\big) = \Phi\big(\widetilde{D}_i\big) \tag{21}$$

which implies,

$$\widetilde{D}_i = \Phi^{-1}\Big(\Xi^{-1}\big(1 - \Phi\big(\widetilde{Q}_i\big)\big)\Big). \tag{22}$$

So the credit defaults when,

$$\widetilde{V}_i < \Phi^{-1}\Big(\Xi^{-1}\big(1 - \Phi\big(\widetilde{Q}_i\big)\big)\Big) \tag{23}$$

14

The assumption of correlation in individual credits' default boundaries might be motivated, for example, as a reduced form approach for modeling time-variation in market liquidity conditions. When markets are liquid, firms in aggregate find it comparatively easy to refinance maturing debt. This ease is captured in this model by a large common factor realization that works to reduce all firms' default boundaries. In contrast, when liquidity is scarce, market participants are less willing to refinance maturing debts and firms find it difficult to issue new debt and avoid default. This feature is captured in the model by large negative common factor realizations that raise all accounts' default boundaries and increase the conditional *ex ante* probability of default.

5.2 Default Rate Distribution of an Asymptotic Portfolio

Consider a portfolio of N homogeneous credits all underwritten to an identical standard, meaning that all credits share the same values for their latent variable correlations, ρ_Q, and ρ_V, and all credits' have unconditional default probabilities drawn from the same underlying distribution, $\widetilde{P}_i \sim \Xi(\cdot) \, \forall \, i$. It can be demonstrated that the distribution of the default rate of an asymptotic portfolio $\widetilde{X}_{RB}$, can be written as an implicit function of the common latent factor[6],

e_M,

$$
\widetilde{X}_{RB} \sim \left\{ \left[\int_{-\infty}^{\infty} \Phi\left(\frac{\Phi^{-1}\left(\Xi^{-1}\left(1 - \Phi\left(\sqrt{p_Q}\, e_M + \sqrt{1-\rho_Q}\, e_{iQ} \right) \right) \right) - \sqrt{\rho_V}\, e_M}{\sqrt{1-\rho_Q}} \right) \phi(e_{iQ}) \delta e_{iQ}, \phi(e_M) \right], e_M \in (-\infty, \infty) \right\}
$$

(24)

Expression (24) can evaluated using numerical methods. Examples are provided in Section 7.4.

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[6] The full derivation is given in Kupiec (2007b).

6. Correlated Stochastic Default Correlation

6.1 Individual Account Default Dynamics

In this section, the Vasicek model is generalized to incorporate stochastic default correlations. The default correlation parameter in the Vasicek model, ρ_V in expression (1), is generalized and modeled as a credit-specific random variable. The credit-specific correlation is represented by $\tilde{\rho}_{di}$ which has a cumulative distribution

$$\tilde{\rho}_{di} \sim \Psi(\rho_{di}), \quad \rho_{di} \in [-1,1]. \tag{25}$$

Assume each credit (credit i) has an associated latent unobserved factor, $\tilde{T}_i$ with the following properties,

$$
\begin{aligned}
\tilde{T}_i &= \tilde{\rho}_{di} \tilde{e}_M + \sqrt{1 - \tilde{\rho}_{di}^2} \; \tilde{e}_{id} \\
\tilde{e}_M &\sim \phi(e_M) \\
e_{id} &\sim \phi(e_{id}), \\
E(\tilde{e}_{id} \tilde{e}_{jd}) &= E(\tilde{e}_M \tilde{e}_{jd}) = 0 \; \forall \, i, j.
\end{aligned}
\tag{26}
$$

We assume that realizations of $\tilde{\rho}_{di}$ are also driven by a latent Gaussian factor $\tilde{W}_i$ through a probability integral transform. This new latent factor has both common ($\tilde{e}_K$) and idiosyncratic sources of risk ($\tilde{e}_{ic}$). Define,

$$
\begin{aligned}
\tilde{W}_i &= \rho_{ci} \tilde{e}_K + \sqrt{1 - \rho_{ci}^2} \tilde{e}_{ic} \\
\tilde{e}_K &\sim \phi(e_K) \\
e_{ic} &\sim \phi(e_{ic}), \\
E(\tilde{e}_{id} \tilde{e}_{jc}) &= E(\tilde{e}_{ic} \tilde{e}_{jc}) = E(\tilde{e}_M \tilde{e}_{jd}) = E(\tilde{e}_K \tilde{e}_{jd}) = E(\tilde{e}_K \tilde{e}_{jc}) = 0 \; \forall \, i, j.
\end{aligned}
\tag{27}
$$

Assume that high (positive) default correlations are associated with low realizations of the latent factor $\tilde{W}_i$. These assumptions imply,

$$\Psi(\rho_{di}) = 1 - \Phi(W_i), \tag{28}$$

or,

$$\widetilde{\rho}_{di} = \Psi^{-1}\left(1 - \Phi\left(\widetilde{W}_i\right)\right). \tag{29}$$

It can be shown that $E(\widetilde{T}_i) = 0$, and $E(\widetilde{T}_i^2) = 1$. Conditional on a value of $\widetilde{W}_i = W_i$,

under specification (25), $\widetilde{T}_i$ has a conditional standard normal distribution,

$\widetilde{T}_i \mid \widetilde{W}_i = W_i \sim \phi(\cdot)$. Because $\widetilde{T}_i \mid \widetilde{W}_i = W_i$ has a standard normal distribution for any

conditioning value W_i, it follows that the unconditional distribution for $\widetilde{T}_i$ is also

standard normal.

