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EVALUATING THE ROBUSTNESS OF TRADE RESTRICTIVENESS INDICES: SOME GOOD AND BAD NEWS

Stephen Tokarick
International Monetary Fund

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Evaluating the Robustness of Trade Restrictiveness Indices: Some Good and Bad News

by

Stephen Tokarick[1]

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Abstract

There is a great deal of interest in measuring whether an economy has become more or less open to international trade. In a series of papers, Anderson and Neary have developed a trade restrictiveness index (TRI) that has a firm basis in theory and that can be implemented empirically using a computable general equilibrium (CGE) model. This paper examines the robustness of TRI calculations and finds that they are generally insensitive to alternative values for the elasticity of substitution among factors of production, but quite sensitive to differences in model structure. Also, this paper points out that economic growth, i.e. factor accumulation, could render a country's trade regime more or less restrictive (in a welfare sense) without any changes in tariff rates. Economists need to be aware of this possibility in reaching conclusions about whether a country has become more or less open to international trade.

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[1] Senior economist, International Monetary Fund, Research Department, Trade and Investment Division, 700 19th Street N.W., Room 9-700A, Washington D.C., 20431. E-mail address: stokarick@imf.org. Phone: 202-623-7590. FAX: 202-623-7590. The views expressed in this paper are those of the author and should not be attributed to the International Monetary Fund, its Executive Board, or its Management.

I. Introduction

Over the last decade, there has been a great deal of interest in the relationship between a country's degree of openness to international trade and economic growth. A central issue related to this question is how to measure openness to trade in an economically meaningful way. In a series of papers and a book, Anderson and Neary (1996, 2005) develop a trade restrictiveness index (TRI) that has a firm foundation in economic theory and that can be implemented in practice. The TRI is defined as the uniform deflator, or scaling factor, applied to imported goods that would produce the same effect on real income as the country's differentiated structure of tariffs. Alternatively, trade restrictiveness is sometimes measured by computing the uniform tariff equivalent (UTE) that is equivalent, in welfare terms, to the country's existing tariff structure. Anderson and Neary (1994), Lloyd and MacLaren (2002), and O'Rourke (1997) have all calculated TRIs using various types of computable general equilibrium (CGE) models, while Kee, Nicita, and Olarreaga (2004) calculated another type of trade restrictiveness index, the mercantilist trade restrictiveness index (MTRI) using a method developed by Feenstra (1995) that makes some simplifying assumptions, but does not require a CGE model.

Unfortunately, there has not been much work examining how robust the estimates of TRIs are to alternative model structures and economic environments. This paper has two purposes. First, it explores the robustness of the TRI to different CGE model structures. Anderson and Neary (1994) calculated TRIs for twenty five countries using a CGE model for each country that was identical in structure, but different in data and parameter values. Their results showed that the resulting values for the TRIs, and the ranking of countries' restrictiveness, were generally insensitive to alternative elasticity values. So far, there has been very little exploration of how alternative model structures affect the calculation of TRIs and this paper addresses this issue. To date, O'Rourke (2002) is the only other paper to address this question, but he explored the sensitivity of the TRI calculation to the specification of consumer demand.

The second purpose of this paper is to demonstrate that the calculated value of a TRI can change as a result of economic growth, i.e. factor accumulation, even though tariff rates remain unaltered. Typically, TRIs are estimated for two points in time and the values compared to reach a judgment regarding whether the country has become more or less open to international trade. In doing so, however, analysts typically do not take into account changes in the structure of the economy that may have taken place between the two time periods—changes that would affect the welfare cost of a country's tariffs and therefore, its TRI. It turns out that a country's TRI could rise or fall with factor accumulation, depending, among other things, on the bias of the accumulation. This paper points out that if this aspect is neglected, it is possible to reach an incorrect conclusion regarding whether an economy has become more or less open.

II. Robustness of TRI Calculations

This section reports the results of some sensitivity tests on calculated TRIs using a CGE model of a hypothetical economy. The objective is to determine how sensitive the calculated TRIs are to alternative values of the elasticity of substitution among factors of production, and to alternative model structures, as there has been little work on each of these issues. Using a CGE model of Columbia, Anderson (1993) presented estimates that showed that changes in Columbia's TRI were relatively insensitive to alternative values of the elasticity of transformation, as well as the elasticities of final and intermediate demand. This conclusion is derived from a particular CGE model—the Australian model—in which there is no local production of the importable good, no domestic consumption of the exportable, and no explicit factor markets. Instead of modeling factor markets explicitly, Anderson's model employs a transformation function between exportables and nontraded goods, and the elasticity of transformation governs how easy it is to shift production between the two types of goods.

O'Rourke (1997) used the same model structure as Anderson to assess how sensitive calculations of TRIs for Britain and France in the 1880s were to alternative specifications of consumer demand. He considered alternative nesting schemes for commodities in consumer demand and found the calculated TRIs to be quite sensitive to alternative commodity groupings and elasticities of substitution. Neither of these papers investigated the sensitivity of the TRI calculations to alternative production structures. This section provides the results from such an exercise.

A. Calculated TRIs and the Elasticity of Substitution Among Factors of Production

This section reports the results from using a CGE model to assess the sensitivity of calculated TRIs to alternative values of the elasticity of substitution among factors of production. The model used in the section consists of three goods: two imports and one exported good. Each good is produced using three factors of production using a constant elasticity of substitution (CES) production function; there are no intermediate inputs to keep the model as simple as possible. A representative consumer receives all factor income plus tariff revenue and is assumed to maximize a cobb-douglass utility function defined over the three goods. The terms of trade are constant and the price of the export good is taken as the numeraire. In a sense, this model is quite similar to the standard general equilibrium model used in international trade, except that there are three goods and factors instead of just two. Two variants of the model are used to conduct sensitivity tests: one with all factors of production mobile across sectors and one that assumes that one factor of production is sector-specific, i.e. immobile across sectors. This permits an evaluation of the sensitivity of the TRI to alternative values of the elasticity of substitution among factors, as well as with respect to model structure.

