### An Unbiased Appraisal of Purchasing Power Parity

PAUL CASHIN and C. JOHN MCDERMOTT\*

Univariate studies of the hypothesis of unit roots in real exchange rates have yielded consensus point estimates of the half-life of deviations from purchasing power parity (PPP) of between three to five years (Rogoff, 1996). However, conventional least-squares-based estimates of half-lives are biased downward. Accordingly, as a preferred measure of the persistence of real exchange rate shocks we use median-unbiased estimators of the half-life of deviations from parity, which correct for the downward bias of conventional estimators. We study this issue using real effective exchange rate (REER) data for 20 industrial countries in the post-Bretton Woods period. The serial correlation-robust median-unbiased estimator yields a cross-country average of half-lives of deviations from parity of about eight years, with the REER of several countries displaying permanent deviations from parity. However, using the median-unbiased estimator that is robust to the moving average and heteroskedastic errors present in real exchange rate data reduces the estimated half-life of parity deviations. Using this unbiased estimator, we find that the majority of countries have finite point estimates of half-lives of parity deviation, which is supportive of PPP holding in the post-Bretton Woods period. We also find that the average bias-corrected half-life of parity deviations is about five years, which is consistent with (but at the upper end of) Rogoff's (1996) consensus estimate of the half-life of deviations from parity. [JEL C22, F31]

Do real exchange rates really display parity-reverting behavior? In summarizing the results from studies using long-horizon data, Froot and Rogoff (1995) and Rogoff (1996) report the current consensus in the literature that the half-life of a shock (the time it takes for the shock to dissipate by 50 percent) to the real exchange rate is about three to five years, implying a slow parity reversion

<sup>\*</sup>Paul Cashin is a Senior Economist in the Macroeconomic Studies Division of the IMF's Research Department; John McDermott is Chief Economist at the National Bank of New Zealand. The authors thank Peter Clark, Robert Flood, Geoffrey Kingston, Sam Ouliaris, Peter Phillips, Kenneth Rogoff, Miguel Savastano, Peter Wickham, seminar participants at the International Monetary Fund, the Sixth Australian Macroeconomics Workshop (University of Adelaide), and New Zealand Exchange Rate Workshop (Victoria University of Wellington), and an anonymous referee for their comments and suggestions. Nese Erbil and Chi Nguyen provided excellent research assistance.

rate of between 13 to 20 percent per year.<sup>1</sup> Such a slow speed of reversion to purchasing power parity is difficult to reconcile with nominal rigidities and, as pointed out by Rogoff (1996), is also difficult to reconcile with the observed large short-term volatility of real exchange rates.

In earlier work, Meese and Rogoff (1983) demonstrated that a variety of linear structural exchange rate models failed to forecast more accurately than a naïve random walk model for both real and nominal exchange rates. If the real exchange rate follows a random walk, then innovations to the real exchange rate persist and the time series can fluctuate without bound. This result is contrary to the theory of purchasing power parity (PPP), which at its most basic level states that there is an equilibrium level to which exchange rates converge, such that foreign currencies should possess the same purchasing power.<sup>2</sup>

Notwithstanding the above-mentioned consensus in the literature on the speed of parity reversion, the conclusion of Meese and Rogoff (1983) has been reached in many subsequent studies of the time-series properties of the real exchange rate. This conclusion has usually been derived using formal statistical tests that failed to reject the null hypothesis of a unit root in the real exchange rate against the alternative of a stationary autoregressive (AR) model. If the unit root model can characterize real exchange rate behavior, then PPP does not hold because there is no propensity to revert back to any equilibrium level.

The empirical literature on testing the existence of PPP has developed in tandem with developments in the unit root econometrics literature, and has taken several paths. First, a standard rationale for the inability of researchers to clearly reject the unit root null, especially in the post–Bretton Woods period, is that unit root tests have low power because of the relatively short sample periods under study. In response, long-run data of a century or more, which span several exchange rate regimes, have been analyzed to improve the power of unit root tests (see Frankel, 1986; and Lothian and Taylor, 1996, among others). Second, panel data methods have been used in an attempt to increase the power of unit root tests (see Frankel and Rose, 1996; and Wu, 1996, among others).<sup>3</sup> Third, as PPP implies cointegration between the nominal exchange rate, domestic price level, and foreign price level, multivariate tests of the null hypothesis of no cointegration between these three variables have been carried out (see Corbae and Ouliaris,

<sup>&</sup>lt;sup>1</sup>Abuaf and Jorion (1990) use data on bilateral real exchange rates between the United States and several industrial countries during the twentieth century, and find average half-lives of deviations from parity of a little over three years. Frankel (1986) and Lothian and Taylor (1996) use two centuries of annual data on the sterling-dollar real exchange rate in calculating half-lives of about five years. Wu (1996) and Papell (1997) use panel data methods on quarterly post–Bretton Woods data to derive half-lives of between two to three years.

<sup>&</sup>lt;sup>2</sup>The version of PPP with the longest pedigree is that of relative PPP, which states that the exchange rate will be proportional to the ratio of money price levels (including traded and nontraded goods) between countries, that is, to the relative purchasing power of national currencies (see Wickham, 1993). For earlier surveys on PPP and exchange rate economics, see Isard (1995), Froot and Rogoff (1995), and Sarno and Taylor (2002).

<sup>&</sup>lt;sup>3</sup>The results of the recent burst of activity in the conduct of univariate tests of PPP have been characterized by Taylor (2001) as either in the "whittling down half-lives" camp (such as Frankel and Rose, 1996; and Wu, 1996) or the "whittling up half-lives" camp (such as Papell, 1997; O'Connell, 1998; and Engel, 2000).

1988; and Edison, Gagnon and Melick, 1997, among others). Both the long-run data and panel approaches have produced results that more frequently reject the unit root null for real exchange rates, while (particularly for post–Bretton Woods data) the results from cointegrating regressions have varied widely in their ability to reject the null of no cointegration (Froot and Rogoff, 1995).

However, one aspect of unit root econometrics that has been largely neglected in the PPP literature is the problem of "near unit root bias," which biases empirical results in favor of finding PPP. An important pitfall in using the autoregressive or unit root model to analyze the persistence of shocks to the real exchange rate is that standard estimators, such as least squares, are significantly downwardly biased in finite samples. This downward bias of least-squares estimates of autoregressive parameters becomes particularly acute when the autoregressive parameter is close to unity—in this case the process is close to being nonstationary and, as the leastsquares estimator minimizes the regression residual variance, it will tend to make the data-generating process appear to be more stationary than it actually is by forcing the autoregressive parameter away from unity. As lower values of the autoregressive parameter imply faster speeds of adjustment following a shock, this will also result in a downward bias to least-squares-based estimates of half-lives of shocks.<sup>4</sup> This near unit root bias is likely to be particularly relevant for real exchange rates, as they are often found to be stationary, yet exhibit shocks that are highly persistent.

In this paper we will generate a transformation from some initial estimator to a median-unbiased estimator, in order to correct for the near unit root bias. Median-unbiased estimators for autoregressive models have been proposed by Andrews (1993), Andrews and Chen (1994), McDermott (1996), and earlier by Rudebusch (1992), and Stock (1991). The median-unbiased estimators employed in this paper are based on initial estimators that include Dickey-Fuller, Augmented Dickey-Fuller, and Phillips-Perron unit root regressions. Importantly, this paper is the first to use the initial estimators proposed by Phillips (1987) and Phillips and Perron (1988), which, given they are robust to weakly dependent and heterogeneously distributed time series, are better suited than alternative median-unbiased estimators for modeling real exchange rates.

Conventional analyses of whether real exchange rates are better modeled as stationary or random walk processes typically focus on whether real exchange rate shocks are mean-reverting (finite persistence) or not (permanent). Such tests of the null hypothesis of a unit root in real exchange rates are rather uninformative as to the speed of parity reversion, because a rejection of the unit root null could still be consistent with a stationary model of real exchange rates that has highly persistent shocks. In contrast, this paper concentrates on measuring the duration of shocks to

<sup>&</sup>lt;sup>4</sup>While this bias is certainly present in standard (least-squares) estimation of the unit root model, panel-data methods (such as those applied by Wu, 1996; Papell, 1997; and Taylor and Sarno, 1998), which pool cross-country information, will also be subject to the near unit root bias that raises the estimated speed of reversion to parity (see Cermeño, 1999). In addition, the use of panel unit root tests has also been criticized because authors typically assume that rejection of the joint null hypothesis of unit root behavior of the whole panel of real exchange rates implies that all real exchange rates are stationary, when in actuality it only implies that at least one of them is stationary (see Taylor and Sarno, 1998). This bolsters the case for using univariate methods, as is done in the present paper.

the real exchange rate and associated confidence intervals, and undertakes no hypothesis tests as to the suitability of the assumption of a unit root as the process governing the evolution of real exchange rates. Instead of unit root tests, in this paper we characterize the extent of parity reversion in terms of point and interval estimates of the half-life of deviations from purchasing power parity, where the half-life is defined as the duration of time required for half the magnitude of a unit shock to the level of a series to dissipate. Point and interval estimators are useful statistics for providing information to draw conclusions about the relevance of PPP, as unlike hypothesis tests they are informative when a hypothesis is not rejected, and will be used in this paper.

The contributions of this paper are fourfold. First, the median-unbiased estimator of Andrews (1993) and Andrews and Chen (1994) is used to obtain point and interval estimates of the autoregressive parameter in the real exchange rate data. These median-unbiased estimators are generated from a transformation of an initial estimator, and correct for the downward bias in conventional (least-squares) estimation of the autoregressive parameter in unit root models. Second, we follow McDermott (1996) and use median-unbiased estimators that allow for initial estimators that display a wider class of error processes (particularly a moving average error structure) than previously considered in the literature. In particular, we follow Baillie and Bollerslev (1989) and Lothian and Taylor (1996, 2000) and are careful to use heteroskedasticity-robust estimation methods, as it is well known that exchange rate series exhibit both serial correlation and time-dependent heteroskedasticity. In particular, our preferred median-unbiased estimator uses an initial estimator that is based on autoregressive models of Phillips (1987) and Phillips and Perron (1988), which are robust to a wide variety of weakly dependent and heterogenously distributed time series. Third, these unbiased estimates of the autoregressive parameter and associated impulse response functions are used to calculate an unbiased scalar measure of the average duration (in terms of halflives) and range of typical real exchange rate shocks. Fourth, using Andrews' (1993) unbiased model-selection rule we can overcome the low power problems inherent in conventional unit root tests of PPP, and be more definitive about our willingness to draw conclusions as to the presence or absence of parity reversion of real exchange rates in the post-Bretton Woods period.