6.2 Default Correlations

Expressions (26), (27), and (29) imply an unconditional correlation between latent

factors $\widetilde{T}_i$ and $\widetilde{T}_j$,

$$\begin{aligned}
Corr(\widetilde{T}_i, \widetilde{T}_j) &= \frac{Cov(\widetilde{T}_i, \widetilde{T}_j)}{\sqrt{Var(\widetilde{T}_i) \, Var(\widetilde{T}_j)}} = E\left(\widetilde{\rho}_{id} \widetilde{\rho}_{jd} \widetilde{e}_M^2\right) \\
&= E\left(\widetilde{\rho}_{id} \widetilde{\rho}_{jd}\right) E\left(\widetilde{e}_M^2\right) \\
&= E\left(\widetilde{\rho}_{id} \widetilde{\rho}_{jd}\right)
\end{aligned} \tag{30}$$

The shape of the probability density function of $\widetilde{\rho}_{id} \widetilde{\rho}_{jd}$ does depend on the values

of the correlation parameters ρ_{ic} and ρ_{jc}. If the density function for the default

correlation parameter, $\Psi(\cdot)$ is symmetric about its mean value, using numerical methods,

it can be shown that,

$$E\left(\widetilde{\rho}_{id} \widetilde{\rho}_{jd}\right) \approx E\left(\widetilde{\rho}_{id}\right) E\left(\widetilde{\rho}_{jd}\right) \tag{31}$$
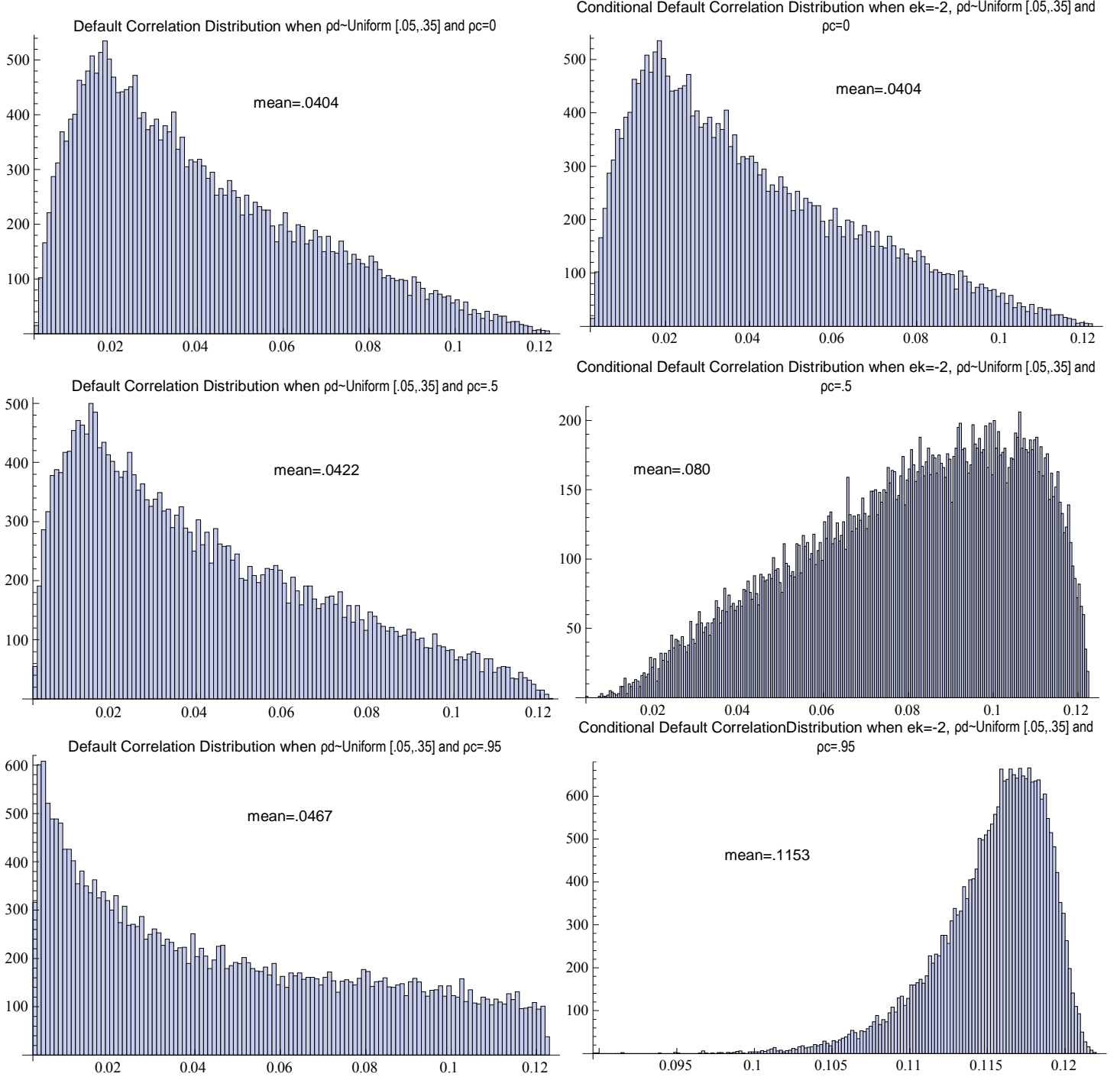
Figure 1: Unconditional (left panels) and conditional (right panels) default correlation density functions for $\widetilde{\rho}_{id} \sim Uniform[0.05, 0.35]$. The first row represents the model $\rho_{ic} = 0$, the case where stochastic default correlation parameters are independent. The middle row correspond to the case $\rho_{ic} = 0.5$; The final row represents the case $\rho_{ic} = 0.95$. The conditional density functions represent the distribution of default correlations conditional on a realized value $\widetilde{e}_K = -2$, the common factor in the latent variable $\widetilde{W}_i$.

Expression (31) indications that the unconditional correlation between $\tilde{T}_i$ and $\tilde{T}_j$ is, for most practical purposes, insensitive to the correlation parameters ρ_{ic} in these credits' latent factors that determine that correlation parameter realizations, $\tilde{w}_i$ and $\tilde{w}_j$. The value of the correlation parameter ρ_{ic} does however change the shape of the unconditional default correlation distribution as well as determine the default correlation between $\tilde{T}_i$ and $\tilde{T}_j$ conditional on a realized value of the common factor, e_K.

Under the model assumptions, a smaller than average value realization of the common factor e_K leads to a higher than average conditional correlation whereas a realization of e_K greater than zero leads to a lower than average conditional correlation between $\tilde{T}_i$ and $\tilde{T}_j$,

$$
\begin{aligned}
Corr(\tilde{T}_i, \tilde{T}_j \mid e_K < 0) &= E(\tilde{\rho}_{id}\tilde{\rho}_{jd} \mid e_K < 0) > E(\tilde{\rho}_{id})E(\tilde{\rho}_{jd}) \\
Corr(\tilde{T}_i, \tilde{T}_j \mid e_K = 0) &= E(\tilde{\rho}_{id}\tilde{\rho}_{jd} \mid e_K = 0) = E(\tilde{\rho}_{id})E(\tilde{\rho}_{jd}) \\
Corr(\tilde{T}_i, \tilde{T}_j \mid e_K > 0) &< E(\tilde{\rho}_{id}\tilde{\rho}_{jd} \mid e_K > 0) < E(\tilde{\rho}_{id})E(\tilde{\rho}_{jd})
\end{aligned}
\tag{32}
$$

These properties are illustrated in Figure 1.

