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# The Inverted Fisher Hypothesis: Inflation Forecastability and Asset Substitution

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*This paper examines the implications of inflation persistence for the inverted Fisher hypothesis that nominal interest rates do not adjust to inflation because of a high degree of substitutability between money and bonds. It is emphasized that the substitutability between nominal assets and capital renders the hypothesis inconsistent with the data when inflation persistence is high. Using a switching regression model, the analysis allows the reflection of inflation in interest rates to vary according to the degree of inflation persistence or forecastability. The hypothesis is supported by U.S. data only when inflation forecastability is below a certain threshold.* [JEL C51, E43]

This paper examines the implications of inflation persistence for the *inverted* Fisher hypothesis (IFH) proposed by Carmichael and Stebbing (1983) (hereinafter referred to as Carmichael and Stebbing). The standard Fisher hypothesis implies that the *real* interest rate is not affected by inflation because of the substitutability between bonds and capital. In contrast, Carmichael and Stebbing argue that this hypothesis becomes inverted if there is a high substitutability between money and bonds and if government regulation precludes the payment of interest

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on money.[1] They propose that the *nominal* interest rate does not adjust to inflation, with the real interest rate moving inversely one-for-one with inflation.

Presupposing that the inflation process is stationary (that is, nonpersistent), Carmichael and Stebbing deemphasize the potential substitutability between nominal assets (money and bonds) and capital, but this substitutability matters when the inflation process is highly persistent (that is, realized inflation is informative in predicting future inflation). As a result, the IFH is likely to be supported only when the inflation process is stationary. Carmichael and Stebbing conjecture that the hypothesis is expected to hold only in countries with moderate or low inflation. Also, Barth and Bradley (1988) attribute the failure of the hypothesis in samples extending beyond 1978 to its crucial dependence on the assumption of moderate inflation.

A plausible interpretation of Carmichael and Stebbing's conjecture is that agents are more concerned about substitutions between money and capital when inflation is high than otherwise, since changes in inflation are usually sizable and thus have important consequences for the return on money. In this case, higher inflation should be associated with less reflection of inflation in real interest rates. This interpretation relies on asset substitutability varying across inflation levels. In contrast, if substitutability among assets is constant, the failure of the IFH is attributable to the effect of inflation persistence on asset substitutions. This paper suggests that if inflation is persistent, and thus a direction or trend in future inflation is largely anticipated, changes in inflation lead to asset substitutions. Thus, higher inflation persistence should be associated with less reflection of inflation in real interest rates.

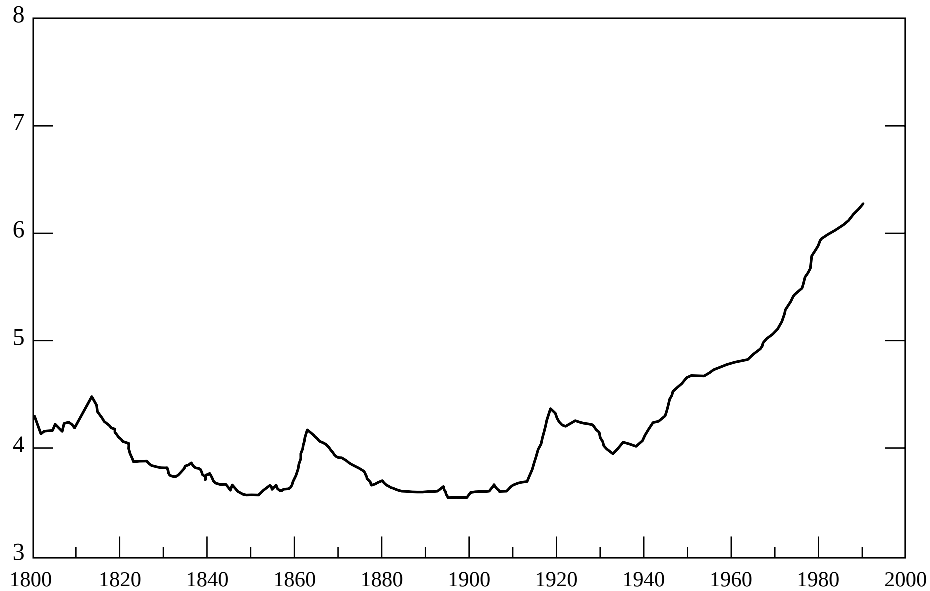
Despite evidence on the persistence of inflation in the post–World War II period (for example, Klein, 1975; and Evans and Wachtel, 1993), no previous studies have explicitly taken into account the influence of inflation persistence in testing the IFH. This paper sheds light on the link between inflation persistence and the effect of asset substitutions on interest rates. Studies of hyperinflations and other rapid inflations (for example, Laidler, 1993) suggest that expected inflation affects money holdings. Although most economies do not have such high inflation, many do experience highly persistent inflation over a substantial length of time. If inflation is highly persistent, economic agents can largely forecast inflation using their recent experience. When higher inflation is expected, agents will substitute capital for nominal assets. The resulting decrease in real balances alters the implicit marginal return on money, which Carmichael and Stebbing, however, assumed to be approximately constant.

The more persistent the inflation rate, the higher its short-term forecastability becomes. Changes in the monetary regime affect the persistence of inflation and thus its forecastability. Friedman (1977) views regime changes as an important source of inflation uncertainty. Klein (1975) provides an appealing explanation, subsequently emphasized by Friedman and Schwartz (1982, chapter 10), of how

[1]Carmichael and Stebbing argue that one should take into account the substitutability between *money* and bonds in determining the effect of inflation on interest rates, especially because the data mainly pertain to the return on financial assets rather than to the return on capital.

Figure 1. U.S. Price Level, 1800–1992

*(logarithm price level, 1950 = ln(100))*




Source: Flood and Mussa (1994).

inflation forecastability is influenced by fundamental changes in the character of the monetary system. Klein argues that with the collapse of the Bretton Woods system and the explicit step toward a fiat money system in the 1970s, changes in institutional arrangements made inflation more predictable in the short term than it had been earlier.[2]

In a similar vein, Flood and Mussa (1994) emphasize the influence of monetary regimes on the behavior of prices and inflation as follows. Prices under the gold standard had a tendency to return to a normal level. With the widespread acceptance of paper money standard after World War II, however, price levels no longer have a tendency to revert to a norm, as is shown in Figure 1. They attribute a noticeable downturn of inflation during the 1980s, which follows an upsurge of inflation beginning in the late 1960s and extending into the 1970s, partly to the shift in policy. They argue that the "well-managed" paper money standard is likely to deliver a low rate of inflation. Also, a shift in emphasis on whether monetary policy targets money growth, interest rates, or inflation results in changes in the

[2]Klein argues that, under specie standards that provided an anchor for the price level, there was considerable short-term unpredictability, but much less long-term unpredictability, of inflation. Under the fiduciary monetary system of the post–World War II period, there is no anchor for the price level. Agents have come to regard prices as largely affected by policy, and short-term unpredictability is less than before. For example, Hutchison and Keeley (1989) show that inflation evolved from a white noise process in the pre–World War I period to a highly persistent, nonstationary process in the post-1960 period, which strengthened the Fisher effect.

inflation process (see Rudebusch and Svensson, 1998). With the anti-inflationary policy by U.S. Federal Reserve Chairman Paul Volcker and subsequent steady policies, U.S. inflation has become lower and less persistent since the mid-1980s than before.

Rather than restricting the IFH by using a chronological time scale, we propose that the IFH holds only when inflation forecastability is below a threshold value. To test whether the validity of the IFH depends on regimes, we estimate *switching regressions* that allow the reflection of inflation in interest rates to vary over regimes. Two regimes are assumed to be distinguishable according to whether inflation forecastability is below or above a threshold estimate. To measure inflation forecastability, we estimate an inflation equation that incorporates an autoregressive (AR) process, reflecting a backward-looking nature, along with the short-term interest rate that carries information about future inflation, reflecting a forward-looking nature. Thus, the inflation equation is based on the efficient use of available information as in Nelson and Schwert (1977), consistent with rational expectations. As the inflation process can involve structural shifts unknown to agents *ex ante*, we use a rolling estimation of the inflation equation, taking into account the role of learning for such shifts.

Carmichael and Stebbing tested the IFH by regressing the actual real interest rate on the actual inflation. This testing procedure does not require the use of arbitrary data about inflation expectation and does not involve, under the IFH, an errors-in-variables problem that makes estimates inconsistent owing to measurement error on the regressor. In this paper, two schemes are considered for switching between regimes. First, a switching regression with a smooth transition is used to reflect the notion that agents assign different weights to possible regimes according to their judgments, with time-varying confidence, about the inflation process. This scheme somewhat controls the errors-in-variables problem in testing the IFH under alternative regimes. Second, a switching regression with perfect discrimination is used to treat two regimes separately. This scheme is immune to the errors-in-variables problem in testing the IFH.

Linearity testing supports the evidence for the threshold effect in the Carmichael and Stebbing equation. This paper finds new evidence that the IFH is supported by U.S. data only when inflation forecastability is below a certain (estimated) threshold. This finding is consistent with the argument that the implicit marginal return on money and the nominal interest rate adjust to inflation through substitution between nominal assets and capital in periods of persistent inflation, but they do not adjust to inflation when inflation is nonpersistent and thus largely unpredictable.

## I. Conceptual Background and the Model

The IFH is based on the premise that optimizing agents hold money and financial assets up to the point at which their after-tax real yields are equal. The after-tax real rate of return on a bill is given by

$$r_{Nt} = i_{Nt} - \pi_{t+1}, \tag{1}$$

where $i_{Nt}$, the after-tax nominal rate, is defined as $i_{Nt} = (1-\theta_t)i_t$ with the marginal tax rate, $\theta_t$, and $\pi_{t+1}$ denotes inflation at the maturity of the bill. The after-tax real return on money is given by $r_{mt} = z_t - \pi_{t+1}$, where $z_t$ is the *implicit* marginal return on money. In equilibrium, $r_{Nt} = r_{mt}$, therefore, $r_{Nt} = z_t - \pi_{t+1}$ or, equivalently, $z_t = i_{Nt}$. Assuming a close substitutability between money and financial assets, Carmichael and Stebbing argue that $z_t$ is approximately constant.

Taking (conditional) expectations of equation (1) on both sides,

$$E(r_{Nt}) = E(i_{Nt}) - E(\pi_{t+1}), \tag{1'}$$

where $E$ is the expectations operator conditional on the information set available in period $t$. Carmichael and Stebbing assume that expectations are unbiased—that is,

$$\pi_{t+1} = E(\pi_{t+1}) + \varepsilon_{t+1}, \tag{2}$$

where $\varepsilon_{t+1}$ is a mean-zero random error. Also, the after-tax nominal interest rate is assumed to be known at all points in time—that is, $E(i_{Nt}) = i_{Nt}$.

To test the IFH, Carmichael and Stebbing present the following equation:

$$E(r_{Nt}) = \alpha_0 + \alpha_2' E(\pi_{t+1}) + \xi_t, \tag{3}$$

where $\alpha_0$ is a constant, and $\xi_t$ is a mean-zero random error and independent of $\varepsilon_{t+1}$. Under the IFH, $E(r_{Nt})$ moves inversely one for one with $E(\pi_{t+1})$—that is, $\alpha_2' = -1$. Thus, the IFH implies that $i_{Nt} = \alpha_0 + \xi_t$, suggesting that the nominal rate should be independent of both $E(\pi_{t+1})$ and $\pi_{t+1}$.[3] Substituting equations (1) and (1′) into equation (3) and eliminating $E(\pi_{t+1})$ using equation (2), Carmichael and Stebbing obtain in the testable form

$$r_{Nt} = \alpha_0 + \alpha_2' \pi_{t+1} + \xi_t - (1 + \alpha_2')\varepsilon_{t+1}. \tag{3'}$$

Under the null hypothesis $\alpha_2' = -1$,[4] an errors-in-variables problem in equation (3′) vanishes. The Carmichael and Stebbing estimation avoids the use of arbitrary data on inflation expectations in determining the effect of inflation on interest rates. This procedure cannot be used to test the standard Fisher hypothesis because of an errors-in-variables problem (see Graham, 1988).

