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ABSTRACT

I construct a model in which infinitely-lived entrepreneurs cannot borrow more than the value of their landholdings. I show how this constraint leads naturally to an equilibrium in which the land's price has a bubble. I demonstrate that bursting bubbles in land prices may have dramatic and persistent distributional and aggregate effects. I discuss appropriate and inappropriate policy interventions in the wake of a bubble collapse.

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Housing and housing price derivatives are important sources of collateral for loans in the United States. From July 2006 to October 2008, the twenty-city Case-Shiller house price index fell by just under 25%. This price decline is often interpreted as representing the bursting of a *bubble*. The decline is blamed for significant changes in credit markets that began in the second half of 2007, and to a recession that is now dated as having begun in December 2007. There has been a massive and varied government response to these events.

Motivated by these observations, in this paper I construct a model in which collateralized borrowing plays an essential role in re-allocating capital to its efficient uses. I show that collateral scarcity can generate a stochastic bubble in the price of collateral. I discuss the implications of this bubble's bursting for aggregates and for welfare. Using the model, I assess several ongoing policy initiatives and propose a specific superior intervention.

The structure of my model closely resembles models described in Angeletos (2007), Kartashova (2008), and Kiyotaki-Moore (2008). A fraction of entrepreneurs have productive investment opportunities, while others do not. The arrival of the desirable projects is i.i.d. over entrepreneurs and over time. Efficient production requires the re-allocation of physical capital from entrepreneurs without good projects to those with good projects. This re-allocation is accomplished via loans. Markets are incomplete in the sense that these loans cannot be made contingent on whether a given entrepreneur gets a good project.

The novel feature of my model relative to theirs is that all entrepreneurs are each endowed with one unit of land.¹ If the borrower defaults, a lender can seize a borrower's land

¹Heathcote and Davis (2007) document that the 2000-06 run-up in home prices was largely driven by a contemporaneous increase in land prices. Hence, I re-interpret the bubble in *home* prices as reflecting one in *land* prices.

In an earlier version of this paper, I assumed that entrepreneurs could augment the stock of collateral (through home construction, for example). As long as there is a resource cost of adjusting the collateral

but no other borrower resources. Hence, a borrower's repayment is bounded from above by the value of his land. I assume that land is an asset that pays no dividend. (I discuss the role of this assumption later in the introduction.)

Given this set of assumptions, it is not surprising that there is an initial value of capital consistent with an equilibrium in which capital is constant over time and land has zero value. In this equilibrium, no borrowing and lending takes place. However, there is also a specification of initial capital that induces an equilibrium in which capital is constant over time and the land price is a positive constant. I interpret this positive price as being a bubble in land prices.

The intuition behind the existence of the bubble is simple, robust, but often ignored. In the model, all entrepreneurs face occasionally binding borrowing constraints. Land, even though it is intrinsically worthless, may have value because it serves to relax this constraint (see Kocherlakota (1992)). In this way, the bubble allows entrepreneurs to re-allocate physical capital more efficiently, which leads to higher wages, output, and consumption for the economy. Remarkably, this re-allocation is so useful that, as long as capital share is sufficiently low, the economy generates these higher aggregates in the bubbly steady-state equilibrium using *less* physical capital.

I use these two equilibria to construct another one with a stochastic bubble. (Such equilibria can be quite complicated; I deliberately focus on one that is simple.) In this third kind of equilibrium, there is some small probability of the bubble's bursting at each date. Before the bubble bursts, the land price is positive and constant. After the bubble bursts, the land price reverts to zero forever. Entrepreneurs can exchange any kind of financial contract

stock, this augmentation possibility may be consistent with a price bubble.

that is explicitly contingent on the price of land. However, this kind of financial market completeness is not all that helpful, given the aggregate nature of the shock.

Immediately after the bubble bursts, the entrepreneurs with good projects have little capital available for investment. Macroeconomic aggregates fall dramatically. Entrepreneurs have to self-finance their projects, and begin to accumulate physical capital for this purpose. In any given date, much of the accumulated capital in society is used inefficiently because the owners do not have useful projects. The economy transits to a new lower level of economic activity. Entrepreneurs and workers alike mourn the collapse of the bubble. Nonetheless, from an ex-ante perspective, a positive stochastic bubble expands the social pie.

I discuss a range of possible interventions in the wake of the bubble collapse. The bubble's collapse creates two related but distinct problems. First, entrepreneurs have lost wealth. Second, entrepreneurs without good projects are accumulating wealth via a low-return savings vehicle. Successful interventions must cure both problems. I argue that several of the current policy moves (including bailing out financial intermediaries) are poorly designed to meet these objectives.

My preferred post-policy intervention is based on an insight of Caballero and Krishnamurthy (2006). As is well-known from overlapping generations economies, governments can replace bubbles by rolling over public debt. Caballero and Krishnamurthy emphasize that, unlike the bubble itself, the debt rollover is not subject to stochastic breakdown. The key is that the government can pledge to fill any shortfalls in the debt rollover with labor income taxes. This (off-equilibrium) commitment is sufficient to ensure that the debt is always repaid, even though no taxes are ever actually collected in equilibrium.

With this argument in mind, I construct a two-part intervention. First, the government

compensates owners of land for their losses by giving them government debt. This debt can be seized by creditors, and injects a new source of collateral into the economy. Second, the government commits to paying a high real interest rate on its debt. I show that this intervention completely eliminates the adverse ex-post impact of the bubble's collapse on aggregate outcomes, without creating undesirable ex-ante incentive effects.

Earlier, I stated that I assumed that land pays no dividend. I make this assumption to deal with an issue emphasized by Santos and Woodford (1997). They point out that there is a fundamental difficulty with generating bubbles in economies with immortal agents. To rule out finite-period arbitrages, bubbles must grow at the rate of interest. At the same time, bubbles cannot grow faster than the rate of growth of the economy. These two considerations together imply that the rate of interest cannot be larger than the economy's growth rate. It follows that no infinitely-lived asset, including any bubbly one, can pay dividends that grow at the same rate as the economy, or its price would be infinite.

Assuming that land produces no dividends, as I do, is *sufficient* to ensure that land prices can exhibit a bubble and still remain finite. However, such a strong assumption is not *necessary*: Bubbles in land prices may exist as long as the growth rate of land income is smaller than the growth rate of output. This assumption is still strong but seems plausible. In reality, as economies grow, they undergo a structural transformation in which the share of the service sector grows relative to the shares of the manufacturing and agricultural sectors. This shift in production induces a systematic decline in the share of income from land. I use the stronger zero-dividend assumption, because it allows me to focus on equilibria without explicit time dependence.

This paper is closely related to that of Kraay and Ventura (2007). They set up a two-

period overlapping generations model in which external firm finance involves a social loss. They use that model to argue that the 1990's dot.com stock price bubble may have led to an improvement in societal welfare by crowding out inefficient investments. They interpret the expansion of government debt in the 2000's as a way to re-create the desirable effects of the bubble in the wake of its collapse. Thus, the policy implications of the papers are somewhat similar.

I make several modelling contributions relative to their work. My model is an infinite horizon setup in which borrowing is limited by land. The structure of the model is designed to mimic the workhorse growth model used in macroeconomics.² It is desirable to be able to work with bubbles in such frameworks, because they are more readily mapped into macroeconomic data. This framework allows me to show that there is a direct connection between an asset's role as collateral and its price having a bubble.

1. Constant Bubbles

In this section, I set up a version of the model without any aggregate shocks. The basic framework closely resembles that in Kiyotaki-Moore (2008). The main difference is that land can be used as collateral. The analysis demonstrates that steady-state equilibria with bubbles are always better in terms of output, consumption, wages, and welfare than steady-state equilibria without bubbles. Nonetheless, physical capital may well be *lower* in the bubbly steady-state equilibrium.

