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Stability and Security.

Stress Testing Market Risks and Derivatives Portfolios

Presentation at the Conference

Macroprudential Supervision: Challenges for Financial Supervisors

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Agenda

I. **Stress tests for market risk: basic concepts**

II. **Maximum Loss: a stress testing method uncovering portfolio-specific worst-case scenarios**

III. **Stress testing: integrating market and credit risk (the methodology of the “Systemic Risk Monitor”)**

An Example

- Swap entered by SK Securities in Jan. 97 (see Gay et al. (1999))
- Payout after 1 year depended on FX rates of THB, IDR, JPY vis-à-vis USD:

$$\text{Payout} = \text{USD 53m} \cdot \left[5 \cdot \left(\frac{THB_0}{THB_2} - 1\right) + \text{Max}\left(0, \frac{3 \cdot IDR_0 - IDR_1 - IDR_2}{IDR_2}\right) + \text{Max}\left(0, 1 - \frac{JPY_0}{JPY_2}\right) - 0.97 \right]$$

If positive: a profit; if negative: a loss

- Decision based on historical volatilities

	THB	IDR	JPY	VaR
Volatility p.a.	1.23%	2.20%	6.88%	USD 16 m

What really happened

	THB	IDR	JPY	Loss
Depreciation (1y)	51.8%	77.9%	2.9%	**USD 189 m**

- How a stress test could have looked like

	THB	IDR	JPY	Loss
Scenario 1: minor crisis	-15%	-15%	0%	USD 58 m
Scenario 2: midsize crisis	-30%	-30%	0%	USD 116 m
Scenario 3: major crisis	-50%	-50%	0%	**USD 184 m**

Stress Testing Market Risk

Ingredients for stress testing

- Portfolio: the trading book (subject to market risk)

  Balance sheet positions
  - Bonds
  - Equity

  Derivatives
  - Interest rate derivatives (swaps, bond-options, caps, floors, …)
  - Equity derivatives (equity options, index futures, …)
  - FX options
- Scenarios: possible future states of the market
  $\boldsymbol{r} = (r_1,...,r_n)$ vector of risk factor values
  r_i are: interest rates, exchange rates, equity indices etc.
- Portfolio valuation function P as a function of $\boldsymbol{r}$: $P = P(\boldsymbol{r})$
- Current state of the market: $\boldsymbol{r}_{CM}$
- Hence, current portfolio value: $P(\boldsymbol{r}_{CM})$

Performing stress tests

1. Select scenarios $\boldsymbol{r}_{stress1}$, $\boldsymbol{r}_{stress2}$,... (according to some selection principle)
2. Calculate portfolio values $P(\boldsymbol{r}_{stress1})$, $P(\boldsymbol{r}_{stress2})$,…
3. Derive some measure of riskiness of the scenarios

How to Select Scenarios

- Standard scenarios
  - E.g. 200 bp interest rate shift
- Historical scenarios
  - Replay a historical crisis
  - Historically observed risk factor changes
- Subjective worst-case scenarios
  - Initial shock is translated into risk factor changes
  - Involvement of a wide range of staff, including senior management

Example: Interest Rate Risk in the Banking Book

- Standardized framework according to the Basel document on the principles for the management and supervision of interest rate risk
- Part of Basel II - Pillar 2
- Coverage: interest rate sensitive positions of the banking book (on- and off-balance sheet)
- Scenario: 200 basispoint shift of yield curves in all currencies
  - Per currency: take the worst case depending on the distribution of assets and liabilities in a re-pricing scheme
- Compare resulting decline in economic value to the sum of Tier 1 and 2 capital
  - Above a 20% threshold: bank considered as outlier

Dangers of Scenario Selection

- A stress scenario for one portfolio might be a lucky strike for another portfolio
- Standard and historical scenarios may nourish a false illusion of safety
- Subjective worst-case scenarios might be too implausible to trigger management action

Requirements for “objective worst-case scenarios”:

- Scenarios should be portfolio-specific
- There should be some “objective” measure of plausibility
- Consider only scenarios with minimal level of plausibility
- Within plausible scenarios, look for the most harmful one

Maximum Loss: Framework for selecting objective worst-case scenarios

Maximum Loss

- Good overview on Maximum Loss in doctoral thesis by Studer (1997)
- Chose **trust region** *TR*: Set of scenarios above a certain minimal plausibility threshold
- Maximum Loss defined as: $\mathrm{MaxLoss}_{TR}(P) := \sup_{\mathbf{r} \in TR} \{P(\mathbf{r}_{\mathrm{CM}}) - P(\mathbf{r})\}$
- "Above the plausibility threshold no loss worse than Maximum Loss can happen"