If inflation is close to a martingale difference sequence with respect to the agents' information set (that is, if the conditional expectation of inflation based on the available information set is almost the same as the unconditional expectation of inflation), there is little persistence in inflation. As such, current changes in

[3]Estimating the nominal rate equation to test the IFH should account for the nominal rate response to $E(\pi_{t+1})$, which involves the use of arbitrary data about expected inflation. Nonetheless, one can check whether the nominal rate is independent of actual inflation (see, for the result of this test, the robustness check section).

[4]Gallagher (1986) points out that this testing pertains to the maintained hypothesis that $(\pi_{t+1})$ and $\xi_t$ are only contemporaneously uncorrelated. Following earlier studies including Carmichael and Stebbing, this study interprets the IFH as claiming that inflation and nominal interest rates are *contemporaneously* uncorrelated in the sense of Gallagher.

inflation have little effect on agents' expectations of future inflation and thus on the substitution between nominal assets and capital. This would fit Carmichael and Stebbing's presumed environment. However, their proposition is consistent with the principles of rational agents only when inflation persistence is low enough. If inflation is highly persistent, substitution between nominal assets and capital occurs when the inflation rate changes.

Now the real rate response to inflation is allowed to vary with inflation persistence. Two regimes are classified according to whether inflation persistence, $\rho_t^\pi$, is below or above a threshold, $\tau$. If the two regimes are perfectly discriminated, then define $D_t = 0$ if $\rho_t^\pi \le \tau$ and $D_t = 1$ otherwise, where $\tau$ is a fixed parameter. Two equations are set up. The first equation states that the implicit marginal return on money is negatively related to real balances:

$$z_t = \Lambda\left(\frac{M_t}{P_t}\right),\ \Lambda_{\frac{M}{P}} < 0. \tag{4}$$

This notion is consistent with the money-in-utility function (MIUF) approach in a monetary general equilibrium framework (Danthine and Donaldson, 1986; Lucas, 2000; Walsh, 1998; and Choi and Oh, forthcoming), which suggests diminishing marginal utility gains from holding real balances. Another equation presents the demand for real balances, $M_t^d/P_t$, as follows:

$$M_t^d/P_t = \Phi\left\{\overset{(+)}{Y_t},\ \overset{(-)}{i_t},\ \overset{(-)}{\pi_{t+1}^e}\cdot D_t\right\}, \tag{5}$$

where $Y_t$ denotes real output and $\pi_{t+1}^e$ represents the expected nominal rate of return on real assets. That is, the demand for real balances increases with real output but decreases with returns on bonds and real assets. Equation (5) introduces a portfolio shift with a threshold effect owing to shifts in the inflation process in the conventional money-demand function that is consistent with the MIUF approach and Friedman and Schwartz's (1982, pp. 37–40) money-demand function. The $\pi_{t+1}^e$ term appears when inflation is persistent ($D_t = 1$), so that expected inflation is reflected in the nominal rate of return; otherwise it does not affect money holdings as in the Carmichael and Stebbing setting.

In equilibrium, $M_t^d = M_t$ and $z_t = \Lambda(M_t/P_t)$. For any $\pi_{t+1}^e$, there is an equilibrium value for $M_t/P_t$ and, hence, for $z_t$. When $\pi$ is highly persistent, if agents expect future $\pi$ to rise, they will hold less money and more capital. With this substitution, $z_t$ rises and, hence, $i_{Nt}$ rises too. Conversely, when inflation persistence is low, agents, who are not expecting $\pi$ to change in a certain direction, will not alter their money holdings. As a result, both $z_t$ and $i_{Nt}$ will be unchanged.

The next step is to set out an augmented version of equation (3′) as

$$r_{Nt} = \alpha_0 + \alpha_2'\pi_{t+1} + \xi_{1t} - \left(1+\alpha_2'\right)\varepsilon_{t+1} \qquad \text{if } \rho_t^\pi \le \tau, \tag{6a}$$

$$r_{Nt} = \alpha_0 + \gamma_2'\pi_{t+1} + \xi_{2t} - \left(1+\gamma_2'\right)\varepsilon_{t+1} \qquad \text{if } \rho_t^\pi > \tau. \tag{6b}$$

Premultiplying equation (6a) by $(1 - D_t)$, equation (6b) by $D_t$, and adding yields

$$r_{Nt} = \alpha_0 + (1 - D_t)\alpha_2'\pi_{t+1} + D_t\gamma_2'\pi_{t+1} + (1 - D_t)\eta_{1t} + D_t\eta_{2t}, \tag{7}$$

where $\eta_{1t} \equiv \xi_{1t} - (1 + \alpha_2')\varepsilon_{t+1}$ and $\eta_{2t} \equiv \xi_{2t} - (1 + \gamma_2')\varepsilon_{t+1}$, which are distributed as $N(0,\sigma_1^2)$ and $N(0,\sigma_2^2)$, respectively. This regime-wide heteroskedasticity assumption reconciles the evidence from Hamilton (1988) and Choi (1999a) that the interest rate process involves heteroskedastic error.[5] In contrast with Carmichael and Stebbing's IFH ($\alpha_2' = \gamma_2' = -1$), it is argued that $\alpha_2' = -1$ holds but $\gamma_2' = -1$ does not. It is expected that $\gamma_2' > -1$ because asset substitutions under the high-persistence regime raise the nominal rate response to inflation.

So far this study has considered how inflation forecastability affects the link between interest rates and inflation, on the presumption that substitutability among assets is fixed. From a different viewpoint, supposing that asset substitutability is positively correlated with inflation, we can also test Carmichael and Stebbing's conjecture that the IFH is expected to hold only when inflation is low by replacing the forecastability index with the level of inflation.

## II. Empirical Methodology

To make estimating equation (7) tractable, we incorporate "deterministic switching based on other variables" as proposed by Goldfeld and Quandt (1972, pp. 258–77; 1973). First, we estimate an inflation forecast model includes lags of the dependent variable and the nominal interest rate, $i_t$, as a predictor of future inflation (Nelson and Schwert, 1977)

$$\pi_{t+1} = b_0 + \sum_{i=1}^{k} b_i\pi_{t-i} + \lambda i_t + \varepsilon_{t+1}. \tag{8}$$

Note that the IFH implies no response of nominal rates to inflation so that $\lambda = 0$ under the IFH, whereas the Fisher effect implies that $\lambda \neq 0$ . To account for structural changes in the inflation process (through shifts in parameters $b$'s and $\lambda$) and to reflect the idea that agents learn about the inflation process by looking back over their recent experience, inflation forecast errors are computed from the rolling estimation of the model. The rolling regression is preferred to an entire-period regression, because the latter would provide forecasts based on information that agents would not have had unless the model parameters were constant.

Second, as a proxy for $\rho_t^\pi$, we use the inflation forecastability index, $F_t$, which is defined as the rolling-weighted $R^2$ of the inflation forecast regression[6]

[5]Switching model (7) assumes heteroskedasticity across regimes but homoskedasticity within a regime, whereas a model of GARCH (generalized autoregressive conditional heteroskedasticity, Bollerslev, 1986) (with regime shifts) allows for autocorrelation in the second moment of errors, implying history-dependent variability in errors.

[6]As inflation moves more persistently, it can be better forecast using the past history of inflation. Autocorrelations up to several orders of the residual, which contain information about persistence, will be reflected in this index. In general, agents would utilize indicators for future inflation, too.

$$F_t = 1 - \sum_{i=0}^{w-1} h^i \hat{\varepsilon}_{t+1-i}^2 \Bigg/ \sum_{i=0}^{w-1} h^i \left(\pi_{t+1-i} - \overline{\pi}\right)^2, \qquad 0 < h \leq 1, \tag{9}$$

where $\hat{\varepsilon}_{t-i}$ (for $i = 0,1,...,w-1$) is the inflation forecast error of the rolling regression with window size $w$, $\overline{\pi}_t = \frac{1}{w}\sum_{i=0}^{w-1}\pi_{t+1-i}$, and $h$ is a weight parameter. Notice that $F_t$ is negatively related to the ratio of the variance of unanticipated inflation (that is, the conditional variance of inflation) to the variance of inflation. If inflation is highly persistent, the variance of inflation far outweighs that of unanticipated inflation—that is, the value of $F_t$ becomes high.

Third, we define the standardized *unobservable* switching index as $F_t^* = \tilde{F}_t + \nu_t$, where $\tilde{F}_t$ is the standardized $F_t$ and $\nu_t$ is an identically and independently distributed (i.i.d.) drawing from $N(0,1)$ and is assumed to be independent of $\eta_{1t}$ and $\eta_{2t}$. Also, using the switching index, the indicator function can be approximated by a continuous function

$$D_t = \int_{-\infty}^{\tilde{F}_t - \tau} \frac{1}{\sqrt{2\pi}} \exp\left(-\tfrac{1}{2}v_t^2\right) dv_t. \tag{10}$$

This smoothing function allocates weights to observations according to forecastability and in effect allows for a continuum of states between two extreme regimes.[7] Thus, the smoothing function reflects the fact that agents, learning from their recent experience, assign different weights to the two regimes based on their judgment, with time-varying confidence, about the inflation process. The log-likelihood function for equation (7) up to a constant term is given by

$$\ln L = \frac{1}{2}\sum_{t=1}^{T} \ln\left[\sigma_1^2\left(1-D_t\right)^2 + \sigma_2^2 D_t^2\right] - \frac{1}{2}\sum_{t=1}^{T} \frac{\left(r_{Nt} - \alpha_0 - \left[\alpha_2'\left(1-D_t\right) + \gamma_2' D_t\right]\pi_{t+1}\right)^2}{\sigma_1^2\left(1-D_t\right)^2 + \sigma_2^2 D_t^2}. \tag{11}$$

Substituting equation (10) into equation (11), a maximum-likelihood estimation (MLE) can be made by maximizing the log-likelihood function with respect to $\alpha_0$, $\alpha_2'$, $\gamma_2'$, $\sigma_1$, $\sigma_2$, and $\tau$.

To determine whether the threshold effect is statistically significant, we conduct the Lagrange multiplier (LM) test for linearity, following a procedure suggested by Granger and Teräsvirta (1993). The procedure is described in Appendix I. Since the threshold parameter is unknown a priori and not identified under the null hypothesis of linearity ($\alpha_2' - \gamma_2' = 0$), classical tests have nonstandard distributions. This is the so-called Davies problem (Davies, 1987). Following Hansen (1996), three types of final statistics that are functionals of the collection of LM test statistics over the grid set are computed: the supremum (SupLM), the average (AveLM), and the exponential average (ExpLM) of all LM statistics.[8]

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[7]Observations with moderate forecastability may be grouped between the two extreme regimes. The smooth transition treats these observations as a mix of the two regimes, weighted by the distance from each. Thereby the degree to which inflation is reflected in interest rates is a monotonic function of inflation persistence.

[8]Davies (1987) and Granger and Teräsvirta (1993) suggest using the supremum of statistics over a grid set, whereas Andrews and Ploberger (1994) suggest using the average and the exponential average of statistics.

The grid set is composed of 101 grids that evenly divide the range from the 10th to the 90th percentile of the empirical distribution of the (standardized) switching index. Their significance levels are calculated using a simulated empirical distribution of these statistics.

