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Abstract

We analyze the cyclical effects of moving from risk-insensitive (Basel I) to risk-sensitive (Basel II) capital requirements in the context of a dynamic equilibrium model of relationship lending in which banks are unable to access the equity markets every period. Banks anticipate that shocks to their earnings as well as the cyclical position of the economy can impair their capacity to lend in the future and, as a precaution, hold capital buffers. We find that the new regulation changes the behavior of these buffers from countercyclical to procyclical. Yet, the higher buffers maintained in expansions are insufficient to prevent a significant contraction in the supply of credit at the arrival of a recession. We show that cyclical adjustments in the confidence level behind Basel II can reduce its procyclical effects without compromising banks' long-run solvency.

Keywords: Banking regulation, Basel II, Business cycles, Capital requirements, Credit crunch, Loan defaults, Relationship banking.

JEL Classification: G21, G28, E43

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1 Introduction

A widespread concern about the new risk-sensitive bank capital regulation, known as Basel II, is that it might amplify business cycle fluctuations, forcing banks to restrict their lending when the economy goes into recession. Even in the old regime of essentially flat capital requirements of the 1988 Basel Accord (Basel I), bank capital regulation has the potential to be procyclical because bank profits may turn negative during recessions, impairing banks' lending capacity. Additionally, the capital requirements prescribed by the Internal Ratings Based (IRB) approach of Basel II are an increasing function of banks' estimates of the probability of default (PD) and loss given default (LGD) of each loan, and these inputs are likely to rise in downturns. So the concern about Basel II is that the increase in capital requirements during downturns might lead to a severe contraction in the supply of credit.

Two key conditions are necessary for these contractionary effects to occur. First, some banks must find it difficult to respond to the higher capital requirements by issuing new equity. Second, some of their borrowers must be unable to switch to other sources of finance.[1] However, these conditions are not sufficient for the existence of significant procyclical effects, since banks anticipate that shocks to their earnings as well as the cyclical position of the economy can impair their capacity to lend in the future and, as a precaution, may hold capital in excess of the regulatory requirements. The critical question is whether capital buffers (that will endogenously respond to regulatory changes) will be sufficient to neutralize the procyclicality added by the new requirements.

This paper analyzes the cyclical effects of Basel II in the context of a tractable dynamic equilibrium model of relationship banking in which the business cycle is modeled as a two-state Markov switching process. At every date a continuum of entrepreneurs enters the market. They demand funds for two consecutive periods, giving rise to an overlapping generations structure. Consistent with the view that relationship banking makes banks privately informed about their borrowers, we assume that (i) borrowers become dependent

[1] These conditions have been noted by Blum and Hellwig (1995) and parallel the conditions in Kashyap, Stein, and Wilcox (1993) for the existence of a bank lending channel in the transmission of monetary policy.

on the banks with which they first start a lending relationship, and (ii) banks with ongoing relationships have no access to the equity market. The first assumption captures the lock-in effects caused by switching costs and the potential lemons problem faced by banks when a borrower is switching from another bank.[2] The second assumption captures the implications of these informational asymmetries for the market for seasoned equity offerings, which can create prohibitive transaction and dilution costs for urgent recapitalizations. It is consistent with the view of the Basel Committee on Banking Supervision (2004, paragraph 757): "It may be costly for banks to raise additional capital, especially if this needs to be done quickly or at a time when market conditions are unfavorable." It can also be seen as a convenient reduced-form for the observed delays in banks' recapitalizations.[3]

The combination of relationship lending and the inability of banks with ongoing relationships to access the equity market establishes a natural connection between the capital shortages of some banks at a given date and the credit rationing of some borrowers at that date. Yet, each cohort of new borrowers is assumed to be funded by banks that renew their lending relationships, have access to the equity market, and hence face no binding limits to their lending capacity.

In order to isolate the potential cyclicality coming from the supply side of the loan market, we abstract from demand-side cyclicality and aggregate feedback effects that might mitigate and exacerbate, respectively, the aggregate implications of the cyclicality in banks' lending capacity. The model, however, could serve as a building block of a more comprehensive dynamic stochastic general equilibrium model in which, say, part of the production comes from entrepreneurial firms that require relationship bank finance.

We define equilibrium under the assumption of free entry into the banking sector, which implies that the banks' net present value at the dates in which they can issue equity must be zero. We characterize the equilibrium loan rates and banks' capital decisions in each

[2]See Boot (2000) for a survey of the relationship banking literature. Several papers explicitly analyze the costs of switching lenders under asymmetric information (e.g., Sharpe, 1990) as well as the trade-offs behind the possible use of multiple lenders as a remedy to the resulting lock-in effects (e.g., Detragiache *et al.*, 2000). We are implicitly assuming that these alternative arrangements are prohibitively costly.

[3]Barakova and Carey (2001) study the time to recovery of US banks that became undercapitalized in the 1984-1999 period, showing that they needed an average of 1.6 years to restore their capital positions.

state of the economy, and derive a number of comparative statics results. Assuming that equity financing is more costly than deposit financing, we show that capital requirements increase equilibrium loan rates, but have an ambiguous effect on capital holdings. On the one hand, the higher prospects of ending up with insufficient capital call for the holding of larger buffers; on the other hand, higher capital requirements reduce the profitability of future lending, and thus the bank's interest in preserving its future lending capacity. Our analytical expressions suggest that the shape of the distributions of loan losses in different states of the economy matter for determining which effect dominates. Since the impact of capital requirements on the supply of second period loans is, therefore, analytically ambiguous, we assess it numerically.

For the numerical analysis, we describe the distributions of loan losses according to the single risk factor model of Vasicek (2002), which provides the foundation for the IRB capital requirements of Basel II. Under this model, capital requirements have an exact value-at-risk interpretation: required capital is such that it can absorb the potential losses of a loan portfolio over a one-year horizon with a probability (or confidence level) of 99.9%.[4]

We find that when the value of the ongoing lending relationships is large enough and the cost of equity capital is not very large, banks optimally choose to keep capital buffers. Under realistic parameterizations, Basel II leads banks to hold buffers that range from about 2% of assets in recessions to about 5% in expansions. The procyclicality of these buffers reflects that banks are concerned about the upsurge in capital requirements that takes place when the economy goes into a recession. We find, however, that these equilibrium buffers are insufficient to neutralize the effects of the arrival of a recession, which may cause a very significant reduction in the supply of credit.[5] Under the flat capital requirements of Basel I, the same economies would exhibit slightly countercyclical buffers and essentially no credit

[4] As shown by Gordy (2003), the single risk factor model also has the feature that the contribution of a given loan to value-at-risk is additive, that is, it depends on the loan's own characteristics and not of those of the portfolio in which it is included.

[5] Supervisors seem aware of this possibility. For instance, Greenspan (2002) claims that "The supervisory leg of Basel II is being structured to supplement market pressures in urging banks to build capital considerably over minimum levels in expansions as a buffer that can be drawn down in adversity and still maintain adequate capital."

crunch effects.[6] For the purposes of comparison, we also compute the equilibrium in a laissez-faire environment without capital regulation, finding that banks' capital buffers (in this world, pure "economic capital") would be of around 5% and not very cyclical, and credit rationing would be more cyclical (and on average higher) than under Basel I, but less cyclical (and on average lower) than under Basel II.

Our results also show that the probabilities of bank failure under Basel II are likely to be substantially lower than under Basel I and, as one would expect, much lower (about 100 folds!) than in the laissez-faire benchmark. This suggests that the business cycle side-effects of Basel II may have a payoff in terms of the long-term solvency of the banking system. It also suggests the possibility of ameliorating the procyclical impact of Basel II by introducing some small adjustments in the IRB capital requirements. Specifically, we consider the possibility of modifying the cyclical profile of confidence levels in a way that keeps their long-term average at 99.9%, but lessens the target in those situations in which credit rationing turns out to be the highest under the Basel II regime. We find that these adjustments may achieve significant reductions in procyclicality without major costs in terms of banks' long-term solvency.

The papers closest to ours are Estrella (2004), Peura and Keppo (2006), and Zhu (2008). Estrella (2004) considers the dynamic optimization problem of a bank when its dividend policy and equity raising processes are subject to quadratic adjustment costs, loan losses follow a second-order autoregressive process, and bank failure is costly. The paper focuses on the comparison between the optimal capital decisions of the bank in the absence of regulation and under a value-at-risk rule, concluding that they are very different. Peura and Keppo (2006) consider a bank with an asset portfolio of exogenous size in the context of a continuous-time model where raising equity takes time. A supervisor checks at random intervals of time whether the bank complies with a minimum capital requirement. The paper finds that the bank may hold capital buffers in order to reduce the risk of being closed for

[6] Some papers, starting with Bernanke and Lown (1991), point out that the introduction of Basel I caused a credit crunch in the US during the months preceding the cyclical peak of 1990. But no credit crunch episode has been detected after banks adjusted their capital holdings to the higher requirements.

holding insufficient capital when audited. Finally, Zhu (2008) adapts the model of Cooley and Quadrini (2001) to the analysis of banks with decreasing returns to scale, minimum capital requirements, and linear equity-issuance costs. Assuming ex-ante heterogeneity in banks' capital positions, the paper finds that for poorly-capitalized banks, risk-sensitive capital requirements increase safety without causing a major increase in procyclicality, whereas for well-capitalized banks, the converse is true. Relative to these three papers, we simplify the details of the banks' dynamic optimization problem and embed such problem in the context of an equilibrium model of relationship banking with endogenous loan rates. Additionally, we adopt the realistic loan default model of the IRB approach of Basel II and focus on the implications for the dynamics of aggregate bank lending.

The rest of the paper is organized as follows. Section 2 presents the model. In Section 3 we analyze the capital decision of a representative bank. Section 4 defines the equilibrium and provides the comparative statics of equilibrium loan rates and banks' capital holdings. In Section 5 we present the numerical results concerning the size and cyclical behavior of capital holdings, capital buffers, credit rationing, and probabilities of bank failure in a number of parameterizations of the model. In Section 6 we examine adjustments of the Basel II framework that reduce its procyclical effects. Section 7 discusses the robustness of our results to changes in some of the key assumptions of the model. Section 8 concludes. Appendix A contains the proofs of the analytical results, and Appendix B discusses the choice of parameter values for the numerical analysis.

2 The Model

Consider a discrete time economy in which time is indexed by $t = 0, 1, 2, ...$ The economy is populated by three classes of risk-neutral agents: entrepreneurs, banks, and investors.

2.1 Entrepreneurs

Entrepreneurs belong to overlapping generations formed by a continuum of measure one of ex-ante identical and penniless individuals who remain active for up to two periods (three dates).

Entrepreneurs have the opportunity to undertake *investment projects* with the following characteristics. The first period project of an entrepreneur born at date t requires a unit investment at that date. At date $t+1$ the project yields $1+a$ if it is successful, and $1-\lambda$ if it fails, with $a>0$ and $0<\lambda<1$. The second period project of an entrepreneur born at date t requires μ units of investment at date $t+1$. The return at date $t+2$ of this project is independent of the return of the initial project, and equals $(1+a)\mu$ if it is successful, and $(1-\lambda)\mu$ if it fails, so parameter μ measures the scale of the second period project.