²There are a number of papers in which borrowing constraints create bubbles in economies with infinitely-lived agents. (See, among many others, Hellwig and Lorenzoni (forthcoming) and Scheinkman and Weiss (1986).) In these papers, bubbles allow agents to achieve better intertemporal allocations. However, as far as I know, the papers do not show a tight connection between the existence of bubbles and productive efficiency.

A. Model Economy

I consider an infinite-horizon economy with a unit measure of entrepreneurs and a unit measure of workers. Workers play little role in this analysis, except to soak up the returns to labor. More specially, each worker supplies one unit of labor inelastically at each date. The workers simply consume their labor income at every date; they do not borrow or lend.

Entrepreneurs maximize the expectation of:

$$(1) \quad \sum_{t=1}^{\infty} \beta^{t-1} \ln(c_t), 0 < \beta < 1$$

where c_t is consumption at date t . Each entrepreneur has a technology that converts k_{t+1} units of capital installed at date t and n_{t+1} units of labor hired at date $t + 1$ into y_{t+1} units of output, according to the production function:

$$(2) \quad y_{t+1} = A_{t+1} k_{t+1}^{\alpha} n_{t+1}^{1-\alpha}$$

Here, total factor productivity A_{t+1} is a random variable that is i.i.d. over both entrepreneurs and over time. It equals 1 with probability π and 0 with probability $(1 - \pi)$. I denote the history of productivity shocks in period t by A^t . A given entrepreneur learns the value of A_{t+1} at date t (at the time that capital for next period is installed). Capital depreciates at rate δ per period, regardless of the value of A .

Each entrepreneur is endowed with a unit of land. He can buy and sell land, which is infinitely divisible. The entrepreneur can borrow or save using one-period risk-free bonds. In borrowing, his land is the only form of collateral. Hence, the entrepreneur's repayment in period $(t + 1)$ is bounded from above by the value of his land in period $(t + 1)$.

Suppose that the entrepreneur faces a land price sequence $(p_t)_{t=1}^{\infty}$, an interest rate sequence $(r_t)_{t=1}^{\infty}$, and a wage sequence $(w_t)_{t=1}^{\infty}$. Then entrepreneur's budget set consists of

(c, L, b, k, n) that satisfy:

$$\begin{aligned} & c_t(A^{t+1}) + p_t L_{t+1}(A^{t+1}) + b_{t+1}(A^{t+1}) + k_{t+1}(A^{t+1}) + w_t n_t(A^{t+1}) \\ \leq & b_t(A^t)(1 + r_t) + A_t k_t(A^t)^\alpha n_t(A^{t+1})^{1-\alpha} + (1 - \delta)k_t(A^t) + p_t L_t(A^t) \text{ for all } t, A^{t+1} \end{aligned}$$

$$b_{t+1}(A^{t+1})(1 + r_{t+1}) \geq -p_{t+1}L_{t+1}(A^{t+1}) \text{ for all } t, A^t$$

$$c_t(A^{t+1}), k_{t+1}(A^{t+1}), L_{t+1}(A^{t+1}) \geq 0 \text{ for all } t + 1, A^{t+1}$$

$$L_1 = 1, k_1(A_1) \text{ and } b_1(A_1) \text{ given}$$

Here, I allow for the possibility that there is a non-trivial joint distribution between the realization of first-period productivity and an entrepreneur's initial level of capital and bonds.

A specification of prices (p, w, r) and entrepreneurial quantities (c, L, b, k, n) form an equilibrium if (c, L, b, k, n) maximizes the entrepreneur's utility among all allocations in his budget set, and markets clear:

$$(3) \quad \sum_{A^{t+1}} \Pr(A^{t+1}) n_t(A^{t+1}) = 1$$

$$\begin{aligned} (4) \quad & \sum_{A^{t+1}} \Pr(A^{t+1}) c_t(A^{t+1}) + \sum_{A^{t+1}} \Pr(A^{t+1}) k_{t+1}(A^{t+1}) \\ = & (1 - \delta) \sum_{A^t} \Pr(A^t) k_t(A^t) + \sum_{A^{t+1}} \Pr(A^{t+1}) A_t k_t(A^t)^\alpha n_t(A^{t+1})^{1-\alpha} \end{aligned}$$

$$(5) \quad \sum_{A^{t+1}} \Pr(A^{t+1}) L_{t+1}(A^{t+1}) = 1$$

$$(6) \quad \sum_{A^{t+1}} \Pr(A^{t+1}) b_{t+1}(A^{t+1}) = 0$$

Here, $\Pr(A^{t+1})$ is the probability of a given sequence A^{t+1} occurring. I assume that a law of large numbers applies in the population, so that the fraction of entrepreneurs in period t

with history A^{t+1} is the same as the unconditional probability of that history.

It is true that few entrepreneurs literally collateralize their loans using land. However, consider the following chain of transactions, given that the interest rate is 0. First, homeowner X borrows \$800000 from bank Y , using a \$1 million home as collateral. Bank Y identifies an investment opportunity with return $r > 0$. Bank Y uses its loan to X as collateral to borrow \$800000 from bank Z , and then invests that \$800000 in the high-return project. In this story, X and Y are jointly operating as an entrepreneur in the model does when $A_{t+1} = 1$. Bank Z is operating like an entrepreneur with $A_{t+1} = 0$. Thus, in the real world, there are many layers of "paper" collateralization that are ultimately grounded in a physical asset. The model abstracts from these multiple layers.

B. Two Steady-State Equilibria

In this section, I construct two equilibria in which all prices and *aggregate* quantities are constant over time. In both equilibria, the distribution of wealth across entrepreneurs is evolving, even though aggregates are not. In the first equilibrium, land prices equal zero, and in the second, land prices are positive.

The construction of these equilibria follows that in KM. The basic idea is that there are two kinds of entrepreneurs at any date t . The first kind knows that his realization of A_{t+1} is 1. His production technology has a high return, and so he wants to borrow as much capital as possible to invest in it. Following KM, I'll label these entrepreneurs "investors". The second kind of entrepreneurs knows that his production technology has a low return, because $A_{t+1} = 0$. I'll label these entrepreneurs "savers".

A critical feature of the model is that the investors can freely adjust labor demand.

This means that any investor who installed capital k_t at date $t - 1$ and faces wage w_t in period t will choose n_t so that:

$$(7) \quad n_t = k_t[w_t/(1 - \alpha)]^{-1/\alpha}$$

Thus, all investors have the same capital-labor ratio. It follows that this capital-labor ratio is the same as that set by the average investor, so that the equilibrium wage:

$$(8) \quad w_t = (1 - \alpha)\bar{k}_t^\alpha(1/\pi)^{-\alpha}$$

where \bar{k}_t is the per-capita³ level of installed capital. We can conclude that an investor who installs k_t in period $(t - 1)$ gets a payoff equal to:

$$(9) \quad (1 - \delta + \alpha(\bar{k}_t\pi)^{\alpha-1})k_t$$

Both investors and savers face standard consumption-savings problems. The gross rate of return of investors is governed by the marginal product of capital for a representative investor. The gross rate of return for savers is $1 - \delta$.

C. No Bubbles

Suppose first that $p_t = 0$ for all t ; in such an equilibrium, neither investors nor savers can borrow. Consider an entrepreneur with wealth W_t defined to be:

$$W_t = A_t k_t^\alpha n_t^{1-\alpha} + (1 - \delta)k_t - w_t n_t + b_t(1 + r_t)$$

(this is the right-hand side of his flow budget constraint in period t). Entrepreneurs consume $c_t = (1 - \beta)W_t$ (according to the usual myopic rule associated with log utility), set $k_{t+1} = \beta W_t$

³Throughout, I use the term "per-capita" to refer to "per-entrepreneur".

and set $L_{t+1} = 1$. Investors earn a high gross rate of return on k_{t+1} equal to $(1 - \delta + MPK_{NB})$, where MPK_{NB} is the constant marginal product of capital for investors in this (non-bubble) equilibrium. Savers earn a low gross rate of return on k_{t+1} equal to $(1 - \delta)$.