Our main results may be summarized briefly. First, using post–Bretton Woods data on the real effective exchange rates of 20 industrial countries and conventional (least-squares) biased estimation of unit root models, we replicate Rogoff's (1996) consensus estimate of the half-life of deviations from purchasing power parity (PPP) of between three to five years. Second, we find that serial correlation-robust median-unbiased point estimates of the half-lives of deviations of real exchange rates from PPP in the post–Bretton Woods period are typically *longer* than the previous consensus allows for, with cross-country average (median) half-lives of parity deviation lasting about eight years. In particular, we find that for at least 5 of the 20 countries in our sample, deviations of the real exchange rate from parity are best viewed as being permanent. Third, notwithstanding this result, when we use the median-unbiased estimator that is robust to the moving average and heteroskedastic errors present in real exchange rate data, we find that the

majority of countries have finite point estimates of half-lives of parity deviation, yet with wide confidence intervals around these point estimates. Using an unbiased model-selection rule, for these countries with finite point estimates of halflives of deviation from parity there is evidence of reversion (albeit slow) of real exchange rates to parity, which is consistent with PPP holding in the post–Bretton Woods period. Fourth, we find that the average heteroskedasticity-robust medianunbiased point estimates of the half-life of parity deviation is about five years, which is *shorter* in duration than those derived from median-unbiased estimators that fail to account for heteroskedasticity. Such a duration of parity reversion is consistent with (but at the upper end of) Rogoff's (1996) consensus estimate of the (downwardly biased) half-life of deviations of real exchange rates from parity. In summary, while median-unbiased methods always increase the estimated half-life of deviations from PPP in comparison with those derived from conventional, downwardly biased (least-squares) methods, allowing for heteroskedasticity reduces the bias-corrected estimated half-life of parity deviations.

#### I. Biased and Unbiased Measures of Half-Lives of Shocks to Parity

The existence of long-run PPP is inconsistent with unit roots (infinite half-lives of parity deviation) in the real exchange rate process. This notion has stimulated the growth of a large literature, using various tests, to resolve whether PPP holds in the post-Bretton Woods period. However, analyses of the trend-stationary or difference-stationary dichotomy of standard unit root tests focus only on whether such shocks are mean-reverting (finite persistence) or not (permanent). For economists, long-run PPP means more than the absence of a unit root-it also means the presence of a sufficient degree of mean reversion in exchange rates (over the horizon of interest) to validate the theoretical predictions of models based on the PPP assumption. For example, using the Dornbusch (1976) overshooting model, which has plausible assumptions about nominal wage and price rigidities, we would expect substantial convergence of real exchange rates to PPP over one to two years. Rather than use unit root tests to evaluate PPP it is preferable to use a scalar measure of the speed of reversion of real exchange rate shocks, and recent papers examining the post-Bretton Woods period have used estimates of the half-life of deviations from PPP to do so (Andrews, 1993; Andrews and Chen, 1994; and Cheung and Lai, 2000b).<sup>5</sup>

To estimate the speed of convergence to purchasing power parity (PPP) most researchers use the first-order autoregressive (AR(1)) model of the univariate time series  $\{q_t: t = 0, ..., T\}$ , assuming independent identically distributed normal errors. The model considered is

$$q_t = \mu + \alpha q_{t-1} + \varepsilon_t \quad \text{for} \quad t = 1, \dots, T, \tag{1}$$

<sup>&</sup>lt;sup>5</sup>Biased and median-unbiased point and interval estimates of the half-life of shocks to economic time series have also been used by Cashin, Liang, and McDermott (2000) in modeling the persistence of shocks to world commodity prices, and by Cashin, McDermott, and Pattillo (forthcoming) in modeling the persistence of terms of trade shocks. Earlier, Stock (1991) considered point and asymptotic confidence intervals for the largest autoregressive root in a time series.

where  $q_t : t = 0, ..., T$  is the real exchange rate,  $\mu$  the intercept,  $\alpha$  the autoregressive parameter (where  $\alpha \in (-1,1]$ ), and  $\varepsilon_t$  are the innovations of the model. A time trend is usually not included in equation (1), as a trend would not be consistent with long-run PPP (which imposes the restriction that real exchange rates have a constant unconditional mean).<sup>6</sup> This model is the same as that used for testing whether there is a unit root in a time series—consequently, this model is often referred to as the Dickey-Fuller (1979) regression. The half-life, which is the time it takes for a deviation from PPP to dissipate by 50 percent, is calculated from the autoregressive parameter,  $\alpha$  (see below for details).

# Problems with Conventional (Least-Squares) Measures of the Half-Life of Shocks to Parity

There are three problems with using these least-squares-based half-lives as evidence of the persistence of PPP deviations: biased autoregressive parameter coefficients; no confidence interval around the half-life measures; and serially correlated and heteroskedastic errors. We discuss each of these problems in turn.

#### Downwardly Biased Autoregressive Parameters

First, it has been known since the work of Orcutt (1948) that least-squares estimates of lagged dependent variable coefficients (such as the autoregressive parameter in the Dickey-Fuller regression) will be biased toward zero in small samples. The literature on the bias of least-squares estimation of autoregressive models is an old one. Marriott and Pope (1954) established the mean bias of the least squares estimator for the stationary AR(1) model, as did Shaman and Stine (1988) for stationary AR(p) models. While least squares will be the best linear unbiased estimator under the Gauss-Markov theorem, in the autoregressive case the assumptions of this theorem are violated, as lagged values of the dependent variable cannot be fixed in repeated sampling, nor can they be treated as distributed independently of the error term for all lags. Marriott and Pope (1954) showed that, ignoring second-order terms, the expected value of the least-squares estimate of the true  $\alpha$  in the AR(1) model of equation (1) can be approximated by:  $E(\hat{\alpha}) = \alpha - (1 + 3\alpha)/N$ , where N = T - 1. Using simulation calculations, Orcutt and Winokur (1969) find that, for T = 40 and true  $\alpha = 1$ , the least-squares mean bias is  $E(\hat{\alpha}) - \alpha = 0.129$ . Similarly, the simulation calculations of Andrews (1993) reveal that the least-squares median bias of equation (1), again for T = 40 and true  $\alpha = 1$ , is slightly smaller at 0.107. In general, the larger the true value of  $\alpha$ , the larger the least-squares bias, and so the bias is largest in the unit root case. The bias shrinks as the sample size grows, as the estimate converges to the true population value.

The downward bias in least-squares estimates of the autoregressive parameter arises because there is an asymmetry in the distribution of estimators of the auto-

<sup>&</sup>lt;sup>6</sup>Time trends are sometimes included in tests of PPP in an attempt to control for the Balassa-Samuelson effect, where the failure of PPP to hold can be due to differential rates of productivity growth in the tradable and nontradable sectors.

regressive parameter in AR models. The distribution is skewed to the left, resulting in the median exceeding the mean. As a result, the median is a better measure of central tendency than the mean in least-squares estimates of Dickey-Fuller models. The exact median-unbiased estimation procedure proposed by Andrews (1993) can be used to correct this bias. The bias correction delivers an impartiality property to the decision-making process, because there is an equal chance of under- or overestimating the autoregressive parameter in the unit root regression. Moreover, an unbiased estimate of  $\alpha$  will allow us to calculate an unbiased scalar estimate of persistence—the half-life of a unit shock.

#### Lack of Confidence Intervals Around Point Estimates of Half-Lives

Second, reporting only point estimates of the half-lives provides an incomplete picture of the speed of convergence toward PPP. To gain a more complete view one should use interval estimates. Fortunately, median-unbiased estimation allows for the calculation of median-unbiased confidence intervals. Moreover, interval estimation addresses the low power problem, usually associated with unit root tests (DeJong and others, 1992), by informing us whether we are failing to reject the null because it is true or because there is too much uncertainty as to the true value of the autoregressive parameter.

#### Serially Correlated and Heteroskedastic Errors

Third, the presence of serial correlation (typical in economic time series) means that the Dickey-Fuller regression will often not be appropriate. In such cases, we can follow Andrews and Chen (1994) and use an AR(p) model, which adds lagged first differences to account for serial correlation. The AR(p) model (also known as an Augmented Dickey-Fuller regression) takes the form

$$q_t = \mu + \alpha q_{t-1} + \sum_{i=1}^{p-1} \psi_i \Delta q_{t-i} + \varepsilon_t \quad \text{for} \quad t = 1, ..., T,$$
 (2)

where the observed real exchange rate series is  $q_t$ : t = -p, ..., T. Andrews and Chen (1994) show how to perform approximately median-unbiased estimation of autoregressive parameters in Augmented Dickey-Fuller regressions.

While Andrews (1993) assumes *iid* errors and Andrews and Chen (1994) assume AR(*p*) errors, neither approach allows for the possibility of a moving average error structure (unless  $p \rightarrow \infty$  as  $T \rightarrow \infty$ ). If moving average and heteroskedastic errors are present, as is typical for real exchange rate data, then even the Andrews-Chen method may not account for the biases arising from these attributes of the data. One method that can deal with more general error processes than those used in previous work is the semi-nonparametric technique of the Phillips-Perron (1988) unit root regression, which estimates the model of equation (1) and accounts for a wide range of serial correlation and heteroskedasticity using nonparametric methods.

It is well known that selection of the lag truncation point for the spectral density at zero frequency can have a large impact on the estimated spectral density and, therefore, on the Phillips-Perron test (Andrews, 1991). Monte Carlo studies by Phillips and Xiao (1998) and Cheung and Lai (1997) find that the small-sample properties of the Phillips-Perron unit root test are significantly improved if prewhitened kernel estimation of the long-run variance parameter (following Andrews and Monahan, 1992) occurs prior to the use of any data-determined bandwidth selection procedure (such as that of Andrews, 1991). In particular, with the combined use of these two procedures, the Phillips-Perron test performs better than (or at least as well as) the Augmented Dickey-Fuller test in terms of comparative power and yields tighter confidence intervals.<sup>7</sup> We implement the bias-corrected Phillips-Perron regression in this paper, using the approach initially set out by McDermott (1996).

