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# PRICING-TO-MARKET, THE INTEREST-RATE RULE, AND THE EXCHANGE RATE

Maurice Obstfeld
University of California, Berkeley

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*University of California, Berkeley*

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**Abstract**

Even when the exchange-rate plays no expenditure-switching role, countries may wish to have flexible exchange rates in order to free the domestic interest rate as a stabilization tool.

The beauty of economics as an intellectual pursuit is its reliance on the interplay of formal theory, statistical analysis, and human events. A master economist must assume away the distracting details of a situation in the interest of mathematical clarity. At the same time, he or she must see how relevant subtleties may affect the interpretation of data and the applicability of different models in real life. Because the ultimate policy decisions at stake are so complex, with such vast potential to do harm or good in the world, economics (and especially macroeconomics) is perpetually unsettled, subject to constant questioning and reassessment. Guillermo Calvo must be ranked as one of the great masters of economics, and one of the most unsettling. Time and again, starting with his classic work on the dynamic inconsistency problem, he has uncovered the hidden crux of a major scientific issue and forced the profession to rethink conventional beliefs.

One area that has undergone extensive rethinking of late is the classic Milton Friedman case for exchange-rate flexibility, according to which floating exchange rates are helpful in cushioning national economies from real

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idiosyncratic shocks. Paradoxically, one influential assault on the Friedman case originates in the idea that the pass-through of exchange-rate changes to domestic prices may be high, another in the idea that pass-through may be very low.

According to Calvo and Reinhart (2002), one factor behind the reluctance of emerging markets to allow large swings in nominally "floating" exchange rates is a relatively rapid pass-through of those swings to *consumer* prices. Rapid pass-though of this kind has two implications. Exchange-rate changes will have a greater potential in the short run to affect domestic inflation, and thereby to impede the pursuit of an inflation target. At the same time, rapid pass-though to all the prices consumers face blunts the exchange rate's impact on international relative prices, and thereby reduces its potential expenditure-switching effects. On both counts, the costs of exchange-rate volatility are higher in emerging markets compared with the benefits that Friedman claimed.

Another way the expenditure-switching effects of the exchange rate can be eliminated is if there is zero pass-through—both to domestic and import prices. This is the polar opposite of the case that Calvo and Reinhart emphasize, but it would prevail if domestic producers and foreign producers of a country's imports both were to preset their prices in terms of the local currency. Devereux and Engel (2003) analyze a formal model that includes this type of local-currency pricing. They show that in their model, welfare-maximizing monetary policies may entail fixed exchange rates. This theoretical analysis is viewed as a major challenge to the Friedman case, and one that is applicable to industrial rather than emerging economies.[1] A foreign-based exporter presetting its price in an emerging-market currency would implicitly be acquiring a contingent asset denominated in that currency while issuing a contingent liability denominated in goods. This practice would therefore contradict the observation of "original sin," which restricts emerging borrowers to issuing liabilities indexed to international currencies.[2] As a result, local-currency pricing of imports is not expected to characterize emerging economies.

[1] See Engel (2002) for further discussion. Obstfeld and Rogoff (2000) argue that while the industrial-country retail or consumer prices of imports indeed appear sticky in domestic currency, wholesale import prices do exhibit some exchange-rate pass-through.

[2] Eichengreen and Hausmann (1999). Calvo and Reinhart (2002) identify the widespread foreign-currency denomination of liabilities as another key factor behind "fear of floating."

The modest goal of the present paper is to reinstate Friedman's case in the industrial-country setting while retaining the Devereux-Engel local-currency pricing framework. A minor modification of their model—the introduction of nontraded goods—is enough to restore the need for exchange flexibility, even when all shocks are real. In my modified model, exchange rate changes still have absolutely no expenditure-switching effects in goods markets. They are necessary, however, to allow countries to pursue independent interest-rate policies in a world of international capital mobility. That is, the rationale for exchange flexibility does not originate in goods markets, as in Friedman, but in asset markets. Divergent interest-rate movements are needed, in turn, to allow the divergent consumption movements implied by idiosyncratic national technology shocks in the presence of nontraded goods. A by-product of my argument is an analysis of equilibria and optimal policies in terms of interest-rate rather than money-supply rules.

# 1 The Model

I adopt the basic setup outlined by Devereux and Engel (2003) but modify it in two ways. First, I model monetary policy as a choice of the nominal interest rate (rather than a monetary aggregate) by the central bank. Second, and more importantly, I introduce nontraded goods in order to illustrate the scope for an independent interest-rate policy in the Devereux-Engel local-currency pricing (LCP) framework. What is the intuition for this last effect? With nontraded goods and flexible prices, a national productivity shock has a disproportionate effect on Home consumption, introducing an ex post asymmetry between the countries. To mimic this response under sticky prices—thereby achieving the best possible (second-best) ex post allocation—authorities must apply a disproportionate interest-rate stimulus in Home.

The basic setup of the model is as follows.

There are two (ex ante) symmetrical countries, Home and Foreign. Each country produces a continuum of tradable goods (Home's indexed by [0,1), Foreign's by [1,2]) and a continuum of nontradable goods (indexed by [0,1]).

Each Home representative agent is an atomistic yeoman producer of one differentiated tradable good $i$ and one differentiated nontradable good $i$, and also supplies labor.[3] The producer of generic goods $i$ maximizes

$$U_0(i) = \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{(C_t(i))^{1-\rho}}{1-\rho} - \kappa L_t(i) \right] \right\},$$

where $C$ is a consumption index, $L$ is labor supply, $\rho > 0$ and $\beta \in (0,1)$. Because of the assumption that the monetary instrument is the nominal interest rate, and that the money supply adjusts endogenously, there is no need to model explicitly the demand for money (see Woodford 2003), and I will assume that any money-demand effects on welfare are negligibly small.

