Monetary Rules for Commodity Traders*

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April 3 2012

Abstract

We study monetary policy rules for open economies with different export-import structures and imported inputs in production, under alternative assumptions on international risk sharing. We describe, in particular, how dynamic responses to alternative shocks and welfare implications of monetary rules interact with imperfections in financial markets. For most parametrizations, a PPI targeting rule emerges as slightly superior to CPI targeting, and considerably better than expected CPI targeting and an export price targeting rule. Imperfect risk sharing is noticeably harmful for welfare and calls for a more aggressive response to inflation.

1. Introduction

The large swings in commodity prices over the past few years have called into question the ability of inflation targeting rules to provide a firm, welfare-enhancing anchor for monetary policy. This is particularly the case for countries that are large commodity traders, and where commodity items, such as food and oil, account for a sizeable share of consumption baskets. As most (if not all) emerging markets fit into at least one of these categories, it is hardly surprising to hear calls for a re-appraisal of standard Taylor rules as the basis for monetary policy in those countries.

A main contention is that strict inflation targeting (IT) – based on an explicit and pre-announced quantitative annual target for headline CPI and low or null weight on the output gap – can be highly destabilizing for an small open and emerging economy (SOEM) (cf. Frankel, 2010, 2011; IMF, 2011). To see why, consider an SOEM that exports commodities but also has non-trivial net imports of food or oil, and where these goods accounts for a substantial share of the CPI. A rise in the world price of oil/food (especially if the magnitudes seen in 2006-08 and 2010-11) would then hurt the country directly via a terms of trade (TOT) deterioration [1]. But this can be

*We thank Marola Castillo for assistance beyond the call of duty.

[1] Or, at best, no significant TOT change if the price of its main exports (e.g. copper) correlates highly with food and oil.

exacerbated by the policy response. As food and oil weigh heavily on the domestic consumption basket, CPI inflation will rise which, under a CPI IT rule, will result in a rise of the policy interest rate. Higher rates will lower domestic demand and cause an appreciation of the nominal exchange rate. And, if the country is a price taker in the world market for its exports, the appreciation will exert downward pressure on home goods prices and on profit margins. Falling domestic consumption and squeezed profits would reinforce the contractionary impact of the TOT deterioration on output and employment so, in this sense, the CPI Taylor rule tends to exacerbate the pro cyclical response of the SOEM to TOT shocks.[2]. The more volatile the world relative price of commodities, the argument goes, the worse is the trade-off between output and inflation stabilization engendered by strict CPI IT.

An alternative would be to target "core" CPI inflation, where "core" means that volatile flex-price goods like food and oil are purged from the pertinent inflation index. This is actually practiced by some SOEMs like Korea and South Africa, so one may ask why other SOEMs have not yet followed. One reservation might be that the purging may be seen by the public as non-transparent and thus detract from policy credibility – a potential Achilles heel for central banks devoid of a long-standing anti-inflationary record. Another easy criticism is that, in countries where commodities accounts for 30 to 50% of CPI, one might wonder what such a "purged" CPI stands for. Still, this proposal has been often taken as a serious contender in academic and policy circles, particularly in light of a sizeable theoretical literature, largely inspired by advanced countries with production is dominated by sticky price sectors, that argues that PPI inflation targeting can reproduce the flex-price equilibrium (Goodfriend and King, 2001; Faia and Monacelli, 2010). It is unclear, however, how applicable the resulting insights are to SOEM contexts, especially given that PPI and core CPI may not display much congruence for countries are large producers and exporters of commodities, in which the aggregate PPI index may have a sizeable flexible price component.

A third alternative, espoused by Frankel (2010, 2011), is to peg a producer-based price measure – either the Export Price (PEP) or Producer Prices (PPT). PEP amounts to stabilizing the domestic currency price of the country's main exporting commodity. PPT is broader and amounts to stabilizing the domestic producer price index (PPI) once the latter is computed with value added weights instead of gross sales weights as in old-fashioned PPI. It is straightforward to see that how either of these alternatives could go some way towards mitigating the pro-cyclical "bias" of CPI IT: when TOT deteriorates, the exchange rate automatically depreciates to stabilize the domestic price of exports or output. If wages and non-tradable prices are sticky, it follows that domestic relative prices and producers' profit margins are also stabilized. Hence the effects of TOT volatility on output and employment

[2] Mutatis mutandis, a similar criticism of IT would be applicable to an advanced country SOE which produces and exports commodities (including certain food staples, and/or gas and oil) but imports mainly manufactured goods, and where manufactured goods weighs heavily on CPI: when commodity prices fall heavily relative to world manufacturing prices, TOT deteriorates while domestic CPI inflation either remains about constant or rises. As per strict application of Taylor rules on headline CPI, no major monetary loosening would be called for. But this would prevent monetary policy to offset - at least in part - the negative impact of the TOT on output and employment.

are offset via exchange rate fluctuations. In a world where financial markets are incomplete and hence the SOEM cannot fully insure its consumers against such fluctuations in commodity TOT, this proposal arguably may some especial traction.

Against this background, one objective of this paper is to examine monetary policy and the relative merits of alternative rules in a formal, optimizing welfare-based framework. The focus on welfare evaluation is key because, as we shall discuss below, each of the mentioned proposals contains important trade-offs in terms of the various elements of conventional utility functions. For instance, standard theory indicates that a benevolent policy maker should try to maximize the consumption of the representative citizen, minimize labor effort, and reduce the volatility of both. Put the first two objectives together, and assuming that production is proportional to employment, this implies that one really wants to maximize the ratio of consumption to domestic output (C/Y), and reduce the variability of C and Y. Yet, it is sometimes the case that the proponents of each targeting proposal focus on a single objective and single out one type of shock. Instead, the purpose of this paper is to look at the overall suitability of each rule in the context of an SOEM that is a commodity exporter as well as importer, and of a central bank that faces conflicting objectives in terms of the distortions it seeks to mitigate and the relative importance of the various shocks hitting such an economy.

To this end, our analytical framework is that of SOE Keynesian model along the lines of Gali and Monacelli (2005) and Catao and Chang (2010), adapting it to the case where the SOEM is both a commodity and a non-commodity exporter (the latter possibly encompassing activities like tourism and lightly processed manufacturing). To frame the discussion in terms of the proposals reviewed above the model lets access international financial markets may be imperfect and allow imports (like oil/energy) not only be consumed but also be an input to production. In this setting, the main contribution of this paper is quantitative: we seek to establish which of these various policy rules are welfare-superior for given a menu of shocks (whose relative size and persistence we calibrate to real world counterparts) and parametrizations of the model.

We build our framework, also, so as to be able to examine how the results depend on fundamental aspects of the environment. One focus here is, in particular, whether and how the relative sizes of the economy's productive sectors make a difference for the comparison of the policy alternatives. Another dimension that we examine is the role of financial integration vis a vis the rest of the world. In both cases, we discuss how differences emerge, and quantify their significance.

The paper proceeds as follows: the next section sets up the baseline model. Section 3 discusses how the features embedded in the model affect a key building block of the analysis, that is, the derivation of the New Keynesian aggregate supply equation. Section 4 examines the responses of the model to different shocks, with an emphasis in how those responses depend on the degree of financial integration. Finally, Section 5 compares alternative monetary rules in terms of welfare. It also discusses the implications of a given policy under alternative assumptions on international risk sharing, imported inputs to production, and the competitive export sector. Section 6 concludes. Some technical material is deferred to an Appendix.

2. The Model

In this paper, we adapt the Catao and Chang's (2010, henceforth CC) small open economy model to the case where the SOEM is mainly a commodity exporter although it can also export other (less-flexible) price goods (like tourism and lightly processed manufacturing). To frame the discussion in terms of the existing proposals, the model lets access international financial markets may be imperfect and allow imports (like oil/energy) not only be consumed but also be an input to production.

2.1. Households

The economy has a representative household with preferences:

$$E\sum_{t=0}^{\infty}\beta^t U_t$$

where $0 < \beta < 1$, $E(.)$ is the expectations operator, and U_t depends on consumption C_t and labor effort N_t:

$$U_t = \frac{C_t^{1-\sigma}}{(1-\sigma)} - \frac{\varsigma N_t{}^{1+\varphi}}{1+\varphi}$$

σ, φ, and ς are parameters.

Consumption is a C.E.S. aggregate of a home good C_h and imports C_m :

$$C_t = \left[(1-\alpha)^{1/\eta} C_{ht}^{(\eta-1)/\eta} + \alpha^{1/\eta} C_{mt}^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}$$

where η is the elasticity of substitution between home and foreign goods, and α measures the degree of openness.

We will see later that C_h is an aggregate of varieties of a differentiated good, while imports can not only be consumed but also used as inputs to production. The latter, in particular, allows us to interpret imports as (possibly an aggregate of) oil or food. For the representative agent, however, these details are irrelevant, as he assumes he can purchase any quantities of C_h and C_m at their prevailing market prices. The minimum cost of a unit of consumption, or *CPI*, expressed in domestic currency, is then

$$P_t = \left[(1-\alpha) P_{ht}^{1-\eta} + \alpha P_{mt}^{1-\eta}\right]^{1/(1-\eta)} \tag{1}$$

where P_{ht} and P_{mt} are respectively the domestic currency prices of the home consumption aggregate and commodity imports. Also, optimal demands for home goods and imports are given by

$$C_{ht} = (1-\alpha)\left(\frac{P_{ht}}{P_t}\right)^{-\eta} C_t \tag{2}$$

$$C_{ft} = \alpha \left(\frac{P_{mt}}{P_t} \right)^{-\eta} C_t$$

Note that, if $P_{ht} = P_{mt} = P_t$, α equals the fraction of all consumption that is imported. In this sense, α is a measure of openness. The case $\alpha < 1/2$ is often associated with "home bias". [3]

The representative agent chooses consumption and labor effort taking prices and wages as given. The agent owns domestic firms and receives their profits (dividends) as well as any transfer from the government.

The resulting maximization problem is well known (see e.g. Gali and Monacelli 2005). If W_t is the domestic wage, optimal labor supply is then given by the equality of the marginal disutility of labor with the marginal utility of the real wage:

$$\varsigma C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \tag{3}$$

We depart from CC in that households have access to international financial markets, but this access may be limited. In CC and others, complete, frictionless markets imply that marginal rates of substitution in consumption at home and abroad are equal up to a real exchange rate correction. In contrast, if home agents are excluded from international financial markets, the trade balance must be zero at all times. The following specification allows for both possibilities as well as for intermediate ones:

$$C_t = \left[\kappa X_t^{1/\sigma} C_t^* \right]^{\psi} \left[V_t / P_t \right]^{1-\psi} \tag{4}$$

where κ is a positive constant, C_t^* is an index of world consumption, X_t is the *real exchange rate* (the ratio of the price of world consumption to the domestic CPI, both measured in a common currency), V_t is domestic *value added* in nominal terms, and ψ is a constant in the unit interval. If $\psi = 1$, the preceding expression reduces to the one implied by complete markets. [4] On the other hand, if $\psi = 0$, we get that $P_t C_t = V_t$, and hence a zero trade balance. Intermediate values of ψ do not have a direct interpretation, but we use this device to examine the robustness of our results to departures from the complete markets paradigm.

We assume, however, that domestic agents can trade a full set of contingent securities among themselves. Then, as well known, any domestic security can be priced observing that the *stochastic discount factor* at t for domestic currency payoffs at $t + k$ is given by

$$\Xi_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+k}} \right)$$

In particular, the domestic safe interest rate is given by

$$\frac{1}{1+i_t} = E_t \Xi_{t,t+1} \tag{5}$$

$$= \beta E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right] \tag{6}$$

[3] We have assumed $\eta \neq 1$. If $\eta = 1$, C_t and P_t are Cobb Douglas.

[4] Assuming that the marginal utility of consumption in the rest of the world is proportional to $C_t^{*-\sigma}$.

2.2. Prices

For simplicity, we assume that the world price of imports is exogenously given in terms of a world currency. Using asterisks to denote prices denominated in world currency, the domestic currency price of imports is then

$$P_{mt} = S_t P_{mt}^*$$

where S_t is the nominal exchange rate (domestic currency per foreign currency). So, there is full pass through from world to domestic imports prices.

