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Phillips Curves, Phillips Lines and the Unemployment Costs of Overheating

Prepared by Peter B. Clark and Douglas Laxton[1]

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Abstract

Most empirical work on the U.S. Phillips curve has had a strong tendency to impose global linearity on the data. The basic objective of this paper is to reconsider the issue of nonlinearity and to underscore its importance for policymaking. After briefly reviewing the history of the Phillips curve and the basis for convexity, we derive it explicitly using standard models of wage and price determination. We provide some empirical estimates of Phillips curves and Phillips lines for the United States and use some illustrative simulations to contrast the policy implications of the two models.

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Authors' E-Mail Address: PClark1@imf.org and DLaxton@imf.org

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[1]Research Department, International Monetary Fund. This paper was written in large part while Clark was on leave at the London School of Economics, Center for Economic Performance. An earlier version of this paper was presented at the Second International Conference on Computing in Economics and Finance, Geneva, June 26–28, 1996. We wish to thank Allan Manning and Alun Thomas for comments on an earlier draft. The opinions expressed in this paper are those of the authors and do not necessarily reflect the views of the IMF.

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SUMMARY

The objective of the paper is to reconsider the issue of nonlinearity of the Phillips curve and to underscore its importance for policymaking. A true Phillips curve implies that the relationship between inflation (adjusted for inflation expectations) and excess demand is asymmetric: a given amount of unemployment below the nonaccelerating inflation rate of unemployment (NAIRU) generates more inflation than the same amount of unemployment above the NAIRU lowers it. This asymmetry implies an important role for stabilization policy that is absent from linear models of the unemployment-inflation process. In particular, there can be significant gains from preventing an overheating of the economy, since the gains when unemployment is below the NAIRU are more than offset by the increase in unemployment needed to stabilize inflation. Moreover, convexity in the Phillips curve by itself (abstracting from inflation adjustment arising from forward-looking expectations and credibility effects) will result in higher cumulative unemployment costs, the faster is the pace of disinflation.

The paper briefly reviews the history of the Phillips curve and the theoretical basis for nonlinearity. It argues that the importance of nonlinearity in the original work of Phillips and Lipsey was lost in subsequent analyses that focused on explaining shifts in the curve. The paper then provides a derivation of the Phillips relationship, showing that the curvature reflects a structural parameter that links in a nonlinear manner the level of the real wage to the unemployment rate. The paper also shows that convexity in the Phillips curve implies that there is an important distinction between the NAIRU and the natural rate of unemployment that must be taken into account in estimation. Finally, estimated equations for the U.S. Phillips curve are used to calculate the unemployment costs of overheating and rapid deflation in linear and nonlinear models of the Phillips relationship.

I. INTRODUCTION

During cyclical recoveries there may be periods during which declines in unemployment are associated with little or no rise in the inflation. Indeed, there have been occasions when both unemployment and inflation fell at the same time. This was the case in the United States from 1984 to 1986 and in Canada, the United States, and the United Kingdom from 1992 to 1994. This inversion of the more typical inverse relationship between inflation and unemployment may reflect simply the fact that the actual unemployment rate is above the NAIRU as well as other factors, such as changes in the NAIRU itself, inflation expectations, or quasi-exogenous elements such as commodity prices.

Such periods of an apparently benign association between economic expansion and inflation may influence the judgement as to when in fact growth in demand relative to capacity output will begin to exert upward pressure on inflation. Such a judgement must necessarily involve an assessment of the benefits of further reductions in unemployment compared with the costs of the possibility of moving into a region of rising inflation. This issue has recently been addressed on page 55 of the 1996 *Economic Report of the President*:

> "A controversial issue in macroeconomics is whether the benefits from further reducing the unemployment rate when the economy is operating near full capacity outweigh the costs of possibly increasing the inflation rate. . . . The view that the unemployment rate must change by more when inflation is reduced than when it is increased, and the related view that a small increase in inflation may spark runaway inflation, have been the basis for cautious policy."

The need for caution in risking rising inflation in return for a reduction in unemployment when an economy is operating in the vicinity of full employment has been amply demonstrated by the many episodes of recession in industrial countries required to undo the inflationary impact of excess demand. However, the *Economic Report of the President* appears to undercut the basis for such caution by asserting on page 55 that "Much empirical work suggests . . . that for small changes, increases and decreases in inflation exhibit the same sacrifice ratio. And, small increases in inflation historically have not triggered runaway inflation." Of course it is true that for sufficiently small changes one can approximate a nonlinear relationship with a linear one. But this presumes that policymakers are so adept at fine-tuning that they can confine the economy in the range within which the linear approximation holds. We believe that the postwar experience with macroeconomic policy provides little support for this sanguine view. Rather, it is preferable to presume that any overshooting of the full employment level of output is likely to be large and that consequently policy decisions should be made with full recognition that any output and employment gains from a rise in inflation resulting from excess demand will be more than offset by losses needed to unwind the higher inflation.

The thrust of this paper is that there is an asymmetry in the relationship between inflation (adjusted for inflation expectations) and excess demand as measured by whether the actual unemployment rate is above or below the NAIRU; in other words, a given amount of unemployment below the NAIRU generates more inflation than the same amount of unemployment above the NAIRU lowers it. Thus the aim here is to re-establish the nonlinearity of the Phillips curve and to underscore its importance for policymaking. As described in Laxton, Meredith and Rose (1995), and Clark, Laxton, and Rose (1996), this nonlinearity implies that there are significant gains from preventing an overheating of the economy, as the additional employment above the NAIRU is more than offset by the reduction in employment below the NAIRU needed to stabilize inflation. Therefore macroeconomic policy that avoids boom and bust cycles can in fact raise the average level of employment and output.

In Section II below we briefly review the history of the Phillips curve, emphasizing that the assumption of nonlinearity that was explicit in the original analysis of Phillips (1958) and Lipsey (1960) tended to become implicit in subsequent analysis. Moreover, the importance of curvature was completely overshadowed by the attention (rightly) given to shifts in the curve in response to endogenous changes in inflation expectations. An important point of this review is that the curvature property further undermines the ability of policymakers to exploit any short-run tradeoff between inflation and unemployment.