All projects operating from date t to date $t+1$ have an identical probability of failure p_t. The outcomes of contemporaneous projects exhibit positive but imperfect correlation, so their aggregate *failure rate* x_t is a continuous random variable with support $[0,1]$ and cumulative distribution function (cdf) $F_t(x_t)$ such that

$$p_t = E_t\left(x_t\right) = \int_0^1 x_t \; dF_t(x_t).$$

For simplicity, we consider the case in which the history of the economy up to date t only affects $F_t(x_t)$ (and, thus, p_t) through an observable *state variable* s_t that can take two values, h and l, and follows a Markov chain with

$$q_h = \Pr\left(s_t = h \mid s_{t-1} = h\right) \text{ and } q_l = \Pr\left(s_t = h \mid s_{t-1} = l\right).$$

Moreover, we assume that the cdfs corresponding to the two states, $F_h(\cdot)$ and $F_l(\cdot)$, are ranked in the sense of first-order stochastic dominance, so that the probabilities of business failure in each state satisfy

$$p_h > p_l.$$

Thus states h and l may be interpreted as recession (high business failure) and expansion (low business failure) states, respectively.

2.2 Banks

Banks are competitive intermediaries specialized in channeling funds from investors to entrepreneurs. Following the literature on relationship banking, we assume that the financing of an entrepreneur in this economy relies on a sequence of one-period loans granted by the single bank from which the entrepreneur obtains his first loan. We also assume that setting up the relationship with the entrepreneur makes the bank incur some cost c, to be subtracted from first period revenue.[7] Finally, for simplicity, we abstract from the possibility that part of the required second period investment μ is internally financed by the entrepreneur.[8]

Banks are funded with deposits and equity capital, both of which are raised from investors. To simplify the analysis we assume that deposits are fully insured (at a zero premium), and their supply is perfectly elastic at a risk-free rate that we normalize to zero.[9] We also assume that investors require an excess return $\delta \geq 0$ on each unit of equity capital. The cost of capital δ is intended to capture in a reduced-form manner distortions (such as agency costs of equity or debt tax shields) that introduce a comparative disadvantage of equity financing relative to deposit financing—in addition to deposit insurance.[10]

We introduce an important imperfection concerning banks' equity financing: While banks entering the market or renewing their portfolio of lending relationships can raise new equity in an unrestricted manner, recapitalization is impossible for banks with ongoing lending relationships. This assumption is intended to capture in a simple way the long delays or prohibitive dilution costs that a bank with opaque assets in place might face when organizing an urgent equity injection.[11]

Banks are managed in the interest of their shareholders, who are protected by limited

[7] This cost might include personnel, equipment and other operating costs associated with the screening and monitoring functions emphasized in the literature on relationship banking.

[8] This simplification is standard in relationship-banking models; see, for example, Sharpe (1990, p. 1072) or von Thadden (2004, p. 14). Moreover, if entrepreneurs' first-period profits are small relative to the required second-period investment (as in our numerical analysis below), the quantitative effects of relaxing this assumption would be negligible.

[9] In our numerical analysis the probability of bank failure is a small fraction of the 0.1% target of Basel II, so the fair deposit insurance premium would be negligible.

[10] Further to the reasons for the extra cost of equity financing offered by the corporate finance literature, Holmström and Tirole (1997) and Diamond and Rajan (2000) provide agency-based explanations specifically related to banks' monitoring role.

[11] These costs are most likely related to asymmetric information. Specifically, in a world in which banks learned about their borrowers after starting a lending relationship (like in Sharpe, 1990) and borrower quality were asymmetrically distributed across banks, the market for seasoned equity offerings might be affected by a lemons problem (like in Myers and Majluf, 1984). Thus, after a negative shock, banks with lending relationships of poorer quality would be more interested in issuing equity at any given price, which explains why the prices at which new equity could be raised may be unattractive to banks with higher-quality relationships and why, in sufficiently adverse circumstances, the market for those SEOs may collapse.

liability. Entry to the banking sector is free at all dates, but banks are subject to a *capital requirement* that obliges them to hold a capital-to-loans ratio of at least γ_s on the loans made when the state of the economy is s. This formulation encompasses both Basel I and Basel II type of regulation, as well as a laissez-faire environment with zero capital requirements. In Basel I the capital requirement is (for corporate loans) a constant ratio $\gamma_l = \gamma_h = 8\%$. Basel II aims at a better alignment of capital requirements with the underlying banking risks, and consequently requires higher capital for riskier loans. In our setup there is no cross-sectional heterogeneity among borrowers but the state of the economy affects the risk of the representative loan, so Basel II amounts to a capital requirement in the high default state, γ_h, higher than the capital requirement in the low default state, γ_l.[12]

To guarantee that the funding of investment projects is attractive to banks at all dates, we assume that

$$(1 - p_s)(1 + a) + p_s(1 - \lambda) - c > (1 - \gamma_s) + \gamma_s(1 + \delta), \tag{1}$$

for $s = h, l$. Thus, in all states of the economy, the expected return per unit of investment, net of the setup cost c, is greater than the cost of funding it with $1 - \gamma_s$ deposits and γ_s capital.

3 Banks' Capital Decision

Consistent with the assumption that banks with ongoing lending relationships may face capital constraints, we assume that entrepreneurs born at date t obtain their first period loans from unrestricted banks that can raise capital at this date. This allows us to analyze the banking industry as if it were made of overlapping generations of banks that operate for two periods, specialize in loans to contemporaneous entrepreneurs, and cannot issue equity

[12] The precise Basel II formula that relates the capital requirement γ_s to the loans' probability of default p_s will be described in Section 5.1. Although Basel II stipulates that estimates of the probability of default "must be a long-run average of one-year default rates" (Basel Committee on Banking Supervision, 2004, paragraph 447), industry practices based on point-in-time rating systems, the dynamics of rating migrations, and composition effects make the effective capital charges on a representative loan portfolio very likely to be higher in recessions than in expansions. See, for example, Kashyap and Stein (2004), Catarineu-Rabell, Jackson, and Tsomocos (2005), Gordy and Howells (2006), and Saurina and Trucharte (2007).

at the interim date.[13]

In this economy, the supply of loans to the entrepreneurs who start up at date t might be affected by the recapitalization constraint faced by their banks at date $t+1$. In fact, banks will be aware of this and, in order to better accommodate the effect of negative shocks to their first period income or possibly higher capital requirements in the case of risk-sensitive capital regulation, they may hold a buffer of equity capital on top of the first period regulatory minimum.

To understand the financing problem faced by each generation of entrepreneurs in this economy, consider a representative bank that lends to the measure one continuum of entrepreneurs starting up at date t, possibly refinances them at date $t+1$, and gets liquidated at date $t+2$.[14] Let s and s' denote the states of the economy at dates t and $t+1$, respectively.

At date t the representative bank raises $1-k_s$ deposits and $k_s \geq \gamma_s$ capital, and invests these funds in a unit portfolio of first period loans. The equilibrium interest rate on these initial loans, denoted r_s, will be determined endogenously, as explained below, but under our perfect competition assumption the bank takes it as given. Since the supply of deposits is perfectly elastic at a zero interest rate, r_s should be interpreted as the spread between initial loan rates and deposit rates.

At date $t+1$ the bank gets $1+r_s$ from the fraction $1-x_t$ of performing loans (that is, those extended to entrepreneurs whose projects are successful) and $1-\lambda$ from the fraction x_t of defaulted loans, and incurs the setup cost c, so its assets are $1+r_s-x_t(\lambda+r_s)-c$, while its deposit liabilities are $1-k_s$. Thus its capital at date $t+1$ is

$$k_s'(x_t) = k_s + r_s - x_t(\lambda + r_s) - c, \tag{2}$$

where x_t is a random variable whose cdf conditional on the state of the economy at date t is $F_s(x_t)$. If $k_s'(x_t) < 0$ the bank fails, while if $k_s'(x_t) \geq 0$ it can operate for a second period. Using the definition of $k_s'(x_t)$, it is immediate to show that bank failure occurs when the

[13] Notice that a bank that can raise capital at date t is essentially identical to a new bank established at that date.

[14] It will become obvious that banks that can issue equity face constant returns relative to the size of their loan portfolio.

default rate x_t exceeds the critical value

$$\widehat{x}_s = \frac{k_s + r_s - c}{\lambda + r_s}. \tag{3}$$

The entrepreneurs that start up at date t demand an amount μ of second period loans at date $t+1$.[15] At this stage entrepreneurs are dependent on the bank, so their demand is inelastic as long as the loan interest rate does not exceed the success return of the projects in the second investment period. Thus the second period loan rate will be a.

Since the bank cannot issue new equity at date $t+1$, its *maximum lending capacity* is given by the ratio between its available capital $k_s'(x_t)$ and the capital requirement $\gamma_{s'}$, which depends on the state of the economy s' at date $t+1$. Thus, whenever $k_s'(x_t) \geq 0$ there are two cases to consider: the case with excess lending capacity, $k_s'(x_t) \geq \gamma_{s'}\mu$, and the case with insufficient lending capacity, $k_s'(x_t) < \gamma_{s'}\mu$. Using the definition of $k_s'(x_t)$ in (2), it is immediate to show that the latter case arises when the default rate x_t exceeds the critical value

$$\widetilde{x}_{ss'} = \frac{k_s + r_s - c - \gamma_{s'}\mu}{\lambda + r_s}, \tag{4}$$

which is obviously smaller than $\widehat{x}_s$, defined in (3). Thus, whenever $0 < \widetilde{x}_{ss'} < \widehat{x}_s < 1$, one can find three different situations at date $t+1$, depending on the realization of the default rate: for $x_t \in [0, \widetilde{x}_{ss'}]$, the representative bank has excess lending capacity; for $x_t \in (\widetilde{x}_{ss'}, \widehat{x}_s]$, the bank has insufficient lending capacity; and for $x_t \in (\widehat{x}_s, 1]$ the bank fails. We next derive the expected continuation payoffs of the bank's shareholders in each of the two cases where the bank does not fail.

When there is *excess lending capacity* at date $t+1$ the bank finances μ loans using $(1-\gamma_{s'})\mu$ deposits and $\gamma_{s'}\mu$ capital. Since $k_s'(x_t) \geq \gamma_{s'}\mu$, the bank pays a dividend of $k_s'(x_t) - \gamma_{s'}\mu$ to its shareholders at date $t+1$.[16] At date $t+2$ the bank gets $1+a$ from the fraction $1-x_{t+1}$ of performing loans and $1-\lambda$ from the fraction x_{t+1} of defaulted loans, so its

[15] Note that this includes entrepreneurs that defaulted on their initial loans. This is because under our assumptions such default does not reveal any information about the entrepreneurs' second period projects.

[16] Since entrepreneurs born at date $t+1$ borrow from banks that can raise equity at that date, the bank may use the excess capital to either pay a dividend to its shareholders or to reduce the deposits to be raised at this date. However, under deposit insurance and $\delta \geq 0$, the second alternative is strictly suboptimal.

assets are $[1+a-x_{t+1}(\lambda+a)]\mu$, while its deposit liabilities are $(1-\gamma_{s'})\mu$. Thus shareholders' expected payoff, conditional on the state of the economy at date $t+1$, can be expressed as $\mu\pi_{s'}$, where

$$\pi_{s'} = \int_0^1 \max\left\{\gamma_{s'} + a - x_{t+1}(\lambda + a),\ 0\right\}\ dF_{s'}(x_{t+1}) \tag{5}$$

is the expected gross equity return on a per-unit-of-loans basis. The value of shareholders' stake in the bank at date $t+1$, inclusive of the dividend $k'_s(x_t) - \gamma_{s'}\mu$, can be written as

$$v_{ss'}(x_t) = (\beta\pi_{s'} - \gamma_{s'})\mu + k'_s(x_t), \tag{6}$$

where $\beta = 1/(1+\delta)$ is the shareholders' discount factor implied by the cost of capital δ. The first term in (6) measures the net present value contribution of the capital that remains invested in the bank up to date $t+2$. Assumption (1) guarantees that $\beta\pi_{s'} > \gamma_{s'}$, so that such contribution is positive.[17]

When there is *insufficient lending capacity* at date $t+1$ the bank finances $k'_s(x_t)/\gamma_{s'}$ loans with $[k'_s(x_t)/\gamma_{s'}] - k'_s(x_t)$ deposits and $k'_s(x_t)$ capital. At date $t+2$ shareholders' expected payoff, conditional on the state of the economy at date $t+1$, can be expressed as $\pi_{s'}k'_s(x_t)/\gamma_{s'}$, where $\pi_{s'}$ is the expected gross equity return on a per-unit-of-loans basis given by (5). When there is insufficient lending capacity at date $t+1$ the bank pays no dividends at that date and, hence, the value of shareholders' stake in the bank is just

$$v_{ss'}(x_t) = \frac{\beta\pi_{s'}}{\gamma_{s'}}k'_s(x_t). \tag{7}$$

As before, assumption (1) implies that $\beta\pi_{s'} > \gamma_{s'}$, and hence shareholders strictly benefit from keeping $k'_s(x_t)$ invested in the bank.

