Optimal Monetary Policy under Dollar Pricing*

Konstantin Egorov

kegorov@NES.ru

Dmitry Mukhin

dmitry.mukhin@Yale.edu

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Abstract

The recent empirical evidence shows that most international prices are sticky in dollars. This paper studies the optimal *non-cooperative* monetary policy and the welfare implications of dollar pricing in a context of an open economy model with nominal rigidities and input-output linkages between firms. We establish the following results: 1) dollar pricing generates asymmetric international spillovers such that other countries partially peg their exchange rates to the dollar giving rise to a "global monetary cycle"; 2) capital controls cannot insulate other countries from U.S. spillovers; 3) the U.S. finds it optimal to deviate from inflation targeting to partially stabilize global economy; 4) the optimal cooperative policy is hard to implement because it generates gains for non-U.S. countries and losses for the U.S.; 5) there are potential gains from dollar pricing for the U.S., while other countries can benefit from forming a currency union such as the Eurozone.

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1 Introduction

The key feature of the international price system is that most prices in global trade are set in dollars and remain sticky in U.S. currency at horizons of up to two years (Gopinath 2016, Goldberg and Tille 2008, Gopinath and Rigobon 2008). A growing empirical literature shows that this fact has important implications for the transmission of monetary shocks between countries. In particular, a depreciation of the U.S. exchange rate decreases import prices relative to prices of domestic goods around the world, which leads to expenditure switching towards foreign goods and increases the volume of global trade (Boz, Gopinath, and Plagborg-Møller 2017). In contrast, a depreciation of other exchange rates does not affect export prices in the currency of destination and hence, has no stimulating effect on exports, so that the trade balance in non-U.S. economies adjusts mostly through changes in imports (Casas, Díez, Gopinath, and Gourinchas 2017). In addition, the countries with a higher share of dollar pricing in trade experience larger spillovers of U.S. monetary policy on output, exchange rates and interest rates (Zhang 2018).[1]

While a lot of progress has been made in understanding the positive implications of dollar currency pricing (DCP), much less is known about its implications for the optimal policy and the welfare effects. These questions are, however, at the heart of recent policy debates: Does U.S. monetary policy generate negative spillover effects on the rest of the world and leads to "currency wars" (Bernanke 2017)? Should the optimal policy focus on price stabilization as in a closed economy or should it also respond to foreign shocks (Engel 2011, Corsetti, Dedola, and Leduc 2018)? Can free floating exchange rates insulate countries from international spillovers (Friedman 1953, Devereux and Engel 2003) and what explains the widespread "fear of floating" in the data (Calvo and Reinhart 2002)? Should countries use capital controls (Blanchard 2017)? Does the U.S. enjoy an "exorbitant privilege" from the dominant status of its currency in global trade (Gourinchas and Rey 2007, Eichengreen 2011)? Are there gains from forming a currency union such as the Eurozone in this case (Mundell 1961)? Do countries benefit from international cooperation (Benigno and Benigno 2003)? Finally, is the current international price system optimal or is it desirable to move to the one with a more symmetric use of currencies?

We answer these questions in a context of a standard open-economy sticky-price model by Gali and Monacelli (2005) — which has been actively used as a workhorse model in the recent normative literature (see e.g. Farhi and Werning 2012, 2017) — by augmenting it with two additional assumptions. First, we replace producer currency pricing (PCP) in the international trade with DCP. Second, we allow for foreign inputs in production, which reflects the fact that most imported goods are used either as intermediates or have to go through wholesale and retail sectors before reaching consumers (see Johnson and Noguera 2012, Burstein, Neves, and Rebelo 2003). Combined together, these two assumptions generate international price linkages between firms that are sticky in dollars and play crucial role

[1] Auer, Burstein, and Lein (2018) show that currency choice of exporters matters not only for prices at the dock, but also determines the pass-through into retail prices.

in the analysis. The rest of the model is standard. Households use a complete set of Arrow-Debreu securities to share risk between countries, while firms set domestic prices in local currency. After the uncertainty is resolved and the monetary policy is announced, the agents make consumption and production decisions.

While there has been much progress recently in solving the optimal *cooperative* monetary policy (see Engel 2011, Corsetti, Dedola, and Leduc 2018), little is known about the optimal *non-cooperative* policy away from the very special limiting cases such as Cole and Obstfeld (1991) parametrization (see Clarida, Galı, and Gertler 2002, Gali and Monacelli 2005).[2] At the same time, as we explain below, the international spillovers of monetary policy are highly asymmetric under DCP and lead to important *strategic interactions* across countries. Therefore, we are mostly interested in the non-cooperative policy and adopt the following strategy to make progress in this direction. To build intuition, we start with a simplified "static" version of the model in Section 3 with fully sticky prices and discretionary monetary policy. This case provides an important benchmark: the planner takes prices as given in their currency of invoicing and can only affect allocation by changing nominal wages and exchange rates. We derive the optimal policy and characterize it in terms of simple targets. Surprisingly, as we show in Section 4, most of these insights remain true in a "dynamic" model with Rotemberg (1982) price adjustment and optimal policy with commitment.

Our first main result is that while price stability remains the optimal target in non-U.S. economies under DCP as under PCP, such policy no longer implements the first-best allocation and leads to an implicit peg to the dollar.[3] The intuition behind this result is straightforward when international prices are fully rigid in dollars: the country's monetary policy cannot generate expenditure switching towards its exports and the quantity exported is exogenous to the policy. Given that country's dollar revenues from exports are exogenous and import prices are also fixed in dollars, it follows from country's budget constraint that the quantity imported is also independent from local monetary policy. As a result, the best outcome the policy can achieve is to implement the optimal production and consumption of domestic goods by stabilizing the prices of local producers. Interestingly, this result holds also in a dynamic setup when prices change gradually and both terms-of-trade and trade adjustment are endogenous to monetary policy. Intuitively, in contrast to the PCP case, exchange rate depreciation does not mechanically generate expenditure switching under dollar pricing. Instead, the planner changes the dollar value of marginal costs, which makes firms to adjust their export prices. This is, however, associated with additional costs of price adjustment under Rotemberg pricing, and as a result, the planner

[2] A few papers that allow for more general parameter values ignore the country's budget constraint in the planner's problem (see e.g. De Paoli 2009, Faia and Monacelli 2008). We find it very hard to rationalize this assumption and show it is not satisfied under either commitment or discretion. More analysis has been done on the non-cooperative *fiscal* policy (see Engel 2016, Farhi and Werning 2012, 2017).

[3] The former result about the optimality of domestic price stabilization generalizes the insight from Casas, Díez, Gopinath, and Gourinchas (2017) (henceforth, CDGG), who show that under complete asset markets and Cole-Obstfeld parametrization, the evolution of country's terms of trade is independent from local monetary policy, which therefore, should focus on domestic targets.

does not want to deviate from stabilizing firms' marginal costs.

While the optimal policy of non-U.S. economies can be formulated in terms of domestic target, it does respond to foreign shocks. Given that all international prices are set in dollars, an appreciation of U.S. exchange rate increases the prices of imported intermediates in other economies and puts upward pressure on local prices.[4] To keep them constant, other countries have to tighten their monetary policy, "leaning against the wind" and effectively introducing a partial peg to the dollar. The result shows that the widespread anchoring of exchange rates to the dollar in the data (see Ilzetzki, Reinhart, and Rogoff 2017) can be due not only to the global financial cycle (Rey 2015), but also to the dominant status of the dollar in international trade. This result is also consistent with the fact that rising import prices are among main factors mentioned by policymakers in emerging economies when explaining their decision to stabilize exchange rates.

Interestingly, we show that additional fiscal instruments such as capital controls cannot insulate countries from foreign spillovers and restore efficient allocation in the global economy. This finding is surprising in light of the result from Farhi and Werning (2016) that the laissez-faire risk sharing is generically inefficient when monetary policy cannot implement the first-best allocation. This discrepancy comes from the fact that even though efficient allocation cannot be achieved in our setting, the optimal policy does eliminate the aggregate demand externality and closes the gap between the social and private value of insurance. Importantly, this result does not rely on the assumption that asset markets are complete and remains true for arbitrary structure of the financial markets. It also shows that the nature of the international spillovers is important for the optimal policy: while capital controls and other macroprudential policies might be efficient in curbing financial spillovers (see e.g. Farhi and Werning 2013, Aoki, Benigno, and Kiyotaki 2016), they are unlikely to help with the spillovers arising from DCP.

We then show that the optimal policy of the U.S. deviates from domestic price stabilization and leads to highly asymmetric spillovers between countries. Indeed, the fact that both domestic and export prices are set in producer currency allows U.S. monetary policy to stabilize not only domestic production and consumption, but also to stimulate exports. Moreover, the stickiness of import prices in dollars partially insulates the U.S. economy from foreign shocks and allows it to set monetary policy independently from the policy of other countries. At the same time, the byproduct of dollar depreciation is that export prices of all other countries go down relative to the prices of domestic goods stimulating the global production and generating large spillovers on other economies. We show that the U.S. should be concerned with such backfiring of its policy (*cf.* Bernanke 2017): the international spillovers arising from movements in markups of exporters from all other countries result in a suboptimal global demand, which in turn affects the U.S. economy (*cf.* Devereux and Engel 1998). Therefore, the optimal U.S. policy with commitment deviates from domestic price targeting to partially stabilize export prices of other

[4] Intermediate goods in production is the key assumption that explains the optimality of peg in our model in contrast to the previous literature (Goldberg and Tille 2009, Casas, Díez, Gopinath, and Gourinchas 2017).

economies. This motive pushes U.S. policy closer to the optimal *cooperative* policy — which would use U.S. monetary instruments to stabilize average export prices in dollars across the world — thought does not fully implement it. While beneficial for other countries and the global economy, the cooperative policy decreases the welfare of the U.S., which therefore has no incentives to coordinate its policy with other countries. This result contrasts with the conventional wisdom that the cooperative policy is Pareto improving as it eliminates the terms-of-trade externality and benefits all countries.

Finally, we argue that while a more efficient monetary policy in the U.S. allows it to achieve higher welfare than in other economies, the welfare ranking is ambiguous under the optimal policy. Indeed, as argued above, U.S. policy can simulatenously stabilize domestic and export production. As a result, the equilibrium welfare is higher in the U.S. than in other countries when they follow the same policy of price targeting. However, U.S. planner internalizes effects of its policy on global demand and finds it optimal to partially sacrifice domestic objectives to improve allocation in world economy. This policy benefits other countries more than the U.S. as it does not require them to deviate from domestic price stabilization. Therefore, under the optimal policy, U.S. welfare can fall below the welfare of other countries, though this is unlikely under realistically low values of openness of U.S. economy.

This analysis also reveals a new source of gains from forming a currency union such as the Eurozone. While the standard critique — that a member of a currency area loses an independent monetary policy and cannot use it to stabilize the economy — still applies in our setting, forming a currency union can boost the welfare if it promotes the status of the common currency in the global trade. In particular, we show that a country (the Eurozone) with imports and exports invoiced in its currency can potentially achieve the same level of welfare as the country (the U.S.), which is the issuer of the dominant currency used in all international trade.

Related literature [TO BE COMPLETED]

2 Environment

This section lays out our baseline analytical model. To emphasize the role of the international price system, we introduce dollar currency pricing (DCP) and global input-output linkages in an otherwise conventional open-economy sticky-price model a la Gali and Monacelli (2005) that has been extensively used in the recent normative literature (see e.g. Farhi and Werning 2017).

The world consists of a continuum of symmetric small open economies. To disentangle the role of the dominant currency, we assume that the U.S. is a small economy and is symmetric to other countries in all respects except for the use of the dollar in international trade. Time is discrete and the horizon is infinite. The international asset markets are complete and agents use Arrow-Debreu securities to share the risk before the realization of shocks. After uncertainty is resolved and the transfers between countries are made, the households decide how much to work and to consume. Prices are set by monopolistic firms and are subject to Rotemberg (1982) adjustment costs in the currency of invoicing.

Motivated by the recent empirical literature, we deviate from the conventional assumptions about the international price system in two ways. First, we assume that all international prices are set in dollars rather than the currency of producer or buyer. While this is an extreme assumption, the empirical evidence shows that it provides a good first-order approximation to the real world (Gopinath 2016).[5] Domestic products are invoiced in local currency. Second, we allow for input-output linkages in production to capture the fact that intermediate goods account for most of the international trade (see e.g. Koopman, Wang, and Wei 2014).

2.1 Households

There is a continuum of countries indexed by $i \in [0, 1]$ with $i = 0$ corresponding to the U.S. A representative household in country i has isoelastic preferences with linear disutility from labor (see Rogerson 1988, Hansen 1985)[6]

$$U_i = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{it}^{1-\sigma}}{1-\sigma} - N_{it} \right] \tag{1}$$

over labor N_{it} and consumption bundle C_{it}:

$$C_{it} = \left[(1-\gamma)^{\frac{1}{\theta}} C_{iit}^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}} \int C_{jit}^{\frac{\theta-1}{\theta}} \mathrm{d}j \right]^{\frac{\theta}{\theta-1}}, \tag{2}$$

where C_{jit} denotes country i's consumption of goods produced in country j, and $1-\gamma$ reflects the home bias in consumption that can arise due to trade costs or preferences for locally produced goods. For simplicity, we assume the same elasticity of substitution θ between goods produced in different countries. Bundle C_{jit} in turn aggregates the continuum of unique varieties $\omega \in [0, 1]$ produced in country j:

$$C_{jit} = \left(\int C_{jit}(\omega)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}\omega \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3}$$

where $\varepsilon > 1$ is the elasticity of substitution between products produced in a given country.

The asset markets are complete, and the agents trade a full set of Arrow-Debreu securities in period $t = 0$ before the shocks are realized subject to the ex-ante budget constraint:

$$\sum_{t,h} Z_t^h B_{it}^h = 0, \tag{4}$$

[5]For simplicity, we take firms' currency choice as exogenous. Mukhin (2018) discusses the interactions between the optimal monetary policy and firms' invoicing decisions. See also Gopinath and Stein (2017), Drenik, Kirpalani, and Perez (2018), Chahrour and Valchev (2017), Rey (2001), Krugman (1980) for models of endogenous currency choice.

[6]The assumption that utility is linear in labor is not essential for our results, but provides much tractability to the analysis: when coupled with the constant returns to scale, it allows to disentangle production for domestic and foreign markets (see Section 3.3 for the discussion).

where B_{it}^h denotes country i's holdings of the security that pays one dollar in state $h \in H$ in period t and zero in all other states of the world, and Z_t^h is the price of such security.[7] Because asset markets are complete, the assumption that Arrow-Debreu securities are denominated in dollars is without loss of generality in most applications below.

After state of the world h is revealed, households face the ex-post budget constraint

$$P_{it}C_{it} = W_{it}N_{it} + \Pi_{it}^f + T_{it} + \mathcal{E}_{it}B_{it}^h, \tag{5}$$

where Π_{it}^f and T_{it} denote the profits of local firms and the net transfers from the government, and $\mathcal{E}_{it}$ is country i's exchange rate against the U.S.: an increase in $\mathcal{E}_{it}$ corresponds to a depreciation of local currency. Let all prices be expressed in the currency of destination and define the price index for products exported from j to i as $P_{jit} = \left(\int P_{jit}(\omega)^{1-\varepsilon}\mathrm{d}\omega\right)^{\frac{1}{1-\varepsilon}}$. The consumer price index is given by

$$P_{it} = \left[(1-\gamma)P_{iit}^{1-\theta} + \gamma\int P_{jit}^{1-\theta}\mathrm{d}j\right]^{\frac{1}{1-\theta}}. \tag{6}$$

The goods market clearing condition states that total output is equal to the sum of local demand Y_{it}^D and foreign demand Y_{it}^F for final goods C_{ijt} and intermediate goods X_{ijt}:

$$Y_{it} = Y_{it}^D + Y_{it}^F = (1-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}(C_{it}+X_{it}) + \gamma\int\left(\frac{P_{ijt}}{P_{jt}}\right)^{-\theta}(C_{jt}+X_{jt})\,\mathrm{d}j. \tag{7}$$

Finally, the market clearing in the asset markets requires that for every $h \in H$,

$$\int B_{it}^h\mathrm{d}i = 0. \tag{8}$$

2.2 Firms

In each country i, there is a continuum of firms, each using a constant returns to scale technology to produce a unique variety ω from labor and a bundle of intermediate goods:

$$Y_{it}(\omega) = A_{it}\left(\frac{X_{it}(\omega)}{\alpha}\right)^{\alpha}\left(\frac{L_{it}(\omega)}{1-\alpha}\right)^{1-\alpha} \tag{9}$$

For simplicity, we assume a roundabout production with the same bundle of goods used in consumption and production:

$$X_{it} = \left[(1-\gamma)^{\frac{1}{\theta}}X_{iit}^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}}\int X_{jit}^{\frac{\theta-1}{\theta}}\mathrm{d}j\right]^{\frac{\theta}{\theta-1}}. \tag{10}$$

It follows that the marginal costs of production are $MC_{it} = \frac{P_{it}^{\alpha}W_{it}^{1-\alpha}}{A_{it}}$.

[7] To simplify the notation, the history index h is suppressed whenever possible.

Firms are monopolistic competitors and are subject to Rotemberg (1982) price-adjustment costs.[8] Because of the constant returns to scale assumption, the price-setting problem of a firm is separable across markets. However, we adopt a standard assumption from the literature that a firm sets a uniform price in all markets of destination with the same currency of invoicing. The optimal domestic price of a representative firm in a non-U.S. economy is set in local currency and solves

$$\max_{\{P_t\}} \mathbb{E}\sum_{t=0}^{\infty} \Theta_{it}\left[(P_t - \tau_i MC_{it})\left(\frac{P_t}{P_{iit}}\right)^{-\varepsilon} Y_{it}^D - (1-\gamma)\frac{\varphi}{2}\tau_{Ri}\left(\frac{P_t}{P_{t-1}} - 1\right)^2 W_{it}\right], \tag{11}$$

and firms are allowed to choose any initial price in period $t = 0$ without paying adjustment costs, i.e. they can choose P_{-1}. In this expression, the local demand shifter Y_{it}^D is defined in (7), τ_i and τ_{Ri} stay respectively for time-invariant production and price-adjustment subsidies, and $\Theta_{it} \equiv \beta^t C_{it}^{-\sigma}/P_{it}$ is the nominal stochastic discount factor. For tractability reasons, we assume that adjustment costs are in terms of labor rather than output and are independent from sales up to a constant $1-\gamma$.[9]

In contrast, the export prices in a representative non-U.S. economy are set in dollars and are uniform across all markets of destination:

$$\max_{\{P_t^*\}} \mathbb{E}\sum_{t=0}^{\infty} \Theta_{it}\left[(\mathcal{E}_{it}P_t^* - \tau_i MC_{it})\left(\frac{\Psi_i P_t^*}{P_{it}^*}\right)^{-\varepsilon} Y_{it}^F - \gamma\frac{\varphi}{2}\tau_{Ri}\left(\frac{P_t^*}{P_{t-1}^*} - 1\right)^2 W_{it}\right], \tag{12}$$

where again, firms are free to choose initial price, the foreign demand shifter Y_{it}^F is defined in (7), $P_{it}^* = \left(\int P_{it}^*(\omega)^{1-\varepsilon}\mathrm{d}\omega\right)^{\frac{1}{1-\varepsilon}}$ is the dollar export price index of country i, and Ψ_i denotes the time-invariant export tax. As explained in detail below, this second fiscal instrument plays an important role in our analysis as it allows the government to eliminate both the monopolistic distortion and the terms-of-trade externality.[10] Given the Ricardian equivalence, we assume without loss of generality that the government runs a balanced budget and transfers the net revenues from taxes to households.

The case of the U.S. is slightly different because local firms set the same price for both domestic and foreign customers and hence, the law of one price holds for all goods produced in the U.S. up to an export tax. Given that $P_{it}^* = \Psi_i P_{iit}$ for the U.S., the individual firm's problem is:

$$\max_{\{P_t\}} \mathbb{E}\sum_{t=0}^{\infty} \Theta_{it}\left[(P_t - \tau_i MC_{it})\left(\frac{P_t}{P_{iit}}\right)^{-\varepsilon} Y_{it} - \frac{\varphi}{2}\tau_{Ri}\left(\frac{P_t}{P_{t-1}} - 1\right)^2 W_{it}\right]. \tag{13}$$

[8]While equivalent to the first-order to the Calvo (1983) pricing, the Rotemberg model provides additional analytical tractability as we discuss in Section 4. In addition, in contrast to the Calvo model, the recent literature has found little evidence in favour of price dispersion as the main source of inflation costs (Nakamura, Steinsson, Sun, and Villar 2018).

[9]The previous literature has explored several alternative assumptions about adjustment costs in the Rotemberg model (*cf.* e.g. Schmitt-Grohé and Uribe 2004, Faia and Monacelli 2008, Kaplan, Moll, and Violante 2018).

[10]One can use import tariff instead of the export tax as the Lerner symmetry holds in the model.

2.3 Equilibrium conditions

The equilibrium is such that households maximize expected utility subject to the ex-ante and ex-post budget constraints, firms maximize expected profits, the government's budget constraint is satisfied, and the markets clear. We next show how the set of equilibrium conditions can be reduced to just a few constraints in the planner's problem (see Appendix A.1 for details). Following Farhi and Werning (2012), it is convenient to divide the equilibrium conditions into the "demand block" that does not depend on the price setting and is the same for all countries and the "supply block" that is different for the U.S. and other economies.

2.3.1 Demand block

The demand block of the model includes the price indices, market clearing, and risk-sharing conditions. Because of the DCP assumption, the import price index is the same for all countries and equals $P_t^* = \left(\int P_{jt}^{*1-\theta} \mathrm{d}j\right)^{\frac{1}{1-\theta}}$. Define for each country i the real exchange rate against the bundle of imported goods Q_{it}, the terms-of-trade S_{it}, and the deviation of export prices from the domestic ones Φ_{it}:[11]

$$Q_{it} \equiv \frac{\mathcal{E}_{it} P_t^*}{P_{it}}, \qquad S_{it} \equiv \frac{P_t^*}{P_{it}^*}, \qquad \Phi_{it} \equiv \frac{\mathcal{E}_{it} P_{it}^*}{P_{iit}}. \tag{14}$$

Combining these definitions with the ideal price index P_{it} from (6), we obtain the first constraint:

$$Q_{it}^{\theta-1} = \gamma + (1-\gamma)\left(\Phi_{it} S_{it}\right)^{\theta-1}. \tag{15}$$

Intuitively, a fall in domestic prices relative to imported ones – i.e. a depreciation of the real exchange rate – should be associated either with higher terms-of-trade (if the law of one price holds) or a larger wedge between domestic and export prices (if the terms-of-trade stay constant).

Using the definitions of the price indices, the market clearing (7) can be rewritten as:

$$\frac{A_{it} X_{it}^{\alpha} L_{it}^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} = (1-\gamma)\left(\Phi_{it} S_{it}\right)^{\theta} Q_{it}^{-\theta}\left(C_{it} + X_{it}\right) + \gamma S_{it}^{\theta} \int Q_{jt}^{-\theta}\left(C_{jt} + X_{jt}\right) \mathrm{d}j, \tag{16}$$

where the global demand shifter $C_t^* \equiv \int Q_{jt}^{-\theta}\left(C_{jt} + X_{jt}\right) \mathrm{d}j$ combines the expenditure-switching effect Q_{jt} and the aggregate demand effect $C_{jt} + X_{jt}$ of foreign countries. Using the labor supply condition to substitute out real wages, firms' demand for intermediate goods can be expressed in terms of consumption and labor as

$$X_{it} = \frac{\alpha}{1-\alpha} C_{it}^{\sigma} L_{it}. \tag{17}$$

A higher consumption C_{it} increases real wages in the economy, making labor more expensive relative to

[11] Notice that the ratio of the real exchange rates (terms-of-trade) of countries i and j satisfies the conventional definition of bilateral real exchange rates (terms-of-trade).

goods, which stimulates demand for intermediates. Also, demand for intermediate inputs is increasing in L_{it} because of the decreasing returns to scale in production. The labor market clearing condition in turn aggregates the labor used in production and on price adjustment:

$$N_{it} = L_{it} + \frac{\varphi}{2}(1-\gamma)\Pi_{iit}^2 + \frac{\varphi}{2}\gamma\Pi_{it}^{*2}, \tag{18}$$

where $\Pi_{iit} \equiv \frac{P_{iit}}{P_{iit-1}} - 1$ and $\Pi_{it}^* \equiv \frac{P_{it}^*}{P_{it-1}^*} - 1$ are inflation rates for domestic and export prices.