Likewise, we assume that the world currency price of the world consumption consumption aggregate is exogenous. Denoting it by P_t^*, the real exchange rate is then:

$$X_t = S_t P_t^* / P_t$$

It is useful also to define the domestic price of imports relative to the price of the home consumption aggregate by

$$Q_t = \frac{P_{mt}}{P_{ht}} = \frac{S_t P_{mt}^*}{P_{ht}} \tag{7}$$

In CC, the home consumption aggregate is also the economy's only export, so Q_t is the *terms of trade.* In that case, the terms of trade and the relative price of home output are essentially the same, since 1 implies that

$$\left(\frac{P_t}{P_{ht}}\right)^{1-\eta} = (1-\alpha) + \alpha Q_t^{1-\eta} \tag{8}$$

A key aspect of CC relative to other models is that fluctuations in the relative world price of imports weakens the association between the real exchange rate and the terms of trade. This has important consequences for welfare and optimal policy, as discussed later on. To see this, note that, using the definitions of Q_t and X_t, the preceding expression can be rewritten as:

$$1 = (1-\alpha)\left(\frac{P_{ht}}{P_t}\right)^{1-\eta} + \alpha X_t^{1-\eta} Z_t^{*1-\eta} \tag{9}$$

where $Z_t^* = P_{mt}^*/P_t^*$ is the world's relative price of imports, which we take as exogenous.

From (8), an improvement in the terms of trade (a fall in Q_t) requires an increase in the relative price of domestic output (P_h/P). Given Z_t^*, 9 then implies that X_t must fall (a real appreciation). Hence X_t and Q_t must always move in the same direction in response to shocks other than Z_t^*. This is indeed an implication of several recent models (e.g. Gali and Monacelli 200x). But it is evident that in our model X_t and Q_t can move in opposite directions when Z_t^* moves.

In what follows we will assume that the home economy can produce a second good, which is only exported. Hence Q_t will not suffice to measure the terms of trade. However, the intuition of the preceding discussion remains.

2.3. Domestic Production

As mentioned, the economy is assume to produce two kinds of goods: varieties of a differentiated good, which can be assembled to obtain an aggregate that can be consumed or exported; and a perfectly competitive, homogenous commodity, which is only exported at exogenous world prices. We describe each production sector in turn.

2.3.1. The imperfectly competitive sector

The first consumption-export good can be obtained by assembling intermediate goods varieties, indexed by $j \in [0,1]$:

$$Y_{ht} = \left[\int_0^1 Y_t(j)^{(\varepsilon-1)/\varepsilon} dj \right]^{\varepsilon/(\varepsilon-1)} \tag{10}$$

where ε is the elasticity of substitution between domestic varieties. Minimizing the cost of producing the aggregate implies that, given Y_{ht}, the demand for each variety is given by

$$Y_t(j) = \left(\frac{P_t(j)}{P_{ht}} \right)^{-\varepsilon} Y_{ht} \tag{11}$$

where $P_t(j)$ is the price of variety j, and P_{ht} is the relevant price index:

$$P_{ht} = \left[\int_0^1 P_t(j)^{1-\varepsilon} dj \right]^{1/(1-\varepsilon)} \tag{12}$$

The intermediate variety j is produced by a single firm using labor $L_t(j)$ and imports $M_t(j)$ according to

$$Y_t(j) = \iota A_t L_t(j)^{1-\varkappa} M_t(j)^{\varkappa} \tag{13}$$

where A_t is a common (country-wide) productivity shock, $\varkappa$ is the commodity share in production, and $\iota = [\varkappa^{\varkappa}(1-\varkappa)^{1-\varkappa}]^{-1}$ is an irrelevant constant.

Variety producers take input prices and wages as given. We allow for the existence of a subsidy to employment *to monopolistic competition firms* at constant rate υ. Hence cost minimization requires each variety producer to choose inputs such that

$$\frac{M_t(j)}{L_t(j)} = \frac{M_t}{L_t} = \frac{\varkappa}{1-\varkappa} \frac{W_t}{P_{mt}} \tag{14}$$

where the first equality emphasizes that all variety firms choose the same relative input mix, and we have defined $L_t = \int_0^1 L_t(j) dj$ and $M_t = \int_0^1 M_t(j) dj$.

Nominal marginal cost is the same for all variety firms and given by

$$\Psi_t = P_{mt}^{\varkappa}((1-\upsilon)W_t)^{1-\varkappa}/A_t \tag{15}$$

Variety producers are monopolistic competitors and set prices in domestic currency as in Calvo (1983): each individual producer is allowed change nominal prices with probability $(1-\theta)$. As is now well known, all variety producers with the opportunity to reset prices in period t will choose the same price, say $\bar{P}_t$, which satisfies:

$$\sum_{k=0}^{\infty} \theta^k E_t \left[\Xi_{t,t+k} Y_{t+k|t} (\bar{P}_t - \frac{\varepsilon}{\varepsilon - 1} \Psi_{t+k}) \right] = 0 \tag{16}$$

where $Y_{t+k|t}$ is the demand in period $t+k$ for a producer that last set her price in period t :

$$Y_{t+k|t} = \left(\frac{\bar{P}_t}{P_{ht+k}} \right)^{-\varepsilon} Y_{ht+k} \tag{17}$$

It also follows (from 12) that the price of the home aggregate is given by:

$$P_{ht} = \left[(1-\theta) \bar{P}_t^{1-\varepsilon} + \theta P_{h,t-1}^{1-\varepsilon} \right]^{1/(1-\varepsilon)} \tag{18}$$

2.3.2. The Competitive Exports Sector

As mentioned, we extend CC by allowing for a competitive export sector, composed of firms that use labor to produce a good that can be exported at exogenous world prices. This addition allows us to explore a number of realistic and policy relevant alternatives, as we will see.

For simplicity, we assume that the production function in the competitive export sector is Cobb Douglas is given by

$$Y_{xt} = A_{xt} L_{xt}^{\varrho}$$

where $0 \leq \varrho \leq 1$, L_{xt} is the amount of labor input in the sector, and A_{xt} a sectorial productivity shock. Note that the case $\varrho = 0$ reduces to the assumption that homogenous exports are given by an exogenous endowment. This case has received some attention in the literature (e.g. Bodenstein and Guerrieri 2011).

Profit maximization requires:

$$\begin{aligned} \varrho A_{xt} L_{xt}^{\varrho - 1} &= \frac{W_t}{S_t P_{xt}^*} \\ &= \frac{W_t}{X_t P_t} \frac{1}{Z_{xt}^*} \end{aligned} \tag{19}$$

where Z_{xt}^* is the export's world relative price. (Note that we have assumed that the wage is subsidized at the same rate as in the differentiated goods sector.)

2.4. Equilibrium

We assume that the foreign demand for the domestic aggregate is given by a function of its price relative to P_t^* and the index C_t^* of world consumption. Hence market clearing for the home good requires:

$$\begin{aligned} Y_{ht} &= C_{ht} + \phi \left(\frac{P_{ht}}{S_t P_t^*} \right)^{-\gamma} C_t^* \\ &= (1-\alpha) \left(\frac{P_{ht}}{P_t} \right)^{-\eta} C_t + \phi \left(\frac{P_{ht}}{S_t P_t^*} \right)^{-\gamma} C_t^* \end{aligned} \tag{20}$$

where ϕ is a constant and γ is the price elasticity of the foreign demand for home exports. Note that we allow the home and foreign elasticities of demand for home goods, γ and η, to differ. Also, the case $\phi = 0$ amounts to the assumption that the home aggregate is a nontraded good.

Equilibrium also requires that the supply of labor equal the demand for labor:

$$N_t = L_t + L_{xt} \tag{21}$$

And in equilibrium, value added in home production must be:

$$\begin{aligned} V_t &= P_{ht} Y_{ht} + S_t P_{xt}^* Y_{xt} - P_{mt} M_t \\ &= P_{ht} Y_{ht} + Z_{xt}^* X_t P_t A_{xt} L_{xt}^{\varrho} - Z_t^* X_t P_t M_t \end{aligned} \tag{22}$$

The description of the model is complete once a monetary policy rule is specified.

3. Aggregate Supply

A main channel through which commodity price shocks affect equilibria in this model is, perhaps unexpectedly, the link between those shocks and aggregate supply. And how that channel works depends on the extent to which international financial markets are complete or not, i.e., the value of ψ in 4. This section elucidates the connections.

It is useful to look at a typical log linear approximation to the model. Following previous work (e.g. Gali 2008) it is not hard to show that 16 and 18 result in a relation between the dynamics of prices in the imperfectly competitive sector and marginal costs there:

$$\pi_{ht} = \beta E_t \pi_{h,t+1} + \lambda (mc_t - mc)$$

where

$$\lambda = \frac{(1-\beta\theta)(1-\theta)}{\theta}$$

is a parameter, lowercase letters denote logs of the corresponding uppercase letters, $\pi_{ht} = p_{ht} - p_{ht-1}$ is usually called "domestic" inflation, mc_t is the log of marginal cost in terms of P_{ht}, or of the inverse of the markup, i.e.

$$mc_t = \log \Psi_t / P_{ht} \tag{23}$$

and mc is its nonstochastic steady state value (equal to $\log(\varepsilon - 1)/\varepsilon$). As is well known, this equation stresses that inflation increases when markups are below their target value, i.e. when current and expected future values of mc_t, mc_{t+1}... are above mc. Price setters then set higher prices at a speed given by the Calvo parameter θ.

Understanding the dynamics of aggregate supply, then, involves understanding the determinants of markups or, equivalently, marginal costs in terms of domestic prices. Now, from 15,

$$\begin{aligned} \frac{\Psi_t}{P_{ht}} &= \frac{P_{mt}^{\varkappa} W_t{}^{1-\varkappa}}{P_{ht} A_t} \\ &= \frac{1}{A_t} Q_t^{\varkappa} \left(\frac{W_t}{P_t} \frac{P_t}{P_{ht}} \right)^{1-\varkappa} \end{aligned}$$

where we have used the definition $Q_t = P_{mt}/P_{ht}$. Hence, taking logs,

$$\begin{aligned} mc_t &= -a_t + \varkappa q_t + (1-\varkappa) \left[(w_t - p_t) + (p_t - p_{ht}) \right] \\ &= -a_t + [\varkappa + \alpha(1-\varkappa)] q_t + (1-\varkappa)(w_t - p_t) \end{aligned}$$

where we have used $p_t - p_{ht} = \alpha q_t$, from the linearization of 8. This expression is intuitive. As usual, real (producer) marginal cost increases with the real (consumer) wage and falls with productivity. The term q_t captures the effect of the price of imports in terms of P_{ht}, and works in two ways. First, an increase in the price of imports raises marginal costs with elasticity $\varkappa$ because imports are an input to production. Second, an increase of the price of imports reduces P_{ht} relative to P_t and, hence, increases the cost to producers to pay a given consumption wage.

As advocated by Woodford (2003), it will be useful to summarize shocks in terms of their effect on *natural* or flexible price values, which we will denote with an n superscript. The equations that implicitly map shocks to natural variables are given by part 7.2 of the appendix. In analyzing the impact of a shock, valuable insights are often obtained by thinking about how the shock first affects natural variables, and then how the changes in the latter affect the deviations or *gaps* of actual variables from the natural counterparts.

Thus, for example,

$$\begin{aligned} mc_t^n &= -a_t + [\varkappa + \alpha(1-\varkappa)] q_t^n + (1-\varkappa)(w_t - p_t)^n \\ &= mc \end{aligned}$$

where the last equality follows from the fact that the log markup is equal to $-mc$ under flexible prices. It follows that

$$mc_t - mc = [\varkappa + \alpha(1-\varkappa)](q_t - q_t^n) + (1-\varkappa) \left[(w - p_t) - (w_t - p_t)^n \right]$$

This says that the dynamic of markups depend on the deviations of q_t and the real wage relative from their natural values, that is, the real wage gap and the terms of trade gap.

To proceed, linearize the market clearing equation for the monopolistic good:

$(1-\alpha)\left(\frac{P_{ht}}{P_t}\right)^{-\eta} C_t + \phi\left(\frac{P_{ht}}{S_t P_t^*}\right)^{-\gamma} C_t^*$

$$
\begin{aligned}
y_{ht} &= \omega\left[\eta(p_t - p_{ht}) + c_t\right] + (1-\omega)\left[\gamma(p_t - p_{ht} + x_t) + c_t^*\right] \\
&= \omega\left[\eta\alpha q_t + c_t\right] + (1-\omega)\left[\gamma(q_t - z_t^*) + c_t^*\right]
\end{aligned}
$$

where ω is the fraction of production that is consumed at home in the steady state, we have used again $p_t - p_{ht} = \alpha q_t$, and also that the real exchange rate can be expressed as

$$x_t = (1-\alpha)q_t - z_t^*$$

It follows that $(q_t - q_t^n)$ can be written as:

$$\left[\omega\eta\alpha + (1-\omega)\gamma\right](q_t - q_t^n) = y_{ht} - y_{ht}^n - \omega(c_t - c_t^n)$$

i.e. as a function of the *output gap* and the *consumption gap.* Exogenous shocks affect $(q_t - q_t^n)$, and hence marginal costs, through their impact on those two gaps.