The model described above differs from the model used by Anderson (1993) and O'Rourke (1997) in several ways. First, the model used in this paper introduces factor markets explicitly, while the others do not. Second, the model allows for consumption of the country's export good, as well as domestic production of the two imported goods. In fact, the model assumes that domestic goods are perfect substitutes for imports, as is common in trade theory. Third, unlike Anderson and O'Rourke, the model has no nontraded goods. Nontraded goods are realistic features of many economies and should be included, but they are excluded here to keep the model as simple as possible and to create a structure that is significantly different from the one used by Anderson and O'Rourke. Furthermore, introducing nontraded goods would introduce an indeterminacy into the model because the model would consist of more goods than factors and some additional structure would be needed to ensure positive outputs of all goods. This indeterminacy is ruled out in many CGE models by introducing product differentiation—treating imports as imperfect substitutes for domestically produced goods.

B. Sensitivity Results From a CGE Model

Table 1 presents the calculated UTEs from the model for alternative values of the elasticities of substitution among the three factors of production, σ_{M1}, σ_{M2}, and σ_E, in the two import sectors and the export sector. UTEs are calculated holding two of the elasticities of substitution constant, while varying the third only, from a value of 2.0 to 0.5. Estimates of the UTEs are presented for two cases: one where one of the factors is sector specific and the other where all three factors are mobile across all sectors.

Table 1 reveals that for *a given model structure*, the calculated UTEs are generally insensitive to changes in the elasticity of substitution among factors, however, they are quite sensitive to the choice of model structure. For example, in the specific-factors model, varying the elasticity of substitution among factors does have an impact on the calculated UTE, however, the magnitude of the change is relatively small. Altering the elasticity of substitution in the first import sector, σ_{M1}, by 75 percent (from 2.0 to 0.5) results in a decline in the UTE of only about 12 percent. Similarly, the same percentage reduction in σ_{M2} raises the UTE by about 13 percent. The largest impact comes in the export sector: reducing σ_X by 75 percent increases the UTE by 30 percent. In all cases, changes in the elasticity of

Table 1. Sensitivity of the Uniform Tariff Equivalents to Changes in Values of the Elasticity of Substitution Between Labor and Capital

			Fixed Factor Model		All Factors Mobile		Percent Difference
σM1	σM2	σX	UTE	Percent Change 1/	UTE	Percent Change 1/	Between Two Models 2/
2.00	1.50	1.50	0.05367016		0.05152938		-3.99
1.50	1.50	1.50	0.05208624	-2.95	0.05115417	-0.73	-1.79
1.25	1.50	1.50	0.05114180	-1.81	0.05092261	-0.45	-0.43
0.75	1.50	1.50	0.04880535	-4.57	0.05033853	-1.15	3.14
0.50	1.50	1.50	0.04731791	-3.05	0.04996629	-0.74	5.60
1.50	2.00	1.50	0.05073410		0.05064263		-0.18
1.50	1.50	1.50	0.05208624	2.67	0.05115417	1.01	-1.79
1.50	1.25	1.50	0.05292666	1.61	0.05141959	0.52	-2.85
1.50	0.75	1.50	0.05535122	4.58	0.05197050	1.07	-6.11
1.50	0.50	1.50	0.05754775	3.97	0.05225622	0.55	-9.20
1.50	1.50	2.00	0.04936698		0.05077284		2.85
1.50	1.50	1.50	0.05208624	5.51	0.05115417	0.75	-1.79
1.50	1.50	1.25	0.05387572	3.44	0.05143039	0.54	-4.54
1.50	1.50	0.75	0.05926470	10.00	0.05222511	1.55	-11.88
1.50	1.50	0.50	0.06411704	8.19	0.05278760	1.08	-17.67

Source: Simulations with CGE model.

substitution among factors translate into a change in the UTE that is far less than one-for-one. The responsiveness of the UTE to changes in the elasticity of substitution among factors of production is even smaller in the version of the model in which all factors of production are intersectorally mobile.

In contrast, calculated UTEs are quite sensitive to model structure. Reading across rows of Table 1, *for given elasticity values*, different assumptions regarding factor mobility significantly alters calculated values for the UTE in most cases—there are only two cases in which the difference between the UTEs across the two models is less than 1 percent. On the other hand, the discrepancies between the values of the UTE for the two models can be substantial, as in the last row of table 1. This finding, although confined to the particular CGE used here, reinforces the point made by O'Rourke (1997) that care should exhibited in choosing a particular model structure to calculate a UTE for a given country. In particular, there is a great deal of interest in policy circles about cross-country comparisons of trade restrictiveness. It should be kept in mind that UTEs depend on model structure and if estimates of UTEs are derived from a identical model structure, one might be skeptical of the rankings that emerge.

III. The Effects of Factor Accumulation on Measures of Trade Restrictiveness

As already noted, economists typically compute TRIs for a given economy at two different points in time. If the TRI has increased, then the country has become less open to international trade or equivalently, the country's trade policy regime has become more restrictive. In calculating the two TRIs, allowance is not typically made for structural changes, as a result of factor accumulation for example, in the economy between the two time periods.

It turns out to be very important for the measurement of trade restrictiveness to take into account the effects of factor accumulation because a country's TRI could rise or fall, depending on the precise nature of economic growth, even if tariff policy remains unchanged. That is, a country's tariff structure could become more or less costly with factor accumulation and this would affect values for the TRI. If the effects of factor accumulation are not taken into account, then it is possible to reach very different conclusions whether a country has become more or less open to trade. This section shows how a country's TRI is affected by factor accumulation and presents results from some simulations that demonstrate that factor accumulation could cause large changes in a country's TRI.

Anderson and Neary derive the TRI using the balance-of-trade function for a small, open economy. In general, the TRI, denoted by Δ, is given implicitly by:

$$B\left(\frac{p_{M1}}{\Delta},\frac{p_{M2}}{\Delta},p_E,p_N,u^0,v\right)=0 \tag{1}$$

where $B(\cdot)$ is the balance-of-trade function, p_{M1} and p_{M2} are the prices of the two imported goods, p_E is the price of the export good, p_N is the price of the nontraded good, u^0 is the initial utility level, and v is the vector of factor supplies (capital and labor). Totally differentiating (1) (noting that $dp_E = 0$), gives:

$$\hat{\Delta} = \frac{B_{M1}dp_{M1} + B_{M2}dp_{M2} + \Delta(B_N dp_N + B_v dv)}{(B_{M1}p_{M1} + B_{M2}p_{M2})} \tag{2}$$

where B_k is the partial derivative of the balance-of-trade function with respect to the three prices and the endowment vector. Equation (2) can be re-written in the following form:

$$\hat{\Delta} = \sum_i \sigma_i \hat{p}_{Mi} + \frac{\Delta(B_N dp_N + B_v dv)}{\sum_i B_{Mi} p_{Mi}} \tag{3}$$

Where $\sigma_i = \dfrac{B_{Mi} p_{Mi}}{\sum_i B_{Mi} p_{Mi}}$, which can be interpreted as the “marginal cost of tariffs”.

