Implementable rules for international monetary policy coordination*

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1 Introduction

What are the key dimensions that monetary policymakers should take into account in a financially integrated world? In principle, international financial integration should provide countries with more efficient means to finance growth, and effective ways to diversify their portfolios.[1] While this should improve welfare, and thus facilitate the policy problem (by reducing the number of inefficiencies), the recent international financial turmoil, and the ensuing literature, emphasize the downside of financial globalization, pointing to an increased vulnerability to foreign shocks, particularly for emerging economies, and thus to an increased complexity of the monetary policy problem (e.g. Rey, 2016 and Obstfeld, 2017).

A substantial literature has analyzed the nature of monetary policy in face of multiple distortions in goods markets and domestic and international financial markets (see citations below). But there are fewer papers that provide practical guidance for the policy-maker on the type of rules that should guide monetary policy when the policy environment is constrained by these types of distortions. This paper aims to address this question.

As an initial reference point, we focus on a modeling framework developed in Banerjee et al. (2016), which compares the international propagation of shocks under different monetary policy regimes in an environment with nominal rigidities and multiple financial frictions in domestic and international capital markets. That study finds that simple Taylor-type rules – traditionally deemed appropriate when the key policy tradeoff stems from nominal rigidities (see e.g. Woodford, 2001) – perform quite poorly in comparison to fully optimal rules (cooperative or non-cooperative) that take into account the structure of international capital flows and financial frictions. That paper does not elaborate on the key dimensions that monetary policymakers should take into account when setting monetary policy in financially integrated economies. We address that question directly in the current paper.

In particular, we aim to provide more precise policy recommendations concerning the set of variables that should guide monetary policy decisions in integrated economies, beyond traditional inflation and output indicators. A large literature has discussed interest rate rules for open economies within models that do not feature a prominent role for salient international capital movements, financial intermediation and financial frictions.[2] Some papers discuss the role of financial integration, but either in small-open-economy models (e.g. Morón and Winkelried, 2005 and Garcia et al. (2011)), or abstracting from the strategic policy dimensions and thus not using welfare-based optimal policies as benchmarks. Most of the focus in this literature has been on the role that the exchange rate should play in policy rules. Our focus is rather on the role that financial factors, ranging from credit spreads to leverage and international capital mobility, should play in informing monetary policy decisions.

To this end, we use the model developed by Banerjee et al. (2016) (BDL), which features stan-

[1] The empirical evidence on these effects remains ambiguous. Prasad et al. (2007) found little support for this theoretical prediction, although they point to confounding factors (governance, lack of regulation etc.) as a key source of meagre empirical evidence on the benefits of globalization.

[2] See for example Ball (1999), Taylor (2001), Laxton and Pesenti (2003), Kollmann (2002), Leitemo and Söderström (2005), Batini et al. (2003), Devereux and Engel (2003a), Adolfson (2007), Svensson (2000), Devereux et al. (2006), Kirsanova et al. (2006), Wollmershäuser (2006), Galí and Monacelli (2005), Pavasuthipaisit (2010)

dard nominal rigidities alongside financial frictions and international financial interdependence. The first dimension gives rise to the traditional inflation stabilization incentives, while the second provides incentives to stabilize credit spreads, as well as to mitigate the negative consequences of international financial spillovers.

BDL have shown that in this model the non-cooperative optimal monetary policies imply responses to shocks that are quite similar to those produced by cooperative policies. On the contrary, "standard" Taylor-type policy rules, responding only to domestic inflation and output, generate allocations that appear quite different from the optimal ones. BDL argue that taking the financial dimension explicitly into account – not done by simple Taylor-type rules – can considerably improve outcomes. These authors, though, do not compute the simple optimal rules that would describe to which extent the financial dimensions should be taken into consideration. Our paper extends the results obtained by BDL by providing exactly these explicit simple rules, and thus by emphasizing the role of financial frictions in shaping the policy tradeoff.

Optimal policy rules in our model, as typical in the DSGE literature and *a fortiori* in the real world, cannot always easily be represented by simple rules (Giannoni and Woodford, 2004). In order to achieve our goal, we borrow from the empirical macroeconomic literature, and try to obtain a simple, albeit approximate description of optimal monetary policy rules through the lens of SVARs. This approach has the appeal to use the same tools and language used in recent econometric studies where monetary policy is shown to respond to financial factors (e.g. Brunnermeier et al., 2017). By treating the DSGE model as the data generating process, and by using a novel extension of recent SVAR identification techniques, we isolate targeting rules for monetary policy that closely replicate the optimal monetary policy outcomes derived in Banerjee et al. (2016). These rules indicate prominent roles for non-traditional factors (such as credit spreads) in the description of optimal monetary policy.

Finally, the focus of our contribution is on welfare-based policy rules, whether fully optimal or approximately so.[3]. In the international monetary policy coordination literature, an important dimension of policy outcomes is the payoff that each country can reap by committing to the optimal cooperative rule relative to "self-oriented" alternative.

2 Example: Recovering rules in the New-Keynesian model

In this section we provide a stylized example of how we aim to approximate optimal policies through simpler rules.[4]

[3]The earlier literature used stylized models with ad-hoc objective functions for the policymakers (e.g. Hamada, 1976, Canzoneri and Henderson, 1992). With a growing emphasis on the choice-theoretic foundations of macroeconomic models the literature has increasingly converged towards the adoption of model-consistent policy objectives, and towards the use of analytic frameworks borrowed from public finance and the optimal taxation literature in particular. For recent studies on the international dimensions of monetary policy see for example Devereux (1990), Devereux and Engel (2003b), Obstfeld and Rogoff (2002), Corsetti and Pesenti (2001), Coenen et al. (2009) Canzoneri et al. (2005) and Benigno and Benigno (2006) among others

[4]This section summarizes the key insights, relevant for out analysis, of Giannoni and Woodford (2003a,b) and of the SVAR literature (e.g. see Rubio-Ramirez et al., 2010, for a recent survey).

Consider the optimal policy problem in a variant of the canonical closed-economy New-Keynesian (NK) model (Woodford, 2003). To illustrate the point at issue we augment the canonical NK model with (external) consumption habits and working capital (e.g. Christiano et al., 2005). The role of these assumptions will appear evident in the derivation of the policy problem.

This simple model (in linearized form) consists of a consumption Euler equation (translated in terms of output gap x_t) and a Phillips curve for inflation (π_t), i.e.

$$E_t x_{t+1} = (1+h)\,x_t - hx_{t-1} + \alpha E_t\left(i_t - \pi_{t+1} - i_t^\star\right) \tag{2.1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa\left(x_t + \nu_t\right) + \rho i_t \tag{2.2}$$

where (i_t) is the nominal interest rate, where $i_t^\star$ is the natural rate of interest (i.e. a convolution of disturbances, including productivity shocks), x_t is the output gap, π_t is the inflation rate, ν_t is a cost-push shock, $\beta \in (0,1)$ is the households' time-preference factor, $\alpha > 0$ is the intertemporal elasticity of substitution, $\kappa > 0$ is a coefficient involving a measure of price rigidity and the discount factor, $\rho > 0$ parametrizes the effect of working capital, and $h \in (0,1)$ measures the importance of consumption habits. To ease our illustration, we assume that all shocks are mean-zero, unit-variance iid stochastic processes.

In order to close the model we must specify the behavior of the monetary authority, as inflation (or equivalently the nominal interest rate) is not pinned down by the decentralized behavioral equations. For simplicity we describe the loss function of the monetary authority as

$$\mathcal{L}_t = E_t \frac{1}{2} \sum_{i=0}^{\infty} \beta^i \left[\left(\pi_{t+i} - \pi^\star\right)^2 + \lambda_x x_{t+i}^2\right] \tag{2.3}$$

where a $\star$ denotes the inflation target. For the sake of illustration, we assume that the interest rate *target* is a stochastic processes (a policy preference shock).

Minimizing equation (2.3) in terms of π_t, x_t and i_t, subject to equations (2.1) and (2.2) yields the following first order conditions (where $\phi_j; j = \{1,2\}$ are Lagrange multipliers),

$$\pi: \quad \pi_t - \pi^\star - \alpha\beta^{-1}\phi_{1,t-1} - \phi_{2,t} + \phi_{2,t-1} = 0 \tag{2.4a}$$

$$x: \quad \lambda_x x_t + (1+h)\,\phi_{1,t} - h\beta E_t \phi_{1,t+1} - \beta^{-1}\phi_{1,t-1} + \kappa\phi_{2,t} = 0 \tag{2.4b}$$

$$i: \quad \alpha\phi_{1,t} + \rho\phi_{2,t} = 0. \tag{2.4c}$$

This system can be further simplified as

$$\pi: \pi_t - \pi^\star - \frac{\alpha}{\rho}\phi_{1,t} + \left(\frac{\alpha}{\rho} - \alpha\beta^{-1}\right)\phi_{1,t-1} = 0 \tag{2.5}$$

$$x: \phi_{1,t} = \frac{\rho}{\beta\left(\rho\left(1+h\right)+\kappa\alpha\right)}\phi_{1,t-1} + \frac{h\beta\rho}{\left(\rho\left(1+h\right)+\kappa\alpha\right)}E_t\phi_{1,t+1} - \frac{\lambda_x\rho}{\left(\rho\left(1+h\right)+\kappa\alpha\right)}x_t \tag{2.6}$$

In the special case of $\rho = 0$, i.e. the canonical NK model, rearranging these equations would yield the well known optimal targeting rule for this model, i.e.

$$\pi_t = -\frac{\lambda_x}{\kappa}(x_t - x_{t-1}) + \pi^\star \tag{2.7}$$

In general however, it might be the case that such simple representation is not feasible, e.g. when $\rho \neq 0$ in our example.

Even in this case though, a further simplification is feasible if $h = 0$. In this case we could still eliminate the Lagrange multipliers, expressing (2.6) as an infinite sum of lagged output gaps,[5] i.e.

$$\phi_{1,t} = -\sum_{j=0}^{\infty}\left(\frac{\rho}{\beta(\rho+\kappa\alpha)}\right)^j \frac{\lambda_x \rho}{(\rho+\kappa\alpha)} x_{t-j} \tag{2.8}$$

By replacing this in equation (2.5) we get a "simple" that excludes Lagrange multipliers, i.e.

$$\pi_t - \pi^\star\alpha + \sum_{j=0}^{\infty}\left(\frac{\rho}{\beta(\rho+\kappa\alpha)}\right)^j \frac{\lambda_x \rho}{(\rho+\kappa\alpha)}\left(\frac{\alpha}{\rho}x_{t-j} - \left(\frac{\alpha}{\rho} - \alpha\beta^{-1}\right)x_{t-1-j}\right) = 0 \tag{2.9}$$

One obvious drawback of this simplified representation of the optimal policy is that it involves an infinite sum. In practice though, a relevant question is how many lags are necessary in order to approximate the optimal rule sufficiently well.[6]

Finally, if $\rho > 0$ and $h > 0$, the simple backward iteration will not eliminate the multipliers. In this case, a practical question is to what extent the optimal rule can be approximated by a simpler rule that omits the multipliers.

The direct substitution approach just described is of limited practical applicability in larger models like ours. We therefore resort to a more general alternative approach based on regression methods that exclude multipliers. There are two appealing features of this alternative, and one drawback. The pros are that i) the coefficients of estimated simple rule are not restricted to be the same as those of the infinite order representation: they can partially compensate the error coming from truncation; even when the simple recursive substitution is not feasible, the estimated rule could compensate for the omitted multipliers using other endogenous variables as proxies; and ii) this method would "filter" the data produced by our model in the same way that we would filter real world data (as in the SVAR literature). The latter aspect would allow us to estimate the same rule using real-world data and compare it with our aproximated rule. The main drawback is that this method is potentially computing intensive.[7]

To preview the estimation based approach using our simple model, consider the case of $\rho = 0$.[8] The New-Keynesian economy under optimal policy is summarized by equations (2.1), (2.2) and (2.7).

We can find the rational-expectation solution of our model using the method of undetermined coefficients. We conjecture that the output gap at time t depends only on exogenous and predetermined

[5]This is obviously possible only if certain parameter restrictions are satisfied, so that the sum converges. This would be the case for small enough values of ρ in our example.

[6]Beyond this simple model, even eliminating all the Lagrange multipliers could leave us with a rule involving a very large number of variables. One further dimension of approximation would then consist of replacing some of the endogenous variables too. In this case, the accuracy of the approximation would inform us about which variables play a more important role in the rule (at the selected lag length).

[7]The estimation error could in principle be controlled by using as many simulated data points as necessary.

[8]Clearly this case would not require any approximation. We use this case as it greatly simplifies the illustration.

variables of the model (minimum state variable solution), i.e.

$$x_t = c_1 x_{t-1} + c_2 \nu_t + c_3 \pi_t^\star. \tag{2.10}$$

By replacing this conjecture into equation (2.2) and making use of equation (2.7) one can find the coefficients of the conjecture that solve the rational expectation equilibrium, and that this is indeed the unique saddle-path solution of the linearized New-Keynesian model under optimal policy. To make our point it suffices to show that we can represent the solution as an SVAR, i.e.

$$A_0 z_t = A_1 z_{t-1} + \varepsilon_t \tag{2.11}$$

where $z_t' = [\pi_t, x_t, i_t]$, $\varepsilon_t' = [\nu_t, \pi_t^\star, i_t^\star]$, and where the matrices of coefficients are

$$A_0 = \begin{bmatrix} 1 & \frac{\lambda_x}{\kappa} & 0 \\ 0 & 1 & 0 \\ 0 & \left(\frac{\lambda_x}{\kappa} - \alpha\right)(c_1 - 1) & 1 \end{bmatrix} \tag{2.12}$$

$$A_1 = \begin{bmatrix} 0 & \frac{\lambda_x}{\kappa} & 0 \\ 0 & c_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{2.13}$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 \\ c_2 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2.14}$$

The reduced form VAR is thus

$$z_t = \widetilde{A}_1 z_{t-1} + u_t \tag{2.15}$$

where $\widetilde{A} = A_0^{-1} A_1$ and $u_t = A_0^{-1} \varepsilon_t$ are the reduced form shocks, i.e. a convolution of the structural shocks. Importantly note that $var\,(u_t) = A_0^{-1\prime} A_0^{-1}$.

Assume that we do not know the structure of the model, and in particular we do not know equation (2.7), but that we have enough observations for z_t, then we can estimate $\widetilde{A}$ and u_t, e.g. by OLS.

Let us represent the estimated version of equation (2.15) as

$$z_t = \widehat{\widetilde{A}}_1 z_{t-1} + C\mu_t \tag{2.16}$$

where μ_t are the reduced form shocks normalized to have unit variance, and C is such that $C'C$ is the estimated covariance matrix of the reduced form shocks.

The identification of the monetary policy shocks could then be achieved by imposing restrictions on the coefficient matrices, and in particular on C.