A critical assumption in the model is that of market segmentation between Home and Foreign. A Home producer of tradables can practice third-degree price discrimination, charging distinct same-currency prices in the Home and Foreign markets. By assumption, Home and Foreign consumers (who are also producers of other goods) face prohibitively high costs of arbitraging the resulting international price differentials.

Let $W_t$ be nominal marketable wealth at the start of period $t$, $P_t$ the nominal price of consumption during the period, $T_t$ transfer payments from the government, and $R_{t+1}$ the nominal ex post return on the agent's portfolio. Furthermore, let $Y_j(i)$ be the level of output that Home producer $i$ supplies to the Home tradables market ($j = \mathrm{H}$) and to the Home nontradables market ($j = \mathrm{N}$); let $P_j(i)$ be the corresponding domestic-currency price charged. To the Foreign market Home producer $i$ supplies $Y_{\mathrm{H}}^*(i)$ at Foreign-currency price $P_{\mathrm{H}}^*(i)$ Then the flow budget constraint for producer $i$ is

$$\begin{aligned} & W_{t+1}(i) - (1 + R_{t+1}(i))W_t(i) \\ = \; & P_{\mathrm{H},t}(i)Y_{\mathrm{H},t}(i) + \mathcal{E}_t P_{\mathrm{H},t}^*(i)Y_{\mathrm{H},t}^*(i) + P_{\mathrm{N},t}(i)Y_{\mathrm{N},t}(i) + T_t - P_t C_t(i), \end{aligned} \tag{1}$$

where $\mathcal{E}$ is the Home-currency price of Foreign currency (the nominal exchange rate). There are isomorphic intertemporal maximization problems for the Foreign agents (whose supplies are denoted by asterisks).

To maximize utility each producer must grasp the nature of Home and Foreign demand, which depend in turn on the form of the consumption index. As in Obstfeld and Rogoff (2000), overall consumption depends on consumption of tradables and nontradables,

$$C = \frac{C_{\mathrm{T}}^{\gamma} C_{\mathrm{N}}^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}},$$

[3]Foreign producer $i$ supplies tradable $1+i$ and nontradable $i$.

where the tradables subindex depends on consumption of Home- and Foreign-produced tradables,

$$C_{\mathrm{T}} = 2C_{\mathrm{H}}^{\frac{1}{2}} C_{\mathrm{F}}^{\frac{1}{2}}.$$

In turn, $C_{\mathrm{H}}$, $C_{\mathrm{F}}$, and $C_{\mathrm{N}}$ are CES functions of the available varieties,

$$C_j = \left[\int_0^1 C_j(i)^{\frac{\theta-1}{\theta}} \mathrm{d}i\right]^{\frac{\theta}{\theta-1}},$$

with substitution elasticity $\theta$.[4] Based on these assumptions, demands for the goods produced by individual $i$ take the forms

$$\begin{aligned}
C_{\mathrm{H}}(i) &= \gamma \left(\frac{P_{\mathrm{H}}(i)}{P_{\mathrm{H}}}\right)^{-\theta} \left(\frac{P_{\mathrm{H}}}{P_{\mathrm{T}}}\right)^{-1} \left(\frac{P_{\mathrm{T}}}{P}\right)^{-1} C, \\
C_{\mathrm{H}}^*(i) &= \gamma \left(\frac{P_{\mathrm{H}}^*(i)}{P_{\mathrm{H}}^*}\right)^{-\theta} \left(\frac{P_{\mathrm{H}}^*}{P_{\mathrm{T}}^*}\right)^{-1} \left(\frac{P_{\mathrm{T}}^*}{P^*}\right)^{-1} C^*, \\
C_{\mathrm{N}}(i) &= (1-\gamma) \left(\frac{P_{\mathrm{N}}(i)}{P_{\mathrm{N}}}\right)^{-\theta} \left(\frac{P_{\mathrm{N}}}{P}\right)^{-1} C.
\end{aligned}$$

The exact price indexes entering the price indexes that Home consumers face are defined as follows:

$$P = P_{\mathrm{T}}^{\gamma} P_{\mathrm{N}}^{1-\gamma},$$

$$P_{\mathrm{T}} = P_{\mathrm{H}}^{\frac{1}{2}} P_{\mathrm{F}}^{\frac{1}{2}},$$

$$P_j = \begin{cases} \left[\int_0^1 P_j(i)^{1-\theta} \mathrm{d}i\right]^{\frac{1}{1-\theta}}, & j = \mathrm{H}, \mathrm{N}, \\ \left[\int_1^2 P_j(i)^{1-\theta} \mathrm{d}i\right]^{\frac{1}{1-\theta}}, & j = \mathrm{F}. \end{cases}$$

There are complete markets in claims on future money payments. As in Backus and Smith (1992), the resulting ex post allocation satisfies the condition

$$\frac{C_t^{-\rho}}{P_t} = \frac{(C_t^*)^{-\rho}}{\mathcal{E} P_t^*} \tag{2}$$

in all dates and states, where $C^*$ is Foreign consumption and $P^*$ is the Foreign price level measured in Foreign currency. Since purchasing power parity need not hold ex post in this model, the preceding condition does not generally equalize marginal utilities of consumption internationally.

[4] For $j = \mathrm{F}$, the integration is over the interval [1,2].