The proportional change in the TRI is the weighted sum of the proportional changes in domestic prices, where the weights are the derivatives of the balance-of-trade function, plus an additional term capturing changes in factor endowments and in the price of the nontraded good induced by the factor accumulation. If there were no factor accumulation and no nontraded good, the second term on the right-hand side of equation (3) would be zero, so (3) would collapse to the expression derived in Anderson and Neary (1994). Obviously, the proportional change in the TRI as a result of factor accumulation depends on the derivative of the balance of trade function with respect to the vector of factor endowments, B_V and the impact of a change in the price of the nontraded good, captured by B_N. Also, equation (3) reveals that the magnitude of the second term on the right-hand side depends on the *level* of the TRI, Δ.

A. Case of No Nontraded Good

To illustrate how factor accumulation affects the TRI, consider first the case where there are two imported goods and one export good, so there are no complications arising from nontraded goods. Nontraded goods are introduced below in sub-section B. In this simple case, equation (3) reduces to:

$$\hat{\Delta} = \sum_i \sigma_i \hat{p}_{Mi} + \frac{\Delta(B_V dV)}{\sum_i B_{Mi} p_{Mi}} \qquad (4).$$

The balance-of-trade function is given by:

$$\begin{aligned} B(p_E, p_{M1}, p_{M2}, t_1, t_2, u, v) &= E(p_E, p_{M1}, p_{M2}, u) - G(p_E, p_{M1}, p_{M2}, v) \\ &- t_1 P^*_{M1}(E_{M1} - G_{M1}) - t_2 P^*_{M2}(E_{M2} - G_{M2}) \end{aligned} \qquad (5)$$

where p_E is the price of exports, p_{M1} is the price of the first import good, p_{M2} is the price of second import good, t_1 and t_2 are the corresponding tariff rates, u is the level of utility, v is the vector of factor endowments, $E(\cdot)$ is the expenditure function, and $G(\cdot)$ is the GDP function. Subscripts next to E or G denote a partial derivative with respect to that variable. For example, $E_{M1} = \frac{\partial E}{\partial P_{M1}}$, which is the compensated demand for imported good 1. The derivative of the balance-of-trade function with respect to factor endowments is:

$$B_V dV = (t_1 P^*_{M1} G_{M1V} + t_2 P^*_{M2} G_{M2V} - G_V) dv \qquad (6)$$

where G_{M1V} and G_{M2V} capture how outputs of the two imported goods change as a result of factor accumulation at constant prices—the Rybczynski terms. These can be positive or negative depending on the nature of the accumulation.[2] Using a basic duality result, G_V equals the vector of factor prices. Equation (6) can be rewritten as

$$B_V dV = -(P^*_{M1} G_{M1V} + P^*_{M2} G_{M2V} + P^*_E G_{EV}) dv \qquad (7).$$

It can be shown (see appendix) that the welfare effect of factor accumulation is given by:

$$dU\left[E_U - t_1 P^*_{M1} E_{M1U} - t_2 P^*_{M2} E_{M2U}\right] = \left[G_V - t_1 P^*_{M1} G_{M1V} - t_2 P^*_{M2} G_{M2V}\right] dV \qquad (8).$$

[2] Appendix II contains expressions for the solutions for all the endogenous variables for the case of four goods (export, two import goods, and a nontraded good) and five factors.

Since the bracketed term on the left-hand side is positive in stable models, the welfare effect of accumulation depends on the sign of the bracketed-term on the right-hand side of (8), which is of opposite sign of equation (6). Alternatively, using the definitions of terms in (8), factor accumulation will raise welfare if:

$$\left[p_E^* \left(\frac{\partial X_E}{\partial v} \right) + p_{M1}^* \left(\frac{\partial X_{M1}}{\partial v} \right) + p_{M2}^* \left(\frac{\partial X_{M2}}{\partial v} \right) \right] > 0 \tag{9}$$

that is, factor accumulation will raise welfare if the change in the value of output, *measured at world prices*, is positive. If the value of output at world prices falls, real income falls. It turns out that equation (9) is the condition that determines whether growth will raise welfare in the presence of protection, and was explained in Johnson (1967) and Caves and Jones (1974). If the pattern of factor accumulation is sufficiently biased toward one of the protected sectors, then it could result in a reduction in welfare.

Thus, the effect of factor accumulation on the TRI depends on the sign of B_V and the sign of $\sum_i B_{Mi} p_{Mi}$, the sum of the derivatives of the balance-of-trade function with respect to the prices of the import good. In the case of two imported goods:

$$\begin{aligned} \sum_i B_{Mi} p_{Mi} &= \left[-t_1 P_{M1}^* (E_{M1M1} - G_{M1M1}) - t_2 P_{M2}^* (E_{M2M1} - G_{M2M1}) \right] p_{M1}^* dt_1 \\ &+ \left[-t_1 P_{M1}^* (E_{M1M2} - G_{M1M2}) - t_2 P_{M2}^* (E_{M2M2} - G_{M2M2}) \right] p_{M2}^* dt_2 \end{aligned} \tag{10}$$

Both $(E_{M1M1} - G_{M1M1})$ and $(E_{M2M2} - G_{M2M2})$ must be negative, but the cross-price terms could be positive or negative depending on whether the two goods are substitutes or complements. Thus, the sign of $\sum_i B_{Mi} p_{Mi}$ is ambiguous.[3] Given these results, factor accumulation will raise the TRI if equations (7) and $\sum_i B_{Mi} p_{Mi}$ are both of the same sign and reduce it if they are of opposite sign.

The above results can be used to determine how factor accumulation affects the TRI and the uniform tariff equivalent (UTE) using a specific-factor's model. The results can be classified into a number of cases, summarized in Table 2, depending on the configuration of the accumulation. It turns out that it is possible to pin down how factor accumulation would affect B_V in two cases—accumulation biased toward the export sector and biased toward labor. In each case, $B_V < 0$ and welfare rises, so the numerator of the second term on the right-hand side of equation (4) is negative. How these two types of factor accumulation affect the TRI, however, depends on the sign of the denominator in the second term on the right-hand side of equation (4). Of course, in the case of balanced growth, in which all factor supplies change by the same proportion, the cost of the tariffs, and therefore the TRI, remains unchanged. Only in the case of unbalanced growth will factor accumulation alter the welfare cost of tariffs.

