APPENDIX I: DATA AND THEORETICAL FRAMEWORK

This Annex provides an overview of the theoretical model and empirical methodology underlying the projections of liquidity and insolvency risks presented in the main text of the note. The description of the empirical methodology focuses on the construction of the COVID-19 shock and on the key equations linking the theoretical model to the balance sheet data of firms in ORBIS.

A. Theoretical Model for SMEs and the COVID-19 Shock

The analysis on SMEs builds on the work by Gourinchas and others (2020). In this model, firms optimize their demand for labor and intermediate inputs (and therefore their output), subject to four types of shocks: an aggregate demand shock affecting all industries, an industry supply shock, an industry demand shock, and an industry productivity shock.

On the supply side, firms produce output combining labor, materials, and a fixed input using a Cobb-Douglas production function. On the demand side, firms face a constant elasticity of substitution (CES) demand function for their differentiated goods. Further, the firms' optimization problem is static, in a partial equilibrium setup, and varies across different sectors of economic activity, depending on the constraints induced by the shocks.

The model provides a closed-form expression of how a firm's cash flow depends on the aggregate demand shock and on the sectoral demand and supply shocks. Accordingly, the expression for the predicted change in the cash for firms in sectors with a constrained labor supply (i.e. in sectors where the shock to the supply of workers is impeding firms to hire the desired number of employees) is

$$CF_{is}' - CF_{is} = p_{is}d_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{\xi}_s \widehat{AD} - 1 \right] - w_{is}l_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^M m_{is} \left[\mathbb{1}(\Delta Profit > 0) \ \hat{x}_s - 1 \right] - p_{is}^$$

and the analogous expression for unconstrained firms is

$$CF_{is}' - CF_{is} = p_{is}d_{is} \left(\mathbb{1}(\Delta Profit > 0)\xi_s \widehat{AD} - 1\right) - \left(w_{is}l_{is} + p_{is}^M m_{is}\right) \left(\mathbb{1}(\Delta Profit > 0)\hat{x_s}^c - 1\right)$$

In these expressions, $CF_{is}' - CF_{is}$ refers to the difference in the cash flow for firm *i* in sector *s* following the COVID-19 shock; $p_{is}d_{is}$ is the nominal demand for firm *i* in sector *s*; ξ_s is the change in the sector-specific demand due to COVID-19; \widehat{AD} is the change in the aggregate demand due to COVID-19; $\widehat{x_s}^c$ is the change in the sector-specific labor supply constraint due to COVID-19, and α and β are the labor (*l*) and material (*m*) shares in production, respectively.

B. Data

The analysis uses data from Orbis, a product of Bureau van Dijk – Moody's Analytics. Orbis provides the most comprehensive cross-country dataset on private firms. Specifically, the dataset provides information on firms' balance sheet and income statements allowing to map the model to the data. The final sample comprises 20 countries: Australia, Austria, Belgium, Czech Republic, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Japan, Korea, Norway, Poland, Portugal, Slovenia, Spain, Sweden, and the United Kingdom. In addition, the following (regional) country groupings are considered:

- Northern Europe: Austria, Belgium, Germany, Finland, France, Ireland, UK, Sweden, Norway.
- Southern Europe: Spain, Greece, Italy, Portugal.
- Eastern Europe: Czech Republic, Hungary, Poland, Slovenia.
- Asia-Pacific: Australia, Japan, South Korea

The data require significant cleaning before conducting the analysis. The cleaning procedure follows closely Kalemli-Özcan and others (2015). Once the data are cleaned, the final dataset is constructed by defining SMEs (the object of interest) as those firms with at most 250 employees. The resulting dataset consists of about 1.25 million firms.

