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Fixed Investment and Capital Flows: A Real Options Approach

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Abstract

This paper draws a link between international capital flows and the real options approach to investment by extending a model of real estate investment. It explains gradual investment, investment booms, and investment during recessions and emphasizes sunk costs, uncertainty, and the value of waiting. The optimal waiting time increases as foreign borrowing becomes more expensive because higher returns are required to cover the sunk costs of investing. The lower the initial level of profitability, the more likely investment will be sequential; conversely, a relatively high initial rate of return will be associated with simultaneous investment.

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SUMMARY

Recent developments in the theory of investment under uncertainty have emphasized the importance of the timing of the decision to investand the value of waiting, as well as the existence of sunk costs that cannot be recouped if it is subsequently decided to reverse the investment. This theory can explain sudden bursts of investment activity, overinvestment, and strategic interaction among investors. As international capital flows in general share a number of these features insofar as they do not always take place smoothly and continuously, the paper applies this approach to sudden movements in capital flows.

The model involves two investors who borrow at the given world interest rate to invest in projects in the domestic economy, where the rate of return varies inversely with the amount invested. Investment involves a fixed cost, but there are assumed to be no subsequent costs of adjusting the capital stock, and capital flows reflect variations in the stock of capital. Uncertainty enters either as affecting the domestic rate of return or the foreign interest rate. The model characterizes the interaction over time of the two investors and the optimal timing of their jointly dependent investment decisions.

The optimal waiting time to invest increases as foreign borrowing becomes more expensive because higher returns are required to cover the fixed cost of investing. The lower the initial level of profitability relative to its future evolution, the more likely investment will follow a sequential pattern; conversely, a relatively high initial rate of return will be associated with simultaneous investment. If the borrowing cost—the foreign interest rate—is constant, there is no correlation between capital flows and the spread between this rate and the domestic rate of return. When the borrowing cost is stochastic, however, there is a negative correlation between the spread and capital flows.

I. Introduction

International capital flows often appear to be subject to sudden changes in sentiment. This is most clearly the case in episodes of speculative attacks against fixed or quasi-pegged exchange rates, such as occurred in the ERM in 1992-93, Mexico in 1994-95, and in many Asian emerging markets in 1997-98.² Much of the analysis of episodes of this kind identifies shifts in economic fundamentals or self-fulfilling speculative bubbles as underlying the change in sentiment generating the fluctuations in capital flows.³ However, another perspective on the episodic nature of the movement of capital internationally can be obtained by looking at the factors underlying the timing of domestic investment and the capital flows that may be associated with the change in the capital stock. Exploring the link between real investment and capital flows can bring out the value of waiting before implementing a decision to invest, and therefore provide an alternative explanation for the sudden movements in capital flows.

One noteworthy feature of the Asian crisis is that it was preceded by remarkably successful economic performance in terms of high output growth, low inflation, and very strong private domestic investment. Indeed, a significant fraction of this investment was financed by foreign capital attracted by the prospect of relatively high returns. For example, for the developing countries in Asia, investment rose from 29 percent of GDP in 1983-90 to 33 percent of GDP in 1991-96, and from 28 percent of GDP to 32 percent of GDP over the same period for the four newly industrialized Asian economies⁴. While one cannot draw a precise link between the capital inflows and domestic investment, a connection certainly exists as neither consumption spending nor the fiscal deficit increased significantly in the region as a whole over this period.

The Asian crisis thus highlights an important role for foreign borrowing in facilitating the expansion of business fixed investment, especially real estate. Thus it would appear useful to explore the extension of theories of real estate investment to a model of capital flows in an open economy. In particular, the model of Grenadier (1996) shows how a gametheoretic approach to the exercise of options can be useful in understanding certain aspects of investment decisions. The exercise of the option to build provides a potential explanation for a number of aspects of the real estate market. For example, some real estate markets have been prone to sudden bursts of investment activity. In addition, certain markets, e.g., those for office buildings, have been characterized by considerable periods of overbuilding where increases in the stock of office space have occurred in the face of a fall in demand and in prices. Finally, concern about the exercise of the option to build preemptively by competitors can result in the simultaneous investment activity by a number of agents.

² See Dornbusch and Werner (1995); Eichengreen, Rose and Wyplosz (1996); Sachs, Tornell and Velasco (1996) and Radelet and Sachs (1998).

³ See Masson (1998).

⁴ Table A44, I.M.F. World Economic Outlook, October 1997.

International capital movements in general share many of these same features in that they do not always take place smoothly and continuously, but rather are often subject to sharp bursts of inflows and outflows. This is not surprising, as capital flows reflect investment activity that requires both an assessment of the relative returns to capital projects in different markets as well as decisions on when to commit the capital to particular alternatives. Recent developments in the theory of investment under uncertainty have emphasized the importance of the timing of the decision to invest and the value of waiting. A key aspect of this theory is the existence of sunk costs associated with the investment that cannot be recouped if a decision is made subsequently to pull out of the activity.

The magnitude of the sunk costs relative to the value of the investment vary widely by type of capital flow. Pure portfolio investment may involve only transactions costs and the time and costs incurred to gather information about the economic prospects of foreign enterprises. These costs would appear to be quite small for individuals, but they may be substantial for investment companies setting up mutual funds that deal, for example, with emerging markets. Similarly, the expansion of bank loans abroad requires gathering information about the prospective solvency of the borrower. Sunk costs are probably largest in the case of foreign direct investment that involves a substantial increase in ownership, either through greenfield investments or by acquisition. In this case the capital flow entails market research, feasibility studies, and legal analysis. The higher the sunk costs incurred as a result of the investment, the more important the real option value of waiting to see how prospects are likely to develop before reaching a decision to proceed with the investment. As the model used in this paper makes extensive use of this concept of the real option value, it is described in some detail below.

The model developed here relaxes the assumption of indivisible investment in Grenadier (1996) by allowing a variable investment level and external borrowing so as to adapt the model to the situation of an open economy. Both the case of monopoly and that of Cournot competition between investors are modelled explicitly and related to the economic fundamentals of the economy where the investment takes place. Imperfect competition captures the strategic interaction of market players, as market share is often cited as an important consideration when investing in a foreign market; in a perfectly competitive environment market share would be irrelevant. This modified model rationalizes such phenomena as gradual investment, investment booms, and investment during recessions, and it emphasizes the role of sunk costs and uncertainty in determining the timing of investment. It also shows that the correlation between capital flows and the spread between the domestic return to capital and the foreign interest rate depends importantly on the source of uncertainty; if the foreign interest rate is stochastic, the correlation is expected to be high, but would be low if the uncertainty affects the domestic rate of return.

