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Optimized Reserve Holdings and Country Portfolios

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Abstract

In this paper, we investigate the impact of the emerging economies, financially less developed, on the allocation of internationally traded assets in financially developed economies. Departing from the standard small open economy assumption, we extend the Devereux and Sutherland (2006) algorithm for solving for the optimal steady state portfolio to a setting with multiple countries, with possibly different potfolio objectives. In a calibrated three-country economy, we discuss the impact of the trade structure and varying portfolio allocation strategies in financially less developed economies on the allocation of internationally traded assets in the rest of the world. Our results highlight that changes in reserves allocation strategies in emerging economies can have substantial effects on the composition of international investment positions in industrial economies.

1 Introduction

In the last two decades, emerging economies have increased remarkably their importance for the global economy. The share of emerging markets in the value of world exports in goods and services have increased from 19 percent in 1990 to over 33 percent in 2007[1]. While the rising importance of emerging economies in goods market trade has been remarkable, these countries participation in global financial markets is still limited. This results in international portfolio investment strategies that do not resemble those usually adopted in industrialized economies. Domestic savings, for example, are sometimes channeled through the government and often passively invested in international reserve holdings. The rise in aggregate savings in emerging economies in Asia, for example, has been mainly reflected in an increase in capital inflows into US dollar-denominated fixed income securities.

There are several interpretations in the literature about the drivers of the increase of international reserves in emerging economies. Aizenman (2007) argues that the rapid hoarding of reserves in the aftermath of the East Asian crisis has been dominated by self insurance against exposure to foreign shocks. Jeanne and Ranciere (2006) characterize the optimal level of reserves for emerging market countries seeking insurance against sudden stops in capital inflows. Aizenman and Lee (2007) discuss the possibility that the increase

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[1]WEO definition of emerging markets and developing economies; values in current USD.

in international reserves is driven either by precautionary or export-promotion motives. Another stream of the literature investigates the effects of the currency denomination of international reserves and the role of the reference currency — e.g., Papaioannu, Portes and Siourounis (2006).

Jeanne (2007) presents a calibrated, welfare-based model in which the optimal level of reserves is chosen to insure against the risk of a "sudden stop" in capital inflows. The results indicate that the insurance model can only account for the observed reserves accumulation in emerging Asia, if the expected cost of a capital account crisis is unrealistically high. The conclusion that most of the current buildup of reserves is not justified by precautionary reasons has some implications for reserve management in these economies. In particular, there is little reason for emerging economies to invest excess reserves in liquid, but low yielding foreign assets in which central bank tend to invest. Rather reserves should be viewed as a component of domestic wealth that is managed by the public sector on behalf of the domestic citizenry, taking full advantage of the portfolio diversification opportunities available abroad. This is a trend that might take on considerable importance looking forward.

Indeed, an increasing number of emerging market countries are transferring a fraction of their reserves to sovereign wealth funds (SWFs), mandated to improve the allocation of foreign reserves along the return-risk profile in contrast to traditional management of foreign exchange reserves. A first group of countries that has established SWFs consists of resource-rich economies, which have recently benefitted from an upward trend in oil and commodity prices.[2] A second group of countries, most notably in Asia, has established SWFs to allocate more profitably reserves in excess of what is needed for forex intervention or balance-of-payment stabilization. The source of reserve accumulation for these countries, rather than to large revenues from commodities, is often related to structural distortions in domestic savings and inflexible exchange rate regimes. As these authorities have become more comfortable with reserve levels, foreign assets have been moved to specialized agencies, which often have explicit return objectives and may invest in more risky assets than central banks.[3] Naturally, key issues are how the described developments will affect not only gross and net international capital flows and external positions between industrialized economies and emerging markets, but also *among industrialized economies.*

These developments have led to an interest in understanding the effects of emerging markets on *global capital flows*, and their corresponding macroeconomic implications. For example, in a recent influential paper Caballero, Farhi and Gourinchas (2007) argue that institutional constraints in emerging Asia in its ability to generate financial assets from real investment might explain the recent decline in US long term real interest rates, and the growing share of US assets in the global portfolio. Dooley, Folkerts-Landau and Garber (2003) argue that the emergence of China as a larger trading nation and its choice of a fixed exchange rate regime vis-a-vis the dollar, driven by the necessity to absorb excess labor supply, has triggered a revived Bretton Wood System

[2] Prominent examples of such SWFs include Norway's Government Pension Fund, investment agencies set up by GCC countries such as the Abu Dhabi Investment Authority (ADIA) which manages the foreign assets of the Emirate of Abu Dhabi (UAE) and the Russian oil stabilization fund which will be partly transformed into a fund for future generations from 2008 onwards.

[3] Prominent examples include funds that have been operating for decades, as e.g. the Singapore Government Investment Company (GIC), but also more recently established funds such as the Korea Investment Corporation (KIC), and the investment portfolio of the Exchange Fund managed by the Hong Kong Monetary Authority. Recently, the Chinese authorities announced the establishment of a new investment agency responsible for the management of a portion of Chinese foreign reserves.

In light of these discussions, this paper looks at the impact of emerging market portfolio allocation strategies on capital flows among industrialized economies. In particular, we explore how the trade structure, the presence of savings distortions and the portfolio allocation strategy in emerging economies might have an impact on the pattern of foreign assets positions in mature economies. For addressing these issues and in light of the systemic importance of emerging economies, we depart from the usual small open economy assumption and adopt a multi-country approach, in which all asset prices and returns are endogenously determined as functions of fundamentals.

Until recently, however, standard open-economy models with incomplete financial markets have been unable to provide an appropriate framework to analyze the implications of emerging markets for global gross asset positions, as they only captured net capital flows. In particular, differences in risk characteristics of assets, as well as different portfolio allocation strategies could not be studied as determinants of capital allocation.

The difficulty in calculating the optimal portfolio with incomplete markets in standard DSGE models were so far of technical nature. Until recently, standard numerical methods could not be applied because portfolio choice is not well-defined in a certainty equivalence setting, as it would depend on higher order moments, like the variance and covariance of asset returns. An alternative, but more restrictive approach in the literature, going back to Lucas (1982), has been to assume complete markets and full risk-sharing between economies and then characterize the portfolio allocation in a decentralized equilibrium with a given set of assets; recent contributions are Engel and Matsumoto (2005) Heathcote and Perri (2005) and Kollmann (2006).

There is, however, a recent literature developing simple methods, applicable using standard solution techniques, to analyze portfolio allocation in dynamics general equilibrium models, on which we build in this paper. This growing literature includes key contributions by Devereux and Sutherland (2006, 2007), Evans and Hnatkovska (2006), and Van Wincoop and Tille (2007).

The contribution of our paper is twofold. First, we show how to extend the Devereux and Sutherland (2006) methods for solving for optimal steady state portfolios in a setting with more than two agents/countries, allowing thereby for certain countries to follow diverse portfolio allocation strategies. Second, we analyze a calibrated three-country open-economy model in which one country, while fully integrated in goods market trading with the rest of the world, faces constraints in its portfolio allocation strategy. In the other two countries comprising the world economy, households will instead optimally trade in a given set of international assets, including bonds and equities. In this set up, we evaluate the sensitivity of international capital flows to the trade structure and the stochastic environment.

We consider several arrangements in which the country characterized by suboptimal saving decisions opens up to partially constrained international financial flows. The source of savings distortions, thereby, is modeled as a suboptimal consumption rule in the spirit of Caballero, Farhi and Gourinchas (2007).[4] In this set up, we will consider the following 3 scenarios : (i) all government asset holdings are mechanically invested in short-term nominal bonds of just one country (resembling holdings of dollar reserves); asset holdings are split, according to mean-variance considerations, (ii) between nominal bonds of the other two countries; and (iii) among all foreign securities including equities. As a useful benchmark,

[4] In the conference version of the paper, we also considered the case in which distortionary taxes are levied on households by country 3 government, which invests in foreign securities all its revenues. Since results are broadly similar to those presented here arising under the saving distiortions implied by the consumption rule assumed below, in the current version of the paper we focus on this latter case only.

we will also look at the case in which households in the emerging economy could maximize a standard intertemporal utility index and freely trade all foreign assets, while still restricting their domestic assets not to be traded internationally.[5]

The remainder of the paper is organized as follows. In the following section, we describe the three-country open economy model that we utilize for our analysis. In Section 3, we discuss our multiple-agent extension of the Devereux-Sutherland (2006) algorithm. Section 4 and 5 present the model calibration and the results of our numerical analysis, while Section 5 concludes.

2 The Model

The world economy consists of three countries. All countries are completely specialized in one traded good, of which they receive an endowment each period. Agents in country 1 and 2 can freely trade domestic and foreign claims on a fraction of their endowment (equity) and borrow and lend in zero-net supply nominal bonds denominated in their respective currencies. In contrast, country 3 is subject to financial restrictions, as domestic financial instruments are assumed not to be traded internationally. Moreover, we will introduce different sources and degrees of financial restrictions below, concerning the type of internationally traded assets that residents can trade and the role of simple investment rules.