Now turn to the real wage. Linearizing the optimal labor supply equation, and omitting constants,

$$w_t - p_t = \sigma c_t + \varphi n_t$$

In turn, labor supply must be a weighted average of labor employed in the two sectors:

$$n_t = \frac{L}{N} l_t + \frac{L_x}{N} l_{xt}$$

Eq. 19 gives

$$(1-\varrho)l_{xt} = a_{xt} - (w - p_t) + (x_t + z_{xt}^*)$$

The flexible price version of this equation indicates how a shock to the price of homogeneous exports affect the economy: they must raise the "natural" levels of employment in that sector, total employment, real wages, and marginal costs. The magnitude of that effect is determined by the elasticity ϱ and the size of the sector L_x/N.

Also, note that a shock to the relative price of homogeneous exports is exactly equivalent as a shock to productivity in that sector: in this model, A_{xt} and Z_{xt}^* enter only through their product.

Labor employed in the monopolistic competition sector is, in turn,

$$(1-\varkappa)l_t = y_{ht} - a_t - \varkappa(w_t - p_t - (z_t^* + x_t))$$

Labor demand increases, naturally, with production, and falls with productivity and the real wage. The novel term is the one in $z_t^* + x_t$, the log of P_t/P_{mt}. Given production and the real wage, a positive shock to the price of imports results in a switch towards more labor usage.

Given the above, in the expression for the real wage gap:

$$(w - p_t) - (w_t - p_t)^n = \sigma(c_t - c_t^n) + \varphi(n_t - n_t^n)$$

one can eliminate the labor gap $n_t - n_t^n$ for terms in the output gap and the real exchange rate gap $(x_t - x_t^n) = (1-\alpha)(q_t - q_t^n)$. The resulting expression is cumbersome and best discussed in reference to the calibrated examples later.

Finally, as mentioned already, the role of the consumption gap will depend on the degree of financial integration. With complete international financial markets $\psi = 1$, so

$$c_t = c_t^* + \frac{1}{\sigma} x_t$$

Hence

$$c_t - c_t^n = \frac{1}{\sigma}(x_t - x_t^n) = \frac{(1-\alpha)}{\sigma}(q_t - q_t^n)$$

In this case, any shock can affect the consumption gap only through the shock's impact on the real exchange gap. And the above expression tells us that no shock other than c_t^* affects the natural level of consumption directly.

To see the impact of incomplete markets, consider the polar case of $\psi = 0$. Then trade must be balanced in every period, or

$$C_t = X_t Z_{xt}^* A_x L_{xt}^{\varrho} + \frac{P_{ht}}{P_t} Y_{ht} - \frac{\varkappa}{1-\varkappa} \frac{W_t}{P_t} L_t$$

This can be used to write the consumption gap in terms of the other gaps. But note that this also indicates that z_{xt}^* and a_{xt} will raise the natural level of consumption directly, which will also push up the natural real wage.

Perhaps the latter part of our discussion is further understood by considering the case in which $\varrho = 0$, i.e. the homogeneous export is a pure endowment, a case that has been considered by others. In that case, an increase, $Y_{xt} = A_{xt}$ and, obviously, $L_{xt} = 0$. With complete markets, A_{xt} and therefore Y_{xt}, does not enter anywhere else in the set of equilibrium equations, so a shock to productivity in that sector, or to the relative world price of the homogeneous export, does not affect equilibrium, except possibly to the extent that the policy rule (which we have not specified yet) reacts to A_{xt} above and beyond its reaction to the other endogenous variables. What explains this? With complete markets, home agents are completely insured against A_{xt} and Z_{xt}^*, so any such shock only results in a transfer from or to abroad. This means that, if markets are complete, A_{xt} and Z_{xt}^* can only effect equilibria if the policy rule reacts to them, or through their impact on the labor market if ϱ is not zero.

With incomplete markets, the preceding expression for C_t shows that A_{xt} and Z^*_{xt} affect equilibria directly, through the increase in consumption resulting from an increase in the value of exports.

4. Dynamics

To further illustrate the workings of the model, here we examine the impulse responses of a calibrated version of the model. The precise details of the calibration are discussed in a later section. For concreteness, we assume in this section that monetary policy is given by a strict PPI Taylor rule with coefficient 1.5 on PPI inflation (and zero on the output gap). The examination of alternative rules is, in fact, the subject of the next section.

In order to highlight the influence of financial integration, the first subsection describes dynamics under complete integration ($\psi = 1$). The second subsection assumes $\psi = 0$, that is, financial autarky.

4.1. Perfect Financial Integration

Figure 1 displays impulse responses to a one standard deviation shock to the world relative price of imports z^*_t, which is assumed to follow an AR(1) process with autocorrelation coefficient 0.75. For ease of interpretation, the figure assumes that there are no imported inputs in production ($\varkappa = 0$). This makes the model comparable to that of Catao and Chang (2010).

The impulse responses in the figure assume $\sigma = 2$ and η values of 0.5, 2, and 5. The case $\sigma = 1/\eta$ is particularly useful for the discussion: as Catao and Chang (2010) show, a z^* shock does not affect the natural (flexible price) level of output in that case, while the natural value of Q_t (the "terms of trade ") deteriorate and the natural real exchange rate appreciates. Under full international risk sharing, the latter implies that natural consumption must fall. Since natural output does not move, under strict PPI targeting the nominal interest rate does not change. This implies, via the New Keynesian Phillips curve, that PPI inflation and the output gap remain at zero. All of these results obtain in Figure 1 for the case $\eta = 1/\sigma = 0.5$. In that case, also, welfare falls because consumption falls, while labor employment does not change. Finally, the real interest rate increases, as it tracks consumption growth.

For $\eta > 1/\sigma$, Catao and Chang (2010) show that natural output must increase in response to a z^* shock. This, too, is evident in Figure 1. In the presence of sticky prices, this means that the demand for home differentiated goods increases, which leads to an increase in PPI inflation. The nominal interest rate increases in response, although not by enough to prevent an increase in output of differentiated goods and labor employment in that sector. Note that labor employment in the competitive export sector must, in turn, fall, since the relevant product wage (not shown) increases. The terms of trade deteriorate by less as η is larger, and the real exchange rate appreciates by more, so that consumption falls more. This, together with the overall increase in employment, leads to lower welfare as η is larger.

The impulse responses to a one standard deviation shock to the world relative price of homogeneous exports (Z_{xt}^*) are given by Figure 2. As discussed at the end of the previous section, the key impact is through the labor market. Employment in the competitive export sector expands and is reflected in an increase in the real wage. A higher real wage increases real marginal costs in the sticky price sector and causes a fall in natural output there. Hence the output gap becomes positive and domestic inflation increases. The interest response is, therefore, positive.

The real exchange rate appreciates, on the other hand. Q_t, or the inverse of P_{ht}/P_t, also falls. Real exchange rate appreciation results in a fall in consumption due to complete markets. The increase in the real wage, on the other hand, results in a fall in employment in the sticky price sector. Total employment, however, increases. Together with the fall in consumption, this implies that welfare $falls$. This may be counterintuitive, but it just reflects the fact that much of the "benefit " of an increase in the relative price of homogeneous exports is transferred abroad because of full international insurance, as discussed at the end of the previous section.

Figure 3 displays the impact of a contractionary shock to the policy rule. The response is quite conventional. The key observation is that, although the nominal interest rate falls (reflecting the contraction of the sticky price sector), the real interest rate increases. At fixed prices, higher real interest rates must be met by higher consumption growth, which means that consumption falls on impact to then grow back to the steady state. With complete markets, this has to be met with a real exchange rate appreciation on impact. The RER appreciation must involve a fall in Q or, equivalently, a fall in the relative price of differentiated goods. Also, the fall in demand mean that output, employment, and inflation in the sticky price sector all fall. The real wage falls, which favors employment in the competitive export sector. The net change in employment is, however, negative.

Since both consumption and labor effort fall in response to the monetary shock, the total effect on welfare is, in principle, ambiguous. For our parametrization, lifetime utility increases. This suggests that the so called *terms of trade externality,* e.g. the country's ability to increase welfare by affecting the world price of its (differentiated) products, is relatively strong in this economy.

4.2. Imperfect Financial Integration

As mentioned, Figures 4 to 6 compare the results under perfect capital markets against those under financial autarky, that is, under $\psi = 0$, to illustrate the differences that financial integration may make.

Figure 4 gives the impulse responses to a shock to the world price of imports, z_t^*. For comparison, the value of η is now fixed at $1/\sigma = 2$. Under financial autarky, natural output increases in response to a positive z_t^* shock in this case. This again leads to PPI inflation, and an expansion of the sticky price sector. In response, the nominal interest rate increases.

Notably, the fall in consumption is much steeper than under complete markets. This reflects the fact that the relative price of home produce falls (roughly by the same amount as the fall in the terms of trade), which drives down the value of domestic income. Note that the real exchange rate depreciates in this case.

As a consequence of the consumption fall, labor supply increases, which results in a fall of the real wage and an expansion of employment in both sectors. Increased employment and falling consumption lead to a fall in welfare that is much larger than under complete markets.

Figure 5 displays the impulse responses to a shock to Z^*_{xt}. Here we see that autarky makes a big difference relative to the complete markets case. First, as discussed at the end of last section, consumption is pushed up because a positive shock to Z^*_{xt} increases the value of exports, part of which results in increased consumption. At the same time, the shock results in higher employment in the homogeneous export sector. Both effects cause a jump in the real wage, which is reflected in a fall in labor employed in the sticky price sector. This means also that output in that sector fall by more, and as a result the output gap becomes *negative.* So domestic inflation turns negative. The policy reaction is to reduce the interest rate.

Total employment falls, which together with higher consumption means that overall utility now goes up, instead of down.

Finally, Figure 6 displays the responses to a contractionary monetary shock. The responses under autarky are qualitatively the same as with complete markets.

5. Evaluating Alternative Monetary Rules

5.1. Monetary Rules

Equilibrium of the proposed model is pinned down once a monetary policy rule is given. If monetary policy is an interest rate rule, we need to append the Euler equation 5 to the equilibrium system. The model is then closed by specifying different rules relating the domestic interest rate to other variables.

We consider rules of the Taylor type, which have been prominent in the literature. The first one is based on a domestic inflation (PPI) target:

$$\log(1 + i_t) = (\rho + \phi_\pi(\Pi_{ht} - 1) + \phi_y(y_{ht} - y^n_{ht}) + v_t)$$

where Y^n_{ht} is natural output[5]. This rule turns out to be optimal in basic closed economy New Keynesian models (Woodford 2003) and delivers a relatively high level of welfare in recent open economy models as well.

A second rule targets current CPI inflation, as given by:

[5] Natural output is obtained, together with other natural variables, by setting $\theta = 0$ in equations **??** to 27 . This yields a static 4-equation system which can be solved for C^n_t, Q^n_t, X^n_t, and Y^n_{ht} given the exogenous variables Z^*_t, C^*_t, and A_t.

$$\log(1+i_t) = (\rho + \phi_\pi(\Pi_t - 1) + \phi_y(y_{ht} - y_{ht}^n) + v_t)$$

The latter is a widely practiced rule among central banks, including those of most emerging markets, several of them being commodity exporters.

We study two other rules that have featured in the recent literature, both of which are in principle much more accommodative of commodity price shocks than the standard Taylor rules with either PPI or CPI targeting. One of them is the targeting of forecast (rather than actual) CPI inflation:

$$\log(1+i_t) = (\rho + \phi_\pi(E(\Pi_{t+1}) - 1) + \phi_y(y_{ht} - y_{ht}^n) + v_t)$$

The last one is a rule recently proposed by Frankel (2010) which stabilizes the domestic price of exports. This entails moving the nominal exchange rate in tandem with the foreign currency price of the export good (measured relative to stable world prices p^*):

$$(1+\Delta s_t)(1+\Delta z_{xt}^*) = 1$$

5.2. Calibration and Computation

Table 1 reports the baseline calibration. The baseline parameter values are reasonably standard and broadly in line what we have used in earlier work (Catăo and Chang, 2010) and assume a quarterly frequency.