The risk-sharing condition under complete markets can be compactly written as

$$C_{it} = \left(\frac{Q_{it}}{\Lambda_i}\right)^{\frac{1}{\sigma}} \bar{C}_t, \tag{19}$$

where $\bar{C}_t \equiv \int Q_{jt}^{-\frac{1}{\sigma}} C_{jt} \mathrm{d}j$ and Λ_i is a state-invariant constant determined from the country's budget constraint. Intuitively, the efficient risk sharing prescribes higher *relative consumption* in country i in states of the world, in which its consumption bundle is cheaper and the real exchange rate is depreciated. The average *level of consumption*, on the other hand, depends on the expected (permanent) income and is captured by Λ_i.

Combining household and government budget constraints and firms' profits, we obtain the country's ex-post budget constraint:

$$NX_{it} + B_{it}^h = 0, \quad NX_{it} = \gamma P_t^* \left[S_{it}^{\theta-1} \int Q_{jt}^{-\theta}\big(C_{jt} + X_{jt}\big)\mathrm{d}j - Q_{it}^{-\theta}\big(C_{it} + X_{it}\big) \right]. \tag{20}$$

The two terms in NX_{it} correspond to the dollar value of country i's exports and imports. The countries use asset markets to transfer resources B_{it}^h across periods and states of the world, but have to respect the ex-ante budget constraint:

$$\mathbb{E}\sum_{t=0}^{\infty} \beta^t \frac{\bar{C}_t^{-\sigma}}{P_t^*} B_{it}^h = 0, \tag{21}$$

where $\beta^t \frac{\bar{C}_t^{-\sigma}}{P_t^*}$ reflects the nominal global stochastic discount factor for dollar-denominated securities, which is the same for all countries under complete markets.

Remark: while generically equations (20)-(21) impose important constraints on the planner's decision, there are two special cases when given the risk-sharing condition (19), the net exports are constant across the states of the world and the budget constraint becomes a side equation that determines the value of Λ_i independent from the policy. In either case, one needs to assume no intermediate goods $\alpha = 0$ — as their demand is not pinned down by the risk-sharing equation — and the Cobb-Douglas aggregator between goods produced in different countries $\theta = 1$, so that changes in the terms of trade have no effect on net exports. In addition, one needs to ensure that net exports are also independent from movements in the real exchange rate. This requires either no home bias in consumption $\gamma = 1$ or the Cole and Obstfeld (1991) parametrization $\frac{1}{\sigma} = \theta = 1$. In the former case the PPP holds and $Q_{it} = 1$ for any realization of shocks, while in the latter case the risk sharing exactly offsets changes in the purchasing power across countries. These two parametrizations have been extensively used in the previous normative literature and, as we discuss below, are behind several important results.

Lemma 1 (Balanced trade) *Assume no intermediate goods $\alpha = 0$, Cobb-Douglas aggregator $\theta = 1$, and either (i) no home bias $\gamma = 1$, or (ii) intertemporal elasticity equal to the intratemporal one $\frac{1}{\sigma} = \theta$. Then trade is always balanced $NX_{it} = 0$ and Λ_i is independent from monetary policy.*

2.3.2 Price-setting block

From now on, assume that the government uses production subsidy $\tau = \frac{\varepsilon-1}{\varepsilon}$ to eliminate the monopolistic distortion in the economy. Using the labor supply condition and the price index definitions, the (non-linear) domestic Phillips curve can be written as:

$$\Pi_{iit}(\Pi_{iit}+1) = -\kappa \left(1 - \frac{\Phi_{it} S_{it}}{Q_{it}} \frac{C_{it}^{\sigma(1-\alpha)}}{A_{it}}\right) \left(\frac{Q_{it}}{\Phi_{it} S_{it}}\right)^{1-\theta} (C_{it}+X_{it}) C_{it}^{-\sigma} + \beta \mathbb{E}_t \Pi_{iit+1}(\Pi_{iit+1}+1), \quad (22)$$

where $\kappa \equiv \frac{\varepsilon-1}{\varphi \tau_{Ri}}$ is the price-adjustment parameter, and $\Pi_{ii0} = 0$, which reflects the optimal choice of the initial price at $t = 0$. As usual, the dynamic price setting equation can be thought as an error-correction model: when the first bracket on the right-hand side is positive, the price exceeds the marginal costs and, other things equal, firms decrease their prices, i.e. $\Pi_{iit} < 0$. The speed of adjustment is faster when these deviations from the optimal price are associated with larger sales and a higher stochastic discount factor (the next few terms in the equation).

Similarly, the optimal price setting of exporters is characterized by

$$\Pi_{it}^*(\Pi_{it}^* + 1) = -\frac{\kappa}{\Psi_i} \left(1 - \frac{\Psi_i S_{it}}{Q_{it}} \frac{C_{it}^{\sigma(1-\alpha)}}{A_{it}}\right) Q_{it} S_{it}^{\theta-1} C_t^* C_{it}^{-\sigma} + \beta \mathbb{E}_t \Pi_{it+1}^*(\Pi_{it+1}^* + 1), \quad (23)$$

and $\Pi_{i0}^* = 0$. While the "weights" in this expression are determined by the global rather than local demand, the first bracket is very similar to the one for domestic prices in (22). Notice that the only difference in the first bracket between (22) and (23) is that Φ_i is replaced with Ψ_i in the second case. Since domestic and export prices are sticky in different currencies, the monetary policy can generate state-dependent deviations from the law of one price. However, under rational expectations, the only source of deviations from the law of one price in the long-run is the export tax Ψ_i, while monetary policy cannot make prices systematically different across markets.

In contrast to other countries, U.S. firms set the same price for local and foreign customers, so that the law of one price holds in the U.S. up to the export tax

$$\Phi_{it} = \Psi_i \quad \text{for} \quad i = 0, \tag{24}$$

and the monetary policy cannot affect the wedge between domestic and export prices. Therefore, the Phillips curve for U.S. producers is

$$\begin{aligned} \Pi_{iit}(\Pi_{iit}+1) = -\frac{\kappa}{\Psi_i}\left(1 - \frac{\Psi_i S_{it}}{Q_{it}}\frac{C_{it}^{\sigma(1-\alpha)}}{A_{it}}\right)\left[(1-\gamma)\,\Psi_i^{\theta} Q_{it}^{-\theta}\left(C_{it}+X_{it}\right)+\gamma C_t^*\right] Q_{it} S_{it}^{\theta-1} C_{it}^{-\sigma} \\ + \beta \mathbb{E}\Pi_{iit+1}\left(\Pi_{iit+1}+1\right), \end{aligned} \tag{25}$$

and $\Pi_{ii0} = 0$. As before, the first bracket corresponds to the deviations of the preset price from the optimal level, while the other terms are demand shifters and the stochastic discount factor.

2.3.3 Equilibrium

In what follows, we adopt the primal approach with the planner choosing the optimal allocation subject to the relevant equilibrium conditions summarized in the next lemma.

Lemma 2 (Implementability) *The allocation* $\{C_{it}, L_{it}, X_{it}, \Lambda_i, B_{it}^h\}$ *and prices* $\{Q_{it}, S_{it}, \Phi_{it}, \Pi_{iit}, \Pi_{it}^*\}$ *constitute a part of the equilibrium if and only if equations* (15)–(25) *hold.*

In words, the allocation is implementable if it satisfies the market clearing and the optimal risk-sharing conditions, respects countries' budget constraints and firms' price-setting behaviour. As is standard in the literature, we abstract from the issues how exactly the optimal policy is implemented. For example, the planner can use money supply as an instrument in the presence of the cash-in-advance constraint or money in the utility, or alternatively, rely on nominal interest rates in a dynamic version of the model (see Atkeson, Chari, and Kehoe 2010).

3 Static Model

We first solve for the optimal policy in a "static" version of the model defined by assumption

Assumption 1 *Prices are fully sticky in currency of invoicing* $\varphi \to \infty$*, policymakers lack commitment, and there is no export tax* $\Psi_i = 1,\ i \in [0, 1]$*.*

Combined together, these assumptions imply that the planner takes prices in their currency of invoicing and the transfers between countries as given and chooses the policy to maximize the welfare state by state. While interesting on its own, this special case provides an important benchmark for the analysis. First, it allows us to derive a non-linear closed-form solution without relying on second-order approximations or no-uncertainty limit. This, hopefully, helps to build the intuition for the results, which remain largely unchanged in a dynamic setting in the next section. Second, we argue below that under condition 1, our results remain robust to a number of alternative assumptions, including more general asset markets, functional forms, and a set of exogenous shocks. To avoid the confusion, we adopt the following definitions of "price stabilization" and "partial peg" for the rest of the section:

Definition *We say that the policy stabilizes domestic prices if it implements $\frac{MC_{it}}{P_{iit}} = 1$ in all states of the world. We say there is a partial peg of currency i to currency j if everything else equal, the monetary policy in country i depreciates local exchange rate in response to a depreciation of currency j.*

3.1 Non-U.S. policy

Given assumption 1, the planner's problem in a representative non-U.S. country is

$$\max_{C_{it},L_{it},X_{it},\Phi_{it},Q_{it}} \frac{C_{it}^{1-\sigma}}{1-\sigma} - L_{it}$$
$$\text{s.t.} \quad (15),\ (16),\ (17),\ (20).$$

Because agents sign international financial contracts before the policy is chosen, the planner takes as given the dollar transfers B_{it}^{h} between countries. Thus, from a perspective of a policymaker, the risk-sharing condition (20) is closer to the case of financial autarky rather than to the complete markets. The dollar denomination of the transfers also implies that local monetary policy cannot affect their real value, i.e. to devalue the foreign debt. Because of the small size of the economy, the planner also takes global demand summarized by C_t^* as given and independent of the policy. Of course, in equilibrium, agents form rational expectations about the monetary policy and take them into account when setting prices and trading Arrow-Debreu securities.

Proposition 1 (Non-U.S.) *Under A1, the optimal policy in a non-U.S. country (i) stabilizes domestic prices, (ii) implements partial peg to the dollar, (iii) gives rise to a global monetary cycle, and (iv) cannot implement the efficient allocation.*

The key property of the planer's problem is that the monetary policy cannot affect country's exports or imports. Indeed, because export prices are fully sticky in dollars, the depreciation of exchange rate does not affect the prices that consumers in other countries face and does not generate expenditure switching towards exported goods. The value of exports is therefore fixed in dollars. Given that import prices are also fully rigid in dollars, the country's budget constraint implies that the volume of imports is also exogenous to monetary policy. Thus, even though monetary policy can generate expenditure switching between domestic and imported goods, in equilibrium, such policy only changes production and consumption of local goods, but leaves the volume of imports unchanged.

Being unable to change the trade flows across the border, the planner focuses on local production and consumption. As in a closed economy, the optimal policy then prescribes stabilization of marginal costs of domestic producers. Intuitively, there is only one rigid local price and the optimal policy uses it as a numeraire to replicate part of the flexible-price allocation. Thus, as first shown by Casas, Díez, Gopinath, and Gourinchas (2017) and generalized here in a static setting, the optimal policy in a non-U.S. economy targets PPI-based inflation and is the same under DCP as under PCP (Gali and Monacelli 2005). This similarity, however, masks important differences between the two currency regimes. First, because of the suboptimal terms of trade and no expenditure switching under dollar pricing, the monetary policy is less efficient under DCP than under PCP and cannot implement the optimal allocation.

Second, though the optimal policy reaction to local productivity shocks is the same under two currency regimes, the dollar pricing generates highly asymmetric response to foreign shocks. Indeed, the stabilization of marginal costs requires that the monetary policy — effectively controlling nominal wages W_{it} — offsets any movements in prices of intermediates P_{it}:

$$\frac{MC_{it}}{P_{iit}} = \frac{P_{it}^{\alpha} W_{it}^{1-\alpha}}{A_{it} P_{iit}} = 1. \tag{26}$$

Given that under dollar invoicing, prices of foreign inputs depend solely on the monetary policy of the U.S., the optimal policy implements a partial peg to the dollar: an appreciation of the dollar exchange rate increases the prices of all internationally traded goods, raises the marginal costs of firms in other countries and induces non-U.S. economies to tighten their monetary policy to stabilize producer prices. This, in turn, gives rise to a global monetary cycle with the monetary policy positively correlated across all countries even when exogenous shocks are completely idiosyncratic.[12]

In sum, Proposition 1 suggests that the wide use of the dollar in the international trade contributes to the fact that the U.S. retains its dominant position in the global monetary system despite the end of the Bretton Woods System. In particular, the model is consistent with the fact that most countries in the world experience a "fear of floating" and use the dollar as an anchor currency in their monetary policy (see Calvo and Reinhart 2002, Ilzetzki, Reinhart, and Rogoff 2017). This mechanism is related and highly complementary to the international financial spillovers of U.S. monetary policy, which are the focus of a growing "Global Financial Cycle" literature (see e.g. Rey 2015, Aoki, Benigno, and Kiyotaki 2016, Giovanni, Kalemli-Ozcan, Ulu, and Baskaya 2017). There are no frictions in the international asset markets in our model, however, and the optimality of the peg is solely due to the dominance of the dollar as the invoicing currency in global trade. The mechanism is consistent with the fact that central banks often mention volatile import prices as a rationale for pegging exchange rate and is supported by the recent empirical literature that shows that both the spillover effects and the tightness of the peg increase in the share of DCP in country's trade (Boz, Gopinath, and Plagborg-Møller 2017, Zhang 2018).

[12]Proposition 1 extends results from Mukhin (2018) to a non-cooperative setting and contrasts with the inward-looking policy in models with no intermediate goods (Goldberg and Tille 2009, Casas, Díez, Gopinath, and Gourinchas 2017).

3.2 U.S. policy

Consider next the planner's problem in the U.S. Given that the law of one price holds for goods produced in the U.S. and Q_{0t} is independent of monetary policy, the problem can be written as

$$\max_{C_{it},L_{it},X_{it},C_t^*} \frac{C_{it}^{1-\sigma}}{1-\sigma} - L_{it}$$
$$\text{s.t.} \quad (16),\ (17),\ (20),\ (26).$$

As in other economies, the policymaker is subject to the market clearing conditions (16)-(17) and the budget constraint (20). The terms of trade are predetermined and cannot be changed by the monetary policy. Moreover, because both international transfers and prices of exports and imports are denominated in dollars, the monetary policy cannot use exchange rate to manipulate the value of foreign debt relative to the net exports.

The important difference from other countries, however, is that despite being a small economy, U.S. policy does affect the global equilibrium — in particular, the global demand C_t^* — because of dollar pricing. The planner's problem above assumes a strategic behaviour of the U.S. à la Stackelberg such that the planner takes into account the best response of other countries to its policy. In contrast, defining the Nash equilibrium is tricky because one needs to take a stand which global aggregates — prices or quantities — the planner takes as given. It turns out, however, that the optimal policy does not depend on the assumptions about the strategic interactions between policymakers, i.e. solution to the planner's problem remains unchanged if one drops constraint (26). To see this, note that global demand C_t^* is sufficient statistics about the global economy for U.S. monetary policy and the planner is free to choose any value of C_t^* consistent with the country's budget constraint independently from the policy of other countries. Therefore, the Nash equilibrium coincides with the Stackelberg equilibrium in this model.

Intuitively, suppose that the U.S. has an easy monetary policy. The effect on exports depends then on the policy in other economies. On the one hand, if it remains passive and does not respond to the shock, the dollar depreciates and generates expenditure switching towards the U.S. goods. On the other hand, if monetary authorities in other countries peg their exchange rates to the dollar and do not allow for the expenditure switching, they need to ease their policy as well. This stimulates aggregate demand and increases U.S. exports. Thus, in both cases U.S. exports go up — either because of the expenditure switching effect or due to a shift in global demand — and the U.S. planner can achieve an increase in C_t^* independently from the response of other economies. It follows that while U.S. policy affects other countries, in a static version of the model, the U.S. planner does not need to be concerned about the backfiring of these spillovers when choosing the optimal policy.

Proposition 2 (U.S.) *Under A1, the optimal policy in the U.S. (i) is independent from shocks and monetary policy in other countries, (ii) stabilizes domestic prices, and (iii) cannot implement the efficient allocation.*

The proposition implies that despite all differences, the optimal target of monetary policy in the U.S. is the same as in other economies, i.e. domestic prices. The intuition for this result can be seen from an envelope argument. Assume for a moment no intermediate goods in production $\alpha = 0$. A monetary shock that increases labor supply by one unit provides additional A_{it} units of output that are split between local and foreign consumers. Because domestic and export prices coincide, the value of the marginal output is $A_{it}P_{iit}$, which can be used to increase consumption by $A_{it}\frac{P_{iit}}{P_{it}}$ units. At the optimum, this marginal rate of transformation between consumption and labor should be equal to the marginal rate of substitution $\frac{\partial U_{it}/\partial L_{it}}{\partial U_{it}/\partial C_{it}} = \frac{W_{it}}{P_{it}}$, which implies the marginal cost stabilization $\frac{MC_{it}}{P_{iit}} = \frac{W_{it}}{A_{it}P_{iit}} = 1$. This logic can then be generalized to the case with intermediate goods.

Despite this similarity of optimal policy between economies, there are two important differences. First, notice that the intuition behind price stabilization is quite different in two cases. The trade flows of non-U.S. countries are exogenous to their monetary policy, which makes policymakers focus on stabilizing domestic margins. In contrast, U.S. policy does change exports and imports, but stabilization of domestic prices simultaneously achieves the optimal admissible trade balance adjustment. To see this, note that U.S. exports can be expressed from the budget constraint as $\gamma S_{it}^{\theta}C_t^* = \gamma S_{it}Q_{it}^{-\theta}(C_{it} + X_{it}) - \frac{S_{it}B_{it}^h}{P_t^*}$. Substitute this expression into the market clearing condition and use the definition of price index (15):

$$Y_{it} = (1-\gamma)S_{it}^{\theta}Q_{it}^{-\theta}(C_{it} + X_{it}) + \gamma S_{it}Q_{it}^{-\theta}(C_{it} + X_{it}) - \frac{S_{it}B_{it}^h}{P_t^*} = \frac{P_{it}}{P_{iit}}(C_{it} + X_{it}) - \frac{S_{it}B_{it}^h}{P_t^*}.$$

Under assumption 1, the last term is exogenous to the policy and the trade-off faced by the planner is similar to the one in a closed economy. This result clearly relies on the fact that the law of one price holds and domestic price stabilization ensures that both local and export prices are aligned with the costs of production. At the same time, the policy cannot achieve the efficient allocation because of the suboptimal terms of trade and no expenditure switching between imported and domestic goods.

The second key difference from non-U.S. economies is that domestic price stabilization in the U.S. does not imply partial peg to other currencies. Because prices of imported goods are fully sticky in dollars, the shocks and monetary policy in other countries have no effect on marginal costs of U.S. producers. As a result, U.S. monetary policy is fully independent and only responds to local productivity shocks. Therefore, DCP generates highly asymmetric international spillovers: while U.S. monetary shocks affect the import prices in all other economies making them optimally "lean against the wind" and partially peg their exchange rates to the dollar, the U.S. has a free floating exchange rate and an independent monetary policy.[13]

[13]It is worth mentioning that in contrast to the optimal policy, the equilibrium allocation in the U.S. does depend on foreign shocks through endogenous transfers B_{it}^h and preset prices.

3.3 Discussion

Before proceeding to the next results, it is worth discussing which ingredients of the model are important for Propositions 1 and 2 and which are not. In particular, while we impose a lot of structure in Section 2 that will be useful in the dynamic version of the model, the static results hold under much weaker assumptions (see Appendix A.2 for the proofs).

Input-output linkages Clearly, DCP is the main source of asymmetric spillovers in the model. At the same time, the peg to the dollar requires in addition that imported goods are used as inputs by local firms, which then set prices sticky in local currency. It is this two-layer price stickiness that makes partial peg optimal in the model. On the other hand, the roundabout production structure is just a simplifying assumption: in practice, most imported goods are used either as inputs or go through the wholesale and retail sectors, and the model can easily accommodate that by assuming that home-bias is stronger for final goods used in consumption than for intermediate goods used in production. While this is consistent with the low pass-through of exchange rate shocks into retail prices (Auer, Burstein, and Lein 2018), the tightness of the peg would only increase with more imported goods used in production than consumption. Note also that neither price stabilization, nor peg depends on the elasticity of substitution between goods θ, which is allowed to be below or above one.

Functional forms Related to the previous point, the parametric assumptions about preferences and production function can be significantly relaxed. To be precise, the propositions hold for arbitrary constant returns to scale technology and any utility function that is linear in labor. The last restriction only required to ensure peg to the dollar, while the marginal cost stabilization is optimal for arbitrary well-behaved preferences. In addition, given that prices are fully sticky, one does not need to take a stand on the type of nominal rigidities: models with state-dependent and time-dependent price adjustment coincide in this limit.

International asset markets Notice also that the price targeting is independent from the particular values of transfers B^h_{it} between countries. Therefore, one can allow for arbitrary values of transfers and completeness of asset markets as long as the planner takes these B^h_{it} as given. This in particular requires that transfers are denominated in foreign currency — so that the planner cannot manipulate their value — and are independent from outcomes that are endogenous to monetary policy, e.g. this excludes domestic equity with profits depending on monetary shocks. To prove the global monetary cycle, on the other hand, one needs to determine how monetary policy affects equilibrium exchange rates, which requires making assumptions about the international asset markets. While we use complete markets as a benchmark, the global monetary cycle requires only that nominal exchange rate depreciates in response to a positive monetary shock — the property that holds in a much larger class of models.

Shocks Finally, while following the previous literature, we focus mainly on productivity shocks, our results hold for a much larger set of shocks. In particular, one can allow for any non-distortionary shocks, i.e. the shocks that keep equilibrium allocation at the first-best under flexible prices, and markup shocks as they do not affect equilibrium under fully sticky prices. More importantly, one can also extend the results to financial shocks and shocks to capital flows — isomorphic to shocks to B^h_{it} — which play important role in the international business cycles (see e.g. Giovanni, Kalemli-Ozcan, Ulu, and Baskaya 2017) and exchange rate dynamics (see e.g. Itskhoki and Mukhin 2019).

3.4 Capital controls

In light of the negative spillovers from the U.S. on other economies discussed above, it is natural to ask whether additional fiscal instruments such as capital controls can be used to improve the allocation and increase country's welfare. The modern conventional wisdom among both policymakers and scholars is that "[the use of capital controls by emerging economies] allows advanced economies to use monetary policy to increase domestic demand, while shielding emerging economies of the undesirable exchange rate effects" (Blanchard 2017). In other words, the U.S. can focus on its domestic objectives when setting the monetary policy, while other countries can insulate their economies from the arising spillovers by using the capital controls. Although this argument is usually made for the spillovers arising from the international financial markets, it might be equally important in a context of DCP in global trade.

To answer this question, we partially relax assumption 1 and allow for semi-commitment: the planner moves after the prices are preset, but before the trade in international asset markets takes place. This implies that the policymakers take prices as given, but internalize the effect of their decisions on the international risk sharing. In addition, we allow the planner in a non-U.S. economy to choose optimally both the monetary policy and a state-contingent tax on capital flows between countries. The latter instrument corresponds to capital controls and effectively allows the planner to choose any risk sharing subject to the ex-ante budget constraint:

$$\max_{\{C_{it}, L_{it}, X_{it}, \Phi_{it}, Q_{it}, B^h_{it}\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\frac{C_{it}^{1-\sigma}}{1-\sigma} - L_{it} \right]$$
$$\text{s.t.} \quad (15),\ (16),\ (17),\ (20),\ (21).$$

Proposition 3 (Capital controls) *Under A1, capital controls do not insulate economies from U.S. spillovers and are not used by the planner, i.e. the optimal allocation is the same with and without capital controls.*

At a first glance, this may look as a surprising result: after all, the general principle in economics is that the planner usually finds it optimal to mitigate distortions in one market by allowing for distorting other margins in the economy. In our model, this would correspond to distorting intertemporal asset markets to improve the allocation in the static goods markets. Proposition 3 is especially surprising given the result from Farhi and Werning (2016) that the laissez-faire risk sharing is *generically* inefficient when monetary policy cannot implement the first-best allocation. Yet, it turns out that capital controls are completely redundant and are not used by the planner in our setting.[14] The international spillovers arising from DCP are therefore very different from the ones arising from the financial frictions and cannot be eliminated with macroprudential policy.

To see the intuition behind this result, note that the reason the risk sharing might be inefficient under sticky prices is the aggregate demand externality: individual agents do not internalize the fact that an international wealth transfer increases aggregate demand, stimulates production and decreases the labor wedge. The optimal monetary policy, however, fully eliminates the labor wedge in our model by stabilizing marginal costs, which removes the aggregate demand externality and closes the gap between private and social value of insurance. This contrasts with the case when monetary policy is constrained by the zero lower bound or fixed nominal exchange rates and cannot close this wedge (*cf.* e.g. Farhi and Werning 2017). Thus, even though the monetary policy cannot implement the first-best allocation under dollar pricing, it is still powerful enough to eliminate the aggregate demand externality and implement the optimal risk sharing between countries. Importantly, this result does not rely on the assumption that asset markets are complete and remains true for arbitrary structure of the international financial markets as long as the pay-off of assets is independent from the monetary policy.