Section III derives an equation for the Phillips curve based on simple models of wage and price determination. Following the standard approach of Layard, Nickell, and Jackman (1991), the real wage is specified as a function of the level of the unemployment rate. This enables us to relate the reduced-form coefficient of the unemployment rate in the Phillips curve to the structural parameter linking the level of the real wage to the unemployment rate. The theoretical and empirical literature on this latter link, including Layard, Nickell, and Jackman (1991), Nickell (1987), and Blanchflower and Oswald (1994), is brought to bear to support the contention that the Phillips relationship is indeed a curve.

Section IV provides some empirical estimates of Phillips curves and straight Phillips lines for the United States because, as noted above, the current cyclical position of the U.S. economy makes this topic a timely one. As already emphasized in the papers cited above where inflation is explained as an asymmetric function of the output gap, the asymmetry has important implications for the specification of the demand pressures emanating from the unemployment rate because it implies that the NAIRU must be distinguished from the natural rate of unemployment. Section V uses the estimated coefficients in Section IV to calculate the unemployment costs of overheating and rapid deflation using both linear and nonlinear models of the unemployment-inflation process. Finally, Section VI provides some concluding comments and suggestions for further research.

II. The Phillips Curve—A Selective Review

In his seminal article, Phillips (1958) examined the relationship in the United Kingdom between the rate of change in nominal wages and the unemployment rate over the period

1861–1957. The evidence he presented appeared to show not only that these two variables were negatively related, but also that this relationship was nonlinear. He did not provide any explicit theory for this relationship, but rather conjectured along the following lines:

> "When the demand for labor is high and there are very few unemployed we should expect employers to bid wage rates up quite rapidly, each firm and each industry being continually tempted to offer a little above the prevailing rates to attract the most suitable labor from other firms and industries. On the other hand it appears that workers are reluctant to offer their services at less than the prevailing rates when the demand for labor is low and unemployment is high so that wage rates fall only very slowly. The relationship is therefore likely to be highly nonlinear." [Phillips (1958), p. 283]

Believing that such a nonlinear link between wage inflation and the unemployment rate was apparent in a scatter diagram of the observations over the subperiod 1861–1913, Phillips attempted to estimate this relationship using a log-linear specification. As the rate of change in nominal wages was negative in many years over this period, he avoided the problem of taking the logarithm of a negative number by combining the observations into six groups based on the level of unemployment and computing the averages of the variables in each group. Using a trial-and-error procedure, he fitted a line through the resulting six points. The resulting line shows considerable curvature, with the rate of change of nominal wages rising sharply as the unemployment rate falls below roughly 3 percent, and declining only very slowly as the unemployment rises above this level. Thus the Phillips relationship was very much a curve from the very beginning of the analysis of the relationship between inflation and unemployment.[2]

In an extensive discussion of the issues raised in this paper, Lipsey (1960) rejected a number of the subsidiary hypotheses proposed by Phillips. Nonetheless, his analysis supported Phillips' basic finding of a negative and nonlinear relationship between the change in wages and the level of unemployment. Wishing to use all 52 observations in the 1861–1913 sample, he estimated an equation in natural units using the inverse of the level and the square of the unemployment rate as explanatory variables. The resulting curve is remarkably similar to that originally estimated by Phillips. Moreover, Lipsey filled the gap in Phillips' paper by providing

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[2]As pointed out in the *Journal of Political Economy*, Vol. 81, No. 2, Part II, March/April, 1973, Irving Fisher was the first to provide a statistical investigation of the relationship between inflation and unemployment. This work originally appeared as an article entitled "A Statistical Relation Between Unemployment and Price Changes," *International Labour Review*, Vol. 13, No. 6 (June 1926), pp. 785–92, and is reprinted in its entirety on pp. 496-502 in the above-mentioned journal. It is interesting to note that Fisher viewed the causation going from inflation to unemployment: ". . . what the figures show is largely, if not mostly, a genuine and straightforward causal relationship; that the ups and downs of employment are the effects, in large measure, of the rises and falls of prices, due in turn to the inflation and deflation of money and credit. (Fisher (1973), p. 502).

a simple model, which is described briefly below, that can account for the negative and nonlinear relationship between the two variables. Many of the issues in subsequent discussions of the Phillips curve were foreshadowed in Lipsey's paper, in particular, that shifts in the curve may be endemic on account of changes in actual and expected inflation. Yet the underlying basis for the curve, namely, that demand conditions in the labor market are a key determinant of wage inflation and that the magnitude of the effect of unemployment on the change in wages depends inversely on the unemployment rate itself, was central in Lipsey's work.

In subsequent research on the Phillips curve the nature and degree of curvature did not receive a great deal of attention. There appears to have been the view that the relationship between wage inflation and unemployment may indeed be nonlinear and that therefore one could estimate wage equations in both linear and nonlinear specifications to ascertain which gave the better fit. This was the approach taken, for example, by Coe (1985), Coe (1988), and Grubb (1986). However, there was more explicit focus on the issue of nonlinearity in Nickell (1987) and Layard, Nickell, and Jackman (1991). Their work provides a discussion of the theoretical basis for this nonlinearity which is a maintained hypothesis in the empirical specification of their wage equations. Moreover, their reading of the literature is that this specification leads to better empirical results in the work of others: ". . . it is commonplace that in many countries wage equations appear to perform better when [the unemployment rate] u is replaced by log(u) or 1/u. . . ." (Layard, Nickell, and Jackman, 1991, p. 201).