For the sake of simplicity, we describe the equilibrium conditions that apply to unrestricted economies 1 and 2, under the benchmark case of symmetry, only for country 1, the numeraire country, neglecting thereby the corresponding country index. Therefore, all real asset prices will be expressed in terms of country 1 consumption basket. Obviously, appropriate arbitrage conditions will assure that the law of one price holds across all internationally traded securities.

2.1 Asset Prices and Returns

In country 1 and 2, four type of financial assets are traded internationally: two nominal one-period bonds and two equities. Following the standard approach in international finance since Lucas (1982), from country 1 perspective, domestic and foreign equities represent claims on the endowments of goods $D_{1,t+1}$ and $D_{2,t+1}$, respectively — specifically, shares in the fixed, positive amount of 'trees' yielding them, for simplicity normalized to 1.

The unitary real payoff of a share of the home equity purchased in period $t-1$ is thus $\frac{P_{1,t}}{P_t} D_{1,t} + Z_{E,t}$, where $Z_{E,t}$ is the real price of country 1 equity, and $\dfrac{P_{1,t}}{P_t}$ is the relative price of good 1 in terms of country 1's CPI (P_t). As a result the ex-post gross rate of return on equity is defined as:

$$r_{1,t+1} = \frac{\frac{P_{1,t+1}}{P_{t+1}} D_{1,t+1} + Z^E_{1,t+1}}{Z^E_{1,t}}. \tag{1}$$

Nominal bonds represent a claim on a unit of currency of country 1 or 2 the next period and are assumed to be in zero aggregate net supply. Assuming that the real price of a claim

[5] In some experiments in the conference version we also assumed that the nominal exchange rate between country 3 and 1 is fixed, so that money supply will have to be adjusted accordingly. Since results are very similar to those with flexible exchange rates, to save on space we do not report them here.

to country 1 currency is denoted by $Z^B_{1,t}$, then the real ex-post return on a nominal bond purchased at time t is therefore:

$$r_{B,t+1} = \frac{1}{Z^B_{1,t} P_{t+1}} \tag{2}$$

Similar conditions hold for assets issued in country 2, with their prices and returns being denoted with $Z^E_{2,t}$ and $Z^B_{2,t}$, and $r_{2,t+1}$ and $r^*_{B,t+1}$, respectively.

2.2 Households

Representative households in country 1 maximize lifetime utility by choosing purchases of the consumption good, C_t given the following quite standard utility index:

$$E_t \left[\sum_{k=0}^{\infty} \beta^k \left(\frac{1}{1-\rho} C_{t+k}^{1-\rho} \right) \right],$$

where β is the discount factor, ρ denotes risk aversion. Aggregate consumption C_t is defined across all home and foreign goods, and its functional form is given by the following constant elasticity aggregator:

$$C_t = \left[(\mu_1)^{\frac{1}{\theta}} (C_{1,t})^{1-\frac{1}{\theta}} + (\mu_2)^{\frac{1}{\theta}} (C_{2,t})^{1-\frac{1}{\theta}} + (\mu_3)^{\frac{1}{\theta}} (C_{3,t})^{1-\frac{1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

where $C_{i,t}$ with $i = 1, 2, 3$ stands for consumption goods originating in the corresponding country i. The parameter θ stands for the elasticity of substitution between goods of different origin, while μ_i measures the importance of different goods in preferences of households, with $\sum \mu_i = 1$. The above aggregator could be easily generalized to the case of country and goods specific shares μ_i, and elasticities θ.

Defining the aggregate price index as the price of one unit of consumption:

$$P_t = \left[\mu_1 (P_{1,t})^{1-\theta} + \mu_2 (P_{2,t})^{1-\theta} + \mu_3 (P_{3,t})^{1-\theta} \right]^{\frac{1}{1-\theta}},$$

the corresponding demand for domestic and foreign goods can be written as follows:

$$\begin{aligned} C_{1,t} &= \mu_1 \left(\frac{P_{1,t}}{P_t} \right)^{-\theta} C_t, \\ C_{2,t} &= \mu_2 \left(\frac{P_{2t}}{P_t} \right)^{-\theta} C_t, \\ C_{3,t} &= \mu_3 \left(\frac{P_{3,t}}{P_t} \right)^{-\theta} C_t. \end{aligned}$$

Utility maximization is subject to following budget constraint in real terms:

$$C_t + NFA_t = \sum_{k=1}^{N} \alpha_{k,t-1} r_{k,t} + \frac{P_{1,t}}{P_t} (Y_{1,t} + D_{1,t}), \tag{3}$$

where N is the number of assets traded in the economy. Each period the representative household has to decide how to split its income between consumption and net savings in financial assets NFA_t. Its sources of income consist of the two stochastic endowments of good 1, $Y_{1,t}$ and $D_{1,t}$, and of the real returns on past net savings, $\sum \alpha_{k,t-1} r_{k,t}$.

The key distinction between the two endowments $Y_{1,t}$ and $D_{1,t}$ is that only claims to the latter could be traded on financial markets before uncertainty about it is resolved, as explained above, while the former could be traded only on spot goods markets after its amount is realized every period.

The variable $\alpha_{k,t-1}$represents the real gross holdings of asset k, positive or negative, brought into period t from the end of the period $t-1$, and $r_{k,t}$ is the realized real return on the same asset, as defined above. Note that by definition, we have:

$$NFA_{t-1} = \sum_{k=1}^{N} \alpha_{k,t-1}$$

indicating that the total period $t-1$ investment of assets must add up to end of period t net wealth.

We wrote the budget constraint (3) in terms of NFA_t, the net foreign asset position of the country, defined as:

$$\begin{aligned} NFA_t &= B_{2t} + s_{2t} Z^E_{2,t} - s^{**}_{1t} Z^E_{1,t} - B^{**}_{1t}, \\ &= B_{1t} + B_{2t} + s_{2t} Z^E_{2,t} + (s_{1t} - 1) Z^E_{1,t}, \end{aligned}$$

where B_{1t} and B_{2t} are country 1 real holdings of bonds denominated in currency 1 and 2; s_{1t} and s_{2t} are holdings of claims on $D_{1,t+1}$ and $D_{2,t+1}$. B^{**}_{1t} and s^{**}_{1t} are overall holdings by residents of rest of the world, always including country 2 agents and, depending on the trading arrangement on international financial markets, country 3 agents.

The second equality is obtained by using the market clearing conditions for country 1 assets, requiring that bonds be in zero net supply and the equity shares sum up to 1, namely:

$$\begin{aligned} B_{1t} + B^{**}_{1t} &= 0 \\ s^{**}_{1,t} + s_{1,t} &= 1. \end{aligned}$$

It is useful to show how to derive the above wealth accumulation equation of country 1 in terms of the net foreign asset position NFA_t from the more standard period by period budget constraint. The latter would be as follows:

$$\begin{aligned} Z^E_{1,t} s_{1,t} + Z^E_{2,t} s_{2,t} + B_{1t} + B_{2t} &= \left(Z^E_{1,t} + \frac{P_{1,t}}{P_t} D_{1,t} \right) s_{1,t-1} + \left(Z^E_{2,t} + D_{2,t} \right) s_{2,t-1} \\ &\quad + r_{B,t} B_{1t-1} + r^*_{B,t} B_{2t-1} + \frac{P_{1,t}}{P_t} Y_{1,t} - C_t, \end{aligned}$$

or, by using the above definition of equity return (1):

$$\begin{aligned} Z^E_{1,t} s_{1,t} + Z^E_{2,t} s_{2,t} + B_{1t} + B_{2t} &= r_{1,t} \left(Z^E_{1,t-1} s_{1,t-1} \right) + r_{2,t} \left(Z^E_{2,t} s_{2,t-1} \right) \\ &\quad + r_{B,t} B_{1t-1} + r^*_{B,t} B_{2t-1} + \frac{P_{1,t}}{P_t} Y_{1,t} - C_t. \end{aligned}$$

It is then easy to rewrite the last expression in terms of NFA_t by adding and subtracting $\frac{P_{1,t}}{P_t} D_{1,t}$ from its right-hand side, and subtracting from both sides $Z^E_{1,t}$, since:

$$r_{1,t}\left(Z^{E}_{1,t-1}s^{1}_{1,t-1}\right)-Z^{E}_{1,t}=Z^{E}_{1,t-1}\frac{\left(Z^{E}_{1,t}+\frac{P_{1,t}}{P_t}D_{1,t}\right)}{Z^{E}_{1,t-1}}\left(s^{1}_{1,t-1}-1\right)+\frac{P_{1,t}}{P_t}D_t.$$

Now adding and subtracting $r_{B,t}NFA_{t-1}$ we get:

$$\begin{aligned} NFA_t &= \left(r_{1,t}-r_{B,t}\right)\alpha_{1,t-1}+\left(r_{2,t}-r_{B,t}\right)\alpha_{2,t-1}+\left(r^{*}_{B,t}-r_{B,t}\right)B_{2,t-1} \\ &\quad +r_{B,t}NFA_{t-1}+\frac{P_{1,t}}{P_t}\left(Y_t+D_t\right)-C_t \end{aligned}$$

So the corresponding gross asset positions for country 1 are defined as follows:

$$\alpha_{1,t-1} = Z^{E}_{1,t-1}\left(s^{1}_{1,t-1}-1\right) \tag{4}$$

$$\alpha_{2,t-1} = Z^{E}_{2,t-1}s_{2,t-1} \tag{5}$$

$$\alpha_{3,t-1} = B_{2,t-1}, \tag{6}$$

namely gross holdings of country 1 equities by the rest of the world, gross holdings of country 2 equities by country 1 residents, and gross holdings of currency 2 bonds by country 1 residents. Notice that, by construction, $B_{1t-1}=NFA_{t-1}-\sum_{k=1}^{N}\alpha_{k,t-1}$.