Putting together the two cases, as well as the case in which the bank fails, we can express the market value of the bank at date $t+1$, inclusive of dividends, as

$$v_{ss'}(x_t) = \begin{cases} (\beta\pi_{s'} - \gamma_{s'})\mu + k'_s(x_t), & \text{if } x_t \leq \widetilde{x}_{ss'}, \\ \dfrac{\beta\pi_{s'}}{\gamma_{s'}}k'_s(x_t), & \text{if } \widetilde{x}_{ss'} < x_t \leq \widehat{x}_s, \\ 0, & \text{if } x_t > \widehat{x}_s, \end{cases} \tag{8}$$

[17]To see this, notice that $\pi_{s'} > \int_0^1 [\gamma_{s'} + a - x_{t+1}(\lambda+a)]\ dF_{s'}(x_{t+1}) = \gamma_{s'} + a - p_{t+1}(\lambda+a)$, but assumption (1) implies $a - p_{t+1}(\lambda + a) > \delta\gamma_{s'}$ and hence $\pi_{s'} > (1+\delta)\gamma_{s'} = \gamma_{s'}/\beta$.

which is a continuous and piecewise linear function of x_t.[18] Going backward one period, the net present value of the representative bank that in state s holds capital k_s and charges an interest rate r_s on its unit of initial loans is

$$v_s(k_s, r_s) = \beta E_t[v_{ss'}(x_t)] - k_s, \tag{9}$$

where the operator $E_t(\cdot)$ takes care of the fact that, at date t, v_s is subject to the uncertainty about both the state of the economy at date $t+1$ (which affects the second period capital requirement $\gamma_{s'}$ and gross equity return $\pi_{s'}$) and the default rate x_t of initial loans (which determines the capital $k'_s(x_t)$ available at that date).

Taking as given the initial loan rate r_s, the representative bank that first lends to a generation of entrepreneurs in state s will choose its capital k_s so as to maximize $v_s(k_s, r_s)$ subject to the constraint $k_s \in [\gamma_s, 1]$. Since $v_s(k_s, r_s)$ is continuous in k_s, for any given interest rate r_s, the bank's capital decision always has a solution. In Appendix A we show that the function $v_s(k_s, r_s)$ is neither concave nor convex in k_s, and that we may have interior solutions or corner solutions with $k_s = \gamma_s$. When the solution is interior, there is a positive probability that the bank has insufficient lending capacity in the high default state $s' = h$ (and possibly also in the low default state $s' = l$), and there is a positive probability that the bank has excess lending capacity in the low default state $s' = l$ (and possibly also in the high default state $s' = h$). The intuition for this result is as follows. If in the two possible states at date $t+1$ the bank had a zero probability of finding itself with insufficient lending capacity, then it would have an incentive to reduce its capital at date t in order to reduce its funding costs at that date. On the other hand, if in the two possible states at date $t+1$ the bank had a zero probability of finding itself with excess lending capacity, then it would have an incentive either to increase its capital at date t and thereby relax the capital constraint at date $t+1$, or to go to the corner $k_s = \gamma_s$.[19]

[18] Note that $\beta\pi_{s'} > \gamma_{s'}$ implies that if the bank does not fail at date $t+1$ the market value of its equity $v_{ss'}(x_t)$ is strictly greater than the accounting value $k'_s(x_t)$.

[19] The possible preference for the corner $k_s = \gamma_s$ is due to the fact that, in this case, the function $v_s(k_s, r_s)$ is (locally) either decreasing or convex in k_s; see Appendix A for the details.

4 Equilibrium

In the previous section we have characterized banks' capital and lending decisions at the dates in which they can raise capital, as well as at the dates in which they cannot. This analysis has taken as given the interest rate r_s at the beginning of a lending relationship in state s, with the continuation loan rate being the success return a of the second period investment projects. In order to define an equilibrium, it only remains to describe how the initial loan rate is determined.

Given our free entry assumption, in equilibrium the pricing of these loans must be such that the net present value of the bank is zero under the bank's optimal capital decision. Were it negative, no bank would extend loans. Were it positive, incumbent banks would have an incentive to expand, and new banks would profit from entering the market. Hence in each state of the economy $s = h, l$ we must have

$$v_s(k_s^*, r_s^*) = 0, \tag{10}$$

for

$$k_s^* = \arg \max_{k_s \in [\gamma_s, 1]} v_s(k_s, r_s^*). \tag{11}$$

Therefore we may define an *equilibrium* as a sequence of pairs $\{(k_t, r_t)\}$ describing the capital-to-loan ratio k_t of the banks that can issue equity at date t and the interest rate r_t charged on their initial loans, such that each pair (k_t, r_t) satisfies (10) and (11) for $s = s_t$, where s_t is the state of the economy at date t.

The existence of an equilibrium is easy to establish. Differentiating (10) we have

$$\frac{dv_s}{dr_s} = \frac{\partial v_s}{\partial k_s^*}\frac{dk_s^*}{\partial r_s} + \frac{\partial v_s}{\partial r_s},$$

where the first term is zero, by the envelope theorem, and the second is positive, because of the higher interest payments at date $t+1$ (see Appendix A for details). So $v_s(k_s^*, r_s)$ is continuous and monotonically increasing in r_s. Moreover, for sufficiently low interest rates we have $v_s(k_s^*, r_s) < 0$, while for $r_s = a$ assumption (1) implies $v_s(k_s^*, r_s) > 0$. Hence we conclude that there is a unique r_s^* that satisfies $v_s(k_s^*, r_s^*) = 0$.[20]

[20] However, since the function $v_s(k_s, r_s)$ is neither concave nor convex in k_s, there may be multiple optimal

4.1 Comparative statics

The structural parameters that describe the economy are the following: The success return a (which determines the interest rate of continuation loans), the loss given default λ, the scale of the second period projects μ, the cost of setting up a lending relationship c, the cost of bank equity capital δ, the probabilities of transition from each state to the high default state q_h and q_l, and the capital requirements γ_h and γ_l. To complete the description of the economy, one must also specify the state-contingent cdfs of the default rate, $F_h(x_t)$ and $F_l(x_t)$.

Table 1 summarizes the comparative statics of the equilibrium interest rates on initial loans r_s^*, which are derived in Appendix A. The table shows the sign of the derivative dr_s^*/dz obtained by differentiating (10) with respect to each exogenous parameter z. The effects of the various parameters on r_s^* are inversely related to their impact on the profitability of banks' lending activity. Other things equal, a and μ impact positively on the profitability of continuation lending; λ affects negatively the profitability of both initial lending (directly) and continuation lending (directly and by reducing the availability of capital in the second period); c has a similar negative effect (with no direct effect on the profitability of continuation loans); δ increases the cost of equity funding in both periods; γ_h and γ_l increase the burden of capital regulation in the corresponding initial state, as well as in the corresponding continuation state (which will be h or l with probabilities q_s and $1-q_s$, respectively); finally, q_s decreases the profitability of continuation lending because, in the high default state h, loan losses are higher and the corresponding capital requirement γ_h may also be higher.

Table 1. Comparative statics of the initial loan rate r_s^*

$z =$	a	λ	μ	c	δ	q_s	γ_h	γ_l
$\dfrac{dr_s^*}{dz}$	$-$	$+$	$-$	$+$	$+$	$+$	$+$	$+$

values of k_s corresponding to r_s^*.

Table 2 summarizes the comparative statics of the equilibrium initial capital k_s^* chosen by the representative bank in an interior solution—obviously, when the solution is at the corner $k_s^* = \gamma_s$, marginal changes in parameters other than the capital requirement γ_s do not change k_s^*. As further explained in Appendix A, the recursive nature of the comparative statics of the system given by (10) and (11) makes it convenient to decompose the effects of the change in any parameter z into a *direct effect* (for constant r_s^*) and a *loan rate effect* (due to the change in r_s^*):

$$\frac{dk_s^*}{dz} = \frac{\partial k_s^*}{\partial z} + \frac{\partial k_s^*}{\partial r_s}\frac{dr_s^*}{dz}.$$

Loan rate effects can be easily determined. Differentiating the first-order condition that characterizes k_s^* in an interior solution gives

$$\frac{\partial^2 v_s}{\partial k_s^2}\frac{\partial k_s^*}{\partial r_s} + \frac{\partial^2 v_s}{\partial k_s \partial r_s} = 0.$$

The coefficient of $\partial k_s^*/\partial r_s$ is negative, by the second-order condition that characterizes k_s^* in an interior solution, and the second term is negative (see Appendix A). Hence $\partial k_s^*/\partial r_s$ is negative, which implies that the signs of loan rate effects are the opposite to those in Table 1. Intuitively, the initial capital k_s and the initial loan rate r_s are substitutes in the role of providing the bank with sufficient capital for its continuation lending (see the definition of $k_s'(x)$ in (2)). In an interior solution, the marginal value of k_s is decreasing in k_s, and thus also in r_s, so a larger r_s reduces the bank's incentive to hold excess capital.

Table 2. Comparative statics of the initial capital k_s^*
(in an interior equilibrium)

	$z =$	a	λ	μ	c	δ	q_s	γ_h	γ_l
$\dfrac{\partial k_s^*}{\partial z}$	(direct effect)	$+$	?	$+$	$+$	$-$	?	?	?
$\dfrac{\partial k_s^*}{\partial r_s}\dfrac{dr_s^*}{dz}$	(loan rate effect)	$+$	$-$	$+$	$-$	$-$	$-$	$-$	$-$
$\dfrac{dk_s^*}{dz}$	(total effect)	$+$	?	$+$	?	$-$	?	?	?

For the parameters a, μ, and δ, the direct and the loan rate effects point in the same direction, so the total effect can be analytically signed. In essence, higher profitability of continuation lending (captured by a and μ) and lower costs of capital (captured by δ) encourage banks to hold larger capital buffers in order to better self-insure against the default shocks that threaten its continuation lending. For the setup cost c, the direct and the loan rate effects have unambiguous but opposite signs, so the total effect is ambiguous. The positive direct effect comes from the fact that, by the definition of $k_s'(x)$ in (2), c subtracts to the bank's continuation lending capacity exactly like k_s adds to it, without affecting the profitability of such lending and hence the marginal gains from self-insuring against default shocks. The direct effects on k_s of the parameters λ, q_s, γ_h, and γ_l have ambiguous signs. Increasing any of these parameters simultaneously reduces the profitability of continuation lending and impairs the expected capital position of the bank when such lending has to be made. The value of holding excess capital in the initial lending period falls, but the prospects of ending up with insufficient capital increase. So the profitability of continuation lending and the need for self-insurance against default shocks move in opposite directions. The resulting ambiguity of the direct effects extends to the total effects.

The details of the analytical expressions suggest that the shape of the distributions of default rates matter for the determination of these effects, which eventually becomes a question to be elucidated either empirically or by numerically solving the model under realistic parameterizations. Since the goal of the paper is to assess the potential impact of the Basel II capital requirements, which are in the process of being implemented, we resort to the second alternative.