C. Calibration of the COVID-19 Shock

The COVID-19 impact on SMEs is modeled as a combination of an aggregate demand shock (calibrated using the WEO baseline on a quarterly basis by country), a sectoral demand shock (which is based on the fraction of employees relying on face-to-face interactions), a sector-specific labor supply shock (related to whether industries are considered essential and to their teleworkability), and an industry productivity shock (related to the productivity differential between working from home and at the office).¹

The analysis assumes that a first 8-week lockdown is implemented from week 9 of 2020 (roughly capturing what was actually observed during March-April), and that a second four-week lockdown takes place in November. During the lockdowns all four shocks are in place. Once the lockdowns end, the sectoral labor supply and technology shocks return to pre-COVID levels, while sectoral demands evolve according to an autoregressive (AR(1)) process with persistence level (autocorrelation coefficient) of 0.5 at a quarterly frequency, reflecting society's (potential) concerns about returning to "normalcy" even after containment measures subside. In addition, the downside scenarios assume that the second lockdown extends for 8 weeks (November and December), higher persistency with a coefficient equal to 0.75, and 1 percent lower GDP growth in 2021 (the analysis explores each assumption separately as well as all of them jointly).

¹ For more details on the construction of these shocks in the data, see Gourinchas and others (2020).

D. Bringing the Model to the Balance Sheet Data

Using the model-implied expression for the change in firms' operating cash flow, the empirical analysis constructs indicators for two types of firms: (i) firms with a projected negative cash stock, and (ii) firms with projected negative equity. These firm-level indicators are then used to compute the share of SME jobs and debt at risk presented in the note.

A firm is projected to have a negative stock of cash whenever, at the end of 2020 or 2021,

Cash Stock_{*i.s.t*-1} + Operating Cash Flow_{*i.s.t*} - Interest Payment_{*i.s.t*} < 0,

where the operating cash flow is constructed as in the cash flow equation from the theoretical model described above.

Moreover, a firm is projected to have negative equity if, at the end of 2020 or 2021,

$$Equit y_{i,s,t-1} + Net \ Income_{i,s,t} < 0,$$

where equity is directly taken from the data and Net Income is built consistently with the cash flow equation presented above (and it is assumed that there is no dividend distribution). By checking for illiquidity or insolvency at the end of the year, the analysis allows firms to have a temporary cash or equity shortage during the year and to close their gaps by the end of the year.

Finally, the law of motion of debt from 2020 to 2021 is determined in the different scenarios by whether it is assumed that illiquid firms at the end of 2020 exit the market or continue operating in the following year. In the latter case, all firms with liquidity shortfalls are allowed to issue debt to exactly cover their liquidity shortfalls. Instead, firms with liquidity surpluses use the proceeds to accumulate cash, rather than to pay off debt, in the face of the COVID-19 shock and high uncertainty. Moreover, in both scenarios it is assumed that existing debt can be rolled over. Finally, it is also assumed that the interest rate on the existing debt remains unchanged, reflecting the declines in interest rates this year and the easing in financing conditions. Interest payments on the new debt issued in 2020 are assumed to be due only in 2021, reflecting that many countries introduced moratoria on interest payments this year.

APPENDIX II: THE IMPACT OF COVID-RELATED SME DEFAULTS ON BANK CAPITAL

This appendix illustrates the methodology used to map measures of firm-level insolvency, illiquidity, or lack of viability (as implied by an ICR below 1) into expected bank losses from lending to SMEs and the corresponding bank capital under COVID, both measured relative to a non-COVID scenario. Due to the lack of firm-bank level data and limited bank level data in some countries, unless specified otherwise, the analysis is conducted at the banking system level. Moreover, for the same reasons, the analysis does not consider differences in accounting standards, coverage ratios, or takes

into account the impact of government measures to support firms. For the sake of expositional simplicity, we frame this description focusing mainly on the impact of firm insolvency.

Conceptually, banking system expected losses from lending to SMEs are given by equation (1)

$$Expected \ Losses_SMEs_{c,t} = PD_SME_{c,t} * LGD_SMEs_{c,t} * EAD_SME_{c,t}$$
(1)

Where c and t denote the country and period, respectively. PD_SME is the probability of default for SMEs, LGD_SME is the loss given default and EAD_SME is the exposure of banks to SMEs (i.e. the share of lending to SMEs).

The revised capital ratio after accounting for the expected extra losses from the SME portfolio follows equation (2)

$$CET1R_{c,t} = \frac{CET1R_{c,t-1} - (Expected \ Losses_{SME_{c,t}} - Normal \ Times \ Losses_{SME_{c}})}{RWAs_{c,t}}$$

Where CET1R is the tier1 capital to risk-weighted assets ratio, *Normal Times Losses*_{SME_c} are the losses expected in a normal year (and mostly already provisioned for), and RWA stands for risk weighted assets

In what follow we describe how we obtain each of the components needed to calculate the equations above for the COVID-19 case.