Figure 1 illustrates the unconditional and conditional default correlation distributions that are generated by this modeling framework. The left panels in Figure 1 plot the unconditional default correlation density functions when for alternative values of ρ_{ic} ($\rho_{ic} = 0, \rho_{ic} = 0.5, {}_{ic}= 0.95$) when the density function for $\tilde{\rho}_{id} \sim Uniform[0.05, 0.35]$. Notice that as the value of ρ_{ic} increases, the unconditional density functions place more density on both very low and very high default correlation realizations, but the mean of the unconditional density is little changed.

The right column panels of Figure 1 plot the conditional default correlation densities that correspond with those in the adjacent left panels conditional on a realized

value of $\tilde{e}_K = -2$ for the common latent factor driving $\tilde{W}_i$ realizations. Notice that the

greater the value of ρ_{ic}, the more substantial the shift in the conditional correlation

density function toward high default correlation values. Notice as well that the mean of

the conditional distribution is increasing in ρ_{ic}. While these charts illustrate a specific

example assuming $\tilde{\rho}_{id}$ is uniformly distributed, the qualitative nature of behavior of

unconditional and conditional default correlation densities are similar for any admissible

symmetric distribution.

It is important to mention that, while this model allows correlation between the

latent factor values that determine individual account default correlation parameters,

$\tilde{\rho}_{id}$, these default correlations are independent of the common factor, $\tilde{e}_M$, that triggers

individual account defaults.

Credit i is assumed to default when its associated latent factor takes on a value

less than a credit-specific stochastic threshold, $\tilde{T}_i < D_i$.   Define a default indicator

function,

$$\tilde{I}_{Ti} = \begin{cases} 1 & if \ \tilde{T}_i < D_i \\ 0 & otherwise \end{cases}. \tag{33}$$

The indicator function, $\tilde{I}_{Ti}$, is a binomial random variable. Let $\tilde{I}_{Ti} \mid e_M, e_K$ represent the

value of the indicator function conditional on a realization of the two common factors,

e_M and e_K,

$$\tilde{I}_{Ti} \mid e_M, e_K = \begin{cases} 1 & if \ \dfrac{\Phi^{-1}(PD) - \left(\Psi^{-1}\left(1 - \Phi\left(\rho_c e_K + \sqrt{1 - \rho_c^2}\, \tilde{e}_{ic} \right) \right) \right) e_M}{\sqrt{1 - \left(\Psi^{-1}\left(1 - \Phi\left(\rho_c e_K + \sqrt{1 - \rho_c^2}\, \tilde{e}_{ic} \right) \right) \right)^2}} - \tilde{e}_{id} < 0 \\ 0 & otherwise \end{cases} \tag{34}$$

6.2 Default Rate Distribution of an Asymptotic Portfolio

The default rate on a portfolio of N homogenous credits, $\tilde{X}_{RC}$ is,

$$\tilde{X}_{RC} = \frac{\sum_{i=1}^{N} \tilde{I}_{Ti}}{N} . \tag{35}$$

The portfolio default rate conditional on realized values of the common factors, e_M, and

e_K, is ,

$$\tilde{X}_{RC} \mid e_M, e_K = \frac{\sum_{i=1}^{N} \tilde{I}_{Ti} \mid e_M, e_K}{N} . \tag{36}$$

It can be shown that[7], as $N \rightarrow \infty$,

$$\lim_{N \rightarrow \infty} \left(\tilde{X}_{RC} \mid e_M, e_K \right) \xrightarrow[a.s.]{} \int_{e_{ic}=-\infty}^{e_{ic}=\infty} \Phi \left(\frac{\Phi^{-1}(PD) - \left(\Psi^{-1} \left(1 - \Phi \left(\rho_c e_K + \sqrt{1-\rho_c^2}\, e_{ic} \right) \right) \right) e_M}{\sqrt{1 - \left(\Psi^{-1} \left(1 - \Phi \left(\rho_c e_K + \sqrt{1-\rho_c^2}\, e_{ic} \right) \right) \right)^2}} \right) \phi(e_{ic}) \tag{37}$$

Under the stochastic correlation assumptions of this section, an asymptotic portfolio's

unconditional credit loss distribution can be derived numerically using Monte Carlo

simulation and expression (37). Examples are provided in Section 7.5.

7. Alternative Asymptotic Portfolio Credit Loss Rate Distributions

7.1 Overview

In this section we illustrate some alternative examples of the alternative

generalized asymptotic portfolio credit loss rate distributions that were derived in

Sections 4-6. We will us the basic Vasicek distribution as the benchmark of comparison.


[7] The derivation is given in Kupiec (2007c).

7.2 Account Level EAD and LGD Distribution Example

For illustrative purposes, we adopt the Beta distribution with the first parameter (alpha) equal to 1.5 and the second parameter (beta) equal to 5 to represent individual account LGD and Draw rate distributions. The Beta distribution is given by,

$$\tilde{\lambda} \sim Beta(1.5, 5, \tilde{\lambda})$$
$$Beta(1.5, 5, \lambda) = \frac{\Gamma(6.5)}{\Gamma(1.5)\Gamma(5)} \lambda^{0.5} (1-\lambda)^4, \quad \text{for} \quad 0 < \lambda < 1 \tag{41}$$

where $\Gamma(b) = \int_0^\infty y^{b-1} e^{-y} dy$, $b > 0$, is the mathematical gamma function. This unconditional distribution, plotted in Figure 2, has a mean of 0.2308 and is skewed right.

The unconditional $Beta(1.5, 5)$ distribution is approximately representative of the random draw rate distribution that might be observed on a revolving corporate credit or perhaps the loss given default rate distribution for corporate bank loans or near-prime mortgages.

