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Correlations Between Real Interest Rates and Output in a Dynamic International Model: Evidence from G-7 Countries

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Abstract

This paper examines the extent to which a dynamic international general equilibrium model can account for observed movements in real interest rates and interest rate differentials. Using data for Group of Seven, the study finds that measured real interest rates are countercyclical in a single country and that the contemporaneous cross-correlations between international real interest differentials and output growth spreads are negative. Predictions of the baseline model are, however, inconsistent with the data. Extending the benchmark model to include habit persistence in consumption improves the match between theory and data.

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SUMMARY

There has been a great deal of interest in the existence of real interest rate differentials across countries. For example, Cumby and Obstfeld (1984), Mark (1985), and Cumby and Mishkin (1986) provided evidence of significant and persistent differentials in international real interest rates. This paper examines the extent to which a dynamic open economy general equilibrium model of a world with international capital mobility can account for observed movements in real interest rates and interest differentials.

Empirical results using data for G-7 countries suggest that measured real interest rates are countercyclical in each country and that the cross-correlations between real interest differentials and output growth spreads across countries are negative. The benchmark model's predictions of a strongly procyclical real interest rate (due to a strong and immediate response of both consumption and investment demand to positive shocks to home technology) and of a positive contemporaneous correlation between interest rate spreads and output growth differentials between the home and foreign country are not consistent with the empirical evidence.

The main result of the paper is that modifying the baseline model to include habit persistence in consumption helps improve the model's performance with respect to the correlations between interest rates and output growth differentials. This modification has the potential of making the model's predictions consistent with empirical observation because the demand response following a positive technology shock will be reduced relative to the base case. The slow response of aggregate demand generates a countercyclical movement in real interest rates, and thereby causes a negative correlation between interest differentials and output growth differentials across countries.

I. INTRODUCTION

The existence of real interest differentials across countries and their correlations with output growth differentials has received considerable recent attention in open economy macroeconomics. Standard models of international economies link consumption growth differentials across countries to changes in relative prices and interest rates. While several studies have documented the relationships between relative price (real exchange rates) movements and consumption and between relative prices and interest rates (uncovered interest rate parity) there are few studies of the correlations between international interest rate differentials and output growth differentials. This paper focuses on the relationship between real interest and output in a single country and between output growth differentials and interest rate spreads across countries using data for the G-7 countries. An objective of this study is to examine the extent to which a dynamic open economy general equilibrium model with international capital mobility can account for observed movements in real interest rates and interest differentials.

The motivations for this paper are the following. First, there has been a great deal of interest in the existence of interest differentials across countries because it has been suggested that they imply imperfections in capital markets. Numerous studies have examined the existence of significant and persistent differentials in real interest rates across countries[1]. Uncovered interest rate parity, however, is only implied by extremely simple theoretical models. The models used to study international properties of aggregate fluctuations (for example, Baxter and Crucini (1993, 1995), Backus, Kydland and Kehoe (1992), and Stockman and Tesar (1995)) have complex predictions for the behavior of interest rate differentials. These studies have paid only limited attention to comparing the observed comovements between real interest rates and macro aggregates with those predicted by the theory[2]. In monetary equilibrium models of the international business cycle, (for example, liquidity effect models developed in Grilli and Roubini (1992), Schlagenhauf and Wrase (1992), and Ho (1993)) interest in confronting price data and relative price movements has not been extended to real interest differentials although interest rates played a significant theoretical role in decentralizing incentives in these models[3].

A recent study by Head and Smith (1998) examines the links between differentials in real interest rates and rates of consumption growth in a multi-country exchange economy

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1 Using Euro-deposit rates for the G-7 countries, Cumby and Obstfeld (1984), Mark (1985) and Cumby and Mishkin (1986) all have shown that these differentials can be large and can last several quarters or more. Modjtahedi (1988) derived a linear dynamic stochastic process for the ex ante real interest differentials between the United States and a number of OECD countries.

2 One of the main reasons for this is that there is no perfect empirical counterpart to the theoretical risk-free interest rate. Measured real interest rates, whether ex-ante or ex-post, may be inappropriate proxies.

3 All of these models rely only on traded goods and thereby might not generate deviations from purchasing power parity, and hence real interest differentials.

with non-traded goods. Their study investigates whether observed interest differentials are consistent with the predictions of a simple dynamic economy. In light of the previous studies, it is important to examine whether the relationships generated among endogenously determined output, consumption, and interest rates in a theoretical economy with complete markets are consistent with their observed movements.

Compared to previous studies, in particular those which focus on an exchange economy (for example, Head and Smith (1998)) there are two major contributions of the present study. First, it incorporates production technologies which depend on both labor and physical capital. This is an important extension because the introduction of technological processes allows the adjustments in both consumption and output, whereas in exchange economies without production, the level of output is exogenous. Secondly, the model can predict explicitly the impact of technology shocks on real interest rate dynamics.

The second motivation for this study comes from the recent work of Beaudry and Guay (1996). Their study documents the incompatibility of the standard closed economy real business cycle model with observed movements in real interest rates. Beaudry and Guay show that the prototypical real business cycle model driven by persistent technology shocks predicts a procyclical movement in real interest rates while the data indicate either acyclical or slightly countercyclical behavior[4]. They argue that in most of the business cycle models a persistent positive technology shock induces an increase in demand for both consumption and investment that, in the short run, overrides the increase in supply. This causes interest rates to increase as output rises, thereby generating procyclical movements in interest rates. Their main finding is that extending the baseline model to include habit persistence in consumption and adjustment costs to capital significantly improves the model's empirical performance with respect to the correlation between interest rate and output since they reduce the responsiveness of demand to technology shocks.

Beaudry and Guay's closed economy exercise motivates us to perform a similar analysis in an open economy model [5]. The predictions regarding the correlations between real interest rate and output of the open economy model may differ from the closed economy model because the latter typically exhibits much higher correlations between domestic consumption and output. Since the real interest rate depends on expected future consumption growth the differences in the correlation of output with the real interest rate in an open vs. a closed economy framework are likely to arise. In this paper I investigate these differences and the implications.

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4 Their empirical results are based on the data for the United States for the period 1954:1-1990:4.

5 In some ways, the question is much more interesting in open economies, since most theories suggest very different movements in real interest rates in closed vs. open economies.

Using a dynamic open economy general equilibrium model of a world with international capital mobility, I consider the extent to which interest rate differentials can predict future output growth. To do this, I focus on two measures: (1) cross-correlations between output and interest rates in a single country and (2) cross-correlations between differentials in output growth rates and interest rates across countries.

Empirical results using data for G-7 countries extend support to Beaudry and Guay's finding (for U.S. data) that measured real interest rates (both ex-post and ex-ante) are countercyclical for all countries. Further, the cross-correlation between real interest differentials and output growth spreadss is found to be negative. In light of this empirical evidence, the second part of the paper examines whether the observed movements of real interest rates and interest differentials are consistent with the predictions of a dynamic open economy general equilibrium model. This model relies on technology shocks to nontraded goods production to generate deviations from purchasing power parity, imperfect correlations of consumption across countries, and real interest differentials. The benchmark model's predictions of a strongly procyclical real interest rate and of a positive contemporaneous correlation between interest spreads and output differentials between the home and foreign country, are not consistent with the empirical evidence.

The main result of the paper is that modifying the baseline model to include habit persistence in consumption contributes to improve the model's performance with respect to the correlations between interest rates and output growth differentials. This modification has the potential of making the model's predictions consistent with empirical observation because the demand response following a technology shock will be reduced relative to the base case. The slow response of aggregate demand generates a countercyclical movement in real interest rates (as in Beaudry and Guay's work), thereby causes a negative correlation between interest differentials and output differentials across countries.

The remaining sections of the paper are organized as follows. Section II presents the empirical results using data for G-7 countries. It analyzes the statistical properties of international real interest rate differentials and output differentials and examines the contemporaneous correlations between output and interest rate in a single country and across countries. In section III we investigate the predictions of a two-country dynamic general equilibrium model for the links between interest rates and output growth. First, we document the predictions of the baseline model and then examine whether introduction of habit formation in consumption reconciles theory with observed movements in interest rates. Finally, section IV offers some concluding remarks.

II. THE EMPIRICAL RELATIONSHIP BETWEEN OUTPUT AND INTEREST RATES

This section explores the empirical link between real interest rates and real output growth for the G-7 data. In the empirical studies (for example, Cumby and Obstfeld (1984), Mark (1984), Mishkin(1984), and Cumby and Mishkin (1986)) focusing on the measurement of real interest differentials across countries it has been a common practice to assume a Fisher equation which relates risk-free returns to forecasts of the intertemporal marginal rate of substitution (i.e. expected future consumption growth). This paper also assumes a Fisher equation in measuring the real interest rates. First I focus on the correlations between real interest rates and output in a single country, extending the analysis of Beaudry and Guay. Next I consider the correlations between interest differentials and output growth differentials across countries. Using simple summary statistics of the data, I investigate the following three questions. *First*, is the negative correlation between output and real interest rate reported by Beaudry and Guay for the U.S. data found in other G-7 countries ? *Second*, is the link between real output growth and real interest differentials equally strong across all (pairs of) countries? *Third*, is this link sensitive to the measurement of the real interest rate?

A. Data

All empirical results are based on quarterly data for G-7 countries from 1970:1 through 1995:4. The countries are labeled as: Canada (C), France (F), Germany (G), Italy (IT), Japan (JA), United States (US) and United Kingdom (UK). Output is measured in terms of real GNP net of real government spending [6]. The price index for output corresponds to the GNP deflator. In each country growth rate of output corresponds to the first difference of the logarithm of real output. Then a measure of growth rates of real output differentials is computed as the difference between growth rates for each country pair. The interest rates are real returns on three-month treasury bills. Real interest rates are computed from the nominal rates using the Consumer Price Index (CPI) as a measure of inflation. The data are taken from International Financial Statistics (IFS), Citibase and Cansim. A detailed description of the data is given in the appendix I.