The question of whether the Phillips relationship was a straight line or a curve was eclipsed in part because in the 1960s and 1970s the dominant issue was the extent to which the relationship was stable. First, the natural rate hypothesis propounded by Friedman (1968) and Phelps (1970) called into question the usefulness of the Phillips curve as an analytical construct. Moreover, attention shifted to the expectations-augmented Phillips curve and the determinants of the NAIRU, as well was the factors generating the apparent rise in the NAIRU during the 1970s and 1980s in many industrial countries. Second, the two oil price shocks and unstable monetary policies in the 1970s led to large shifts in inflation relative to the level of the unemployment rate, so that as discussed below, the connection between these two variables was considerably obscured. Partly as a result of these developments, the important policy implications of the nonlinearity of the Phillips curve were never explored.[3]

The perception of the relative unimportance of the curvature of the Phillips curve does not appear to reflect theoretical qualms. No one has to our knowledge taken issue with Lipsey's original formulation, which bears restating. His point of departure is the fairly

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[3]It is noteworthy that Phillips (1958) pointed out the implication of nonlinearity for wage inflation in the penultimate paragraph of his paper on p. 299: "Because of the strong curvature of the fitted relation in the region of low percentage unemployment, there will be a lower average rate of increase of wage rates if unemployment is held constant at a given level than there will be if unemployment is allowed to fluctuate about that level." However, this insight was not developed further by Phillips.

standard hypothesis about dynamic adjustment, which in this case is that the rate of change in wages is related to excess demand. Lipsey assumes that this is a simple linear relationship. If one had a direct measure of excess demand, that would be the end of the matter. However, the commonly used proxy for excess demand is the inverse of unemployment rate, which is bounded at zero or some small value greater than zero. Consequently, as the unemployment rate declines from the NAIRU level and approaches zero, excess demand for labor approaches infinity and—by the dynamic adjustment hypothesis—so does the rate of change in wages. This implies a nonlinear relationship between the level of the unemployment rate and wage inflation.[4]

A nonlinear relationship between the level of the unemployment rate and the level of the wage rate can be derived from the shirking model of wage determination described by Shapiro and Stiglitz (1984). In this model employees dislike work, but they are willing to do so in order to receive compensation rather than incur the risk of becoming unemployed. However, workers cannot be forced to work by firms, who thus need to make work sufficiently attractive that the employee does not shirk. But to prevent a wage-price spiral as firms bid for workers, there must be enough unemployment so that workers cannot be sure of getting another job, i.e., unemployment functions as a "worker-disciplining device." Consequently, the real wage and the unemployment rate will be negatively related, and as shown by Layard, Nickell, and Jackman (1991), p. 162, this relationship will be nonlinear. As described in Section III below, if one posits a dynamic adjustment for prices similar to that described in the preceding paragraph, then one can derive a negative and nonlinear relationship between the rate of change of prices and the level of the unemployment rate.

Such a relationship has also been derived quite independently by Evans (1985) using a disaggregated model of bottlenecks across separate labor markets. At any point in time workers are located in specific labor markets where the base wage adjusts only slowly to excess demand via an expectations-augmented Phillips curve, and this base wage puts a short-term floor below which the actual wage cannot fall. Labor is otherwise fully employed, i.e., there is a "bottleneck," and the wage adjusts flexibly to clear the market in each sector. While labor supply is predetermined at each moment of time, the net labor flow into each sector is a function of the market clearing wage in that sector and the economy-wide average. This model yields a nonlinear relationship between inflation and aggregate demand, as indexed by the proportion of sectors characterized by bottlenecks. Moreover, it also results in a natural rate of output determined where the upward pressure on base wages in bottleneck sectors is just offset by downward pressure in sectors with unemployment. These imbalances between sectors are maintained, on average, by random shocks notwithstanding the operation of equilibrating forces.

There would thus appear to be sufficient theoretical grounds for assuming that the relationship between inflation and excess demand is in fact nonlinear. Moreover, the U.S.

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[4]For a compact mathematical restatement of this argument, see Nickell (1987), pp. 110–11.

experience of the 1960s would appear to bear this out. Figure 1 plots the unemployment rate and the CPI inflation rate from 1961 to 1994. The points plotted for the 1960s show inflation rising at an increasing rate as unemployment declined during the decade. Thus one can "see" the Phillips curve in the U.S. data over this period, especially the inflationary consequences of approaching a kind of "wall" as the unemployment rate is progressively reduced toward zero.

This apparently plausible Phillips curve was subsequently obscured by shifts in the relationship between inflation and unemployment during the 1970s and 1980s, as can be seen in Figure 1. Part of the reason for these shifts was no doubt the two oil price shocks in the 1970s. However, an endemic factor was the shift in inflation expectations, as pointed out by Friedman (1968) in his critique of the Phillips curve. He argued that any attempt by policymakers to reduce the level of unemployment below the natural rate is bound to fail because workers will ultimately seek to adjust their nominal wages in order to regain fully the loss in purchasing power arising from higher inflation. The search model of Phelps (1970) explains the short-run tradeoff between unemployment and inflation on the basis of inflows and outflows of frictional unemployment, but it comes to the same conclusion, namely, that a fall in unemployment below the natural rate reflects mistaken perceptions by workers about their real wage. The correction of these mistakes as workers adjust their wage demands involves a shift in the unemployment rate back to the natural level.

The apparent immutability of the natural rate led Summers (1988) to ask "Should Keynesian Economics Dispense with the Phillips Curve?" He answers in the affirmative because he is "dispirited" by the implication of the standard natural rate Phillips curve that macroeconomic policy is incapable of influencing the average level of output and unemployment. We would argue that this possibly depressing implication is not well founded. The reason is that it is based on the standard linear Phillips relationship, i.e., what is in fact a straight Phillips line. With the few exceptions noted above, most economists have estimated and used linear Phillips relationships. However, as discussed above, this specification is at odds with both the original Phillips-Lipsey approach and strong theoretical considerations pointing to a nonlinear relationship. Moreover, as emphasized in earlier papers, we have found not only that there is empirical evidence in favor a Phillips *curve*, but also that this curvature has the important implication that stabilizing macroeconomic policy can raise the average level of output and employment.[5] Therefore this paper can be viewed as rehabilitating the original Phillips curve in a form that takes proper account of the natural rate hypothesis.