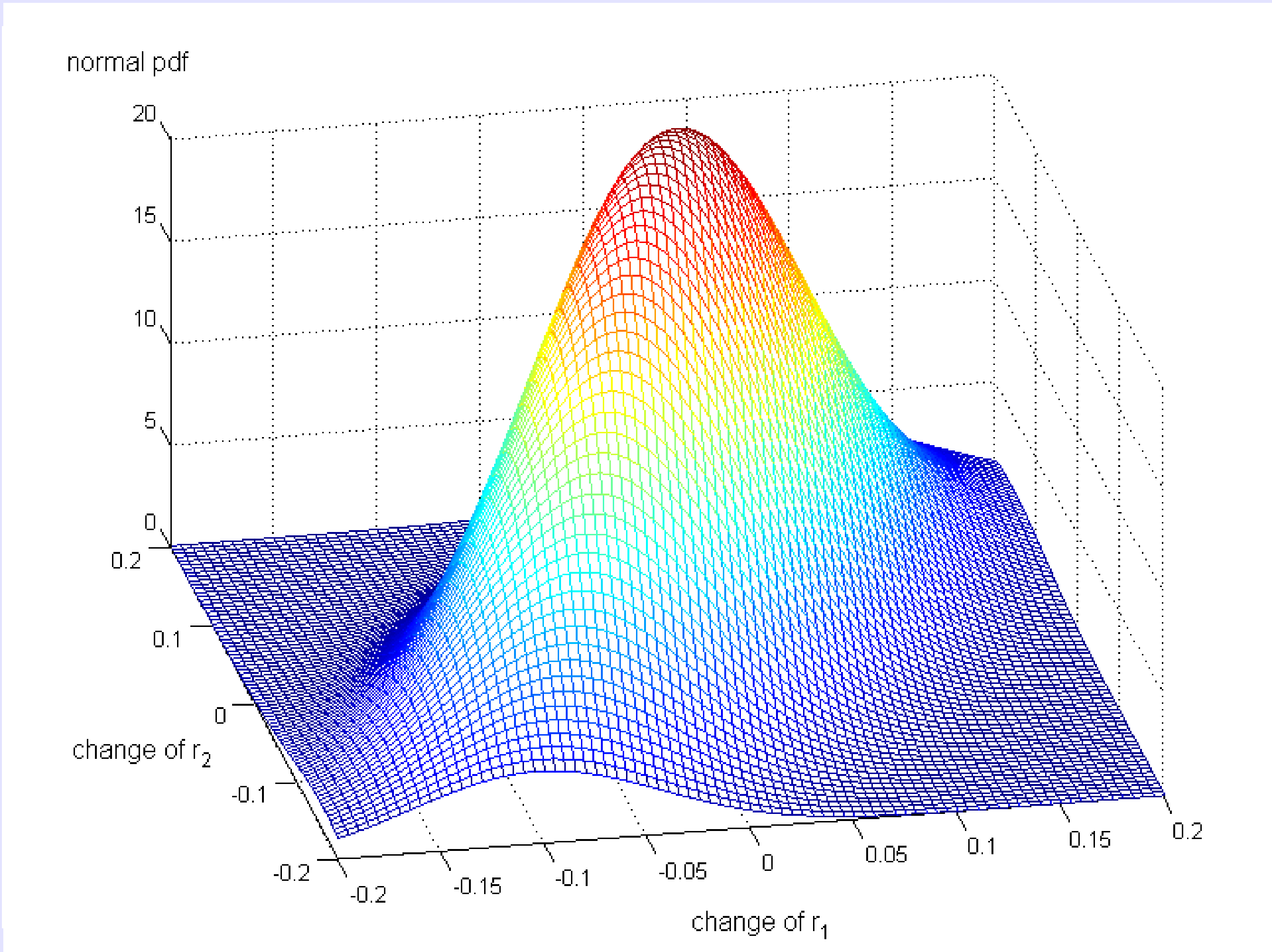
Choice of trust region

- By means of the multivariate risk factor distribution
- Trust region shall have some predefined probability (p) and contain only scenarios with "highest density"
- In case risk factors have an elliptic distribution (e.g. multivariate normal, Student-t): Trust region is an ellipsoid of scenarios with Mahalanobis distance to $\mathbf{r}_{\mathrm{CM}}$ below some threshold k_p:

