Value Added and Productivity Linkages Across Countries*

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Abstract

What is the relationship between international trade and business cycle synchronization? Using data from 40 countries, I find that trade in intermediate inputs plays a significant role in synchronizing GDP fluctuations across countries while trade in final goods is not significant. Motivated by this new fact, I build a model of international trade in intermediates that is able to replicate more than 85% of the empirical trade-comovement slope, offering the first quantitative solution for the "Trade Comovement Puzzle". The model relies on two key assumptions: (i) price distortions due to monopolistic competition and (ii) fluctuations in the mass of firms serving each country. The combination of those ingredients creates a link between domestic measured productivity and foreign shocks through trade linkages. Finally, I provide evidence for the importance of those elements in the link between foreign shocks and domestic GDP.

Keywords: International Trade, International Business Cycle Comovement, Networks, Input-Output Linkages

JEL Classification Numbers: F12, F44, F62

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1 Introduction

The "Trade Comovement Puzzle", uncovered by Kose and Yi (2001 and 2006), refers to the inability of international business cycle models to quantitatively account for the positive empirical relationship between international trade and GDP comovement.[1] Using different versions of the workhorse international real business cycle (IRBC) model, several authors have succeeded to qualitatively replicate the positive link between trade and GDP comovement but fall short of the quantitative relationship by an order of magnitude.[2]

This paper has three main contributions. First, it contributes to empirical investigations of the association between bilateral trade and GDP comovement and shows that trade in intermediate inputs plays a significant role in synchronizing GDP fluctuations across countries. Second, it proposes a general equilibrium dynamic model of trade in inputs with monopolistic pricing and firms entry/exit. In the benchmark calibration, the model is able to replicate more than 85% of the trade-comovement slope, hence offering the first quantitative solution for the "Trade Comovement Puzzle". Finally, it provides new empirical support for the role of the two key ingredients generating the quantitative results, namely price distortions and fluctuations of the mass of firms serving every market.

Empirics Since the seminal paper by Frankel and Rose (1998), a large empirical literature has studied cross countries' GDP synchronization, showing that bilateral trade is an important and robust determinant of GDP correlation. I update those findings and show that business cycle synchronization is associated with trade in intermediate inputs while trade in final good is found insignificant. The paper refines previous analysis by constructing a panel dataset of 40 countries consisting of four 10-years windows ranging from 1970 to 2009, which allows for dyadic as well as time windows fixed effects. In this setting, I show that the positive relationship between trade and comovement is solely driven by trade in intermediates whereas trade in final good is found insignificant. This new finding suggests that the rise in global value chains plays a particular role in the synchronization of GDP across countries.

[1] For empirical studies on this topic, among many others, see Frankel and Rose (1998), Clark and van Wincoop (2001), Imbs (2004), Baxter and Kouparitsas (2005), Kose and Yi (2006), Calderon, Chong, and Stein (2007), Inklaar, Jong-A-Pin, and Haan (2008), di Giovanni and Levchenko (2010), Ng (2010), Liao and Santacreu (2015), di Giovanni, Levchenko and Mejean (2016) and Duval et al (2016)

[2] For quantitative studies, see Kose and Yi (2001, 2006), Burstein, Kurz and Tesar (2008), Johnson (2014) or Liao and Santacreu (2015)

Theory As discussed in Kehoe and Ruhl (2008), international production linkages alone are not sufficient to generate a strong link between domestic GDP and foreign shocks. The intuition is as follows: GDP is the sum of value added produced within a country and is computed by statistical agencies as the difference between final production and imports, measured using base prices. When imports are used in production, price taking firms choose a quantity of imported input that equalizes their marginal cost and their marginal revenue. Up to a first order approximation, changes in the quantity of imported input yields exactly as much benefit as it brings costs. Hence, foreign shocks have an impact on domestic value added *only* to the extent that they impact the supply of domestic factors. This "negative result" is at the heart of the Trade-Comovement Puzzle. In this paper, I incorporate two ingredients that create an endogenous link between domestic productivity and foreign shock through trade linkages.

First, when firms chose their price, they do not equalize the marginal cost and the marginal revenue product of their inputs. As noted previously by Hall (1988) and discussed in Basu and Fernald (2002), Gopinath and Neiman (2014) or Llosa (2014), this wedge between marginal cost and marginal product of inputs implies that any change in intermediate input usage is associated with a first order change in value added, over and beyond changes domestic factors.[3]

Second, fluctuations along the extensive margin have the potential to create an additional link between domestic productivity and foreign shocks. With love of variety, a firm with more suppliers produces a higher level of output for the same level of inputs. Hence, any variation in the mass of suppliers leads to a first order productivity change. Love for variety is a form of increasing return: a firm with more suppliers is more efficient at transforming inputs into output, which allows measured value added to react over and beyond variations in domestic factor supply.

Quantitative analysis Motivated by the discussion above, I propose a multi-country dynamic general equilibrium a model of international trade in inputs that relies on (i) monopolistic competition and (ii) fluctuations in the mass of firms serving each country. Production is performed by a continuum of heterogeneous firms combining in a Cobb-Douglas fashion labor, capital and a nested CES aggregate of intermediate inputs bought from domestic and foreign firms.

I calibrate the model to 14 countries and a composite "rest-of-the-world" and assess its ability to replicate the strong relationship between trade in inputs and GDP synchronization. Fixed effect regressions on this simulated dataset shows that the model is able to replicate more than

[3]Related to this point, Burstein and Cravino (2015) show that if all firms take prices as given, a change in trade costs can affect aggregate productivity only to the extent that it changes the production possibility frontier at constant prices. This can be interpreted as saying that shocks to the *foreign* trading technology have no impact on aggregate *domestic* TFP if all firms have constant returns to scale and take prices as given.

85% of the trade-comovement *slope* observed in the data, a significant improvement compared to previous studies. Decomposing the role of each ingredient, I show that trade in intermediates alone is not sufficient to replicate the trade-comovement relationship. The addition of monopolistic pricing and extensive margin adjustments increase the simulated trade-comovement slope by a factor seven and allow the model to better fit the data.

Further empirical evidence In the last part of the paper, I provide evidence supporting the modeling assumptions. First, using the Price Cost Margin as a proxy for monopoly power and OECD data at the industry level, I find that countries with higher markups have a GDP that is more systematically negatively correlated with terms-of-trade movements, meaning that they experience a larger GDP decrease when the price of their imports rises. Second, I construct the extensive and intensive margins of trade and regress country-pair GDP correlations on those indexes. A higher degree of business cycle synchronization is associated with an increase in the range of goods traded and is not associated with an increase in the quantity traded for a given set of goods.[4]

Relationship to the literature Starting with Frankel and Rose (1998), a number of papers have studied and confirmed the positive association between trade and comovement in the cross-section.[5] The empirical part of this paper is mostly related to two recent contributions. First, Liao and Santacreu (2015) is the first to study the importance of the extensive margin for GDP and TFP synchronization. Second, di Giovanni et al (2016) uses a *cross-section* of French firms and presents evidence that international input-output linkages at the micro level are a important drivers of the value added comovement observed at the macro level. Their evidence is in line with the findings of this paper and supports the mechanism I develop in the quantitative part but also add elements relative to multinational firms that I do not model.[6]

If the empirical association between bilateral trade and GDP comovement has long been known, the underlying economic mechanism leading to this relationship is still unclear. Using the workhorse IRBC with three countries, Kose and Yi (2006) have shown that the model can

[4]This result is in line with the analysis in Liao and Santacreu (2015) which emphasizes the role of the extensive margin. Compared to them, I am adding the panel dimension by performing fixed effect regression which allows me to control for country-pair fixed effects that can be correlated with trade intensity.

[5]see papers cited in footnote 2.

[6]Relatedly, Ng (2010) uses cross-country data from 30 countries and shows that bilateral production fragmentation has a positive effect on business cycle comovement. The concept of bilateral production fragmentation used is different from this paper as it takes into account only a subset of trade in intermediates, namely imported inputs that are then further embodied in exports. Moreover, the cross section nature of the analysis does not allow neither for dyadic nor time windows fixed effects.

explain at most 10% of the *slope* between trade and business cycle synchronization, leading to what they called the "Trade Comovement Puzzle". Since then, many papers have refined the puzzle, highlighting different ingredients that could bridge the gap between the data and the predictions of classic models.

Burstein, Kurz and Tesar (2008) show that allowing for production sharing among countries can deliver tighter business cycle synchronization if the elasticity of substitution between home and foreign intermediate inputs is extremely low.[7] Arkolakis & Ramanarayanan (2009) analyse the impact of vertical specialization on the relationship between trade and business cycle synchronization. Their Ricardian model with perfect competition does not generate significant dependence of business cycle synchronization on trade intensity, but they show that the introduction of price distortions that react to foreign economic conditions allows their model to better fit the data. Incorporating trade in inputs in an otherwise standard IRBC, Johnson (2014) shows that the puzzle cannot be solved by adding the direct propagation due to the international segmentation of supply chains. Compared to those papers, I add firm entry and exit as well as monopolistic competition and argue that those are key ingredients for the model to deliver quantitative results in line with the data. Liao and Santacreu (2015) build on Ghironi & Melitz (2005) and Alessandria & Choi (2007) to develop a two-country IRBC model with trade in differentiated intermediates. Compare to this paper, I add multinational production with global supply chains which creates a strong interdependency in firms' pricing end export decisions. Furthermore, I extend the quantitative analysis to many countries and show the international propagation of shocks is affected by the whole network of input-output linkages.[8] Finally, a complementary approach has been developed by Drozd, Kolbin and Nosal (2014) which model the dynamics of trade elasticity. Building on Drozd and Nosal (2012), their model features customers accumulation with matching frictions between producers and retailers. Changes in relative marketing capital across country-specific goods create time variations in the trade elasticity which allows the model to better match the data. Similar to my paper, the setup gives rise to a wedge between the price of imports and their marginal product in final good production, but in their case it is driven by the Nash bargaining process over the surplus generated by matches between producers and retailers. The role of firms heterogeneity in international business cycles has been pioneered

[7] In their benchmark simulations, the authors take the value of 0.05 for this elasticity.

[8] In their model, no firm is both an importer and an exporter. While this assumption simplifies the resolution, it prevents any network effect. The absence of production linkages makes it essentially a model of trade in final good only in which domestic and foreign goods are substitutes rather than complements. This, in turn, creates forces toward negative GDP correlation as is illustrated by the fact that when they use an elasticity of substitution equal to 3.1, their model generates a *negative* association between trade and GDP comovement. This underlines the importance of modeling trade in intermediates which creates complementaries in production.

by Ghironi & Melitz (2005) and the business cycle implication of firms' heterogeneity is studied in Fattal-Jaef & Lopez (2014).

The rest of the paper is organized as follows: the second section studies empirically the relationship between trade and GDP synchronization and highlights the important role of trade in intermediate inputs. Section three presents a simple static model of small open economy that provides clear intuitions for the role of markups and entry/exit in generating a link between trade and GDP fluctuations. The fourth section proposes a quantitative model of international trade in intermediate goods with heterogeneous firms and monopolistic competition. In the fifth section, I present the calibration strategy and the quantitative results are presented in section six. Section seven provides further empirical evidence supporting the model while section eight concludes.