A. Probability of Default

The first step is to rely on firm-level projections to estimate the probability of default on the SME lending portfolio of banks. If loan level data are available, the probability of default can be estimated as²:

$$PD_SME_{c,t} = \left(\frac{N \text{ loans defaults between } t - 1 \text{ and } t}{N \text{ performing loans at end of period } t - 1}\right)$$

While the firm-level analysis presented in the paper (e.g. paragraph 13) does not provide information on whether firms default on their loan obligations, it reveals which companies are projected to have negative equity with or without the COVID-19 shock. It also provides an estimate of the projected debt held by these firms. It is therefore possible to estimate the change in the share of debt at risk of default, $FlowDaR_t$, as:

$$FlowDaR_SME_{t} = \left(\frac{\sum_{f \in I_{t}, f \in S_{t-1}} D_{t,f}}{\sum_{f \in S_{t-1}} D_{t,f}}\right)$$

² Chan-Lau, Jorge, Daniel, Christina, Liliana Schumacher, and Mindaugas Leika (2015). "Treatment of Credit Risk in Stress Tests", Guidance Note on Stress Testing #3, 2015.

Where *f* is any firm in our sample, $D_{t,f}$ is their debt at the end of period *t*, I_t is the set of insolvent firms at time t, and S_{t-1} is the set of firms which were NOT insolvent at time t-1. That is, the numerator is the total projected debt of firms which were solvent at the end of the previous year but are projected to be insolvent at the end of this year. The denominator is the total debt of firms which are solvent at the beginning of the year. This share can be computed in a COVID (*FlowDaR*_{t,COVID}) and non-COVID (*FlowDaR*_{L,no-COVID}) scenario. These measures are different with respect to the share of debt-at-risk (Figures 9 to 10) as they consider only firms that enter the period in a solvent status and not all firms.

Under the assumptions that (i) firms default if they are insolvent and that (ii) the share of bank loans over total firm debt is the same for all firms within a country,³ $FlowDaR_SME_{t,COVID}$ calculated under the COVID scenario could proxy for the probability of default. However, there are some reasons why this measure may overstate or understate actual defaults

- A firm which is considered insolvent according to the corporate model (illustrated above in Appendix I) may still pay its current obligations. For instance, the owners may use some of their personal wealth as they may envision the firm will become profitable in the near future, or because they have posted their own real estate as collateral; business owners may also receive other (monetary or non-monetary) benefits from the firm, such as wages. Analogously, a firm which is projected to be illiquid could still service its debt by borrowing.
- A firm which is projected to be solvent may not be able or willing to service its debt, for instance due to liquidity shortfalls.
- The ORBIS sample may not be perfectly representative of bank SME borrowers

To overcome these potential biases, this analysis assumes that the probability of default on banks' SME portfolios (that is, the flow of loans into non-performing status) and probability of insolvency of ORBIS firms (that is, the flow of firms entering insolvency) are linked in a linear relationship which is not impacted by COVID-19 itself. That is, for each country c, and each year t=2020,2021:

$$PD_SME_{c,t} = \rho_{c,t} + FlowDaR_SME_{c,t}$$

To calibrate the parameters ρ this analysis relies on the fact that the flow of debt at risk in the no-COVID scenario corresponds to the PD actually observed in 2018. In fact, given that the ORBIS data used to compute the debt-at-risk measures refer to 2017 and the projections are one year ahead, then $FlowDaR_{t,no-COVID}$ is also equivalent to the projected $FlowDaR_{2018}$. That is, given that there was no COVID in 2018, then $FlowDaR_{t,no-COVID}$ is the flow of debt at risk the firm-level analysis predicts for 2018. In other words, pre-COVID (2018 in particular) and no-COVID scenarios are equivalent. Given these considerations, for each country, and year t, the parameters are calibrated as:

³ Or, under the milder assumption, that, at the firm level, the share of bank loans over firm debt is uncorrelated with the probability of being insolvent.

$$\rho_{c,t} = PD_SME_{c,2018} - FlowDaR_SME_{c,t,no-COVID}$$

Where $PD_SME_{c,2018}$ is the weighted average probability of default in the banks' portfolio of SME loans in 2018, according to data from the European Banking Authority (EBA).⁴ The increase in PD is constrained to be non-negative, under the assumption that COVID-19 cannot be a positive shock for any country in 2020 or 2021.