A main addition relative to our earlier calibration pertains to parameters of the export sector. As baseline, we have set the volatility and persistence of export price shocks to be the same as those of import prices 0.05 (also on a quarterly frequency), but then raising it later to as high as 0.1, which is an upper range for the conditional standard deviation of an AR(1) regression of the HP-detrended world price of metal commodities (as per the IMF commodity sub-index for metals over 1990-2011). Regarding labor share and TFP, it is reasonable to assume that if the export sector comprises highly capital intensive industries, such as mining, the labor share in output is small. On the other hand, the value of the output of such industries can be substantial. We choose parameters so that employment in the export sector is about one percent of total employment, and the value the sector's output is about ten percent of total value added. These numbers are in line with UN data for Chile and Peru. The resulting parameters are $A_x = 0.15$ and $\varrho = 0.1$.

Another parameter that is new relative to (Catao and Chang, 2010) is the share of imported inputs in production. Blanchard and Gali (2007) use 0.02 for the imported oil input share in the US but it given that our imported inputs are not limited to oil and that a typically emerging market economy is less energy efficient than its advanced country counterpart, we raise this to 0.1. Finally, we set the parameter ψ governing the degree of market completeness to vary between zero (complete financial autarky) and one (perfect international financial markets).

Given a parametrization of the model, we use the numerical procedures and programs of Schmitt Grohe and Uribe (2004), as implemented via DYNARE, to calculate a second order approximation of the equilibrium, and the resulting utility, associated with each policy under study. We follow Schmitt-Grohe and Uribe (2007), Wang (2006), and Catao and Chang (2010) in computing welfare comparisons that are "conditional " on the same starting point which, as in these previous studies, is the non-stochastic steady state. Besides the obvious theoretical appeal of this metric, Wang (2006) shows that it also simplifies the computational burden considerably. Specifically, he shows that the second order approximation of the instantaneous discounted value of the representative agent's welfare, conditional on such a starting point, has the form $V_t = V + \frac{1}{2} g_{\sigma\sigma}(\bar{x}, 0)\sigma$, where V is steady state conditional utility, x is a vector of the model state variables at the non-stochastic steady state, $g_{\sigma\sigma}$ is the second derivative of the g function with respect to the variance of the shocks (once the system is set so that the vector of control variables takes the form $y_t = g(x_t, \sigma)$), and σ is a scalar governing the variance of the shocks. Intuitively, given each policy, the last term in the above equation captures the adjustment of initial positions due to uncertainty.

Computationally, this calculation amounts to a simple addition of a control variable V_t to the system of equations entering the Schmitt Grohe and Uribe algorithm, which will evolve according to the law of motion $V_t = U(C_t, N_t) + \beta E_t V_{t+1}$. As standard, we measure welfare losses in each case as a percentage of steady state consumption that the representative home agent would be willing to give up to live in the resulting steady state. For CPI for instance is:

$$\lambda_{cpi} = 1 - \frac{(1-\sigma)^{1/(1-\sigma)}}{C_{ss}} \left[U_{cpi}(1-\beta) + \zeta \frac{N_{ss}^{1+\varphi}}{1+\varphi} \right]^{1/(1-\sigma)}$$

5.3. Alternative Monetary Rules

Table 2 presents results on welfare comparisons across the four rules. Start with the complete markets case and strict IT (i.e. zero weight on the output gap). The table shows that for low substitutability between the imported commodity and the domestic varieties ($\eta < 1$), the PPI rule is marginally superior to the CPI rule, delivering a relative gain of between 0.05% to 0.06% of steady state consumption. Table 2 also shows that the gap narrows as η or σ rise, i.e., as the domestic variety becomes a better substitute for the imported commodity good or the inter-temporal substitution elasticity ($1/\sigma$) declines.

The second panel in Table 2 also shows that the CPI rule is welfare superior to the Frankel's EPT rule and by not so trivial margins (relative to the typically tiny sizes of such welfare gaps). In particular, when $\eta = 0.25$ and $\sigma = 2$ the representative agent loses 0.18% of consumption in perpetuity by foregoing CPI IT in favor of the EPT rule. Perhaps more surprising is that targeting expected CPI in the face of commodity price shocks delivers lower welfare relative to targeting current CPI for low values of η. Once again, the gains are not so trivial – some 0.3% of steady state consumption, with the gap between the two rules also declining as η goes larger. In fact, when η approaches values for "normal' varieties (5 in the table), expected CPI wins out for $\sigma > 2$.

The remainder of Table 2 further highlights that the PPI rule is typically superior to others, with more sizeable gaps than that relative to headline CPI IT. The exception is for $\eta = 5$ and $\sigma > 2$, when the expected CPI rule wins out.

Table 3 considers complete financial autarky ($\psi = 0$). Two main results emerge. First, imperfect integration with world capital markets take much of the lustre of CPI targeting – the welfare superiority of PPI rises relative to the complete market baseline case – except when η is unrealistically high at 5. Second, the superiority of CPI IT relative to other rules such as EPT is not a preserve of complete markets. However, the expected CPI rule fares much worse. In fact, the welfare gap between PPI and expected CPI IT rises to staggering numbers of the order of 5 to 6% of steady state consumption when η is low.

Tables 4 and 5 basically repeat the exercise but allowing for some positive weight on the output gap. A typical value of the Taylor coefficient on the output gap on quarterly data is 0.125 (amounting 0.5 on annual data), so we stick to that number. In this case, a comparison between the first panel of Table 4 and the first panel of Table 2 show that a higher weight on the output gap further narrows the gap between PPI and CPI. This is consistent with the intuition that the higher weight on the gap, makes the CPI rule less draconian in its strong reaction to imported commodity inflation – a point often played down in discussions of the pitfalls of CPI targeting (as in, e.g., Frankel, 2010). Also consistent with this argument, the wedge between the superiority of the CPI rule and other alternatives generally increase on the higher weight on the output gap. Overall, transitivity means that PPI generally wins all other rules too, and particularly so the commodity price targeting rule. Furthermore, these gains are higher, the lower the intra-temporal substitution elasticity between imported commodities and domestic varieties.

Table 6 goes back to the baseline parameterization with complete markets and strict IT except that now the Taylor coefficient on inflation is twice as high, at 3.0. Then we can see that, once again, the gap between PPI and headline CPI is narrow and in fact even a tad narrower than under a more moderate Taylor coefficient. Once again, both PPI and CPI are far superior to other rules, except the expected CPI rule when η and σ are high.

Finally, Table 7 moves the export and import price volatility parameters, setting the export price one to 0.1 and lowering the import price one to 0.03. This is roughly consistent with the case of a country that exports highly volatile metal commodities like copper and imports less volatile commodities like food and/or other goods. Table 7 shows that the PPI is again the winner and increases its edge over its main contender in previous tables – headline CPI IT. As before, both PPI and CPI IT continues to beat the EPT rule, with non-trivial welfare gains relative to what is commonly found in the literature.

5.4. Monetary Policy and Risk Sharing

Our model also allows for an examination of the performance of the different rules being considered under alternative assumptions on the economic environment. Given that PPI targeting emerged in the last subsection as the winner of the across rules comparisons, this subsection delves more deeply into the welfare implications of the PPI rule when we change some of the assumptions maintained so far.

The first question that one might want to ask is whether the PPI rule should be more or less aggressive than we have assumed, and how the answer may depend on the incompleteness of international financial markets. Recall that our baseline case assumed that a rule with parameter 1.5 on PPI inflation. But some have argued that a more aggressive response to inflation is welfare improving. Kollman (20xx), for example, has found that a value of 3 for the PPI inflation parameter is closer to optimal.

To examine this issue, Table 8 describes welfare implications of several scenarios. The numbers in the Table are welfare losses, again measure in terms of the equivalent percentage loss in steady state consumption. The top panel of the Table refer to our baseline case, which features full international risk sharing. For the baseline policy rule, the panel stresses that welfare losses fall with η, the elasticity of substitution between domestic and foreign output in home consumption. This is natural, as a higher η means that domestic agents find it easier to adjust to relative prices and, hence, maybe less affected by relative price distortions. On the other hand, increases in σ have ambiguous effects on welfare, depending on η. In particular, for $\eta = 5$ an increase in σ lowers welfare, while it raises welfare for the other η values in the table.

The next panel in Table 8 shows the implications of raising the Taylor rule coefficient on PPI inflation from 1.5 to 3. The panel shows that, notably, a more aggressive response to inflation raises welfare in most cases, often significantly (although admittedly the welfare effects are quite small in absolute terms). The exception is for large values of η and σ together. Overall, our results are in line with those of Kollman (xx).

The third panel of the table shows the implications of flexible targeting, assuming a coefficient of 0.125 on the output gap. In terms of welfare, a positive coefficient on the output gap does qualitatively the same job as a more aggressive response to PPI inflation. Welfare increases, except for the largest (η, σ) pair in the table.

The fourth panel returns to the baseline Taylor rule, but assumes that international risk sharing is incomplete. To do this, the assumption is that the parameter ψ in 4 is zero. Notably, welfare falls considerably relative to the base case (in which $\psi = 1$). The fall in welfare is clearly stronger as σ increases. This, of course, reflects the fact that the reduction in risk sharing is more costly for the representative agent the higher the coefficient of relative risk aversion.

The last panel assumes $\psi = 0$ but an aggressive response to PPI inflation ($\varphi_\pi = 3$ instead of 1.5). This reduces significantly the welfare losses associated with incomplete risk sharing. In fact, for some parameter combinations (i.e. low σ and large η) the losses fall to almost zero.

While a more thorough investigation is warranted, it is fair to say that, overall, the model confirms some intuitive prescriptions. Incomplete risk sharing across countries results in lower welfare, an effect that is exacerbated if risk aversion is high. A relatively aggressive monetary policy response to inflation seems desirable, especially when risk sharing is imperfect.

5.5. Policy and Competitive Exports

Table 9 examines the implications of a more influential competitive exports sector. To look at this issue, the bottom three panels of the table assume that the standard deviation of the relative price of the competitive sector, $\sigma_{z_x^*}$, is ten percent, while it was five percent in the baseline scenario. For convenience, the top panel in the table gives the baseline case.

The second panel of Table 9 assumes full international risk sharing. Comparing it against the base case (the top panel) confirms that a more volatile process for z_x^* lowers welfare. However, the quantitative effect is minimal. This is as expected, in particular given our discussion in section 3. With complete markets, a higher world price of the competitive export does not have direct effects on the representative agent's consumption (which is pinned down by international risk sharing) but has to put more hours into the production of the competitive export.

As also discussed in section 3, the increase in $\sigma_{z_x^*}$ has more impact if markets are incomplete. The third panel of Table 9 presents this case, by assuming also that $\psi = 0$. Clearly, and as expected, welfare is clearly lower than in the base case. A comparison with the fourth panel of Table 8 reveals that the lion's share of the fall in welfare is due to imperfect risk sharing, not to a higher $\sigma_{z_x^*}$. But a higher $\sigma_{z_x^*}$ can by itself change welfare significantly, by 0.1 percent or more, if η is small and σ large.

The final panel in table 8 displays the implications of a more aggressive policy stance. Again, the improvement on welfare is relatively large.

Overall, increasing the volatility of the relative price of the competitive exports sector lowers welfare relative to the base case, as expected. But the effect is quantitatively small for most parameter configurations, regardless of the degree of risk sharing imperfections.

5.6. The Role of Imported Inputs

Finally, and for completeness, the bottom three panels of Table 10 show the implications of reducing the share of imported inputs in domestic production of differentiated varieties from $\varkappa = 0.1$ to $\varkappa = 0$. Again, the baseline case is included as the first panel for ease of comparison.

With full international risk sharing, the result is an increase in welfare. This is expected because shocks to relative imports prices, Z_t^*, are transmitted to marginal costs and welfare via the imported input. The quantitative impact seems, however, small.

Under autarky, welfare is lower than in the baseline, as shown by the first and third panels of the table. But a comparison with the fourth panel of Table 8 shows that the fall in welfare is due to market incompleteness. If financial autarky is a given, then a smaller share of imports in production does raise welfare is all cases.

Finally, comparing the two last panels of Table 10 shows that, when $\varkappa = 0$, a more aggressive rule raises welfare under financial autarky. The impact is quantitatively large.

6. Final Remarks

To be written

7. Appendix

7.1. Solving the Model

It is useful to rewrite the pricing function 16 in a recursive way. Divide both sides by P_{ht} and express it as

$$\frac{\bar{P}_t}{P_{ht}} J_t = \frac{\varepsilon}{\varepsilon - 1} H_t \tag{24}$$

where

$$J_t = E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} Y_{t+k|t}$$

and

$$H_t = E_t \sum_{k=0}^{\infty} \theta^k \Xi_{t,t+k} Y_{t+k|t} F_{t+k} \Pi^h_{t,t+k}$$

where

$$\begin{aligned} F_t &= \Psi_t / P_{ht} \\ &= (P_{mt}/P_t)^{\varkappa} ((1-\upsilon)W_t/P_t)^{1-\varkappa} (P_t/P_{ht})/A_t \end{aligned} \tag{25}$$

is marginal cost in terms of the home aggregate, and

$$\Pi^h_{t,t+k} = P_{ht+k}/P_{ht}$$

is domestic (home goods) inflation between t and $t+k$.