I am looking to construct an equilibrium in which per-capita capital is constant. In such an equilibrium, per-capita wealth is constant at some level \bar{W}_{NB} . Hence:

$$(10) \quad \bar{W}_{NB} = \pi\beta(MPK_{NB} + 1 - \delta)\bar{W}_{NB} + (1 - \pi)\beta(1 - \delta)\bar{W}_{NB}$$

This restriction pins down MPK_{NB} to be:

$$(11) \quad MPK_{NB} = \frac{[1 - \beta + (1 - \pi)\beta\delta]}{\pi\beta} + \delta$$

We can then solve for per-capita wealth using the restriction::

$$(12) \quad MPK_{NB} = \alpha(\beta\bar{W}_{NB})^{\alpha-1}(1/\pi)^{1-\alpha}$$

where we exploit the equilibrium condition that investors hire $1/\pi$ units of labor each.

It is now straightforward to solve for the rest of the equilibrium. Workers earn a constant wage $w_{NB} = (1 - \alpha)(\beta\bar{W}_{NB})^\alpha(1/\pi)^{-\alpha}$. In this equilibrium, per-capita capital is constant at $k_{NB} = \beta\bar{W}_{NB}$. Entrepreneurs' wealths evolve over time in response to their idiosyncratic shocks, according to the rule:

$$(13) \quad W_{t+1} = \beta(1 - \delta + MPK_{NB})W_t \text{ if } A_{t+1} = 1$$

$$(14) \quad = \beta(1 - \delta)W_t \text{ if } A_{t+1} = 0$$

Then, at each date, they set k_{t+1} and c_t as above. Note that even though aggregates are constant, the cross-sectional variance of logged entrepreneurial consumption is growing.

The equilibrium interest rate r^* lies in $(-\infty, -\delta]$ to ensure that no entrepreneur ever wants to buy bonds, instead of holding depreciating capital. Throughout this paper, equilibrium real interest rates turn out to be non-positive. This property of equilibrium is an implication of my assumption that there is no growth in the economy. If I instead generalize the analysis to include trend growth in total factor productivity, then equilibrium interest rates can be positive as long as they are bounded above by the equilibrium growth rate of output in the economy.

D. Constant Bubble

I now construct an equilibrium in which land prices are constant at a positive level p^* . In this equilibrium, land and bonds are completely equivalent assets. This means that $r_t = 0$ for all t . Without loss of generality, I assume that all entrepreneurs keep their holdings of land at $L_t = 1$. In this fashion, the role of land is serve as collateral.

Now at each date, we define entrepreneurial wealth W_t to be:

$$(15) \quad W_t = A_t k_t^\alpha n_t^{1-\alpha} + (1 - \delta)k_t + p^* L_t + b_t(1 + r_t) - w_t n_t$$

With this change of adding $p^* L_t$ to wealth, entrepreneurial decision rules are basically the same as above. At date t , investors have an investment opportunity with a gross rate of return equal to $(1 - \delta + MPK_{BUB})$, where MPK_{BUB} is the steady-state marginal product of capital. In response, they each borrow as much as possible (set b_t equal to p^*), set $k_{t+1} = \beta W_t$, and set consumption equal to $(1 - \beta)W_t$. It is suboptimal for savers to choose a positive level of k_{t+1} , because this plan offers a lower gross rate of return than holding bonds or land. Instead, savers set bond-holdings b_{t+1} equal to $\beta W_t - p^*$ (given that they set $L_{t+1} = 1$) and set consumption equal to $(1 - \beta)W_t$. Note that some savers might actually be borrowers

(against the values of their land).

In a steady-state, per-capita wealth must be constant at some level \bar{W}_{BUB} . This level satisfies:

$$(16) \quad \bar{W}_{BUB} = \pi\beta(MPK_{BUB} + 1 - \delta)\bar{W}_{BUB} + (1 - \pi)\beta\bar{W}_{BUB}$$

which implies that:

$$(17) \quad MPK_{BUB} = \frac{[1 - \beta]}{\beta\pi} + \delta$$

We can then solve for per-capita wealth as before to satisfy:

$$(18) \quad MPK_{BUB} = \alpha(\beta\bar{W}_{BUB})^{\alpha-1}(1/\pi)^{1-\alpha}$$

Per capita wealth for savers and investors is the same. Hence, bond market-clearing implies that:

$$(19) \quad (1 - \pi)(\beta\bar{W}_{BUB} - p^*) - \pi p^* = 0$$

so that $p^* = (1 - \pi)\beta\bar{W}_{BUB}$.

We can now readily solve for the rest of the equilibrium. Workers earn a constant wage $w_{BUB} = (1 - \alpha)(\beta\bar{W}_{BUB})^\alpha(1/\pi)^{-\alpha}$. Entrepreneurs' wealths evolve over time in response to their idiosyncratic shocks, according to the rule:

$$(20) \quad \begin{aligned} W_{t+1} &= \beta(1 - \delta + MPK_{BUB})W_t \text{ if } A_{t+1} = 1 \\ &= \beta W_t \text{ if } A_{t+1} = 0 \end{aligned}$$

They then set c_t, k_{t+1}, b_{t+1} and L_{t+1} as described earlier.

The existence of the positive bubble hinges on the nature of the borrowing constraint faced by entrepreneurs. With a positive bubble, the equilibrium interest rate r^* cannot exceed the economy's growth rate (here, 0). This restriction implies that the present value of entrepreneurial future income is infinite at each date. The entrepreneur's decision problem only has a solution if entrepreneurs cannot access that full future present value because of binding borrowing constraints.

Woodford (1986) offers one way to understand this connection between bubbles and borrowing constraints. Models with borrowing constraints resemble overlapping generations (OG) economies. In the current model, investors want to dump all of their financial assets. In this sense, they resemble the old agents in an overlapping generations framework. (The difference is that investors want to use their financial assets to fund investment, not consumption.) In contrast, savers are happy to hold financial assets. In this sense, they resemble the young agents in an OG setting. In the no-bubble steady-state, the equilibrium interest rate 0 is less than the growth rate of the economy. As in an OG economy, this dynamic inefficiency suggests that a bubble equilibrium may exist.

E. Discussion

The behavior of aggregates are determined by the steady-state levels of wealth. It is easy to see that:

$$(21) \quad MPK_{NB} > MPK_{BUB}$$

which implies in turn that:

$$(22) \quad \bar{W}_{BUB} > \bar{W}_{NB}$$

There is more wealth with bubbles, which means that per-capita consumption, output, and wages are all higher in the bubble steady-state.

It is true that investors receive a lower return in the bubble steady-state, because the marginal product of capital is lower. However, it is simple to exploit the concavity of the log function to show that:

$$\begin{aligned}
(23) \quad & \pi \ln(1 - \delta + MPK_{BUB}) \\
& > \pi \ln(1 - \delta + MPK_{NB}) + (1 - \pi) \ln(1 - \delta)
\end{aligned}$$

It follows that an entrepreneur, with a given level of wealth, would be happier in the bubble steady-state.