As a result, optimal savings and portfolio decisions can be characterized by the following standard first order conditions:

$$C_t^{-\rho}=\beta\,E_t\left[C_{t+1}^{-\rho}\,r_{N,t+1}\right], \tag{7}$$

$$E_t\left[C_{t+1}^{-\rho}\left(r_{k,t+1}-r_{N,t+1}\right)\right]=0 \tag{8}$$

where $k=1...N-1$.

2.3 The financially constrained economy

We consider several arrangements in which country 3 opens up to partially constrained international financial flows, but both saving and portfolio investment decisions are suboptimal. Precisely, we consider an economy in which saving distortions are modeled in the spirit of Caballero, Farhi and Gourinchas (2007), by assuming that due to domestic financial frictions country 3 households consumption and saving decisions are characterized by the following simple consumption rule:

$$RER^{*}_t C^{*}_t=\gamma\left(NFA^{*}_{t-1}+Z^{E}_{3,t}\right) \tag{9}$$

where C^{*}_t is aggregate consumption in country 3, RER^{*}_t is the bilateral real exchange rate between country 1 and 3, defined as the relative price of C^{*}_t in terms of country 1 consumption C_t, while $Z^{E}_{3,t}$ is the price of equity in country 3, and $\gamma=\frac{1-\beta}{\beta}\frac{D^{SS}}{Y^{SS}+D^{SS}}$.[6] Notice that by assumption only residents in country 3 can own domestic equities. The price

[6] This is necessary to ensure a well-defined steady state.

of this equity will be determined by assuming that its expected return (including dividends $D_{3,t}$) has to equal that on the portfolio of international securities traded by the country, R_t^*.[7]

Net wealth accumulation in country 3 will thus obey:

$$RER_t^* C_t^* + NFA_t^* = R_t^* NFA_{t-1}^* - \frac{P_{3,t}}{P_t} \left(Y_{3,t} + D_{3,t} \right), \tag{10}$$

where R_t^* is the gross return on net foreign assets, which will depend on the portfolio choice in country 3. Thus, as in Caballero et al. (2007), a shock that will increase non-dividend output relative to total output will trigger an increase in savings, and thus a current account surplus. Although in an admittedly stylized way, this situation is meant to capture the reserve accumulation strategy of some emerging economies, in which domestic saving surpluses arising from distorted decisions are channelled into holdings of international assets by local governments.

Concerning the portfolio choice of foreign securities in country 3, we consider the following three scenarios: (i) all asset holdings are mechanically invested in country 1 nominal bonds, namely $R_t^* = r_{B,t+1}$; asset holdings are split, according to mean-variance principles (ii) between country 1 and 2 bonds; and (iii) among all foreign securities including equities. The first scenario represents passive strategies of reserve management, while the second and the third capture in a stylized way a movement towards more active investment strategies. In modeling the latter, we broadly follow the insights from the literature on optimal reserve management — e.g. Jeanne and Ranciere (2006) and Papaioannou et al. (2006) — and assume that the government allocates its portfolio of securities by minimizing its variance.

As useful benchmarks, however, we consider also the two polar cases of financial integration, financial autarky and full integration. The most extreme form of restriction, financial autarky, implies that the period-by-period budget constraint in terms of country 1 real currency becomes:

$$RER_t^* C_t^* = \frac{P_{3,t}}{P_t} \left(Y_{3,t} + D_{3,t} \right).$$

In the case of full financial integration, rather than follow the consumption rule (9) above, country 3 households maximize an intertemporal utility index similar to that of the other countries, but are restricted to hold all their domestic assets, while being able to freely trade all country 1 and 2 securities, including issuing foreign debt in the form of bonds denominated in either foreign currency.

2.4 Exogenous Processes

We assume that endowment components Y_{it} and D_{it} follow AR(1) processes with country specific parameters and innovations correlated across shocks and countries:

$$\log Y_{it} = \phi_{Yi} \log Y_{it-1} + \varepsilon_{Y_i,t}, \tag{11}$$

$$\log D_{it} = \phi_{Di} \log D_{it-1} + \varepsilon_{D_i,t}, \tag{12}$$

[7] In the conference version of the paper, we also considered the case in which country 3 government invests in foreign securities all its revenues generated by distortionary taxes levied on households consumption. Since results where bradly similar to those arising under the saving distiortions implied by the consumption rule (9), in the current version of the paper we focus on the latter case only.

where ϕ_Y and ϕ_D are the persistence parameters, and $\varepsilon_{Y,t}$ and $\varepsilon_{D,t}$ are country specific iid innovations. In all countries, the monetary authority is assumed to follow an exogenous money supply rule

$$\log M_{it} = \phi_{Mi} \log M_{it-1} + \varepsilon_{M_i,t},$$

where the term $\varepsilon_{M,t}$ represents a country specific iid innovation. The variance-covariance matrix of all the shocks will be denoted as Σ.

Finally, as in Devereux and Sutherland (2007) we assume that the following quantity equation pins down the aggregate price level:

$$M_{i,t} = P_{i,t}\left(Y_{i,t} + D_{it-1}\right)$$

The model is closed by defining resource constraints for the demand of individual goods.[8]

3 Solving for steady-state portfolios

In this section we describe our solution procedure which extends the methods in Devereux and Sutherland (2006) to an economy with more than 2 agents, potentially solving different maximization problems. We first outline the procedure for the 2-country case, and then proceed to extend it to the 3-country case, where country 3 portfolio choice follows from utility-maximization or other criteria like portfolio variance minimization.[9]

3.1 The 2-country case

To apply the procedure we first need to rewrite the equations governing the evolution of net wealth in terms of excess returns with respect to country 1 nominal bond. As shown above, for country 1 this equation is as follows:

$$NFA_t = \sum_{k=1}^{N-1} \alpha_{k,t-1}\left(r_{k,t} - r_{B,t}\right) + \frac{P_{1,t}}{P_t}\left(Y_{1,t} + D_{1,t}\right) + r_{B,t} NFA_{t-1} - C_t.$$

Under some general conditions, Devereux and Sutherland (2006) show that since expected excess returns are equal to zero up to first order, the term $\sum_{k=1}^{N-1} \alpha_{k,t-1}\left(r_{k,t} - r_{B,t}\right)$ will only be a function of the unexpected shocks in the approximate solution around the steady state — in the case of our model economy, the vector of innovations of exogenous processes ε. Moreover, they show that in the case of 2 countries the (near-stochastic) steady-state optimal portfolio, $\overline{\alpha}$, will be implicitly defined by the following moment conditions obtained

[8] In some experiments we also assumed that the nominal exchange rate between country 3 and 1 is fixed, so that money supply will have to be adjusted accordingly, but results are broadly similar to those with flexible exchange rates.

[9] We do not study how portfolio allocations evolve over time in this version of the paper, for the following two reasons. First, the model does not have any endogenous sources of dynamics; second, as is well-known, it displays a unit root in the repsonse of the wealth distribution to shocks. However, it would be straightforward to apply the methods developed by Dedola and Lombardo (2008), extending the 2 agents-2 assets solution in Devereux and Sutherland (2007) to the case of multiple assets and multiple agents, while taking care of the nonstationarity by assuming a different class of preferences.

by taking a second order approximation of the portfolio first order conditions around a nonstochastic steady state:

$$E_{t-1}\left[\left(\widehat{C}_t - \widehat{C}_t^{**} - \frac{1}{\rho}\widehat{RER}_t\right)\widehat{Rx}_{k,t}\right] = 0, \tag{13}$$

where $\widehat{Rx}_{k,t} = \widehat{r_{k,t}} - \widehat{r_{B,t}}$, $k = 1...N-1$, $\widehat{C}_t^{**}$ denotes consumption in country 2. Under the assumption of homoschedastic shocks, the above conditions will hold for any period.