3.5 Gains from cooperation

Given the spillover effects of U.S. monetary policy, it is natural to ask whether there are welfare gains from international cooperation? The important benchmark in the literature is the case of PCP when international cooperation precludes countries from "beggar-thy-neighbour" policies that exploit the terms-of-trade externality and generates Pareto improvement for all countries (see Corsetti and Pesenti 2001). The situation is quite different under dollar pricing: the terms of trade are predetermined in this case and are taken as given by the discretionary monetary policy. Instead, the externalities arise from the asymmetric spillovers of the U.S. policy on other economies. The U.S. has an infinitely small weight in the objective function of the global planner, but its monetary policy affects all international prices. It follows that the optimal cooperative solution is to use the U.S. policy to bring export prices of all countries closer to the optimal level rather than to target the local U.S. producers. In other words, the U.S. monetary instruments are only used in response to global shocks instead of idiosyncratic ones.[15] At the same time, the optimal policy in other countries does not change relative to the non-cooperative case and targets local prices.

[14] While related to the "approximate efficiency" of the risk sharing in Fanelli (2017), the laissez-faire portfolio choice is *exactly* optimal in our setting.

[15] This is, of course, an extreme result that is due to the assumption that the U.S. is a small economy. Under a more realistic assumption that the U.S. account for a significant fraction of global GDP, the U.S. optimal policy under cooperation targets a weighted average of local and global shocks.

Proposition 4 (Cooperation) *Under A1, the optimal cooperative policy stabilizes domestic prices in non-U.S. economies and uses U.S. monetary policy to stabilize global dollar export prices, i.e.* $\int \frac{MC_{it}}{\mathcal{E}_{it} P^*_{it}} \mathrm{d}i = 1$.

It follows that there is a conflict of interests between countries. The non-U.S. countries unambiguously win from the cooperation: while their policy does not change, the negative spillovers from the U.S. policy are lower under the cooperation. In contrast, there are no gains from cooperation for the U.S.: its domestic objectives are sacrificed for the world economy and there is nothing that the country gets in return as other economies do not change their policy. This contrasts with the quid pro quo under PCP when each country restrains the terms-of-trade manipulation to be compensated by lower negative spillovers from other countries. In other words, if a country can choose between joining others in international cooperation or play non-cooperative equilibrium with them, it would voluntary choose the former option under PCP. On the other hand, under DCP, only non-U.S. countries would prefer the cooperative option, while the U.S. has no interest in participating in such agreement.

Lastly, note that countries' interests are perfectly aligned in response to global shocks: the U.S. does not have to choose between local and international shocks to respond to, and the stabilization of marginal costs in all countries achieves the optimal allocation in this case. This prediction is consistent with the high level of cooperation between central bankers around the world during the global financial crisis of 2008–2009. The model also explains why it is much harder to sustain the global cooperation if the crisis ends in the U.S., but other countries have not fully recovered yet.

4 Dynamic Model

While the static model provides an important benchmark, one might be concerned about its assumptions. In particular, the fact that prices are fully sticky implies that monetary policy cannot change the terms of trade and partially explains why optimal policy targets domestic prices. In addition, the discretionary policy does not internalize its effect on firms' price setting behavior abstracting from another potentially important motive of monetary policy. This section relaxes both assumptions and extends the analysis to a dynamic setup with sluggish price adjustment and the optimal monetary policy under commitment. To this end, we replace assumption 1 with

Assumption 2 *Prices are partially sticky* $\varphi < \infty$*, policymakers have commitment technology, and there are optimal export tax* $\Psi_i = \frac{\theta}{\theta-1}$ *and price-adjustment subsidy* $\tau_{Ri} = \frac{\varepsilon-1}{\theta}$.

As is standard in the normative sticky-price literature, we rely on time-invariant taxes and subsidies to ensure that the steady state of the economy is efficient. To understand why, note that there are potentially two sources of distortions in the economy — the ones that would arise even under flexible prices and the ones that are purely due to nominal rigidities. The former imply that under sticky prices, the monetary policy would try to improve upon the flexible-price allocation and mitigate the flexible-price distortions. The classical example of such distortions are monopolistic markups in a closed economy, which give rise to an inflationary bias with monetary policy trying to boost the output towards efficient level. One fiscal instrument, e.g. labor subsidy, is sufficient to eliminate time-invariant markups and ensure that the flexible-price equilibrium is efficient. The monetary policy then targets exclusively distortions that are due to nominal rigidities and aims to replicate the flexible-price equilibrium.

In contrast, there are two sources of distortions under flexible prices in an open economy — monopolistic markups and the terms-of-trade externality. While labor subsidy can still be calibrated to ensure the efficient steady state by balancing these two margins (see e.g. Farhi and Werning 2012), it is not generically sufficient to guarantee the efficiency of the flexible-price equilibrium away from the steady state. As a result, the monetary policy deviates from replicating the flexible-price equilibrium in this case (Benigno and Benigno 2003). We therefore deviate from the previous literature and introduce an optimal export tax $\Psi_i = \frac{\theta}{\theta-1}$, which together with labor subsidy allows to eliminate both distortions and ensure the efficiency of the flexible-price equilibrium. Our rationale for this assumption is threefold. First and most important, it allows us to disentangle the new motives of monetary policy that arise due to DCP from the standard terms-of-trade effects (see e.g. De Paoli 2009). Second, while export and import tariffs are rarely used in practice because of the WTO rules, there is large empirical evidence that exporters do charge markups over marginal costs in foreign markets (see e.g. De Loecker, Goldberg, Khandelwal, and Pavcnik 2016). Finally, our assumption is arguably not much less realistic than the conventional one with a labor subsidy balancing two distortions in the steady state.[16]

Lastly, while price-adjustment subsidy is irrelevant under flexible prices, we introduce it for similar reasons. Intuitively, while the relevant demand elasticity for an individual firm is the one between products ε, the relevant elasticity from a social perspective is the one between goods from different countries θ. This introduces a wedge between private and social costs of price adjustment, which are eliminated with subsidy $\tau_{Ri} = \frac{\varepsilon-1}{\theta}$.

4.1 Non-U.S. policy

Under assumption 2, the planner's problem in a non-U.S. economy is

$$\max_{\{C_{it},L_{it},X_{it},\Phi_{it},Q_{it},B^h_{it},S_{it},\Pi_{iit},\Pi^*_{it}\}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\frac{C_{it}^{1-\sigma}}{1-\sigma} - L_{it} - \frac{\varphi}{2}(1-\gamma)\Pi_{iit}^2 - \frac{\varphi}{2}\gamma\Pi_{it}^{*2}\right]$$
$$\text{s.t.} \quad (15) - (23).$$

In contrast to the static case, the planner internalizes the effect of its policy on price-setting and risk-sharing decisions of agents in the economy. In addition to targeting domestic production and consumption, in the dynamic setup, the planner should also take into account the effect of monetary policy on

[16]Note, however, that this assumption comes at a cost of an additional restriction $\theta > 1$.

terms of trade and inflation. Importantly, while similar to the setup in CDGG, our model features terms of trade that are endogenous to the monetary policy. Indeed, note that the marginal costs of exporters in dollar terms can be expressed using the risk-sharing condition as

$$\frac{MC_{it}}{\mathcal{E}_{it}} = \frac{P_{it}^{\alpha} W_{it}^{1-\alpha}}{A_{it}\mathcal{E}_{it}} = C_{it}^{-\alpha\sigma} \frac{\bar{C}_t^{\sigma} P_t^*}{\Lambda_i A_{it}}.$$

Without intermediate goods $\alpha = 0$, the C_{it}-term goes away and the only remaining variable endogenous to monetary policy is a constant Λ_i. Intuitively, monetary shocks increase nominal wages and depreciate the nominal exchange rate by the same amount in this case, leaving the dollar value of marginal costs unchanged. As a result, the planner takes the dynamics of the terms of trade as given and focuses on domestic margins. This mechanism is similar to the static case and has been described by CDGG.[17] This is, however, no longer true in the presence of intermediate goods $\alpha > 0$, which implies that monetary policy can adjust the country's terms of trade. Intuitively, because exporters use domestic intermediates with prices sticky in local currency, a depreciation of exchange rate does change the dollar value of marginal costs and firms' export prices. Surprisingly, given this sharp differences between two models, we find that the optimal monetary policy still stabilizes domestic prices.

Proposition 5 (Non-U.S.) *Under A2, the optimal non-cooperative policy in a non-U.S. country stabilizes domestic prices. Therefore, results from Propositions 1 and 3 remain true as well.*

The proposition therefore generalizes the result that non-U.S. economies target domestic inflation from the static model above and CDGG. At the same time, the intuition is quite different and, to the best of our knowledge, is new to the literature. In contrast to the PCP case when exchange rate depreciation automatically decreases the prices of exported goods in the currency of destination and generates expenditure switching towards them, under DCP, monetary policy cannot directly change export prices in currency of destination. The only way, in which it does change prices of exported goods, is through changing the dollar value of production costs: a depreciation of national currency decreases the prices of local intermediates in dollar terms, which makes firms adjust their export prices downwards. This change in prices is, however, associated with additional price-adjustment costs. Given the optimal value of τ_{Ri}, the firms' adjustment decisions are socially optimal and the planner finds it suboptimal to deviate from marginal cost stabilization to generate additional terms-of-trade depreciation.

The corollary of domestic price stabilization is that all our results from Propositions 1 and 3 remain true in the dynamic setup. In particular, non-U.S. economies partially peg their exchange rates to the dollar, giving rise to the global monetary cycle. Moreover, the capital controls cannot insulate countries from U.S. monetary spillovers and are not used under the optimal policy. As before, the reason is that despite suboptimal allocation, the optimal monetary policy eliminates the aggregate demand externality by equalizing the private and social value of insurance.

[17] While the analysis of CDGG relies on the additional assumption that $\sigma = \theta = 1$, one can show that it also goes through for arbitrary values of these elasticities under the optimal export tax.

4.2 U.S. policy

Under commitment, the planner's problem in the U.S. is quite different than in the static case:

$$\max_{\{C_{it},L_{it},X_{it},C_t^*,\bar{C}_t,Q_{it},B_{it}^h,S_{it},\Pi_{iit},\{\Pi_{jt}^*,S_{jt}\}\}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\frac{C_{it}^{1-\sigma}}{1-\sigma}-L_{it}-\frac{\varphi}{2}\Pi_{iit}^2\right]$$
$$\text{s.t.}\quad (15)-(26).$$

As a Stackelberg leader, the planner internalizes the effect of its policy on global equilibrium and in particular, the price-setting behaviour of exporters around the world, the ex-ante risk sharing of households, and the optimal response of other countries to its policy. As a result, the optimal policy does not any longer stabilize domestic prices or any other simple target — instead, the policy simultaneously trades off several objectives. To make progress in understanding these trade-offs, we make additional simplifying assumptions and follow the standard approach in the literature of deriving a quadratic loss function (see e.g. Engel 2011).

Assumption 3 *There are no intermediate goods in production $\alpha = 0$ and the intratemporal elasticity is equal to the intertemporal elasticity $\theta = \frac{1}{\sigma}$.*

These parameter restrictions correspond to the Faia and Monacelli (2008) specification, which is frequently used in the sticky-price open economy literature. While this assumption comes not without costs, it is still much weaker than the standard Cole-Obstfeld specification and in particular does not imply balanced trade in every state of the world and irrelevance of international asset markets (see Lemma 1). Moreover, we also experimented with several alternative specifications of the model, which allow for intermediate goods and obtained qualitatively similar results.

We follow the standard steps to derive the loss function by taking the second-order approximation to the demand block and sequentially substituting out the linear terms. In contrast to the usual case, however, this is not enough to get rid of all endogenous linear terms. The reason is that under DCP, U.S. monetary policy can implement allocations that the country cannot unilaterally achieve when prices are flexible. As we explain in detail below, the U.S. can increase its welfare by depreciating the exchange rate. However, under commitment, the planner internalizes the fact that exporters from all other countries form rational expectations about U.S. monetary policy and set their prices accordingly. Therefore, to derive a quadratic objective function, we augment demand block with the second-order approximation to the ex-ante price-setting of exporters (23).[18]

[18] The ex-ante pricing setting under Rotemberg pricing can be obtained by iterating forward equation (23) and puts a restriction on the expected discounted sum of output gaps of exporters:

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(1-\frac{\Psi_i S_{it}}{Q_{it}}\frac{C_{it}^{\sigma(1-\alpha)}}{A_{it}}\right)Q_{it}S_{it}^{\theta-1}C_t^*C_{it}^{-\sigma}=0.$$

Intuitively, it says that U.S. monetary policy cannot systematically generate deviations of prices from firms' marginal costs.

Proposition 6 (U.S.) *Under A2 and A3, U.S. loss function is*

$$\mathcal{L}^{US} = \mathbb{E}\sum_{t=1}^{\infty}\beta^t\left[\frac{L}{2\theta}\tilde{y}_{it}^2 + \frac{\varphi}{2}\pi_{iit}^2 + \frac{\omega}{2}\frac{\theta^2 L}{\theta-1}\int\tilde{s}_{jt}^2\mathrm{d}j\right] + t.i.p. + \mathcal{O}(\epsilon^3), \tag{27}$$

where $\tilde{y}_{it} \equiv y_{it} - \theta a_{it}$ and $\tilde{s}_{it} \equiv s_{it} + \sigma\bar{c}_t - a_{it}$ are respectively the output gap and the terms-of-trade gap, L and ω are the steady-state labor supply and openness of the economy corrected for export tax. The optimal non-cooperative policy in the U.S. (i) is independent from policy in other countries, (ii) responds to both local and global shocks.

The first two terms in the loss function (27) are completely standard: as in a closed economy, the welfare is decreasing in output gap and inflation. In particular, similarly to the case of a small open economy under PCP, the terms-of-trade gap for the U.S. is proportional to the output gap and does not appear as a separate term in the objective function (*cf.* Gali and Monacelli 2005). In contrast to the PCP case, however, the welfare of the U.S. does depend on the terms-of-trade gaps of other countries, which are endogenous to U.S. monetary policy. Intuitively, a depreciation of the dollar decreases prices and markups of exporters in other countries leading to a global economic boom. This in turn increases demand for U.S. goods. Given an optimal export tax, the prices of U.S. goods are higher than marginal costs, and an increase in global demand allows the country to get additional "profits" relative to the flexible price allocation and increase its welfare. This explains a linear term in the U.S. objective function after using all demand-block constraints and why one needs to use the second-order approximation to the export prices of other countries.

The latter constraint implies, on the other hand, that firms' markups depend on their expectations about the U.S. monetary policy. In particular, large deviations of export prices from marginal costs make firms set higher markups and lead to suboptimally low global demand.[19] This distortion is lowest when U.S. monetary policy stabilizes the average export prices of other countries, though one monetary instrument is clearly not enough to close the terms-of-trade gap in all economies.

It follows that U.S. planner faces a trade-off between stabilizing domestic margins — output gap and inflation — and targeting global prices to attain the optimal global demand. Notice that the planner cares about export prices of other countries not because of the benevolence to other countries, but because they have direct effect on U.S. exports and imports and hence, affect U.S. welfare. Thus, in contrast to the static model, the U.S. optimal policy is no longer fully inward-looking and responds to both domestic and global shocks. The planner under commitment therefore internalizes the backfire of its policy through import prices and deviates from the optimal target in a static setup in the direction of the optimal cooperative policy.

At the same time, just as in the static case, U.S. policy remains independent from monetary policy of other countries. This is not a general result as it relies on the Faia-Monacelli parametrization under

[19]This mechanism was first described by Devereux and Engel (1998) in a context of PCP and LCP.

assumption 3. However, we still find it instructive that this property may hold in a dynamic version of the model. The intuition behind this result comes from the fact that demand for imported goods is independent from monetary policy in the Faia-Monacelli case: the expenditure-switching effect of exchange rate depreciation in (16) cancels out with the higher aggregate consumption due to a wealth transfer associated with the real exchange rate depreciation from (19).

4.3 Gains from DCP

Are there gains for the U.S. from the dominant status of its currency in international trade? To address this question, we proceed under assumption 3 and derive the loss function for non-U.S. countries:

$$\mathcal{L}^{non-US} = \mathbb{E}\sum_{t=1}^{\infty}\beta^t\left[\frac{L}{2\theta}\tilde{y}_{it}^2 + \frac{L\theta}{2}\omega(1-\omega)\tilde{\phi}_{it}^2 + \frac{\varphi}{2}(1-\gamma)\pi_{iit}^2 + \frac{\varphi}{2}\gamma\pi_{it}^{*2} + \frac{\omega}{2}\frac{\theta^2 L}{\theta-1}\int\tilde{s}_{jt}^2\mathrm{d}j\right] + t.i.p. + \mathcal{O}(\epsilon^3), \tag{28}$$

where $\tilde{\phi}_{it} = \phi_{it}$ is the deviation from the law of one price. As for the U.S., the welfare of other countries depends on output gap, inflation, and global terms-of-trade gap. At the same time, because domestic and export prices are sticky in different currencies, domestic inflation does not coincide with the inflation for exported goods and the two enter separately in the objective function. More importantly, there are additional losses associated with the violations of the law of one price: even if the planner closes the output gap, the distribution of output between domestic consumers and exports is not efficient when $\tilde{\phi}_{it} \neq 0$. This, in turn, results in suboptimal consumption of local and imported goods and decreases the welfare of households. Notice that given that the law of one price holds for the U.S. $\tilde{\phi}_{it} = 0$ and $\pi_{iit} = \pi_{it}^*$ and the loss function (28) simplifies to the U.S. welfare (27).

Proposition 7 (Welfare) *Under A3, U.S. welfare is higher than in other economies under domestic price stabilization, but can be lower under the optimal policy with commitment.*

Consider first the case of domestic price stabilization. Given that global demand (and hence, global terms-of-trade gap) has the same effect on all countries, it does not affect the difference between U.S. and non-U.S. welfare. At the same time, price stabilization closes both the output gap and inflation in the U.S. economy. In contrast, the same policy in other countries only ensures zero domestic inflation, while export inflation, output gap and law-of-one-price distortions cannot be fully eliminated. It follows immediately that U.S. welfare is higher than the welfare of other countries under such policy. Intuitively, the fact that both domestic and export prices are set in dollars makes U.S. monetary policy more efficient by allowing it simultaneously to stabilize domestic and external margins. On the other hand, the monetary policy in other countries is constrained by the fact that export prices are in dollars.[20]

[20]There are two reasons why our results contrast with the conclusion of Devereux, Shi, and Xu (2007) that the U.S. has always lower welfare than the rest of the world under DCP: (i) in our setting, the planner is subject to the country's budget

While U.S. planner can achieve a higher welfare than in other countries, her objective instead is to maximize the absolute rather than relative welfare of the economy. From Proposition 6, this requires that U.S. monetary policy deviates from targeting domestic price level and partially stabilizes the global terms-of-trade gap. This has two implications for the welfare. On the one hand, the positive effect of smaller global gap is symmetric across countries and benefits non-U.S. economies as much as the U.S. without changing their relative welfare. On the other hand, this policy comes at a cost of larger output gap and inflation in the U.S. economy, which decrease its relative welfare. In sum, the gains from DCP for the U.S. go down under the optimal policy and can even become negative. Thus, despite higher efficiency of U.S. monetary policy, it is not necessarily true that it is more beneficial to be an issuer of the dominant currency rather than to stay under the umbrella of another country that partially stabilizes the global economy.

4.4 Currency union

Our model has also important implications for the optimal currency area (Mundell 1961). While there is much debate about the costs of having a common currency, the benefits of currency unions are less well understood. The new insight that emerges from Proposition 7 is that forming a currency union such as the Eurozone can improve the welfare of its members if it helps to promote the common currency in the international trade.[21] Indeed, a similar model with endogenous currency choice from Mukhin (2018) implies that exporters within the union are more likely to use the local currency instead of the dollar as the currencies of producer and buyer coincide in this case.[22] Moreover, because of strategic complementarities in currency choice, the trade flows between the union and other countries are more likely to switch to the common currency as well. Thus, if the euro manages to replace the dollar at the global stage, the Eurozone monetary policy becomes more efficient in generating the optimal expenditure switching towards the exported goods. These new gains from trading with countries outside of the union can potentially outweigh the costs of less efficient stabilization within the union.

A more empirically relevant question, however, is whether the union gains from a common currency when it is used for invoicing of the Eurozone's imports and exports, but does not replace the dollar as a vehicle currency in trade between third countries.[23] Interestingly, the next proposition shows that the Eurozone can achieve the same level of welfare in this case as the U.S.

Proposition 8 (Eurozone) *Assume that the Eurozone is a small economy with imports and exports invoiced in euros and all other international trade is in dollars. Then under A2 and A3, the planner's problem for the Eurozone is isomorphic to the U.S. problem and hence, achieves the same welfare.*

constraint, (ii) we assume that all countries are small and abstract from strategic interactions between countries that arise in their model with two countries.

[21] See Chari, Dovis, and Kehoe (2013) for other benefits of a currency union arising from a higher level of commitment.

[22] Cavallo, Neiman, and Rigobon (2014) provide supporting evidence that the deviations from the law of one price are smaller for countries with a common currency.

[23] Gopinath (2016) shows that the share of the Euro in global trade is close to the trade share of the Eurozone.

Intuitively, the Eurozone is similar to the U.S. with its imports, domestic prices and exports all set in its own currency. As a result, the efficiency of monetary policy is also similar in two countries. The main difference is, of course, that depreciation of the euro has no global effects: since a zero measure of imports and exports of other countries are invoiced in euros, the international spillovers of the Eurozone monetary policy are trivial and there is no peg to the Euro. However, the result from Proposition 6 that monetary policy of other economies is irrelevant for the U.S. under the Faia-Monacelli parametrization holds for the Eurozone as well. Therefore, the optimal policy in the Eurozone coincides with the one in the U.S. and trades off output gap, inflation and import prices. This, in turn, implies that the welfare is exactly the same as in the U.S. The important corollary of this result is that the use of the dollar as a vehicle currency does not generate any additional gains for the U.S. relative to the case when the dollar is used in bilateral trade flows between the U.S. and the rest of the world.

5 Conclusion

This paper characterizes the optimal non-cooperative monetary policy of the U.S. and other economies in a world with international prices sticky in dollars. We show that the dominance of the dollar results in asymmetric spillovers and policies across countries. As under PCP, domestic prices stabilization remains the optimal target for non-U.S. economies under a wide range of assumptions, even though it is much less efficient in the world with DCP and does not achieve the efficient external adjustment. Moreover, given that a large fraction of imported goods are used as intermediates in production or go through the wholesalers and retailers, this policy results in a partial peg to the dollar. We show that capital controls do not insulate countries from U.S. spillovers and are not used under the optimal policy.

On the other hand, U.S. monetary policy is more efficient and has global implications. Because both domestic and export prices are sticky in dollars, the policymakers can simultaneously stabilize domestic margins and adjust the trade balance. The optimal policy under commitment, however, also targets the global terms-of-trade gap, which through import prices and global demand affects the U.S. economy. Therefore, it is optimal for the U.S. to deviate from a strict price targeting and partially stabilize the global economy. While this policy increases the welfare of other economies, it remains far from the optimal cooperative policy. In contrast to other countries, the U.S. has little incentives to implement cooperative policy, given that it benefits others, but generates no additional benefits for the U.S.

We show that the U.S. can always achieve a higher welfare than other countries. At the same time, the issuer of the dominant currency has more objectives to target and in particular, finds it in his interests to partially sacrifice price stability in favor of lower volatility of the global economy. Because this policy benefits other economies as well, the welfare of the U.S. can be lower in equilibirum than the welfare of other countries. We also point to a new source of gains from forming a currency union: if larger monetary union helps to promote its currency in trade with other economies, then it can potentially increase the welfare of its members.

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A Appendix

A.1 Equilibrium Conditions

A.1.1 Demand block

Households choose C_{it} and N_{it} in each state h to maximize (1)

$$U_i = \mathbb{E}\sum_{t=0}^{\infty} \beta^t \left[\frac{C_{it}^{1-\sigma}}{1-\sigma} - N_{it}\right]$$

subject to the ex-ante budget constraint (4)

$$\sum_{t,h} \frac{Z_t^h}{\mathcal{E}_{it}} \left[P_{it}C_{it} - W_{it}N_{it} - \Pi_{it} - T_{it}\right] = 0,$$

where we have plugged in the ex-post budget constraint (5) . The optimality conditions imply the labor supply condition

$$\frac{W_{it}}{P_{it}} = C_{it}^{\sigma}, \tag{A1}$$

and the equilibrium price of an Arrow-Debreu security Z_t^h,

$$Z_t^h = \mathcal{E}_{it}\lambda_i^{-1} Pr_{ht}\Theta_{it}, \tag{A2}$$

which has to be equal to the nominal stochastic discount factor $\Theta_{it} \equiv \beta^t C_{it}^{-\sigma}/P_{it}$ adjusted for the nominal exchange rate $\mathcal{E}_{it}$, for the state- and time-invariant Lagrange multiplier λ_i, and for the probability of state h, Pr_{ht}.