Figure 2: *Beta* (1.5, 5) Density Function
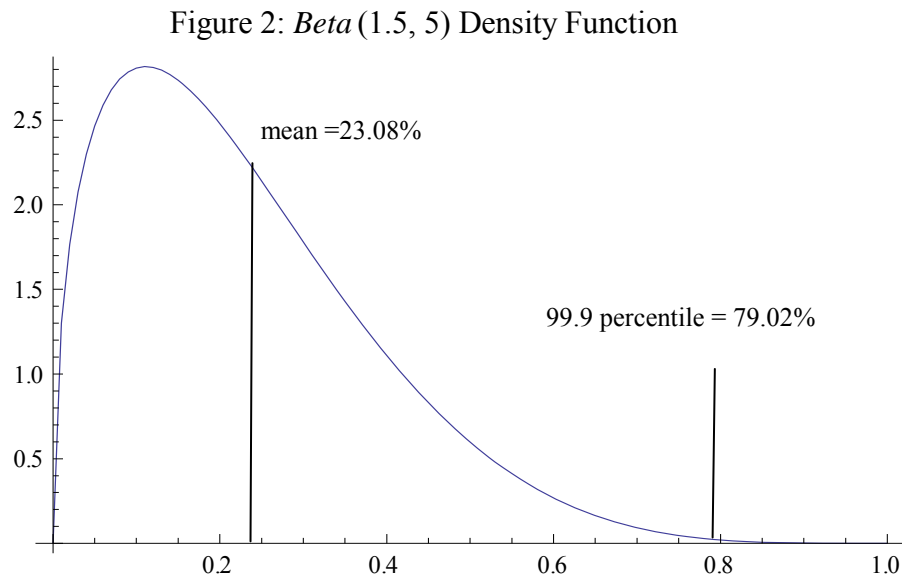
7.3 Asymptotic Portfolio EAD and LGD Distribution

Expression (13) implies that an asymptotic portfolio's *LGD* probability density function is given by,

$$LGD_p \sim \left\{ E\left(\Theta^{-1}\left(1 - \Phi\left(\widetilde{Y}_i \mid e_M\right)\right)\right), \phi\left(e_M\right) \right\} \quad e_M \in (-\infty, \infty) \tag{42}$$

Similarly, from expression (13), an asymptotic portfolio's EAD density function can be written as,

$$EAD_P \sim \left\{ d_0 + (1 - d_0) \, E\left(\Omega^{-1}\left(1 - \Phi\left(\widetilde{Z}_i \mid e_M\right)\right)\right), \phi\left(e_M\right) \right\} \quad e_M \in (-\infty, \infty) \tag{43}$$

The example in this section will focus on illustrating the portfolio *LGD* density function, but the discussion and results are analogous for the portfolio *EAD* density function.

Under the assumption that individual account *LGD*s are distributed *Beta*(1.5,5), numerical techniques can be used to derive the asymptotic portfolio's *LGD* density function given a correlation value, ρ_Y, that drives the *LGD* latent Gaussian factor, $\widetilde{Y}_i$.

Figure 3 plots an asymptotic portfolio's *LGD* probability density function under alternative correlation assumptions for the latent factors that drive individual account *LGD* realizations. If *LGD* realizations are uncorrelated, the asymptotic portfolio loss distribution would converge to 23.08 percent, the mean of an individual accounts' *LGD* distribution under the *Beta*(1.5,5) assumption. When *LGD*s are correlated, *LGD* risks are not completely diversified and considerable LGD risk may remain at the portfolio level.

Figure 3 plots an asymptotic portfolio's *LGD* density for two *LGD* correlation assumptions, $\rho_Y = 0.20$, and $\rho_Y = 0.50$. The portfolio *LGD* rate consistent with a 99.9 percent cumulative loss density is 47.12 percent when $\rho_Y = 0.20$. This critical value represents the true model-consistent "downturn *LGD*" estimate that should be used in the Basel II AIRB when $\rho_Y = 0.20$. When *LGD* correlations are 50 percent ($\rho_Y = 0.50$), the

99.9 percent cumulative *LGD* rate is 62.66 percent. If account *LGD*s are perfectly

correlated ($\rho_Y = 1$), the asymptotic portfolio *LGD* density converges to the *LGD* density

for an individual account, and the 99.9 percent critical value is 79.02 percent. As

correlation approaches 1, the ability to diversify *LGD* risk within the portfolio diminishes

and, in this example, the asymptotic portfolio LGD distribution becomes increasing right

skewed, approaching the *Beta*(1.5,5) density as $\rho_Y \to 1$.

Figure 3: Asymptotic Portfolio LGD Density Under Alternative LGD
Correlation Assumptions
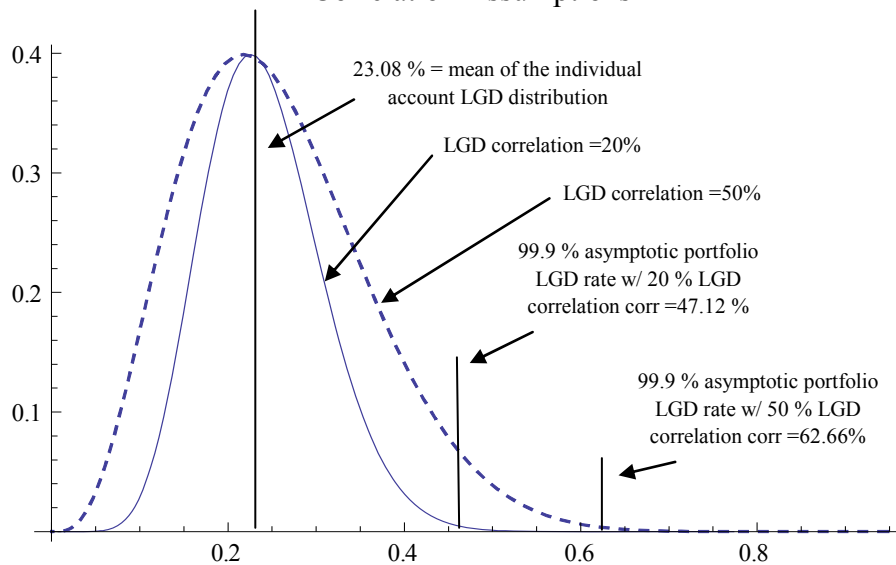
7.3 Asymptotic Portfolio Credit Loss Distribution with Correlated LGDs

Consider a portfolio of fully drawn one-year term-loans. Assume each loan has a

0.50 percent probability of default. In addition, assume the loans are to corporate

creditors and that a default correlation assumption of $\rho_V = 0.20$ is appropriate. Assume

as well that individual account *LGD*s are stochastic and distributed *Beta*(1.5,5).

