From Just in Time, to Just in Case, to Just in *Worst*-Case

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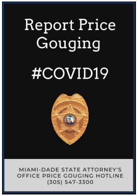
October 25, 2020

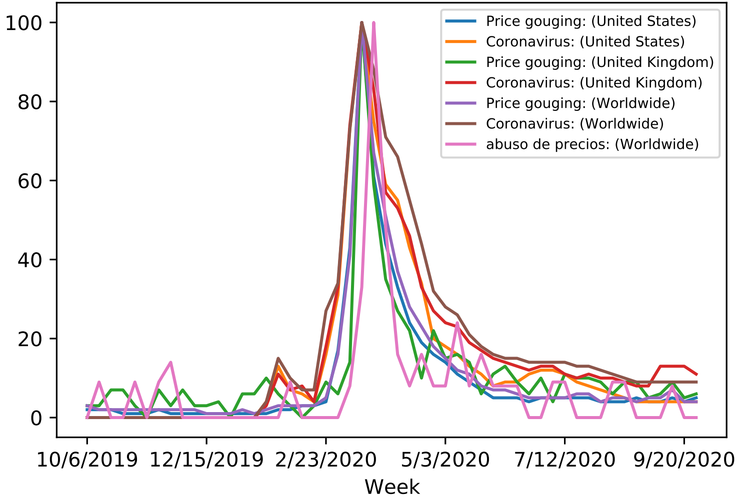
Covid-19, supply chain disruptions, and prices

- Many products suffer supply disruptions:
  - Well known: Essential products, Personal protective equipment
  - Less known: Beer, Electronic products,
- Price gouging
  - Consumer reaction
  - Law enforcement reaction

Price gouging



Covid and Price gouging



Motivation

- Management Literature:
  - Just-in-Time: Search for efficiency
  - Just-in-Case: Dealing with idiosyncratic risk
  - Aspects we would like to include in the analysis:
    - Ownership: Price gouging implies the impossibility of markets to work, or contracts to be fully specified.
    - Uncertainty: What to do with uncertain aggregate shocks?
- Just-in-*worst* case
  - What does a robust strategy to supply chains look like?
  - How can policy help?

Simplifying Assumptions

- Location problem: two locations (Valley and Mountain)
- Only aggregate shocks (not idiosyncratic)
- One global manufacturer and many small suppliers (N_t)
- Prices and costs are constant and exogenously given
- Concentrate exclusively on survival probabilities, not on product availability
    - Survival is good because small suppliers grow
    - No contract with suppliers can be written conditioning on their location.

Basic Framework

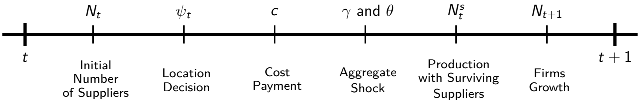
- Aggregate Shock: γ and θ
- Need at least one firm to produce.

$$\Pi_t = \begin{cases} pN_t^s & \text{if } N_t^s \geq 1 \\ 0 & \text{o.w.} \end{cases}$$

- Cost paid at the beginning of the period, and surplus is split equally among all suppliers. The multinational has zero profits.
- Expected Flow Profits are:

$$\begin{aligned} \Pi_{\text{Valley}} &= ((1-\gamma)+\gamma\theta)p - c \\ \Pi_{\text{Mountain}} &= ((1-\gamma)+\gamma(1-\theta))p - c \end{aligned}$$

- Growth of firms: $N_{t+1} = A \cdot (N_t^s)^{1-\mu}$



Survival Problem

- Survival problem is particularly interesting from the behavioral point of view: Experimental research shows people tend to settle on probability matching behavior
- Example:
    - Assume $\gamma = 0.2$. (Prob. aggregate shock hits)
    - Assume $\theta = 0.6$. So conditional on an aggregate shock, Valley survives with probability 60% and the Mountain survives with probability 40%
    - In our setting, people would choose to locate in the Valley 60 percent of the time, and in the Mountain 40 percent of the time, even though profit maximization implies a corner solution.
- Why is this relevant to supply chain?

Six Cases

Relationship between modeling choices and characteristics of the policy function.

	Baseline	Risk	Uncertainty
Decentralized			
Centralized			
$\widetilde{\theta}$	θ	$\sim U[\bar{\theta}-\Delta, \bar{\theta}+\Delta]$	$\in [\bar{\theta}-\Delta, \bar{\theta}+\Delta]$

Baseline: Independent Producers

- We study the value functions for each individual supplier. Suppliers only care about the continuation value of their own survival.

$$
\begin{aligned}
V_t^v &= ((1-\gamma)+\gamma\theta)p \qquad - c + \frac{1}{1+\beta}((1-\gamma)+\gamma\theta)V_{t+1} \\
V_t^m &= ((1-\gamma)+\gamma(1-\theta))p - c + \frac{1}{1+\beta}((1-\gamma)+\gamma(1-\theta))V_{t+1}.
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$$

- Rationality implies a corner solution

$$
V_t^v - V_t^m = \gamma(2\theta-1)\left(p + \frac{1}{1+\beta}V_{t+1}\right) > 0
$$

Every supplier goes to the Valley!