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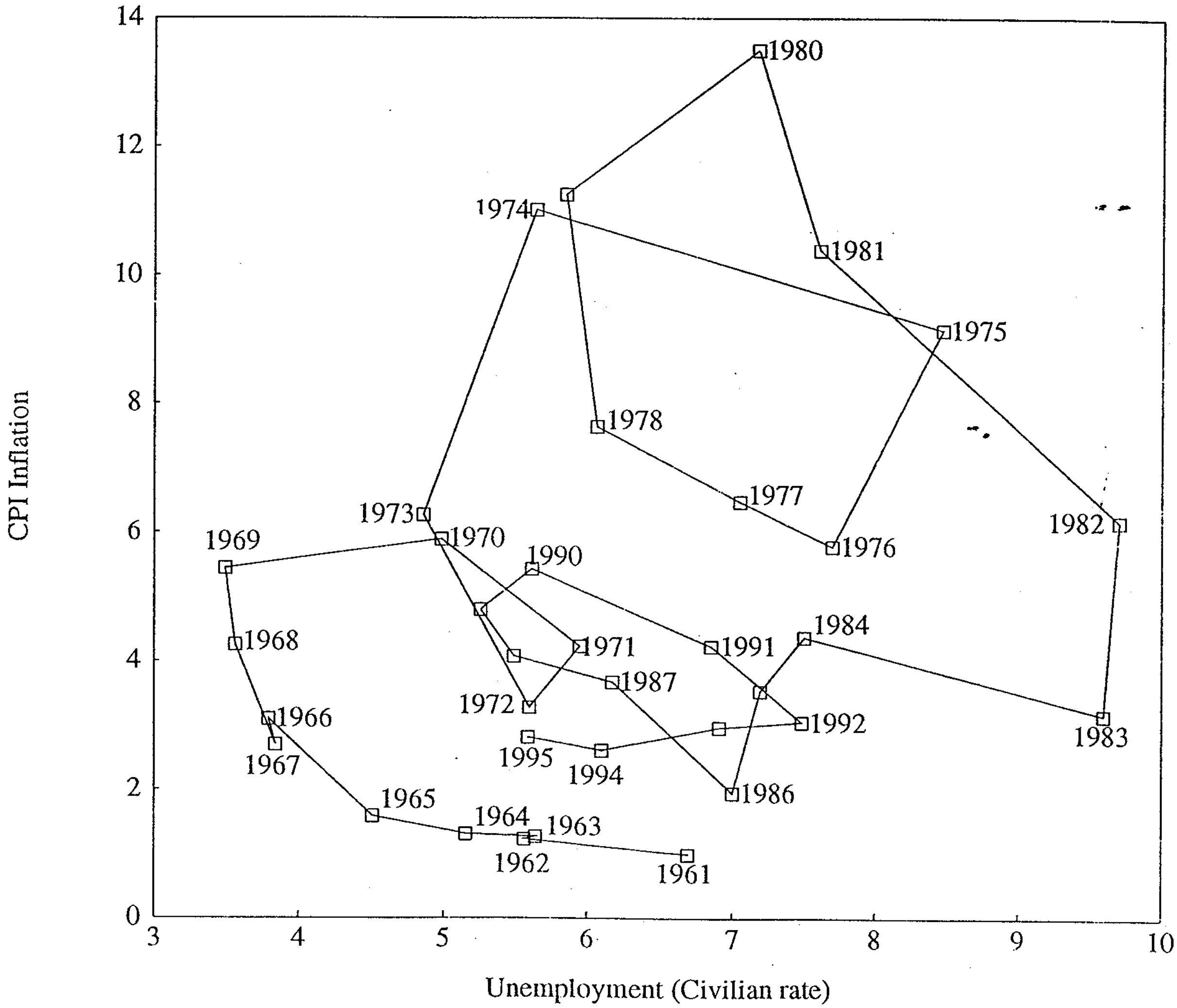
[5]In his article, Summers recognizes (p. 25, footnote 2) that nonlinearity in the relationship between inflation and output or unemployment gaps could justify activist policy, but notes that he is unaware of strong empirical evidence demonstrating the existence of such nonlinearity.

Figure 1. Inflation and Unemployment in the United States, 1961-95



III. The Basic Model

In this section we derive a reduced-form equation for the Phillips curve in which the inflation rate is a function of expected inflation, the unemployment rate relative to the NAIRU, and import prices relative to the domestic price level. The key point of this derivation is to show that the reduced-form coefficient of the unemployment rate has embedded in it a structural parameter that involves a nonlinear relationship between the level of the *real* wage and the unemployment rate. The micro and macro evidence on this relationship is brought to bear in the discussion of this specification. By combining a fairly standard model of real wage setting with a simple model of price setting by firms, we are able to show that inflation is related in a nonlinear manner to the unemployment rate.[6] In order to simplify the analysis all equations are specified in log-linear form. Therefore all variables are expressed in logs and are represented by lowercase letters. As we are primarily interested in long-run relationships, the only attention given to dynamics relates to price expectations and price adjustment; otherwise, the relationship between variables is assumed to the contemporaneous.[7]

The part of the model dealing with wage determination consists of three equations that determine the level of the target real wage, the nominal wage, and price expectations:

$$w_t^* = -\alpha_1 u_t + \alpha_2 z_t + \alpha_3 pr_t + \alpha_4 (pm_t - p_t) \tag{1}$$

$$w_t = w_t^* + \bar{p}_t^e \tag{2}$$

$$\bar{p}_t^e = \delta p_t^e + (1-\delta) p_{t-1} \tag{3}$$

where
w = nominal wage
w* = target *real* wage
p = value added (GDP) deflator
pm = price of imports
pr = trend productivity (level)
u = unemployment rate
z = wage pressure variables
$\bar{p}_t^e$ = aggregate expected inflation
p_t^e = forward-looking component of expected inflation

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[6]For an extremely elegant derivation of the Phillips curve from underlying real wage and the price setting equations, see Ireland and Wren-Lewis (1995).