B. Measuring Real Interest Differentials

The two measures of interest rate differentials considered are ex-post and ex-ante three month real yield differentials. The ex ante real rate of interest for a one-period bond is derived from the Fisher equation as the difference between the nominal interest rate and expected

[6] In the RBC literature it has been a common practice to adjust the data to exclude the government sector, particularly when the model does not include the government sector. For examples, see Beaudry and Guay (1996), Watson (1991) and King, Plosser, Stock, and Watson (1991).

inflation. Computing an empirical measure of the ex ante real interest differential therefore requires a measure of expected inflation. In previous studies, different approaches have been used to generate a measure of expected inflation[7]. One of the common methods is to compute an inflation forecast from a time-series model such as an ARIMA (Autoregressive integrated moving average) model. Following Baxter (1994), in this paper I compute one-quarter-ahead expected inflation as the forecast from an ARIMA (4,1) model of inflation. Then the ex ante interest rate differentials are computed as the nominal interest differentials minus this measure of the expected inflation differentials[8].

Alternatively, the ex post (i.e. realized) real interest rate, which is defined as the difference between the nominal rate and the actual inflation rate, is frequently used as a proxy for ex ante rates. So these two measures of inflation differentials differ by the forecast error for the inflation differential. Under rational expectations, the forecast error of inflation is a white noise process (i.e. a mean zero, serially uncorrelated random variable). Therefore, if there is a relationship between ex ante real interest rate differentials and real output growth differentials, it should be evident using ex post real rates as well. However, the ex post rates are expected to be more volatile and less persistent than the ex ante rates because of the inclusion of an additional orthogonal random component in the ex post rates. I compute the ex post real interest differentials as the nominal return differentials at the end of quarter t minus the actual inflation differential between the end of quarter t and quarter t+1.

C. Summary Statistics

Table 1 provides summary statistics for the ex ante and ex post measures of real interest differentials for the 21 country pairs. The results suggest three important features of the time series properties of real rates. *First*, ex post real interest differentials appear to be more volatile than ex ante differentials for all country pairs. On the average, the standard deviation of ex-post rate is 3.21 compared to 2.64 for the ex-ante rate. *Second*, ex ante real rate differentials exhibit significant persistence[9] in all cases in the range of 0.87-0.94. Ex post real rate differentials are also positively serially correlated; however the degree of persistence is much less than that of the ex post differential in each case. The average persistence for ex-post rate is only 0.71 compared to 0.91 for the ex-ante measure. These results are not surprising, since under rational expectations ex post differentials contain the serially uncorrelated forecast

[7] Three common approaches are: (1) to use survey data on inflationary expectations, (2) to compute an inflation forecast from a time series model such as an ARMA model, or (3) to compute expected inflation by 'smoothing' the inflation series using, for example, a long moving average or an exponential smoother (this can be one- or two-sided).

[8] Differences in tax rates across countries may have some impact on interest rate differentials. However, in this study we focus only on fluctuations in interest differentials at high frequencies. The effects from tax-differentials are not likely to be significant for movements at high frequencies.

[9] The persistence measure is the first-order autocorrelation coefficient.

error for the inflation differentials.[10] *Third*, the ex ante and ex post measures of real interest differentials are positively correlated for all country pairs. These features of real interest differential are similar and consistent with those reported by Baxter (1994) and Head and Smith (1998).

Similar summary statistics for output differentials are presented in Table 2. In measuring the output differentials across countries I use two estimates. *First*, the differentials in the levels of output across countries defined as $Y_t^d = (Y_t^i - Y_t^j)$ where Y_t^d is a measure of output differentials and Y_t^i and Y_t^j are actual levels of output in each country pair with $i \neq j$. Second, the differentials in the growth rates of output in each country, defined as $g_t^d = g_t^i - g_t^j$ where g_t^d measures output grow differentials across countries.

The main results from this table are the following: *first*, the volatility of output differential is much higher when it is measured in levels of output in each country. *second*, output differentials in levels also show strong persistence. For the measures in growth rates (first-differenced data) the persistence of output differential is much lower and therefore suggests that this series is first-differenced stationary.

Comparing the summary statistics for output growth differentials with those of interest rate differentials we find two distinctions in the behavior of these two series over time. *First*, output differentials are more volatile than real interest spreads across countries. *Second*, real interest rate differentials are more persistent than those of real output growth differentials. These results are in line with those reported by Head and Smith (1998) for consumption growth differentials and interest differentials across the G-7 countries.

D. Correlations Between Real Interest Rates and Output

This section investigates the cross correlations between output and real interest rates for the G-7 countries. Following Beaudry and Guay I estimate these correlations for a single country using the ex-post and ex-ante measures of interest rates. In addition, I examine the cross correlations between differentials in interest rates and differentials in output growth rates across countries. In each contemporaneous correlations, current output is related to current, past and future values of real interest rate. In particular we focus on five predicted correlations, $corr(Y_t, r_{t-k})$, where $k = \{-10, -5, 0, 5, 10\}$.

[10] Since the ex post differential combines the ex ante differential with the inflation forecast error (i.e. it combines a persistent process with a white noise process), it is expected to exhibit lower persistence than the ex ante differential.

1. Cross-correlations in a single country

Table 3 and 4 report the contemporaneous correlations between output and two measures of interest rates for a single country. In each table panel A presents sample moments where output is measured relative to linear trend. In the panel B, output is in growth rates and in panel C the HP-filter is applied to output and interest rates.

In general, the point estimates suggest that the cross-correlations between output and interest rate are negative for all countries in the sample. Table 3 indicates that for linear detrended output, the correlation is reported highest for Canada (-0.44) and lowest for Italy (-0.20). The plot of contemporaneous correlations between linear detrended output and ex-post return exhibits the distinctive patterns of relationship in each country. In both tables, the magnitude of correlations are relatively smaller for panel B (output measured in growth rates) and panel C (HP-filtered data). In particular , for HP-filtered data[11] the sample moments are close to zero.[12] These results are consistent with the findings of Beaudry and Guay and therefore confirm that the countercyclical real interest rate reported by them for the United States is also found in other G-7 countries. In standard macroeconomic theory, interest rates are perhaps considered the best predictor of future economic activity. High interest rates typically predict falling output in the future. In dynamic models of international risk-sharing, the risk-free rate is a function of expected future consumption growth.

The economic intuition for this negative correlation is clear: empirically real interest rates tend to be low during a period of economic boom (normally characterized by higher output growth) and high during a period of recession (normally associated with lower output growth). Beaudry and Guay reported that this characterization of the real interest rate remains apparent when long run restrictions (as in King, Plosser, Stock and Watson (1991)) are used to identify correlations induced by technology shocks.

2. Cross-correlations across countries

Table 5 and 6 report the cross correlations between output growth differentials and two measures of interest rate differentials for the 21 pairs of countries in the G-7. The contemporaneous correlations are estimated for actual output differentials in levels, $corr(Y_t^d, r_{t-k}^d)$ and for output differentials in growth rates, $corr(g_t^d, r_{t-k}^d)$. In general, the results suggest that output growth differentials are negatively correlated with contemporaneous value of interest differentials for majority of country pairs. These negative co-movements

[11] The HP-filter is a stationary-inducing transformation. It tends to remove nonstationary components that are integrated of order 4 and less. See Hodrick and Prescott (1980) and King and Rebelo (1993) for details regarding HP-filter.

[12] Beaudry and Guay (1996) and Precott, Guenther, Kehoe, and Manuelli (1983) also reported that the empirical correlation between HP-filtered output and the real interest rate is only slighly negative.

suggest that a rise(fall) in output differentials across countries causes international real interest differentials to fall(rise). This pattern is consistent with the country specific evidence of a negative correlation between real interest rates and output for each of the G-7 countries. The economic intuition is clear: suppose the home country experiences a period of economic boom (normally associated with higher output and lower interest rate). Foreign output and interest rate remaining the same, it causes the output differentials (between home and foreign country) to rise and the real interest differentials to fall . Therefore, if real interest is countercyclical empirically (i.e. a lower real interest rate is associated with higher growth rate of output) then differentials in the output growth across countries are expected to be negatively correlated with the interest differentials.

In summary, the empirical examination of the comovements between output and real interest for G-7 data indicate a countercyclical pattern for real interest in a single country and an inverse relationship between differentials in output growth rates and interest differentials across countries.

III. INTEREST RATE MOVEMENTS IN AN ARTIFICIAL ECONOMY

This section develops a two-country, two-sector dynamic general equilibrium model of a world with international capital mobility. This model is similar to the open economy model developed by Stockman and Tesar (1995). In light of the major finding of Beaudry and Guay (1996) that the introduction of habit persistence in consumption and of adjustment costs to capital significantly improves their model's empirical performance, it seems warranted to add these two features to the dynamic intertemporal model in this paper. Then it is straight forward to specify the benchmark model (without habit persistence in consumption) as a special case of the complete model.

Introduction of habit formation in consumption and of adjustment costs to capital accumulation seems warranted for two reasons. *First*, there is substantial empirical evidence for both adjustment costs to capital and habit persistence in consumption.

Habit formation

Habit formation has a long history in the study of consumption and it has been increasingly incorporated into dynamic economic modeling. Deaton and Muelbauer (1980) survey early work in the area, Deaton (1992) gives a more recent overview. Recent research on asset-pricing (Constantinides (1990), Ferson and Constantinides (1991), Heaton (1995) and Campbell and Cohrane (1995)) has found that habit formation in consumption is empirically plausible and may help explain certain asset-pricing puzzles. In a sense, habit persistence is a fundamental feature of psychology: repetition of a stimulus diminishes the response (since an individual's habit level depends on the history of aggregate consumption rather than the individual's own past consumption) and even the perception of the stimulus.

