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Soft Exchange Rate Bands and Speculative Attacks: Theory, and Evidence from the ERM since August 1993

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Abstract

We present a model of a "soft" exchange rate target zone and interpret it as a stylized description of the post-August 1993 ERM. Our central bank targets a moving average of the current and past exchange rates, rather than the exchange rate's current level, thus allowing the rate to move within wide margins in the short run, but within narrow margins in the long run. For realistic parameters, soft target zones are significantly less vulnerable to speculative attacks than "hard" target zones. These predictions are consistent with the ERM's experience and the abatement of speculative pressure in European markets since the bands' widening in 1993.

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SUMMARY

Despite the interest and active research generated by target zone models, much work remains to be done to capture the institutions and stylized facts describing the experience of countries that have adopted target-zone-like arrangements in recent years. In this study, we present a model of a soft target zone with two main goals: to extend research on target zones to include a "soft-band" policy that has long interested scholars of currency markets, but has never received rigorous treatment; and to capture some key aspects of the wide-band ERM intervention policy and explain its resilience against speculation after the post-August 1993 period.

The main feature of our model is the assumption that exchange rate intervention responds not only to the level of the exchange rate at a given point in time, but also to its behavior over the recent past. We develop our model as a direct extension of standard target zone models and show that, for reasonable parameters, soft target zones should be considerably more resilient to speculative attacks than hard target zones. This prediction is consistent with the sharp abatement of speculative pressure in Europe since August 1993: at that time, ERM central banks adopted a policy that, while committing to a narrow exchange rate target over long horizons, has tolerated wider fluctuations over short horizons. The new policy regime has brought greater stability to European currency markets: after enduring at least two dozen speculative attacks from August 1992 to July 1993, the ERM has been threatened on only a couple of occasions since August 1993. Our analysis suggests that this performance can be traced-at least in part-to European central banks' "soft" approach to exchange rate intervention, and points to the usefulness of this policy for other countries that target their exchange rates.

I. Introduction

If one feature of European currency markets since the crisis of 1993 stands out for lack of a clear theoretical explanation, it is the sudden achievement of speculative peace after the turbulent 1992-93 period. After enduring at least two dozen speculative attacks from August 1992 to July 1993, the European Exchange Rate Mechanism (ERM) has been threatened on only a couple of occasions since August 1993, mainly in early 1995, when the Spanish peseta and the Portuguese escudo were realigned.[1]

Clearly, not much of an explanation would be needed to account for this experience had ERM currencies been allowed to float freely within their new margins when the bands were widened from ±2¼ percent (±6 percent for the Spanish peseta and Portuguese escudo) to ±15 percent in August 1993. However, that is hardly what happened. As Figure 1 shows, ERM exchange rates have never come close to their new limits after August 1993, as European central banks have continued to confine them within narrower margins, usually intervening when the rates hovered around their former intervention limits. Indeed, European officials have repeatedly insisted that the formal shift to a wide-band ERM did not reflect a relaxation of their commitment to narrow exchange rate bands—except possibly in the short run, under strong market pressure, and with the understanding that "the room for maneuver allowed by the new ERM margins would only be used to absorb self-reversing market movements" (European Monetary Institute, 1995, p. 18). In this light, it is not surprising that the financial press and central bank watchers have continued to focus on the old ±2¼ percent ERM bands, as indicators of likely official intervention, even after the shift to wide bands in August 1993.

Similarly, had European "fundamentals" sharply converged after the shift to wide bands, one would normally expect speculative pressure to subside. However, it is widely acknowledged that substantial macroeconomic convergence among EU countries was not achieved until late-1996 as reflected, for instance, by these countries' exchange rate volatility finally returning to its pre-crisis levels (see European Monetary Institute, 1996b). Furthermore, while the approaching EMU deadline of January 1999 can explain EU fundamentals' gradual harmonization, it can hardly explain the overnight calming of speculative pressure on August 1, 1993.

By common account, the main reason for the speculative truce achieved in August 1993 lies in ERM central banks' adoption of a policy that, while committing to a narrow exchange rate target over long horizons, has tolerated wider fluctuations over short horizons. Having dropped the commitment to keep exchange rates within narrow margins *at all times*, European central banks have not been forced to offer "one-way bets" to short-lived

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[1]See European Monetary Institute (1995, 1996a) and Begg et al. (1997) for a survey of developments in European currency markets since the advent of the wide-band ERM.

Figure 1

ERM Exchange Rates Against the DM

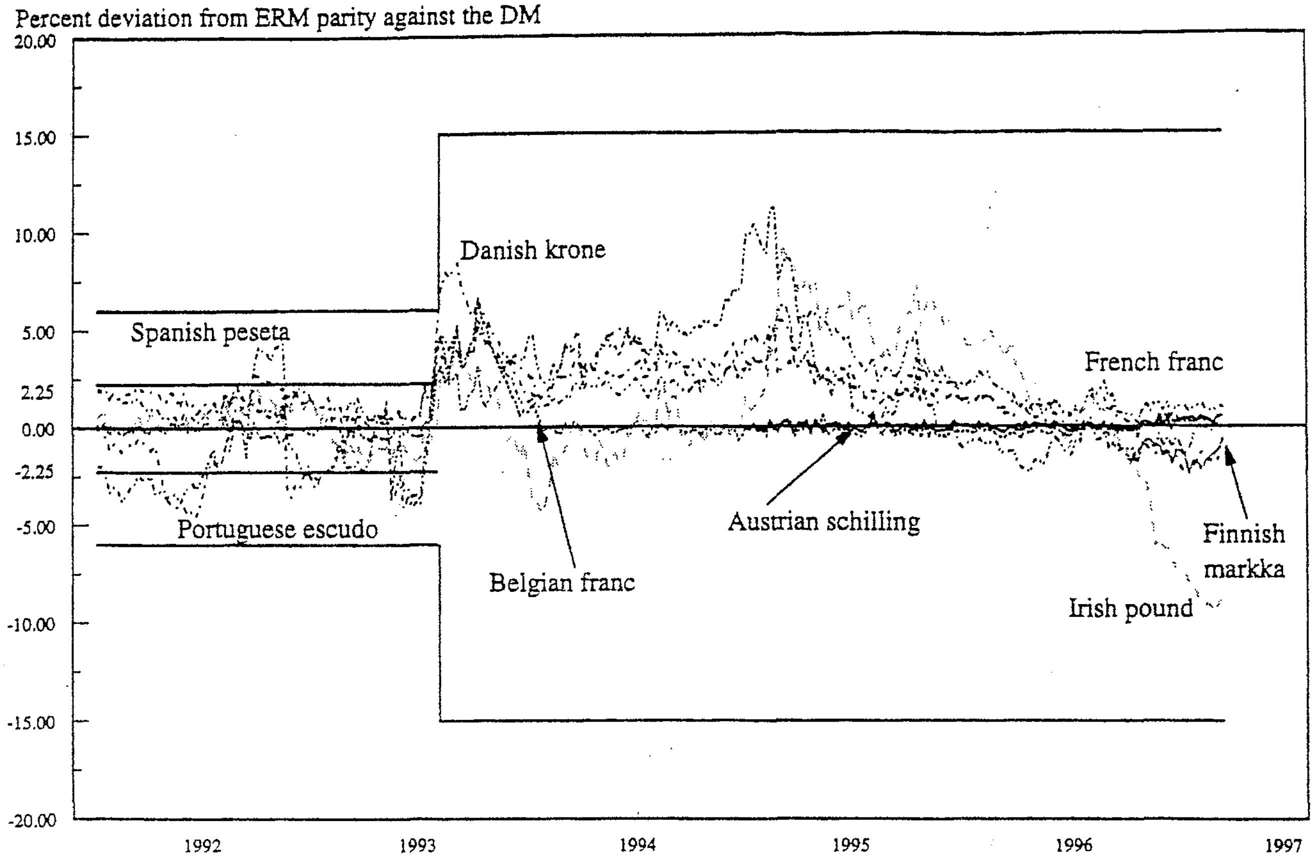
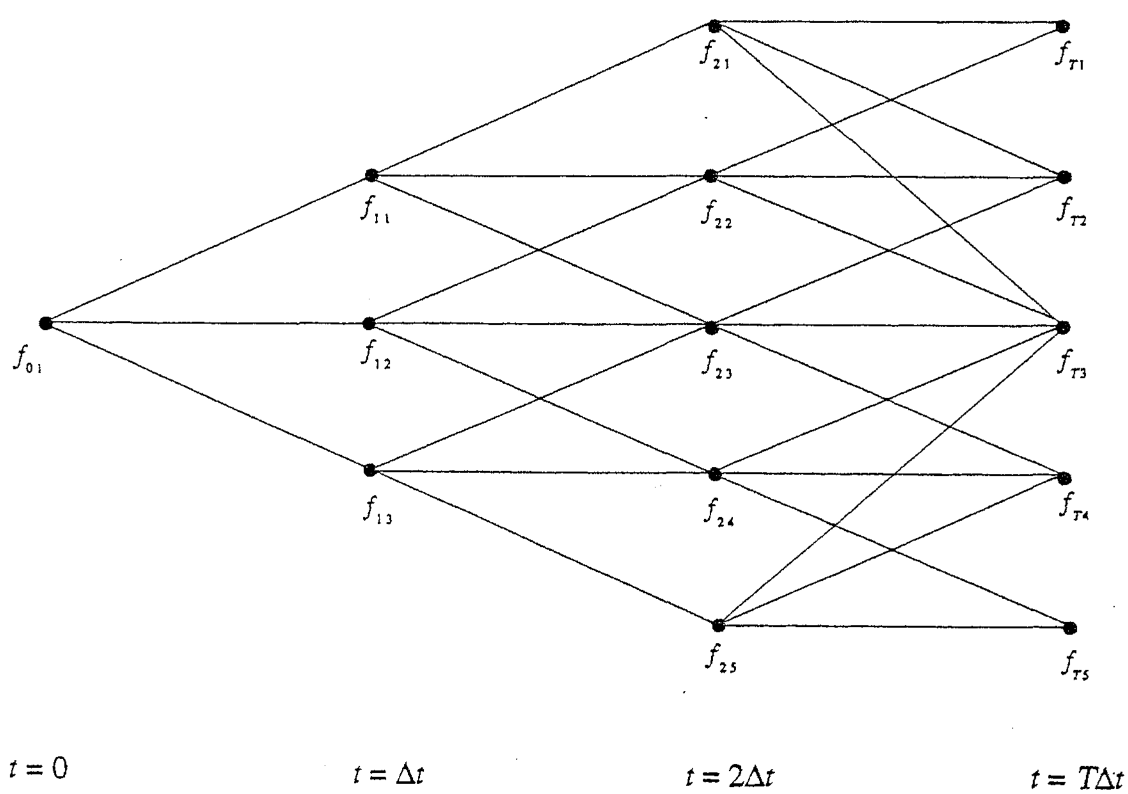