The estimated probability of default in the COVID scenario is thus:

$$PD_SME_{c,t,COVID} = \rho_{c,t} + FlowDaR_SME_{c,t,COVID}$$

The same procedure can also be applied to the flow of firms that are projected to be illiquid, providing an alternative assessment of the impact of COVID-19 on banks.

B. From Probability of Default to Losses and Capital Ratios

Two additional elements are required to go from the probability of default to bank losses: the loss given default and banks' exposure to SME lending. Following a rule-of-thumb used in IMF stress tests, the loss given default (LGD) during a downturn is calculated as:

$$LGD_{c,downturn} = 0.08 + 0.92 * LGD_{c,2018}$$

Where $LGD_{c,2018}$ is the weighted average LGD in the banks' portfolios of SME loans as reported by EBA for 2018.

Thus, for each country *c* and year t (=2020, 2021), the expected losses over assets from the SME portfolio are:⁵

Expected losses_SMEs over assets

$$= PD_SME_{c,t,COVID} * LGD_{c,downturn} * \left(\frac{Loans_SME_{c}}{Total \ Loans_{c}}\right) * \left(\frac{Total \ Loans_{c}}{Assets_{c}}\right)$$

Where the $\frac{Loans_{s}SME_{c}}{Total Loans_{c}}$ is the share of bank loans to SMEs according to EBA (which is available for Euro

area and some non-euro area countries in our sample) and $\left(\frac{Total Loans_c}{Assets_c}\right)$ is the share of loans over

⁴ EBA collects this probability for some countries that are not under EBA's jurisdiction, such as Korea, Russia, and the United States.

⁵ Every equation hereafter is derived under the assumption that a country's banking system can be treated as one representative bank.

assets at the country level. The product of these two terms is the banking system' loan exposure to SME as a share of assets. This is 9.4%, on average, in the sample of countries we consider. This analysis focuses on the additional losses coming from the COVID-19 pandemic which are captured by the formula:

Extra losses_SMEs over assets_{ct.Covid}

$$= \left(PD_{SME_{c,t,COVID}} * LGD_{c,downturn} - PD_{SME_{c,2018}} * LGD_{c,2018}\right) * \left(\frac{Loans_{SME_{c}}}{Total \ Loans_{c}}\right) * \left(\frac{Total \ Loans_{c}}{Assets_{c}}\right)$$

Figures A.1 report, under the baseline scenario, the sum for 2020-21 of the extra losses as per the formula above, computed using the calibration ($PD = \rho + FlowDaR$) applied to each of the criteria used to measure firm financial weakness.

The difference between the total losses on SME lending and the additional losses coming from COVID are determined by the normal times losses, defined as:

$$Normal\ losses_{SMEs}\ over\ assets_{c} = PD_{SME_{c,2018}} * LGD_{c,2018} * \left(\frac{Loans_{SME_{c}}}{Total\ Loans_{c}}\right) * \left(\frac{Total\ Loans_{c}}{Assets_{c}}\right)$$

and plotted in Figure A.2, which shows that these losses are most sizeable in Southern Europe. To investigate the impact of SME insolvency on banking system' capital ratios, it is also useful to express these losses in terms of risk-weighted assets:

$$Extra \ losses \ SMEs \ to \ RWAs_{c,t,Covid} = Extra \ losses \ SMEs \ over \ assets_{c,t,Covid} / \left(\frac{RWAs_c}{Assets_c}\right)$$

It is further assumed that SME loans have a risk weight of 1 that increases to 1.5 if some loans default. Thus, the increase in risk-weighted assets due to higher risks as a result of loan defaults is given by:

Increase RWAs due to risk_{c,t,Covid}

$$= \frac{1}{2} * PD_SMEs_{ct,COVID} * (1 - LGD_{c,downturn}) * \left(\frac{Loans_SME_{c}}{Total \ Loans_{c}}\right) \\ * \left(\frac{Total \ Loans_{c}}{Assets_{c}}\right) / \left(\frac{RWAs_{c}}{Assets_{c}}\right)$$

