HOW TO SPEND A WINDFALL

Dealing with volatility and capital scarcity

Ton S. van den Bremer and Frederick van der Ploeg[1]

University of Oxford

ABSTRACT

Intergenerational funds smooth expected consumption across generations in face of an oil windfall. Precautionary buffers or liquidity funds cope with oil price volatility and are a politically more acceptable alternative to hedging. The magnitude of these buffers depends on the volatility of oil prices, the degree of prudence, intergenerational inequality aversion and the GDP share of oil rents. For a net creditor, asset return uncertainty depresses saving unless prudence is large and risk aversion small. For a net debtor, oil price and asset return uncertainty depress borrowing below what is necessary for consumption smoothing. Uncertain returns on domestic investment offers an alternative explanation why capital scarce, developing countries are big savers and small investors. Allowing for infinite horizons, our ballpark estimate of the optimal liquidity buffer for Ghana is very small relative to its intergenerational fund of 24 billion USD even for very high degrees of prudence, for Norway our rough estimate of the optimal liquidity fund is 156 billion USD (given relative prudence of 3) compared with 1.39 trillion USD for the intergenerational fund. Iraq should build an intergenerational fund of 2.81 trillion USD; even for a very low coefficient of relative prudence (1.025) it accumulates an enormous liquidity fund of 1.92 trillion USD. Given capital scarcity and inefficient adjustment of public capital, we argue that Ghana should be concerned about using its windfall for investment rather than hedging against oil price volatility.

Keywords: oil price volatility, hedging, asset return uncertainty, sovereign wealth fund, precautionary buffers, intergenerational fund, liquidity fund, public investment, adjustment costs, PIMI, economic development, Norway, Ghana, Iraq

JEL code: E21, E22, D91, Q32

26 March 2012

[1] This research was support by the BP funded Oxford Centre for the Analysis of Resource Rich Economies and has been prepared for the CBRT-IMFER Conference on "Policy Responses to Commodity Price Movements", Istanbul, 6-7 April, 2012. Correspondence address: Department of Economics, University of Oxford, Manor Road Building, Manor Road, Oxford OX1 UQ, U.K. We thank our colleagues Torfinn Harding and Samuel Wills for their help with the oil windfall data for Ghana and Iraq, respectively, and are grateful for the comments of Pierre-Olivier Gourinchas, Thomas Helbing, Ayhan Kose and three reviewers.

1. Introduction

Many countries have substantial oil wealth and thus experience oil revenue windfalls.[2] Some countries are able to harness these windfalls for growth and development, especially if their institutions are good, but many other countries have miserable growth performance despite large resource bonanzas (e.g., Sachs and Warner, 1997; Mehlum et al., 2006; Boschini et al., 2007; van der Ploeg, 2011). It has been argued that developing countries facing capital scarcity should use their oil windfalls not to accumulate sovereign wealth according to some permanent income or bird in hand rule, but use them to invest in the domestic economy and boost economic development (e.g., Collier, et al., 2010; van der Ploeg and Venables, 2011, 2012). Given the many political and institutional failures that are present in many oil rich developing economies, it is a tough job to transform subsoil wealth into productive, growth enhancing physical and human capital. However, a much bigger challenge facing oil rich countries is the notorious volatility of oil prices. For a quintessential feature of the natural resource curse is the adverse growth effect of the resulting macroeconomic volatility, especially in countries that have poor financial systems, are landlocked and ethnically fractionalized, and have restrictions on international trade and unrestricted capital flows (e.g., Blattman et al., 2007; van der Ploeg and Poelhekke, 2009, 2010; Aghion et al., 2009).

Our prime objective is therefore to address the vexed question of how economies can put their oil windfalls to good use and how they should cope with the historically high volatility of oil prices.[3] To address this, one must also take account of asset return uncertainty and the massive uncertainty about the return on domestic investment projects. Indeed, the latter type of uncertainty may explain why many developing, resource rich countries are big savers but invest rather little in their domestic economy (e.g., Cherif and Hasanov, 2011). We want to understand why volatility weakens the argument for investing in the domestic economy instead of in sovereign wealth. We also want to obtain an order of magnitude of the optimal shares of the windfalls to be allocated to intergenerational saving on the one hand and precautionary saving on the other hand, and thus how much is left to boost consumption.

Our other main objective is to understand why capital scarcity provides a reason to spend part of the windfall on domestic investment and consumption. We are interested to find out how these shares depend on how volatile oil prices are, how safe or risky investment in foreign assets is, how volatile the interest paid on foreign debt is and on how risky domestic investment projects are. We build on earlier work on

[2] Whenever we refer to oil, it should be interpreted to refer more generally to natural resources (gas, diamonds, copper, bauxite, phosphate, etc.) and could also be interpreted as foreign aid. Oil is thus used as a short-hand for natural resources and foreign aid and oil price as a short-hand for commodity prices.

[3] Earlier work uses a model of a small open economy that exports exhaustible resources to quantify optimal precautionary saving in response to volatile resource prices and demonstrates that current account balances of countries with a greater weight of resource revenue to future income are bigger (Bems and de Carvalho Filho, 2011).

capital scarcity and adjustment costs for adjusting the stock of public capital (van der Ploeg and Venables, 2011, 2012), which suggests that using windfalls of foreign exchange to bring down public and publicly guaranteed debt helps to boost economic development. This is related to the empirical fact that fast growth often goes together with reductions in net foreign liabilities – the 'allocation puzzle' (e.g., Gourinchas and Jeanne, 2007; Aizenman et al., 2004; Pradad et al., 2006). This puzzle can be explained by the idea that capital will not be invested in high debt economies due to the higher risk of expropriation. Limited commitment incentivizes the government to pay down external debt along the adjustment path (e.g., Cohen and Sachs, 1986; Aguiar and Amador, 2011). Debt overhang can exacerbate volatility (e.g., Aguiar et al., 2009). Political economy frictions and the resulting debt dynamics may thus jointly explain the empirically observed negative relationship between volatility and growth. Our model of capital scarcity is inspired by such considerations. We do not give micro foundations of lack of commitment, but instead postulate a relationship between sovereign debt and the risk premium and explore how this affects economic development.

It is important to be clear at the outset that our analysis starts from the premise that the private sector cannot achieve the first best outcome due to various market failures: private agents do not have as good access to international capital markets as governments in many economies and are thus less able to smooth consumption; derivatives and hedging may be too costly, unavailable or politically infeasible; the private sector does not internalize the interest spread externality to do with sovereign risk and capital scarcity; public goods such as infrastructure, education or health are inadequately provided by the market so the government is needed to provide these goods and ensure economic development takes off; the economy is not necessarily able to absorb a rapid buildup of public capital. Furthermore, even if rates of return on domestic capital and foreign assets are equalized, the marginal product of public capital may be higher and the supply of public capital thus lower due to various market and non-market distortions unless the government corrects for these distortions. We use the metaphor of capital scarcity to capture this. There are then two decisions to consider: the first is how much to save; the second is whether to invest at home or abroad.

To get a better grasp of how volatility impacts on the optimal management of oil bonanzas, we start with a simple two period model of an oil rich economy, which is perfectly integrated into world financial markets and has no capital scarcity. We show that whereas the intergenerational fund depends on how temporary the windfall is the optimal amount of precautionary saving depends on the degree of prudence and the aversion to intergenerational inequality. We then extend the model with risky investment in domestic investment projects and introduce capital scarcity to show that it is then optimal to spend part of the windfall on investment.

We have two other objectives. The first one is to extend our insights into managing oil windfalls in stochastic environments to infinite horizon, continuous time frameworks using a novel method for solving the stochastic saddlepoint system of differential equations (van den Bremer and van der Ploeg, 2012) defining the economy and the optimal policy rules. The second one is to apply these techniques to three oil rich countries: Norway, Ghana and Iraq.

Norway does not suffer from capital scarcity. Hence, investing in sovereign wealth according to some precautionary version of the permanent income rule should be optimal. Norway's precautionary buffer or liquidity fund for a coefficient of relative prudence of 3 is about 11.2 percent of its intergenerational fund. For a higher degree of prudence of 10, Norway's liquidity fund is 37.5 percent of the intergenerational fund and the buildup of total assets and the consumption increment during the first three decades look similar to the often advocated bird in hand rules.

Iraq has much larger reserves and they last much longer (at current and projected rates of production), hence its intergenerational fund is much smaller than its liquidity fund. Even for a very low degree of prudence of 1.025, Iraq's liquidity buffer adds another 68 percent to its intergenerational fund. For more realistic degrees of prudence, Iraq's liquidity buffer exceeds its intergenerational buffer but this is at the expense of falling consumption to make room for precautionary saving in the early years of the windfall.

Whilst Iraq is highly vulnerable to volatile oil prices, Ghana's small and short windfall requires a much greater part of the windfall to be saved and thus less vulnerability to oil price volatility. This implies that Ghana needs to add hardly any liquidity buffer to complement its intergenerational fund. However, it seems likely that Ghana does suffer from capital scarcity which holds back domestic investment and development. In that case, we show that it should spend part of its windfall on domestic investment. This speeds up development, albeit that the ramping up of public investment goes together with an increased inefficiency of delivery of public investments.

The outline of the paper is as follows. Section 2 discusses the pragmatic obstacles in using options and structured derivatives to hedge against the risk of oil price volatility. Section 3 discusses the basics of windfall management under volatile oil prices and risky returns on foreign assets in economies that are well integrated into the world financial markets. An appendix discusses how allowing for temporal risk aversion can introduce an additional element of prudence in the management of windfalls. Section 4 deals with oil rich developing economies with capital scarcity and needs to invest in public capital. It shows how the optimal harnessing of oil windfalls in volatile environments is affected by capital scarcity and sheds light on the puzzle why many developing oil rich countries appear to be big savers and small investors. Section 5 uses an infinite horizon, continuous time approach to estimate the sizes of the optimal precautionary buffers or liquidity funds, intergenerational funds and consumption increments for the qualitatively and quantitatively very different windfalls of Norway, Ghana and Iraq. Section 6 introduces capital scarcity and adjustment costs for public investment within an infinite horizon, continuous time framework and uses this to calculate how much of Ghana's windfall should be used for investment, how much to build an intergenerational buffer and how much to boost consumption. Finally, section 7 concludes and offers some policy suggestions.

2. Hedging against volatile oil prices versus other instruments

The most natural way for an oil exporter to deal with the volatility of future oil prices is to hedge against it and transfer the risk to those who are better able to bear it (e.g., Daniel, 2001; Stulz, 2002). An interesting recent example is Mexico, which started hedging in 2009 after oil prices reached heights of almost 140 US dollar per barrel in 2008 and started to decline. The Ministry of Finance bought a put option at a strike price of 70 US dollar per level. When the oil price went significantly below this strike price, the Ministry of Finance pocketed a profit on its option of 8 billion US dollars. The costs of the option were 1.5 billion US dollar. Fig. 1 shows how the hedge worked. Effectively, the loss in oil revenue has been compensated to a large extent by the profits on the option. The option thus provides an insurance policy against the risk of future oil price volatility. Ecuador, Columbia, Algeria, Texas and Louisiana have also used options to protect themselves against volatile oil and gas prices.

Fig. 1: Hedging against oil price volatility

The plain vanilla put option discussed above has relatively high cost. For example, at the money and 20 percent out of the money commodity options for insuring an underlying portfolio of 100 million US dollars will cost 5 and 1 million US dollar, respectively. For gold and copper the hedging costs are much higher. Lu and Neftci (2008) therefore argue for structured reverse reversal option products which lower the cost of plain-vanilla options by selling other options simultaneously (e.g., a so called zero premium collar) but warn that such products can lead to huge financial losses if the commodity prices rises above its cap. Barrier options (e.g., an up and out put option or a knockout option) are a cheaper alternative (Lu and Neftci, 2008). Indeed, many developing countries use commodity derivatives markets to hedge against commodity price risk (e.g., Larson et al., 1998).

Although options and other structured derivative products may be of some help in managing oil price volatility, they are expensive and risky. Furthermore, for most commodities (including oil) maturities are too short, and the financial markets are too thin and lack sufficient depth to provide adequate protection. Also, for many poor countries the political cost-benefit analysis does not work in favor of using options to hedge against oil price volatility. There are huge political risks when a lot of money has been spent on insuring against volatility with plain-vanilla options and the options need not be exercised. Structured products carry even greater risks. Political opponents will be quick to point out that this money could have been better spent on primary education, health care or other worthy causes. If the commodity option is exercised with a profit, opponents will denounce it as speculation. Another complication is that big commodity exporters which hedge against commodity price volatility, especially if they have private information, can influence the market price and stand to be accused of speculation rather than insurance.

We therefore focus in the rest of our paper on the potential use of precautionary buffers or liquidity funds for coping with oil price volatility. Such funds are designed to self insure against periods where the oil price and oil revenue are low. Of course, government should also make sure that goods, labor and capital markets are flexible as this helps the domestic economy to deal better with oil price volatility. It also helps for the government to avoid irreversible commitments which cannot be kept if oil prices fall by a large amount. Independent liquidity funds reduce the need for such politically difficult measures. Furthermore, the country should avoid being dependent on one export commodity such as oil and thus attempt to diversify its economy into sectors whose fortunes are orthogonal to those of the oil sector. Finally, the government can, as an alternative to a liquidity fund, use debt instruments that relate the debt payment to the oil price or use GNI linked bonds to protect itself against oil price volatility. The idea is that, in the event of a crash in oil prices, the government's debt burden falls as well.