Furthermore, if we know what the true response of z_t to monetary policy shocks is, then we can identify the policy shock in (2.16) by looking for C that yields (approximately) the same IRFs as those produced by the data generating process (2.4) for one of the elements of μ_t. The Appendix summarizes more in details how this is done in the literature and the special application pursued here.

If the identification is approximately successful, it must be that $C \approx A_0^{-1}$, so that we can recover the structural representation (2.11), and thus the policy equation. The latter is the equation associated with the policy shock (Leeper et al., 1996).

3 The Model

Our results are structured around the two country core-periphery model developed by Banerjee et al. (2016).[9] We denote the emerging economy with the superscript 'e' and the centre country with the superscript 'c'.

The schemata for our model is described in Figure 1.

[Figure 1 about here.]

In the centre country there are households, global financiers (banks or asset managers[10]), capital goods producers, production firms, and a monetary authority. There is a global capital market for one-period risk free bonds. In the emerging market country there are also households, local borrowers (banks or financial managers), capital goods producers, production firms, and a monetary authority. The centre country households make deposits with global financiers at the centre country risk free rate, and can hold centre country one-period nominal government debt, which may also be traded on international capital markets. The global banks receive deposits from households in the centre country, and invest in risky centre country technologies, as well as in emerging market banks. Banks in both countries finance purchases of capital from capital goods producers, and rent this capital to goods producers. The borrowing banks in the emerging market economy are funded through loans from global banks/financiers.[11] There are two levels of agency constraints; global banks must satisfy a net worth constraint in order to be funded by their domestic depositors, and local EME banks in turn must have enough capital in order to receive loans from global banks. In both countries, the production goods firms use capital and labour to produce differentiated goods, which are sold to retailers. Retailers are monopolistically competitive and sell to final consuming households, subject to a constraint on their ability to adjust prices. This set of assumptions constitutes the minimum arrangement whereby capital flows from advanced economies to EMEs have a distinct directional pattern, financial frictions act to magnify capital slow spillovers, and

[9]More details about this model, including the list of all equations can be found in Banerjee et al. (2015).

[10]In the remainder of the paper, to simplify the discussion, we will refer to capital goods financiers in both the centre and peripheral countries as banks. It should be noted however that the key thing that distinguishes them is that they make levered investments, and are subject to contract-enforcement constraints. In this sense, they need not be literally banks in the strict sense.

[11]This assumption is meant to capture the feature that within-country financial intermediation between savers and investors is more difficult in EMEs than in advanced economies. See e.g. Mendoza Quadrini and Rios-Rull 2009. We could relax the extreme assumption that EME households did not directly finance EME banks. Under the reasonable assumption that frictions in intermediation within EMEs exceeded those in the centre country, our qualitative results would remain unchanged.

(due to sticky prices) the monetary policy and exchange rate regime may have real consequences for the nature of spillovers and economic fluctuations.

The emerging country is essentially a mirror image of the centre country, except that households in the emerging country do not finance local banks, but instead engage in inter-temporal consumption smoothing through the purchase and sale of centre currency denominated nominal bonds.[12] Banks in the emerging market use their own capital and financing from global financiers to make loans to local entrepreneurs. The net worth constraints on banks in both the emerging market and centre countries are motivated along the lines of Gertler and Karadi (2011).

The goal of monetary policymakers is to respond to domestic and foreign shocks in order to maximize welfare of the domestic households. This amounts to reducing the welfare losses emerging from two frictions: nominal rigidities and agency problems in the financial sector.

3.1 The Emerging Market Economy (EME)

A fraction n of the world's households live in the emerging economy. Households consume and work, and act separately as bankers. A banker member of a household has probability θ of continuing as a banker, upon which she will accumulate net worth, and a probability $1-\theta$ of exiting to the status of a consuming working household member, upon which all net worth will be deposited to her household's account. In every period, non-bank household members are randomly assigned to be bankers so as to keep the population of bankers constant.

While EME households do not have access to the local financial market, they can trade in international bonds (B_t^e) with foreign agents.[13] These bonds can be thought of as T-bills of the core country (rebated directly to core-country households), deposits at core-country financial intermediaries, or simply bonds traded directly with core-country households. Under the assumptions of our model these three alternatives are equivalent.[14]

Households in the EME have preferences over (per capita) consumption C_t^e and labor H_t^e supply given by:

$$E_0 \sum_{t=0)}^{\infty} (\chi_t^e \beta)^t \left(\frac{C_t^{e(1-\sigma)}}{1-\sigma} - \frac{H_t^{e(1+\psi)}}{1+\psi} \right)$$

where consumption is broken down further into consumption of home (C_{et}^e) and foreign (C_{ct}^e) baskets as

$$C_t^e = \left(v^{e\,\frac{1}{\eta}} C_{et}^{e1-\frac{1}{\eta}} + (1-v^e)^{\frac{1}{\eta}} C_{ct}^{e1-\frac{1}{\eta}} \right)^{\frac{1}{1-\eta}}$$

[12]We assume that the market for centre country nominal bonds is frictionless. Adding additional frictions that limit the ability of emerging market households to invest in centre country nominal bonds would just exacerbate the impact of financial frictions that are explored below.

[13]This is clearly an extreme case. In reality EME households' saving does reach domestic firms via the local banks too. Since, in our model, EME households can lend to domestic firms indirectly, via the international financial market, our assumption emphasizes the strong influence that core-country financial conditions exert on EME financial markets.

[14]In particular we are not assuming a special role for government debt, nor asymmetries in the degree to which the contract between depositors and core-country banks can be enforced.

Here $\eta > 0$ is the elasticity of substitution between home and foreign goods, $v^e \geq n$ indicates the presence of home bias in preferences,[15] and we assume in addition that within each basket, goods are differentiated and within country elasticities of substitution are $\sigma_p > 1$. χ_t^e is a log-normally distributed preference shock.

The price index for EME households is

$$P_t^e = \left(v^e P_{et}^{1-\eta} + (1-v^e)P_{ct}^{1-\eta}\right)^{\frac{1}{1-\eta}}.$$

Then the household budget constraint is described as follows

$$P_t^e C_t^e + S_t B_t^e = W_t^e H_t^e + \Pi_t^e + R_{t-1}^* S_t B_{t-1}^e.$$

Households purchase dollar (centre country) denominated debt (B_t^e). S_t is the nominal exchange rate (price of centre country currency). They consume home and foreign goods. W_t^e is the nominal wage, and Π_t^e represents profits earned from banks and firms, net of new capital infusion into banks. R_t^* is the centre country rate on bonds. Households have the standard Euler conditions and labor supply choices described by

$$E_t \Lambda_{t+1}^e \frac{R_t^*}{\pi_{t+1}^e} \frac{S_{t+1}}{S_t} = 1$$

$$\frac{W_t^e}{P_t^e} = C_t^{e\sigma} H_t^{e\psi}$$

where $\Lambda_t^e \equiv \frac{\chi_t^e}{\chi_{t-1}^e} \beta \left(\frac{C_t^e}{C_{t-1}^e}\right)^{-\sigma}$, and $\pi_{t+1}^e \equiv \frac{P_{t+1}^e}{P_t^e}$.

Given two-stage budgeting, it is straightforward (and omitted here) to break down consumption expenditure of households into home and foreign consumption baskets.

3.1.1 Capital goods producers

Capital producing firms in the EME buy back the old capital from banks at price Q_t^e (in units of the consumption aggregator) and produce new capital from the final good in the EME economy subject to the following adjustment cost function:

$$P_t^e I_t^e \left(1 + \zeta \left(\frac{P_t^e I_t}{P_{t-1}^e I_{t-1}} - 1\right)^2\right)$$

where I_t^e represents investment in terms of the EME aggregator good.

EME banks then finish the capital goods and rent them to intermediate goods producers.[16]

$$K_t^e = I_t^e + (1-\delta) K_{t-1}^e$$

[15] Home bias is adjusted to take into account of country size. In particular, for a given degree of openness $x \leq 1$, $v^e = 1 - x(1-n)$, and a similar transformation for the centre country home bias parameter.

[16] Equivalently, we could assume that the bank provides risky loans to intermediate goods producers, who use the funds to purchase capital. The only risk of this loan concerns the (real) gross return on the underlying capital stock.

where K_t^e is the capital stock in production.

3.1.2 EME banks

EME banks begin with some bequeathed net worth from their household, and continue to operate with probability θ, as described above. We also follow Gertler and Karadi in the nature of the incentive constraint. Ex ante, EME banks have an incentive to abscond with borrowed funds before the investment is made. Consequently, conditional on their net worth, their leverage must be limited by a constraint that ensures that they have no incentive to abscond.

At the end of time t a bank i that survives has net worth given by $N_{t,i}^e$ in terms the EME good. It can use this net worth, in addition to debt raised from the global bank, to invest in physical capital at price Q_t^e in the amount $K_{t+1,i}^e$. Debt raised from the global bank is denominated in centre country currency (although later we will experiment with local-currency denomination). In real terms (in terms of the centre country CPI), we denote this debt as $V_{t,i}^e$. Thus, EME bank $i's$ balance sheet is given by

$$N_{t,i}^e + RER_t V_{t,i}^e = Q_t^e K_{t+1,i} \tag{3.1}$$

where $RER_t = \frac{S_t P_t^c}{P_t^e}$ is the real exchange rate.

Bank $i's$ net worth is the difference between the return on previous investment and its debt payments to the global bank.

$$N_{t,i}^e = R_{k,t}^e Q_{t-1}^e K_{t,i}^e - RER_t R_{ct-1} V_{t-1,i}^e$$

where R_{ct-1} is the ex-post real interest rate received by the global bank, equal to the predetermined nominal interest rate adjusted by ex-post inflation in the centre country, and $R_{k,t}^e$ is the gross return on capital.

Because it has the ability to abscond with the proceeds of the loan and its existing net worth, the loan from the global bank must be structured so that the EME bank's continuation value from making the investment exceeds the value of absconding. Following Gertler and Karadi (2011), we assume that the latter value is κ_t^e times the value of existing capital (κ_t^e is a random variable, and represents the stochastic degree of the agency problem). Hence denoting the bank's value function by $J_{t,i}^e$, it must be the case that

$$J_{t,i}^e \geq \kappa_t^e Q_t^e K_{t+1,i}^e. \tag{3.2}$$

This is the incentive compatibility constraint faced by the bank.

Once the bank has made the investment, at the beginning of period $t+1$ its return is realized.

The problem for an EME bank at time t is described as follows:

$$\text{Max } \; J_{t,i\;[K_{t+1,i}^e, V_{t,i}^e]}^e = E_t \Lambda_{t+1}^e \left[(1-\theta)(R_{k,t+1}^e Q_t^e K_{t+1,i}^e - RER_{t+1} R_{ct} V_{t,i}^e) + \theta J_{t+1,i}^e\right]$$

subject to (3.1) and (3.2).

The full set of first order conditions for this problem are set out in Banerjee et al. (2015).

The evolution of net worth averaged across all EME banks, taking account that banks exit with probability $1-\theta$, and that new banks receive infusions of cash from households at rate δ_T times the existing value of capital, can be written as:

$$N^e_{t+1} = \theta\left((R^e_{kt+1} - \frac{RER_{t+1}}{RER_t} R_{c,t})Q^e_t K^e_t + \frac{RER_{t+1}}{RER_t} R_{c,t} N^e_t\right) + \delta_T Q^e_t K^e_{t-1}$$

The first term on the right hand side captures the increase in net worth due to surviving banks, given their average return on investment. The second term represents the 'start-up' financing given to newly created banks by households.

Firms in the EME hire labour and capital to produce retail goods. Since a central aim of our analysis is to explore the role of monetary policy and the exchange rate regime for capital flows and macroeconomic spillovers, we assume that firms in both countries have Calvo-style sticky prices with Calvo re-set parameter $1-\varsigma$. The representative EME firm has production function given by:

$$Y^e_t = A^e_t H^{e(1-\alpha)}_t K^{e(\alpha)}_{t-1}$$

Given this, then we can define the aggregate return on investment for EME banks (averaging across idiosyncratic returns) as

$$R^e_{kt+1} = \frac{R^e_{zt+1} + (1-\delta)Q^e_{t+1}}{Q^e_t}$$

where R_{zt+1} is the rental rate on capital and δ is the depreciation rate on capital.

The representative EME firm chooses labour and capital so as to minimize costs. We can then define the EME firm's real marginal cost implicitly by the conditions

$$MC_{et}(1-\alpha)A^e_t H^{e(-\alpha)}_t K^{e(\alpha)}_{t-1} = W^e_{rt} \tag{3.3}$$

$$MC_{et}\alpha H^{e(1-\alpha)}_t K^{e(\alpha-1)}_{t-1} = R^e_{zt}$$

The Calvo pricing formulation implies the following specification for the PPI rate of inflation π^{PPI}_{et} in the EME. Here Π^*_{et} denotes the inflation rate of newly adjusted goods prices, F_{et} and G_{et} are implicitly defined, and $\frac{\sigma_p}{\sigma_p-1}$ represents the optimal static markup of price over marginal cost:

$$\Pi^*_{et} = \frac{\sigma_p}{\sigma_p-1}\frac{F_{et}}{G_{et}}\pi^{PPI}_{et} \tag{3.4}$$

$$F_{et} = Y_{et}MC_{et} + \mathbb{E}_t\left[\beta\varsigma\Lambda^e_{t,t+1}\pi^{PPI\eta}_{et+1}F_{et+1}\right] \tag{3.5}$$

$$G_{et} = Y_{et}P_{et} + \mathbb{E}_t\left[\beta\varsigma\Lambda^e_{t,t+1}\pi^{PPI-(1-\eta)}_{et+1}G_{et+1}\right] \tag{3.6}$$

$$\pi^{PPI1-\eta}_{et} = \varsigma + (1-\varsigma)\left(\Pi^*_{et}\right)^{1-\eta} \tag{3.7}$$

3.2 Center country

Except for banks, the center country sectors are identical to the peripheral country. Banks differ instead both in terms of funding and investments.

3.2.1 Centre country banks

A representative global bank j has a balance sheet constraint given by

$$V_{jt}^{e} + Q_t^c K_{jt}^c = N_{jt}^c + B_t^c$$

where V_{jt}^e is investment in the EME bank, and $Q_t^c K_{jt}^c$ is investment in the centre country capital stock. N_{jt}^e is the bank's net worth, and B_t^c are deposits received from households. All variables are denominated in real terms, (in terms of the centre country CPI).

The global bank's value function can then be written as:

$$J_{jt}^c = E_t \max_{K_{j,t+1}^c, V_{jt}^e, B_t^c} \Lambda_{t+1}^c \left[(1-\theta)(R_{kt+1}^c Q_t^c K_{jt}^c + R_{ct} V_{jt}^e - R_t^* B_t^c) + \theta J_{jt+1}^c \right]$$

Here, Λ_{t+1} is the stochastic discount factor for centre country households, R_{ct} is, as described above, the return on the global bank's loans to the EME bank, and R_t^* is the risk-free rate paid to domestic depositors.