Figure 2
The State-Space for Fundamentals



speculative spurts, thus undermining the scope for speculators' expectations to coalesce on devaluations.[2]

Although this view of the post-August 1993 ERM experience is widespread in policy circles, it has no formal counterpart in economic analysis. In this paper, we take one step toward bridging this gap by developing a model of a "soft" target-zone, which we view as a reasonable—if stylized—characterization of the ERM since the 1993 crisis. We study the properties of this model and its implications for exchange rate dynamics and the vulnerability of exchange rate targets to speculative attacks. Upon calibrating the model with data from the French franc/Deutsche mark (FF/DM) and Spanish peseta/Deutsche mark (SP/DM) markets, we find that soft intervention could go a long way toward explaining the dramatic soothing of speculative pressure in European currency markets after August 1993.

The key feature of our soft target zone model is that it shifts the reference for exchange rate intervention from the level of the exchange rate at each instant to the behavior of the exchange rate over a time interval. In particular, we assume that the central bank intervenes to keep within an assigned band not the *current level* of the exchange rate, as in standard target zone models, but rather *a moving average* of the current and past exchange rates. The main implication of this policy is that it allows exchange rates to fluctuate within a potentially wide band over short horizons, while keeping them on average within a narrow band over longer horizons. This property implies that the target zone becomes significantly less vulnerable to speculative pressure, as the central bank can often postpone the erosion of foreign exchange reserves (or base-money, more generally) until the exchange rate shocks have worn out.

Analytically, our study lies at the crossroads of the literature on exchange rate target zones originated by Krugman (1991) and the literature on the pricing of history-dependent options.

From the target zone literature (see Svensson, 1992, for a review), we draw much of our conceptual framework, including our basic model of exchange rate determination and speculative attacks. In particular, our work is inspired by Williamson's (1985) early discussion of target zones with "soft buffers," and can, in fact, be viewed as a formalization of Williamson's idea and of subsequent discussions of the likely properties of his proposed

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[2]Even when the Spanish peseta's and Portugese escudo's central parities were realigned in early-1995, these currencies' standing with respect to the old ERM limits seem to have been the primary consideration: in 1994 "the French franc and the Danish krone remained stable, *on average,* around their former lower bilateral limits" while "the Spanish peseta and the Portugese escudo stayed above their former limits" (European Monetary Institute, 1995, p. 18; our italics).

policy.[3] Our work is also related to that of Klein (1992), Klein and Lewis (1993), and Lewis (1995), authors with whom we share a concern for developing more realistic models of target zone intervention, and from whose work we draw upon to extend our basic model in a later section.

From the option literature, we draw a solution method suggested by the analogy between our exchange rate problem and the problem of pricing so-called Asian options, namely, options whose payoff depends on the average price of a security over a time interval. We build on a method developed by Hull and White (1993) to solve a problem that, as discussed below, is substantially more difficult than that of pricing Asian options.

The paper is structured as follows: Section II presents the model and Section III describes its solution. Section IV calibrates the model using post-August 1993 ERM data, and Section V studies its implications for exchange rate dynamics and for the vulnerability of a soft target zone to speculative attacks. Section VI outlines some extensions, and Section VII concludes.

II. A Model of a Soft Target Zone

Apart from technicalities such as its discrete-time set-up, our basic model is standard in the target zone literature. Consider the familiar forward-looking exchange rate equation

$$s_t = f_t + \alpha \mathrm{E}_t\left[\Delta s_t / \Delta t \right], \tag{1}$$

where s_t denotes the (log) exchange rate, defined as the domestic price of a unit of foreign currency, f_t denotes variables "fundamental" to the determination of exchange rates, $E_t[.]$ is the usual rational expectation operator, and Δt is the time interval.

We treat fundamentals, $f_t \equiv m_t + v_t$, as the sum of two components: a component controlled by the central bank, the "monetary base" m_t, and a residual, all-inclusive "velocity" term, v_t. (More precisely, m_t is the difference between domestic and foreign base-moneys.) A standard no-bubble assumption, $\lim_{i\to\infty} \left(\frac{\alpha}{1+\alpha}\right)^i E_t[S_{t+i\cdot\Delta t}]$, allows us to solve (1) forward solely in terms of f_t and of the average exchange rate that—as we shall see—determines the intervention policy.

Let velocity follow the auto-regressive process

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[3]See, for instance, Frenkel and Goldstein (1986) and, more recently, Arrowsmith (1966). See also Labhard and Wyplosz (1996), who also interpret the post-August 1993 ERM as a narrow target zone with soft margins and provide evidence for this interpretation.

$$\Delta v_t = \lambda\left[\theta_t - v_t\right]\Delta t + \sigma\epsilon_t\sqrt{\Delta t}, \tag{2}$$

where θ_t is the reversion level of v_t and ϵ_t is a stochastic shock to velocity.

Equation (2) is a flexible characterization of the velocity process, whose limit for $\Delta t \rightarrow 0$ is a mean-reverting process widely adopted in the target zone literature (see, for instance, Delgado and Dumas, 1992). According to (2), velocity follows a process with a tendency to return to θ_t at the rate λ. When $\lambda=0$, (2) degenerates into a martingale with no tendency to revert to any value. Equation (2) can also be written as

$$\Delta f_t \equiv \Delta v_t + \Delta m_t = \lambda\left[\left(\theta_t + m_t\right) - f_t\right]\Delta t + \Delta m_t + \sigma\epsilon_t\sqrt{\Delta t}. \tag{3}$$

If, as suggested by Delgado and Dumas (1992), the central bank adjusts θ_t to keep the target rate for f_t, $\theta_t + m_t$, constant at, say, μ/λ (except, possibly, at realignments), then equation (3) can be written as

$$\Delta f_t = \left[\mu - \lambda f_t\right]\Delta t + \Delta m_t + \sigma\epsilon_t\sqrt{\Delta t}, \tag{4}$$

or

$$\Delta v_t = \left[\mu - \lambda\left(v_t + m_t\right)\right]\Delta t + \sigma\epsilon_t\sqrt{\Delta t}. \tag{5}$$

Equation (5) is a tractable specification that will be estimated in Section 4.

Given equation (1) and the forcing process (2) (or (5)), the model is closed by specifying the intervention policy at the edges of the band. It is here where our model departs from previous target zone models. Instead of assuming the central bank to intervene to keep the exchange rate within a band $[-U,U]$ at all times, we assume the central bank to intervene to keep only a moving average $\bar{s}_t$ of past exchange rates within the band $[-U,U]$. The central bank does so by creating or destroying just enough base-money to prevent $\bar{s}_t$ from straying outside the band $[-U,U]$.[4]

The main feature of this policy—which, following Williamson (1985), we refer to as a "soft" target zone—is that it lets the central bank care about the behavior of the exchange rate over a *time interval*, rather than just its current level. The particular way in which past exchange rates are brought into play has no important qualitative implications for our analysis. For this reason, we begin by focusing on a definition of $\bar{s}_t$ that seems both simple and reasonably realistic, in that it assumes the central bank to be more concerned with the exchange rate's recent behavior than with its long-past behavior. (We consider alternative

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[4]It is here immaterial whether changes in m_t reflect domestic or foreign intervention. For simplicity, we refer to intervention as being performed only by the domestic central bank.

specifications of $\bar{s}_t$ in Section VI.) We define $\bar{s}_t$ as an average of the current exchange rate and the previous period's average, or

$$\bar{s}_t \equiv \delta s_t + (1-\delta)\bar{s}_{t-\Delta t} = \delta \sum_{i=0}^{\infty} (1-\delta)^i s_{t-i\cdot\Delta t}, \quad \delta = (0, 1] \tag{6}$$