[7]Ireland and Wren-Lewis (1995) are very explicit about the dynamics of their model, but they do not deal with the issue of nonlinearity.

The level of real wages is set in Equation (1), following the analysis of Layard, Nickell, and Jackman (1991), as negatively related to the unemployment rate and a function of a range of wage pressure variables, z, that includes, for example, the level and duration of unemployment benefits, measures of union power, the tax wedge, and the long-term unemployed. Real wages are also assumed to be positively related to labor productivity, which is proxied by Layard, Nickell and Jackman (1991) by the capital-labor ratio.[8] Finally, as real wages are expressed in terms of the value added deflator, an increase in the relative price of imports would tend to raise this measure of real wages as workers attempt to compensate for the loss in purchasing power.

Equation (2) embodies the assumption that nominal wages reflect both the target real wage and the aggregate price level expected in the period that corresponds to the nominal wage bargain. As shown in Equation (3), this aggregate expected price level has a forward-looking component, p^e, and a backward-looking component, p_{t-1}. The weights on these two components sum to unity, so that the natural rate hypothesis holds.

The coefficient of primary interest in this wage block is α_1. As Equation (1) is specified in log-linear terms, this coefficient is a measure of the elasticity of the real wage with respect to the unemployment rate and therefore embodies the nonlinear relationship between wages and the unemployment rate described above. There are two main sources of empirical estimates for α_1. One constitutes macroeconomic evidence on the effect of unemployment on the level of the real wage in the United Kingdom, which is contained in Nickell (1987) and Layard, Nickell, and Jackman (1991). Their specification of the real wage equation is the same as Equation (1) above, but the z variably has been spelled out in considerable detail in terms of its components, and variables involving dynamic adjustment have been included. Nickell (1987) estimates a number of different equations that include and exclude a number of different variables. In all cases, the unemployment rate enters significantly with a coefficient that clusters around -0.1. In Layard, Nickell, and Jackman (1991), the estimated coefficient of the unemployment rate for the equation reported on p. 441 is -0.091. These are, of course, not two independent results because the methodology and data sets are the same.

A completely different data set and methodology have been employed by Blanchflower and Oswald (1994) to analyze the effect of unemployment on real wages across regions using microeconomic data involving roughly 3½ million people in 12 countries. They estimate an equation similar to Equation (1) above, except that the unemployment rate refers to the worker's area and the other variables control for the personal characteristics of the worker, such as age, gender, race and years of schooling, and for "fixed effects" such as regions and industries. In cross-section, the nominal wage is roughly the same as the real wage as prices for at least tradable goods do not vary that much from region to region within a country. The

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[8]As pointed out by Manning (1993), the inclusion of a productivity variable in a structural real wage equation is problematic. However, as described below, the productivity term drops out of our reduced-form equation.

estimated elasticity of wages with respect to the unemployment rate is generally around -0.1. For example, for Britain, Canada and the United States, the estimates of the unemployment elasticity lie in a range from -0.08 to -0.11 in nearly all specifications. It is remarkable that these results are very close to those reported above using macroeconomic data.[9]

Blanchflower and Oswald provide an extensive discussion of the theoretical basis for this "wage curve," as they denote it. They consider a number of noncompetitive labor market theories as providing plausible explanations for the empirical relationship they have uncovered, but they do not claim that there is one unambiguous theoretical model of the labor market that is generating the wage curve. What is important here is that the macroeconomic wage-setting approach propounded in Layard, Nickell and Jackman (1991) can be seen as also generating the link between wages and the unemployment rate at the microeconomic level documented by Blanchflower and Oswald.

Turning now to price-setting behavior, we have two equations: one that determines the target price of the firm and another that specifies the price adjustment rule:

$$p_t^* = w_t + \beta_1 cu_t + \beta_2 x_t - \beta_3 pr_t \tag{4}$$

$$p_t - p_{t-1} = \gamma\left(p_t^* - p_{t-1}\right) \tag{5}$$

where:
cu = capacity utilization
p^* = firm's target price
x = price pressure variables

Equations (1) to (5) imply the following reduced-form equation for inflation, π_t, which is measured by the first difference in the log of the price level, $p_t - p_{t-1}$:

$$\pi_t = \gamma\delta\pi_t^e + \gamma(\beta_1 cu_t - \alpha_1 u_t) + \gamma(\alpha_3 - \beta_3)pr_t + \gamma\alpha_2 z_t + \gamma\beta_2 x_t + \gamma\alpha_4(p_m - p)_t \tag{6}$$

where expected inflation is equal to $\pi_t^e = p_t^e - p_{t-1}$

[9]This "remarkable" correspondence between the macroeconomic and microeconomic estimates of the real wage unemployment elasticity may, however, be spurious. Blanchard and Katz (1996) have recently questioned the empirical results of Blanchflower and Oswald (1994) on the grounds that their estimate of the coefficient of the lagged dependent variable is biased toward zero. Using panel data for 51 U.S. states from 1980 to 1991, the Blanchard and Katz estimates of this coefficient range from 0.258 to 0.978 and the impact elasticity of unemployment on the wage rate varies from -0.019 to -0.062. As a result, the implied long-run elasticity of the wage rate with respect to the unemployment rate is in most cases considerably larger than the -0.1 estimate of Blanchflower and Oswald.

Equation (6) can be simplified by assuming that α_3 and β_3 are of the same order of magnitude, so that the productivity variable drops out of the equation.