The bank faces the no-absconding constraint:

$$J_{jt} \geq \kappa_t^c \left(V_{jt}^e + Q_{ct} K_{jt}^c \right)$$

where, as in the EME case, κ_t^c measures the degree of the agency problem, and as before we assume it is subject to exogenous shocks.

The full set of first order conditions for the global bank are discussed in detail in (Banerjee et al., 2015). As in the case of the EME banks, we can describe the dynamics of net worth for the global banking system by averaging across surviving banks, and including the 'start-up' funding provided by centre country households. We then get the following law of motion for net worth

$$N_{t+1}^c = \theta \left(\left(R_{kt+1}^c - R_t^* \right) Q_t^c K_t^c + \left(R_{c,t} - R_t^* \right) V_t^e + R_t^* N_t^c \right) + \delta_T Q_{t+1}^c K_{t-1}^c. \tag{3.8}$$

Again, the details of the production firms and price adjustment in the centre country are identical to those of the EME economy.[17]

3.3 Exogenous shocks

We consider four different types of exogenous shocks per country: TFP, preference, financial and policy shocks. We also include a UIP shock, which captures the "excess volatility" of the real exchange rate,

[17]See Banerjee et al. (2015), the working paper version of this article, for the full set of equations.

not fully explained by our model.

All stochastic processes, except preference shocks, are modeled as autoregressive of order one. Preference shocks are modeled AR(1) processes in (log) differences (i.e. $\log(\chi_t^j) - \log(\chi_{t-1}^j) \approx AR(1)$, $j = \{e, c\}$).

3.4 Monetary policy

We use various characterizations of monetary policy, depending on our specific objective. For the calibration of the model we assume that central banks in each country follow an interest rate rule similar to those used in estimated DSGE models (see in particular Del Negro et al., 2013), i.e.

$$\log R_{t,j} = \lambda_{r,j} \log R_{t-1,j} + (1 - \lambda_{r,j}) \left(\frac{\lambda_{\pi,j}}{4} \sum_{i=0}^{3} \log \left(\frac{\pi_{t-i}^j}{\pi_{ss}^j} \right) + \frac{\lambda_{y,j}}{4} \log \left(\frac{Y_t^j}{Y_{t-3}^j} \right) \right) + \varepsilon_{r,t}^j : \; j = \{e, c\}, \quad (3.9)$$

where $\varepsilon_{r,t}^j$ is a monetary policy shock.

For the policy exercises we first derive the optimal cooperative and non-cooperative policies as discussed further below, and then derive simple targeting rules that can approximate the fully optimal rules sufficiently well.

3.4.1 Optimal monetary policies

Following BDL we consider two alternative optimal policy regimes: i) Optimal policy under cooperation; and ii) Optimal policy under non-cooperation (Nash equilibrium). Only after confirming that under our calibration, as in BDL, the Nash and cooperative policies generate very similar responses of the economies to shocks, we will focus our analysis on the derivation of simple approximately optimal rules under cooperation.

The non-cooperative frameworks require an explicit definition of policy instruments. The "Taylor-rule" framework consists of setting the nominal interest rate in response to a subset of variables. The optimal non-cooperative framework cannot be set in terms of policy rates, for reasons akin to the "indeterminacy" of Sargent and Wallace (1975).[18] We focus on the case in which PPI inflation is the instrument of the non-cooperative game.

The two optimal-policy frameworks can be defined more precisely as follows:

Definition 1 (Cooperative policy problem)**.** *Under the cooperative policy (CP) problem both policymakers choose the vector of all endogenous variables Θ_t excluding the policy "instruments", and the policy instruments $\pi_{GDP,t}^e$ and $\pi_{GDP,t}^c$ in order to solve the following problem*

$$\mathcal{W}_{CP,0} \equiv \max_{\Theta_t, \pi_{GDP,t}^e, \pi_{GDP,t}^c} \left[\varsigma n \mathcal{W}_0^c + (1 - \varsigma)(1 - n) \mathcal{W}_0^e \right], \quad (3.10)$$

[18] See for example , and the discussion in Blake and Westaway (1995) and Coenen et al. (2009).

subject to

$$E_t F\left(\Theta_{t+1}, \Theta_t, \Theta_{t-1}, \pi^e_{GDP,t+1}, \pi^c_{GDP,t+1}, \pi^e_{GDP,t}, \pi^c_{GDP,t}, \pi^e_{GDP,t-1}, \pi^c_{GDP,t-1}, \Phi_{t+1}, \Phi_t, \Phi_{t-1}; \varphi\right) = 0, \tag{3.11}$$

where households welfare is defined as

$$\mathcal{W}^j_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{j(1-\sigma)}}{1-\sigma} - \frac{H_t^{j(1+\psi)}}{1+\psi} \right), \tag{3.12}$$

where C_t is consumption, H_t is labor, $\beta \in (0,1)$ is the discount factor (common across countries and shared by policymakers), $\psi > 0$ and $\sigma > 0$ are preference parameters, Φ_t, is the vector of all exogenous shocks, φ is the parameter measuring the importance (loading) of the exogenous shocks in the model ($\varphi = 0$ implies that the model is deterministic), $\varsigma \in (0,1)$ is the Pareto weight, and $F(\cdot)$ is the set of equations representing all the private sector resource constraints, the public-sector constraints, and all first-order conditions solving the private sector optimization problems.

Furthermore, the policymaker is subject to the "timeless-perspective" constraint, which defines the $t = 0$ range of possible policy interventions (see Benigno and Woodford, 2011a).

Denoting by the conformable vector Γ_t the vector of Lagrange multipliers on the constraint (3.11), the first order conditions of this problem can be defined as

$$E_t \mathcal{P}\left(\Theta_{t+1}, \Theta_t, \Theta_{t-1}, \pi^e_{GDP,t+1}, \pi^c_{GDP,t+1}, \pi^e_{GDP,t}, \pi^c_{GDP,t}, \pi^e_{GDP,t-1}, \pi^c_{GDP,t-1}, \Phi_{t+1}, \Phi_t, \Phi_{t-1}, \Gamma_{t+1}, \Gamma_t, \Gamma_{t-1}; \varphi\right) = 0, \tag{3.13}$$

We can then state the following:

Definition 2 (Cooperative Equilibrium)**.** *The cooperative equilibrium is the set of endogenous variables (quantities and relative prices) and policy instruments, such that given any exogenous process Φ_t equations* (3.11) *and* (3.13) *are jointly satisfied $\forall t$.*

The (open-loop) non-cooperative policy problem can be defined as follows:

Definition 3 (Non-cooperative policy problem)**.** *Under the non-cooperative policy (NP) problem , each policymaker chooses independently all endogenous variables and her own instrument in order to solve the following problem*

$$\mathcal{W}^j_{NP,0} \equiv \max_{\Theta_t, \pi^j_{GDP,t}} \mathcal{W}^j_0 : \ j = \{e, c\}, \tag{3.14}$$

subject to

$$E_t F\left(\Theta_{t+1}, \Theta_t, \Theta_{t-1}, \pi^e_{GDP,t+1}, \pi^c_{GDP,t+1}, \pi^e_{GDP,t}, \pi^c_{GDP,t}, \pi^e_{GDP,t-1}, \pi^c_{GDP,t-1}, \Phi_{t+1}, \Phi_t, \Phi_{t-1}; \varphi\right) = 0. \tag{3.15}$$

Furthermore, the policymaker is subject to the "timeless-perspective" constraint, which defines the $t = 0$ range of possible policy interventions.

The first order conditions of this problem can be defined as

$$E_t\mathcal{P}^j\left(\Theta_{t+1},\Theta_t,\Theta_{t-1},\pi^e_{GDP,t+1},\pi^c_{GDP,t+1},\pi^e_{GDP,t},\pi^c_{GDP,t},\pi^e_{GDP,t-1},\pi^c_{GDP,t-1},\Phi_{t+1},\Phi_t,\Phi_{t-1},\Gamma^j_{t+1},\Gamma^j_t,\Gamma^j_{t-1};\varphi\right)=0, \tag{3.16}$$

where Γ^j_t is a vector of Lagrange multipliers related to the constrained maximization problem of the j policymaker, where $j = \{e, c\}$.

We can then state the following definition:

Definition 4 (Nash Equilibrium)**.** *The non-cooperative (Nash) equilibrium is the set of endogenous variables (quantities and relative prices) and policy instruments, such that, for any exogenous process* Φ_t*, equations* (3.15) *and* (3.16)*, both for* $j = e$ *and* $j = c$*, are jointly satisfied* $\forall t$*.*

4 Quantitative analysis

4.1 Calibration

We calibrate the model using data for the US (AE) and New Zealand (EM). We chose the latter on the basis of (quarterly) data availability. The model can deliver second moments that are in the ball park of empirical estimates. Tables A.1 and A.2 report model and data unconditional moments under the Taylor rule and under an optimal cooperative monetary policy, using as an example macro moments from the US and New Zealand. In this version of the paper, the approximation of the optimal policy is done assuming a uniform persistence of the shocks of 0.5.

4.2 Spillovers and monetary policy trade-offs

We have assumed that policymakers aim to maximize households' welfare, nationally or globally depending on the cooperative regime. This amounts to tackling inefficiency wedges that drive a gap between the efficient allocation and the actual equilibrium.

There are four kinds of inefficiencies in our model. First, prices do not adjust efficiently to shocks. A price-dispersion wedge ensues, which is the classical inefficiency in New-Keynesian models. Second, financial markets are imperfect, generating a wedge between the policy rate and the cost of capital: the credit spread. Third, households' risk sharing is incomplete, creating a wedge between households' marginal utilities across countries. Fourth, international goods markets are not perfectly competitive, creating a demand externality, and thus a wedge between the actual allocation and the distribution of global demand that is optimal from a social planner point of view. When all these wedges are closed, there is no further role for monetary policy. Of these wedges, nominal rigidities play a crucial role, since in their absence monetary policy would have no traction, except for its effect on nominal debt contracts.

To gain intuition on how these wedges affect the economy, we compare the response to shocks of the inefficiency-ridden economy with the response of the frictionless economy: essentially captured by an international real business cycle model. When nominal rigidities are present, the response of the economy to shocks depends on policy. In this case our benchmark is a NK model with only price rigidities and a simple rule that links the nominal rate to current inflation.

4.2.1 Flexible prices: The effect of financial frictions

Figure 2 shows the response of a **TFP shock** in AE, with and without financial frictions. The dashed line shows the response of the frictionless economy. Relative to a standard IRBC, in this economy there is still a round-about of funds to finance EME's capital. This round-about is frictionless, so that it does not constitute an incentive for a social planner to change allocations. Upon the TFP shock AE's GDP increases while EME's GDP contracts. Real rates (i.e. the policy rate with constant inflation) fall in both countries.

Financial frictions alter this picture particularly for the EME. Credit spreads in the center country fall, thanks to the positive development in asset prices. The ensuing fall in the cost of capital generates a further boost in aggregate activity (only marginally for AE). The international financial interdependence implies an easing of the financial condition in EME too, and due to the double-layer of financial friction this easing is even stronger than in the epicenter country. As a consequence GDP in the EME increases considerably and persistently (after the second quarter) instead of contracting. While CPI inflation is constant, the relative price of domestic output changes reflecting relative productivity: it falls on impact in the center country and increases in the EME. After the initial impact of the unexpected TFP improvement in AE, relative-price changes switch sign and adjust quickly to their long run equilibrium.

Figure 3 shows the response of the two economies to a pure (contractionary) **financial shock** (a positive κ shock). The most remarkable difference, relative to TFP shocks, is the non-monotonic path of most variables. Spreads, on the contrary increase persistently.

What should policy do in this case? To reproduce the frictionless allocation, policymakers should counter the fall in credit spreads, e.g. by generating a (relatively more) contractionary path of real rates. Under flexible prices this could still be achieved to some extent by exploiting the fact that debt contracts are nominal: the ex-post value of debt repayments depends on inflation and on the real exchange rate. Under sticky prices, the traction of monetary policy is much stronger, but policymakers must now confront the price-dispersion wedge too. We turn to this case in the next section.

[Figure 2 about here.]

[Figure 3 about here.]

4.3 The optimal cooperative and non-cooperative policy

How close does the standard Taylor rule come to producing optimal responses of the economy to a TFP shock? Figure 4 shows the response of the world economy to the TFP shock in the center country, under three different monetary policy regimes: Cooperation (solid line); Nash (dashed line); and Taylor rule (circled line). The response under the two optimizing policies is very similar (except on impact). Optimal policy engineers a larger contraction in spreads that overstimulates production. This appears to be achieved by rising real rates markedly on impact. Spreads contract much more under the optimal policies and for a protracted period of time.

What is apparent from the figure is that the standard Taylor rule does not come close to replicating the optimal response to the TFP shock, either the optimal cooperative solution or the non-cooperative (Nash) solution. A similar result is shown for a variety of shocks in Banerjee et al. (2016). This then raises the key question of the present paper - how to characterize an operational monetary policy rule which is implied by the optimal policy outcomes described above?

[Figure 4 about here.]

5 Identification of simple optimal policy rules

We have seen that simple Taylor rules imply adjustments to shocks that are considerably different from those implied by the cooperative allocation. On the contrary, the non-cooperative optimal policies do comparatively better. It is reasonable thus to believe that the simple Taylor rule performs poorly because it neglects important dimensions of our relatively more complex model. In particular the baseline Taylor rule does not react to variables that are better indicators of the financial inefficiency wedge. We thus want to look for simple rules that mimic as close as possible either the cooperative or the non-cooperative policies, i.e. rules that generate IRFs very close to those obtained under optimal policies.

In searching for *simple* good approximations to the truly optimal policies we apply two main criteria. First the rules should strike a balance between limiting the number of variables and lags involved in the rule on the one hand, and approximation error on the other. Second, following Giannoni and Woodford (2004), we wish to characterize the optimal policy in terms of market allocations and prices only, i.e. excluding the Lagrange multipliers of the policy problem. Self-evidently, a practical rule for monetary policy implementation should include only economically observable variables that are available to the policy-maker in real time.

Deriving simple (approximately optimal) rules analytically is not feasible in our model. One alternative would be to assume that policy is conducted using simple linear feedback rules on a subset of variables, and then choose the coefficients of the rules by maximizing an objective function (e.g. global welfare or country specific welfare). This is the strategy commonly used in the literature (e.g. Coenen et al., 2009). A practical drawback of this approach is that without prior knowledge of the number of variables and lags necessary to yield good results, experimentation can be very computationally intensive.[19] While this remains an avenue worth exploring, in this paper we follow an alternative route. In particular we follow a strategy inspired by the empirical monetary macroeconomics literature. The

[19]Note that with this approach the model must be evaluated (to second order) for each draw of the parameters.