The essential implication of this policy is that, while it commits the rate s_t *on average* to the range $[-U, U]$, it allows it to stray outside this range in any given period t, the more so the smaller is the weight δ attached to the current rate. In particular, the band for the exchange rate s_t varies as a function of the history of the exchange rate. To see this clearly, re-write (6) as

$$s_t = \frac{\bar{s}_t}{\delta} - \frac{1-\delta}{\delta}\bar{s}_{t-\Delta}, \tag{7}$$

from which the single-period band for s_t is defined as a function of $\bar{s}_{t-\Delta t}$ as

$$\frac{-U - (1-\delta)\bar{s}_{t-\Delta t}}{\delta} \leq s_t \leq \frac{U - (1-\delta)\bar{s}_{t-\Delta t}}{\delta}. \tag{8}$$

Thus, for instance, the lower $\bar{S}_{t-\Delta t}$ is (i.e., the more appreciated has the exchange rate been recently), the greater the tolerance is for a rate temporarily exceeding the upper target U, and vice versa. Hence, a central bank that succeeds in keeping the rate mostly in the middle of the band $[-U, U]$, enjoys more flexibility to allow short-lived fluctuations of s_t outside that band in any given period t. Despite this flexibility—and this is the key feature of our model—the exchange rate remains committed on average to the same band $[-U, U]$ as in a hard target zone.

For our subsequent discussion, it is helpful to summarize the intervention policy by the "mean intervention lag" L implicit in the moving average $\bar{s}_t$, namely $L \equiv \delta\sum_{i=0}^{\infty} (1-\delta)^i \, i\Delta t = \frac{1-\delta}{\delta}\Delta t$ The mean intervention lag L captures the central bank's backward horizon when intervening, that is, the relative importance the bank attaches to short-lived and persistent changes in exchange rates. L varies with δ between zero and infinity. When $\delta = 1$, for instance, then $L = 0$. In this case, intervention depends only on the current exchange rate. The exchange rate is then constrained to remain within the band $[-U, U]$ at all times, and our soft-band collapses into the standard "hard"-band modeled by Krugman (1991). In contrast, when $\delta<1$, then $L>0$, and the past behavior of the exchange rate matters for intervention: temporary deviations of the rate from the band $[-U, U]$ are tolerated, the more so the higher is L.

Finally, we establish a basis to compare the resilience of soft and hard target zones to speculation. We introduce in our model a motive for speculative attacks in the spirit of Krugman (1979), as adapted to target zone models by Krugman and Rotemberg (1992) and Delgado and Dumas (1993), and compare the vulnerability of soft and hard target zones to speculative attacks in terms of their expected lifetime until such an attack occurs.[5]

As in the classic speculative attack literature, we assume that there is a minimum acceptable limit to a country's monetary base, below which the central bank is unwilling (or unable) to dip. (Without loss of generality, we normalize this lower bound at zero.) The presence of this limit—combined with the no-arbitrage requirement that the shadow (i.e., post-collapse) exchange rate never exceeds the market exchange rate when the central bank intervenes in defense of the target zone—causes a speculative attack when the domestic base-money dwindles to an endogenous, strictly positive level.[6] At that point, the exchange rate flows smoothly out of the target zone, while the domestic monetary base suddenly drops to zero.

III. Solution of the Model

A. Outline

Despite its conceptual simplicity, our model is difficult to solve. The exchange rate s_t depends on future rates through equation (1) and on past rates through the intervention policy. Thus, all exchange rates at all dates are linked in a highly non-linear fashion.

Our problem resembles that of valuing so-called Asian options, whose pay-off depends on the average price of an underlying security over a certain time interval. This analogy suggests a solution method, widely employed in the finance literature, that involves casting the state-space of f_t in the form of a tree, assigning transition probabilities to each node to replicate the stochastic properties of (5), and solving the model backward, beginning with an arbitrary solution for the terminal period.[7] (Following Hull and White, 1993, in particular, we

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[5]To minimize the departure from standard analysis of target zones, we focus on speculative attacks driven by the dynamics of fundamentals. In Bartolini and Prati (1997) we discuss informally self-fulfilling speculative attacks on soft exchange rate targets.

[6]To rule out a "gold standard paradox," whereby a central bank under attack gains reserves during a crisis, we assume that the post-collapse regime is a one-side target zone in which intervention resumes if $\bar{s}_t$ falls again to $-U$ after the attack—see Krugman and Rotemberg (1992) and Delgado and Dumas (1993), for a discussion.

[7]Attempt to solve the model analytically encounters formidable problems, mainly in the form of a non-standard, non-linear partial differential equation, expressing s_t as a function of f_t and

(continued...)

work with a trinomial tree, which allows us to replicate a mean-reverting fundamental process.)

However, our problem is more complicated than that of valuing Asian options. Here the exchange rate is a function of past and expected future *exchange rates*, not of past fundamentals alone. (The option-pricing analog would be to value a derivative whose payoff depends on its own past and future prices.) We overcome this problem by defining an additional state variable—the average exchange rate $\bar{s}_t$—, solving the model backward contingent on each period's possible inherited averages, and assuring consistency between the forward solution for the exchange rate and the transition equation for its average by requiring the exchange rate to satisfy a fixed point for $\bar{s}_t$ in each period. This solution can be made arbitrarily accurate by shortening the time- and $\bar{s}_t$- steps and by increasing the fundamental tree's horizon.

The remainder of this sections gives details of this solution.

B. Definition of the State-Space

The definitions of the fundamentals' state-space and of the transition probabilities follow Hull and White (1993). We define a state-space for f_t in the form of a trinomial tree, with $T+1$ periods, $t = 0, \Delta t, \ldots, T{\cdot}\Delta t$, and n_t fundamental positions in each period (see Figure 2 for illustration).[8] Fundamentals in each period are denoted by f_{ti}, with $i=1, 2, \ldots, n_t$. The fundamental step is Δf (with $\Delta f \equiv \Delta m \equiv \Delta v$) and the time step is Δt. Δt is chosen independently, while Δf is set at $\Delta f = \sigma\sqrt{3\Delta t}$ to assure positive transition probabilities.

Along the tree, f_t starts from f_{01} at time zero and emanates along three branches at each node, taking $n_t \equiv 1+2(t/\Delta t)$ possible values at t, spaced Δf apart. The transition probabilities are defined so that the volatility and time-varying drift of the trinomial process match those of (5). Absent intervention, in each period fundamentals follow the dynamics of v_t and move from f_{ti} to

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[7](...continued)
$\bar{s}_t$. Additional complications include the difficulty of separating this equation's boundary conditions and the need to evaluate numerically the power series arising with mean reversion. Details on the set up of the analytical solution of our model are available upon request.

[8]In Figure 2, note how the tree branches out: for fundamentals far away from their reversion point, the mid branch is shifted toward the middle of the tree to assure positive transition probabilities as $f_t \ll \mu/\lambda$ or $f_t \gg \mu/\lambda$. See Hull and White (1993) for a complete discussion.

$$
\begin{cases}
f_{t+\Delta t,k_i} + \Delta f, & \text{with probability} \quad \dfrac{\sigma^2 \Delta t}{2(\Delta f)^2} + \dfrac{\eta_{ti}^2}{2(\Delta f)^2} + \dfrac{\eta_{ti}}{2\Delta f}, \\[2ex]
f_{t+\Delta t,k_i}, & \text{with probability} \quad 1 - \dfrac{\sigma^2 \Delta t}{(\Delta f)^2} - \dfrac{\eta_{ti}^2}{(\Delta f)^2}, \\[2ex]
f_{t+\Delta t,k_i} - \Delta f, & \text{with probability} \quad \dfrac{\sigma^2 \Delta t}{2(\Delta f)^2} + \dfrac{\eta_{ti}^2}{2(\Delta f)^2} - \dfrac{\eta_{ti}}{2\Delta f},
\end{cases}
\tag{9}
$$

where $\eta_{ti} \equiv \mu_{ti}\Delta t + (k_i - i - 1)\Delta f$, and $\mu_{ti} \equiv [\mu - \lambda f_{ti}]$ is the drift at f_{ti}. The integer k_i is chosen so that the fundamental reached at $t+\Delta t$ by the middle branch, $f_{t+\Delta t,k_i} = f_{ti} + k_i\Delta f$, lies as close as possible to $f_{ti} + \mu_{ti}\Delta t$, the fundamental expected at $t + \Delta t$ as of t. Thus, η_{ti} is the fundamental drift at f_{ti}, net of the drift induced by shifting the branches down by $(k_i - i - 1)\Delta f$.

We also construct a state-space for the exchange rate average $\bar{s}_t$, which allows us to solve for each exchange rate at t contingent on the inherited average $\bar{s}_{t-\Delta t}$. We consider a vector of M possible averages, $\{\bar{s}_1, \ldots, \bar{s}_j, \ldots, \bar{s}_M\}$, which yields $n_t M$ exchange rates s_{tij} to be determined in each period t, each conditional on a fundamental $i = 1,\ldots, n_t$ and an average $j = 1,\ldots,M$. We denote by $\mathbf{S}_t$ the exchange rate matrix at t, with typical element s_{tij}. Our task is to solve for all s_{tij}, $t = 0,\ldots, T\cdot\Delta t$, $i=1,\ldots, n_t$, $j=1,,\ldots, M$, and use the matrix $\mathbf{S}_0$ as the model's solution.