[3] Complementarity between import goods is possible, but all goods cannot be complements for each other if markets are stable.

Table 2. Summary of the Effects of Factor Accumulation on the TRI in the Specific-Factor's Model

Nature of Factor Accumulation	Effects on Sectoral Outputs at Constant Prices	Effects on: $B_V = (t_1 P_{M1}^* G_{M1V} + t_2 P_{M2}^* G_{M2V} - G_V)$ or $B_V = -(P_{M1}^* G_{M1V} + P_{M2}^* G_{M2V} + P_E^* G_{EV})$
Export-Biased Growth: $\hat{K}_E > 0$	$G_{EV} > 0$ $G_{M1V} < 0$ $G_{M2V} < 0$	$B_V < 0$
Growth Biased Toward the First Import Sector: $\hat{K}_{M1} > 0$	$G_{EV} < 0$ $G_{M1V} > 0$ $G_{M2V} < 0$	Sign of B_V is ambiguous: depends on Rybczynski terms
Growth Biased Toward the Second Import Sector: $\hat{K}_{M2} > 0$	$G_{EV} < 0$ $G_{M1V} < 0$ $G_{M2V} > 0$	Sign of B_V is ambiguous: depends on Rybczynski terms
Biased Toward Labor: $\hat{L} > 0$	$G_{EV} > 0$ $G_{M1V} > 0$ $G_{M2V} > 0$	$B_V < 0$

B. Factor Accumulation and the TRI With Nontraded Goods

This section explores how factor accumulation affects welfare and the TRI in the presence of a nontraded sector. The model used in this section consists of four goods (an export, two import goods, and a nontraded) produced by labor, which is mobile across all sectors and a sector-specific factor, i.e. capital. As a consequence, the wage rate will be the same across all sectors, but the returns to capital will differ. Formally, the model equations are given in appendix II and it shows how factor accumulation affects the cost of protection.

The balance-of-trade function is given by:

$$B(p_E, p_{M1}, p_{M2}, p_N, t_1, t_2, u, v) = E(p_E, p_{M1}, p_{M2}, p_N, u) - G(p_E, p_{M1}, p_{M2}, p_N, v)$$
$$-t_1 P^*_{M1}(E_{M1} - G_{M1}) - t_2 P^*_{M2}(E_{M2} - G_{M2}) \tag{11}$$

where p_N is the price of the nontraded good.

Differentiating equation (11) with respect to factor endowments gives:

$$B_V dV = (t_1 P^*_{M1} G_{M1V} + t_2 P^*_{M2} G_{M2V} - G_V) dV \tag{12}$$

which is similar to equation (6). But with a nontraded good, altering factor endowments will change the price of the nontraded good. Differentiating (11) with respect to the price of the nontraded good gives:

$$B_N dp_N = -\left[t_1 P^*_{M1}(E_{M1N} - G_{M1N}) + t_2 P^*_{M2}(E_{M2N} - G_{M2N})\right] dp_N \qquad (13).$$

Using (12) and (13), the term $(B_N dp_N + B_V dV)$, which appears in the numerator of the second term on the right-hand side of equation (3) can be written:

$$B_N dp_N + B_V dV = -\left[t_1 P_{M1}^*(E_{M1N} - G_{M1N}) + t_2 P_{M2}^*(E_{M2N} - G_{M2N})\right] dp_N$$
$$+(t_1 P_{M1}^* G_{M1N} + t_2 P_{M2}^* G_{M2N} - G_V) dV \qquad (14).$$

Finally, using the condition that the market for the nontraded good must clear (see equation (14a) in the appendix), $(B_N dp_N + B_V dV)$, which equals $-B_u du$, is:

$$-B_u du = (t_1 P_{M1}^* G_{M1V} + t_2 P_{M2}^* G_{M2V} - G_V) dV$$
$$-\left[t_1 P_{M1}^*(E_{M1N} - G_{M1N}) - t_2 P_{M2}^*(E_{M2N} - G_{M2N})\right]$$
$$\left[\frac{G_{NV}(E_U - t_1 P_{M1}^* E_{M1U} - t_2 P_{M2}^* E_{M2U}) + E_{NU}(t_1 P_{M1}^* G_{M1V} + t_2 P_{M2}^* G_{M2V} - G_V)}{(E_{NN} - G_{NN})(E_U - t_1 P_{M1}^* E_{M1U} - t_2 P_{M2}^* E_{M2U}) + E_{NU} t_1 P_{M1}^*(E_{M1N} - G_{M1N}) + E_{NU} t_2 P_{M2}^*(E_{M21N} - G_{M2N})}\right] dV$$

(15),

which is the negative of the welfare effect of factor accumulation in the presence of a nontraded good (see appendix II).

The effect of factor accumulation on the TRI in obviously much more complicated due to the fact that the accumulation will alter the price of the nontraded good. For example, suppose the accumulation raises the price of the nontraded good. If both imported goods are substitutes in demand for the nontraded good, the demand for imports will increase and this will be welfare improving, since the tariff reduced imports below the optimum. If accumulation reduces the price of the nontraded good, the demand for imports will decline, which worsens the welfare loss induced by the tariffs. These effects are in addition to those on production described above in section B—the effects of accumulation at constant prices.

In addition to the effects arising through changes in the price of the nontraded good, factor accumulation will alter outputs of each good. Using the definitions for G_{jv}, equation (12) can be written:

$$B_V = -\left[p_E^* \left(\frac{\partial X_E}{\partial v} \right) + p_{M1}^* \left(\frac{\partial X_{M1}}{\partial v} \right) + p_{M2}^* \left(\frac{\partial X_{M2}}{\partial v} \right) + p_N \left(\frac{\partial X_N}{\partial v} \right) \right] \tag{16}$$

using $G_{jv} = \dfrac{\partial X_j}{\partial v}$, where X_j is output of good j. Thus, the effect of accumulation on B_V depends on the Rybczynski terms. Equation (16) is the same as equation (9), except that it includes a term that captures how the output of the nontraded good changes when factor endowments change. In sum, the effect of factor accumulation on the TRI is given by substituting equation (15) into equation (3). As can be readily seen, expression (3) becomes quite complicated and no clear results emerge. Therefore, the next section uses a simple simulation model to uncover results.