In this equation the variables determining the NAIRU are the wage and price pressure variables, x_t and z_t. To derive an expression for the NAIRU and express demand pressures as the difference between the NAIRU and the current unemployment rate, it is necessary to make a number of simplifications. First, assume that both γ and δ are close to 1.0, so that $\pi_t - \gamma\delta\pi^e_t = 0$. Second, in order to abstract from the effects of import prices on the NAIRU, assume that $pm_t = p_t$. Third, define capacity utilization as the difference between actual output (y) and potential output (y^*), i.e., $cu = y_t - y^*_t$, and assume that at the NAIRU that cu = 0. On the basis of these simplifying assumptions, the (log of the) NAIRU is equal to:

$$nairu_t = (\alpha_2/\alpha_1)\ (z_t) + (\beta_2/\alpha_1)\ (x_t) \tag{7}$$

Using this expression for the NAIRU in the general equation for the inflation rate gives:

$$\pi_t = \gamma\delta\pi^e_t + \gamma\beta_1 cu_t + \gamma\alpha_1(nairu - u_t) \tag{8}$$

Finally, one can make an Okun's Law type assumption that excess demand in the product market is proportional to that in the labor market, i.e., $cu = y - y^* = \theta$ (nairu - u). Making this substitution for cu in Equation (8) gives:

$$\pi_t = \gamma\delta\pi^e_t + \gamma\ (\alpha_1 + \beta_1\theta)\ (nairu_t - u_t) \tag{9}$$

Equation (9) shows that embedded in the reduced-form coefficient $\gamma(\alpha_1 + \beta\theta)$ of the excess demand variable is the structural parameter α_1 that relates the level of the real wage to the unemployment rate in a nonlinear manner. This provides support for the view that the inflation rate is also a nonlinear function of excess demand, as measured by the difference between the log of the NAIRU and the unemployment rate. The following section provides empirical estimates of this relationship that are based on both linear and nonlinear versions of Equation (9).

IV. The U.S. Phillips Curve, the NAIRU and the Natural Rate of Unemployment

A. The NAIRU and the Natural Rate

In his presidential address on the role of monetary policy, Friedman (1968) defined the natural rate to be "the level of unemployment that would be ground out by the Walrasian system of general equilibrium equations, provided that there is embedded in them the actual structural characteristics of labor and commodity markets, including market imperfections, stochastic variability in demands and supplies, the cost of gathering information about job vacancies and labor availabilities, the costs of mobility, and so on." Friedman was clearly defining the natural rate to be the level of unemployment that the economy would tend to generate, on average, given an average set of shocks.[10] The basic idea that Friedman was propounding was that monetary policy could not be used to stimulate economic activity to induce a reduction in the unemployment rate that was permanently below the natural rate. However, while the long-run natural rate hypothesis implies that a permanent increase in inflation cannot be expected to reduce the unemployment rate in the long run, it does not imply that the natural rate of unemployment will be independent of the way in which monetary policy is conducted. Indeed, as shown in Clark, Laxton and Rose (1995), policy rules that increase the variance of aggregate demand and unemployment, thereby allowing large boom and bust cycles, will result in a lower average level of real GDP and a higher natural rate of unemployment.

This result follows from the fact that in nonlinear models of the Phillips curve that embody the restriction of convexity in the short-run aggregate supply curve, there is an important distinction between the NAIRU and the natural rate of unemployment. This distinction can be illustrated by considering a stylized representation of the expectations augmented Phillips curve.

$$\pi_t = \pi_t^e + F(NAIRU_t - UNR_t) \qquad (10)$$

where: π_t is inflation, π_t^e is expected inflation, UNR is the unemployment rate, and F(NAIRU-UNR) is some convex functional form that represents the effects of excess aggregate demand and supply conditions. An important implication of convexity in the Phillips curve is that in a stochastic steady-state equilibrium, the natural rate of unemployment (NAT) must be greater than the NAIRU by a constant amount α—see Debelle and Laxton (1996) for a mathematical proof. The basic intuition why the natural rate, which is the average level of unemployment, must be greater than the NAIRU is straightforward. If excess demand in the labor market is more inflationary than excess supply is disinflationary, then the mean value of excess demand conditions (NAIRU-UNR) that enters a convex Phillips curve must be negative for inflation to be bounded.

Figure 2 provides a simple diagrammatical representation of the Phillips curve in order to illustrate the implications of convexity more formally. For purely illustrative purposes

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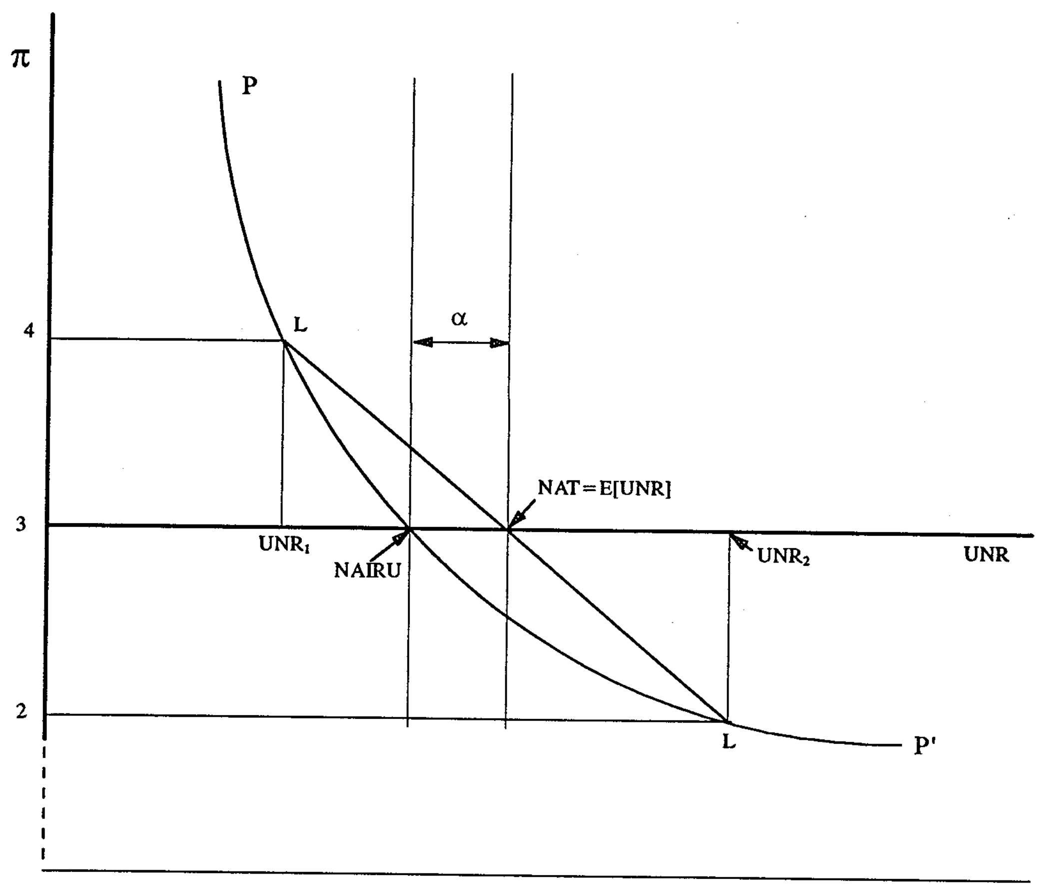
[10]Friedman (1968) did not believe that the natural rate of unemployment was in any sense optimal or invariant to changes in institutional factors that affected incentives to demand or the supply of labor services. In particular, he argued that government policies that directly affect minimum wages or unemployment insurance schemes should be expected to shift the natural rate over time.