C. Solution

The solution proceeds backward from $t = T\cdot\Delta t$ to $t=0$, beginning with an arbitrary solution for the terminal period (which we set at the standard target zone solution with mean-reverting fundamentals presented by Delgado and Dumas, 1992) and choosing T large enough to make the importance of the terminal solution negligible.

Given a complete solution for $S_{t+\Delta t}$, the first step to solve for $\mathbf{S}_t$ is to apply the pricing equation (1) to each element (i, j) of $\mathbf{S}_t$, with probabilities defined by (9). This calculation yields M different solutions for each s_{tij}, each conditional on an average $\bar{s}_j$ that can, in principle, be reached at t starting from the history $\bar{s}_j$ inherited at t from $t-\Delta t$. Among these M solutions, however, only one is consistent with the transition equation (6) and the average $\bar{s}_j$ inherited from $t-\Delta t$. This solution is given by the unique fixed point for j' of the equation

$$
\bar{s}_{j'} \approx \delta \left(\frac{f_{ti}}{1+\alpha} + \frac{\alpha}{1+\alpha} \mathrm{E}_t[s_{t+\Delta t,.,j'}] \right) + (1-\delta)\bar{s}_j \tag{10}
$$

In equation (10), obtained by combining (1) and (6), the expectation $E_t[s_{t+\Delta t,\,.\,,j'}]$ is taken over fundamentals i in accord with (9), while the sign $\approx$ indicates that a fixed point is sought within a $\bar{s}_j$ -step. Uniqueness of the fixed point follows from the fact that as j' rises, the left-hand side of (10) also rises, by construction, while its right-hand side falls, since $E_t[s_{t+\Delta t,\,.\,,j'}]$ does, as future intervention in support of the currency becomes more likely as $\bar{s}_j$ rises, and vice versa.[9] The fixed-point calculation can be made arbitrarily precise by reducing the $\bar{s}_t$-step.

Finally, intervention at time t is incorporated by truncating s_{tij} along each column (history) j in $\boldsymbol{S}_t$ to the range admissible by (8), reflecting the increase or decrease in m_t(and, hence, in $f_t \equiv v_t + m_t$) necessary to support an average exchange rate within the band $[-U, U]$.

This completes the solution for S_t. The procedure is repeated for $\boldsymbol{S}_{t-\Delta t}, \ . \ . \ . \ , \boldsymbol{S}_0$, where S_0 is a 1xM vector of solutions conditional on a single initial fundamental, f_{01}, and M initial histories. The whole procedure is repeated for all elements of the initial fundamental vector.

D. Speculative Attacks and Expected Lifetime of the Target Zone

To determine the lifetime of the target zone, we first obtain the solution $\boldsymbol{S}_0^\infty$ for a one-sided soft target zone, conditional on base-money being at its lowest admissible level of zero. This solution is obtained exactly as described above, as a special case of a two-sided zone S_0 with band $[-U,\infty]$ and m_t=0. The one-sided solution $\boldsymbol{S}_0^\infty$ is then used as the shadow exchange rate (see footnote 6), against which the market rate S_0 must be checked when the central bank intervenes at the edge U. The first time the shadow rate exceeds the market rate, a speculative attack strips the central bank of its remaining base-money, while the exchange rate flows smoothly into a one-sided target zone in which intervention resumes if $\bar{s}_t$ falls as low as -U.[10]

We compute the expected lifetime of the target zone by Montecarlo, as follows. We endow the central bank with an initial stock of base-money m_0; we initialize v_t at $v_0=\mu/\lambda$ (so that $f_0=m_0+\mu/\lambda$), and $\bar{s}_t$ at μ/λ; and we generate 1,000 paths for v_t in accordance with (9).

Along each path, in each period we draw v_t and compute $f_t \equiv v_t + m_t$, which, together with the inherited $\bar{s}_{t-\Delta t}$, uniquely identifies the current exchange rate $s_t = s_{0ij}$. We then use s_t to update $\bar{s}_t$, we draw $v_{t+\Delta t}$, and so on. If, in any period, intervention is required to keep the average exchange rate within the band, fundamentals and base-money are shifted up or down, as necessary, by Δf. If intervention is in support of the currency (i.e., if m_t falls), the post-

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[9]The assumption that the central bank intervenes only by the minimal amount $\Delta m \equiv \Delta f$ at a time, is important, for otherwise the exchange rate would not be a monotonic function of j.

[10]A necessary and sufficient condition for the target zone's expected lifetime to be finite is that $\mu>0$, which makes intervention at U unconditionally more likely than intervention at -U.

intervention exchange rate is checked against the corresponding shadow rate. If the latter exceeds the former, a speculative attack terminates the path. The expected lifetime of the target zone is then computed as the sample average of the paths' lifetimes.

IV. Model calibration

Our model involves four structural parameters: α, σ, μ, and λ, which we calibrate using data from the FF/DM and SP/DM markets, the two largest ERM markets in our post-August 1993 sample. We emphasize that, despite our attempt to calibrate the model with realistic parameters, we are aware of its stylized nature. Our goal is not to insist on the model's empirical accuracy, but rather to use its predictions as suggestive of the main qualitative features of soft target zones and to verify that these features are, in principle, numerically significant.

Note also that the goal of this empirical section is not to test the soft target zone model against alternative interpretations of the policies implemented in the ERM since August 1993. Our approach is more in the vein of standard calibration exercises, whereby reasonable parameters are chosen for a model which is *maintained* as true. However, the issue of whether the post-August 1993 regime could be described by alternative policy regimes is clearly of interest. A natural alternative would be to interpret this period as a managed-float regime, whereby the central bank intervenes with increasing intensity as the exchange rate departs from its central parity, but without commitment to an explicit band. This interpretation could be easily nested in our model, by letting $U \to \infty$ and by interpreting—as often done in the target zone literature—mean reversion in fundamentals as reflecting the central bank's effort to steer exchange rates towards their central parity. An interesting implication of this treatment is that exchange rates generated by this managed-float regime should be then distributed unconditionally normally if fundamental shocks are themselves normally distributed. In contrast, exchange rates from our soft regime should have a distribution with truncated tails and greater probability mass towards the middle of the band than in a normal distribution. This contrast suggests that, in principle, one may try to discriminate between these two regimes by examining the unconditional distribution of exchange rates, even though—as discussed by Svensson (1992) and Lindberg and Söderlind (1996)—this effort may rarely be successful: unconditional distributions of mean-reverting exchange rates *with* and *without* bands differ too little, for reasonable degrees of mean-reversion, for empirical tests to be able to pick up the difference with confidence.

As a matter of fact, for our data the hypothesis of normality can be rejected with 97 percent confidence for the FF/DM rate and with 91 percent confidence for the SP/DM rate, lack of weight in the probability tails being one apparent cause of rejection. (The kurtosis for the FF/DM fell short of the normal value of 3 by 0.56 and that for the SP/DM rate by 0.24.) However, it is clear that this test provides only very indirect support of our model, in that it only rejects the joint hypotheses of managed float and normality of the exchange rates. On the other hand, using a methodology aimed at picking up tail behavior with greater precision, Labhard and Wyplosz (1996) found evidence of "soft" narrow bands being implemented during the post-August 1993 ERM (see footnote 15). Overall, we take this preliminary evidence as confirming the anecdotal evidence cited in Section I on the behavior of ERM central banks since August 1993. Clearly, more thorough tests should be conducted if the goal is to provide conclusive evidence on the policies prevailing in the ERM since August 1993.

Our calibration exercise involves three main steps:

First, following Flood, Rose, and Mathieson (1991) we use equation (1) and the uncovered interest parity (UIP) condition, $E_t[s_{t+1}-s_t]=i_t-i_t^*$, to estimate f_t as $f_t=s_t-\alpha(i_t-i_t^*)$, where i_t and i_t^* are the domestic and foreign interest rates on one-month contracts.[11] The only non-observable input to complete this step is the parameter α. Although α could be estimated, previous attempts to this end have yielded such a broad and imprecise array of estimates (see, for instance, Flood, Rose and Mathieson, 1991, and Bartolini and Bodnar, 1992) that an agnostic approach seemed safer. We considered a representative set of values for α, namely, α=0.1, α=0.5, and α=1, viewing α=0.1 and α=1 as representative of the low and high ranges, respectively, of possible values of this parameter. Following Flood and Rose (1995), we view α=0.5 as a reasonable baseline value. After conducting our empirical analysis for all these three values, we verified that our results were robust to the specific choice of α.

In the second step of our calibration, we subtract from the calculated series f_t the portion of fundamentals controlled by the central bank, namely, the base-money differentials between France and Germany, and between Spain and Germany, to obtain an estimate of the velocity component of fundamentals as a residual (i.e., $v_t \equiv f_t-m_t$). To this end, we obtained data for base-money for the three countries in our sample (seasonally adjusted and corrected for changes in reserve requirements that occurred in the sample) and subtracted (the logarithm of) French/ German and Spanish/German base-money differentials from our previous estimates of f_t.