Although most of the remainder focuses at how the government should buffer against oil price volatility, the government may also help the private sector to hedge against this volatility and prevent changes in world oil prices being passed on fully to domestic consumers, especially if households are risk averse and face high adjustment costs, credit markets and self-insurance are imperfect and hedging opportunities for private individuals using futures contracts and options are limited (e.g., Federico et al., 2001).[4] Of course, the political benefits of smoothing retail petroleum prices must be set against the associated deadweight losses. Although the tradeoff between retail oil price stability and consumer welfare, on the one hand, and government fiscal stability, on the other hand, poses important challenges given the high volatility of oil prices, it is beyond the scope of the present paper to discuss these.

3. How to spend a windfall without capital scarcity: case for intergenerational and liquidity funds

To highlight the challenge of oil exporters facing volatile revenue streams and asset returns, we first analyze a simple two-period model of saving without investment in physical capital and a power utility function with constant relative prudence and constant relative risk aversion. The power utility function has a positive third derivative, which is a necessary condition for precautionary saving (Kimball, 1990).[5]

In period 1 income consists of exogenous production income Y plus known oil income O_1, which can be either consumed, C_1, or saved in financial assets, A. In period 2, consumption, C_2, equals the gross return on financial assets, $(1 + r + \varepsilon_r) A$, plus oil income, $O_2 + \varepsilon_O$, where ε_r and ε_O denotes the stochastic components of the return on assets and future oil income, respectively. We assume future oil income is uniformly distributed, $\varepsilon_O \sim U(0,\sigma_O^2)$, so $\varepsilon_O \in \left[-\sigma_O\sqrt{3},\sigma_O\sqrt{3}\right]$. The standard deviation σ_O is restricted to be small enough to ensure that consumption is never negative. The return on financial assets is also uniformly distributed, $\varepsilon_r \sim U\left(0,\sigma_r^2\right)$, so $\varepsilon_r \in\left[-\sigma_r\sqrt{3},\sigma_r\sqrt{3}\right]$. The standard deviation σ_r is small enough to ensure that, with negative returns, assets are never fully wiped out, which imposes an upper limit on the range of asset return volatility that can be considered. We thus have the following budget constraints:

(1a) $$C_1 = Y + O_1 - A,$$

(1b) $$C_2 = (1+r+\varepsilon_r)A + Y + O_2 + \varepsilon_O, \quad \varepsilon_r \sim U\left(-\sqrt{3}\sigma_r, \sqrt{3}\sigma_r\right), \quad \varepsilon_O \sim U\left(-\sqrt{3}\sigma_O, \sqrt{3}\sigma_O\right).$$

The rate of time preference is $\rho > 0$. Saving in financial assets is chosen to maximize expected utility, E[V], subject to the budget constraints (1), where the present discounted value of utility is:

(2) $$V(C_1,C_2) \equiv U(C_1) + \frac{1}{1+\rho}U(C_2), \quad U(C) = \frac{C^{1-1/\sigma}}{1-1/\sigma},$$

[4] In many poor countries the share of petroleum consumption in household income is high, income and price elasticities for petroleum demand are low and households are relatively risk averse in which case the risk aversion effect dominates the effect of substituting away from petrol if its price is high and towards petrol if its price is low so that consumers benefit from petrol price stability (e.g., Turnovsky et al., 1980).

[5] Alternatives which rely less on the positive third derivative to obtain prudent saving are discussed in appendix A.

with $\sigma > 0$ the elasticity of intertemporal substitution. *CRRA* = $1/\sigma$ corresponds to the coefficient of relative risk aversion or the coefficient of relative inequality aversion and *CRP* = $1+1/\sigma$ to the coefficient of relative prudence. The Euler equation for this problem is:

$$(3) \qquad U'(C_1) = \frac{1}{1+\rho} \mathrm{E}\left[(1+r+\varepsilon_r)U'\left((1+r+\varepsilon_r)A+Y+O_2+\varepsilon_O\right)\right].$$

To focus attention on the effects of uncertainty, we suppose that the mean asset return equals the rate of time preference, $r = \rho$.

3.1. No uncertainty about asset returns or oil income

Without income volatility or asset return volatility ($\sigma_r = \sigma_O = 0$), (1) and (3) can be solved to give the optimal amount of saving and optimal levels of present and future consumption:

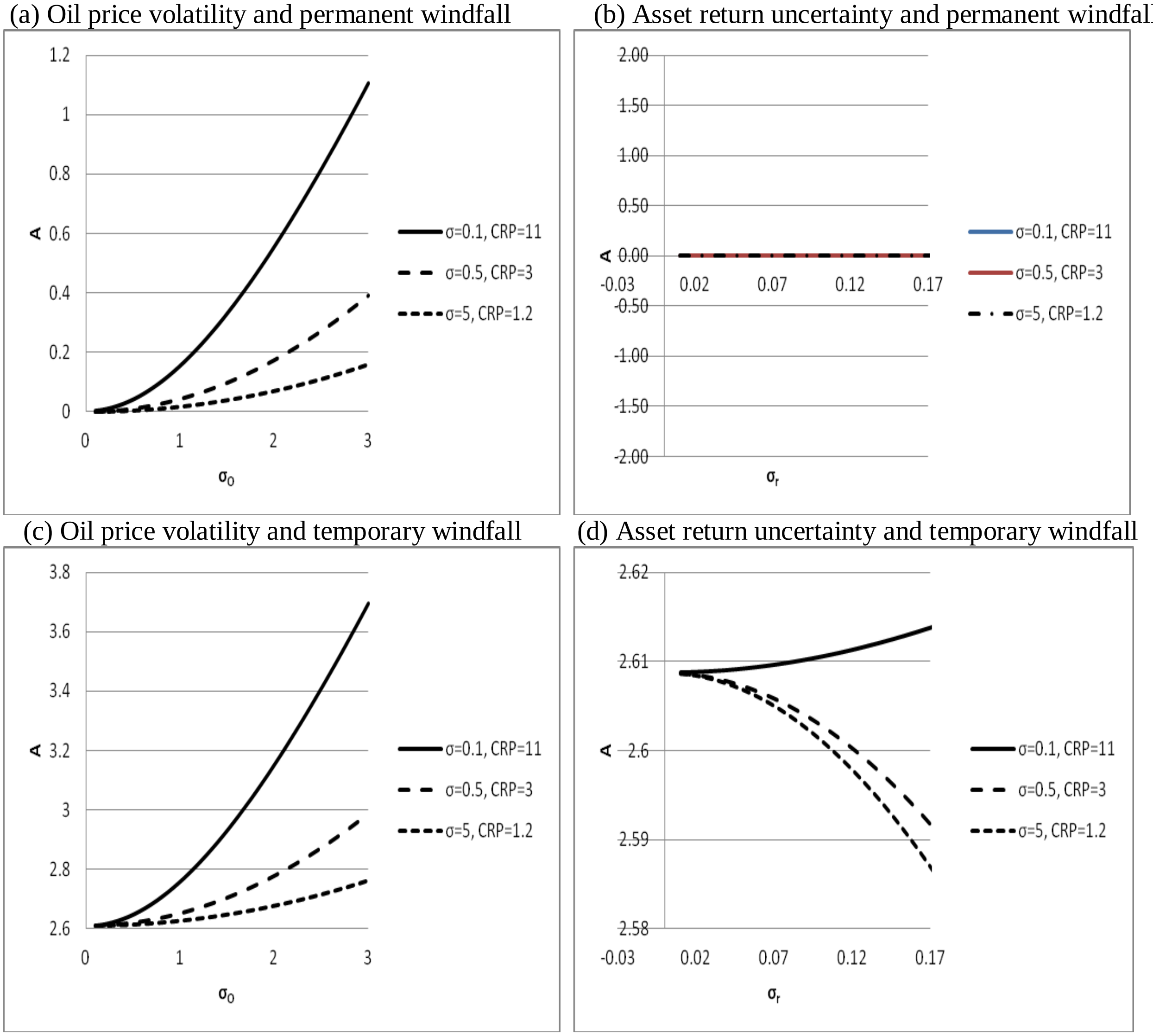
$$(4) \qquad A = \frac{O_1 - O_2}{2+\rho}, \quad C_1 = C_2 = Y + O, \quad O \equiv \frac{(1+\rho)O_1 + O_2}{2+\rho},$$

where O denotes the permanent value of the oil windfall (also called the annuity value of the windfall). Whatever the intertemporal stream of oil revenue, the intertemporal stream of consumption is fully smoothed. Consumption thus equals production income plus the permanent value of the oil windfall. If current and future oil income are the same, there is no saving, $A = 0.$ If the stream of oil revenue is expected to increase in the future, $O_2 > O_1$, the country borrows using future oil revenue as collateral. If oil revenue is expected to decline, the country must save to achieve the same consumption increment in all time periods.

3.2. Stochastic asset returns and volatile oil income

With stochastic asset returns and oil income $(\sigma_r > 0, \sigma_O > 0)$, the optimal value of C_1 and A can easily be calculated from (1) and (3). Fig. 2 shows the optimal saving response for a *sustained* oil windfall (panels (a) and (b)) and a *declining* windfall (panels (c) and (d)). Panel (a) indicates that oil income uncertainty induces precautionary saving: a financial buffer to cope with oil income volatility. This is driven by prudence. The size of the resulting liquidity fund increases with the volatility of oil income. Precautionary saving also increases with the coefficient of relative prudence, which for power utility functions equals $CRP \equiv 1 + 1/\sigma > 1.$ Panel (b) confirms that, if the expected stream of oil income is constant, there is no precautionary saving in response to asset return uncertainty. As there is no motive for intergenerational saving or borrowing, asset return realizations are irrelevant.

Figure 2: Oil price volatility, asset return uncertainty and precautionary saving



Key: $Y = 10$, $r = \rho = 0.3$, $O_1 = 8 > O_2 = 2$.

Proposition 1: A second order approximation of the stochastic Euler equation (3) yields:

$$\text{(5)}\quad \frac{U'(C_1)-U'(\mathrm{E}[C_2])}{U'(\mathrm{E}[C_2])} = \frac{CRP \times CRRA}{2\mathrm{E}[C_2]^2}\left(A^2\sigma_r^2 + \sigma_O^2 + 2A\,\mathrm{cov}(\varepsilon_r, \varepsilon_O)\right) - \frac{CRRA}{(1+\rho)\mathrm{E}[C_2]}\left(A\sigma_r^2 + \mathrm{cov}(\varepsilon_r, \varepsilon_O)\right).$$

With only oil income uncertainty, precautionary saving is more substantial if σ is small and $\sigma_O/\mathrm{E}[C_2]$ is large, irrespective of the size of the intergenerational fund. With only asset return uncertainty, there is only a precautionary buffer if $0.5CRP > \frac{(1+r)A + Y + O_2}{(1+r)A}$. Else, saving is reduced.

Proof: Substituting a second order Taylor series expansion of $U'\left((1+r+\varepsilon_r)A+Y+O_2+\varepsilon_O\right)$ with respect to ε_O and ε_r into (3) and taking expectations, we obtain (5). If $\varepsilon_r = 0$, equation (5) becomes $\frac{U'(C_1)-U'(\mathrm{E}[C_2])}{U'(\mathrm{E}[C_2])} = 0.5(1+\sigma^{-1})\sigma^{-1}\sigma_O^2 / \mathrm{E}[C_2]^2 > 0.$ Hence, C_2/C_1 is large if σ is small and $\sigma_O/\mathrm{E}[C_2]$ is large. If $\varepsilon_O = 0$, (5) becomes $\frac{U'(C_1)-U'(\mathrm{E}[C_2])}{U'(\mathrm{E}[C_2])} = \left\{0.5\times CRP - \frac{(1+r)A+Y+O_2}{(1+r)A}\right\}\times CRRA \times \frac{A^2\sigma_r^2}{\mathrm{E}[C_2]^2}.$ This is only positive if $0.5\times CRP > \frac{(1+r)A+Y+O_2}{(1+r)A}$. Q.E.D.

Turning our attention to managing a temporary windfall under asset return uncertainty, panel (d) confirms the result of proposition 1 that the effect of asset return uncertainty on the precautionary saving buffer is indeed ambiguous. To understand this, one must realize that for the class of power utility functions the parameter σ captures *both* risk aversion and prudence. But risk aversion and prudence act in opposing directions. Risk aversion acts to reduce saving to zero to minimize the impact of an uncertain income on assets that have been accumulated to achieve the job of intergenerational consumption smoothing (the second term on the right hand side of (5)). In contrast, prudence acts to increase saving to build up precautionary buffers (see the first term on the right hand side of (5)). The risk aversion effect goes back to Marshall and boils down to ‘those who save a lot have a lot to lose’; the prudence effect echoes Boulding and yields precautionary saving provided the third derivative of utility is positive (i.e., $U''' > 0$, $V''' > 0$) (Sandmo, 1970). Depending on which effect is bigger, asset return uncertainty has a negative or positive effect on saving in financial assets.

Both the negative risk aversion effect ($1/\sigma$) and the positive prudence ($1+1/\sigma$) effect are strong if σ is small. From panel (d) it is clear that for small values of σ, say 0.1, the net effect of asset return uncertainty on saving is positive and this effect is larger if asset returns are more volatile (the prudence effect dominates). For larger values of σ, say 0.5 and above, the net effect of asset return uncertainty on saving is negative and more negative if asset return volatility is larger (the risk aversion effect dominates). The country then saves *less* than is necessary to smooth the expected fall in future oil income.

We note from (5) that, if asset holdings are zero, there is no effect of asset return uncertainty on saving. If the country is a net asset holder, $A > 0$, the prudence effect is positive whilst the risk aversion effect in (5) is negative. Hence, for small enough values of σ, the prudence effect dominates and saving will be higher; for large values of σ, the risk aversion effect dominates and saving will be smaller (see right panel of fig. 2). However, if the country is a debtor, $A < 0$, both the risk aversion and the prudence effects are unambiguously positive. It follows that a higher degree of risk aversion (lower σ) always induces less borrowing for debtor countries.