Testing either $\alpha_2' = -1$ or $\gamma_2' = -1$ in equation (7) may involve a measurement-error problem. That is, a correlation remains between the regressor and the error term under the null hypothesis of $\alpha_2' = -1$ or of $\gamma_2' = -1$ if the two regimes are imperfectly discriminated ($0 < D_t < 1$). To deal with the errors-in-variables problem in the testing, we first test $H_0(\alpha_2' = \gamma_2' = -1)$ and then $H_0(\gamma_2' = -1 \mid \alpha_2' = -1)$, assuming that the IFH holds under the low-forecastability regime. Both hypotheses should not be rejected if Carmichael and Stebbing's IFH holds.

Next, to test either $\alpha_2' = -1$ or $\gamma_2' = -1$ separately under the assumption of perfect discrimination, we estimate a piecewise linear regression for equation (7), setting $D_t = 0$ if $\tilde{F}_t \leq \tau$, and $D_t = 1$ otherwise. This testing procedure is robust to the errors-in-variables problem because the estimator under different regimes uses mutually exclusive observations. Estimating this threshold model should account for the Davies problem, as $\tau$ is not identified under the null hypothesis of no threshold effect.

This study tests the IFH under alternative regimes, controlling the nuisance parameter problem in the context of Hansen (1999, 2000); see also Appendix II. First, we estimate the threshold model by weighted least squares (WLS) to control heteroskedastic errors and obtain a consistent estimate for $\tau$ by minimizing the sum of squared residuals over a grid set. Second, we perform the likelihood-ratio test for $\alpha_2' = \gamma_2'$ using *p*-values constructed from a bootstrap procedure. Third, we form the confidence interval for $\tau$ by forming the no-rejection region using the likelihood-ratio statistic for tests on $\tau$. Finally, as in Hansen (1999), we make an inference about the slope estimate as if the threshold estimate were the true value, since Chan (1993) and Hansen (2000) show that the dependence on the threshold estimate is not of first-order asymptotic importance.

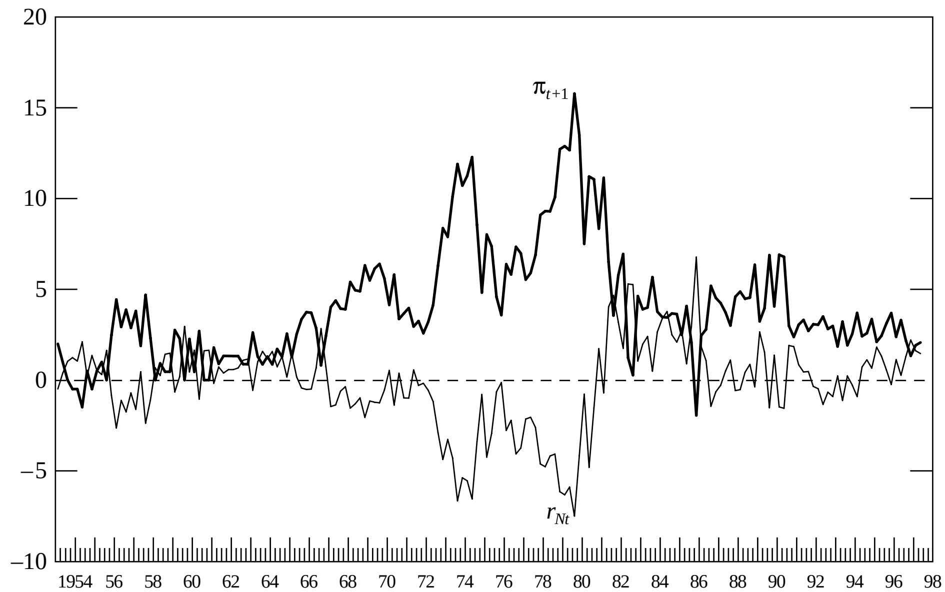
## III. Empirical Results

### Measuring Variables

This study uses the U.S. time-series data taken from Federal Reserve Economic Data (FRED) over the period from January 1947 (1947:01) to 1997:12. The three-month treasury bill rate is used as the nominal interest rate. To measure inflation, the study uses the consumer price index (CPI) with no adjustment for housing costs for the whole period or, alternatively, the implicit price deflator for personal consumption expenditure (PPC) from 1959:01 to 1997:12.[9] In keeping with earlier studies on the IFH, we estimate a quarterly model to match the maturity of treasury

[9]Using the mnemonics on the FRED of the Federal Reserve Bank of St. Louis, the variable definitions are TB3MS (the three-month treasury bill rate), CPIAUSL (the CPI-U: whole items), and PCE/PCEC92 (the personal consumption expenditure deflator). The average for each quarter is used to measure the quarterly series. All except interest rates are seasonally adjusted data.

Figure 2. Short-Term Real Interest Rates and Inflation, 1953:Q2–1997:Q4
*(percent)*




bills, although both interest and inflation rates are available monthly.[10] The inflation rate in period $t$ is defined as $\pi_t \equiv 400(P_t/P_{t-1} - 1)$, where $P_t$ is the price level in period $t$. To account for the tax effect on interest rates (Darby, 1975; and Feldstein, 1976), we assume that the marginal tax rate on interest income is fixed at 0.3, following Evans and Lewis (1995). For comparison purposes, we also use Sahasakul's (1986) marginal tax rate data.

Figure 2 depicts the tax-adjusted (ex post) real rate of return on three-month treasury bills and the CPI inflation rate at the maturity of the bill. It is quite discernible that inflation was rather persistent in the late 1960s, and high and highly persistent from the 1973 oil shock until 1981. Paul Volcker's anti-inflation policy kept inflation in check in 1982 and, thereafter, policy has consistently aimed at keeping inflation low. The real rate dipped in the 1970s but changed in the opposite direction by less than one for one with inflation. The downward movement in inflation and financial deregulation in the early 1980s resulted in increases in cash and interest-earning deposits. This may have led to a sharp rise in the real rate and a decrease in the implicit marginal return on money. After the mid-1980s, the movement in the real rate became quite steady.

[10]The use of a monthly model may cause serial correlation in errors because the maturity of yield on a bill is longer than the data frequency. Suppose that the error term of equation (3′) (in a difference form) for the one-month rate is serially uncorrelated. Based on this equation, the monthly model of the three-month rate can be expressed—for example, via the expectations hypothesis. Time aggregation results in serial correlation in errors.

The forecastability index is measured by the quarterly average of a monthly forecastability index to capture the variability of inflation within a quarter. Inflation forecast errors are computed from the rolling estimation of a monthly inflation model ($w = 60$), which includes nine lags of the dependent variable and the nominal interest rate as a predictor of future inflation.[11] Figure 3 shows the rolling estimates of $\Sigma_{i=1}^{9} b_i$ and $\lambda$ over time in equation (8): inflation persistence and forecastability can be positively related to $\Sigma_{i=1}^{9} b_i$ and also to $\lambda$, because information about future inflations is more likely to be reflected in the nominal rate with a higher inflation persistence; the predictive power of the interest rate in regressions, however, tends to complement that of lagged inflation rates. The monthly forecastability index is then measured by equation (9) by setting $h = 0.9$ for the CPI ($h = 0.98$ for the PPC); when the CPI is used, the most recent year accounts for the 72 percent of the weight given to all observations within each window (the corresponding figure is 31 percent for the PPC). For comparison purposes, the switching variable is alternatively defined as the lagged quarterly inflation rate.

Figure 4 displays the standardized inflation forecastability index ($\tilde{F}_t$) along with the smoothing function ($D_t$) given the $\tau$ estimate (reported in Table 3 below). The indices for both the CPI and the PPC show relatively high values, particularly in the late 1960s, for a few years after the first (1973–74) oil shock, and during 1979–86. These were mostly periods of relatively high and persistent inflation; the indices show quite low values in most periods after 1986. Movements in the real interest rate, inflation, and the forecastability index suggest that the IFH can hardly be reconciled with the data during persistent inflationary eras.

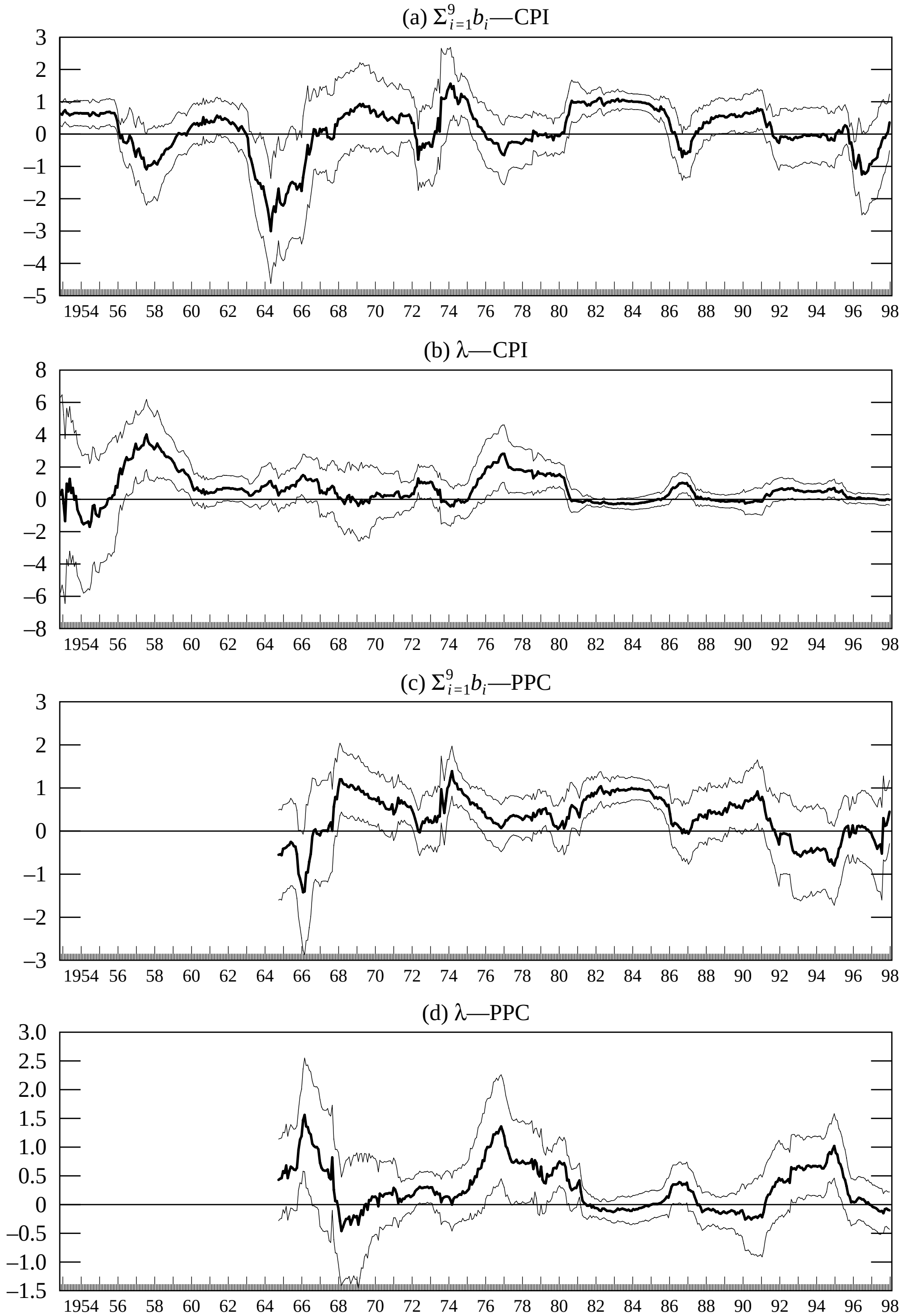
This study uses first differences of the variables to test the IFH, following Carmichael and Stebbing (1983), Barth and Bradley (1988), and Gupta (1991), since estimating the level-form equation involves the use of nonstationary time series. The first-difference-form regression does not include an intercept unless indicated; regressions with the intercept provide almost identical results. The starting date of regressions with the CPI is 1953:Q2 (second quarter of 1953), whereas that of regressions with the PPC is either 1959:Q3 or 1965:Q2, depending upon the starting date of the switching index. The ending date of the regressions is either 1997:Q3 or 1982:Q4, depending upon the availability of Sahasakul's tax rate data.