## Bias-Correcting Estimates of the Autoregressive Parameter and the Model Selection Rule

Andrews (1993) presents a method for median-bias correcting the least-squares estimator. To calculate the median-unbiased estimator of  $\alpha$ , suppose  $\hat{\alpha}$  is an estimator of the true  $\alpha$  whose median function ( $m(\alpha)$ ) is uniquely defined  $\forall \alpha \in (-1,1]$ . Then  $\hat{\alpha}_u$  (the median-unbiased estimator of  $\alpha$ ) is defined as:

$$\hat{\alpha}_{u} = \begin{cases} 1 \text{ if } \hat{\alpha} > m(1), \\ m^{-1}(\hat{\alpha}) \text{ if } m(-1) < \hat{\alpha} \le m(1), \\ -1 \text{ if } \hat{\alpha} \le m(-1), \end{cases}$$
(3)

where  $m(-1) = \lim_{\alpha \to -1} m(\alpha)$ , and  $m^{-1}:(m(-1), m(1)] \to (-1, 1]$  is the inverse function of m(.) that satisfies  $m^{-1}(m(\alpha)) = \alpha$  for  $\alpha \in (-1, 1]$ . That is, if we have a function that for each true value of  $\alpha$  yields the median value (0.50 quantile) of  $\hat{\alpha}$ , then we can simply use the inverse function to obtain a median-unbiased estimate of  $\alpha$ . Intuitively, we find the value of  $\alpha$  that results in the least-squares estimator having a median value of  $\hat{\alpha}$ . For example, if the least-squares estimate of  $\alpha$  equals 0.8, then we do not use that estimate, but instead use that value of  $\alpha$  that results in the least-squares estimator having a median of 0.8. The extent of the median bias rises with the persistence of the innovations, which is particularly important for near unit root series such as the real exchange rate, which in the literature have previously (using point estimates of  $\alpha$ ) been found to be stationary, yet with shocks that are rather persistent.<sup>8,9</sup>

<sup>&</sup>lt;sup>7</sup>In contrast, the results of De Jong and others (1992)—that the semi-parametric Phillips-Perron unit root test has low power when there is positive serial correlation—were obtained using Phillips-Perron estimators with arbitrarily fixed bandwidth selection and without prewhitening. Choi and Chung (1995) also find that using the Andrews (1991) automatic bandwidth selection procedure for the Phillips-Perron test results in the Phillips-Perron test being more powerful than the Augmented Dickey-Fuller test.

<sup>&</sup>lt;sup>8</sup>The size of the bias correction can be large, especially when  $\alpha$  is close to one. For example, for a sample size of 60 observations using the AR(1) model of equation (1), a least-squares estimate of  $\alpha = 0.80$  would correspond to a median-unbiased estimate of  $\alpha = 0.85$ .

<sup>&</sup>lt;sup>9</sup>Other sources of bias in estimation of unit root regressions have been examined in the large literature on PPP, and will not be examined in this paper. These include large size bias in univariate tests for long-run PPP, due to a significant unit root component in the relative price of nontraded goods (Engel, 2000); size bias in multivariate tests for long-run PPP, due to a failure to control for cross-sectional correlation (O'Connell, 1998); and sample-selection bias of the countries analyzed, which biases the results toward understating the general relevance of parity reversion (Cheung and Lai, 2000a).

#### Model Selection Rule

The median-unbiased estimator can also be used to derive an unbiased modelselection rule, where for any correct model the probability of selecting the correct model is at least as large as the probability of selecting each incorrect model (Andrews, 1993; Andrews and Chen, 1994).<sup>10</sup> Suppose the problem is to select one of two models defined by  $\alpha \in I_a$  and  $\alpha \in I_b$ , where  $I_a$  and  $I_b$  are intervals partitioning the parameter space (-1, 1] for  $\alpha$ , with  $I_a = (-1, 1)$  and  $I_b = \{1\}$ . Then the unbiased model selection rule would indicate that model  $I_m$  should be chosen if  $\hat{\alpha}_u \in I_m$ , for m = a, b. This is also a valid level 0.50 (unbiased) test of the  $H_0$ :  $\alpha \in I_a$ versus  $H_1$ :  $\alpha \in I_b$ .

Importantly, the median-unbiased estimator  $\hat{\alpha}_u$  is the lower and upper bounds of the two one-sided 0.5 confidence intervals for the true  $\alpha$  when m(.) is strictly increasing (Andrews, 1993, p. 152). These confidence intervals have the property that their probabilities of encompassing the true  $\alpha$  are one-half. That is, there is a 50 percent probability that the confidence interval from minus one to  $\hat{\alpha}_u$  contains the true  $\alpha$ , and a 50 percent probability that the confidence interval from  $\hat{\alpha}_u$ to one contains the true  $\alpha$ . For example, if  $\hat{\alpha}_u = 0.90$ , then the probability that the true  $\alpha$  is less than 0.90 is one-half, and the probability that the true  $\alpha$  exceeds 0.90 is also one-half.

Based on the median-unbiased estimate of  $\alpha$ , other tests with different size and power properties can also be constructed. Using the 0.05 and 0.95 quantile functions of  $\hat{\alpha}$  we can construct two-sided 90 percent confidence intervals or one-sided 95 percent confidence intervals for the true  $\alpha$ . These confidence intervals can be used either to provide a measure of the accuracy of  $\hat{\alpha}$  or to construct the conventional exact one- or two-sided tests of the null hypothesis that  $\alpha = \alpha_0$ . In this paper we use such confidence intervals only to provide a measure of the accuracy of  $\hat{\alpha}$ .

In a Monte Carlo study of the AR(p) model, Andrews and Chen (1994, p. 194) demonstrate that the unbiased model-selection rule has a probability of correctly selecting the unit root model (when the true  $\alpha = 1$ ) of about 0.5. This is much lower than the corresponding probability for a (two-sided) level 0.10 test or (one-sided) level 0.05 test of a unit root null hypothesis, as the unbiasedness condition does not (unlike the level 0.10 or 0.05 tests) give a bias in favor of the unit root model. The greater size of Andrews' unbiased model selection rule, in comparison with conventional tests, increases the probability of rejecting the unit root null. This indicates that if the true  $\alpha < 1$ , then the probability of a type II error (failure to reject the unit root model when it is false) is smaller for Andrews' model selection rule than for conventional tests, especially for the near unit root case.

<sup>&</sup>lt;sup>10</sup>The unbiased model selection procedure based on the median-unbiased estimate of the AR(1) model is an exact test, as are its associated confidence intervals. However, the unbiased model selection procedure based on the median-unbiased estimate of the AR(*p*) model is an approximate test, as are its associated confidence intervals. This is because the distribution of  $\hat{\alpha}_u$  calculated from the AR(*p*) model depends on the true values of the  $\psi_i$  terms in equation (2), which are unknown. Andrews and Chen (1994) demonstrate that the approximately median-unbiased point and interval estimates of  $\alpha$  in the AR(*p*) model are very close to being median-unbiased. A similar unbiased model selection procedure can be invoked for the Phillips-Perron regression, and this is also an approximate test because of the need to estimate the serial correlation correction.

#### **Calculating Half-Lives**

Our interest in this paper concerns the persistence of shocks to economic time series. In this connection, the impulse response function of a time series  $\{q_t: t = 1, 2, ...\}$  measures the effect of a unit shock occurring at time *t* (that is,  $\varepsilon_t \rightarrow \varepsilon_t + 1$  in equations (1) and (2)) on the values of  $q_t$  at the future time periods t+1, t+2, ... This function quantifies the persistence of shocks to individual time series. For the AR(1) model the impulse response function is given by

$$IR(h) = \alpha^h \text{ for } h = 0, 1, 2, \dots$$
 (4)

For an AR(p) model the impulse response function is given by

$$IR(h) = f_{11}^{(h)}$$
 for  $h = 0, 1, 2, ...,$  (5)

where  $f_{11}^{(h)}$  denotes the (1,1) element of  $\mathbf{F}^h$  and where  $\mathbf{F}$  is the  $(p \times p)$  matrix

	$\alpha_1$	$\alpha_2$	$\alpha_3$	•••	$\alpha_{p-1}$	$\alpha_p$
	1	0	0		0	0
$F \equiv$	0	1	0		0	0
	:	÷	÷		$lpha_{p-1} \ 0 \ ec{0} \ ec{0}$	÷
		0	0		1	0

However, rather than consider the whole impulse response function to gauge the degree of persistence, we use a scalar measure of persistence that summarizes the impulse response function: the half-life of a unit shock (HLS). For the AR(1) model (with  $\alpha \ge 0$ ), the HLS gives the length of time until the impulse response of a unit shock is half its original magnitude, and is defined as HLS = ABS(log(1/2)/log( $\alpha$ )). Since median-unbiased estimates of  $\alpha$  have the desirable property that any scalar measures of persistence calculated from them (such as half-lives) will also be median unbiased, we can calculate the median-unbiased estimate of HLS by inserting the median-unbiased estimate of  $\alpha$  in the formula for HLS. Similarly, the 90 percent confidence interval of the exactly median-unbiased Dickey-Fuller estimate of the HLS is calculated using the 0.05 and 0.95 quantiles of  $\hat{\alpha}$  in the formula for HLS.

Median-unbiased point and confidence intervals for the HLS are calculated in a similar fashion for the Phillips-Perron estimator of equation (1), under the assumption that the impulse response function can be approximated by an AR(1) process.<sup>11</sup> The median-unbiased estimate of the HLS is calculated using the

<sup>&</sup>lt;sup>11</sup>In calculating the point and interval estimates of the HLS it is assumed that the nonparametric adjustment of the Phillips-Perron regression removes the serial correlation, and what is left is a pure AR(1) process. Given this assumption holds, the HLS can then be calculated from the "noise reduced" impulse response function. This approximation is required because the true impulse response function is unobservable, as a parametric form of the time-series model does not exist. The results from the Augmented Dickey-Fuller regression suggest that this approach is a reasonable one, because the non-monotonicity of the impulse response function occurs at low lags and the shape of the impulse response is dominated by the AR(1) component.

median-unbiased Phillips-Perron estimate of  $\alpha$  (following the approach of McDermott, 1996) in the formula for the HLS. Similarly, the 90 percent confidence interval of the median-unbiased Phillips-Perron estimate of the HLS is calculated using the 0.05 and 0.95 quantiles of  $\hat{\alpha}$  in the formula for the HLS.<sup>12</sup>

The half-life derived from the values of  $\alpha$  assumes that shocks decay monotonically. While appropriate for the AR(1) model, this assumption is inappropriate for an AR(*p*) model (with *p* > 1), since in general shocks to an AR(*p*) will not decay at a constant rate. The approximately median-unbiased point estimate of the half-life for AR(*p*) models (such as the Augmented Dickey-Fuller regression) can be calculated from the impulse response functions of equation (5), with the halflife defined as the time it takes for a unit impulse to dissipate permanently by onehalf from the occurrence of the initial shock (Cheung and Lai, 2000b). Similarly, the 90 percent confidence interval of the approximately median-unbiased estimate of the half-life is calculated using the 0.05 and 0.95 quantiles of  $\hat{\alpha}$ , calculated again as the time it takes for a unit impulse to dissipate permanently by one-half from the occurrence of the initial shock.