This latter situation is relevant for an anticipated windfall. Prudence and risk aversion now act in the same direction as the country is borrowing to smooth consumption. To reduce the effect of future oil income uncertainty, prudence dictates that future consumption is increased by borrowing less. The net effect of asset return uncertainty on saving is higher for an anticipated than for a temporary windfall. The reason is that for the anticipated windfall the country must take on debt to smooth consumption so that risk aversion and prudence act in the same direction, whereas for a temporary windfall the country accumulates assets and thus risk aversion and prudence act in opposing directions (see panel (d)).

3.3. What assets should the fund invest in?

Proposition 1 also shows that by a careful choice of the sovereign wealth portfolio an oil rich country can hedge oil income risk. The key question is whether one should choose equity holdings in companies whose fortunes move inversely with the world price of oil, $\text{cov}(\varepsilon_r, \varepsilon_O) < 0$, or companies who are not affected by or even benefit from increases in the oil price. Examples of the former are energy intensive users such as aluminum smelters, steel producers, oil companies, etc. whilst examples of the latter are companies that offer substitutes for fossil fuels, produce energy efficient cars, etc. The prudence effect in equation (5) indicates that net asset holders that invest in companies whose share prices vary inversely with the price of oil need to hold less precautionary buffers. Net debtors need to hold bigger precautionary buffers. This suggests the hypothesis that net debtors should invest less in energy-intensive companies and more in companies whose fortunes do not vary inversely with the price of oil if they are unable or unwilling to cut their debt. Indeed, if , $A\,\varepsilon_r + \varepsilon_O = 0$ the prudence term drops out completely from (5).

The risk aversion term in (5) indicates that investing in assets that are negatively correlated with the world price of oil increases saving as the Marshallian argument that there is more at risk is less relevant. For net asset holders and high coefficients of relative prudence, this effect is outweighed by the prudence effect.

4. Capital scarcity and investing to invest: the volatility curse in developing economies

To capture that developing economies often experience capital scarcity and sometimes have substantial sovereign debt ($D = -A > 0$) before enjoying an oil bonanza, we take a shortcut. We suppose that countries have to pay a risk premium on their sovereign debt, $\pi(d) > 0,\ \pi' > 0,\ d \equiv D/Y, \forall D > \bar{D},$ and take this as a metaphor for capital scarcity and for a country not being well integrated into world capital markets (cf., van der Ploeg and Venables, 2011, 2012). So the risk premium is high if either the debt is high or the ability to pay the debt burden (as indicated by permanent production income) is low. Since there is no conclusive empirical support for oil windfalls alleviating the debt premium paid on international capital markets, we abstract from this possibility.

The so called separation theorem states that the optimal level of domestic investment does not depend on the size of a windfall of foreign exchange. It holds if an economy can borrow freely on international capital markets and does not suffer from capital scarcity (e.g., van der Ploeg and Venables, 2012). However, this theorem does not hold when there is capital scarcity, so that the optimal level of domestic investment does depend on the amount of saving in the country and thus on the size of the oil windfall. To capture this, we introduce domestic investment *I*, which is used to produce additional output under decreasing returns to scale, $F(I), F'>0, F''<0,$ and multiplicative uncertainty given by the factor $1+\varepsilon_I$, where ε_I is uniformly distributed with zero mean and standard deviation σ_I. The economy thus has two assets, sovereign wealth and public capital, and has to decide how much of the windfall to invest in each. The government maximizes expected utility $\mathrm{E}[V(C_1,C_2)]$ subject to (2) and the budget constraints:

(1a′) $$C_1 = Y + O_1 + D - I,$$

(1b′) $$C_2 = Y + (1+\varepsilon_I)F(I) + O_2 + \varepsilon_O - \left[1+\rho+\pi(D)+\varepsilon_r\right]D,$$

where $\varepsilon_i \sim U\left(-\sqrt{3}\sigma_i, \sqrt{3}\sigma_i\right)$ with mean zero and variance $\sigma_i^2, i = r, I,$ and $\varepsilon_O \sim U\left(-\sqrt{3}\sigma_O O_2, -\sqrt{3}\sigma_O O_2\right)$ with mean zero and variance $(\sigma_O O_2)^2$. This gives the familiar Euler equation for determining the optimal level of sovereign debt and the optimality condition for domestic investment:

(4′) $$U'(C_1) = \mathrm{E}\left[\left(\frac{1+\rho+\pi(D)+D\pi'(D)+\varepsilon_r}{1+\rho}\right)U'\left(C_2\right)\right],$$

(6) $$U'(C_1) = \mathrm{E}\left[\left(\frac{(1+\varepsilon_I)F'(I)}{1+\rho}\right)U'\left(C_2\right)\right],$$

where C_1 and C_2 are given by (1a′) and (1b′), respectively. The relevant social cost of borrowing exceeds the private cost of borrowing, since the government in contrast to the private sector internalizes the higher cost of borrowing resulting from having debt. This results in a corresponding increase in the cost of public investment and explains why the separation theorem breaks down. From a policy perspective, the relevant question is what to do with the additional windfall income that arises from resource extraction.

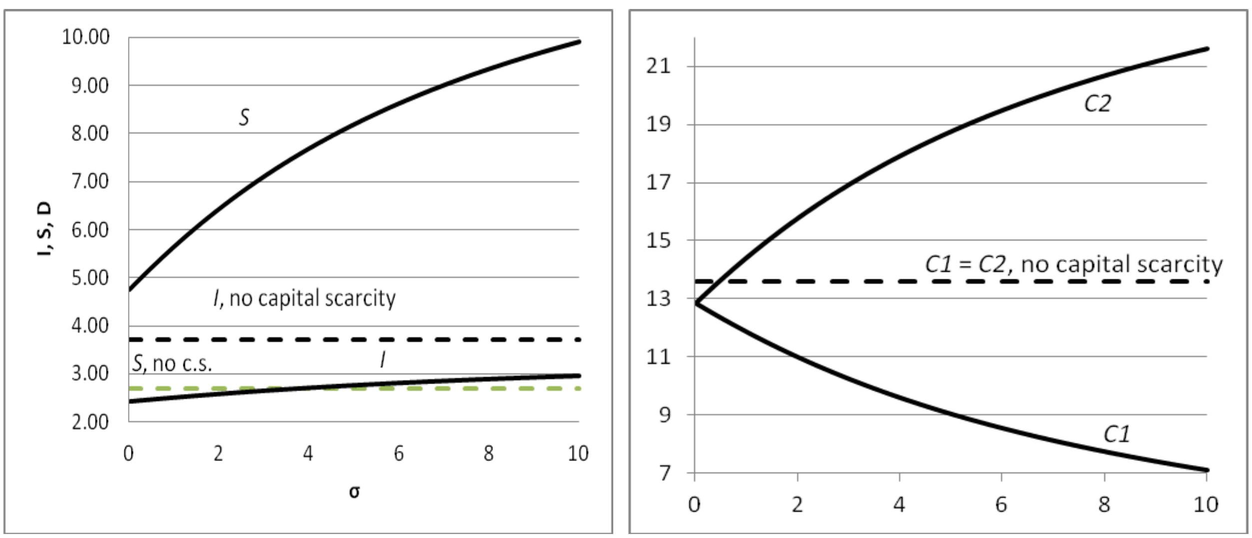
4.1. No uncertainty: the case for spending part of the windfall on domestic investment

Without uncertainty, investment follows from setting the marginal cost of public investment to the social cost of borrowing, $F'(I) = 1+\rho+\pi(D)+D\pi'(D)$), so that the optimal level of public investment decreases in the level of public debt. If there is no capital scarcity, investment follows from $F'(I)=1+\rho$ and does not depend on the size of the windfall, so consumption is fully smoothed and there is no savings response regardless of the degree of intergenerational inequality aversion, $1/\sigma$ (see dashed lines in fig. 3).

The solid lines in fig. 3 show the effects of capital scarcity if there is no windfall. The right panel indicates that consumption of the two generations now diverges. Public investment is lower as it is stifled by the higher interest rate. This is what we mean by capital scarcity. If $\sigma \to \infty$, society is indifferent between consumption of different generations and the interest rate immediately adjusts to the discount rate. All debt is paid off. Society thus prefers greater returns to intergenerational equality, which shows up in the biggest discrepancy between consumption of the present and the future generation. If society has more aversion to intergenerational inequality ($1/\sigma$ higher), then it does not allow consumption levels between generations to diverge so much. This requires more debt to smooth consumption, so that the interest rate is higher and public investment is lower than if society is less concerned about intergenerational inequality.

Fig. 3: Intergenerational inequality aversion σ and how to spend the windfall – no uncertainty

(a) Investment, saving and debt **(b) Present and future consumption**



Key: Intergenerational inequality aversion is measured by $1/\sigma$. $Y_1 = Y_2 = 15, \bar{r} = \rho = 0.3, \beta = 0.5, E = 5, D_0 = 15$. S is how much is paid off in period 1, $S \equiv D_0 - D$. The remaining debt D is paid off in period 2.

A windfall of foreign exchange does not affect the qualitative nature of these effects. If there is no capital scarcity, the windfall boosts consumption in both generations equally but does not affect investment. There is perfect consumption smoothing. The fraction that is saved out of the windfall is zero for a permanent windfall and positive and less than one for a temporary windfall. For an anticipated windfall, the economy must borrow to smooth consumption.

With capital scarcity, a temporary windfall in period 1 is used to pay off some debt and reduce capital scarcity and boost public investment. In general, society is willing to accept intergenerational inequality in exchange for the efficiency gains resulting from paying off debt, reducing the debt premium and thus increasing public investment. However, with diminishing returns to public investment, these efficiency gains are less for smaller degrees of intergenerational inequality aversion (higher values of σ) for which initial investment without the windfall is higher. For $\sigma \to \infty$, the efficiency argument always wins: there is no concern about intergenerational inequality, so all the debt is paid off to get rid of the interest burden. Having paid off all the debt, society is indifferent when the windfall is consumed and thus the savings increment goes to zero as $\sigma \to \infty$. For infinite inequality aversion ($\sigma = 0$), the windfall induced boost to consumption is allocated equally to both generations. Furthermore, the boost to consumption exceeds that if there is no capital scarcity, $\Delta C_1 = \Delta C_2 = \Delta O_1 + \left(\frac{1}{1+r}\right)\Delta O_2 + \left(\frac{D\Pi'}{1+r}\right)\Delta I \ > \Delta O_1 + \left(\frac{1}{1+r}\right)\Delta O_2$ if $D > 0$.

Turning attention to a permanent windfall in a capital scarce economy, we note that with infinite inequality aversion ($\sigma = 0$), perfect consumption smoothing is simply obtained by consuming the windfall in the period that it arises. After all, the windfall in each period is the same. If there is less concern with intergenerational inequality (σ higher), saving and investment rise as the debt premium falls. Consumption of future generations increases by more than consumption of the present generation falls; efficiency is thus improved at the expense of more inequality.

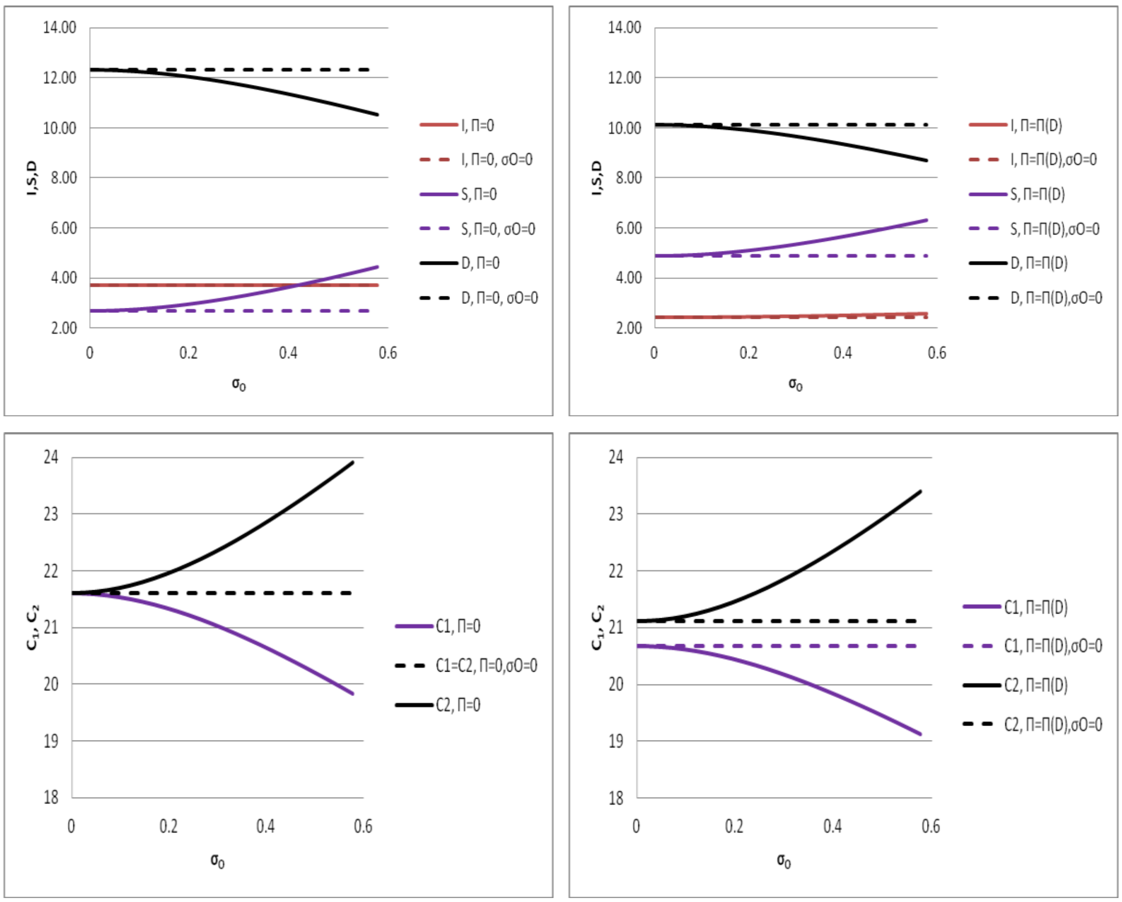
4.2. Windfall volatility and capital scarcity

Fig. 4 shows the effects of a volatile permanent windfall on investment, borrowing and consumption. Panel (a) applies if there is no capital scarcity. Saving is increased for more volatile windfalls for precautionary reasons, so present consumption falls at the expense of future consumption and there is no perfect smoothing of consumption. The separation theorem holds, so public investment is unaffected by windfall volatility. Panel (b) applies if there is capital scarcity. Even without windfall uncertainty (dashed lines), we see that now both present and future consumption are lower than if there is no capital scarcity. Present consumption is also lower than future consumption. The reason is that capital scarcity prevents public debt being high enough to fully smooth consumption. The higher social cost of borrowing resulting from capital scarcity holds back public investment and thus economic development.