IV. Simulations Using a CGE Model

Given the inability to make definitive statements about how factor accumulation affects the TRI and the UTE, this section reports the results of some simulations using a simple general equilibrium model, as used in section II of this paper and described in appendix I. The simulations examine how the UTE for a small, open economy would be affected by growth that is biased toward each of the sectors (exports, imports, and the nontraded) as well as labor-biased growth.

Briefly, the model consists of four sectors (exports, two import goods, and a nontraded good). Output of each sector is produced using labor, which is mobile across all sectors and a sector-specific factor. Thus, the wage rate is the same in every sector, but the return to capital differs. The country is taken to be "small", that is it unable to influence its terms of trade through changes in tariff rates.

The model is used to calculate how five types of factor accumulation affect the calculated UTE (and the TRI). First, for hypothetical values of parameters and elasticities, the model is used to calculate the UTE of the existing tariff structure, which serves as a benchmark. Second, a UTE is calculated assuming that the endowment of capital specific to each sector and the economy-wide endowment of labor increases by one percent. This experiment generates five new values for the UTE, as the endowment changes are taken one at a time. Third, UTEs are calculated based on the post-growth structure of the economy, that is, the base against which the UTEs are calculated is the structure of the economy *after the growth takes place.* Thus, this type of calculation isolates how the structure of the economy would affect the calculation of the UTE, that is, how the UTE would be affected by a different configuration of factor endowments. Finally, steps two and three are repeated for a three percent change in factor endowments. The results are summarized in table 3.

The first row of table 3 gives the "no-growth" UTE of the existing tariff structure. As can be seen from the second row, a one-percent increase in the amount of sector-specific capital (holding all other factor endowments unchanged) results in a higher UTE, relative to the no-growth UTE, and in general, the deviation from no-growth UTE is quite large: from a low of 39.1 percent to 184 percent. Factor accumulation raises the UTE in all cases because it raises welfare. Therefore, a higher UTE is required to keep utility constant at its pre-growth level. These conclusions, also apply to these same experiments when the endowment of the sector-specific factor rises by three percent and when the mobile factor—labor—increases.

Table 3 also presents the effects on the calculated UTE of altering the structure of the economy, but "taking out" the growth effects. This is done by calculating a UTE relative to a new base—an equilibrium in which just the factor endowments differ. Thus, the UTE discussed above can be thought of as an "uncompensated UTE", while the UTE presented in this section is a "compensated UTE"—compensated for the effect of economic growth. Anderson and Neary (1996) discuss this issue in the context of how growth affects the restrictiveness of a quantitative restriction on imports.

Table 3 demonstrates that structural change alone has a small effect on the calculated UTEs: the "compensated UTEs" hardly change relative to the no-growth UTEs. In no case does the compensated UTE change by more than 0.3 percent, relative to the no-growth UTE.

As well, the changes in the compensated UTEs are not uniform relative to the no-growth UTE. For the specific model structure used here, factor accumulation biased toward the first import sector and the export sector results in a higher UTE, while accumulation biased toward the second import sector and the nontraded sector reduce the UTE. Growth in the supply of labor reduces the UTE.

Table 3. Effects of Different Types of Growth on Calculated Uniform Tariff Equivalents

Tariff rate on good 1= .10 Tariff rate on good 2= .05	Biased toward Import 1	Biased toward Import 2	Biased toward Export	Biased toward Nontraded	Biased toward Labor
sigmaM1= 1.5					
sigmaM2= 2.5					
No-Growth UTE	0.07764731	0.07764731	0.07764731	0.07764731	0.07764731
1% Increase in Factor Endowment:					
UTE Relative to initial utility level	0.11362378	0.10796766	0.13208957	0.19001286	0.22072171
Percent change relative to no-growth UTE	46.33	39.05	70.11	144.71	184.26
UTE Relative to post-growth utility level	0.07773624	0.07762241	0.07765337	0.07763496	0.07759027
Percent change relative to no-growth UTE	0.11	-0.03	0.01	-0.02	-0.07
Welfare effect of factor accumulation (EV)	424.9	350.1	698.8	1765.4	2452.6
3% Increase in Factor Endowment					
UTE Relative to initial utility level	0.16578290	0.15312651	0.20471900	0.32390205	0.38675210
Percent change relative to no-growth UTE	113.51	97.21	163.65	317.15	398.09
UTE Relative to post-growth utility level	0.07791261	0.07757377	0.07766826	0.07761057	0.07747763
Percent change relative to no-growth TRI	0.34	-0.09	0.03	-0.05	-0.22
Welfare effect of factor accumulation (EV)	1273.5	1048.8	2088.1	5272.1	7332.6

Source: Model simulations.

V. Conclusions

There is a great deal of interest in measuring the restrictiveness of a country's trade policy and Anderson and Neary have derived an extremely useful way of doing this—computing a trade restrictiveness index that has a firm basis in economic theory. Little work has been done to date, however, on how sensitive the calculated TRIs and UTEs are to assumed elasticity values and to changes in exogenous variables. This paper has evaluated the robustness of the Anderson-Neary TRI and the UTE with a small-scale CGE model to changes in the value of the elasticity of substitution between factors of production and to the pattern of economic growth.

The simulations in this paper demonstrate that changes in the value of the elasticity of substitution between labor and capital can result in fairly large changes in the UTE. Perhaps more striking is the result that the UTE is quite sensitive to the assumed model structure used in the CGE. For example, the differences between UTEs calculated with a specific-factor's model and an all factors mobile model could be quite large. In all cases examined, a specific-factor's model generated UTEs that were always larger than the UTEs calculated with a all factors mobile model.

In calculating trade restrictiveness, Anderson and Neary recommend that a UTE be calculated for two alternative years and that these years be close to each other. This paper points out that between the two years, the structure of the economy is likely to have changed as a result of economic growth. Therefore, in measuring trade restrictiveness, it is an advantage to have the UTE capture the changing structure of the economy. Rapidly growing economies, such as China which may be growing around ten percent a year, could experience large changes in their UTEs.

This paper has shown that it is very difficult analytically to decompose the effect of changes in a country's TRI (and UTE) into parts that are simply due to changes in trade policy and the part from economic growth. Therefore, in calculating UTEs, changes in exogenous variables, such as factor accumulation, need to be taken into account.