VAR literature (e.g. Sims, 1980, and Sims, 1998, Leeper et al., 1996) tells us that identifying monetary policy shocks amounts to identifying the structural policy equation for monetary policy. Furthermore, we know that identification can be achieved (among other approaches) by sign restrictions on IRFs (e.g Uhlig, 2005, Canova and De-Nicoló, 2002, Baumeister and Hamilton, 2015, and Rubio-Ramirez et al. (2010)). Our strategy for identifying optimal monetary policy builds on these insights. In particular we treat our model as the data generating process (DGP) and simulate long series for a subset of variables. We then estimate the VAR on the simulated data, where the structural policy equation is identified by matching the VAR response to a monetary policy shock with the response implied by the DSGE model under optimal (cooperative or non-cooperative) policy. On practical grounds, this approach allows us to consider various policy specifications (i.e. number of variables and lags) relatively more quickly. One additional advantage of this method is that it uses the same lenses through which we read empirical data. In principle we could estimate the same VAR on our simulated data and on real-world data and compare the implied policy rules. We leave this as an extension for future research.

To summarize our procedure (detailed in the Appendix) we use the DSGE under optimal policy (cooperative or not) as our DGP. We draw 800000 realizations for each variable (dropping the first 10%) when all shocks are active. We then estimate a VAR(p) in our 9 variables, where we set p=2 for the sake of parsimony.[20]

We then identify, for each country, a policy shock. In this regard it is important to note that under the optimal policy, typically there would not be policy shocks. We add policy shocks to the first order conditions relating to the policy instruments under Nash, i.e. domestic (PPI) inflation. In particular we assume that the FOCs of the policymakers with respect to domestic PPI inflation are affected by random fluctuations.[21] This choice is motivated on technical grounds, but it is consistent with the idea that a policymaker that would be prone to random deviations from a Taylor rule, could be prone to random deviations from any rule, optimal or not.[22]

The identification of the shock is achieved by minimizing the distance between the IRFs generated by our DGP and those generated by our DSGE when policy is conducted following our approximated rules.

Our aim is to summarize optimal policies by sufficiently simple rules. In pursuing this goal, we have to strike a balance between simplicity and accuracy. On the one hand, omitting variables from the policy rule can result in a deterioration of accuracy, and thus in welfare losses. On the other hand, including too many variables (and lags) can reduce our ability to provide an economic rationale supporting the policy prescription.

We start with a baseline subset of 9 variables: GDP, producer price index (PPI) inflation, real exchange rate (RER_t), credit spreads ($R_{j,k,t+1} - R_{j,t}$, $j = \{e, c\}$), and the two TFP shocks ($A_{e,t}$ and $A_{c,t}$), i.e.

$$x_t = [Y_{e,t}, Y_{c,t}, \pi_{e,t}^{PPI}, \pi_{c,t}^{PPI}, RER_t, spread_{e,t}, spread_{c,t}, A_{e,t}, A_{c,t}] \tag{5.1}$$

[20]To gain intuition on the sources of approximation error, we study cases with $p > 2$.
[21]More precisely in those equations inflation appears as $\pi_{i,t}^{PPI} + \varepsilon_{R_i,t}$; $i = \{e, c\}$, where $\varepsilon_{R_i,t}$ is an i.i.d. shock.
[22]See also Leeper et al. (1996, p. 12).

This selection of variables is inspired by the literature on optimal policies in open economies, as well as the need to be parsimonious. Corsetti et al. (2010), for example, show that in a two-country model the loss function of the policymakers would depend on domestic and foreign output, domestic and foreign inflation, the terms of trade, and a measure related to capital market inefficiency. We then consider possible extensions (e.g. including state variables) and assess how large is the improvement in accuracy.

The policy rules that we back out with this approach may not be unique. This is because there might be other (combinations of) variables perfectly collinear with the variables that we use in the estimation. This problem is shared by theoretical optimal policy rules too. One related additional complication, in the estimation-based approach, consists of the need to ensure stochastic non-singularity of the VAR. This means that we cannot estimate a VAR with more variables than exogenous driving shocks in the DGP.

Our baseline set of variables does not include interest rates. It is well known that optimal (Ramsey) policies cannot always be represented as interest rate rules: their general form is a targeting rule (Svensson, 2010). As shown by Giannoni and Woodford (2003a) in a linear-quadratic context, the optimal targeting rule will display variables that appear in the objective (loss) function, together with the Lagrange multipliers associated to the policy constraints. This implies that optimal rules might not display the interest rate explicitly (pure targeting rules à la Giannoni and Woodford, 2003b). A further problem that can arise by approximating optimal rules with simple interest rate rules relates to implementability and the existence of saddle-path (i.e. unique) equilibria. To the extent that the inferred rule is an approximation of the optimal rule, it might fail to generate uniqueness of equilibria. While this problem can in principle be solved (e.g. aiming for higher accuracy of the approximation) we avoid the issue altogether by resorting to pure targeting rules.

Inclusion of exogenous shocks

Optimal targeting rules establish relationships between "gaps" or efficiency wedges and other variables (e.g. Lagrange multipliers). This is immediately evident in the linear-quadratic approach (e.g. Benigno and Woodford, 2011b) or in general when the reduced-form policy objective function is derived from the equilibrium conditions of the model (e.g. Woodford, 2003 or Corsetti et al., 2010). This means that targeting rules depend both on endogenous variables and exogenous shocks, i.e. equilibrium outcomes under the "natural", frictionless allocations. For example, in the basic New-Keynesian model, driven by TFP shocks only, the optimal targeting rule relates inflation to the output gap (and its lag), where the gap is the distance between output under sticky prices and output under flexible prices. Backing out this policy rule in a VAR would be difficult. The VAR should contain three variables: inflation, output and TFP. Yet there is only one shock, so that stochastic singularity could emerge.[23] Adding measurement errors would be of little help, as it would introduce serially correlated errors in the VAR.

In view of this constraint we consider alternative policy specifications, under different assumptions

[23] Whether it emerges in practice depends on the correct specification of the lag structure too. In the basic New-Keynesian model inflation and output depends only on current TFP so that perfect multicollinearity would make estimation of the tree variable VAR impossible.

concerning the "observability" of exogenous shocks. We limit this analysis to the two key shocks of interest in our study: TFP and financial shocks.

5.1 Identification of optimal cooperative policies

Before describing optimal cooperative policies approximated by simple rules, it is important to stress that under cooperation, and in contrast to the non-cooperative case, we cannot associate one rule to e.g. the EM country and the other rule to the AE country. While each country implements it's mandate with "error", both share the same objective, and this is reflected by their systematic response to endogenous variables and exogenous shocks. This said, we label rules by country name for convenience and consistency with the non-cooperative case.

5.1.1 Using TFP shocks in the VAR

We now assume that that policymakers act cooperatively, up to idiosyncratic shocks to their decision rules. As described above, we estimate pure targeting rules for both countries using the 9 variable VAR as described above. Note in particular that the VAR includes TFP shocks, but no other exogenous shocks. Under the assumption that the DGP for the true model is driven by cooperative policy-making, the identified approximate policy rules can be derived separately for the emerging economy and the advanced economy. The rules take the following form:

$$\sum_{i=1}^{9}\sum_{j=0}^{2}\gamma_{ij}x_{i,t-j}=0$$

where $x_{i,t-j}$ represents the variable i evaluated at time $t-j$, and γ_{ij} is the estimated targeting rule coefficient. In the following tables, we list the coefficients, where we have normalized so that the coefficient on contemporaneous PPI inflation is unity.

Table 1: EM targeting rule: TFP shocks in VAR

Variable	t	t-1	t-2
GDP EM	0.003	-0.031	0.022
GDP AE	-0.01	0.024	-0.012
Spread EM	-0.003	0.002	0
Spread AE	-0.029	0.02	-0.003
PPI infl. EM	1	-0.26	-0.094
PPI infl. AE	0.286	-0.302	0.061
Real exch. rate	0.034	-0.031	-0.003
Shock TFP EM	-0.022	0.049	-0.02
Shock TFP AE	0.057	-0.064	0.014

While these rules are somewhat arbitrary, as discussed earlier they are inspired by the optimal target rules of simpler models. They are history dependent, and functions of variables relevant for the inefficiency wedges (except for the shock).

Table 2: AE targeting rule: TFP shocks in VAR

Variable	t	t-1	t-2
GDP EM	-0.019	0.076	-0.057
GDP AE	0.098	-0.034	-0.061
Spread EM	-0.002	0.001	0
Spread AE	-0.027	0.037	-0.009
PPI infl. EM	0.129	-0.393	0.266
PPI infl. AE	1	-0.964	0.458
Real exch. rate	0.008	-0.015	0.007
Shock TFP EM	0.019	-0.073	0.054
Shock TFP AE	-0.015	-0.076	0.086

Relative to the simple targeting rules of New Keynesian open economy models, the estimated rules here give a role to financial variables; the interest rate spreads for both countries appear in the rules of both countries. In addition, the rules indicate an important interaction between the center country and the emerging economy. For instance, inflation in the center country should react changes in GDP, spreads, and TFP shocks in the emerging markets, and a similar interdependence characterizes the targeting rule for the emerging economy.

Targeting rules inform us about the *trade off*, or optimal sacrifice ratio, faced by policymakers. For example equation (2.7) tells us that a positive output growth must be accompanied by inflation falling below its target. To gain intuition along this perspective Figures 5 and 6 displays the coefficients of the approximated rules (1 and 2) respectively. In particular, these figures relate the weighted average of three quarters of domestic inflation to the normalized coefficients of the other variables appearing in the targeting rule.[24] To improve readability, shaded areas represent different groups of variables.

These graphs suggest that the trade-off between PPI inflation and spreads is very weak, while the major source of trade-off is between domestic and foreign inflation.

[Figure 5 about here.]

[Figure 6 about here.]

We then compare the response of the economy to TFP shocks: the truly optimal cooperative policy implied by the simulated model, and the "estimated" optimal cooperative policy. For comparison purposes, we add the response of the economy under the non-optimized Taylor rule. Figures 7 and 8 show the impulse responses for a subset of variables: seven target variables plus net-worth in the two economies.

We see that in the response to TFP shocks in the advanced economy, the simple targeting rule delivers responses that are very close, if not identical, to those produced by the fully optimal rule. GDP rises in the advanced economy, but falls in the emerging market, which experiences a significant real exchange rate appreciation. Spreads fall in both countries, as does PPI inflation, and net worth rises. Clearly, the targeting rules outlined above capture very closely the responses of the true optimal policy.

[24] This is obtained rewriting the targeting rule as $\frac{a_1\pi^j_{GDP,t} + a_2\pi^j_{GDP,t-1} + a_3\pi^j_{GDP,t-2}}{a_1+a_2+a_3} = \frac{1}{a_1+a_2+a_3}$ (rest of terms), where $j = \{e, c\}$ and where $a_{1\ldots3}$ are the coefficients on inflation.

[Figure 7 about here.]

[Figure 8 about here.]

5.1.2 Using financial shocks in the VAR

The previous figures estimated the VAR including productivity shocks. As an alternative we can estimate the VAR using the following set of variables

$$x_t = [Y_{e,t}, Y_{c,t}, \pi^{PPI}_{e,t}, \pi^{PPI}_{c,t}, RER_t, spread_{e,t}, spread_{c,t}, \kappa_{e,t}, \kappa_{c,t}] \quad (5.2)$$

Here, the VAR includes financial shocks, as defined by shocks to the ICC constraint in the model, described in Section 3 above. Proceeding in the same way as before, we obtain the following estimates for the targeting rules, for the emerging economy and the advanced economy, respectively:

Table 3: EM targeting rule: Financial shocks in VAR

Variable	t	t-1	t-2
GDP EM	-0.021	0.011	0.01
GDP AE	-0.009	0.025	-0.021
Spread EM	-0.105	-0.049	0.129
Spread AE	-0.227	0.736	-0.453
PPI infl. EM	1	-0.498	0.011
PPI infl. AE	0.234	-0.186	0.104
Real exch. rate	0.036	-0.02	-0.018
Shock Fin. EM	0.058	0.025	-0.072
Shock Fin. AE	0.118	-0.366	0.242

Table 4: AE targeting rule: Financial shocks in VAR

Variable	t	t-1	t-2
GDP EM	-0.007	0.024	-0.027
GDP AE	0.071	-0.058	0.022
Spread EM	0.041	0.381	-0.381
Spread AE	-0.152	-0.218	0.293
PPI infl. EM	0.203	-0.104	0.064
PPI infl. AE	1	-0.469	-0.025
Real exch. rate	0.003	-0.036	0.044
Shock Fin. EM	-0.023	-0.208	0.215
Shock Fin. AE	0.06	0.077	-0.147

Interpreted as before, these results imply a significant degree of inflation smoothing. But the important difference now is that conditional on a financial shock, there is a large positive weight on spreads, and most notably on the spread for the advanced economy. This is true for the inflation response of both the advanced economy and the emerging economy. Figures 9 and 10 offer a graphical representation of these results.

[Figure 9 about here.]

[Figure 10 about here.]

Figures 11 to 12 display the response of the economy to shocks, under the three policy constellations, the ad-hoc Taylor benchmark, the fully optimal rule and the approximation obtained by controlling for financial shocks in the VAR. Here we focus on financial shocks in the advanced economy. In this case, we find that the estimated targeting rule does a good job at replicating the optimal response for both shocks. Now we also find that the response to the financial shock in the advanced economy, under the same targeting rule is also highly accurate - spreads increase in both countries, and the response of the spread in the advanced economy is exactly that produced by the optimal policy derived from the DSGE model. But the response of GDP, inflation, the real exchange rate, and net worth is captured very closely using the target rule.

These results give us some confidence in the approximation methods used here to capture optimal policy rules. While it is still the case that in simpler models, targeting rules that omit Lagrange multipliers (which is our key desiderata) may be recovered directly from the model, that is not the case for our two country model with financial frictions. In this case, the simple empirical approach we have outlined here represents a promising way to capture the key features of optimal policy that could be used to guide policy-makers in dealing with the domestic and spillover effects of shocks.

[Figure 11 about here.]

[Figure 12 about here.]

5.1.3 Controlling simultaneously for TFP and financial shocks in the VAR

Controlling for each shock separately, as done in the previous subsections, shows that that the role of credit spreads is not invariant to the specification. Therefore, in this subsection we include both types of shocks at once in the VAR. In order to maintain the number of shocks equal to the number of variables in the VAR we have to trade-off the inclusion of two sets of shocks with the omission of two other variables. The results showed above indicate that domestic GDP plays a relatively minor role in the rule, which makes this variable a good candidate for the omission.