As with the estimation of  $\alpha$ , the median-unbiased half-lives and confidence intervals can be interpreted in two ways. Using the Andrews unbiased modelselection rule, there is a 50 percent probability that the confidence interval from zero to the estimated median half-life contains the true half-life of a shock to any given time series, and a 50 percent probability that the confidence interval from the estimated median half-life to infinity contains the true half-life of a shock to any given time series. Alternatively, we can use the 90 percent confidence interval to indicate the range that has a 90 percent probability of containing the true halflife of a shock to any given time series.

#### II. Data and Empirical Results

In this section we will investigate the persistence properties of the real exchange rate. The theory of relative PPP holds that the exchange rate will be proportional to the ratio of money price levels (including traded and nontraded goods) between countries, which implies that changes in relative price levels will be offset by changes in the exchange rate. By examining the persistence properties of real exchange rates we can determine whether real exchange rates do converge to their equilibrium relative PPP value in the long run, and thus determine whether PPP is consistent with the data.

The data used to estimate the near unit root model are monthly time series of the real exchange rate obtained from the International Monetary Fund's *International Financial Statistics (IFS)* over the sample 1973:4 to 2002:4 (the post–Bretton Woods period), which gives a total of 349 observations. The definition of the real exchange rate is the real effective exchange rate (REER) based on consumer prices (line *rec*), for which 20 industrial countries were selected. As such,

<sup>&</sup>lt;sup>12</sup>Both the Dickey-Fuller and Phillips-Perron median-unbiased measures of persistence and associated confidence intervals can be compared with their least-squares counterparts, where the least-squares point and interval estimates will (given they are functions of a downwardly biased  $\alpha$ ) tend to understate the actual amount of persistence in shocks to economic time series (see Section II).

we will examine the behavior of REER based on (i) the nominal effective exchange rate, which is the trade-weighted average of bilateral exchange rates vis-à-vis trading partners' currencies; (ii) the domestic price level, which is the consumer price index; and (iii) the foreign price level, which is the trade-weighted average of trading partners' consumer price indices. We analyze effective rather than bilateral real exchange rates as the effective rate measures the international competitiveness of a country against all its trade partners, and helps to avoid potential biases associated with the choice of base country in bilateral real exchange rate analyses.

The REER indices measure how nominal effective exchange rates, adjusted for price differentials between the home country and its trading partners, have moved over a period of time. The consumer price index (CPI)-based REER indicator is calculated as a weighted geometric average of the level of consumer prices in the home country relative to that of its trading partners, expressed in a common currency. The IMF's CPI-based REER indicator (base 1995 = 100) of country *i* is defined as

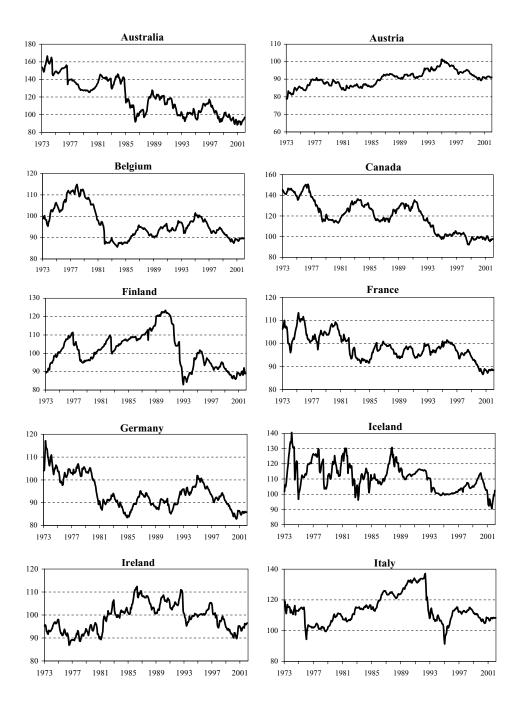
$$q_i = \prod_{j \neq i} \left[ \frac{P_i R_i}{P_j R_j} \right]^{W_{ij}},$$

where *j* is an index that runs over country *i*'s trade partner (or competitor) countries;  $W_{ij}$  is the competitiveness weight attached by country *i* to country *j*, which is based on 1988–90 average data on the composition of trade in manufacturing, non-oil primary commodities, and tourism services<sup>13</sup>;  $P_i$  and  $P_j$  are the seasonally adjusted consumer price indices in countries *i* and *j*; and  $R_i$  and  $R_j$  are the nominal exchange rates of the currencies of countries *i* and *j* in U.S. dollars. As shown by McDermott (1996), alternative measures of the real exchange rate, such as real bilateral exchange rates based on consumer prices and the *IFS*'s REER based on normalized unit labor costs, are both highly correlated with the *IFS*'s CPI-based REER index.

The REER data for all 20 countries are set out in Figure 1—an increase in the REER series indicates a real appreciation of the country's currency.<sup>14</sup> Several features of the data stand out. First, a cursory inspection of the REER series indicates that most countries have real exchange rates that appear to exhibit symptoms of drift or nonstationarity. There appear to be substantial and sustained deviations from PPP (that is, nonstationarity in the REER). The evolution of REER appears to be a highly persistent, slow-moving process; for most countries the REER does not appear to cycle about any particular equilibrium value, especially for Japan (the general appreciation of its exchange rate is typical of a process with a unit

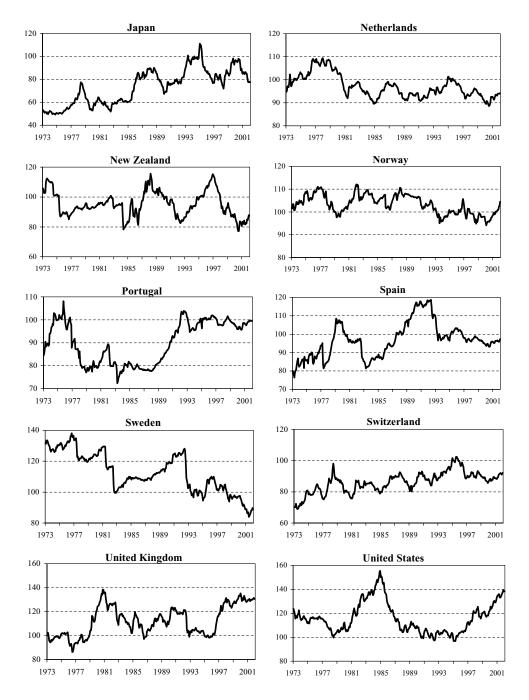
 $<sup>^{13}</sup>W_{ij}$  can be interpreted as the sum over all markets of a gauge of the degree of competition between producers of country *i* and *j*, divided by the sum over all markets of a gauge of the degree of competition between producers of country *i* and all other producers.

<sup>&</sup>lt;sup>14</sup>The 20 countries (and *IFS* country numbers) are Australia (193), Austria (122), Belgium (124), Canada (156), Finland (172), France (132), Germany (134), Iceland (176), Ireland (178), Italy (136), Japan (158), Netherlands (138), New Zealand (196), Norway (142), Portugal (182), Spain (184), Sweden (144), Switzerland (146), United Kingdom (112), and the United States (111). A decline (depreciation) in a country's REER index indicates a rise in its international competitiveness (defined as the relative price of domestic tradable goods in terms of foreign tradables). For a detailed explanation and critique of how the IMF's REER indices are constructed, see Zanello and Desruelle (1997) and Wickham (1993).



#### Figure 1. Real Effective Exchange Rate, (base 1995 = 100), Industrial Countries, April 1973-April 2002

Source: International Monetary Fund, International Financial Statistics.



#### Figure 1. Real Effective Exchange Rate, (base 1995 = 100), Industrial Countries, April 1973-April 2002 (concluded)

Source: International Monetary Fund, International Financial Statistics.

	and Heteros	skedasticity	
Country	Breusch- Godfrey	White	ARCH
Australia	21.3*	10.9*	0.8
Austria	17.8*	21.0*	5.1*
Belgium	37.6*	3.0	5.7*
Canada	17.4*	8.7*	4.9*
Finland	11.7*	1.2	2.6
France	349*	349	321*
Germany	30.6*	25.0*	53.6*
Iceland	9.8*	25.0*	1.8
Ireland	29.1*	11.8*	0.0
Italy	29.7*	7.7*	41.4*
Japan	347*	348*	326*
Netherlands	31.4*	3.6	49.2*
New Zealand	30.1*	0.9	16.7*
Norway	21.2*	2.4	2.3
Portugal	4.5	1.7	2.4
Spain	13.3*	0.1	3.5
Sweden Switzerland	29.6* 28.7* 40.4*	1.8 2.3 2.9	8.6* 22.6* 11.6*
United Kingdom	40.4**	2.9	0.6
United States	28.2*	25.2*	

# Table 1. Real Effective Exchange Rates: Tests for Serial Correlation and Heteroskedasticity

Notes: Breusch-Godfrey is a Lagrange Multiplier test of the hypothesis of no serial correlation in the residuals of the AR(1) model of equation (1); the test statistic is distributed as a  $\chi^2(n)$ , where *n* is the order of the autocorrelations (here n = 2, so the 5 percent critical value is 5.99). White is White's (1980) test of the null hypothesis of homoskedasticity; the test statistic is distributed as a  $\chi^2(s)$ , where *s* is the number of regressors, which here include the square of the regressors only (so s = 2, and the 5 percent critical value is 5.99). ARCH is the ARCH LM test for autoregressive conditional heteroskedasticity, where the null is that the coefficient on lagged squared residuals are all zero; the test is distributed as a  $\chi^2(q)$ , where *q* is the number of squared residuals (here q = 1, so the 5 percent critical value is 3.84). An asterisk denotes significance at the 5 percent level.

root). Second, sharp movements in the REER during the 1980s and 1990s are a relatively frequent occurrence, especially for countries such as Australia, Italy, New Zealand, the United Kingdom, and the United States. Third, tests carried out on the residuals from the least-squares regression of equation (1) indicate that the majority of REER regressions have residuals that exhibit serial correlation and heteroskedasticity (see Table 1). We now describe the results for our analysis of the persistence of parity deviations for the REER series, using both biased least-squares and median-unbiased estimators.

