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Inflation and Sovereign Default

TURALAY KENC, WILLIAM PERRAUDIN, and PAOLO VITALE*

Recent research has highlighted the role that the government budget constraint plays in determining the consumer price level. According to the fiscal approach to price determination, prices adjust so that the discounted value of future real government primary surpluses equals the current real value of public debt. An important implication is that the probability of a crisis involving default on public debt may directly affect consumer prices. This paper examines the interaction of prices and sovereign insolvency crises using simple, continuous-time models of the government budget constraint. [JEL E40, E63, F34, H63]

Recent research on the fiscal approach to price determination (see Leeper, 1991; Sims, 1994; Woodford, 1995, 1996; Canzoneri and Diba, 1996; and Cochrane, 1998) suggests a radical new view of monetary policy and price level determination. If the resources available to service government debt are predetermined and do not automatically adjust to satisfy the government's long-run solvency constraint, other variables, most notably the price level, will adjust. Policies of this type are called non-Ricardian in contrast to more traditional, Ricardian policies in which governments always adjust their real debt service flows to satisfy their intertemporal budget constraint.

The interest of the new approach is its strong and novel implications. For example, many governments follow monetary policies based on interest rate targets, and such targets are often recommended by international agencies such as

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the IMF. Some have argued, on the basis of some well-established results of macroeconomic theory (Sargent and Wallace, 1975), that fixing time paths for nominal interest rates leaves the price level indeterminate and is therefore an ill-conceived policy. In a non-Ricardian policy regime, interest rate targets make perfect sense, however, since the price level is pinned down by the nominal government debt.

Furthermore, the fiscal approach provides justification for the view of some policymakers that fiscal rectitude is a prerequisite for price stability. IMF price stabilization programs place heavy emphasis on securing improvements in budget balance as a way of reducing inflation. The stability pact agreed by participants in the new single European currency may also be viewed as an attempt to contain price pressures by limiting fiscal expansion.

Surprisingly, the new literature on the fiscal approach has tended to ignore how prices and inflation are affected by actual defaults. In most models, prices or other variables are assumed to adjust so that, in equilibrium, the government budget constraint does hold and there is no insolvency. In this paper, we seek to fill this gap. We analyze simple models of the government budget constraint under different assumptions about the hazard that default will occur.

Section I of this paper reviews the basic arguments behind the fiscal approach, contrasting it with more standard analysis of Ricardian policies, and then describes what happens if defaults can occur in equilibrium. Section II describes some simple continuous time models of the government budget constraint. Section III solves for the paths followed by prices and discusses comparative dynamics. Section IV considers alternative assumptions regarding what happens when default takes place. Section V concludes the paper. Proofs are provided in the Appendix.

I. The New Fiscal Approach and Sovereign Debt

Fiscal Policy and Price Determination

The main hypothesis of the new fiscal approach to price determination is that, forgiven time paths of interest rates and prices, a government may adopt a fiscal policy which involves never repaying debt. Instead, it plans to roll over the debt forever. For the private sector to be willing to hold the current debt stock, however, the present discounted value of future government surpluses must equal the market value of the debt stock. If the government's fiscal instruments are predetermined, market prices must adjust to restore equilibrium.

In flexible price models that explore this idea (see, for example, Woodford, 1994), the variable that equilibrates shifts in discounted future government surpluses is the aggregate price level. If a fiscal shock cuts the real value of discounted primary surpluses, the price level rises to reduce the current real value of the government's nominal debt. If prices or wages are sticky reflecting the presence of overlapping nominal contracts, for example, both the price level and real interest rates will adjust to equilibrate shocks to government surpluses (see Woodford, 1996).

The assumption that the government's fiscal instruments are predetermined and outside the influence of the monetary authorities is crucial to the above argument. If fiscal surpluses were automatically increased whenever government debt rose so as to balance the present value (intertemporal) budget constraint for any time path of prices and real interest rates, then the government budget constraint would play no role in determining prices. Policies of this type are termed Ricardian by Woodford and others, while policies in which the government's fiscal instruments are predetermined are called non-Ricardian.

One should notice that the connection between fiscal and monetary policy explored by Woodford, Sims, and others is distinct from the well-known linkages discussed by Sargent and Wallace (1985). In the Sargent-Wallace analysis, money issuance adjusts so that discounted real seigniorage equals future discounted primary deficits. Sargent and Wallace (1985) assume for simplicity that all government debt is indexed, thereby precluding gains to the government from erosion in the real value of debt when prices rise. In contrast, the Leeper-Sims-Woodford analysis turns crucially on this latter kind of gain and supposes that the flow of seigniorage from money issuance adjusts passively so as to maintain equilibrium in the money market.

Implications of the Fiscal Approach

According to the simple quantity theory if the interest rate is pegged, the price level (P_t) is indeterminate since the money supply (M_t) is adjusted passively and M_t and P_t only enter the model in the form of a ratio. However, as mentioned above, if a non-Ricardian policy is followed, wealth effects of the kind examined by Woodford, Sims, and the other authors cited, are introduced, and the price level is once again determinate, since for a given nominal bond stock, variations in prices affect real behavior through a wealth effect and only one set of prices will then be consistent with equilibrium.

Indeed, as Woodford (1995) points out, under non-Ricardian policies, changes in the monetary policy affect the price level only insofar as they interfere with the present value budget constraint, that is, through seigniorage. Cochrane (1998) suggests that monetary effects have only a second-order impact on the price level. He even advocates that one abstract from monetary considerations in the study of price determination.[1]

Another implication of the fiscal approach is that governments may need to coordinate fiscal and monetary policies in order to control inflation. Woodford (1996) and Canzoneri and Diba (1996) argue that the Maastricht criteria for entry into the projected European currency union that included limits on government debt and fiscal deficits are justified on these grounds.

A further consequence of the fiscal approach is that one may observe regime changes as a government changes from a Ricardian to a non-Ricardian policy or vice versa. The regime which prevails will be determined by market perceptions

[1]In this paper, we follow Cochrane's advice in that we construct an equilibrium model where money supply is set passively in order to satisfy its current demand.

of the rules followed by fiscal and monetary authorities and, hence, may be quite unstable.[2]

The Fiscal Approach and Sovereign Default

Surprisingly, given the basic thrust of the fiscal approach literature, there has been no consideration of how actual defaults affect outcomes. In the papers of Woodford and others, default is something that occurs only on off-equilibrium paths. So, although it affects the evolution of prices (which change in order to bring the economy back into equilibrium), defaults take place with probability zero. In this paper, we consider *what happens if default is possible.*

Our analysis yields novel links between sovereign default and inflation. To seethe novelty, note that in standard monetary models of price determination, a country's debt burden and possible default have no direct impact on nominal policies. Furthermore, to the extent that it deals with macroeconomic effects, the sovereign debt literature (summarized by Obstfeld and Rogoff, 1996, Chapter 6; and surveyed,for example, by Eaton and Fernandez, 1995) concentrates on real distortions that may follow from heavy indebtedness, without tracing, through the government budget constraint, how macroeconomic policy more generally may be affected.