Adjustment Cost

Many empirical studies of investment support the presence of adjustment costs in capital (see for example Shapiro (1986)). Complete markets international models without adjustment costs are likely to be inappropriate because they display excess volatility in investment. Recent empirical studies of aggregate fluctuations find that introduction of adjustment costs to capital accumulation reduces the response of investment demand to exogenous shocks and therefore allows for more persistent movements in standard deviation of investment which matches the observed relative volatility of investment and output[13].

Second, the introduction of these features has the potential to significantly improve the model's performance. Beaudry and Guay (1996) argued that the prototypical real business cycle model predicts a procyclical interest rate due to a strong and immediate response of both consumption and investment to persistent technology shocks. However, the type of response of demand to technology shocks depends on the particular specification of preferences and production possibilities adopted. For example, if firms find it costly to adjust capital or if the marginal utility of present consumption depends positively on past consumption, the demand response following a technology shock would be diminished relative to the base case. Therefore, these two modifications have the potential of improving the model's performance (driven by persistent technology shocks) with respect to the correlation between interest rate and output since they reduce the responsiveness of aggregate demand (consumption demand + investment demand) to technology shocks.

A. The Model with Habit Persistence and Adjustment Costs

I develop a dynamic open economy intertemporal model with habit formation in consumption and adjustment costs to capital accumulation. The model is formulated in discrete time with an infinite horizon. In the world economy, each country produces the same internationally traded good, and one nontraded good for domestic consumption and investment, in each period. Each good is produced by labor and sector specific capital, and the production possibilities are stochastic. The countries have identical preferences and follow identical production techniques, but each country's techniques are subjected to country-specific productivity shocks. Let the countries be labeled as 'home' and 'foreign' and denote all foreign variables with an asterisk.

[13] Baxter and Crucini (1993), Baxter (1988), Crucini (1991), and Backus, Kehoe and Kydland (1992) noted that adjustment cost to capital is one type of friction that inhibits capital mobility which is necessary to prohibit extreme and highly unrealistic swings in national capital stocks in response to exogenous shocks.

The preferences of a representative agent in each country are assumed to exhibit habit persistence,[14] to be time-separable and to depend on consumption of traded good, C_{Tt} and nontraded good, C_{Nt} and leisure L_t, according to

$$U = \sum_{t=0}^{\infty} \beta^t E_t \left[\frac{1}{1-\gamma} \left(C_{Tt}^{\alpha} C_{Nt}^{1-\alpha} - \phi C_{Tt-1}^{\alpha} C_{Nt-1}^{1-\alpha} \right)^{1-\gamma} L_t^{a} \right], \phi > 0 \tag{1}$$

where $\beta \epsilon$ (0,1) is the discount factor; γ is the risk aversion parameter showing the intertemporal elasticity of substitution ; α is the share of the traded good in aggregate consumption; ϕ is the habit persistence parameter; a is the intertemporal elasticity of substitution for leisure; and E_t is the expectation operator. Similarly, the preferences in the foreign country are given by

$$U^* = \sum_{t=0}^{\infty} \beta^t E_t \left[\frac{1}{1-\gamma} \left(C_{Tt}^{*\alpha} C_{Nt}^{*1-\alpha} - \phi C_{Tt-1}^{*\alpha} C_{Nt-1}^{*1-\alpha} \right)^{1-\gamma} L_t^{*a} \right], \phi > 0 \tag{2}$$

The technologies used to produce the traded and non-traded goods are assumed to be of the Cobb-Douglas form with constant returns to scale.

Traded Sector:

$$Y_{Tt} = Z_{Tt} K_{Tt}^{\theta_T} N_{Tt}^{(1-\theta_T)} \tag{3}$$

$$Y_{Tt}^* = Z_{Tt}^* K_{Tt}^{*\theta_T} T_{Tt}^{*(1-\theta_T)} \tag{4}$$

Non-traded Sector:

$$Y_{Nt} = Z_{Nt} K_{Nt}^{\theta_N} N_{Nt}^{(1-\theta_N)} \tag{5}$$

$$Y_{Nt}^* = Z_{Nt}^* K_{Nt}^{*\theta_N} N_{Nt}^{*(1-\theta_N)} \tag{6}$$

where Y_{Tt}, Y_{Tt}^*,Y_{Nt}, Y_{Nt}^* are output levels for home and foreign country in the traded and non-traded sector respectively; K_{Tt}, K_{Tt}^*,K_{Nt}, K_{Nt}^* are capital stocks at the beginning of period t; N_{Tt}, N_{Tt}^*,N_{Nt}, N_{Nt}^* are the amount of work supplied at period t; Z_{Tt}, Z_{Tt}^*,Z_{Nt}, Z_{Nt}^* are technology shocks; θ_T and θ_N are the shares of capital in each sector. It is assumed that

[14] We consider this very simple form of habit persistence since such parsimony has the advantage of limiting the number of parameters to estimate and therefore allows for more powerful tests of the model.

capital is perfectly mobile across countries in the traded sector, but there is no inter-sectoral mobility within the country.

The installation of new capital is assumed to be associated with some adjustment costs so that firms find it costly to adjust capital in response to exogenous shocks (e.g. technology shocks). The stocks of capital in each sector evolve according to the accumulation equations

$$K_{(t+1)T} = (1-\delta)K_{Tt} + \left(\frac{I_{Tt}}{K_{Tt}}\right)^{\mu} K_{Tt} \tag{7}$$

$$K_{(t+1)N} = (1-\delta)K_{Nt} + \left(\frac{I_{Nt}}{K_{Nt}}\right)^{\mu} K_{Nt} \tag{8}$$

$$K^*_{(t+1)T} = (1-\delta)K^*_{Tt} + \left(\frac{I^*_{Tt}}{K^*_{Tt}}\right)^{\mu} K^*_{Tt} \tag{9}$$

$$K^*_{(t+1)N} = (1-\delta)K^*_{Nt} + \left(\frac{I^*_{Nt}}{K^*_{Nt}}\right)^{\mu} K^*_{Nt} \tag{10}$$

where I_{Tt} and I_{Nt} are gross investment in the traded and nontraded sectors, δ is the depreciation rate, and $1 \geq \mu > 0$ is the adjustment cost parameter. Under this specification the capital accumulation equations are concave in adjustment cost i.e.$\frac{\partial K_{t+1}}{\partial I_t} > 0$, and $\frac{\partial^2 K_{t+1}}{\partial I_t^2} < 0$. This precise formulation of adjustment costs has been chosen so that a steady state exists and is invariant to the size of the adjustment costs. From these accumulation equations we can derive the following,

$$I_{Tt} = \left[\frac{K_{T(t+1)}}{K_{Tt}} - (1-\delta)\right]^{\frac{1}{\mu}} K_{Tt} \tag{11}$$

$$I_{Nt} = \left[\frac{K_{N(t+1)}}{K_{Nt}} - (1-\delta)\right]^{\frac{1}{\mu}} K_{Nt} \tag{12}$$

$$I^*_{Tt} = \left[\frac{K^*_{T(t+1)}}{K^*_{Tt}} - (1-\delta)\right]^{\frac{1}{\mu}} K^*_{Tt} \tag{13}$$

$$I^*_{Nt} = \left[\frac{K^*_{N(t+1)}}{K^*_{Nt}} - (1-\delta)\right]^{\frac{1}{\mu}} K^*_{Nt} \tag{14}$$

In equilibrium the goods and labor markets clear. In the market for traded goods, the aggregate resource constraint requires that the world supply must be exhausted by world consumption and investment demand,

$$C_{Tt} + C^*_{Tt} + I_{Tt} + I^*_{Tt} = Y_{Tt} + Y^*_{Tt} \tag{15}$$

The equilibrium conditions for the non-traded good sectors imply that the domestic supply of the good be equal to domestic consumption and investment demand,

$$C_{Nt} + I_{Nt} = Y_{Nt} \tag{16}$$

$$C^*_{Nt} + I^*_{Nt} = Y^*_{Nt} \tag{17}$$

It is assumed that labor is perfectly mobile between the traded and non-traded sectors within a country, but immobile internationally. We normalize each country's population and the endowment of time of the representative household in each country, to one. Therefore, the labor market clearing conditions are given in,

$$N_{Tt} + N_{Nt} + L_t = 1 \tag{18}$$

$$N_{Tt} + N_{Nt} + L_t = 1 \tag{19}$$

The productivity shocks are assumed to follow the first-order autoregressive process, AR(1) ,

$$\begin{bmatrix} Z_{T(t+1)} \\ Z_{N(t+1)} \\ Z^*_{T(t+1)} \\ Z^*_{N(t+1)} \end{bmatrix} = \begin{bmatrix} \rho^T & \nu^T_N & \nu^T_{T*} & \nu^T_{N*} \\ \nu^N_T & \rho^N & \nu^N_{T*} & \nu^N_{N*} \\ \nu^{*T}_T & \nu^{*T}_N & \rho^{*T} & \nu^{*T}_{N*} \\ \nu^{*N}_T & \nu^{*N}_N & \nu^{*N}_{T*} & \rho^{*N} \end{bmatrix} \begin{bmatrix} Z_{Tt} \\ Z_{Nt} \\ Z^*_{Tt} \\ Z^*_{Nt} \end{bmatrix} + \begin{bmatrix} \varepsilon_{Tt} \\ \varepsilon_{Nt} \\ \varepsilon^*_{Tt} \\ \varepsilon^*_{Nt} \end{bmatrix} \tag{20}$$

where $E(\varepsilon_{it}) = E(\varepsilon^*_{it}) = 0$. Under this specification, there are "spillovers" of productivity shocks across countries, i.e., innovations that originate in one country (ε_{it} or ε^*_{it}) are transmitted to the other country, if the "diffusion" parameter ν^i_j , is nonzero. The "persistence" parameter, ρ^i controls the serial correlation of the technology variable within a country.