#### Biased Least-Squares Estimates of Half-Lives of Parity Reversion

Table 2 sets out the results for the half-life of the duration of shocks to the REER, which are calculated from the least-squares estimates of  $\alpha$  in the Dickey-Fuller (DF) regression of equation (1), as set out in Section I. Across all countries, the

Table 2. Half-Lives of Parity Deviations: Biased Least-Squares and Median-Unbiased Estimation of Dickey-Fuller Regressions	Biased Least Squares Median Unbiased	I Half-life 90 percent CI $\alpha$ (months) (months)	$^{\circ}$	United Kingdom 0.989 0.994 115 $[27, \infty]$ 1.000 $\infty$ $[72, \infty]$ 1.000 $[77, \infty]$ United States 0.994 115 $[38, \infty]$ $[38, \infty]$ $1.000$ $\infty$ $[77, \infty]$ $[77, \infty]$ Notes: <i>Least Squares</i> —The results in columms 2–4 of this table are based on least-squares estimation of the Dickey-Fuller regression of equation (1). The half- life is the length of time it takes for a unit impulse to dissipate by half. It is derived using the formula: HLS = ABS(log(1/2)/log( $\alpha$ )), where $\alpha$ is the autoregressive parameter. The least-squares estimate of the HLS is calculated using the least-squares estimate of $\alpha$ in the formula. The half-lives and the 90 percent confidence intervals (CI) of the half-life of parity deviations are calculated by inserting $\hat{\alpha} \pm 1.65 \times se$ ( $\hat{\alpha}$ ) into the HLS formula. The half-lives and the 90 percent confidence intervals are measured in months. <i>Median Unbiased</i> —The results in columns 5–7 of this table are based on the median-unbiased estimates of the Dickey-Fuller regression of equation (1), as given by Andrews (1993). The half-life is the length of time it takes for a unit impulse to dissipate by half. It is derived using the formula: HLS = ABS(log(1/2)/log( $\alpha$ ), where $\alpha$ is the median-unbiased autoregressive parameter. The median-unbiased estimate of the HLS. Similarly, the 90 percent confidence intervals (CI) of the half-life of parity deviations are derived using the 0.05 and 0.95 quantiles of the median-unbiased estimate of $\hat{\alpha}$ in the formula for the HLS. The quantile functions of $\hat{\alpha}$ were generated by numerical simulation (using 10,000 iterations) for <i>T</i> = 349 observations. The half-lives and the 90 percent confidence intervals are
		$\begin{array}{ccc} Half-life & 90 \ percent \ CI \\ \alpha & (months) & (months) \end{array}$	8 8 2 2 3 8 3 3 1 2 3 8 3 2 3 8 3 1 2 3 8 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3	United Kingdom 0.999 0.9115 124, $\approx 1$ 1.000 United States 0.994 115 138, $\approx 1$ 1.000 United States 0.994 115 15 138, $\approx 1$ 1.000 If is the length of time it takes for a unit impulse to dissipate by half. It is derived using the formula: HLS = Al parameter. The least-squares estimate of the HLS is calculated using the least-squares estimate of $\alpha$ in the formula (CI) of the half-life of parity deviations are calculated by inserting $\tilde{\alpha} \pm 1.65 \times se$ ( $\tilde{\alpha}$ ) into the HLS formula. The hal measured in months. <i>Median Unbiased</i> —The results in columns 5–7 of this table are based on the median-unbia quation (1), as given by Andrews (1993). The half-life is the length of time it takes for a unit impulse to d equation (1), as given by Andrews (1993). The half-life is the length of time it takes for a unit impulse to d HLS = ABS(log(1/2)/log( $\alpha$ )), where $\alpha$ is the median-unbiased autoregressive parameter. The median-unbiased estimate of $\alpha$ in the formula for the HLS. Similarly, the 90 percent confidence intervals (CI) of the half- and 0.95 quantiles of the median-unbiased estimate of $\hat{\alpha}$ in the formula for the HLS. The quantile functions of $\hat{\alpha}$ were iterations) for $T = 349$ observations. The half-lives and the 90 percent confidence intervals are measured in moths
		Country	Australia Austria Belgium Canada Finland France Germany Iceland Ireland Ireland Italy Japan New Zealand Norway Norway Portugal Sweden Sweden	United Kngdom United States Notes: Least Squares life is the length of time parameter. The least-squa (CI) of the half-life of pa measured in months. Me- equation (1), as given by HLS = ABS(log(1/2)/log unbiased estimate of $\alpha$ in and 0.95 quantiles of the 1 iterations) for $T = 349$ ob

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mean half-life of parity reversion is 60 months and the median half-life is 53 months.<sup>15</sup> This result is consistent with Rogoff's (1996) consensus of half-lives of parity reversion of between 36 to 60 months (three to five years).

We also report 90 percent confidence intervals for the least-squares half-life of PPP deviations, in order to gauge the variability of the persistence of shocks to the real exchange rate. The confidence intervals are quite wide and encompass half-lives that are consistent with both PPP holding and not holding in the long run. This indicates that there is a high level of uncertainty about the "true" value of the half-life of PPP deviations. For seven countries the upper bound of the 90 percent confidence interval is finite, indicating that these countries have finitely persistent shocks to their real exchange rates. It is important to note that the confidence intervals used here are formed assuming that the estimated autoregressive parameter has a *t*-distribution, which we know to be incorrect, and further biases hypothesis tests toward rejecting the unit root null.

As an example of how to interpret the table, we take the particular cases of Iceland and Canada. For Iceland, the least-squares (LS) estimate of  $\alpha$  is 0.941. Moreover, the time it takes for half of the shock to the REER of Iceland to dissipate is 11 months, while the length of the 90 percent confidence interval for the LS estimate of the half-life of deviations from parity is 7 to 23 months. Accordingly, shocks to the REER of Iceland do not appear to be very persistent, at least relative to the persistence found in other countries' real exchange rates.

For Canada, the LS estimate of  $\alpha$  is 0.994. Moreover, the time it takes for half of the shock to Canada's REER to dissipate is 115 months, while the length of the 90 percent confidence interval for the LS estimate of the half-life of deviations from parity is 47 to  $\infty$  months. Accordingly, shocks to the REER of Canada do appear to be very persistent, especially since the lower bound of the 90 percent confidence interval indicates that there is only a 5 percent chance that the true halflife of parity reversion of the REER is shorter than 47 months.

The DF regression results presented in Table 2 do not attempt to account for the presence of serial correlation. Tests for serial correlation carried out on the residuals from the least-squares regression of equation (1) indicate that all REER regressions (except Portugal) have residuals with serial correlation (see column 2 of Table 1). Accordingly, least-squares estimates of the Augmented Dickey-Fuller (ADF) regressions, which do account for serial correlation, are set out in Table 3. In examining for the presence of serial correlation, the general-to-specific lag selection procedure of Ng and Perron (1995) and Hall (1994) is used, with the maximum lag set to 14.<sup>16</sup> For all countries at least one lag (p = 2) of the first difference of the REER is significant, which ensures that the ADF half-lives will differ from the DF half-lives. The ADF half-lives are typically shorter in duration than those derived from the DF regression, ranging from 11 months (Iceland) to

<sup>&</sup>lt;sup>15</sup>These half-life results are comparable to those obtained by Cheung and Lai (2000b) using leastsquares estimation on monthly bilateral (post–Bretton Woods) dollar real exchange rates, which calculated an average half-life of 3.3 years.

<sup>&</sup>lt;sup>16</sup>Starting with the maximum lag, first-differences of the logarithm of the REER ( $q_t$ ) were sequentially removed from the AR model until the last lag was statistically significant (at the 5 percent level). At that point all lag lengths smaller than or equal to p - 1 are included in the AR(p) regression of equation (2).

		•				,		
		Biased Least Squares			Median Unbiased			
Country	р	α	Half-life (months)	90 percent CI (months)	α	Half-life (months)	90 percent CI (months)	
Australia Austria Belgium Canada Finland France Germany Iceland Ireland Italy Japan Netherlands New Zealand Norway Portugal Spain Sweden Switzerland	2 8 8 14 11 12 2 14 5 2 12 11 3 2 2 2 2 2 14	0.987 0.986 0.993 0.991 0.983 0.983 0.927 0.979 0.981 0.986 0.981 0.968 0.962 0.990 0.982 0.990 0.968	$55 \\ 52 \\ 99 \\ 81 \\ 45 \\ 43 \\ 11 \\ 33 \\ 37 \\ 52 \\ 40 \\ 22 \\ 19 \\ 71 \\ 39 \\ 73 \\ 20 $	$[29, \infty]$ $[29, \infty]$ $[43, \infty]$ $[40, \infty]$ $[30, 140]$ $[20, \infty]$ $[24, 214]$ $[9, 19]$ $[18, 155]$ $[21, 160]$ $[32, 199]$ $[25, 130]$ $[13, 64]$ $[12, 44]$ $[31, \infty]$ $[22, 148]$ $[34, \infty]$ $[13, 56]$	0.993 0.993 1.000 1.000 0.989 0.993 0.993 0.993 0.993 0.985 0.989 0.993 0.989 0.993 0.989 0.974 0.967 1.000 0.989 1.000 0.980	97 98 $\infty$ 67 96 97 11 47 65 95 65 27 22 $\infty$ 64 $\infty$ 35	$[37, \infty]$ $[37, \infty]$ $[48, \infty]$ $[53, \infty]$ $[36, \infty]$ $[35, \infty]$ $[27, \infty]$ $[9, 28]$ $[21, \infty]$ $[24, \infty]$ $[38, \infty]$ $[33, \infty]$ $[14, \infty]$ $[12, 97]$ $[46, \infty]$ $[30, \infty]$ $[43, \infty]$ $[12, \infty]$	
United Kingdom United States	2 2	0.984 0.991	45 77	[24, ∞] [34, ∞]	0.993 1.000	97 ∞	[36, ∞] [46, ∞]	