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Appendix: Structure of the Applied General Equilibrium Model

Model Structure

This paper uses an applied general equilibrium model of the Egyptian economy that consists of six sectors (oil, service exports, manufactured exports, agriculture, imported manufactures, and a nontraded good) and eight factors of production (labor, capital, and a sector-specific factor). Labor and capital are mobile across all sectors. A representative household receives all factor income, as well as all revenue collected from taxation. Egypt is assumed to be a small country, so the terms of trade are exogenous. The price of nontraded goods adjusts to bring about equilibrium in the goods market.

Production Structure

Value added in each sector VA_j is produced by combining a labor input L_j, with capital K_j and a specific factor F_j according to a constant elasticity of substitution (CES) production function:

$$X_j = A_j[\alpha_j L_j^{-\rho_j} + \beta_j K_j^{-\rho_j} + (1-\alpha_j-\beta_j)F_j^{-\rho_j}]^{(-1/\rho_j)} \qquad (1)$$

where A_j, α_j, and β_j, are constants, and $\rho_j = \frac{(1-\sigma_j)}{\sigma_j}$ where σ_j is the elasticity of substitution between factors in sector j. Note that this specification assumes that the elasticity of substitution among all three factors is the same within a given sector. The allocation of the mobile factors—labor and capital—across sectors is determined by equating the value of the marginal product of each factor with its factor price. For labor, this is where the value of the marginal product of labor equals the aggregate wage rate:

$$W = \frac{\partial X_j}{\partial L_j} PD_j \qquad (2)$$

where PD_j is the consumption price of the jth good and W is the wage rate. Similarly for capital:

$$R = \frac{\partial X_j}{\partial K_j} PD_j \qquad (3)$$

where R is the rental rate on capital. Each factor must be fully employed, so

$$\sum_j L_j = \bar{L} \qquad (4)$$

and

$$\sum_j K_j = \bar{K} \tag{5}$$

The return to the specific factor in each sector, f_j, is determined as a residual (since F_j is fixed) so as to satisfy a zero-profit condition:

$$PS_j VA_j = WL_j + RK_j + f_j F_j \qquad (6),$$

where PS_j is the producer price of good j.

Aggregate income and demand

Aggregate income available for spending by the representative consumer (Y) equals the sum of factor income, government revenue, and foreign borrowing, B, which is assumed to be fixed in terms of the numeraire:

$$Y = W\bar{L} + R\bar{K} + \sum_j f_j F_j + GR + B \qquad (7).$$

Government revenue equals indirect tax revenue plus tariff revenue:

$$GR = \sum_j tx_j PS_j X_j + \sum_j tm_j PW_j MD_j \tag{8}$$

where tx_j is the indirect tax (or subsidy rate if negative) on good j, tm_j is the tariff rate on good j, PW_j is the international price of good j, and MD_j are imports of good j. As imports are treated as perfect substitutes for domestically produced goods, imports equal the difference between domestic demand and production.

Aggregate demand

Absent information on elasticities of demand in Egypt, we assume that a representative consumer maximizes a Cobb-Douglass utility function defined over the six goods. The resulting demand functions are:

$$DD_j = \frac{s_j Y}{PD_j} \tag{9}$$

where PD_j is the consumer price (inclusive of taxes or tariffs), DD_j is the demand for good j, and s_j is the budget share of good j. Of course, with this demand structure, the own-price elasticity of demand is -1, the cross-price elasticities are zero, and the income elasticity of demand is 1.

The prices paid by the consumer differ from the prices received by the producer, due to indirect taxes. Furthermore, for the traded goods, prices paid by the consumer and received by the producer differ from world prices as a result of tariffs on imports. For imported goods:

$$PS_j = PW_j(1 + tm_j) \tag{10}$$

while for exported goods, the producer price equals the world price, since there are no export taxes or subsidies:

$$PS_j = PW_j \qquad (11).$$

For commodities subject to a consumption tax, the price paid by the consumer differs from the price received by the producer according to:

$$PD_j = PS_j(1 + tx_j) \qquad (12).$$

Equilibrium

Equilibrium in the model is achieved when a set of factor prices is found that generates zero profits in each sector and is consistent with full employment of each factor. In this model, the terms of trade are given exogenously, so the price of the nontraded good adjusts to achieve equilibrium. In the nontraded sector, demand must equal supply:

$$DD_N = X_N \qquad (13).$$

For the imported good:

$$DD_M = X_M + MD_M \qquad (14),$$

while for the exported good:

$$DD_X + E_X = X_X \tag{15}$$

where E_j are exports of good j.

Data, Elasticities, and Parameter Values

All data for each of the twenty countries studied are taken from version 6 of the Global Trade Analysis Project (GTAP) database, except for the data on Egypt, which are obtained from Löfgren and El-Said (1999). The simulation results in Tables 2 through 5 were generated using hypothetical values for factor intensities and the substitution elasticities. Parameter values are determined by the technique of calibration, described in Mansur and Whalley (1984). Calibration entails using data on exogenous and endogenous variables in the base year to "solve for" unknown parameter values. Because of this technique, the model will replicate the base year data exactly, that is, the model will produce values for all the endogenous variables that match the observed values.

The results from the simulations in Table 6 and Figure 3 are based on data for the Egyptian economy for 1998, taken from a social accounting matrix compiled by Löfgren and El-Said (1999). Parameter values are determined by the technique of calibration (described above), and thus, the model replicates the structure of the Egyptian economy in 1998. The rates of growth in the capital stock and the labor force are taken from Kheir-El-Din and Moursi (2002). In production, values for the elasticity of substitution are taken from Dimaranan and McDougall (1997). The tariff rate on agricultural goods is taken to be 6.5 percent and 27.2 percent on manufacturing goods. We also model an excise tax of 5 percent on the nontraded good.