Since prices of the Arrow-Debreu securities have to be the same in all countries, households' optimality conditions imply the perfect risk-sharing,

$$\frac{\mathcal{E}_{it}\lambda_i^{-1}}{P_{it}C_{it}^{\sigma}} = \frac{\mathcal{E}_{jt}\lambda_j^{-1}}{P_{jt}C_{jt}^{\sigma}}.$$

Multiply both sides by the import price index P_t^*, use the definition of the real exchange rate (14),

$$\lambda_j^{-\frac{1}{\sigma}} C_{it} = \left(\frac{Q_{it}}{\lambda_i}\right)^{\frac{1}{\sigma}} Q_{jt}^{-\frac{1}{\sigma}} C_{jt},$$

integrate it over all countries j, and define a constant $\Lambda_i \equiv \lambda_i \left(\int \lambda_j^{-\frac{1}{\sigma}} \mathrm{d}j\right)^{\sigma}$ to arrive at the risk-sharing (19)

$$C_{it} = \left(\frac{Q_{it}}{\Lambda_i}\right)^{\frac{1}{\sigma}} \int Q_{jt}^{-\frac{1}{\sigma}} C_{jt}\mathrm{d}j.$$

To derive (15), start with the price index (6)

$$P_{it}^{1-\theta} = (1-\gamma)P_{iit}^{1-\theta} + \gamma \int P_{jit}^{1-\theta} \mathrm{d}j,$$

divide everything by $(\mathcal{E}_{it}P_t^*)^{1-\theta}$, and rewrite it as

$$Q_{it}^{\theta-1} = (1-\gamma)\left(\frac{P_{iit}}{\mathcal{E}_{it}P_{it}^*}\frac{P_{it}^*}{P_t^*}\right)^{1-\theta} + \gamma\frac{\int P_{jt}^{*1-\theta}\mathrm{d}j}{P_t^{*1-\theta}},$$

where we have used the definition of the real exchange rate Q_{it} from (14) and converted export prices to dollars, $P_{jit}/\mathcal{E}_{it} = P_{jt}^*$. Then, use the definition of the import price index, $P_t^* = \left(\int P_{jt}^{*1-\theta}\mathrm{d}j\right)^{\frac{1}{1-\theta}}$, and the definitions of the terms of trade S_{it} and the deviation of prices Φ_{it} from (14) to arrive at (15),

$$Q_{it}^{\theta-1} = \gamma + (1-\gamma)\left(\Phi_{it}S_{it}\right)^{\theta-1}.$$

Next, note that the definitions of prices (14) imply that

$$\frac{P_{iit}}{P_{it}} = \frac{Q_{it}}{\Phi_{it}S_{it}}. \tag{A3}$$

Use this condition to rewrite the market clearing condition (7) as

$$\begin{aligned}
Y_{it} &= (1-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}(C_{it}+X_{it}) + \gamma\int\left(\frac{P_{ijt}}{P_{jt}}\right)^{-\theta}(C_{jt}+X_{jt})\,\mathrm{d}j \\
&= (1-\gamma)\left(\Phi_{it}S_{it}\right)^{\theta}Q_{it}^{-\theta}(C_{it}+X_{it}) + \gamma\int\left(\frac{P_{it}^*}{P_t^*}\frac{P_t^*\mathcal{E}_{jt}}{P_{jt}}\right)^{-\theta}(C_{jt}+X_{jt})\,\mathrm{d}j \\
&= (1-\gamma)\left(\Phi_{it}S_{it}\right)^{\theta}Q_{it}^{-\theta}(C_{it}+X_{it}) + \gamma S_{it}^{\theta}\int Q_{jt}^{-\theta}(C_{jt}+X_{jt})\,\mathrm{d}j,
\end{aligned}$$

which turns to (16) after we plug in the production function (9).

To find firm's demand for intermediate inputs, recall that with the Cobb-Douglas production function (9), firms spend share α on intermediate goods and share $1-\alpha$ on labor,

$$\frac{P_{it}X_{it}}{W_{it}L_{it}} = \frac{\alpha}{1-\alpha}.$$

Plug in the labor supply condition (A1) and arrive at (17).

To derive the country's ex-post budget constraint (20), combine the households' budget constraint (5)

$$P_{it}C_{it} = W_{it}N_{it} + \Pi_{it} + T_{it} + \mathcal{E}_{it}B_{it}^h,$$

firms' domestic and export profits (11) and (12)

$$\Pi_{it} = \left(P_{iit} - \tau_i MC_{it}\right) Y_{it}^D + \left(\mathcal{E}_{it} \Psi_i^{-1} P_{it}^* - \tau_i MC_{it}\right) Y_{it}^E - \frac{\varphi}{2} \tau_{Ri} \left(\left(1-\gamma\right) \Pi_{iit}^2 + \gamma \Pi_{it}^{*2}\right) W_{it},$$

and the government budget constraint

$$T_{it} = \left(\tau_i - 1\right) MC_{it} Y_{it} + \left(\Psi_i - 1\right) \mathcal{E}_{it} \Psi_i^{-1} P_{it}^* Y_{it}^E + \left(\tau_{Ri} - 1\right) \frac{\varphi}{2} \left(\left(1-\gamma\right) \Pi_{iit}^2 + \gamma \Pi_{it}^{*2}\right) W_{it},$$

to get

$$P_{it} C_{it} = W_{it} N_{it} + \mathcal{E}_{it} B_{it}^h + \left(P_{iit} - MC_{it}\right) Y_{it}^D + \left(\mathcal{E}_{it} P_{it}^* - MC_{it}\right) Y_{it}^E - \frac{\varphi}{2} \left(\left(1-\gamma\right) \Pi_{iit}^2 + \gamma \Pi_{it}^{*2}\right) W_{it}.$$

Note that the firms' expenditures are equal to wages and purchases of intermediate goods,

$$MC_{it} \left(Y_{it}^D + Y_{it}^E\right) = W_{it} L_{it} + P_{it} X_{it},$$

and use the labor market balance (18)

$$N_{it} = L_{it} + \frac{\varphi}{2} \left(1-\gamma\right) \Pi_{iit}^2 + \frac{\varphi}{2} \gamma \Pi_{it}^{*2}$$

to simplify the budget constraint to

$$P_{it} \left(C_{it} + X_{it}\right) = \mathcal{E}_{it} B_{it}^h + P_{iit} Y_{it}^D + \mathcal{E}_{it} P_{it}^* Y_{it}^E.$$

Plug in the households' and firms' demand for goods, and note that domestic revenues of firms are equal to households' and firms' expenditures on domestic goods, so that we are left with exports minus imports plus international transfers,

$$0 = \gamma \mathcal{E}_{it} P_{it}^* \int \left(\frac{P_{ijt}}{P_{jt}}\right)^{-\theta} \left(C_{jt} + X_{jt}\right) \mathrm{d}j - \gamma \int P_{jit} \left(\frac{P_{jit}}{P_{it}}\right)^{-\theta} \mathrm{d}j \left(C_{it} + X_{it}\right) + \mathcal{E}_{it} B_{it}^h.$$

Divide everything by $\mathcal{E}_{it}$ to express all payments in dollars, convert export prices to dollars $P_{jit} = \mathcal{E}_{it} P_{jt}^*$, use the definition of the import price index $P_t^{*1-\theta} \equiv \int P_{jt}^{*1-\theta} \mathrm{d}j$ to get

$$0 = \gamma P_t^* \left(\frac{P_{it}^*}{P_t^*}\right)^{1-\theta} \int \left(\frac{\mathcal{E}_{jt} P_t^*}{P_{jt}}\right)^{-\theta} \left(C_{jt} + X_{jt}\right) \mathrm{d}j - \gamma \mathcal{E}_{it}^{-\theta} P_t^{*1-\theta} P_{it}^{\theta} \left(C_{it} + X_{it}\right) + B_{it}^h,$$

and finally use the definitions of the real exchange rate Q_{it} and the terms of trade S_{it} from (14) to arrive at (20),

$$0 = \gamma P_t^* \left[S_{it}^{\theta-1} \int Q_{jt}^{-\theta} \left(C_{jt} + X_{jt}\right) \mathrm{d}j - Q_{it}^{-\theta} \left(C_{it} + X_{it}\right)\right] + B_{it}^h.$$

If instead we started from the households' ex-ante budget constraint (4), $\sum_{t,h} Z_t^h B_{it}^h = 0$, we would arrive at

$$\sum_{t,h} Z_t^h NX_{it} = 0,$$

and with Z_t^h plugged in from (A2) we would get

$$\mathbb{E}\sum_t \frac{\mathcal{E}_{it}P_t^*}{P_{it}C_{it}^{\sigma}}\beta^t \left[S_{it}^{\theta-1}C_t^* - Q_{it}^{-\theta}\left(C_{it}+X_{it}\right)\right] = 0.$$

Use the definition of the real exchange rate Q_{it} from (14), the risk sharing (19), and arrive at (21)

$$\mathbb{E}\sum_t \beta^t \bar{C}_t^{-\sigma}\left[S_{it}^{\theta-1}C_t^* - Q_{it}^{-\theta}\left(C_{it}+X_{it}\right)\right] = 0.$$

Proof of Lemma 1 Consider the net exports (20). Plug in the risk sharing (19), use $\alpha = 0$ and $\theta = 1$ to express it as

$$NX_{it} = \gamma P_t^*\left[C_t^* - Q_{it}^{\frac{1}{\sigma}-\theta}\Lambda_i^{-\frac{1}{\sigma}}\bar{C}_t\right].$$

Further, recall the definition of the global demand shifter $C_t^* \equiv \int Q_{jt}^{-\theta}\left(C_{jt}+X_{jt}\right)\mathrm{d}j$. Plug in the risk sharing sharing (19) and $\alpha = 0$ to get

$$C_t^* \equiv \int Q_{jt}^{\frac{1}{\sigma}-\theta}\Lambda_j^{-\frac{1}{\sigma}}\bar{C}_t\mathrm{d}j.$$

Under $\gamma = 1$, the real exchange rate constraint (15) implies that $Q_{it} = 1$. Under $\sigma\theta = 1$, neither expression from above depends on the real exchange rate. Combine the two expressions under either $\gamma = 1$ or $\sigma\theta = 1$,

$$NX_{it} = \gamma P_t^*\bar{C}_t\left[\int \Lambda_j^{-\frac{1}{\sigma}}\mathrm{d}j - \Lambda_i^{-\frac{1}{\sigma}}\right].$$

Note that the sign of NX_{it} is state- and time-invariant, and thus it has to be the case that $NX_{it} = 0$ for all t and h. ■

Finally, the world trade should be balanced at all times t, $\int NX_{it}\mathrm{d}i = 0$. Plug in the expression for NX_{it} from (20)

$$\int \gamma P_t^*\left[S_{it}^{\theta-1}\int Q_{jt}^{-\theta}\left(C_{jt}+X_{jt}\right)\mathrm{d}j - Q_{it}^{-\theta}\left(C_{it}+X_{it}\right)\right]\mathrm{d}i = 0,$$

rearrange the order of integration, and arrive at

$$\int S_{it}^{\theta-1}\mathrm{d}i = 1. \tag{A4}$$

Note that this condition follows immediately from the import price index definition, $P_t^* = \left(\int P_{jt}^{*1-\theta} \mathrm{d}j\right)^{\frac{1}{1-\theta}}$, and the definition of the terms of trade, (14).

A.1.2 Price-setting block

Domestic Phillips curve To derive the domestic Phillips curve, take the FOC from the firm's problem (11),

$$0 = \Theta_{it}\left(1-\varepsilon\right)\left(1 - \frac{\varepsilon}{\varepsilon-1}\tau_i \frac{MC_{it}}{P_t}\right)\left(\frac{P_t}{P_{iit}}\right)^{-\varepsilon} Y_{it}^D - \left(1-\gamma\right)\varphi\tau_{Ri}\left(\frac{P_t}{P_{t-1}}-1\right)\frac{1}{P_{t-1}} W_{it}\Theta_{it}$$
$$+\left(1-\gamma\right)\varphi\tau_{Ri}\mathbb{E}\left(\frac{P_{t+1}}{P_t}-1\right)\frac{P_{t+1}}{P_t^2} W_{it+1}\Theta_{it+1}.$$

Use the value of the production subsidy $\tau_i = \frac{\varepsilon-1}{\varepsilon}$, symmetry across all domestic firms $P_t = P_{iit}$, and the definition of inflation rate $\Pi_{iit} \equiv P_{iit}/P_{iit-1} - 1$ to rewrite it as

$$\Pi_{iit}\left(\Pi_{iit}+1\right) W_{it}\Theta_{it} = \frac{\Theta_{it}}{1-\gamma}\frac{1-\varepsilon}{\varphi\tau_{Ri}}\left(1-\frac{MC_{it}}{P_{iit}}\right) P_{iit} Y_{it}^D + \mathbb{E}\Pi_{iit+1}\left(\Pi_{iit+1}+1\right) W_{it+1}\Theta_{it+1}.$$

Denote $\kappa \equiv \frac{\varepsilon-1}{\varphi\tau_{Ri}}$, plug in the SDF $\Theta_{it} \equiv \beta^t C_{it}^{-\sigma}/P_{it}$, the local demand shifter $Y_{it}^D = \left(1-\gamma\right)\left(P_{iit}/P_{it}\right)^{-\theta}\left(C_{it}+X_{it}\right)$ from (7), the nominal wage $W_{it} = P_{it}C_{it}^{\sigma}$ from the labor supply condition (A1), and the marginal costs $MC_{it} = P_{it}^{\alpha} W_{it}^{1-\alpha}/A_{it}$

$$\Pi_{iit}\left(\Pi_{iit}+1\right) = -\kappa\left(1 - \frac{P_{it}C_{it}^{\sigma(1-\alpha)}}{A_{it}P_{iit}}\right)\frac{P_{iit}}{P_{it}C_{it}^{\sigma}}\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}\left(C_{it}+X_{it}\right) + \beta\mathbb{E}\Pi_{iit+1}\left(\Pi_{iit+1}+1\right)$$

Use definitions of various prices from (14) and (A3) to arrive at (22)

$$\Pi_{iit}(\Pi_{iit}+1) = -\kappa\left(1 - \frac{\Phi_{it}S_{it}}{Q_{it}}\frac{C_{it}^{\sigma(1-\alpha)}}{A_{it}}\right)\left(\frac{Q_{it}}{\Phi_{it}S_{it}}\right)^{1-\theta}(C_{it}+X_{it})C_{it}^{-\sigma} + \beta\mathbb{E}_t\Pi_{iit+1}(\Pi_{iit+1}+1).$$

The optimal choice of the initial price at $t=0$, P_{-1}, leads to $P_0 = P_{-1}$ or, the same, $\Pi_{ii0} = 0$.

Export Phillips curve Similarly, start at the firm's problem (12), and take the FOC

$$0 = \Theta_{it}\left(1-\varepsilon\right)\left(\mathcal{E}_{it} - \frac{\varepsilon}{\varepsilon-1}\tau_i\frac{MC_{it}}{P_t^*}\right)\left(\frac{\Psi_i P_t^*}{P_{it}^*}\right)^{-\varepsilon} Y_{it}^E - \gamma\varphi\tau_{Ri}\left(\frac{P_t^*}{P_{t-1}^*}-1\right)\frac{1}{P_{t-1}^*}W_{it}\Theta_{it}$$
$$+\gamma\varphi\tau_{Ri}\mathbb{E}\left(\frac{P_{t+1}^*}{P_t^*}-1\right)\frac{P_{t+1}^*}{P_t^{*2}}W_{it+1}\Theta_{it+1}.$$

Use the value of the production subsidy $\tau_i = \frac{\varepsilon-1}{\varepsilon}$, symmetry across all firms $\Psi_i P_t^* = P_{it}^*$, and the definition of inflation rate $\Pi_{it}^* \equiv P_{it}^*/P_{it-1}^* - 1$ to rewrite it as

$$\Pi_{it}^* \left(\Pi_{it}^* + 1\right) W_{it}\Theta_{it} = \frac{\Theta_{it}}{\gamma\Psi_i}\frac{1-\varepsilon}{\varphi\tau_{Ri}}\left(\mathcal{E}_{it} - \Psi_i\frac{MC_{it}}{P_{it}^*}\right) P_{it}^* Y_{it}^E + \mathbb{E}\Pi_{it+1}^*\left(\Pi_{it+1}^* + 1\right) W_{it+1}\Theta_{it+1}.$$

Denote $\kappa \equiv \frac{\varepsilon-1}{\varphi\tau_{Ri}}$, plug in the SDF $\Theta_{it} \equiv \beta^t C_{it}^{-\sigma}/P_{it}$, the export demand shifter $Y_{it}^E = \gamma S_{it}^\theta C_t^*$ from (16), the nominal wage $W_{it} = P_{it}C_{it}^\sigma$ from the labor supply condition (A1), and the marginal costs $MC_{it} = P_{it}^\alpha W_{it}^{1-\alpha}/A_{it}$

$$\Pi_{it}^* \left(\Pi_{it}^* + 1\right) = -\frac{\kappa}{\Psi}\left(1 - \Psi_i\frac{P_{it}C_{it}^{\sigma(1-\alpha)}}{A_{it}\mathcal{E}_{it}P_{it}^*}\right)\frac{\mathcal{E}_{it}P_{it}^*}{P_{it}C_{it}^\sigma}S_{it}^\theta C_t^* + \beta\mathbb{E}\Pi_{it+1}^*\left(\Pi_{it+1}^* + 1\right).$$

Use definitions of various prices from (14) to arrive at (23)

$$\Pi_{it}^* \left(\Pi_{it}^* + 1\right) = -\frac{\kappa}{\Psi}\left(1 - \frac{\Psi_i S_{it}}{Q_{it}}\frac{C_{it}^{\sigma(1-\alpha)}}{A_{it}}\right) Q_{it}S_{it}^{\theta-1}C_t^* C_{it}^{-\sigma} + \beta\mathbb{E}\Pi_{it+1}^*\left(\Pi_{it+1}^* + 1\right).$$

Again, the optimal choice of the initial price leads to $\Pi_{i0}^* = 0$.

Phillips curve for the U.S. Note that the problem (13) differs from the problem of a non-U.S. domestic producer (12) in two ways: there is the global demand shifter Y_{it} instead of the domestic Y_{it}^D, and Rotemberg costs are not multiplied by $(1-\gamma)$. Then, following the same steps as before, one can derive the following optimality condition

$$\Pi_{iit}\left(\Pi_{iit} + 1\right) = -\kappa\left(1 - \frac{P_{it}C_{it}^{\sigma(1-\alpha)}}{A_{it}P_{iit}}\right)\frac{P_{iit}}{P_{it}C_{it}^\sigma}Y_{it} + \beta\mathbb{E}\Pi_{iit+1}\left(\Pi_{iit+1} + 1\right).$$

Let's plug in the demand shifter Y_{it} from (16), use definitions of the real exchange rate and the terms of trade from (14) , keep in mind that for the U.S. $\Phi_{it} = \Psi_i$,

$$\Pi_{iit}(\Pi_{iit} + 1) = -\frac{\kappa}{\Psi_i}\left(1 - \frac{\Psi_i S_{it}}{Q_{it}}\frac{C_{it}^{\sigma(1-\alpha)}}{A_{it}}\right)\left[(1-\gamma)\Psi_i^\theta Q_{it}^{-\theta}\left(C_{it} + X_{it}\right) + \gamma C_t^*\right] Q_{it}S_{it}^{\theta-1}C_{it}^{-\sigma} + \beta\mathbb{E}\Pi_{iit+1}\left(\Pi_{iit+1} + 1\right),$$

and the optimal choice of the initial price leads to $\Pi_{ii0} = 0$.

Ex-ante price setting Finally, each of the Phillips curves above, or the ex-post price setting conditions, should be complemented by the corresponding ex-ante price setting condition, which reflects the optimal choice of prices at time -1, P_{-1}.

First, consider the non-U.S. domestic price setting, where the optimal choice of P_{-1} implies $\Pi_{ii0} = 0$. Plug it in the ex-post price setting and iterate forward to get the ex-ante price setting condition

$$0 = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left(1 - \frac{\Phi_{it} S_{it} C_{it}^{\sigma(1-\alpha)}}{A_{it} Q_{it}}\right) \left(\frac{Q_{it}}{\Phi_{it} S_{it}}\right)^{1-\theta} (C_{it} + X_{it}) C_{it}^{-\sigma}. \tag{A5}$$

Similary, the ex-ante non-U.S. export price setting and the U.S. price setting conditions are

$$0 = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left(1 - \frac{\Psi_i S_{it}}{Q_{it}} \frac{C_{it}^{\sigma(1-\alpha)}}{A_{it}}\right) Q_{it} S_{it}^{\theta-1} C_t^* C_{it}^{-\sigma}, \tag{A6}$$

$$0 = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left(1 - \frac{\Psi_i S_{it}}{Q_{it}} \frac{C_{it}^{\sigma(1-\alpha)}}{A_{it}}\right) \left((1-\gamma) \Psi_i^{\theta} Q_{it}^{-\theta} (C_{it} + X_{it}) + \gamma C_t^*\right) Q_{it} S_{it}^{\theta-1} C_{it}^{-\sigma}. \tag{A7}$$

A.1.3 Equilibrium

Proof of Lemma 2 Let's list all equilibrium conditions that for given policies constitute an equilibrium. These are (14)–(25) (where the price setting conditions (22), (23), (25) implicitly contain their ex-ante versions (A5), (A6), (A7) as well), the labor supply condition (A1), the global trade balance (A4), 3 definitions of inflation rates Π_{iit}, Π_{it}^*, Π_{00t}, and the definition of the import price index.

Note that the latter condition, $P_t^{*1-\theta} \equiv \int P_{jt}^{*1-\theta} \mathrm{d}j$, is equivalent to the terms of trade normalization (A4), and thus we can drop it.

Implicitly, we have already used the labor supply condition (A1) to get rid of nominal wages W_{it} in all other equilibrium conditions. Similarly, let's use definitions of Q_{it}, Φ_{it}, and S_{it} to get rid of P_{iit}, P_{it}^*, and P_{it}. All of the conditions (15)–(25) already do not contain P_{iit}, P_{it}^*, P_{it}. Definition of Q_{it} is the only condition that contains P_{it} and thus it could be dropped.

For the non-U.S. countries, the definition of Φ_{it} is the only condition with $\mathcal{E}_{it}$, and thus could be dropped as well. Let's plug in S_{it} to the export inflation

$$\Pi_{it}^* = \frac{P_{it}^*}{P_{it-1}^*} - 1 = \frac{P_t^*}{P_{t-1}^*} \frac{S_{it-1}}{S_{it}} - 1.$$

Then the definition of S_{it} could be dropped as it is the only condition with P_{it}^*. The definition of domestic inflation $\Pi_{iit} = P_{iit}/P_{iit-1} - 1$ could be dropped as it is the only condition with P_{iit}.

For the U.S., plug in $\Phi_{0t} = \Psi_0 = P_{0t}^*/P_{00t}$ and the definition of S_{0t} to the definition of inflation

$$\Pi_{00t} = \frac{P_{00t}}{P_{00t-1}} - 1 = \frac{P_t^*}{P_{t-1}^*} \frac{S_{0t-1}}{S_{0t}} - 1.$$

Then these definitions of Φ_{0t} and S_{0t} could be dropped since no other conditions contain either P_{00t} or P_{0t}^*.

Finally, by combining definitions of Π_{it}^* and Π_{00t}, we can get rid of P_t^* and get

$$\Pi_{it}^* = (\Pi_{00t} + 1) \frac{S_{0t}}{S_{0t-1}} \frac{S_{it-1}}{S_{it}} - 1. \tag{A8}$$

In the end, we can reduce the system of equilibrium varaibles to the allocation $\{C_{it}, L_{it}, X_{it}, \Lambda_i, B_{it}^h\}$ and prices $\{Q_{it}, S_{it}, \Phi_{it}, \Pi_{iit}, \Pi_{it}^*\}$. Then, all equilibrium conditions are represented by equations (15)–(25), and definitions of various prices, which result in the global terms of trade normalization (A4) and the dynamic equation for inflation rates and terms of trade (A8). ■

A.2 Static Model

In a static model, there is no inflation, and thus the labor market clearing (18) collapses to $N_{it} = L_{it}$.