Of course, these are two steady-state equilibria, with different initial levels of capital. It is useful to understand to what extent these differences can be attributed to these different levels of capital. Toward that end, note that in the non-bubble steady state, all entrepreneurs hold capital and so per-capita capital is equal to $\bar{k}_{NB} = \beta \bar{W}_{NB}$. We can write \bar{k}_{NB} in terms of primitives as:

$$\begin{aligned}
(24) \quad \bar{k}_{NB} &= \frac{[MPK_{NB}/\alpha]^{1/(\alpha-1)}}{\pi} \\
&= \frac{\{[1 - (1 - \pi)\beta(1 - \delta)]\pi^{-1}\beta^{-1} - 1 + \delta\}^{1/(\alpha-1)}}{\alpha^{1/(\alpha-1)}\pi}
\end{aligned}$$

In contrast, in the bubble steady-state, only investors hold capital and so per-capita capital is equal to $\bar{k}_{BUB} = \beta\pi\bar{W}_{BUB}$. We can write \bar{k}_{BUB} in terms of primitives as:

$$(25) \quad \bar{k}_{BUB} = \frac{\{[1 - (1 - \pi)\beta]\pi^{-1}\beta^{-1} - 1 + \delta\}^{1/(\alpha-1)}}{\alpha^{1/(\alpha-1)}}$$

Remarkably, these formulae imply that if α is sufficiently low, then steady-state capital is actually *higher* in the non-bubble steady-state.⁴ This result underscores the productive efficiency role of the land bubble in this economy. Without a bubble, entrepreneurs are forced to self-finance. In many periods, they are unable to exploit their capital effectively because they don't have a good project. They end up accumulating a lot of (wasted) capital just to take advantage of those dates in which their project is actually operational. With a land price bubble, investors can borrow, and so resources flow readily from savers to investors. There is no need for entrepreneurs to accumulate as much capital.

2. Stochastic Bubble

I now consider the behavior of the economy in an equilibrium in which the price of land has a stochastic bubble of the following form. At each date, the coin is flipped which has a probability ε of coming up heads. If the outcome of the coin flip is tails, the land price equals a positive number p^+ ; if the outcome is heads, then the land price equals zero at date and thereafter. I interpret the "heads" outcome as the bursting of a bubble.

⁴It is easy to see that $\bar{k}_{BUB} \leq \bar{k}_{NB}$ if and only if:

$$\begin{aligned} \pi^{\alpha-1} &\geq \frac{\{[1 - (1 - \pi)\beta(1 - \delta)]\pi^{-1}\beta^{-1} - 1 + \delta\}}{\{[1 - (1 - \pi)\beta]\pi^{-1}\beta^{-1} - 1 + \delta\}} \\ &= [1 + (\delta\beta - \delta\pi\beta)/(1 - \beta + \pi\beta\delta)] \end{aligned}$$

The right-hand side is increasing in β and equals $1/\pi$ when $\beta = 1$. Hence, the inequality is satisfied as long as α is sufficiently near 0.

A. Model Economy With A Sunspot

I change the above economy in two ways. First, there is a Markov chain z with support $\{0, 1\}$ such that:

$$(26) \quad z_1 = 1$$

$$\Pr(z_t = 1 | z_{t-1} = 1) = (1 - \varepsilon), \varepsilon > 0$$

$$\Pr(z_t = 1 | z_{t-1} = 0) = 0$$

Thus, z_t equals one until it switches to being equal to zero forever. The probability of a switch is ε .

Second, in period $(t - 1)$, entrepreneurs can buy and sell *two* kinds of one-period assets that are available in zero net supply. The first asset is a risk-free asset that pays off one unit of consumption regardless of the realization of z . The second asset is an Arrow security that pays off one unit of consumption if $z_t = 1$ and pays off zero units of consumption if $z_t = 0$. In this market structure, financial markets are complete with respect to the shocks involved in the evolution of z . Note that the Arrow security is valueless if $z_{t-1} = 0$.

It is straightforward to extend the definition of an equilibrium in light of these changes in the economy. Instead of a single risk-free rate, we need to price z_{t+1} -contingent consumption. Let $q_t(z^{t+1})$ be the price of z_{t+1} -contingent consumption in history z^t . Then, the budget set of the entrepreneur in this equilibrium, given (p, w, q) , is defined by two key constraints. The first is the flow constraint:

$$c_t(A^{t+1}, z^t) + p_t(z^t)L_{t+1}(A^{t+1}, z^t) + \sum_{z_{t+1} \in \{0,1\}} q_t(z^{t+1})b_{t+1}(A^{t+1}, z^{t+1})$$

$$\begin{aligned}
& +k_{t+1}(A^{t+1}, z^t) + w_t(z^t)n_t(A^{t+1}, z^t) + p_t(z^t)L_t(A^t, z^{t-1}) \\
\leq & b_t(A^t, z^t) + A_t k_t(A^t, z^{t-1})^\alpha n_t(A^{t+1}, z^t)^{1-\alpha} + (1 - \delta)k_t(A^t, z^{t-1}) \text{ for all } t, A^{t+1}, z^t \\
& L_1 = 1 \\
& k_1(A_1), b_1(A_1) \text{ given}
\end{aligned}$$

The second is the borrowing constraint:

$$\begin{aligned}
(27) \quad & b_{t+1}(A^{t+1}, z^{t+1}) \geq -p_{t+1}(z^{t+1})L_{t+1}(A^{t+1}, z^t) \text{ for all } t, A^{t+1}, z^{t+1} \\
& c_t(A^{t+1}, z^t), k_{t+1}(A^{t+1}, z^t), L_{t+1}(A^{t+1}, z^t) \geq 0 \text{ for all } t, A^{t+1}, z^t
\end{aligned}$$

This borrowing constraint requires that the entrepreneur cannot be required to repay more than the value of his land in any date and state. An equilibrium is a specification of (p, w, q, c, L, k, b, n) such that (c, L, k, b, n) solves the entrepreneur's problem given (p, w, q) and markets clear for all (t, z^t) .

B. A Simple Stochastic Equilibrium

In this subsection, I construct an equilibrium in which $p_t(z^t) = 0$ if $z_t = 0$ and in which if $z_t = 1$, the economy has a positive constant land price p^+ . The interpretation of this equilibrium is that there is a bubble in collateral prices when $z_t = 1$ and this bubble bursts when $z_t = 0$. As before, on any loan from one entrepreneur to another, repayment is bounded from above by the market value of collateral. Since this market value is stochastic, the loan itself is intrinsically risky. I refer to these loans as being *collateralized debt*. These loans pay off zero when $z_t = 0$ and pay off a positive amount when $z_t = 1$. Hence, they are isomorphic to the Arrow security in the market structure described above. As we shall see,

while entrepreneurs can trade risk-free assets, their borrowing constraints imply that no such trade takes place in equilibrium.

Before the Bubble Bursts

I begin by analyzing what happens during the initial periods in which $z_t = 1$. As before, let W_t represent the right-hand side of the entrepreneur's budget constraint in date t . Investors (who have $A_{t+1} > 0$) act much like they do in a deterministic bubbly steady-state. They set $c_t = (1 - \beta)W_t$, $k_t = \beta W_t$ and set (b, L) so that in history (A^{t+1}, z^{t+1}) :

$$(28) \quad b_{t+1}(A^{t+1}, z^{t+1}) + p_{t+1}(z^{t+1})L_{t+1}(A^{t+1}, z^t) = 0$$

Because land has zero value when $z_{t+1} = 0$, the borrowing constraint implies that $b_{t+1}(A^{t+1}, z^t, 0) = 0$. In words, investors can only borrow by issuing *collateralized* debt (not risk-free debt).

The savers also set $c_t = (1 - \beta)W_t$, but their portfolio problem is more complicated than in the deterministic bubbly steady-state. Saving through land or lending delivers no payoff if $z_{t+1} = 0$. Hence, savers now want to hedge themselves either by buying some claims that pay off when $z_{t+1} = 0$ or by holding physical capital. However, the investors cannot provide any claims that pay off when $z_{t+1} = 0$ because their land is worthless in that state. In equilibrium, savers can only hedge themselves by holding some physical capital.

More specifically, savers invest $(1 - \gamma)\beta W_t$ by buying land and buying Arrow securities that pay off when $z_t = 1$. They also invest $\gamma\beta W_t$ in physical capital. Here, γ satisfies the first-order condition:

$$(29) \quad \frac{(1 - \varepsilon)\delta}{(1 - \gamma) + \gamma(1 - \delta)} - \frac{\varepsilon}{\gamma} = 0$$

It follows that:

$$(30) \quad \gamma = \frac{\varepsilon}{\delta}$$

I impose the restriction that ε is sufficiently low:

$$(31) \quad \varepsilon < \delta$$

to ensure that γ is less than one.