### Table 3. Half-Lives of Parity Deviations:Biased Least-Squares and Median-Unbiased Estimation of Augmented Dickey-Fuller Regressions

Notes: *Least Squares*—The results of columns 3–5 of this table are based on least-squares estimation of the Augmented Dickey-Fuller regression of equation (2). The half-life for AR(p) models is calculated from the impulse response functions (equation (5)), and is defined as the time it takes for a unit impulse to dissipate permanently by one-half from the occurrence of the initial shock. Similarly, the 90 percent confidence interval (CI) is calculated using the 0.05 and 0.95 quantiles, calculated again as the time it takes for a unit impulse to dissipate permanently by one-half from the occurrence of the initial shock. The half-lives of parity deviations and the 90 percent confidence intervals are measured in months. In examining for the presence of serial correlation, the general-to-specific lag selection procedure of Ng and Perron (1995) and Hall (1994) is used, with the maximum lag (p) set to 14. First-differences of the REER ( $q_i$ ) were sequentially removed from the AR model until the last (p - 1) lag was statistically significant (at the 5 percent level). The optimal lag length (p) is listed in column 2. *Median Unbiased*—The results of columns 6–8 of this table are based on the median-unbiased estimates of the Augmented Dickey-Fuller regression of equation (2), as given by Andrews and Chen (1994). The median half-life for AR(p) models is calculated from the impulse response functions (equation (5)), and is defined as the time it takes for a unit impulse to dissipate permanently by one-half from the occurrence of the initial shock. Similarly, the 90 percent confidence interval (CI) is calculated using the 0.05 and 0.95 quantiles, calculated again as the time it takes for a unit impulse to dissipate permanently by one-half from the occurrence of the initial shock. Similarly, the 90 percent confidence interval (CI) is calculated using the 0.05 and 0.95 quantiles, calculated again as the time it takes for a unit impulse to dissipate permanently by one-half from the occurrence of the initial shock

99 months (Belgium). Across all countries, the mean half-life of parity reversion is 48 months and the median half-life is 45 months. While for several countries the 90 percent confidence interval is narrower than for the DF regression (such as New Zealand and Norway), in several cases the variability of shocks to the REER is so wide as to include infinity as the upper bound of the confidence interval (such as Canada and the United States).<sup>17</sup>

The ADF regressions presented in Table 3 do not attempt to account for the presence of heteroskedasticity, and so will be invalid when there are departures from the maintained hypothesis of AR(p) errors. Tests for heteroskedasticity carried out on the residuals from the least-squares regression of equation (1) indicate that most REER regressions have residuals with heteroskedasticity (see columns 3–4 of Table 1). Accordingly, given the presence of heteroskedasticity and serial correlation (including moving-average error structures) in the real exchange rate series, the results of Phillips-Perron (PP) regressions, which are valid in the presence of serial correlation and heteroskedasticity, are presented in Table 4. The duration of PP half-lives are typically lower again than those derived from the ADF and DF regressions, ranging from 9 months (Iceland) to 79 months (Canada). Across all countries, the mean half-life of parity reversion from the PP regression is 35 months and the median half-life is 30 months. For those countries with finite upper bounds for the 90 percent confidence interval of the half-lives of deviations from parity, the confidence intervals based on PP regressions are typically tighter than those derived from the ADF and DF regressions, yet continue to encompass a wide range of half-lives. Controlling for the serial correlation present in the data lowers considerably the estimated half-life of parity deviations-the PP regressions detect more serial correlation than the ADF and DF regressions, and thus produce lower estimated half-lives of deviations from parity.

Broadly, the three least-squares results of Tables 2–4 indicate that across all countries the median half-life of parity deviations is finite, with an average length of about four years, and a lower bound on the confidence intervals for the true half-lives of about two years. However, the upper bound of the confidence interval for many REER is greater than ten years (and in many cases is infinity).

#### Median-Unbiased Estimates of Half-Lives of Parity Reversion

The half-lives of PPP deviations calculated above (using the least-squares estimator) are reasonably close to Rogoff's (1996) consensus of three to five years (36 to 60 months). However, as noted in Section I above, the least-squares estimator of the autoregressive parameter in each of the DF, ADF, and PP regressions is biased downward. As a result, the above calculations of the duration of deviations from PPP are also likely to be biased downward (and in favor of finding that PPP holds in the REER data). Consequently, we remove this bias by

<sup>&</sup>lt;sup>17</sup>Consistent with Papell (1997), we find that accounting for serial correlation in the disturbances weakens the evidence against a null hypothesis of a unit root in the real exchange rate series, as the point estimates of the autoregressive parameter are typically lower in the AR(p) case than for the AR(1) regression.

blaced Least ofdates and median enblaced Lannanen er himper ener kegresions									
		В	iased Least Squar	es		Median Unbiased			
Country	b	α	Half-life (months)	90 percent CI (months)	α	Half-life (months)	90 percent CI (months)		
Australia Austria Belgium Canada Finland France Germany Iceland Italy Japan Netherlands New Zealand Norway Portugal Spain Sweden Switzerland United Kingdom United States	$\begin{array}{c} 1.38\\ 0.65\\ 0.14\\ 0.39\\ 0.72\\ 0.98\\ 1.32\\ 1.14\\ 0.43\\ 1.91\\ 1.60\\ 1.22\\ 1.95\\ 1.44\\ 0.62\\ 0.21\\ 0.05\\ 1.62\\ 1.53\\ 0.63\\ \end{array}$	0.982 0.977 0.989 0.991 0.986 0.973 0.976 0.922 0.967 0.972 0.982 0.976 0.976 0.949 0.951 0.989 0.978 0.978 0.986 0.975 0.987	$     \begin{array}{r}       39\\       30\\       61\\       79\\       50\\       25\\       29\\       9\\       20\\       25\\       38\\       28\\       13\\       14\\       60\\       31\\       48\\       16\\       27\\       52     \end{array} $	$ \begin{bmatrix} 21, 329 \\ [18, 79] \\ [27, 264] \\ [35, 311] \\ [24, \infty] \\ [14, 113] \\ [16, 143] \\ [6, 16] \\ [12, 64] \\ [14, 98] \\ [20, 278] \\ [16, 146] \\ [8, 29] \\ [9, 31] \\ [27, 286] \\ [18, 123] \\ [23, \infty] \\ [10, 36] \\ [15, 160] \\ [23, 226] \\ \end{bmatrix} $	0.995 0.989 1.000 1.000 0.984 0.988 0.931 0.977 0.984 0.994 0.994 0.994 0.9988 0.959 0.960 1.000 0.990 1.000 0.968 0.987 1.000	$     \begin{array}{r}       136 \\       61 \\                          $	$ \begin{bmatrix} 26, \infty \end{bmatrix} \\ \begin{bmatrix} 20, 96 \end{bmatrix} \\ \begin{bmatrix} 41, \infty \end{bmatrix} \\ \begin{bmatrix} 52, \infty \end{bmatrix} \\ \begin{bmatrix} 33, \infty \end{bmatrix} \\ \begin{bmatrix} 17, 128 \end{bmatrix} \\ \begin{bmatrix} 19, 100 \end{bmatrix} \\ \begin{bmatrix} 6, 22 \end{bmatrix} \\ \begin{bmatrix} 14, 247 \end{bmatrix} \\ \begin{bmatrix} 17, 133 \end{bmatrix} \\ \begin{bmatrix} 25, \infty \end{bmatrix} \\ \begin{bmatrix} 19, 103 \end{bmatrix} \\ \begin{bmatrix} 9, 74 \end{bmatrix} \\ \begin{bmatrix} 9, 87 \end{bmatrix} \\ \begin{bmatrix} 40, \infty \end{bmatrix} \\ \begin{bmatrix} 21, 91 \end{bmatrix} \\ \begin{bmatrix} 32, \infty \end{bmatrix} \\ \begin{bmatrix} 11, \infty \end{bmatrix} \\ \begin{bmatrix} 11, \infty \end{bmatrix} \\ \begin{bmatrix} 18, 108 \end{bmatrix} \\ \begin{bmatrix} 34, \infty \end{bmatrix} $		

### Table 4. Half-Lives of Parity Deviations:Biased Least-Squares and Median-Unbiased Estimation of Phillips-Perron Regressions

Notes: *Least Squares:*—The results of columns 3–5 of this table are based on least-squares estimation of the Phillips-Perron (PP) regression of equation (1). The half-life is the length of time it takes for a unit impulse to dissipate by half from the occurrence of the initial shock. It is derived using the formula: HLS = ABS(log(1/2)/log( $\alpha$ )), where  $\alpha$  is the autoregressive parameter. The PP estimate of the HLS is calculated using the PP estimate of  $\alpha$  in the formula for the HLS. The 90 percent confidence intervals (CI) of the half-life of parity deviations are calculated by inserting  $\hat{\alpha} \pm 1.65 \times \text{se}(\hat{\alpha})$  into the HLS formula, where se ( $\hat{\alpha}$ ) is calculated using a long-run variance estimator. The half-lives and the 90 percent confidence intervals are measured in months. To control the amount of serial dependence allowed in the Phillips-Perron regression, the bandwidth parameter needs to be selected—we have used the automatic bandwidth selector of Andrews (1991), where the bandwidth (number of periods of serial correlation included) is indicated by *b*, and is reported in column 2. Prewhitened kernel estimation of the long-run variance parameter (following Andrews and Monahan, 1992) is used prior to the implementation of the data-determined bandwidth selection procedure. *Median Unbiased*—The results in columns 6–8 of this table are based on the median-unbiased estimates of the Phillips-Perron regression of equation (1). The half-life is the length of time it takes for a unit impulse to dissipate by half from the occurrence of the initial shock. It is derived using the formula: HLS = ABS(log(1/2)/log( $\alpha$ )), where  $\alpha$  is the autoregressive parameter. The median-unbiased estimate of the HLS is calculated using the median-unbiased estimate of  $\alpha$  in the formula for the HLS. Similarly, the 90 percent confidence intervals (CI) of the half-life of parity deviations are derived using the 0.05 and 0.95 quantiles of the median-unbiased estimate of  $\hat{\alpha}$  in the formula for the HLS. Similarly, the 90 percen