A.2.1 Generalization

In this section, we generalize our framework.

Consumers maximize

$$U_i = \mathbb{E} \sum_{t=0}^{\infty} \beta^t u (C_{it}, L_{it})$$

instead of (1), subject to

$$\sum_{t,h} \frac{Z_t^h}{\mathcal{E}_{it}} [P_{it} C_{it} - W_{it} L_{it} - \Pi_{it} - T_{it}] = 0.$$

Then the labor supply condition (A1) changes to

$$-\frac{u_L (C_{it}, L_{it})}{u_C (C_{it}, L_{it})} = \frac{W_{it}}{P_{it}}, \tag{A9}$$

and the risk-sharing (A2) becomes

$$Q_{it} u_C (C_{it}, L_{it}) = \lambda_i \beta^{-t} \frac{Z_t^h P_t^*}{Pr_{ht}}. \tag{A10}$$

The firms' production function is

$$Y_{it} = A_{it} F (L_{it}, X_{it}) \tag{A11}$$

instead of (9), where function F exhibits constant returns to scale. A cost minimization problem

$$\min_{L_{it}, X_{it}} W_{it} L_{it} + P_{it} X_{it}$$
$$\text{s.t.} \quad A_{it} F (L_{it}, X_{it}) = Y_{it},$$

yields the following optimality condition

$$\frac{W_{it}}{P_{it}} = \frac{F_L\left(L_{it}, X_{it}\right)}{F_X\left(L_{it}, X_{it}\right)}. \tag{A12}$$

Next, express the total costs as

$$TC_{it} \equiv W_{it}L_{it} + P_{it}X_{it} = P_{it}\left(\frac{W_{it}}{P_{it}}L_{it} + X_{it}\right),$$

plug in the optimality condition (A12),

$$TC_{it} = \frac{P_{it}}{F_X\left(L_{it}, X_{it}\right)}\left(F_L\left(L_{it}, X_{it}\right)L_{it} + F_X\left(L_{it}, X_{it}\right)X_{it}\right) = \frac{P_{it}Y_{it}}{A_{it}F_X\left(L_{it}, X_{it}\right)}, \tag{A13}$$

where the last equality is due to Euler's theorem. Then, the marginal costs are just $MC_{it} \equiv P_{it}/\left(A_{it}F_X\left(L_{it}, X_{it}\right)\right)$.

Finally, for conveniency of notation, plug in the labor supply condition (A9) into the firm's optimality condition (A12), and express its solution as $X_{it} = X\left(C_{it}, L_{it}\right)$, which we will use instead of condition (17).

A.2.2 Efficient allocation

To solve for the efficient allocation in one country, we allow the planner to choose all quantities in this country directly. However, the planner has to take international prices as given and respect the country's budget constraint as well as the foreign demand for the domestic goods. Thus, the social planner's problem can be written as

$$\begin{aligned} &\max_{C_{it}, L_{it}, X_{it}, S_{it}, \{C_{jit}, X_{jit}\}_j} u\left(C_{it}, L_{it}\right) \\ \text{s.t.}\quad & A_{it}F\left(L_{it}, X_{it}\right) = C_{iit} + X_{iit} + \gamma S_{it}^{\theta}C_t^*, \\ & P_t^*\gamma S_{it}^{\theta-1}C_t^* - \int P_{jt}^*\left(C_{jit} + X_{jit}\right)\mathrm{d}j + B_{it}^h = 0, \\ & C_{it} = \left[(1-\gamma)^{\frac{1}{\theta}}C_{iit}^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}}\int C_{jit}^{\frac{\theta-1}{\theta}}\mathrm{d}j\right]^{\frac{\theta}{\theta-1}}, \\ & X_{it} = \left[(1-\gamma)^{\frac{1}{\theta}}X_{iit}^{\frac{\theta-1}{\theta}} + \gamma^{\frac{1}{\theta}}\int X_{jit}^{\frac{\theta-1}{\theta}}\mathrm{d}j\right]^{\frac{\theta}{\theta-1}}. \end{aligned}$$

Here the planner can choose any export price in dollars by choosing S_{it}, but all import prices in dollars, P_t^* and P_{jt}^*, are taken as given. Plug in the last two conditions to get rid of C_{it} and X_{it}, and denote the Lagrange multiplier for the market clearing as η_{it} and for the budget constraint as ρ_{it}. Then the FOCs are

$$0 = u_C\left(C_{it}, L_{it}\right)\frac{\mathrm{d}C_{it}}{\mathrm{d}C_{iit}} - \eta_{it}$$

$$0 = u_C\left(C_{it}, L_{it}\right)\frac{\mathrm{d}C_{it}}{\mathrm{d}C_{jit}} - \rho_{it}P^*_{jt}$$

$$0 = u_L\left(C_{it}, L_{it}\right) + \eta_{it}A_{it}F_L\left(L_{it}, X_{it}\right)$$

$$0 = \eta_{it}A_{it}F_X\left(L_{it}, X_{it}\right)\frac{\mathrm{d}X_{it}}{\mathrm{d}X_{iit}} - \eta_{it}$$

$$0 = \eta_{it}A_{it}F_X\left(L_{it}, X_{it}\right)\frac{\mathrm{d}X_{it}}{\mathrm{d}X_{jit}} - \rho_{it}P^*_{jt}$$

$$0 = -\eta_{it}\gamma\theta S_{it}^{\theta-1}C^*_t + \rho_{it}\gamma P^*_t\left(\theta-1\right)S_{it}^{\theta-2}C^*_t$$

Use the FOC with respect to L_{it} to substitute for η_{it} in all other conditions,

$$\eta_{it} = \frac{-u_L\left(C_{it}, L_{it}\right)}{A_{it}F_L\left(L_{it}, X_{it}\right)}.$$

Similarly, use the FOC with respect to S_{it} to substitute for ρ_{it},

$$\rho_{it} = \frac{-u_L\left(C_{it}, L_{it}\right)}{A_{it}F_L\left(L_{it}, X_{it}\right)P^*_t}\frac{\theta}{\theta-1}S_{it}.$$

Divide the first FOC by the second (and the fourth by the fifth) to find the relative demand for domestic and foreign varieties,

$$\frac{C_{iit}}{C_{jit}} = \frac{X_{iit}}{X_{jit}} = \frac{1-\gamma}{\gamma}\left(\frac{\theta}{\theta-1}S_{it}\frac{P^*_{jt}}{P^*_t}\right)^{\theta}. \tag{A14}$$

Next, combine the first two FOCs and use the consumption aggregator to derive

$$\left(\frac{-u_L\left(C_{it}, L_{it}\right)}{u_C\left(C_{it}, L_{it}\right)A_{it}F_L\left(L_{it}, X_{it}\right)}\right)^{\theta-1} = \left(1-\gamma\right) + \gamma\left(\frac{\theta}{\theta-1}S_{it}\right)^{1-\theta}.$$

Similarly, combine the the fourth and the fifth FOCs and use the intermediates aggregator to get

$$\left(A_{it}F_X\left(L_{it}, X_{it}\right)\right)^{1-\theta} = \left(1-\gamma\right) + \gamma\left(\frac{\theta}{\theta-1}S_{it}\right)^{1-\theta}, \tag{A15}$$

and therefore the two conditions together imply

$$\frac{-u_L\left(C_{it}, L_{it}\right)}{u_C\left(C_{it}, L_{it}\right)} = \frac{F_L\left(L_{it}, X_{it}\right)}{F_X\left(L_{it}, X_{it}\right)}. \tag{A16}$$

Lemma A0 *Assume that prices are flexible. Then the optimal non-cooperative allocation can be implemented with* $\tau_i = \frac{\varepsilon - 1}{\varepsilon}$ *and* $\Psi_i = \frac{\theta}{\theta - 1}$.

Proof of Lemma A0 The labor supply condition (A9) and the firms' optimality condition (A12) together yield the last of the planner's conditions, (A16).

With CES demand, as in (7), we get

$$\frac{C_{iit}}{C_{jit}} = \frac{X_{iit}}{X_{jit}} = \frac{1-\gamma}{\gamma} \left(\frac{P_{iit}}{P_{it}} \right)^{-\theta} \left(\frac{P^*_{jt}}{P^*_t} \frac{\mathcal{E}_{it} P^*_t}{P_{it}} \right)^{\theta},$$

and with definition of prices (14) it becomes

$$\frac{C_{iit}}{C_{jit}} = \frac{X_{iit}}{X_{jit}} = \frac{1-\gamma}{\gamma} \left(\Phi_{it} S_{it} \frac{P^*_{jt}}{P^*_t} \right)^{\theta}. \tag{A17}$$

Compare this condition with the planner's (A14), and the two coincide when $\Phi_{it} = \Psi_i = \frac{\theta}{\theta - 1}$.

Under flexible prices, the price setting conditions (22) and (23) collapse to[24]

$$A_{it} F_X (L_{it}, X_{it}) = \frac{S_{it} \Phi_{it}}{Q_{it}}, \qquad A_{it} F_X (L_{it}, X_{it}) = \frac{S_{it} \Psi_i}{Q_{it}}.$$

The two together once again imply $\Phi_{it} = \Psi_i = \frac{\theta}{\theta - 1}$, but combing the latter one with the price index constraint (15) yields

$$\left((1-\gamma) + \gamma \left(\Psi_i S_{it} \right)^{1-\theta} \right)^{\frac{1}{\theta - 1}} = \frac{1}{A_{it} F_X (L_{it}, X_{it})},$$

which is the same condition as planner's (A15). ■

A.2.3 Non-U.S. policy

Proof of Proposition 1 The policy problem is

$$\begin{aligned}
& \max_{C_{it}, L_{it}, X_{it}, \Phi_{it}, Q_{it}} u(C_{it}, L_{it}) \\
\text{s.t.} \quad & A_{it} F(L_{it}, X_{it}) = (1-\gamma) Q_{it}^{-\theta} \Phi_{it}^{\theta} S_{it}^{\theta} (C_{it} + X_{it}) + \gamma S_{it}^{\theta} C^*_t, \\
& \gamma P^*_t \left[S_{it}^{\theta - 1} C^*_t - Q_{it}^{-\theta} (C_{it} + X_{it}) \right] + B^h_{it} = 0, \\
& X_{it} = X(C_{it}, L_{it}), \\
& Q_{it}^{\theta - 1} = \gamma + (1-\gamma) (\Phi_{it} S_{it})^{\theta - 1}.
\end{aligned}$$

Plug in the last two conditions to get rid of X_{it} and Q_{it}, and denote the Lagrange multiplier for the

[24] Recall that flexible prices lead to the same condition as stabilization of domestic prices, that is $MC_{it}/P_{iit} = 1$. And (A13) implies $MC_{it} \equiv P_{it} / (A_{it} F_X (L_{it}, X_{it}))$. For the export prices, the corresponding condition is $\Psi_i MC_{it} = \mathcal{E}_{it} P^*_{it}$.

market clearing as η_{it} and for the budget constraint as ρ_{it}. Then the FOCs are

$$\begin{aligned}0 &= u_C(C_{it}, L_{it}) + \eta_{it} A_{it} F_X(L_{it}, X_{it}) X_C(C_{it}, L_{it}) \\ &- \eta_{it}(1-\gamma) Q_{it}^{-\theta} \Phi_{it}^{\theta} S_{it}^{\theta} (1 + X_C(C_{it}, L_{it})) - \rho_{it} \gamma P_t^* Q_{it}^{-\theta} (1 + X_C(C_{it}, L_{it}))\end{aligned}$$

$$\begin{aligned}0 &= u_L(C_{it}, L_{it}) + \eta_{it} A_{it} (F_L(L_{it}, X_{it}) + F_X(L_{it}, X_{it}) X_L(C_{it}, L_{it})) \\ &- \eta_{it}(1-\gamma) Q_{it}^{-\theta} \Phi_{it}^{\theta} S_{it}^{\theta} X_L(C_{it}, L_{it}) - \rho_{it} \gamma P_t^* Q_{it}^{-\theta} X_L(C_{it}, L_{it})\end{aligned}$$

$$\begin{aligned}0 &= -\eta_{it}(1-\gamma)\left(-\theta Q_{it}^{1-2\theta}(1-\gamma) S_{it}^{\theta-1} \Phi_{it}^{2\theta-2} + Q_{it}^{-\theta} \theta \Phi_{it}^{\theta-1}\right) S_{it}^{\theta} (C_{it} + X_{it}) \\ &+ \rho_{it} \gamma P_t^* \theta Q_{it}^{1-2\theta} (1-\gamma) S_{it}^{\theta-1} \Phi_{it}^{\theta-2} (C_{it} + X_{it})\end{aligned}$$

Use the real exchange rate constraint (15) to simplify the last FOC to

$$\rho_{it} P_t^* = \eta_{it} S_{it} \Phi_{it}.$$

Plug in ρ_{it} from this condition to the first two FOCs, and similarly simplify them to

$$0 = u_L(C_{it}, L_{it}) + \eta_{it} A_{it} (F_L(L_{it}, X_{it}) + F_X(L_{it}, X_{it}) X_L(C_{it}, L_{it})) - \eta_{it} S_{it} \Phi_{it} Q_{it}^{-1} X_L(C_{it}, L_{it}),$$

$$0 = u_C(C_{it}, L_{it}) + \eta_{it} A_{it} F_X(L_{it}, X_{it}) X_C(C_{it}, L_{it}) - \eta_{it} S_{it} \Phi_{it} Q_{it}^{-1} (1 + X_C(C_{it}, L_{it})).$$

Divide one by another, use the labor supply (A9), the firms' optimality condition (A12), and rearrange to get

$$0 = \left(A_{it} F_X(L_{it}, X_{it}) - S_{it} \Phi_{it} Q_{it}^{-1}\right) \left[\frac{W_{it}}{P_{it}} (1 + X_C(C_{it}, L_{it})) + X_L(C_{it}, L_{it})\right].$$

Under well-behaved $u(C_{it}, L_{it})$ and $F_X(L_{it}, X_{it})$, both $X_C(C_{it}, L_{it})$ and $X_L(C_{it}, L_{it})$ are non-negative, and thus it has to be the case that

$$A_{it} F_X(L_{it}, X_{it}) = \frac{S_{it} \Phi_{it}}{Q_{it}}. \tag{A18}$$

Plug in (A3) and note that due to (A13) it collapses to the stabilization of domestic prices, $MC_{it}/P_{iit} = 1$. This is the first part of the Proposition.

For the second part, note that the firm's optimality condition (A12) and its constraint (A11), could be jointly solved for L_{it} and X_{it} as functions of A_{it}, W_{it}, P_{it}, and Y_{it} only. Then, the marginal costs from (A13) could be expressed as a function of just two prices and productivity, in particular denote it as $MC_{it} = G(W_{it}, P_{it})/A_{it}$. Then, with linear disutility from labor, $u_L(C_{it}, L_{it}) = -1$, and separable utility function, $u_{CL}(C_{it}, L_{it}) = 0$, the stabilization of domestic prices could be expressed as

$$G\left(P_{it} u_C (C_{it})^{-1}, P_{it}\right) / A_{it} = P_{iit},$$

where we have used the labor supply condition (A9).

Next, combine the risk-sharing (A10) for a non-U.S. country and for the U.S. (using the definition of the real exchange rate from (14)),

$$\mathcal{E}_{it} = \frac{P_{it} u_C (C_{it})^{-1}}{P_{0t} u_C (C_{0t})^{-1}} \frac{\lambda_i}{\lambda_0}.$$

Then, rewrite the price index (6) as

$$P_{it}^{1-\theta} = (1-\gamma) P_{iit}^{1-\theta} + \gamma \mathcal{E}_{it}^{1-\theta} P_t^{*1-\theta}.$$

Finally, consider a system of these three conditions, where P_{iit} and P_t^* are fixed. Suppose that the U.S. depreciates and $P_{0t} u_C (C_{0t})^{-1}$ rises. All else equal, this will lead (through a nominal exchange rate $\mathcal{E}_{it}$ appreciation) to lower price index P_{it} and lower marignal costs for a non-U.S. country. Then, to stabilize domestic prices, the non-U.S. policymaker has to increase its nominal demand, $P_{it} u_C (C_{it})^{-1}$. Since this nominal demand enters the marignal costs through wages as well as through the price index, an increase in $P_{it} u_C (C_{it})^{-1}$ will be smaller than an increase in $P_{0t} u_C (C_{0t})^{-1}$. This constitutes a partial peg to the dollar and gives rise to a global monetary cycle.

For the last part of the Proposition, look at the planner's optimality conditions. (A16) is true as it depends only on the labor supply (A9) and the firms' optimality condition (A12). As with flexible prices, the CES demand leads to (A17), which coincide with (A14) only under $\Phi_{it} = \Psi_i = \frac{\theta}{\theta-1}$. Finally, plug in the price index constraint (15) to the domestic price stabilization condition (A18) to get

$$(A_{it} F_X (L_{it}, X_{it}))^{1-\theta} = (1-\gamma) + \gamma (\Phi_{it} S_{it})^{1-\theta},$$

which is almost the same as the planner's (A15). However, the planner's terms of trade S_{it} move with shocks and $\Phi_{it} = \Psi_i$ is constant, while in the current condition S_{it} is fixed and Φ_{it} responds to shocks. ■

A.2.4 U.S. policy

Proof of Proposition 2 To set up the U.S. policy problem, start with the U.S. market clearing (16), the demand for intermediates (17), and the budget constraint (20). Note that this system depends on the global demand C_t^*,

$$C_t^* \equiv \int Q_{jt}^{-\theta} (C_{jt} + X_{jt}) \, \mathrm{d}j, \tag{A19}$$

which the U.S. does not take as given. To find how the global demand C_t^* depends on the non-U.S. policy, consider the set of equilibrium conditions for each non-U.S. country j,

$$Q_{jt}^{\theta-1} = \gamma + (1-\gamma)\left(\Phi_{jt} S_{jt}\right)^{\theta-1}$$

$$A_{jt} F\left(L_{jt}, X_{jt}\right) = (1-\gamma) Q_{jt}^{-\theta} \Phi_{jt}^{\theta} S_{jt}^{\theta}\left(C_{jt} + X_{jt}\right) + \gamma S_{jt}^{\theta} C_t^*$$

$$\gamma P_t^* \left[S_{jt}^{\theta-1} C_t^* - Q_{jt}^{-\theta}\left(C_{jt} + X_{jt}\right)\right] + B_{jt}^h = 0$$

$$X_{jt} = X\left(C_{jt}, L_{jt}\right)$$

$$A_{jt} F_X\left(L_{jt}, X_{jt}\right) = \frac{S_{jt}\Phi_{jt}}{Q_{jt}}$$

These conditions are the same as the constraints of the non-U.S. policy problem, except that we have added the optimal policy condition (A18).

This system of the non-U.S. equilibrium conditions contains 5 equations and 5 non-U.S. variables (Q_{jt}, Φ_{jt}, L_{jt}, X_{jt}, C_{jt}). One can solve this system and find all non-U.S. variables as functions of predetermined variables $\left\{S_{jt}, B_{jt}^h, P_t^*\right\}$, shocks $\{A_{jt}\}$, and the global demand C_t^*. Plug these solutions back to the definition of C_t^*, (A19), and get an additional constraint for the U.S. policy problem.

However, this additional constraint will always be satisfied by construction. To see that, integrate the budget constraint (20) over all non-U.S. countries j,

$$\gamma P_t^* \left[\int S_{jt}^{\theta-1} \mathrm{d}j C_t^* - \int Q_{jt}^{-\theta}\left(C_{jt} + X_{jt}\right) \mathrm{d}j\right] + \int B_{jt}^h \mathrm{d}j = 0.$$

Note that the sum of all international transfers has to be equal to zero, $\int B_{jt}^h \mathrm{d}j = 0$, and the global terms of trade normalization (A4) implies $\int S_{jt}^{\theta-1} \mathrm{d}j = 1$. Then this condition collapses to (A19).

Thus, there are no additional constraints for the U.S., and its policy problem could be written as

$$\max_{C_{it}, L_{it}, X_{it}, C_t^*} u\left(C_{it}, L_{it}\right)$$

$$\text{s.t.} \quad A_{it} F\left(L_{it}, X_{it}\right) = (1-\gamma) Q_{it}^{-\theta} \Psi_i^{\theta} S_{it}^{\theta}\left(C_{it} + X_{it}\right) + \gamma S_{it}^{\theta} C_t^*,$$

$$\gamma P_t^* \left[S_{it}^{\theta-1} C_t^* - Q_{it}^{-\theta}\left(C_{it} + X_{it}\right)\right] + B_{it}^h = 0,$$

$$X_{it} = X\left(C_{it}, L_{it}\right).$$

This proves the first part of the Proposition.

As before, denote the first two Lagrange multipliers as η_{it} and ρ_{it}, and plug in the last condition to get rid of X_{it}. The resulting FOCs are:

$$\begin{aligned} 0 &= u_C\left(C_{it},L_{it}\right)+\eta_{it}A_{it}F_X\left(L_{it},X_{it}\right)X_C\left(C_{it},L_{it}\right) \\ &- \eta_{it}\left(1-\gamma\right)Q_{it}^{-\theta}\Psi_i^{\theta}S_{it}^{\theta}\left(1+X_C\left(C_{it},L_{it}\right)\right)-\rho_{it}\gamma P_t^{*}Q_{it}^{-\theta}\left(1+X_C\left(C_{it},L_{it}\right)\right) \end{aligned}$$

$$\begin{aligned} 0 &= u_L\left(C_{it},L_{it}\right)+\eta_{it}A_{it}\left(F_L\left(L_{it},X_{it}\right)+F_X\left(L_{it},X_{it}\right)X_L\left(C_{it},L_{it}\right)\right) \\ &- \eta_{it}\left(1-\gamma\right)Q_{it}^{-\theta}\Psi_i^{\theta}S_{it}^{\theta}X_L\left(C_{it},L_{it}\right)-\rho_{it}\gamma P_t^{*}Q_{it}^{-\theta}X_L\left(C_{it},L_{it}\right) \end{aligned}$$

$$0=-\eta_{it}\gamma S_{it}^{\theta}+\rho_{it}\gamma P_t^{*}S_{it}^{\theta-1}$$

Use the last FOC to substitute for ρ_{it}, and use the price index constraint (15) along with $\Psi_i = 1$ to rewrite the first two FOCs as

$$0=u_C\left(C_{it},L_{it}\right)+\eta_{it}A_{it}F_X\left(L_{it},X_{it}\right)X_C\left(C_{it},L_{it}\right)-\eta_{it}S_{it}Q_{it}^{-1}\left(1+X_C\left(C_{it},L_{it}\right)\right)$$

$$0=u_L\left(C_{it},L_{it}\right)+\eta_{it}A_{it}\left(F_L\left(L_{it},X_{it}\right)+F_X\left(L_{it},X_{it}\right)X_L\left(C_{it},L_{it}\right)\right)-\eta_{it}S_{it}Q_{it}^{-1}X_L\left(C_{it},L_{it}\right)$$

Divide one by another, use the labor supply (A9), the firms' optimality condition (A12), and rearrange to get

$$0=\left(A_{it}F_X\left(L_{it},X_{it}\right)-S_{it}Q_{it}^{-1}\right)\left(X_L\left(C_{it},L_{it}\right)+\frac{W_{it}}{P_{it}}\left(1+X_C\left(C_{it},L_{it}\right)\right)\right)$$

As before, under well-behaved $u\left(C_{it},L_{it}\right)$ and $F_X\left(L_{it},X_{it}\right)$, both $X_C\left(C_{it},L_{it}\right)$ and $X_L\left(C_{it},L_{it}\right)$ are non-negative, and thus it has to be the case that

$$A_{it}F_X\left(L_{it},X_{it}\right)=\frac{S_{it}}{Q_{it}}$$

Plug in (A3) and note that due to (A13) it collapses to the stabilization of domestic prices, $MC_{it}/P_{iit}=1$. This is the second part of the Proposition.

To show that this allocation does not coincide with the efficient allocation, it's enough to note that in the efficient allocation $\Phi_{it}=\frac{\theta}{\theta-1}$, while for the U.S. $\Phi_{it}=1$. ■

A.2.5 Capital controls

Proof of Proposition 3 The policy problem can be rewritten as

$$
\begin{aligned}
&\max_{C_{it},L_{it},X_{it},\Phi_{it},Q_{it}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t u\left(C_{it},L_{it}\right) \\
\text{s.t.} \quad & A_{it}F\left(L_{it},X_{it}\right)=\left(1-\gamma\right)Q_{it}^{-\theta}\Phi_{it}^{\theta}S_{it}^{\theta}\left(C_{it}+X_{it}\right)+\gamma S_{it}^{\theta}C_t^{*}, \\
&\sum_{t,h}Z_t^h\left[S_{it}^{\theta-1}C_t^{*}-Q_{it}^{-\theta}\left(C_{it}+X_{it}\right)\right]=0, \\
&X_{it}=X\left(C_{it},L_{it}\right), \\
&Q_{it}^{\theta-1}=\gamma+\left(1-\gamma\right)\left(\Phi_{it}S_{it}\right)^{\theta-1}.
\end{aligned}
$$

where we expressed the budget constraint through the prices of Arrow-Debreu securities, as in (4). $\left\{Z_t^h\right\}$ are taken as given by the non-U.S. policymaker.