The final step of our calibration consists of analyzing statistically the filtered series v_t with results documented below.

Clearly, the method we use to filter the unobservable "velocity" shocks from exchange and interest rate data leaves the exact nature of these shocks behind the curtains. Our filtered v_t series, for instance, ends up capturing factors as diverse as exogenous money demand shocks and central bank activities (such as sterilized intervention) not captured by changes in m_t. If interest differentials include a risk premium, this would also be automatically embedded into v_t.

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[11]UIP usually captures the behavior of free-floating exchange rates rather poorly. However, it can be expected to hold better for target zone exchange rates (see Svensson, 1990, for the standard theoretical argument, and Rose and Svensson, 1995, for evidence in favor of UIP in the FF/DM market).

None of these problems seems so severe to suggest pursuing the natural alternative to our non-structural approach, namely, to specify a model underlying (1) and make specific assumptions on the source of the shocks v_t. In fact, given the disappointing empirical record of structural exchange rate models, we view an approach that does not require us to impose more structure on model (1) as an advantage. Our approach reflects the view that although many factors may contribute to the short-run behavior of fundamentals (including sterilized intervention, exchange rate response to shocks in a world with sticky prices, and transitory changes in risk premia), the sustainability of an exchange rate target in an integrated capital market ultimately depends on central banks' willingness to alter relative money supplies. Accordingly, our econometric approach implies that the auto-regressive parameter λ in (5) captures the short-run dynamics of velocity induced by factors other than changes in base-money m_t, while the evolution of base-money determines the long-run sustainability of the target.

We analyzed the stochastic properties of the filtered v_t series by estimating an error-correction equation using a standard general-to-specific approach. We chose a monthly frequency to offset the large intra-month volatility in base-money induced by payment flows unrelated to monetary policy,[12] focusing on the period from August 1993 to December 1996.[13]

We estimated equation (5) separately for FF/DM and SP/DM data, and generally obtained simple and well-behaved specifications. For instance, we found no evidence of serial correlation in SP/DM data, and found an error-correction model with only one lag of the dependent variable sufficient to eliminate serial correlation from FF/DM data. We found only two significant outliers in FF/DM data, responsible for increasing the estimated coefficients' volatility while minimally affecting the point estimates: one in correspondence of the December 1995 French public sector strikes, which caused a temporary plunge in m_t, and one in correspondence of the May 1995 French elections, which caused a temporary surge in v_t. Our estimates in Table 1 include dummies for these observations. We included a single dummy in the SP/DM sample for March 1995, in correspondence of the realignment of the peseta in this month. Interestingly, we found no evidence of a structural break between the pre- and post-realignment SP/DM periods.

Estimation results are summarized in Table 1, which also reports the static version of the error-correction models estimated from FF/DM data, which is in the form of (5).

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[12]In France and Germany, commercial banks are subject to average monthly reserve requirement, which allows them to accommodate intra-month shocks unrelated to monetary policy through changes in reserves; the same requirement holds in Spain, through over shorter, ten-day reserve-maintenance periods.

[13]Estimates from weekly data gave very similar point estimates, but significantly higher standard errors.

Table 1. Benchmark Model $v_t - v_{t-1} = \mu - \lambda\,(v_{t-1} + m_{t-1}) + \epsilon_t$

Exchange Rate	α	λ (White std. errors in parenthesis)	μ (White std. errors in parenthesis)	σ	R^2	Serial Corr.n (p-value in parenthesis)	Normality (p-value in parenthesis)	Implied Half-Life (in months)
FF/DM	0.1	0.191 0.280[(1)] (0.095)	0.238 0.348 (0.117)	0.007	0.37	0.527 (0.667)	0.721 (0.697)	2.1
	0.5	0.176 0.281[(1)] (0.096)	0.218 0.348 (0.118)	0.006	0.42	0.291 (0.831)	0.146 (0.929)	2.1
	1.0	0.152 0.248[(1)] (0.109)	0.188 0.306 (0.306)	0.006	0.42	0.262 (0.852)	6.508 (0.039)	2.4
SP/DM	0.1	0.241 (0.062)	1.068 (0.276)	0.010	0.48	0.169 (0.917)	3.197 (0.202)	2.5
	0.5	0.266 (0.067)	1.176 (0.295)	0.011	0.52	0.250 (0.861)	3.577 (0.167)	2.2
	1.0	0.290 (0.074)	1.275 (0.326)	0.011	0.56	0.389 (0.762)	4.272 (0.118)	2.0

Table Notes: (1) Static estimates implied by the error-correction model (the FF/DM model corresponds to the benchmark model plus one lag of the dependent variable). Samples: FF/DM: monthly data from 1993:8 to 1996:11; SP/DM: monthly data from 1993:8 to 1996:12. The serial correlation test is an F(3,35) test for correlation up to the 4th order. The normality test is distributed as a $\chi^2(2)$. The FF/DM regressions include dummies for the December 1995 public sector-strikes and for the April 1995 presidential elections. The SP/DM regressions include a dummy for the March 1995 realignment.

Table 1 shows that, despite the rather short sample, the coefficient λ is quite significant for SP/DM data and marginally significant for FF/DM data, implying half-lives of the velocity shocks between 2 and 2½ months (or annualized values of λ between 3 and 4). This evidence of mean-reversion may seem surprising in light of the known difficulty of identifying mean-reversion for the major free-floating exchange rates. However, first note that our analysis focuses on the statistical behavior of fundamentals, not on that of exchange rates.[14] Furthermore, convergence among ERM fundamentals is bound to be stronger than among fundamentals for free-floating currencies. Finally, besides factors such as the mean-reverting effect of price stickiness and transitory risk-premia, the estimated behavior of ERM fundamentals likely reflects also sterilized intervention undertaken by ERM central banks to influence exchange rates in the short run without sacrificing control of national money supplies.

The estimated standard errors of v_t (i.e., of $\sigma\epsilon_t\sqrt{\Delta t}$) range between 0.006 and 0.011 per month (or 2 and 4 percent per year). Finally, the estimates of μ and λ from Table 1 can be combined to yield the implied mean-reversion points for f_t (i.e., μ/λ). These average 1 percent above the central SP/DM exchange rate parity and 2.8 percent above the central FF/DM parity.

In light of these results, we calibrate our model conservatively and solve it around an annualized baseline with α=0.5, λ=3, σ=0.04, and μ/λ=0.028, and examine a wide range of values around this baseline. Consistent with the results of Labhard and Wyplosz (1996), we set the band-width at ±3 percent.[15] We examine mean intervention lags L ranging from 0 to 6 months, using L=3 months as a benchmark, since with this value the model-simulated exchange rate spends ¾ of the time within a ±3 percent band, about the same as did the ERM currencies (excluding the Dutch guilder) from August 1993 to March 1997.[16]

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[14]Indeed, the tendency for asset prices to be much more volatile and less stable than their underlying fundamentals (e.g., stock dividends) is well known in the financial literature. See, for instance, the survey by Gilles and LeRoy (1991). In the exchange rate literature, our estimated *fundamentals'* half-lives are very similar to those obtained by Lindberg and Söderlind (1994) for the Swedish krona—the only comparable study that we are aware of.

[15]Labhard and Wyplosz (1996) estimate soft ERM bands of unconstrained width for the period from August 1993 to November 1995, and obtain bands of total width averaging 6 percent across currencies, with values of almost exactly 6 percent for the FF/DM and the SP/DM rates. (Labhard and Wyplosz allow their bands to be asymmetric around zero; for simplicity, we normalize our band symmetrically around zero.)

[16]We set the time unit Δt at 1/200 years, and T=200, which was more than sufficient for approximation purposes. We normalized our Montecarlo experiment by endowing the central bank with an initial stock of base-money yielding a life expectancy of about 6½ years (the

(continued...)

V. Exchange Rate Dynamics in a Soft Target Zone

A. Fundamentals' vs. Exchange Rates' Changes

Figure 3 presents solutions for the soft-band exchange rate, plotted against fundamentals. Four of these solutions are drawn, conditional on inherited exchange rate averages of zero, one, two, and three percent, and are labeled $\bar{s}_{t-\Delta t}=\{0.00,\ 0.01,\ 0.02,\ 0.03\}$, respectively. Figure 3 also plots the solution for the hard-band exchange rate, labeled $L=0$. All rates are measured as deviations from the center of the band. The curves are asymmetric around $s_t=0$ due to the positive drift of fundamentals, $\mu/\lambda=0.028$.

Several properties of the solution are apparent in Figure 3. First, the figure highlights the dependency of the soft-band exchange rate on its history, summarized by the inherited average $\bar{s}_{t-\Delta t}$. This dependency implies that, while a unique exchange rate is defined as a function of fundamentals for a hard target zone, a family of such curves, each parameterized by $\bar{s}_{t-\Delta t}\in[-U,\ U]$, is defined for a soft target zone.

The figure also illustrates that the gain in flexibility granted by a soft target zone depends on the history of past exchange rates. For instance, when the inherited exchange rate average equals zero, a soft target zone allows substantial room for the exchange rate to sway, temporarily, outside both of the band's edges. In contrast, when the inherited average equals U (i.e., 0.03, in our example), a soft target zone provides no more room than a hard target zone for the exchange rate to rise above the band's upper edge, but substantially more room for it to fall below the band's lower edge.