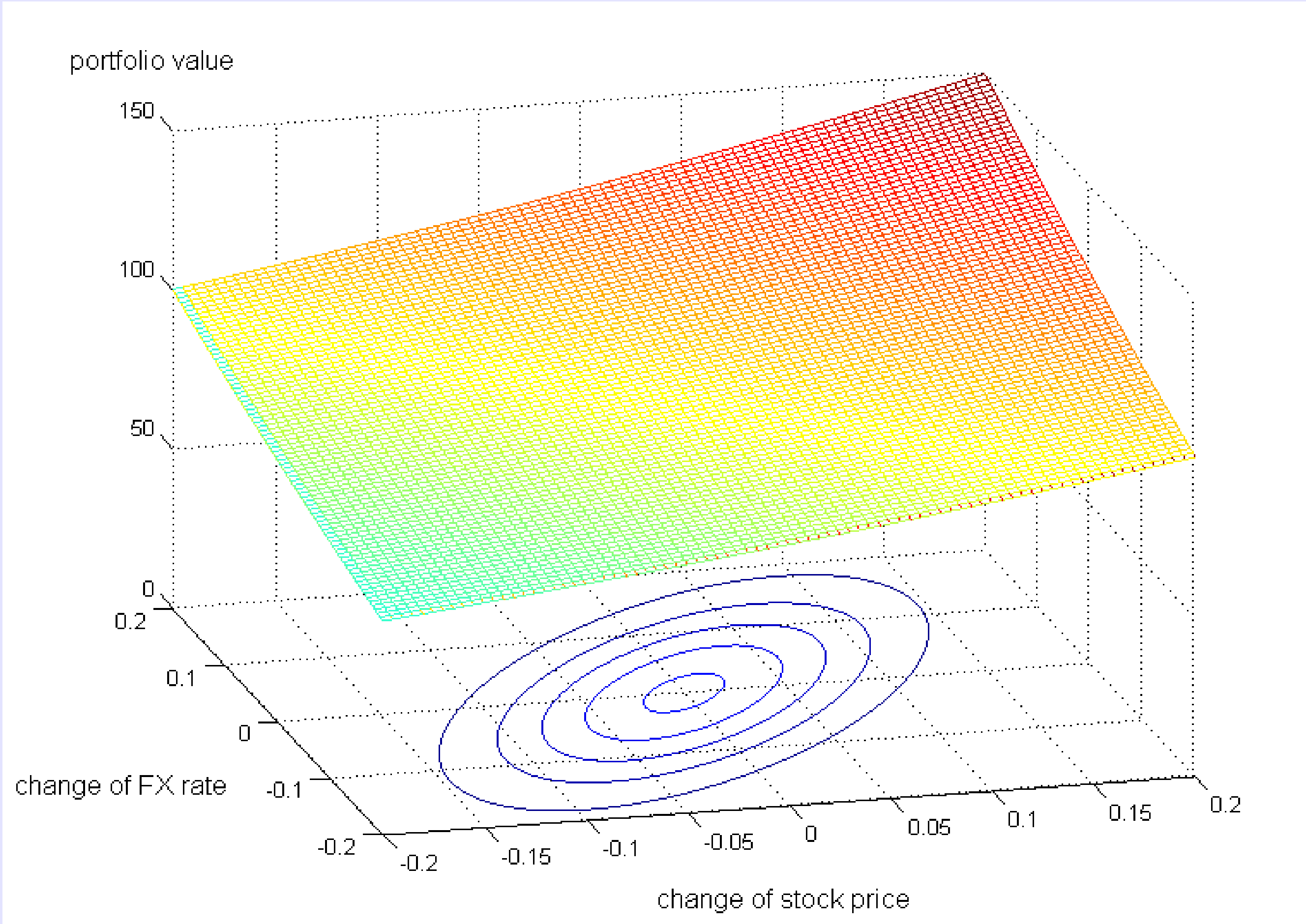
$$TR = \left\{ \mathbf{r} : (\mathbf{r} - \mathbf{r}_{\mathrm{CM}})' \, \Sigma^{-1} (\mathbf{r} - \mathbf{r}_{\mathrm{CM}}) \le k_p^{\,2} \right\}$$

(Σ is the co-variance-matrix)

Trust Region: Area of Highest Density



Within Trust Region: Find Scenario with Smallest Portfolio Value (= Maximum Loss)