5 Numerical Results

To further explore the forces that affect banks' initial capital buffers as well as to assess the implications for the dynamics of lending under different regulations, we numerically solve the model in a number of plausible scenarios. Importantly, in all scenarios we assume that the state-contingent probability distributions of the default rate, described by the cdfs $F_h(x_t)$ and $F_l(x_t)$, conform to the single risk factor model that underlies the capital requirements associated with the IRB approach of Basel II.[21] This means that we assess the implications of the new capital requirements under the assumption that the supervisor's model of reference is correct.[22] In line with the one-year ahead value-at-risk perspective of Basel II, the parameterization assumes that each model period corresponds to one calendar year.

5.1 The single risk factor model

Suppose that the project undertaken by entrepreneur i at date t fails if $y_{it} < 0$, where y_{it} is a latent random variable defined by

$$y_{it} = \alpha_t + \sqrt{\rho_t}\, u_t + \sqrt{1-\rho_t}\, \varepsilon_{it},$$

where α_t is a parameter determined by the state of the economy at date t, u_t is a single factor of systematic risk, ε_{it} is an idiosyncratic risk factor, and $\rho_t \in (0,1)$ is a state-contingent

[21] The single factor model is due to Vasicek (2002) and its use as a foundation for the capital requirements of Basel II is due to Gordy (2003).

[22] Of course, the model could be similarly solved under alternative specifications of the relevant cdfs, but in that case the requirements set under the regulatory formula described below would not have the direct value-at-risk interpretation implied by our parameterization.

parameter that determines the correlation among project failures. It is assumed that u_t and ε_{it} are $N(0,1)$ random variables, independently distributed from each other and over time, as well as, in the case of ε_{it}, across projects. Let $\Phi(\cdot)$ denote the cdf of a standard normal random variable. Conditional on the information available at date t, the probability of failure of project i is $p_t = \Pr(y_{it} < 0) = \Phi(-\alpha_t)$, since $y_{it} \sim N(\alpha_t, 1)$, which implies $\alpha_t = -\Phi^{-1}(p_t)$.

With a continuum of projects, the aggregate failure rate x_t is only a function of the realization of the single risk factor u_t. Specifically, by the law of large numbers, x_t coincides with the probability of failure of a (representative) project i conditional on the information available at t and the realization of u_t:

$$x_t = g_t(u_t) = \Pr\left(-\Phi^{-1}(p_t) + \sqrt{\rho_t}\, u_t + \sqrt{1-\rho_t}\, \varepsilon_{it} < 0 \mid u_t\right) = \Phi\left(\frac{\Phi^{-1}(p_t) - \sqrt{\rho_t}\, u_t}{\sqrt{1-\rho_t}}\right).$$

Using the fact that $u_t \sim N(0,1)$, the cumulative distribution function of the aggregate failure rate can be expressed as

$$F_t(x_t) = \Pr\left(g_t(u_t) \leq x_t\right) = \Pr\left(u_t \leq g_t^{-1}(x_t)\right) = \Phi\left(\frac{\sqrt{1-\rho_t}\, \Phi^{-1}(x_t) - \Phi^{-1}(p_t)}{\sqrt{\rho_t}}\right).$$

In Basel II the correlation parameter ρ_t is assumed to be a decreasing function of the state-contingent probability of default p_t. Hence we postulate the following state-contingent probability distributions of the default rate

$$F_s(x) = \Phi\left(\frac{\sqrt{1-\rho_s}\, \Phi^{-1}(x) - \Phi^{-1}(p_s)}{\sqrt{\rho_s}}\right), \tag{12}$$

where, as stipulated by Basel II for corporate loans, the correlation parameter ρ_s is decreasing in the probability of default p_s according to the formula

$$\rho_s = 0.12\left(2 - \frac{1-e^{-50p_s}}{1-e^{-50}}\right). \tag{13}$$

In the IRB approach of Basel II, capital must cover the one-year ahead losses due to loan defaults with a probability of 99.9%. Hence the capital requirement in state s is given by $\gamma_s = \lambda F_s^{-1}(0.999)$, where $F_s^{-1}(0.999)$ is the 99.9% quantile of the distribution of the default rate. Using (12), the Basel II capital requirement becomes

$$\gamma_s = \lambda\Phi\left(\frac{\Phi^{-1}(p_s) + \sqrt{\rho_s}\, \Phi^{-1}(0.999)}{\sqrt{1-\rho_s}}\right), \tag{14}$$

where ρ_s is given by (13). This is the formula for corporate exposures of a one-year maturity that appears in Basel Committee on Banking Supervision (2004, paragraph 272).[23] It should be noted that Basel II establishes that the expected losses, λp_s, should be covered with general loan-loss provisions, while the remaining charge, $\lambda(\gamma_s - p_s)$, should be covered with capital. However, from the perspective of our analysis, provisions are just another form of equity capital, so the distinction between the expected and unexpected components of loan losses is immaterial.

5.2 Benchmark scenarios

Table 3 shows the set of parameter values that define the three benchmark scenarios considered in our numerical analysis. The scenarios differ in the volatility of the state-contingent probabilities of default p_l and p_h. We briefly comment on them here, relegating further discussion in the light of available data on the US banking sector to Appendix B. Note that because of our normalization of the risk-free rate to zero, all interest rates and rates of return in the parameterization should be interpreted as spreads over the risk-free rate.

Panel A of Table 3 contains the parameters that are common to the three scenarios. The value of the success return $a = 0.04$ implies that the interest rate of continuation loans is 4%.[24] The loss given default (LGD) parameter $\lambda = 0.45$ is taken from the Basel II "foundation IRB" formula for unsecured corporate exposures.[25] The scale of the second period projects $\mu = 1$ provides a neutral starting point—fine-tuning this parameter would require some empirical estimation of the growth rate of loan exposures along a typical corporate lending relationship or, alternatively, of the asset growth rate in a representative business financed by banks. The cost of setting up a lending relationship $c = 0.03$ is chosen so as yield realistic initial loan rates. The cost of bank capital $\delta = 0.04$ is intended to capture the tax disadvantages of equity financing relative to deposit financing. The probabilities of

[23] The Basel II formula incorporates an adjustment factor that is increasing in the maturity of the exposure, and equals one for a maturity of one year.

[24] The success return could be higher, as long as the part that can be pledged to the bank without destroying the entrepreneur's incentives is set at 4%. See Holmström and Tirole (1997) for a discussion of the concept of pledgeable return.

[25] In the "advanced IRB" approach banks are allowed to use their own internal models to estimate λ.

transition to the high default state, $q_l = 0.20$ and $q_h = 0.64$, imply expected durations of 5 years for the low default state and 2.8 years for the high default state, which we calibrate according to the observed behavior of the charge-off ratio of FDIC-insured commercial banks in the US during the period 1970-2004.[26]

Table 3. Parameter values in the benchmark scenarios

A. Common parameters

a	μ	λ	c	δ	q_h	q_l
0.04	1.00	0.45	0.03	0.04	0.64	0.20

B. Probability of default (PD) scenarios

Scenarios		Benchmark PDs p_s (%)	Basel II requirements γ_s (%)
Low volatility	$s = l$	1.20	6.87
	$s = h$	2.91	10.01
Medium volatility	$s = l$	1.10	6.60
	$s = h$	3.26	10.51
High volatility	$s = l$	1.00	6.31
	$s = h$	3.62	11.00

The three PD scenarios are defined so as to keep the expected capital charge under Basel II equal to 8%, which is the capital requirement under Basel I. Appendix B discusses the choice of parameter values in the light of available data on US banks.

Panel B of Table 3 shows our choices for the probabilities of default (PDs) in each state, p_l and p_h, and the corresponding Basel II capital requirements, γ_l and γ_h, implied by (14). In each scenario we have chosen the PDs such that the long-run average capital requirement

[26] Expected durations are computed as $q_l + 2(1 - q_l)q_l + 3(1 - q_l)^2 q_l + ... = 1/q_l = 5$ for state l, and $(1 - q_h) + 2q_h(1 - q_h) + 3q_h^2(1 - q_h) + ... = 1/(1 - q_h) \simeq 2.8$ for state h. It should be noted that, in line with the empirical findings of Bruche and González-Aguado (2006), in our parameterization credit cycle downturns last longer than typical business cycle downturns.

under Basel II (given the underlying unconditional probabilities of visiting each state) is 8%, as under the risk-insensitive Basel I regulation.[27] The idea is to allow for a comparison of the cyclical effects of Basel I and Basel II that is not affected by a change in the long-run average level of the capital requirements. The three scenarios only differ in the magnitude of the cross-state variation in the PDs—and all of them are within a range that can be considered empirically plausible.

It is important to note that we are assuming that the LGD parameter λ does no depend on the state s. Altman et al. (2005), using data on bond defaults, find that LGDs are positively correlated with default rates. In the context of our model, however, cyclical variation in LGDs has a similar effect on banks' profits and Basel II capital requirements as an increase in the cyclical variation in PDs. Hence, the effects of our conservative assumption could be offset by placing more weight on the results corresponding to the scenarios with more volatile PDs.

5.3 Capital buffers and procyclicality

Table 4 shows initial loan rates r_s^*, initial capital k_s^*, and the implied capital buffers $\Delta_s^* = k_s^* - \gamma_s$, for $s = h, l$, in each of the scenarios described in Table 3 and under the two regulatory frameworks that we want to compare: Basel I, with a flat capital requirements of 8%, and Basel II, with the requirements given by (14). As a reference, we also include the results in a laissez-faire environment without capital requirements ($\gamma_h = \gamma_l = 0$).

The results show that initial loan rates are always higher in the high default state, reflecting the need to compensate banks for both a higher PD and a lower prospective profitability of continuation lending (since the high default state h is more likely to occur after state h than after state l). These rates are very similar in the two Basel frameworks, confirming previous results from static models predicting that the loan pricing implications of Basel II will be small.[28] Basel II slightly increases loan rates in the high default state, and

[27] The unconditional probabilities of the low and the high default state, denoted ϕ_l and ϕ_h, can be obtained by solving the system of equations $q_l\phi_l + q_h\phi_h = \phi_h$ and $\phi_l + \phi_h = 1$, which gives $\phi_l = (1 - q_h)/(1 - q_h + q_l) \simeq 0.64$ and $\phi_h = q_l/(1 - q_h + q_l) \simeq 0.36$.

[28] See Repullo and Suarez (2004).

induces no significant change in loan rates in the low default state. These effects may be explained by the fact that Basel II significantly increases banks' capital in the high default state, but has a smaller impact on capital in the low default state.

Table 4. Initial loan rates, capital, and capital buffers
(all variables in %)

Scenarios		Basel I			Basel II			Laissez-faire		
		r_s^*	k_s^*	Δ_s^*	r_s^*	k_s^*	Δ_s^*	r_s^*	k_s^*	Δ_s^*
Low	$s=l$	1.2	11.0	3.0	1.2	11.9	5.0	0.7	5.2	5.2
volatility	$s=h$	2.4	11.2	3.2	2.5	12.2	2.2	1.8	5.4	5.4
Medium	$s=l$	1.2	11.0	3.0	1.2	11.7	5.1	0.6	5.1	5.1
volatility	$s=h$	2.7	11.2	3.2	2.8	12.5	1.9	2.1	5.3	5.3
High	$s=l$	1.1	10.9	2.9	1.1	10.7	4.3	0.5	5.0	5.0
volatility	$s=h$	3.0	11.1	3.1	3.1	12.6	1.6	2.3	5.2	5.2

The parameters that define each of the three scenarios and the associated Basel II capital requirements are described in Table 3. The Basel I capital requirement is always 8%.