The term $\sum_{k=1}^{N-1} \alpha_{k,t-1}\left(r_{k,t} - r_{B,t}\right)$ in the budget constraint depicting the wealth effect arising from realized excess returns on foreign assets and liabilities, can thus be replaced with the auxiliary, iid variable ξ_t. Therefore, a solution for the approximated equilibrium around the nonstochastic steady state will yield policy rules for the vector of excess returns $\widehat{R}_{x,t}$ and for $\Delta_t = \left(\widehat{C}_t - \widehat{C}_t^{**} - \frac{1}{\rho}\widehat{RER}_t\right)$ which will be functions of innovations ε_t, but also of the ξ_t.[10]

Since up to first order $\xi_t = \overline{\alpha}'\widehat{Rx}_t$, the auxiliary variable could be substituted out yielding expressions in terms of fundamentals innovations ε_t for Δ_t and $\widehat{R}_{x,t}$. Formally,

$$\begin{aligned}
\widehat{R}_{x,t} &= R_1\xi_t + R_2\varepsilon_t \\
\Delta_t &= d_1\xi_t + D_2'\varepsilon_t,
\end{aligned}$$

so that substituting out ξ_t yields

$$\begin{aligned}
\widehat{R}_{x,t} &= R_1\overline{\alpha}'\widehat{R}_{x,t} + R_2\varepsilon_t = \widetilde{R}\varepsilon_t \\
\Delta_t &= d_1\overline{\alpha}'\widehat{R}_{x,t} + D_2'\varepsilon_t = \widetilde{D}\varepsilon_t,
\end{aligned}$$

where:

$$\begin{aligned}
\widetilde{R} &= \left(I - R_1\overline{\alpha}'\right)^{-1} R_2 \\
\widetilde{D} &= d_1\overline{\alpha}'\left(I - R_1\overline{\alpha}'\right)^{-1} R_2 + D_2'
\end{aligned}$$

In the case of two agents, k excess returns and e fundamentals shocks with variance-covariance matrix Σ, the time-invaryING portfolio moment conditions (13) will amount to the following matrix equation:

$$\begin{aligned}
\underbrace{0}_{kx1} &= \underbrace{\widetilde{R}\Sigma\widetilde{D}'}_{(kxe)(exe)(ex1)} \\
0 &= \left(I - R_1\overline{\alpha}'\right)^{-1} R_2\Sigma\left[d_1\overline{\alpha}'\left(I - R_1\overline{\alpha}'\right)^{-1} R_2 + D_2'\right]',
\end{aligned}$$

defining the steady-state unknown elements of the vector $\overline{\alpha}$, representing the gross holdings of foreign assets and liabilities for country 1, excluding the reference asset. The latter's position will be derived from the assumed level of steady state net foreign assets — we will assume throughout that this is zero for all countries.

[10] The real marginal utility differential Δ will also be a function of state variables, like the wealth distribution.

The above formula yields the following closed form solution for $\overline{\alpha}$:

$$\overline{\alpha}' = -\frac{(D_2'\Sigma R_2')(R_2\Sigma R_2')^{-1}}{\left(d_1 - (D_2'\Sigma R_2')(R_2\Sigma R_2')^{-1} R_1\right)}, \tag{14}$$

which can be shown to be equal to the expression derived by Devereux and Sutherland (2006).[11]

3.2 The 3 country case

In the case of more than two agents, to take into account the effects of asset returns on the wealth distribution across agents, we will have to first keep track of the holdings of n-1 agents, and include the relevant moment conditions from their portfolio optimization problems. In our 3-country economy, in addition to country 1, it will be enough to characterize foreign asset holdings by country 3, replacing the term $\sum_{k=1}^{N-1} \alpha^*_{k,t-1}(r_{k,t} - r_{B,t})$ in the corresponding budget constraint with the auxiliary, iid variable ξ_t^*, and taking into account the effects of both ξ_t and ξ_t^* on country 2 wealth. By Walras's law, it must be the case that net foreign assets in country 2 will be equal to the negative of the sum of net foreign assets in country 1 and 3, NFA_t and NFA_t^*. Therefore, the negative of $(\xi_t + \xi_t^*)$ will capture the effects of the steady state gross holdings on country 2 net wealth.

Secondly, with a first order approximation of the model in hand, we will solve for excess returns $\widehat{R}_{x,t}$, Δ_t, defined as above, and its counterpart Δ_t^* for country 3, depending on the specific portfolio problem solved by this country, as a function of fundamentals innovations ε and of ξ_t and ξ_t^*. Since as before

$$\begin{bmatrix} \xi_t \\ \xi_t^* \end{bmatrix} = \overline{\alpha}'\widehat{Rx}_t,$$

the moments conditions from the specific portfolio optimization problem, will be a function of the optimal steady state portfolio matrix $\overline{\alpha}$.

[11] The moment conditions can be written also as:

$$\begin{aligned} R_2\Sigma D_2 + R_2\Sigma R_2'\left(\left(I - R_1\overline{\alpha}'\right)^{-1}\right)'\overline{\alpha}d_1 &= 0 \\ \left(\left(I - R_1\overline{\alpha}'\right)'\right)^{-1}\overline{\alpha}d_1 &= -\left(R_2\Sigma R_2'\right)^{-1}\left(R_2\Sigma D_2\right); \end{aligned}$$

because of the following equality,

$$\left(\left(I - R_1\overline{\alpha}'\right)^{-1}\right)'\overline{\alpha} = \overline{\alpha}\left(\left(I - \overline{\alpha}'R_1\right)^{-1}\right)',$$

and noticing that $\overline{\alpha}'R_1$ is actually a scalar in this case, the above moment conditions simplifies to

$$\overline{\alpha}d_1 = -\left(R_2\Sigma R_2'\right)^{-1}\left(R_2\Sigma D_2\right)\left(1 - R_1'\overline{\alpha}\right),$$

from which it is easy to derive the solution for $\overline{\alpha}$ provided by Devereux and Sutherland (2006):

$$\overline{\alpha} = \left(R_2\Sigma D_2 R_1' - d_1 R_2\Sigma R_2'\right)^{-1}\left(R_2\Sigma D_2\right).$$

Utility maximization In the case of full utility based portfolio optimization by country 3, it is possible to obtain a closed form solution for $\overline{\alpha}$. The portfolio optimality conditions will be the counterpart of (13)

$$E_{t-1}\left[\left(\widehat{C}_t - \widehat{C}_t^* - \frac{1}{\rho}\widehat{RER}_t^*\right)\widehat{Rx}_{k,t}\right] = 0,$$

implying that $\Delta_t^* = \left(\widehat{C}_{1,t} - \widehat{C}_t^* - \frac{1}{\rho}\widehat{RER}_t^*\right)$. Up to first order, the relevant solutions for $\widehat{R}_{x,t}$, Δ_t, and Δ_t^* can be expressed as follows:

$$\begin{aligned}
\widehat{R}_{x,t} &= R_1\begin{bmatrix}\xi_t\\ \xi_t^*\end{bmatrix} + R_2\varepsilon_t\\
\Delta_t &= D_1'\begin{bmatrix}\xi_t\\ \xi_t^*\end{bmatrix} + D_2'\varepsilon_t + ...,\\
\Delta_t^* &= D_1^{*\prime}\begin{bmatrix}\xi_t\\ \xi_t^*\end{bmatrix} + D_2^{*\prime}\varepsilon_t + ...
\end{aligned}$$

Substituting out ξ_t and ξ_t^* :

$$\begin{aligned}
\widehat{R}_{x,t} &= \widetilde{R}\varepsilon_t\\
\Delta_t &= \widetilde{D}\varepsilon_t\\
\Delta_t^* &= \widetilde{D}^*\varepsilon_t,
\end{aligned}$$

where:

$$\begin{aligned}
\widetilde{R} &= \left(I - R_1\overline{\alpha}'\right)^{-1}R_2\\
\widetilde{D} &= D_1'\overline{\alpha}'\left(I - R_1\overline{\alpha}'\right)^{-1}R_2 + D_2'\\
\widetilde{D}^* &= D_1^{*\prime}\overline{\alpha}'\left(I - R_1\overline{\alpha}'\right)^{-1}R_2 + D_2^{*\prime}.
\end{aligned}$$