Figure 2: Implications of Convexity in the Phillips Curve



assume that the desired inflation rate is 3 percent and that the monetary authorities are capable of achieving this outcome, on average. The PP′ curve in the diagram is drawn convex to the origin and it crosses the horizontal axis—the line corresponding to the 3 percent inflation target—at the point where unemployment is equal to the NAIRU. We refer to this level of unemployment as the NAIRU because, in the absence of shocks, there will be no tendency for the inflation rate to rise or fall from its desired level of 3 percent. In order to see the implications of convexity, suppose that aggregate demand shocks arrive periodically that move the economy to a low unemployment state, UNR_1, and this results in a 4 percent inflation rate. To achieve their 3 percent inflation target on average, the monetary authorities must therefore move the economy to a high unemployment state UNR_2 in the following period in order to generate an outcome of 2 percent inflation so that the average inflation rate over both periods will be equal to 3 percent. Because the Phillips curve is convex to the origin, the average value of unemployment in the presence of shocks—the natural rate—must be greater than the point where the Phillips curve crosses the horizontal axis. As noted above, we refer to the point where the Phillips curve crosses the horizontal axis as the NAIRU because this is where there is no tendency for inflation to depart from its steady-state value in the *absence of shocks*. By contrast, following Friedman's definition in his 1968 presidential address, the natural rate of unemployment, NAT, is the average level of unemployment that will be generated by an average set of shocks. This is denoted by the expected level of unemployment, E[UNR], in Figure 2, which is where the straight line LL crosses the horizontal axis denoting the inflation target.

Obviously, the gap between the NAIRU and the natural rate—which is equal to α in Figure 2—will depend on the degree of convexity and the variance of aggregate demand shocks. It is easily seen that a symmetrical increase in size of these shocks, which is represented by a shift upward and to the right of the LL line, would result in an increase in α. Thus, the difference between the NAIRU and the natural rate will depend on the degree of convexity in the Phillips curve and the variance of aggregate demand and supply shocks. By contrast, in a purely linear world, there is no distinction between the NAIRU and the natural rate, so that larger fluctuations in demand would not raise the mean value of the unemployment rate.

B. On Choosing a Functional Form for the Phillips Curve

The distinction between these two concepts has some important implications for interpreting estimation results and for policy analysis. For example, it has been quite common for researchers to employ techniques that are derived from an objective function that imposes that the mean value of the NAT is equal to the mean value of the unemployment rate, or that the mean value of excess demand pressures is zero over the sample. Laxton, Meredith, and Rose (1995) show that relying on such measures of central tendency to measure excess demand pressure and then imposing the condition that α is equal to zero will make it difficult to find evidence of nonlinearity. Moreover, previous attempts to measure costs of overheating may have been biased because researchers have had a strong tendency to place the linear model on a pedestal and place the burden of proof on the nonlinear model. For example, it has been quite common over the last two decades for researchers to specify the linear model as the null hypothesis—see for example, Gordon (1994)—and then test if positive or negative gaps provide additional explanatory power.

This research strategy is not particularly informative for models designed for policy analysis, as it pays insufficient attention to the policy errors that would be induced if policymakers took actions on the basis of the wrong model. Indeed, as we will demonstrate in Section V below, linear models of the unemployment-inflation process suggest that there are very small costs associated with boom and bust cycles, while models with convexity imply that these costs could be potentially quite large. This is an important practical issue because econometric tests will have low power for rejecting the null hypothesis of moderate convexity against an alternative hypothesis of strict linearity. This will mean that our statistical inferences will be based to a large extent on which model we place on the pedestal. If the linear model is set up as the null hypothesis, it will be difficult to reject it in circumstances where tests have low power. Alternatively, if a model with modest convexity is set up as the null hypothesis, it will also be difficult to reject it against an alternative hypothesis of global linearity.

Given these difficulties, the emphasis in this paper is not to test for asymmetry against an alternative hypothesis of linearity, as was the case in earlier papers. Rather, the objective in this paper is to *impose* moderate convexity on the unemployment-inflation process and then compare the predictions of this estimated model with an alternative model that imposes global linearity. One advantage of focusing on the unemployment-inflation process instead of the output-inflation process is that it may be easier to motivate a functional form to impose some structure on the data. The particular functional form employed in this paper for the Phillips curve is:

$$F(NAIRU\text{-}UNR) = \gamma\ (NAIRU\text{-}UNR)\ /\ (UNR\text{-}\phi), \tag{11}$$

where ϕ is the absolute minimum level of unemployment that can be achieved through expansionary policies and γ is a parameter to measure the degree of convexity in the Phillips curve.