## Carmichael and Stebbing Regressions and Linearity Testing

To begin with, the single-regime Carmichael and Stebbing regression for equation (3′) is estimated by the ordinary least squares (OLS) method for the period 1953:Q2–1997:Q3 when the CPI is used (1959:Q3–1997:Q3 when the PPC is used). Table 1 summarizes the results. Despite the difference in the sample periods, the results with a constant marginal tax rate are virtually the same as those

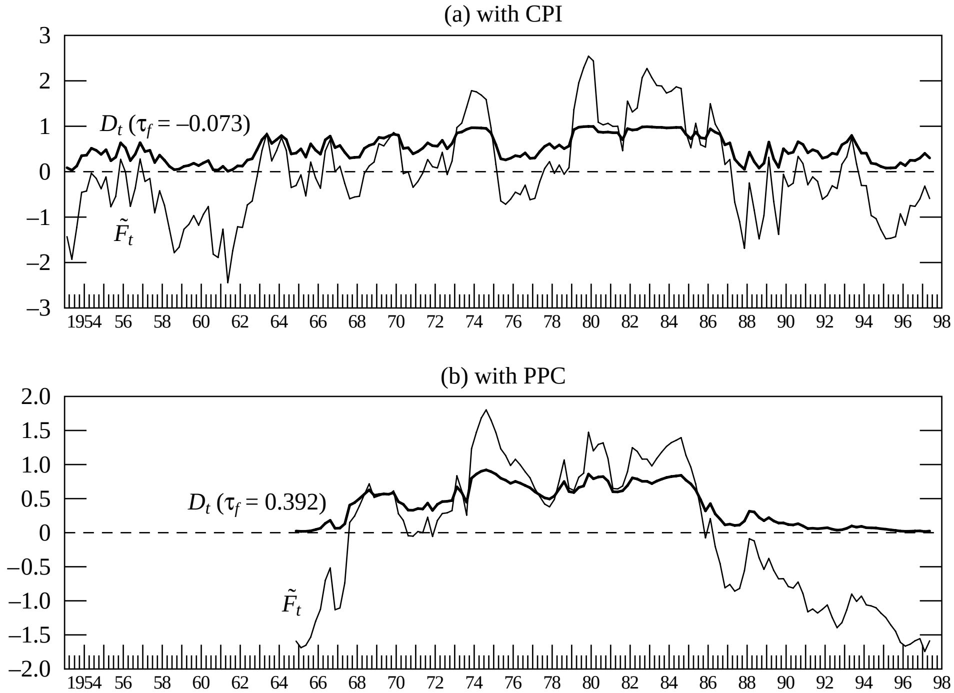
[11] Different lag lengths are suggested by different information criteria (over rolling samples). However, choosing an alternative lag length (13 lags, for example) does not affect the results of this study qualitatively.

**Figure 3. Rolling Estimates of Inflation Forecasting Regression, 1953:01–1998:12**




Notes: The top two (bottom two) panels pertain to the inflation equation using CPI (PPC). Thin lines represent two-standard-error bands.

Figure 4. Standardized Forecastability Index and Smoothing Function Values, 1953:Q2–1997:Q4




with a time-varying tax rate.[12] The IFH ($\alpha_2' = -1$) is rejected at the 5 percent significance level in all cases. For example, with a constant marginal tax rate adjustment, the $\alpha_2'$ estimate is –0.896 with the CPI (–0.931 with the PPC). The null hypothesis of $\alpha_2' = -1$ is rejected by the Wald test at the 5 percent level, although it is less strongly rejected in the PPC case. The LM tests for the null hypothesis of homoskedastic errors against an alternative specification in which the variance of residuals depends on the lagged nominal rate (as in Marsh and Rosenfeld, 1983, and Hamilton, 1988) provide strong evidence against homoskedasticity. The Carmichael and Stebbing equation estimated by WLS to account for such heteroskedasticity (bottom panel) also provides evidence against the IFH.

Next, linearity testing is performed to determine whether there exists a threshold effect. Table 2 reports the test results assuming a constant marginal tax rate. The top panel reports results with the forecastability index as the switching variable, and the bottom panel reports results with the lagged inflation level. Errors are assumed to be either homoskedastic (upper row) or heteroskedastic (lower row). In the case of heteroskedastic errors, the variance of errors is assumed

[12]Barth and Bradley (1988) and Gupta (1991) find that estimation results for equation (3′) are not sensitive to whether or not the real rate is adjusted for taxes. Indeed, as Barth and Bradley point out, "since taxes are levied on the nominal rate, irrespective of tax adjustments, the real interest rate should vary inversely and one-for-one with the inflation rate (under the hypothesis)," unless the tax rate is correlated with inflation.

**Table 1. Single-Regime Regressions**

| Parameter | Constant Marginal Tax Rate[1] CPI (53:Q2–97:Q3) | Constant Marginal Tax Rate[1] PPC (65:Q2–97:Q3)[2] | Time-Varying Marginal Tax Rate[1] CPI (53:Q2–82:Q4) | Time-Varying Marginal Tax Rate[1] PPC (65:Q2–82:Q4)[2] |
|---|---|---|---|---|
| OLS | | | | |
| $\alpha_2'$ | –0.896 (0.037)** | –0.933 (0.045)** | –0.853 (0.044)** | –0.872 (0.070)** |
| $\bar{R}^2/D\text{-}W$ | 0.89 / 1.72 | 0.79 / 1.67 | 0.88 / 1.89 | 0.73 / 1.84 |
| LM(1)[3] | 9.37 (0.002) | 6.61 (0.010) | 8.48 (0.004) | 5.40 (0.020) |
| WLS | | | | |
| $\alpha_2'$ | –0.928 (0.021)** | –0.957 (0.033)** | –0.905 (0.027)** | –0.904 (0.056)** |
| $\bar{R}^2/D\text{-}W$ | 0.92 / 1.54 | 0.84 / 1.57 | 0.91 / 1.65 | 0.78 / 1.71 |

Notes: The results of estimating equation (3) in first-difference form. Standard errors (in parentheses) for the OLS estimator are based on White's correction for heteroskedasticity. The WLS estimator is based on the weight that equals the square root of the lagged nominal interest rate. The superscript ** denotes significance at the 0.01 level. *D-W* denotes the Durbin-Watson statistic.

[1]For the tax-adjusted interest rate, a constant average marginal tax rate ($\theta = 0.3$) is assumed in the first and second columns, and the average marginal tax rate ($\theta_t$) from Sahasakul (1986) is used in the third and fourth columns.

[2]The starting date is set the same as those for switching regressions with the forecastability index.

[3]Lagrange multiplier statistic for the null hypothesis that the variance of residuals depends on the lagged nominal interest rate (*p*-values are in parentheses).

**Table 2. Tests for Linearity**

| Switching Variable | Test Statistic SupLM | AveLM | ExpLM | $LM(\hat{\tau}_f)$[3] |
|---|---|---|---|---|
| *Forecastability index*[1] | | | | |
| CPI | 6.81 (0.012)[2] | 6.05 (0.005) | 3.08 (0.006) | 6.26 (0.012) |
| (53:Q2–97:Q3) | 6.26 (0.045) | 5.35 (0.024) | 2.73 (0.028) | 5.31 (0.021) |
| PPC | 3.06 (0.148) | 2.75 (0.086) | 1.39 (0.095) | 3.00 (0.083) |
| (65:Q2–97:Q3) | 2.95 (0.158) | 2.66 (0.097) | 1.34 (0.106) | 2.93 (0.087) |
| *Inflation level* | | | | |
| CPI | 14.78 (0.000) | 13.75 (0.000) | 7.03 (0.000) | 15.11 (0.000) |
| (53:Q2–97:Q3) | 14.50 (0.000) | 12.46 (0.000) | 6.56 (0.000) | 13.39 (0.000) |
| PPC | 7.53 (0.011) | 5.38 (0.016) | 2.91 (0.013) | 5.04 (0.019) |
| (59:Q3–97:Q3) | 7.54 (0.015) | 5.01 (0.017) | 2.79 (0.017) | 5.04 (0.025) |

Notes: Regressions are in first-difference form, assuming $\theta = 0.3$.

[1]Measured from the monthly inflation forecast model that includes nine lags of the dependent variable and the three-month treasury bill rate (two-period-lagged monthly figures).

[2]Figures in the upper row (lower row) for each statistic are based on the homoskedasticity (heteroskedasticity) assumption. The asymptotic *p*-values (in parentheses) for SupLM, AveLM, and ExpLM are computed from simulations ($J = 1{,}000$) over the grid set ($\#\Gamma = 102$).

[3]The statistic follows a $\chi^2(1)$ distribution under the null hypothesis that the coefficient on inflation is constant across the subsamples grouped by $\hat{\tau}$, obtained from the joint estimation of equations (7) and (9) by MLE.

### Table 3. Switching Regressions with Smooth Transitions

| | Constant Marginal Tax Rate[1] | | Time-Varying Marginal Tax Rate[1] | |
|---|---|---|---|---|
| Parameter | CPI (53:Q2–97:Q3) | PPC (65:Q2–97:Q3) | CPI (53:Q2–82:Q4) | PPC (65:Q2–82:Q4) |
| *Forecastability*[2] | | | | |
| $\alpha_2'$ | –1.021 (0.024)** | –1.042 (0.034)** | –1.028 (0.032)** | –1.119 (0.100)** |
| $\gamma_2'$ | –0.834 (0.035)** | –0.810 (0.082)** | –0.767 (0.049)** | –0.656 (0.145)** |
| $\tau$ | –0.073 (0.202) | 0.392 (0.275) | –0.036 (0.218) | 0.167 (0.573) |
| $\sigma_1$ | 0.255 (0.032)** | 0.291 (0.028)** | 0.293 (0.037)** | 0.433 (0.083)** |
| $\sigma_2$ | 0.852 (0.088)** | 1.165 (0.184)** | 0.949 (0.120)** | 1.247 (0.340)** |
| $\bar{R}^2/D\text{-}W$ | 0.90 / 1.70 | 0.79 / 1.68 | 0.88 / 1.82 | 0.73 / 1.77 |
| $[\gamma_2' = \alpha_2' = -1]$[3] | 22.7 (0.000) | 5.39 (0.067) | 22.8 (0.000) | 9.56 (0.008) |
| $[\gamma_2' = -1 \mid \alpha_2' = -1]$ | 22.3 (0.000) | 3.79 (0.051) | 22.7 (0.000) | 5.13 (0.024) |
| Number of observations (low, high) | 85, 93 | 75, 55 | 60, 59 | 41, 30 |
| *Inflation level* | | (59:Q3–97:Q3) | | (59:Q3–82:Q4) |
| $\alpha_2'$ | –1.029 (0.021)** | –1.066 (0.037)** | –1.010 (0.032)** | –1.059 (0.055)** |
| $\gamma_2'$ | –0.765 (0.051)** | –0.838 (0.070)** | –0.762 (0.050)** | –0.802 (0.079)** |
| $\tau$ | 0.493 (0.172)** | 0.145 (0.144) | 0.157 (0.222) | –0.165 (0.177) |
| | <5.65>[4] | <4.60> | <4.99> | <4.20> |
| $\sigma_1$ | 0.294 (0.021)** | 0.259 (0.028)** | 0.300 (0.032)** | 0.239 (0.046)** |
| $\sigma_2$ | 1.078 (0.129)** | 1.054 (0.105)** | 1.002 (0.128)** | 1.063 (0.116)** |
| $\bar{R}^2/D\text{-}W$ | 0.90 / 1.75 | 0.80 / 1.67 | 0.88 / 1.83 | 0.75 / 1.75 |
| $[\gamma_2' = \alpha_2' = -1]$ | 21.1 (0.000) | 5.77 (0.056) | 24.2 (0.000) | 6.33 (0.042) |
| $[\gamma_2' = -1 \mid \alpha_2' = -1]$ | 17.3 (0.000) | 2.52 (0.113) | 23.6 (0.000) | 5.10 (0.024) |
| Number of observations (low, high) | 135, 43 | 100, 53 | 77, 42 | 47, 47 |

Notes: Estimation of equations (7) and (9) in first-difference form by MLE. The switching variable is the forecastability index (upper panel) or lagged inflation (lower panel). Standard errors are in parentheses. The superscript ** denotes significance at the 0.01 level. *D-W* denotes the Durbin-Watson statistic.