Besides the work of Grenadier, there are only a few examples in the literature that address the strategic aspect of the investment decision, which include Kulatilaka and Perotti (1992), Lambrecht and Perraudin (1996) and Smets (1993). Kulatilaka and Perotti study the case

of strategic investment in a two-period duopoly model with certain fixed costs but uncertain variable costs. They consider both the Cournot and the Stackelberg cases. In our model, only Cournot competition is considered, because of analytical tractability. Lambrecht and Perraudin (1994) show that the value of the option to invest by a single firm is reduced drastically when there exists the possibility of strategic entry by competing firms, weakening the hysteresis effect. In contrast to the model proposed here and in Grenadier (1996), their analysis assume that there can be only one firm in the market. Smets (1993) uses a duopolistic competition setup similar to that of Grenadier, and hence, similar to that used in this paper, to analyze the strategic decision of firms to switch from exporting to foreign direct investment as a means of exploiting profits in a foreign market, which arises from lower variable labor costs in the foreign country. In Smets' model there are two equilibria. In the first one firms invest jointly, while in the second one foreign direct investment proceeds sequentially. A common feature of these models is that firms and agents decisions can be modelled as optimal stopping problems. A detailed exposition of the related theory can be found in Dutta and Rustichini (1993) and Lambrecht and Perraudin (1994).

While it does not consider strategic behavior, the paper of Kulatilaka and Kogut (1996) is also related. It builds upon earlier work by Dixit (1989) on the persistence of trade deficits as resulting from the decision of exporters to enter a domestic market during a period of exchange rate overvaluation, while keeping the option to withdraw when the exchange rate moves unfavorably. They introduce the possibility of shifting production to the domestic economy as an alternative to exporting, emphasizing foreign direct investment as the source of hysteresis instead of trade as posed originally by Dixit.

Clearly, there are alternative ways to model investment and capital flows besides the one proposed here. In particular, there is the approach based on asymmetric and incomplete information. For example, Chen and Khan (1997) analyze the pattern of portfolio capital flows to emerging markets building on a previous model by Dygvig and Zender (1991). Cabral (1997) studies both the problem of overinvestment and underinvestment as a preemption game between two firms entering a market that can only support one of them. We argue that the approach proposed here has the advantage that there are no asymmetries among investors as each one has access to all the information available and knows how his investment decision affects the domestic return to capital.

The organization of the paper is a follows. Section II provides a brief description of the key determinants of the value of waiting and hence the timing of investment. The following Section III develops the model of the real option approach to capital flows. Section IV presents the results for the case of domestic exogenous shocks and Section V considers the case of exogenous shocks to the borrowing rate. Section VI describes the relationship between the interest rate spread and capital flows. Section VII provides some concluding remarks.

II. THE VALUE OF WAITING TO INVEST

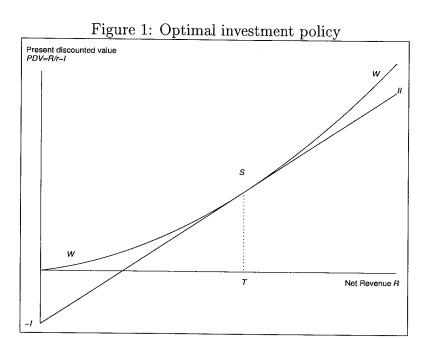
There are three aspects of investment decisions that affect the value of waiting. First, as noted above, most investment involves some sunk costs, i.e., expenditures that are largely irreversible. This reflects the fact that much capital is industry or firm specific, so that if a decision were made to reverse the investment decision, the capital could not be used productively elsewhere. This is most obviously the case of investments made in marketing and advertising, but it is also true of business fixed investment. Even equipment that is not firm specific, such as office equipment, computers, cars and trucks, will typically have resale value considerably below acquisition cost. Second, investment activity is inherently risky, as it requires the commitment of resources today for a particular purpose based on expectations of future outcomes for many variables that in general will not in fact be realized. Third, there is typically discretion in the timing of investment, as capital allocation involves not only a decision on the amount but also on when to invest. Waiting to commit resources may be the preferred option because more information will become available that bears on the profitability of the investment.

When all three features are present, the standard present discounted value (PDV) criterion for investment, which is that one should invest at the time when the present value of expected cash flows of a unit of capital equals or exceeds the purchase price and installation cost of the capital, needs to be modified. The reason is that when a firm makes an irreversible investment outlay, it gives up the option of waiting for new information that affects the profitability of the investment. This lost option value is an opportunity cost that needs to be incorporated with the other costs of investment. Consequently, it is appropriate to actually undertake a project only when its present discounted value exceeds the purchase and installation costs by an amount equal to the value of keeping the option open and not committing the expenditures.

The value of waiting can be illustrated using a diagram described by Dixit (1992). This involves a single project with irreversible expenditure, I, that yields a stream of net revenue, R, which lasts forever. This revenue stream is uncertain with a given probability distribution and is discounted by a positive interest rate, r. The standard PDV approach implies that one should invest whenever R/r exceeds I. This involves the implicit assumption that the choice is between investing now or never. However, if one adds the additional possibility of waiting, then this alternative can be better than that of either not investing at all or proceeding immediately to invest.

This can be seen by comparing these two alternative strategies explicitly. On the one hand, the latter alternative has an expected value of zero, as it involves either not investing at all or investing exactly at the point where the NPV criterion is satisfied. The former strategy involves waiting for a given interval of time to see what the outurn will be, where there are two possibilities: if R/r is less than I, then profits are zero as there is no investment, whereas

profits are positive if R/r is greater than I. As the expected value of these two possibilities it itself positive, the option of waiting is preferable to a strategy of either using the PDV criterion or not investing at all. Waiting over the given interval enables the investor to reduce the risk of a low-return outcome. Of course, if the revenue stream turns out to be sufficiently high, it is no longer optimal to wait any longer. There is an optimal trigger point for the value of R which exceeds the usual PDV criterion.



The optimal waiting time and therefore the optimal trigger point, is determined where the marginal value of waiting is equal to the marginal value of investing. The former is equal to the slope of the value of investing schedule shown as WW in Figure 1, where net revenue, R, is on the horizontal axis and the present discounted value of the entire investment project, R/r-I, is on the vertical axis. When the current value of R is very low, the present discounted value of future receipts also very low, and the WW schedules goes to zero from above as R goes to zero. Increasing current values of R raises the present discounted value of the project, resulting in the convex curve WW. The marginal value of investing is equal to 1/r and is equal to the slope of the II schedule, which shows the value of net revenue, R/r-I as a function of R. The optimal value for the net revenue is given by the trigger point, T, which is where the two schedules are tangent to each other at point S. This is known as the smooth pasting condition which equates the marginal value of waiting with the marginal value of investing. This condition, which determines the optimal timing of investment, is a key feature of the model of capital flows described below.