Actual exchange rate paths will typically exhibit patterns intermediate to those captured by the curve $\bar{s}_{t-\Delta t}=0$ and the curve $\bar{s}_{t-\Delta t}=0.03$. For instance, as a currency gradually depreciates from the middle of the band toward its upper edge, so does the average exchange rate. Thus, a slow depreciation would eventually bring the currency in a situation similar to that captured by the curve $\bar{s}_{t-\Delta t}=0.03$. In contrast, a currency subject mostly to sharp, short-lived fluctuations would typically be in a situation more similar to that of the curve $\bar{s}_{t-\Delta t}=0$. Hence, the effective gain in flexibility granted by a soft target zone depends on the statistical properties of fundamentals: the stronger is the transitory component of fundamentals' shocks, the broader is the scope for the exchange rate to stray outside its long-run target in the short run. In contrast, a soft band grants little extra flexibility when fundamentals rise or fall along a steady trend.

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[16](...continued)
interval between the ERM realignments of January 1987 and the July 1993 crisis) for a hard 2¼-percent band with baseline parameters.

Figure 3
Exchange Rates as a Function of Fundamentals

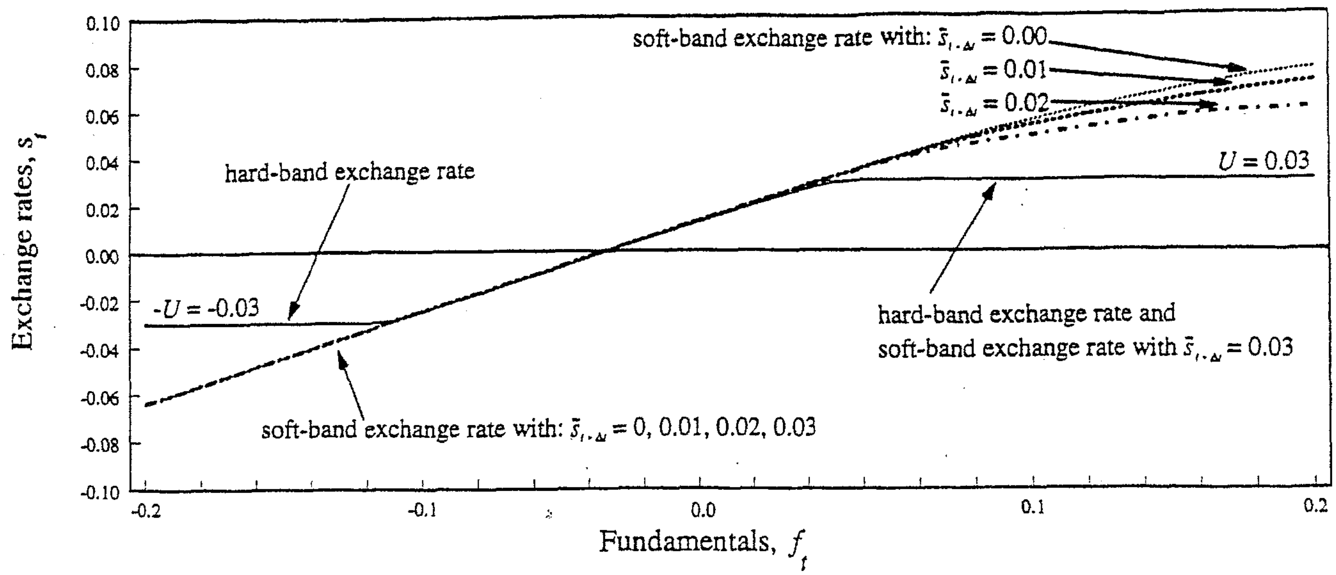

Figure 4
Effect of Changes in L

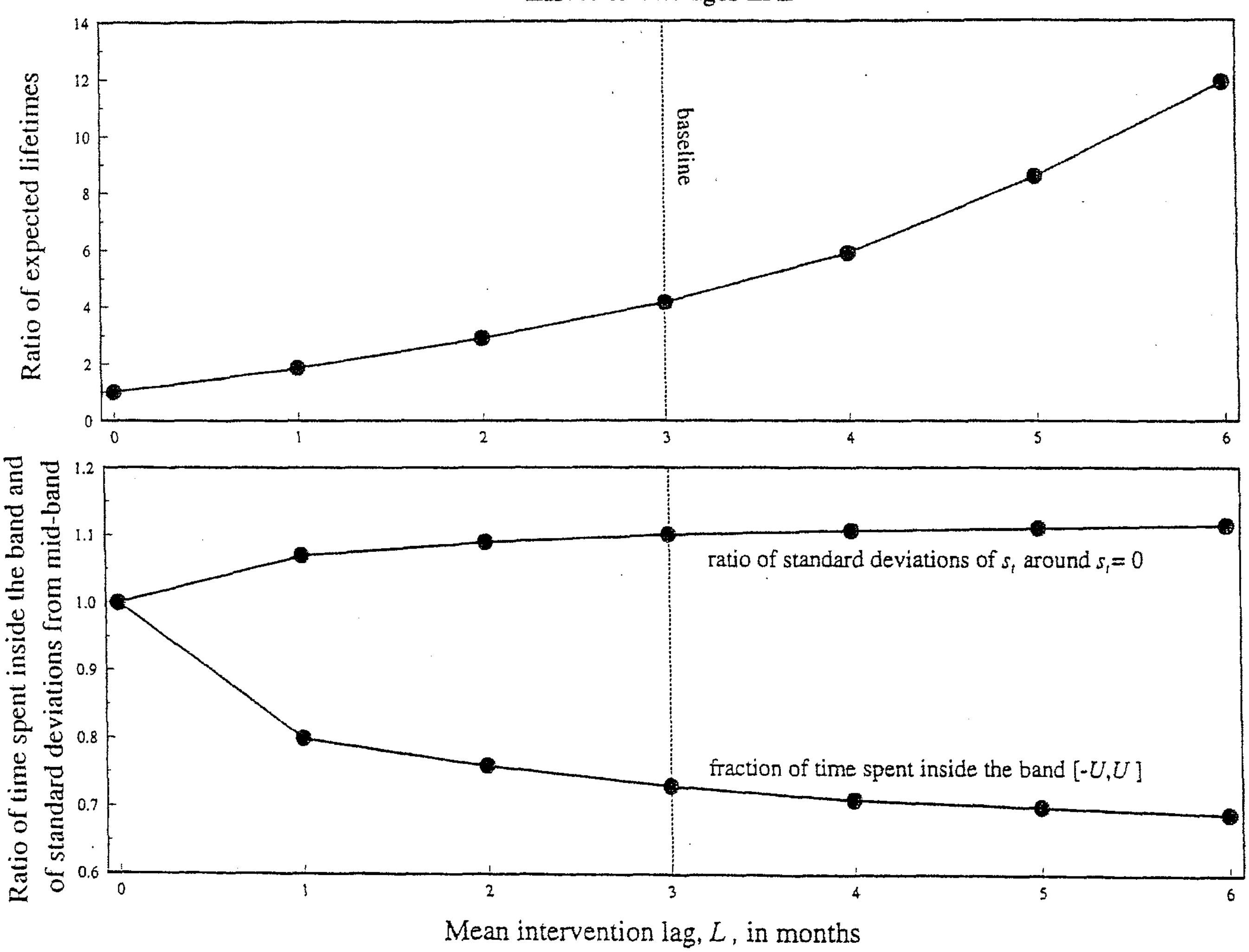

Figure 3 displays other interesting features. For instance, all curves are almost identical in the inner-band region, but the soft-band curves become steeper than the hard-band curve as the exchange rate approaches the band's edges $[-U,\ U]$. (This feature is not very visible in Figure 3, however, due to fairly small baseline value of σ and the scaling of the axes.) This is an intuitive feature of our model. As the exchange rate approaches the band's edges, investors know that the central bank is less likely to intervene in a soft target zone than in a hard one. Hence, Krugman's (1991) "honeymoon effect" (the stabilizing effect of anticipated future intervention on current exchange rates) is mitigated, and a given fundamental shock maps into a larger change in exchange rates in a soft target zone than in a hard target zone.

B. Vulnerability to Speculative Attacks and Comparative Statics

The most important property of a soft target zone is that it is less vulnerable to speculative attacks than a hard target zone. Figure 4 illustrates this property, by plotting the ratio of the expected lifetimes of a soft target zone to that of a hard target zone, as a function of the mean intervention lag L. The ratio of expected lifetimes rises monotonically with L, from 1 for $L=0$ (the value of L for which a soft band collapses into a hard one) to infinity for $L\to\infty$. Note that the ratio of expected lifetimes rises fairly quickly. For instance, the expected lifetime of our baseline soft target zone increases more than four-fold as the mean intervention lag rises up to only 3 months.