2 Empirical Evidence

In this section, I use a sample of 40 countries[9] during period stretching from 1970 to 2009 and update the initial Frankel and Rose (1998) analysis on the relationship between bilateral trade and GDP comovement as well as provide empirical support for the specific role of trade in intermediate inputs.

There are two main findings. First, the empirical association between business cycle synchronization and international trade is significantly reduced when controlling for country-pair and time windows fixed effects. Second, trade in intermediate goods play a high and significant role in explaining GDP comovement, while the explanatory power of trade in final good is found not significant. I first describe the data, then I explain the econometric strategy and finally I present the results in details.

I use annual data on real GDP from the Penn World Table which is transformed in two ways: (i) HP filter with smoothing parameter 6.25 to capture the business cycle frequencies and (ii) log first difference. Trade data come from Johnson and Noguera (2016) who combine data on trade, production, and input use to construct trade flows from 1970 to 2009 separating between trade in final good from the trade in intermediate inputs.

[9] The list of countries is: Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, China, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, India, Indonesia, Ireland, Israel, Italy, Japan, Korea, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Russia, Slovak Republic, Slovenia, Spain, Sweden, Switzerland, Thailand, Turkey, United Kingdom, United States and Vietnam

In a first set of regressions, I construct a symmetric measure of bilateral trade intensity between countries i and j using total trade flows as: $\text{Total}_{ij}=\frac{Total\ Trade_{ij}}{GDP_i+GDP_j}$, which measure the importance of the trade relationship relative to total GDP.[10]

In order to disentangle the influence of trade flows in inputs from the final goods, I further construct the indexes Final_{ij} and Intermediate_{ij} with the same formulation but taking into account only the trade flows in final and intermediate goods respectively.[11]

The extent to which countries have correlated GDP can be influenced by many factors beyond international trade, including correlated shocks, financial linkages, monetary policies, etc... Because those other factors can themselves be correlated with the index of trade proximity in the cross section, using cross-section identification could yield biased results. In order to separate the effect of trade linkages from other elements, I construct a panel dataset by creating four periods of ten years each. In every time window, I compute GDP correlation for all country pairs as well as trade indexes as defined above. The trade index relative to a given time window is then constructed by taking the average of all yearly indexes. Using panel data allows me to control for time invariant country-pair specific factors that are not observables.

I estimate the following equations:

$$(1) \qquad \text{corr}(GDP_{it}^{filtered}, GDP_{jt}^{filtered}) = \alpha_1 + \beta_T \log(\text{Total}_{ijt}) + controls + \epsilon_{1,ijt}$$

$$(2) \qquad \text{corr}(GDP_{it}^{filtered}, GDP_{jt}^{filtered}) = \alpha_2 + \beta_I \log(\text{Intermediate}_{ijt}) + \beta_F \log(\text{Final}_{ijt}) + controls + \epsilon_{2,ijt}$$

In the rest of this section I present two facts that characterize the relationship between GDP synchronization and international trade. Results are gathered in tables 1 and 1

Finding 1: The trade-comovement slope loses significance when controlling for country-pair and time windows fixed effect

The initial Frankel and Rose (1998) finding that bilateral trade correlates with business cycle synchronization does not answer the question of trade's role in transmitting shocks. Using cross-sectional variation shows that country-pairs with higher trade linkages experience more correlated GDP fluctuations but does not rule out omitted variable bias such as, for example, the fact that

[10] In a supplemental appendix, I also used an index defined as $\text{Total}_{ij}=\max\left(\frac{Total\ Trade_{ij}}{GDP_i}, \frac{Total\ Trade_{ij}}{GDP_j}\right)$. This measure has the advantage to take a high value whenever one of the two countries depends heavily on the other for its imports or exports. The empirical and simulated results hold when I use this index.

[11] In appendix, I verify the robustness of my findings using an alternative dataset and method of separating intermediate from final goods. In the STAN database of the OECD, input-output tables have been used at the country level to disentangle trade flows in intermediate and final goods from 1995 to 2014. All results are robust using this categorization.

neighboring countries have at the same time more correlated shocks and larger trade flows. By constructing a panel dataset and controlling for both country-pair and time windows fixed effects, this paper relates to recent studies that try to control for unobserved characteristics.[12]

Using only *within* country-pair variations and controlling for aggregate time windows fixed effects, the positive relationship between trade in GDP correlation still holds for HP filter and first differences as shown in columns (1) and (3) table 1, but they are not as significant as previously found.[13]

Finding 2: Trade in Intermediate inputs plays a strong role in GDP comovement

To investigate further the relationship between trade and GDP comovement at business cycle frequency, columns (2) and (4) of 1 disentangle the effect of trade in intermediate inputs from trade in final goods. The results highlight a specific role for trade in intermediate inputs.[14] With HP filter as well as log difference, the index of trade proximity in intermediate goods is high and significant. According to the point estimates in 1, moving from the 25th to the 75th percentile of trade proximity in intermediate inputs is associated with a GDP comovement increase between 0.1 (column (6)) and 0.12 (column (2)). These findings are robust when using two time windows of 20 years each, as shown in table 2. These results strongly suggest that international supply chains are an important determinant of the degree of business cycle synchronization across countries.[15]

[12]Di Giovanni and Levchenko (2010) includes country pair fixed effects in a large cross-section of industry-level data with 55 countries from 1970 to 1999 in order to test for the relationship between sectoral trade and *output* (not *value-added*) comovement at the industry level. Duval et al (2016) includes country pair fixed and year effects in a panel of 63 countries from 1995 to 2013 and test the importance of *value added* trade in GDP comovement.

[13]See footnote 1 for papers finding a high and robust association between total trade and business cycle comovement using cross-sectional settings.

[14]Di Giovanni and Levchenko (2010) investigate the role of vertical linkages in *output* synchronization (not *value added*) using input-output matrices from the BEA. Their estimates imply that vertical production linkages account for some 30 percent of the total impact of bilateral trade on the business cycle correlation

[15]The results presented here used a fixed effect specification. One might also consider that the variation across country-pairs are assumed to be random and uncorrelated with trade proximity indexes, in which case a random effect treatment would be a better fit. To discriminate between fixed or random effects, I run a Hausman specification test where the null hypothesis is that the preferred model is random effects against the fixed effects. This tests whether the error terms ϵ_{ijt} are correlated with the regressors, with the null hypothesis being they are not. Results display a significant difference ($p < 0.001$), indicating that the two models are different enough to reject the null hypothesis, and hence to reject the random effects in favor of the fixed effect model.

	dependent variable: corr($GDP_i^{filtered}$,$GDP_j^{filtered}$)			
	HP filter		First Difference	
	(1)	(2)	(3)	(4)
log(Total)	0.022**		0.027**	
	(2.07)		(2.55)	
log(Intermediate)		0.053**		0.042*
		(2.18)		(1.87)
log(Final)		-0.030		-0.016
		(-1.25)		(-0.70)
Country-Pair FE	yes	yes	yes	yes
Time Window FE	yes	yes	yes	yes
R-squared (within)	0.153	0.155	0.141	0.142
R-squared (overall)	0.137	0.135	0.132	0.129
N	————————- 2900 ————————-			

t stat. in parentheses, *** means $p < 0.01$, ** means $p < 0.05$ and * means $p < 0.10$

Table 1: Trade and GDP correlation with 10 years time windows

	dependent variable: corr($GDP_i^{filtered}$,$GDP_j^{filtered}$)			
	HP filter			First Difference
	(1)	(2)	(3)	(4)
log(Total)	0.019*		0.017	
	(1.67)		(1.39)	
log(Intermediate)		0.073**		0.074**
		(2.20)		(2.40)
log(Final)		-0.053		-0.057*
		(-1.55)		(-1.85)
Country-Pair FE	yes	yes	yes	yes
Time Window FE	yes	yes	yes	yes
R-squared (within)	0.068	0.075	0.009	0.018
R-squared (overall)	0.115	0.091	0.117	0.050
N	———————————- 1450 ———————————-			

t stat. in parentheses, *** means $p < 0.01$, ** means $p < 0.05$ and * means $p < 0.10$

Table 2: Trade and GDP correlation with 20 years time windows

3 A simple model

In this section, I show in a simple framework why the inclusion of input-output linkages across countries is not sufficient for a model to generate a strong relationship between trade and GDP comovement, and how the inclusion of two new elements (monopolistic pricing and extensive margin adjustments) goes a long way toward generating a link between a shock in a trading partner's economy and domestic GDP. Section 4 will then present a quantitative general equilibrium model with many countries that is able to replicate 85% of the trade-comovement relationship observed in the data, hence proposing the first quantitative solution for the trade comovement puzzle.

For the sake of exposition, I consider here a static small open economy. In such a world, Kehoe and Ruhl (2008, henceforth KR) showed that a change in the price of imported inputs has no impact, up to a first order approximation, on measured productivity. This means that any change in GDP is due to variations in domestic factors supply. I start by briefly reviewing this important result.

3.1 The Kehoe and Ruhl (2008) negative result

The economy produces a final good y, used for consumption and exports, which is produced by combining imported inputs x and domestic factors of production ℓ (possibly a vector) according to:

$$y = F(\ell, x) \tag{1}$$

where $F(.,.)$ has constant returns to scale and is concave with respect to each of its arguments. The final good producer chooses intermediate and imported inputs to maximize its profit taking as given all prices. Optimality requires that factors are paid their marginal product:

$$p_y F_\ell(\ell, x) = w \qquad \text{and} \qquad p_y F_x(\ell, x) = p_x \tag{2}$$

with p_y the final good price, p_x the price of imported inputs x and w the price of domestic factors. Gross Domestic Product is the sum of value added in the country, which is simply the value of final goods minus the value of imported inputs. Importantly, many statistical agencies (and in particular the OECD database used in the empirical analysis above) use *base period prices* when valuing estimated quantities in the construction of GDP.[16] Since prices are kept constant at their base value, we denote them with the superscript b to emphasize the fact that they are treated as parameter and not as endogenous objects:

$$GDP = p_y^b F(\ell, x) - p_x^b . x \tag{3}$$

Let us now compute the first order change in GDP when the Terms-of-Trade ($\equiv p_x$) change:

$$\frac{dGDP}{dp_x} = \underbrace{p_y^b F_\ell(\ell, x) \frac{\partial \ell}{\partial p_x}}_{\text{Factor Supply Effect}} + \underbrace{\frac{\partial x}{\partial p_x}(p_y^b F_x(\ell, x) - p_x^b)}_{\text{Input-Output Effect}} \tag{4}$$

The first term in equation (4) captures the value added change due to variations in factor

[16] In the US, the Bureau of Economic Analysis uses a Fisher chain-weighted price index to construct GDP at time t relative to GDP at time $t-1$ according to:

$$\frac{GDP_t}{GDP_{t-1}} = \left(\frac{\sum_k p_{t-1}^k q_t^k}{\sum_k p_{t-1}^k q_{t-1}^k} \right)^{0.5} \left(\frac{\sum_k p_t^k q_t^k}{\sum_k p_t^k q_{t-1}^k} \right)^{0.5}$$

where k indexes all components of GDP. Intuitively, the Fisher index is a mix between two base period pricing methods where the base price is alternatively the price at $t-1$ and at t.