Investigating links between sovereign default and monetary policy seems natural. For governments in the throes of stabilization and transition to market economies, for example, the two most pressing macroeconomic issues are how to control prices and how to manage indebtedness to the foreign and private sectors. In principle, the new fiscal approach to price determination might suggest that these two issues maybe closely connected.

To examine these issues, we develop simple continuous-time models of price determination under different assumptions about the government's default probability.From the government budget constraint, we obtain equilibrium conditions that prices must satisfy. These conditions, which are nonlinear differential equations, are then solved subject to boundary conditions implied by the government's long run solvency.

Whether the default crisis involves all or some of the debt and whether fiscal policy after default still determines the price level have crucial impacts on the results. When the post crisis fiscal policy is still predetermined, we find that the risk of default creates positive pressure on prices before the crisis takes place.This pressure is greater the larger is the fraction of the debt on which the sovereign would default. Again, when post crisis fiscal policy is Ricardian, default itself has no instantaneous impact on prices which follow a continuous path.

[2] Several recent empirical studies have examined whether the U.S. postwar fiscal policy satisfies long-run solvency conditions. These include Hamilton and Flavin (1986), Kremers (1989), and Wilcox (1989). In general, these studies conclude that U.S. fiscal policy prior to the 1980s was sustainable and hence consistent with solvency if the fiscal deficit was adjusted for the impact of inflation on nominal government debt. However, Kremers (1989) and Wilcox (1989) raise doubts about the sustainability of policies in the 1980s.

The results are quite different if default is partial and fiscal policy continues to be non-Ricardian after the crisis. In this case, possible future default places no pressure on prices before the crisis takes place. Default itself is deflationary, however, in that prices jump down discretely when the crisis occurs. The interest rate on nominal debt which is known not to be subject to default will actually be lower than the inflation rate since its holders know that they will benefit in real terms from the deflationary consequences of default on other debt.

II. A Fiscal Approach to Inflation

Basic Assumptions

We begin by setting out notation and basic assumptions about government behaviour. Suppose that the government issues short-term nominal debt with value D_t, money of value M_t, and indexed debt with nominal value $P_t\tilde{D}_t$. Here, P_t is the level of consumer prices.

At any given time, let the Quantity Theory hold in that money market equilibrium is equivalent to

$$M_t V = P_t Y_t, \tag{1}$$

where V is a constant velocity parameter and Y_t denotes total real output. For simplicity, in the remainder of the paper we shall suppose that $Y_t = Y$ is constant.[3]

Suppose that at a random date, T, a crisis occurs and the government defaults on its debt. The probability that such a crisis takes place between t and $t + \Delta$, given that it has not so far occurred, is assumed to be

$$\text{Prob}\{T \le t+\Delta \mid T > t\} = 1 - \exp\left[-\int_t^{t+\Delta} \lambda_s ds\right], \tag{2}$$

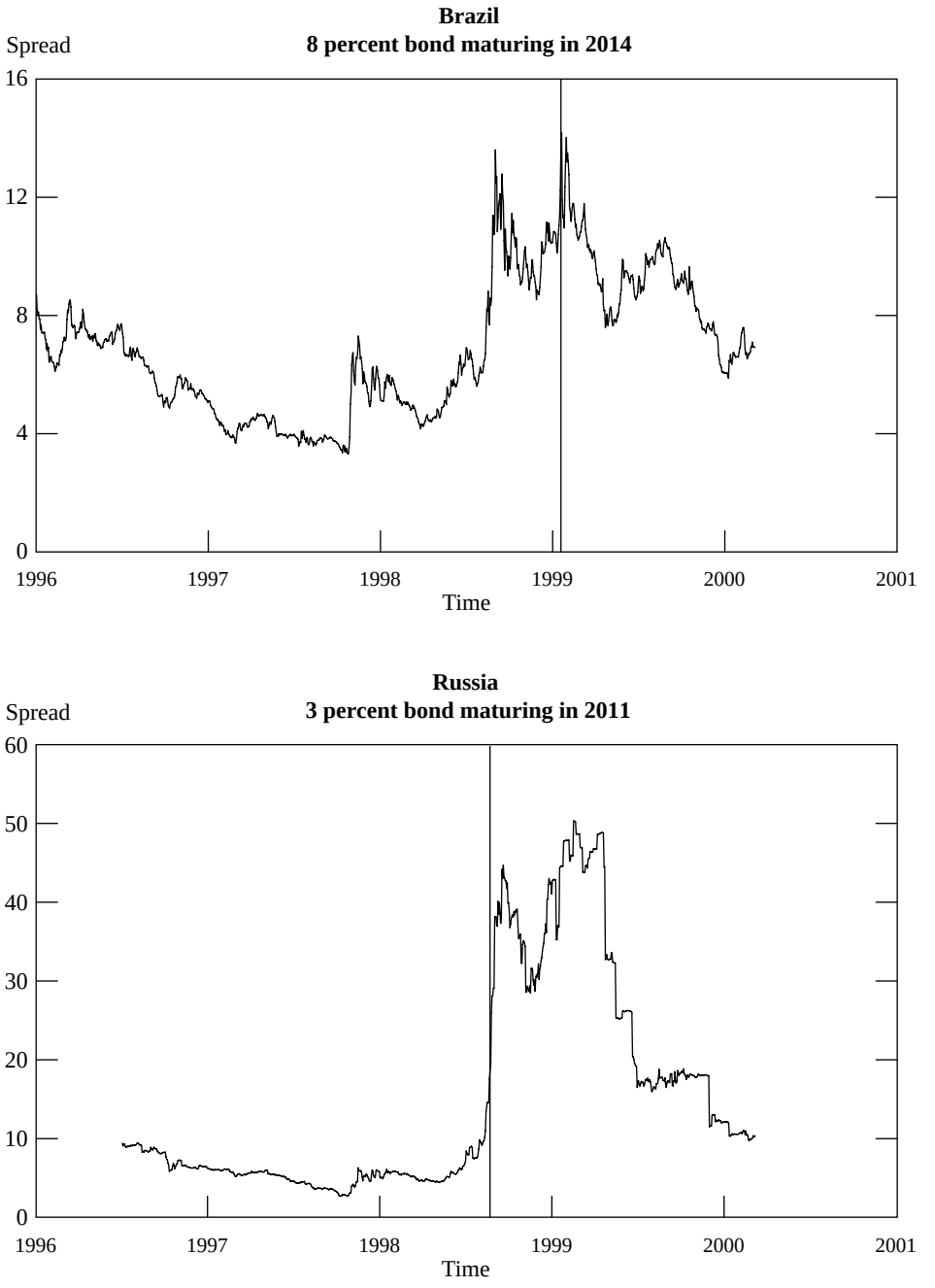
where $\lambda_s \equiv \lambda(s)$ is a real-valued, non-negative function defined for all $s \ge 0$.