Expression (13) implies that an asymptotic portfolio's credit loss density can be written

as an implicit function,

$$\tilde{\Lambda}_p \sim \left\{ \Phi\left(\frac{\Phi^{-1}(0.005) - \sqrt{.2}\, e_M}{\sqrt{.8}} \right) \cdot E\left(\Theta^{-1}\left(1 - \Phi\left(\tilde{Y}_i \mid e_M\right)\right)\right) \phi(e_M) \right\} \quad e_M \in (-\infty, \infty) \qquad (44)$$

where $\Theta(\cdot) \sim Beta(1.5,5)$.

Figure 4 plots the asymptotic portfolio's credit loss probability distribution function under alternative assumptions ($\rho_Y = .20, \quad \rho_Y = .50$) about the correlation among individual credit *LGD*s. For reference, the figure includes the Vasicek portfolio loss rate distribution which is equivalently is equivalent to assuming that *LGD* is stochastic with 0 correlations among individual credits. The Vasicek and Basel II AIRB loss distributions would be identical except for AIRB "fine tuning" through the regulatory correlation function. When *PD*=0.5 percent, the wholesale regulatory correlation function sets correlation slightly higher (21.3 percent) than the correlation used in Figure 2.

Figure 4 illustrates the importance of *LGD* correlation as a determinant of an asymptotic portfolio's credit loss distribution and the 99.9 percent unexpected loss estimate used to calculate AIRB minimum capital requirements. Even modest levels of *LGD* correlation result in significantly higher unexpected credit loss rates when measured at the 99.9 percentile. For example, when *LDG* correlation is 20 percent (the estimated value in Frye (2000b)) unexpected credit losses more than double, from 0.51 under the Vasicek model, to 1.07 percent under the stochastic *LGD* model. When *LGD* correlation is 50 percent, unexpected credit losses increase to 1.43 percent. Clearly stronger *LGD* correlation leads to higher unexpected credit losses.

Figure 4: Asymptotic Portfolio Credit Loss Rate Distribution for
Wholesale Term Loans when PD=0.5%, default correlation = 20%, and
Account LGD~Beta(1.5,5)
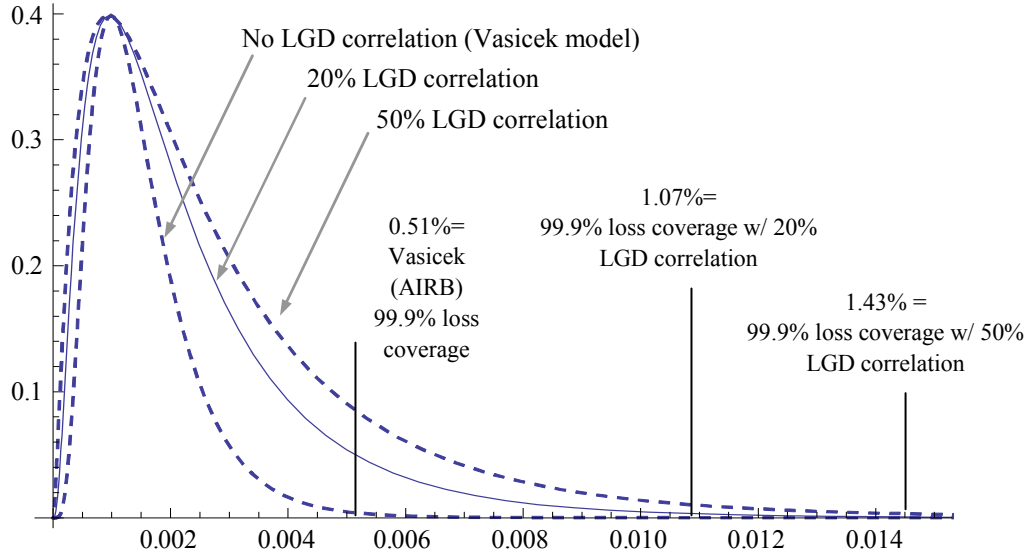
7.4 Asymptotic Portfolio Credit Loss Distribution with Correlated LGD and EAD

This section models the credit loss distribution for a hypothetical portfolio of wholesale one-year credit lines. As before, we assume each loan had a 0.50 percent probability of default and the lines are to corporate creditors with a default correlation of $\rho_V = 0.20$. Assume that individual account *LGD*s are stochastic and correlated with *Beta*(1.5,5) distributions. We also assume that individual account draw rates are stochastic and correlated with *Beta*(1.5,5) distributions. For simplicity, we assume that no accounts have any initial drawn exposure. Expression (13) implies that the asymptotic portfolio credit loss distribution is given by,

$$\tilde{\Lambda}_p \sim \left\{ \Phi\left(\frac{\Phi^{-1}(0.005) - \sqrt{.20}\, e_M}{\sqrt{.8}} \right) \cdot E\left(\Omega^{-1}\left(1 - \Phi\left(\tilde{Z}_i \mid e_M\right)\right)\right) E\left(\Theta^{-1}\left(1 - \Phi\left(\tilde{Y}_i \mid e_M\right)\right)\right) \phi(e_M) \right\} \tag{45}$$

where $\Theta(\cdot) \sim Beta(1.5,5)$ and $\Omega(\cdot) \sim Beta(1.5,5)$. We will derive the portfolio credit loss

distribution under different assumptions for ρ_Z and ρ_Y, the *EAD* and *LGD* correlation

parameters.