supply and depends heavily on the degree of substitutability or complementarity between foreign and domestic inputs[17] as well as on the elasticity of factor supply. The second term captures the direct impact that changes of imported input usage have on GDP. With perfect competition, profit maximization insures that $p_y F_x(\ell, x) = p_x$ when using current prices. When base period prices p_y^b and p_x^b are close to their current value,[18] this term disappears. In such a model, any first order change in GDP following a terms of trade shock is solely driven by variations in domestic factor supply. This is the negative result presented in KR: when firms take prices as given, profit maximization insures that the marginal benefit of using an additional unit of imported input x ($p_y F_x(\ell, x)$) is equal to its marginal cost (p_x). Hence, up to a first order approximation, domestic value added is affected by a foreign technological shock only through a change in factor supply. In other words, the measured productivity is not affected to foreign shocks.[19]

Equation (4) encapsulates in a simple way the reasons why even sophisticated RBC models cannot generate a quantitatively important link between trade linkages and GDP comovement. In models with perfect competition and constant returns to scale, the change in GDP after a foreign shock is solely driven by variations in domestic factors supply. Such a change, in turn, is disciplined by (i) the elasticity of labor supply and (ii) the complementarity between domestic and foreign inputs.[20]

3.2 Markups and Love for variety

Consider now a variant of the economy described above with an additional production step: inputs are imported by a continuum of intermediate producers with a linear production function $m = x$. Critically, I now add two new elements: (1) a *price wedge* for intermediate producers $\mu \neq 1$ so that the price of intermediates m is given by $p_m = \mu \times p_x$, and (2) love for variety in the final good production technology in the form of a Dixit-Stiglitz aggregation of intermediates.[21]

[17] The role of complementarity is discussed at length in Burstein et al (2008) or in Boehm et al (2015).

[18] With a Fisher chain-weighted price index in the construction GDP, base period prices are always close to current prices.

[19] Note that an important part of the reasoning rests upon the fact that GDP is constructed using constant base prices. If the prices used to value final goods and imported inputs were to change due to the shock, one would have an additional term in equation (4).

[20] If domestic and foreign inputs are strongly complements, any shock that increases foreign input usage also rises demand for domestic inputs, which increases GDP.

[21] In many models, the elasticity of substitution in the CES aggregation governs at the same time the markup charged by monopolistic competitors and the love degree of love for variety. In order to clearly differentiate the sheer effect of markup from the love for variety, I assume here that the markup μ can take any value, including the case where $\mu = \sigma/(\sigma - 1)$.

The production function in the final good sector is:

$$y = F(\ell, \mathcal{I}) \qquad \text{with } \mathcal{I} = \left(\int_0^{\mathcal{M}} m_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \tag{5}$$

This production function displays love for variety in the following sense: for a given amount of total imports, the larger the mass of input suppliers $\mathcal{M}$, the higher the amount of final production obtainable.

For each variety m_i, there is a producer with a linear technology using imports only:

$$\forall \; i \; \in [0, \mathcal{M}], \quad m_i = x_i \tag{6}$$

All intermediate producers are completely symmetric and I denote by m their (common) production and by x their (common) import levels. The bundle $\mathcal{I}$ can then be simply expressed as $\mathcal{I} = \mathcal{M}^{\sigma/(\sigma-1)} m$ and the price index dual to the definition of the bundle is $\mathcal{P} = \mathcal{M}^{1/(1-\sigma)} p_m$, which is also equal to $F_{\mathcal{I}}(\ell, \mathcal{I})$, the marginal productivity of the input bundle in final good production. Finally, taking the derivative of GDP with respect to p_x while keeping prices constant at their base period value, we obtain:

$$\frac{dGDP}{dp_x} = \underbrace{p_y^b F_\ell(\ell, \mathcal{I}) \frac{\partial \ell}{\partial p_x}}_{\text{Factor Supply Effect}} + \underbrace{\left(\mathcal{M} \frac{\partial m}{\partial p_x^b} + \frac{\partial \mathcal{M}}{\partial p_x} m \right) . (\mu - 1) \, p_x}_{\text{Markup Effect}} + \underbrace{\frac{1}{\sigma - 1} p_m m \frac{\partial \mathcal{M}}{\partial p_x}}_{\text{Entry/Exit Effect}} \tag{7}$$

Equation (7) is the counterpart of (4) in a model with extensive margin adjustments and where importing firms are not price takers. Crucially, the introduction of those two elements create a link between a foreign shock and domestic value added variations, over and beyond any change in domestic factor supply.

First, the existence of a price wedge $\mu \neq 1$ means that the first term does not vanish. With $m'(p_x) < 0$,[22] a decrease in the price of imported inputs leads to an increase in GDP. When firms are price setters and earn a positive profit, the marginal revenue generated by an additional unit of imported input x is larger than its marginal cost p_x. Hence, cheaper inputs means more sales, more profit and more value added.

Moreover, any change in the mass of firms $\mathcal{M}$ also impacts domestic value added. One can model many reasons why the mass of producing firms would change, including a free entry con-

[22] This can be easily proved if assuming that $F(.)$ is a Cobb Douglas aggregation of domestic factors and intermediates.

dition with initial sunk cost or any reason that changes the supply of potential entrepreneurs.[23] A change in the number of price setting firms gives a time varying element to the effect described above, triggering a greater reaction of GDP after a foreign shock. Note that this effect is not governed by the love for variety which is captured by the parameter σ. Overall, the key idea governing this first term can be expressed as follows: firms that charge a markup have a disconnect between the marginal cost and the marginal revenue product of their inputs. The difference between those two is accounted as value added in the form of profits. Any change in input usage leading to a change in profits triggers a change in value added produced.

Second, when $\sigma < +\infty$, another effect arises. When the production function exhibits love for variety, any change in the mass of firms implies an additional reaction for the input bundle $\mathcal{I}$. If the decrease of p_x is accompanied by an increase in the mass of producing firm,[24] the bundle $\mathcal{I}$ increases not only because each intermediate producer will tend to produce more, but also because an increase in the mass of firms mechanically increases $\mathcal{I}$ even for a fixed amount of intermediates.

With love for variety, a producer that has access to more suppliers can produce more output for the same level of input, and a change in the mass of firms impacts the marginal cost of producing final goods over and beyond the change in input prices. Another way of saying this is that the set of feasible combinations of output $\mathcal{I}$, and inputs $\int_0^{\mathcal{M}} m_i di = X$ is not independent of the mass of producers $\mathcal{M}$: a change of $\mathcal{M}$ has an effect on the production possibility frontier. Interestingly, this channel is at work independently of the price distortion channel discussed previously. Even in the absence of monopoly pricing, the sheer fluctuation in the mass of producing firms coupled with a love for variety in final good production creates a link between import price and GDP fluctuation even with fixed factor supply.

Finally, note that the introduction of markups and love for variety allows GDP to change over and beyond changes in the domestic factors of production. Using a "growth accounting" perspective, this means that the introduction of those two elements makes domestic productivity change after a foreign shock, even with a fixed technology. Two countries that have important trade flows in intermediate inputs should then have correlated measured TFP, a prediction I test in the data in section 7.

[23] In an additional appendix available upon request, I model the free condition and show that indeed an increase of import price leads to a decrease in the mass of firms.

[24] If the mass of firms is pinned down by a free entry condition, the increase in profits of each intermediate producer when the price of imported input goes up leads to a increase in the mass of firms.

4 A model of International Trade in Inputs

4.1 Setup

In this section, I build a quantitative model of international trade in inputs with *monopolistic competition* and *firm entry/exit* and assess its ability to replicate the strong relationship between trade and business cycle synchronization.[25] The model is related to Ghironi and Melitz (2005) and Alessandria and Choi (2007), extended with multiple asymmetric countries and global value chains with intermediate goods crossing borders multiple times. The combination of international input-output linkages, price distortions and extensive margin adjustments allows the model to give a quantitative account of the relationship between trade and GDP movements.[26]

I consider an environment with N countries indexed by k. In each country, there is a representative agent with preferences over leisure and the set of available goods Ω_k described by

$$\begin{aligned} U_{k,0} \quad &= \mathbb{E}_0 \left[\sum_{t=0}^{+\infty} \beta^t \left(\log\left(C_{k,t}\right) - \psi_k \frac{L_{k,t}^{1+\nu}}{1+\nu} \right) \right] \\ \text{with} \qquad\qquad & C_t = \left(\int_{\Omega_k} q_{i,t}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

where ψ_k is a scaling parameter, ν is the inverse of the Frisch elasticity of labor supply and σ the elasticity of substitution between different varieties of final goods. The agent chooses consumption, investment and labor in each period subject to the budget constraint:[27]

$$P_{k,t}\left(C_{k,t} + K_{k,t+1} - (1-\delta)K_{k,t}\right) = w_{k,t}L_{k,t} + r_{k,t}K_{k,t}$$

Production is performed by a continuum of heterogeneous firms combining in a Cobb-Douglas fashion labor ℓ_k, capital k_k and intermediate inputs $I_{k,t}$ bought from other firms from their home country as well as from abroad. Firms' productivity is the product of an idiosyncratic part φ and a country specific part $Z_{k,t}$. Firms maximize their static profit taking as given all input prices. Omitting time indexes for now, the intermediate input index in country k, I_k is an Armington aggregation of country specific bundles $M_{k',k}$ for all k', with the Armington elasticity denoted ρ.

[25] In section 6, I present a decomposition of the role that each of the novel ingredients has on the quantitative results.

[26] Alternatively, the model presented here can be though of as an extension of the IRBC model presented in Johnson (2014) with two new elements: markups and extensive margin adjustments. Again, section 6 shows that those two ingredients are quantitatively essentials in generating a link between trade and GDP comovement.

[27] Note that the right hand side of this equation include firms' profits since, as explained below, firms pay entry costs using domestic labor. It should then be understood that $L_{k,t}$ includes both production and "entry cost" workers.

In order to introduce a rationale for markups and for love for variety, each country specific bundle is itself a CES aggregation of many varieties, with the elasticity of substitution σ (which governs both the markup firms charge and the degree of love for variety). The production function is:

$$
\begin{aligned}
Q_k(\varphi) \quad &= Z_k.\varphi \ . \ I_k(\varphi)^{1-\eta_k-\chi_k} \ . \ \ell_k(\varphi)^{\chi_k} \ . \ k_k(\varphi)^{\eta_k} \\
\text{with} \qquad & I_k(\varphi) = \left(\sum_{k'} \omega_k(k')^{\frac{1}{\rho}} M_{k',k}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \\
\text{and} \qquad & M_{k',k} = \left(\int\limits_{\Omega_{k',k}} m_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}
\end{aligned}
$$

where $\omega_k(k')$ is the share of country k' in the production process of country k with $\sum_{k'} \omega_k(k') = 1$ and $\Omega_{k',k}$ is the endogenous set of firms based in k' and exporting to k. For later use, I define notations for the ideal price indexes dual to the two layers of the nested CES aggregation. $\mathcal{P}_{k,k'}$ denotes the price of the country-pair specific bundle $M_{k',k}$ and IP_k the unit price of the intermediate input bundle I_k. The unit cost of the Cobb Douglas bundle aggregating I_k, k_k and ℓ_k (called the "input bundle") is PB_k and represents the price of the basic production factor in country k. The exact expressions of those objects are standard and can be found in the appendix.