Table 5: EM targeting rule: Financial and TFP shocks in VAR

Variable	t	t-1	t-2
Shock Fin. EM	-0.01	-0.002	0.01
Shock Fin. AE	-0.04	-0.046	0.067
Spread EM	0.016	0.006	-0.016
Spread AE	0.042	0.109	-0.125
PPI infl. EM	1	-0.411	0.012
PPI infl. AE	0.355	-0.173	0.032
Real exch. rate	0.036	-0.032	0.002
Shock TFP EM	-0.019	0.018	-0.001
Shock TFP AE	0.065	-0.034	-0.001

Table 6: AE targeting rule: Financial and TFP shocks in VAR

Variable	t	t-1	t-2
GDP EM	0.001	-0.074	0.07
GDP AE	0.015	-0.137	0.106
Spread EM	-0.001	0.134	-0.122
Spread AE	-0.041	0.249	-0.188
PPI infl. EM	0.02	-0.039	0.013
PPI infl. AE	1	-0.418	0.054
Real exch. rate	0	-0.002	0.011
Shock TFP EM	0.002	0.001	-0.007
Shock TFP AE	0.037	-0.002	-0.003

Figures 13 and 14 show again the same information graphically. Similarly to the case of a VAR controlling for financial shocks only, spreads play an important role here too.

[Figure 13 about here.]

[Figure 14 about here.]

Figures 15 and 16 compare the responses to TFP shocks in AE and EM respectively, under three sets of rules: the approximated rules, the fully optimal cooperative policy, and (as benchmark) the ad-hoc Taylor rule. Figures 17 and 18 display the policy comparison under financial shocks in AE and EM respectively. All four cases display a very tight match between the approximated rules and the fully optimal rules.[25]

[Figure 15 about here.]

[Figure 16 about here.]

[Figure 17 about here.]

[Figure 18 about here.]

6 Conclusion

Our paper addresses the question of how practical rules for monetary policy can be characterized in financially integrated economies, with multiple conflicting distortions, so as to maximize welfare. We contribute to the recent literature on the international dimensions of monetary policy by providing more insights concerning simple, implementable rules that can best approximate fully optimal, but complex, rules. Financial factors are a key channels through which shocks and policies propagate across countries above and beyond exchange rates. The latter have been shown to provide little additional usefulness in designing monetary policies for open economies. International capital flows (and the associated credit

[25] Under financial shocks, the responses reveal a zig-zag behaviour of the rules. It should be noted though that the scale of these oscillating responses is very small, and that this behavior is insignificant for responses sufficiently different from zero.

spreads) are more likely to play an independent role in shaping the response of economies to shocks. Our results indicate that (relatively) simple targeting rules for monetary policy can replicate welfare based optimal monetary policy very closely. These targeting rules differ considerably from commonly described rules for New Keynesian open economy models, and in particular contain non-traditional variables, such as credit spreads. These results are consistent with the basic economic principle that are widely discussed in the related literature: Targeting rules display relationships among efficiency wedges that matter for policymakers, i.e. the "gap" variables that would appear in the loss-function of the policymaker. With sufficiently simple models, the policy objective can be expressed analytically in terms of these gaps, and simple rules can be explicitly and exactly derived. More in general, the richer, and presumably the more interesting are the features of the model, the less likely might be that explicit simple rules can be derived analytically. This is the case of our model. One practical contribution of our paper is to show that we could still provide more economic intuition on the policy tradeoffs by approximating the truly optimal rule using regression methods. In particular we point out that SVAR methods have been used to identify policy in the real world, where less information is available to the econometrician than it is available to the modeler. By exploiting those insights we show that relatively simple rules, that trade off inflation volatility and spread volatility, can bring about allocations very similar to those produced by truly, more complex optimal policies. Our paper can be seen as a first step in the direction of simplifying the communication of policy prescriptions in complex models. One of the appealing features of optimal policy analysis in New-Keynesian models is its intuitive simplicity. The research agenda to which this paper contributes aims at maximizing communicability while expanding the number of real-world frictions that policymakers are likely to face in the real world.

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Appendix

A Data moments and calibration

Table A.1 compares the standard deviation of a subset of variables in the data and in the model. Table A.2 shows the same set of models under the cooperative Ramsey policy.

Table A.1: Standard deviations (%)

Variable/Country	Model	Data
$\Delta\log(GDP)$ EM	1.7	1.8
$\Delta\log(GDP)$ AE	0.92	0.72
$\Delta\log(\frac{I}{GDP})$ EM	4.7	3.6
$\Delta\log(\frac{I}{GDP})$ AE	2.1	1.2
Short term Rate EM	3.2	5.6
Short term Rate AE	2.1	4
Spread AE	5	2.5
$\Delta\log(RER)$	2.8	5.5
Inflation EM	5.6	5.2
Inflation AE	1.9	2.8

Table A.2: Standard deviations under optimal cooperation (%)

Variable/Country	Model	Data
$\Delta\log(GDP)$ EM	1.1	1.8
$\Delta\log(GDP)$ AE	3.2	0.72
$\Delta\log(\frac{I}{GDP})$ EM	4.5	3.6
$\Delta\log(\frac{I}{GDP})$ AE	3.4	1.2
Short term Rate EM	5	5.6
Short term Rate AE	12	4
Spread AE	1	2.5
$\Delta\log(RER)$	3	5.5
Inflation EM	4.3	5.2
Inflation AE	2.3	2.8

B Identification procedure

B.1 VAR representations of the DGP and of the data

The solution of the system (3.11)-(3.13) (or 3.16) can be represented in VAR form as

$$\begin{bmatrix} x_t \\ \Gamma_t \end{bmatrix} = D \begin{bmatrix} x_{t-1} \\ \Gamma_{t-1} \end{bmatrix} + A_0^{-1}\Sigma \begin{bmatrix} \varepsilon_t \\ \varepsilon_{R,t} \end{bmatrix} \tag{B.1}$$

where $\varepsilon_{R,t}$ is the vector of the two policy innovations, and where for simplicity we assume that the shocks are i.i.d. or that they are observable (and thus are included in x_t). Σ is a diagonal matrix of standard deviations of the innovations.

The "empirical" VAR fitted on simulated data takes the form

$$\underbrace{\widetilde{x}_t}_{n\times 1} = F\widetilde{x}_{t-1} + u_t.$$

Excluding multipliers amounts to potentially inducing an omitted variable problem in the estimation of the rule.

B.2 Identification via IRFs matching

Let us recall that in order to determine the policy equation (rule) in the VAR, it suffices to identify the monetary policy shock: The equation corresponding to the identified policy shock is the policy rule.

We achieve identification by minimizing the distance between the "true" IRFs to a policy shock (based on the DSGE under optimal policy) and the IRFs to a policy shock obtained from the VAR estimated on simulated data.

First note that DGP IRFs can be written as

$$x_{t+h} = SD^h A_0^{-1}\Sigma \begin{bmatrix} \varepsilon_t \\ \varepsilon_{R,t} \end{bmatrix} \tag{B.2}$$

where S selects the first block of the h power of the matrix D.

The empirical counterpart is

$$\widetilde{x}_{t+h} = F^h u_t. \tag{B.3}$$

where u_t is a reduced form shock, i.e. a convolution of the structural shocks that we want to disentangle.

Define

$$E(u_t u_t') \equiv \Sigma_u = PQQ'P' \tag{B.4}$$

where P is a factorization of Σ_u, (e.g. Cholesky) and Q is an orthonormal rotation matrix such that $QQ' = I$ (e.g. Canova and De-Nicoló, 2002).

Note that Σ_u has $\frac{n\,(n+1)}{2}$ independent parameters, while the matrix $\widetilde{A}_0^{-1} \equiv PQ$ has n^2 parameters. In order to determine the $n^2 - \frac{n\,(n+1)}{2}$ parameters we need $\frac{n\,(n-1)}{2}$ restrictions (order condition).

Note that although we are only interested in one row of the VAR, we need the full $\widetilde{A}_0$ matrix. Our strategy, borrowed from the SVAR literature on sign-restrictions, consists of deriving the missing restrictions from matching DSGE and VAR IRFs.

Since we have n variables, with h IRFs periods per variable we have $n \times h$ IRF-gap points. Then we need $h^* = n/2 - 1/2$ periods per variable for an exact identification (i.e. h such that $n \times h = \frac{n\,(n-1)}{2}$).

Thus, for example, with 9 variables we can reach exact identification with 4 periods per variable. These points do not need to be contiguous. For example, with 9 variables, we choose to match the first two periods together with the fourth and sixth period.

Let's define the difference between IRFs as

$$\nu_h \equiv \left[\left(x_1 - \widetilde{x}_1 \right), \ldots, \left(x_h - \widetilde{x}_h \right) \right] \tag{B.5}$$

so that ν_h is a $n \times h$ matrix. If $h = h^*$, ν_h contains exactly $\dfrac{n(n-1)}{2}$ elements: the missing restrictions.

Let's assume that the policy shock of interest is the last element of the vector $\varepsilon_{R,t}$, *wlog.*[26] Then we can write

$$\nu_h = \left[SD^H A_0^{-1} \Sigma - F^H \widetilde{A}_0^{-1} \right] \begin{bmatrix} 0 \\ \\ 0 \\ 1 \end{bmatrix} \tag{B.6}$$

where $X^H \equiv \left[I, X, \ldots X^h \right]$, $X = \{D, F\}$.

Since an exact solution ($\nu_h = 0$) is unlikely to exist, due to omitted variables in the VAR, we look for a solution that minimizes the IRFs gap, i.e.

$$(P, Q) = \arg \min_{P,Q} \left\| \mathbb{W} \nu_h \right\|_N \tag{B.7}$$

subject to

$$PP' = \Sigma_u \tag{B.8}$$

where $\|\cdot\|_N$ is the N norm of a vector and $\mathbb{W}$ is a weighting matrix used to give more prominence to particular horizons.[27] We choose $\mathbb{W}$ to penalize sign discrepancies in the first two periods. In particular we do so by multiplying the IRF-gap in the first two periods by 1000, if the signs do not coincide.

The matrix P can be obtained in different ways. The two most common consist of i) the Cholesky factorization of the estimated variance of the residuals u_t, and ii) the spectral decomposition ($E\left(u_t u_t'\right) = PDP'$, where P is the eigenvector matrix and D the diagonal eigenvalue matrix).

Fore example under Cholesky factorization, the whole set of n^2 restrictions would be

$$restrictions = \begin{cases} \arg \min_{P,Q} \left\| \mathbb{W} \nu_h \right\|_N \\ P = chol(\Sigma_u) \end{cases} \tag{B.9}$$

This implies, for example, that the restrictions we are looking for, amount to $\dfrac{n(n-1)}{2}$ possible pairwise rotations of the rows of a Givens (Jacobi) matrix Q, for a given rotation angle θ.

As there are uncountable orthogonal rotation matrices Q, it is difficult to to search for the best

[26] The rotation matrix Q, will make sure that this assumption is satisfied in the empirical model, when the identification conditions are met.

[27] We consider $N = \infty$ and $N = 2$.

identification effectively. A number of methods have bee suggested to make this search practical. Canova and De-Nicoló (2002) and Canova and De Nicolò (2003) suggest the following algorithm (based on Jacobi eigenvalue method for symmetric matrices, Golub and Van Loan, 1996, p 426). First they recall that the eigenvector matrix P can be represented as a product of Jacobi (Givens) rotations

$$P = \prod_{m,n} Q_{m,n}(\theta) \tag{B.10}$$

where $Q_{m,n}(\theta)$ rotates (pairwise) rows m and n by an angle θ, where $\theta \in (0, \pi)$.

Alternatively P could be obtained from the Cholesky factorization of $E(u_t u_t')$ and Q can be obtained from Jacobi (Givens) rotation matrices.

Finally another alternative consists of the following algorithm discussed in Rubio-Ramirez et al. (2010, Algorithm 2, p 688) (RWZ henceforth):

Algorithm 1 (RWZ)**.**

1. *Factorize the covariance of estimated residuals (e.g. Cholesky). Call it* P
2. *Draw* M *matrices* $S^{(m)}$ *from a* $N(0,1)$, $m = 1, \ldots, M$
3. *Generate a QR factorization* $S^{(m)} = Q^{(m)} R^{(m)}$ *(normalizing diag(R) to be positive)*
4. *Generate IRFs using* $PQ^{(m)}$
5. *Chose* $Q^{(m)}$ *such that* $Q^{(m)} = \underset{m}{arg\,min}\ \|\mathbb{W}\nu_h\|_N$

The Givens-based method has the advantage of being parametrized by θ. This implies that if $\|\mathbb{W}\nu_h\|_N$ is (piece-wise) continuous in θ, optimization algorithms can be used (e.g. simulated annealing).

While considering all these alternative orthogonalization procedures, we use as benchmark Givens rotations with optimization based on simulated annealing.

Once the optimal P and Q have been found, the model can be rewritten as

$$\widetilde{A}_0 \widetilde{x}_t = \widetilde{A}_0 F \widetilde{x}_{t-1} + \begin{bmatrix} \varepsilon_t' \\ \varepsilon_{R,t}' \end{bmatrix}. \tag{B.11}$$

where

$$\widetilde{A}_0 = Q' P^{-1} \tag{B.12}$$

The policy equation is then the equation corresponding to the identified policy shock (Leeper et al., 1996, p. 9).[28]

[28] Recently Arias et al. (2015) exploit this fact to impose identifying restrictions consisting of priors about the way policy responds to endogenous variables.

B.3 Using simulated annealing to match IRFs

By using Givens rotations (Golub and Van Loan, 1996) we can parametrize the optimization problem (IRFs matching) in terms of the vector of rotation angles θ. This vector has length $n_\theta \equiv \frac{n(n-1)}{2}$, since there are exactly n_θ possible permutations of the elements of the innovation vector, and there is one angle per permutation. The matching function is appears smooth and continuous along each element of θ, but it is highly non-linear along moving across angles, with a large number of local minima. Using the simulated annealing optimization method seems thus appropriate for our case. We use the GenSA package in R (Xiang et al., 2013) to minimize our matching function (i.e. ν_h).