As before, denote the Lagrange multipliers η_{it} and ρ_i (ρ_i now is state- and time-invariant), and plug in the last two constraints. Then the FOCs are:

$$
\begin{aligned}
0=\,&u_C\left(C_{it},L_{it}\right)+\eta_{it}A_{it}F_X\left(L_{it},X_{it}\right)X_C\left(C_{it},L_{it}\right) \\
&-\eta_{it}\left(1-\gamma\right)Q_{it}^{-\theta}\Phi_{it}^{\theta}S_{it}^{\theta}\left(1+X_C\left(C_{it},L_{it}\right)\right)-\rho_i\frac{Z_t^h}{\beta^t Pr_{ht}}Q_{it}^{-\theta}\left(1+X_C\left(C_{it},L_{it}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
0=\,&u_L\left(C_{it},L_{it}\right)+\eta_{it}A_{it}\left(F_L\left(L_{it},X_{it}\right)+F_X\left(L_{it},X_{it}\right)X_L\left(C_{it},L_{it}\right)\right) \\
&-\eta_{it}\left(1-\gamma\right)Q_{it}^{-\theta}\Phi_{it}^{\theta}S_{it}^{\theta}X_L\left(C_{it},L_{it}\right)-\rho_i\frac{Z_t^h}{\beta^t Pr_{ht}}Q_{it}^{-\theta}X_L\left(C_{it},L_{it}\right)
\end{aligned}
$$

$$
\begin{aligned}
0=\,&-\eta_{it}\left(1-\gamma\right)\left(-\theta Q_{it}^{1-2\theta}\left(1-\gamma\right)S_{it}^{\theta-1}\Phi_{it}^{2\theta-2}+Q_{it}^{-\theta}\theta\Phi_{it}^{\theta-1}\right)S_{it}^{\theta}\left(C_{it}+X_{it}\right) \\
&+\rho_i\frac{Z_t^h}{\beta^t Pr_{ht}}\theta Q_{it}^{1-2\theta}\left(1-\gamma\right)S_{it}^{\theta-1}\Phi_{it}^{\theta-2}\left(C_{it}+X_{it}\right)
\end{aligned}
$$

Use the price index constraint (15) to simplify the last FOC to

$$
\rho_i\frac{Z_t^h}{\beta^t Pr_{ht}}=\eta_{it}\gamma\Phi_{it}S_{it}.
$$

Plug in ρ_i from this condition to the first two FOCs, and similarly simplify them to

$$
0=u_C\left(C_{it},L_{it}\right)+\eta_{it}A_{it}F_X\left(L_{it},X_{it}\right)X_C\left(C_{it},L_{it}\right)-\eta_{it}\Phi_{it}S_{it}Q_{it}^{-1}\left(1+X_C\left(C_{it},L_{it}\right)\right)
$$

$$0 = u_L(C_{it}, L_{it}) + \eta_{it} A_{it} \left(F_L(L_{it}, X_{it}) + F_X(L_{it}, X_{it}) X_L(C_{it}, L_{it}) \right) - \eta_{it} \Phi_{it} S_{it} Q_{it}^{-1} X_L(C_{it}, L_{it})$$

Note that they are exactly the same as in the non-U.S problem without capital controls (Appendix A.2.3), and thus they also lead to the domestic price stabilization,

$$A_{it} F_X(L_{it}, X_{it}) = \frac{S_{it}\Phi_{it}}{Q_{it}}.$$

Next, plug in this domestic price stabilization to the first FOC, and use the third FOC to go back from η_{it} to ρ_i,

$$Q_{it} u_C(C_{it}, L_{it}) = \rho_i \beta^{-t} \frac{Z_t^h}{\gamma P r_{ht}}.$$

Compare this with the private risk-sharing (A10), and note that they are the same (P_t^* is state-invariant).

Thus, the non-U.S. monetary policy still stabilizes domestic prices, and the capital controls achieve the same allocation of transfers across states as the private risk-sharing. ■

A.2.6 Gains from cooperation

Proof of Proposition 4 The global policy problem is

$$\max_{\{C_{it}, L_{it}, X_{it}, \Phi_{it}, Q_{it}\}_i} \int u(C_{it}, L_{it}) \, \mathrm{d}i$$

$$\text{s.t.} \quad A_{it} F(L_{it}, X_{it}) = (1-\gamma) Q_{it}^{-\theta} \Phi_{it}^{\theta} S_{it}^{\theta} (C_{it} + X_{it}) + \gamma S_{it}^{\theta} \int Q_{jt}^{-\theta} (C_{jt} + X_{jt}) \, \mathrm{d}j,$$

$$\gamma P_t^* \left[S_{it}^{\theta-1} \int Q_{jt}^{-\theta} (C_{jt} + X_{jt}) \, \mathrm{d}j - Q_{it}^{-\theta} (C_{it} + X_{it}) \right] + B_{it}^h = 0,$$

$$X_{it} = X(C_{it}, L_{it}),$$

$$Q_{it}^{\theta-1} = \gamma + (1-\gamma)(\Phi_{it} S_{it})^{\theta-1},$$

$$\Phi_{0t} = 1.$$

Here we have a continuum of constraints, each corresponding to a different country. All terms of trade $\{S_{it}\}_i$ and transfers $\{B_{it}^h\}_i$ are taken as given, and they have to satisfy global balances $\int S_{jt}^{\theta-1} \mathrm{d}j = 1$ and $\int B_{jt}^h \mathrm{d}j = 0$. P_t^* is taken as given as well.

As before, plug in the last three constraints, and denote the remaining Lagrange multipliers as η_{it} and ρ_{it}. Then the FOCs are:

$$\begin{aligned} 0 = {} & u_C(C_{it}, L_{it}) + \eta_{it} A_{it} F_X(L_{it}, X_{it}) X_C(C_{it}, L_{it}) \\ & - \eta_{it} (1-\gamma) Q_{it}^{-\theta} \Phi_{it}^{\theta} S_{it}^{\theta} (1 + X_C(C_{it}, L_{it})) - \int \eta_{jt} S_{jt}^{\theta} \mathrm{d}j \gamma Q_{it}^{-\theta} (1 + X_C(C_{it}, L_{it})) \\ & + \int \rho_{jt} S_{jt}^{\theta-1} \mathrm{d}j \gamma P_t^* Q_{it}^{-\theta} (1 + X_C(C_{it}, L_{it})) - \rho_{it} \gamma P_t^* Q_{it}^{-\theta} (1 + X_C(C_{it}, L_{it})) \end{aligned}$$

$$
\begin{aligned}
0 = {} & u_L\left(C_{it}, L_{it}\right) + \eta_{it} A_{it}\left(F_L\left(L_{it}, X_{it}\right) + F_X\left(L_{it}, X_{it}\right) X_L\left(C_{it}, L_{it}\right)\right) \\
& - \eta_{it}(1-\gamma) Q_{it}^{-\theta} \Phi_{it}^{\theta} S_{it}^{\theta} X_L\left(C_{it}, L_{it}\right) - \int \eta_{jt} S_{jt}^{\theta} \mathrm{d}j \gamma Q_{it}^{-\theta} X_L\left(C_{it}, L_{it}\right) \\
& + \int \rho_{jt} S_{jt}^{\theta-1} \mathrm{d}j \gamma P_t^* Q_{it}^{-\theta} X_L\left(C_{it}, L_{it}\right) - \rho_{it} \gamma P_t^* Q_{it}^{-\theta} X_L\left(C_{it}, L_{it}\right)
\end{aligned}
$$

$$
\begin{aligned}
0 = {} & -\eta_{it}(1-\gamma)\left(-\theta Q_{it}^{1-2\theta}(1-\gamma) S_{it}^{\theta-1} \Phi_{it}^{2\theta-2} + Q_{it}^{-\theta} \theta \Phi_{it}^{\theta-1}\right) S_{it}^{\theta}\left(C_{it} + X_{it}\right) \\
& + \int \eta_{jt} S_{jt}^{\theta} \mathrm{d}j \gamma \theta Q_{it}^{1-2\theta}(1-\gamma) S_{it}^{\theta-1} \Phi_{it}^{\theta-2}\left(C_{it} + X_{it}\right) \\
& - \int \rho_{jt} S_{jt}^{\theta-1} \mathrm{d}j \gamma P_t^* \theta Q_{it}^{1-2\theta}(1-\gamma) S_{it}^{\theta-1} \Phi_{it}^{\theta-2}\left(C_{it} + X_{it}\right) + \rho_{it} \gamma P_t^* \theta Q_{it}^{1-2\theta}(1-\gamma) S_{it}^{\theta-1} \Phi_{it}^{\theta-2}\left(C_{it} + X_{it}\right)
\end{aligned}
$$

Use the real exchange rate constraint (15) to simplify the last FOC (which holds for all i except for the U.S.) to

$$
\rho_{it} P_t^* = \eta_{it} \Phi_{it} S_{it} - \int \eta_{jt} S_{jt}^{\theta} \mathrm{d}j + \int \rho_{jt} P_t^* S_{jt}^{\theta-1} \mathrm{d}j
$$

Plug in ρ_{it} from this condition to the first two FOCs, and similarly simplify them to

$$
0 = u_C\left(C_{it}, L_{it}\right) + \eta_{it} A_{it} F_X\left(L_{it}, X_{it}\right) X_C\left(C_{it}, L_{it}\right) - \eta_{it} \Phi_{it} S_{it} Q_{it}^{-1}\left(1 + X_C\left(C_{it}, L_{it}\right)\right)
$$

$$
0 = u_L\left(C_{it}, L_{it}\right) + \eta_{it} A_{it}\left(F_L\left(L_{it}, X_{it}\right) + F_X\left(L_{it}, X_{it}\right) X_L\left(C_{it}, L_{it}\right)\right) - \eta_{it} \Phi_{it} S_{it} Q_{it}^{-1} X_L\left(C_{it}, L_{it}\right)
$$

Note that they are exactly the same as in the non-U.S. non-cooperative problem (Appendix A.2.3), and thus they also lead to the domestic price stabilization in the non-U.S. countries,

$$
A_{it} F_X\left(L_{it}, X_{it}\right) = \frac{S_{it} \Phi_{it}}{Q_{it}}.
$$

For the U.S., there is no third FOC. Then, there are always two values of Lagrange multipliers, η_{0t} and ρ_{0t}, to support any allocation in the U.S.

Next, plug in the domestic price stabilization to the first FOC,

$$
0 = u_C\left(C_{it}, L_{it}\right) - \eta_{it} \Phi_{it} S_{it} Q_{it}^{-1},
$$

Plug this in the third FOC, multiply by $S_{it}^{\theta-1}$, integrate over i, and use the terms of trade normalization $\int S_{jt}^{\theta-1} \mathrm{d}j = 1$,

$$
0 = \int Q_{jt} u_C\left(C_{jt}, L_{jt}\right) S_{jt}^{\theta-1} \mathrm{d}j - \int \frac{Q_{jt} u_C\left(C_{jt}, L_{jt}\right)}{\Phi_{jt}} S_{jt}^{\theta-1} \mathrm{d}j.
$$

Use the risk-sharing across countries (A10), $Q_{it} u_C\left(C_{it}, L_{it}\right) \lambda_i^{-1} = Q_{jt} u_C\left(C_{jt}, L_{jt}\right) \lambda_j^{-1}$, and the ex-ante symmetry across all non-U.S. countries, which implies $\lambda_i = \lambda_j$ and $S_{jt} = 1$. Then

$$1 = \int \frac{1}{\Phi_{jt}} \mathrm{d}j.$$

Finally, plug in the domestic marginal cost stabilization one more time, and use definitions of various prices (14) along with the marginal cost definition $MC_{it} \equiv P_{it} / \left(A_{it} F_X \left(L_{it}, X_{it}\right)\right)$,

$$1 = \int \frac{S_{jt}}{Q_{jt} A_{jt} F_X \left(L_{jt}, X_{jt}\right)} \mathrm{d}j = \int \frac{MC_{jt}}{\mathcal{E}_{jt} P^*_{jt}} \mathrm{d}j.$$

■

A.3 Dynamic Model

A.3.1 Non-U.S. policy

Proof of Proposition 5 The non-U.S. policy problem with capital controls could be written as

$$\max \mathbb{E}\sum_{t=0}^{\infty}\beta^t \left[\frac{C_{it}^{1-\sigma}}{1-\sigma} - N_{it}\right]$$

$$\text{s.t.} \quad \frac{A_{it}}{1-\alpha}C_{it}^{\sigma\alpha}L_{it} = (1-\gamma)\,Q_{it}^{-\theta}\Phi_{it}^{\theta}S_{it}^{\theta}\left(C_{it} + \frac{\alpha}{1-\alpha}C_{it}^{\sigma}L_{it}\right) + \gamma S_{it}^{\theta}C_t^*,$$

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\bar{C}_t^{-\sigma}\left[S_{it}^{\theta-1}C_t^* - Q_{it}^{-\theta}\left(C_{it} + \frac{\alpha}{1-\alpha}C_{it}^{\sigma}L_{it}\right)\right] = 0,$$

$$N_{it} = L_{it} + \frac{\varphi}{2}(1-\gamma)\,\Pi_{iit}^2 + \frac{\varphi}{2}\gamma\Pi_{it}^{*2},$$

$$C_{it} = \left(\frac{Q_{it}}{\Lambda_{it}}\right)^{\frac{1}{\sigma}}\bar{C}_t,$$

$$Q_{it}^{\theta-1} = (1-\gamma)\left(\Phi_{it}S_{it}\right)^{\theta-1} + \gamma,$$

$$\Pi_{iit}\left(\Pi_{iit}+1\right) = \beta\mathbb{E}_t\Pi_{iit+1}\left(\Pi_{iit+1}+1\right) + \frac{1-\varepsilon}{\varphi\tau_{Ri}}\left(1 - \frac{\Phi_{it}S_{it}}{Q_{it}}\frac{C_{it}^{\sigma-\sigma\alpha}}{A_{it}}\right)\left(\frac{Q_{it}}{\Phi_{it}S_{it}}\right)^{1-\theta}\left(C_{it}^{1-\sigma} + \frac{\alpha}{1-\alpha}L_{it}\right),$$

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(1 - \frac{\Phi_{it}S_{it}}{Q_{it}}\frac{C_{it}^{\sigma-\sigma\alpha}}{A_{it}}\right)\left(\frac{Q_{it}}{\Phi_{it}S_{it}}\right)^{1-\theta}\left(C_{it}^{1-\sigma} + \frac{\alpha}{1-\alpha}L_{it}\right) = 0,$$

$$\Pi_{it}^*\left(\Pi_{it}^*+1\right) = \beta\mathbb{E}_t\Pi_{it+1}^*\left(\Pi_{it+1}^*+1\right) + \frac{1-\varepsilon}{\varphi\tau_{Ri}}\Psi_i^{-1}\left(1 - \frac{\Psi_iS_{it}}{Q_{it}}\frac{C_{it}^{\sigma-\sigma\alpha}}{A_{it}}\right)Q_{it}S_{it}^{\theta-1}C_t^*C_{it}^{-\sigma},$$

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(1 - \frac{\Psi_iS_{it}}{Q_{it}}\frac{C_{it}^{\sigma-\sigma\alpha}}{A_{it}}\right)Q_{it}S_{it}^{\theta-1}C_t^*C_{it}^{-\sigma} = 0,$$

$$Q_{it} = \frac{\mathcal{E}_{it}P_t^*}{P_{it}}, \quad S_{it} = \frac{P_t^*}{P_{it}^*}, \quad \Phi_{it} = \frac{\mathcal{E}_{it}P_{it}^*}{P_{iit}},$$

$$\Pi_{iit} = \frac{P_{iit}}{P_{iit-1}} - 1, \quad \Pi_{it}^* = \frac{P_{it}^*}{P_{it-1}^*} - 1.$$

Policymaker chooses C_{it}, N_{it}, L_{it}, Q_{it}, Φ_{it}, S_{it}, Π_{iit}, Π_{it}^*, Λ_{it}, $\mathcal{E}_{it}$, P_{it}, P_{it}^*, P_{iit} state-by-state. The presence of capital controls is expressed in the fact that Λ_{it} can change state-by-state and across time (but subject to the country's budget constraint). C_t^*, $\bar{C}_t$, and P_t^* are taken as given.

Here we included the market clearing (16) (with demand for intermediates (17) plugged in), the ex-ante budget constraint (21), the labor balance (18), the risk-sharing (19), the price index constraint (15), the ex-post and the ex-ante domestic price setting (22), the ex-post and the ex-ante export price setting (23), as well as definitions of various prices (14) and of inflation rates. Recall that in order to get the ex-ante domestic price setting constraint, we write down the ex-post constraint (22), iterate it forward until the infinity, and then impose $\Pi_{ii0} = 0$. The ex-ante export price setting constraint is similar.

Let's use the primal approach. Use $S_{it} = \frac{P_t^*}{P_{it}^*}$ to plug in P_{it}^* in all other constraints, and drop this constraint. P_{it} enters only in the definition of Q_{it}, and thus we can drop both this variable and this constraint. Similarly, $\mathcal{E}_{it}$ enters only the definition of Φ_{it}, and we again drop this constraint. The, P_{iit} enters only the definition of Π_{iit}, which because of that could be dropped. Finally, the last two lines of our policy problem collapse just to

$$\Pi_{it}^* = \frac{P_t^*}{P_{t-1}^*} \frac{S_{it-1}}{S_{it}} - 1, \tag{A20}$$

where the policymaker takes the path of P_t^* as given. (Note that this constraint is equivalent to (A8).) Then, let's plug in the risk-sharing instead of C_{it}, and the labor balance instead of N_{it}, and arrive at the final policy problem

$$\max \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\frac{Q_{it}^{\frac{1}{\sigma}-1} \Lambda_{it}^{1-\frac{1}{\sigma}} \bar{C}_t^{1-\sigma}}{1-\sigma} - L_{it} - \frac{\varphi}{2} (1-\gamma) \Pi_{iit}^2 - \frac{\varphi}{2} \gamma \Pi_{it}^{*2} \right]$$

$$\text{s.t.} \quad \frac{A_{it}}{1-\alpha} Q_{it}^{\alpha} \Lambda_{it}^{-\alpha} \bar{C}_t^{\sigma\alpha} L_{it} = (1-\gamma) \Phi_{it}^{\theta} S_{it}^{\theta} Q_{it}^{\frac{1}{\sigma}-\theta} \Lambda_{it}^{-\frac{1}{\sigma}} \bar{C}_t + \frac{\alpha}{1-\alpha} (1-\gamma) Q_{it}^{1-\theta} \Phi_{it}^{\theta} S_{it}^{\theta} \Lambda_{it}^{-1} \bar{C}_t^{\sigma} L_{it} + \gamma S_{it}^{\theta} C_t^*,$$

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \bar{C}_t^{-\sigma} \left[S_{it}^{\theta-1} C_t^* - Q_{it}^{\frac{1}{\sigma}-\theta} \Lambda_{it}^{-\frac{1}{\sigma}} \bar{C}_t - \frac{\alpha}{1-\alpha} Q_{it}^{1-\theta} \Lambda_{it}^{-1} \bar{C}_t^{\sigma} L_{it} \right] = 0,$$

$$Q_{it}^{\theta-1} = (1-\gamma) (\Phi_{it} S_{it})^{\theta-1} + \gamma,$$

$$\begin{aligned} \Pi_{iit} (\Pi_{iit} + 1) = {} & \beta \mathbb{E}_t \Pi_{iit+1} (\Pi_{iit+1} + 1) \\ & + \frac{1-\varepsilon}{\varphi \tau_{Ri}} \left(1 - \frac{\bar{C}_t^{\sigma(1-\alpha)}}{A_{it}} Q_{it}^{-\alpha} \Lambda_{it}^{\alpha-1} \Phi_{it} S_{it} \right) \left(\frac{Q_{it}}{\Phi_{it} S_{it}} \right)^{1-\theta} \left(Q_{it}^{\frac{1}{\sigma}-1} \Lambda_{it}^{1-\frac{1}{\sigma}} \bar{C}_t^{1-\sigma} + \frac{\alpha}{1-\alpha} L_{it} \right), \end{aligned}$$

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \left(1 - \frac{\bar{C}_t^{\sigma(1-\alpha)}}{A_{it}} Q_{it}^{-\alpha} \Lambda_{it}^{\alpha-1} \Phi_{it} S_{it} \right) \left(\frac{Q_{it}}{\Phi_{it} S_{it}} \right)^{1-\theta} \left(Q_{it}^{\frac{1}{\sigma}-1} \Lambda_{it}^{1-\frac{1}{\sigma}} \bar{C}_t^{1-\sigma} + \frac{\alpha}{1-\alpha} L_{it} \right) = 0,$$

$$\Pi_{it}^* (\Pi_{it}^* + 1) = \beta \mathbb{E}_t \Pi_{it+1}^* \left(\Pi_{it+1}^* + 1 \right) + \frac{1-\varepsilon}{\varphi \tau_{Ri}} \Psi_i^{-1} \left(1 - \Psi_i S_{it} Q_{it}^{-\alpha} \Lambda_{it}^{\alpha-1} \frac{\bar{C}_t^{\sigma(1-\alpha)}}{A_{it}} \right) \Lambda_{it} S_{it}^{\theta-1} C_t^* \bar{C}_t^{-\sigma},$$

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \left(1 - \Psi_i S_{it} Q_{it}^{-\alpha} \Lambda_{it}^{\alpha-1} \frac{\bar{C}_t^{\sigma(1-\alpha)}}{A_{it}} \right) \Lambda_{it} S_{it}^{\theta-1} C_t^* \bar{C}_t^{-\sigma} = 0,$$

$$\Pi_{it}^* = \frac{P_t^*}{P_{t-1}^*} \frac{S_{it-1}}{S_{it}} - 1.$$

Plug in the price index (15) to get rid of Q_{it}, and denote the Lagrange multipliers for the remaining constraints as η_t, ρ, μ_t, ν, μ_t^*, ν^*, ζ_t. Then the FOCs are:

- with respect to Φ_{it}

$$
\begin{aligned}
0 = & \frac{1}{\sigma} Q_{it}^{\frac{1}{\sigma}-1} \Lambda_{it}^{1-\frac{1}{\sigma}} \bar{C}_t^{1-\sigma} (1-\gamma) Q_{it}^{1-\theta} \Phi_{it}^{\theta-2} S_{it}^{\theta-1} + \eta_t \frac{A_{it}}{1-\alpha} \alpha Q_{it}^{\alpha} (1-\gamma) Q_{it}^{1-\theta} \Phi_{it}^{\theta-2} S_{it}^{\theta-1} \Lambda_{it}^{-\alpha} \bar{C}_t^{\sigma\alpha} L_{it} \\
& - \eta_t (1-\gamma) \theta \Phi_{it}^{\theta-1} S_{it}^{\theta} Q_{it}^{\frac{1}{\sigma}-\theta} \Lambda_{it}^{-\frac{1}{\sigma}} \bar{C}_t - \eta_t (1-\gamma) \Phi_{it}^{\theta} S_{it}^{\theta} \left(\frac{1}{\sigma} - \theta \right) Q_{it}^{\frac{1}{\sigma}-\theta} (1-\gamma) Q_{it}^{1-\theta} \Phi_{it}^{\theta-2} S_{it}^{\theta-1} \Lambda_{it}^{-\frac{1}{\sigma}} \bar{C}_t \\
& - \eta_t \frac{\alpha}{1-\alpha} (1-\gamma) Q_{it}^{1-\theta} \theta \Phi_{it}^{\theta-1} S_{it}^{\theta} \Lambda_{it}^{-1} \bar{C}_t^{\sigma} L_{it} \\
& - \eta_t \frac{\alpha}{1-\alpha} (1-\gamma)(1-\theta) Q_{it}^{1-\theta} (1-\gamma) Q_{it}^{1-\theta} \Phi_{it}^{\theta-2} S_{it}^{\theta-1} \Phi_{it}^{\theta} S_{it}^{\theta} \Lambda_{it}^{-1} \bar{C}_t^{\sigma} L_{it} \\
& - \rho \bar{C}_t^{-\sigma} \left[\left(\frac{1}{\sigma} - \theta \right) Q_{it}^{\frac{1}{\sigma}-\theta} \Lambda_{it}^{-\frac{1}{\sigma}} \bar{C}_t + \frac{\alpha}{1-\alpha} (1-\theta) Q_{it}^{1-\theta} \Lambda_{it}^{-1} \bar{C}_t^{\sigma} L_{it} \right] (1-\gamma) Q_{it}^{1-\theta} \Phi_{it}^{\theta-2} S_{it}^{\theta-1} \\
& + \mu_t [...] + \nu [...] + \mu_t^* [...] + \nu^* [...]
\end{aligned}
$$