Below, we shall consider cases in which λt is constant or increasing over time. The latter case corresponds to situations in which the market is increasingly fearful of default. To motivate our modeling of a default hazard as a function of time, we show, in Figure 1, spreads over U.S. Treasuries of U.S.-dollar-denominated Brazilian and Russian debt.[4] When defaults and interest rates are independent (as they are in our model since interest rates are assumed to be

[3]In other related models, notably Woodford (1994, 1995), an equilibrium demand for money is obtained from the solution of a representative agent's utility maximization problem. Instead, in the present context we choose to simply employ the quantity theory, as we follow Cochrane's claim that a simple fiscal theory of price determination can abstract from monetary considerations. Indeed, in the solution for the equilibrium path for the price level, we introduce the rather stark assumption that the velocity of circulation of money in (1) is very high. This is equivalent to the cashless economy employed by Cochrane (1998).

[4]The spreads are yields to maturity on the Brazilian 8 percent government bond maturing in 2014, and the Russian 3 percent bond issue maturing on 2011, less the yield to maturity on the U.S. Treasury 8 1/8 of 2019. The data source is Bloomberg.

Figure 1. Crisis Country Spreads Over U.S. Treasury Yields



Note: The difference between the yield-to-maturity of the indicated bond and that of the U.S. Treasury 8 1/8 of 2019 is plotted against time. The vertical lines indicate the date of the Brazilian devaluation (i.e., 13th January, 1999) and the date of the Russian default on domestic debt (i.e. 17th August,

nonstochastic), and agents are risk neutral (or risks are diversifiable), the spread on short bonds equals the instantaneous hazard of default.

The vertical lines in each chart show the dates of the January 1999 Brazilian currency crisis and the August 1998 default by the Russian government. It is noticeable that the spreads in Figure 1 fluctuate substantially over time. In the case of Brazilian spreads, there is a clear trend upwards in spreads in the period before the crisis as the market became concerned about possible default (which, of course, did not transpire).In the case of the Russian bonds, the spread is noticeably at in the period prior to the crisis.

Modeling default by specifying an exogenously given default rate, λ_t, is an approach followed by a series of recent papers in the finance literature, see Jarrow and Turnbull (1995) and Duffie and Singleton (1997) and the papers cited therein. The alternative that Duffie and Singleton and others refer to as the structural approach is to suppose that default is triggered when some process (e.g., firm value or sovereign borrower ability-to-pay) crosses a threshold either at the maturity of the debt or before.

Papers on sovereign debt valuation that take this latter "structural" approach include Bartolini and Dixit (1991) and Cohen (1993). Ideally, one would like the moment of default to be determined by the optimizing decision of the government. Modeling this explicitly is difficult, however, and even so-called structural models specify the default triggers in an ad hoc fashion.

The fact that the default decision is not endogenized in our model is not a serious drawback. Various of the results we obtain hold for different specifications of $\lambda(t)$ so it is legitimate to hold the functional form constant and then to analyze the impact of possible future default on prices and inflation. The one important restriction that we impose is the assumption that default comes at least in part as a surprise. This seems to us realistic and indeed the instantaneously predictable nature of default in the Bartolini-Dixit and Cohen models is questionable.[5]

Returning to our description of government behaviour, let W_t denote the total nominal value of government bonds, i.e., $W_t \equiv D_t + P_t\tilde{D}_t$, and let X_t denote total government liabilities, i.e., $X_t \equiv D_t + M_t + P_t\tilde{D}_t$. Suppose that the government's debt issuance policy is such that

$$D_t = \xi W_t \qquad \text{and} \qquad P_t\tilde{D}_t = (1-\xi)W_t, \tag{3}$$

for constant ξ. Prior to default, suppose that W_t grows at an exogenously specified rate, μ_w, so that[6]

[5]By ruling out surprises, their models imply that the default premium of short debt is zero except when the sovereign ceases to service the debt fully. In principle, their models could be extended to allow for surprise defaults and non-zero default premia on short debt by incorporating jump components into their state variables.

[6]The assumption that there exists an exogenously given growth rate for the nominal value of the government bonds is analogous to the assumption of Cochrane (1998) that the face value of government liabilities is given. This appears justified in the case of developing countries in which the dynamics of public spending and tax revenues are often outside the control of central authorities.

$$W_t = \exp[\mu_w t]W_0. \tag{4}$$

The Government's Flow Financing Constraint

Let R_t and G_t denote the government's tax revenues and spending expressed in real terms. The real primary surplus is then $g_t \equiv R_t - G_t$. Let r be an exogenously specified real interest rate, assumed fixed.[7]

We now adopt three important assumptions.

1. Suppose the default is total (involving all the debt) and that the government cannot subsequently borrow in the bond market.[8]
2. Assume that prior to default, the real primary surplus is an exogenously given function of time, $g_t = g_0 \exp[\mu_g t]$, while money is adjusted passively to ensure that the Quantity Equation holds.
3. Suppose that after default, M_t is adjusted according to an exogenously given growth rule while the real primary surplus, g_t, is adjusted to maintain the government's long-run solvency (in a sense to be defined precisely below).[9]

These assumptions imply that prior to the default crisis, fiscal policy is exogenous and monetary policy reacts passively, while after default fiscal policy is endogenous and monetary policy is exogenously given. When fiscal policy or monetary policy are exogenous, we shall say that prices are respectively fiscally or monetarily determined.

As we demonstrate below, the effect of our three assumptions is that prior to default the price level is fiscally determined in the way described above, while afterwards it is determined by the classic Quantity Theory. In Section IV, we shall discuss what happens if prices are fiscally determined both before and after default (i.e., as an alternative to assumption 3). We shall also examine the consequences of default on only a fraction of the total debt stock (i.e., relaxing assumption 1).

As we show in the Appendix, subject to the above assumptions, prior to default,the government's flow financing constraint may be written as:

$$P_t g_t + \mu_w W_t - \frac{dP_t/dt}{P_t}(1-\xi)W_t + \frac{dM_t}{dt} = \left(r + \lambda_t + \frac{dP_t/dt}{P_t}\right)\xi W_t + (r+\lambda_t)(1-\xi)W_t. \tag{5}$$

[7]For simplicity, we shall not include real capital in the model. The assumption of a fixed real interest rate may not, therefore, be justified in the usual way by supposing a constant returns to scale "storage-type" technology. However, the real interest rate will still be constant under several sets of assumptions. For instance, if one assumes that agents have linear utility functions with constant subjective rates of time preference, or that their utilities are simply separable over time and consumption is constant. In these cases, the real interest rate will then be equal to the time preference rate.

[8]This might seem an extreme assumption, but it is meant to reflect the difficulties developing countries encounter when they try to access international bond markets after default.

[9]Notice that in order to determine the equilibrium dynamics of the price level in the Ricardian regime prevailing after default, we need to assume that the money supply follows a continuous path.