If the windfall income is volatile (solid lines), present and future consumption diverge even more with associated welfare losses. The reason is that for precautionary reasons public debt is being cut back and public investment is increased a little as the windfall becomes more volatile. The precautionary buffer illustrates the tradeoff between precaution and intergenerational equity. With capital scarcity, the burden of smoothing consumption between the generations is shared, so for precautionary reasons both the amount that is borrowed and public investment is reduced. This follows from the arbitrage condition between public capital and debt.

Figure 4: Windfall uncertainty and capital scarcity (permanent windfall, $O_1 = O_2 = 8$, $\sigma = 0.1$)

(a) Without capital scarcity **(b) With capital scarcity**

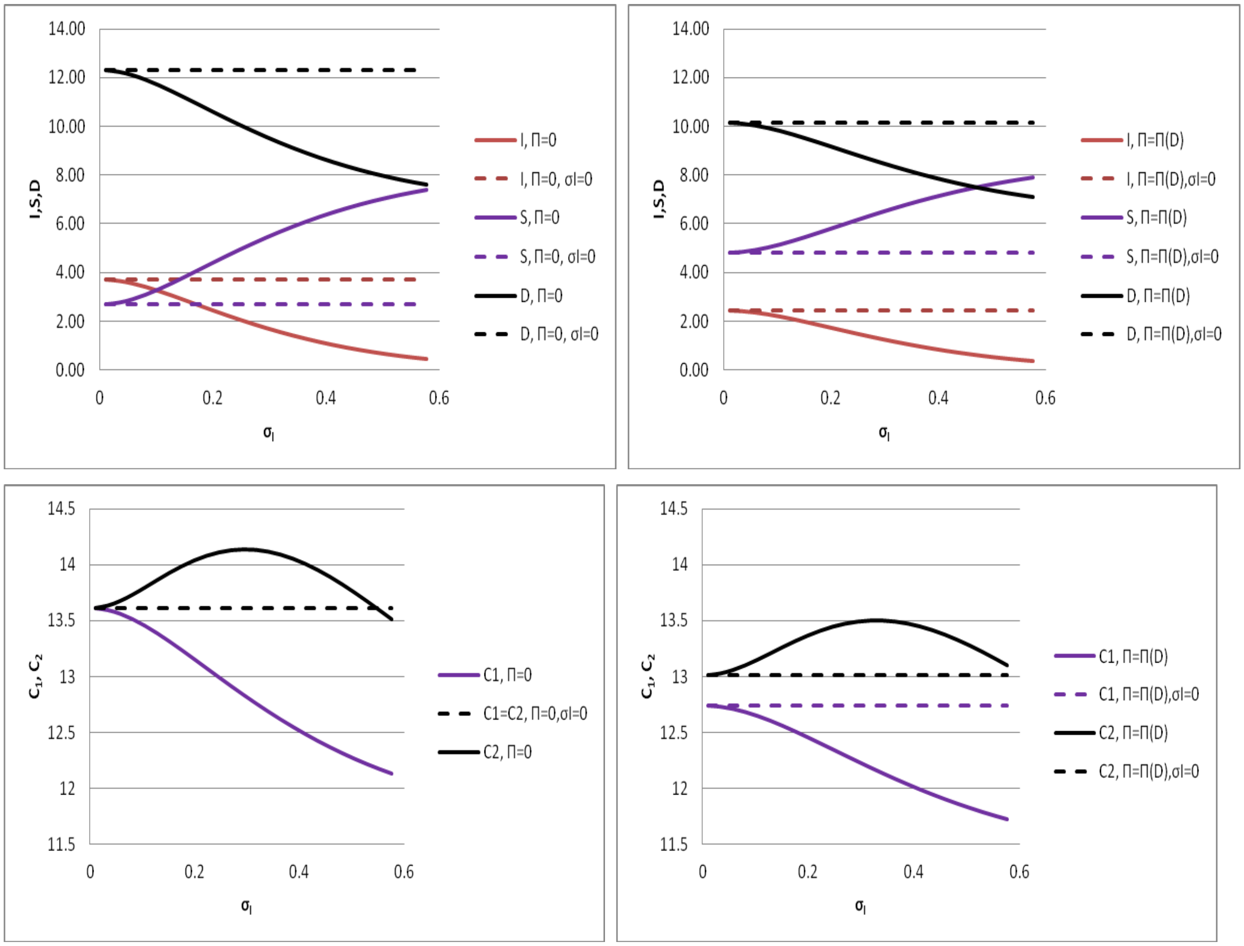


4.3. Investment returns uncertainty and capital scarcity

Fig. 5 displays the effects of investment returns uncertainty on investment, borrowing and consumption. Panel (a) shows what happens if there is no capital scarcity. Uncertainty about the returns on public investment depresses the rate of public investment, but also increases the amount of debt that is paid off and thus reduces borrowing. The latter effect is due to prudence, whereas the former effect is induced by risk aversion. The net effect of both is to depress present consumption and increase future consumption. For high degrees of investment returns uncertainty, future consumption becomes smaller again. This reflects that there are decreasing returns to public investment. Hence, the fall in production income becomes relatively large for high enough variance of investment returns uncertainty and the country remains poor as a result.

Figure 5: Effect of uncertainty about public investment returns on the economy

(a) Without capital scarcity **(b) With capital scarcity**



Panel (b) shows the effects when there is capital scarcity. The vertical axes of panel (b) indicate that capital scarcity lowers public investment, debt and average consumption and forces a wedge between present and future consumption even if there is no investment returns uncertainty. Because of risk aversion, the rate of public investment falls as the returns to public investment become more uncertain. Consequently, less is invested, more of public debt is redeemed, and inequality between present and future consumption rises. There is more redemption of public debt for two reasons. First, as the risk premium drives a wedge between the interest and discount rates, there is more concern about intergenerational inequality and thus saving increases to compensate for the fall in public investment. Second, prudence acts to shift more income to the future. In panel (b) the second effect is much stronger; due to the desire for precautionary saving, intergenerational inequality even moves in the opposite direction. The reduction in public investment is stronger if there is no capital scarcity, because the initial investment level is higher and the effect of uncertainty is multiplicative. With capital scarcity, saving is more powerful. It curbs both the debt and the interest to be paid on it, so the country needs to do less of it.

A higher degree of uncertainty about public investment returns thus implies that countries save more and invest less. This offers an alternative explanation of the view that many oil rich countries facing capital scarcity are big savers and small investors (Cherif and Hasanov, 2011). This view also accords well with an economy without capital scarcity and high degrees of uncertainty about returns on investment.

We summarize the insights of section 4 in the following proposition.

Proposition 2: Capital scarcity depresses public investment below its socially optimal level and retards economic development. A windfall foreign exchange alleviates capital scarcity and allows the economy to bring public investment closer to its optimal level, especially if intergenerational inequality aversion is weak. Oil income uncertainty reduces for precautionary reasons both borrowing and public investment. Driven by risk aversion, investment returns uncertainty depresses public investment. Prudence curbs borrowing and worsens inequality between generations. Both effects are smaller with capital scarcity.

5. Prudent saving in volatile economies

To apply the insights of sections 3 and 4 to real economies, we adopt a continuous-time, infinite-horizon approach abstracting from asset return uncertainty and focusing on oil price uncertainty (cf., the discrete time approach in Bems and de Carvalho Filho, 2011). We postpone the treatment of investment under capital scarcity to section 6. All oil rich countries have to cope with the volatility of oil prices, so we first consider the stochastic dynamics of the oil price and then the optimal saving responses in an infinite-horizon framework. We then apply the framework to the oil windfalls of Norway, Ghana and Iraq.

5.1. Stochastic dynamics of the oil price

The time path of the world oil price is portrayed in fig. 6 and shows substantial volatility.[6] It has been suggested that the log of the real oil price follows a random walk without drift and even wild swings lie comfortably within the 'normal range' (Hamilton, 2009).

We thus describe the oil price by a geometric Brownian motion (the continuous time version of a discrete time random walk for the log of the oil price):

(7) $$dP(t) = \upsilon_P P(t) + \sigma_P P(t) dW(t),$$

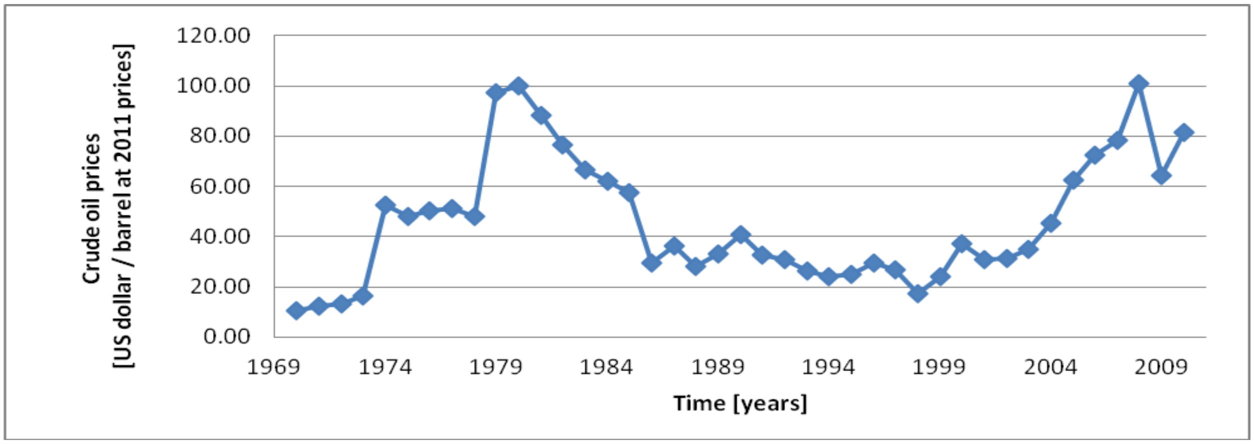
[6] We use a historical annual series for the price of crude oil (BP, 2011), expressed in 2011 prices using CPI data for the US (from OECD Economic Outlook No. 89), for the period 1970-2010.

where $W(t)$ is a Wiener process which satisfies $W(t) - W(s) \sim N(0, t - s)$ for $t \geq s$ and the constants υ_P and σ_P are the percentage drift and the percentage volatility, respectively. The solution to (7) is:

$$(7') \qquad P(t) = P_0 \exp\left(\left(\upsilon_P - 0.5\sigma_P^2\right)t + \sigma_P W(t)\right)$$

with expectation and variance given by $E[P(t)] = P_0 e^{\upsilon_P t}$ and $\mathrm{var}[P(t)] = P_0^2 e^{2\upsilon_P t}(e^{\sigma_P^2 t} - 1)$, respectively.

Figure 6: Crude oil prices 1970-2010.



We obtain ML estimates of the drift and volatility parameters: $\hat{\upsilon}_P \approx 0.009$ and $\hat{\sigma}_P \approx 0.28$ (similar to the values used by Bems and de Carvalho Filho (2011) in their discrete time approach). Since $\hat{\upsilon}_P \approx 0.009$ is statistically insignificant, we set $\upsilon_P = 0$ in the simulations and thus ignore long run trends in the oil price.