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  The pursuit of efficiency implies vulnerability!

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Every supplier goes to the Valley!

- This implies that almost surely (with probability 1) the supply chain will collapse.
  The pursuit of efficiency implies vulnerability!
- Notice that because linearity of expectations, the decision is the **SAME** for certainty and risk.

Baseline: Multinational

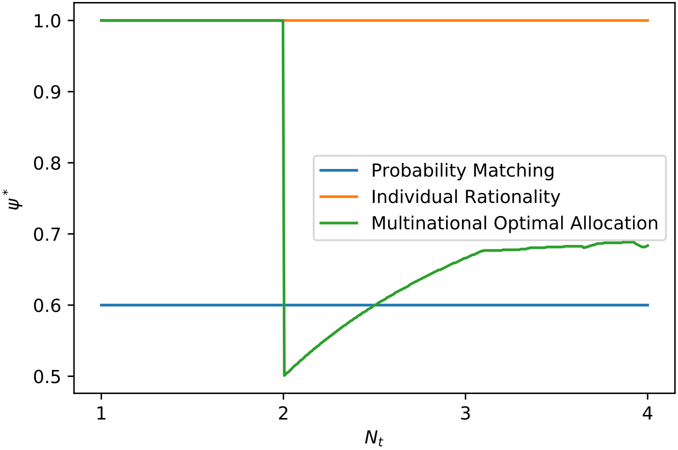
The multinational cares about the survival probability because they benefit from the continuation value of at least one supplier.

$$V(N_t) = \max_{\psi_t} \left\{ \left[\begin{array}{cccc} (1-\gamma) & \cdot & (pN_t & +\frac{1}{1+\beta} V\left(A \cdot (N_t)^{1-\mu}\right) \quad) \\ +\gamma\theta & \cdot & (p\psi_t N_t & +\frac{1}{1+\beta} V\left(A \cdot (\psi_t N_t)^{1-\mu}\right) \quad) \\ +\gamma(1-\theta) & \cdot & (p(1-\psi_t)N_t & +\frac{1}{1+\beta} V\left(A \cdot ((1-\psi_t)N_t)^{1-\mu}\right) \quad) \end{array} \right] - cN_t \right\}$$

where

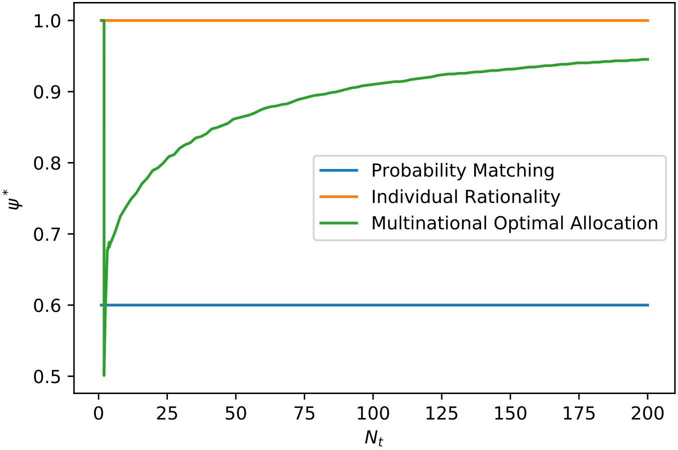
$$\lim_{N \to 1^-} V(N) = 0$$

Simulation



1.0
0.9
0.8
0.7
0.6
0.5
ψ^*
1
2
3
4
N_t
Probability Matching
Individual Rationality
Multinational Optimal Allocation

Simulation



1.0
0.9
0.8
0.7
0.6
0.5
ψ^*
0
25
50
75
100
125
150
175
200
N_t
Probability Matching
Individual Rationality
Multinational Optimal Allocation

Risk: Multinational

- Value Functions:

$$V(N_t) = \max_{\psi_t} E_\theta \left\{ \left[\begin{array}{ccc} (1-\gamma) & (pN_t & +\frac{1}{1+\beta} V\left(A \cdot (N_t)^{1-\mu}\right)) \\ +\gamma\widetilde{\theta} & (p\psi_t N_t & +\frac{1}{1+\beta} V\left(A \cdot (\psi_t N_t)^{1-\mu}\right)) \\ +\gamma(1-\widetilde{\theta}) & (p(1-\psi_t)N_t & +\frac{1}{1+\beta} V\left(A \cdot ((1-\psi_t)N_t)^{1-\mu}\right)) \end{array} \right] - cN_t \right\}$$

$$\lim_{N \to 1^-} V(N) = 0$$

- Because of linearity of expectations, solution is identical:
    - The value function under Risk is identical to the Baseline setting, and the multinational chooses as if $\theta = \bar{\theta}$

Six Cases

Relationship between modeling choices and characteristics of the policy function.