Figure 5: Asymptotic Portfolio Credit Loss Rate Distribution for Wholesale Credit
Facilities when PD=0.5%, Default Correlation = 20%, Account LGD~Beta(1.5,5),
and Account EAD~Beta(1.5,5)
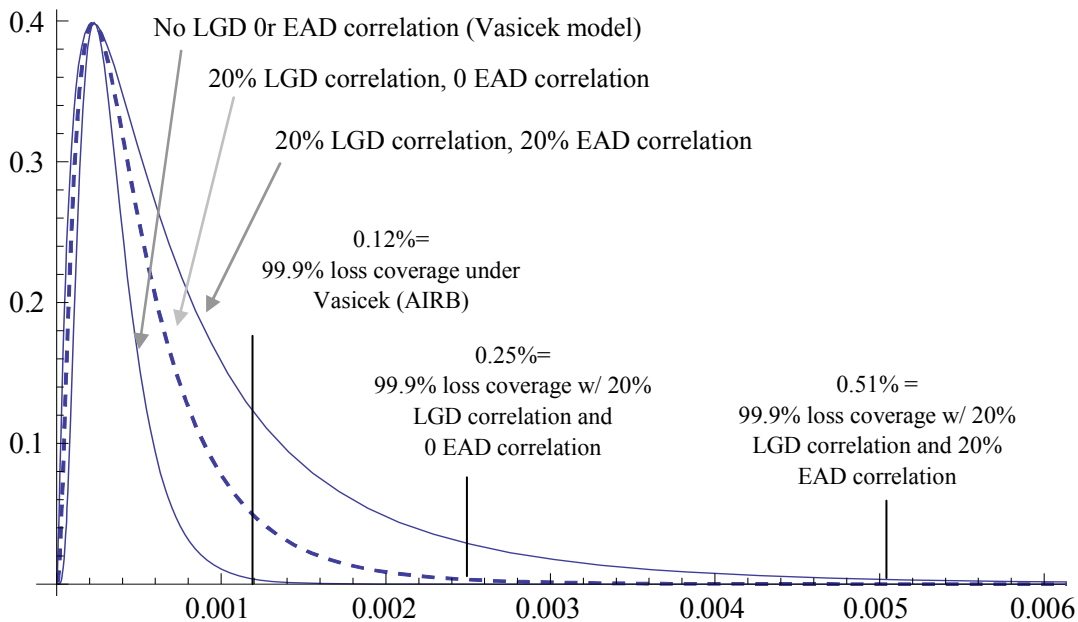
Figure 5 plots the asymptotic portfolio credit loss distribution for $\rho_Z = 0.20$

and $\rho_Y = 0.20$. For comparison purposes, the figure also includes the portfolio loss

distribution for $\rho_Z = 0$ and $\rho_Y = 0.20$, and $\rho_Z = 0$ and $\rho_Y = 0$. The final is equivalent to

using the mean values of the individual account *EAD* and *LGD* distributions, which

corresponds to the Vasicek model and the AIRB framework when later excludes a

"downturn LGD" adjustment. As the Figure 5 indicates, the recognition of *EAD* and *LGD*

correlation increases unexpected credit loss estimates. Under the Vasicek framework, the

99.9 percent unexpected credit loss is 0.12 percent. When individual account *LGD* and

EAD realizations both have 20 percent correlation among their latent factors, the 99.9

percent portfolio unexpected credit loss increases to 0.51 percent. Positive correlation among *LGD* and *EAD* realizations will magnify the capital needs over those set using the basic Vasicek (Basel AIRB) framework.

The numerical results in Figures 4 and 5 depend on the assumption that individual account *EAD*s and *LGD*s are distributed *Beta*(1.5,5), but the qualitative features of this example hold for any distribution assumptions provided *EAD*s and *LGD*s are positive correlated (and positively correlated with portfolio default rates). Each type of credit facility will have a signature *EAD* and *LGD* distribution and correlation pattern that can be used to duplicate the analysis of this section. Kupiec (2007a) discusses some of the possible distributions that might characterize alternative wholesale and retail credit portfolios and illustrates the corresponding asymptotic portfolio credit loss distributions.

7.4 Portfolio Credit Loss Distribution when Default Boundaries are Stochastic

Expression (24) represents the asymptotic portfolio conditional default rate distribution for a portfolio in which individual credits have a stochastic probability of default instead of a fixed default boundary. Depending on the modeling framework, the asymptotic portfolio credit loss rate is calculated by multiplying the conditional portfolio default rate by a fixed EAD and LGD estimate (Vasicek and Basel AIRB), or alternatively by the corresponding conditional value (conditional on the value for e_M) from the portfolio's asymptotic *EAD* and *LGD* distribution. In this section we will focus on the portfolio default rate in isolation.

Figure 6 plots the default rate probability density function for an asymptotic portfolio of credits in which all credits have an unconditional expected probability of default equal to 1 percent, but individual credit's *ex ante* unconditional probability of

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default is stochastic and drawn from a normal distribution with a mean of 1 percent and a variance of 0.2 percent. Figure 1 is based on an assumption of 20 percent correlation between the latent factors $\tilde{V}_i$ and among the latent factors that drive the default boundary realizations, $\tilde{Q}_i$. The solid line in Figure 1 plots the probability density function for the portfolio default rate from the stochastic default boundary model, while the dashed line plots the default rate density function for the same asymptotic portfolio under Vasicek (AIRB) model assumptions.

Figure 6: Asymptotic Portfolio Default Rate Probability Densities for the Stochastic Default Boundry and Vasicek Models
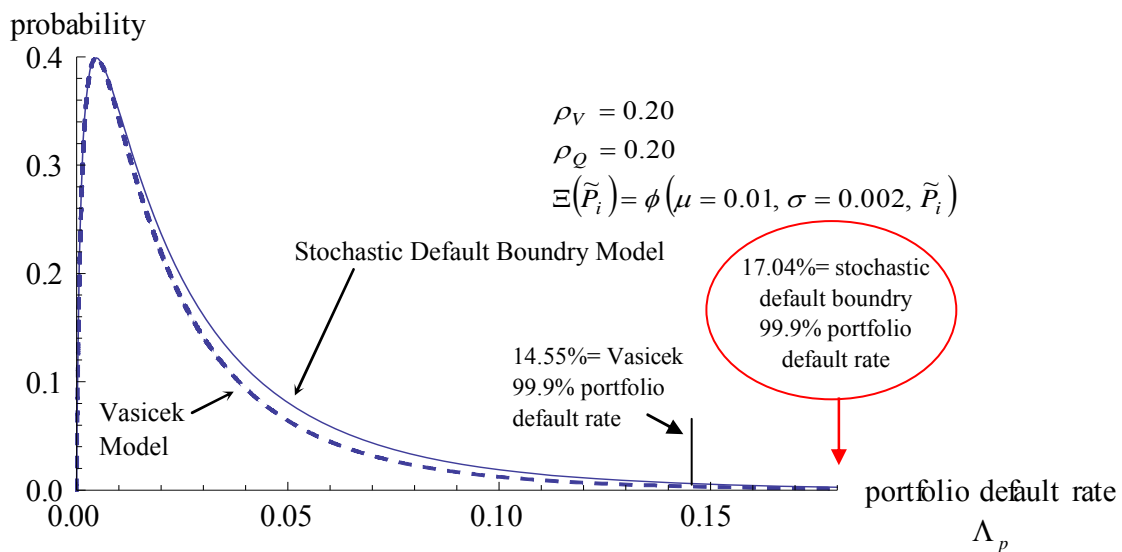
The plots in Figure 1 show that the stochastic character and positive correlation among individual portfolio accounts' *ex ante* default boundaries increases the positive skew of the asymptotic portfolio's default rate distribution. It can be shown that the positive skew becomes more pronounced as the strength of the correlation among account default boundaries increases. In this example, with *ex ante PD*s distributed $\phi(\mu = 0.01, \sigma = 0.002)$ and with latent factor $\tilde{Q}_i$ correlations of 20 percent, the 99.9 percent

cumulative portfolio default rate is 14.55 percent under the Vasicek model ($\rho_Q = 0$), and 17.05 percent when the stochastic properties of the default boundaries are formally modeled.