The portfolio moment conditions for Δ_t (and likewise for Δ_t^*) can thus be written as

$$\begin{aligned}
\underbrace{0}_{1xk} &= \underbrace{\widetilde{D}'\Sigma\widetilde{R}'}_{(1xe)(exe)(exk)}\\
0 &= \left[\underbrace{D_1'}_{1x(n-1)}\underbrace{\overline{\alpha}'}_{(n-1)xk}\underbrace{\left(I - R_1\overline{\alpha}'\right)^{-1}}_{kxk}\underbrace{R_2}_{kxe} + \underbrace{D_2'}_{1xe}\right]'\Sigma R_2'\left(\left(I - R_1\overline{\alpha}'\right)^{-1}\right)',
\end{aligned}$$

where as indicated above, $\overline{\alpha}$ is now a $kx(n-1)$, matrix, where in the 3-country case $n-1 = 2$. Rearranging the latter expression yields:

$$\begin{aligned}
D_1'\overline{\alpha}' &= -\left(D_2'\Sigma R_2'\right)\left(R_2\Sigma R_2'\right)^{-1}\left(I - R_1\overline{\alpha}'\right)\\
\left(D_2'\Sigma R_2'\left(R_2\Sigma R_2'\right)^{-1}R_1 - D_1'\right)\overline{\alpha}' &= \left(D_2'\Sigma R_2'\right)\left(R_2\Sigma R_2'\right)^{-1},
\end{aligned}$$

or in more compact notation:

$$\underbrace{\left(D_2'\mathcal{R}R_1 - D_1'\right)}_{1x(n-1)}\underbrace{\overline{\alpha}'}_{(n-1)xk} = \underbrace{D_2'}_{1xe}\underbrace{\mathcal{R}}_{exk},,$$

where $\mathcal{R} = \Sigma R_2' \left(R_2 \Sigma R_2'\right)^{-1}$. Collecting all the conditions for Δ_t and Δ_t^* yields the following system:

$$\underbrace{\begin{bmatrix} (D_2'\mathcal{R}R_1 - D_1') \\ (D_2^{*\prime}\mathcal{R}R_1 - D_1^{*\prime}) \end{bmatrix}}_{2x2} \overline{\alpha}' = \underbrace{\begin{bmatrix} D_2' \\ D_2^{*\prime} \end{bmatrix}}_{2xe} \underbrace{\mathcal{R}}_{exk};$$

the latter can be readily solved in closed form for the steady state portfolio holdings if the right-hand side matrix matrix has a (generalized) inverse:[12]

$$\overline{\alpha}' = \begin{bmatrix} (D_2'\mathcal{R}R_1 - D_1') \\ (D_2^{*\prime}\mathcal{R}R_1 - D_1^{*\prime}) \end{bmatrix}^{-1} \begin{bmatrix} D_2' \\ D_2^{*\prime} \end{bmatrix} \mathcal{R}. \tag{15}$$

Optimized portfolios In the case of optimized portfolios, it will not generally possible to obtain closed form solutions for portfolio holdings. For concreteness, consider the case in which country 3 foreign reserves are invested in the two nominal bonds, by minimizing the variance of the return on this portfolio; the relevant budget constraint is thus:

$$NFA_t^* \equiv B_{1t}^* + B_{2t}^* = \frac{P_{3,t}}{P_t}\left(Y_{3,t} + D_{3,t}\right) + r_{B,t} NFA_{t-1}^* + \left(r_{B^*,t} - r_{B,t}\right) B_{2t-1}^* - RER_t^* C_t^*,$$

implying that $\left(r_{B^*,t} - r_{B,t}\right) B_{2t-1}^* = \xi_t^*$. The relevant moment conditions Δ_t^* obtained from minimization of the variance of the real dollar-portfolio are as follows:

$$E_{t-1}\left[\left(\widehat{r}_{B^*,t} - \widehat{r}_{B,t}\right)\left(\overline{\alpha}^*\left(\widehat{r}_{B^*,t} - \widehat{r}_{B,t}\right) + \widehat{r}_{B,t}\right)\right] = 0,$$

where $\overline{\alpha}^*$ is the steady state value of gross holdings of country 2 nominal bonds, B_2^*.

Ordering the vector of excess returns in such a way that $\left(\widehat{r}_{B^*,t} - \widehat{r}_{B,t}\right)$ is last, the system of moments conditions can thus be written as

$$\begin{aligned} \widetilde{R}\Sigma\widetilde{D}' &= \underbrace{0}_{kx1} \\ \begin{bmatrix} 0 \\ \ldots \\ 0 \\ 1 \end{bmatrix}' \widetilde{R}\Sigma \left(\widetilde{R}' \begin{bmatrix} 0 \\ \ldots \\ 0 \\ \overline{\alpha}^* \end{bmatrix} + \widetilde{\varrho} \right) &= \underbrace{0}_{1x1}, \end{aligned} \tag{16}$$

where $\widetilde{R}$ and $\widetilde{D}$ are the same as above in full optimization case, and $\widetilde{\varrho}$ is obtained from the first order approximation to $\widehat{r}_{B,t}$ as follows:

$$\begin{aligned} \widehat{r}_{B,t} &= \varrho_1' \begin{bmatrix} \xi_t \\ \xi_t^* \end{bmatrix} + \varrho_2' \varepsilon_t = \widetilde{\varrho}' \varepsilon_t, \\ \widetilde{\varrho} &= \varrho_1' \overline{\alpha}' \left(I - R_1 \overline{\alpha}'\right)^{-1} R_2 + \varrho_2'. \end{aligned}$$

[12] Dedola and Lombardo (2008) formally solve for the general case of k excess returns and n utility-maximizing agents, showing that the closed form solution for the optimal steady state portfolio matrix $\overline{\alpha}$ will be:

$$\overline{\alpha}' = \underbrace{\begin{bmatrix} \left(D_2^{1\prime}\mathcal{R}R_1 - D_1^{1\prime}\right) \\ \ldots \\ \left(D_2^{i\prime}\mathcal{R}R_1 - D_1^{i\prime}\right) \\ \ldots \\ \left(D_2^{n-1\prime}\mathcal{R}R_1 - D_1^{n-1\prime}\right) \end{bmatrix}^{-1}}_{(n-1)x(n-1)} \underbrace{\begin{bmatrix} D_2^{1\prime} \\ \ldots \\ D_2^{i\prime} \\ \ldots \\ D_2^{n-1\prime} \end{bmatrix}}_{(n-1)xe} \underbrace{\mathcal{R}}_{exk}.$$

The latter system will allow to compute the gross holdings of assets that fully characterize country portfolios, namely holdings by country 1 of equities, and holdings of nominal bonds in currency 2 by both country 1 and 3; however, unfortunately, it cannot be solved in closed form.

More generally, in the case of optimized portfolios in which holdings of $k^* + 1$ assets are chosen relative to the reference j-th return, we will have to solve for the following system of equations:

$$\begin{aligned} \widetilde{R}\Sigma\widetilde{D}' &= \underbrace{0}_{kx1} \\ \underbrace{\widetilde{R}^*}_{k^*xe}\Sigma\left[\widetilde{R}'\left[\begin{array}{c}\underbrace{0}_{(k-k^*)x1} \\ \underbrace{\overline{\alpha}^*}_{k^*x1}\end{array}\right] + \widetilde{\varrho}_j\right] &= \underbrace{0}_{k^*x1}, \end{aligned} \quad (17)$$

where $\widetilde{R}^*$will reflect the appropriate selection from $\widetilde{R}$ involving the relevant k^* excess returns.

4 Calibration

A crucial ingredient in the numerical exercises below is the dynamics of the exogenous processes, including the variance-covariance matrix of their innovations. In our model, the exogenous stochastic forces contain two endowments shocks denoted by $Y_{i,t}$ and $D_{i,t}$, and an exogenous process for money denoted by $M_{i,t}$. Note that D_t represents the fraction of endowment that is tradable in financial markets, so that Y_t can be interpreted as an exogenous process for non-financial income. Therefore, we identify the empirical counterparts to the model's exogenous variables D_i and Y_i with the real dividends, and the real compensation of employees, respectively; we use data on the broad aggregate M3 for M_i.

We start by computing the covariance structure of exogenous processes in a two-country model, representing the US and the euro area, by abstracting from the existence of a third (emerging) economy. We obtained the US data for the real compensation of employees and real dividend income for from the BLS, while M3 is from the IFS dataset of the IMF. For the euro area, we use the compensation of employees definition of the area wide dataset (see Fagan et al.(2001)), while M3 is obtained from the European Central Bank's data base. Dividends for the euro area are calculated using data from Datastream Global Equity index, by multiplying dividend yields by the market value of euro area stock markets. We took logs and linearly detrended all variables to obtain the empirical counterparts of Y_i and D_i, and M_i. To obtain an estimate of the variance-covariance matrix of the exogenous shocks, we estimated the system of 6 variables by OLS, for simplicity restricting the lag length to one and assuming a diagonal autoregressive matrix, as in the theoretical model. The sample contains quarterly data from 1980:Q1 to 2007:Q2.