	Baseline	Risk	Uncertainty
Decentralized	Corner Solution **Valley**	Corner Solution **Valley**	
Centralized	Internal Solution $\psi(N_t)$	Internal Solution $\psi(N_t)$	

Uncertainty: Independent Producers

- Parameter $\theta \in [\bar{\theta} - \Delta, \bar{\theta} + \Delta]$ implies that nature chooses $\delta \in [-\Delta, \Delta]$ to pick the worst possible case for the supplier.

$$V_t^v = \min_{\delta \in [-\Delta, \Delta]} ((1-\gamma) + \gamma(\bar{\theta} + \delta))p \qquad - c + \frac{1}{1+\beta}((1-\gamma) + \gamma(\bar{\theta} + \delta))V_{t+1}$$

$$V_t^m = \min_{\delta \in [-\Delta, \Delta]} ((1-\gamma) + \gamma(1 - \bar{\theta} - \delta))p - c + \frac{1}{1+\beta}((1-\gamma) + \gamma(1 - \bar{\theta} - \delta))V_{t+1}$$

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- What is the "worst" case?
  The Worst-Case maximizes the probability of disappearing conditional on an aggregate shock.
  - Worst case for the Valley is when $\delta = -\Delta$
  - Worst case for the Mountain is when $\delta = \Delta$

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  The Worst-Case maximizes the probability of disappearing conditional on an aggregate shock.
  - Worst case for the Valley is when $\delta = -\Delta$
  - Worst case for the Mountain is when $\delta = \Delta$
- For $\bar{\theta} > 0.5$, we have an identical solution!

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Relationship between modeling choices and characteristics of the policy function.

	Baseline	Risk	Uncertainty
Decentralized	Corner Solution **Valley**	Corner Solution **Valley**	Corner Solution **Valley**
Centralized	Internal Solution $\psi(N_t)$	Internal Solution $\psi(N_t)$	

Uncertainty: Multinational

Problem of the multinational

$$V(N_t) = \max_{\psi_t(N_t)} \min_{\delta \in [-\Delta, \Delta]} \left\{ \left[\begin{array}{c} (1-\gamma)\left(pN_t + \frac{1}{1+\beta} V\left(A \cdot (N_t)^{1-\mu}\right)\right) + \\ \gamma(\bar{\theta}+\delta) \cdot \left(p\psi_t N_t + \frac{1}{1+\beta} V\left(A \cdot (\psi_t N_t)^{1-\mu}\right)\right) + \\ \gamma(1-\bar{\theta}-\delta) \cdot \left(p(1-\psi_t)N_t + \frac{1}{1+\beta} V\left(A \cdot ((1-\psi_t)N_t)^{1-\mu}\right)\right) \end{array} \right] - cN_t \right\}$$

subject to

$$\lim_{N \to 1^-} V(N) = 0.$$

What is the "worst" case?

Uncertainty: Multinational (Intuition)

- Assume that $\bar{\theta} = 0.6$
- If there is no uncertainty ($\Delta = 0$), then the multinational chooses ψ which coincides with the baseline optimal solution.
- When $\Delta > 0$ but small, the multinational needs to choose assuming the worst possible case occurs.
  - For relatively large N_t's, the optimal ψ is close to one, and therefore, the worst case is for $\delta = -\Delta$
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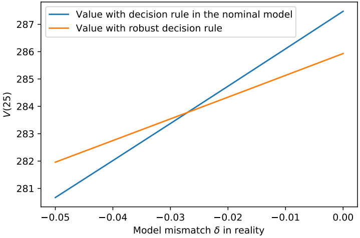
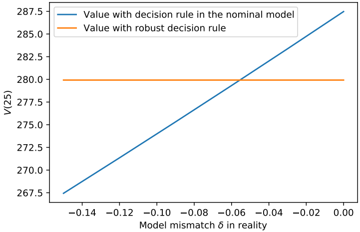
  - For relatively large N_t's, the optimal ψ is close to one, and therefore, the worst case is for $\delta = -\Delta$
  - The multinational then treats $\theta = \bar{\theta} - \Delta$
- However, for Δ big enough — such that the support of θ includes 0.5 — the optimal solution is to always assume $\theta = 0.5$
- The robust strategy is

$$\theta^* = \begin{cases} \bar{\theta} - \Delta & \text{if } \bar{\theta} - \Delta > 1/2 \\ 1/2 & \text{if } \bar{\theta} - \Delta <= 1/2 \end{cases}$$

Uncertainty: Multinational (Simulation)



(a) Δ=0.05



(b) Δ=0.15

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Conclusions

- What does it mean to have a robust supply chain?
  - Clearly not what we had before 2020
  - Management Literature: Just-in-Time and Just-in-Case
    - Some hedging: Deals with idiosyncratic shocks and risk
    - But unprepared to extreme realizations

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  - Support *ex-post*: Price gouging at the retail level need support at the supply chain level
  - Support *ex-ante*: Compensate diversification ex-ante
- Uncertain nature of Future Shocks:
  - Environmental disasters
  - Social unrest (specially in the aftermath of COVID)
  - Other public health threats
  - Geopolitical crises

Conclusions

- Just in....

	Baseline	Risk	Uncertainty
Decentralized	No Hope	No Hope	No Hope
Centralized	Time	Case	*Worst*-Case