Benefits of Maximum Loss

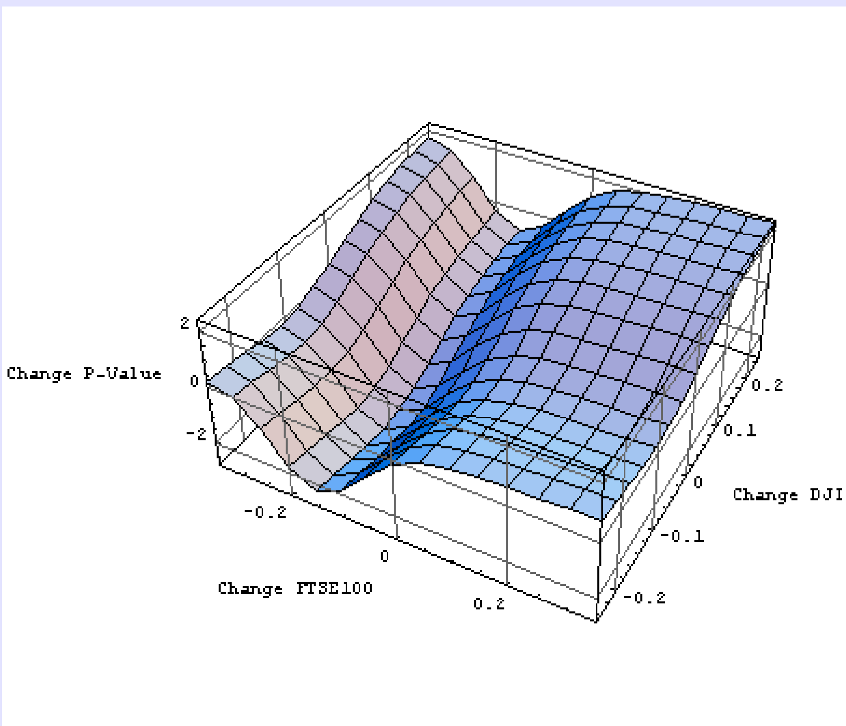
- Maximum Loss not only quantifies risks but also identifies a worst case-scenario
- Searching for worst-case scenarios yields more harmful and more plausible scenarios than other ways of identifying stress scenarios
- Sample portfolio consisting of options on different international stock indices
  - Stress scenarios are identified in different ways
    - Worst-case according to the recommendations of the Derivatives Policy Group
    - Recurrence of Black Friday in October 1987
    - Worst-case scenario implied by Maximum Loss

	Relative Loss	Plausibility
Worst DPG	- 183%	once in 10 yrs
Black Friday	- 154%	once in 19 yrs
Worst Case (ML)	**- 279%**	once in **8** yrs

Benefits of Maximum Loss

Identifying key risk factors of the worst case scenario = Locating the vulnerable spots of a portfolio

Example: Again option portfolio



	Risk Factors	Rel. Changes	Loss	Explanatory Power
Report 1	**FTSE100**	**-13%**	**206** %	**74%**
Report 2	**FTSE100** **DJI**	**-13%** **-8%**	**264** %	**94%**

$$\text{Explanatory Power} = \frac{\text{Loss}\,(\mathbf{r}_{\text{report}})}{\text{Loss}\,(\mathbf{r}_{\text{worst case}})}$$