III. THE MODEL

We start with a description of the model⁵ and proceed to explain the mechanism generating the results. There are two prospective foreign investors in the world economy⁶. Starting operations in the domestic economy requires spending a considerable amount of resources to gather information about the country's economic and legal conditions. The cost of obtaining this information can be represented by a fixed cost, I. After the information has been collected, it is necessary to obtain an operation permit, which we will assume is granted after δ years, where δ is an exogenous parameter. Once operations start, the investor rents capital from the world market at a given rate, r, and obtains the domestic rate of return, P, which is decreasing in the total amount of capital invested in the economy. There are no adjustment costs to changing the stock of capital in any given period. Capital flows in the model are simply captured by the temporal evolution of the stock of capital over time.

We are interested in capturing the idea that first movers have an advantage over late comers, as it appears that market share is an important determinant in business decisions and one of the factors motivating investment in emerging markets. A simple way to model this behavior is to assume that investors are not price takers in the domestic economy: if there is only one active project, the firm behaves as a monopolist. In the case of two active projects, the total amount of capital is determined in a Cournot equilibrium. Therefore, the advantage of being a first mover is the higher profit obtained as a monopolist. We will refer to the first mover as the Leader and to the late comer as the Follower.

As explained above, both fixed costs and uncertainty are essential ingredients in the real options approach to investment. We adopt the standard assumption in models of this kind regarding the treatment of uncertainty, namely, that is characterized by an exogenous shock, X, that follows a geometric brownian motion, a process frequently used to model economic and financial variables:

$$dX = \mu X dt + \sigma X dz \tag{1}$$

where μ is the expected growth rate of X, σ is sometimes referred to as the instantaneous variance of X, and dz is the increment of a standard Wiener Process. It is assumed that μ and σ are constants. Under this assumption, X can take values in the interval $[0,\infty)$ and is lognormally distributed with mean $(\mu-1/2\sigma^2)t$ and variance σ^2t . Modelling the shock as a geometric brownian motion process has the advantage of making possible the derivation of closed-form solutions. However, we recognize that it would be more appropriate to assume that X is governed by a mean-reverting process, as such a process would appear to be more appropriate for the economic fundamentals that affect the domestic rate of return.

⁵ The model is a simple modification of the one described in Grenadier (1996).

⁶ The basic assumption is that there exist only two investors. Their nationality is irrelevant to the analysis.

It remains to specify where the uncertainty comes from. From the point of view of an investor, there are two possible sources of uncertainty that can affect profits and hence his investment decisions. Firstly, there exists uncertainty related to the country itself that affects the domestic return on the project. Secondly, fluctuations in the borrowing terms on the world capital market will also affect profits. Clearly, in the real world uncertainty related to both domestic and world economic and political conditions are very important. To keep the analysis simple, we restrict the analysis to the two extreme cases of domestic uncertainty and foreign uncertainty. Domestic and foreign uncertainty are characterized in Assumptions 1 and 2 below, respectively.

Assumption 1 The investors can borrow capital from abroad at the risk free rate, r, and the domestic rate of return is stochastic and given by

$$P(t) = X(t)[Q(t)]^{\theta}$$

where P is the gross domestic return to one unit of capital, Q is the total capital supplied in the domestic economy, $\theta \in (-1,0)$ is the elasticity of the rate of return with respect to capital and X, defined above, is a multiplicative shock.

Under Assumption 1, returns in the domestic economy exhibit decreasing returns to scale in total capital stock, since $\operatorname{sign}(\partial P/\partial Q) = \operatorname{sign}(\theta) < 0$. As $\theta \to 0$, returns to scale become constant. Also, the domestic return is an increasing function of the shock, X. The next assumption characterizes the case in which there is uncertainty surrounding the cost of borrowing.

Assumption 2 The domestic return to capital is given by

$$P(t) = [Q(t)]^{\theta}$$

while the borrowing rate from abroad is given by

$$r_t = RX_t$$

where R > 0 is a constant and X has been defined above.

Both the domestic rate of return and the borrowing cost are unbounded processes because of the assumption that X follows a geometric brownian motion process. As stated above, a more appropriate assumption would be to model the shock as a mean-reverting process but this would be at the expense of analytical tractability. We have chosen to emphasize tractability rather than realism, while recognizing that the assumption of geometric brownian motion has certain shortcomings.

In this setup, the main problem facing investors is to decide when is the right time to start investing in the domestic economy. This decision can be decomposed in two related problems:

- If no one has entered the market, when is the right time to start investing and to become the Leader?
- Once there is Leader in the market, when is optimal to be a Follower?

Answering these questions require a two-step strategy: we must first solve the Follower's problem and then proceed to solve the Leader's problem, as the latter depends on the former.

The Follower's profits depend on the amount of aggregate capital, Q, in the domestic economy and the exogenous shock X. As is shown below, under Assumption 1 and Cournot competition, the total amount of capital in the domestic economy as well as the Follower's profits can be expressed as a function of the exogenous shock. Consequently, the problem can be reduced to that of finding the threshold or trigger value of the exogenous shock that prompts entering the domestic market, which is defined as the *optimal stopping time*. This is the standard value-of-waiting problem that we described above in section II and can be solved in a closed-form solution.

The Leader's problem is more complicated because he faces a different pattern of profits. If he decides to enter the market he enjoys monopoly profits until the Follower enters, at which point profits are reduced because of Cournot competition. Because the Leader behaves strategically, he needs to know the optimal strategy of the Follower to estimate how long the monopoly stage will last. This problem is more difficult to solve, and while it is not possible to find a closed-form solution, we can prove that there exists a solution and characterize it. In the next section, we proceed to analyze the case in which the domestic rate of return is affected by the exogenous shock.

IV. SHOCKS TO THE DOMESTIC RATE OF RETURN

When the domestic rate of return is subject to exogenous shocks, the analysis is simplified if we express every function in terms of $Y = X^{\gamma}$, where $\gamma = -1/\theta$, as agents' profits can be expressed as linear functions of Y either both in the monopoly case or the Cournot case. Our setup is therefore analogous to that of Grenadier (1996), and the results derived therein can be translated in to our model. The basic problem faced by both investors is whether profits are enough to cover the sunk costs necessary to start operating in the country. In the case of the Follower, he knows that upon entering the market profits will be derived as the result

of Cournot competition. The entry decision of the Leader is more complicated: there is a period of time during which he operates alone and collects monopoly profits. Afterwards, the Follower enters and profits are reduced to Cournot levels. Therefore the Leader's decision to enter depends upon the Follower's entry decision. In consequence, we proceed to analyze what the instantaneous profits are under the two possible market structures, monopoly and Cournot competition, because this information is necessary to determine the optimal strategies of the Follower and the Leader.