The results also show that, in order to preserve their future lending capacity, banks hold sizeable capital buffers. Under Basel I, the cyclical variation in PDs has a rather small impact on capital decisions, although excess capital tends to be larger in the high default state (where loan losses can be expected to cause a larger reduction in future lending capacity) than in the low default state.[29] Under Basel II the cross-state variability in PDs visibly translates into greater variability of both total capital and capital buffers. Interestingly, the cyclical pattern of the buffers gets reversed, from slightly countercyclical in Basel II to strongly procyclical in Basel II. The main reason for this reversal is that, under Basel II, banks in the low default state l anticipate that if the economy switches to the high default state h the capital requirement will increase from γ_l to γ_h. This jump in capital requirements implies

[29] This is consistent with the existing evidence about the behavior of capital buffers under Basel I—including Ayuso et al. (2004) with Spanish data, Lindquist (2004) with Norwegian data, and Bikker and Metzemakers (2004) with data from 29 OECD countries—and raises doubts about the interpretation that such evidence reflects banks' myopia.

a reduction in their lending capacity so, to preserve continuation lending, they have an incentive to hold larger precautionary capital buffers than under Basel I, where the capital requirement stays at 8%. Symmetrically, under Basel II, banks in the high default state h anticipate that if the economy switches to the low default state l the capital requirement will decrease from γ_h to γ_l, so they have an incentive to hold smaller capital buffers than under Basel I. The numerical results for the three scenarios show that the first effect (higher buffers in state l) turns out to be more important than the second effect (lower buffers in state h), which implies that the move from Basel I to Basel II will increase the long-run average level of the capital buffers (computed with the unconditional probabilities of visiting each state).[30]

As for the laissez-faire environment, the results in Table 4 confirm that, under our parameterization, the "economic capital" chosen by banks starting lending relationships is well-below the regulatory capital of any of the two Basel frameworks, but significantly different from zero (and very similar across states), which reflects banks' interest in preserving their valuable lending during the second period.[31] The lower initial interest rates in both states (relative to the Basel frameworks) reflect the savings on the costs of equity financing due to the use of less capital in the two lending periods.

Table 5 compares the cyclical behavior of credit rationing under Basel I and Basel II, as well as in the laissez-faire environment. Lending in any given period is made up of initial loans, whose quantity is always one, and continuation loans, whose quantity varies with the lending capacity of the banks that are unable to issue equity in that period. We denote by *credit rationing* the expected percentage of continuation projects that cannot be undertaken because of banks' insufficient lending capacity. Table 5 shows credit rationing in state $s' = l, h$ when it is reached from state $s = l, h$ according to any of the four possible sequences (s, s'). In our simple model, investment and hence expected gross output (the returns from the funded investment projects) are linearly related to total credit, given the

[30] The increase in the medium volatility scenario is of 0.88 percentage points.

[31] Elizalde and Repullo (2007) discuss the concept of economic capital (and its relationship with the regulatory capital of Basel II) in a model where banks are concerned about the loss of charter values when they become insolvent.

state of the economy, so we can use credit rationing as a summary statistic of aggregate economic activity.

Table 5. Credit rationing
(all variables in %)

	Credit rationing in state s'		
Scenarios	Basel I	Basel II	Laissez-faire
Low volatility			
$(s, s') = (l, l)$	1.4	0.3	2.2
$(s, s') = (l, h)$	1.4	4.9	2.2
$(s, s') = (h, h)$	2.5	3.8	4.5
$(s, s') = (h, l)$	2.5	0.7	4.5
Unconditional	1.8	1.7	3.0
Medium volatility			
$(s, s') = (l, l)$	1.4	0.3	2.1
$(s, s') = (l, h)$	1.4	10.7	2.1
$(s, s') = (h, h)$	2.7	4.5	5.2
$(s, s') = (h, l)$	2.7	0.6	5.2
Unconditional	1.9	2.6	3.2
High volatility			
$(s, s') = (l, l)$	1.3	0.4	2.0
$(s, s') = (l, h)$	1.3	24.4	2.0
$(s, s') = (h, h)$	3.0	5.3	6.1
$(s, s') = (h, l)$	3.0	0.5	6.1
Unconditional	1.9	4.6	3.5

The parameters that define each of the scenarios and the associated Basel II capital requirements are described in Table 3. The Basel I capital requirement is 8%. Credit rationing is the expected percentage of continuation projects that cannot be undertaken because of banks' insufficient lending capacity. The rows show credit rationing in state s' when it is reached from state s according to the sequence (s,s') in the first column. Rows labeled 'unconditional' show weighted averages based on the unconditional probabilities of each state.

In Basel I (as well as in the laissez-faire environment) credit rationing does not depend on whether the arrival state s' is a high or a low default state, since the capital requirement is constant (at 8% or 0%, respectively). Rationing only depends on the profits realized during the previous period, which determine the capital available for continuation lending. The distribution of this random variable depends on the state s of the economy in the previous period. This explains why the figures for Basel I (and the laissez-faire environment) in Table 5 only vary with s in each scenario, and are smaller for $s = l$ than for $s = h$.

Under Basel II, the impact of bank profits is also present, but the overall effects on credit rationing are dominated by the cross-state variation of the capital requirements, and its endogenous effects on capital buffers. Thus the sequences with $(s, s') = (l, h)$, and then those with $(s, s') = (h, h)$, systematically exhibit the largest credit rationing. Intuitively, in the low default state l the transition to the high default state h is less likely than continuing in h after being in h; additionally, in state l the required capital is lower than in state h. For both reasons, banks hold less capital in $s = l$ than in $s = h$ (see Table 4). But then if the economy ends up in $s' = h$, the combination of a lower capitalization in the previous period and a higher current requirement explains the very sizable contractions in lending capacity shown in Table 5.[32] In particular, for the medium volatility scenario, when the economy goes from the low to the high default state (and despite of the fact that banks hold a capital buffer of 5.1% in the low default state) an average of 10.7% of the continuation projects are rationed, a figure that goes up to 24.4% in the high volatility scenario.

Thus Basel II implies significantly larger cyclical variation in credit rationing (and consequently in investment and output) than Basel I. Its incidence on the average level of credit rationing, shown in the rows labeled 'unconditional' in Table 5, depends on the volatility of PDs along the cycle. For the medium volatility scenario, the extra cost of Basel II in terms of long-run average credit rationing amounts to about 0.7% of the potential continuation investment.

The behavior of credit rationing in the laissez-faire environment is very much an amplified version of what is observed under Basel I, with levels of rationing that are between 50% and 100% larger. In relation with Basel II, however, the comparison depends on the volatility

[32] Interestingly, for the sequences with $s' = l$ (which entail the lowest credit rationing under Basel II), the effect of bank profits becomes visible again, producing lower rationing in the (l, l) sequence than in the (h, l) sequence.

of PDs along the cycle: except in the low volatility scenario, the laissez-faire exhibits lower cyclicality and, in the high volatility scenario, it even exhibits lower unconditional expected credit rationing.

5.4 Banks' solvency

We next compare the various regulatory regimes in terms of banks' solvency. Table 6 reports the probability of failure of the representative bank for each of the scenarios described in Table 3 and each of the possible states of the economy. These probabilities are different for banks making initial loans (that in state s start with capital k_s, earn interest r_s on performing loans, and pay the cost c of starting up their lending relationships) and banks making continuation loans (that in state s start with capital γ_s, earn interest a on performing loans, and do not pay c). Unlike in the results on credit rationing, these probabilities are purely forward-looking (i.e., they do not depend on the state of the economy in the previous period) and hence we only report their conditional-on-s and unconditional values.

Table 6 shows that the probabilities of bank failure are much more uniform across states under the risk-sensitive capital requirements of Basel II than under the constant 8% capital requirement of Basel I. Conditional on the state of the economy, the link between the level of the requirements and the level of solvency of second period banks is direct (since these banks hold no capital buffers and the net interest income earned on performing loans is the same in both states), so not surprisingly Basel II implies a significant improvement in solvency in the high default state h and a reduction in solvency in the low default state l, with the unconditional effect being clearly positive. For first period banks there are additional effects coming from the endogenous capital buffers and loan interest rates. Our results in Table 4 show that the latter effects are very small, so solvency is inversely related to the total holdings of capital. Hence Basel II increases the solvency of first period banks in state h, and (in the low and medium volatility scenario) it also increases their solvency in state l, despite imposing lower requirements than the 8% of Basel I. The unconditional effect of Basel II on the solvency of first period banks is positive in the three scenarios. All in all, Basel II roughly halves the probabilities of bank failure associated with Basel I, and makes the risk of failure more evenly distributed over time. This suggests that the risk-sensitive capital requirements of Basel II have a payoff in terms of the long-term solvency of the banking system.

Table 6. Banks' solvency
(all variables in %)

Scenarios		Probability of bank failure		
		Basel I	Basel II	Laissez-faire
Low volatility				
1st-period banks:	$s = l$	0.025	0.016	2.185
	$s = h$	0.094	0.051	4.492
	Unconditional	0.050	0.028	3.013
2nd-period banks:	$s = l$	0.008	0.014	1.023
	$s = h$	0.054	0.018	5.721
	Unconditional	0.024	0.015	2.710
Medium volatility				
1st-period banks:	$s = l$	0.022	0.014	2.080
	$s = h$	0.115	0.054	5.210
	Unconditional	0.056	0.029	3.203
2nd-period banks:	$s = l$	0.006	0.014	0.867
	$s = h$	0.074	0.019	7.195
	Unconditional	0.030	0.015	3.139
High volatility				
1st-period banks:	$s = l$	0.019	0.023	1.968
	$s = h$	0.140	0.059	6.126
	Unconditional	0.063	0.036	3.461
2nd-period banks:	$s = l$	0.005	0.013	0.723
	$s = h$	0.099	0.019	8.895
	Unconditional	0.039	0.015	3.657

The parameters that define each of the scenarios and the associated Basel II capital requirements are described in Table 3. The Basel I capital requirement is 8%. Rows labeled 'unconditional' show weighted averages based on the unconditional probabilities of each state.

It is worth noting that the probabilities of bank failure are very small under both regulations—unconditionally, they range between 0.024% and 0.063% under Basel I, and between 0.015% and 0.036% under Basel II. Interestingly, the combination of capital buffers and net interest income earned on performing loans makes the latter much lower than the 0.1% implied by the 99.9% confidence level of Basel II. This combination also explains the fact that the probabilities of bank failure under the laissez-faire environment are not very high—unconditionally, they range between 2.710% and 3.657%.

6 Policy analysis

Our previous results show that the move from Basel I to Basel II is very likely to imply an increase in the cyclicality of the supply of bank credit. Specifically, we predict a particularly strong reduction in banks' lending capacity (and a rise in credit rationing) when the economy goes into a recession. The results also suggest that banks' solvency will be enhanced by the introduction of Basel II.

Consequently, the comparison of Basel I and Basel II in welfare terms is not trivial and will crucially depend on the (structural or reduced-form) imputation of a social cost to bank failures.[33] Although the model could be extended to perform such a welfare analysis, the discussion in this section will be based on the (simpler) argument that it is possible to ameliorate the procyclical impact of Basel II by introducing some small adjustments in the confidence levels set by the regulator. In particular, we consider the possibility of modifying the cyclical profile of confidence levels in such a way that keeps their long-term average at 99.9%—the current level—but lessens the target in those states (or sequences of states) where credit rationing turns out to be the highest under Basel II regulation.