It is not too hard to express J_t and H_t in recursive form:

$$J_t = \left(\frac{\bar{P}_t}{P_{ht}}\right)^{-\varepsilon} Y_{ht} + \beta\theta E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) \left(\frac{\bar{P}_t}{\bar{P}_{t+1}}\right)^{-\varepsilon} J_{t+1} \tag{26}$$

$$H_t = F_t \left(\frac{\bar{P}_t}{P_{ht}}\right)^{-\varepsilon} Y_{ht} + \beta\theta E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) \left(\frac{\bar{P}_t}{\bar{P}_{t+1}}\right)^{-\varepsilon} \Pi^h_{t+1} H_{t+1} \tag{27}$$

(Abusing notation, we have defined $\Pi^h_{t+1} = \Pi^h_{t,t+1} = P_{t+1}/P_t$)

Equations 3, 4, 9, 14, 18, 19, 20, 24, 25, 26, 27, 21 and 22 are thirteen equations in the fifteen variables W_t/P_t, C_t, N_t, X_t, P_{ht}/P_t, $\bar{P}_t/P_{ht}$, Y_{ht}, J_t, H_t, F_t, Π^h_{t+1}, L_{xt}, L_t, M_t, V_t To complete the system, we need to specify a monetary rule (possibly in conjunction with the Euler equation 5) and solve for an aggregate production function.

To derive an aggregate production function, rewrite 13 as:

$$Y_t(j) = \iota A_t L_t(j) (M_t/L_t)^{\varkappa}$$

where we have used $M_t(j)/L_t(j) = M_t/L_t$. Integrating over j :

$$\int_0^1 Y_t(j) dj = \iota A_t L_t^{1-\varkappa} M_t^{\varkappa}$$

But the LHS is not equal to Y_{ht}. Instead,

$$\int_0^1 Y_t(j)dj = Y_{ht}(1+\Delta_t)$$

where

$$1+\Delta_t = \int_0^1 \left(\frac{P_t(j)}{P_{ht}}\right)^{-\varepsilon} dj$$

is a measure of price dispersion.

Close to the nonstochastic steady state, Δ_t is of second order, so for a first order analysis one can use the approximate production function:

$$Y_{ht} = \iota A_t L_t^{1-\varkappa} M_t^{\varkappa} \tag{28}$$

to complete the solution.

In a second order approximation, one needs to find a second order approximation to Δ_t. Following the steps in pages 62-63 of Gali (2008), to second order around a zero inflation steady state,

$$\Delta_t = \frac{\varepsilon}{2} var_j p_t(j)$$

where $p_t(j) = \log P_t(j)$.

On the other hand, the analysis in Woodford, p.695, gives:

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$$var_j p_t(j) = \theta var_j p_{t-1}(j) + \frac{\theta}{1-\theta}\left(\log \Pi_{ht}\right)^2$$

Combining the two gives:

$$\Delta_t = \theta \Delta_{t-1} + \frac{\varepsilon}{2}\frac{\theta}{1-\theta}\left(\log \Pi_{ht}\right)^2$$

In a second order approximation, one must use this equation together with:

$$(1+\Delta_t) Y_{ht} = \iota A_t L_t^{1-\varkappa} M_t^{\varkappa}$$

Summarizing, the equilibrium system is given by:

$$C_t = \left[\kappa X_t^{1/\sigma} C_t^*\right]^{\psi} \left[V_t/P_t\right]^{1-\psi} \tag{29}$$

$$Y_{ht} = (1-\alpha)\left(\frac{P_{ht}}{P_t}\right)^{-\eta} C_t + \phi X_t^{\gamma}\left(\frac{P_{ht}}{P_t}\right)^{-\gamma} C_t^* \tag{30}$$

$$1 = \left[(1-\theta)\left(\frac{\bar{P}_t}{P_{ht}}\right)^{1-\varepsilon} + \theta \Pi_{h,t}^{\varepsilon-1}\right] \tag{31}$$

$$\frac{\bar{P}_t}{P_{ht}} J_t = \frac{\varepsilon}{\varepsilon - 1} H_t \tag{32}$$

$$J_t = \left(\frac{\bar{P}_t}{P_{ht}}\right)^{-\varepsilon} Y_{ht} + \beta\theta E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_{ht+1}}{P_{t+1}}\right) \left(\frac{P_t}{P_{ht}}\right) \Pi_{h,t+1}^{\varepsilon-1} \left(\frac{\frac{\bar{P}_t}{P_{ht}}}{\frac{\bar{P}_{t+1}}{P_{ht+1}}}\right)^{-\varepsilon} J_{t+1} \tag{33}$$

$$H_t = F_t \left(\frac{\bar{P}_t}{P_{ht}}\right)^{-\varepsilon} Y_{ht} + \beta\theta E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_{ht+1}}{P_{t+1}}\right) \left(\frac{P_t}{P_{ht}}\right) \left(\frac{\frac{\bar{P}_t}{P_{ht}}}{\frac{\bar{P}_{t+1}}{P_{ht+1}}}\right)^{-\varepsilon} \Pi_{h,t+1}^{\varepsilon} H_{t+1} \tag{34}$$

$$\frac{1}{1+i_t} = \beta E_t \left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{\frac{P_t}{P_{ht}}}{\frac{P_{t+1}}{P_{ht+1}}}\right) \Pi_{h,t+1}^{-1}\right] \tag{35}$$

$$F_t = Q_t^{\varkappa} \left(\frac{(1-\upsilon)\frac{W_t}{P_t}}{\frac{P_{ht}}{P_t}}\right)^{1-\varkappa} \frac{1}{A_t} \tag{36}$$

$$\frac{W_t}{P_t} = \varsigma C_t^{\sigma} N_t^{\varphi} \tag{37}$$

$$1 = (1-\alpha)\left(\frac{P_{ht}}{P_t}\right)^{1-\eta} + \alpha \left(X_t Z_t^*\right)^{1-\eta} \tag{38}$$

$$(1+\Delta_t) Y_{ht} = \iota A_t L_t \left(\frac{M_t}{L_t}\right)^{\varkappa} \tag{39}$$

$$\frac{1-\varkappa}{\varkappa}\frac{M_t}{L_t} = \frac{(1-\upsilon) W_t}{P_t} \left(X_t Z_t^*\right)^{-1} \tag{40}$$

$$\Delta_t = \theta\Delta_{t-1} + \frac{\theta}{1-\theta}\frac{\varepsilon}{2}\left(\log \Pi_{ht}\right)^2 \tag{41}$$

$$Q_t \frac{P_{ht}}{P_t} = X_t Z_t^* \tag{42}$$

$$\varrho A_{xt} L_{xt}^{\varrho - 1} = \frac{W_t}{X_t P_t}\frac{P_t^*}{P_{xt}^*} \tag{43}$$

$$V_t = P_{ht} Y_{ht} + Z_{xt}^* X_t P_t A_{xt} L_{xt}^{\varrho} - Z_t^* X_t P_t M_t \tag{44}$$

$$N_t = L_t + L_{xt} \tag{45}$$

With 18 variables $\frac{W_t}{P_t}, C_t, N_t, L_t, L_{xt}, V_t, X_t, \frac{P_{ht}}{P_t}, \frac{\bar{P}_t}{P_{ht}}, Y_{ht}, J_t, H_t, F_t, \Pi_{t,t+1}^{h}, \Delta_t, M_t, Q_t, i_t,$ and 17 equations,[6] the system closes with the specification of a monetary policy rule.

NB Using 42 in 36 gives

$$F_t = \left(X_t Z_t^*\right)^{\varkappa} \left((1-\upsilon)\frac{W_t}{P_t}\right)^{1-\varkappa} \frac{1}{A_t(\frac{P_{ht}}{P_t})} \tag{46}$$

so we can drop Q_t and 42 and 36, using 46. This gives 17 eqs with 16 vars.

[6]The remaining variables in t+1 can be obtained by forwarding and lagging the behavioral equations as appropriate.

7.2. Natural System of Equations

With "natural" we mean "flexible price equilibrium". Let a superscript n denote the natural variables. Also, W_t^n and P_{ht}^n will denote the natural levels of W_t/P_t and P_{ht}/P_t (since it is as if we normalized P_t to one). Then the natural equations are:

$$W_t^n = \varsigma \left(C_t^n\right)^{\sigma} \left(L_t^n + L_{xt}^n\right)^{\varphi} \tag{47}$$

$$A_t P_{ht}^n = \frac{\varepsilon}{\varepsilon - 1} (X_t^n Z_t^*)^{\varkappa} ((1-\upsilon) W_t^n)^{1-\varkappa} \tag{48}$$

$$1 = (1-\alpha) \left(P_{ht}^n\right)^{1-\eta} + \alpha \left(X_t^n Z_t^*\right)^{1-\eta} \tag{49}$$

$$C_t^n = \left[\kappa \left(X_t^n\right)^{1/\sigma} C_t^*\right]^{\psi} \left[V_t^n\right]^{1-\psi} \tag{50}$$

$$Y_{ht}^n = (1-\alpha) \left(P_{ht}^n\right)^{-\eta} C_t^n + \phi (X_t^n)^{\gamma} \left(P_{ht}^n\right)^{-\gamma} C_t^* \tag{51}$$

$$Y_{ht}^n = \iota A_t L_t^n \left(\frac{\varkappa}{1-\varkappa} (1-\upsilon) W_t^n \left(X_t^n Z_t^*\right)^{-1}\right)^{\varkappa} \tag{52}$$

$$\varrho A_{xt} \left(L_{xt}^n\right)^{\varrho - 1} = \frac{W_t^n}{X_t^n Z_{xt}^*} \tag{53}$$

$$V_t^n = P_{ht}^n Y_{ht}^n + Z_{xt}^* X_t^n A_{xt} (L_{xt}^n)^{\varrho} - Z_t^* X_t^n M_t^n \tag{54}$$

$$\frac{1-\varkappa}{\varkappa} \frac{M_t^n}{L_t^n} = (1-\upsilon) W_t^n \left(X_t^n Z_t^*\right)^{-1} \tag{55}$$

These are nine equations in the nine variables $W^n, C^n, L^n, L_x^n, V^n, P_h^n, X^n, Y_h^n, M_t^n$.

8. Steady State and Calibration

Rewrite the natural equations in ss as:

$$W = \varsigma C^{\sigma} N^{\varphi} \tag{56}$$

$$N = L_x + L \tag{57}$$

$$AP_h = \frac{\varepsilon}{\varepsilon - 1} X^{\varkappa} ((1-\upsilon)W)^{1-\varkappa} \tag{58}$$

(Assuming $Z^* = 1$)

$$1 = (1-\alpha)P_h^{1-\eta} + \alpha X^{1-\eta} \tag{59}$$

$$C = \left[X^{1/\sigma}\right]^{\psi} V^{1-\psi} \tag{60}$$

(We have assumed $\kappa C^* = 1$.)

$$Y_h = (1-\alpha)P_h^{-\eta} C + \phi X^{\gamma} P_h^{-\gamma} \tag{61}$$

(The previous ϕ is replaced by ϕ/κ. This is just a normalization.)

$$Y_h = \iota AL \left(\frac{\varkappa}{1-\varkappa}(1-\upsilon)W/X\right)^{\varkappa} \tag{62}$$

$$\varrho A_x L_x^{\varrho - 1} = \frac{W}{X} \tag{63}$$

(assuming $Z_x^* = 1$)

$$V = P_h Y_h + X A_x L_x^{\varrho} - XM \tag{64}$$

$$\frac{1-\varkappa}{\varkappa} \frac{M}{L} = (1-\upsilon)\, W/X \tag{65}$$

Table 1: Calibration of the Model's Second-Order Representation and Policy Rules

Discount Factor	β	0.99
Coefficient of risk aversion	σ	[2,6]
Inverse of elasticity of labor supply	φ	[1]
Degree of Openness	α	0.25
Average period between price adjustments	θ	0.66
Coefficient on domestic inflation in Taylor Rule	ϕ_π	[1.5,3.0]
Coefficient on output gap in Taylor Rule	ϕ_y	0-0.5
Parameter of persistence associated with persistent monetary policy shock	ρ_v	0.6
Parameter of persistence associated with persistentce of exrpot and import price shocks	ρ_z	0.75
Elasticity of substitution between varieties produced within any given country	ϵ	6
Elasticity of substitution between domestic and foreign goods	η	[0.25,5]
Index of foreign demand	C^*	1
Share of imported inputs in production		0.1
Share of Labor in Export Sector		0.1
Relative Size (TFP) coefficient on Export Sector		0.15
Price Elasticity of Foreign Demand for the home goods	γ	[0.2,5]
Standard Deviation associated with monetary policy shock	σ_v	0.006
Standard Deviation associated with relative import price shock	σ_z	[0.03 0.05]
Standard Deviation associated with relative export price shock	σ_z	[0.05 0.10]
Standard Deviation associated with productivity shock	σ_a	0.012