The variance-covariance matrix for the innovations to the productivity process is

$$E(\varepsilon_{Tt,}, \varepsilon_{Nt,}\varepsilon^*_{Tt,}, \varepsilon^*_{Nt})(\varepsilon_{Tt,}, \varepsilon_{Nt,}\varepsilon^*_{Tt,}, \varepsilon^*_{Nt})' = \begin{bmatrix} \sigma^2_{\varepsilon_T} & \psi^T_N & \psi^T_{T*} & \psi^T_{N*} \\ \psi^N_T & \sigma^2_{\varepsilon_N} & \psi^N_{T*} & \psi^N_{N*} \\ \psi^{T*}_N & \psi^{T*}_T & \sigma^2_{\varepsilon^*_T} & \psi^{T*}_{N*} \\ \psi^{N*}_T & \psi^{N*}_N & \psi^{N*}_{T*} & \sigma^2_{\varepsilon^*_N} \end{bmatrix} \tag{21}$$

where the variance of the innovations to technology shocks is measured by the parameter, $\sigma^2_{\varepsilon i}$ and the contemporaneous correlation of the innovations to productivity is determined by the parameter ψ^i_j.

B. The Benchmark Model

As a benchmark case we concentrate on a simple model without habit persistence in consumption. Under this formulation, we assume the habit persistent parameter, ϕ equals 0. Then the utility function takes the simple form,

$$U = \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\gamma} \left(C^{\alpha}_{Tt} C^{1-\alpha}_{Nt} \right)^{1-\gamma} L^a_t \right] \tag{22}$$

The baseline model, still, imposes costs of adjustment to capital accumulation[15]. Therefore, technologies and capital accumulation equations (3)-(10) remain same in the baseline model. Also, the market clearing conditions, equations (15) - (19) and the technology shock process, equation (20) remain the same. For this two-country world economy we can define and derive a competitive equilibrium. Since there are no distortions in the economy, and production exhibits constant returns to scale, the equivalence between competitive equilibrium and a social planning optimum allows us to solve instead a social planner's problem in which a weighted sum of national utilities is maximized, subject to technologies and the aggregate resource constraint. Treating each country symmetrically (so that the weights in the social planner's objective function are equal) the equilibrium allocation for the world economy are found by maximizing the expected utility of the representative agent in each country subject to the production technology (3)-(6) and the constraints (11) to (20).

Given the equilibrium stochastic processes for quantities, the stochastic processes for prices can be obtained from the Euler equation linking real interest rates and intertemporal

[15] Adjustment costs to capital have been incorporated to slow the response of investment to location-specific shocks. Because there is a single good produced in two different countries, capital owers have strong incentive to locate new investment in the more-productive location (so long as productivity shocks are persistent). Without some firction in the capital adjustment process, the model would display excessive volatility of investment (see, Baxter and Crucini (1995)).

marginal rate of substitution between the present and future consumption. Using this definition, the one-period risk-free rate of interest is given by

$$\frac{1}{r_t} = E_t \left[\beta \left(\frac{C_t}{C_{t+1}} \right)^{\gamma} \left(\frac{L_{t+1}}{L_t} \right)^{a} \right] \tag{23}$$

where C_t denotes the composite consumption index for the home country at time t. This index can be derived by using Euler's Theorem and the first-order conditions, and is given here by

$$C\left(C_{T,}C_N\right) = C_T^{\alpha} C_N^{1-\alpha} \tag{24}$$

In equation (23) r_t is the one-period return on a risk-free bond and E_t is the conditional expectation operator based on information available at time t. As usual in consumption-based asset-pricing, riskless returns are forecasts of consumption growth. Therefore, under rational expectations, they are predicted to have similar time-series properties to those of consumption growth[16]. In this model, r_t also depends on the intertemporal substitution in leisure because the specification of utility function is non-separable in composite consumption and leisure.

C. Model Parameterization and Solution

The model specified above must be solved, parameterized, and simulated to evaluate and compare their quantitative implications with the empirical evidence discussed in section II. Solving the model involves maximizing domestic and foreign household utility $(U + U^*)$ subject to the technology and the resource constraints. Since closed-form solutions to this problem are not readily available, some approximation method must be adopted. We follow the quadratic approximation method described in Hansen and Prescott (1991). [17]

Quantitative predictions of this theoretical model depend, to a large extent, on the values of the model's parameters of preference and technology and the parameters

[16] This relationship is obtained not only in the two-country model considered here, but also in a broader class of multi-country economies with traded and nontraded goods. For example, production would have no effect on this relationship.

[17] Using techniques described in Hansen and Prescott (1991) the simulation methods for solving the social planner's problem consists of undertaking the following steps: (1) set economy parameters; (2) compute the deterministic steady state; (3) derive quadratic approximation of social planner's return function around the steady state; (4) iterate on the Ricatti equation until policy function converges; (5) simulate the economy. The final products from this procedure are the stochastic processes for $Y_t, C_t, K_t, I_t,$ and N_t that approximately characterize the optimal allocation.

of the technology shock processes. With the exception of the adjustment cost parameter, the parameters describing preferences, production possibilities and the technology shock processes are set equal to those in Stockman and Tesar (1995) and are presented in table 7[18]. The parameters of preferences suggest that the rate of time discount (β) is set equal to 0.96, the intertemporal elasticity of substitution ($1/\gamma$) to 0.5, and the intertemporal elasticity of substitution in leisure ($1/a$) to -0.315.[19] The remaining parameter to be chosen is the share of traded goods in consumer's aggregate consumption bundle, α. This share is difficult to estimate directly from the data. However, this share can be inferred from data on percentage of total consumption and percentage of non-traded goods consumption in GDP. Stockman and Tesar reported that total consumption is approximately 80 percent of GDP and consumption of non-traded goods is roughly 40 percent of GDP since non-traded goods production makes up about 50 percent of aggregate output. [20] This implies that share of traded goods is approximately half of composite consumption bundle. Based on this information α is set equal to 0.5 which implies that aggregate consumption is made up of equal shares of traded and non-traded goods.

The parameters of technology suggest that the depreciation rate of capital is set equal to 10 percent per annum, the share of labor in traded-good industry ($1 - \theta^T$) to 61 percent and the share of labor in non-traded industry ($1 - \theta^N$) to 56 percent.[21] In the empirical studies the standard way to calibrate the adjustment cost parameter is to choose it so as to match the observed relative volatility of investment and output. In this study the adjustment cost parameter ranges mainly between 0.35 and 0.55 [22].

The stochastic processes governing technology shocks in (20) is a quadrivariate autoregressive process. The estimated autocorrelation matrix for the vector of shocks $[Z_T, Z_N, Z_T^*, Z_N^*]$ employed by Stockman and Tesar is

$$\Omega = \begin{bmatrix} 0.154 & 0.040 & -0.199 & 0.262 \\ -0.150 & 0.632 & -0.110 & 0.125 \\ -0.199 & 0.262 & 0.154 & 0.040 \\ -0.110 & 0.125 & -0.015 & 0.632 \end{bmatrix} \tag{25}$$

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18 The primary reason for sticking to Stockman and Tesar's (1995) parameters is because the specification of the model in this study is closest to their formulation.

19 This is consistent with a steady-state allocation of 20 percent of the time endowment to work effort and 80 percent to leisure.

20 Stockman and Tesar equate the steady-state values of outputs of the traded and non-traded industries so that nontraded goods comprise half of the aggregate output.

21 The production parameter ($1 - \theta^i$) is estimated from the average labour share in the seven OECD countries.

22 Within this range for adjustement cost parameter, the standard deviation of investment relative to that of output is approximately equal to 3 i.e. ($\frac{\sigma_I}{\sigma_Y} \cong 3$) which is supported by data.

where Ω is a 4x4 matrix describing the autoregressive component of the disturbance. The parameter values display a low degree of persistence, especially in the traded-good sector. The estimated variance-covariance matrix of the shocks reported as

$$V[\varepsilon] = \begin{bmatrix} 3.62 & 1.23 & 1.21 & 0.51 \\ 1.23 & 1.99 & 0.51 & 0.27 \\ 1.21 & 0.51 & 3.62 & 1.23 \\ 0.51 & 0.27 & 1.23 & 1.99 \end{bmatrix} \tag{26}$$

The estimates of $V[\varepsilon]$ suggest that the variance of the productivity shocks in the traded good sector is about twice as high as in the non-traded good sector. The correlation between innovations to traded goods industry is reported as 0.33, while for the non-traded industry such correlations is 0.14. Country-specific innovations (across sectors within a country) are slightly larger, with a cross-sector correlation of 0.46. Stockman and Tesar estimated the autocorrelation and variance-covariance matrices from the Hodrick-Prescott filtered data.

D. Predictions of the Baseline Model Regarding Interest Rates and Output

This section examines the predictions of the baseline model (without habit formation in consumption) regarding the theoretical cross-correlations between interest rates and aggregate output[23].

Estimation of real interest rate

The issue of measurement of the interest rate has posed a more serious problem than for most other macroeconomic variables because there is no perfect empirical counterpart to the theoretical risk- free rate which depends on the forecast (i.e. conditional expectations) of future consumption growth. Measured real interest rates, whether ex-ante or ex-post, may be inappropriate proxies. In this paper, we use the following approximation technique to estimate real return on a risk-free bond. First, simulate the model for a fairly large period (e.g. 10,000) and generate observations on intertemporal marginal rate of substitution defined as, $x_t = \left\{\frac{C_{t+1}}{C_t}\right\}$, and other variables (e.g. intertemporal substitution is leisure, exogenous shocks, aggregate output etc.). Second, estimate the equation: $x_t = \varphi x_{t-1} + \tau S_{t-1} + \varepsilon_t$ where S_t includes growth rate of leisure, exogenous shocks and aggregate output and ε_t is the error term. Then under rational expectations, $E_t(x_{t+1}) = \widehat{\varphi}\, x_t + \widehat{\tau} S_t$ which can be used as a proxy for risk-free interest rate , r_t.