The CET1 ratio in the COVID scenario is estimated by deducting the extra losses from SME lending due to COVID-19 from both the numerator and the denominator. The increase in risk-weighted assets is, instead, added to the denominator. The losses are added across 2020 and 2021. The capital ratio under COVID-19 is calculated as:

CET1R_{c.2021,due to Covid}

 $= \frac{CET1R_{c} - \sum_{t=2020,2021} (Extra \ Losses_{SMEs} \ to \ RWAs_{c,t,Covid})}{1 + \sum_{t=2020,2021} (-Extra \ Losses_{SMEs} \ to \ RWAs_{c,t,Covid} + Increase \ RWAs \ due \ to \ risk_{c,t,Covid})}$

Where $CET1R_c$ is the pre-COVID risk-weighted common equity tier 1 capital ratio. The change in capital due to COVID (i.e. the difference between $CET1R_c$ and $CET1R_{c,2021,due to Covid}$) for each country group,⁶ is reported in Figures A.3 (under the baseline) and Figure A.4 (under the adverse scenario) using several criteria to define the flow of debt-at-risk. Figures 12 and 13 are similar but focus on negative equity and liquidity gaps only. The distribution of country-level changes in capital ratios is illustrated by Figures A.5 and A.6.

Within this framework, it is easy to simulate the impact of possible supporting policies for SMEs on bank losses and capital. This can be achieved by substituting the measure of flow of debt-at-risk with the counterfactual flow under the COVID scenario, conditional on the policy under consideration. Then, the probability of default can be derived as:

 $PD_SME_{ct,COVID,policy} = \rho_{c,t} + FlowDaR_SME_{c,t,COVID,policy}$

Where the calibration parameters $\rho_{c,t}$ are the same as in the no policy scenarios. Estimates for losses and capital ratios also follow the same formulas as above.

The impact of policies on bank losses depends on both the nature of the policy and on which of the criteria is used to calibrate the probability of default. The proposed equity injection and the debt injection have a similar impact on firms' liquidity. However, the equity injection leads to a significant improvement in firms' solvency position (as it increases its net equity), while debt injections have a negligible impact on insolvency. The difference between these two policies is mirrored by their different impact on bank capital, as illustrated by Figure A.7, which focuses on SME insolvency, and Figure A.8, which focuses on SME illiquidity.

C. Heterogeneity

The estimated losses on SME loans are on average small, leading to a decrease of CET1R below 1 percentage point in all regions except for Southern Europe. This is due to the fact that SME loans represent a small fraction of the average bank's loan portfolio. However, these weighted averages mask significant heterogeneity between countries and banks.

The distribution of the capital losses at the country level (illustrated by Figures A5 and A6) reveals that some countries experience a decrease in capital ratio much larger than the average impact on any group – and up to ten times larger than the least exposed countries. In fact, the most exposed

⁶The groups are defined as follow. Asia-Pacific includes Australia, Japan, and Korea. Eastern Europe includes Czech Republic, Hungary, Poland, and Slovenia. Northern Europe includes Austria, Belgium, Finland, France, Germany, Ireland, Norway, Sweden, and the UK. Southern Europe includes Greece, Italy, Portugal, and Spain. Section I. illustrates how the analysis can be extended to countries which are not in the EBA and/or ECB CBD datasets.

country experiences an average drop between 1.5 and 2 percentage points in the baseline scenario (and up to 3 percentage points in the adverse scenario). This is both because the impact of the COVID-19 shock on SMEs is larger in those countries and because their banks are more exposed to SME risk. In fact, the size of the shock and banks' exposure to SME are positively correlated, magnifying heterogeneity within each geographical area.

Even within a country, some banks may be impacted by the rise in SME insolvency more than others: SME lending may represent a significantly larger share of the portfolio of some banks, and lending to the most harshly hit industries may not be equal across all banks. A bank-level analysis is thus performed to quantify such heterogeneity. The analysis focuses on a sample of large European banks from the EBA Transparency Exercise. This dataset provides information on the portfolio of 130 large banks in Europe which EBA monitors. It contains information on each bank's lending to SME and on lending to specific non-financial industry.⁷ A bank-level flow of debt-at-risk is constructed as:

$$FlowDaR_SME_{b,t} = \sum_{s} FlowDaR_SME_{c,t,s}\theta_{b,s}$$

Where $FlowDaR_SME_{c,t,s}$ is the flow of debt-at-risk (described above) for country c, year t, and industry s, and $\theta_{b,s}$ is bank b's share of corporate lending to industry s. The bank-specific flow of debt at risk is used to compute bank-specific probability of defaults and then, relying on the procedure described above for the country-level analysis, to estimate the bank-specific change in CET1R.