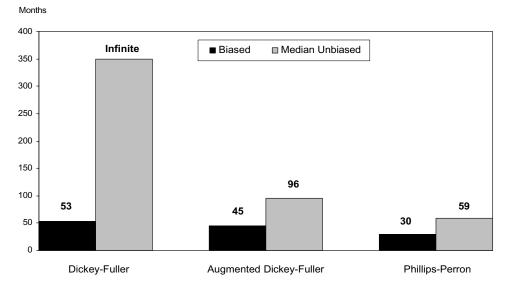
calculating median-unbiased point estimates and confidence intervals for the autoregressive parameter in equations (1) and (2).<sup>18</sup>

Median-unbiased estimates of the half-life of PPP deviations for the DF regressions are set out in Table 2. In comparison with the median-unbiased estimates of  $\alpha$  in DF regressions, the least-squares estimates of  $\alpha$  are biased downward by between 0.005 and 0.013. While this is a small difference in absolute terms, it has important implications for the half-life measures of the persistence of the REER. The median-unbiased point estimates of the half-lives are much greater than their least-squares counterparts for every country, with 11 of the countries having a half-life of infinity. Across all countries, the average (median) bias-corrected half-life of parity reversion is infinity, clearly exceeding the average downwardly biased least-squares AR(1) half-life of 53 months (Figure 2). This implies no parity reversion, rather than the 15 percent per year calculated using biased DF methods. In addition, the 90 percent confidence intervals for the median-unbiased estimates of half-lives of parity reversion are typically much wider than for their LS counterparts, and the REER of all countries in Table 2 (except Iceland) have an upper bound to the confidence interval of the unbiased half-lives that embrace infinity.<sup>19</sup>

The results in Table 2 indicate that, using the Andrews unbiased model-selection rule, 9 of the countries are subject to REER shocks that are finitely persistent, while 11 of the countries experience permanent shocks to their REER series. The interpretation of this rule is that for any given country there is a 50 percent probability that the confidence interval from zero to the estimated median-unbiased half-life contains the true half-life of shocks to its REER, and a 50 percent probability that the confidence interval from the estimated median-unbiased half-life to infinity contains the true half-life of shocks to its REER. Let us again take the examples of Iceland (short-lived half-life) and Canada (infinite half-life). While there is a 50 percent probability that the confidence interval from 13 months to infinity contains the true half-life of shocks to the REER of Iceland, there is a 50 percent probability that the confidence interval from 13 months to infinity contains the true half-life of shocks to its REER of Iceland, while there is a 50 percent probability that the confidence interval from 13 months to infinity contains the true half-life of shocks to the REER of Iceland. For Canada, while there is a 50 percent probability that the confidence interval with a finite upper bound contains the true half-life of shocks to its REER, there is a 50 percent probability that the confidence interval with a finite upper bound contains the true half-life of shocks to its REER, there is a 50 percent probability that the true half-life of shocks to its REER, there is a 50 percent probability that the true half-life of shocks to its REER of Iceland.

<sup>&</sup>lt;sup>18</sup>Median-unbiased estimates (and confidence intervals) of the half-life of a shock (for T = 349 observations (1973:4–2002:4)) were determined using quantile functions of  $\hat{\alpha}$  generated by: numerical simulation (using 10,000 iterations), following the method suggested by Appendix B of Andrews (1993) for the DF regression of equation (1) (see Table 2); numerical simulation (using 10,000 iterations), following the method suggested by McDermott (1996) for the PP regression of equation (1) (see Table 4); and numerical simulation (using 2,500 iterations), following the method suggested by Andrews and Chen (1994) for the ADF regression of equation (2) (see Table 3).

<sup>&</sup>lt;sup>19</sup>Our results for the median-unbiased Dickey-Fuller regression are similar to those of Andrews (1993), who calculated point and interval estimates of the half-life of monthly bilateral dollar real exchange rates for several industrial countries over the period 1973 to 1988. He found that, using least squares, the half-life of PPP deviations for each real exchange rate was finite, with an average half-life of about 31 months. However, using the median-unbiased procedure (for an AR(1) model) only three of the eight real exchange rates had finite half-lives of PPP deviations, with an average half-life of about 60 months. The remaining five exhibited permanent parity deviations. Similarly, while all of Andrews' median-unbiased lower bounds of the 90 percent confidence interval were less than 36 months (as is the case for the majority of countries in the present study), the upper bounds were all infinite (as is the case for all but one country in the present study).





shocks to its REER will be infinite (Table 2). Using the Andrews unbiased modelselection rule, the finite (Iceland) and infinite (Canada) point estimates of the halflives of deviations from parity indicate that while shocks to Iceland's REER are transitory, shocks to Canada's REER are best viewed as being permanent.

Using the alternative loss function implicit in conventional two-sided (level 0.10) hypothesis tests, there is another way to interpret our findings. Again, taking the examples of Iceland and Canada, we find that the estimated 90 percent confidence intervals of the bias-corrected half-life of deviations from parity range from 8 months to 42 months, and from 77 months to infinity, respectively (Table 2). There is a 90 percent probability that the above confidence intervals contain the true half-life of shocks to each country's REER. Accordingly, there is a 5 percent probability that the confidence interval from zero to 8 months contains the true half-life of shocks to the REER of Iceland, and a 95 percent probability that the confidence interval from 8 months to infinity contains the true half-life of shocks to the REER of Iceland. As found in the biased DF regression, shocks to the REER of Iceland do not appear to be very persistent. In contrast, while there is a 5 percent probability that the confidence interval from zero to 77 months contains the true half-life of shocks to the REER of Canada, there is a 95 percent probability that the confidence interval from zero to 77 months contains the true half-life of shocks to the REER of Canada, there is a 95 percent probability that the confidence interval from 77 months to infinity contains the true half-life of shocks to the REER of Canada.

The median-unbiased estimates of the autoregressive parameter in ADF regressions control for serial correlation, and are reported in Table 3. Again, in comparison with their least-squares counterparts, the median-unbiased half-lives of deviations from PPP are typically much longer, ranging from 11 months (Iceland) to infinity (Canada and the United States, among others). Across all countries, the aver-

Source: Authors' calculations.

age bias-corrected half-life of parity reversion is 96 months, in excess of the average downwardly biased least-squares AR(p) half-life of 45 months (Figure 2). This implies a rate of parity reversion of only 8 percent per year, rather than the 17 percent per year calculated using biased ADF methods. Similarly, the median-unbiased confidence intervals are much wider than their least-squares counterparts. The Andrews unbiased model-selection rule indicates that all but 5 of the 20 countries have finitely persistent shocks to their REER, which is consistent with the reversion of REER to parity. However of all 20 countries, only Iceland and Norway produced a 90 percent confidence interval for the unbiased half-life of deviations from parity that did not include infinity. Taking the United Kingdom as an example, while there is a 50 percent probability that the confidence interval from zero to 97 months contains the true half-life of shocks to its REER, there is also a 50 percent probability that the confidence interval from 97 months to infinity contains the true half-life. In addition, while there is a 5 percent probability that the confidence interval from zero to 36 months contains the true half-life of shocks to the REER of the United Kingdom, there is a 95 percent probability that the confidence interval from 36 months to infinity contains the true half-life (Table 3). The width of this confidence interval for the half-life indicates there is a great deal of uncertainty as to the duration of the true half-life of parity reversion of the United Kingdom's REER.

Our bias-corrected ADF regression results accord with those obtained by Murray and Papell (2000), who follow Andrews and Chen (1994) in calculating median-unbiased estimates of half-lives for bilateral dollar real exchange rates. They find that the average bias-corrected half-life is about three years, but with confidence intervals that are typically so large that the point estimates of bias-corrected half-lives from ADF regressions provide virtually no information regarding the true size of the half-lives. However, an important deficiency of the Murray-Papell analysis is the inability of their ADF bias-correction method to account for timedependent heteroskedasticity, which is a common feature of real exchange rate series. Once allowance is made for a wider class of serial correlation and heteroskedasticity, as is done in this paper, the speed of parity reversion is typically faster than that found with other median-unbiased models (see Figure 2).

The regression that corrects for the least-squares downward bias, and controls for serial correlation and heteroskedasticity, is the median-unbiased PP regression, the results of which are reported in Table 4. Across all countries, the average bias-corrected half-life of parity reversion is 59 months, clearly exceeding the average downwardly biased least-squares PP half-life of 30 months (see Figure 2). This implies a rate of parity reversion of only 13 percent per year, rather than the 24 percent per year calculated using biased PP methods. The broad pattern found in the biased least-squares estimates of half-lives of parity reversion, with the estimation method that controls for serial correlation and heteroskedasticity (the PP regression) clearly yielding the smallest half-lives of deviations from parity.<sup>20</sup>

<sup>&</sup>lt;sup>20</sup>In the context of biased least-squares estimation, Lothian and Taylor (2000) also find much smaller estimates of the half-lives of shocks to the dollar-sterling real exchange rate when using heteroskedasticity-robust PP regressions rather than ADF regressions.

As shown in Figure 2, implementation of methods of bias correction that do not account for *both* the serial correlation and heteroskedasticity present in real exchange rate data will tend to overestimate the duration of the half-life of deviations from parity, and erroneously indicate that purchasing power parity does not hold in the post–Bretton Woods period. Interestingly, the average half-life of parity deviations derived from median-unbiased PP methods is very close to the average half-life of parity deviations derived from (conventional) biased DF estimation methods.<sup>21</sup> However, the downwardly biased least-squares DF estimates of the half-lives of deviations from parity yield no cases of infinite half-lives; in contrast, 6 of the 20 countries experience non-finite half-lives using bias-corrected PP estimation methods (see Tables 2 and 4).

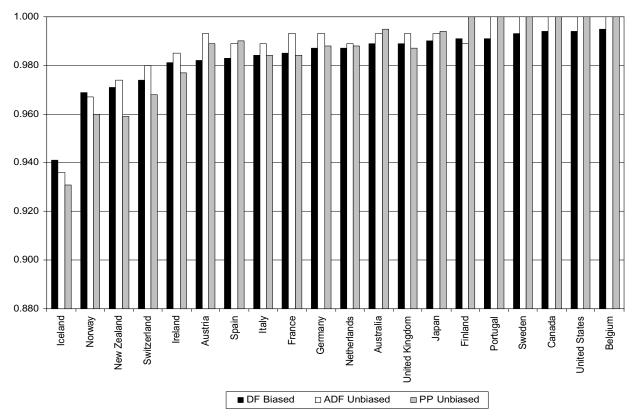
While the cross-country average half-lives of parity reversion based on the median-unbiased PP regression are slightly longer than the cross-country average based on the biased least-squares DF regression (see Figure 2), the country-specific results display quite a deal of heterogeneity. For those countries that have relatively small (biased) DF estimates of the autoregressive parameter (such as Iceland), the bias-corrected PP estimates of the autoregressive parameter tend to be lower than the biased DF estimates—this is consistent with faster speeds of parity reversion and greater evidence in favor of purchasing power parity (see Figure 3). In contrast, for those countries that have relatively large (biased) DF estimates of the autoregressive parameter (such as Australia), the bias-corrected PP estimates of the autoregressive parameter speeds of the autoregressive parameter (such as Australia), the bias-corrected PP estimates of the autoregressive parameter for the biased DF estimates of the autoregressive parameter (such as Australia), the bias-corrected PP estimates of the autoregressive parameter (such as Australia), the bias-corrected PP estimates of the autoregressive parameter (such as Australia), the bias-corrected PP estimates of the autoregressive parameter tend to be greater than the biased DF estimates.