Here, the left hand side represents sources of nominal funds including (i) the nominal primary surplus, $P_t g_t$, (ii) plus the increase in debt, $\mu_w W_t$, (iii) minus the cost of revaluing indexed debt, $(1-\xi)W_t(dP_t/dt)/P_t$, and (iv) plus the change in the nominal money stock, dM_t/dt.

The right hand side shows the uses to which these resources are put, namely (i) interest payments on the nominal debt, $(r + \lambda_t + (dP_t/dt)/P_t)\xi W_t$, and (ii) interest payments on the indexed debt, $(r + \lambda_t)(1 - \xi)W_t$. Note that both interest rates include compensation for the fact that if default occurs the values of the debt stocks, D_t and $P_t\widetilde{D}_t$, jump to zero (see the λ_t terms).

Rearranging and substituting for dM_t/dt using the Quantity Equation, one obtains:

$$P_t g_t + \mu_w W_t + \frac{Y}{V}\frac{dP_t}{dt} = \left(r + \lambda_t + \frac{dP_t/dt}{P_t}\right)W_t. \tag{6}$$

Since g_t and W_t are exogenously given functions of time, equation (6) is a non-linear differential equation determining the price level, P_t. This equation may be solved subject to a boundary condition that follows from the government's long-run solvency condition.

Long-Run Solvency

For simplicity, one may think of there being just two sectors in the economy, the government and the private sector. In the Appendix, we show that if the private sector is on its long run intertemporal budget constraint (i.e., satisfies a transversality condition on its borrowing), the government's ability to borrow is also constrained. This generates a long-run solvency condition for the government, namely

$$E_0\left\{\int_0^\infty \left(P_s g_s + \left[r + \left(dP_s/dt\right)/P_s\right]M_s\right)\exp\left[-\int_0^s \left[r + \left(dP_\tau/dt\right)/P_\tau\right]d\tau\right]ds\right\} = X_0. \tag{7}$$

Equation (7) says that the total nominal liabilities of the government, X_0, must equal the expected discounted resources it has available to service them. The latter resources comprise the exogenous flow of nominal primary surpluses, $P_t g_t$, plus the flow of profits that the central bank makes on its monopoly control of money issuance,$(r + (dP_t/dt)/P_t)M_t$.

Now, recall our assumptions that default is complete so that debt values jump to zero when default occurs and that after default, the economy shifts into a monetarily determined equilibrium. If T is the default date, we deduce that

$$\int_T^\infty g_\tau \exp\left[-r(\tau - T)\right]d\tau + \int_T^\infty \left[r + \frac{dP_\tau/dt}{P_\tau}\right]\frac{M_\tau}{P_\tau}\exp\left[-r(\tau - T)\right]d\tau = \frac{M_T}{P_T}. \tag{8}$$

Using equation (8), one may evaluate the expectation in equation (7) to obtain:

$$\int_0^\infty \left(P_s g_s + \left[r + \left(dP_s/dt\right)/P_s\right]M_s\right)\exp\left[-\int_0^s \left[r + \left(dP_\tau/dt\right)/P_\tau + \lambda_\tau\right]d\tau\right]ds + \tag{9}$$
$$\int_0^\infty \lambda_T \exp\left[-\int_0^T \left[r + \left(dP_\tau/dt\right)/P_\tau + \lambda_\tau\right]d\tau\right]M_T\,dT = X_0,$$

where P_t is now not the actual price level but instead is the price level under the assumption that default has not yet occurred at t. Substituting for M_s using the money market equilibrium condition and rearranging, we finally have

$$\int_0^\infty \left(g_s + \left[r + \lambda_t + \left(dP_s/dt\right)/P_s\right]\frac{Y}{V}\right)\exp\left[-r\,s - \int_0^s \lambda_\tau d\tau\right]ds = \frac{X_0}{P_0}. \tag{10}$$

This is the fundamental equation that the price level must satisfy when default is complete and prices are monetarily determined after default.

III. Price Solutions

Constant Default Rate

Explicitly solving the price determination equation, (6), subject to the boundary condition, (10), is difficult because of their highly nonlinear nature. However, if velocity is high, so that the terms involving $Y = V$ are negligibly small, or the economy is cashless, so that these terms disappear altogether, fully analytic solutions maybe obtained. In this section, we study these analytic solutions. It is important to note that, in assuming high velocity, we are merely supposing that ow of receipts from seigniorage is small *compared with the size of the government deficit.* For many countries, this is true although probably not for those experiencing hyperinflation.[10]

To begin with, suppose that the default rate is constant in that $\lambda_t = \bar{\lambda}$. This case corresponds to the situation experienced by Russia in the run up to their default as illustrated by the plot in Figure 1. As we show in the Appendix, under the assumption of high velocity, the general solution to equation (6) is

$$P_t = -\frac{\left(W_0/g_0\right)\exp\left[\left(\mu_w - \mu_g\right)t\right]\eta_2 \exp[\eta\, t]}{A + \exp[\eta\, t]}, \tag{11}$$

where A is a free constant to be determined from the boundary condition and $\eta \equiv (r + \bar{\lambda} - \mu_g)$. Under the assumption of high velocity, the government's long-run solvency condition is:

$$\int_0^\infty g_s \exp\left[-\left(r + \bar{\lambda}\right)s\right]ds = \frac{W_0}{P_0}. \tag{12}$$

[10] Between the mid 1960s and the late 1980s seigniorage levels, as reported by Little, Cooper, Corden, and Rajapatirana (1991), do not exceed 2/3 percent for most developing countries. Argentina,with levels close to 10 percent, has been the main exception.

Substituting for P_0 and solving, one may show (see the Appendix) that $A = 0$. Hence,the price level is:

$$P_t = \left(W_0/g_0\right)\exp\left[\left(\mu_w - \mu_g\right)t\right]\left(r + \bar{\lambda} - \mu_g\right). \tag{13}$$

This simple solution is log-linear in time and implies a constant inflation rate:

$$\frac{dP_t/dt}{P_t} = \mu_w - \mu_g. \tag{14}$$

Though the inflation rate is independent of the default rate, from equation (13), the price level, P_t, is increasing in $\bar{\lambda}$. The intuition here is that increasing the default rate lowers the expected present discounted value of the flow of real debt service. Since (subject to our simplifying assumptions), this has to equal the current real value of the debt, the price level must be higher.

Comparative Dynamics of the $\lambda_t = \bar{\lambda}$ Case

It is helpful to understand the geometry of the price solutions. Qualitative features of the phase diagram for log prices are illustrated in Figure 2. For each value of the free constant A, prices follow a different trajectory. When $A = 0$, log P_t is linear in t with a slope $\mu_w - \mu_g$ and intercept $\log(-\eta W_0/g_0)$. When $A > 0$, for large t, log P_t asymptotes to a linear function with slope $\mu_w - \mu_g + \eta$ and intercept $\log(-\eta W_0/g_0)$. If $-1 < A < 0$, the trajectory explodes to infinity in finite time. The trajectory corresponding to $A = 0$ appears as a straight line in Figure 2.