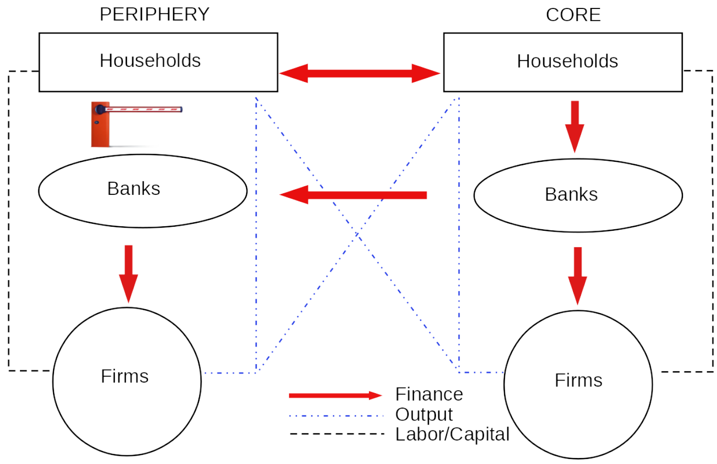


Figure 1: The two-country world

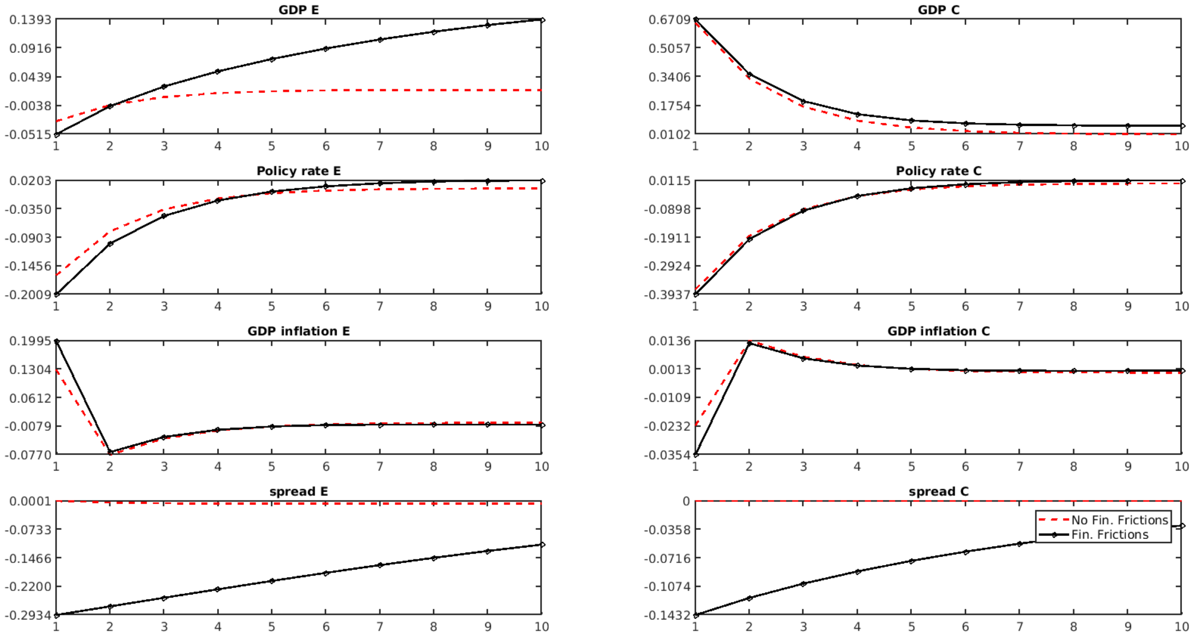


Figure 2: Flexible prices: AE TFP shock

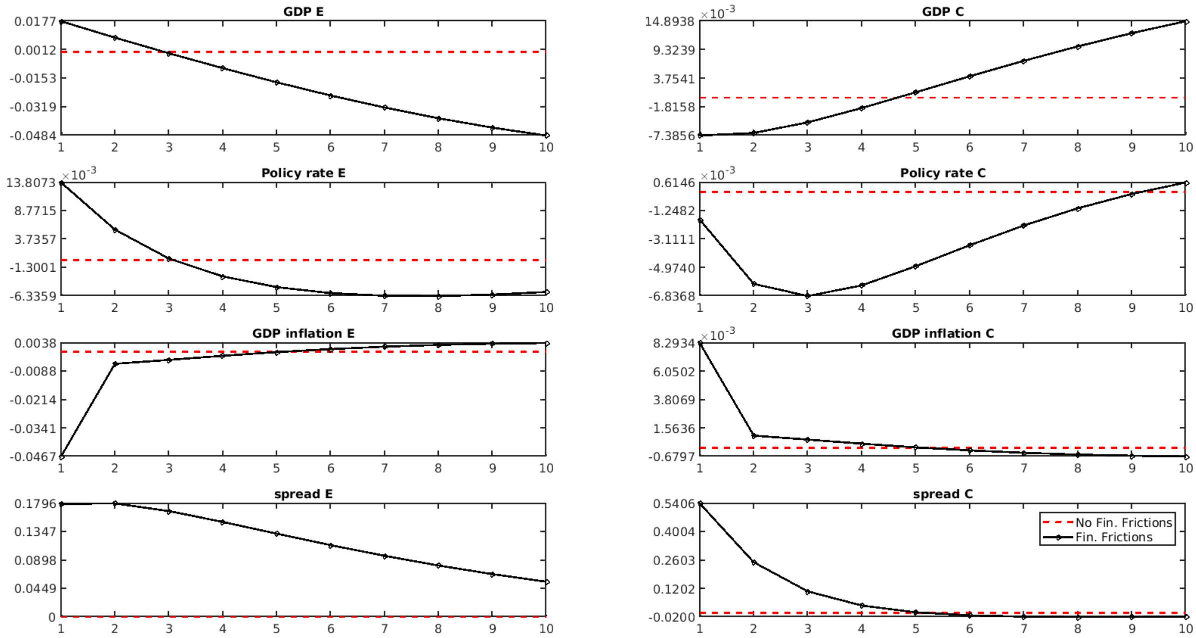
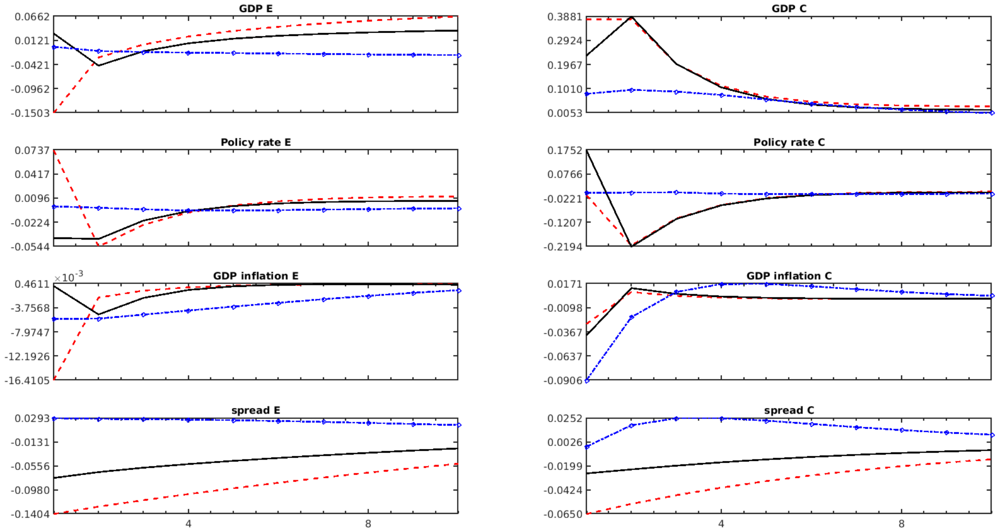


Figure 3: Flexible preices: Financial shock

Figure 4: TFP shock: Alternative policy regimes



Note: Circled line = Taylor rule; Dashed line = Nash; Solid line=Cooperation.

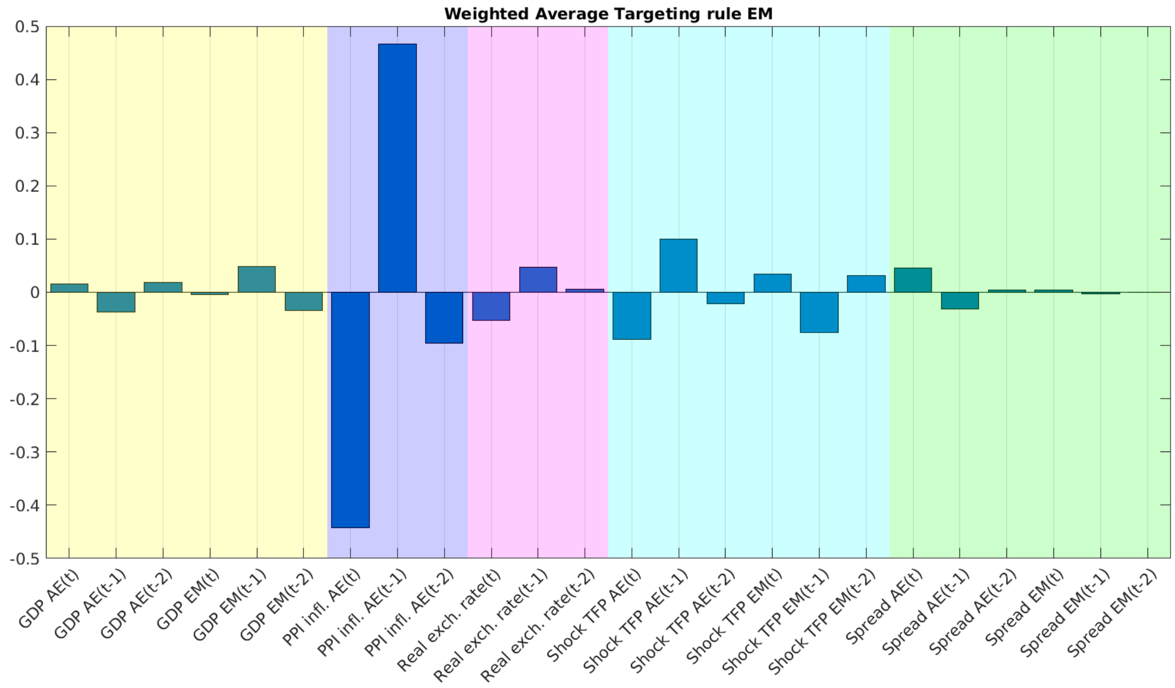


Figure 5: Normalized coefficients of EM targeting rule 1: VAR controlling for TFP shocks.

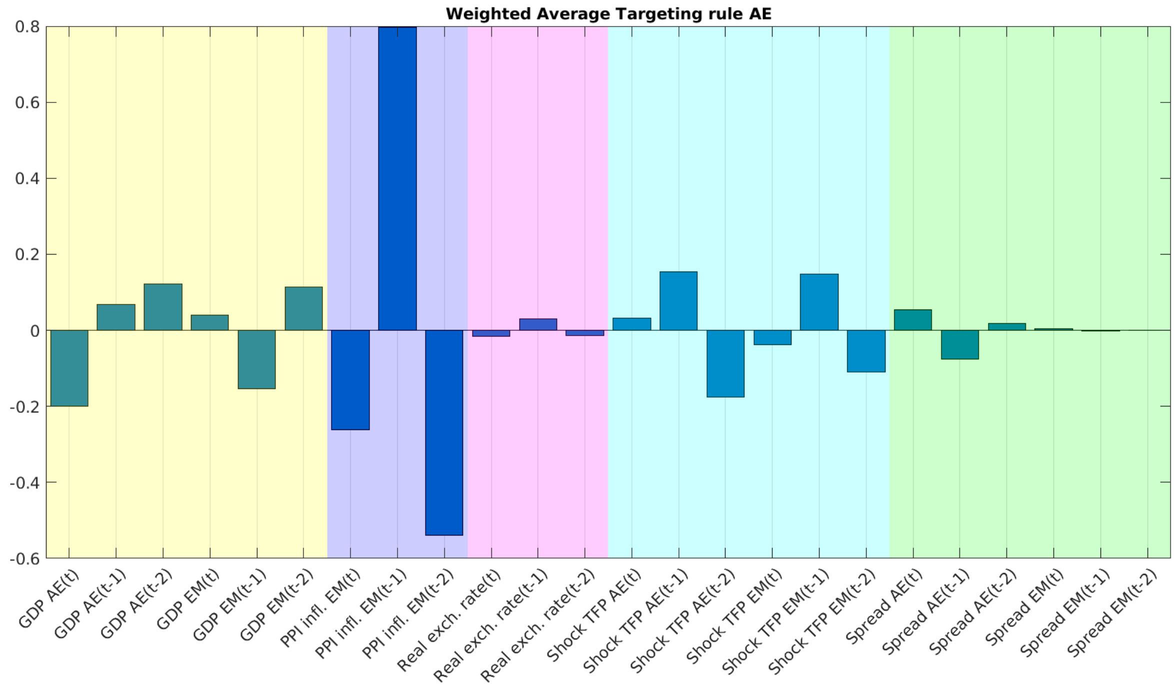


Figure 6: Normalized coefficients of AE targeting rule 2: VAR controlling for TFP shocks.

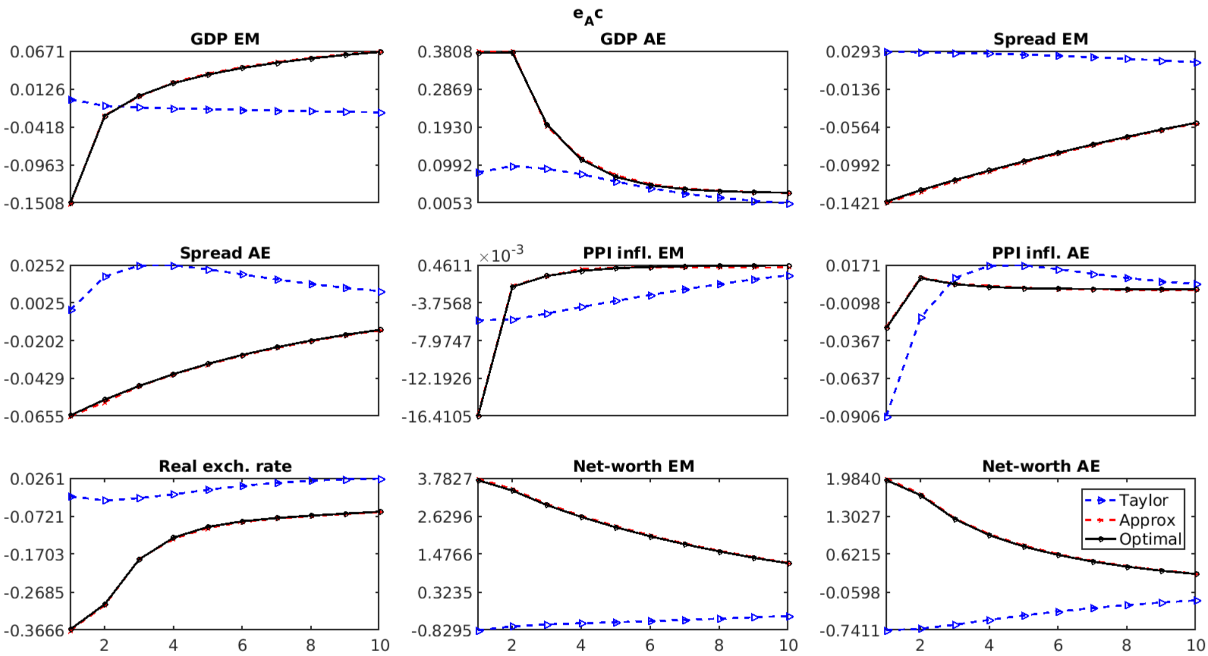


Figure 7: Responses under truly optimal and approximated policies, controlling for TFP shocks in VAR: TFP shock in AE.