5.2. An infinite-horizon model of prudent saving

The government maximizes expected social welfare

$$(8) \qquad \mathrm{E}_0\left[\int_0^\infty U\left(C(t)\right)e^{-\rho t}dt\right]$$

subject to the stochastic dynamics of the oil price (7) and the asset accumulation equation:

$$(9) \qquad \frac{dA}{dt} = rA + Y - C - (P - \Psi)O, \quad A(0) = A_0,$$

where Y is exogenous (non-oil) production income and $\Psi > 0$ denotes the constant extraction cost per barrel of oil. We use Itō calculus to solve this stochastic optimization problem. It follows that the optimal change in consumption must satisfy the stochastic Euler equation:

$$(10) \qquad \frac{1}{dt}\mathrm{E}_t\left[dC\right] = \sigma\left[r - \rho\right]C + \frac{1}{2}CRP\left[O - \frac{\partial S}{\partial P}\right]^2\left(\frac{\sigma_P P}{C}\right)^2 C,$$

where $O - \partial S/\partial P = \partial C/\partial P$ is the effect of an oil price shock on consumption (cf., the 'marginal propensity to consume' out of the wealth generated by an oil price shock) and $CRP = 1 + 1/\sigma > 1$ is the coefficient of relative prudence. Since we suppose $r = \rho$, the first term on the right hand side of (10) drops out so that the expected time path of consumption slopes upwards. Consumption is thus initially low to allow more prudent saving. This prudence effect is high if the coefficient of relative prudence and oil price uncertainty (as measured $\sigma_P\, P/C$) are high. Furthermore, under the permanent income hypothesis, countries with a more temporary windfall save a greater proportion of the windfall than those with a more permanent windfall, and therefore have a smaller prudence effect (as $\partial S/\partial P$ is high).[7] Precautionary saving only plays a role if oil shocks result in consumption shocks and the more so if windfall is more permanent. We solve this problem consisting of a number of coupled differential equations (one of which is a partial differential equation) using a multiple shooting algorithm (van den Bremer and van der Ploeg, 2012). For completeness, the extension of the model to allow for asset return uncertainty as well oil price uncertainty is given in appendix B.

5.3. Norway: Declining oil windfall

For the period 2010-2030 we use official production forecasts provided by the Norwegian Ministry of Petroleum and Energy (2009). Thereafter, we assume a linearly declining production profile and choose the time horizon during which all the reserves are exhausted based on estimated total reserves of 7,313 billion Sm^3 of oil equivalent or 45,999 billion barrels of oil equivalent (Norwegian Ministry of Petroleum and Energy, 2009). These reserves estimates include potentially recoverable and undiscovered reserves and are therefore subject to uncertainty, but we abstract from this type of uncertainty. We adopt an oil price of 80 USD per barrel – the crude oil price in 2010 - and assume extraction costs of 30 USD per barrel. The high extraction cost reflects that offshore oil extraction is relatively costly. Annual oil rents are then 73 billion USD in 2010 (18% of 2010 GDP). Panel (a) of fig. 7 indicates that the time paths of projected oil rents follows a flat profile for the first ten years and then tapers off to zero in the next five decades. We suppose that $r = \rho = 3$ percent.

The blue line in panel (b) shows that gradually over a period of six decades 1.39 trillion USD of additional sovereign wealth is accumulated under the permanent income hypothesis (PIH) if oil price volatility is absent. Panel (c) shows that this saving response permits a permanent increase in consumption of 41.82 billion USD (10 percent of 2010 GDP). This amounts to an annual annuity of 8387 USD for each Norwegian citizen. This buffer of 1.39 trillion USD is what amounts to the

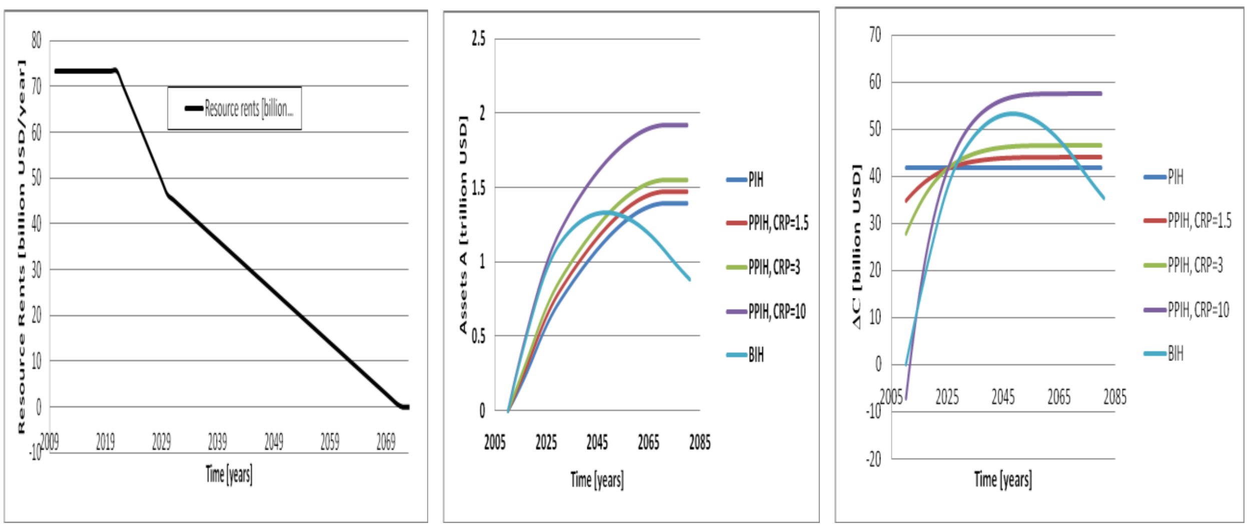
---

[7] We calculate $\partial S / \partial P = O(t) - r\int_t^T O(\tau) e^{-r(\tau - t)} d\tau$ from the deterministic solution.

intergenerational equity or generational fund. Taking a ballpark figure for the coefficient of relative prudence of 3 and the estimated stochastic dynamics of the world oil price, the green line in panel (b) indicates the prudent saving response. Norway should thus accumulate not 1.39 trillion USD but 1.55 trillion USD. The difference of 155.7 billion USD is the volatility buffer or stabilization fund. It adds 11.2 percent to Norway's generational fund, which does not seem large but is significant nonetheless. If the coefficient of relative prudence is 10, the purple line in panel (b) indicates that a total of 1.92 trillion USD are built up, so that the liquidity fund rises to 522.4 billion USD or 37.5 percent of the generational fund. With a prudence coefficient at the lower end of the plausible range, the red line in panel (b) shows that the buffer is only 77.7 billion USD or 5.6 percent of the generations fund.

Fig. 7: Optimal response to Norwegian windfall

(a) Projected oil rents, *N* (b) Build up of assets, Δ*A* (c) Consumption, Δ*C* increment



Panel (c) of fig. 7 shows the corresponding consumption increments. As we have seen, in the absence of volatility, there is a sustained increase in consumption of 41.8 billion USD (8387 USD per citizen). Taking account of oil price volatility and using the ballpark coefficient of relative prudence of 3, consumption increases initially by only 27.7 billion USD and then rises to 46.5 USD. The prudent saving response thus implies less consumption today (–33 percent with *CRP* = 3) and more consumption (+ 11 percent with *CRP* = 3) in the long run. These prudential tilts in the consumption profile are bigger if the coefficient of relative prudence is large. For example, the purple line in panel (c) corresponding to *CRP* = 10 indicates that consumption initially actually falls by 7.1 billion USD, thus permitting a long-run increase of 57.9 billion USD.

Interestingly, we see from panels (b) and (c) that for high degrees of prudence the optimal response during the first twenty years or so is very close to the so-called bird-in-hand (BIH) rule, which effectively puts all oil revenue in the fund and takes a given percentage (4 percent) from the fund to finance the government budget. However, the difference is that in the very long run the fund and the consumption increment under the BIH rule dwindle away to zero whilst the consumption increment is permanently sustained under the optimal rule.

Finally, the temporary nature of Norway's windfall implies that the marginal propensity to consume falls monotonically from 0.84 billion USD for every dollar increase in price a barrel of oil in 2005 to zero around 2065. Equation (10) thus implies therefore that the prudence effect falls with time as well and thus that the upward tilt of the path for the consumption increment reduces with time as can be seen from panel (c) of fig. 7.

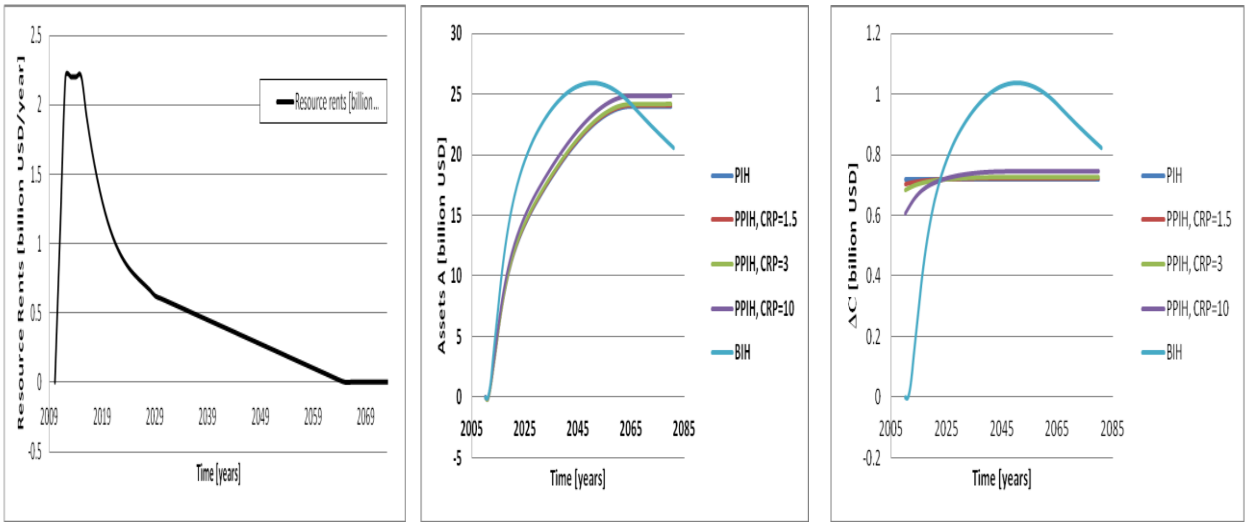
5.4. Ghana: Temporary and small oil windfall

In contrast to Norway, Ghana has only very recently been enjoying an oil windfall. The base case extraction scenario used in a recent comprehensive report by the World Bank is based on proven recoverable oil reserves of 490 million barrels (Dessus et al., 2009) and probably underestimates Ghana's future production and projects an oil production profile until 2029. We use a more recent but perhaps still conservative estimate of 700 million barrels (CIA World Fact Book) and assume a linearly declining production profile after 2029. Because Ghana's oil reserves are only offshore, we adopt an extraction cost of 30 USD/barrel as for Norway. At peak production in 2013, annual resource rents are approximately 1.8 billion USD, 6% of 2010 GDP (31.3 billion USD) or 74 USD per citizen. Abstracting from oil price uncertainty, the permanent component of this oil rent stream is 718.7 million USD per year or an annual annuity of a mere 29.5 USD per Ghanaian citizen; see the blue line in panel (b) of fig. 8. This is only 0.35 percent of the annual annuity that the Norwegians will continue to enjoy. The annuity value of the windfall, 2.3 percent of GDP (compared with 10 percent for Norway), is small, because the windfall is small and of short duration. Abstracting from oil price uncertainty, Ghana needs to build up a fund of 24.0 billion USD to sustain the permanent boost to consumption of 718.7 million USD per year.

Since the windfall is relatively small and temporary and the marginal propensity to save is high, (10) indicates that the volatility effect is small and thus the upward tilt of the time path of consumption resulting from prudence is much smaller than for Norway (see panel (c) of fig. 8). The size of the stabilization fund or volatility buffer is thus almost irrelevant compared to the size of the generational fund. Ghana should accumulate minimally 24 and with maximum prudence 25 billion USD to sustain its increment in consumption. This amounts to sovereign wealth fund of 1000 USD per Ghanaian. Also taking account of the significant uncertainty about discoveries of new reserves will affect the size of the optimal liquidity fund (van der Ploeg, 2010). Still, taking account of Ghana's capital scarcity would lower the generational fund in favor of investment in infrastructure, education, health, etc. as the return on these assets is higher (see section 6).

Figure 8: Optimal response to Ghanaian windfall

(a) Projected oil rents, *N* (b) Build up of assets, ΔA (c) Consumption increment, ΔC



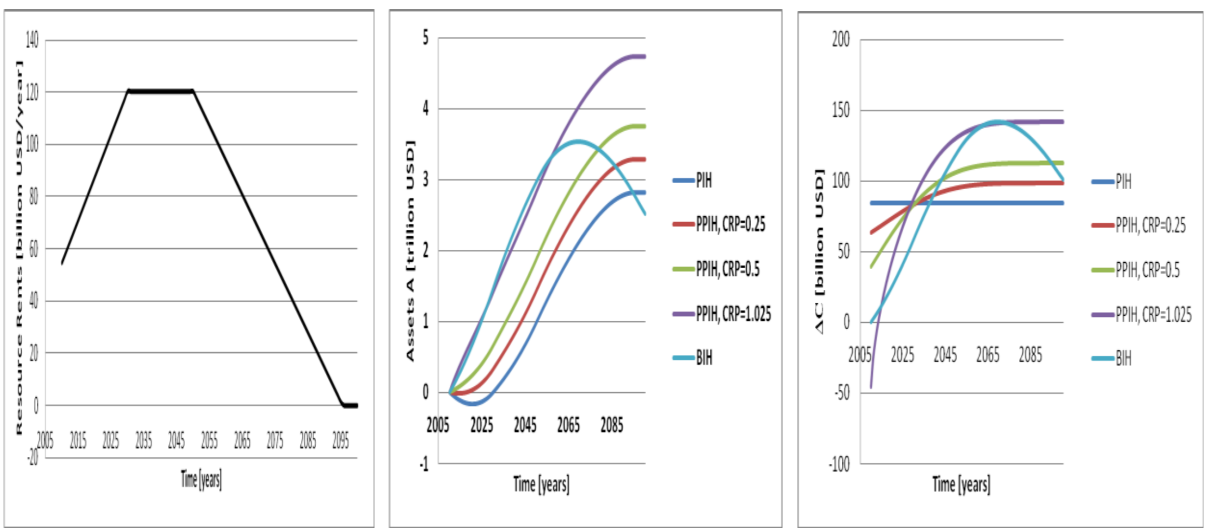
5.5. Iraq: Huge and rising oil windfalls

Our oil rent projects for Iraq are based on the BP Energy Outlook 2030, so we let production increase linearly from its 2010 production of 2.5 million barrels/day to 5.5 million barrels/day in 2030. We then keep production constant until 2050 followed by a linear decrease. The proven reserves estimate for 2010 is 115 trillion barrels (BP, 2011). With these reserves and assumed production profile, oil will continue to flow in Iraq until at late as 2100. Being onshore and easily accessible, we guess oil extraction costs for Iraq to be lower than for Norway and Ghana, namely 20 USD/barrel. Our conservative estimate of oil rents in 2010 is thus 55 billion USD, almost half of 2010 GDP (116 billion USD). Panel (a) of fig. 9 indicates that oil rents are projected to rise to 120 billion USD in 2030 and stay at that level until 2050. Not taking account of oil price volatility and setting $r = \rho = 0.03$, Iraq should accumulate sovereign wealth amounting to a staggering 2.81 trillion USD or 92,500 USD per Iraq citizen. This ensures a sustained annual consumption increment of 84.4 billion USD. This amounts to an ever-lasting annual annuity of 2775 USD for each Iraqi citizen (compared to 29.5 USD for each Ghanaian or 8387 USD for each Norwegian). So in permanent USD/capita terms, the windfall is almost 100 times bigger than Ghana's windfall and a third of Norway's windfall.