[1]For the tax-adjusted interest rate, a constant average marginal tax rate ($\theta = 0.3$) is assumed in columns 1 and 2, and the average marginal tax rate ($\theta_t$) from Sahasakul (1986) is used in columns 3 and 4.

[2]Measured from the monthly inflation forecast model that includes nine lags of the dependent variable and the three-month treasury bill rate (two-period-lagged monthly figures).

[3]The Wald test for the null hypothesis is indicated within the square brackets (*p*-values are in parentheses).

[4]The threshold value of the (annual) inflation rate (in percent, shown in angled brackets) is converted from the $\tau$ estimate.

to depend on the lagged nominal rate. Since the *p*-values for the test statistics tend to be higher under the heteroskedasticity assumption, one can conservatively decide, based on those *p*-values, whether linearity is rejected.

All the test statistics—SupLM, AveLM, and ExpLM—indicate that linearity is strongly rejected in most cases except for the case of the PPC with the forecastability index, where the evidence against linearity is rather weak (with a 9–16 percent significance level). The evidence against linearity becomes stronger with the level of inflation as the switching variable: in particular, for the PPC, linearity is rejected at the 5 percent level. Also, a specification test for parameter constancy across given subsamples is performed as if the threshold estimate ($\tau$ reported in Table 3) were the true value as in Durlauf and Johnson (1995) and Choi (1999b). The last column of Table 2 provides evidence against parameter constancy across the subsamples. Measuring the after-tax real rate with the time-varying marginal tax rate provides qualitatively the same results (not reported). Taken together, these results give credence to a significant threshold effect in the relationship between interest rates and inflation.

## Switching Regressions with Smooth Transition

The switching regression with smooth transition defined by equations (7) and (10) is estimated by MLE. Table 3 summarizes the results using a forecastability index (upper panel) and, alternatively, using the inflation level (lower panel). The first and second columns pertain to the tax rate adjustment with a constant marginal tax rate, and the third and fourth columns pertain to that with a time-varying marginal tax rate. The $\alpha_2'$ estimate ranges from –1.12 to –1.01, whereas the $\gamma_2'$ estimate ranges from –0.84 to –0.76, indicating that the reflection of inflation in the real rate varies across regimes. The estimate of $\tau$, for example, is –0.073 with the CPI (0.392 with the PPC) when the forecastability index and constant marginal tax rate are used: the observations for the high-forecastability regime are 93 out of 178 for the CPI (55 out of 130 for the PPC). The estimates of $\sigma_1$ and $\sigma_2$ indicate that the conditional variance of the real rate under the high-forecastability regime is about three to four times as high as that under the other regime, consistent with the assumption of regime-wide heteroskedasticity.

The Wald test results run counter to the IFH when inflation forecastability is above a threshold value. Specifically, in the upper panel of Table 3, the Wald test strongly rejects the null hypothesis of $\gamma_2' = \alpha_2' = -1$ in most cases, indicating that the IFH does not hold for the whole sample. In addition, the Wald test rejects the null hypothesis of $\gamma_2' = -1$ given $\alpha_2' = -1$ at the 1 percent level for the CPI and at about the 5 percent level for the PPC. Considering how close the $\alpha_2'$ estimate is to –1, these results suggest that the data do not support the IFH under the high-forecastability regime. This finding is insensitive to the choice of the price variable and marginal tax rate adjustment. The same implications can be drawn from the results with the inflation level as the switching index (lower panel), although the *p*-value of the Wald test for the null hypothesis of $\gamma_2' = -1$ given $\alpha_2' = -1$ is rather high (0.113) for the PPC. The threshold value for inflation ranges from 4 to 6 percent.

## Switching Regressions with Perfect Discrimination

Table 4 reports the results of switching regressions with perfect discrimination, which are estimated by WLS to account for heteroskedasticity, using the square root of the nominal rate as the weight applied to each observation. The likelihood-ratio test statistic for a threshold effect, $F_1$, is highly significant in all cases, as shown by the *p*-values, strongly indicating a threshold effect. The samples are perfectly discriminated according to the LS estimate of $\tau$, which, however, involves some degree of uncertainty as indicated by a confidence interval (90 percent) that is not very tight.[13] With the inflation level as the switching index, more observations tend to be assigned to the low-inflation regime.

The point estimate of $\alpha_2'$ is very close to –1, and the *t*-value for $\alpha_2' = -1$ is too low to reject the IFH under the low-forecastability regime. In contrast, the point estimate of $\gamma_2'$ is less than 0.90 and the *t*-value for $\gamma_2' = -1$ is greater than 2.8, indicating that the IFH is rejected only under the high-forecastability regime. In addition, the IFH is rejected under the high-inflation regime but not under the low-inflation regime (lower panel of Table 4).

In sum, linearity tests provide the evidence of the threshold effect in Carmichael and Stebbing's regressions. The switching regression results indicate that the IFH is supported by U.S. data only when inflation forecastability is below an estimated threshold. The results are robust to the measurement of the price variable, the marginal tax rate adjustment, and the switching index.

## IV. Robustness Checks and Some Evidence for Other Countries

### Robustness Checks

This section examines, first, whether the results are affected when the analysis follows Carmichael and Stebbing's timing convention that $r_{Nt}$ is the after-tax real return on a three-month treasury bill held from the beginning to the end of quarter *t*. The interest rate should reflect inflation expectations over the life of the bill. Since data for inflation do not pertain to a specific date in the month, the underlying maturity cannot be exactly matched with the inflation horizon. Also, the interest rate with this timing is subject to daily seasonality (see, for example, Hamilton, 1997) as reserve requirements with the two-week maintenance period affect the daily funds rate and thus other short-term rates. For comparison purposes, switching regressions following the Carmichael and Stebbing timing (with the CPI) are estimated.[14] The starting date of regressions (1962:Q3) is dictated by the availability of daily data for the three-month treasury bill rate. As shown in Table 5, the results obtained using the forecastability index and the inflation level deliver the same message as those shown in Tables 3 and 4.

We also examine whether accounting for shifts in institutional factors affects the results. First, to account for financial deregulation in the early 1980s, we estimate the switching regression separately for the pre-deregulation

[13]The plots of the likelihood ratio as a function of $\tau_f$ indicate a unique major dip in the likelihood ratio around the estimate, suggesting that two regimes are sufficient to describe the nature of the threshold effect.

[14]To deal with the seasonality issue, we use the period-average rate of interest rather than the rate at a specific date for quarter *t*, taking the rate of inflation as the period-average figure.

## Table 4. Switching Regressions with Perfect Discriminations

| | Constant Marginal Tax Rate[1] | | Time-Varying Marginal Tax Rate[1] | |
|---|---|---|---|---|
| Parameter | CPI (53:Q2–97:Q3) | PPC (65:Q2–97:Q3) | CPI (53:Q2–82:Q4) | PPC (65:Q2–82:Q4) |
| *Forecastability*[2] | | | | |
| $F_1$[3] | 9.47 (0.047) | 8.15 (0.067) | 11.2 (0.017) | 6.61 (0.136) |
| $\tau$ | –0.168 | 0.230 | –0.205 | –0.386 |
| | [–0.600, 0.092][4] | [–0.012, 0.689] | [0.532, 0.671] | [n.a.,[5] 0.907] |
| $\alpha_2'$ | –1.005 (0.032)** | –1.036 (0.049)** | –1.006 (0.040)** | –1.107 (0.096)** |
| $\gamma_2'$ | –0.877 (0.026)** | –0.829 (0.054)** | –0.833 (0.034)** | –0.808 (0.066)** |
| $\bar{R}^2$/*D-W* | 0.92 / 1.54 | 0.83 / 1.54 | 0.91 / 1.64 | 0.80 / 1.65 |
| *t*-value for $\alpha_2' = -1$ | –0.15 (0.561) | –0.74 (0.770) | –0.15 (0.562) | –1.11 (0.860) |
| *t*-value for $\gamma_2' = -1$ | 4.68 (0.000) | 3.16 (0.001) | 4.98 (0.000) | 2.89 (0.003) |
| Number of observations (low, high) | 78, 100 | 69, 61 | 47, 72 | 21, 50 |
| *Inflation level* | | (59:Q3–97:Q3) | | (59:Q3–82:Q4) |
| $F_1$ | 15.5 (0.000) | 15.8 (0.004) | 9.70 (0.028) | 10.7 (0.025) |
| $\tau$ | 0.018 <4.18>[6] | 1.095 <7.06> | –0.099 <4.03> | 0.759 <7.04> |
| | [–0.116, 0.542] | [1.008, 1.257] | [–0.510, 0.955] | [0.690, 0.965] |
| $\alpha_2'$ | –0.989 (0.026)** | –1.006 (0.034)** | –0.974 (0.034)** | –1.004 (0.049)** |
| $\gamma_2'$ | –0.826 (0.032)** | –0.644 (0.085)** | –0.812 (0.039)** | –0.651 (0.097)** |
| $\bar{R}^2$/*D-W* | 0.92 / 1.53 | 0.85 / 1.45 | 0.91 / 1.64 | 0.84 / 1.52 |
| *t*-value for $\alpha_2' = -1$ | 0.42 (0.338) | –0.19 (0.575) | 0.75 (0.227) | –0.09 (0.535) |
| *t*-value for $\gamma_2' = -1$ | 5.31 (0.000) | 4.19 (0.000) | 4.75 (0.000) | 3.60 (0.001) |
| Number of observations (low, high) | 114, 64 | 133, 20 | 68, 51 | 74, 20 |

Notes: Estimation of equation (7) in first-difference form by WLS. Standard errors are in parentheses. The superscript ** denotes significance at the 0.01 level. *D-W* denotes the Durbin-Watson statistic.

[1]For the tax-adjusted interest rate, a constant average marginal tax rate ($\theta = 0.3$) is assumed in the first and second columns, and the average marginal tax rate ($\theta_t$) from Sahasakul (1986) is used in the third and fourth columns.

[2]Measured from the monthly inflation forecast model that includes nine lags of the dependent variable and the three-month treasury bill rate (two-period-lagged monthly figures).

[3]The likelihood-ratio test for the null hypothesis of no threshold effect. *P*-values in square brackets are obtained from a bootstrap procedure with 1,000 replications of bootstrap samples, following Hansen (1999).

[4]The 90 percent confidence interval for $\tau_f$ is computed using the likelihood-ratio statistics (Hansen, 1999).

[5]The lower bound is not available (n.a.), since no statistics on $\tau < -0.386$ are greater than its critical value.