Reduced vulnerability to speculative attacks does not come for free. The central bank must let the exchange rate sway occasionally outside the target band $[-U,\ U]$. However, Figure 4 shows that the fraction of time spent inside the band falls slowly with L. With a mean intervention lag of 3 months, for instance, the exchange rate still spends about 75 percent of the time inside the band $[-U,\ U]$, and its standard deviation from the band's central parity is only about 10 percent higher than in a hard target zone. For practical reference, from August 1993 to March 1996, ERM exchange rates against the DM (excluding the Dutch guilder/DM rate, which was explicitly targeted within a hard ±2¼ percent band) spent 77 percent of their time within a narrow ±3 percent band,[17] with no rate deviating more than ±2½ percent on average over the whole period. Hence, from the perspective of our model, a mean intervention lag of about three months seems to capture reasonably well the commitment of ERM central banks to a narrow fluctuation band after August 1993.[18]

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[17]72 percent, for the FF/DM and SP/DM rates.

[18]Clearly, central banks' mean intervention lag and their target band-width cannot be identified separately through a simple calculation of this sort. (These two parameters could be identified separately using data on official intervention, but these data are seldom released by central banks.) Here we treat the band-width as exogenous, as our goal is not to provide an exhaustive characterization of the post-August 1993 ERM policy, but only to use our soft target zone model as one possible interpretation of this episode, consistent with the

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How does the ratio of expected lifetimes respond to changes in fundamentals' parameters? The most important of these effects is illustrated in Figure 5: a soft target zone's gain in life expectancy becomes stronger as the mean-reversion parameter λ becomes higher. The main reason for this link is intuitive: a higher λ shortens the lifetime of fundamental shocks, thus raising the likelihood that the exchange rate will return to within its band before the country's base-money is depleted. This effect is already substantial when $\lambda=3$, a value suggested by our FF/DM and SP/DM estimates. It becomes stronger for higher values of λ, and vice versa.

Note that, although the trade-off highlighted in Figure 5 is intuitive overall, it is the result of several contrasting effects that make the overall effect less than obvious. For instance, at the same time that a higher λ stabilizes the behavior of v_t, it also increases the level of base-money that triggers a speculative attack, thus reducing the stock of base-money that must be depleted before triggering a speculative attack, and making the net effect of λ on the ratio of lifetimes ambiguous a priori. Despite this overall ambiguity, in Figure 5—drawn for plausible parameters—the most intuitive link between λ and the band's expected lifetime prevails.

Changes in μ also have interesting and quantitatively significant effects. As Figure 6 shows, a higher μ reduces a soft band's benefits: targeting the exchange rate on average is of little help when fundamentals pull the rate steadily away from the middle of the band. As in other models of speculative attacks on target zones (see, for instance, Dumas and Svensson, 1994), the expected lifetimes and their ratio are very sensitive to changes in μ.

Finally, Figure 7 shows that the ratio of expected lifetimes falls as the volatility of fundamentals, σ, rises. Although this link is intuitive—a higher σ offsets fundamentals' tendency to revert to their mean—this prediction may be disappointing for policy purposes, suggesting that the counter-speculation role of soft target zones is enhanced when fundamentals are *least* volatile. This may be the opposite effect to what European central banks were hoping for, when they adopted a more flexible exchange rate policy in August 1993.

In summary, given our model's stylized nature, it is clear that our numerical examples cannot be relied upon for accurate predictions on the dynamics of exchange rates in a soft target zone. Nonetheless, the analysis of this section delivers a fairly clear message: soft target zones are more resilient to speculative attacks than standard hard target zones, significantly so for most plausible ranges of parameter values.

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[18](...continued)
widespread view that a narrow/soft exchange rate band was indeed implemented in this period.

Figure 5
Effect of Changes in λ

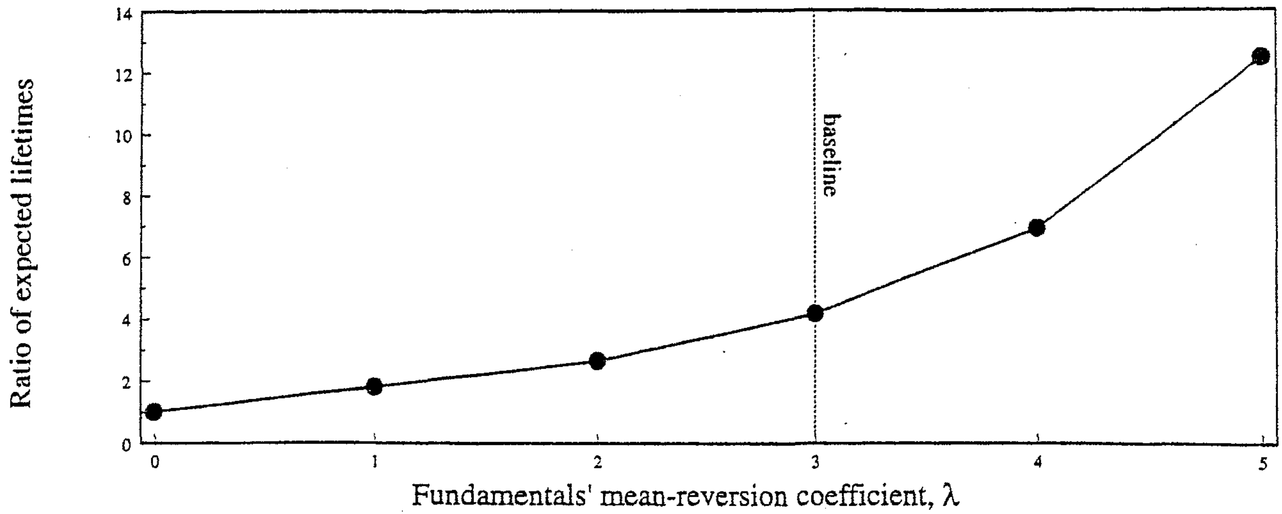


Figure 6
Effect of Changes in μ/λ

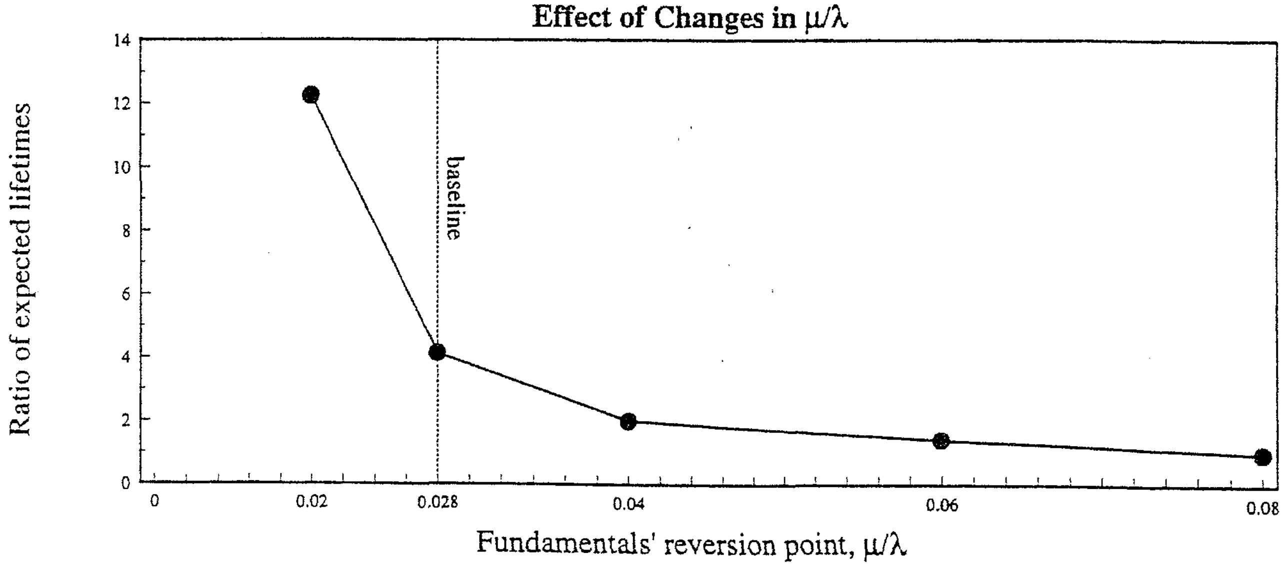
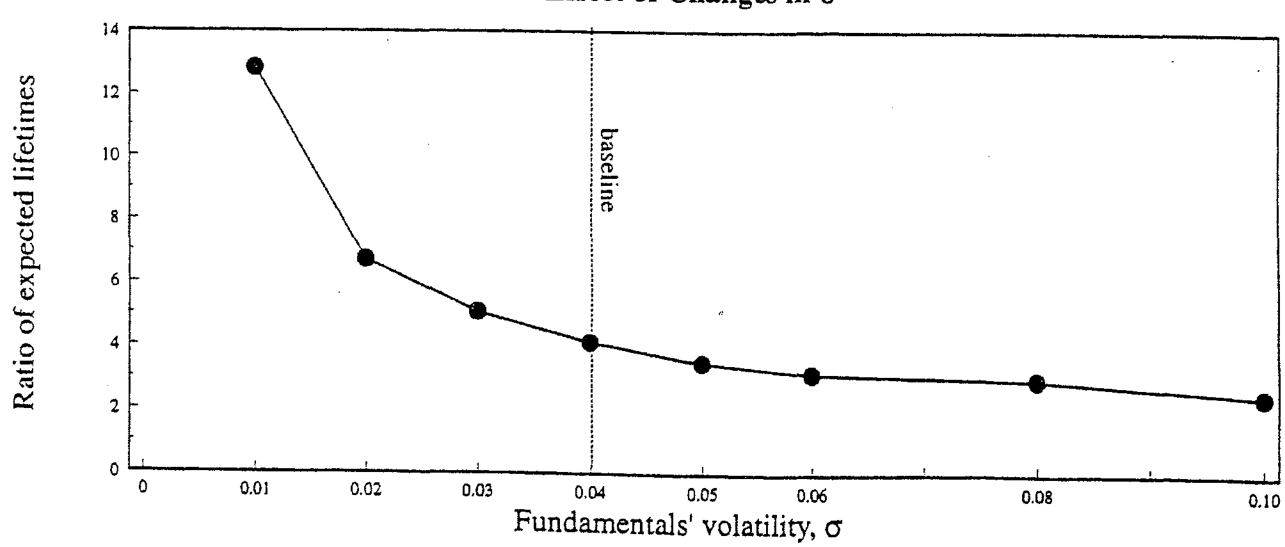