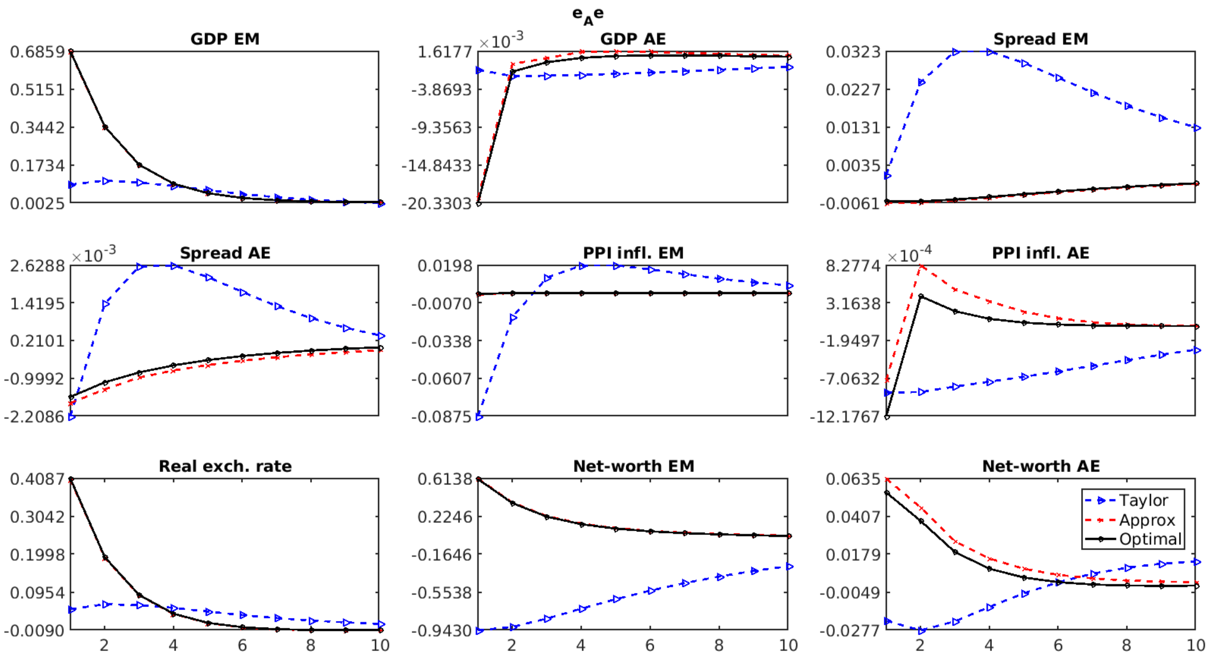


Figure 8: Responses under truly optimal and approximated policies, controlling for TFP shocks in VAR: TFP shock in EM.

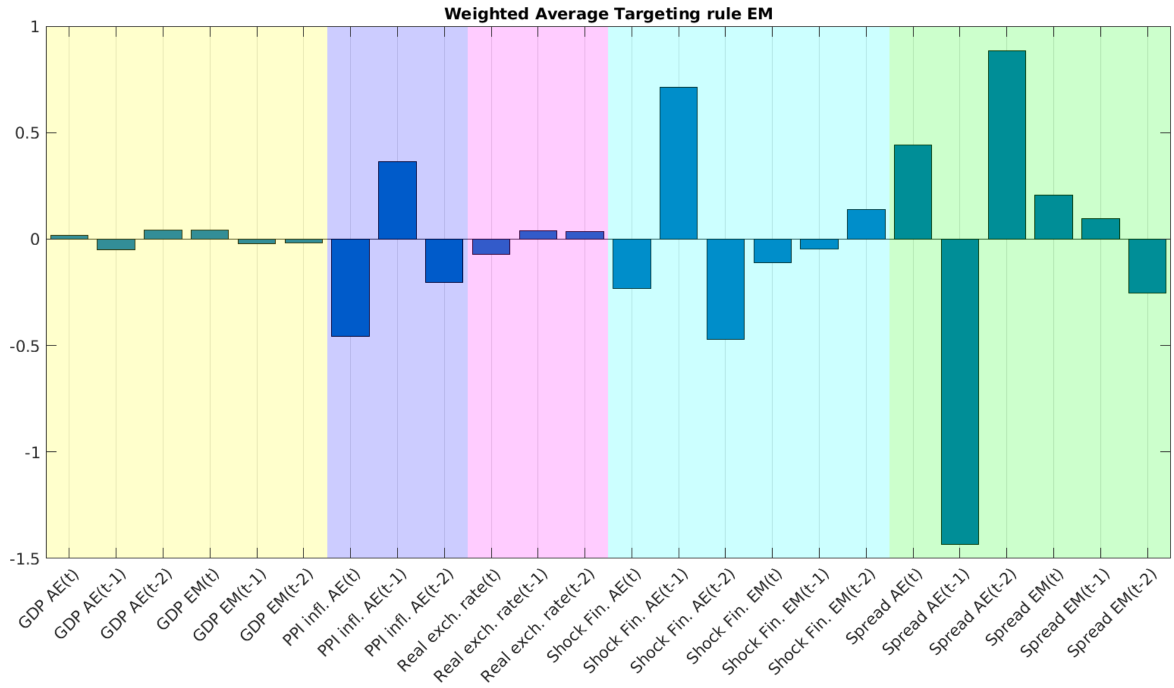


Figure 9: Coefficients of EM targeting rule 3: VAR controlling for financial shocks.

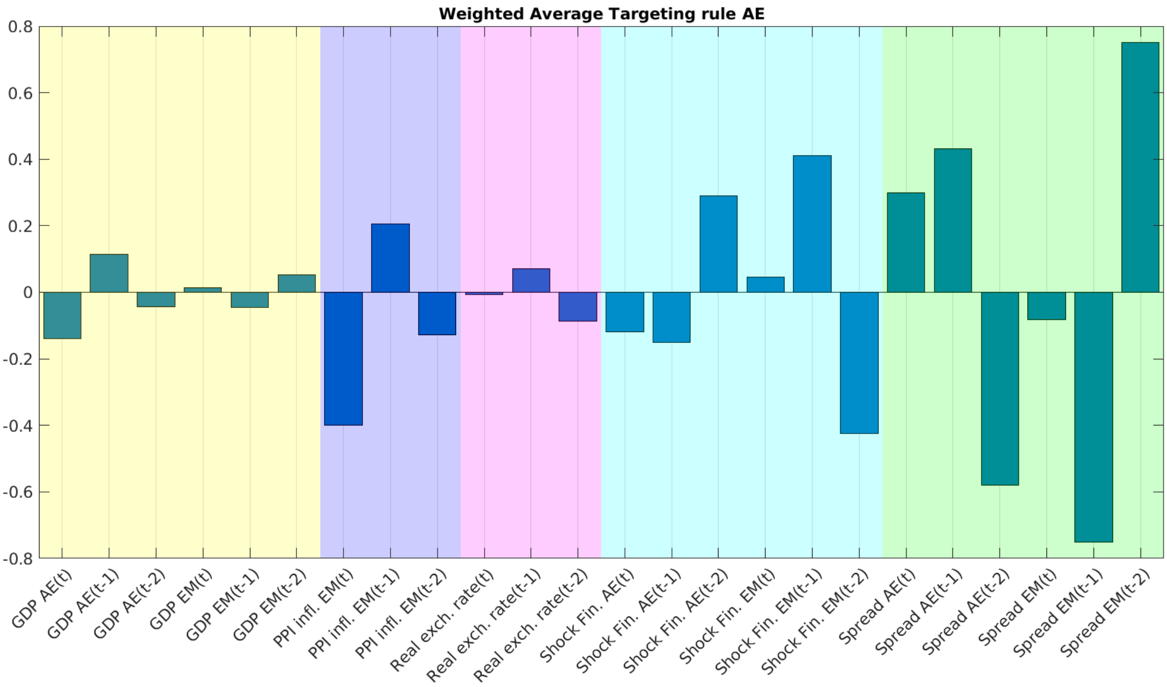


Figure 10: Coefficients of AE targeting rule 4: VAR controlling for financial shocks.

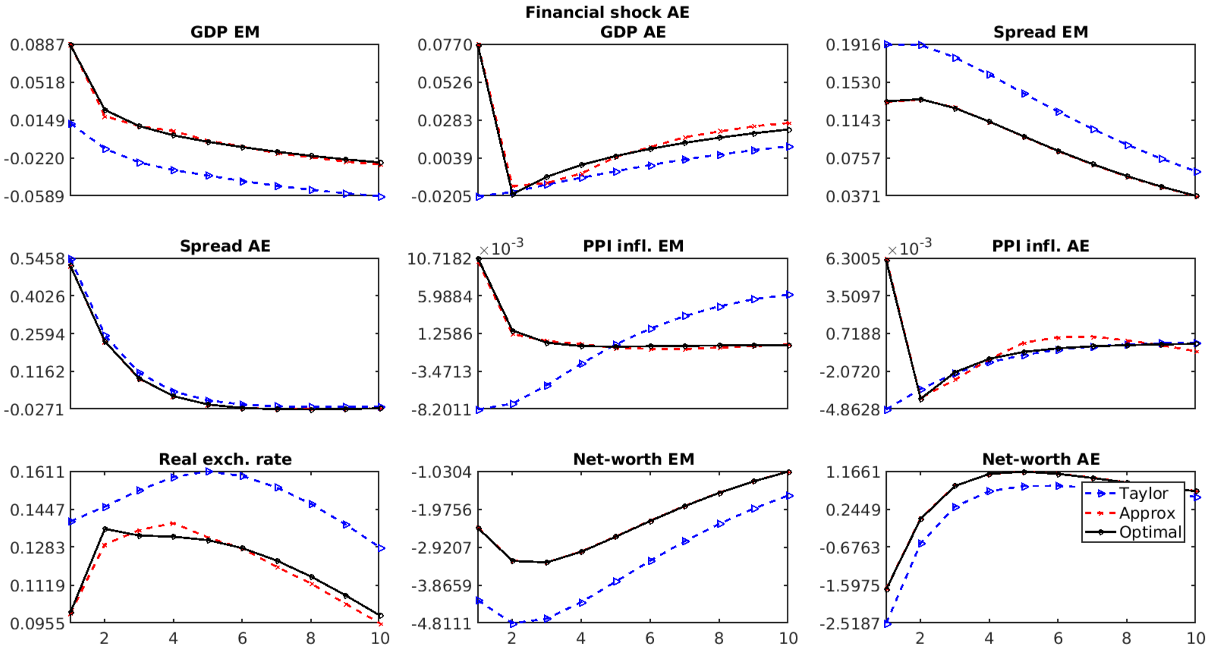


Figure 11: Responses under truly optimal and approximated policies, controlling for financial shocks in VAR: Financial shock in AE.

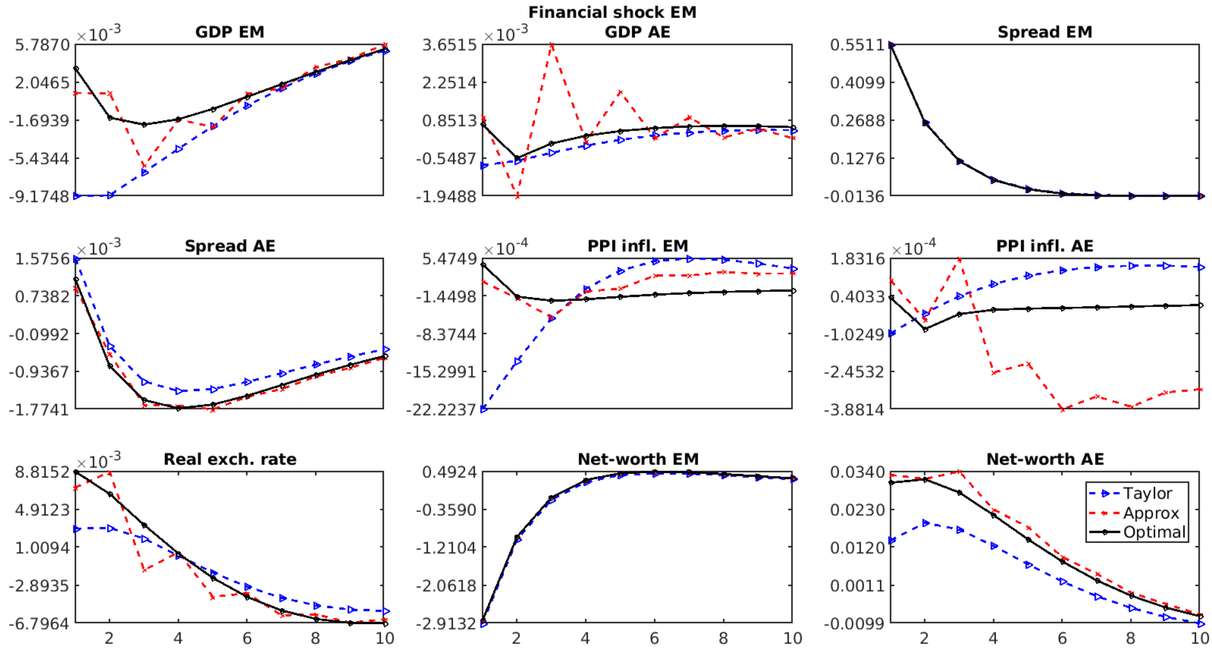


Figure 12: Responses under truly optimal and approximated policies, controlling for financial shocks in VAR: Financial shock in EM.

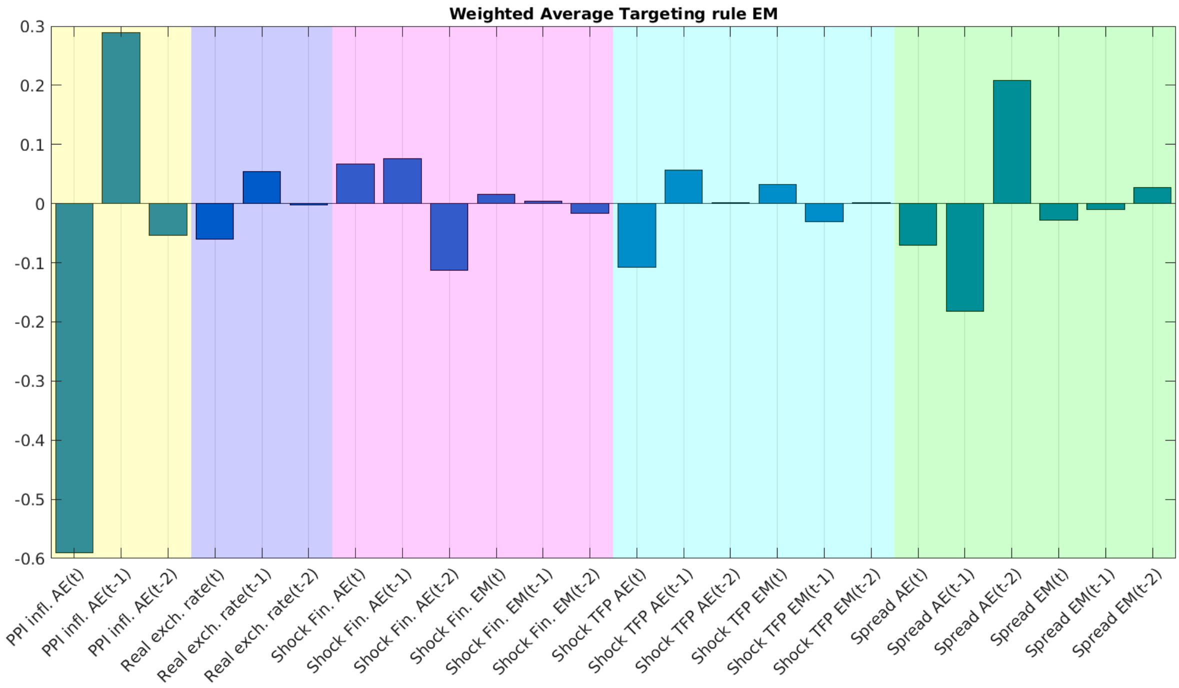


Figure 13: Coefficients of EM targeting rule 5: VAR controlling for TFP and financial shocks.