Table 7 shows the results of two specific policy experiments of this kind. Both are performed under the parameterization of the medium volatility scenario described in Table 3. Policy 1 reduces the confidence level in the high default state h to 99.8% and increases the confidence level in the low default state l to $\eta > 99.9\%$ so as to maintain the long-run

[33] Repullo and Suarez (2004) perform this type of welfare analysis in a static setup where procyclicality is not a concern, but capital requirements imply a deadweigh loss due to the extra cost of equity financing.

average at 99.9%. Thus η solves:

$$\eta \times \phi_l + 0.998 \times \phi_h = 0.999,$$

where ϕ_l and ϕ_h are, respectively, the unconditional probabilities of the low and the high default state.[34] Such a small adjustment causes a relevant change in capital requirements (from 6.6% to 7.9% for γ_l and from 10.5% to 9.3% for γ_h), modifying banks' optimal buffers (which become less procyclical), and smoothing the cyclicality of credit rationing. As shown in Panel A of Table 7, credit rationing in the sequences (l, h) and (h, h) falls from 10.7% and 4.5%, respectively, to less than 4% in both sequences. Unconditionally, it falls from 2.6% to 1.9%, which is its unconditional value under Basel I. Interestingly, although the probabilities of bank failure in the high default state h obviously increase, they remain lower than 0.08% in all cells, and their unconditional average only increases from 0.029% to 0.040% for first period banks, and from 0.015% to 0.017% for second period banks.

In Policy 2 we confine the reduced 99.8% confidence level to periods where state h occurs after state l. The objective is to reduce the credit rationing detected in the second period of sequences with $(s, s') = (l, h)$. The capital requirement when h occurs after h is left unchanged, while the confidence level applied to the second period of the sequences (l, l) and (h, l) is increased so as to to keep the long-run average confidence level at 99.9%. By construction, Policy 2 makes smaller adjustments in the Basel II capital requirements, so it will be less effective than Policy 1 in terms of smoothing credit rationing, but it will also be less significant in terms of its implications for banks' solvency. The results in Table 7 show that credit rationing in the second period of the (l, h) sequence gets substantially reduced, but not as much as with Policy 1, while the unconditional probabilities of bank failure are almost unchanged relative to those of Basel II.

All in all, our policy experiments show the feasibility of achieving significant gains in terms of credit rationing without major costs in terms of banks' long-term solvency. This can be achieved with cyclical adjustments that preserve the value-at-risk foundation of the Basel II requirements. The choice between Policy 1 and Policy 2 (or the fine tuning of

[34] See Footnote 27 for the expressions of ϕ_l and ϕ_h in terms of the transition probabilities q_l and q_h.

their details) should eventually depend on the trade-off between the gains in terms of a smoother and lower credit rationing, and the losses in terms of less smooth and slightly higher probabilities of bank failure.

Table 7. Procyclicality correction
(all variables in %)

A. Credit rationing

	Credit rationing in state s'				
$(s,s')=$	(l,l)	(l,h)	(h,h)	(h,l)	Unconditional
Basel I	1.4	1.4	2.7	2.7	1.9
Basel II	0.3	10.7	4.5	0.6	2.6
Policy 1	0.8	3.7	3.6	1.6	1.9
Policy 2	0.5	4.4	4.4	0.6	1.9

B. Banks' solvency

	Probability of failure of 1st-period banks in state s'				
$(s,s')=$	(l,l)	(l,h)	(h,h)	(h,l)	Unconditional
Basel I	0.022	0.115	0.115	0.022	0.056
Basel II	0.014	0.054	0.054	0.014	0.029
Policy 1	0.017	0.079	0.079	0.017	0.040
Policy 2	0.019	0.054	0.054	0.019	0.031

	Probability of failure of 2nd-period banks in state s'				
$(s,s')=$	(l,l)	(l,h)	(h,h)	(h,l)	Unconditional
Basel I	0.006	0.074	0.074	0.006	0.030
Basel II	0.014	0.019	0.019	0.014	0.015
Policy 1	0.007	0.035	0.035	0.007	0.017
Policy 2	0.011	0.035	0.019	0.014	0.016

The parameters and associated Basel II capital requirements are those of the medium volatility scenario described in Table 3. The Basel I capital requirement is 8%. Policy 1 reduces the confidence level of the Basel II formula to 99.8% in state h and increases it in state l so as to keep the unconditional average at 99.9%. Policy 2 sets the confidence level at 99.8% only when state h occurs after state l, compensating it when state l occurs so as to keep the unconditional average at 99.9%. Columns labeled 'unconditional' show weighted averages based on the unconditional probabilities of each state.

7 Discussion

In this section we discuss some simplifying features of our model, including the distribution of banks' market power in the first and second period of the lending relationships, the use of short-term loan contracts, and the assumption that banks with ongoing relationships have no access to the equity market. We consider the possible effects of relaxing these assumptions, noting that in some cases our conclusions about the procyclicality of Basel II would be strengthened.

7.1 Competition and market power

In our model banks are perfectly competitive in the market for first period loans and act as monopolists in the market for second period loans. This is justified by the fact that borrowers get locked-in after one period due to some (unmodeled) asymmetric information problem (e.g., one that makes borrowers searching for a new bank after one period look like "rejected" by the initial bank, which leads to a standard lemons problem in the refinancing market). We have opted for an all-or-nothing modeling of the lock-in effect for several reasons. The most obvious one is tractability: if the market for continuation loans were more competitive, the degree of effective competition and the resulting loan rates would vary with the degree of credit rationing. Even in the polar case of perfect competition in the market for second period loans, over the range of poor realizations of the risk factor that lead to rationing, banks would be able to appropriate scarcity rents from their (non-rationed) borrowers.[35] Relative to the current modeling, the more competitive market for second period loans would entail lower continuation rents for banks, and hence lower incentives for them to keep capital buffers. This uncovers a second justification for our assumption: it yields a conservative estimate of the procyclicality induced by capital regulation.

[35] In the context of the single risk factor model of Basel II, either all banks or no bank are capital constrained in the market for second period loans, so the second period loan rate would be either the monopoly rate a or some break-even rate that makes continuation lending a zero net present value investment for the banks.

7.2 Short-term loan contracts

We have described the relationship between entrepreneurs and banks as instrumented by a sequence of one-period loans. One might wonder whether focusing on short-term contracts builds in the "imperfection" that drives our main results. The answer is yes and no. With long-term contracts, or more generally contracts that differ from the sequence of one-period loans on which we focus, there might be room for improving over the credit allocation outcomes obtained in our analysis. For example, for given capital buffers, setting higher loan rates in the first period and lower loan rates in the second would reduce the incidence of credit rationing, since banks would have more capital to support their second period lending. But in the context of our model, long-term contracts pose important commitment and enforcement problems. In particular, they would have to specify the loan rates in the first and the second period, as well as the rationing scheme to be used in those cases where the bank ends up with insufficient lending capacity (since otherwise the bank might try to renegotiate the terms of the second period loans by threatening entrepreneurs to put them in the pool of rationed borrowers). This means that default rates in the first period would have to be verifiable, and banks would have to be restricted in their ability to pay dividends. They might also have to commit to maintain a capital buffer in the first period. The reason for this is that when competing for borrowers in the first period, banks internalize the whole surplus that the underlying investment projects generate over the two periods, but once the relationship starts, banks only internalize the return of their current and future lending. If the second period loan rate is lower than a, then banks do not take into account part of the continuation surplus when deciding how much capital to hold—recall that first period capital buffers are held for the purpose of reducing the expected credit rationing in the second period. All in all, the previous considerations suggest that there are serious commitment and enforcement problems that limit the possible improvements in the allocation of credit that long-term contracts might bring about.

7.3 Imperfect access to the equity market

The assumption that banks with ongoing relationships have no access to the equity market is obviously crucial for our results. With perfect, frictionless access to the equity market in the interim period, there would be no credit rationing among second period borrowers, except in the rare event that their bank fails (and it is not recapitalized by its shareholders). Banks in such a context would most likely hold no buffers, at least under the high capital requirements of Basel I or Basel II. Given the ample evidence of equity-issuance frictions, the key question is whether the specificities of our approach—that ties these frictions to the informational asymmetries associated with relationship lending—drive the results.

A more general way of capturing equity-issuance frictions would be to assume that access can occur with some (exogenous) probability $\upsilon < 1$. Changes in parameter υ could then be used to evaluate the "marginal" effects of the friction on capital buffers and credit rationing. One could also explore situations in which υ is contingent on the state s' of the economy at the interim date. This extension would probably confirm the results obtained in the current setup and might even reinforce our conclusions about the procyclicality of Basel II.[36]

It should also be noted that the really strong assumption that we are making is that banks can frictionless access the equity market when they renew their stock of lending relationships. This assumption is instrumental to achieving a tractable OLG structure in the context of an infinite horizon problem that otherwise would be characterized by longer memory and more complex dynamics (namely, credit rationing might also affect first period loans and the pricing of these loans might depend on the full history of the economy). A modeling alternative would be to assume a structure similar to the one in the popular Calvo (1983) model of staggered price setting, i.e., that in each period a fraction of the banks can issue new equity. In this context one would have to discuss the allocation of the newly born entrepreneurs to the existing banks. Would they demand loans to the recapitalizing banks only? If not, how would the pricing of the new loans be determined?

[36] If banks can with some probability access the equity market in the interim period, they would have lower incentives to keep capital buffers, so depending of parameter values, the incidence of credit rationing might be higher.

7.4 Other extensions

The framework used in this paper could also be extended in a number of other directions. First, we could consider lending relationships that extend over more than two periods. If relationships last for T periods and banks cannot raise equity for the whole length of the relationship, the qualitative results should be very similar to ours. Such a model would, of course, yield richer dynamics, as the effect of a shock would propagate over several periods. Second, we could consider alternative distributions $F_s(x_t)$ of the default rate x_t, keeping the Basel II capital requirements as in (14), but allowing for thicker tails in the loan loss distribution. Third, we could incorporate cyclically-varying demand. One easy way of doing this would be to let the scale μ of second period projects vary with the state s of the economy. A bigger challenge would be to introduce some "downward sloping" aggregate demand for loans, which could be done by assuming that entrepreneurs are heterogeneous in their opportunity cost of becoming active in the first period. The already existing variability in projects' success probabilities would tend to produce (procyclical) variability in the demand for loans. Further variability might be introduced by replacing the current success return a for some a_s. Finally, one could allow for feedback effects from constrained to unconstrained entrepreneurs by letting $a_t = a(I_t)$ instead of a, where $a(I_t)$ is an increasing (and possibly concave) function of I_t, the aggregate investment at date t. This will capture demand externalities or technological complementarities similar to those studied in endogenous growth theory. Analyzing these alternative models is beyond the scope of this paper.

8 Concluding Remarks

In many supervisory and industry reports on the implications of Basel II, it is standard to first recognize the potential cyclical effects of the new risk-sensitive capital requirements and then qualify that, given than most banks hold capital in excess of the regulatory minima, the practical incidence of the procyclicality problem is likely to be small if not negligible. While some of these reports do not have the extension or the technical nature required to elaborate on the foundations of their claim, others unveil two related misconceptions at the heart of it.

The first misconception is that the holding of capital buffers means that capital requirements are "not binding." Under a purely static perspective this would be tautologically true. In a convex optimization problem, it would also be true that small changes in the level of the requirements would not alter the optimal capital holdings. In a dynamic problem, however, this need not be the case: banks may hold capital buffers in a given period because they wish to reduce the risk of facing a statically "binding" requirement in the future. Perhaps these precautions make future requirements "not binding" when the time comes, but clearly their presence alters banks' capital decisions and the whole development of future events. So observing that banks hold capital buffers does not mean that capital requirements do not matter.

A second, related misconception is to accept that the cyclical behavior of capital buffers under Basel II can be somehow predicted from the empirical behavior of capital buffers in the Basel I era. If buffers are endogenously affected by the prevailing bank capital regulation (even if they appear not to "bind"), reduced-form extrapolations from the Basel I world to the Basel II world do not resist the Lucas' critique.