A. Monopoly

In this case only the Leader is active in the domestic economy. Instantaneous profits, π^L , are given by

$$\pi_t^L = P_t Q_t - r Q_t
= X_t Q_t^{\theta+1} - r Q_t$$

Profit maximization yields the following expressions for the amount of capital invested, the domestic return and profits:

$$Q_t = \left(\frac{1+\theta}{r}\right)^{\gamma} X_t^{\gamma} \tag{2}$$

$$P_t = \frac{r}{1+\theta} \tag{3}$$

$$\pi^L = K_1 X_t^{\gamma} \tag{4}$$

where $\gamma = -1/\theta$ and $K_1 = -\theta \left(\frac{1+\theta}{r}\right)^{\gamma-1} > 0$. From equation (2) It can be observed that the amount of capital is positively correlated with the exogenous shock, X, since $\operatorname{sign}(\partial Q/\partial X) = \operatorname{sign}(\gamma) > 0$. The markup charged by the monopolist over his borrowing costs is:

$$\mu_M \equiv \frac{P_t}{r} = \frac{1}{1+\theta} \tag{5}$$

From equation (4) it can be seen that profits vary stochastically, as they are a function of the random disturbance term, X, and they vary inversely with the foreign interest rate, as $\partial K_1/\partial r < 0$. However, because there are assumed to be no adjustment costs, it follows that the monopolist always varies his capital stock optimally in response to shocks and thereby maintains the domestic rate of return, P, at a fixed level for a given r, as shown in equation (3). Consequently, the interest rate spread, P-r, is also constant and is equal to $-r\theta/(1+\theta) > 0$.

B. COURNOT COMPETITION

When the Follower enters the market, equilibrium is a result of Cournot competition. In this case, the instantaneous profits are given by:

$$\pi_t^i = X_t Q_t^\theta Q_t^i - r Q_t^i$$

for i = L, F, where $Q_t = Q_t^L + Q_t^F$. Profit maximization yields the following first-order conditions:

$$\theta X_t Q_t^{\theta - 1} Q_t^i + X_t Q_t^{\theta} = r \qquad i = L, F$$

Adding up both first-order conditions and taking into account that in equilibrium $Q_t^L = Q_t^F$, yields the following relationships:

$$Q_t = \left(\frac{2+\theta}{2r}\right)^{\gamma} X_t^{\gamma} \tag{6}$$

$$Q_t^L = Q_t^F = Q_t/2 (7)$$

$$P_t = \left(\frac{2r}{2+\theta}\right) \tag{8}$$

$$\pi_t^i = K_2 X_t^{\gamma} \qquad i = L, F \tag{9}$$

where the constant K_2 is given by:

$$K_2 = rac{1}{2} \left(rac{- heta r}{2+ heta}
ight) \left(rac{2+ heta}{2r}
ight)^{\gamma}$$

As in the case of monopoly, K_2 is a decreasing function of r, implying that ceteris paribus, an increase in the cost of borrowing results in lower net profits. Again, the domestic rate of return to capital, P, is constant and the interest rate spread, P-r, is equal to $-\theta r/(2+\theta)$, which is lower than the monopoly spread. The markup under Cournot competition is equal to:

$$\mu_C \equiv \frac{P_t}{r} = \frac{1}{1 + \theta/2} \tag{10}$$

which is less than μ_M .

With Cournot competition, for any given value of X, the capital stock is larger than under monopoly while the domestic return is smaller. In general, in both regimes there is no correlation between the level of capital stock and the interest rate spread. They are negatively correlated only when the regime changes; for example, when the Follower enters the market, the capital stock increases as the spread falls.

We proceed to show that this model is equivalent to Grenadier's by analyzing first the Follower's strategy and then the Leader's strategy.

C. Follower's Strategy

Once the Leader has already invested in the domestic economy, it is necessary to find the trigger value, X_F , of the domestic return that makes it attractive for the Follower to exercise the option to invest in the domestic economy. The value of being the Follower is equal to the amount of capital, I, owned by the investor and the option to start the project, with the value of this option denoted by W(X). Standard arguments⁷ show that W(X) must be the solution of the following second-order differential equation:

$$\frac{1}{2}\sigma^2 X^2 W''(X) + \mu X W'(X) - rW(X) = 0$$

However, W(X) can also be expressed in terms of Y instead of X because, as noted above, there is a one-to-one relationship between these variables. In this case, the option to invest in the domestic economy will be given by the solution W(Y) of:

$$\frac{1}{2}\sigma'^2 Y^2 W''(Y) + \mu' Y W'(Y) - rW(Y) = 0$$
(11)

where application of Ito's lemma to $Y = X^{\gamma}$ shows that Y is also a geometric brownian motion that satisfies:

$$dY = \mu' Y dt + \sigma' Y dz$$

where the parameters μ' and σ' are defined by:

$$\mu' = \gamma \mu + \frac{1}{2} \gamma (\gamma - 1) \sigma^2$$
$$\sigma' = \gamma \sigma$$

Inspection of equations (4) and (9) shows that net profits are linear functions of Y both in the monopoly case and the Cournot competition case:

$$d\pi = KdY$$

$$= K\mu'Ydt + K\sigma'Ydz$$

$$= \mu'\pi dt + \sigma'\pi dz$$

Therefore, we can reinterpret the parameters μ' and σ' as the expected growth rate and the instantaneous variance of profits net of the borrowing costs, respectively. The expected payoff from investing at time t is given by

$$\Pi_t = \mathbf{E} \int_t^\infty \pi_s e^{-r(s-t)} ds$$
$$= \frac{\pi_t}{r - \mu'}$$

Positive profits require the following assumption:

⁷ See Dixit and Pindyck (1994) for a detailed explanation.

Assumption 3 $\mu' < r$

The general solution W(Y) of equation (11) is given by

$$W(Y) = AY^{\beta} + BY^{\beta_1}$$

where β and β_1 are the positive and negative solutions, respectively, of the quadratic equation:

 $\frac{1}{2}\sigma'^{2}\eta(\eta - 1) + \mu'\eta - r = 0$

Because we impose the condition that the value to invest is zero when Y = 0, the constant B must be equal to zero, so that the solution reduces to:

$$W(Y) = AY^{\beta} \tag{12}$$

where

$$\beta = \frac{1}{2} - \frac{\mu'}{\sigma'^2} + \sqrt{\left(\frac{\mu'}{\sigma'^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma'^2}} > 0$$

The value of the constant, A, as well as the value of Y_F must be determined from the boundary conditions, which are derived below.