Table 2. Welfare Gaps between Policy Rules measured in % of Steady State Consumption

Case 1: Strict IT (zero weight on ygap) with Complete Markets (ψ=1)

PPI-CPI

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.0678	0.0655	0.0609	0.0532
4	0.0576	0.055	0.0499	0.0411
6	0.0539	0.0511	0.0457	0.0366

CPI-Frankel

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.1836	0.1624	0.1299	0.1154
4	0.1119	0.0962	0.0737	0.0979
6	0.0912	0.0773	0.058	0.0947

CPI-Exp(CPI)

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.3578	0.3537	0.3382	0.0599
4	0.3012	0.294	0.2727	-0.0281
6	0.2809	0.2726	0.2492	-0.0595

PPI-Frankel

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.2565	0.2333	0.1964	0.1546
4	0.1742	0.1561	0.1283	0.1218
6	0.1496	0.1331	0.1082	0.1132

PPI-EXP(CPI)

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.4309	0.4249	0.4049	0.0991
4	0.364	0.3544	0.3278	-0.0043
6	0.34	0.329	0.3	-0.0411

Frankel-EXP(CPI)

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.1736	0.1908	0.2077	-0.0553
4	0.1885	0.1971	0.1985	-0.1255
6	0.1887	0.1943	0.1905	-0.1533

Ranking matrix

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	PPI	PPI	PPI	PPI
4	PPI	PPI	PPI	EXP(CPI)
6	PPI	PPI	PPI	EXP(CPI)

Table 3. Welfare Gaps between Policy Rules measured in % of Steady State Consumption

Case 2: Strict IT (zero weight on ygap) with Financial Autarky (ψ=0)

PPI-CPI

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.3147	0.2577	0.1667	0.0264
4	0.6315	0.4581	0.257	0.0332
6	0.858	0.5843	0.3067	0.0365

CPI-EPT

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.1125	0.1695	0.1931	0.0304
4	0.0369	0.19	0.2563	0.0343
6	-0.0028	0.224	0.311	0.0381

CPI-Exp(CPI)

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	2.7094	1.597	0.8206	0.2531
4	4.0166	2.1106	0.9732	0.2541
6	4.9457	2.4168	1.0552	0.2568

PPI-EPT

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.4279	0.4281	0.3605	0.0568
4	0.6694	0.6515	0.516	0.0676
6	0.855	0.8163	0.6235	0.0747

PPI-EXP(CPI)

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	3.0415	1.863	0.9901	0.2796
4	4.7571	2.6089	1.2404	0.2877
6	6.1002	3.0926	1.382	0.2939

EPT-EXP(CPI)

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	2.591	1.4226	0.6251	0.2226
4	3.9735	1.9057	0.7095	0.2195
6	4.9495	2.1619	0.7302	0.2182

Ranking matrix

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	PPI	PPI	PPI	PPI
4	PPI	PPI	PPI	PPI
6	PPI	PPI	PPI	PPI

Table 4. Welfare Gaps between Policy Rules measured in % of Steady State Consumption

Case 3: Flexible IT (zero weight on ygap) with Complete Markets (ψ=1)

PPI-CPI

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.0623	0.0602	0.0557	0.0324
4	0.0546	0.052	0.0467	0.0206
6	0.0516	0.0488	0.0433	0.0163

CPI-EPT

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.203	0.1822	0.1501	0.1242
4	0.126	0.1107	0.0884	0.0989
6	0.1035	0.09	0.071	0.0928

CPI-Exp(CPI)

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.258	0.2509	0.2319	0.0207
4	0.2314	0.2215	0.1978	-0.0277
6	0.2207	0.2098	0.1843	-0.0457

PPI-EPT

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.2656	0.2426	0.2059	0.1567
4	0.1809	0.1629	0.1353	0.1195
6	0.1554	0.1391	0.1144	0.1092

PPI-EXP(CPI)

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.3207	0.3113	0.2878	0.0531
4	0.2865	0.274	0.2449	-0.0071
6	0.2729	0.2592	0.2281	-0.0295

EPT-EXP(CPI)

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.0548	0.0685	0.0815	-0.1033
4	0.1049	0.1103	0.1089	-0.126
6	0.1164	0.1191	0.1129	-0.1378

Ranking matrix

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	PPI	PPI	PPI	PPI
4	PPI	PPI	PPI	EXP(CPI)
6	PPI	PPI	PPI	EXP(CPI)

Table 5. Welfare Gaps between Policy Rules measured in % of Steady State Consumption

Case 4: Flexible IT (zero weight on ygap) with Financial Autarky (ψ=0)

PPI-CPI

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.2731	0.2198	0.1398	0.0216
4	0.5523	0.3987	0.2236	0.029
6	0.7551	0.5152	0.272	0.0327

CPI-EPT

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.1747	0.2269	0.2384	0.0529
4	0.1504	0.2768	0.3109	0.053
6	0.1399	0.3237	0.3669	0.0544

CPI-Exp(CPI)

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	1.8001	1.0819	0.5637	0.1771
4	2.8845	1.5694	0.7441	0.201
6	3.6377	1.8638	0.8426	0.2131

PPI-EPT

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.4488	0.4477	0.3789	0.0745
4	0.706	0.6799	0.5373	0.0821
6	0.9015	0.8491	0.645	0.0873

PPI-EXP(CPI)

sigma/eta	**0.25**	**0.5**	**1**	**2**
2	2.0832	1.3065	0.7052	0.1987
4	3.5039	1.9938	0.9745	0.2302
6	4.5775	2.4402	1.1287	0.2462

EPT-EXP(CPI)

sigma/eta	**0.25**	**0.5**	**1**	**2**
2	1.6197	0.8511	0.3238	0.124
4	2.717	1.2781	0.4278	0.1476
6	3.4659	1.5094	0.4652	0.1581

Ranking matrix

sigma/eta	**0.25**	**0.5**	**1**	**2**
2	PPI	PPI	PPI	PPI
4	PPI	PPI	PPI	PPI
6	PPI	PPI	PPI	PPI

Table 6. Welfare Gaps between Policy Rules measured in % of Steady State Consumption

Case 5: Strict IT with complete Markets and Aggressive Taylor (ϕ_π= 3.0)

PPI-CPI

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.0395	0.0359	0.0314	0.0407
4	0.0291	0.0264	0.0232	0.0324
6	0.0259	0.0234	0.0205	0.0296

CPI-EPT

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.2715	0.25	0.2147	0.1202
4	0.1935	0.1762	0.1481	0.0802
6	0.1698	0.1538	0.128	0.0683

CPI-Exp(CPI)

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.0923	0.0884	0.0769	-0.0404
4	0.0814	0.0763	0.0632	-0.0491
6	0.0769	0.0714	0.0579	-0.0521

PPI-EPT

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.3112	0.2861	0.2462	0.161
4	0.2229	0.2028	0.1714	0.1127
6	0.196	0.1775	0.1487	0.098

PPI-EXP(CPI)

sigma/eta	**0.25**	**0.5**	**1**	**2**
2	0.1318	0.1243	0.1084	0.0003
4	0.1106	0.1027	0.0864	-0.0167
6	0.1029	0.0949	0.0785	-0.0226

EPT-EXP(CPI)

sigma/eta	**0.25**	**0.5**	**1**	**2**
2	-0.1783	-0.1609	-0.1371	-0.1602
4	-0.1113	-0.0993	-0.0844	-0.1289
6	-0.092	-0.0817	-0.0696	-0.1199

Ranking matrix

sigma/eta	**0.25**	**0.5**	**1**	**2**
2	PPI	PPI	PPI	PPI
4	PPI	PPI	PPI	EXP(CPI)
6	PPI	PPI	PPI	EXP(CPI)

Table 7. Welfare Gaps between Policy Rules measured in % of Steady State Consumption

Case 6: Strict IT with Complete Markets and High Export Price Volatility

PPI-CPI

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.3019	0.2921	0.2714	0.1521
4	0.2618	0.2499	0.2254	0.0915
6	0.2466	0.2339	0.208	0.0698

CPI-EPT

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.8429	0.7519	0.6107	0.4788
4	0.5363	0.4673	0.3654	0.3901
6	0.4472	0.3853	0.2961	0.3716

CPI-Exp(CPI)

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.9986	1.0009	0.9753	0.1282
4	0.8686	0.858	0.8088	-0.1367
6	0.8212	0.806	0.748	-0.2325

PPI-EPT

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	1.1499	1.0484	0.8855	0.6324
4	0.8038	0.7218	0.5941	0.483
6	0.7005	0.6246	0.5078	0.443

PPI-EXP(CPI)

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	1.3066	1.2989	1.252	0.2807
4	1.1396	1.1166	1.0416	-0.0457
6	1.0803	1.0514	0.9655	-0.1637

EPT-EXP(CPI)

sigma/eta	**0.25**	**0.5**	**1**	**5**
2	0.1532	0.2453	0.3602	-0.3472
4	0.3252	0.3835	0.437	-0.5187
6	0.3641	0.411	0.4439	-0.591

Ranking matrix

sigma/eta	**0.25**	**0.5**	**1**	**2**
2	PPI	PPI	PPI	PPI
4	PPI	PPI	PPI	EXP(CPI)
6	PPI	PPI	PPI	EXP(CPI)

Table 8: PPI Targeting, International Risk Sharing, and Welfare

Baseline Rule (φ_π = 1.5), Full International Risk Sharing

SIG\ETA	0.25	0.5	1	5
2	0.1694	0.1472	0.1031	-0.0605
4	0.1486	0.1321	0.0970	-0.0376
6	0.1401	0.1252	0.0925	-0.0337

Aggressive Response to Inflation (φ_π = 3.0), Complete Markets

SIG\ETA	0.25	0.5	1	5
2	0.0821	0.0629	0.0315	-0.0653
4	0.0678	0.0583	0.0399	-0.0199
6	0.0626	0.0559	0.0415	-0.0074

Flexible Inflation Targeting (φ_π = 1.5, φ_y = 0.125), Full International Risk Sharing

SIG\ETA	0.25	0.5	1	5
2	0.1500	0.1271	0.0837	-0.0693
4	0.1347	0.1175	0.0834	-0.0386
6	0.1280	0.1124	0.0810	-0.0320

Baseline Rule (φ_π = 1.5), Incomplete International Risk Sharing (ψ = 0)

SIG\ETA	0.25	0.5	1	5
2	0.6182	0.2916	0.1658	0.0903
4	1.1997	0.4704	0.2468	0.1260
6	1.5626	0.5663	0.2876	0.1428

Aggressive Response to Inflation (φ_π = 3.0), Incomplete International Risk Sharing (ψ = 0)

SIG\ETA	0.25	0.5	1	5
2	0.2632	0.0883	0.0237	-0.0142
4	0.4828	0.1534	0.0539	0.0021
6	0.6227	0.1902	0.0699	0.0101

Table 9: PPI Targeting, International Risk Sharing, and Competitive Exports Prices

Baseline Rule (φ_π = 1.5), Full International Risk Sharing

SIG\ETA	0.25	0.5	1	5
2	0.1694	0.1472	0.1031	-0.0605
4	0.1486	0.1321	0.0970	-0.0376
6	0.1401	0.1252	0.0925	-0.0337

Full Risk Sharing, More Volatile Prices for Competitive Exports (σ_{z*} = 0.1)

SIG\ETA	0.25	0.5	1	5
2	0.1775	0.1547	0.1100	-0.0549
4	0.1562	0.1391	0.1034	-0.0324
6	0.1475	0.1320	0.0988	-0.0287

Baseline Rule (φ_π = 1.5), Incomplete Risk Sharing, More Volatile Prices for Competitive Exports

SIG\ETA	0.25	0.5	1	5
2	0.6978	0.2908	0.1304	0.0309
4	1.4554	0.5306	0.2402	0.0784
6	1.9192	0.6580	0.2951	0.1008

Aggressive Rule (φ_π = 3), Incomplete Risk Sharing, More Volatile Prices for Competitive Exports