Production functions for every variety are given by

$$Y_{\rm H} = AL_{\rm H},\ Y_{\rm N} = AL_{\rm N}$$

in Home and

$$Y_{\rm F}^{*} = A^{*}L_{\rm F}^{*},\ Y_{\rm N}^{*} = A^{*}L_{\rm N}^{*}$$

in Foreign, so that $A$ and $A^*$ are economy-wide productivity shocks. Letting lower-case letters (except for interest rates) denote natural logarithms, I write the stochastic processes followed by the productivity shocks as

$$a_t = \lambda a_{t-1} + u_t,\ a_t^{*} = \lambda a_{t-1}^{*} + u_t^{*}, \tag{3}$$

where $\lambda \in [0, 1]$ and the shocks $u$ and $u^*$ are normally distributed with means of zero and a common variance of $\sigma_u^2$.

Finally, the economy's nominal anchor is provided by the nominal interest-rate setting rule followed by the central bank,

$$\log(1 + i_t) = \bar{\imath} + \psi p_t - \alpha_{\rm H} u_t - \alpha_{\rm H}^{*} u_t^{*}, \tag{4}$$

where $i_t$ is the nominal interest earned between dates $t$ and $t+1$. Foreign's central bank has a corresponding rule,

$$\log(1 + i_t^{*}) = \bar{\imath} + \psi p_t^{*} - \alpha_{\rm F} u_t - \alpha_{\rm F}^{*} u_t^{*}. \tag{5}$$

It would be possible, in general, to add "noise" to these reactions functions by positing that central banks observe with error some of the variables to which they respond. But I will not pursue that generalization at this stage.

# 2 The Flexible-Price Equilibrium

Consider next the model's equilibrium when all prices are flexible and the central banks do not respond to productivity innovations (that is, the $\alpha$ coefficients in the interest-rate rules are all zero). Under flexible-prices, producers set domestic-money prices at a fixed gross markup, $\theta/(\theta - 1)$, over nominal marginal cost (equal to $W/A$ in Home and $W^*/A^*$ in Foreign), where $W$ and $W^*$ are the Home and Foreign nominal wage rates. Using the conditions for the optimal labor-consumption tradeoff,

$$\frac{W}{P} C^{-\rho} = \kappa = \frac{W^{*}}{P^{*}} \left(C^{*}\right)^{-\rho}, \tag{6}$$

along with the price-index definitions, one can derive the flex-price levels of overall consumption for the two countries:

$$C = \left[ \left( \frac{\theta - 1}{\theta \kappa} \right) A^{1-\frac{\gamma}{2}} (A^*)^{\frac{\gamma}{2}} \right]^{\frac{1}{\rho}}, \; C^* = \left[ \left( \frac{\theta - 1}{\theta \kappa} \right) (A^*)^{1-\frac{\gamma}{2}} A^{\frac{\gamma}{2}} \right]^{\frac{1}{\rho}}. \qquad (7)$$

Observe that $C = C^*$ always in the flex-price equilibrium when all goods are tradable (i.e., when $\gamma = 1$). But the equality need not hold when $\gamma < 1$. In the latter case, a country's flex-price consumption depends disproportionately on its own productivity shock. The reason is simple: that shock affects the nontradable as well as the tradable sector.

The formulas in eq. (7) suggest already that a differential response of national interest rates to global and national productivity shocks—and hence, exchange-rate flexibility—will be necessary under sticky prices to mimic the flexible-price consumption responses to productivity shocks.

Using the price-index definitions and eq. (6), one can also establish that in the flex-price equilibrium,

$$\frac{P_{\mathrm{H}}}{P_{\mathrm{F}}} = \frac{A^*}{A} = \frac{P^*_{\mathrm{H}}}{P^*_{\mathrm{F}}}.$$

Thus, despite discriminatory price setting, in this flexible-price setting consumers in Home and Foreign face the same international relative prices in equilibrium, and the global allocation of resources is efficient (subject to the nontradability of nontradable goods). Of course, $P_{\mathrm{N}} = P_{\mathrm{H}}$, and similarly in Foreign.

Equilibrium real interest rates must be consistent with the path of expected consumption growth described by eqs. (3) and (7). Nominal interest rates and the resulting path for the overall money price level must, in turn, be consistent with the required path of equilibrium real interest rates.

Nominal interest rates have their relevant impact on the economy through the intertemporal Euler equation for nominal bond holdings,

$$\frac{C_t^{-\rho}}{P_t} = (1 + i_t)\beta \mathrm{E}_t \left\{ \frac{C_{t+1}^{-\rho}}{P_{t+1}} \right\}. \qquad (8)$$

(There is a parallel equation for Foreign.) Taking logs of the preceding equality and noting that consumption is lognormally distributed, I derive

$$-\rho c_t - p_t = \log(1 + i_t) + \log \beta - \rho \mathrm{E}_t c_{t+1} - \mathrm{E}_t p_{t+1} + \frac{\rho^2}{2}\sigma_c^2 + \frac{1}{2}\sigma_p^2 - \rho \sigma_{cp}.$$

The variances above are endogenous, but because they will be constant over time, it is simple to compute them once we have solutions for the equilibrium levels of $c$ and $p$ in terms of current shocks and the means and variances of future variables. After substituting the policy rule (4) for $\log(1+i_t)$ above, we obtain a difference equation with the unique stable price-level solution:

$$p_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+\psi}\right)^{s+1-t} \rho \left(\mathrm{E}_t \left\{c_{s+1} - c_s\right\}\right) - \frac{1}{\psi}\left(\log\beta + \bar{\iota} + \frac{\rho^2}{2}\sigma_c^2 + \frac{1}{2}\sigma_p^2 - \rho\sigma_{cp}\right). \tag{9}$$

Above, consumption can be expressed in terms of the underlying technology shocks using eq. (7), allowing one to compute directly the equilibrium values of $\sigma_c^2$, $\sigma_p^2$,and $\sigma_{cp}$. I will carry out the analogous calculation for the sticky-price case, and therefore omit it here. At this point I observe only that higher current and expected future consumption growth rates are associated with a higher price level today. The reason is that higher consumption growth requires higher real interest rates. A higher price level raises the real interest rate through a policy channel—a higher nominal interest rate—and through a lowering of inflation expectations. Once the nominal price levels $P$ and $P^*$ are determined, nominal wages and the nominal exchange rate, which is given by $\mathcal{E} = W/W^*$ in this model, are likewise pinned down.