[6]The threshold value of the (annual) inflation rate (in percent, shown in angled brackets) is converted from the $\tau$ estimate.

(before 1980) and post-deregulation (after 1980) periods. Qualitatively the same result is found from the post-deregulation period regression; similar implications are obtained from the pre-deregulation period regression, in which most observations belong to the low-forecastability regime, so that the result is more favorable to the Carmichael and Stebbing argument. Second, to account for price controls that affect inflation and possibly the interest rate setting, we included a Korean War dummy (set to equal 1 for the years 1951–53) in the inflation forecast equation. A step function taken from Gordon (1990), which was designed to capture the effects of imposing and then eliminating price controls during the Nixon era (1971–74), was also used. Using the resulting forecastability measure has little effect on the results, as shown in the third column of Table 5. Alternatively, the regressions were estimated leaving out the Korean War period and the era of the Nixon price controls. The results are very similar except that the cutoff value rises.

The forecastability index is alternatively measured with a quarterly inflation regression that includes the three-month treasury bill rate and four lags of the dependent variable, where the lag length of four for the CPI (PPC) is selected by the information criteria of Akaike, Schwarz, and Hannan-Quinn (that of Akaike and Hannan-Quinn criteria) that are described in Lütkepohl (1991). With this alternative index, linearity tests gave similar results, and, as reported in the last column of Table 5, switching regressions with the CPI provides qualitatively the same results. Also, alternative window sizes (four or six years) or alternative values of $h$ provide similar results and support the main conclusion in most cases. Further, the forecastability measure is largely consistent with Evans and Wachtel's (1993) result, obtained from a Markov switching model, that the probability of being in the random-walk state hovered near 100 percent in the late 1970s and did not fall appreciably until 1985.

To seek a more direct measure of inflation persistence, we estimate the fractional difference process for inflation using a method from Phillips (1998).[15] Figure 5 depicts the fractional integration parameter ($d$) with two-standard-error bands from the rolling estimation of the quarterly inflation process (window size = 20) with the CPI and the PPC. The rolling estimate of $d$ for 1967–86 hovers close to a unit root ($d = 1$) or a nonstationary fractional root ($d > 0.5$). The inflation process is stationary with short memory for the first half of the 1960s and the most recent two years. It is marginally stationary but with long memory (for example, $d > 0.3$) for some fraction of the other years, although confidence intervals include some short-memory alternatives for the second half of the 1980s. This measure, although less volatile, is largely consistent with the inflation forecastability index, with a correlation coefficient of 0.40. Table 6 reports the regression results using the rolling estimate of $d$ in place of $F_t$. The results are similar to the main results. Specifically, the Wald test results suggest that the IFH under high inflation persistence ($d > 0.40$ with

[15]We consider the fractional difference process of the form $(1 - L)^d(\pi_t - \mu) = v_t$, where $L$ is the lag operator, $d$ is the fractional integration parameter, and $v_t$ is a mean zero stationary error. This process is stationary for $|d| < 0.5$.

**Table 5. Switching Regressions with Alternative Measures of Timing and Inflation Forecast**

| Parameter | Carmichael and Stebbing Timing[1]: Forecastability index (62:Q3–97:Q3) | Carmichael and Stebbing Timing[1]: Inflation level (62:Q3–97:Q3) | Inflation Forecast[2]: Monthly forecast with price controls (53:Q2–97:Q3) | Inflation Forecast[2]: Quarterly forecast model (53:Q2–97:Q3) |
|---|---|---|---|---|
| *Smooth transition* | | | | |
| $\alpha_2'$ | –0.987 (0.035)** | –1.018 (0.034)** | –1.025 (0.024)** | –1.005 (0.035)** |
| $\gamma_2'$ | –0.858 (0.033)** | –0.788 (0.043)** | –0.835 (0.034)** | –0.851 (0.032)** |
| $\tau$ | –0.533 (0.199) | 0.087 (0.146) | –0.122 (0.181) | –0.112 (0.207) |
| $\sigma_1$ | 0.216 (0.056)** | 0.217 (0.030)** | 0.246 (0.030)** | 0.461 (0.037)** |
| $\sigma_2$ | 0.801 (0.070)** | 1.023 (0.109)** | 0.836 (0.079)** | 0.728 (0.067)** |
| $\bar{R}^2$/*D-W* | 0.88 / 1.75 | 0.89 / 1.71 | 0.90 / 1.70 | 0.90 / 1.73 |
| $[\gamma_2' = \alpha_2' = -1]$[3] | 25.3 (0.000) | 24.8 (0.000) | 24.5 (0.000) | 26.5 (0.000) |
| $[\gamma_2' = -1 \mid \alpha_2' = -1]$ | 24.1 (0.000) | 23.1 (0.000) | 23.1 (0.000) | 26.5 (0.000) |
| Number of observations (low, high) | 38, 103 | 93, 48 | 80, 98 | 83, 95 |
| *Perfect discrimination* | | | | |
| $F_1$[4] | 6.03 (0.148) | 8.50 (0.052) | 8.52 (0.075) | 9.02 (0.037) |
| $\tau$ | –0.363 | 0.320 <5.79>[6] | –0.097 | 0.330 |
| | [–0.918, 0.598][5] | [–0.703, 0.864] | [–0.885, 0.303] | [–0.242, 0.598] |
| $\alpha_2'$ | –1.031(0.059)** | –0.969 (0.034)** | –0.997 (0.031)** | –0.983 (0.028)** |
| $\gamma_2'$ | –0.873 (0.027)** | –0.826 (0.035)** | –0.877 (0.027)** | –0.860 (0.030)** |
| $\bar{R}^2$/*D-W* | 0.91 / 1.65 | 0.91 / 1.63 | 0.92 / 1.53 | 0.92 / 1.53 |
| *t*-value for $\alpha_2' = -1$ | –0.53 (0.701) | 0.91 (0.183) | 0.09 (0.464) | 0.60 (0.275) |
| *t*-value for $\gamma_2' = -1$ | 4.67 (0.000) | 4.90 (0.000) | 4.58 (0.000) | 4.59 (0.000) |
| Number of observations (low, high) | 53, 88 | 100, 41 | 82, 96 | 96, 82 |

Notes: Regressions in first-difference form with the CPI, assuming $\theta = 0.3$. Equations (7) and (9) are estimated by MLE (upper panel), and equation (7) with perfect discrimination is estimated by WLS (lower panel). Standard errors are in parentheses. The superscript ** denotes significance at the 0.01 level. *D-W* denotes the Durbin-Watson statistic.

[1]The interest rate is the yield on the first business day of a quarter (available from 1962:Q3), and inflation ($\pi_{t+1}$) is the annualized rate in the last month of quarter *t*. The switching variable is the last month's value in each quarter from the monthly forecastability index, and the inflation level is the last month's inflation in quarter $t-1$.

[2]The monthly forecast model includes nine lags of the dependent variable, the three-month treasury bill rate (two-month-lagged figures), a dummy variable for the Korean War period, and a step function for the Nixon-era price controls (Gordon, 1990). The *quarterly* forecast model includes four lags of the dependent variable and the one-quarter-lagged three-month treasury bill rate ($h = 0.72$).

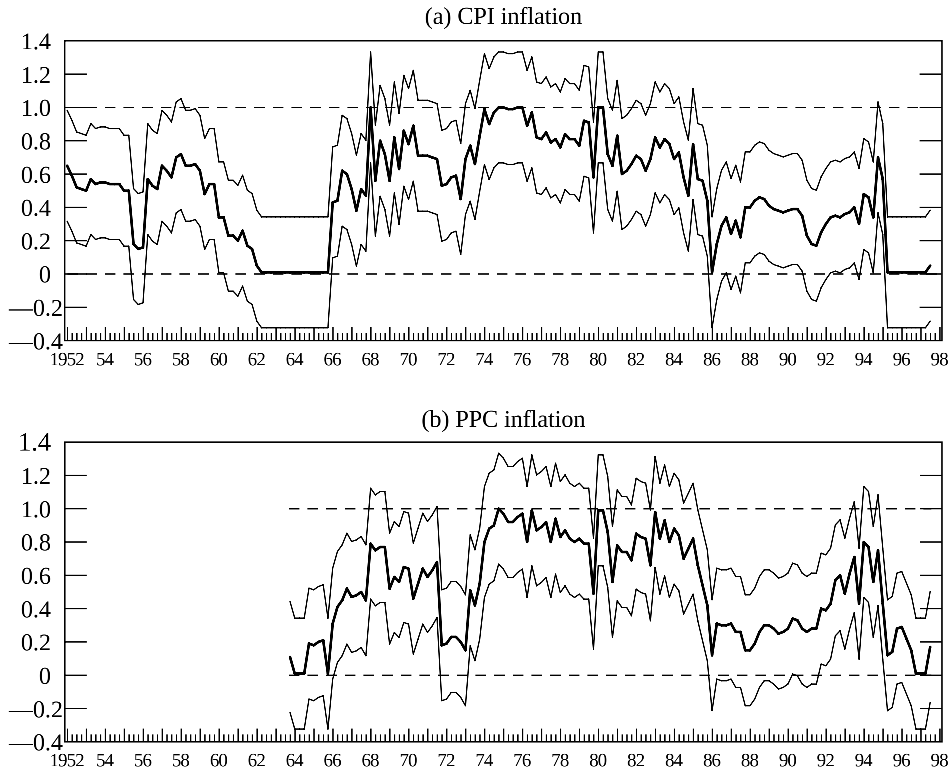
[3]The Wald test for the null hypothesis is indicated within the square brackets (*p*-values are in parentheses).

[4]The likelihood-ratio test for the null hypothesis of no threshold effect. *P*-values in square brackets are obtained from a bootstrap procedure with 1,000 replications of bootstrap samples, following Hansen (1999).

[5]The 90 percent confidence interval for $\tau_f$ is computed using the likelihood-ratio statistics (Hansen, 1999).

[6]The threshold value of the (annual) inflation rate (in percent, shown in angled brackets) is converted from the $\tau$ estimate.

**Figure 5. Rolling Estimate of the Fractional Integration Parameter for Quarterly Inflation, 1953:Q2–1997:Q4**




Note: Dashed lines represent two-standard-error bands.

the CPI and $d > 0.54$ with the PPC) is strongly rejected with the CPI but weakly rejected with the PPC. When the perfect discrimination scheme is employed, for both the CPI and the PPC, the IFH under high inflation persistence ($\gamma_2' = -1$) is strongly rejected, whereas the IFH under low inflation persistence ($\alpha_2' = -1$) is not rejected.

Further, we also estimate directly the nominal rate equation $i_{Nt} = \alpha_0 + \alpha_1 \pi_{t+1} + \xi_t$, in which $\alpha_1 = 0$ should hold under the IFH. The estimated results suggest that $\alpha_1 = 0$ is not rejected under the low-forecastability regime but strongly rejected under the high-forecastability regime, robust to the choice of the switching index, consistent with the results obtained using equation (7).