SIG\ETA	0.25	0.5	1	5
2	0.2367	0.0307	-0.0457	-0.0907
4	0.5250	0.1193	-0.0042	-0.0693
6	0.7081	0.1694	0.0177	-0.0589

Table 10: PPI Targeting, International Risk Sharing, and Imported Inputs

Baseline Rule (φ_π = 1.5), Full International Risk Sharing

SIG\ETA	0.25	0.5	1	5
2	0.1694	0.1472	0.1031	-0.0605
4	0.1486	0.1321	0.0970	-0.0376
6	0.1401	0.1252	0.0925	-0.0337

Baseline Rule (φ_π = 1.5), Full International Risk Sharing, Smaller Imported Inputs Share ($\varkappa$ = 0.0)

SIG\ETA	0.25	0.5	1	5
2	0.1605	0.1345	0.0826	-0.0891
4	0.1434	0.1235	0.0821	-0.0560
6	0.1367	0.1186	0.0802	-0.0489

Baseline Rule (φ_π = 1.5), No International Risk Sharing, Smaller Imported Inputs Share ($\varkappa$ = 0.0)

SIG\ETA	0.25	0.5	1	5
2	0.5048	0.1978	0.1086	0.0600
4	0.6356	0.2698	0.1487	0.0826
6	0.6662	0.3041	0.1682	0.0932

Aggresive Rule (φ_π = 3), No International Risk Sharing, Smaller Imported Inputs Share ($\varkappa$ = 0.0)