# 3 The Model with Preset Nominal Prices

In the sticky-price version of the model, producers of tradables set their domestic and export prices a period in advance of sales, and must meet all demand that materializes at that price. Prices can be rest fully after one period, but again must be maintained for a period thereafter. Exporters set prices in the currency of the purchaser—there is local-currency pricing (LCP) as in Devereux and Engel (2003). Nontradables producers simply set prices in their respective domestic currencies. While these price dynamics would be oversimplified for many purposes, they do allow us to consider the qualitative stabilization roles of interest and exchange rates in a usefully transparent setting.

Let's consider the price-setting problem of a generic Home producer $i$ who sets prices for date $t$ on date $t-1$. Because the decision has no repercussions beyond date $t$, we may imagine that the producer chooses prices $P_{\mathrm{H},t}$, $P^*_{\mathrm{H},t}$, and $P_{\mathrm{N},t}$ so as to maximize

$$\mathrm{E}_{t-1}\left\{\frac{C_t(i)^{1-\rho}}{1-\rho}-\kappa L_t(i)\right\}$$

subject to eq. (1),

$$L_t(i)=\frac{Y_{\mathrm{H},t}(i)+Y_{\mathrm{F},t}(i)+Y_{\mathrm{N},t}(i)}{A_t},$$

and the demand equations

$$\begin{aligned}
Y_{\mathrm{H},t}(i) &= \gamma\left(\frac{P_{\mathrm{H},t}(i)}{P_{\mathrm{H},t}}\right)^{-\theta}\left(\frac{P_{\mathrm{H},t}}{P_{\mathrm{T},t}}\right)^{-1}\left(\frac{P_{\mathrm{T},t}}{P_t}\right)^{-1}C_t,\\
Y^*_{\mathrm{H},t}(i) &= \gamma\left(\frac{P^*_{\mathrm{H},t}(i)}{P^*_{\mathrm{H},t}}\right)^{-\theta}\left(\frac{P^*_{\mathrm{H},t}}{P^*_{\mathrm{T},t}}\right)^{-1}\left(\frac{P^*_{\mathrm{T},t}}{P^*_t}\right)^{-1}C^*_t,\\
C_{\mathrm{N},t}(i) &= (1-\gamma)\left(\frac{P_{\mathrm{N}}(i)}{P_{\mathrm{N}}}\right)^{-\theta}\left(\frac{P_{\mathrm{N}}}{P}\right)^{-1}C_t.
\end{aligned}$$

In performing this maximization the atomistic producer takes macro quantities and price indexes (not indexed by $i$ above) as given.

Because all domestic prices are preset and, I assume, known as of date $t-1$, the first-order condition for $P_{\mathrm{H},t}(i)$, for example, is

$$P_{\mathrm{H},t}(i)=\frac{\theta}{\theta-1}\cdot\frac{\kappa P_t\mathrm{E}_{t-1}\left\{C_t/A_t\right\}}{\mathrm{E}_{t-1}\left\{C_t(i)^{-\rho}C_t\right\}}.$$

(I am making an assumption of common knowledge on the part of price setters.) All producers are symmetric, so $C_t(i)=C_t$ in equilibrium, for all $i$, and therefore all set their Home tradable prices equal to

$$P_{\mathrm{H},t}=\frac{\theta}{\theta-1}\cdot\frac{\kappa P_t\mathrm{E}_{t-1}\left\{C_t/A_t\right\}}{\mathrm{E}_{t-1}\left\{C_t^{1-\rho}\right\}}. \tag{10}$$

Similar reasoning leads to the Home firms' pricing formulas for exports and nontradables, respectively:

$$P^*_{\mathrm{H},t}=\frac{\theta}{\theta-1}\cdot\frac{\kappa P_t\mathrm{E}_{t-1}\left\{C^*_t/A_t\right\}}{\mathrm{E}_{t-1}\left\{\mathcal{E}_tC_t^{-\rho}C^*_t\right\}}=\frac{\theta}{\theta-1}\cdot\frac{\kappa P^*_t\mathrm{E}_{t-1}\left\{C^*_t/A_t\right\}}{\mathrm{E}_{t-1}\left\{(C^*_t)^{1-\rho}\right\}}, \tag{11}$$

$$P_{\mathrm{N},t} = \frac{\theta}{\theta - 1} \cdot \frac{\kappa P_t \mathrm{E}_{t-1}\left\{C_t/A_t\right\}}{\mathrm{E}_{t-1}\left\{C_t^{1-\rho}\right\}} = P_{\mathrm{H},t}. \tag{12}$$

The second equality in eq. (11) above is derived using eq. (2). These three pricing equations have isomorphic counterparts for Foreign producers.