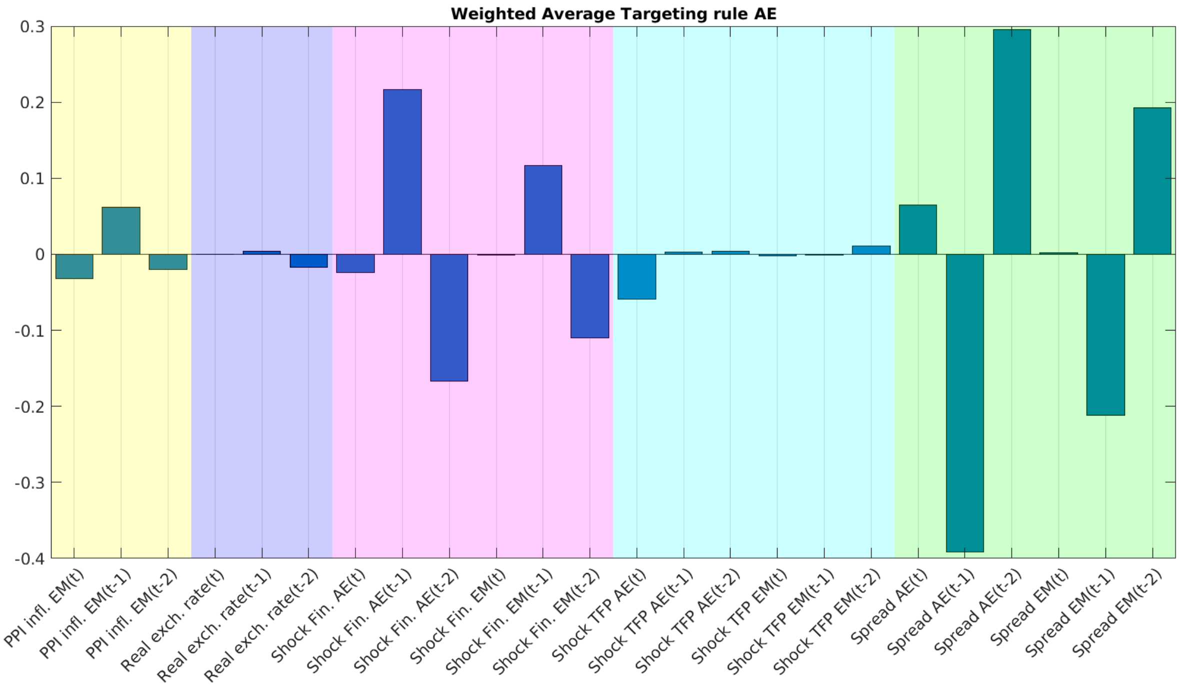


Figure 14: Coefficients of AE targeting rule 6: VAR controlling for TFP and financial shocks.

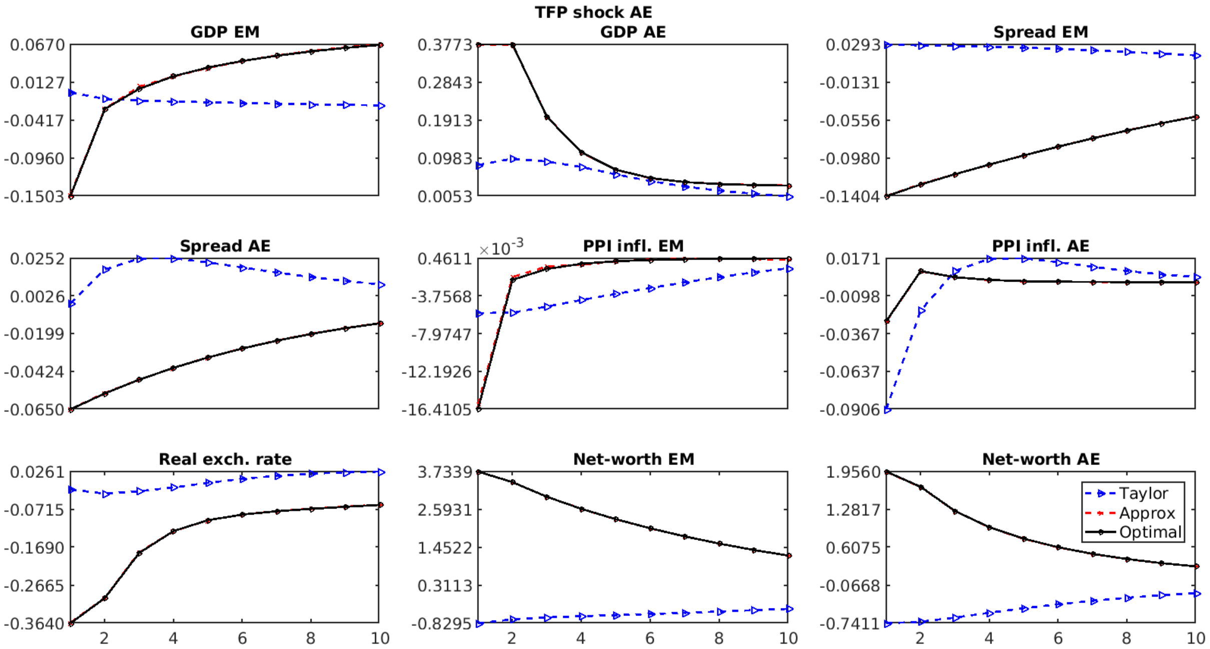


Figure 15: Responses under truly optimal and approximated policies, controlling for TFP and financial shocks in VAR: TFP shock in AE.

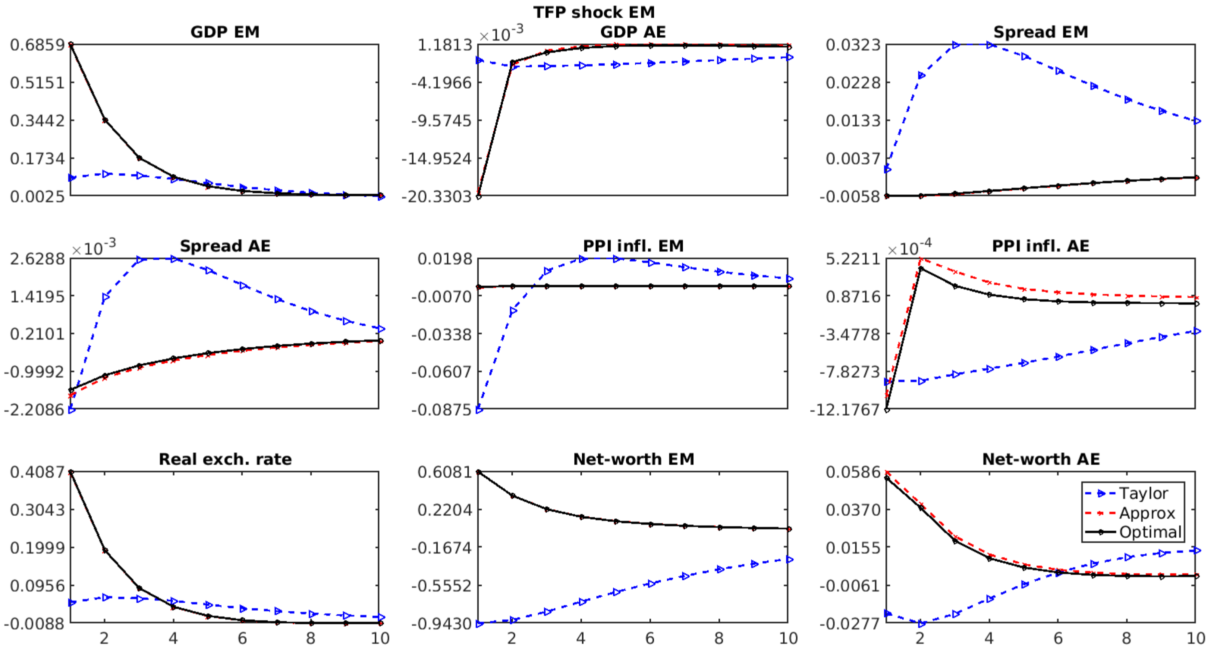


Figure 16: Responses under truly optimal and approximated policies, controlling for TFP and financial shocks in VAR: TFP shock in EM.

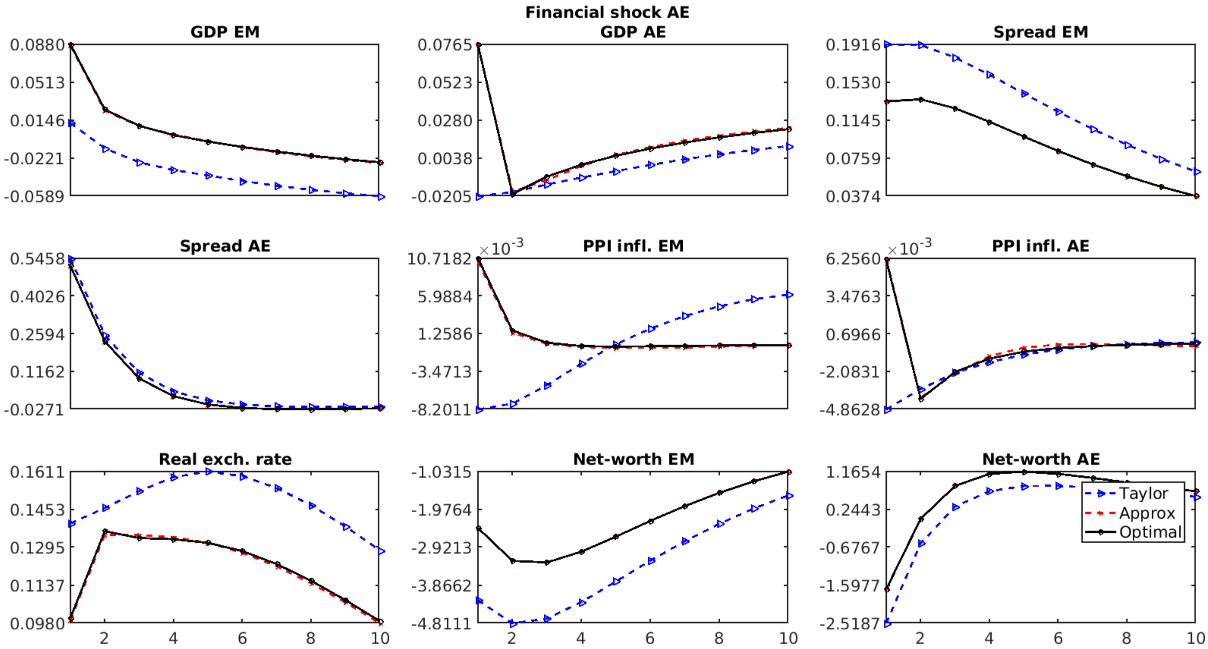


Figure 17: Responses under truly optimal and approximated policies, controlling for TFP and financial shocks in VAR: Financial shock in AE.

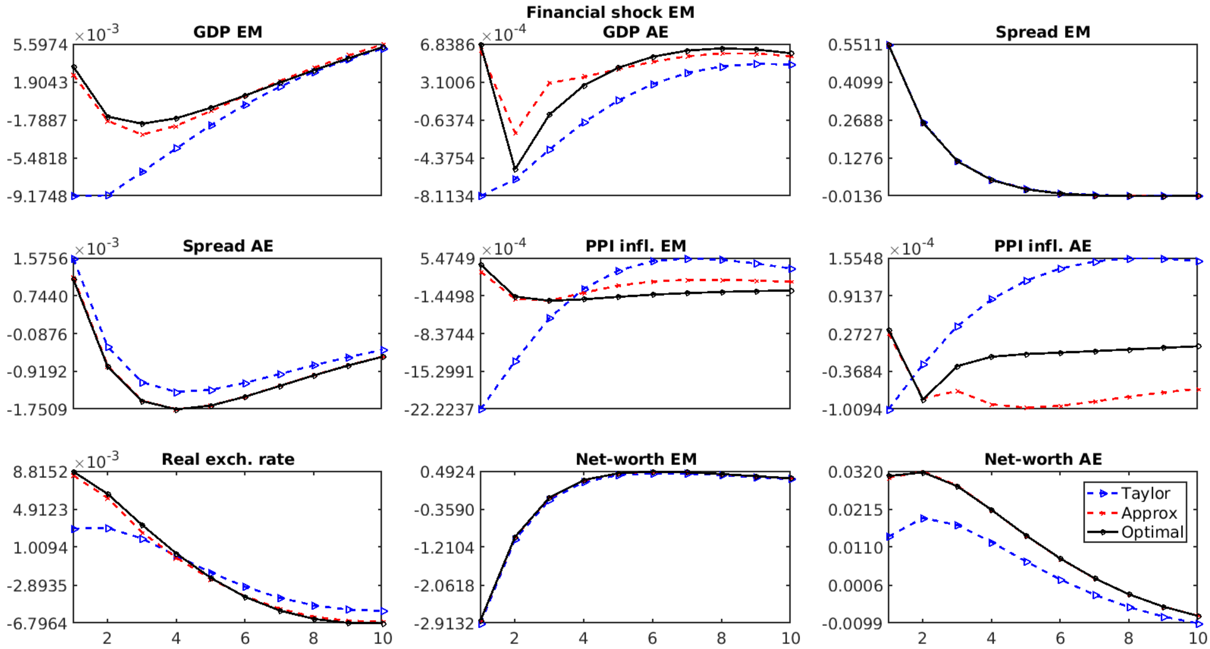


Figure 18: Responses under truly optimal and approximated policies, controlling for TFP and financial shocks in VAR: Financial shock in EM.