Our model provides a tractable framework in which it is possible to evaluate the cyclical effects of Basel II without incurring in these misconceptions. To keep the analysis as transparent as possible, we have simplified on a number of dimensions. For example, we abstract from demand side fluctuations and aggregate feedback effects that might mitigate and exacerbate, respectively, the supply-side effects that we identify. As we have discussed above, one could take our model as a building block for a fuller dynamic stochastic general equilibrium model with a production sector partly composed of entrepreneurial firms that rely on relationship bank lending. One could also think about extensions that generalize our modeling of the frictions related to banks' access to equity financing. Our contribution, from this perspective, is to show that the interaction of relationship lending (which makes some borrowers dependent on the lending capacity of the specific bank with which they establish a relationship) with frictions in banks' access to equity markets (which makes some banks' lending capacity a function of their historically determined capital positions and the capital requirements imposed by regulation) has the potential to cause significant cyclical swings in the supply of credit.

Under realistic parameterizations, the model produces capital buffers and equilibrium loan rates whose levels and cyclicality in the Basel I regulatory environment are in line with those observed in the data. The same parameterizations when applied to the Basel II environment suggest that the new requirements imply a substantial increase in the procyclicality induced by bank capital regulation. Specifically, despite banks taking precautions and holding larger buffers during expansions in order to have a reserve of capital for the next recession (when capital requirements rise), the arrival of recessions is normally associated with a sizeable credit crunch, as capital-constrained banks are induced to ration credit to some of their dependent borrowers.

Having a model that explicitly accounts for the endogenous determination of capital buffers and equilibrium loan rates is also important for policy analysis. We have shown that some cyclical adjustments in the confidence level of Basel II can substantially reduce the incidence of credit rationing over the business cycle without compromising the long-run solvency targets implied by the new regulation.

Appendix

A Proofs of analytical results

Solutions to the representative bank's capital decision Using the definition of $v_{ss'}(x_t)$ in (8), the net present value $v_s(k_s, r_s)$ of the representative bank that in state s holds capital k_s and charges an interest rate r_s on its unit of initial loans may be written as

$$v_s(k_s, r_s) = q_s v_{sh}(k_s, r_s) + (1 - q_s) v_{sl}(k_s, r_s), \tag{15}$$

where

$$v_{ss'}(k_s, r_s) = \beta \left[\int_0^{\widetilde{x}_{ss'}} [(\beta\pi_{s'} - \gamma_{s'})\mu + k_s'(x)]\, dF_s(x) + \frac{\beta\pi_{s'}}{\gamma_{s'}} \int_{\widetilde{x}_{ss'}}^{\widehat{x}_s} k_s'(x)\, dF_s(x) \right] - k_s. \tag{16}$$

By the definitions (4) and (3) of $\widetilde{x}_{ss'}$ and $\widehat{x}_s$, the function $v_{ss'}(k_s, r_s)$ has the following properties:

1. For $k_s \leq c - r_s$ we have $\widetilde{x}_{ss'} < \widehat{x}_s \leq 0$, so

$$\frac{\partial v_{ss'}}{\partial k_s} = -1 < 0.$$

2. For $c - r_s < k_s \leq c - r_s + \gamma_{s'}\mu$ we have $\widetilde{x}_{ss'} \leq 0 < \widehat{x}_s$, so

$$\frac{\partial v_{ss'}}{\partial k_s} = \frac{\beta^2 \pi_{s'}}{\gamma_{s'}} F_s(\widehat{x}_s) - 1 \lesseqgtr 0, \text{ and } \quad \frac{\partial^2 v_{ss'}}{\partial k_s^2} = \frac{\beta^2 \pi_{s'} F_s'(\widehat{x}_s)}{\gamma_{s'}(\lambda + r_s)} > 0.$$

3. For $c - r_s + \gamma_{s'}\mu < k_s < c + \lambda + \gamma_{s'}\mu$ we have $0 < \widetilde{x}_{ss'} < 1$, so

$$\frac{\partial v_{ss'}}{\partial k_s} = \frac{\beta}{\gamma_{s'}} \left[\beta\pi_{s'} F_s(\widehat{x}_s) - (\beta\pi_{s'} - \gamma_{s'}) F_s(\widetilde{x}_{ss'})\right] - 1 \lesseqgtr 0,$$

and

$$\frac{\partial^2 v_{ss'}}{\partial k_s^2} = \frac{\beta}{\gamma_{s'}(\lambda + r_s)} [\beta\pi_{s'} F_s'(\widehat{x}_s) - (\beta\pi_{s'} - \gamma_{s'}) F_s'(\widetilde{x}_{ss'})] \lesseqgtr 0.$$

4. For $c + \lambda + \gamma_{s'}\mu \leq k_s$ we have $1 \leq \widetilde{x}_{ss'} < \widehat{x}_s$, so

$$\frac{\partial v_{ss'}}{\partial k_s} = \beta - 1 < 0.$$

Hence the function $v_{ss'}(k_s, r_s)$ is linearly decreasing or strictly convex for $k_s \leq c - r_s + \gamma_{s'}\mu$, linearly decreasing for $k_s \geq c + \lambda + \gamma_{s'}\mu$, and may be increasing or decreasing, and concave or convex for $c - r_s + \gamma_{s'}\mu < k_s < c + \lambda + \gamma_{s'}\mu$. Introducing the constraint $k_s \in [\gamma_s, 1]$ (and assuming that parameter values are such that $c + \lambda + \gamma_{s'}\mu < 1$) it follows that the problem $\max_{k_s \in [\gamma_s, 1]} v_{ss'}(k_s, r_s)$ has either a corner solution with $k_s = \gamma_s$, or an interior solution with $k_s \in (c - r_s + \gamma_{s'}\mu, c + \lambda + \gamma_{s'}\mu)$. In the latter case we have $0 < \widetilde{x}_{ss'} < 1$, so there is a positive probability $F_s(\widetilde{x}_{ss'})$ that the bank has excess lending capacity in state s', and a positive probability $1 - F_s(\widetilde{x}_{ss'})$ that the bank has insufficient lending capacity in state s'. Since $\gamma_l \leq \gamma_h$ implies $c - r_s + \gamma_l\mu \leq c - r_s + \gamma_h\mu$ and $c + \lambda + \gamma_l\mu \leq c + \lambda + \gamma_h\mu$, we conclude that the problem $\max_{k_s \in [\gamma_s, 1]} [q_s v_{sh}(k_s, r_s) + (1 - q_s) v_{sl}(k_s, r_s)]$ has either a corner solution with $k_s = \gamma_s$, or an interior solution with $k_s \in (c - r_s + \gamma_l\mu, c + \lambda + \gamma_h\mu)$. In the latter case, there must be a positive probability that the bank has insufficient lending capacity in state $s' = h$ (and possibly also in state $s' = l$), and a positive probability that the bank has excess lending capacity in state $s' = l$ (and possibly also in state $s' = h$).

Comparative statics of the initial loan rate The sign of dr_s^*/dz for $z = a, \lambda, \mu, c, \delta, q_s, \gamma_h, \gamma_l$ can be obtained by total differentiation of (10):

$$\frac{\partial v_s}{\partial k_s}\frac{dk_s^*}{dz} + \frac{\partial v_s}{\partial r_s}\frac{dr_s^*}{dz} + \frac{\partial v_s}{\partial z} = 0. \tag{17}$$

When k_s^* is interior, the first-order condition for a maximum that follows from (11) gives $\partial v_s/\partial k_s \mid_{(k_s^*, r_s^*)} = 0$, so the first term in (17) vanishes. Moreover when k_s^* is interior it must be the case that $0 < \widetilde{x}_{ss'} < 1$ for at least one state s', so differentiating (15) and (16) we have

$$\frac{\partial v_s}{\partial r_s} = q_s \frac{\partial v_{sh}}{\partial r_s} + (1 - q_s)\frac{\partial v_{sl}}{\partial r_s} > 0,$$

since

$$\frac{\partial v_{ss'}}{\partial r_s} = \beta \left[\int_0^{\widetilde{x}_{ss'}} (1 - x)\ dF_s(x) + \frac{\beta \pi_{s'}}{\gamma_{s'}} \int_{\widetilde{x}_{ss'}}^{\widehat{x}_s} (1 - x)\ dF_s(x) \right] \geq 0,$$

with strict inequality for at least one state s'. Hence we are left with:

$$\frac{dr_s^*}{dz} = -\left(\frac{\partial v_s}{\partial r_s}\right)^{-1} \frac{\partial v_s}{\partial z}. \tag{18}$$

Similarly, in a corner solution with $k_s^* = \gamma_s$ we have $dk_s^*/dz = 0$ for all $z \neq \gamma_s$, in which case the first term in (17) also vanishes and (18) obtains again. Finally, for $z = \gamma_s$, we have $dk_s^*/d\gamma_s = 1$ and, thus,

$$\frac{dr_s^*}{d\gamma_s} = -\left(\frac{\partial v_s}{\partial r_s}\right)^{-1}\left(\frac{\partial v_s}{\partial \gamma_s} + \frac{\partial v_s}{\partial k_s}\right),$$

where $\partial v_s/\partial k_s \mid_{(k_s^*,r_s^*)} \leq 0$, since otherwise fixing $k_s^* = \gamma_s$ would not be optimal. With these expressions in mind, the results in Table 1 can be immediately related to the (self-explanatory) signs of the partial derivatives of $v_s(k_s^*, r_s^*)$ that we summarize in Table A1 (and whose detailed expressions we omit, for brevity).

Table A1. Effects on the net present value of the bank

$z =$	r_s	a	λ	μ	c	δ	q_s	γ_h	γ_l
$\frac{\partial v_s}{\partial z}$	$+$	$+$	$-$	$+$	$-$	$-$	$-$	$-$	$-$

Comparative statics of the initial capital When the optimal initial capital in state s is at the corner $k_s^* = \gamma_s$, with $\partial v_s/\partial k_s \mid_{(k_s^*,r_s^*)} < 0$, marginal changes in any parameter other than γ_s will have no impact on k_s^*, while obviously $dk_s^*/d\gamma_s = 1$. Thus, in what follows we focus on the more interesting interior solution case.[37]

The sign of dk_s^*/dz for $z = a, \lambda, \mu, c, \delta, q_s, \gamma_h, \gamma_l$ can be obtained by total differentiation of the first-order condition $\partial v_s/\partial k_s = 0$ that characterizes an interior equilibrium:

$$\frac{\partial^2 v_s}{\partial k_s^2}\frac{dk_s^*}{dz} + \frac{\partial^2 v_s}{\partial k_s \partial r_s}\frac{dr_s^*}{dz} + \frac{\partial^2 v_s}{\partial k_s \partial z} = 0. \tag{19}$$

By the second-order condition we have $\partial^2 v_s/\partial k_s^2 < 0$, which gives

$$\frac{dk_s^*}{dz} = -\left(\frac{\partial^2 v_s}{\partial k_s^2}\right)^{-1}\left(\frac{\partial^2 v_s}{\partial k_s \partial z} + \frac{\partial^2 v_s}{\partial k_s \partial r_s}\frac{dr_s^*}{dz}\right).$$

[37] The case with $k_s^* = \gamma_s$ and $\partial v_s/\partial k_s \mid_{(k_s^*,r_s^*)} = 0$ is a mixture of both cases since, depending on the sign of the effect of the marginal variation in a parameter, the optimal decision might shift from being at the corner to being interior. A similar complexity may occur if the change in a parameter breaks some underlying indifference between an interior and a corner solution (or between two interior solutions). We will omit the discussion of these cases, for simplicity.