The only stochastic elements of this model are the country specific technological shocks (Z_k) which follow an AR(1) process. In all countries, the distribution of productivity φ is Pareto with shape parameter γ and density $g(\varphi) = \gamma\varphi^{-\gamma-1}$. For simplicity and in line with the empirical results in section 2, I restrict trade to be only between firms which means that I consider only trade in intermediate inputs.

In order to be allowed to sell its variety to a country j, a firm from country i must pay a fixed cost f_{ij} (labeled in unit of the "input bundle") as well as a variable (iceberg) cost τ_{ij}. Firms choose which countries they enter (if any), affecting both the level of competition and the marginal cost of all firms in the country. As will be clear below, profits are strictly increasing with productivity φ so that equilibrium export decisions are defined by country-pair specific thresholds $\varphi_{k,k'}$ above which firms from k find it profitable to pay the fixed cost $f_{kk'}$ and serve country k'. Finally there is an overhead entry cost $f_{E,k}$, sunk at the production stage, to be paid before firms know their actual productivity. Based on their expected profit in all markets, firms enter the economy until the expected value of doing so equals the overhead entry cost. This process determines the mass of firms M_k actually drawing a productivity, some of which optimally decide to exit the market before production due to the presence of fixed costs.

4.2 Equilibrium

In this section, I present the key conditions that characterize the equilibrium of the model. Introducing X_k the aggregate consumers' revenue in k and S_k the total firms' spendings (including fixed costs payments) in country k respectively, total demand faced by firm φ is given by

$$q(\varphi) = \left(\frac{p_{k,k}(\varphi)}{\mathcal{P}_k}\right)^{-\sigma} \frac{X_k}{\mathcal{P}_k} + \sum_{k'} \left(\frac{p_{k,k'}(\varphi)}{\mathcal{P}_{k,k'}}\right)^{-\sigma} \left(\frac{\mathcal{P}_{k,k'}}{IP_{k'}}\right)^{-\rho} \frac{\omega_{k'}(k)(1-\eta_{k'}-\chi_{k'})S_{k'}}{IP_{k'}} \tag{8}$$

where $p_{k,k'}(\varphi)$ is the price charged by a firm from country k, with productivity φ, when selling in country k' and the summation is done over all markets that are served by a firm with productivity φ. Firms are monopolists within their variety and choose their price at a constant markup over marginal cost and the markup depends on the price elasticity of demand. In this case, the only elasticity that is relevant to firms' pricing is σ, capturing the fact that firms compete primarily with other firms coming from their home country since their individual pricing decision has no impact on the country-specific price index in every market.[28] The marginal cost of a firm with productivity φ in country k is $PB_k/(Z_k\varphi)$ and its optimal price is given by:

$$p_{k,k'} = \tau_{k,k'} \frac{\sigma}{\sigma-1} \frac{PB_k}{Z_k\varphi} \tag{9}$$

Unlike in the canonical Krugman (1980), Melitz (2003) or Ghironi and Melitz (2005) models, one cannot solve for prices for each firm independently. Through PB_k, the price charged by firm φ in country k depends on the prices charged by all firms supplying country k (both domestic and foreign) which in turn depend on the prices charged by their suppliers and so on and so forth. The input-output linkages across firms create a link between the pricing strategies of all firms and one needs to solve for all those prices at once. Doing so requires solving for all country-pair specific price indexes $\mathcal{P}_{k,k'}$.

The definitions of price indexes give rise to a simple relationship between the price of the country k specific bundle at home, $\mathcal{P}_{k,k}$, and its counterpart in country k', $\mathcal{P}_{k',k}$:

$$\mathcal{P}_{k,k'} = \tau_{kk'} \left(\frac{\varphi_{k,k'}}{\varphi_{k,k}}\right)^{\frac{\sigma-\gamma-1}{1-\sigma}} \times \mathcal{P}_k \tag{10}$$

[28] With a finite number of firms, both elasticities σ and ρ would appear in the pricing strategy. In such a case, every firm would take into account the fact that its own price has an impact on the unit cost of the corresponding country-specific bundle. Therefore, when decreasing its price a firm would attract more demand compare to firms from its own country but also increase the share of total demand that goes to every other firms from the its country.

Intuitively, the ratio between the price of a country specific bundle in two different markets depends on the relative iceberg costs as well as the relative entry thresholds. Using this relation in the definition of price indexes in every country yields a system of N equations which jointly defines all price indexes:

$$\mathcal{P}_k^{1-\rho} = \mu_k \left(\sum_{k'} \omega_k(k') \left(\tau_{k'k} \left(\frac{\varphi_{k',k}}{\varphi_{k',k'}} \right)^{\frac{\sigma-\gamma-1}{1-\sigma}} \mathcal{P}_{k'} \right)^{1-\rho} \right)^{1-\eta_k-\chi_k} \quad , \ k = 1, ..., N \tag{11}$$

with μ_k depending on entry thresholds, the mass of firms and parameters.[29] For given thresholds and mass of firms, this system admits a unique non negative solution.[30]

Turning now to the determination of export strategies, the productivity thresholds above which firms from country k optimally decide to pay the fixed cost and serve market k' are simply given by:

$$\pi_{k,k'}(\varphi_{k,k'}) = \frac{PB_k}{Z_k}.f_{k,k'} \quad \text{for all } k \text{ and } k' \tag{12}$$

where $\pi_{k,k'}(\varphi)$ is the variable profit earned by a firm with productivity φ in market k'. I assume that the fixed cost $f_{k,k'}$ is paid in units of the basic production factor in country k deflated by the aggregate level of productivity, as is the case in Ghironi and Melitz (2005) for example.

The mass of firms deciding to enter the market in each period is finally determined by the free entry condition. With the assumption that $f_{E,k}$ is labeled in units of labor, the condition writes:

$$\Pi_k = M_k \frac{w_k}{Z_k}.f_{E,k} \quad \text{for all } k \tag{13}$$

where Π_k denotes aggregate profits of all firms in country k. An expression of Π_k can be found using a property first noted by Eaton and Kortum (2005) according to which total profit in country k are proportional to total revenues. Defining R_k the total sales of firms from country k made on all markets, we have the following result:

Lemma 1 : Total profit in country k are proportional to total revenues:

$$\Pi_k = \frac{\sigma - 1}{\gamma\sigma} R_k \tag{14}$$

[29] $\mu_k^{\frac{1-\sigma}{1-\rho}} = \frac{\gamma\varphi_{k,k}^{\sigma-\gamma-1}}{\gamma-(\sigma-1)} M_k \left(\frac{\sigma}{\sigma-1} \frac{w_k^{\chi_k} \times r_k^{\eta_k}}{\chi_k^{\chi_k} \times \eta_k^{\eta_k} \times (1-\eta_k-\chi_k)^{1-\eta_k-\chi_k}} \frac{1}{Z_k} \right)^{1-\sigma}$

[30] Following Kennan (2001) and denoting $G_k = \mathcal{P}_k^{1-\rho}$ and G the associated $N \times 1$ vector, it suffices to show that the system is of the form $G = f(G)$ with $f : \mathbb{R}^N \to \mathbb{R}^N$ a vector function which is strictly concave with respect to each argument, which is obvious as long as $0 < \eta_k + \chi_k < 1$. This argument stresses the importance of decreasing return to scale with respect to intermediate inputs in order to ensure unicity of the equilibrium.

Proof: see Appendix.

Closing the model involves market clearing conditions for capital, labor and goods. Labor can be used either for production (L_k^p) or for the entry cost (L_k^e) so that $L_k = L_k^p + L_k^e$. Classic properties of Cobb-Douglas production functions imply that total labor and capital payments for production are equal to a fraction $\eta_k + \chi_k$ of firms' variable spendings. Since total profit are used in the entry fixed cost payment, total consumer's spending is defined as $X_k = w_k L_k + r_k K_k = (\eta_k + \chi_k) S_k + \Pi_k$. Moreover, the investment Euler Equation (capital supply) is given by

$$\frac{1}{C_{k,t}} = \beta \mathbb{E}_t \left[\frac{1}{C_{k,t+1}} \times \left(\frac{r_{k,t+1}}{P_{k,t+1}} + (1-\delta) \right) \right] \tag{15}$$

while labor supply is:

$$\psi_k L_{k,t}^{\nu} = \frac{w_{k,t}}{P_{k,t}} \frac{1}{C_{k,t}} \tag{16}$$

Finally, trade being allowed in intermediate goods only, revenues in foreign countries come from other firms' spending while domestic revenues also include consumers' spendings. Total revenues of all firms from country k are:

$$R_k = X_k + \left[\sum_{k'} \left(\frac{\mathcal{P}_{k,k'}}{IP_{k'}} \right)^{1-\rho} \omega_{k'}(k)(1 - \eta_{k'} - \chi_{k'}) S_{k'} \right] \tag{17}$$

This formula has a simple interpretation: firms in country k receive revenues from their final good sales to their home consumers (for a total amount of X_k) as well as from sales as intermediate goods on all markets. In every country k', firms allocate a constant fraction $1 - \eta_k - \chi_k$ of their total spendings to intermediate inputs, which is then scaled by the weight $\omega_{k'}(k)$ representing the importance of country k in the production process of country k'. Finally, since country k specific bundle in k' is in competition with other country specific bundles in the input market, total revenues of k-firms when selling in k' also depend on the ratio of $\mathcal{P}_{k,k'}$ to $IP_{k'}$ to a power reflecting the relevant the price elasticity, in this case the macro (Armington) one ρ. For later use, it is useful to define total trade between k and k' as

$$T_{k \to k'} = \left(\frac{\mathcal{P}_{k,k'}}{IP_{k'}} \right)^{1-\rho} \omega_{k'}(k)(1 - \eta_k - \chi_k) S_{k'}$$

Using $X_k = (\eta_k + \chi_k) S_k + \Pi_k$, the good market clearing condition can be written in compact form as

$$\left\{ \underbrace{\left(\mathcal{I}_N - \left(W^T \circ P\right)\right)}_{=M} \begin{pmatrix} (1-\eta_1-\chi_1).S_1 \\ \vdots \\ (1-\eta_N-\chi_N).S_N \end{pmatrix} = 0_{\mathbb{R}^N} \right. \tag{18}$$

where W the weighting matrix defined as $W_{ij} = \omega_i(j)$, P a matrix defined by $P_{ij} = \left(\frac{\mathcal{P}_{i,j}}{IP_i}\right)^{1-\rho}$ and $\circ$ is the element-wise (Hadamard) product. To gain intuitions, one can note that the matrix P scales the weights $\omega_{k'}(k)$ in order to account for the competition across country-specific bundles. If the Armington elasticity ρ is above unity (country specific bundles are substitutes) then a country i which is able to charge low prices in some market j (a low $\mathcal{P}_{i,j}$) will attract a higher share of total expenditures from all firms in this country. Classically, this effect completely disappears in the case of a Cobb-Douglas aggregation of country specific bundles, because in such a case the spending shares are fixed.

The solutions of this system form a one dimensional vector space.[31] Setting $w_1 = 1$, implying $S_1 = L_1^p/\chi_k$, provides a unique solution for all variables by solving together the price system (11), the threshold system (12), the Spending system (18), the Free Entry system (13) as well as the labor and capital market equilibrium conditions.