With these decision rules in hand, we can solve for per-capita quantities much as we did in the deterministic bubbly steady-state. The constant MPK_{stoch} for investors must satisfy:

$$(32) \quad 1 = \beta\pi(1 - \delta + MPK_{stoch}) + \beta(1 - \pi)[(1 - \varepsilon\delta^{-1}) + \varepsilon\delta^{-1}(1 - \delta)]$$

We can then solve for period 1 per-capita wealth \overline{W}_{stoch} :

$$(33) \quad MPK_{stoch} = \alpha(\beta\pi\overline{W}_{stoch})^{\alpha-1}$$

(Note that $\overline{W}_{stoch} < \overline{W}_{bub}$, because for $\varepsilon > 0$, entrepreneurs hold some wealth in the form of storage.) Given \overline{W}_{stoch} , we can solve for the initial levels of the other aggregate quantities as in the deterministic bubbly steady-state case:

$$(34) \quad \overline{c}_{stoch} = (1 - \beta)\overline{W}_{stoch}$$

$$(35) \quad \overline{y}_{stoch} = (\beta\pi\overline{W}_{stoch})^{\alpha-1}$$

$$(36) \quad \overline{w}_{stoch} = (1 - \alpha)\overline{y}_{stoch}$$

$$(37) \quad \overline{k}_{stoch} = \beta\pi\overline{W}_{stoch}$$

$$(38) \quad p^+ = \beta(1 - \pi)(1 - \varepsilon\delta^{-1})\overline{W}_{stoch}$$

As in the deterministic bubbly steady-state case, I assume that all entrepreneurs hold the per-capita levels of land. Intertemporal trade is conducted through the trade of collateralized debt.

These calculations imply that if ε is near zero, equilibrium per-capita quantities when $z_t = 1$ are well-approximated by the deterministic bubbly steady-state. In contrast, the deterministic steady-state is a poor guide to the behavior of asset prices. When $z_t = 1$, the risk-free return is $-\delta$. Intuitively, the savers hold a positive amount of physical capital and are not borrowing-constrained. For them, physical capital is a risk-free asset that has gross return equal to $1 - \delta$, and so arbitrage pins down the gross risk-free rate at 0.

However, in this stochastic equilibrium, collateralized debt is not risk-free. Land is worthless when $z_t = 0$, and so is any debt that is backed by land. Thus, collateralized debt's expected gross return is given by $(1 - \varepsilon)$. It follows that there is a spread in expected returns between collateralized debt and risk-free debt equal to:

$$(39) \quad (1 - \varepsilon) - (1 - \delta) = \delta - \varepsilon$$

Note that as the probability of the bubble's bursting becomes larger, the expected return on collateralized debt falls and the risk-free rate stays unchanged. The risk premium on collateralized debt is *decreasing* as a function of ε .

Moreover, the risk premium is not continuous with respect to ε . If $\varepsilon = 0$, collateralized debt is risk-free and its risk premium is zero. For ε positive but near zero, the gap in expected returns is close to δ .

After the Bubble Bursts

Now suppose that z^t is a history in which $z_t = 0$ and $z_{t-1} = 1$. Along this sample path, the bubble bursts and land becomes worthless at date t . Entrepreneurs who were investors in period $(t - 1)$ are unaffected by this realization of z_t ; their obligations are wiped out but so is the land backing those loans. The savers from period $(t - 1)$ are greatly affected. They had invested $(1 - \varepsilon\delta^{-1})\beta\overline{W}_{stoch}$ in collateralized debt and land. All of this wealth is wiped out.

Despite this massive redistribution, there is no immediate impact on aggregate output. The investors at date t take their accumulated capital, hire workers, and produce output. Wages in period t are unaffected by the bubble's bursting, because they are fully pinned down by the fixed quantities of capital and labor. However, there *is* an immediate effect on aggregate consumption. Recall that consumption is equal to a fraction $(1 - \beta)$ of wealth. A fraction p^*/\overline{W}_{stoch} of per-capita wealth vanishes because land value has vanished. Hence, consumption also falls by this same fraction.⁵

From period $(t + 1)$ onwards, the situation with respect to production changes dramatically. The economy's aggregate dynamics are determined by the evolution of per-capita wealth \overline{W}_{t+s} , which is governed by the (nonlinear) difference equation:

$$(40) \quad \overline{W}_{t+s} = \beta\pi(1 + MPK_{t+s} - \delta)\overline{W}_{t+s-1} + \beta(1 - \pi)(1 - \delta)\overline{W}_{t+s-1}, s \geq 1$$

$$(41) \quad MPK_{t+s} = \alpha(\beta\pi\overline{W}_{t+s-1})^{\alpha-1}$$

⁵This fall in consumption may seem puzzling. Installed capital and output don't change. Entrepreneurs are now investing less into collateral. So what happens to all of the previously consumed output? The answer is that period $(t + 1)$ savers hold a lot more physical capital than in period t . This increase in physical capital holdings is enough to rationalize the fall in consumption.

$$(42) \quad \bar{W}_t = \bar{W}_{stoch}[1 - \beta(1 - \pi)(1 - \varepsilon\delta^{-1})]$$

Once we know the per-capita level of wealth at each date, it is straightforward to compute the other per-capita variables in each period. Their time paths are given by:

$$(43) \quad \bar{c}_{t+s} = (1 - \beta)\bar{W}_{t+s}$$

$$(44) \quad \bar{w}_{t+s} = (1 - \alpha)(\beta\pi\bar{W}_{t+s})^{-\alpha}$$

$$(45) \quad \bar{y}_{t+s} = (\beta\pi\bar{W}_{t+s})^\alpha$$

All of these variables fall sharply from period t to period $(t + 1)$ as a result of the bubble collapse, and then transit to a new, lower, steady-state level.⁶

After the bubble collapses, the interest rate on risk-free debt is still (bounded from above by) 0. However, the interest rate on collateralized debt changes. An issuer of collateralized debt cannot raise any funds today, regardless of what the issuer promises to repay next period (because the underlying collateral is now worthless). Hence, the interest rate on collateralized debt is now (essentially) infinite (its price is zero). The spread between collateralized debt and risk-free debt spikes greatly in the wake of the bubble's collapse.

If $z_{t-1} = 1$, investors who issue collateralized debt at that date make positive payments if $z_t = 1$ and make zero payments if $z_t = 0$. It is natural to interpret the latter state as being one in which these investors *default* on their obligations. After these wide-scale defaults, no further borrowing takes place. All projects are fully self-financed. There is a very real sense in which financial markets shut down after a bubble collapse.

⁶Wages are fully flexible in this model. If wages could only adjust slowly, then unemployment would jump up when the bubble bursts and slowly decrease over time.

Intermediation is direct in this model. Suppose instead that savers lent and investors borrowed from a common, zero-profit, intermediary. At date t in a bubbly steady-state, the investors each owe p^* to the intermediaries in the form of debt backed by land. After the bubble collapses, the investors will give this (worthless) land to the intermediaries. The intermediaries are now insolvent: they owe p^* to savers and have no resources with which to make this repayment. One critical feature of this model is that restoring financial health to the intermediaries is not enough to restore intermediation. The bubble's collapse induces disintermediation because entrepreneurs lose collateral, not because the intermediaries are in trouble.

Welfare Effects

The bubble's collapse has large adverse effects on aggregate variables. However, this economy features a great deal of heterogeneity in its population. In this subsection, I discuss how these different agents are affected by the bubble's collapse. The impact on workers is easy to see. After the bubble bursts in period t , the level of capital installed in productive technologies is permanently lower. Wages are permanently lower and so workers are made worse off by the bubble's collapse.