[23] For simplicity we focus on the relationship between real return and aggregate output in each country. Aggregate output is the price weighted sum of traded sector output and non-traded output: $Y_t = Y_t^T + \frac{P_t^N}{P_t^T} Y_t^N$, where $\frac{P_t^N}{P_t^T}$ is the relative price of non-traded good.

Table 8 reports cross-correlations between interest rates and output in a single country implied by the above baseline model. To be consistent with the empirical results in section II, these correlations are estimated for three detrending methods of output: linear detrended, first-differenced and HP-filtered. The first two rows represent correlations between detrended output and the interest rate, while in the subsequent rows output is measured in growth rates. For simplicity it is assumed that the exogenous productivity shocks occur only in the home country. However, "spillovers" of technology shocks from home to foreign country are allowed. For all three measures of output, Table 8 reflects a strong positive correlation between real interest and output in the home country. The economic intuition is clear: with a positive shock to home technology, the rise in the marginal product of capital in the home country increases investment demand and simultaneously increases consumption demand through an increase in permanent income. These two induced shifts in aggregate demand offset the shift in aggregate supply [24]causing current goods to become relatively expensive and thus inducing a rise in interest rate. For home country, the theoretical correlations between linear detrended output (i.e. a trend-stationary process) and interest rate is 0.45, which suggests a procyclical pattern for the interest rates. The model also predicts a strong positive correlation of 0.49, between output growth (i.e. the technology process is difference stationary) and interest rate in the home country. Finally, the contemporaneous correlation between output and interest rate after applying HP-filter to the data is 0.89. In the foreign country, the corresponding cross-correlations between output and domestic interest rate are positive but not very significant.

It is important to examine the sensitivity of the results reported in Table 8 with respect to changes in parameter values in order to further explore the predictions of the model regarding the pattern of correlations between interest rate and output. In particular, I investigate whether results change for reasonable changes in the intertemporal elasticity of substitution or the depreciation rate. In general, the pattern of correlations is found to be robust to changes in parameters. For example, doubling the intertemporal elasticity of substitution in consumption from 0.5 to 1 has the impact of raising the contemporaneous correlations between output and interest rates only marginally from 0.45 to 0.50 for linear detrended output. Similarly, when the yearly depreciation rate is changed from 10 percent to 20 percent, the correlation between output growth differentials and interest differentials only falls from 0.65 to 0.55.

Overall, the open economy dynamic general equilibrium model in this paper predicts a strongly procyclical real interest rate in the home country for a positive shock to technology. This pattern extends support to the predictions of the closed economy version of Beaudry and Guay's exercise and again indicates an inconsistency between theory and observed movements in real interest rates.

[24] This paper refers to aggregate demand and aggregate supply as located in the real-interest-rate output space, which is the most relevant space for describing goods market equilibrium in the neo-classical framework.

Table 9 presents cross-correlations between differentials in output and interest rate between home and foreign country. I look at both differentials in artificial data as well as differentials in growth rates of output in the home and foreign country. The main result from table 9 is that for a positive shock to home technology, the underlying model predicts a strong positive correlation between the interest rate spreads and the output differentials between the home and foreign country. This pattern is consistent with the procyclical real interest rate movement implied by the model. The intuition is straightforward: a positive exogenous shock to home technology leads to a rise in marginal product of capital and causes both investment and consumption demand to rise. The increase in aggregate demand causes real interest to rise as output rises at home. Under the assumption of international capital mobility, the level capital investment falls in the foreign country[25]; this will cause the level of output to decline and real interest rate to decline as well. Thus, the standard model suggests a positive correlation between interest rate differentials ($r_t^d = r_t - r_t^*$) and output growth differentials ($g_t^d = g_t - g_t^*$) between home and foreign country. This result again indicates that the prediction of this dynamic general equilibrium model are inconsistent with observed differentials (discussed in section II).

In summary, the predictions of the baseline model regarding interest rate movements are incompatible with the empirical observations presented in section II. This discrepancy between theory and data (following Beaudry and Guay's exercise) suggest investigating whether the modification of the benchmark model with habit persistence in consumption help reconcile the theory with the data.

E. Predictions of the Extended Model Regarding Interest Rates and Output

This section examines whether modification of the baseline model with habit persistence in consumption can potentially improve the match between the predictions of the model and the data. The model with habit formation is solved using the same quadratic approximation method described earlier. A complete solution to the social planner's problem including the first-order conditions and the steady state equilibrium are presented in appendix II. The model is simulated and calibrated using the same values (as in baseline model) for all parameters except for the habit persistent parameter.

Habit formation parameter

Given the specification of the utility function, the values of habit persistence parameter, ϕ ranges between 0 (for base line model) and 0.285. These estimates are consistent with the selected values of adjustment parameter that match the observed relative volatility of investment and output. In order to evaluate whether the parameter values for the degree of

[25] This is because of the shift of capital from foreign to the more productive home industries.

habit persistence are of reasonable magnitude, it is useful to refer to some of the previous studies. For example, the estimates of habit persistence parameter in Beaudry and Guay ranges between 0.3 and 0.5. Using quarterly data, Ferson and Constantinides (1991) reported the estimate of habit persistence to be between 0.2 and 0.7. These comparisons suggest that our estimates of habit persistence is relatively low compared to the previous two studies. One reason could be the specification of preference in our model which is dissimilar to their formulation.

Table 10 presents the cross-correlations between output and interest rates implied by the extended model. As before these correlations between output and interest rates of a single country are estimated for all three detrending methods: linear detrended, in growth rates and HP-filtered. The main result is that the introduction of habit persistence in consumption to the baseline model (with adjustment costs to capital formation) leads the extended model to predict a negative contemporaneous correlation between output and real interest rates in the home country. The estimates of cross-correlation between output and interest rate ranges between -0.39 and -0.65. These results in this dynamic open economy model are consistent with what Beaudry and Guay's find for their closed economy framework. The economic intuition is simple: in the extended model a positive shock to home technology leads to a fall in real interest rate as output rises. This occurs because the presence of habit persistence in consumption combined with the adjustment costs to capital generates a friction which slows the response of consumption and investment demand to technology shocks compared to the base case; this causes the real interest rate to decline as output rises, thereby generating a countercyclical movements in real interest rates which match the data.

Next I focus on the correlations between differentials in output growth rates and interest differentials between home and foreign country in the extended model. The results are presented in Table 11. The important feature of this table is that now the extended model displays a negative contemporaneous correlations between interest differentials and output growth differentials. Therefore with habit persistence the observed differentials are consistent with dynamic general equilibrium model.

These negative correlations between output and interest differentials across countries are consistent with the countercyclical movements in real interest rate in a single country. The economic intuition is clear: in the extended model a positive shock to home productivity leads to a fall in real interest rate as output rises at home. The foreign country still experiences a decline in both output and real interest rate. However, the magnitude of decline will be much smaller compared to the base case. Because the presence of habit persistence and adjustment costs in capital accumulation may inhibit the mobility of capital across countries. In this situation spillover of technology shocks from home country will cause capital investment, output, and the interest rate to fall to a smaller extent in the foreign country. Thus, the augmented model generates a negative correlation between output differentials and interest differentials across countries.

In summary, extending the baseline model (with adjustment costs) to include habit persistence in consumption improves the match between theory and the observed co-movements between output and interest rates in a single country and between interest differentials and output growth differentials across countries.

IV. CONCLUSION

An objective of this paper has been to examine the extent to which a dynamic open-economy general equilibrium model can account for observed movements in real interest rates and international interest rate differentials. There are two major findings of this study. First, the predictions of the simple dynamic open economy model are not consistent with the observed co-movements between output and interest rates in a single country and between differentials in output growth and interest rate spreads across countries. In effect, the baseline model driven by a positive exogenous shock to home productivity predicts a highly procyclical real interest rate and a positive correlation between output growth spreads and real interest differentials across countries. However, the data suggest exactly the opposite for these correlations. This discrepancy between the theory and data may be attributed to the immediate response of aggregate demand to technology shocks in the standard model that are too strong to be consistent with the observations.

The second finding of the paper is that modification of the benchmark model to include habit persistence in consumption improves the model's ability to match the observed comovement between output growth differentials and interest differentials across countries. These sorts of findings extend the analysis of Beaudry and Guay to a dynamic open economy model. The second result is also consistent with the macro-international finance literature (driven by Campbell, Cochrane and Abel) on the behavior of asset prices. Therefore the results of this study suggest that introduction of habit formation in consumer's preference have a significant impact on the behavior of open economy models and thus contributes to our understanding of the mechanism by which technology shocks are propagated through the economy.

The analysis on the dynamic international model in this paper offers some directions for future research. First, it is perhaps interesting to examine the predictions of the model regarding the Uncovered Interest Rate Parity (UIRP) i.e. whether measured real interest differentials across countries can account for expected exchanges in the real exchange rates. Second, extending the model to include government sector is also an important area of future research. In a model in which Ricardian equivalence fails, fiscal policy will effect the dynamic properties of real interest rates and interest differentials because governments in practice use distortionary taxation. It would be interesting to model the actions of the government and examine the impact of fiscal policy shocks on cross country correlations between output growth differentials and real interest differentials.

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Table 1. Summary Statistics for Real Interest Differentials[1], Quarterly Data: 1970:01-1995:04

Country pair	Standard Deviation		Persistence[2]		Correlation[3]
	Ex-post	Ex-ante	Ex-post	Ex-ante	
US-C	2.33	1.51	0.68	0.88	0.51
US-UK	3.93	2.77	0.71	0.91	0.59
US-G	2.26	3.02	0.74	0.94	0.46
US-IT	3.23	2.82	0.66	0.86	0.14
US-F	2.42	1.88	0.65	0.85	0.62
US-JA	2.16	2.62	0.71	0.91	0.37
C-UK	3.43	2.23	0.67	0.87	0.36
C-G	2.58	2.11	0.73	0.93	0.47
C-IT	2.71	2.54	0.66	0.86	0.19
C-F	2.29	2.17	0.69	0.89	0.76
C-JA	4.04	3.13	0.68	0.88	0.42
UK-G	4.36	3.21	0.72	0.92	0.33
UK-IT	3.77	3.51	0.72	0.92	0.32
UK-F	3.41	2.48	0.69	0.89	0.29
UK-JA	3.83	2.87	0.72	0.92	0.37
G-IT	4.77	4.27	0.72	0.92	0.26
G-F	2.98	2.34	0.71	0.91	0.34
G-JA	2.78	2.63	0.67	0.87	0.19
IT-F	1.93	1.73	0.74	0.94	0.13
IT-JA	3.49	3.29	0.73	0.93	0.21
F-JA	4.77	2.22	0.73	0.93	0.24
Average	3.21	2.64	0.71	0.90	0.34

[1]/ Units are annualized percent per quarter.