GDP definition An important feature of GDP construction in OECD data is the use of base prices and quantity estimates. In order to be as close as possible to the method used in the construction of the data used in the empirical analysis, I define GDP using steady state prices as base prices.[32] GDP is obtained by deflating nominal spendings using steady-state price indexes that are corrected from product variety effects, such that:

$$GDP_k = \underbrace{\widehat{\mathcal{P}_k^{ss}} \frac{X_k}{\widehat{\mathcal{P}_k}}}_{\text{Consumption + Investment}} + \underbrace{\sum_{k'} \widehat{\mathcal{P}_{k,k'}^{ss}} \frac{T_{k\to k',t}}{\widehat{\mathcal{P}_{k,k',t}}}}_{\text{Exports}} - \underbrace{\sum_{k'} \widehat{\mathcal{P}_{k',k}^{ss}} \frac{T_{k'\to k,t}}{\widehat{\mathcal{P}_{k',k,t}}}}_{\text{Imports}} \tag{19}$$

where we defined $\widehat{\mathcal{P}_{k,k'}} = \left(M_k.\varphi_{k,k'}^{-\gamma}\right)^{1/(\sigma-1)} \mathcal{P}_{k,k'}$ in order to be consistent with the way actual data are collected.[33] Note that the first term is equal to total of final goods of domestic firms in

[31] One can easily show that the matrix M is non invertible: summing all rows results in a column of zero.

[32] In the data, GDP is defined using the Fisher ideal quantity index which is a geometric mean of the Laspeyres and Paasche indexes. Hence, for all periods t, the base period price is a geometric mean between period t and period $t+1$.

[33] Since both consumers' utility and production functions have a CES component, it is well known that the associated price indexes can be decomposed into components reflecting average prices (captured by statistical agencies) and product variety (which is not taken into account in national statistics). See Feenstra (1994) or Ghironi and Melitz (2005) for a discussion of this

the domestic market, which is also equal to total consumers' spending on final good purchases. Hence, it can be seen as the Gross National Income (GNI_k) since there is no trade in assets across countries.

4.3 GNI elasticity in a simplified case

In order to investigate the mechanics driving the propagation of shocks across countries in the model, let us study a special case with $\rho = 1$ and fixed labor, capital and mass of potential entrants.[34] The goal of this section is to compute the elasticity of GNI in any country i with respect to a technology shock in country 1:

$$\eta_{GNI_i,Z_1} = \frac{\partial \log(GNI_i)}{\partial \log(Z_1)}$$

where Gross National Income as computed in national statistics ($GNI_k = (w_k L_k + r_k K_k)/\widehat{\mathcal{P}_k}$). Computing the elasticity of all endogenous variable with respect to technological shocks leads to the closed-form formula in lemma 2.

Lemma 2 : In the Cobb-Douglas ($\rho = 1$) case and fixing both labor and capital supply, the elasticity of GNI in all countries with respect to a technology shock in country 1 are given by:

$$\begin{pmatrix} \eta_{GNI_1,Z_1} \\ \vdots \\ \eta_{GNI_N,Z_1} \end{pmatrix} = \left(\frac{\gamma - (\sigma - 1)}{\sigma\gamma - (\sigma - 1)} \right) .(\mathcal{I}_N - \widehat{W} - T)^{-1} \begin{pmatrix} 1 \\ 0 \\ \vdots \end{pmatrix} \tag{20}$$

with $\widetilde{W}_{i,j} = (1 - \eta_i - \chi_i)\omega_{i,j}$ the matrix of scaled weights $\omega_{i,j}$ representing the intensive margin adjustments and T a "Transmission" matrix[35], function of γ and σ, accounting for extensive margin movements.

Proof: see Appendix.

These expressions are reminiscent of what can be found in static Cobb-Douglas network models such as Acemoglu et al (2012) for example, with an additional effect coming from firm heterogeneity and the extensive margin adjustments captured by the matrix T. In this context,

[34] Without capital supply, the model is completely static. A fixed mass of potential entrants does not mean a fixed mass of actual producers because entry thresholds $\varphi_{k,k}$ are not fixed.

[35] $T = \Lambda \mathcal{I}_N$, with $\Lambda = \frac{1}{\sigma + \frac{(\sigma-1)^2}{\gamma - (\sigma-1)}}$

the international propagation pattern of country specific shocks runs through two channels. First, for fixed spending share, the matrix $\widetilde{W}$ records the input-output linkages if the export decision of firms are kept constant. Second, the change in prices and revenues in all markets triggers a change in the productivity thresholds $\varphi_{k,k'}$. This channel is characterized by the matrix T which is a function of σ and γ which govern the adjustments along the extensive margin.

The computations leading to the expressions of the GNI elasticities in this lemma are greatly simplified by the assumption that factors of production (labor and capital) are fixed. In the general model, however, this constitute an important amplification channel through two effects. First, as in many macro models, a positive productivity shock in any country contributes to the decrease of prices all over the world and hence an increase in real wage. This triggers an increase in labor supply that amplifies the benefits of the shock in terms of output.[36] In addition, there is a second channel going through the change in the mass of active firms in every country. With the assumption that the mass of potential entrepreneurs is proportional to the labor size, an increase in labor supply results in a proportional increase in the mass of potential entrants. Whether the mass of actual producing firms goes up or down in any country k will also be determined by the changes in the thresholds φ_{ik} for all i which in turns crucially depends on the value of the Armington elasticity ρ. In the Cobb-Douglas case where the expenditure shares are fixed, a positive technological shock will result in a decrease of all entry thresholds in every market. Putting pieces together, a positive shock triggers at the same time more potential entrepreneurs and a decrease of the entry threshold in every market. As a result, the new structure of input-output linkages amplifies the benefits of the shock.

5 Calibration

The goal of this section is to quantitatively assess the model's ability to match the strong empirical relationship between trade proximity in intermediate input and GDP synchronization. The model is calibrated to 14 countries and a composite rest-of-the-world for the time period 1989 to 2008. In what follows, I explain in detail the calibration strategy while the simulation results are gathered in the next section.

For a calibration with N countries, there are $3 \times N^2 + N + 6$ parameters to determine, on top of which one needs to set parameters relative to the technological shocks. For each country-pair

[36] This increase in labor supply is tempered by the wealth effect.

(i,j), one needs values for the weights $\omega_i(j)$, the iceberg trade costs τ_{ij} and the fixed costs $f_{i,j}$, then for every country i one needs values for "value added" share in production $(\eta_k + \chi_k)$ and scaling parameter ψ_i. The set of common parameters is given by $\chi_k/(\chi_k + \eta_k)$ the labor share in value added, ν for the (inverse) elasticity of labor supply, γ for the distribution of productivity draws, σ for the within country (micro) elasticity of substitution across varieties and ρ for the (macro) elasticity of substitution of country-specific bundles. Finally, we will also need to set the volatility, covariance and auto-correlation of the TFP shocks in all countries, as discussed in detail below.

My calibration is a mixture of estimations from micro data (taken from the literature as well as re-estimated) and a matching of macro moments. The goal is to match exactly the relative GDP across all country pairs, the volatility, persistence and level of GDP co-movement as well as the trade proximity in intermediate goods in order to give a reasonable account of the ability of the model to generate a strong link between bilateral trade and GDP synchronization despite the fact that typical trade flows between two given countries are very low compare to their GDPs.

From micro data

The discount factor β is 0.99. The (inverse) elasticity of labor supply ν is 2/3 leading to a Frisch elasticity of 1.5. The sunk entry cost $f_{E,k}$ in each country is set in order to get a ratio of total number of *potential* (not actual) firms divided by the population of 10%, in line with US estimates taking into account that not all potential entrepreneurs enter the economy in equilibrium. The variable (iceberg) trade costs are taken from the ESCAP World Bank: "International Trade Costs Database"[37]. This database features symmetric bilateral trade costs in its wider sense, including not only international transport costs and tariffs but also other trade cost components discussed in Anderson and van Wincoop (2004).

As in di Giovanni and Levchenko (2013), fixed access costs are computed from the "Doing Business Indicators".[38] In particular, I measure the relative entry fixed costs in domestic markets by using the information on the amount of time required to set up a business in the country relative to the US.[39] If according to the Doing Business Indicators database, in country i it takes

[37]See at http://artnet.unescap.org/

[38]The World Bank's Doing Business Initiative collected data on regulations regarding obtaining licenses, registering property, hiring workers, getting credit, and more. See http://francais.doingbusiness.org/data/exploretopics/trading-across-borders and http://francais.doingbusiness.org/data/exploretopics/starting-a-business

[39]As argued in di Giovanni and Levchenko (2013), using the time taken to open a business is a good indicator because it measures entry costs either in dollars or in units of per capita income, because in the model $f_{i,i}$ is a

10 times longer to register a business than in the U.S., then $f_{i,i} = 10 \times f_{US,US}$. I normalize the lowest entry fixed cost so that no entry threshold lies below the lower bound o the productivity distribution, which is taken to be one in every country. To measure the fixed costs associated with entry in a foreign market, I use the Trading Across Borders module of the Doing Business Indicators. I choose the number of days it takes to import to a specific country, using the same normalization as for the domestic entry cost.[40]

In the benchmark simulations, I choose the macro (Armington) elasticity ρ to be equal to unity while the micro elasticity σ is equal to 5. There are many papers estimating those elasticities for intermediate or final goods. Saito (2004) provides estimations from 0.24 to 3.5 for the Armington elasticity[41] and Anderson and van Wincoop (2004) report available estimates for the micro elasticity in the range of 3 to 10. Following Bernard, Eaton, Jensen, and Kortum (2003), Ghironi and Melitz (2005) choose a micro elasticity of 3.8. Recently, papers such as Barrot and Sauvagnat (2015) or Boehm, Flaaen and Pandalai-Nayar (2015) argue that firms' ability to substitute between their suppliers can be very low. The choice of a value of $\sigma = 5$ leads to markups of 25%. The aggregate profit rate, however, is only of 17.4% since firms have to pay fixed cost in order to access any market. There is also a theoretical convenience to use $\rho = 1$, as it allows the model to take the same form as other network models such as Acemoglu (2012), Bigio and La'O (2015) and many others. Finally, the capital and labor shares in value added are fixed at 2/3 and 1/3 respectively and I set $\gamma = \sigma - 0.4$ as described in Fattal-Jaef and Lopez (2010).

Matching of macro moments

For the remaining parameters, I use data on 14 countries from 1989 to 2008 and chose parameter values in order to match specific targets. More precisely, I jointly set the country size parameters $(\psi_i)_{i=1,\dots,N}$, the value added share $\chi_k + \eta_k$ as well as the spending weights $\omega_i(j)$ (the matrix W) in order to match all countries relative GDP and all relative trade flows in real terms. I normalize the real GDP of the composite rest-of-the-world to 100 and set all other real GDPs so that the ratio of their real GDP to the one of the rest-of-the-world in the simulated economy matches exactly its counterpart in the data for the time window 1989 to 2008. My targets are

quantity of inputs rather than value.

[40] This approach means that the fixed cost associated with trade from France to the US is the same as the one from Germany to the US. One must keep in mind, however, that the iceberg variable cost will differ.