Appendix II

This appendix presents an analysis of how factor accumulation affects real income in the presence of protection. The complete model used in the second section of the paper is given here. The model contains an export good, two import goods, and a nontraded good. Each good is produced using labor and sector-specific capital. The zero-profit conditions are:

$$wa_{LE} + r_E a_{KE} = p_E^* \tag{1a}$$

$$wa_{LM1} + r_{M1} a_{KM1} = p_{M1}^*(1+t_1) \tag{2a}$$

$$wa_{LM2} + r_{M2} a_{KM2} = p_{M2}^*(1+t_2) \tag{3a}$$

$$wa_{LN} + r_N a_{KN} = p_N \tag{4a}$$

where p_E^* is the world price of exports, p_{M1}^* and p_{M2}^* are the world prices of imports, t_1 and t_2 are the ad-valorem tariff rates applied to imports, p_N is the price of the nontraded good, a_{ij} is the amount of factor i used per unit of good j, w is the wage rate, and r_j is the return to capital in sector j. The full-employment conditions are:

$$a_{LE} X_E + a_{LM1} X_{M1} + a_{LM2} X_{M2} + a_{LN} X_N = L \tag{5a}$$

$$a_{KE} X_E = K_E \tag{6a}$$

$$a_{KM1} X_{M1} = K_{M1} \tag{7a}$$

$$a_{KM2} X_{M2} = K_{M2} \tag{8a}$$

$$a_{KN} X_N = K_N \tag{9a}$$

where L is the endowment of labor, K_j is the amount of capital used in sector j, and X_j is output of good j. The price of the nontraded good is determined by the requirement that the quantity of the nontraded good demanded equal the quantity supplied:

$$E_N(p_E, p_{M1}, p_{M2}, p_N, U) = G_N(p_E, p_{M1}, p_{M2}, p_N, V) \qquad \text{(10a).}$$

The demand for the nontraded good E_N, equals the derivative of the expenditure function with respect to the price of the nontraded good, p_N, while G_N, the supply of the nontraded good, equals the derivative of the GDP function with respect to the price of the nontraded good. Real income, or utility, is given by U and V is the vector of factor supplies.

The budget constraint for the economy is:

$$G(P_E, P_{M1}, P_{M2}, P_N, V) + t_1 P_{M1}^*(E_{M1} - G_{M1}) + t_2 P_{M2}^*(E_{M2} - G_{M2}) = E(P_E, P_{M1}, P_{M2}, P_N, U), \qquad \text{(11a)}$$

where $G(P_E, P_{M1}, P_{M2}, P_N, V)$ is the economy's GDP function, $E(P_E, P_{M1}, P_{M2}, P_N, U)$ is the expenditure function, and P_j and P_j^* are the domestic and world prices of good j respectively. The subscripts E, M, and N denote the exportable, importable, and nontraded sector respectively and a subscript next to the expenditure or GDP function represents partial differentiation with respect to that variable. The terms $t_1 P_{M1}^*(E_{M1} - G_{M1})$ and $t_2 P_{M2}^*(E_{M2} - G_{M2})$ measure tariff revenue on imports of good 1 and 2 respectively.

Totally differentiating (11a) gives the welfare effect of a change in factor endowments, *dV:*

$$dU\left[E_U - t_1P^*_{M1}E_{M1U} - t_2P^*_{M2}E_{M2U}\right] = \left[G_V - t_1P^*_{M1}G_{M1V} - t_2P^*_{M2}G_{M2V}\right]dV$$
$$+\left[t_1P^*_{M1}\left(E_{M1N} - G_{M1N}\right) + t_2P^*_{M2}\left(E_{M2N} - G_{M2N}\right)\right]dp_N \tag{12a}$$

where E_{M1N} and E_{M2N} capture how domestic demand for each imported good (E_{M1} and E_{M2}) changes as a result of a change in the price of the nontraded good and (G_{M1N} and G_{M2N}) measure how output of each imported good (G_{M1} and G_{M2}) changes as a result of changes in the price of the nontraded good.

To see how p_N is affected by a change in endowments, totally differentiate (10a), which gives

$$dP_N = \frac{1}{(E_{NN} - G_{NN})}\left[G_{NV}dV - E_{NU}dU\right] \tag{13a}$$

Substituting the expression for *dU* from (12a) into (13a) gives the effect of a change in factor endowments on the price of the nontraded good:

$$dP_N = \left[\frac{G_{NV}(E_U - t_1 P^*_{M1} E_{M1U} - t_2 P^*_{M2} E_{M2U}) + E_{NU}(t_1 P^*_{M1} G_{M1V} + t_2 P^*_{M2} G_{M2V} - G_V)}{(E_{NN} - G_{NN})(E_U - t_1 P^*_{M1} E_{M1U} - t_2 P^*_{M2} E_{M2U}) + E_{NU} t_1 P^*_{M1}(E_{M1N} - G_{M1N}) + E_{NU} t_2 P^*_{M2}(E_{M2N} - G_{M2N})} \right] dV$$

(14a).

Substituting (14a) for dP_N in equation (12a) gives the welfare effect of a change in endowments dV:

$$dU(E_U - t_1 P^*_{M1} E_{M1U} - t_2 P^*_{M2U} E_{M2U})$$

$$= (G_V - t_1 P^*_{M1} G_{M1V} - t_2 P^*_{M2} G_{M2V}) dV$$

$$+ t_1 P^*_{M1}(E_{M1N} - G_{M1N}) + t_2 P^*_{M2}(E_{M2N} - G_{M2N})$$

$$\left[\frac{G_{NV}(E_U - t_1 P^*_{M1} E_{M1U} - t_2 P^*_{M2} E_{M2U}) + E_{NU}(t_1 P^*_{M1} G_{M1V} + t_2 P^*_{M2} G_{M2V} - G_V)}{(E_{NN} - G_{NN})(E_U - t_1 P^*_{M1} E_{M1U} - t_2 P^*_{M2} E_{M2U}) + E_{NU} t_1 P^*_{M1}(E_{M1N} - G_{M1N}) + E_{NU} t_2 P^*_{M2}(E_{M21N} - G_{M2N})} \right] dV$$

(15a).

Solutions For Endogenous Variables:

Using the model in equations (1a) through (9a), the changes in sectoral outputs are:

$$\hat{X}_E = \left[\frac{\lambda_{LE}\sigma_E\theta_{KE}\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}}{\lambda_{KE}\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]}\right]\hat{K}_E$$

$$+\left[\frac{-\theta_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN}\lambda_{LM1}}{\lambda_{KM1}\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]}\right]\hat{K}_{M1}$$

$$+\left[\frac{-\theta_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN}\lambda_{LM2}}{\lambda_{KM2}\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]}\right]\hat{K}_{M2}$$

$$+\left[\frac{-\theta_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN}\lambda_{LN}}{\lambda_{KN}\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]}\right]\hat{K}_N$$

$$+\left[\frac{\theta_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN}}{\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]}\right]\hat{L}$$

$$+\left[\frac{\theta_{LE}\sigma_E(\lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2})}{\theta_{KE}\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]}\right]\hat{P}_E$$

$$+\left[\frac{-\theta_{KM2}\theta_{KN}\lambda_{LM1}\sigma_{M1}\theta_{LE}\sigma_E}{\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]}\right]\hat{P}_{M1}$$

$$+\left[\frac{-\theta_{KM1}\theta_{KN}\lambda_{LM2}\sigma_{M2}\theta_{LE}\sigma_E}{\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]}\right]\hat{P}_{M2}$$

$$+\left[\frac{-\theta_{KM1}\theta_{KM2}\lambda_{LN}\sigma_N\theta_{LE}\sigma_E}{\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]}\right]\hat{P}_N$$