Using the bias-corrected PP regression results, the Andrews unbiased modelselection rule indicates that all but 6 of the 20 countries have finitely persistent shocks to their REER, which is consistent with the reversion of REER to parity (Table 4). For example, the United Kingdom exhibits a median-unbiased half-life of deviation from parity of 52 months. Using the Andrews unbiased model-selection rule, this indicates that the United Kingdom experiences finitely persistent (transitory) shocks to its REER. However, nine countries have 90 percent confidence intervals for their half-lives of deviations from parity that embrace infinity. Taking the United States as an example, while there is a 5 percent probability that the confidence interval from zero to 34 months contains the true half-life of shocks to its REER, there is a 95 percent probability that the confidence interval from 34 months to infinity contains the true half-life (Table 4). Using the Andrews unbiased model-selection rule, the unit root model is the most appropriate representation of the United States REER, and so shocks to its REER are best viewed as being permanent.<sup>22</sup>

<sup>&</sup>lt;sup>21</sup>The presence of moving average and heteroskedastic errors in the real exchange rate data (as set out in Table 1) is most likely attributable to outliers in the real exchange rate data. The Phillips-Perron regression controls for this form of heteroskedasticity, while the Augmented Dickey-Fuller and Dickey-Fuller regressions do not. As a result, estimated half-lives calculated from the latter two models (derived from both biased least-squares and median-unbiased estimators) will be greater than those derived from the biased and bias-corrected Phillips-Perron model. In addition, the Phillips-Perron regression picks up more serial correlation (particularly of the moving average type), which would also reduce the estimated half-lives of parity deviations.

<sup>&</sup>lt;sup>22</sup>As a check of the robustness of our results, for the same sample of countries as studied in this paper, McDermott (1996) found that (using the bias-corrected Phillips-Perron method) the persistence properties of shocks to the real effective and real bilateral exchange rate series were very similar. In addition, a comparison of the two versions of the real exchange rate for Canada, Japan, and the United Kingdom (using the bias-corrected Dickey-Fuller method) again found that the median-unbiased persistence results for real bilateral exchange rates are very similar to those derived using the real effective exchange rate series.

#### Figure 3. Biased Dickey-Fuller (DF), Unbiased Augmented Dickey-Fuller (ADF), and Unbiased Phillips-Perron (PP) Estimates of Autoregressive Parameter



Autoregressive parameter

Source: Authors' calculations.

For the majority of countries, these typically finite half-lives of deviations from parity, and associated slow speeds of parity reversion, are consistent with the findings of Flood and Taylor (1996). They find that evidence in favor of PPP is hard to discern from analyses of the short-run behavior of real exchange rates. However, once the data are averaged over 10 and 20 years, the regression coefficient (in a pooled regression of the exchange rate change on the inflation differential averages) is statistically significant and close to unity, providing stronger evidence of PPP.

In contrast to the above results, Taylor (2001) argues that there are two sources of *upward* bias in the conventional estimation of the speed of parity reversion: first, temporal aggregation bias, whereby sampling data at low frequencies does not allow one to identify a high-frequency adjustment process; and second, the linear AR(1) specification of the standard (Dickey-Fuller) unit root model, which assumes that reversion occurs monotonically, regardless of how far the process is from parity. However, as the present paper uses monthly REER data, the temporal aggregation bias is likely to be minimal. In addition, our use of AR(p) models allows for shocks to the REER to decline at a rate that is not necessarily constant.<sup>23</sup> Taylor finds that when both sources of upward bias are present, then the estimated half-life can be between 1.5 to 2.2 times the true half-life, for the case when monthly averaged data are being used to estimate a monthly (or greater) half-life nonlinear threshold autoregressive process. Taylor (2001, p. 491) also acknowledges that month-to-month variation in nominal exchange rates is likely to dwarf the variation in prices, and that the former are accurately measured in the International Monetary Fund's IFS data. In comparison, the results from Tables 3 and 4 indicate that even after controlling for serial correlation and heteroskedasticity, the bias-corrected half-life of parity deviations is typically about twice as large as the downwardly biased half-life.24

#### III. Conclusion

This paper has reexamined whether purchasing power parity (PPP) holds during the post–Bretton Woods period, by investigating the time-series properties of the real effective exchange rate for 20 industrial countries. The theory of relative PPP holds that the exchange rate will be proportional to the ratio of money price levels (including traded and nontraded goods) between countries, which implies that

<sup>&</sup>lt;sup>23</sup>While median-unbiased estimation allowing for AR(p) behavior does engender a non-monotonic impulse response function, the shape of that impulse response function will be unaffected by the size of the shock to the real exchange rate. In contrast, methods that are robust to nonlinear adjustment to equilibrium allow for such an effect, yet do not correct for downwardly biased autoregressive parameters (see Taylor, Peel, and Sarno, 2001, for a discussion of these issues).

<sup>&</sup>lt;sup>24</sup>In addition to temporal aggregation bias, Imbs and others (2002) demonstrate that cross-sectional aggregation bias raises the persistence of conventionally measured real exchange rate shocks. Cross-sectional aggregation bias arises from the failure of conventional estimation of the speed of parity reversion to take account of cross-sectoral heterogeneity in the dynamic properties of the typical components of aggregate price indices. This failure to allow for the persistence of relative prices to vary across sectors induces an upward bias in aggregate half-life measures, with the bias rising with the extent of cross-sectoral heterogeneity in the speed of parity reversion.

changes in relative price levels will be offset by changes in the exchange rate. In assessing whether real exchange rates do converge to their equilibrium relative PPP value in the long run, we eschew undertaking hypothesis tests of whether real exchange rates follow a unit root process. Instead, we follow Andrews (1993) and use point and interval statistics of the half-life of deviations from parity as our preferred measure of the persistence of real exchange rate shocks.

Univariate studies of the hypothesis of unit roots in real exchange rates have yielded consensus point estimates of the half-life of deviations of real exchange rates from purchasing power parity of between three to five years (Rogoff, 1996). Using conventional (least-squares) biased estimation of unit root models, we replicate the consensus finding in the literature. However, using median-unbiased estimation techniques that are robust to the presence of serial correlation and remove the downward bias of standard estimators, we find that the half-lives of parity reversion are longer than the consensus point estimate, with the cross-country average of unbiased half-lives of deviations from parity lasting about eight years.

We concentrate on the results derived from the regression that allows for the broadest error structure—median-unbiased estimates of Phillips-Perron regressions that are robust to the serial correlation and heteroskedasticity commonly found in real exchange rate series. In the post–Bretton Woods period, the majority of countries are found to have finitely persistent shocks to their real effective exchange rates, which is consistent with the reversion of exchange rate deviations from PPP. Averaging across all countries, the point estimate of the half-life of parity deviations is about five years, which is consistent with (but at the upper end of) Rogoff's (1996) consensus estimate of the half-life of deviations from purchasing power parity. In summary, while median-unbiased methods increase the estimated half-life of deviations from PPP in comparison with downwardly biased least-squares-based methods, allowing for heteroskedasticity reduces the bias-corrected estimated half-life of parity deviations.

Using conventional two-sided hypothesis tests, the confidence interval of the bias-corrected half-life of deviations from parity is typically extremely wide, indicating that there is a great deal of uncertainty as to the "true" speed of parity reversion. When using the results from the heteroskedasticity-robust Phillips-Perron regression, 11 of the 20 countries in our sample have finite upper bounds to the confidence intervals of their half-lives of parity deviation, indicating that for just under half the countries the associated confidence intervals around the half-lives are too wide to provide much information as to whether PPP holds in the long run. Although the null hypothesis of no PPP is not rejected at the 5 percent level for 9 of the 20 countries (as the upper bound of the bias-corrected confidence interval for the half-life contains infinity), failure to reject the null hypothesis conveys little information as to the validity of PPP, as such a failure may occur either because PPP is true, or because there is too much uncertainty as to the speed of reversion to PPP.

In contrast, using the median-unbiased point estimates of the half-lives of deviations from parity and the Andrews (1993) unbiased model-selection rule, we can be more definitive about our willingness to draw conclusions as to the presence or absence of parity reversion of real exchange rates in the post–Bretton Woods period. When using the results from the heteroskedasticity-robust Phillips-Perron regression, we find that 14 of the 20 countries in our sample have finite bias-corrected half-lives of parity reversion. This indicates that for these countries there is a better than even chance that shocks to their real exchange rates are transitory. Consequently, for these countries we can conclude that there is reversion of real exchange rates to parity, and so PPP holds in the post–Bretton Woods period.

Our results confirm Rogoff's (1996) "PPP puzzle"—that while PPP holds for the majority of countries, the speed of reversion of real exchange rates to parity is, in many cases, rather slow. In comparison with conventional least-squares-based estimators, our use of serial-correlation robust median-unbiased estimators typically doubles the estimated half-life of parity deviations. However, we find that using (biased and median-unbiased) estimators that are robust to the heteroskedasticity typically present in real exchange rate series reduces the estimated half-life of parity deviations. In particular, the cross-country average half-life derived from heteroskedasticity-robust median-unbiased methods is close to the conventionally estimated (biased) average half-life of parity deviations.

While our results increase our understanding of the parity-reverting behavior of real exchange rates over the post–Bretton Woods period, as median-unbiased models are superior to biased least-squares-based models of exchange rates in several respects, there is scope for improvement. In particular, the implications for exchange rate modeling of median-unbiased methods, in the presence of nonlinearities in the adjustment of real exchange rates toward long-run equilibrium, have yet to be explored. This challenge awaits future research.

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