[41] Feenstra et al (2014) studies the macro and micro elasticities for final goods and reports estimates between -0.29 and 4.08 for the Armington elasticity. They find that for half of goods the macro elasticity is significantly lower than the micro elasticity, even when they are estimated at the same level of disaggregation.

Parameter	Value	Counterpart
β	0.99	Discount factor – Annual discount rate of 4%
ρ	1	Macro (Armington) Elasticity of substitution (from Literature)
σ	5	Micro Elasticity of substitution – 25% markup, average profit of 17.4%
ν	2/3	Labor Curvature – Frisch elasticity of 1.5
$f_{E,i}$	[1 - 10]	$M/L = 0.1$ – Mass of plants over working population
τ_{ij}	[1 - 3]	Iceberg trade cost – From ESCAP - World Bank
f_{ij}	[1 - 10]	Fixed trade cost – "Doing Business Indicators"
γ	4.6	Pareto shape – (Fattal-Jaef & Lopez (2014))
$\chi_k/(\chi_k + \eta_k)$	0.7	Labor share – 70% of value added.

Table 3: Parameters fixed using micro data

then N real GDP targets as well as N^2 directed trade flows (over GDP), to which one must add the constraint that spending shares $\omega_i(j)$ sum to one for each country, which leads to $(N^2 + 2N)$ equations for an equal number of parameters to match. Since complete financial autarky is inconsistent with the trade balances observed in the data, I calibrate the model to match steady-state trade imbalances, and then hold those nominal imbalances constant. Note that in order to be as close as possible to the data used in the empirical analysis, I construct the quantity estimates by deflating the nominal spendings by the price index that do not take into account love for variety, as described in section 4.2.

Finally, I need to calibrate the persistence and the variance-covariance matrix for the country-level TFP shocks $(Z_i)_{i=1,\dots,N}$. In order not to overestimate the impact of idiosyncratic shocks, I chose to set their volatility (the diagonal elements of the variance-covariance matrix) so that the model can replicate GDP volatility (de-trended using HP filtering) in every country. This allows me to generate fluctuations in the simulated economy that are similar to those observed in the data. Similarly, I set the off diagonal elements (the covariance terms) so that the average correlation of GDP in the model match the one observed in the data, which is 0.475 for the 1989-2008 time window. Recall that the goal of this exercise is not to explain the *level* of comovement across countries, but its *slope* when there is a change in trade. Hence, I set the *level* at the observed value and will vary parameters governing trade in order to evaluate the *slope*. Finally, I set a common value for auto-correlation of shocks so that the GDP series generated by the model is exactly 0.84 which is the value of GDP autocorrelation observed in the data.

6 Quantitative results

Trade Comovement Slope

The goal of this section is to assess the ability of the model to replicate the strong empirical relationship between trade proximity in intermediate inputs and GDP synchronization. The calibration procedure presented in the previous section yields values for all parameters so that the model economy matches the data for the period 1989 to 2008. With those values, I simulate a sequence of 5,000 shocks and record the correlation of HP-filtered GDP as well as the average index of trade proximity. Since my goal is to use within country-pair variations in order to perform a fixed-effect estimation of the effect of trade on GDP comovement, I then recalibrate the model with different targets for trade proximity across countries, decreasing and increasing the target by 10%. For each configuration, I feed the economy with the *exact same* sequence of 5,000 shocks and record the correlation of HP-filtered GDP as well as the average index of trade proximity. This gives rise to a panel dataset in which I have $14 \times 13/2 = 91$ observations for each of the 3 configurations, hence a total of 273 observations. I then perform fixed effect regressions on the simulated dataset and find that the model is able to explain more than 85% of the trade-comovement slope.

	dependent variable: corr(GDP_i^{HP},GDP_j^{HP})	
	Data	Model
log(Intermediate)	0.053**	0.047***

Decomposition - Role of the ingredients

In order to assess the role of each ingredient in the quantitative results, I then turn off one by one the key elements of the model. Results are presented in table 4 and yield interesting insights. First, the sole addition of price distortions to an otherwise classic IRBC model with input-output linkages increases the trade comovement slope from 0.007 to 0.032. Finally, the amplification coming through the fluctuation in the mass of firms serving all markets increases the slope from 0.032 to 0.047, showing that adjustments along the extensive margin is a powerful way to generate quantitative results in line with the empirical link between trade in inputs and GDP comovement.

	Trade-Comovement Slope
I/O linkages + Markups + Extensive Margin	0.047***
I/O linkages + Markups	0.031***
I/O linkages	0.007***

Table 4: Decomposition of the result

7 Further Empirical Evidence

In section 6, it has been shown that the combination of global value chains with price setting firms and extensive margin adjustments went a long way toward providing a quantitative solution for the trade comovement puzzle. While the empirical relevance of international input-output linkages as been uncovered in section 2, it is also interesting to test for the empirical relevance of markups and firms' entry/exit. In this section, I go back to the data and provide empirical support for the role of markups and entry/exit in creating a link between trade and GDP fluctuations.

First, building on Liao and Santacreu (2015), I disentangle trade flows into their intensive and extensive margins and show that, controlling for both country-pair and time window fixed effects, the empirical association between trade and business cycle synchronization is almost only driven by the extensive margin. Next, turning to the importance of price setting, I start by using sector level data to construct measures of markups that are then aggregated at the country level. I then show that countries with larger markups are also more sensitive to terms of trade shocks.

7.1 The Role of Extensive Margin of Trade

Following Hummels & Klenow (2005) as well as Feenstra & Markusen (1994), I construct the Extensive and Intensive margins of trade between countries j and m using the Rest-of-the-World as a reference country k. The extensive margin (EM) is defined as a weighted count of varieties exported from j to m relative to those exported from k to m. If all categories are of equal importance and the reference country k exports all categories to m, then the extensive margin is simply the fraction of categories in which j exports to m. More generally, categories are weighted by their importance in k's exports to m. The corresponding intensive margin (IM) is defined as the ratio of nominal shipments from j to m and from k to m in a common set of goods. With this construction, the product of both margins of trade between j and m is equal to the ratio of total trade flows between j and m to total trade flows from the reference country k to m, which is usually denoted as OT. Formally, the margins are defined as:

$$\text{Extensive Margin} \quad EM_{jm} = \frac{\sum\limits_{i \in I_{jm}} p_{kmi} q_{kmi}}{\sum\limits_{i \in I} p_{kmi} q_{kmi}}$$

$$\text{Intensive Margin} \quad IM_{jm} = \frac{\sum\limits_{i \in I_{jm}} p_{jmi} q_{jmi}}{\sum\limits_{i \in I_{jm}} p_{kmi} q_{kmi}}$$

$$\text{Trade Ratio} \quad OT_{jm} = \frac{X_{j \to m}}{X_{k \to m}} = EM_{jm} \times IM_{jm}$$

Where I_{jm} is the set of observable categories in which j has a positive shipment to m, I is the set of all categories exported by the reference country which is supposed to be the universe of all categories and $X_{j \to m}$ is total trade flows from country j to country m. Since those measures are not symmetric within every country-pair, I define for a given country pair (i, j) as the sum of the margins from i to j and from j to i, which are then averaged over the time window.

Constructing four 10-years time window ranging from 1970 to 2009, I estimate the following equation

$$\text{corr}(Y_{it}^{HP}, Y_{jt}^{HP}) = \alpha + \beta_{EM} \log(\text{EM}_{ijt}) + \beta_{IM} \log(\text{IM}_{ijt}) + controls + \epsilon_{ijt} \tag{21}$$

Results are gathered in 5 and show that the extensive margin of trade is an important determinant of GDP comovement. This result is particularly striking given that most of the variation in trade is explained by variations along the intensive margin. Indeed, performing a Shapley value decomposition of OT on the intensive and extensive margins, one finds that only one fourth of the total variance in OT is explained by the variation of the extensive margin. Put simply: even though EM does not vary too much (compare to IM), its variations are strongly correlated with the variations of GDP comovement.[42]

[42] Those results are in line with the similar analysis in Liao and Santacreu (2015). Compare to them, I add both dyadic fixed effects and time window fixed effects.

	dependent variable: corr($GDP_i^{filtered}$,$GDP_j^{filtered}$)	
	HP filter	First Difference
	(1)	(2)
log(EM)	0.184**	0.013
	(2.00)	(0.57)
log(IM)	-0.000***	-0.021
	(-4.12)	(-1.63)
Country-Pair FE	yes	yes
Time Window FE	yes	yes
R-squared (within)	0.146	0.144
R-squared (overall)	0.151	0.095
N	———————— 2666 ————————	

t stat. in parentheses, *** means $p < 0.01$, ** means $p < 0.05$ and * means $p < 0.10$

Table 5: Trade and GDP correlation with 10 years time windows

7.2 Terms of Trade and GDP: the role of Markups

Using data from 22 countries from 1971 to 2010,[43] I assess the role of markups in generating a link between terms of trade and GDP fluctuations.

In the model presented above, markups play an important role to make GDP react to foreign shocks, as shown in the decomposition in table 4. In order to find empirical support for the role of markup, I depart from a direct test of the model and test the following hypothesis: countries where markups are high experience a larger decrease in GDP when experiencing an increase in their terms-of-trade. This is not a test of the model presented in previous sections since the model does not feature markup across countries. However, it is an indication of the role markups can play in generating a link between foreign shocks and domestic GDP. In order to test this hypothesis, I compute the correlation of filtered GDP with the terms of trade and regress this correlation on markups estimates. Results show that markups have a significant impact on GDP-Terms of Trade correlation, with higher markups associated with lower correlation between GDP

[43]The list of countries is: Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Iceland, Israel, Italy, Japan, Korea, Luxembourg, Mexico, the Netherlands, Portugal, Spain, Sweden, the United-Kingdom and the United-States

and the terms of trade.

Data on real GDP and terms of trade at the annual frequency are both taken from the OECD database and filtered according to two different procedure. I first apply the Hodrick and Prescott filter with a smoothing parameter of 6.25 which captures the business cycle frequencies. I also apply the Baxter and King band pass filter and keep fluctuations between 8 and 25 years in order to capture medium-term business cycles (Comin and Gertler (2006)). Using the detrended series, I compute the correlation between filtered GDP and filtered terms-of-trade for two 20-years time windows from 1971 to 2010, hence creating a panel dataset where each country appears two times.

I use Price Cost Margin (PCM) as an estimate of markups within each industry. Introduced by Collins and Preston (1969) and widely used in the literature, PCM is the difference between revenue and variable cost, i.e. the sum of labor and material expenditures, over revenue:

$$PCM = \frac{\text{Sales} - \text{Labor expenditure} - \text{Material expenditure}}{\text{Sales}} \tag{22}$$

Data at the industry level come from the OECD STAN database, an unbalanced panel covering 107 sectors for 34 countries between 1970 and 2010. Due to missing data for many countries in the earliest years, I restrict the analysis for 22 countries.[44] I compute PCM for each industry-country-year and then construct an average of PCM within each country-year by taking the sales-weighted average of PCM over each industry. Finally, the average PCM for a given time window is simply the mean of country-year PCM over all time periods. Results are presented in table 6.

	dependent variable: corr($GDP_i^{filtered}$,$ToT_i^{filtered}$)			
	HP-filter	BK-filter	HP-filter	BK-filter
Average PCM	-1.507***	-2.049***	-2.650***	-3.705***
	(-2.70)	(-3.11)	(-2.87)	(-4.10)
Country FE	no	no	yes	yes
Time FE	no	no	yes	yes
N	43			

Note: t stat. in parentheses.