Finally, the sensitivity of the results to serial correlation in the residual was checked. The single-regime regression results are affected little by the use of MLE, allowing for serial correlation, or by the Newey-West correction for serial correlation and heteroskedasticity. Unfortunately, if there is serial correlation in the errors, existing procedures are not readily applicable to linearity testing and statistical inference for a threshold parameter. Nonetheless, equations (7) and (10) were estimated by MLE, allowing for the first-order serial correlation following

Table 6. Switching Regressions with Fractional Root Estimate as Inflation Persistence Measure

| Parameter | CPI (53:Q2–97:Q3) | PPC (64:Q2–97:Q3) |
|---|---|---|
| *Smooth transition* | | |
| $\alpha_2'$ | –1.090 (0.020)** | –1.021 (0.033)** |
| $\gamma_2'$ | –0.824 (0.028)** | –0.871 (0.068)** |
| $\tau < d >$[1] | –0.351 (0.157)* <0.396> | 0.051 (0.347) <0.543> |
| $\sigma_1$ | 0.145 (0.023)** | 0.311 (0.034)** |
| $\sigma_2$ | 0.752 (0.058)** | 0.993 (0.150)** |
| $\bar{R}^2/D\text{-}W$ | 0.90 / 1.73 | 0.79 / 1.68 |
| $[\gamma_2' = \alpha_2' = -1]$[2] | 50.8 (0.000) | 3.77 (0.152) |
| $[\gamma_2' = -1 \mid \alpha_2' = -1]$ | 21.8 (0.000) | 3.38 (0.066) |
| Number of observations (low, high) | 63, 115 | 68, 67 |
| *Perfect discrimination* | | |
| $F_1$[3] | 19.1 (0.001) | 10.8 (0.017) |
| $\tau < d >$ | 0.350 <0.600> | 0.633 <0.711> |
| | [0.251, 0.701][4] | [0.393, 0.941] |
| $\alpha_2'$ | –1.000 (0.026)** | –1.021 (0.042)** |
| $\gamma_2'$ | –0.824 (0.031)** | –0.774 (0.063)** |
| $\bar{R}^2/D\text{-}W$ | 0.93 / 1.52 | 0.83 / 1.55 |
| $t$-value for $\alpha_2' = -1$ | –0.01 (0.504) | –0.50 (0.691) |
| $t$-value for $\gamma_2' = -1$ | 5.67 (0.000) | 3.61 (0.000) |
| Number of observations (low, high) | 110, 68 | 88, 47 |

Notes: Equations (7) and (9) are estimated by MLE (upper panel), and equation (7) with perfect discrimination is estimated by WLS (lower panel). A constant average marginal tax rate ($\theta = 0.3$) is assumed. The switching variable is the rolling estimate of the fractional root of an inflation process (Phillips, 1998). Standard errors are in parentheses. The superscripts ** and * denote significance at the 0.01 and 0.05 level, respectively. *D-W* denotes the Durbin-Watson statistic.

[1]The threshold value of $d$ (in angled brackets) is converted from the $\tau$ estimate.

[2]The Wald test for the null hypothesis is indicated within the square brackets ($p$-values are in parentheses).

[3]The likelihood-ratio test for the null hypothesis of no threshold effect. $P$-values in square brackets are obtained from a bootstrap procedure with 1,000 replications of bootstrap samples, following Hansen (1999).

[4]The 90 percent confidence interval for $\tau_f$ is computed using the likelihood-ratio statistics (Hansen, 1999).

Goldfeld and Quandt (1972, pp. 258–77). Table 7 shows that this procedure yields strong evidence for this study's argument regarding the IFH, although the significance level is rather higher. The serial correlation estimate is around 0.5 under the low-forecastability (inflation) regime and nil under the other regime, perhaps indicating a threshold effect in serial correlation as well. Also, since the WLS point estimates under the perfect discrimination scheme are consistent, the $t$-test with the Newey-West correction was applied for the IFH under alternative regimes. Again, the results are little affected.

**Table 7. Switching Regressions with Serial Correlation**

| | Switching Index | | | |
|---|---|---|---|---|
| Parameter | Forecastability (53:Q2–97:Q3) | Forecastability (53:Q2–82:Q4) | Fractional root (53:Q2–97:Q3) | Inflation level (53:Q2–97:Q3) |
| $\alpha_2'$ | –1.027 (0.016)** | –1.042 (0.024)** | –1.057 (0.012)** | –1.035 (0.019)** |
| $\gamma_2'$ | –0.893 (0.036)** | –0.820 (0.049)** | –0.843 (0.036)** | –0.786 (0.047)** |
| $\tau$ | 0.182 (0.145) | 0.104 (0.183) | –0.140 (0.143) | 0.341 (0.194) |
| | | | $< 0.539 >^1$ | $<5.16>^1$ |
| $\sigma_1$ | 0.228 (0.022)** | 0.261 (0.033)** | 0.143 (0.015)** | 0.249 (0.028)** |
| $\sigma_2$ | 0.951 (0.097)** | 0.988 (0.122)** | 0.906 (0.088)** | 1.008 (0.119)** |
| $\varphi_1{}^2$ | 0.572 (0.091)** | 0.471 (0.125)** | 0.715 (0.090)** | 0.451 (0.096)** |
| $\varphi_2{}^2$ | 0.029 (0.118) | –0.037 (0.151) | –0.124 (0.132) | –0.088 (0.141) |
| $\bar{R}^2$ | 0.89 | 0.88 | 0.90 | 0.90 |
| $[\gamma_2' = \alpha_2' = -1]^3$ | 8.92 (0.012) | 13.5 (0.001) | 30.6 (0.000) | 20.8 (0.000) |
| $[\gamma_2' = -1 \mid \alpha_2' = -1]$ | 5.96 (0.015) | 10.4 (0.001) | 6.83 (0.009) | 15.0 (0.000) |
| Number of observations (low, high) | 103, 74 | 62, 56 | 86, 91 | 129, 48 |

Notes: Regressions in the first-difference form with the CPI. Switching regressions with smooth transition are estimated by MLE. A constant average marginal tax rate ($\theta = 0.3$) is assumed for the first, third, and fourth columns, and a time-varying marginal tax rate is assumed for the fourth column. Standard errors are in parentheses. The superscript ** denotes significance at the 0.01 level.

[1]The threshold value of *d* or inflation (in angled brackets) is converted from the $\tau$ estimate.

[2]The serial correlation coefficients are $\varphi_1$ and $\varphi_2$ under the low-forecastability (persistence of inflation) regime and the high-forecastability (persistence of inflation) regime, respectively.

[3]The Wald test for the null hypothesis is indicated within the square brackets (*p*-values are in parentheses).

## IFH Tests for Germany, Argentina, and Brazil

The IFH is tested for three countries with different inflation processes. Consider first a low-inflation economy with significant regulation on money. During the 1958:Q1–1997:Q4 period, Germany has experienced low inflation (between –1.3 and 8.5 percent a year) and this inflation showed no remarkable persistence. We find that linearity testing provides no evidence for nonlinearity in the Carmichael and Stebbing equation and that the IFH is not rejected for the whole sample period.

Next, consider two economies that shifted from high inflation to low inflation. Argentina experienced high inflation before 1991 (which averaged about 88 percent in the 1970s and 237 percent for 1980:Q1–1990:Q4) and single digit inflation rates since 1991 with the adoption of a hard currency link to the dollar (a currency board). Brazil also experienced high inflation (252 percent for 1980:Q2–1994:Q3) and single-digit inflation rates since 1997. In both countries, interest rates show excessive sensitivity to inflation changes when inflation rises sharply during the 1980s (which results in a sharp changes in the real rate). We exclude the samples with excessive real interest rate changes—by more than 2000 percent (200 percent) per annum for Argentina (Brazil)—for the 1979:Q3 (1980:Q3)–2000:Q3 period, the starting period of which is dictated by the availability of the interest rate (inflation) series.

**Table 8. Switching Regressions for Argentina and Brazil**

| Parameter/ Test Statistics | Argentina (1979:Q3–2000:Q3) | | | Brazil (1980:Q3–2000:Q3) | | |
|---|---|---|---|---|---|---|
| | SupLM | AveLM | ExpLM | SupLM | AveLM | ExpLM |
| | 7.58 (0.021) | 3.26 (0.099) | 2.29 (0.051) | 3.74 (0.093) | 2.58 (0.122) | 1.38 (0.110) |
| Linearity test statistics[1] | 4.87 (0.088) | 2.58 (0.172) | 1.47 (0.148) | 3.80 (0.059) | 3.06 (0.051) | 1.58 (0.052) |
| $F_1$[2] | 10.9 (0.108) | | | 12.4 (0.082) | | |
| $\tau < d >$[3] | 0.533 <0.643>[3] | | | 0.172 <84.3>[3] | | |
| | [0.509, 1.126][4] | | | [–0.210, 0.865] | | |
| $\alpha_2'$ | –1.343 (0.393)** | | | –0.933 (0.197)** | | |
| $\gamma_2'$ | –0.062 (0.193) | | | 0.134 (0.238) | | |
| $\bar{R}^2$ | 0.13 | | | 0.27 | | |
| $t$-value for $\alpha_2' = -1$ | –1.24 (0.890) | | | 0.34 (0.368) | | |
| $t$-value for $\gamma_2' = -1$ | 4.86 (0.000) | | | 4.76 (0.000) | | |
| Number of observations (low, high) | 52, 27 | | | 35, 18 | | |

Notes: Equation (7) in a first-difference form including an intercept under perfect discrimination is estimated by WLS, assuming $\theta = 0$. Inflation and interest rates are measured by the quarterly CPI (seasonally adjusted) and money market rates, respectively (data source: IMF, *International Financial Statistics* (Washington), various issues). The switching variable is the rolling estimate of the fractional root of an inflation process for Argentina and the lagged inflation level for Brazil. Standard errors are in parentheses. The superscript ** denotes significance at the 0.01 level.

[1]Figures in the upper row (lower row) for each statistic are based on the homoskedasticity (heteroskedasticity) assumption. The asymptotic $p$-values (in parentheses) for SupLM, AveLM, and ExpLM are computed from simulations ($J$ = 1,000) over the grid set (#$\Gamma$ = 102).

[2]The likelihood-ratio test for the null hypothesis of no threshold effect. $P$-values in square brackets are obtained from a bootstrap procedure with 1,000 replications of bootstrap samples, following Hansen (1999).

[3]The threshold value of $d$ (in angled brackets) is converted from the $\tau$ estimate.

[4]The 90 percent confidence interval for $\tau_f$ is computed using the likelihood-ratio statistics (Hansen, 1999).

Table 8 reports the results of linearity test and switching regressions with perfect discrimination. Linearity in the Carmichael and Stebbing equation is rejected for both countries, although less significantly under the heteroskedasticity assumption for Argentina. Using the rolling estimate of the fractional root (or the inflation level) as the switching index, the result suggests that the IFH is supported by the data for Argentina only when inflation persistence is below a certain threshold. Using the inflation level as the switching index to include most samples during the 1980s in the regression, the result suggests that the IFH is supported by the data for Brazil if inflation is below a certain threshold but not otherwise. The excessive sensitivity of interest rates to accelerated inflation in Argentina and Brazil suggests that there can be another regime of very high inflation in which the real rate responds positively to inflation changes.

## V. Related Studies and Discussions

### Related Recent Studies on Testing the (Standard) Fisher Hypothesis

Crowder and Hoffman (1996) suggest the validity of the Fisher hypothesis on the basis of a long-run relationship between the nominal interest rate and inflation. King and Watson (1997), however, suggest that the evidence about the Fisher equation over their entire period is inconclusive, showing that their results support the Fisher hypothesis for only a range of certain identifying restrictions. Mishkin (1992) finds that empirical evidence supports the existence of a long-run Fisher effect only during periods of high inflation when inflation and interest rates exhibit trends.[16] In a similar vein, Barsky (1987) and Hutchison and Keeley (1989) show the presence of a strong Fisher effect under monetary regimes with high inflation forecastability. These results on the Fisher hypothesis or equation reconcile the current study's finding that the IFH is not supported under a regime with high inflation forecastability.