(16a)

$$
\begin{aligned}
\hat{X}_{M1} = {} & \left[\frac{\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}}{\lambda_{KM1}\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]} \right] \hat{K}_{M1} \\
& + \left[\frac{-\theta_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN}\lambda_{LE}}{\lambda_{KE}\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]} \right] \hat{K}_E \\
& + \left[\frac{-\theta_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN}\lambda_{LM2}}{\lambda_{KM2}\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]} \right] \hat{K}_{M2} \\
& + \left[\frac{-\theta_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN}\lambda_{LN}}{\lambda_{KN}\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]} \right] \hat{K}_N \\
& + \left[\frac{\theta_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN}}{\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]} \right] \hat{L} \\
& + \left[\frac{\theta_{LM1}\sigma_{M1}(\lambda_{KE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2})}{\theta_{KM1}\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]} \right] \hat{P}_{M1} \\
& + \left[\frac{-\theta_{LM1}\sigma_{M1}\theta_{KM2}\theta_{KN}\lambda_{LE}\sigma_E}{\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]} \right] \hat{P}_E \\
& + \left[\frac{-\theta_{LM1}\sigma_{M1}\theta_{KE}\theta_{KN}\lambda_{LM2}\sigma_{M2}}{\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]} \right] \hat{P}_{M2} \\
& + \left[\frac{-\theta_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\lambda_{LN}\sigma_N}{\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]} \right] \hat{P}_N
\end{aligned}
$$

(17a).

$$\hat{X}_{M2} = \left[\frac{\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}}{\lambda_{KM2}\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]} \right] \hat{K}_{M2}$$

$$+ \left[\frac{-\theta_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN}\lambda_{LE}}{\lambda_{KE}\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]} \right] \hat{K}_E$$

$$+ \left[\frac{-\theta_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN}\lambda_{LM1}}{\lambda_{KM1}\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]} \right] \hat{K}_{M1}$$

$$+ \left[\frac{-\theta_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN}\lambda_{LN}}{\lambda_{KN}\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]} \right] \hat{K}_N$$

$$+ \left[\frac{\theta_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN}}{\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]} \right] \hat{L}$$

$$+ \left[\frac{\theta_{LM2}\sigma_{M2}(\lambda_{KE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2})}{\theta_{KM2}\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]} \right] \hat{P}_{M2}$$

$$+ \left[\frac{-\theta_{LM2}\sigma_{M2}\theta_{KN}\lambda_{LM1}\sigma_{M1}}{\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]} \right] \hat{P}_{M1}$$

$$+ \left[\frac{-\theta_{LM2}\sigma_{M2}\theta_{KM1}\theta_{KN}\lambda_{LE}\sigma_E}{\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]} \right] \hat{P}_E$$

$$+ \left[\frac{-\theta_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\lambda_{LN}\sigma_N}{\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]} \right] \hat{P}_N$$

(18a).

$$\hat{X}_N = \left[\frac{\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\theta_{KN}}{\lambda_{KN}\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]}\right]\hat{K}_N$$

$$+\left[\frac{-\theta_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\lambda_{LE}}{\lambda_{KE}\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]}\right]\hat{K}_E$$

$$+\left[\frac{-\theta_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\lambda_{LM1}}{\lambda_{KM1}\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]}\right]\hat{K}_{M1}$$

$$+\left[\frac{-\theta_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\lambda_{LM2}}{\lambda_{KM2}\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]}\right]\hat{K}_{M2}$$

$$+\left[\frac{\theta_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}}{\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]}\right]\hat{L}$$

$$+\left[\frac{\theta_{LN}\sigma_N(\lambda_{KE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN})}{\theta_{KN}\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]}\right]\hat{P}_N$$

$$+\left[\frac{-\theta_{LN}\sigma_N\theta_{KE}\theta_{KM2}\lambda_{LM1}\sigma_{M1}}{\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]}\right]\hat{P}_{M1}$$

$$+\left[\frac{-\theta_{LN}\sigma_N\theta_{KE}\theta_{KM1}\lambda_{LM2}\sigma_{M2}}{\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]}\right]\hat{P}_{M2}$$

$$+\left[\frac{-\theta_{LN}\sigma_N\theta_{KM1}\theta_{KM2}\lambda_{LE}\sigma_E}{\left[\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2}\right]}\right]\hat{P}_E$$

(19a),

where σ_j is the elasticity of substitution between labor and capital in sector j, θ_{ij} is the cost-share of factor i in good j, λ_{ij} is the share of factor i employed in sector j, and a "^" denotes proportional change, i.e. $\hat{X}_M = \dfrac{dX_M}{X_M}$.

Finally, the solution for the wage rate is:

$$\hat{w} = \frac{1}{(\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN} + \lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN} + \lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN} + \lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2})}$$

$$\left[\begin{array}{l} (-\theta_{KE}\theta_{KM1}\theta_{KM2}\theta_{KN})\hat{L} + (\lambda_{LE}\sigma_E\theta_{KM1}\theta_{KM2}\theta_{KN})\hat{P}_E + (\lambda_{LM1}\sigma_{M1}\theta_{KE}\theta_{KM2}\theta_{KN})\hat{P}_{M1} \\ +(\lambda_{LM2}\sigma_{M2}\theta_{KE}\theta_{KM1}\theta_{KN})\hat{P}_{M2} + (\lambda_{LN}\sigma_N\theta_{KE}\theta_{KM1}\theta_{KM2})\hat{P}_N + \\ \left(\frac{\lambda_{LE}}{\lambda_{KE}}\right)\hat{K}_E + \left(\frac{\lambda_{LM1}}{\lambda_{KM1}}\right)\hat{K}_{M1} + \left(\frac{\lambda_{LM2}}{\lambda_{KM2}}\right)\hat{K}_{M2} + \left(\frac{\lambda_{LN}}{\lambda_{KN}}\right)\hat{K}_N \end{array}\right]$$

(20a).