Since the Iraqi windfall is so large and lasts so long, the marginal propensity to consume out of oil wealth is relatively large and thus the prudence effect shown in (10) is very large. Iraq is thus very vulnerable to oil price volatility and needs to build up a relatively large volatility buffer or stabilization fund compared to its generational fund. In fact, it is so vulnerable that with the coefficients of relative prudence used in figures 7 and 8, consumption will initially become very negative. The purple lines in panels (b) and (c) show what happens with a coefficient of relative prudence equal to 1.025. This corresponds to a very high elasticity of intertemporal substitution of 40, but this does not matter as the intertemporal substitution effect is inoperative if $r = \rho$. With such aversion to intergenerational inequality, the drop in consumption to allow prudent saving would normally be moderated. The prudent gradual accumulation of financial assets over eight decades leads to a volatility buffer of 1.92 trillion USD, which amounts to 68 percent of the generational fund. This brings the total sovereign wealth fund up to 4.73 trillion USD or 155,587 USD per Iraqi citizen. To achieve this amount of prudent saving, consumption has to fall initially by 45.15 billion USD (1485 USD per citizen) and then rises to 141.9 billion USD in the long run. Interestingly, the cumulated assets path is not that different from the BIH path. The need to borrow in view of the anticipated increase in oil rents is eliminated by prudential considerations even at the lowest values of the coefficient of relative prudence.

Figure 9: Optimal response to Iraq oil windfall

(a) Projected oil windfall, N (b) Build up of assets, ΔA (c) Consumption increment, ΔC



Since initial falls in consumption to make room for prudent saving seem politically infeasible, panels (b) and (c) of fig. 9 also plot the effects for even tinier degrees of prudence. We then get a modest increase in consumption today followed by a gradual rise in consumption over the next eighty years. The required build up of sovereign wealth is less. This also occurs if the government is more impatient ($\rho > 0.03$).

6. Prudent saving and investment: capital scarcity and adjustment costs

The main message of section 5 is that for Iraq the main challenge is not how to harness its windfall for development but how to cope with the notorious volatility of oil prices. Norway's challenge is to spread its wealth fairly across generations and to a lesser extent than Iraq cope with oil price volatility. In contrast to Ghana, Norway is well integrated into world capital markets and does not suffer from capital scarcity. Norway therefore should not allocate a part of its windfall to investment in domestic capital. For Ghana, oil price volatility is quantitatively of little significance but spreading wealth towards future generations is important. However, given that Ghana is likely to suffer from capital scarcity, it should perhaps allocate part of its windfall not to sovereign wealth but to investment in the domestic economy (cf., van der Ploeg and Venables, 2012). In general, the optimal level of present consumption is below that of future consumption for two reasons. The first reason is that, if there is capital scarcity, it is optimal to pay off debt and reduce the interest burden. This effect is especially strong if intergenerational inequality aversion is not so large ($1/\sigma$ small) and capital scarcity is substantial (as discussed in section 4). The second reason arises from the need to build a precautionary saving buffer. The size of this buffer is bigger if the coefficient of relative prudence ($1+1/\sigma$) is high, oil prices are more uncertain relative to the level of consumption and the windfall is more permanent in which case a smaller proportion of the windfall is saved. Given the relatively small effects of prudence we found in section 5, we abstract from oil price uncertainty and focus on capital scarcity.

We now also model that scaling up investment leads to absorption problems, so investment is more costly in the early stages of economic development when investment is high. Recent survey evidence suggests that only 40 to 60 percent of spending on public investment leads to effective accumulation of public sector capital (Dabla-Norris et al., 2011; Gupta et al., 2011). As public investment is ramped up, the efficiency of public investment deteriorates (cf., Berg, et al., 2011; van der Ploeg, 2012). We capture this by introducing internal costs of adjustment, which has two advantages. First, they capture that absorption problems frustrate rapid economic development. Second, they generate bigger returns on public investment and thus allow a more realistic calibration of the model to developing economies.

Allowing for capital scarcity and adjustment costs in ramping up public investment, the government maximizes social welfare (9) subject to the debt and capital accumulation dynamics:

$$\text{(11)} \qquad \dot{D} = \left[r + \Pi(D)\right]D + C + \left[1 + 0.5\phi I / K\right]I - F(K) - PO, \quad D(0) = D_0,$$

$$\text{(12)} \qquad \frac{dK}{dt} = I - \delta K, \quad K(0) = K_0,$$

where δ is the depreciation rate of the public capital stock K, I is public investment and ϕ is the adjustment cost parameter. For simplicity, we abstract from oil extraction costs ($\Psi = 0$). The production function has decreasing returns to scale and is given by $F(K), F' > 0, F'' < 0.$ The production function contains a private sector response, but hiring of labor and capital from world markets (FDI from abroad) are for simplicity suppressed in $F(K)$. We take the Cobb-Douglas production function $F(K) = EK^{\beta}$ and total factor productivity E is set to match Ghana's GDP in 2010. The ratio of investment that delivers public capital and total investment spending is called the 'public investment measure of inefficiency' or *PIMI* for short. We see from (11) that $PIMI = 1/(1+0.5\phi I/K)$.

The optimality conditions for this problem are (van der Ploeg, 2012):

(13a) $\dot{C} = \sigma C\left[\Pi(D) + \Pi'(D)D\right], \quad C(0)$ free,

(13b) $\dot{K} = \left[\frac{1}{\phi}(q-1) - \delta\right]K, \quad K(0) = K_0,$

(13c) $\dot{q} = \left[r + \Pi(D) + \Pi'(D)D + \delta\right]q - (1-\alpha)\beta EK^{\beta-1} - \frac{1}{2\phi}(q-1)^2, \quad q(0)$ free,

(13d) $\dot{D} = \left[r^* + \Pi(D)\right]D + C + \frac{1}{2\phi}(q^2-1)K - EK^{\beta} - N, \quad D(0) = D_0,$

(13e) $\dot{N} = -\eta N, \quad N(0) = N_0,$

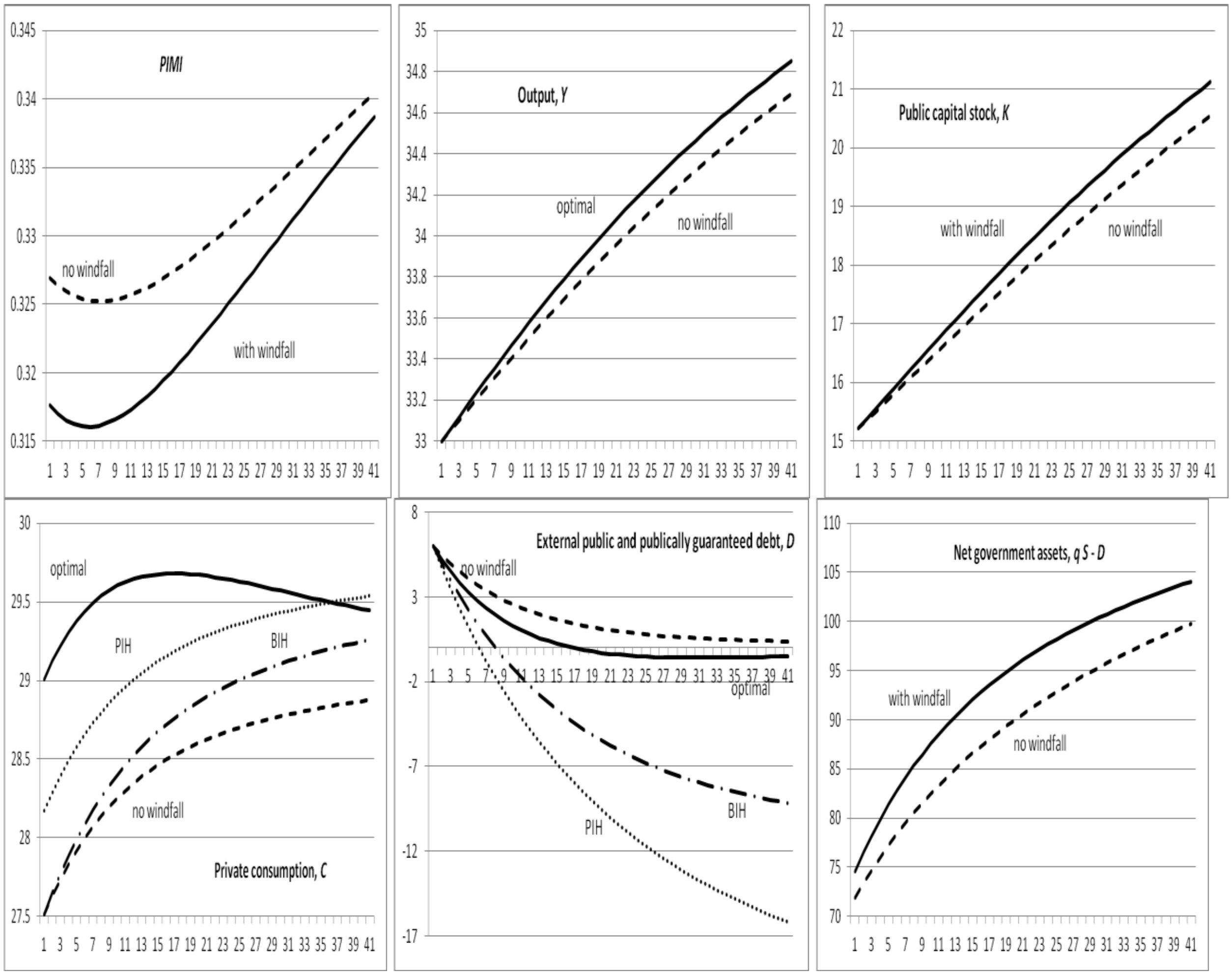
where q denotes the social value of public capital and $0 < \alpha < 1$ the share of private capital in value added. Equation (13a) is a modified version of the Keynes-Ramsey rule. The market does not internalize the interest spread externality and thus borrows too much from a social perspective. In contrast, the social planner modifies the interest rate (the world interest rate plus interest premium) to include $\Pi'(D)D$ to correct for the interest spread externality. For an economy with capital scarcity, it is thus optimal to have a rising path of consumption; the economy consumes less upfront to pay of debt and lower the risk premium. Equation (13b) gives the public sector capital stock dynamics, where q indicates the social value of public capital. The rate of public investment is proportional to its social value, $I/K = (q-1)/\phi$. Equation (13c) gives the intertemporal efficiency condition for public sector investment, which states that the marginal product of public capital plus the marginal reduction in adjustment cost must equal the social cost of borrowing (the market interest rate plus the interest premium on government debt, Π, plus the correction term to allow for the rising cost of public debt, $\Pi'D$, plus the depreciation charge. Equation (13d) gives the dynamics of government debt with the cost of public sector investment, transfers and output substituted. Finally, Ghana's temporary resource windfall is reasonably well captured by (13e).

The five-dimensional system (13) has predetermined state variables D, S and N and non-predetermined variables C and q. Hence, $C(0)$ and $q(0)$ adjust instantaneously to ensure that the economy is on its three-dimensional stable manifold. Based on the empirical evidence (van der Ploeg and Venables, 2011), we use $\Pi(D) = 10^{-4}\exp(6.294)\left[\exp\left(1.9D/31.3\right) - 1\right]$ for the interest spread schedule where 6.294 is the mean log of the spread. This implies that a 10%-point increase in the debt-GDP ratio pushes up the interest differential by 6.9%-points if the economy starts out with a debt-GDP ratio of 100 percent (or 1.3%-points if it starts off with zero foreign debt). This specification ensures that the steady state has zero debt: $D(\infty) = 0,\ q(\infty) = 1 + \phi\delta,\ K(\infty) = \left(\dfrac{\beta(1-\alpha)E}{(r+\delta)(1+\phi\delta) - 0.5\phi\delta^2}\right)^{\frac{1}{1-\beta}}$ and $C(\infty) = EK(\infty)^{\beta} - \delta(1+0.5\phi\delta)K(\infty).$ Long run public capital decreases with $(r^* + \delta)(1+\phi\delta) - 0.5\phi\delta^2$, which exceeds the rental plus depreciation charge, especially if cost of adjusting public capital ϕ is high. So a high value of ϕ corresponds to an absorption constraint in that it requires higher marginal returns on public capital. Investment is thus relatively inefficient in the early stages of economic development when public investment rates have to be high. In the steady state $PIMI = 1/(1+0.5\phi\delta) = 0.4$, but in the early stages of development and during the windfall less of investment outlays is delivered (the *PIMI* falls) as public investment rates (I/K) will be higher. A ballpark estimate for the output elasticity with respect to the stock of public capital is 0.17 (Bom and Ligthart, 2009). In line with this evidence, we have $\beta = 0.15$ $(1-\alpha)$. We set $\alpha = 0.1$ and thus $\beta = 0.167$. With an expected lifetime of public capital of 40 years, $\delta =$ 0.025. We also set the elasticity of intertemporal substitution to $\sigma = 0.5$ and set $r^* = \rho = 0.03$ as before.