In order to estimate processes for country 3 shocks , we use IFS data for China on profits, as dividend data are not available, real wages and M3. Unfortunately, Chinese data is only available annually from 1980 to 2006, making it impossible to directly estimate the correlation of the shocks at quarterly frequency as in the case of the US and the Euro area. We thus proceeded as follows. Using the annual counterparts of the euro area and US data discussed above, as well as Chinese data, we first estimated the variance-covariance matrix at annual frequency from a VAR(1) system of 9 variables by OLS. We then assumed that quarterly correlations of innovations are well approximated by annual correlations, that can obtained from the annual variance-covariance matrix. In order to identify quarterly variances of Chinese innovations, we assumed that their ratios relative to US variances (but not the variance itself) resemble those at the annual frequency. For instance, to obtain an estimate of the quarterly variances of shocks to country 3 non-financial income Y_3, we multiplied the quarterly variance of ε_{Y_1} obtained from the US-euro area system by the ratio of the annual variance of Chinese innovations to real wages to the annual variance of ε_{Y_1}, as obtained from the estimated VAR(1) system for US, China and the euro area.

The estimated variance-covariance matrix is given in Table 1 below, where the diagonal reports the relative variances of the shocks using ε_{Y_1} as the benchmark (the latter shock's standard deviation is equal to 0.91 percent), while off-diagonal elements indicate the correlations between the shocks — we will use this matrix for the calculation of the optimal portfolio holdings in the next section. Note that all shocks turn out to be substantially more volatile for country 3, reflecting the properties of Chinese data.

However, the estimated persistence of shocks in all countries turns out to be quite similar. The persistence of real compensation of employees for US, euro area and China are $\rho_{Y_1} = 0.92$, $\rho_{Y_2} = 0.91$, $\rho_{Y_3} = 0.92$, respectively, expressed on a quarterly frequency. Correspondingly, we estimated the autoregressive parameters for the dividend process being $\rho_{D_1} = 0.92$, $\rho_{D_2} = 0.90$, $\rho_{D_3} = 0.92$. Finally, the parameters for the money processes are $\rho_{M_1} = 0.92$, $\rho_{D_2} = 0.91$, and $\rho_{D_3} = 0.92$; the persistence of money shocks, however, is irrelevant for portfolio decision as they have only one period real effects in our model.

Table1: Estimated Covariance Matrix

	ε_{Y_1}	ε_{Y_2}	ε_{Y_3}	ε_{M_1}	ε_{M_2}	ε_{M_3}	ε_{D_1}	ε_{D_2}	ε_{D_3}
ε_{Y_1}	**1**	0.27	0.08	0.19	0.00	0.12	-0.02	0.08	-0.14
ε_{Y_2}	0.27	**0.47**	0.01	-0.01	0.23	-0.11	-0.01	0.08	-0.11
ε_{Y_3}	0.08	0.01	**2.08**	0.03	-0.06	0.07	-0.03	0.03	0.33
ε_{M_1}	0.19	-0.01	0.03	**12.14**	0.10	-0.29	-0.02	-0.10	-0.16
ε_{M_2}	0.00	0.23	-0.06	0.10	**13.50**	-0.22	-0.15	-0.09	-0.08
ε_{M_3}	0.12	-0.11	0.07	-0.29	-0.22	**43.83**	0.22	0.03	0.10
ε_{D1}	-0.02	-0.01	-0.03	-0.02	-0.14	0.22	**3.77**	0.05	0.23
ε_{D_2}	0.08	0.08	0.03	-0.10	-0.09	0.03	0.05	**0.05**	-0.03
ε_{D_3}	-0.14	-0.11	0.33	-0.16	-0.08	0.10	0.23	-0.03	**129.21**

To provide some empirical benchmarks for our numerical results, we also report a few stylized facts concerning country portfolios of G7 economies. As shown in column 1 of Table 2 below, the share of domestic holdings of domestic equities ranges from 90 percent in the US to 65.3 percent in the UK. Moreover, financial home bias, defined as the difference between the share of foreign equities in the domestic portfolio and the weight the foreign equity market has in the world market, is substantial, as shown in the last column of Table 2.

In our model, given the assumed symmetry between country 1 and 2, these measures would correspond to s_1 and $s_1 - 1/2$, respectively. In the next section, we will show that the simple calibrated model described in the previous section is able to broadly replicate some of the features of international portfolio holdings.

Table 2: Equity Holdings		
	Financial Domestic Share (in percent)	Financial Home bias (in percent)
United States	90.0	25.6
Japan	85.0	17.7
Germany	68.9	18.8
France	70.4	23.6
UK	65.3	20.7
Italy	77.2	19.7
Canada	79.8	19.2

5 Numerical results

In this section we report our main results. We first compute the optimal steady-state portfolio in a baseline two-country model with no restrictions on financial trading by domestic households. Then we investigate how exposing these economies to a third-country with distorted savings and different degrees of financial restrictions changes their portfolio choices. The scenario that we have in mind here intends to resemble, in an obviously simplified set-up, the situation the world economy has been facing in the recent years.

On the one hand, emerging Asia/ China, as well as oil-exporters are playing an increasing role as trade partners of industrialized economies such as the US and the euro area. On the other hand, these economies are still less integrated financially, and their international portfolio allocation policies are not entirely driven by the decision of freely optimizing households. Instead, national domestic savings are sometimes intermediated by governments and often invested following simple rule-of -thumb policies (like more or less mechanical accumulation of US Treasuries), or more sophisticated strategies of sovereign wealth funds, taking into account the risk and return trade-offs of the different assets (like mean-variance policies).

To put our the results in some perspective, especially those derived in the two country model, it is helpful to compare our results to those obtained by Lucas (1982), Baxter and Jerman (1997), Heathcote and Perri (2008), Engel and Matsumoto (2005) and Couerdacier et al. (2007 and 2008). In the two-country model by Lucas (1982), national representative agents have identical preferences across countries and are endowed only with stochastic financial income payments, similar to our D_i component (in the form of a tree yielding stochastic fruits). In such an environment, perfect risk-sharing can be obtained when agents of each economy are perfectly pooled, owning half of the claims to home and foreign endowments. Baxter and Jerman (1997) consider the role of non diversifiable labor income, similar to our Y_i component. In their set up, if assets returns and labor income are highly correlated, the optimal portfolio is then characterized by extreme foreign bias, being short in domestic assets.

Allowing for preference heterogeneity and imperfect substitutability between domestic and foreign traded goods in a production economy with capital accumulation, Heathcote and Perri (2008) show that a standard two-country/ two-good RBC model can generate substantial home bias in equity holdings, because relative returns to domestic stocks move inversely with relative labor income in response to productivity shocks. In a similar vein, Engel and Matsumoto (2005) argue that sticky prices are an additional feature that is able to generate a negative correlation between labor income and profits of domestic firms, also tilting portfolio shares in favor of home equities.

In a paper that is most closely related to ours, Coeurdacier, Kollmann and Martin (2007) show that in an endowment economy with multiple goods, substantial home bias in equities can be generated if assets traded in the model include not only equities but also real one-period bonds. In a second paper, Coeurdacier, Kollmann and Martin (2008) introduce shocks to investment demand and bond trading in the Heathcote and Perri (2008) model, with effectively complete asset markets, and are able to rationalize the observed pattern of international portfolios. In contrast to the former paper, our model assumes incomplete markets in an dynamic economy with nominal bonds, and, while also abstracting from the specification of the production function, provides an empirical calibration of the exogenous driving forces of our model.

5.1 Baseline two-country setting

We first report the composition of the optimal portfolio in a baseline set-up that assumes that the financially constrained economy is completely closed to intertemporal and intratemporal trade, and has therefore no effect at all on the country 1 and 2 consumption and portfolio decisions. We adopt the following symmetric parametrization. For the preference parameters, we assume the following values: the risk aversion parameter $\rho = 2.0$; the elasticity of substitution parameter $\theta = 3$; and the time preference rate $\beta = 0.98$. The share of financially nontradable income in the steady-state Y^{SS} is set at 85 percent , so that D^{SS} equals 15 percent of total income – a similar proportion holds in the US data between dividend and nonfinancial personal income. Furthermore, we assume that the degree of home bias in goods markets in country 1 is $\mu_1 = 0.8$, and is the same in country 2, so that imports are 20% of GDP on average — we also assume in the first step that NFA positions are zero in the steady-state; subsequently we will look at a situation in which country 1 has a negative long-run international investment position. Shock processes are set according to the two-country VAR estimates presented in the previous section, obtained with US and euro area data.

Table 3 presents our main findings, reporting gross asset positions as a ratio to steady state output across different experiments for the 2-country setting.