The impact of the bubble's collapse on entrepreneurs is more difficult to sort through. There are three distinct effects. First, from period t onwards, entrepreneurs who become savers have a low rate of return ($-\delta$ instead of 0). Second, period $(t - 1)$ savers are virtually wiped out in terms of wealth, but period $(t - 1)$ investors lose no wealth. The third effect is a consequence of the fall in equilibrium wages: investors in period t and thereafter earn higher rates of return on their installed capital.

The overall impact on entrepreneurs in period t depends on how these effects cumulate. It is straightforward to show that if δ is near 1, then both period $(t - 1)$ investors and period $(t - 1)$ savers are made worse off by the bubble's collapse. They have a positive probability of earning a low return on their wealth, and can be made arbitrarily worse off by making that return arbitrarily small.

The story is different if δ is sufficiently small, so that the returns of future savers are not that greatly affected. In that case, period $(t - 1)$ investors are certainly made better off by the bubble's collapse, because they lose no wealth and they earn higher returns whenever they are investors in the future. Period $(t - 1)$ savers lose wealth, but gain from the higher returns that accrue to future investors. If ε/δ is sufficiently small, though, the period $(t - 1)$ savers are certainly worse off because they lose so much wealth.

To summarize, workers always lose from the bubble's collapse, because their wages fall. If $\varepsilon\delta^{-1}$ is sufficiently small, then period $(t - 1)$ savers definitely lose from the bubble's collapse. If δ is sufficiently large, period $(t - 1)$ investors also lose. However, if δ is sufficiently small, period $(t - 1)$ investors actually gain from the bubble's collapse.

This ex-post analysis is also highly revealing about the ex-ante welfare effects of the stochastic bubble. Having a bubble, even a stochastic one, increases the amount of installed capital, and increases wages in all periods before the bubble bursts. Entrepreneurs' wealths are higher with the bubble (until it bursts), and they earn higher returns as savers. They do earn lower returns as investors. On net, though, both entrepreneurs and workers are better off with the bubble as long as δ is sufficiently large.

C. Numerical Example

In this subsection, I numerically simulate the results of a bubble collapse in this model, assuming that ε is small (10^{-7}). I set $\alpha = 1/3$, $\delta = 0.1$, and $\beta = 0.95$. These settings are standard in annual macroeconomic models. Finally, I set $\pi = 0.6$, so that 40% of the entrepreneurs have useless projects in any given year. (I pick this last parameter somewhat arbitrarily, so as to generate a fall in entrepreneurial wealth of about 40%.) I plot the effects of the shock on logged output; the effects on wages, consumption, and per-capita capital are quite similar.

The computed path is depicted in Figure 1. In this figure, logged output falls by 16% in the first year after the bubble bursts. It rises slightly thereafter, but the shock is highly persistent. In the long run, logged output remains 15% below its original steady-state.

The initial fall in output is attributable to the share of wealth that disappears when the bubble collapses. Given these parameter settings, around 40% of per-capita wealth in the initial, bubbly, steady-state is in the form of land. This fall in wealth is translated one-for-one into a fall in the capital that gets used by investors. Because capital share is $1/3$, this 40% fall in capital translates into a 16% fall in logged output. The law of motion (40) of $\ln \bar{W}$ is basically a unit root, and so this shock is highly persistent.

The long-run impact of the shock is shaped by a different force. Before the bubble bursts, savers have a rate of return of 0. After the bubble bursts, savers have an interest rate equal to $-\delta = -0.1$. This change in the savers' rates of return is responsible for the long run fall in wealth and output.

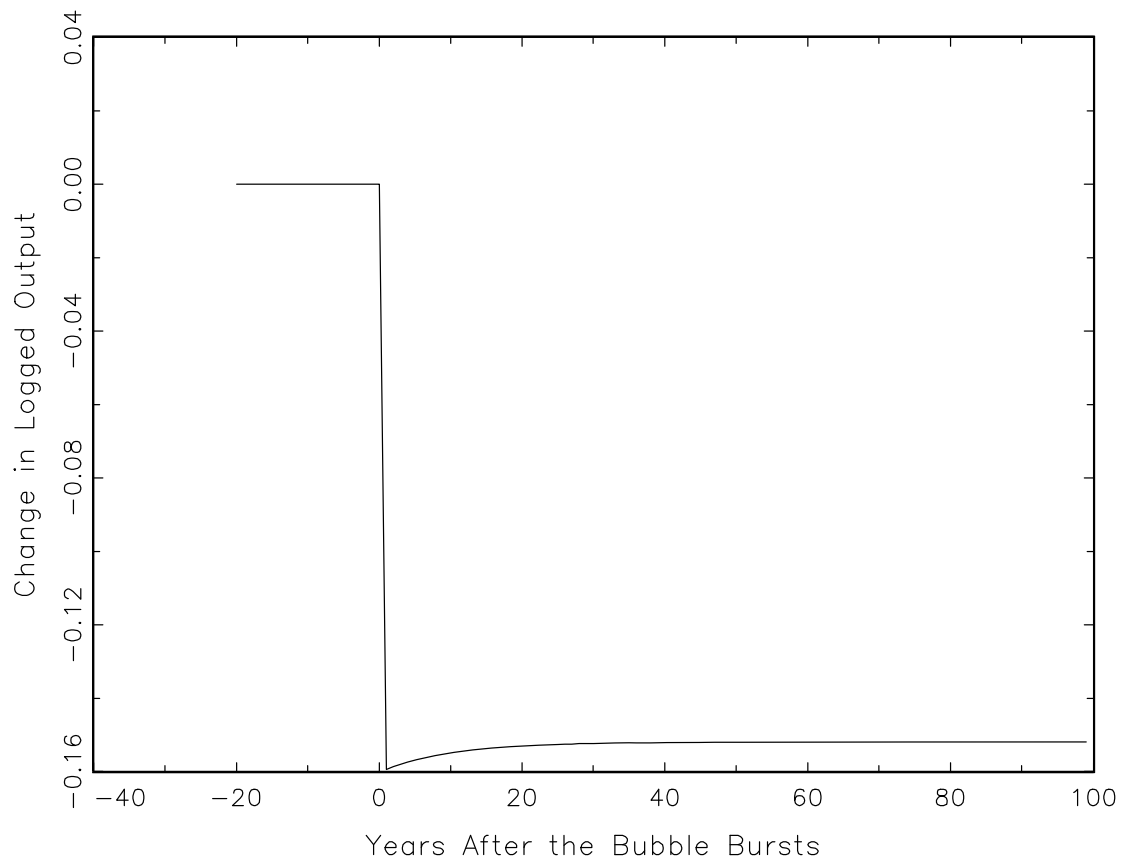


Figure 1: **Output Before and After A Bubble Burst**

3. Government Interventions

In this economy, bubble years are good years. The collapse of the bubble triggers a precipitous fall in output which has permanent adverse consequences. Suppose the economy is in the first period after a bubble collapse. What can the government do, if anything, to restore better long-run economic health? I first discuss a class of desirable interventions, and then compare them to what is currently (late 2008-early 2009) being done in the United States to deal with the ongoing financial crisis.

A. Government Debt as Collateral

In the model economy, the bubble is useful because it expands borrowing capacity. Once the bubble collapses, there is no source of collateral, and entrepreneurs are forced to self-finance their projects. To help the economy, the government must provide some other source of collateral to the entrepreneurs.

Caballero and Krishnamurthy (2006) (CK) provide useful insights about what these other forms of collateral might be. They analyze an overlapping generations model of an incomplete markets open economy. They use the overlapping generations model to generate bubbles in real estate prices, and study the consequences of these bubbles. My model and theirs differ in important respects, but their discussion of government debt is highly relevant.