Figure 2. Phase Portrait for log P_t when $\lambda_t = \bar{\lambda}$

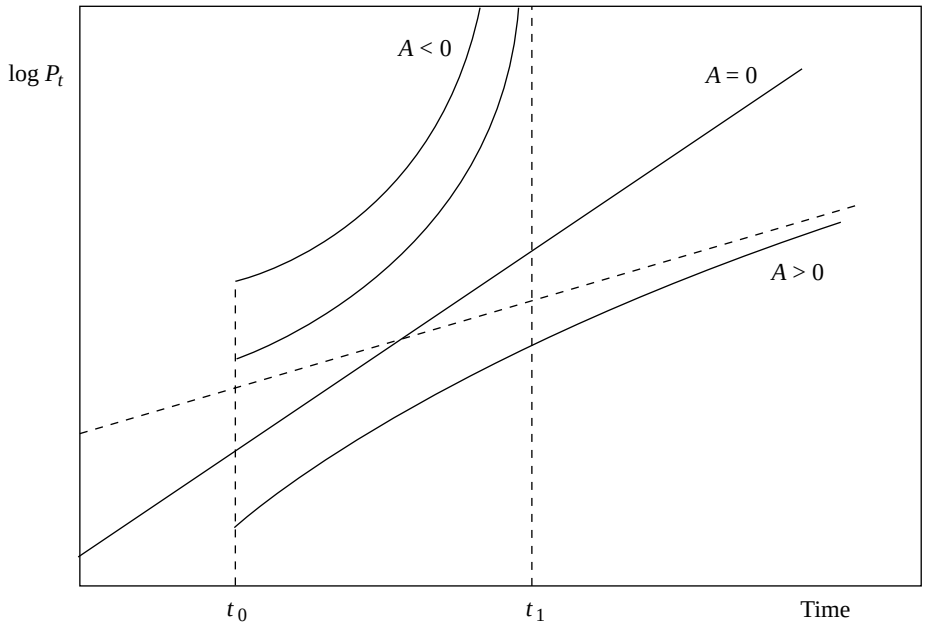
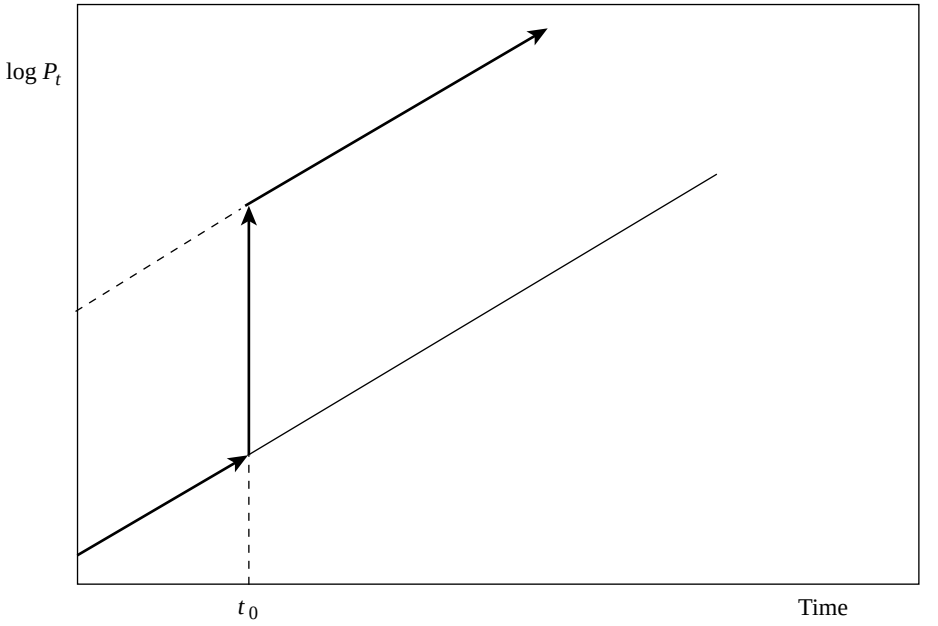


Figure 3. Rise in Crisis Rate when $\lambda_t = \bar{\lambda}$



Using our knowledge of the phase portrait, one may analyze the comparative dynamics of the price level and inflation. Consider a discrete increase in λ_t from $\bar{\lambda}_1$ to $\bar{\lambda}_2 > \bar{\lambda}_1$ occurring at time t_0. The effect of this is shown in Figure 3. The log price level jumps instantaneously at t_0 from the old trajectory to a new higher trajectory, parallel to the first. The jump size is simply $\log((r + \bar{\lambda}_2 - \mu_g)/(r + \bar{\lambda}_1 - \mu_g))$.

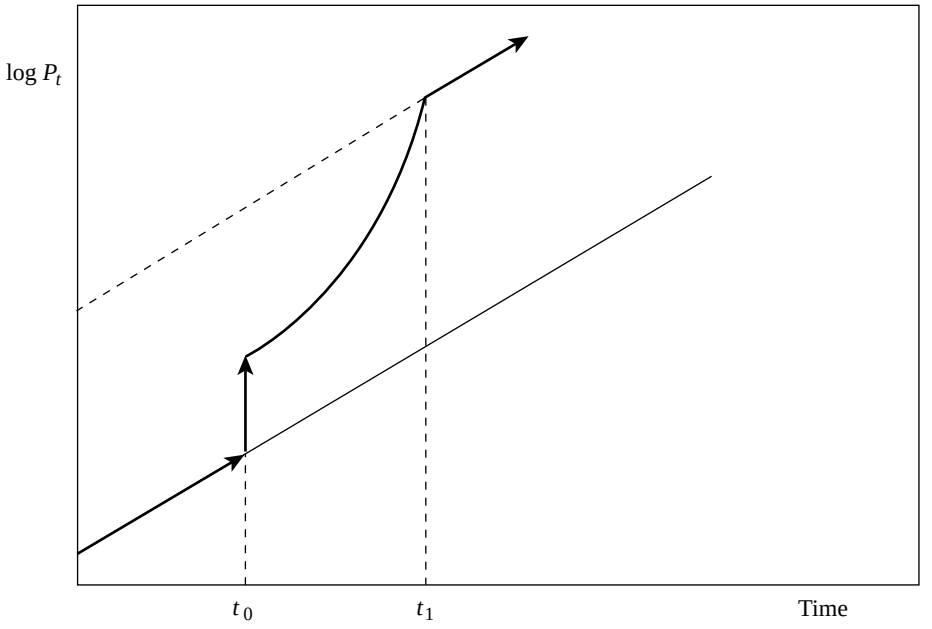
Now, consider an anticipated future rise in the rate of default, $\lambda_t = \bar{\lambda}$. Suppose that the increase takes place at t_1 and is announced at $t_0 < t_1$. The resulting dynamics are shown in Figure 4. Initially, there is a positive jump in the price level, followed by movement along one of the explosive trajectories shown in Figure 2. At t_1, prices hit a higher linear trajectory and then follow this for $t > t_1$.