The qualitative characteristics of this example generalize to other probabilistic characterizations of the stochastic default boundary. If the true *ex ante* unconditional probability of a default of individual accounts in a portfolio are random, and if the default boundaries realizations are positively correlated, as for example might occur when creit market are vulnerable to liquidity shocks, then unexpected portfolio default rates (and unexpected credit losses) will be magnified relative to the estimates generated by the Vasicek (Basel II AIRB) model.

7.5 Portfolio Credit Loss Distribution when Default Correlation is Stochastic

Expression (37) can be used to numerically estimate the asymptotic default rate distribution for a portfolio in which individual credits have a stochastic default correlation parameter, $\widetilde{\rho}_{di}$ in expression (26). Similar to the analysis in Section 7.4, the analysis in this section focuses on an asymptotic portfolio's default rate probability distribution in isolation. This default rate must be augmented with assumptions about the asymptotic portfolio's *EAD* and *LGD* rates in order to arrive at an estimate of an asymptotic portfolio's credit loss rate distribution.

The examples in this section will assume that all credits in the portfolio have correlation parameters $\widetilde{\rho}_{di}$ (in expression (26)) that are distributed uniformly over the range $[.05, .35]$. This implies an unconditional expected correlation parameter of 0.20, which in turn, implies an expected default correlation, $Corr(\widetilde{T}_i, \widetilde{T}_j)$, of 4 percent. The

equivalent Vasicek model formulation would use $\rho_V = 0.04$ (in expression (1)). This parameterization is also equivalent the Basel II AIRB model for retail credits. Recall that Figure 1 (see Section 6.2) illustrates the shape of the implied correlation distribution and how it varies with the parameter value ρ_{ic} in expression (27).

Figure 7: Portfolio Credit Loss Distribution PD=0.01, ρc=.5, and $\rho d \sim$ Uniform[.05,.35]
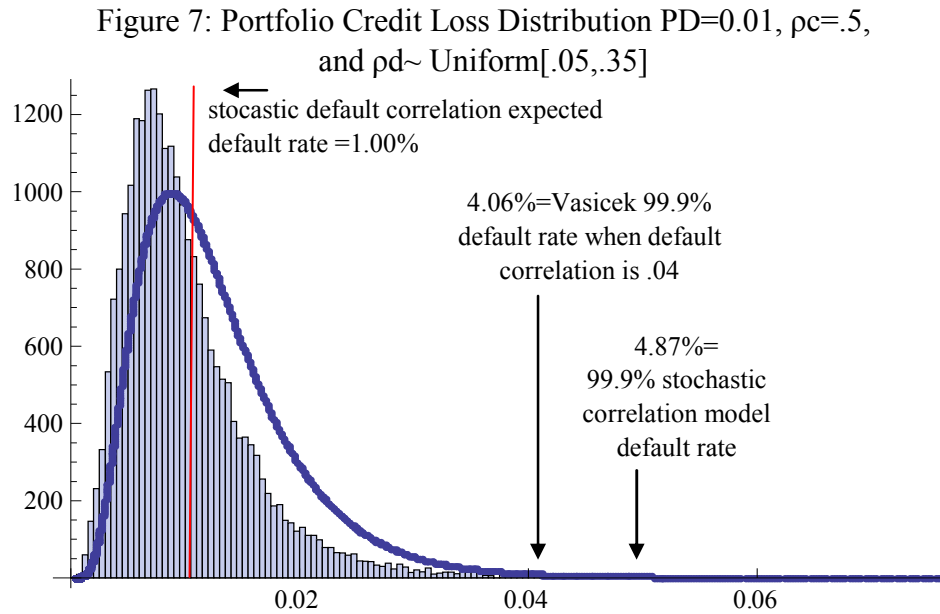

Figure 7 plots the implied asymptotic portfolio default rate under the stochastic default correlation model assumptions when $\rho_{ic} = 0.5$. Each credit has is assumed to have a 1 percent unconditional probability of default. Under the stochastic model assumptions, the latent Gaussian factors, $\tilde{w}_i$, have pair wise correlations of 25 percent . Recall that these factors in turn drive the default correlation parameter $\tilde{\rho}_{id}$ through the relationship $\tilde{\rho}_{di} = \Psi^{-1}\left(1 - \Phi(\tilde{w}_i)\right)$.

Figure 7 shows that relaxing the Vasicek assumption of constant default correlation, and allowing correlation realizations to be stochastic and positively correlated results in a substantial increase in the 99.9 percent asymptotic portfolio default rate. When the default correlation is 4 percent in the Vasicek model, the 99.9 percent

cumulative default rate is 4.06 percent. When stochastic correlation is formally

recognized in the model, the 99.9 percent cumulative default rate increases to 4.87

percent. Recall that this effect occurs notwithstanding the fact that the model assumes

independence between $\tilde{T}_i$ and $\tilde{w}_i$, and so the latent factors that drive default rates and

default correlations are independent.

Figure 8 repeats the calculations of Figure 7 under the assumption $\rho_{ic} = 0.9$. The

plots in Figure 8 shows that the increase in the correlation among the $\tilde{w}_i$ Gaussian factors

results in an increase in the 99.9 percent cumulative portfolio default rate. When

$\rho_{ic} = 0.9$, the 99.9 percent critical value of portfolio default rate distribution is 5.606

percent, 1.60 percent larger than the equivalent Vasicek (Basel AIRB) model estimate.