[2]/ Persistence measure is the first-order autocorrelation coefficient.

[3]/ Correlation is the comovement between the *ex ante* and *ex post* measure of real interest differentials.

Table 2. Summary Statistics of Output Differentials
Quarterly Data: 1970:01-1995:04

Country pair	Standard Deviation		Persistence	
	Differentials in levels[1]	Differentials in growth rates[2]	Differentials in levels	Differentials in growth rates
US-C	7.81	3.11	0.91	0.16
US-UK	6.49	5.44	0.92	-0.07
US-G	7.57	6.32	0.93	-0.28
US-IT	9.02	6.51	0.94	-0.13
US-F	7.08	6.56	0.95	-0.13
US-JA	7.48	5.64	0.89	-0.07
C-UK	6.47	5.43	0.91	-0.04
C-G	5.58	4.42	0.91	-0.33
C-IT	6.83	4.43	0.92	-0.10
C-F	4.54	3.65	0.94	-0.05
C-JA	4.41	3.07	0.86	-0.51
UK-G	5.19	4.79	0.92	0.19
UK-IT	4.43	3.11	0.93	-0.09
UK-F	8.15	4.78	0.95	-0.28
UK-JA	9.97	6.24	0.87	-0.15
G-IT	9.61	3.96	0.91	0.12
G-F	9.76	1.92	0.94	0.13
G-JA	7.15	5.27	0.91	-0.07
IT-F	5.51	3.57	0.86	-0.10
IT-JA	6.58	7.02	0.84	0.14
F-JA	7.79	5.77	0.93	-0.39

[1]/ Difference between the level of output in each country.
[2]/ Difference between the growth rate of output in each country.

Table 3. Sample Correlations Between Output and Ex-post Real Interest Rates, 1970:01-1995:04

Variable		Country						
	k	Canada	France	Germany	Italy	Japan	U.K.	U.S.
(A) Linear Detrended output:								
	-10	0.05	-0.43	-0.11	-0.25	-0.32	-0.35	-0.37
	-5	-0.04	-0.39	-0.04	-0.26	-0.44	-0.24	-0.50
	0	**-0.44**	**-0.28**	**-0.38**	**-0.20**	**-0.35**	**-0.28**	**-0.33**
	5	-0.35	-0.15	-0.25	-0.01	-0.17	0.07	-0.06
	10	-0.37	0.04	-0.38	0.19	-0.05	0.06	0.13
(B) First-differenced Output:								
	-10	-0.11	-0.03	0.00	-0.01	-0.05	0.01	-0.14
	-5	-0.13	0.03	0.03	0.00	-0.06	0.10	-0.01
	0	**-0.19**	**-0.25**	**-0.13**	**-0.14**	**-0.17**	**-0.13**	**-0.15**
	5	-0.08	0.17	-0.08	0.14	0.05	0.03	0.07
	10	0.06	0.18	-0.05	0.12	0.05	-0.05	0.07
© HP-filtered data:								
	-10	-0.09	-0.01	0.02	0.01	-0.03	0.03	-0.12
	-5	-0.11	0.05	0.05	0.02	-0.04	0.12	0.01
	0	**-0.08**	**-0.14**	**-0.02**	**-0.03**	**-0.05**	**-0.11**	**-0.04**
	5	-0.06	0.19	-0.06	0.16	0.07	0.05	0.09
	10	0.08	0.20	-0.03	0.14	0.07	-0.03	0.09

Table 4. Sample Correlations Between Output and Ex-ante Real Interest Rates, 1970:01-1995:04

Variable	Country						
k	Canada	France	Germany	Italy	Japan	U.K.	U.S.
(A) Linear Detrended Output:							
-10	0.29	-0.02	0.24	0.17	0.30	-0.20	0.16
-5	0.30	0.03	0.34	0.03	0.28	0.08	0.10
0	**-0.32**	**-0.33**	**-0.22**	**-0.38**	**-0.24**	**-0.25**	**-0.27**
5	-0.14	-0.59	-0.19	-0.63	-0.39	-0.05	-0.60
10	-0.38	-0.57	-0.37	-0.59	-0.16	-0.15	-0.35
(B) First-differenced Output:							
-10	0.05	-0.08	0.09	-0.03	0.13	0.21	-0.01
-5	-0.04	-0.15	-0.06	-0.16	-0.09	0.15	-0.08
0	**-0.09**	**-0.18**	**-0.11**	**-0.12**	**-0.13**	**-0.10**	**-0.12**
5	-0.14	-0.09	-0.12	-0.06	0.02	-0.10	0.04
10	-0.09	0.07	-0.06	0.11	0.18	-0.02	0.17
© HP-filtered data:							
-10	0.09	-0.04	0.13	0.00	0.17	0.24	0.02
-5	-0.07	-0.12	-0.03	-0.12	-0.06	0.18	-0.04
0	**-0.06**	**-0.14**	**-0.07**	**-0.08**	**-0.09**	**-0.07**	**-0.09**
5	-0.10	-0.06	-0.08	-0.03	0.06	-0.07	0.07
10	-0.06	0.10	-0.02	0.15	-0.21	0.01	0.21

Table 5. Sample Correlations Between Output Differentials and Ex-post Real Interest Differentials

	Country pair																				
k	US-C	US-UK	US-G	US-IT	US-F	US-JA	C-UK	C-G	C-IT	C-F	C-JA	UK-G	UK-IT	UK-F	UK-JA	G-IT	G-F	G-JA	IT-F	IT-JA	F-JA
(A) Differentials in levels of output:																					
-10	0.20	-0.36	-0.05	-0.11	-0.30	-0.14	-0.13	0.06	-0.09	-0.21	-0.19	0.04	0.16	0.05	0.13	0.12	-0.41	-0.31	0.17	0.04	-0.35
-5	0.20	-0.18	0.40	0.07	0.04	-0.10	0.02	0.17	-0.01	-0.08	-0.27	0.21	0.39	0.29	-0.05	0.15	-0.26	-0.15	0.17	0.05	-0.16
0	**-0.25**	**-0.11**	**-0.72**	**-0.24**	**-0.37**	**-0.17**	**-0.09**	**-0.30**	**-0.19**	**-0.18**	**-0.21**	**-0.28**	**-0.43**	**0.02**	**-0.17**	**-0.15**	**-0.29**	**-0.34**	**-0.12**	**-0.07**	**-0.40**
5	0.03	0.31	-0.56	0.32	0.48	0.14	0.16	0.13	0.21	0.29	0.02	-0.41	-0.43	0.57	0.21	0.20	0.15	0.29	0.10	0.09	0.27
10	-0.14	0.19	0.25	0.39	0.54	0.03	-0.01	-0.64	0.38	0.38	-0.10	0.51	-0.35	0.63	0.16	0.38	0.31	0.33	0.21	0.21	0.45
(B) Differentials in growth rates of output:																					
-10	-0.06	0.04	0.20	0.09	0.21	0.07	-0.02	-0.02	-0.47	0.04	-0.03	0.11	0.18	0.18	-0.06	0.02	0.11	0.09	-0.01	0.09	0.14
-5	-0.04	0.14	0.26	0.11	0.26	-0.02	0.14	0.10	0.10	0.15	-0.09	0.14	0.14	0.22	-0.09	0.01	0.19	0.06	-0.07	-0.06	0.08
0	**-0.14**	**-0.18**	**-0.15**	**-0.19**	**-0.24**	**0.06**	**-0.08**	**0.05**	**-0.05**	**-0.18**	**-0.15**	**-0.07**	**0.05**	**-0.15**	**-0.03**	**-0.26**	**-0.16**	**-0.12**	**-0.03**	**-0.04**	**-0.15**
5	-0.16	0.00	-0.16	0.05	0.10	-0.06	-0.08	-0.15	0.06	0.04	-0.13	0.09	-0.01	0.10	0.02	0.08	0.28	0.02	0.02	0.04	0.15
10	-0.01	-0.13	-0.18	0.06	0.08	-0.02	-0.09	-0.04	0.17	0.14	0.05	-0.01	-0.08	-0.03	-0.06	0.08	0.28	0.02	0.07	0.10	0.16

Table 6. Sample Correlations Between Output Differentials and Ex-ante Real Interest Differentials