1. Boundary conditions

To derive the smooth pasting conditions described above in section II, it is necessary to analyze the Cournot competition case when both the Follower and the Leader are investing in the market. Expected discounted profits at the time that the project is completed, for a given shock value of Y, are equal to:

$$\frac{K_2Y}{r-\mu'}$$

The boundary conditions are then given by

$$W(Y_F) = \frac{e^{-(r-\mu')\delta}}{r-\mu'} K_2 Y_F - I \tag{13}$$

$$W'(Y_F) = \frac{e^{-(r-\mu')\delta}}{r - \mu'} K_2 \tag{14}$$

Condition (13) is the value matching condition, that simply states that at the trigger value Y_F , the value of the option must be equal to the discounted cash flow obtained from exercising it. Equation (14) is the smooth pasting condition which is necessary to assure maximization of the option to invest. The left-hand side is the marginal value of waiting and the right-hand side is the marginal value of the investment.

2. Optimal strategy of the Follower

We can find an expression for the value of the option to invest using equations (12), (13) and (14) to determine both the constant, A, and the trigger value, Y_F :

$$W(Y) = \begin{cases} \frac{I}{\beta - 1} \left(\frac{Y}{Y_F}\right)^{\beta} & \text{if } Y < Y_F \\ \frac{e^{-(r - \mu')\delta}}{r - \mu'} K_2 Y - I & \text{if } Y \ge Y_F \end{cases}$$
 (15)

where the trigger value is given by

$$Y_F = \left(\frac{\beta I}{\beta - 1}\right) \left(\frac{r - \mu'}{K_2}\right) e^{(r - \mu')\delta} \tag{16}$$

When stated in terms of X, the trigger value is $X_F = Y_F^{1/\gamma}$. Being the follower thus has a value equal to:

$$F(Y) = I + W(Y) \tag{17}$$

The Follower will enter the market only when there is a wedge between the present value of the decision and the sunk cost I, which can be seen from the fact that $F(Y_F) = I\beta/(\beta-1)$.

The following proposition is analogous to Proposition 1 in Grenadier (1996):

Proposition 1 Conditional on the Leader having committed to operate in the country, the optimal Follower strategy is to pay the sunk cost I and enter the domestic market the first moment that X_t equals or exceeds the trigger value X_F , that is, the optimal entry time of the Follower, T_F , is given by

$$T_F = \inf \left\{ t \ge 0 : X_t \ge \left[\left(\frac{\beta I}{\beta - 1} \right) \left(\frac{r - \mu'}{K_2} \right) e^{(r - \mu')\delta} \right]^{1/\gamma} \right\}$$

The expected optimal entry time of the follower, T_F , increases in r as borrowing becomes more expensive, because higher domestic returns are required so that the present discounted value of the project, which is decreasing in r, is enough to cover the fixed cost, I. For the same reason, an increase in the latter increases the expected entry time. If the time to obtain an operating permit, δ , lengthens, the expected entry time also increases since it takes longer to start operating in the domestic country and recover the sunk cost. In contrast, when σ increases and domestic return becomes more volatile, the expected entry time decreases because the effective drift component, μ' , increases. The latter is equivalent to the deterministic growth rate of the domestic return, so a higher value makes the investment project more attractive. In addition, we can interpret $r - \mu'$ as the opportunity cost of postponing operations. As this cost decreases, the optimal stopping time T_F decreases, leading to a smaller delay. In the limit case, $T_F \to 0$ as $\mu' \to r$: because the opportunity cost is null, it is optimal to start operations immediately.

3. Leader's strategy

As noted above, the Leader's strategy is complicated by the fact that it must take account of the Follower's behavior. Assume that in the year t the Leader's is still τ years from obtaining the operation permit and define $L(Y,\tau)$ as the value of the Leader's project at that specific time. If $Y \geq Y_F$, the value of the Leader's project is equal to

$$\frac{e^{-(r-\mu')\tau}}{r-\mu'}K_1Y + \frac{e^{-(r-\mu')\delta}}{r-\mu'}(K_2 - K_1)Y$$

where the first term is the expected discounted present value of holding a monopoly position forever and the second term is the necessary correction once the Follower starts operations.

If $X < X_F$, then the Follower will not enter the market until year T_F , where T_F is the first time in which $X > X_F$. Grenadier shows that the Leader's payoff is equivalent to the payoff of the following portfolio:

- 1. Purchase a call option on an asset that pays a perpetual dividend rate of K_1Y , with zero exercise price and fixed expiration date of τ .
- 2. Purchase a call option on an asset that pays a perpetual dividend rate of $(K_2 K_1)Y$, with zero exercise price and a stochastic expiration date $T_F + \delta$.

Notice that although our setup allows for a variable capital stock, our assumptions reduce the problem to a setup similar to that of Grenadier's, where his constants D(2) < D(1) have been replaced by our constants $K_2 < K_1$ respectively, provided that the original stochastic variable X is replaced by Y. Therefore, the solution of $L(Y, \tau)$ is given by

$$L(Y,\tau) = \begin{cases} \frac{e^{-(r-\mu')\tau}}{r-\mu'} K_1 Y + \frac{\beta I}{\beta-1} \left(\frac{K_2 - K_1}{K_2}\right) \left(\frac{Y}{Y_F}\right)^{\beta} & \text{if } Y < Y_F \\ \frac{e^{-(r-\mu')\tau}}{r-\mu'} K_1 Y + \frac{e^{-(r-\mu')\delta}}{r-\mu'} (K_2 - K_1) Y & \text{if } Y \ge Y_F \end{cases}$$

Thus all the results from Grenadier (1996) follow through. In particular, we have an analog of his Proposition 2 which establishes when it is more valuable to be the Leader or the Follower, and that drives all the results with respect to the investment strategies:

Proposition 2 (Grenadier 1996): Assume that the Leader is just starting to gather the information after paying the sunk cost, that is $\tau = \delta$. Then there exists a unique point $Y_L \in (0, Y_F)$ with the following properties:

a)
$$L(Y, \delta) - I < W(Y)$$
 if $Y < Y_L$

b)
$$L(Y, \delta) - I = W(Y)$$
 if $Y = Y_L$

c)
$$L(Y, \delta) - I > W(Y)$$
 if $Y_L < Y < Y_F$

d)
$$L(Y, \delta) - I = W(Y)$$
 if $Y \ge Y_F$

Clearly, there is one X_L that corresponds to Y_L such that $Y_L = X_L^{\gamma}$, so that we can either refer to trigger points with respect to the original geometric brownian motion or the modified one. The trigger points X_L and X_F , $X_L < X_F$ that correspond to Y_L and Y_F in the previous proposition, determine three possible regions. In the first one, when $X < X_L$, it is better to be the follower, as property a) indicates. The second region corresponds to the case $X_L < X < X_F$, where the value of being the Leader dominates the value of being the follower. Finally, when $X \ge X_F$, it is indifferent to be either the Leader or the Follower. The next section explores the implications of this proposition in more detail.