### Forecastability and Uncertainty

Inflation tends to be positively related to short-term inflation forecastability (or persistence): for example, based on the CPI, the correlation between the forecastability index (rolling estimate of fractional root) and lagged inflation is 0.51 (0.57). Friedman (1977) suggests that high inflation leads to greater inflation uncertainty. As Ball and Cecchetti (1990) point out, however, the inflation-uncertainty link depends on the forecast horizon. Indeed, high inflation owing to policy under a fiduciary monetary system raises long-term uncertainty but may make it easier to predict short-term inflation, as argued by Klein (1975). High inflation in the United States during the 1970s and early 1980s raises the variability of inflation (or long-term inflation uncertainty).[17] Recall that our forecastability measure is inversely related to the ratio of short-term to long-term uncertainty. It is highly plausible that long-term uncertainty outweighs short-term uncertainty during high-inflation periods and, hence, that inflation is positively related to inflation forecastability.

### Direct Effects of Inflation Uncertainty on Interest Rates

An existing strand of the literature examines the direct effect of inflation uncertainty on interest rates. For example, Lahiri, Teigland, and Zaporowski (1988) regress interest rates on the moments of the probability distribution of forecasts constructed from the American Statistical Association—National Bureau of Economic Research

[16]As noted by King and Watson (1997), Mishkin (1992) and Evans and Lewis (1995) find that nominal rates do not respond fully to permanent changes in inflation and attribute this to a small-sample bias associated with shifts in the inflation process.

[17]Fischer (1981) argues that high inflation raises the variability, but not necessarily the uncertainty, of inflation. According to Ball and Cecchetti (1990), inflation uncertainty pertains to the variance of unanticipated inflation, whereas inflation variability pertains to the variance of inflation. Since high inflation tends to be largely anticipated, unanticipated inflation will not be large relative to inflation, supporting Fischer's argument. Also, Ball and Cecchetti show that inflation has much larger positive effects on inflation uncertainty at long horizons.

survey of the implicit GNP deflator. Others (for example, Makin, 1983) do so on the semiannual Livingston uncertainty (or disagreement) measure, computed as the standard deviation of the inflation forecasts of respondents to the Livingston survey. In contrast, the present study examines how the link between interest rates and inflation is altered by inflation forecastability (persistence) with a threshold effect.

### Link Between Inflation Level and Inflation Effect on Interest Rates

Azariadis and Smith (1998) suggest a threshold effect in the relation between inflation and returns. Since real balances and bank deposits are substitutes in households' portfolios, the zero nominal return on real balances anchors the rate of return on bank deposits. In a low-inflation economy in which credit is not rationed, an increase in inflation leads to lower real rates (and a higher capital-output ratio).[18] Conversely, in a high-inflation economy in which credit must be rationed, an increase in inflation leads to a smaller capital stock. Thus, whether or not the economy has credit rationing is crucial for the link of the inflation level to the inflation effect on returns. Also, its assumption of a constant implicit real rate of return on real balances anchors the nominal rate. Using cross-country data, Barnes, Boyd, and Smith (1999) find that inflation and nominal rates are only weakly positively correlated for low-to-moderate-inflation economies, whereas inflation has a positive effect on nominal rates for high-inflation economies. This line of research reconciles our framework, in that asset substitutability is emphasized. Our framework, however, emphasizes the link between the inflation process and asset substitutions, assuming that the implicit real returns to real balances rise with inflation if inflation is persistent.

### Inflation Forecastability and Inflation Level

On the one hand, this study argues that the relative importance of the substitutability between money and bonds and that between nominal assets and capital varies with inflation forecastability (or persistence). On the other hand, defining the switching variable by the inflation level is motivated by the possibility of varying substitutability over regimes. The estimated results reconcile both approaches and do not distinguish one from the other, given that high inflation is highly correlated with the persistence of inflation.

## VI. Concluding Remarks

The IFH, strongly supported by Carmichael and Stebbing's finding using U.S. data for 1953–78, was confirmed later by Gupta's (1991) finding for 1968–85. Barth and Bradley (1988), however, report that the hypothesis no longer holds when samples are extended beyond 1978. They attribute this reversal to the crucial dependence of the hypothesis on relatively stable inflation (and moderate regulatory changes), but they

[18]This is similar to the Mundell-Tobin effect that represents a portfolio-substitution effect of inflation on the steady-state capital-labor ratio. The Mundell-Tobin effect, however, assumes that *real balances and capital are substitutes* (direct financing for capital investments) so that an increase in inflation increases the portfolio demand for capital and thus the capital-labor ratio, which, in turn, lowers the real rate of return on assets.

do not directly test the effect of inflation or its persistence. The current study finds that the estimation of equation (3′) using quarterly U.S. data for 1953–97 lends little support to the IFH.

These results, however, do not imply a complete rejection of the IFH, since support for the hypothesis depends on inflation persistence. Agents can largely forecast changes in inflation during persistently inflationary periods, and asset substitutions brought about by such changes alter the implicit marginal return on money and, hence, the nominal rate. As a result, the IFH will receive less support from the entire sample, which includes persistently inflationary periods. Taking into account that the reflection of inflation in interest rates varies with inflation forecastability, this study's analyses with U.S. data reject linearity in the Carmichael and Stebbing equation and provide new evidence that the IFH is supported only if inflation forecastability is below a certain threshold. Further, our argument for the validity of the IFH is also consistent with the results obtained for Germany, Argentina, and Brazil.

This study provides policy implications as follows. A discretionary monetary policy results in an inflation bias, which tends to involve persistence in inflation. To reduce the inflation bias, the central bank may adopt inflation targeting (with a Taylor-type policy rule) or exchange rate targeting, both of which will reduce inflation persistence and increase credibility. Also, emphasis on reputations by central bankers will lead the economy to a less persistent or low-inflation equilibrium. Thus, credibility and reputation is (inversely) related to inflation persistence.[19] Shifts in monetary regimes can alter the nature of the nominal anchor, as emphasized by Flood and Mussa (1994). A regime shift towards less persistent inflation or more stable prices may favor the IFH. Further, suppose that economic activities are sensitive to changes in the real interest rate. Then the one-for-one inverse relationship between real interest rate and inflation (that is, the validity of the IFH) reconciles a trade-off between inflation and unemployment that becomes weak when a discretionary monetary policy makes inflation persistent.

## APPENDIX I. Linearity Testing

Equation (7) in a difference form can be rewritten for testing linearity:

$$\Delta r_{Nt} = \alpha_2' \Delta\pi_{t+1} + D_t(\gamma_2' - \alpha_2')\Delta\pi_{t+1} + (1 - D_t)\eta_{1t} + D_t\eta_{2t}.$$

Testing the null hypothesis of $\alpha_2' = \gamma_2'$ proceeds in the following steps, as in Granger and Teräsvirta (1993). First, run the following regression by least squares (LS) $\Delta r_{Nt} = \beta\Delta\pi_{t+1} + e_t$; then compute the residual ($\hat{e}_t = \Delta r_{Nt} - \hat{\beta}\Delta\pi_{t+1}$) and the sum of squared residuals $SSR_0 = \Sigma\hat{e}_t^2$. Next run the following regression by LS: $\hat{e}_t^* = \delta\Delta\pi_{t+1}^* + \lambda D_t \Delta\pi_{t+1}^* + \varsigma_t$, where $\hat{e}_t^* = \hat{e}_t/\sqrt{g_t}$, $\pi_{t+1}^* = \pi_{t+1}/\sqrt{g_t}$ and, given $\tau$, $D_t$ is defined by equation (7). Define $g_t = 1$ if homoskedastic error is assumed, and $g_t = i_{t-1}$ if heteroskedastic error is assumed. Compute $SSR_1 = \Sigma\hat{\varsigma}_t^2$. Finally, compute the test statistic $LM = T(SSR_0 - SSR_1)/SSR_0$, where $T$ is the number of observations.

[19] The change points of inflation persistence can also be related to shifts in the Fed's operating mechanism (see, for the Fed's operating mechanism, Choi, 1999a). In particular, the nonborrowed reserve targeting for 1979:10–1982:10, which involves high and volatile money growth, matches the persistent inflation regime, whereas federal funds rate targeting since 1987 matches the nonpersistent inflation regime. (Under the Bretton Woods system and before the Vietnam War, interest rates were the focus of monetary policy and inflation persistence was low.)

Following Hansen (1996), $J$ realizations of the LM statistics are generated for each grid in the grid set $\Gamma$. We generate ($\omega_t^j$, $t=1,...T$) i.i.d. $N(0,1)$ random variables; under the assumption of homoskedastic error, generate $\Delta\tilde{r}_{Nt}^j = \hat{\beta}\Delta\pi_{t+1} + \omega_t^j\hat{\sigma}_e$, where $\hat{\sigma}_e$ is the standard deviation of $\hat{e}_t$; or, under the assumption of heteroskedastic error, generate $\Delta\tilde{r}_{Nt}^j = \hat{\beta}\Delta\pi_{t+1} + \omega_t^j\{\sqrt{g_t}/\hat{\sigma}_g\}\hat{\sigma}_e$, where $\hat{\sigma}_g$ is the standard deviation of $g_t$; and the steps in the preceding paragraph are repeated for $j$ = 1, ..., $J$ over $\Gamma$. Then we construct empirical distributions for three functionals of the collection of the statistics

$$SupLM = \sup_{\tau\in\Gamma} LM(\tau),\ AveLM = \tfrac{1}{\#\Gamma}\sum_{\tau\in\Gamma} LM(\tau),\ ExpLM = \ln\left\{\tfrac{1}{\#\Gamma}\sum_{\tau\in\Gamma}\exp\big(LM(\tau)/2\big)\right\},$$

where $\#\Gamma$ is the number of grid points in $\Gamma$.

## APPENDIX II. Estimating Switching Regressions with Perfect Discrimination

The switching regression with perfect discrimination in a difference form is given by

$$\Delta r_{Nt}^* = \alpha_2'\Delta\pi_{t+1}^* \cdot I_{(F_t\le\tau)} + \gamma_2'\pi_{t+1}^* \cdot I_{(F_t>\tau)} + u_t^*,$$

where $\Delta r_{Nt}^* = \Delta r_{Nt}^*/\sqrt{g_t}$; $\pi_{t+1}^* = \pi_{t+1}/\sqrt{g_t}$; $I_{(\bullet)}$ is an indicator function; and $g_t = i_{t-1}$. As in Hansen (1999, 2000), testing and estimating of the threshold model proceed as follows. First, the regression, $\Delta r_{Nt}^* = \alpha_2'\Delta\pi_t^* + u_t$, is run by LS, and the residual $\tilde{u}_t = \Delta r_{Nt}^* - \tilde{\alpha}_2'\Delta\pi_{t+1}^*$ and the sum of squared residuals $S_0 = \Sigma\tilde{u}_t^2$ are computed. Next, for any given $\tau$, the slope coefficient is estimated by LS. Compute $\hat{u}_t^*(\tau) = \Delta r_{Nt}^* - \hat{\alpha}_2'\Delta\pi_{t+1}^* \cdot I_{(F_t\le\tau)}$ and $S_1(\tau) = \Sigma\hat{u}_t^*(\tau)$. The LS estimate of $\tau$ is given by $\hat{\tau} = \arg\min_{\tau\in\Gamma} S_1(\tau)$. Then, the likelihood-ratio test for the null hypothesis of no threshold effect is based on $F_1 = (T-1)[S_0 - S_1(\hat{\tau})]/S_1(\hat{\tau})$. $P$-values for $F_1$ are constructed by a bootstrap procedure (Hansen, 1996, 1999). Finally, to test the null hypothesis of $\tau = \tau_0$, construct the likelihood-ratio statistic to reject for large values of $LR_1(\tau_0)$, where $LR_1(\tau) = (T-1)[S_1(\tau) - S_1(\hat{\tau})]/S_1(\hat{\tau})$. The no-rejection region of $\tau$ at confidence level $1-\alpha$ is the set of values of $\tau$ such that $LR_1(\tau) \le c_\alpha$, where $c_\alpha = -2\ln(1-\sqrt{1-\alpha})$.

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