Table 6: Markups and GDP-ToT correlation

[44]For Germany, data are available only from 1991 onward (after the reunification), which is why the total number of observation in the regressions is 43.

The first two columns of table 6 show the results of pooled cross-section analysis where I do not use the panel dimension of the dataset. In the last two columns, I perform fixed effect regression and add time dummies to control for time specific factors that might affect the correlation of GDP and terms-of-trade. In each of those cases, regressions are performed using the two filtering methods and are found to be statistically significant, implying that countries with higher markups also experience a larger decrease in their GDP when the relative price of their import rises.

8 Conclusion

This paper analyzes the relationship between international trade and business cycle synchronization across countries. I start by refining previous empirical studies and show that higher trade in intermediate input is associated with an increase in GDP comovement, while trade in final good is found insignificant.

Motivated by this new fact, I propose a model of trade in intermediates with two key ingredients: (1) monopolistic pricing and (2) firms' entry/exit. Both elements are necessary in order for foreign shocks to have a first order impact on domestic productivity through trade linkages. The propagation of technological shocks across countries depends on the worldwide network of input-output linkages, which emphasize the importance of going beyond two-country models to understand international GDP comovement.

I calibrate this model to 14 countries and assess its ability to replicate the empirical findings. Overall, the quantitative exercise suggests that the model is able to replicate more than 85% of the trade comovement slope, offering the first quantitative solution for the "Trade Comovement Puzzle".

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A Empirical Appendix

A.1 Data description

I focus the empirical analysis on 40 OECD countries and major emerging markets, which account for around 90% of world GDP. Trade data comes from Johnson and Noguera (2016) who constructed bilateral trade flows separated between final and intermediate goods for 42 countries between 1970 and 2009[45]. According to their data appendix A.2, here is the method used for data construction: for bilateral goods trade, they use the NBER-UN Database [http://cid.econ.ucdavis.edu] for 1970-2000 and the CEPII BACI Database [http://www.cepii.fr] for 1995-2009. This data is reported on a commodity-basis. They assign commodities to end uses and industries using existing correspondences from the World Bank [http://wits.worldbank.org]. To assign commodities to end uses, they use correspondences between SITC (Revision 2) 4-digit or HS (1996 Revision) 6-digit commodities and the BEC end use classifications. To assign commodities to industries, they use correspondences between SITC and HS categories and ISIC (Revision 2) industries. GDP data comes from the Penn World Tables version 9.0 (http://www.rug.nl/ggdc/productivity/pwt/).

In Johnson and Noguera (2016)'s data for Russia starts only in 1990 while data for Estonia, Slovak Republic, Slovenia and Czech Republic start only in 1993. All country-pairs involving one of those five countries appears only for times in the case of 10 years time-windows and cannot be used at all in the case of 20 years time-windows. In total, I have 630 country-pairs appearing 4 times and 190 pairs appearing 2 times (both in the case of 10 years time windows), leading to a dataset with a total of 2900 observations.

A.2 Robustness Checks

As a robustness check, I also use the STAN Bilateral Trade Database by Industry and End-Use data (BTDIxE).[46] BTDIxE consists of values of imports and exports of goods, broken down by end-use categories. Estimates are expressed in nominal terms, in current US dollars for all OECD member countries. The trade flows are divided into capital goods, intermediate inputs and consumption. For the sake of comparison with the results in the main text, I first group the capital and intermediate goods together and create the index of trade proximity as explained in the main text. Due to data availability, I use the data from 1995 to 2014 which allows me to create four time windows of 5 years each (tables 7 and 8). With 20 countries, the dataset contains 190 pairs, for a total of 760 observations with four time windows. The tables below

[45] I drop Romania and South Africa from their sample because of lack of GDP series in the Penn World Tables

[46] See at http://www.oecd.org/trade/bilateraltradeingoodsbyindustryandend-usecategory.htm.

present the robustness results using both the HP filter (for business cycle frequencies) and then the Baxter and King filter (for medium term frequencies).

	dependent variable: corr(GDP_i^{HP},GDP_j^{HP})					
	(1)	(2)	(3)	(4)	(5)	(6)
log(Total)	0.064		-0.009		0.103	
	(5.94)		(-0.14)		(1.53)	
log(Intermediate)		0.044		0.146		0.209
		(1.88)		(1.77)		(2.59)
log(Final)		0.021		-0.152		-0.107
		(1.06)		(-2.04)		(-1.39)
Country-Pair FE	no	no	yes	yes	yes	yes
Time Trend	no	no	no	no	yes	yes
N	760					

t stat. in parentheses

Table 7: Trade and HP-Filtered GDP - STAN database (1995 to 2014)

	dependent variable: corr(GDP_i^{BK},GDP_j^{BK})					
	(1)	(2)	(3)	(4)	(5)	(6)
log(Total)	0.075		0.433		0.397	
	(5.23)		(3.86)		(3.16)	
log(Intermediate)		0.115		0.562		0.538
		(3.71)		(3.71)		(3.60)
log(Final)		-0.036		-0.106		-0.122
		(-1.32)		(-0.76)		(-0.83)
Country-Pair FE	no	no	yes	yes	yes	yes
Time Trend	no	no	no	no	yes	yes
N	760					

t stat. in parentheses.

Table 8: Trade and BK-Filtered GDP - STAN database (1995 to 2014)

B Theoretical Appendix

B.1 Equilibrium Conditions in the general CES case

Price Indexes and Pricing System

$$\mathcal{P}_{k,k'} = \left(\int\limits_{\Omega_{k,k'}} p_{k,k'}(\varphi)^{1-\sigma} g(\varphi) d\varphi \right)^{\frac{1}{1-\sigma}} \quad \text{and} \quad IP_k = \left(\sum_{k'=1,\dots,N} \omega_k(k') \mathcal{P}_{k',k}^{1-\rho} \right)^{\frac{1}{1-\rho}}$$

$$PB_k = \chi_k^{-\chi_k} \times \eta_k^{-\eta_k} \times (1-\eta_k-\chi_k)^{(\eta_k+\chi_k-1)} \times IP_k^{1-\eta_k-\chi_k} \times w_k^{\chi_k} \times r_k^{\eta_k}$$

Using the optimal pricing strategy $p_{k,k'} = \tau_{k,k'} \frac{\sigma}{\sigma-1} \frac{PB_k}{Z_k \varphi}$ with the definition of the price index relative to each country specific bundle, we have the pricing system:

$$\mathcal{P}_k^{1-\rho} = \mu_k \left(\sum_{k'} \omega_k(k') \left(\tau_{k'k} \left(\frac{\varphi_{k',k}}{\varphi_{k',k'}} \right)^{\frac{\sigma-\gamma-1}{1-\sigma}} \mathcal{P}_{k'} \right)^{1-\rho} \right)^{1-\eta_k-\chi_k} \quad , \; k = 1, \dots, N$$

with $\mu_k^{\frac{1-\sigma}{1-\rho}} = \frac{\gamma \varphi_{k,k}^{\sigma-\gamma-1}}{\gamma-(\sigma-1)} M_k \left(\frac{\sigma}{\sigma-1} \frac{w_k^{\chi_k} \times r_k^{\eta_k}}{\chi_k^{\chi_k} \times \eta_k^{\eta_k} \times (1-\eta_k-\chi_k)^{1-\eta_k-\chi_k}} \frac{1}{Z_k} \right)^{1-\sigma}$.

Entry Thresholds In very market, entry occurs until the profit of the least productive firms is equal to the fixed cost of accessing the market. Denoting by X_k total final good spending by consumers ($X_k = P_k(C_k + I_k) = w_k L_k + r_k K_k + \Pi_k$), we get

- At Home

$$\pi_{k,k}(\varphi_{k,k}) = f_{k,k} \frac{PB_k}{Z_k}$$

$$\Leftrightarrow \quad \varphi_{k,k} = \left(\frac{\sigma}{\sigma-1} \frac{PB_k}{Z_k} \frac{1}{\mathcal{P}_k} \right) \times \left(\frac{\sigma f_{k,k} \frac{PB_k}{Z_k}}{X_k + \left(\frac{\mathcal{P}_k}{IP_k} \right)^{1-\rho} \omega_k(k)(1-\eta_k-\chi_k) S_k} \right)^{\frac{1}{\sigma-1}}$$

- Abroad

$$\pi_{k,k'}(\varphi_{k,k'}) = f_{k,k'} \frac{PB_k}{Z_k}$$

$$\Leftrightarrow \quad \varphi_{k,k'} = \left(\tau_{kk'} \frac{\sigma}{\sigma-1} \frac{PB_k}{Z_k} \frac{1}{\mathcal{P}_{k,k'}} \right) \times \left(\frac{\sigma f_{k,k} \frac{PB_k}{Z_k}}{\left(\frac{\mathcal{P}_{k,k'}}{IP_{k'}} \right)^{1-\rho} \omega_{k'}(k)(1-\eta_k-\chi_k) S_{k'}} \right)^{\frac{1}{\sigma-1}}$$

Spending System Total revenue of all firms from country k can be written as

$$R_k = X_k + \left[\sum_{k'} \left(\frac{\mathcal{P}_{k,k'}}{IP_{k'}}\right)^{1-\rho} \omega_{k'}(k)(1-\eta_k-\chi_k)S_{k'}\right]$$

Free entry insures that variable profits are exactly equal, on aggregate, to fixed costs and entry costs payment, implying that $R_k = S_k + \Pi_k$. Capital and labor demand impose $r_k K_k + w_k L_k = (\eta_k + \chi_k)S_k + \Pi_k$. Finally, the spending system can be simply written as

$$\left\{ \underbrace{\left(\mathcal{I}_N - \left(W^T \circ P\right)\right)}_{=M} \begin{pmatrix} S_1 \\ \vdots \\ S_N \end{pmatrix} = 0_{\mathbb{R}^N} \right.$$

where W the weighting matrix defined as $W_{ij} = \omega_i(j)$, P a matrix defined by $P_{ij} = \left(\frac{\mathcal{P}_{i,j}}{IP_i}\right)^{1-\rho}$ and $\circ$ is the element-wise (Hadamard) product.

Labor and Capital Market Equilibrium Using the labor supply equation, L_k is simply

$$L_k^{\nu} = \frac{1}{\psi_k}\frac{w_k}{\mathcal{P}_k}$$

Equipped with S_k the total spending of all firms in k, wages w_k and rental rate r_k are defined simply by $w_k = \chi_k \frac{S_k}{L_k}$ and $r_k = \eta_k \frac{S_k}{K_k}$.