- with respect to S_{it}

$$
\begin{aligned}
0 = & \frac{1}{\sigma} Q_{it}^{\frac{1}{\sigma}-1} \Lambda_{it}^{1-\frac{1}{\sigma}} \bar{C}_t^{1-\sigma} (1-\gamma) Q_{it}^{1-\theta} S_{it}^{\theta-2} \Phi_{it}^{\theta-1} + \eta_t \frac{A_{it}}{1-\alpha} \alpha Q_{it}^{\alpha} (1-\gamma) Q_{it}^{1-\theta} S_{it}^{\theta-2} \Phi_{it}^{\theta-1} \Lambda_{it}^{-\alpha} \bar{C}_t^{\sigma\alpha} L_{it} \\
& - \eta_t (1-\gamma) \Phi_{it}^{\theta} \theta S_{it}^{\theta-1} Q_{it}^{\frac{1}{\sigma}-\theta} \Lambda_{it}^{-\frac{1}{\sigma}} \bar{C}_t - \eta_t (1-\gamma) \Phi_{it}^{\theta} S_{it}^{\theta} \left(\frac{1}{\sigma} - \theta \right) Q_{it}^{\frac{1}{\sigma}-\theta} (1-\gamma) Q_{it}^{1-\theta} S_{it}^{\theta-2} \Phi_{it}^{\theta-1} \Lambda_{it}^{-\frac{1}{\sigma}} \bar{C}_t \\
& - \eta_t \frac{\alpha}{1-\alpha} (1-\gamma) Q_{it}^{1-\theta} \Phi_{it}^{\theta} \theta S_{it}^{\theta-1} \Lambda_{it}^{-1} \bar{C}_t^{\sigma} L_{it} \\
& - \eta_t \frac{\alpha}{1-\alpha} (1-\gamma)(1-\theta) Q_{it}^{1-\theta} (1-\gamma) Q_{it}^{1-\theta} S_{it}^{\theta-2} \Phi_{it}^{\theta-1} \Phi_{it}^{\theta} S_{it}^{\theta} \Lambda_{it}^{-1} \bar{C}_t^{\sigma} L_{it} \\
& - \eta_t \gamma \theta S_{it}^{\theta-1} C_t^* + \rho \bar{C}_t^{-\sigma} (\theta - 1) S_{it}^{\theta-2} C_t^* \\
& - \rho \bar{C}_t^{-\sigma} \left[\left(\frac{1}{\sigma} - \theta \right) Q_{it}^{\frac{1}{\sigma}-\theta} \Lambda_{it}^{-\frac{1}{\sigma}} \bar{C}_t + \frac{\alpha}{1-\alpha} (1-\theta) Q_{it}^{1-\theta} \Lambda_{it}^{-1} \bar{C}_t^{\sigma} L_{it} \right] (1-\gamma) Q_{it}^{1-\theta} S_{it}^{\theta-2} \Phi_{it}^{\theta-1} \\
& + \zeta_t \frac{S_{it-1}}{S_{it}^2} \frac{P_t^*}{P_{t-1}^*} - \beta \mathbb{E}_t \zeta_{t+1} \frac{1}{S_{it+1}} \frac{P_{t+1}^*}{P_t^*} + \mu_t [...] + \nu [...] + \mu_t^* [...] + \nu^* [...]
\end{aligned}
$$

- with respect to L_{it}

$$
\begin{aligned}
0 = & -1 + \eta_t \frac{A_{it}}{1-\alpha} Q_{it}^{\alpha} \Lambda_{it}^{-\alpha} \bar{C}_t^{\sigma\alpha} - \eta_t \frac{\alpha}{1-\alpha} (1-\gamma) Q_{it}^{1-\theta} \Phi_{it}^{\theta} S_{it}^{\theta} \Lambda_{it}^{-1} \bar{C}_t^{\sigma} \\
& - \rho \bar{C}_t^{-\sigma} \frac{\alpha}{1-\alpha} Q_{it}^{1-\theta} \Lambda_{it}^{-1} \bar{C}_t^{\sigma} + \mu_t [...] + \nu [...]
\end{aligned}
$$

- with respect to Λ_{it}

$$0 = -\frac{1}{\sigma} Q_{it}^{\frac{1}{\sigma}-1} \Lambda_{it}^{-\frac{1}{\sigma}} \bar{C}_t^{1-\sigma} - \alpha \eta_t \frac{A_{it}}{1-\alpha} Q_{it}^{\alpha} \Lambda_{it}^{-\alpha-1} \bar{C}_t^{\sigma\alpha} L_{it} + \eta_t \frac{1}{\sigma} (1-\gamma) \Phi_{it}^{\theta} S_{it}^{\theta} Q_{it}^{\frac{1}{\sigma}-\theta} \Lambda_{it}^{-\frac{1}{\sigma}-1} \bar{C}_t$$
$$+ \eta_t \frac{\alpha}{1-\alpha} (1-\gamma) Q_{it}^{1-\theta} \Phi_{it}^{\theta} S_{it}^{\theta} \Lambda_{it}^{-2} \bar{C}_t^{\sigma} L_{it} + \rho \bar{C}_t^{-\sigma} \left[\frac{1}{\sigma} Q_{it}^{\frac{1}{\sigma}-\theta} \Lambda_{it}^{-\frac{1}{\sigma}-1} \bar{C}_t + \frac{\alpha}{1-\alpha} Q_{it}^{1-\theta} \Lambda_{it}^{-2} \bar{C}_t^{\sigma} L_{it} \right]$$
$$+ \mu_t [...] + \nu [...] + \mu_t^* [...] + \nu^* [...]$$

- with respect to Π_{iit}

$$0 = -\varphi (1-\gamma) \Pi_{iit} + \mu_t [...]$$

- with respect to Π_{it}^*

$$0 = -\varphi \gamma \Pi_{it}^* + \zeta_t + \mu_t^* [...]$$

Now let's guess and verify that the optimal policy stabilizes domestic prices, $\Pi_{iit} = 0$, and thus the ex-post domestic price setting has to hold state-by-state,

$$1 = \frac{\bar{C}_t^{\sigma(1-\alpha)}}{A_{it}} Q_{it}^{-\alpha} \Lambda_{it}^{\alpha-1} \Phi_{it} S_{it},$$

that no capital controls are used $\Lambda_{it} = 1$,[25] and that all 4 price setting constraints do not bind, $\mu_t = \nu = \mu_t^* = \nu^* = 0$.

Then the FOC with respect to Π_{it}^* implies $\zeta_t = \varphi\gamma\Pi_{it}^*$. The FOC with respect to L_{it} can be reduced to

$$\rho \bar{C}_t^{-\sigma} \frac{\alpha}{1-\alpha} = -\bar{C}_t^{-\sigma} Q_{it}^{\theta-1} + \eta_t \left((1-\gamma) \Phi_{it}^{\theta} S_{it}^{\theta} + \gamma \frac{1}{1-\alpha} \Phi_{it} S_{it} \right).$$

The FOC with respect to Λ_{it} can be used to find η_t, $\eta_t \Phi_{it} S_{it} = \bar{C}_t^{-\sigma}$, and then the FOC with respect to L_{it} determines ρ, $\rho = \gamma$. The FOC with respect to Φ_{it} can be shown to be satisfied. The last remaining FOC, the one with repsect to S_{it}, can be then reduced to

$$\Pi_{it}^* (\Pi_{it}^* + 1) = \beta \mathbb{E}_t \Pi_{it+1}^* (\Pi_{it+1}^* + 1) + \frac{\theta-1}{\varphi} \left(\frac{\theta}{\theta-1} \Phi_{it}^{-1} - 1 \right) S_{it}^{\theta-1} \bar{C}_t^{-\sigma} C_t^*.$$

Note that under the domestic price stabilization, the export price setting becomes

$$\Pi_{it}^* (\Pi_{it}^* + 1) = \beta \mathbb{E}_t \Pi_{it+1}^* (\Pi_{it+1}^* + 1) + \frac{1-\varepsilon}{\varphi \tau_{Ri}} \Psi_i^{-1} \left(1 - \Psi_i \Phi_{it}^{-1}\right) S_{it}^{\theta-1} \bar{C}_t^{-\sigma} C_t^*,$$

[25]No capital controls implies $\Lambda_{it} = const$, and then the ex-ante symmetry of the non-U.S. countries implies $\Lambda_{it} = 1$.

and the two coincide under the appropriate level of the price-adjustment subsidy, $\tau_{Ri} = \frac{\varepsilon-1}{\theta}$.

Thus, we have shown that if this policy (domestic price stabilization and no capital controls) is feasible, that is it satisfies all the constraints, then it is optimal, that is it satisfies all the FOCs. ■

A.3.2 U.S. policy

Proof of Proposition 6 To derive the welfare loss function, let's start with the utility function (1), the market clearing (16), the ex-ante budget constraint (21), and the price index constraint (15), rewritten as

$$U_i = \mathbb{E}\sum_{t=0}^{\infty}\beta^t \left[\frac{C_{it}^{1-\sigma}}{1-\sigma} - (1-\gamma)\, A_{it}^{-1}\Psi_i^{\theta} S_{it}^{\theta} Q_{it}^{-\theta} C_{it} - \gamma A_{it}^{-1} S_{it}^{\theta} C_t^{*} - \frac{\varphi}{2}\Pi_{iit}^2\right],$$

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t \bar{C}_t^{-\sigma}\left[S_{it}^{\theta-1}C_t^{*} - Q_{it}^{-\theta}C_{it}\right] = 0,$$

$$Q_{it}^{\theta-1} = (1-\gamma)\,(\Psi_i S_{it})^{\theta-1} + \gamma.$$

Let's take second-order approximations around the non-stochastic steady state to all three conditions,

$$U_i = \mathbb{E}\sum_{t=0}^{\infty}\beta^t L \Bigg[\omega\Psi c_{it} - \theta(1-\omega)\, s_{it} + \theta(1-\omega)\, q_{it} - \omega c_t^{*} - \omega\theta s_{it} + \frac{1}{2}(1-\omega+\omega\Psi)(1-\sigma)\, c_{it}^2$$
$$-\frac{1}{2}(1-\omega)(c_{it} - a_{it} + \theta s_{it} - \theta q_{it})^2 - \frac{1}{2}\omega\,(c_t^{*} - a_{it} + \theta s_{it})^2 - L^{-1}\frac{\varphi}{2}\Pi_{iit}^2 + a_{it} + \mathcal{O}\left(\epsilon^3\right)\Bigg],$$

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t \left[(\theta-1)\, s_{it} + c_t^{*} + \theta q_{it} - c_{it} + \frac{1}{2}\left((\theta-1)\, s_{it} + c_t^{*} - \sigma\bar{c}_t\right)^2 - \frac{1}{2}\left(-\theta q_{it} + c_{it} - \sigma\bar{c}_t\right)^2\right] = \mathcal{O}\left(\epsilon^3\right),$$

$$q_{it} = \frac{1-\omega}{1-\omega+\omega\Psi}s_{it} + \frac{1}{2}(\theta-1)\frac{(1-\omega)\,\omega\Psi}{(1-\omega+\omega\Psi)^2}s_{it}^2 + \mathcal{O}\left(\epsilon^3\right),$$

where L is the steady state-level of L_{it}, ω is the steady-state measure of openness

$$\omega \equiv \frac{\gamma A_i^{-1} S_i^{\theta}\bar{C}}{L} = \frac{\gamma}{(1-\gamma)\,\Psi_i^{\theta} + \gamma},$$

and "small-letter" variables are log-deviations of "big-letter" variables. In the price index constraint, we have used its first-order version to get rid of q_{it}^2-term.

Use the budget constraint to express the first-order consumption, $\mathbb{E}\sum_{t=0}^{\infty}\beta^t c_{it}$, as a function of other variables, and plug it in the welfare along with the price index constraint to get

$$
\begin{aligned}
U_i = \mathbb{E}\sum_{t=0}^{\infty}\beta^t L \Big[& \omega\left(\Psi-1\right)c_t^* - L^{-1}\frac{\varphi}{2}\Pi_{iit}^2 \\
& -\frac{1}{2}\left(1-\omega+\omega\Psi\right)\sigma\left(c_{it}-\frac{\omega\Psi}{1-\omega+\omega\Psi}\bar{c}_t-\frac{1-\omega}{1-\omega+\omega\Psi}\theta a_{it}\right)^2 \\
& -\frac{1}{2}\theta\omega\left(1+\frac{\left(1-\omega\right)\Psi}{1-\omega+\omega\Psi}\right)\left(s_{it}+\sigma\bar{c}_t-a_{it}\right)^2 \\
& +\frac{1}{2}\omega\left(\Psi-1\right)c_t^{*2}+\frac{1}{2}\omega\frac{\omega\Psi^2+1-\omega+\Psi}{1-\omega+\omega\Psi}\sigma\bar{c}_t^2 \\
& -\omega\Psi\sigma c_t^*\bar{c}_t+\omega c_t^* a_{it}-\omega\bar{c}_t a_{it}+\frac{1}{2}\left(\theta-1\right)a_{it}^2+\mathcal{O}\left(\epsilon^3\right)\Big].
\end{aligned}
$$

Note that under flexible prices, output is equal to θa_{it}, $y_{it}^f = \theta a_{it}$, and thus we define output gap as

$$
\tilde{y}_{it} \equiv y_{it} - \theta a_{it}. \tag{A21}
$$

Similarly, if we consider a world equilibrium with sticky prices and DCP, and then allow all prices in one non-U.S. country to be flexible, then in that particular country $s_{it}^f = -\sigma\bar{c}_t + a_{it}$ and $c_{it}^f = \frac{\omega\Psi}{1-\omega+\omega\Psi}\bar{c}_t + \frac{1-\omega}{1-\omega+\omega\Psi}\theta a_{it}$, and thus we adopt the following definitions of gaps,

$$
\tilde{s}_{it} \equiv s_{it} + \sigma\bar{c}_t - a_{it}, \quad \tilde{c}_{it} \equiv c_{it} - \frac{\omega\Psi}{1-\omega+\omega\Psi}\bar{c}_t - \frac{1-\omega}{1-\omega+\omega\Psi}\theta a_{it}. \tag{A22}
$$

Next, let's take the first-order approximation to the risk-sharing (19),

$$
q_{it} - \sigma c_{it} = -\sigma\bar{c}_t + \mathcal{O}\left(\epsilon^2\right),
$$

and to the market clearing (16),

$$
A_{it}L_{it} = Y_{it} = \left(1-\gamma\right)Q_{it}^{-\theta}\Psi_i^{\theta}S_{it}^{\theta}C_{it} + \gamma S_{it}^{\theta}C_t^*,
$$

$$
y_{it} = -\left(1-\omega\right)\theta q_{it} + \left(1-\omega\right)c_{it} + \theta s_{it} + \omega c_t^* + \mathcal{O}\left(\epsilon^2\right).
$$

Together with the first-order approximation to the price index constraint, these 3 conditions imply that

$$
\tilde{s}_{it} = \sigma\tilde{y}_{it} + \mathcal{O}\left(\epsilon^2\right), \quad \tilde{c}_{it} = \frac{1-\omega}{1-\omega+\omega\Psi}\tilde{y}_{it} + \mathcal{O}\left(\epsilon^2\right). \tag{A23}
$$

Then we can rewrite the welfare loss function as

$$U_i = \mathbb{E}\sum_{t=0}^{\infty}\beta^t L\left[\omega\left(\Psi-1\right)c_t^* - \frac{1}{2}\sigma\tilde{y}_{it}^2 - L^{-1}\frac{\varphi}{2}\Pi_{iit}^2\right.$$
$$+\frac{1}{2}\omega\left(\Psi-1\right)c_t^{*2} + \frac{1}{2}\omega\frac{\omega\Psi^2+1-\omega+\Psi}{1-\omega+\omega\Psi}\sigma\bar{c}_t^2$$
$$\left.-\omega\Psi\sigma c_t^*\bar{c}_t + \omega c_t^* a_{it} - \omega\bar{c}_t a_{it} + \frac{1}{2}\left(\theta-1\right)a_{it}^2 + \mathcal{O}\left(\epsilon^3\right)\right].$$

Next, recall the definition of the global demand C_t^*,

$$C_t^* \equiv \int Q_{jt}^{-\theta} C_{jt}\mathrm{d}j,$$

and plug in the risk-sharing (19) to get

$$C_t^* = \bar{C}_t,$$

where we have used the ex-ante symmetry of the non-U.S. countries, $\Lambda_i = 1$, and the Faia-Monacelli parametrization $\sigma\theta = 1$. Recall the global terms of trade normalization (A4), $\int S_{jt}^{\theta-1}\mathrm{d}j = 1$. Finally, recall the ex-ante export price setting of the non-U.S. countries (23)

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left(1-\Psi_i S_{it}\frac{\bar{C}_t^\sigma}{A_{it}}\right)S_{it}^{\theta-1}C_t^*\bar{C}_t^{-\sigma} = 0,$$

where again we have used the ex-ante symmetry of the non-U.S. countries, $\Lambda_i = 1$, but have not used their optimal policy.

Now let's take second-order approximations to the last two conditions,

$$\int\left[s_{jt} + \frac{1}{2}\left(\theta-1\right)s_{jt}^2\right]\mathrm{d}j = \mathcal{O}\left(\epsilon^3\right),$$

$$\mathcal{O}\left(\epsilon^3\right) = \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[-\sigma\bar{c}_t - \int s_{jt}\mathrm{d}j + \frac{1}{2}\sigma^2\bar{c}_t^2 + \frac{1}{2}\left(1-2\theta\right)\int s_{jt}^2\mathrm{d}j - \frac{1}{2}\int a_{jt}^2\mathrm{d}j\right.$$
$$\left.-\sigma\bar{c}_t c_t^* - \int c_t^* s_{jt}\mathrm{d}j - \int\sigma\left(\theta-1\right)s_{jt}\bar{c}_t\mathrm{d}j + \int c_t^* a_{jt}\mathrm{d}j + \theta\int s_{jt}a_{jt}\mathrm{d}j\right].$$

Use the first condition to substitute for $\int s_{jt}\mathrm{d}j$ and the global demand constraint $\bar{c}_t = c_t^*$ to reduce the price setting to

$$\mathcal{O}\left(\epsilon^3\right) = \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[-c_t^* - \frac{1}{2}\theta^2\int\left(s_{jt}+\sigma\bar{c}_t - a_{jt}\right)^2\mathrm{d}j + \frac{1}{2}\theta\left(\theta-1\right)\int a_{jt}^2\mathrm{d}j + \frac{1}{2}\left(\sigma-1\right)\bar{c}_t^2\right].$$

Finally, express the linear global demand, $\mathbb{E}\sum_{t=0}^{\infty}\beta^t c_t^*$, as a function of other variables, and plug it in the welfare to arrive at

$$U_i = -\mathbb{E}\sum_{t=0}^{\infty}\beta^t \frac{L}{2}\left[\sigma\tilde{y}_{it}^2 + L^{-1}\varphi\Pi_{iit}^2 + \omega\frac{\theta^2}{\theta-1}\int \tilde{s}_{jt}^2 \mathrm{d}j + t.i.p. + \mathcal{O}\left(\epsilon^3\right)\right].$$

This is the U.S. welfare loss function (27).

Now we can formulate the linear-quadratic policy problem for the U.S. One way to do this is by rewriting the loss function through the terms of trade gap instead of the output gap (remember from (A23) that these gaps are proportional to each other). Ultimately, the policy problem can be expressed as

$$\min_{\left\{\tilde{s}_{it},\tilde{s}_{jt},\Pi_{iit},\Pi_{jt}^*\right\}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t \frac{L}{2}\left[\theta\tilde{s}_{it}^2 + L^{-1}\varphi\Pi_{iit}^2 + \omega\frac{\theta^2}{\theta-1}\int \tilde{s}_{jt}^2 \mathrm{d}j + t.i.p. + \mathcal{O}\left(\epsilon^3\right)\right]$$

$$\Pi_{iit} = \beta\mathbb{E}_t\Pi_{iit+1} + \frac{\varepsilon-1}{\tau_{Ri}\varphi}\left[(1-\gamma)+\gamma\Psi_i^{-\theta}\right]A_i^{\theta-1}\tilde{s}_{it} + \mathcal{O}\left(\epsilon^2\right),$$

$$\Pi_{jt}^* = \beta\mathbb{E}_t\Pi_{jt+1}^* + \frac{\theta-1}{\varphi}\Psi_i^{1-\theta}A_i^{\theta-1}\tilde{s}_{jt} + \mathcal{O}\left(\epsilon^2\right),$$

$$\Pi_{iit} - \Pi_{jt}^* = \tilde{s}_{it-1} - \tilde{s}_{it} - \tilde{s}_{jt-1} + \tilde{s}_{jt} + a_{it-1} - a_{it} - a_{jt-1} + a_{jt} + \mathcal{O}\left(\epsilon^2\right).$$

All ex-ante constraints have dropped out since, ultimately, all "small-letter" variables are linear functions of shocks, and shocks are zero mean. Out of the ex-post constraints, we have used the price index constraint (15) to get rid of q_{it}, the risk-sharing (19) to get rid of c_{it}, the global terms of trade normalization (A4) to get rid of the global demand $\bar{c}_t$. Then all we are left with is the ex-post price setting for the U.S. firms (25), the ex-post export price setting for the non-U.S. countries (23), and the dynamic constraint (A8), which links the inflation definitions with the terms of trade dynamics.

Note that this problem does not depend on the optimal policy of the non-U.S. countries. This is due to the fact that the global demand constraint, $C_t^* \equiv \int Q_{jt}^{-\theta}C_{jt}\mathrm{d}j = \int Q_{jt}^{\frac{1}{\sigma}-\theta}\mathrm{d}j\bar{C}_t$, has collapsed to just $C_t^* = \bar{C}_t$. Then, similar to the logic from the static model (Appendix A.2.4), this constraint will follow by construction from the budget constraints of non-U.S. countries and global balances. Even though the U.S. problem also depends on $\bar{C}_t$ through the risk-sharing (19) (and thus, in general, $\bar{C}_t$ depends not only on C_t^*, but also on Q_{jt}, which, in turn, are determined by the non-U.S. policies), under the Faia-Monacelli parametrization $\sigma\theta = 1$, $\bar{C}_t$ is equivalent to C_t^*, and thus can be freely chosen by the U.S.

Finally, it is straightforward to see that the U.S. can not simultaneously close both the domestic and the global gaps, $\tilde{s}_{it}^2$ and $\int \tilde{s}_{jt}^2\mathrm{d}j$, thus the optimal policy will balance between the two and depend on foreign shocks a_{jt}. ■

A.3.3 Gains from DCP

Proof of Proposition 7 First, let's derive the non-U.S. welfare loss function (28).

Once again, let's start with the utility function (1), the market clearing (16), the ex-ante budget constraint (21), and the price index constraint (15), rewritten as

$$U_i = \mathbb{E}\sum_{t=0}^{\infty}\beta^t \left[\frac{C_{it}^{1-\sigma}}{1-\sigma} - (1-\gamma) A_{it}^{-1}\Phi_{it}^{\theta}S_{it}^{\theta}Q_{it}^{-\theta}C_{it} - \gamma A_{it}^{-1}S_{it}^{\theta}C_t^{*} - \frac{\varphi}{2}(1-\gamma)\Pi_{iit}^2 - \frac{\varphi}{2}\gamma\Pi_{it}^{*2}\right],$$

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t \bar{C}_t^{-\sigma}\left[S_{it}^{\theta-1}C_t^{*} - Q_{it}^{-\theta}C_{it}\right] = 0,$$

$$Q_{it}^{\theta-1} = (1-\gamma)(\Phi_{it}S_{it})^{\theta-1} + \gamma.$$

The key difference from the U.S. is that now we have state- and time-dependent Φ_{it} instead of a constant Ψ_i and that we have two inflation rates instead of one. Let's take the second-order approximations around the non-stochastic steady state to all three conditions,

$$\begin{aligned} U_i = \mathbb{E}\sum_{t=0}^{\infty}\beta^t L &\left[\omega\Psi c_{it} - \theta(1-\omega)s_{it} + \theta(1-\omega)(q_{it}-\phi_{it}) - \omega c_t^{*} - \omega\theta s_{it}\right. \\ &+\frac{1}{2}(1-\omega+\omega\Psi)(1-\sigma)c_{it}^2 - \frac{1}{2}(1-\omega)(c_{it}-a_{it}+\theta s_{it}-\theta(q_{it}-\phi_{it}))^2 \\ &\left.-\frac{1}{2}\omega(c_t^{*}-a_{it}+\theta s_{it})^2 - L^{-1}\frac{\varphi}{2}(1-\gamma)\Pi_{iit}^2 - L^{-1}\frac{\varphi}{2}\gamma\Pi_{it}^{*2} + a_{it} + \mathcal{O}(\epsilon^3)\right], \end{aligned}$$

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[(\theta-1)s_{it} + c_t^{*} + \theta q_{it} - c_{it} + \frac{1}{2}((\theta-1)s_{it}+c_t^{*}-\sigma\bar{c}_t)^2 - \frac{1}{2}(-\theta q_{it}+c_{it}-\sigma\bar{c}_t)^2\right] = \mathcal{O}(\epsilon^3),$$

$$q_{it} = \frac{1-\omega}{1-\omega+\omega\Psi}(s_{it}+\phi_{it}) + \frac{1}{2}(\theta-1)\frac{(1-\omega)\omega\Psi}{(1-\omega+\omega\Psi)^2}(s_{it}+\phi_{it})^2 + \mathcal{O}(\epsilon^3),$$

where L is again the steady state-level of L_{it}, ω is the steady-state measure of openness.