D. Hysteresis, Sequential Development and the Speed of Development

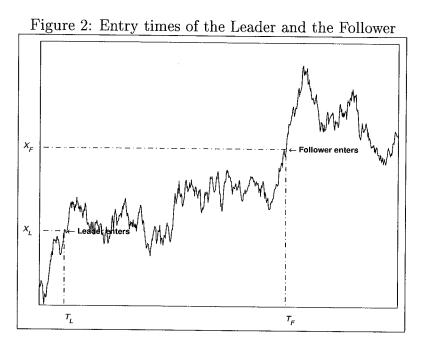
In this section we illustrate graphically the timing of the investment as well as the evolution of the capital stock and therefore capital flows. In our model with only two investors, sequential development is the process in which one investor starts to invest in the domestic economy, and then only after some time elapses the second investor takes the same decision. This investment pattern corresponds to one of the two possible equilibria of the model; while the other equilibrium corresponds to the process of simultaneous investment, which is described in the next section⁸. In the context of the domestic economy where the stochastic shock X is positively correlated with the domestic return in the country, the higher X is, the better are the domestic economic conditions. Proposition 2 shows that there are two important points, X_L and X_F , satisfying $X_L < X_F$, which summarize the economic conditions required to have either one, two or no investor operating in the country. Sequential investment occurs only if the country's initial economic conditions are lower than X_F and is characterized in the following proposition.

Equilibrium with sequential investment: Assume that the initial shock value satisfies $X_0 < X_F$. If your competitor has not started his project, then start the project when $X_t \ge X_L$ for the first time. If the competitor is already in the market, wait until $X_t \ge X_F$ to start your project.

If domestic conditions are lower than the value associated with X_L , no investor would start a project in the country economy. Why? Starting a project requires paying a sunk cost of I and because of the uncertainty surrounding the domestic rate of return, waiting to invest has an economic value. Therefore, it is possible that during long periods no new projects are started, despite a positive trend of the country's economic conditions. This is the so called **hysteresis effect.** When the level X_L is reached, then one of the investors starts his project and enjoys monopoly profits. For economic conditions between X_L and X_F , it is

⁸ See proof in Grenadier (1996).

⁹ A readable explanation is Dixit (1992).

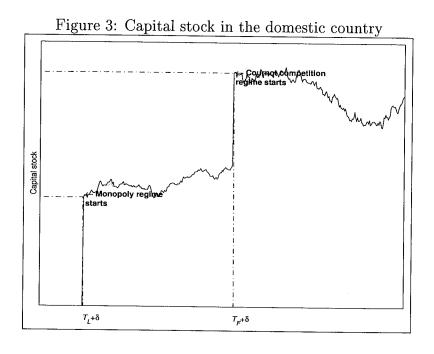


not profitable for the remaining investor to follow¹⁰. Figure 2 illustrates these two threshold values of X. As the exogenous shock evolves over time, the Leader enters when $X = X_L$ for the first time at $t = T_L$. The Follower does not invest until the economic conditions reach a new higher level associated with X_F .

Figure 3 illustrates how the capital stock evolves over time. Before time $T_L + \delta$, the capital stock is zero. Afterwards, it jumps to an amount equal to $Q_{T_L+\delta} = (1+\theta/r)^{\gamma} X_{T_L+\delta}^{\gamma}$ and varies continuously as the monopolist adjusts the capital stock to keep the optimal markup. This process continues until time $T_F + \delta$, the point at which the Leader and the Follower engage in Cournot competition. At that time, the total capital stock jumps from $Q_{T_L+\delta}$ to $Q_{T_F+\delta} = (2+\theta/2r)^{\gamma} X_{T_F+\delta}^{\gamma}$. Afterwards, the capital stock changes continuously as both duopolists adjust their stocks to maximize profits. Therefore, this model can explains discrete upward jumps of capital inflows. Increased competition at $T_F + \delta$ is also associated with a decrease in the domestic interest rate markup, as shown in Figure 4.

The speed of development is associated to the time elapsed between the start of the first and second projects, that is $T_F - T_L$. Grenadier (1996) shows that the median of this quantity tends to decrease the more volatile the domestic conditions, since it would be quicker for the economy to reach the required level, X_F . The needed time to obtain the permit, δ , plays no role in the speed of development. Therefore, this analysis suggests that countries characterize by a more volatile economic environment would tend to have the faster development of new projects.

¹⁰The Leader could be either one of the investors. Because there are no particular preferences, who gets to be the Leader can be decided by a simple lottery, i.e. a coin toss.



E. Investment Booms and Startup of New Projects during Recessions

We have already examined what happens in an economy when the initial economic conditions are such that $X_0 \leq X_F$, so it remains to explain what happens when $X_0 \geq X_F$. The proposition below describes the other equilibrium that gives rise to simultaneous entry by both competitors.

Proposition 3 (Grenadier 1996) Equilibrium with simultaneous investment or investment boom: Assume that the initial shock value satisfies $X_0 \geq X_F$. Then there exists $X_J > X_F$ such that the optimal strategy is to start the project at once the moment $X_t \leq X_F$ or $X_t \geq X_J$. If the competitor enters first, then enter instantaneously thereafter.

If the shock is in the range corresponding to the interval $[X_F, X_J]$, the best policy for both investors is inaction. However, once the shock moves outside this interval in either direction, it is in the interests of both investors to start projects immediately. Therefore there are two possible cases of simultaneous entry which are shown in Figures 5 and 6. In the case of improving economic fundamentals, for any given $\widehat{X} > X_F$, the equilibrium strategy is to enter immediately because the value of being the Leader is greater than value of being the Follower, as shown in Proposition 2. In particular, there is a value X_J such that it maximizes the value of both competitors is maximized when they enter simultaneously. Therefore, both investors enter at the same time when X reaches the value X_J as long as X has not fallen below X_F . This is the case illustrated in Figure 5.

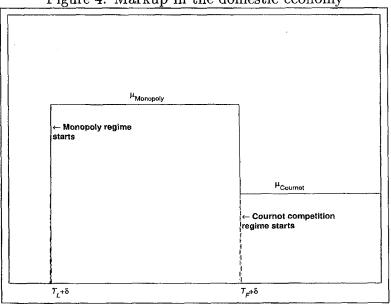


Figure 4: Markup in the domestic economy

The other case of simultaneous entry is related to a deterioration of the economic fundamentals, as shown in Figure 6. When the economic fundamentals in the interval (X_L, X_F) , and none of the investors have started projects, there is no advantage in being either the Leader or the Follower. Therefore there is no entry. Nevertheless, once the fundamentals are such that $X < X_F$, simultaneous entry occurs. This is a consequence of Proposition2, which shows that it is more valuable to be the Leader than the Follower when $X \in (X_L, X_F)$. This last case rationalizes what it would otherwise seem to be the irrational start of new projects when the economy starts to slowdown.