Hence the sign of dk_s^*/dz coincides with the sign of the second term in brackets, which has two components: the direct effect of z on k_s^* (for constant r_s^*) and the loan rate effect (due to the effect of z on r_s^*). The signs of the direct effects shown in the first row of Table 2 coincide with the signs of the cross derivatives $\partial^2 v_s/\partial k_s \partial z$ summarized in Table A2 (whose detailed expressions we omit, for brevity).

Table A2. Effects on the marginal value of capital

$z =$	r_s	a	λ	μ	c	δ	q_s	γ_h	γ_l
$\dfrac{\partial^2 v_s}{\partial k_s \partial z}$	$-$	$+$	?	$+$	$-$	$-$	?	?	?

The signs of the loan rate effects shown in the second row of Table 2 can be simply obtained from the results summarized on Table 1 and the fact that by differentiating (15) and (16) one can show that

$$\frac{\partial^2 v_s}{\partial k_s \partial r_s} = q_s \frac{\partial^2 v_{sh}}{\partial k_s \partial r_s} + (1 - q_s) \frac{\partial^2 v_{sl}}{\partial k_s \partial r_s} < 0,$$

where

$$\frac{\partial^2 v_{ss'}}{\partial k_s \partial r_s} = \frac{\beta}{\gamma_{s'}(\lambda + r_s)} [\beta \pi_{s'} (1 - \widehat{x}_s) F_s'(\widehat{x}_s) - (\beta \pi_{s'} - \gamma_{s'})(1 - \widetilde{x}_{ss'}) F_s'(\widetilde{x}_{ss'})].$$

To check this notice that the second-order condition $\partial^2 v_s/\partial k_s^2 < 0$ implies

$$\beta f_s(\widehat{x}_s) \left[q_s \frac{\pi_h}{\gamma_h} + (1 - q_s) \frac{\pi_l}{\gamma_l} \right] < q_s \frac{\beta \pi_h - \gamma_h}{\gamma_h} F_s'(\widetilde{x}_{sh}) + (1 - q_s) \frac{\beta \pi_l - \gamma_l}{\gamma_l} F_s'(\widetilde{x}_{sl}).$$

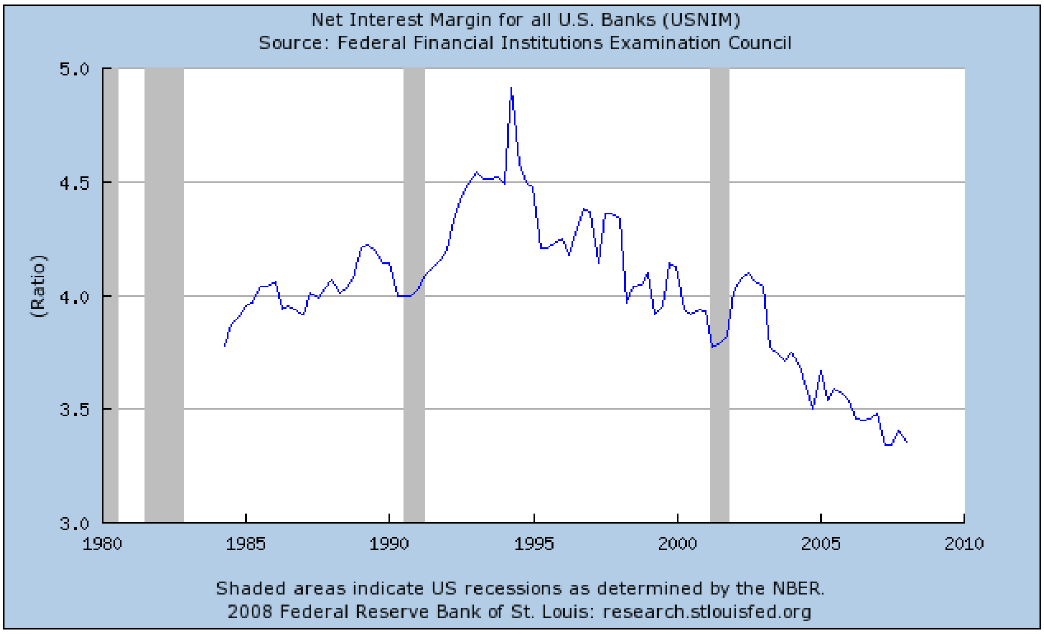
Hence using the definitions (3) and (4) of $\widehat{x}_s$ and $\widetilde{x}_{ss'}$, together with the fact that $\gamma_l \leq \gamma_h$, we have $1 - \widehat{x}_s < 1 - \widetilde{x}_{sl} \leq 1 - \widetilde{x}_{sh}$, so we conclude that

$$\beta(1 - \widehat{x}_s) f_s(\widehat{x}_s) \left[q_s \frac{\pi_h}{\gamma_h} + (1 - q_s) \frac{\pi_l}{\gamma_l} \right] < q_s \frac{\beta \pi_h - \gamma_h}{\gamma_h} (1 - \widetilde{x}_{sh}) F_s'(\widetilde{x}_{sh}) + (1 - q_s) \frac{\beta \pi_l - \gamma_l}{\gamma_l} (1 - \widetilde{x}_{sl}) F_s'(\widetilde{x}_{sl}),$$

which after some reordering proves the result.

B Discussion of parameter values

Interest rate on continuation loans: $a = 0.04$. The interest rates on banks' marginal lending and borrowing activities are not available in standard statistical sources. A common approach is to proxy them with implicit average rates computed from accounting figures. According to the FDIC Statistics on Banking for the years 2004 to 2007 (available at http://www2.fdic.gov/SDI/SOB/), *Total interest income* of all US commercial banks represents, on average, 5.74% of *Earning assets*, while *Total interest expense* represents 2.32% of *Total liabilities*. This yields an average net interest margin of 3.42%. Yet *Service charges on deposit accounts* are 0.55% of *Total deposits*, which implies that deposit-funded activities yield an average intermediation margin of 3.97%. This number is very close to our assumed 4%. (See Figure 1 for quarterly data on the net interest margin of US banks over a longer period.)

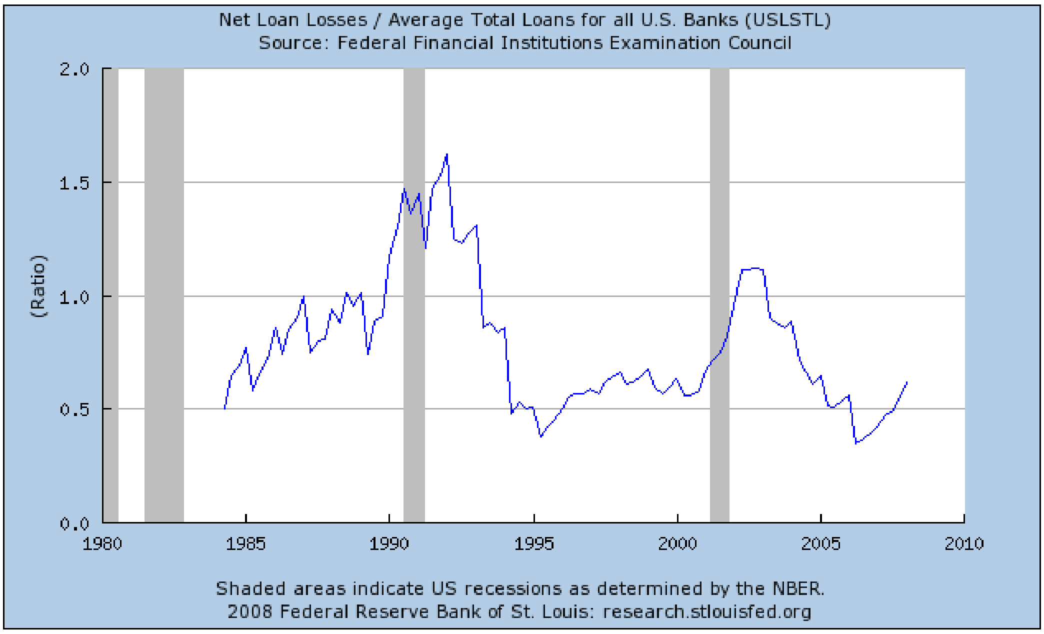


Cost of setting up a lending relationship: $c = 0.03$. This is a rather conservative estimate of the importance of intermediation costs. According to the FDIC Statistics on Banking for the years 2004 to 2007, *Total non-interest expense* of all US commercial banks represented an average of 3.97% of *Total assets*.

Cost of bank capital: $\delta = 0.04$. Based on the estimates of Graham (2000) for non-financial corporations, an annual discount rate of 4% is a rather conservative estimate for the tax disadvantage of equity financing. To see this, consider the standard measure of the marginal tax shield of debt financing, net of personal taxes: $MTS = [(1-\tau_i)-(1-\tau_c)(1-\tau_e)]/(1-\tau_i)$, where τ_i, τ_c, and τ_e are the marginal tax rates on personal interest income, corporate income, and personal equity income, respectively. As in Hennessy and Whited (2007), set $\tau_i = 0.29$ and consider $\overline{\tau}_c = 0.40$ as an upper bound to τ_c (based on the combination of the top statutory federal rate and the average state rate as reported by Graham, 2000). Suppose, conservatively, that $\tau_e = 0$, so that the marginal investor manages to make its equity income fully exempt from personal taxation. Then we get $MTS \simeq 0.04$ for $\tau_c = 0.32$, where this last choice is consistent with US data. In particular, for US commercial banks over the period 2004-2007, *Applicable income taxes* represented, on average, 31.7% of *Pre-tax net operating income*. As in Hennessy and Whited (2007), this number can be seen as the expected tax rate in a situation in which the representative bank earns positive taxable income and hence faces an effective corporate tax rate of $\overline{\tau}_c$ with a probability of 80%, while it faces an effective zero tax rate with a probability of 20%.

Probabilities of transition to the high default state: $q_l = 0.20$ and $q_h = 0.64$. In our Markov switching setup, the expected durations of states l and h are $1/q_l$ and $1/(1-q_h)$, respectively. We calibrate these durations using data from the FDIC Historical Statistics on Banking (available at http://www2.fdic.gov/hsob/index.asp). Specifically, we compute the annual ratio of *Net loan and lease charge-offs* to *Gross loans and leases* for FDIC-insured commercial banks over the period 1969-2004, and we detrend the series using the standard HP-filter for annual data. The resulting series includes 20 below-average observations in 4 complete low default phases (implying an average duration of $20/4 = 5$ years) and 14 above-average observations in 5 complete high default phases (implying an average duration of $14/5 \simeq 2.8$ years). The observations corresponding to 1969 and 2004 belong to censored below-average phases that are not taken into account. The imputed expected durations are in line with Koopman et al. (2005), that identify a stochastic cycle in US business failure rates with a period of between 8 and 11 years.

PD scenarios: $p_l \in [1.00, 1.30]$ and $p_h \in [2.88, 3.63]$. In the Special Report "Commercial Banks in 1999" (available at http://www.philadelphiafed.org/files/bb/bbspecial.pdf), the Federal Reserve Bank of Philadelphia offers data on the experience of US commercial banks during the full cycle of the 1990s. Following the 1990-1991 recession, the aggregate ratio of *Non-performing loans* to *Total loans* was slightly above 3% in years 1990-1993, declined to slightly above 2% in 1993, and remained below 1.5% (with a downward trend) for the rest of the decade. It is also possible to check the realism of our PD scenarios by looking at the ratio of *Loan losses* to *Total loans*, whose quarterly evolution over recent years appears in Figure 2. Notice that under our assumption about the value of the LGD parameter, $\lambda = 0.45$ (which we take from the "foundation IRB" approach of Basel II), the average default rate behind the series in Figure 2 should be $1/0.45 \simeq 2.22$ times the ratio depicted there, which again suggests the realism of our choice of PDs slightly above 1% in low default states and around 3% in high default states.



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