Table 3: Steady-State Portfolios in a Two-Country Model (measured in output units)

	Benchmark (1)	No correlation (2)	Standardized (3)	$\phi^{y,1} = 0.1$ (4)	$\mu = 0.02$ (5)	$\theta = 0.5$ (6)
$Z_1^E(s_1 - 1)$	-1.2161	-0.9421	0.5709	-2.2455	-3.4319	-4.1211
$Z_2^E s_2$	1.1166	0.8690	-0.5889	2.3844	3.5690	4.1301
B_2	-0.1555	0.0437	1.1023	0.0067	0.3824	0.0817

As indicated by results in column 1, the baseline 2-country economy generates a degree of equity home bias broadly in line with the data. As shown in Section 2, $Z_1^E(s_1 - 1)$ represents gross holdings of country 1 equities by country 2, where recall that s_1 stands for the share of country 1 equities held by country 1. Similarly, $Z_2^E s_2$ represents the gross holdings of foreign equities by country 1. Holdings of foreign equities amount then to over 100% of GDP in the steady state, a number slightly larger than in the data. In terms of shares, the benchmark parameterization implies that $s_1 = 0.8345$, and $s_2 = 0.1519$, and replicates reasonably closely the stylized facts on portfolio holdings in the data presented in Table 2.

The reason behind the large domestic home bias is as follows. Domestic households wish to insure against consumption risk by choosing assets that provide a good hedge against fluctuations in the bulk of their income, the nonfinancial component Y. In our set up, given the small estimated correlation between innovations to domestic Y and D, the main source of comovements between equity returns and shocks to Y is due to endogenous real exchange rate movements. Consider a negative shock to Y_1 leading to an appreciation of the real exchange rate in country 1; the latter makes claims on domestic equities more valuable compared to claims on foreign equity and a better hedge against fluctuations in non-financial endowment, thus accounting for the substantial of home bias in our results.

As shown in Table 3, the real holdings of domestic foreign bonds B_2 are negative, implying a (slightly larger) positive gross position in domestic bonds. This is driven by the procyclical reaction of prices following a rise in domestic endowments (implied by the positive correlation of shocks to money and Y), thus making domestic nominal bonds a better hedge against income fluctuations than foreign bonds.

The finding that under a reasonable parameterization the model is broadly consistent with the equity home bias in the data is important, as generally the portfolio composition turns out to be quite sensitive to assumed structural parameters. The other columns in Table 3 illustrate the impact of changing the calibrated correlation and persistence of the shock processes, as well as preference parameters determining trade patterns. First, we set the correlation among the shocks to zero, while keeping their variances at the estimated values. As shown in the second column of Table 3, the degree of home bias is now slightly larger than in the baseline calibration, increasing to $s_1 = 0.87$ in country 1. Thus, the estimated non-zero correlations of shocks act as to actually reduce the home bias. The results are due to the estimated positive correlation between within-country endowment shocks, ε_{Y_2} and ε_{D_2}, in Table 1, which adversely affects the hedging properties of domestic equities in country 2 against shocks to non-financial income.

In the next experiment, we again assumed that shocks are not correlated, but this time we also set all variances to identical values — note that for the portfolio allocation what matters is the relative size of variances, rather than their levels. Results for this experiment, reported in the third column of Table 3, show that this parameterization reduces the gross positions in foreign equities while increasing those in foreign bonds, thus generating an even larger degree of home bias than under the benchmark calibration. Precisely, in this case we have that $s_1 = 1.07$, and $s_2 = -0.08$, indicating a leveraged long position in domestic equities in the portfolio of households, financed by shortening foreign equities. This result is driven by the fact that financial endowment in country 1 is now less volatile than in the estimated benchmark, making domestic equities an even more attractive hedging instrument against shocks to non-financial income.

By the same token, a decline in the persistence of the process for the non-financial endowment (setting $\phi^y = 0.1$ in both countries instead of the estimated values of 0.92 and 0.91, respectively), brings about a reduction in domestic equity holdings ($s_1 = 0.69$ and $s_2 = 0.32$), reflecting the reduction of hedging demand, as shown in the fourth column of Table 3. Conversely, a decline in the persistence of dividend endowment (setting $\phi^D = 0.1$ in both countries instead of the estimated values of 0.9 and 0.92), triggers a rise in gross domestic equity positions, making domestic equities more attractive — domestic and foreign equity shares, not shown in the Table, are $s_1 = 3.3679$ and $s_2 = -2.3768$. Both results reflect the change in the relative unconditional volatility of financial and non-financial income, and its corresponding effects on the size of gross equity holdings.

Further sensitivity analysis results are reported in the last two columns of Table 3, showing the effects of changes in key preference parameters on the optimal long-run portfolio. A lower elasticity of substitution between domestic and foreign goods ($\theta = 0.5$) implies lower holdings of home equities than under the benchmark scenario ($s_1 = 0.4393$). Similar results we observe in the case of a bias in preferences for foreign goods ($\mu = 0.02$). Note that in this latter case, the terms of trade and real exchange rate are negatively correlated because of the prevalence of imported goods in the consumption basket.

An interesting implication of these results is that in our setting with incomplete markets, the time series properties of the shock processes, like their persistence and covariance structure, as well as preference parameters like the elasticity of substitution, matter for the composition of the equilibrium long-run portfolio. This stands in contrast to the complete market cases analyzed by Coeurdacier, Kollmann and Martin (2007), in which these structural parameters do not affect the degree of equity home bias. We deviate from that set up by having nominal bonds and, in addition to four types of endowment shocks, two monetary shocks. Because of flexible prices, in our economy money shocks have real effects only on impact, when inflation surprises affect the value of nominal bonds. Nevertheless, these impact effects are sufficient to break the complete markets environment in Coeurdacier, Kollmann and Martin (2007), and to produce substantial effects of changes in key structural parameters.

5.2 Three-country setting under financial constraints

We turn to the main object of our analysis, investigating the effects on international optimal portfolios when we allow unfettered trade in goods with a third country — country 3 — but different degrees of financial integration. In the first experiment, we consider the emergence of a third country that, while fully integrated in the goods market with the other economies, does not engage in trade in financial assets at all — we dubbed this case financial autarky in Section 2.3. In the benchmark parameterization, we set the preference parameters so that in all three economies the exports-to-GDP ratio is 20% as before, divided equally between the two foreign countries — for instance, in the case of country 1 we set $\mu_1 = 0.8$ again, and $\mu_2 = \mu_3 = 0.1$. As an implication, in the steady state, both the bilateral and overall trade is balanced. We leave all other parameters at the values discussed in the previous subsection.

The corresponding optimal steady state gross assets holdings are presented in Table 4, where again gross positions can be interpreted relative to steady state GDP. Compared to the two-country set-up, the emergence of intratemporal goods trade with a third-country marginally reduces the foreign gross holdings of domestic equities. In particular, column 1 and 3 show that gross liabilities in domestic equities amount to 120.8 percent of GDP, relative to 121.6 percent in the two-country case. Interestingly, the results are not driven by the stochastic structure of the shocks of country 3, but are entirely due to the existence of trade linkages.[13]

[13] We verified this claim by assuming different calibrations for the covariance structure and relative variances in country 3 — results are not reported here to save on space.

Table 4: Steady-State Portfolio in a Three-Country Model (measured in output units)					
	2- country		3-country		
			Financial Autarky		Financial Constraints
	Benchmark	Openness	Benchmark	Openness	Benchmark
	(1)	(2)	(3)	(4)	(5)
$Z_1^E(s_1-1)$	-1.2161	-0.9523	-1.2083	-1.1832	-1.1813
$Z_1^E s_2$	1.1166	0.8518	1.1130	1.0846	1.1288
B_2	-0.1555	-0.1474	-0.1568	-0.1524	-0.1343

In the next experiment, we assume that country 3 is more open than country 1 and 2, reflecting the larger importance of external demand for emerging economies . In particular, for country 3 we set $\mu_3^* = 0.4$ and $\mu_1^* = \mu_2^* = 0.3$, thus implying that its exports are 60% of GDP, equally distributed towards the other 2 countries — note that to ensure consistency of the trade matrix and satisfy the restriction that all NFA positions remain balanced, we also have to set $\mu_1 = 0.6$ and $\mu_3 = 0.3$ in country 1 and country 2, thus making these two countries more open as well.

As shown in Table 4, column 4, the results indicate only a small impact of the direction of trade for the optimal portfolio, beyond the what arises from a change in home bias. To facilitate comparison of the latter results with the one of the two-country setting, and to abstract from the effects of a reduction in consumption home bias in country 1 and 2, we also report results for the two-country setting in which $\mu_1 = 0.6$. In particular, gross liabilities of domestic equities amount to 118.3 percent of GDP, compared with 95.2 percent in the two-country case (fourth and second column of Table 4, respectively).

Finally, we analyze the consequences of country 3 opening up to intertemporal trade in the rest of the world, but subject to financial constraints similar to the ones presented in the previous section. Note that under this scenario, country 3 is constrained to invest its net savings in country 1's nominal bonds only. This could be interpreted as a requirement to minimize the variance of the nominal ('dollar') return of the foreign assets portfolio in terms of country 1 currency. The results indicate that there are only small changes in the optimal portfolio allocation in the rest of the world, compared to the financial autarky case. Precisely, home equity bias slightly increases in country 1 and 2. The results mainly reflect the estimated covariances of endowment shocks in country 3, especially to its financial component, with shocks in country 1 and 2, as reported in Table 1.