The Increasing Default Rate Case

Now consider the case in which the default rate is increasing over time. This case corresponds to the situation experienced by Brazil in the period prior to their devaluation crisis as illustrated by the plot in Figure 1. When the default rate is a function of time, the analysis is slightly more complicated. For example, when $\lambda_t = \bar{\lambda} t$, as we show in the Appendix,[11] the general solution for the price level may be written as:

[11] One may also obtain solutions when $\lambda_t = \bar{\lambda} = (\bar{\lambda}_0 + t)$ In this case, the general solution of the differential equation comprises confluent hypergometric functions.

Figure 4. Anticipated Rise in Crisis Rate when $\lambda_t = \bar{\lambda}$



$$P_t = -\frac{-\left(W_0/g_0\right)\exp\left[\left(\mu_w-\mu_g\right)t\right]\exp\left[-\left(r-\mu_g\right)t-\frac{\bar{\lambda}t^2}{2}\right]}{A-\int_0^t\exp\left[-\left(r-\mu_g\right)s-\frac{\bar{\lambda}s^2}{2}\right]ds}, \tag{15}$$

where A is a free constant to be determined from the government's long-run solvency condition. Using the latter and substituting for A yields:

$$P_t = \frac{W_0}{g_0}\frac{\exp\left[\left(\mu_w-\mu_g\right)t\right]}{\int_0^\infty\exp\left[-\left(r-\mu_g\right)s-\left(\bar{\lambda}/2\right)\left[s^2+2st\right]\right]ds}. \tag{16}$$

Examining the solution, one may conclude that prices are an increasing function of time so long as $\mu_w > \mu_g$. As in the constant rate of default case, the price level P_t is increasing in $\bar{\lambda}$, other things being equal. The rate of inflation is no longer constant in the $\lambda_t = \bar{\lambda}_t$ case, however. Numerical simulation of the model suggests that the inflation rate decreases over time.

The qualitative nature of the phase portrait for log P_t when $\lambda_t = \bar{\lambda}t$ resembles that of the standard case except that the convergent path that satisfies the boundary condition is convex. (One may show that when $A < \int_0^\infty \exp[-(r-\mu_g)s - \bar{\lambda}s^2/2]ds$, there exists a time, t, when the denominator in equation (15) is null and hence the price level explodes. When the inequality on A is reversed, the price path is non-

explosive.) Hence, the comparative dynamics of the log price level in this case are broadly similar to those in the constant default rate case.

IV. Alternative Assumptions About Default

Partial Default

Recall our important assumptions (i) that default is complete and involves the entire debt stock, and (ii) that, after default, prices are 'monetarily determined' in that the real primary surplus, g_t, is adjusted to maintain government solvency and money growth is exogenously specified. Let us consider the effects of relaxing these assumptions.

If the government defaults only on part of the debt, it is probably more realistic to suppose that the default is on the indexed debt. In our flexible price model, indexed debt may be thought of as foreign currency borrowings. Countries appear to default to a greater extent on their external debt which is commonly denominated in foreign currency.

Modifying our previous analysis, we find that if the government defaults at T on its indexed debt, $(1-\xi)W_T$, and that prices are monetarily determined for $t > T$, the financing constraint in equation (5) becomes

$$P_t g_t + \mu_w W_t + \frac{dM_t}{dt} = \left(r + (1-\xi)\lambda_t + \frac{dP_t/dt}{P_t} \right) W_t. \tag{17}$$

Thus, the only change is that the default rate λ_t is scaled down by a factor $1-\xi$. When velocity is high so the terms involving money disappear, the general solutions for prices in our two cases, $\lambda_t = \bar{\lambda}$ and $\lambda_t = \bar{\lambda}_t$, are of exactly the same form as in equations (11) and (15) except with $\bar{\lambda}$ replaced by $\bar{\lambda}(1-\xi)$. Effectively, the phase portrait for the constant default rate case shown in Figure 2 is shifted down, with the convergent linear path moving down in a strictly parallel fashion.

The modification to the boundary condition in equation (10) required when we assume partial default is more substantial in that the price solution must satisfy

$$\int_0^\infty \left(g_s + \left[r + \left(dP_s/dt \right)/P_s \right] \frac{Y}{V} \right) \exp\left[-r\,s - \int_0^s \lambda_\tau d\tau \right] ds + \tag{18}$$
$$\int_0^\infty \lambda_\tau \exp\left[-\int_0^T \left[r + \lambda_\tau \right] d\tau \right] \left(\frac{Y}{V} + \frac{W_T}{P_T} \right) dT = \frac{X_0}{P_0}.$$

If we again restrict attention to the high velocity case, it is clear that the additional term involving $W_T = P_T$ on the left hand side is positive and hence the constant of integration, A, implied by this boundary condition is higher than in the case with full default. (Recall that this was $A = 0$).

To summarize, the effect of introducing partial default is (i) to shift the convergent linear solution for log prices downwards and (ii) to generate a positive constant of integration. We may conclude in the constant λ_t case that with partial default prices are unambiguously lower than with full default.

Fiscal Price Determination After Default

Finally, consider how the results are affected if, following default, the government's primary surplus continues to follow an exogenously given path and money is adjusted passively to maintain money market equilibrium. In other words, prices are fiscally determined for t greater than the random default time, T. In fact, default must be partial for such a post-default equilibrium to make sense. (In such an equilibrium, in the absence of nominal bonds, prices only appear in the government solvency condition and the Quantity Equation either as a ratio to M_t or in the inflation rate. Since money is assumed endogenous, it is easy to show that the price level is not pinned down.) So, we shall assume that the government defaults completely on its indexed debt, $P_t\tilde{D}_t$, but not on the nominal debt, D_t.

It might appear that the analysis will be more complicated under these assumptions, since in general the price level will now jump at default and holders of the nominal debt must be compensated for this possibility by a new term in nominal interest rates. For the simpler high velocity case, the financing constraints for the government thus become:

$$P_t g_t + \mu_w W_t = \left[r + \frac{dP_t/dt}{P_t} + \lambda_t\left(\xi J_t + (1-\xi)\right)\right] W_t, \quad \text{for} \quad t \in [0,T), \tag{19}$$

$$P_t g_t + \mu_w W_t = \left[r + \frac{dP_t/dt}{P_t}\right] W_t, \quad \text{for} \quad t \in [T,\infty), \tag{20}$$

where J_t is minus the proportional jump in the real value of nominal bonds that occurs when default takes place at T. Since the government primary surplus, g_t, is exogenously given for all t, it follows that

$$\frac{W_t}{P_t} = \int_t^\infty g_\tau \exp\left[-r(\tau - t)\right] d\tau = \frac{g_t}{r - \mu_g} \quad \text{for all} \quad t \geq 0. \tag{21}$$

Since the right hand side of equation (21) is continuous in time, the jump in the real value of the debt at t should default occur at that date is

$$-J_t = \left(\frac{P_{t-}}{P_{t+}} - 1\right) = \left(\frac{W_{t-}}{W_{t+}} - 1\right) = \frac{1}{\xi} - 1. \tag{22}$$

Substituting for J_t in equation (20), we find that $\lambda_t(\xi J_t + (1 - \xi)) = 0$ and the financing constraint is unaffected by the default rate λ_t both before and after the default date, T. Since the boundary condition does not directly depend on λ_t, it is clear that the path of prices prior to default is independent of λ_t.