	Country Pair																				
k	US-C	US-UK	US-G	US-IT	US-F	US-JA	C-UK	C-G	C-IT	C-F	C-JA	UK-G	UK-IT	UK-F	UK-JA	G-IT	G-F	G-JA	IT-F	IT-JA	F-JA
(A) Differentials in levels of output:																					
-10	0.11	-0.15	-0.05	0.28	-0.05	0.26	-0.06	-0.04	0.27	-0.01	0.19	-0.26	0.18	0.11	-0.33	-0.42	-0.08	0.20	-0.42	0.23	0.31
-5	0.48	-0.01	0.10	0.15	-0.01	0.19	0.02	0.14	0.03	0.08	0.23	-0.25	0.11	0.12	-0.10	-0.57	-0.35	0.13	-0.35	0.20	0.13
0	**-0.39**	**-0.19**	**-0.23**	**-0.11**	**-0.17**	**-0.12**	**-0.21**	**-0.29**	**-0.34**	**-0.15**	**-0.12**	**-0.09**	**-0.15**	**-0.16**	**-0.27**	**-0.61**	**-0.63**	**-0.18**	**-0.25**	**-0.19**	**-0.25**
5	0.10	0.22	0.26	-0.39	0.01	0.31	0.31	0.26	-0.39	0.18	0.23	0.03	-0.15	0.11	0.31	-0.54	-0.74	-0.16	-0.18	-0.42	-0.43
10	-0.26	0.31	0.24	-0.24	0.17	0.42	0.34	0.17	-0.17	0.22	0.14	0.20	-0.21	0.04	0.27	-0.35	-0.67	-0.17	-0.04	-0.42	-0.51
(B) Differentials in growth rates of Output:																					
-10	0.21	0.10	0.10	-0.01	0.08	0.04	0.06	0.06	-0.08	0.01	0.05	0.03	-0.04	-0.04	0.06	-0.03	-0.26	0.14	0.07	0.08	0.01
-5	0.04	0.10	-0.08	-0.14	0.01	-0.07	0.09	0.09	-0.14	0.10	-0.05	0.09	-0.03	0.11	0.08	-0.07	-0.29	-0.13	0.06	-0.10	-0.24
0	**-0.05**	**-0.11**	**-0.07**	**-0.21**	**-0.02**	**-0.03**	**-0.10**	**-0.07**	**-0.22**	**-0.35**	**-0.04**	**0.07**	**-0.08**	**-0.04**	**0.08**	**0.02**	**-0.29**	**-0.03**	**0.06**	**-0.16**	**-0.23**
5	-0.19	0.02	0.02	-0.01	0.22	-0.03	0.03	0.00	0.03	0.05	-0.06	0.14	-0.05	0.01	0.02	0.11	-0.12	-0.02	0.18	-0.04	-0.08
10	-0.14	0.06	0.02	0.21	0.22	0.09	0.04	0.02	0.25	0.18	-0.02	0.13	-0.01	-0.05	0.03	0.16	-0.03	0.04	0.24	0.09	0.02

Table 7. Parameter Values

Parameter	Definition
Technology:	
$\delta = 0.10$	Depreciation rate
$(1-\theta^T) = 0.61$	Labor Share in traded good industry
$(1-\theta^N) = 0.56$	Labor Share in nontraded good industry
$\mu = 0.35 - 0.55$	Adjustment cost in capital accumulation
Preference:	
$\beta = 0.96$	Rate of time preference
$1/\gamma = 0.5$	Intertemporal elasticity of substitution
$1/a = -.315$	Intertemporal elasticity of substitution in leisure
$\phi = 0 - 0.285$	Habit persistence in consumption

Table 8. Theoretical Correlations Between Real Output and Real Interest Rates in the Baseline Model

Variable	Cross-correlation with domestic output				
	k				
	-10	-5	0	5	10
Linear detrended output:					
Home country	0.11	0.29	0.45	0.35	0.32
Foreign country	0.05	0.07	0.11	0.06	0.03
First-differenced output:					
Home country	0.33	0.42	0.49	0.17	0.13
Foreign country	0.04	0.05	0.09	0.07	0.06
HP-filtered data:					
Home country	0.39	0.58	0.89	0.68	0.53
Foreign country	0.09	0.11	0.14	0.11	0.08

Table 9. Theoretical Correlations Between Real Output Differentials and Real Interest Rate Differentials in the Baseline Model

Variable	Cross-correlation				
	k				
	-10	-5	0	5	10
Differentials in levels of output:	0.34	0.42	0.51	0.41	0.31
Differentials in growth rates of output:	0.37	0.58	0.81	0.67	0.52

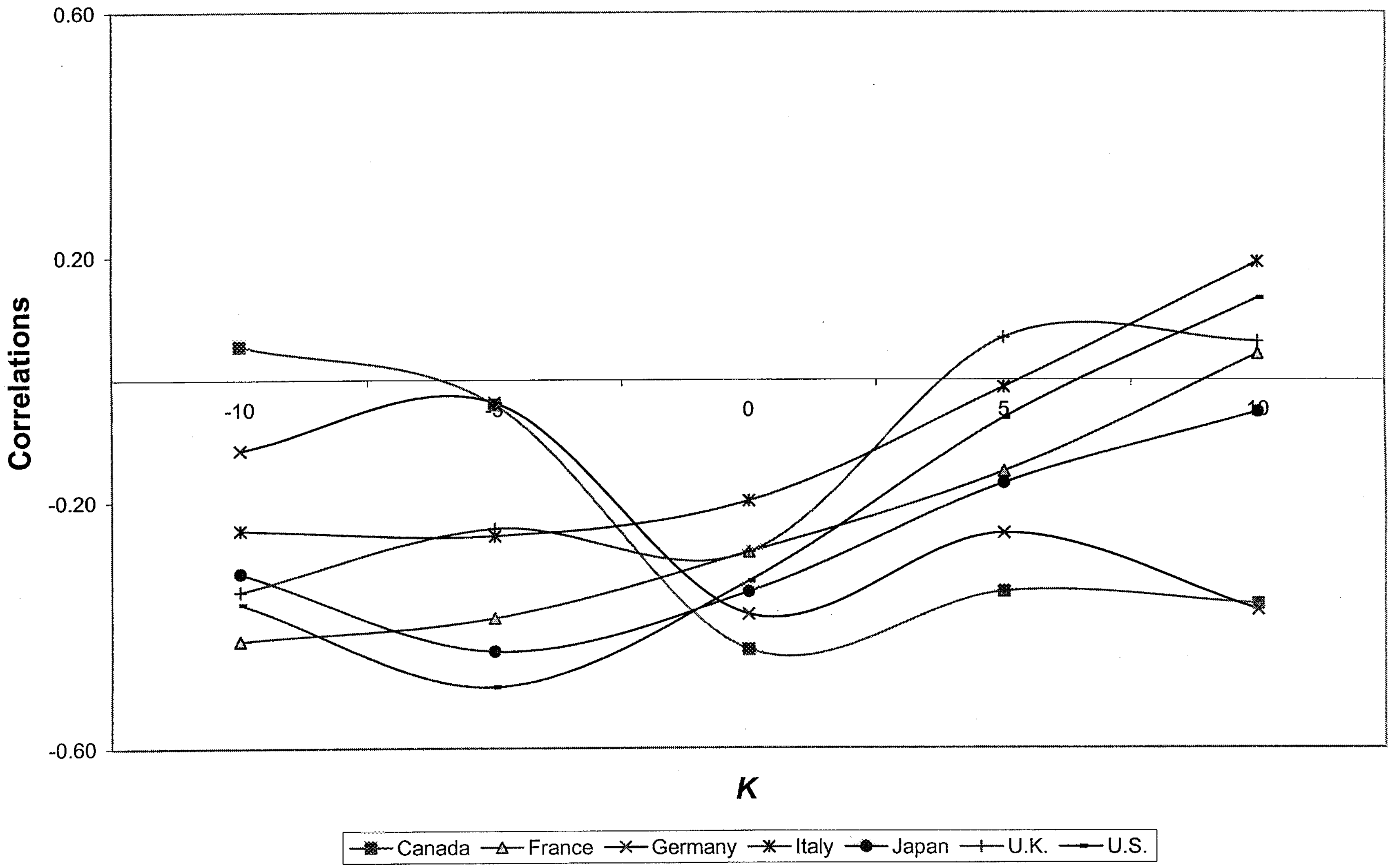
Table 10. Theoretical Correlations Between Real Output and Real Interest Rate in the Extended Model

Variable	Cross-correlation with domestic output				
	k				
	-10	-5	0	5	10
Linear detrended output:					
Home country	-0.21	-0.32	-0.39	-0.35	-0.23
Foreign country	0.03	0.04	0.05	0.06	0.03
First-differenced data:					
Home country	-0.33	-0.39	-0.58	-0.46	-0.33
Foreign country	0.03	0.07	0.08	0.02	0.05
HP-filtered data:					
Home country	-0.29	-0.43	-0.65	-0.44	-0.35
Foreign country	0.03	0.07	0.09	0.02	0.08

Table 11. Theoretical Correlations Between Real Output Differentials and Real Interest Rate Differentials in the Extended Model

Variable	Cross-correlation				
	k				
	-10	-5	0	5	10
Differentials in levels of output:	-0.14	-0.17	-0.29	-0.33	-0.19
Differentials in growth rates of output:	-0.22	-0.31	-0.41	-0.24	-0.13

Figure 1. Cross-Correlations Between Output and Real Interest Rates in G-7 Countries



Description of the Variables and Sources

Variable name	Definition	Source
Output	Real GNP net of government expenditure	Citibase, Cansim and IFS
Output Growth	Rate of growth of real GNP	Citibase, Cansim and IFS
Price index for output	GNP deflator	IFS
Inflation	Rate of change of Consumer Price Index (CPI)	IFS
Nominal interest rate	Nominal returns on three-month T-bills	IFS
Real interest rate	Nominal interest rate minus the contemporaneous rate of Inflation	IFS