Figure 7
Effect of Changes in σ



VI. Extensions of the Model

A. Uncertainty about Intervention

The model studied in the previous sections could be extended in several directions to provide a more realistic characterization of actual intervention policies, particularly of those implemented in the ERM since August 1993. Many of these extensions, especially those introducing "realignment risk" of target zones, are by now standard in the literature and need not be repeated here (see Svensson, 1992, for a review). However, it may be useful to note a simple way in which uncertainty over the future direction of monetary policy can be embedded in our model, so as to capture more realistically the recent ERM experience. Although greater uncertainty over intervention is an unlikely contributor to the ERM's favorable performance since August 1993, it would be unrealistic to view the post-1993 regime as clearly understood by market participants as the previous narrow-band regime.

The easiest way to extend our model in this direction is in the spirit of research by Klein and Lewis (1993) and Lewis (1995). Besides intervening to defend the band's soft margins, let the central bank intervene in every period as follows:

$$\begin{cases} \text{increase } m_t \text{ by } \Delta f \text{ with probability} & \dfrac{\sigma_m^2 \Delta t}{2(\Delta f)^2} + \dfrac{\phi_t^2}{2(\Delta f)^2} + \dfrac{\phi_t}{2\Delta f}, \\[2ex] \text{leave } m_t \text{ unchanged with probability} & 1 - \dfrac{\sigma_m^2 \Delta t}{(\Delta f)^2} - \dfrac{\phi_t^2}{(\Delta f)^2}, \\[2ex] \text{decrease } m_t \text{ by } \Delta f \text{ with probability} & \dfrac{\sigma_m^2 \Delta t}{2(\Delta f)^2} + \dfrac{\phi_t^2}{2(\Delta f)^2} - \dfrac{\phi_t}{2\Delta f}, \end{cases} \tag{11}$$

where Δf and Δt are defined in the same way as in Section III.

In equation (11), the parameter Φ_t determines the instantaneous drift of monetary policy (i.e., $E_t[\Delta m_t \,/\Delta t]=\Phi_t$). For instance, to model leaning-against-the-wind intervention, let $\phi_t \equiv \gamma[\rho - f_t]$, where $\gamma>0$ and ρ is the reversion point of m_t; alternatively, $\Phi_t \equiv \Phi$ yields a persistent bias toward either tighter or looser money, depending on whether $\phi<0$ or $\phi>0$.

With this specification (which could, in principle, be estimated, since it relies on the observable behavior of m_t), in each period the public is uncertain about the exact direction of monetary policy, but knows its drift. This model of stochastic unsterilized intervention can be superimposed on our basic model, leaving its qualitative results unchanged.[19] Fundamentals, , $f_t \equiv v_t + m_t$, would be subject to frequent shocks (the sum of (9) and (11)) and to infrequent (soft) intervention at the band's margins. The model can then be solved exactly as before.

Following Klein (1992), one can also easily introduce uncertainty about U, assuming that the public does not know the exact value of U, but holds prior beliefs on its possible values. As in Klein (1992), the public would progressively refine its knowledge of U, as it observes that the central bank does not intervene as the average $\bar{s}_t$ drifts into previously uncharted territory. Uncertainty over the true value of U is resolved when the central bank finally intervenes.[20]

B. Alternative Definitions of $\bar{s}_t$

Our model's main implications do not depend on the specific definition of $\bar{s}_t$ adopted in Section III. Indeed, our main assumption—that intervention depends on the behavior of the exchange rate over an interval of time, rather than on its value at a given point in time—can be implemented in a variety of ways, all of which are likely to lead to similar conclusions. The assumption of a geometric-weighted average reflects the need for tractability (with this specification, the inherited average $\bar{s}_{t-1}$ is a sufficient statistic for the history of exchange rates) and its likely greater realism (the central bank is assumed to be more concerned about the exchange rate's recent behavior than its long-past behavior).

The natural alternative to our definition of $\bar{s}_t$ would be to aggregate the last, say, N exchange rates, into an equal-weighted average, e.g., $\bar{s}_t \equiv \frac{s_t + s_{t-\Delta t} + \ldots + s_{t-(N-1)\Delta t}}{N}$. This definition would yield the same qualitative results discussed above, but it is daunting from a computational viewpoint.[21] The inherited average $\bar{s}_{t-1}$ would no longer be a sufficient statistic for the exchange rate history: since in each period $\bar{s}_t$ is computed by substituting the exchange rate at $t-(N-1)\Delta t$ with the rate at $t+\Delta t$, the detailed behavior of s_t over the last N periods becomes relevant to forecast future intervention. In this case, solution of the model requires defining a state-space whose dimension increases with M^N, so that it becomes

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[19]Essentially, this extension amounts to assuming a stochastic process for m_t similar to that assumed for v_t, usually in continuous time, in the target zone literature.

[20]One could extend this model further by letting the band-width be re-sampled, so as to avoid complete resolution of uncertainty about U. However, this extension would take the model even further away from the intuition and techniques of standard target zone models.

[21]We have explored this specification in previous work, but found it impossible to solve the model accurately, due to the problems of dimensionality that arise with an equal-weighted average. We thank Giuseppe Bertola and Charles Bean for suggesting the use of a geometric-weighted average as the key to a complete solution of our model.

practically impossible to solve the model when the average is computed over intervals longer than $4 \cdot \Delta t$. This is problematic, since a small Δt is necessary for accuracy. In contrast, our basic model can be solved for any horizon with great accuracy, with arbitrary choice of the mean intervention lag.[22]

Despite these caveats, the properties of our model are unlikely to depend on how exactly the average $\bar{s}_t$ is defined. For instance, the response of the ratio of lifetimes of soft and hard target zones to changes in the mean intervention lag L (which equals $N/2$ with an equal-weighted average), is qualitatively the same as that of our basic model, as well as quantitatively comparable. With a mean intervention lag of one month and baseline parameters, for instance, the model with an equal-weighted average yields a ratio of expected lifetimes between soft and hard target zones of 1.8, fairly similar to the value of 2.1 obtained with a geometric-weighted average on the same state space.[23] The response of the model to changes in the other parameters is also similar between the two specifications (results are available upon request).

VII. CONCLUDING REMARKS

Despite the interest and active research generated by target zone models, much work remains to be done to capture the institutions and stylized facts describing the experience of countries that have adopted target-zone-like arrangements in recent years. The difficulty of this agenda partly reflects the fact that central banks seem to be moving the target for scholarly research, by continuously implementing new breeds of exchange rate policies, including crawling bands in countries such as Chile, Israel, and Mexico, and ingenious arrangements such as the "soft/narrow bands within wide bands" adopted in Europe since August 1993. This last regime presents an interesting challenge for academic research, because it entails a policy that several scholars of currency markets have long advocated, and because it calls for a consistent explanation for the sharp decline in speculative pressure recorded in Europe since the system was implemented in August 1993. This challenge is hardly diminished by the incipient prospect of European Monetary Union. Should EMU be successfully established, the regime that has permitted such a miraculous convergence after the troubled 1992-93 period can only become more appealing, both to scholars of currency markets and to policymakers in other countries striving for exchange rate stability in the face of constant speculative threat.

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[22]An equal-weighted average is also more vulnerable to echo-effects in the intervention policy, whereby the central bank must intervene at time t in response to fluke exchange rate realizations at $t-N \cdot \Delta t$.

[23]To accommodate a mean intervention lag of one month with an equal-weighted average we set N=4 and the time unit at Δt=3 weeks. This choice yields a state-space much coarser than required for solution accuracy.

In this study, we have presented a model of a soft target zone with two main goals: to extend research on target zones to include a "soft-band" policy that has long interested scholars of currency markets, but has never received rigorous treatment; and to capture some key aspects of the wide-band ERM intervention policy and explain its resilience against speculation after the post-August 1993 period. The main feature of our model is the assumption that exchange rate intervention responds not only to the level of the exchange rate at a given point in time, but also to its behavior over the recent past. We have developed our model as a direct extension of standard target zone models and showed that, for reasonable parameters, soft target zones should be considerably more resilient to speculative attacks than hard target zones—a prediction consistent with the sharp abatement of speculative pressure in Europe since August 1993. In trying to keep exchange rate models up to date with central banks' creativity for developing new policies of exchange rate management, this should be a step in the right direction.

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