SIG\ETA	0.25	0.5	1	5
2	0.3116	0.0843	0.0164	-0.0212
4	0.3793	0.1127	0.0315	-0.0103
6	0.3991	0.1274	0.0394	-0.0051

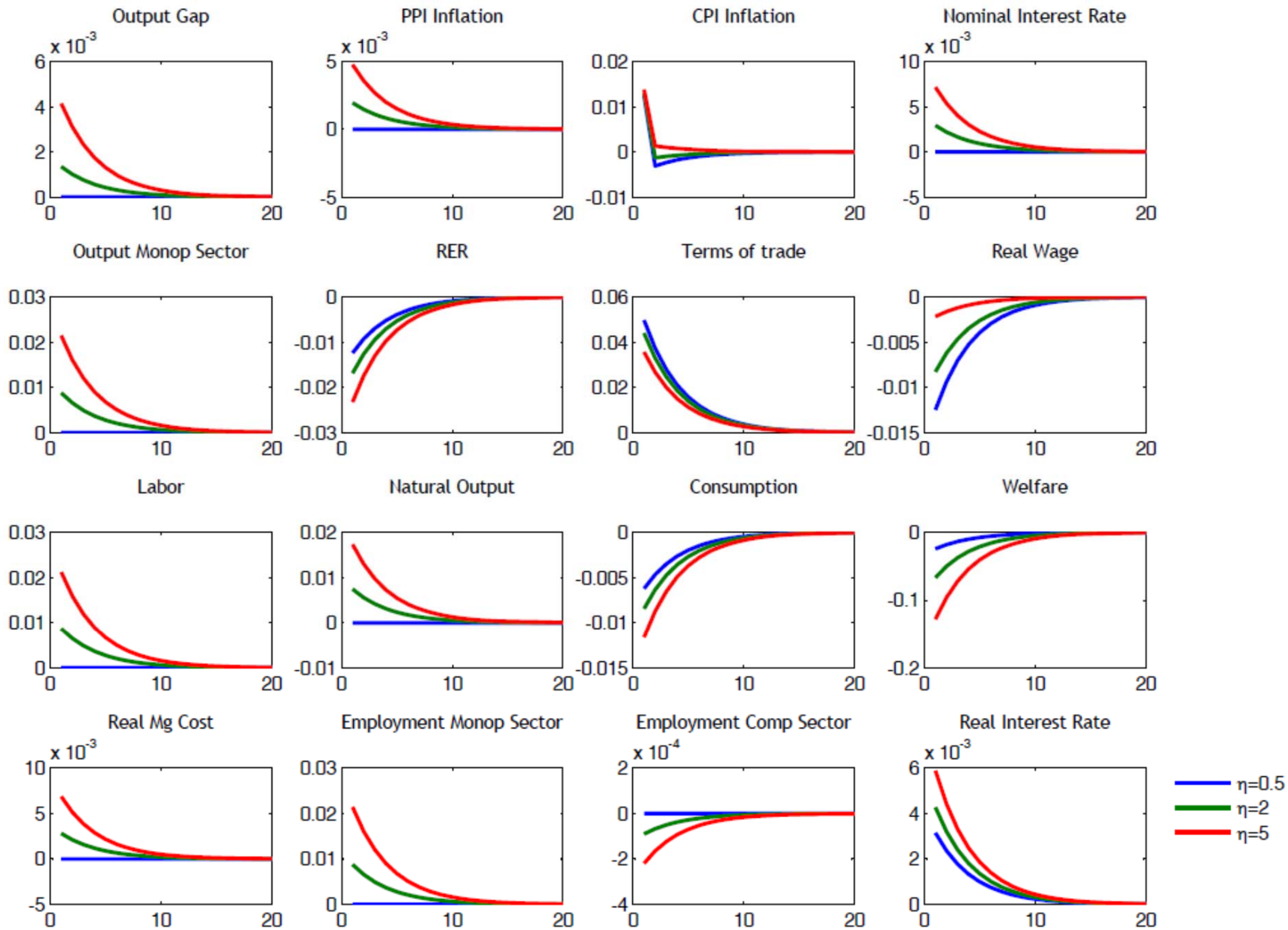

Figure 1: Shock to Import Prices, Perfect Capital Markets

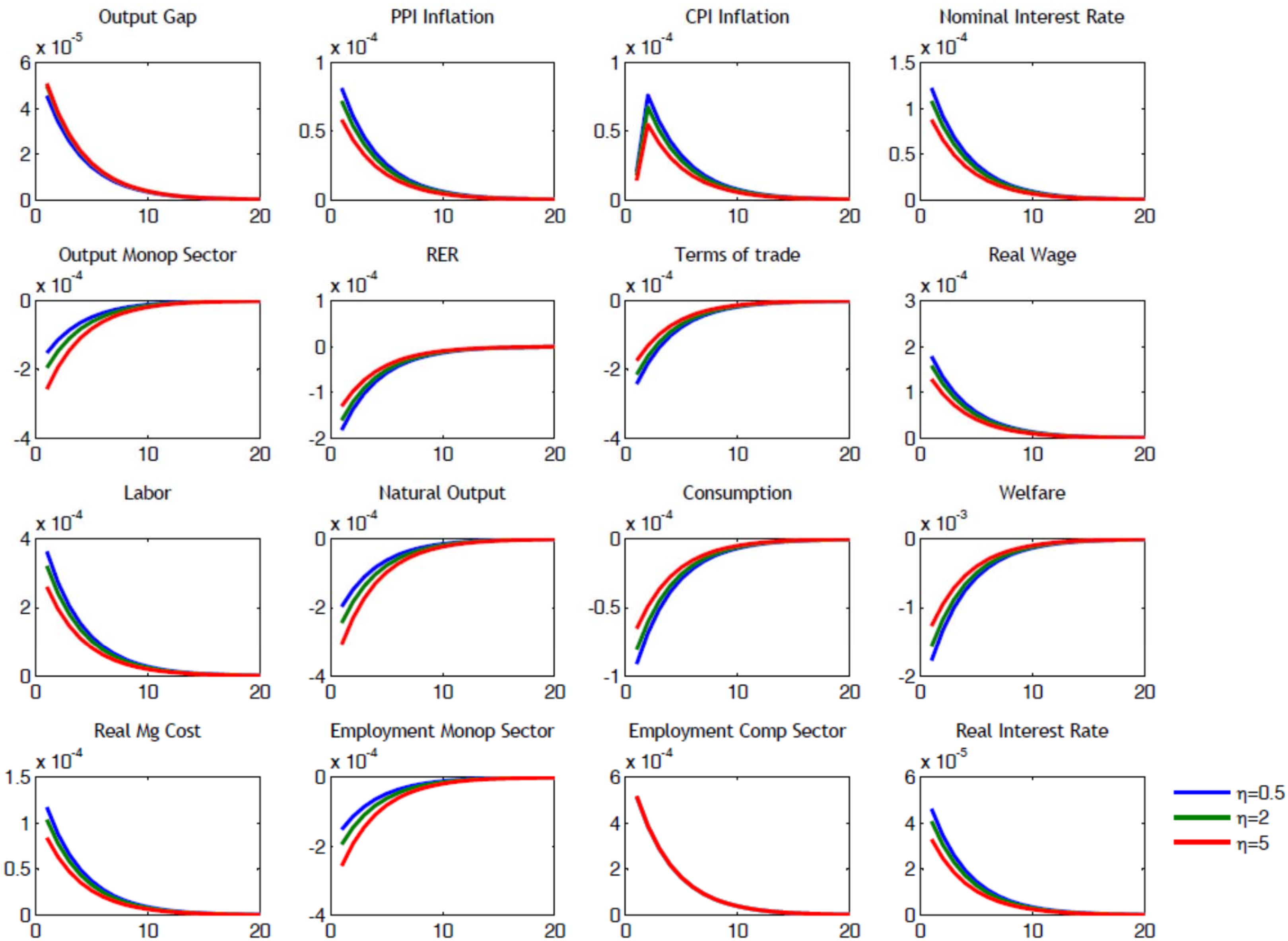

Figure 2: Shock to Export Prices, Perfect Capital Markets

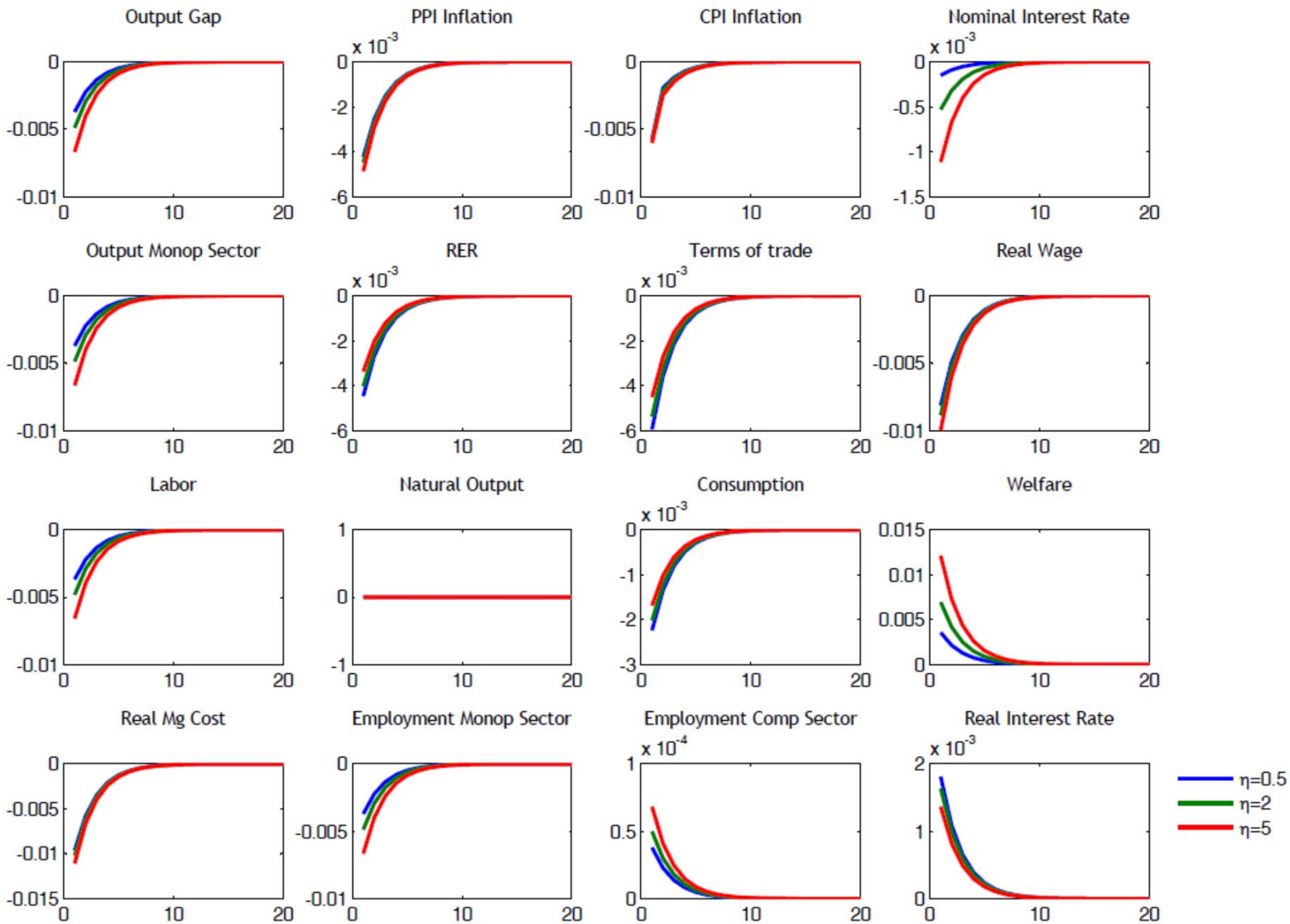

Figure 3: Contractionary Monetary Shock, Perfect Capital Markets

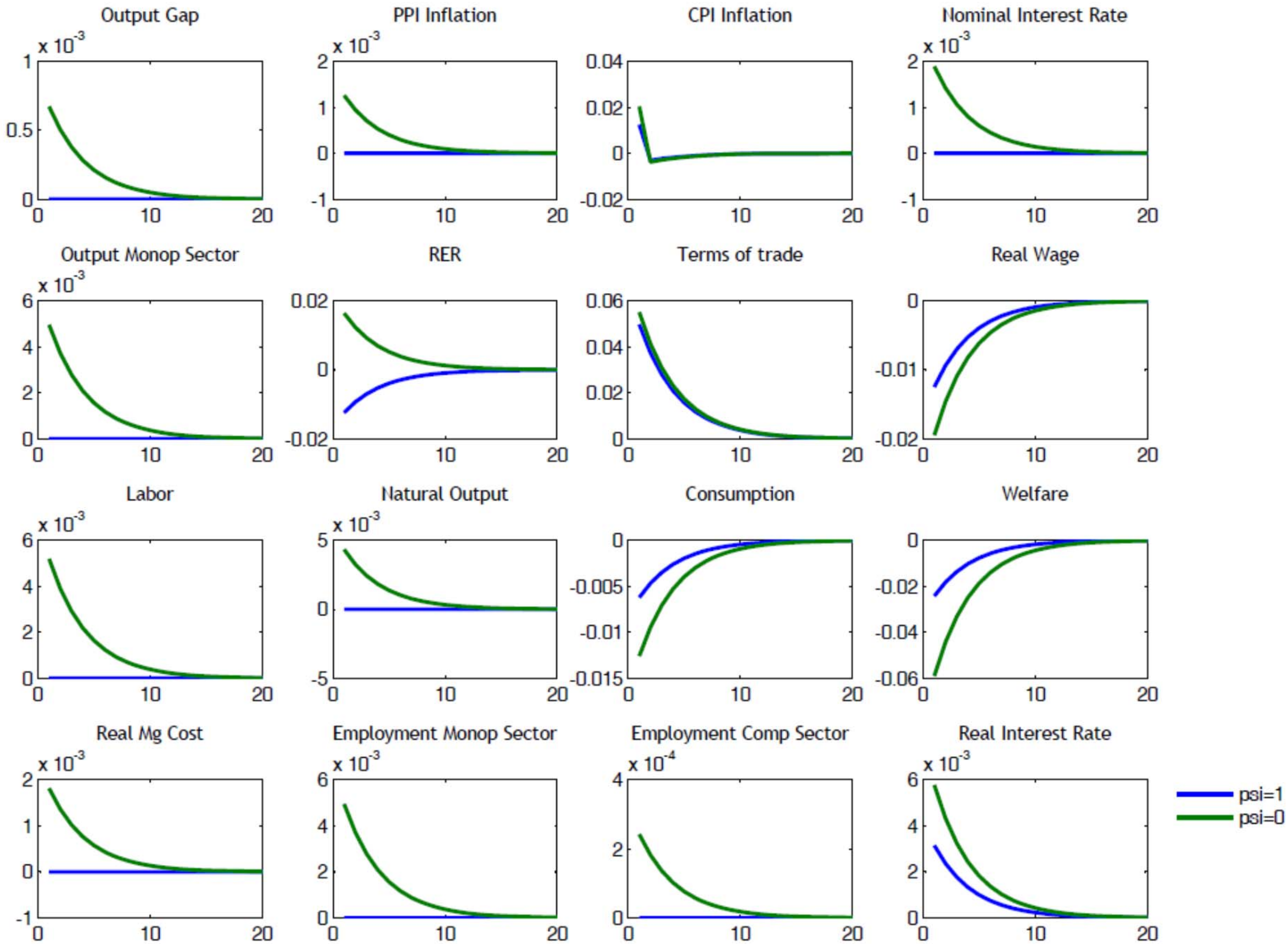

Figure 4: Shock to Import Prices, Perfect Capital Markets vs Autarky

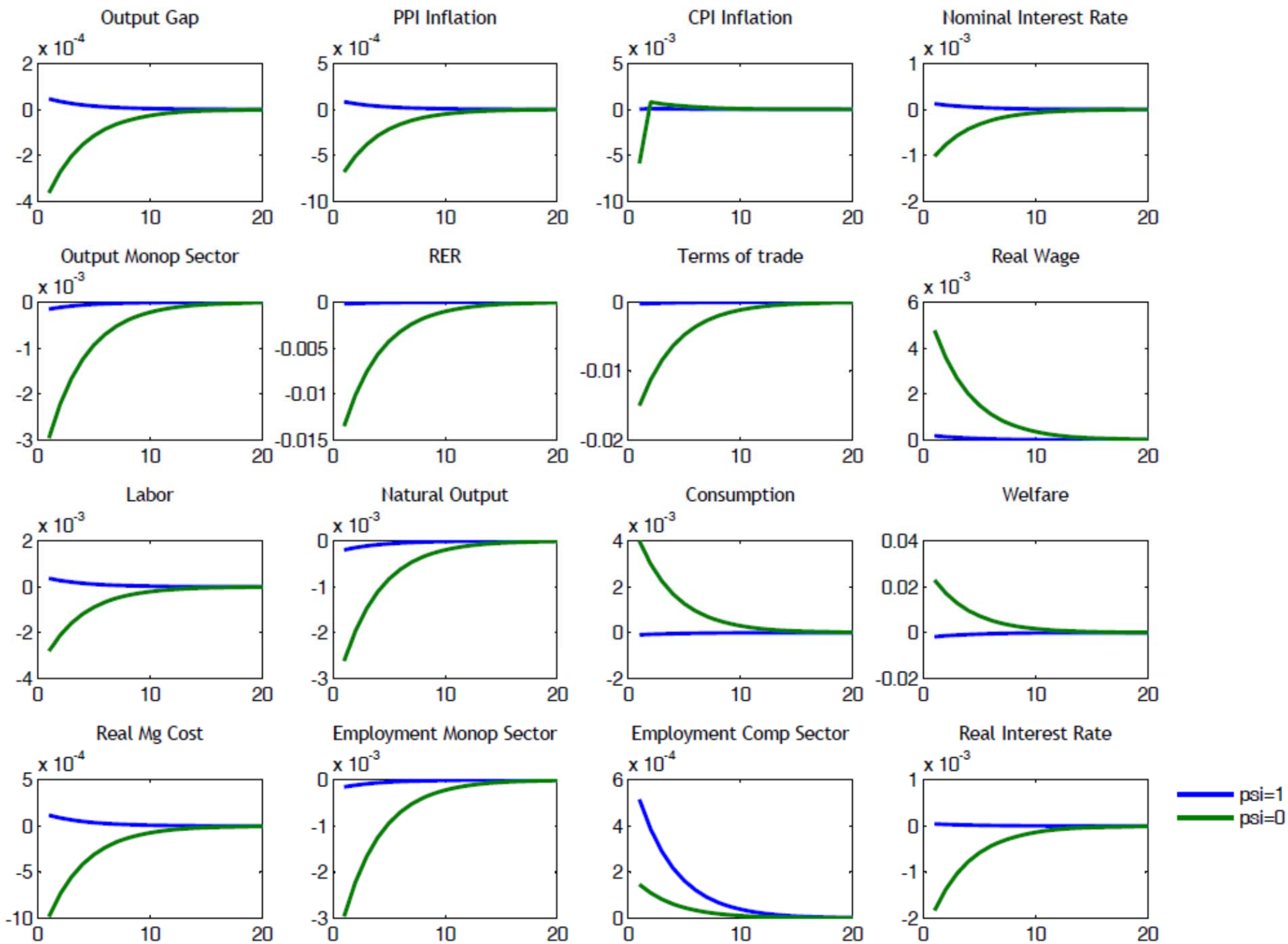

Figure 5: Shock to Export Prices, Perfect Capital Markets Versus Autarky

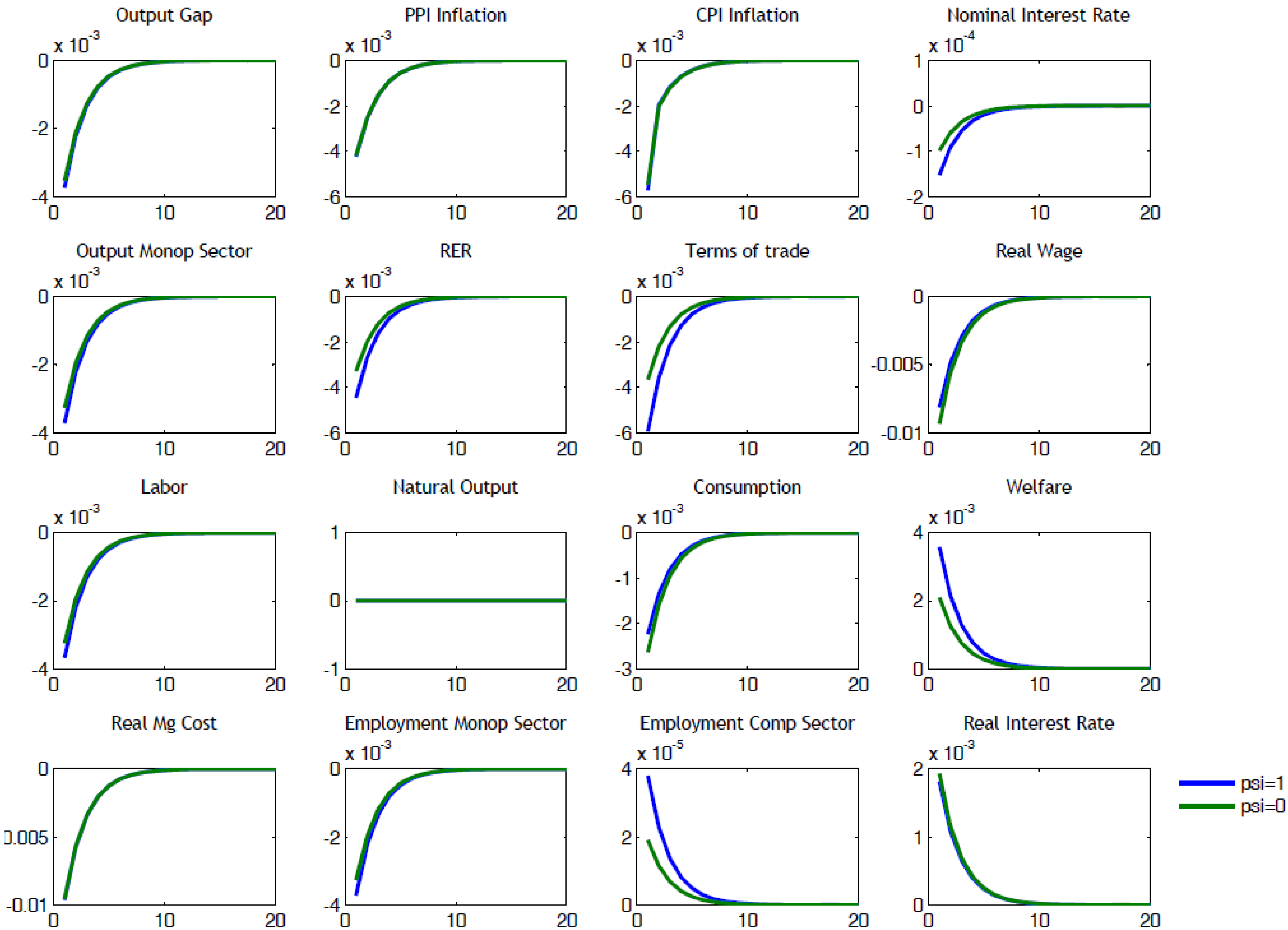

Figure 6: Contractionary Monetary Shock, Perfect Capital Markets Versus Autarky