From eqs. (10)–(12), the relative tradables prices Home consumers face are

$$\frac{P_{\mathrm{H},t}}{P_{\mathrm{F},t}} = \frac{\mathrm{E}_{t-1}\left\{C_t/A_t\right\}}{\mathrm{E}_{t-1}\left\{C_t/A_t^*\right\}}.$$

(Recall that $P_{\mathrm{F},t}$ is preset, in domestic-currency terms, by Foreign exporters to Home.) Correspondingly Foreigners face relative tradables prices

$$\frac{P_{\mathrm{H},t}^*}{P_{\mathrm{F},t}^*} = \frac{\mathrm{E}_{t-1}\left\{C_t^*/A_t\right\}}{\mathrm{E}_{t-1}\left\{C_t^*/A_t^*\right\}}.$$

The pricing formulas, eqs. (10)–(12), also yield useful information about expected consumption levels. Because $P_{\mathrm{N}} = P_{\mathrm{H}}$, the overall price level is $P = P_{\mathrm{H}}^{1-(\gamma/2)} P_{\mathrm{F}}^{\gamma/2}$. Thus, eq. (10) can be written as

$$\left(\frac{P_{\mathrm{H},t}}{P_{\mathrm{F},t}}\right)^{\frac{\gamma}{2}} = \frac{\theta}{\theta - 1} \cdot \frac{\kappa \mathrm{E}_{t-1}\left\{C_t/A_t\right\}}{\mathrm{E}_{t-1}\left\{C_t^{1-\rho}\right\}}. \tag{13}$$

Likewise, using the formula for the Foreign exporter's price, $P_{\mathrm{F}}$, one finds that

$$\left(\frac{P_{\mathrm{F},t}}{P_{\mathrm{H},t}}\right)^{1-\frac{\gamma}{2}} = \frac{\theta}{\theta - 1} \cdot \frac{\kappa \mathrm{E}_{t-1}\left\{C_t/A_t^*\right\}}{\mathrm{E}_{t-1}\left\{C_t^{1-\rho}\right\}}. \tag{14}$$

Combination of the last two expressions to eliminate relative prices yields:

$$1 = \left(\frac{\theta\kappa}{\theta - 1}\right)^{\frac{1}{\gamma(2-\gamma)}} \frac{\left(\mathrm{E}_{t-1}\left\{C_t/A_t\right\}\right)^{\frac{1}{2\gamma}} \left(\mathrm{E}_{t-1}\left\{C_t/A_t^*\right\}\right)^{\frac{1}{2(2-\gamma)}}}{\left(\mathrm{E}_{t-1}\left\{C_t^{1-\rho}\right\}\right)^{\frac{1}{\gamma(2-\gamma)}}}.$$

One similarly derives the corresponding expression involving expected Foreign consumption,

$$1 = \left(\frac{\theta\kappa}{\theta - 1}\right)^{\frac{1}{\gamma(2-\gamma)}} \frac{\left(\mathrm{E}_{t-1}\left\{C_t^*/A_t^*\right\}\right)^{\frac{1}{2\gamma}} \left(\mathrm{E}_{t-1}\left\{C_t^*/A_t\right\}\right)^{\frac{1}{2(2-\gamma)}}}{\left(\mathrm{E}_{t-1}\left\{(C_t^*)^{1-\rho}\right\}\right)^{\frac{1}{\gamma(2-\gamma)}}}.$$

Lognormality implies the following solution for the expected logarithm of Home consumption:

$$
\begin{aligned}
\mathrm{E}_t c_{t+1} \;=\;& \frac{1}{\rho}\log\left(\frac{\theta-1}{\theta\kappa}\right)+\frac{\gamma(2-\gamma)}{\rho}\left\{\frac{\mathrm{E}_t a_{t+1}+\sigma_{cu}}{2\gamma}+\frac{\mathrm{E}_t a^*_{t+1}+\sigma_{cu^*}}{2(2-\gamma)}\right\}-\frac{\sigma_u^2}{2\rho} \\
& -\left(1-\frac{\rho}{2}\right)\sigma_c^2 \qquad (15)
\end{aligned}
$$

(with a parallel solution for $\mathrm{E}_t c^*_{t+1}$). This expression is critical ingredient in the welfare analysis of monetary policies, because it contributes (via the consumption Euler equation, eq. (8)) to the contemporaneous innovation in consumption, $c_t$, and hence to the variance of consumption and its covariance with technology shocks. In the present setting the overall price level is known a period in advance, so the log of the Euler eq. (8) is

$$
c_t=\mathrm{E}_t c_{t+1}-\frac{1}{\rho}\left[\log\beta+\log(1+i_t)-(p_{t+1}-p_t)+\frac{\rho^2}{2}\sigma_c^2\right]. \qquad (16)
$$

To solve for the price level now, substitute the interest-rate rule, eq. (4), into eq. (16), and take date $t-1$ expectations to derive a difference equation for $p_t=\mathrm{E}_{t-1}p_t$,

$$
p_t=\frac{1}{1+\psi}\left\{\mathrm{E}_{t-1}p_{t+1}+\rho\left(\mathrm{E}_{t-1}c_{t+1}-\mathrm{E}_{t-1}c_t\right)-\left(\log\beta+\bar{\imath}+\frac{\rho^2}{2}\sigma_c^2\right)\right\}.
$$

I am now allowing the $\alpha$ coefficients in the interest-rate rule to differ from zero, but because the price level for date $t$ is determined a period earlier, the $\alpha$ values do not enter its solution, which is:

$$
p_t=\sum_{s=t}^{\infty}\left(\frac{1}{1+\psi}\right)^{s+1-t}\rho\left(\mathrm{E}_{t-1}\left\{c_{s+1}-c_s\right\}\right)-\frac{1}{\psi}\left(\log\beta+\bar{\imath}+\frac{\rho^2}{2}\sigma_c^2\right).
$$

This is the natural extension of eq. (9) to the case in which $p_t$ is a function only of information dated $t-1$ or earlier. Using eqs. (3) and (15) to substitute for the expected consumption terms above, one finds that

$$
p_t=\frac{\gamma\lambda(2-\gamma)(\lambda-1)}{1+\psi-\lambda}\left[\frac{a_{t-1}}{2\gamma}+\frac{a^*_{t-1}}{2(2-\gamma)}\right]-\frac{1}{\psi}\left(\log\beta+\bar{\imath}+\frac{\rho^2}{2}\sigma_c^2\right). \qquad (17)
$$