Whether investment follows a sequential process or a boom depends on the initial conditions, summarized by the value of the exogenous shock at t = 0, X_0 . If X_0 is below X_F , we would observe sequential investment. However, it could be the case that structural reforms, such as the removal of capital controls or the liberalization of the financial sector, improve the profitability of investment, shifting X_0 to a value in the range $[X_F, X_J]$. In this case, investment would be characterized by an initial period of inaction followed by a boom.

V. SHOCKS TO THE FOREIGN BORROWING RATE

In this section we analyze the case in which the exogenous shock affect the borrowing cost. We will prove that this case is analogous to that of a stochastic domestic return and that all the results derived previously hold in this setup. Under Assumption 2 and normalizing

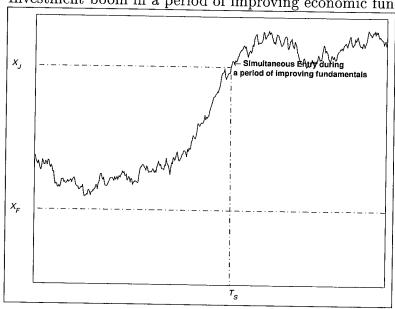


Figure 5: Investment boom in a period of improving economic fundamentals

 $R \equiv 1$, it is not difficult to show that in the case of monopoly, the capital stock, profits and the domestic return to capital are given by:

$$Q_t = \left(\frac{X_t}{1+\theta}\right)^{1/\theta} \tag{18}$$

$$P_t = \frac{X_t}{1+\theta} \tag{19}$$

$$\pi_t^L = -\theta (1+\theta)^{-\frac{1+\theta}{\theta}} X_t^{\frac{1+\theta}{\theta}}$$
$$= K_1 Y_t^{\omega}$$
(20)

where $\omega = \frac{1+\theta}{\theta} < 0$. The interest rate spread, $P_t - r_t$, is equal to

$$P_t - r_t = -X_t \theta / (1 + \theta) \tag{21}$$

and the markup is constant and equal to $1/(1+\theta)$. The relationship between total capital stock and the exogenous shock is exactly opposite to that described in the previous section: total capital stock increases when the borrowing rate decreases, a result that we would expect. This result is derived by taking the partial derivative $\partial Q/\partial X$ in equation (18) and noting that $sign(\partial Q/\partial X) < 0$. On the other hand, the interest rate spread is positively correlated with X, a result obtained by taking the partial derivative $\partial (P_t - r_t)/\partial X$ in equation (21). Therefore, there exists a negative correlation between the capital stock and the interest rate spread:

$$\operatorname{sign}\left[\frac{\partial Q}{\partial (P_t - r_t)}\right] = \operatorname{sign}\left[\frac{\partial Q/\partial X}{\partial (P_t - r_t)/\partial X}\right] < 0$$

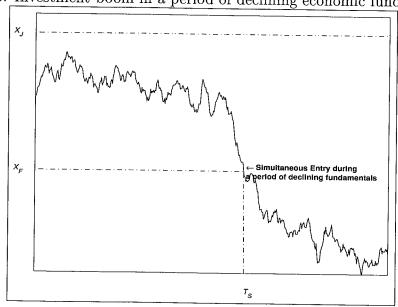


Figure 6: Investment boom in a period of declining economic fundamentals

This result explains partially the observed negative correlation between the interest rate spread and the amount of capital flows to emerging markets, as documented in Cline and Barnes (1997).

In the case of Cournot competition, capital stock, profits for each investor and the domestic rate of return are given by:

$$Q_t = \left(\frac{X_t}{1 + \theta/2}\right)^{1/\theta} \tag{22}$$

$$P_t = \frac{X_t}{1 + \theta/2} \tag{23}$$

$$\pi_t^L = -\frac{\theta}{2} (1 + \frac{\theta}{2})^{-\frac{1+\theta}{\theta}} X_t^{\frac{1+\theta}{\theta}}$$

$$= K_2 Y_t^{\omega}$$
(24)

Again, capital stock is negatively correlated with both increases in the borrowing rate, as represented by an increase in X and with the interest rate spread. Therefore, the analysis of the previous case of shocks to the domestic rate of return applies here. However, we must recall that $\omega < 0$, therefore, the analogous propositions must take into account that if $Y^* > Y^{**} \Rightarrow X^* < X^{**}$. This implies that the correct ordering of the trigger points, if stated in terms of the original brownian motion X should be $X_L > X_F > X_J$. This makes sense, since an increase in X reflects a higher borrowing cost, instead of a higher return. It is necessary to reformulate the equilibrium investment strategies as follows:

- 1. Equilibrium with sequential investment: Assume that the initial shock value satisfies $X_0 > X_F$. If your competitor has not started his project, then start the project when $X_t \leq X_L$ for the first time. If the competitor is already in the market, wait until $X_t \leq X_F$ to start your project.
- 2. Equilibrium with simultaneous investment: Assume that the initial shock value satisfies $X_0 < X_F$. Then it is possible to show that there exists a trigger value $X_J < X_F$ such that the optimal strategy is to start the project at once the moment $X_t \ge X_F$ or $X_t \le X_J$. If the competitor enters first, then enter instantaneously thereafter.

VI. THE RELATIONSHIP BETWEEN THE INTEREST RATE SPREAD AND CAPITAL FLOWS

In this section we analyze the relationship between the interest rate spread, defined here as the difference between the domestic rate of return, P_t , and the borrowing rate, r; and the capital flows, represented by changes in Q.

Profit maximization under imperfect competition implies that firms charge a constant markup over their borrowing costs. The markups and interest rate spreads for the cases of Monopoly and Cournot competition are summarized below.

Monopoly Cournot Markup:
$$P_t/r$$
 $1/(1+\theta)$ $1/(1+\theta/2)$ Spread: P_t-r $-r\theta/(1+\theta)$ $-r\theta/(2+\theta)$

Notice that the interest rate spread is proportional to the borrowing cost, r. Therefore the interest rate spread varies only when the borrowing rate changes.

When the borrowing cost is constant, as in the case analyzed in section IV, the interest rate spread remains constant in despite changes in the multiplicative shock that affect the domestic rate of return. But changes in the domestic rate of return do affect the capital stock, Q, as it adjusts continously such that the optimal markup is preserved. In this environment, in which the cost of capital remains fixed, the interest rate spread does not provide information about changes in the capital stock, implying an almost always zero correlation between these two quantities. The only exception is the case in which the market regime changes from Monopoly to Cournot, that was illustrated in Figures 3 and 4. With the entry of a new competitor, the interest rate spread decreases while at the same time the capital stock jumps.