The distribution of capital losses is illustrated by Figures A.9. The interquartile range and the sample range highlight large differences between banks. The 10% most exposed banks experience an estimated capital drop of 2.5 to 3.4 percentage points or more in the baseline scenario, which is significantly larger than the average decline in the most exposed country (Figure A.5). Unreported analysis reveals that the main factor explaining heterogeneity across banks is the share of SME lending, while other variables, such as the exposure to the hardest hit sectors (e.g. Food and Accommodation) or the ratio of loans over total assets have a more limited role.

The bank-level analysis allows for a useful check. Aggregating from bank-level data to country-level capital losses produces similar estimates than the analysis conducted at the country-level data, as illustrated by Figure A.10.

⁷ Industries are defined according to NACE 1-digit classification. Data on lending by industry is not provided separately for SME borrowers. Thus, it is here assumed that the share of lending to each industry is the same across both SME and other corporate borrowers. Exposure to a non-financial industry is proxied by the gross carrying amount reported by EBA for each reporting bank. The final sample includes 89 banks from 16 European countries, data refers to December 2019 as NACE breakdown of lending is not available for other years.

D. Smaller Banks

The bank-level analysis of section C focuses only on large banks. However, previous literature argues that smaller banks specialize in serving smaller firms⁸ as larger organizations may be less suited to exploit the soft information that is important in relationship lending to smaller companies.

To investigate the degree to which smaller banks are more impacted than others, bank-level data from the EBA Transparency Exercise (2018 Q4) are used. First, a simple linear equation is estimated to shed light on the cross-sectional correlation between bank size (measured by the log of assets) and SME lending:

 $\frac{SME \ loans}{Total \ loans_{b,c}} = \alpha_c + \beta * \log(Assets)_b + \varepsilon_b$

where *b* is a bank in country *c* and α_c is a country fixed effect. The OLS estimate of the parameter β is -2.31 (heteroskedasticity-robust standard error = 1.06, t-stat = -2.19). Thus, a doubling of the size of a bank is associated with a decrease in SME lending of -2.31 * log(2) = .70 percentage points, which is almost 5% of the sample mean (15 percentage points).

As the data refers to large banks, additional data sources are needed to extrapolate to smaller banks. To this aim, unconsolidated end-of-2018 data on bank assets for all commercial banks are obtained from Fitch Connect.⁹ Under the assumption that the relationship between size and SME lending holds with the same parameters for much smaller banks that the ones in the EBA transparency exercise, ¹⁰ the estimated coefficients β and α_c can be used to predict SME lending for all banks.

Figure A.11 illustrates the average SME lending (median is similar) for banks of different size. SME loans are estimated to be more than 40% of total loans for banks in the lowest quintile in terms of assets, and almost 30% for banks in the second to lowest quintile. As the average share of SME loans to total loans among EBA banks is about 15%, and the impact of the SME insolvency is proportional to the share of SME lending, these estimates imply that small banks are expected to suffer large losses.

⁸ See, for instance, Mkhaiber and Werner (2020) "The relationship between bank size and the propensity to lend to small firms: New empirical evidence from a large sample" *Journal of International Money and Finance*

⁹ The final sample includes 2,800 unconsolidated banks' balance sheets from all the 20 countries, representing a median of 65% of total assets in the median country.

¹⁰ As a partial test for this assumption, the square of $log(Assets)_b$ is added to the estimating equation. The related coefficient is estimated to be small (-0.095) and not statistically distinguishable from zero (t-stat = -0.58), suggesting a linear-log relationship fits the EBA data well. Moreover, the estimated relationship between lending to SME and bank size is less strong than what found, using data on US banks of all size, by Mkhaiber and Werner (2020). Thus, it is unlikely that this analysis is overestimating the amount of SME lending of small banks because of the extrapolation from large banks in the EBA data to other banks.