The intuition for this finding is as follows. Equation (21) shows that the government is made neither worse nor better off by default since the amount of discounted resources it devotes to debt service (the right hand side of equation (21)) does not jump at *T*. However, default represents a tax on defaultable debt holders. Thus, when default occurs, the real value of remaining debt must rise as the gains to investors who hold the non-defaultable nominal debt must precisely offset the losses of investors who hold the defaultable, indexed debt. This is possible only if the price level jumps down making the holders of the nominal debt better off in real terms.[12]

V. Conclusion

This paper has investigated the behaviour of prices and inflation when the real primary surplus is exogenously given. In these circumstances, for investors to be willing to hold government debt, consumer prices must adjust so that the real value of the government debt equals the discounted value of the primary surplus. The novelty of our analysis is the interaction that may arise between this fiscal theory of price determination and possible crises involving default on the government debt.

For most of the paper, we concentrate on models in which the fiscally determined price regime switches to a more standard monetarily determined regime at default. In this case, our analysis suggests that prices are increasing in the rate of default. The intuition is that a higher default rate reduces the expected discounted value of real primary surpluses. To compensate, consumer prices must increase so as to ensure the government's long-run solvency.

The dependence of the inflation rate on the default rate is more complex. We show that unanticipated changes in the default rate generate intuitively reasonable dynamics. When the rate of default is a given function, inflation is independent of the default rate when the latter is constant but may be decreasing when, for example, $\lambda_t = \bar{\lambda} t$.

We find that when nominal and indexed debt (for which one may read domestic and foreign currency debt if purchasing power parity holds) are treated symmetrically in the event of default (so for example the same fraction of each is subject to default), then the two forms of debt enter the model symmetrically. The implication is that what matters for inflationary pressure is the total debt rather than the nominal (or domestic currency) debt alone.

This result follows from the fact that the costs of servicing indexed (or foreign) debt crowd out resources which would otherwise have been available for the servicing of nominal debt. Hence, higher indexed (or foreign) debt has the same inflationary impact as nominal (domestic currency) debt.

When default is partial in that the government defaults only on indexed debt, we find that the basic analysis continues to apply except that prices are somewhat

[12]This effect somewhat resembles the impact of buy-backs on debt values highlighted by Bulow and Rogoff (1988).

lower. Pushing up the fraction of debt on which the authorities might default therefore results in higher prices and inflation.

If default is partial and prices are fiscally rather than monetarily determined after the crisis date, T, then the price level jumps down at default, i.e., default is deflationary. The proportional price jump is equal to the fraction of debt on which default occurs and the real value of government debt is continuous at the crisis date T. Prior to the crisis, however, the possibility of default has no impact on the path followed by inflation or prices.

Again, with partial default followed by a fiscally determined equilibrium, investors holding debt on which default will not occur are actually willing to accept an interest rate lower than the sum of the inflation rate and the real interest rate. In other words, the interest rate on non-defaultable, nominal debt contains a discount to allow for the jump in the real value that takes place at default.

APPENDIX

Proof of Equation (5)

Assume that time periods are discrete and of length Δ. The period by period budget constraint faced by the government prior to default is:

$$P_t\left(R_t - G_t\right)\Delta + \left(\exp\left[\mu_w \Delta\right] - 1\right)W_t - \frac{P_{t+\Delta} - P_t}{P_t}\left(1-\xi\right)W_t + M_{t+\Delta} - M_t = \tag{23}$$
$$\left(\exp\left[r\Delta + \int_t^{t+\Delta}\lambda_\tau d\tau\right]\frac{P_{t+\Delta}}{P_t} - 1\right)\xi W_t + \left(\exp\left[r\Delta + \int_t^{t+\Delta}\lambda_\tau d\tau\right] - 1\right)\left(1-\xi\right)W_t.$$

So far, we have developed our model in discrete time. However, difference equations are not particularly easy to analyse. Taking a continuous time limit, one may obtain simple, explicit, closed-form solutions. Dividing through by Δ and letting the time interval shrink to zero (i.e., $\Delta \downarrow 0$) in equation (23) yields equation (5) from the text.

Proof of Equation (7)

Let us start with the latter's period by period financing constraint:

$$P_t C_t \Delta + M_t + \left(D_t + P_t \tilde{D}_t\right)1\{t\} \le P_t\left(Y_t - R_t\right)\Delta + X_t \quad \text{for} \quad t = \Delta, 2\Delta, \ldots \tag{24}$$

where $1\{t\}$ is an indicator function for whether or not the default crisis has occurred, defined as

$$1\{t\} = \begin{cases} 0 \text{ if default occurs at } t \text{ or earlier} \\ 1 \text{ otherwise} \end{cases}, \tag{25}$$

$$X_t \equiv M_{t-\Delta} + 1\{t\}\exp\left[r\Delta + \int_{t-\Delta}^{t}\lambda_s ds\right]\frac{P_t}{P_{t-\Delta}}\left(D_{t-\Delta} + P_{t-\Delta}\tilde{D}_{t-\Delta}\right). \tag{26}$$

Define i_t and q_t as follows:

$$1 + i_t \equiv \exp\left[r\Delta\right]P_t / P_{t-\Delta},\ q_0 \equiv 1 \text{ and } q_{t+\Delta} \equiv q_t \big/ \left(1 + i_{t+\Delta}\right) \text{ for } t = \Delta, 2\Delta, \ldots. \tag{27}$$

Using the goods market equilibrium condition:

$$C_t + G_t = Y_t, \tag{28}$$

one may derive

$$q_t P_t g_t \Delta + \left(q_t - q_{t+\Delta}\right) M_t + \left(q_t 1\{t\} - q_{t+\Delta} 1\{t+\Delta\} \exp\left[\int_t^{t+\Delta} \lambda_s ds\right]\left(1 + i_{t+\Delta}\right)\right) \tag{29}$$
$$\times \left(D_t + P_t \tilde{D}_t\right) + q_{t+\Delta} X_{t+\Delta} \leq q_t X_t,$$

where recall that $X_t \equiv D_t + M_t + P_t \tilde{D}_t$. Since

$$E_t\left[q_t 1\{t\} - q_{t+\Delta} 1\{t+\Delta\} \exp\left[\int_t^{t+\Delta} \lambda_s ds\right]\left(1 + i_{t+\Delta}\right)\right] = 0, \tag{30}$$

it follows that:

$$\mathrm{E}_0\left[\sum_{j=0}^{n}\left(q_{j\Delta} P_{j\Delta} g_{j\Delta} \Delta + q_{j\Delta}\left[\frac{q_{j\Delta} - q_{(j+1)\Delta}}{q_{j\Delta}\Delta}\right] M_{j\Delta}\Delta\right)\right] \leq X_0 - \mathrm{E}_0\left(X_{(n+1)\Delta} q_{(n+1)\Delta}\right). \tag{31}$$