CK emphasize the role of government debt as an extra source of collateral. They first contemplate unbacked government debt, in which the government re-finances existing debt simply by rolling it over. Such debt is isomorphic to the fiat money in KM. and to housing in my model. Suppose that, immediately after the bubble collapses, the government gives each entrepreneur promises to future consumption. This promise is unbacked, in the sense

that the government will simply default on this promise if it cannot roll it over. There is an equilibrium in which this debt is valued and held. However, there are also other equilibria in which this debt is priced at zero (just like land is). Once debt is completely unbacked, its ability to operate as collateral is up to the self-fulfilling beliefs of private agents. There is no way for the government to guarantee that its debt will function as a form of collateral.

CK then consider debt that is explicitly backed by the taxation authority of the government. The fundamental premise here is that the government is a superior collection agency than are private agents. From a strict theoretical point of view, this premise is hard to defend (why not just use those great collection powers on behalf of private lenders?). But in reality, it is true that people who do not pay their taxes can go to jail, while debtors cannot. The government seems to have collection powers that it is unwilling to let private creditors use.

It is clear that if the government commits to using taxes to repay its bonds, the bonds are fundamentally different from intrinsically useless collateral. More subtly, CK argue that, in equilibrium, the government will never need to collect the taxes. Instead, the government can commit to a strategy under which it commits to rolling over the existing debt, and then levies taxes if agents fail to buy the issued debt. Given such a commitment, there is a unique equilibrium in which agents are always willing to buy the issued debt, without any taxes ever being levied. Note though that the government's ability to collect the taxes is necessary to its being able to rule out collapses in the value of this debt. Limitations on this ability to collect taxes (say, because of distortions) curtail the effectiveness of this proposed policy.

B. A Desirable Intervention

The previous subsection argues that the government bonds can serve as collateral. In this subsection, I describe how the government can intervene after a bubble's collapse to restore the economy's health. Throughout, I assume that ε is near zero, so that the path of aggregate variables before the bubble's collapse is well-approximated by the deterministic bubbly steady-state.

Bail Out Landowners And Raise Interest Rates

There are two related, but distinct, problems in the economy after the bubble's collapse. First, entrepreneurs have lost wealth. This fall is responsible for the *immediate* adverse impact on aggregate variables. Second, the equilibrium interest rate has fallen from 0 to $-\delta$. This fall is responsible for the *long-term* adverse impact on aggregate variables. The government can readily fix both these problems with a two-part intervention.

Suppose the bubble bursts at date t . The per-capita wealth loss of the entrepreneurial sector equals p^* . The government hands out bonds to entrepreneurs. The distribution of this handout across entrepreneurs is really irrelevant, as long as the bonds promise to pay p^* next period per-capita. Assuming that the interest rate is 0, this injection restores per-capita entrepreneurial wealth to what it would have been in the absence of the bubble's bursting.

The above injection of wealth cures the first problem created by the bubble's collapse. The second problem is that savers are accumulating wealth through a low rate of return vehicle (storable capital goods). The government can cure the second problem by committing itself to borrow at the real interest rate 0. The savers will lend to the government at this high interest rate, but the investors will not. Now the law of motion of per-capita wealth in the

economy mimics that in the deterministic bubbly steady-state:

$$W_{t+s} = \beta\pi W_{t+s-1}(1 - \delta + \alpha(\beta\pi W_{t+s-1})^{\alpha-1}) + \beta(1 - \pi)W_{t+s-1}$$

Assuming ε is small (so that \overline{W}_{stoch} is close to \overline{W}_{bub}), this law of motion implies that, as in the deterministic bubbly steady-state, wealth is constant at \overline{W}_{bub} . To implement this policy, the government has to repay $\beta(1 - \pi)W_{t+s}$ at each date $(t + s)$. It can readily afford this repayment because in period $(t + s)$, it raises $\beta(1 - \pi)W_{t+s}$ in new funds by its debt issue.

This policy of *raising* interest rates may strike some readers as counterintuitive. It is in fact standard in economies with borrowing constraints. By their very nature, borrowing constraints choke off the demand for loans and thereby force down interest rates. A supply of outside government debt allows agents to avoid borrowing constraints by accumulating enough saving. However, this extra supply of loans also necessarily raises interest rates. (See KM (2008) for a similar argument.)

Ex-Ante Effects

I motivated the above intervention (bailout plus interest rate increases) through its desirable post-bubble effects. In this subsection, I discuss the effect on the economy *before* the bubble bursts if asset traders anticipate a bailout of landowners and a real interest rate peg of 0 once $z_t = 0$.

The ex-post benefits of injecting wealth did not depend on the distribution of this injection across the entrepreneurs because their decision rules are linear. However, there is a particular distribution that has desirable ex-ante effects. Consider an entrepreneur who has L_t units of land in period t when the bubble bursts. The government gives that entrepreneur bonds that promise p^*L_t next period. Thus, the bailout is proportional to the entrepreneur's

holdings of land.

This proportional intervention has desirable ex-ante properties. In the stochastic bubble equilibrium, land is risky. As a hedge, savers hold a portfolio of both debt collateralized by land, and risk-free storable physical capital. This portfolio behavior implies that per-capita wealth is lower in the stochastic bubble equilibrium, even before the bubble bursts, than in the deterministic bubbly steady-state. Savers *under-accumulate* collateral because the bubble is risky.

Now suppose that people are aware of the government's bailout plan. Lenders can now expect their loans to pay a rate of return equal to 0 regardless of the realization of z . Land is risk-free⁷, and so savers no longer hold any physical capital. The law of motion of entrepreneurial wealth is exactly the same as in the deterministic bubbly steady-state throughout the lifetime of the economy, not just after the bubble bursts. As a result, aggregate variables and welfare are higher before the bubble bursts because of the government's post-bubble intervention.

In reaching this conclusion, it is important to keep in mind two critical aspects of the intervention. First, the government explicitly allows the bonds used for the bailout to be seized by creditors when the bubble bursts. This feature of the intervention means that the lenders face no ex-ante risk in making their loans against the borrower's collateral. Second, the government only compensates owners of land. The government does not bail out all holders of Arrow securities.

⁷Kocherlakota (2001) and Kocherlakota and Shim (2007) consider economies in which contracts are enforced solely through the loss of collateral. As in the model in this paper, they show that ex-ante optimal contracts may well require collateral insurance.

C. Current Interventions

As of this writing (early 2009), the federal government is intervening in a massive way in financial markets. The model is clearly simplistic in a number of important ways. Nonetheless, it is useful to think through the effects of the current federal interventions in the context of the model.

One current government policy is to hand out backed debt to banks. The model does not speak directly to the efficacy of this policy. However, if entrepreneurs cannot credibly commit to repaying their loans to banks, then giving backed debt to banks is useless. The policy needs to get extra wealth into the hands of entrepreneurs with desirable projects if it is to be effective.

The Federal Reserve has also adopted a policy by which it will purchase *unsecured* commercial paper. In the context of the model, suppose the government sells backed debt to each entrepreneur in exchange for that entrepreneur's promise to make a repayment next period. This policy has no intrinsic impact on land prices and so land prices may well remain zero. In this equilibrium, there is no viable collateral, and the government will get *nothing* back for its loan. This sounds like a bad policy. But it is essentially an indirect way for the government to give each entrepreneur extra wealth. If the government also pegs the real interest rate at 0, then this policy has equivalent ex-post outcomes to my preferred intervention described above.⁸

The Federal Reserve is currently *lowering* interest rates. This policy does not work well in the model economy. Lowering interest rates increases the demand for loans. But all

⁸The ex-ante effects of this policy are not the same as mine though. My preferred intervention explicitly insures land owners against the fall in the value of land. This policy makes collateral risk-free from an ex-ante perspective. Buying unsecured commercial paper does not do so.

of the potential borrowers are already on their borrowing constraint. As discussed above, the correct solution is to *raise* interest rates after bailing out holders of land. In this fashion, the savers are encouraged to lend to the government, and the government can serve as an intermediary between savers and investors.⁹

Finally, there are many economic actors who have issued securities that implicitly or explicitly pay off contingent on house price movements, without owning an actual physical object called land. In the context of the model, these agents have issued an Arrow security that pays off when $z_t = 0$. With the collapse in the price of land, the agents are not able to make their commitments. At least in some instances, the government has provided sufficient funding to such agents to allow them to make their commitments. Note that this kind of bailout is distinct from the bailout proposed above, in which only owners of land receive compensation for their losses.