The Problem of Dimensional Dependence

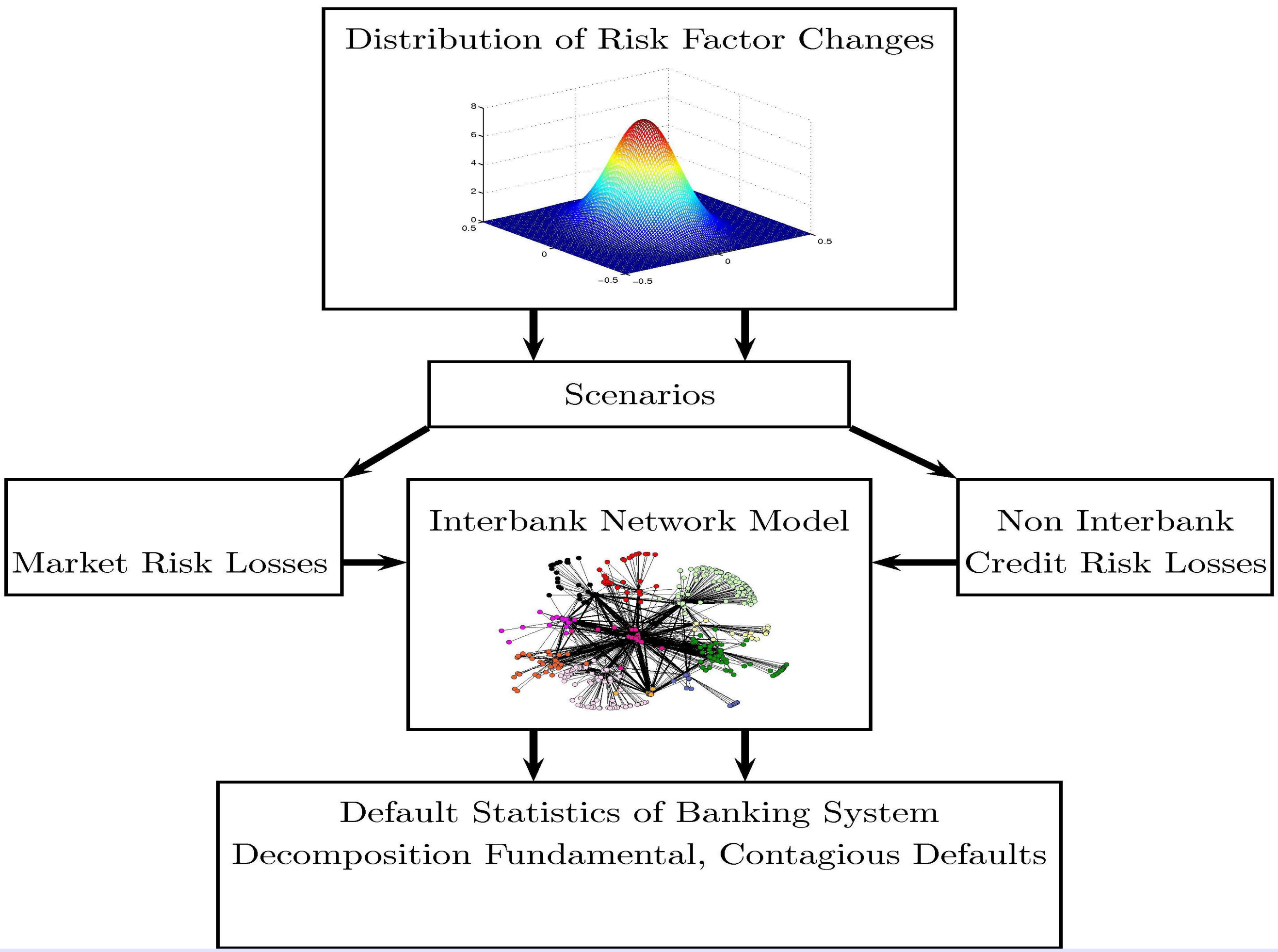
- n : number of risk factors
- Consider elliptic risk factor distributions; then trust regions are ellipsoids
- Trust region shall have probability p
- k : radius of ellipsoid
- n, p, and k depend on each other: e.g. p depends on k and n
  In case of the normal distribution:

$$p(k,n) = 1 - F_{\chi_n^2}(k^2) = 1 - \frac{1}{2^{n/2}\Gamma(n/2)} \int_0^{k^2} s^{\frac{n}{2}-1} e^{-\frac{s}{2}} ds$$

- To keep p fixed: k has to increase as n increases
- If we add an "empty risk factor" (i.e. a factor on which the portfolio value does not depend),
  k has to increase in order to hold p fixed
- We therefore search for MaxLoss within a larger trust region when we add an empty risk factor
- Also MaxLoss is likely to be larger once having added an empty risk factor

==> Makes it hard to compare Maximum Loss across portfolios

Systemic Risk Monitor (SRM) – Basic Structure



Stress Testing in SRM

- 26 market risk factors + 8 credit risk factors = 34 risk factors
- The time horizon in SRM is 3 months
- These factors are modeled statistically
  - Allows for a Monte Carlo-simulation for **analyzing the actual situation** (sampling from the un-conditional distribution)
  - Allows for a Monte Carlo-simulation for **stress testing** (sampling from the conditional distribution)
- For stress testing, a set of risk factors is set to some predefined values
- Remaining factors are sampled from the conditional distribution