The formulas of section B are then used to compute capital losses at the bank-level. Because we do not have bank-firm level data we still rely on country-level estimates of probability of default, computed as described in section A. Loans, assets, risk-weighted assets, and capital ratios for each bank are obtained from Fitch Connect. The ratio of SME lending over total loans is calculated by extrapolating from the size-SME lending relationship estimated with EBA bank-level data.¹¹

Results are illustrated by Figures 14 in the main text (relying on negative equity in the baseline scenario, while A.13 refers to the adverse scenario) and A.12 (relying on liquidity gap): they reveal a weak tail of harshly hit banks, especially among the smaller ones. Among the banks in the smallest quintile of assets, 25% experience a capital drop of 3 percentage points or more in the baseline scenario (negative equity). The estimates indicate also a reduction of capital of approximately 7 percentage points for the 10% of small banks that are hit the hardest. While such losses are small in absolute value, as small banks are small, they are nonetheless indicative that a weak tail of banks may need extra capital injections. Also, this analysis estimates losses on SME loans only, and it thus abstracts from losses on all other assets.

E. Additional details on data sources

The data used to map the debt-at-risk measures on banks' balance sheets are taken from two main data sources: the EBA risk-dashboard, and the ECB Consolidated Bank Statistics Database 2. As explained above, for each country, the 2018 averages are taken for consistency with the ORBIS data. Focusing on other years, such as 2017 or 2019 does not significantly impact the results.

For countries which are not included in the datasets mentioned above, data gaps are filled relying on other sources: Financial Stability Indicators from IMF, 1-year ahead expected probability of default from Moody's, recovery rates from WB's Doing Business survey, and data on share of lending to SME from OECD.

When a variable (which is missing from our main data sources) is directly available from a different data source, then the gap is filled by using this alternative data source. When, instead, the alternative data source reports a related variable, then a simple imputation procedure is performed. For instance, Moody's reports the expected probability of default for listed firms but not for SME. Therefore, for all countries where both PD for SME from EBA and Moody's EPD are present (2018 values), we calculate the ratio between these two variables and then compute the median among these ratios. Then, for the two countries for which EBA does not report PDs, the imputed PD for SME in 2018 is equal to the Moody's EPD multiplied by the median ratio of PDs from EBA over Moody's EPD for the other countries.

Bank-level data are obtained from the EBA transparency exercise dataset and Fitch Connect.

¹¹ While small banks may also lend to different industries than larger ones, such dimension of bank-level heterogeneity is ignored as industry-specific lending data are available only for EBA banks. However, the country-level probabilities of default take into account the sectorial composition of the different economies.



Figure A.1: Extra losses on SME loans due to COVID-19 over total assets (Baseline scenario)

Figure A.2: Estimates of pre-COVID losses on SME lending (over total assets)



Sources: EBA, ECB, and IMF staff calculation





Figure A.4: Change in banks' CET1R from additional losses on SME lending due to COVID-



19 (Adverse scenario)





Figure A.6: Heterogeneity across countries in the change in banks' CET1R from additional losses on SME lending due to COVID-19 (Adverse scenario)



The horizontal line inside each box represents the median; the upper and lower edges of each box show the top and bottom quartiles, respectively; and the top and bottom markers denote the maximum and the minimum, respectively

Figure A.7: Average change in banks' CET1R from additional losses on SME lending due to COVID-19's impact on firms' solvency (Baseline scenario)



Figure A.8: Average change in banks' CET1R from additional losses on SME lending due to COVID-19's impact on firms' liquidity (Baseline scenario)



Figure A.9: Heterogeneity across banks (within the EBA sample) in the change in CET1R from additional losses on SME lending due to COVID-19



Figure A.10: Country-level estimates of CET1R drops from country-level data vis-a-vis banklevel data, Negative Equity (left) and Liquidity Gap (right)



Sources: Orbis, EBA, ECB, and IMF staff calculations





Figure A.12: Heterogeneity across banks of different size in the change in CET1R from additional losses on SME lending due to COVID-19 (Baseline scenario, Liquidity Gap)



Figure A.13: Heterogeneity across banks of different size in the change in CET1R from additional losses on SME lending due to COVID-19 (Adverse scenario, Negative equity)