A complete solution of the model requires an expression for realized consumption, $c_t$, in terms of the date-$t$ shocks. That expression, in turn, allows computation of the equilibrium values of the key moments $\sigma_c^2$, $\sigma_{cu}$,and $\sigma_{cu^*}$. Combination of eq. (16) with eqs. (3), (15), and (17) yields

$$c_t = \left(\frac{\psi\lambda}{1+\psi-\lambda}\right)\left[\left(\frac{2-\gamma}{2\rho}\right)a_t + \frac{\gamma}{2\rho}a_t^*\right] + \frac{1}{\rho}\left(\alpha_{\mathrm{H}}u_t + \alpha_{\mathrm{H}}^* u_t^*\right) + \Omega, \tag{18}$$

where $\Omega$ is a function only of date $t-1$ (or earlier) information.

Equations (7) and (18) disclose the key difference in consumption dynamics between the flexible and fixed-price cases. In the flexible-price case, assuming that $\alpha_{\mathrm{H}} = \alpha_{\mathrm{H}}^* = 0$, the responses of consumption to technology shocks are given by

$$\frac{\mathrm{d}c_t}{\mathrm{d}a_t} = \frac{2-\gamma}{2\rho},\ \frac{\mathrm{d}c_t}{\mathrm{d}a_t^*} = \frac{\gamma}{2\rho}. \tag{19}$$

With sticky prices, however, eq. (18) shows that the responses of consumption are muted whenever $\lambda < 1$. Why? For $\lambda = 1$, technology follows a random walk and so does log consumption; according to eq. (16), current consumption therefore can adjust fully with no change in the real rate of interest. When $\lambda < 1$, however, consumption is mean-reverting and current consumption can adjust to its flex-price level only if the real interest rate falls. In the flexible-price case, $p_t$ indeed does fall, creating a lower real interest rate both through higher expected inflation and through the associated policy-induced fall in the nominal interest rate $i_t$. In contrast, if $p_t$ is rigid in the short run, the required real interest rate response is muted and so is the rise in $c_t$. By appropriate choice of the policy response coefficients $\alpha_{\mathrm{H}}$ and $\alpha_{\mathrm{H}}^*$ in eq. (4), however, the central bank can induce the full flex-price consumption responses, and I show below that it will wish to do so.

That result also holds in the Devereux-Engel (2003) model with no nontraded goods, as the authors show. Because $\gamma = 1$ in their setting, however, flex-price consumption responses to technology shocks are symmetrical, and so central banks' policy responses are absolutely symmetrical as well. That is not the case when $\gamma < 1$, for then, a relatively more forceful Home interest rate intervention is needed to mimic the flexible-price consumption response. Variable international interest-rate differentials imply exchange-rate variation, however, even though the exchange rate has no expenditure switching effects between Home and Foreign goods in this model.

As a last step before a formal welfare analysis of policy rules, I derive the endogenous covariances entering into the model. To simplify the algebra, let us assume that the productivity shocks are independent, $\sigma_{u\,u^*} = 0$. From eq.

(18),

$$\sigma_c^2 = \left\{ \left[ \left( \frac{\psi\lambda}{1+\psi-\lambda} \right) \left( \frac{2-\gamma}{2\rho} \right) + \frac{\alpha_{\rm H}}{\rho} \right]^2 + \left[ \left( \frac{\psi\lambda}{1+\psi-\lambda} \right) \left( \frac{\gamma}{2\rho} \right) + \frac{\alpha_{\rm H}^*}{\rho} \right]^2 \right\} \sigma_u^2, \tag{20}$$

$$\sigma_{cu} = \left[ \left( \frac{\psi\lambda}{1+\psi-\lambda} \right) \left( \frac{2-\gamma}{2\rho} \right) + \frac{\alpha_{\rm H}}{\rho} \right] \sigma_u^2, \tag{21}$$

and

$$\sigma_{cu^*} = \left[ \left( \frac{\psi\lambda}{1+\psi-\lambda} \right) \left( \frac{\gamma}{2\rho} \right) + \frac{\alpha_{\rm H}^*}{\rho} \right] \sigma_u^2. \tag{22}$$

The expressions involving Foreign consumption $c^*$ are analogous but involve the Foreign interest-rate policy coefficients, $\alpha_{\rm F}$ and $\alpha_{\rm F}^*$. Those coefficients do not enter the Home covariances because of the highly insulating role of exchange rate changes in this particular LCP setting. Exchange rate changes merely facilitate independent monetary policies in a world of international capital mobility.

# 4 Welfare and Optimal Monetary Policy Rules

To assess welfare, observe that Home labor supply must be consistent with

$$\begin{aligned} {\rm E}_t L_{t+1} &= \left(1-\frac{\gamma}{2}\right) \left( \frac{P_{{\rm H},t+1}}{P_{t+1}} \right)^{-1} {\rm E}_t \left\{ \frac{C_{t+1}}{A_{t+1}} \right\} + \frac{\gamma}{2} \left( \frac{P_{{\rm H},t+1}^*}{P_{t+1}^*} \right)^{-1} {\rm E}_t \left\{ \frac{C_{t+1}^*}{A_{t+1}} \right\} \\ &= \left(1-\frac{\gamma}{2}\right) \left( \frac{P_{{\rm F},t+1}}{P_{{\rm H},t+1}} \right)^{\frac{\gamma}{2}} {\rm E}_t \left\{ \frac{C_{t+1}}{A_{t+1}} \right\} + \frac{\gamma}{2} \left( \frac{P_{{\rm F},t+1}^*}{P_{{\rm H},t+1}^*} \right)^{1-\frac{\gamma}{2}} {\rm E}_t \left\{ \frac{C_{t+1}^*}{A_{t+1}} \right\}. \end{aligned}$$