When the borrowing cost, r, is affected by exogenous shocks, as assumed in Assumption 2, the correlation between the interest rate spread and the borrowing rate is 1. As the

analysis in section V points out, changes in the capital stock are negatively correlated with changes in the interest rate spread.

Summarizing, this model is able to explain the following facts:

- 1. Fixed investment can be either gradual or explosive. Both cases would depend on the value of an underlying exogenous factor.
- 2. New projects can be started despite a decreasing trend of economic fundamentals. This is a consequence of the equilibrium with simultaneous investment. In this case, because of proposition 2, it is optimal for the investors to exercise their option to invest in the domestic economy although the returns to capital are following a decreasing trend.
- 3. If the exogenous factor does not affect the borrowing rate, the interest rate spread is constant under both Monopoly and Cournot competition. In addition, the spread is not correlated most of the time with changes in the capital stock, which does react to shocks to the exogenous factor. The exception is the precise point in time in which the economy shifts from the monopoly regime to the Cournot competition regime: then, we can observe a drop in the interest rate spread associated with an increase in the capital stock. When the exogenous factor affects the borrowing rate, the interest rate spread is perfectly correlated with the borrowing rate and negatively correlated with changes in the capital stock. The empirical implications of the model are clear: with market power, if uncertainty is driven mostly by factors that do not affect the borrowing rate or the cost of capital, there should not be a clear relationship between capital flows and the interest rate spread. However, if uncertainty is due mainly to factors that affect the foreign cost of capital, then the interest rate spread is negatively correlated with capital flows.

VII. CONCLUDING REMARKS

This paper has shown that some stylized facts of capital flows can be rationalized in an irreversible investment framework, where first movers can benefit more by obtaining monopoly profits investing capital in a small country. In particular the model explains why sometimes fixed investment proceeds gradually and sometimes an investment boom, rationalizes the startup of new projects during a period of declining fundamentals and shows that the interest rate spread can provide little information about capital flows. We proceed to discuss some of the assumptions and omissions in the model.

Though much attention has been paid to entry strategies and some simple results have been obtained, the model is mute with respect to the possibility of exit. In the model, capital flows follow a continuous process characterized by discrete upward jumps, which seem to characterize the observed pattern of capital flows. The introduction of exit decisions would introduce also discrete downward jumps in the capital stock, that can be interpreted as drastic capital outflows.

We have considered only the case of Cournot competition among both investors. because of its analytical tractability. However, an important extension of the model should consider the case of Stackelberg competition. Our guess is that the value of being the Leader would increase while the value of being the Follower would decrease. This would weaken the hysteresis effect, reducing the elapsed time until the first investor enters the market but increasing the period of time until the start of the second project. The relationship between capital flows and economic conditions would not be as clear as in the Cournot case.

We have assumed that the exogenous shock, X, is characterized by a geometric brownian motion process. However, when this shock represents either borrowing costs or the profitability of investment in a given country, this assumption implies that these variables can wander far away from their starting points. A more realistic assumption would be to model X as a mean-reverting process, so that X can deviate from its mean in the short run, but goes back in the long run. The disadvantage of this approach is that closed-form solutions are not available and it would be necessary to derive the optimal strategies numerically.

REFERENCES

- Cabral, Luis, 1997, "Entry mistakes," Discussion Paper No. 1729 (London: CEPR).
- Chen, Zhaohui, and Mohsin Khan, 1997, "Patterns of capital flows to emerging markets: a theoretical perspective," Working paper No. 97/13 (Washington: International Monetary Fund).
- Cline, William R., and K.J.S. Barnes, 1997, "Spreads and risk in emerging markets lending," Research Paper No.97-1 (Washington: Institute for International Finance).
- Dixit, Avinash, 1989, "Hysteresis, import penetration and exchange rate pass-trough," Quarterly Journal of Economics 104, pp. 205-228.
- ----, 1992, "Investment and hysteresis," Journal of Economic Perspectives 6, pp. 107-132.
- ----, 1993, "The art of smooth pasting," (London; New York: Harwood Press).
- ----, and Robert Pindyck, 1994, Investment under uncertainty (New Jersey: Princeton University Press).
- Dornbusch, Rudiger, and Alejandro Werner, 1994, "Mexico: Stabilization, Reform, and No Growth," *Brookings Papers on Economic Activity: 1*, pp. 253-315.
- Dutta, Prajit K., and Aldo Rustichini, 1993, "A theory of stopping time games with applications to product innovation and asset sales," *Economic Theory 3*, pp. 743-763.
- Dygvig, Philip H., and Jaime F. Zender, 1991, "Capital structure and dividend irrelevance with asymmetric information," *Review of Economic Studies 4*, pp. 201-219.
- Eichengreen, Barry, Andrew Rose, and Charles Wyplosz, 1996, "Speculative attacks on pegged exchange rates: an empirical exploration with special reference to the European Monetary System," in *Transatlantic Economic Issues*, ed. by Matthew Canzoneri, Paul Masson, and Vittorio Grilli (Cambridge: Cambridge University Press).
- Grenadier, Steve R., 1996, "The strategic exercise of options: development cascades and overbuilding in real estate markets," *Journal of Finance 51*, pp. 1653-1679.
- Kulatilaka, Nalin, and Bruce Kogut, 1996, "Direct investment, hysteresis and real exchange rate volatility," *Journal of the Japanese and International Economies* 10, pp. 12-36.
- ----, and Enzo Perotti, 1992, "Strategic investment timing under uncertainty," Financial Markets Group, Discussion paper No. 145 (London: London School of Economics).
- Lambrecht, Bart, and William Perraudin, 1994, "Option games," DAE Working Paper No. 9414 (Cambridge, United Kingdom: Cambridge University).
- ----, and William Perraudin, 1996, "Real options and preemption," Institute for Financial Research, Working Paper No. IFR26 (London: Birbeck College).
- Masson, Paul, 1998, "Contagion: Monsoonal Effects, Spillovers, and Jumps between Multiple Equilibria," Mimeo (Washington: International Monetary Fund).

- Radelet, Steve, and Jeffrey Sachs, 1998, "The Onset of the East Asian Financial Crisis," Mimeo (Cambridge, Massachusetts: Harvard Institute for International Development).
- Sachs, Jeffrey, Aaron Tornell, and Andrés Velasco, 1996, "Financial crises in emerging markets: the lessons from 1995," *Brookings Papers on Economic Activity 1*, pp. 147-215.
- Smets, Frank R., 1993, "Exporting vs. foreign direct investment: the effect of uncertainty, irreversibilities and strategic interactions," Unpublished Ph.D. dissertation (New Haven: Yale University).