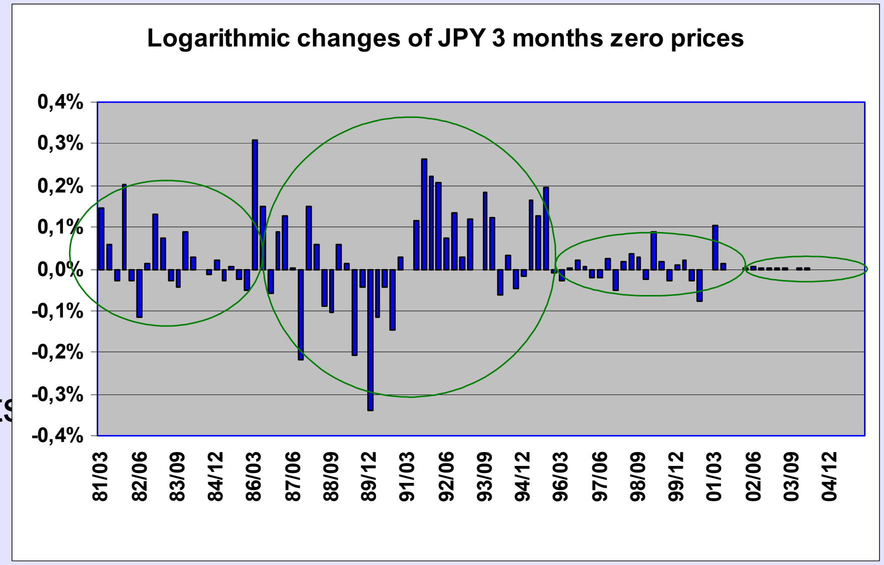
- Stress is considered in two ways
  1. **Direct stress** from the stressed risk factors
  2. **Indirect stress** (“statistical feedback”) from the remaining risk factors that are influenced by the stressed risk factors

Statistical Modeling of Risk Factors

- Multivariate distribution of risk factors is estimated in a 2-step procedure:
  - Step 1: Modeling of **marginal distribution** of each risk factor by models which are optimized with respect to their out-of sample density forecast
  - Step 2: Modeling of **dependencies** between individual risk factors by a grouped t-copula
- Our goal is to have enough flexibility in order to capture
  - Marginal distributions of the various risk factors
  - Patterns of dependence between risk factors
- Market risk factors and credit risk factors are treated in a common statistical model

Marginal Distributions: Model Selection

- No aggregation of higher frequency data, i.e. use quarterly data directly



- GARCH
  - Testing procedure favors consideration of GARCH effects
  - Makes sense for analysis of current situation
  - Should be used with care for stress tests

- Distribution of Residuals
  - Extreme value distribution performs best in the test procedures
  - Simulations show that extreme value distribution leads to too extreme movements
  - SRM now uses t-distribution as marginals

Modeling Dependencies: Grouped t-Copula

- **Copula** models dependencies between risk factors
  - Copula is the part of the multivariate distribution which is not contained in the marginal distributions
- Concept of t**ail-dependence** for assessing dependencies
  - The coefficient of tail-dependence λ between two variables is defined as:

$$\lambda := \lim_{v \to 1^-} \mathbb{P}(X_1 > G_1^{-1}(v) \mid X_2 > G_2^{-1}(v)) > 0;$$

  - Is roughly speaking the probability that one variable is very large (small) given the other variable is very large (small)
  - In case $\lambda > 0$, "one variable can pull up (down) the other variable"
- For the multivariate normal distribution we have $\lambda = 0$ (no tail-dependence)
  - Real data show tail-dependence
- An alternative is given by the t-copula
  - There is tail-dependence between risk factors ($\lambda > 0$)
  - Scenarios can be generated easily in a Monte Carlo-simulation
  - Drawback: between all risk factors there is the same tail-dependence

Modeling Dependencies: Grouped t-Copula

- As an alternative to the t-copula the **grouped t-copula** was introduced by Daul et al. (2003)
  - Risk factors are arranged into groups
  - Within each group risk factors have the same tail-dependence
  - Each group is characterized by a parameter (degrees of freedom)
- Grouped t-copula was adopted for SRM
  - Is suited equally well for MC-simulations as the plain t-copula
  - In SRM risk factors were arranged into 4 groups (in parentheses: estimated degrees of freedom)
    - Credit risk factors (20)
    - FX (14)
    - Equity (5)
    - Interest rates (11)

Literature

Basel Committee on Banking Supervision (2004): “Principles for the Management and Supervision of Interest Rate Risk”

Daul S., E. DeGeorgi, F. Lindskog, A. McNeil (2003): “The grouped t copula with an application to credit risk”, *RISK* Vol. 16, pp 73-76

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