With strictly positive marginal utility, a necessary condition for consumers to be in equilibrium is the transversality condition:

$$\lim_{n\to\infty} \mathrm{E}_0\left(X_{n\Delta} q_{n\Delta}\right) = 0. \tag{32}$$

If this did not hold, consumers could always raise their utility by increasing consumption. It follows that

$$\mathrm{E}_0\left[\sum_{j=0}^{\infty}\left(q_{j\Delta} P_{j\Delta} g_{j\Delta} \Delta + q_{j\Delta}\left[\frac{q_{j\Delta} - q_{(j+1)\Delta}}{\Delta q_{j\Delta}}\right] M_{j\Delta}\Delta\right)\right] = X_0. \tag{33}$$

Again, we wish to take a continuous time limit since this will permit us to obtain tractable solutions. So, consider what happens as $\Delta \downarrow 0$. Since

$$\lim_{\Delta\downarrow 0} \frac{q_{t+\Delta} - q_t}{\Delta q_t} = \lim_{\Delta\downarrow 0} \frac{1}{\Delta}\left(\frac{1}{1 + i_{t+\Delta}} - 1\right) = \frac{d}{d\Delta} \exp\left[-\int_t^{t+\Delta}\left[r + \left(dP_s/dt\right)/P_s\right] ds\right] \tag{34}$$
$$= -\left[r + \left(dP_t/dt\right)/P_t\right],$$

it follows that in the limit the government solvency constraint becomes equation (7) from the main text.

Proof of Equation (13)

Suppose that velocity V is large so that equation (6) simplifies and the behavior of prices prior to default is determined by

$$\frac{dP_t}{dt} = \left[P_t \frac{g_t}{W_t} + \mu_w - r - \lambda_t\right] P_t. \tag{35}$$

The right-hand side of equation (35) is quadratic in P_t. To eliminate the quadratic term, apply the Ricatti transformation, defining a new variable $\tilde{P}_t$ implicitly by the equation:

$$P_t = \frac{-\left(d\tilde{P}/dt\right)\xi_1}{\left(g_0/w_0\right)\tilde{P} \exp\left[\left(\mu_g - \mu_w\right)t\right]}. \tag{36}$$

Substituting in equation (35), one then obtains

$$P_t = \frac{-\left(d^2\tilde{P}/dt^2\right)}{\tilde{P}_t\left(g_0/W_0\right)\exp\left[\left(\mu_g-\mu_w\right)t\right]} + \left(\frac{\left(d\tilde{P}/dt\right)}{\tilde{P}}\right)^2 \frac{1}{\left(g_0/W_0\right)\exp\left[\left(\mu_g-\mu_w\right)t\right]} \tag{37}$$

$$+\frac{\left(d\tilde{P}/dt\right)}{\tilde{P}_t}\frac{\left(\mu_g-\mu_w\right)}{\left(g_0/W_0\right)\exp\left[\left(\mu_g-\mu_w\right)t\right]} - \frac{\left(r-\lambda_t-\mu_w\right)d\tilde{P}/dt}{\left(g_0/W_0\right)\tilde{P}_t\exp\left[\left(\mu_g-\mu_w\right)t\right]}$$

$$-\frac{g_0}{W_0}\exp\left[\left(\mu_g-\mu_w\right)t\right]\left(\frac{\left(d\tilde{P}/dt\right)}{\tilde{P}}\right)^2 \frac{1}{\left[\left(g_0/W_0\right)\exp\left[\left(\mu_g-\mu_w\right)t\right]\right]^2} = 0.$$

Cancelling terms, one obtains the relatively simple linear equation:

$$\frac{d^2\tilde{P}_t}{dt^2} + \frac{d\tilde{P}_t}{dt}\left(r+\lambda_t-\mu_g\right) = 0. \tag{38}$$

Different assumptions about the time paths of the rate of default $\lambda_t = \lambda(t)$ then yield different solutions for the price level. There are two cases which we examine below, namely (i) constant crisis rate, $\lambda_t = \bar{\lambda}$, and (ii) increasing crisis rate, $\lambda_t = \bar{\lambda}t$.

Now, suppose that the crisis rate is constant (i.e., $\lambda_t = \bar{\lambda}$), henceforth referred to as the standard case. Then the differential equation satisfied by prices has the well-known solution

$$\tilde{P}_t = A_1\exp\left[\eta_1 t\right] + A_2\exp\left[\eta_2 t\right], \tag{39}$$

where η_i, $i = 1, 2$ are the roots of

$$\eta^2 + \eta\left[r+\bar{\lambda}-\mu_g\right] = 0, \tag{40}$$

$$\text{i.e.,}\quad \eta_1 = 0 \quad\text{and}\quad \eta_2 = -\left(r+\bar{\lambda}-\mu_g\right). \tag{41}$$

Substituting back yields equation (11) given in the text where $A \equiv A_1/A_2$ (assume $A_2 \neq 0$) To pin down the free constant, A, consider the private sector budget constraint, (12). Integrating in equation (12) and substituting for P_0, we obtain:

$$A = \frac{\eta_2}{r+\bar{\lambda}-\mu_g} - 1 = 0. \tag{42}$$

Hence, the price level is simply equation (13) as it appears in the text.

Proof of Equation (16)

If $\lambda_t = \bar{\lambda}t$, then the differential equation followed by the transformed price function $\tilde{P}_t$ is

$$\frac{d^2\tilde{P}_t}{dt^2} + \left[r-\mu_g+\bar{\lambda}t\right]\frac{d\tilde{P}_t}{dt} = 0 \tag{43}$$

Dividing through by $d\tilde{P}/dt$ and integrating, one obtains:

$$\frac{d}{dt}\log\left[\frac{d\tilde{P}_t}{dt}\right] = -\left[r-\mu_g+\bar{\lambda}t\right] \tag{44}$$

$$\frac{d\tilde{P}_t}{dt} = A_1 \exp\left[-\left(r-\mu_g\right)t - \frac{\bar{\lambda}t^2}{2}\right]. \tag{45}$$

Integrating a second time yields:

$$\tilde{P}_t = A_1 \int_0^t \exp\left[-\left(r-\mu_g\right)t - \frac{\bar{\lambda}t^2}{2}\right]ds + A_2. \tag{46}$$

Reversing the Ricatti transformation gives equation (15) from the main text where $A \equiv -A_2/A_1$. To determine the free parameter A, use the solvency condition to obtain:

$$A = \int_0^\infty \exp\left[-\left(r-\mu_g\right)s - \frac{\bar{\lambda}s^2}{2}\right]ds. \tag{47}$$

Rearranging yields equation (16) from the text.

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