Use the budget constraint to express the first-order consumption, $\mathbb{E}\sum_{t=0}^{\infty}\beta^t c_{it}$, as a function of other variables, and plug it in the welfare along with the price index constraint to get

$$\begin{aligned} U_i = \mathbb{E}\sum_{t=0}^{\infty}\beta^t L &\left[\omega(\Psi-1)c_t^{*} - L^{-1}\frac{\varphi}{2}(1-\gamma)\Pi_{iit}^2 - L^{-1}\frac{\varphi}{2}\gamma\Pi_{it}^{*2}\right. \\ &-\frac{1}{2}(1-\omega+\omega\Psi)\sigma\tilde{c}_{it}^2 - \frac{1}{2}\theta\frac{\omega\Psi}{1-\omega}(1-\omega+\omega\Psi)\tilde{q}_{it}^2 - \frac{1}{2}\theta\omega\tilde{s}_{it}^2 \\ &\left.+\frac{1}{2}\omega(\Psi-1)c_t^{*2} + \frac{1}{2}\omega\frac{\omega\Psi^2+1-\omega+\Psi}{1-\omega+\omega\Psi}\sigma\bar{c}_t^2 - \omega\Psi\sigma c_t^{*}\bar{c}_t + \omega c_t^{*}a_{it} - \omega\bar{c}_t a_{it} + \frac{1}{2}(\theta-1)a_{it}^2 + \mathcal{O}(\epsilon^3)\right]. \end{aligned}$$

Here the consumption and terms of trade gaps, $\tilde{c}_{it}$ and $\tilde{s}_{it}$, are defined in (A22), and the real exchange rate gap is the deviation of q_{it} from its flexible price value,

$$\tilde{q}_{it} \equiv q_{it} + \sigma \frac{1-\omega}{1-\omega+\omega\Psi}\bar{c}_t - \frac{1-\omega}{1-\omega+\omega\Psi}a_{it} = \frac{1-\omega}{1-\omega+\omega\Psi}\left(\tilde{s}_{it} + \tilde{\phi}_{it}\right), \tag{A24}$$

where the last equality follows from the first-order approximation to the price index constraint (15). The law of one price deviation gap is just equal to its level, $\tilde{\phi}_{it} \equiv \phi_{it}$, since the law of one price holds in the flexible price equilibrium.

Using these definitions, and the first-order approximations to the market clearing (16) and to the risk-sharing (19), one can show that

$$\tilde{y}_{it} = \theta\left(1-\omega\right)\tilde{\phi}_{it} + \theta\tilde{s}_{it} + \mathcal{O}\left(\epsilon^2\right), \quad \tilde{q}_{it} = \sigma\tilde{c}_{it} + \mathcal{O}\left(\epsilon^2\right), \tag{A25}$$

instead of (A23), where the output gap is still defined in (A21). Then, the welfare can be rewritten as

$$U_i = \mathbb{E}\sum_{t=0}^{\infty}\beta^t L\left[\omega\left(\Psi-1\right)c_t^* - \frac{1}{2}\sigma\tilde{y}_{it}^2 - \frac{1}{2}\omega\left(1-\omega\right)\theta\tilde{\phi}_{it}^2 - L^{-1}\frac{\varphi}{2}\left(1-\gamma\right)\Pi_{iit}^2 - L^{-1}\frac{\varphi}{2}\gamma\Pi_{it}^{*2}\right.$$
$$\left. + \frac{1}{2}\omega\left(\Psi-1\right)c_t^{*2} + \frac{1}{2}\omega\frac{\omega\Psi^2+1-\omega+\Psi}{1-\omega+\omega\Psi}\sigma\bar{c}_t^2 - \omega\Psi\sigma c_t^*\bar{c}_t + \omega c_t^* a_{it} - \omega\bar{c}_t a_{it} + \frac{1}{2}\left(\theta-1\right)a_{it}^2 + \mathcal{O}\left(\epsilon^3\right)\right].$$

After that, once again, one can repeat the same steps as for the U.S. welfare loss function (Appendix A.3.2), that is use second-order approximations to the global terms of trade normalization (A4), to the ex-ante export price setting of the non-U.S. countries (23), and use the global demand constraint $C_t^* = \bar{C}_t$ to arrive at the non-U.S. welfare loss function (28)

$$U_i = -\mathbb{E}\sum_{t=0}^{\infty}\beta^t\frac{L}{2}\left[\sigma\tilde{y}_{it}^2 + \omega\left(1-\omega\right)\theta\tilde{\phi}_{it}^2 + L^{-1}\varphi\left(\left(1-\gamma\right)\Pi_{iit}^2 + \gamma\Pi_{it}^{*2}\right) + \omega\frac{\theta^2}{\theta-1}\int\tilde{s}_{jt}^2\mathrm{d}j + t.i.p. + \mathcal{O}\left(\epsilon^3\right)\right].$$

Now, let's show that under the domestic price stabilization in all countries, the U.S. welfare is always higher.

First, note that under the domestic price stabilization in the U.S., $\Pi_{00t} = 0$ by definition, and then the ex-post price setting (25) implies $\tilde{s}_{0t} = 0$, which then from (A23) leads to $\tilde{y}_{0t} = 0$. Thus, under this policy, the U.S. can close all of its domestic gaps.

Second, under the domestic price stabilization in the non-U.S. countries, once again $\Pi_{jjt} = 0$ by definition, but the ex-post domestic price setting (22) then implies $\tilde{\phi}_{jt} + \tilde{s}_{jt} = 0$, which then from (A24) leads to $\tilde{q}_{jt} = 0$. Still, as it can be seen from (A25), $\tilde{y}_{jt} \neq 0$, and both $\tilde{\phi}_{jt} \neq 0$ and $\tilde{s}_{jt} \neq 0$. It can be shown from the ex-post export price setting (23) that $\tilde{s}_{jt} \neq 0$ leads to $\Pi_{jt}^{*2} \neq 0$.

Importantly, it can be shown that both the U.S. loss function (27) and the non-U.S. loss function (28) have the same "terms independent of policy", "$t.i.p.$" Then, under symmetric distribution of shocks in all countries, we can take a difference, and arrive at

$$U_0 - U_j = \mathbb{E}\sum_{t=0}^{\infty}\beta^t \frac{L}{2}\left[\sigma\tilde{y}_{jt}^2 + \omega(1-\omega)\theta\tilde{\phi}_{jt}^2 + L^{-1}\varphi\gamma\Pi_{jt}^{*2} + \mathcal{O}\left(\epsilon^3\right)\right],$$

which then implies that $U_0 > U_j$.

Finally, the U.S. policy under commitment balances between closing the domestic gaps, $\tilde{y}_{0t}^2$ and Π_{00t}^2, and the global gap $\int \tilde{s}_{it}^2 \mathrm{d}i$. In doing so, the U.S. improves the expected welfare of all countries. In particular, all countries get to benefit from lower $\mathbb{E}\int \tilde{s}_{0t}^2 \mathrm{d}j$. Note also that for a non-U.S. country it can be shown that

$$\sigma\tilde{y}_{jt}^2 + \omega(1-\omega)\theta\tilde{\phi}_{jt}^2 = \theta(1-\omega)\left(\tilde{s}_{jt} + \tilde{\phi}_{jt}\right)^2 + \theta\omega\tilde{s}_{jt}^2 + \mathcal{O}\left(\epsilon^3\right),$$

and the optimal non-U.S. policy leads to $\tilde{s}_{jt} + \tilde{\phi}_{jt} = 0$. Then, the only "open" gap for a non-U.S. country is the terms of trade gap $\tilde{s}_{jt}^2$ and associated with it export inflation Π_{jt}^{*2}. As long as the U.S. policy helps to reduce the expected global terms of trade gap, $\mathbb{E}\int \tilde{s}_{it}^2 \mathrm{d}i = \int \mathbb{E}\tilde{s}_{it}^2 \mathrm{d}i = \mathbb{E}\tilde{s}_{it}^2$ (where the last equality follows from the ex-ante symmetry of the non-U.S. countries), it can only help to close the non-U.S. terms of trade gap. Thus, the U.S. policy under commitment increases the U.S. welfare by less than it increases the non-U.S. welfare (relative to the domestic price stabilization). Then it's possible that under optimal policy the U.S. welfare could be smaller than the non-U.S. welfare. ■

A.3.4 Currency union

Preliminaries First, let's derive all the equilibrium conditions for the Eurozone problem.

Most importantly, the import price index for the Eurozone, which we denote as $\tilde{P}_{it}^*$, is different from the global import price index P_t^*. While the latter is set and is sticky in dollars, the former is set and is sticky in euros. Then, the Eurozone monetary policy is going to cause the law of one price deviation for the bundle of import goods, which we denote as $\Phi_{it} = \mathcal{E}_{it}P_t^*/\tilde{P}_{it}^*$. Then we define the real exchange rate for the Eurozone as $Q_{it} \equiv \tilde{P}_{it}^*/P_{it}$ and the terms of trade as $S_{it} \equiv \tilde{P}_{it}^*/\left(\Psi_i P_{iit}\right)$ (here we have used the law of one price between domestic and export goods for the Eurozone, $\Psi_i P_{iit} = \mathcal{E}_{it}P_{it}^*$). Then the price index (6) becomes

$$Q_{it}^{\theta-1} = \gamma + (1-\gamma)\left(\Psi_i S_{it}\right)^{\theta-1}.$$

The risk-sharing changes to

$$C_{it} = \Phi_{it}^{\frac{1}{\sigma}} Q_{it}^{\frac{1}{\sigma}} \Lambda_i^{-\frac{1}{\sigma}} \bar{C}_t,$$

the ex-ante budget constraint becomes

$$\mathbb{E}\sum_t \beta^t \bar{C}_t^{-\sigma}\left[S_{it}^{\theta-1}\Phi_{it}^{\theta-1}C_t^* - \Phi_{it}^{\frac{1}{\sigma}-1}Q_{it}^{\frac{1}{\sigma}-\theta}\Lambda_i^{-\frac{1}{\sigma}}\bar{C}_t\right] = 0.$$

The market clearing is

$$Y_{it} = A_{it}L_{it} = (1-\gamma)\,\Psi_i^{\theta} S_{it}^{\theta} Q_{it}^{\frac{1}{\sigma}-\theta} \Lambda_i^{-\frac{1}{\sigma}} \Phi_{it}^{\frac{1}{\sigma}} \bar{C}_t + \gamma S_{it}^{\theta} \Phi_{it}^{\theta} C_t^{*}.$$

Next, the individual firm's problem in the Eurozone becomes very similar to the problem of U.S. firms (13), since both domestic and export prices are set in producer currency,

$$\max_{\{P_t\}} \mathbb{E}\sum_{t=0}^{\infty}\Theta_{it}\left[\left(P_t - \tau_i MC_{it}\right)\left(\frac{P_t}{P_{iit}}\right)^{-\varepsilon} Y_{it} - \frac{\varphi}{2}\tau_{Ri}\left(\frac{P_t}{P_{t-1}} - 1\right)^2 W_{it}\right].$$

Taking the optimality conditions leads to the ex-post price setting

$$\begin{aligned}\Pi_{iit}\left(\Pi_{iit}+1\right) = & -\frac{\kappa}{\Psi_i}\left(1 - \frac{\Psi_i S_{it}}{Q_{it}}\frac{C_{it}^{\sigma}}{A_{it}}\right)\left(\left(1-\gamma\right)\Psi_i^{\theta} Q_{it}^{-\theta} C_{it} + \gamma \Phi_{it}^{\theta} C_t^{*}\right) Q_{it} S_{it}^{\theta-1} C_{it}^{-\sigma} \\ & + \beta\mathbb{E}\Pi_{iit+1}\left(\Pi_{iit+1}+1\right),\end{aligned}$$

and the corresponding ex-ante price setting $\Pi_{ii0} = 0$.

Moreover, now the non-U.S. firms that export to the Eurozone set their prices in euros. An individual firm's problem is

$$\max_{\{P_t\}} \mathbb{E}\sum_{t=0}^{\infty}\Theta_{jt}\left[\left(\frac{\mathcal{E}_{jt}}{\mathcal{E}_{it}}P_t - \tau_j MC_{jt}\right)\left(\frac{\Psi_j P_t}{P_{jit}}\right)^{-\varepsilon}\left(\frac{P_{jit}}{\tilde{P}_{it}^{*}}\right)^{-\theta} Q_{it}^{-\theta} C_{it} - \gamma\frac{\varphi}{2}\tau_{Rj}\left(\frac{P_t}{P_{t-1}} - 1\right)^2 W_{jt}\right],$$

where P_t is set in currency i, the euro, and the bilateral nominal exchange rate, $\mathcal{E}_{jt}/\mathcal{E}_{it}$, converts it to the producer currency j. Next, let's define the special terms of trade for a non-U.S. country j, $S_{jit} \equiv \mathcal{E}_{it}P_t^{*}/P_{jit}$, which reflect the price of j's imports relative to its export to the Eurozone, i. Then, taking the optimality conditions leads to the following ex-post price setting

$$\Pi_{jit}\left(\Pi_{jit}+1\right) = \beta\mathbb{E}\Pi_{jit+1}\left(\Pi_{jit+1}+1\right) - \frac{\kappa}{\Psi_j}\left(1 - \Psi_j S_{jit}\frac{\bar{C}_t^{\sigma}}{A_{jt}}\right)\Phi_{it}^{-\theta} S_{jit}^{\theta-1} Q_{it}^{-\theta} C_{it} \bar{C}_t^{-\sigma},$$

and the corresponding ex-ante price setting $\Pi_{ji0} = 0$, where the inflation rate is defined as $\Pi_{jit} \equiv P_{jit}/P_{jit-1} - 1$. Note that the key difference between this price setting and the non-U.S. export price setting (23) is that the former depends on the Eurozone demand specifically, while the latter does not.

Proof of Proposition 8 Now we can set up the Eurozone policy problem,

$$\max \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\frac{C_{it}^{1-\sigma}}{1-\sigma} - L_{it} - \frac{\varphi}{2}\Pi_{iit}^2\right]$$

$$\text{s.t.}\quad A_{it}L_{it} = (1-\gamma)\,\Psi_i^{\theta} S_{it}^{\theta} \Lambda_i^{-\frac{1}{\sigma}} \Phi_{it}^{\frac{1}{\sigma}} \bar{C}_t + \gamma S_{it}^{\theta} \Phi_{it}^{\theta} C_t^*,$$

$$\mathbb{E}\sum_{t=0}^{\infty} \beta^t \bar{C}_t^{-\sigma} \left[S_{it}^{\theta-1} \Phi_{it}^{\theta-1} C_t^* - \Lambda_i^{-\frac{1}{\sigma}} \Phi_{it}^{\frac{1}{\sigma}-1} \bar{C}_t \right] = 0,$$

$$C_{it} = Q_{it}^{\frac{1}{\sigma}} \Lambda_{it}^{-\frac{1}{\sigma}} \Phi_{it}^{\frac{1}{\sigma}} \bar{C}_t,$$

$$Q_{it}^{\theta-1} = (1-\gamma)\,(\Psi_i S_{it})^{\theta-1} + \gamma,$$

$$\Pi_{iit}\,(\Pi_{iit}+1) = \beta\mathbb{E}\Pi_{iit+1}\,(\Pi_{iit+1}+1) - \frac{\kappa}{\Psi_i}\left(1 - \frac{\Psi_i S_{it}}{Q_{it}}\frac{C_{it}^{\sigma}}{A_{it}}\right)\left((1-\gamma)\,\Psi_i^{\theta} Q_{it}^{-\theta} C_{it} + \gamma \Phi_{it}^{\theta} C_t^*\right)\frac{Q_{it} S_{it}^{\theta-1}}{C_{it}^{\sigma}},$$

$$\mathbb{E}\sum_{t=0}^{\infty} \beta^t \left(1 - \frac{\Psi_i S_{it}}{Q_{it}}\frac{C_{it}^{\sigma}}{A_{it}}\right)\left((1-\gamma)\,\Psi_i^{\theta} Q_{it}^{-\theta} C_{it} + \gamma \Phi_{it}^{\theta} C_t^*\right)\frac{Q_{it} S_{it}^{\theta-1}}{C_{it}^{\sigma}} = 0,$$

$$\Pi_{jit}\,(\Pi_{jit}+1) = \beta\mathbb{E}\Pi_{jit+1}\,(\Pi_{jit+1}+1) - \frac{\kappa}{\Psi_j}\left(1 - \Psi_j S_{jit}\frac{\bar{C}_t^{\sigma}}{A_{jt}}\right)\Phi_{it}^{-\theta} S_{jit}^{\theta-1} Q_{it}^{-\theta} C_{it} \bar{C}_t^{-\sigma},$$

$$\mathbb{E}\sum_{t=0}^{\infty} \beta^t \left(1 - \Psi_j S_{jit}\frac{\bar{C}_t^{\sigma}}{A_{jt}}\right)\Phi_{it}^{-\theta} S_{jit}^{\theta-1} Q_{it}^{-\theta} C_{it} \bar{C}_t^{-\sigma} = 0,$$

$$Q_{it} = \frac{\tilde{P}_{it}^*}{P_{it}},\quad S_{it} = \frac{\tilde{P}_{it}^*}{\Psi_i P_{iit}},\quad \Phi_{it} = \frac{\mathcal{E}_{it} P_t^*}{\tilde{P}_{it}^*},\quad S_{jit} = \frac{\mathcal{E}_{it} P_t^*}{P_{jit}},$$

$$\Pi_{iit} = \frac{P_{iit}}{P_{iit-1}} - 1,\quad \Pi_{jit} = \frac{P_{jit}}{P_{jit-1}} - 1,$$

$$\Psi_i = \frac{\mathcal{E}_{it} P_{it}^*}{P_{iit}},\quad C_t^* = \bar{C}_t,$$

$$\tilde{P}_{it}^* = \left(\int (P_{jit})^{1-\theta}\,\mathrm{d}j\right)^{\frac{1}{1-\theta}}.$$

Policymaker chooses C_{it}, L_{it}, Q_{it}, Φ_{it}, S_{it}, Π_{iit}, Λ_i, $\mathcal{E}_{it}$, P_{it}, P_{it}^*, P_{iit}, $\tilde{P}_{it}^*$, P_{jit}, Π_{jit} state-by-state, while C_t^*, $\bar{C}_t$, and P_t^* are taken as given.

Here we included the market clearing, the ex-ante budget constraint, the risk-sharing, the price index constraint, the ex-post and the ex-ante domestic price setting, the ex-post and the ex-ante import price setting, as well as definitions of various prices and of inflation rates, the law of one price, the global demand constraint, and the definition of the import price index.

Crucially, the Eurozone can not affect global demand C_t^* or $\bar{C}_t$. However, note that in this policy problem, the global demand often enters multiplicatively with the law of one price deviation, Φ_{it}, which the Eurozone can affect. In fact, let's denote the "effective" global demand as $Z_{it} \equiv \Phi_{it}^{\theta} C_t^*$ and adjusted terms of trade as $\tilde{S}_{jit} \equiv \tilde{P}_{it}^*/P_{jit}$. Then, following the primal approach, we can ultimately rewrite this policy problem as

$$\max \mathbb{E}\sum_{t=0}^{\infty}\beta^{t}\left[\frac{Q_{it}^{\frac{1}{\sigma}-1}\Lambda_{i}^{1-\frac{1}{\sigma}}Z_{it}^{1-\sigma}}{1-\sigma}-(1-\gamma)\,A_{it}^{-1}\Psi_{i}^{\theta}S_{it}^{\theta}\Lambda_{i}^{-\frac{1}{\sigma}}Z_{it}-\gamma A_{it}^{-1}S_{it}^{\theta}Z_{it}-\frac{\varphi}{2}\Pi_{iit}^{2}\right]$$

$$\text{s.t.}\quad \mathbb{E}\sum_{t=0}^{\infty}\beta^{t}Z_{it}^{1-\sigma}\left[S_{it}^{\theta-1}-\Lambda_{i}^{-\frac{1}{\sigma}}\right]=0,$$

$$Q_{it}^{\theta-1}=(1-\gamma)\left(\Psi_{i}S_{it}\right)^{\theta-1}+\gamma,$$

$$\Pi_{iit}\left(\Pi_{iit}+1\right)=-\kappa\left(1-\Psi_{i}\frac{S_{it}Z_{it}^{\sigma}}{\Lambda_{it}A_{it}}\right)\left[(1-\gamma)\,\Psi_{i}^{\theta-1}S_{it}^{\theta-1}\Lambda_{i}^{1-\frac{1}{\sigma}}Z_{it}^{1-\sigma}+\gamma\Psi_{i}^{-1}\Lambda_{i}S_{it}^{\theta-1}Z_{it}^{1-\sigma}\right]$$
$$+\beta\mathbb{E}\Pi_{iit+1}\left(\Pi_{iit+1}+1\right),$$

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^{t}\left(1-\Psi_{i}\frac{S_{it}Z_{it}^{\sigma}}{\Lambda_{it}A_{it}}\right)\left[(1-\gamma)\,\Psi_{i}^{\theta-1}S_{it}^{\theta-1}\Lambda_{i}^{1-\frac{1}{\sigma}}Z_{it}^{1-\sigma}+\gamma\Psi_{i}^{-1}\Lambda_{i}S_{it}^{\theta-1}Z_{it}^{1-\sigma}\right]=0,$$

$$\Pi_{jit}\left(\Pi_{jit}+1\right)=\beta\mathbb{E}\Pi_{jit+1}\left(\Pi_{jit+1}+1\right)-\frac{\kappa}{\Psi_{j}}\left(1-\frac{\Psi_{j}Z_{it}^{\sigma}}{A_{jt}}\tilde{S}_{jit}\right)\Lambda_{i}^{-\frac{1}{\sigma}}Z_{it}^{1-\sigma}\tilde{S}_{jit}^{\theta-1},$$

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^{t}\left(1-\frac{\Psi_{j}Z_{it}^{\sigma}}{A_{jt}}\tilde{S}_{jit}\right)Z_{it}^{1-\sigma}\tilde{S}_{jit}^{\theta-1}=0,$$

$$\Pi_{iit}=\frac{S_{it-1}}{S_{it}}\frac{\tilde{P}_{it}^{*}}{\tilde{P}_{it-1}^{*}}-1,\quad \Pi_{jit}=\frac{\tilde{S}_{jit-1}}{\tilde{S}_{jit}}\frac{\tilde{P}_{it}^{*}}{\tilde{P}_{it-1}^{*}}-1,$$

$$\int\tilde{S}_{jit}^{\theta-1}\mathrm{d}j=1.$$

Note that this policy problem is almost identical to the U.S. policy problem. The global demand C_t^* or $\bar{C}_t$ does not appear in this problem at all once we have introduced the "effective" global demand Z_{it}. So, it turns out, that the Eurozone can achieve the same outcomes as the U.S. by manipulating the law of one price deviation $\mathit{\Phi}_{it}$ instead of manipulating the global demand C_t^*.

The only difference between this problem and the U.S. problem is that the import price setting in the former depends on Λ_i, while the import price setting in the latter did not. However, it can be shown that this constant is of the second order, $\Lambda_i = \mathcal{O}\left(\epsilon^2\right)$, and thus one can repeat all the steps to derive the welfare loss function. Crucially, one needs to take the second-order approximation to the non-U.S. export price setting (23), and not to the Eurozone import price setting, and so this difference in two problems will not affect the welfare loss function derivation. Then, once the linear-quadratic problem is set up, we need to take only the first-order approximation to the import price setting, and thus Λ_i will not show up there. Thus, the linear-quadratic problems of the U.S. and the Eurozone are isomorphic to each other. It then follows that the two countries achieve the same welfare up to the second order. ■