Free Entry At each date, firms enter the model until total profits are equal to total sunk cost payment:

$$\Pi_k = M_k \frac{PB_k}{Z_k}.f_{E,k} \quad \text{for all } k$$

B.2 Proof of Lemma 1

Reminder of Lemma 1 : Total profit in country k are proportional to total revenues:

$$\Pi_k = \frac{\sigma-1}{\gamma\sigma} R_k$$

Proof

First, since firms charge a constant markup $\sigma/(\sigma-1)$ over marginal cost, we know that variable profits are a fraction $1/\sigma$ of total revenues. Hence, total profits net of fixed costs for all firms in

k are simply

$$\Pi_k = \frac{R_k}{\sigma} - \sum_{k'} FC_{k\to k'}$$

where $FC_{k\to k'}$ is the sum of fixed cost payment from all firms from country k serving market k'. Then, note that total fixed cost payment for all firms in country k is

$$\begin{aligned} FC_{k\to k'} &= M_k \int_{\varphi_{k,k'}}^{+\infty} f_{kk'} \times \frac{PB_k}{Z_k} \times \gamma\varphi^{-\gamma-1} \times d\varphi \\ &= M_k f_{kk'} \frac{PB_k}{Z_k} \times \varphi_{k,k'}^{-\gamma} \end{aligned}$$

- If $k \neq k'$, we can also express total revenues (sales) from k to k' as

$$\begin{aligned} R_{k,k'} &= M_k \int_{\varphi_{k,k'}}^{+\infty} \left(\tau_{kk'} \frac{\sigma}{\sigma-1} \frac{PB_k}{Z_k} \frac{1}{\mathcal{P}_{k,k'}}\right)^{1-\sigma} \times \omega_{k'}(k) S_{k'} \varphi^{\sigma-1} g(\varphi) d\varphi \\ &= \frac{\gamma M_k}{\gamma - (\sigma-1)} \times \left(\tau_{kk'} \frac{\sigma}{\sigma-1} \frac{PB_k}{Z_k} \frac{1}{\mathcal{P}_{k,k'}}\right)^{1-\sigma} \times \omega_{k'}(k) S_{k'} \varphi_{k,k'}^{\sigma-\gamma-1} \end{aligned}$$

  Next, using the expression for the threshold $\varphi_{k,k'}^{\sigma-1}$ derived above (as a function of $\mathcal{P}_{k,k'}$), we get

$$R_{k,k'} = \frac{\gamma M_k}{\gamma - (\sigma-1)} \times \sigma f_{k,k'} \frac{PB_k}{Z_k} \varphi_{k,k'}^{-\gamma}$$

  And we recognize finally that

$$R_{k,k'} = \frac{\gamma}{\gamma - (\sigma-1)} \times \sigma FC_{k\to k'}$$

- For domestic revenues, we can show using the same steps that

$$X_k + R_{k,k} = \frac{\gamma}{\gamma - (\sigma-1)} \times \sigma FC_{k\to k}$$

Combining those expressions, we get

$$\begin{aligned} \sum_{k'} FC_{k\to k'} &= \frac{\gamma - (\sigma-1)}{\gamma\sigma} \times \left(X_k + \sum_{k'} R_{k,k'}\right) \\ &= \frac{\gamma - (\sigma-1)}{\gamma\sigma} \times R_k \end{aligned}$$

Using this expression of $\sum_{k'} FC_{k\to k'}$ in the definition of profits completes the proof.

B.3 Proof of Lemma 2

Reminder of Lemma 2 : In the Cobb-Douglas ($\rho = 1$) and fixed labor supply case, the elasticity of every GNI with respect to a technology shock in country 1 is given by:

$$\begin{pmatrix} \eta_{GNI_1,Z_1} \\ \vdots \\ \eta_{GNI_N,Z_1} \end{pmatrix} = (\mathcal{I}_N - (1-\eta_k-\chi_k)W - T)^{-1} \begin{pmatrix} 1 \\ 0 \\ \vdots \end{pmatrix}$$

with W the weighting matrix defined above and T a "Transmission" matrix function of γ and σ.

<u>*Proof*</u>:

In this simplified case ($\rho = 1$ and fixed labor and capital supply), the labor and capital demand schedules $w_k L_k = \chi_k S_k$ and $r_k K_k = \eta_k S_k$ provide a one to one mapping between total spendings S_k and the wages w_k and the interest rate r_k. Moreover, inspecting the spending system (18) when $\rho = 1$ reveals that once a choice of numeraire is done (that is, taking $w_1 = 1$ and hence fixing $S_1 = L_1/\chi_1$), the vector of spendings $(S_i)_{i=1,\dots,N}$ is independent of the technology level. Using lemma 1 and the fact that labor and capital supply are fixed, we can then show that total consumers' spending X_i also independent of technology level. Thus, since $GNI_k = X_k/\widehat{\mathcal{P}_k}$ the GNI elasticity is simply the opposite of the elasticity of the country's consumers price index. Moreover, with fixed labor supply and the assumption that the mass of potential entrepreneurs is proportional to labor size, the mass of firms M_i is fixed for every country i. In the next sections, I compute elasticities of all endogenous variables step by step until I can solve for the price index elasticities.

B.3.1 Ideal Price Indexes

Home Price Index at home $\mathcal{P}_k$ Using the definitions of price indexes, we can easily show that

$$\frac{\partial \log(\mathcal{P}_k)}{\partial \log(Z_k)} = -1 + \frac{\partial \log(PB_k)}{\partial \log(Z_k)} + \left(\frac{\gamma-(\sigma-1)}{\sigma-1}\right)\frac{\partial \log(\varphi_{k,k})}{\partial \log(Z_k)}$$

Foreign Price Index "at their home" $\mathcal{P}_{k'}$

$$\frac{\partial \log(\mathcal{P}_{k'})}{\partial \log(Z_k)} = \frac{\partial \log(PB_{k'})}{\partial \log(Z_k)} + \left(\frac{\gamma-(\sigma-1)}{\sigma-1}\right)\frac{\partial \log(\varphi_{k',k'})}{\partial \log(Z_k)}$$

Export Price indexes $\mathcal{P}_{i,j}$ The price index relative to varieties from i sold on j's market is affected by the shock according to:

$$\frac{\partial \log(\mathcal{P}_{i,j})}{\partial \log(Z_k)} = \frac{\partial \log(\mathcal{P}_i)}{\partial \log(Z_k)} + \left(\frac{\gamma - (\sigma - 1)}{\sigma - 1}\right)\left(\frac{\partial \log(\varphi_{i,j})}{\partial \log(Z_k)} - \frac{\partial \log(\varphi_{i,i})}{\partial \log(Z_k)}\right)$$

Input Bundle Price $PB_{k'}$ Abroad Using the fact that wages are not affected by technology shocks, I can compute the elasticity of the input bundle price with respect to a technology shock at home as follow:

$$\frac{\partial \log(PB_{k'})}{\partial \log(Z_k)} = (1 - \eta_k - \chi_k) \sum_j \omega_{k'}(j) \left[\frac{\partial \log(\mathcal{P}_j)}{\partial \log(Z_k)} + \left(\frac{\gamma - (\sigma - 1)}{\sigma - 1}\right)\left(\frac{\partial \log(\varphi_{j,k'})}{\partial \log(Z_k)} - \frac{\partial \log(\varphi_{j,j})}{\partial \log(Z_k)}\right)\right]$$

B.3.2 Thresholds

Home Entry Threshold $\varphi_{k,k}$ at Home Using the definition of the thresholds from above and replacing $\frac{\partial \log(PB_k)}{\partial \log(Z_k)} - 1$ by its expression in the expression of the elasticity of the Home price index at home, we get

$$\frac{\partial \log(\varphi_{k,k})}{\partial \log(Z_k)} = \frac{1}{\sigma - 1 + \kappa\sigma} \times \frac{\partial \log(\mathcal{P}_k)}{\partial \log(Z_k)}$$

Export Entry Threshold $\varphi_{k,k'}$ for Home firms exporting to k' Using the second definition of the export thresholds from above, we get

$$\left(\frac{\gamma}{\sigma - 1}\right)\frac{\partial \log(\varphi_{k,k'})}{\partial \log(Z_k)} = \left(1 + \frac{1}{\sigma - 1}\right) \times \left(\frac{\partial \log(PB_k)}{\partial \log(Z_k)} - 1\right) + \kappa \frac{\partial \log(\varphi_{k,k})}{\partial \log(Z_k)} - \frac{\partial \log(\mathcal{P}_k)}{\partial \log(Z_k)}$$

Moreover, replacing $\frac{\partial \log(PB_k)}{\partial \log(Z_k)} - 1$ by its expression we get and using the fact that $1 + \kappa = \frac{\gamma}{\sigma - 1}$, we get

$$\frac{\partial \log(\varphi_{k,k'})}{\partial \log(Z_k)} = \frac{1}{\sigma - 1 + \kappa\sigma} \times \frac{\partial \log(\mathcal{P}_k)}{\partial \log(Z_k)}$$

Home Entry Threshold $\varphi_{k',k'}$ Abroad Using the definition of the thresholds from above and replacing $\frac{\partial \log(PB_{k'})}{\partial \log(Z_k)}$ by its expression, we get

$$\frac{\partial \log(\varphi_{k',k'})}{\partial \log(Z_k)} = \frac{1}{\sigma - 1 + \kappa\sigma} \times \frac{\partial \log(\mathcal{P}_{k'})}{\partial \log(Z_k)}$$

Export Entry Threshold $\varphi_{k',j}$ for Foreign firms exporting to j With the "second" definition of the threshold and using the expression of $\eta_{\varphi_{k',k'},Z_k}$, we get:

$$\frac{\partial \log(\varphi_{k',j})}{\partial \log(Z_k)} = \frac{1+\kappa}{\frac{\gamma}{\sigma-1}} \times \frac{\partial \log(\varphi_{k',k'})}{\partial \log(Z_k)}$$

Finally, using the expression for $1+\kappa$, we get

$$\frac{\partial \log(\varphi_{k',j})}{\partial \log(Z_k)} = \frac{\partial \log(\varphi_{k',k'})}{\partial \log(Z_k)} = \frac{1}{\sigma - 1 + \kappa\sigma} \times \frac{\partial \log(\mathcal{P}_{k'})}{\partial \log(Z_k)}$$

Price indexes as constructed by statistical agencies Using results above together with the definition of $\widehat{\mathcal{P}_k}$, we get:

$$\frac{\partial \log(\widehat{\mathcal{P}_{k'}})}{\partial \log(Z_k)} = \frac{\gamma - (\sigma-1)}{\sigma\gamma - (\sigma-1)} \frac{\partial \log(\mathcal{P}_{k'})}{\partial \log(Z_k)}$$

B.3.3 Final Expression

Using the expression for the elasticity of all thresholds as functions of the elasticities of price indexes, we can gather the results. Introducing $\Lambda = \frac{1}{\sigma + \frac{(\sigma-1)^2}{\gamma-(\sigma-1)}}$, I define a matrix T (for Transmission) as $T = \text{diag}(\Lambda, ..., \Lambda)$. This matrix characterizes the additional propagation mechanism due to the change in the mass of firms in all markets. Then, the price index elasticities are defined by

$$\begin{pmatrix} \eta_{\widehat{\mathcal{P}_1,Z_1}} \\ \vdots \\ \eta_{\widehat{\mathcal{P}_N,Z_1}} \end{pmatrix} = \left(\frac{\gamma - (\sigma-1)}{\sigma\gamma - (\sigma-1)} \right) \cdot (\mathcal{I}_N - (1-\eta-\chi)W - T)^{-1} \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$$

Finally, noting that for all i, $\eta_{\widehat{\mathcal{P}_i,Z_1}} = -\eta_{GNI_i,Z_1}$ concludes the demonstration.