Using eq. (13) and the Foreign analog of eq. (14) to eliminate the relative price terms above, one finds that

$${\rm E}_t L_{t+1} = \left( \frac{\theta-1}{\theta\kappa} \right) \left[ \left(1-\frac{\gamma}{2}\right) {\rm E}_t \left\{ C_{t+1}^{1-\rho} \right\} + \frac{\gamma}{2} {\rm E}_t \left\{ \left( C_{t+1}^* \right)^{1-\rho} \right\} \right].$$

As a result, period Home expected utility can be written as

$${\rm E}_t \left\{ \frac{C_{t+1}^{1-\rho}}{1-\rho} - \kappa L_{t+1} \right\}$$

$$
\begin{aligned}
&= \mathrm{E}_t\left\{\frac{C_{t+1}^{1-\rho}}{1-\rho} - \left(\frac{\theta-1}{\theta}\right)\left[\left(1-\frac{\gamma}{2}\right)\mathrm{E}_t\left\{C_{t+1}^{1-\rho}\right\} + \frac{\gamma}{2}\mathrm{E}_t\left\{\left(C_{t+1}^{*}\right)^{1-\rho}\right\}\right]\right\} \\
&= \frac{\theta-\left(1-\frac{\gamma}{2}\right)(\theta-1)(1-\rho)}{(1-\rho)\theta}\mathrm{E}_t\left\{C_{t+1}^{1-\rho}\right\} - \frac{\gamma}{2}\left(\frac{\theta-1}{\theta}\right)\mathrm{E}_t\left\{\left(C_{t+1}^{*}\right)^{1-\rho}\right\} \\
&= \frac{\theta-\left(1-\frac{\gamma}{2}\right)(\theta-1)(1-\rho)}{\theta(1-\rho)}\exp\left\{(1-\rho)\mathrm{E}_t c_{t+1} + \frac{(1-\rho)^2}{2}\sigma_c^2\right\} \\
&\quad -\frac{\gamma}{2}\left(\frac{\theta-1}{\theta}\right)\exp\left\{(1-\rho)\mathrm{E}_t c_{t+1}^{*} + \frac{(1-\rho)^2}{2}\sigma_{c^*}^2\right\}.
\end{aligned}
$$

I have already noted that the distribution of Foreign consumption, $C^*$, does not depend on the Home interest-rate rule. Therefore, in considering Home's optimal interest-rate rule, I need only consider maximization of the first summand in the last equation with respect to the feedback coefficients $\alpha_{\mathrm{H}}$ and $\alpha_{\mathrm{H}}^*$. Moreover, it is sufficient to maximize

$$
\begin{aligned}
\mathrm{E}_t c_{t+1} + \frac{(1-\rho)}{2}\sigma_c^2 &= \frac{1}{\rho}\log\left(\frac{\theta-1}{\theta\kappa}\right) + \frac{\gamma(2-\gamma)}{\rho}\left\{\frac{\mathrm{E}_t a_{t+1} + \sigma_{cu}}{2\gamma}\right. \\
&\quad \left. + \frac{\mathrm{E}_t a_{t+1}^{*} + \sigma_{cu^*}}{2(2-\gamma)}\right\} - \frac{\sigma_u^2}{2\rho} - \frac{\sigma_c^2}{2},
\end{aligned}
$$

or even more simply, to maximize

$$
V \equiv \frac{(2-\gamma)\,\sigma_{cu}}{2\rho} + \frac{\gamma\sigma_{cu^*}}{2\rho} - \frac{\sigma_c^2}{2}.
$$

Equations (20)–(22) imply that

$$
\begin{aligned}
V \;\propto\; & (2-\gamma)\left[\left(\frac{\psi\lambda}{1+\psi-\lambda}\right)\left(\frac{2-\gamma}{2}\right) + \alpha_{\mathrm{H}}\right] + \gamma\left[\left(\frac{\psi\lambda}{1+\psi-\lambda}\right)\left(\frac{\gamma}{2}\right) + \alpha_{\mathrm{H}}^{*}\right] \\
& -\left\{\left[\left(\frac{\psi\lambda}{1+\psi-\lambda}\right)\left(\frac{2-\gamma}{2}\right) + \alpha_{\mathrm{H}}\right]^2 + \left[\left(\frac{\psi\lambda}{1+\psi-\lambda}\right)\left(\frac{\gamma}{2}\right) + \alpha_{\mathrm{H}}^{*}\right]^2\right\}.
\end{aligned}
$$

Maximization with respect to the policy parameters yields the procyclical responses

$$
\alpha_{\mathrm{H}} = \frac{2-\gamma}{2}\left(1-\frac{\psi\lambda}{1+\psi-\lambda}\right),\ \alpha_{\mathrm{H}}^{*} = \frac{\gamma}{2}\left(1-\frac{\psi\lambda}{1+\psi-\lambda}\right).
$$

The Foreign response coefficients in eq. (5) are analogous,[5] with

$$\alpha_{\mathrm{F}} = \frac{\gamma}{2}\left(1 - \frac{\psi\lambda}{1+\psi-\lambda}\right),\ \alpha_{\mathrm{F}}^{*} = \frac{2-\gamma}{2}\left(1 - \frac{\psi\lambda}{1+\psi-\lambda}\right).$$

A comparison of eqs. (18) and (19) reveals that these policy responses yield a response of consumption to *innovations* in technology that is identical to the flex-price response. They make the variance of consumption equal to its flex-price variance, and induce the flex-price covariances with the shocks to technology. But interest-rate intervention alone cannot bring the world economy to the flex-price consumption levels. Policy optimization thus yields a strictly second-best allocation, with welfare below the flexible-price level. In particular, as Devereux and Engel (2003) note, consumers will in general face the wrong relative prices in the preset-price equilibrium, prices that do not reflect true levels of relative economic scarcity.