Figure 8: Portfolio Credit Loss Distribution PD=0.01, ρc=.9,
and ρd~ Uniform[.05,.35]
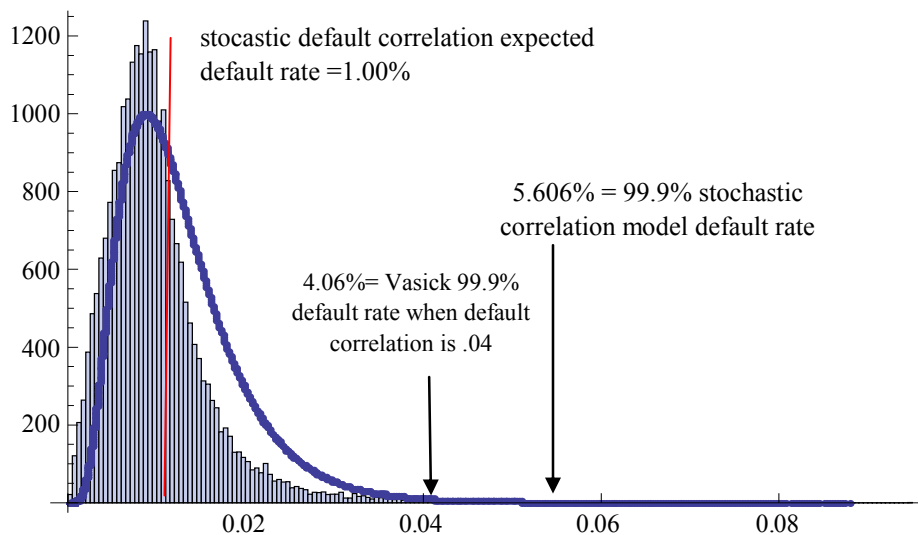
The potential need for a stochastic default correlation model may require some

economic motivation. One possible application is for sub-prime mortgages where history

suggests that default correlations are minor. History may however include only time

frames in which housing prices (an analog for the common factor $\tilde{e}_K$) were increasing but without a strong common trend.. Should a period of strong housing price appreciation become publicized and alter investor expectations, the trend in housing price appreciation could be reinforced and default correlations could fall on a given rated class of mortgages. Should the price trend reverse and alter investor expectations, it is likely that default correlations could increase significantly among this same class of credits.

8. Conclusion

The Basel II framework makes extensive use of supplemental stress testing requirements both to enhance minimum capital requirements under Pillar I and to provide supervisors with blanket powers to help ensure a "failsafe" capital adequacy standard should they actively exercise Pillar II supervisor powers. Notwithstanding the importance placed on stress testing, The Basel II framework provides only minimal guidance on the types of supplemental stress tests banks and supervisors should be conducting under Basel II.

This article has attempted to provide insight into the potential limitations of the Basel II AIRB framework and thereby highlight specific issues and exposures that may merit attention when designing stress tests. It identifies credit exposure features that may lead to significant undercapitalization under AIRB minimum standards. The analysis suggests that it would be prudent to design stress tests to sharpen credit loss exposure estimates whenever Basel II AIRB assumptions regarding a credit portfolio's *LGD*, *EAD*, *PD*, and default correlations are at issue, or alternatively when only a limited sample of outcomes that limits the ability to calibrate the AIRB models parameters given its underlying assumption that *PD, EAD, LGD* and correlation are nonstochastic.

References

Altman, Edward (2006). "Default Recovery Rates and LGD in Credit Risk Modeling and Practice: An Updated Review of the Literature and Empirical Evidence." NYU Solomon Center, November.

Altman, Edward, Brooks Brady, Andrea Resti, and Andrea Sironi (2004). "The Link between Default and Recovery Rates: Theory, Empirical Evidence and Implications." *Journal of Business,* Vol. 78, No. 6, pp. 2203–2228.

Araten, Michel, and Michael Jacobs, Jr. (2001). "Loan Equivalents for Revolving Credits and Advised Lines." *RMA Journal*, May, pp. 34–39.

Araten, Michel, Michael Jacobs, Jr., and Peeyush Varshney (2004). "Measuring LGD on Commercial Loans: An 18-Year Internal Study." *Journal of Risk Management Association,* May, pp. 28–35.

Basel Committee on Banking Supervision (2005), *Guidance on Paragraph 468 of the Framework Document* (July).

Basel Committee on Banking Supervision (2006). *International Convergence of Capital Measurement and Capital Standards: A Revised Framework Comprehensive Version,* June.

Emery, Kenneth, Richard Cantor, and Robert Avner, "Recovery Rates on North American Syndicated Bank Loans, 1989-2003," Moody's Investor Service, March.

Frye, Jon (2000a). "Depressing Recoveries." *Risk*, No. 11, pp. 108–111.

——— (2000b). "Collateral Damage." *Risk*, April, pp. 91–94.

Hamilton, David, Praveen Varma, Sharon Ou, and Richard Cantor (2004). "Default and Recovery Rates of Corporate Bond Issuers: A Statistical Review of Moody's Ratings Performance, 1920–2003." Special Comment, Moody's Investor Service.

Hu, Yen-Ting, and William Perraudin (2002). "The Dependence of Recovery Rates and Defaults." Birckbeck College, Working Paper.

Jiménez, Gabreil, Jose Lopez, and Jesús Saurina (2006). "What Do One Million Credit Line Observations Tell Us about Exposure at Default? A Study of Credit Line Usage by Spanish Firms." Draft working paper, Banco de España.

Kupiec, Paul (2007a). "A Generalized Single Factor Model of Portfolio Credit Risk," FDIC CFR Working Paper No. 2007-06. Available at:
http://www.fdic.gov/bank/analytical/cfr/2007/wp2007/CFRWP_2007_06_KupiecJune07.pdf

Kupiec, Paul (2007b). "Portfolio Credit Losses under an Unbiased Ratings System," memo, FDIC.

Kupiec, Paul (2007c). "Stochastic Default Correlation in an Asymptotic Structural Model of Portfolio Credit Risk," memo, FDIC.

Pykhtin, M. (2003). "Unexpected Recovery Risk." *Risk*, August, pp. 74–78.

Schönbucher, P. (2001). "Factor Models: Portfolio Credit Risks When Defaults Are Correlated." *Journal of Risk Finance,* Vol. 3, No. 1, pp. 45–56.

Schuermann, Til (2004). "What Do We Know about Loss-Given-Default?" In D. Shimko, ed., *Credit Risk Models and Management,* 2nd ed. London: Risk Books.

Tasche, Dirk (2004). "The Single Risk Factor Approach to Capital Charges in Case of Correlated Loss Given Default Rates." SSRN, Working Paper 510982.

Vasicek, O. (1987). "Probability of Loss on a Loan Portfolio." KMV, Working Paper. Published (2003) as "Loan Portfolio Value." *Risk*, December, pp. 160–162**.**