Within the model, insuring all owners of Arrow securities is problematic from both an ex-post and ex-ante point of view. From an ex-post perspective, there is no natural cap on how much the government will be injecting into the economy. This difficulty can be solved (with sufficient political will) by the government's restricting the aggregate size of the bailout to match the total size of the fall in the value of land. The ex-ante deficiency is insurmountable though. Suppose agents anticipate that the government will fully insure all issuers of Arrow securities. Their optimal response is to create enormous amounts of such securities. (It is definitely tempting to speculate that these motives were responsible for the creation of the giant credit default swap market.)

⁹Of course, the Federal Reserve is lowering *nominal* interest rates, not *real* interest rates. However, given that prices are rigid in the short-run, the policy will also lower real interest rates

The collapse of a bubble creates two problems. First, it robs entrepreneurs of wealth. Second, it lowers the real interest rate. A successful intervention needs to resolve both of these difficulties. Not all current government interventions do so.

D. The Ex-Ante Role for Government: Dynamic Efficiency Revisited

The above discussion emphasizes the ability of the government to intervene usefully *after* a bubble collapses. However, in this model, there is a useful ex-ante role for government as well. Bubbles occur in this economy because, without them, the real interest rate is lower than the growth rate of the economy. Bubbles eliminate this gap and allow savers to accumulate wealth more rapidly. However, the possible collapse of the bubble creates a problematic source of aggregate risk. The government has the ability to eliminate this risk. In particular, the government can use its ability to tax ensures that there will never be a collapse in the value of its debt. With this power, the government can use debt rollover to substitute for private sector collateral bubbles.

This analysis is reminiscent of the discussions of dynamic efficiency in overlapping generations economies pioneered by Diamond (1965). However, there is a key difference. In Diamond's setup, the interest rate equals the marginal product of capital net of depreciation ($MPK - \delta$). Hence, one can check for dynamic efficiency by comparing the interest rate or ($MPK - \delta$) to the growth rate. In the deterministic bubbly equilibrium in my model, the existence of financing constraints means that $MPK - \delta$ is larger than the interest rate (and also larger than zero). To assess dynamic efficiency, one has to use a comparison of interest rates and growth rates.¹⁰ In United States data from 1989-2005, the average real

¹⁰Abel, Mankiw, Summers, and Zeckhauser (1989) consider dynamic efficiency in overlapping generations economies with aggregate shocks. They show that in these models, one cannot reliably evaluate dynamic

return on Treasury bills is about 1.3%. The average growth rate of nondurable consumption and services is about 1.8%. The gap is considerably larger in the 1934-2005 period, because the average interest rate falls to zero. (See Mehra and Prescott (2008).)

4. Conclusions

In this paper, I examine a model economy in which capital re-allocation is critical. This re-allocation is accomplished via collateralized lending backed by land. However, land is scarce and so all entrepreneurs face borrowing constraints that bind infinitely often into the future. These two ingredients imply that equilibrium bubbles naturally emerge in the price of land. The resulting bubbles expand entrepreneurial borrowing capacity and generate more output, consumption, and welfare. In this framework, the collapse of a bubble has a dramatic and immediate adverse impact on aggregate variables, which thereafter never fully recover.

The model provides a justification for interventions similar to (but definitely distinct from) those that have been considered and implemented in recent months. Suppose the economy requires a large amount of capital re-allocation on an annual basis. The collapse of a bubble, and the concomitant disintermediation, can greatly disrupt this re-allocation, and then aggregate output can be severely affected within the course of only one year. In this way, the model provides a *possible* justification for the need for speed apparently perceived by the federal government. On the other hand, the model also makes clear that the details of interventions (like whether to raise or lower interest rates!) matter a great deal.

efficiency by comparing average risk-free interest rates to average growth rates. This conclusion applies to my model as well. Their solution is to use the sign of net inflows into firms. This solution is valid in their context, but is unreliable in my model with financial constraints. (Kraay and Ventura (2007) make a similar point.)

Other recent papers draw connections between collateral scarcity and the existence of bubbles (Caballero and Krishnamurthy (2006) and Araujo, et al (2005)). However, to gauge the empirical relevance of these theoretical connections, we need to have good measures of entrepreneurial risk and collateral scarcity. As KM point out, models like theirs (and Angeletos (2007)) are specifically constructed to mimic standard macroeconomic frameworks. For this reason, I believe that it will be relatively easy to augment the model in this paper so that it is well-suited for a serious quantitative analysis of bubbles in the macroeconomy.

Real estate prices fell dramatically in Japan in the 1990's and in the United States in the 2000's. In both settings, the fall in an asset's price had a significant impact on collateralized lending. The theme of this paper is that these kinds of bubble collapses may well be an inevitable part of private sector collateral provision. The taxation power of the government (in the United States and other developed countries) means that its debt is a less risky form of collateral than what is available to the private sector. The government can provide better outcomes if it is able to give this debt to people with investment opportunities, and is willing to pay a high rate of return.

References

- [1] Angeletos, G.-M., 2007, Uninsured idiosyncratic investment risk and aggregate saving, *Review of Economic Dynamics* 10, 1-30.
- [2] Araujo, A., Pascoa, M., and Torres-Martinez, J. P., 2005, Bubbles, collateral, and monetary equilibrium, PUC-Rio working paper.
- [3] Caballero, R., and Krishnamurthy, A., 2006, Bubbles and capital flow volatility: Causes and risk management, *Journal of Monetary Economics* 53, 35-53.
- [4] Diamond, P. A., 1965, National debt in a neoclassical growth model, *American Economic Review* 55, 1126-1150.
- [5] Heathcote, J., and Davis, M., 2007, The price and quantity of residential land in the United States, *Journal of Monetary Economics* 54, 2595-2620.
- [6] Hellwig, C., and Lorenzoni, G., forthcoming, Bubbles and self-enforcing debt, *Econometrica*.
- [7] Kartashova, K., 2008, Improving public equity markets? No pain, no gain, University of Minnesota working paper.
- [8] Kiyotaki, N., and Moore, J., 2008, Liquidity, business cycles, and monetary policy, Princeton University working paper.
- [9] Kocherlakota, N., 1992, Bubbles and constraints on debt accumulation, *Journal of Economic Theory* 57, 245-256.

- [10] Kocherlakota, N., 2001, Risky collateral and deposit insurance, *Advances in Macroeconomics* 1, Article 2.
- [11] Kocherlakota, N, and Shim, I., 2007, Forbearance and prompt corrective action," *Journal of Money, Credit, and Banking* 39, 1107-29.
- [12] Kraay, A., and Ventura, J., 2007, The dot-com bubble, the Bush deficits, and the US current account, in *Current Account Imbalances: Sustainability and Adjustment*, ed. R. Clarida, Chicago: University of Chicago Press.
- [13] Mehra, R., and Prescott, E. C., 2008, The equity premium: ABCs, in *Handbook of the Equity Risk Premium*, ed. by R. Mehra, Amsterdam: Elsevier.
- [14] Santos, M., and Woodford, M., 1997, Rational asset pricing bubbles, *Econometrica* 65, 19-57.
- [15] Scheinkman, J., and Weiss, L., 1986, Borrowing constraints and aggregate economic activity, *Econometrica* 54, 23-46.
- [16] Woodford, M., 1986, Stationary sunspot equilibria in a finance-constrained economy, *Journal of Economic Theory* 40, 128-137.