The Social Planner's Problem

This section contains a full description of the social planner's problem , the first-order conditions and the steady-state equilibrium. The social planner maximizes:

$$\max \beta^t \sum_{t=0}^{\infty} \left[U\left(C_t, C_{t-1}, L_t\right) + U^*\left(C_t^*, C_{t-1}^*, L_t^*\right) \right]$$

over

$$\left(C_{Tt}, C_{Nt,} L_t, N_{Tt,} N_{Nt}, I_{Tt}, I_{Tt}, K_{T(t+1)}, K_{N(t+1)}\right)$$

in the home country and over

$$\left(C_{Tt}^*, C_{Nt,}^* L_t^*, N_{Tt,} N_{Nt}^*, I_{Tt}^*, I_{Tt}^*, K_{T(t+1)}^*, K_{N(t+1)}^*\right)$$

in the foreign country subject to the market clearing conditions for traded and non-traded sector:

$$C_{Tt} + C_{Tt}^* + I_{Tt} + I_{Tt}^* = Y_{Tt} + Y_{Tt}^* \tag{27}$$

$$C_{Nt} + I_{Nt} = Y_{Nt} \tag{28}$$

$$C_{Nt}^* + I_{Nt}^* = Y_{Nt}^* \tag{29}$$

the four equations describing the evolution of the capital stock:

$$I_{Tt} = \left[\frac{K_{T(t+1)}}{K_{Tt}} - (1-\delta) \right]^{\frac{1}{\mu}} K_{Tt} \tag{30}$$

$$I_{Nt} = \left[\frac{K_{N(t+1)}}{K_{Nt}} - (1-\delta) \right]^{\frac{1}{\mu}} K_{Nt} \tag{31}$$

$$I_{Tt}^* = \left[\frac{K_{T(t+1)}^*}{K_{Tt}^*} - (1-\delta) \right]^{\frac{1}{\mu}} K_{Tt}^* \tag{32}$$

$$I_{Nt}^* = \left[\frac{K_{N(t+1)}^*}{K_{Nt}^*} - (1-\delta) \right]^{\frac{1}{\mu}} K_{Nt}^* \tag{33}$$

and the labor constraints in each country:

$$N_{Tt} + N_{Nt} + L_t = 1 \tag{34}$$

$$N_{Tt} + N_{Nt} + L_t = 1 \tag{35}$$

The first-order conditions are:

$$\begin{aligned} K_{T(t+1)} \quad &: \quad -\beta^t \frac{\partial U_t}{\partial C_{Tt}} + \beta^{t+1} \left[\frac{\partial U_{t+1}}{\partial C_{T(t+1)}} \left(\frac{\partial Y_{T(t+1)}}{\partial K_{T(t+1)}} + (1-\delta) \right) - \frac{\partial U_{t+1}}{\partial C_{Tt}} \right] \\ & \quad +\beta^{t+2} \left[\frac{\partial U_{t+2}}{\partial C_{t+1}} \left(\frac{\partial Y_{T(t+1)}}{\partial K_{T(t+1)}} + (1-\delta) \right) \right] \\ &= \quad 0 \end{aligned} \tag{36}$$

$$\begin{aligned} K_{N(t+1)} \quad &: \quad -\beta^t \frac{\partial U_t}{\partial C_{Nt}} + \beta^{t+1} \left[\frac{\partial U_{t+1}}{\partial C_{Nt+1}} \left(\frac{\partial Y_{N(t+1)}}{\partial K_{N(t+1)}} + (1-\delta) \right) - \frac{\partial U_{t+1}}{\partial C_{Nt}} \right] \\ & \quad +\beta^{t+2} \left[\frac{\partial U_{t+2}}{\partial C_{N(t+1)}} \left(\frac{\partial Y_{N(t+1)}}{\partial K_{N(t+1)}} + (1-\delta) \right) \right] \\ &= \quad 0 \end{aligned} \tag{37}$$

$$\begin{aligned} K^*_{T(t+1)} \quad &: \quad -\beta^t \frac{\partial U^*_t}{\partial C^*_{Tt}} + \beta^{t+1} \left[\frac{\partial U^*_{t+1}}{\partial C^*_{T(t+1)}} \left(\frac{\partial Y^*_{T(t+1)}}{\partial K^*_{T(t+1)}} + (1-\delta) \right) - \frac{\partial U^*_{t+1}}{\partial C^*_{Tt}} \right] \\ & \quad +\beta^{t+2} \left[\frac{\partial U^*_{t+2}}{\partial C^*_{t+1}} \left(\frac{\partial Y^*_{T(t+1)}}{\partial K^*_{T(t+1)}} + (1-\delta) \right) \right] \\ &= \quad 0 \end{aligned} \tag{38}$$

$$\begin{aligned} K^*_{N(t+1)} \quad &: \quad -\beta^t \frac{\partial U^*_t}{\partial C^*_{Nt}} + \beta^{t+1} \left[\frac{\partial U^*_{t+1}}{\partial C^*_{Nt+1}} \left(\frac{\partial Y^*_{N(t+1)}}{\partial K^*_{N(t+1)}} + (1-\delta) \right) - \frac{\partial U^*_{t+1}}{\partial C^*_{Nt}} \right] \\ & \quad +\beta^{t+2} \left[\frac{\partial U^*_{t+2}}{\partial C^*_{N(t+1)}} \left(\frac{\partial Y^*_{N(t+1)}}{\partial K^*_{N(t+1)}} + (1-\delta) \right) \right] \\ &= \quad 0 \end{aligned} \tag{39}$$

$$N_{Tt} : \frac{\partial U_t}{\partial C_{Tt}} \frac{\partial Y_{Tt}}{\partial N_{Tt}} = \frac{\partial U_t}{\partial L_t} \tag{40}$$

$$N_{Nt} : \frac{\partial U_t}{\partial C_{Nt}} \frac{\partial Y_{Tt}}{\partial N_{Nt}} = \frac{\partial U_t}{\partial L_t} \tag{41}$$

$$N_{Tt}^* : \frac{\partial U_t^*}{\partial C_{Tt}^*} \frac{\partial Y_{Tt}^*}{\partial N_{Tt}^*} = \frac{\partial U_t^*}{\partial L_t^*} \tag{42}$$

$$N_{Nt}^* : \frac{\partial U_t^*}{\partial C_{Tt}^*} \frac{\partial Y_{Tt}^*}{\partial N_{Nt}^*} = \frac{\partial U_t^*}{\partial L_t^*} \tag{43}$$

where

$$\frac{\partial U_t}{\partial C_{Tt}} = \frac{\alpha C_t}{C_{Tt}} \left[(C_t - \phi C_{t-1})^{-\gamma} L_t^a \right]$$

$$\frac{\partial U_t}{\partial C_{Nt}} = \frac{\alpha C_t}{C_{Nt}} \left[(C_t - \phi C_{t-1})^{-\gamma} L_t^a \right]$$

$$\frac{\partial U_t^*}{\partial C_{Tt}^*} = \frac{\alpha C_t^*}{C_{Tt}^*} \left[\left(C_t^* - \phi C_{t-1}^*\right)^{-\gamma} L_t^{*a} \right]$$

$$\frac{\partial U_t^*}{\partial C_{Tt}^*} = \frac{\alpha C_t^*}{C_{Tt}^*} \left[\left(C_t^* - \phi C_{t-1}^*\right)^{-\gamma} L_t^{*a} \right]$$

$$\frac{\partial U_t}{\partial C_{Tt-1}} = -\phi \frac{\alpha C_{t-1}}{C_{Tt-1}} \left[(C_t - \phi C_{t-1})^{-\gamma} L_t^a \right]$$

$$\frac{\partial U_t}{\partial C_{Nt-1}} = -\phi \frac{\alpha C_{t-1}}{C_{Nt-1}} \left[(C_t - \phi C_{t-1})^{-\gamma} L_t^a \right]$$

$$\frac{\partial U_t^*}{\partial C_{Tt-1}^*} = -\phi \frac{\alpha C_{t-1}^*}{C_{Tt-1}^*} \left[\left(C_t^* - \phi C_{t-1}^*\right)^{-\gamma} L_t^{*a} \right]$$

$$\frac{\partial U_t^*}{\partial C_{Tt-1}^*} = -\phi \frac{\alpha C_{t-1}^*}{C_{Tt-1}^*} \left[\left(C_t^* - \phi C_{t-1}^*\right)^{-\gamma} L_t^{*a} \right]$$

$$\frac{\partial U_t}{\partial L_t} = \frac{a \left(C_t - \phi C_{t-1}\right)^{1-\gamma} L_t^{a-1}}{1-\gamma}$$

$$\frac{\partial U_t^*}{\partial L_t^*} = \frac{a(C_t^* - \phi C_{t-1}^*)^{1-\gamma} L_t^{*a-1}}{1-\gamma}$$

We approximate the first-order conditions near the stationary point.

Steady State Equilibrium

The steady state of the economy (in the home country) depends on structural parameters by the following relations:

$$\frac{Y_T}{K_T} = \frac{1}{\beta\theta_T}\left[\frac{1-\beta}{\mu}\delta^{\frac{1-\mu}{\mu}} + \beta\delta^{\frac{1}{\mu}}\right] \tag{44}$$

$$\frac{Y_N}{K_N} = \frac{1}{\beta\theta_T}\left[\frac{1-\beta}{\mu}\delta^{\frac{1-\mu}{\mu}} + \beta\delta^{\frac{1}{\mu}}\right] \tag{45}$$

$$\frac{C_T}{Y_T} = 1 - \delta^{\frac{1}{\mu}}\frac{K_T}{Y_T} \tag{46}$$

$$\frac{C_N}{Y_N} = 1 - \delta^{\frac{1}{\mu}}\frac{K_N}{Y_N} \tag{47}$$

$$N_T = \frac{1}{1 + \frac{a(1-\phi)C_T}{\alpha(1-\gamma)(1-\beta\phi)(1-\theta_T)Y_T} + \frac{(1-\alpha)(1-\theta_N)Y_N C_T}{\alpha(1-\theta_T)Y_T C_N}} \tag{48}$$

$$N_N = \frac{1}{1 + \frac{a(1-\phi)C_N}{\alpha(1-\gamma)(1-\beta\phi)(1-\theta_N)Y_N} + \frac{(1-\alpha)(1-\theta_N)Y_T C_N}{\alpha(1-\theta_T)Y_N C_T}} \tag{49}$$

$$I_T = \delta^{\frac{1}{\mu}} K_T \tag{50}$$

$$I_N = \delta^{\frac{1}{\mu}} K_N \tag{51}$$

$$1 + r = \frac{1}{\beta} \tag{52}$$

The steady state is thus affected by the habit persistence parameter, ϕ and the adjustment cost parameter μ.