We suppose that the initial investment rate is at its steady-state rate, $I/K = \delta$, and use a ballpark estimate of the *PIMI* of 0.4 to back out the adjustment cost parameter $\phi = 120$. Ghana's GDP for 2010 is 31.2 billion USD. We set it at 33 billion USD for 2012, which implies $EK_0^{\beta} = 33$. We suppose that the initial public capital stock at only half its steady-state level, $K_0 = K(\infty)/2$. Together with the steady state of (13c), $(r+\delta)(1+\phi\delta) = (1-\alpha)\beta EK^{\beta-1} + 0.5\phi\delta^2$, we can then back out $E = 20.13$, $K_0 = 15.22$ and $K(\infty) = 30.44$. The implied steady state levels of output and consumption are 37.0 and 29.4 billion USD, respectively. Ghana's external stock of public and publicly guaranteed external debt was 5.7 billion USD. We set the initial debt level for 2012 equal to $D_0 = 6$ billion USD (18 percent of GDP). We approximate Ghana's oil rents from 2012 onwards (when oil starts to flow) by $N(t) = 2.199\exp(-0.07(t-2012))$, so $N_0 = 2.199$ billion USD and $\eta = 0.07$. This matches 2012 situ oil wealth of 22 billion USD and the permanent value of oil wealth (the annuity value) as 3 percent of that, i.e., $N^P(0) = 0.660$ billion USD/year.

Fig. 10 portrays the development paths of Ghana without the windfall (dashed lines) and with the windfall (solid lines) for variables of endogenous variables with time measured as years from 2011. Without the windfall, Ghana is expected to grow along its development path. The gradual rise in public capital and output will lead to a temporary fall in the efficiency of public investment (lower *PIMI*). In the very long run output grows from 33 to 34.69 billion USD whilst the public and publically guaranteed debt vanishes.

Figure 10: Harnessing Ghana's windfall for domestic investment



Key: *Y*, *C*, *D* and *q S* – *D* are in billions USD. The *PIMI* is the ratio of investment spending to *I*.

The effects of the windfall are to allow a more rapid buildup of public investment (signaled by a temporary higher social value of public investment, *q*) which is inevitably leads to a temporary deterioration of the efficiency of public investment (lower *PIMI*). In the end the stock of public capital is higher than without the windfall which leads to a permanently higher level of output in the long run, i.e., 34.85 instead of 34.69 billion USD. Given that Ghana's windfall is fairly short and not so large, it is a modest increase. Still, the windfall allows for an increase in consumption from 27.51 to 29.01 billion USD at the start of the windfall to billion and for an increase in consumption from 28.53 to 29.68 billion USD in 2028. The windfall allows external public and publically guaranteed debt to be paid off more rapidly, so that the social cost of borrowing falls more rapidly and public investment is stimulated. Net government assets (value of public capital minus public and publically guaranteed debt) jump up from 71.90 to 74.62 billion USD on impact due to the jump increase in the social value of public capital, *q*. Afterwards, net assets continue to grow. The long run value of assets is 104.07 billion USD with the windfall and 99.79 billion USD without the windfall. The long run difference, 4.28 billion USD, is not much more than the short run difference, 3.72 billion USD, which reflects that much of the increase is capitalized at the beginning in the upward jump in the social value of capital.

Simple application of the permanent income hypothesis (PIH) and the bird in hand (BIH) rules shows that these do not affect capital formation and output of the economy. Compared with the no windfall trajectory, the PIH rule leads to a permanent increase in consumption of 660 million USD and a long run size of the intergenerational fund of 16 billion USD. The BIH rule leads to a temporary buildup of more than 9 billion USD. Both of these rules do little to stimulate the economy and thus lead to much lower consumption in the next three or four decades than the optimal 'investing to invest' trajectories. Although we advocate our 'investing to invest' strategy over the PIH and BIH rules, the calibration is rough and it is only the qualitative nature of the optimal paths portrayed in fig. 7 that matter.

7. Concluding remarks

Apart from an intergenerational fund for smoothing consumption across generations in the face of time varying windfalls (deterministic volatility), we have made the case for a liquidity fund to protect oneself against (stochastic) oil price volatility in addition to the usual arguments in favor of more political stability and flexibility of the economy. Such a fund is an important alternative for hedging against oil price volatility, since hedging and related structured products have too many economic costs and political risks. The size of the liquidity funds should be larger if oil income volatility is higher and governments are more prudent. More notably, its size also depends on the marginal propensity to consume out of a windfall: only if oil price shocks lead to consumption shocks, do they necessitate precautionary buffers. If the windfall is temporary, the oil rents are largely saved and little precautionary saving is needed (Ghana). If the windfall is permanent combined with the random walk behavior of the oil price, shocks in the oil price lead directly to shocks in consumption and large precautionary buffers are required (Iraq). Furthermore, the relative size of oil rents to GDP matters for prudence and precautionary saving. With all windfalls ultimately being temporal, countries accumulate assets in a sovereign wealth fund. Asset return uncertainty then has two effects on the size of the liquidity fund: a risk aversion effect which tends to depress saving to minimize exposure to risk and a prudence effect which tends to increase saving. With an anticipated windfall, borrowing is required to smooth consumption. In that case, the two effects of asset uncertainty operate in the same direction so that borrowing is unambiguously reduced.

For developed countries with good access to world capital markets, nothing of the windfall should be spent on domestic investment. However, many developing oil rich countries suffer from capital scarcity and are not well integrated into the world economy. For such countries it does not make sense to channel their windfalls of foreign exchange into a sovereign wealth fund if the prospected return on domestic investment and the cost of borrowing is much more than the meager return on such funds. However, the expected return on domestic investment projects may be large but the outcome highly uncertain. The high degree of uncertainty about domestic investment projects relative to that the uncertain returns on sovereign wealth and even the volatility of oil windfalls then helps to explain why poor oil rich countries are often big savers, but small investors. Still, the share of an oil windfall allocated to public investment does not respond much to higher uncertainty about public investment returns. The share allocated to public investment does depend on capital scarcity. A key feature of many developing economies is that not all of the windfall induced extra consumption and investment demand for non-tradables can be immediately absorbed if absorptive capacity is limited (e.g., van der Ploeg and Venables, 2012). In that case, there is a rationale for a parking fund in addition to an intergenerational and a liquidity fund.

Our illustrative calculations indicate that the optimal liquidity buffer for Ghana is very small relative to its intergenerational fund of 24 billion USD even for very high degrees of prudence, for Norway a ballpark measure for the optimal liquidity fund is 156 billion USD compared with 1.39 trillion USD for the intergenerational fund. Iraq should build an intergenerational fund of 2.81 trillion USD; even for a very low coefficient of relative prudence (1.025) it accumulates an extra massive liquidity fund of 1.92 trillion USD. Given capital scarcity and inefficient adjustment of public capital, we argue that Ghana should aim to use part of its small and declining windfall for public investment rather than hedging against oil price volatility. This gives a boost to the economy and more consumption in the next few decades than with a permanent income or bird in hand rule. Iraq does suffer much less from capital scarcity, but might have a real problem absorbing its large and growing windfall. Iraq should therefore have a relatively large parking fund. Iraq's main challenge is to deal with a very volatile and growing stream of oil revenues.

It is important to stress the different objectives of the three types of sovereign wealth funds that an oil rich country needs. 21 out of 31 oil producers have funds of which 10 focus on stabilization and 8 focus on stabilization and saving (IMF, 2005). Stabilization or liquidity funds are typically contingent on the oil price or oil revenue. For example, Trinidad and Tobago specify that 60 percent of 'excess revenue' over and above a long run moving average. In practice, the size of the liquidity fund should depend on the features highlighted in our analysis, but also on the costs of volatility to the domestic economy or the opportunities for borrowing in the downturn. The political risk of such funds being looted also matters. If this is a serious risk, government will have a bias towards partisan, illiquid investment projects at the expense of saving in liquid sovereign assets and/or growth enhancing neutral investment projects.

In future work it is important to analyze the optimal size of precautionary saving buffers in face of such economic and political distortions. It is also important in case studies of particular countries to allow for Dutch disease effects of oil windfalls (e.g., Corden, 1984). The appreciation of the real exchange rate, the decline of the traded sectors and the accompanying loss in output is especially strong if the windfall is temporary and not smoothed. The bird in hand policy and permanent income policies fail to deliver an optimal response to Dutch disease. The optimal policy should thus strike a balance between smoothing real exchange rate fluctuations and consumption, investing to invest and mitigating Dutch disease, which is tougher if a greater part of consumption and public investment has to be produced at home as adjustment is then more sluggish.

References

Aghion, P, P. Bachetta, R. Rancière and K. Rogoff (2009). Exchange rate volatility and productivity growth: the role of financial development, Journal of Monetary Economics, 56, 494-513.

Aguiar, M. and M. Amador (2011). Growth in the shadow of expropriation, The Quarterly Journal of Economics, 126, 651-697.

Aguiar, M., M. Amador and G. Gopinath (2009). Investment cycles and debt overhang, Review of Economic Studies, 76, 1-31.

Ahn, C.M. (1989). The effect of temporal risk aversion on optimal consumption, the equity premium, and the equilibrium interest rate, Journal of Finance, XLIV, 5, 1411-1420.

Aizenman, J., B. Pinto and A. Radziwill (2004). Sources for financing domestic capital – is foreign saving a viable option for developing countries, Working Paper 10624, NBER, Cambridge, Mass.

Borensztein, E., O. Jeanne and D. Sandri (2009). Macro hedging for commodity exporters, Working Paper 15452, NBER, Cambridge, Mass.

Bems, R. and I. de Carvalho Filho (2011). The current account and precautionary savings for exporters of exhaustible resources, Journal of International Economics, 84, 1, 48-64.

Berg, A., R. Portillo, S.-C. S. Yang and L.-F. Zanna (2011). Government investment in resource abundant low-income countries, mimeo., International Monetary Fund, Washington, D.C.

Blattman, C., J. Hwang, and J.G. Williamson (2009). Winners and losers in the commodity lottery: the impact of terms of trade growth and volatility in the Periphery 1870-1939, Journal of Development Economics, 82, 156-179.

Bom, P.R.D. and J.E. Ligthart (2010). How productive is public capital? A meta regression analysis, GSU Paper 0912, Andrew Young School of Policy Studies, Georgia State University.

Bommier, A. and J.-C. Rochet (2006). Risk aversion and planning horizons, Journal of the European Economic Association, 4, 4, 708-734.

Boschini, A.D., J. Petterson and J. Roine (2007). Resource curse or not: a question of appropriability, Scandinavian Journal of Economics, 109, 593-617.

Boulding, K.E. (1966). Economic Analysis, Volume I: Microeconomics, 4th edition. Harper & Row, New York.

BP (2011). Statistical Review, BP, London.

Bremer, T.S. van den and Ploeg, F. van der (2012) Uncertainty, prudence and precautionary saving: a novel algorithm for continous-time, infinite-horizon problems, University of Oxford.

Cherif, R. and F. Hasanov (2011). The volatility trap: why do big savers invest relatively little?, mimeo., International Monetary Fund, Washington, D.C.

Cohen, D. and J. Sachs (1986). Growth and external debt under risk of debt repudiation, European Economic Review, 30, 529-560.

Collier, P., M. Spence, F. van der Ploeg and A.J. Venables (2010). Managing resource revenues in developing economies, IMF Staff Papers, 57, 1, 84-118.

Cordon, W.M. (1984). Booming sector and Dutch disease economics: survey and consolidation, Oxford Economic Papers, 36, 359-380.

Dabla-Norris, E., J. Brumby, A. Kyobe, Z. Mills and C. Papageorgiou (2011). Investing in public investment: an index of public investment efficiency, Working Paper 11/37, International Monetary Fund, Washington, D.C.

Daniel, J.A. (2001). Hedging government oil price risk, International Monetary Fund, Working Paper WP 01/185, Washington, D.C.

Dessus, S., D. Medvedev, B. Land, K. Bain, S. Kwawukume, M. Andrade, C. Calvacanti, M. Stevens, V. Foster, M. Vagliasindi, C. Costain, C. Jackson, and D. Byerlee (2009). Economy-wide impact of oil discovery in Ghana. 47321-gh, World Bank, Washington, D.C.

Durdu, C., E. Mendoza and M. Terrones (2009). Precautionary demand for foreign assets in sudden stop economies: an assessment of the new mercantilism, Journal of Development Economics, 89, 194-209.

Epstein, L.G. and S. Zin (1989). Substitution, risk aversion, and the temporal behavior of asset returns: a theoretical framework, Econometrica, 57, 4, 937-969.

Federico, G., J.A. Daniel and B. Bingham (2001). Domestic petroleum price smoothing in developing and transition economies, Working Paper WP/01/75, International Monetary Fund, Washington, D.C.

Ghosh, A.R. and J.D. Ostry (1997). Macroeconomic uncertainty, precautionary savings, and the current account, Journal of Monetary Economics, 40, 1, 121-139.