5.3 Diversified Reserves vs Optimal Portfolio

In the last set of experiments, we relax the assumption that in country 3 foreign reserves are mechanically invested in country 1 nominal bonds, investigating their optimal allocation according to different objective functions. In particular, we assume that country 3 foreign assets holdings or reserves are allocated by minimizing the variance of a portfolio, consisting of: (i) country 1 and 2 bonds only (diversified bond portfolio); and (ii) all foreign securities including equities (fully diversified portfolio). Furthermore, we distinguish two cases, depending on whether the real return on the target portfolio is expressed in terms of: (a) country 1 or (b) country 3 consumption basket.[14] We compare the minimum variance

[14] As discussed in the previous section, assuming the target return of the foreign assets portfolio to be expressed in nominal terms in the currency of country 1 ("dollars"), would imply that under variance minimization country 3 hold the corresponding risk free asset only (see also Papaioannou et al. (2007)).

allocation strategy, which we label the diversified reserves portfolio, with the fully optimal strategy aiming at maximizing country 3 households' utility when choosing foreign assets. Thereby, we apply the extensions of the Devereux-Sutherland methods presented in Section 3, particularly the formula (15) derived for the case of utility maximization.

Throughout these exercises we use the benchmark parameterization, but now we assume that country 1 runs a steady-state negative net foreign asset position of 30% of its GDP vis-à-vis country 3. This assumption is necessary to obtain reasonable results with the minimum variance criteria derived in Section 3 — with zero net savings, it would be necessary to go short in at least one asset to hold a positive amount of the other assets, therefore it is possible to achieve a zero variance portfolio with a trivial investment strategy that refrains to invest in foreign assets at all.

The results are presented in Table 5. Holdings of country 1 and country 2 nominal bonds by country 3 are denoted by B_1^* and B_2^*; holdings of country 1 and country 2 equities by s_1^* and s_2^*, respectively. For the sake of comparison with the results in the previous subsection, in which we assumed balanced bilateral current account positions, we start by reporting in the first column of Table 5 the optimal country 1 and 2 portfolios, when country 3 is constrained to hold only country 1 bonds, while running a positive NFA position towards country 1. With respect to the zero NFA case presented in the last column of Table 4, now country 1 displays a larger negative position in domestic equities vis-à-vis country 2, implying a slightly lower home bias. However, the gross holdings of foreign equities of country1 have also decreased, absorbing the larger part of the decrease in its international investment position, so that country 2 features a higher home bias in domestic equities.

Table 5: Steady State Portfolio with non-zero NFA in country 1 and 3

	Benchmark (Financial constraints)	Diversified Reserves				Optimal Portfolio
		Bond Portfolio		Full portfolio		
		(a)	(b)	(a)	(b)	
$Z_1^E\,(s_1-1)$	-1.1934	-1.1934	-1.1934	-0.8968	-1.0708	-3.5712
$Z_2^E s_2$	0.9839	0.9839	0.9839	0.2259	0.4061	3.6709
B_2	-0.0797	-0.3108	-0.3131	-0.0949	-0.0998	-0.2232
$Z_1^E s_1^*$	0.0	0.0	0.0	-0.1784	-0.0737	4.5704
$Z_2^E s_2^*$	0.0	0.0	0.0.	0.4560	0.3476	-5.2787
B_1^*	0.3	0.1610	0.1596	0.133	0.0140	0.6748
B_2^*	0.0	0.1390	0.1404	0.0091	0.0121	0.3335

Allowing country 3 to diversify its reserves between country 1 and country 2 bonds turns out to have a significant impact on cross-holdings of nominal bonds in the rest of the world, but has no impact on international equity holdings (columns under the headings 'Bond Portfolio' in Table 5). Precisely, the debt position of country 1 in country 2 currency increases from 8% to over 30% of GDP. Whether the real return of country 3 reserves is defined in terms of country 1 or its own consumption basket, gross holdings of country 1 bonds always exceeds those of country 2 bonds in country 3 by a marginal amount only. Thus, foreign reserves in country 3 are realistically characterized by positive holdings of both currency 1 and 2 bonds. However, this difference is lowest when the portfolio return is expressed in terms of country 3 consumption basket, as holdings of country 1 bonds are now relatively more risky.[15]

Conversely, under the fully diversified reserve portfolio, in which foreign equities can also be held by country 3, we observe significant changes in cross-border equity holdings in country 1 and 2 (columns under the headings 'Full Portfolio' in Table 5). Compared to the benchmark case, gross holdings of foreign equities by country 1 and 2 decrease, while the amount of equities in country 3 portfolio does not make up for these reductions. Interestingly, the latter country holds a long position in country 2 equities only, reflecting the lower volatility of dividend shocks and then returns relative to country 1, and a substantially larger amount of country 1 currency bonds, relative to country 2 currency bonds. Thus, surprisingly, it seems that the possibility of full diversification of foreign reserves by country 3 actually brings about higher rather than lower equity home bias in the rest of the world. This effect is only slightly reduced if the portfolio return is expressed in country 3 consumption basket, when basically only country 2 equities are held. It is also interesting to notice that a short-sale constraint on reserve holdings would turn out to be effectively binding only in the case when full portfolio returns are expressed in real currency of country 1.

These results, however, are overturned when country 3 foreign assets are allocated benevolently, by maximizing utility of the representative household, as shown in the last column of Table 5. Country 3 runs very large gross positions in foreign equities, of the order of five times its GDP, with opposite signs. These large gross positions mainly reflect the fact that, as shown in Table 1, fundamental shocks are a lot more volatile in country 3 than in the rest of the world, and generally negatively correlated with foreign shocks, thus causing a large demand of foreign assets, especially country 1 equities, for insurance motives. Country 2 equities are shortened as they turn out to provide a worse hedge to country 3 shocks, because of the low volatility of their dividends shocks. Similarly, holdings of foreign equities substantially increase in country 1, up to point of almost perfect diversification, holding roughly 50% of either equities. Conversely, country 2 ends up shortening country 1 equities, against a very large position in domestic equities, in excess of 100% of its total value.[16]

6 Concluding remarks

This paper investigated the effects of the emergence of financially constrained ("emerging") economies on the long run portfolio allocation of financially liberalized ("mature") economies. To do so, we depart from the usual small open economy assumption and adopt a multi-country model. We extend the Devereux and Sutherland (2006) algorithm, solving for optimized portfolios in a macroeconomic model with more than two agents, possibly with different objective functions. While we were able to obtain closed form solutions in the case of utility optimization by all countries representative agents, unfortunately closed from solutions are not generally available in other cases, and the model has to be solved numerically. In this setting, we have conducted several experiments, assuming that country 3, though fully integrated in goods market trading with the rest of the world — the financially

[15] In a previous version of the paper, we also verified that assuming country 3 pegs the exchange rate with country 1 does not substantially affect these results.

[16] In experiments not reported here, we verify that the assumption of non-zero NFA has only a marginal impact on the latter results.

integrated country 1 and 2, one country — is subject to varying degrees of international financial restrictions.

First, however, we have shown that our calibrated two-country DSGE model, though simple, is consistent with the degree of domestic equity homes bias observed in the data. Furthermore, we have also shown that under incomplete markets, the time series properties of the shock processes, like their persistence and covariance structure, as well as preference parameters like the elasticity of substitution, matter for the equilibrium long-run portfolio, in contrast to the complete markets invariance results obtained in the literature.

Second, we have shown that, even the trade pattern with a financially autarkic third country is an important determinant of gross asset positions in the rest of the world. Under our benchmark symmetric calibration, the emergence of trade with country 3 reduces the role of domestic and foreign assets in providing hedging against consumption fluctuations in mature economies, resulting in a decline of gross assets positions and an increase in equity home bias. The latter result is amplified by the degree of openness.

Third, assuming inefficiencies in savings decisions in country 3, we have explored the consequences of different foreign reserves allocation strategies by this country. Interestingly, we have shown that there are substantial differences in the international assets allocation depending on the objective of reserves allocation pursued by country 3. In particular, adoption of an objective of return variance minimization increases the degree of home bias in unconstrained economies (country 1 and 2), compared to mechanical investment of reserves in country 1 bonds. Moreover, we have shown that in this case the denomination of the real return on the reserves portfolio in country 3 has only limited effects on asset allocations internationally. Interestingly, pursuing a benevolent reserve allocation strategy, based on domestic households utility maximization, will lead to very different results, as in this case country 3 holds very large gross positions in foreign equities, reflecting the large hedging demand caused by the relative high domestic shock volatility. In turn, a substantial reshuffling of equity holdings in the rest of the world will ensue.

These results are interesting, as they highlight the relevance of a change in portfolio allocation strategy in systematically important financially underdeveloped economies on international capital markets. As a result, policy changes in reserves allocation strategies may have substantial effects on international holdings of foreign assets not only in emerging economies, but also in financially developed industrial economies.

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