# 5 The Need for Exchange-Rate Flexibility

A key point about the preceding second-best interest-rate rules is that they predict *asymmetric* national responses to technology shocks—except in the special case of no nontradables ($\gamma = 1$) that Devereux and Engel analyze. A useful way to think about this asymmetry is to define the mutually orthogonal global and idiosyncratic shocks

$$u_{\mathrm{W}} \equiv \frac{u+u^{*}}{2},\ u_{\mathrm{D}} \equiv \frac{u-u^{*}}{2}.$$

Then one can express the second-best interest-rate rules for Home and Foreign, respectively, in the simple forms

$$\begin{aligned}\log(1+i_t) &= \bar{\imath} + \psi p_t - \left(1 - \frac{\psi\lambda}{1+\psi-\lambda}\right) u_{\mathrm{W},t} - (1-\gamma)\left(1 - \frac{\psi\lambda}{1+\psi-\lambda}\right) u_{\mathrm{D},t}, \\ \log(1+i_t^{*}) &= \bar{\imath} + \psi p_t^{*} - \left(1 - \frac{\psi\lambda}{1+\psi-\lambda}\right) u_{\mathrm{W},t} + (1-\gamma)\left(1 - \frac{\psi\lambda}{1+\psi-\lambda}\right) u_{\mathrm{D},t}.\end{aligned}$$

[5] It is easily verified that these coefficients also maximize the equal-weights "world planner" welfare function $\frac{1}{2}U + \frac{1}{2}U^{*}$, as Devereux and Engel (2003) also find. Thus, the Nash equilibrium in a policy-rule setting game between the countries is efficient—there is no coordination failure in this model, though that is a model-specific result (Obstfeld and Rogoff 2002, Benigno and Benigno 2003). It may also be checked that at this optimum, the value that $\psi$ (the response to the price level) takes in $(0, \infty)$ is irrelevant for welfare. That is, the optimal choice of the $\alpha$ coefficients fully offsets any welfare effect of $\psi$.

The countries respond identically to the global shock in all cases, but have oppositely signed responses to the idiosyncratic shock when there are nontradable goods and, consequently, $\gamma < 1$. I noted the intuition for this result in the introduction: when $\gamma = 1$, productivity shocks in either country have perfectly symmetrical consumption effects in the flex-price equilibrium, so internationally symmetrical interest-rate responses always suffice to induce the flex-price response to any shock. That is, when all goods are tradable, it is optimal for central banks to respond only to global shocks and to respond with equal interest-rate changes. Nontradables change this. Because a domestic technology shock has a stronger effect on domestic than on foreign consumption, a relatively more forceful domestic interest-rate response is required.

This asymmetry has implications for exchange rates, because, given an interest-rate parity condition, divergent interest-rate movements will call for exchange-rate changes. To see this formally, observe that the Home and Foreign bond Euler equations of a Home investor may be combined to yield the exchange-rate equation

$$\mathcal{E}_t = \left(\frac{1+i_t^*}{1+i_t}\right) \frac{\mathrm{E}_t\left\{\mathcal{E}_{t+1} C_{t+1}^{-\rho}\right\}}{\mathrm{E}_t\left\{C_{t+1}^{-\rho}\right\}}.$$

After taking logs, substituting the optimal interest-rate rules, the international risk-sharing condition, and the equations for ex post consumption levels, one concludes that, apart from additive constants, the log exchange rate under optimal monetary policies is given by

$$e_t = 2(1-\gamma)\left(1 - \frac{\psi\lambda}{1+\psi-\lambda}\right) u_{\mathrm{D},t} + \frac{2(1-\gamma)\psi}{1+\psi}\left(\frac{\psi\lambda}{1+\psi-\lambda}\right) a_{\mathrm{D},t} + \frac{\mathrm{E}_t e_{t+1}}{1+\psi}.$$

This expression makes it clear that idiosyncratic technology shocks will induce exchange-rate movements through the asymmetric response of consumption, something that does not occur in this model when $\gamma = 1$ and all goods are tradable.

Exchange-rate changes do not emerge from optimal second-best monetary policies because they switch expenditure in commodity markets. Instead, the rationale for exchange-rate flexibility lies in the asset markets. Exchange-rate adjustment makes room for expenditure-changing interest-rate policies, and they do so by offsetting the incipient expected return differentials that divergent interest-rate movements would otherwise cause. To enjoy the benefits of both activist monetary policy and open capital markets, governments must allow the exchange rate to move.

# 6 Conclusion

Even when the exchange rate plays no expenditure-switching role, as is true in the model of Devereux and Engel (2003), countries may wish to have flexible exchange rates in order to free the domestic interest rate as a stabilization tool. This can be true even when all shocks are real. Why does no need for exchange flexibility arise in the Devereux-Engel model with exclusively tradable goods? There, national consumptions move in a perfectly synchronized fashion when all shocks are real, whether prices are flexible or preset. Optimal monetary policy simply raises the sticky-price consumption response to its flexible-price level, a job that can be accomplished through globally symmetric monetary policies that maintain asset-market equilibrium without the need for exchange-rate changes. In contrast, nontraded goods make national consumption responses to asymmetric real shocks asymmetric themselves. In that case, optimal monetary policy requires a relatively greater monetary stimulus in the country experiencing the shock, and consequently, a change in the international nominal interest-rate differential and in the exchange rate.

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