Gourinchas, P.-O. and O. Jeanne (2007). Capital flows to developing countries: the allocation puzzle, Working Paper 13602, NBER, Cambridge, Mass.

Gupta, S., A. Kangur, C. Papageorgiou and A. Wane (2011). Efficiency-adjusted public capital and growth, Working Paper 11/217, International Monetary Fund, Washington, D.C.

Hamilton, J.D. (2009). Understanding crude oil prices, Energy Journal, 30, 2, 179-206.

Heuvel, S.J. (2008). Temporal risk aversion and asset prices, Finance and Economics Discussion Series 2008-37, Federal Reserve Board, Washington, D.C.

Kimball, M.S. (1990). Precautionary saving in the small and in the large, Econometrica, 58, 53-73.

Larson, D.F., P. Varangis and N. Yabuki (1998). Commodity risk management and development, Policy Research Working Paper WPS 1963, World Bank, Washington, D.C.

Lehavari, D. and T.N. Srinivasan (1969). Optimal savings under uncertainty, Review of Economic Studies, 36, 153-163.

Leland, H.E. (1968). The precautionary demand for saving, Quarterly Journal of Economics, 82, 3, 465-473.

Lu, Y. and S. Neftci (2008). Financial instruments to hedge commodity price risk for developing countries, Working Paper WP/08/6, International Monetary Fund, Washington, D.C.

Mehlum, H., K. Moene and R. Torvik (2006). Institutions and the resource curse, Economic Journal, 116, 1-20.

Norwegian Ministry of Petroleum and Energy (2009) The Norwegian Petroleum Sector: FACTS.

Pindyck, R.S. (1980). Uncertainty and exhaustible resource markets, Journal of Political Economy, 88, 6, 1203-1225.

Pindyck, R.S. (1981). The optimal production of an exhaustible resource when price is exogenous and stochastic, Scandinavian Journal of Economics, 83, 2, 277-288.

Ploeg, F. van der (1993). A closed-loop solution for a model of precautionary saving, Review of Economic Studies, 60, 2, 385-396.

Ploeg, F. van der (2010). Aggressive oil extraction and precautionary saving: coping with volatility, Journal of Public Economics, 94, 5-6, 421-433.
Ploeg, F. van der (2011). Natural resources: curse or blessing?, Journal of Economic Literature, 49, 2, 366-420.
Ploeg, F. van der (2012). Bottlenecks in ramping up public investment, International Tax and Public Finance, forthcoming.
Ploeg, F. van der and S. Poelhekke (2009). Volatility and the natural resource curse, Oxford Economic Papers, 61, 727-760.
Ploeg, F. van der and S. Poelhekke (2010). The pungent smell of "red herrings": subsoil assets, rents, volatility, and the resource curse, Journal of Environmental Economics and Management, 60, 1, 44-55.
Ploeg, F. van der and A.J. Venables (2011). Harnessing windfall revenues: optimal policies for resource-rich developing economies, Economic Journal, 121, 1-31.
Ploeg, F. van der and A.J. Venables (2012). Natural resource wealth: the challenge of managing a windfall, Annual Review of Economics, 4.
Prasad, E., R. Rajan and A. Subramanian (2006). Patterns of international capital flows and their implications for economic development, Working Paper, IMF, Washington, D.C.
Richard, S.F. (1975). Multilateral risk aversion, utility independence and separable utility functions, Management Science, 22, 1, 12-21.
Roitman, A. (2011). Precautionary savings in a small open economy revisited, Working Paper WP/11/253, International Monetary Fund, Washington, D.C.
Sachs, J.D. and A.M. Warner (1997). Natural resource abundance and economic growth, in G. Meier and J. Rauch (eds.), Leading Issues in Economic Development, Oxford University Press, Oxford.
Sandmo, A. (1970). The effect of uncertainty on saving decisions, Review of Economic Studies, 37, 3, 353-360.
Stulz, R. (2002). Risk Management and Derivatives, South-Western College Publishing.
Turnovsky, S.J., H. Shalit and A. Schmitz (1998). Consumer's surplus, price instability, and consumer welfare, Econometrica, 48, 135-152.
Valencia, F. (2010). Precautionary reserves: an application to Bolivia, Working Paper 10:54, International Monetary Fund, Washington, D.C.
Ventura, L. and J.G. Eisenhauer (2006). Prudence and precautionary saving, Journal of Economics and Finance, 30, 2, 155-168.
Weil, P. (1993). Precautionary saving and the permanent income hypothesis, Review of Economic Studies, 60, 367-383.
Zeldes, S.P. (1989). Optimal consumption with stochastic income: deviations from certainty equivalence, Quarterly Journal of Economics, 104, 2, 275-298.

Appendix A: Alternative approaches to prudent saving

The power utility function has a positive third derivative and thus allows for prudence, but the parameter σ has to do many jobs at once: it characterizes the constant elasticity of intertemporal substitution; $1/\sigma$ measures relative intergenerational inequality aversion as well as relative risk aversion; and prudence is measured by the $CRP = 1 + 1/\sigma$. This is too much to ask of one parameter.[8] The additively separable utility function $V(C_1, C_2)$ allows for *temporal risk aversion* if $V_{12} > 0$ (Richard, 1975). Hence, the country when considering outcomes (C_1 , C_2) prefers the gamble leading to outcomes (low, high) and (high, low) with equal probability to the gamble leading to outcomes (low, low) and (high, high) with equal

[8] Epstein-Zin preferences allow one to introduce two parameters, one to capture the elasticity of intertemporal intertemporal substitution and another one to capture the coefficient of relative risk aversion. This is beyond the scope of the present paper.

probability; temporal risk neutrality implies indifference between these two gambles. Past choices thus matter for tradeoffs between current and future consumption. With the utility function

(A1) $$V(C_1,C_2) \equiv W\left(U(C_1)+\frac{1}{1+\rho}U(C_2)\right), \quad U(C)=\frac{C^{1-1/\sigma}}{1-1/\sigma},\ W(U)=\frac{U^{1-\theta}}{1-\theta},$$

$\theta > 0$ is the coefficient of relative temporal risk aversion.[9] (A1) becomes (2) if $\theta = 0$. The transformation $W(.)$ ensures that the Von Neumann-Morgenstern axioms are satisfied and preferences are time consistent. The parameter θ injects an extra element of prudence into the decision-making process. The coefficients of relative *atemporal* and *temporal* prudence are given by $1+1/\sigma$ and $1+\theta$, respectively. The long run coefficient of relative risk aversion $\zeta \equiv 1-(1-\theta)(1-1/\sigma)$ pertains to gambles with constant streams of consumption outcomes and σ is the elasticity of intertemporal substitution.[10] Numerically solving our two period model for a declining windfall, we see that for the case of oil income volatility the precautionary saving buffer increases with the degree of temporal prudence, $1+\theta$. Asset return uncertainty tends to reduce precautionary saving, and more so if temporal risk aversion is larger.[11]

An interesting example of preferences that generates precautionary saving has a constant and invariant coefficient of relative prudence but does not require a positive third derivative (Roitman, 2011). On the one hand, risk aversion provides incentives to increase savings, but on the other hand imprudence provides incentive to save less in the face of uncertainty. This class of preferences allows one to isolate how changes in risk aversion affect precautionary saving without changing the degree of prudence (see equation (5)). This is not possible with the class of power utility functions used in this paper.

Instead of introducing an extra parameter to capture temporal risk aversion and an additional motive for prudence, one might capture extra prudence using a value-at-risk approach. One way is to introduce a probabilistic constraint that second period consumption is not to fall below a certain fraction of the certainty equivalent level. For our model, this amounts to the constraint:

(A2) $$\text{prob}\left[C_2 < \eta(Y+O)\right] = \text{prob}\left[(1+r)A+\varepsilon_O < -(1-\eta)(Y+O)\right] \le \upsilon, \quad O \equiv [(1+\rho)O_1 + O_2]/(2+\rho),$$

where $\upsilon > 0$ is the tolerance risk. If the policy maker wants the probability that future consumption falls below 95 percent of its certainty equivalent level to be less than 1 percent, $\eta = 0.95$ and $\upsilon = 0.01$. If there is no asset return uncertainty, the optimal value of saving A follows from:

(A3) $$A = \frac{-F^{-1}(\upsilon)-(1-\eta)(Y+O)}{1+r},$$

where F(.) is the cumulative normal density function for ε_O. The first term in the numerator is positive for tolerance risks less than 50 percent and shows the precautionary saving buffer as a result from that. The second term in the numerator shows that this buffer is less if the policy makers puts less value at risk (i.e., η is less than one). To make the problem more interesting, we need another policy variable. This could be done by enriching the model with an optimal level of private as well as public consumption.

---

[9] A similar approach has been used before in different contexts (e.g., Ahn, 1989; Bommier and Rochet, 2006; van den Heuvel, 2008). An alternative way to introduce temporal risk aversion is to maximize a CARA transformation of the present value of quadratic utility. Indeed, if utility $U(.)$ is quadratic and $W(.)$ is a double negative exponential, we have $W(U) = -\exp(-\upsilon U)/\upsilon$, where the coefficient of absolute prudence is given by the constant $CAP \equiv -V''/V' = \upsilon > 0$. Analytical closed form solutions can then be found (van der Ploeg, 1993, 2010).

[10] This is a different way of separating risk aversion from intertemporal substitution than preferences (Epstein and Zin, 1989; Weil, 1993), which are non-expected utility preferences and imply a preference for early or late resolution of uncertainty (van den Heuvel, 2008).

[11] Simulations are available upon request.

Appendix B: Prudent saving and investment in an infinite horizon framework

Asset return and oil price uncertainty

The government maximizes expected social welfare (8) subject to the stochastic oil price dynamics (7) and a now also stochastic asset accumulation equation. We allow for non-zero correlation in the bivariate Brownian motion for oil price P and assets A:

$$\text{(A4)} \qquad \begin{pmatrix} dP \\ dA \end{pmatrix} \sim \mathrm{N}\left(\begin{pmatrix} \upsilon P \\ rA + S \end{pmatrix} dt, \begin{pmatrix} (rA\sigma_A)^2 & \gamma_{rP} O \\ \gamma_{rP} O & (P\sigma_P)^2 \end{pmatrix} dt \right)$$

where the covariance is proportional to oil rents O to that it disappears when oil is exhausted. For simplicity, we assume that γ_{rP} is not a function of A and the world interest rate r is not stochastic. The optimal change in consumption must satisfy the stochastic Euler equation:

$$\text{(A5)} \qquad \frac{1}{dt}\mathrm{E}_t[dC] = \sigma[r-\rho]C + \frac{1}{2}CRP\left(\frac{\left(\frac{\partial C}{\partial P}\right)^2 \sigma_P^2 P^2 + \left(\frac{\partial C}{\partial A}\right)^2 \sigma_A^2 r^2 A^2 + 2\left(\frac{\partial C}{\partial P}\right)\left(\frac{\partial C}{\partial A}\right)\gamma_{rP} O}{C} \right)^2 C.$$

The first term on the right hand side of (A5) corresponds to the deterministic Euler equation. It equals zero if $r = \rho$. The second term is the *positive* prudence term, which indicates that there is an additional upward tilt of the time profile of the expected consumption path leading to precautionary saving buffers if oil price volatility and asset return uncertainty are high. In contrast to the stochastic Euler equation (5) for the two period model of section 4, there is no *negative* risk aversion term. In the two period model, risk aversion (except if prudence is high) combined with risky assets provides incentive to lower these assets to avoid this risk. This effect arose, because whatever is saved and has an uncertain yield in period 2 has to be consumed in period 2. In the infinite horizon model, Marshall's risk aversion effect no longer exists. The shocks affect the asset stock, but not consumption. Equation (A5) indicates that prudence continues to have an effect after oil has been exhausted.

Investment returns uncertainty and capital scarcity

We abstract from adjustment costs for public capital and focus at investment return rather than asset return uncertainty. We capture investment return uncertainty by replacing output by $F(BK)$, where the stochastic productivity shock B has mean 1 and standard deviation σ_B. We get the optimality conditions:

$$\text{(A6)} \qquad \frac{1}{dt}\mathrm{E}_t[dC] = \sigma[\Pi(D) + D\Pi'(D)]C + \frac{1}{2}CRP\left[1 - \frac{\partial S}{\partial H} - \frac{\partial I}{\partial H}\right]^2 \left(\frac{(\sigma_P P)^2 + (\sigma_B K)^2}{C} \right),$$

$$\text{(A7)} \qquad BF'(BK) = \rho + \Pi(D) + D\Pi'(D) + \delta \quad \rightarrow \quad K = K(\overset{-}{D}, \overset{+}{B}),$$

where total income is defined by $H \equiv F(BK) + PO = C + I + S$ and $MPC = 1 - \partial S / \partial H - \partial I / \partial H$ is the marginal propensity to consume out of a windfall.

If there is no capital scarcity ($\Pi(D) = 0$), public investment does not respond to the windfall in which case $\partial I/\partial H = 0$ (see section 5). The effect of prudence term on the rate of change in consumption (A6) is strengthened by a second source of uncertainty, namely uncertainty about the returns on public investment. With capital scarcity, it is optimal to allocate a proportion of windfall income to public investment so $\partial I/\partial H > 0$. This attenuates the effect of prudence on the rate of change in consumption. Hence, uncertainty about the returns on public investment increases whilst capital scarcity decreases the precautionary buffer. Equation (A7) shows that the marginal product of public capital must be set to the social cost of capital plus the depreciation rate of public capital. Hence, the optimal stock of public capital decreases in the debt of the nation and increases in total factor productivity.