Most economists and policymakers would agree that monetary policy would be incapable of reducing the unemployment rate below some minimum unemployment rate, ϕ, without igniting extreme instabilities in inflation expectations. However, there would be considerable difficulties in estimating ϕ in small samples because successful monetary policies would avoid situations where the unemployment rate was in the neighborhood of ϕ. The strategy in this paper is to impose an extremely weak prior for ϕ, namely, that ϕ is equal to zero and then study the policy implications of the estimated model. As this functional form imposes moderate convexity on the data, we would expect that the estimated model would be *biased* toward underestimating the unemployment costs of overheating.

C. On Measuring Inflation Expectations

The traditional approach in estimating the expectations augmented Phillips curve has assumed that inflation expectations can be modeled with a backward-looking fixed-parameter univariate autoregressive process.

$$\pi_t^{be} = A_0 + A_1 \pi_{t-1} + A_2 \pi_{t-2} + A_3 \pi_{t-3} + \ldots + A_n \pi_{t-n} \tag{12}$$

where: π_t^{be} is a backward-looking measure of inflation expectations, A_0 is a constant, π is a measure of inflation, and the A_i coefficients determine the weights on the lagged inflation terms. In practice, in most empirical work on the Phillips curve researchers have imposed A_0 to be equal to zero and the sum-of-the-coefficients on the lagged inflation terms to be equal to one. In many cases, these two conditions have been interpreted as both necessary and sufficient to ensure that there is no long-run tradeoff between unemployment and inflation—for example, see Gordon (1970), Braun (1984), Cozier and Wilkinson (1990), Gordon (1994), Debelle and Stevens (1995), Turner (1995) and Gordon(1996).

There are several problems with this fixed-parameter reduced-form methodology in designing models for policy analysis. First, these restrictions imply that there is a unit root in the inflation process.[11] Second, the estimated A parameters in the equation above should not be expected to be invariant to changes in the underlying process that governs monetary policy. For example, in countries where the monetary authorities have been successful in providing an anchor for inflation expectations, one should expect that the parameters would be different than for countries where there has been a tendency for inflation to drift periodically. For countries that have made systematic errors in offsetting inflationary shocks, it has been a fairly difficult and costly process to re-establish credibility in low inflation—see, for example, Laxton, Ricketts and Rose (1993) and Isard and Laxton (1996).

In such circumstances, reliable measures of inflation expectations may be critical for testing for convexity in the Phillips curve because it may be overly restrictive to employ a fixed-parameter reduced-form model. This measurement problem would appear to be particularly severe in countries that have announced ambitious inflation objectives that are not perceived to be fully credible by market participants. In such cases one might expect to see persistent deviations between actual and expected inflation until credibility in low inflation has been firmly established.

However, as the objective of this paper is not to test for convexity in the U.S. unemployment-inflation data, reliable measures of inflation expectations are not crucial for our purpose. Moreover, such an approach facilitates comparisons with other empirical studies that assume inflation expectations can be modeled with linear fixed-parameter reduced-form filters,

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[11]Sargent (1971) pointed out years ago that this restriction has nothing whatsoever to do with the long-run natural rate hypothesis.

such as Gordon (1996) and Braun and Chen (1996). Finally, as the costs of disinflation are obviously sensitive to the extent to which inflation expectations are forward looking, the assumption that these expectations are purely backward looking allows us to concentrate solely on the implications of convexity for measuring these costs. For these reasons we have adopted the specification embodied in Equation (12).

D. A Simple Time-Varying Parameter Model for the NAIRU

Recent papers by Gordon (1996), Stock, Staiger and Watson (1996), and Debelle and Laxton (1996) have suggested measuring the NAIRU as a time-varying parameter model. This methodology has several benefits relative to more traditional detrending techniques for measuring the NAIRU. First, it is possible to obtain measures of uncertainty in our estimates of the NAIRU. Second, the estimates of the NAIRU are derived in a manner that is consistent with the specification of the Phillips curve. This is particularly important in work on the inflation-unemployment relationship because nonlinear models of the Phillips curve imply that business cycles can have significantly different policy implications than what is implied by linear models. In particular, the latter suggest that the benefits of lower unemployment from inflating the economy are exactly equal to the unemployment costs associated from returning inflation back to some initial level. Indeed, for this reason Summers (1986) refers to the linear model as the integral gap model because it imposes a strong adding-up restriction on the gaps. By contrast, because models with significant convexity in the Phillips curve predict that it may be very costly to reduce inflation, they have some chance of explaining the existence of large protracted recessions that follow periods of overheating.

The two models of the U.S. unemployment inflation process considered in this paper are:

Nonlinear model: $$\pi_t = \alpha(L)\,\pi_t + \gamma\,(NAIRU_t - UNR_t) / UNR_t + \epsilon_t^{\pi} \qquad (13)$$

Linear model: $$\pi = \alpha(L)\,\pi_t + \delta\,(NAIRU_t - UNR_t) + \epsilon_t^{\pi} \qquad (14)$$

These models can be estimated by specifying a stochastic process for the NAIRU that allows for permanent shifts in it. In particular, we assume that the NAIRU follows a random walk and then use the Kalman filter to build up model-consistent estimates.[12]

NAIRU process: $$NAIRU_t = NAIRU_{t-1} + n_t, \quad n_t \sim N(0, \theta_n^2) \qquad (15)$$

Specifying this process for the NAIRU allows us to estimate both models as time-varying parameter models. In the case of the nonlinear model, if we define $Z_t = \gamma\ NAIRU_t$ we can

[12]Obviously, the NAIRU cannot literally follow a random walk. However, this may not be too bad a specification for measurement of historical drift in the NAIRU over a small sample of observations.

estimate the model with a time-varying parameter on the inverse of the unemployment rate and a fixed constant γ. In the case of the linear model, we can define $X_t = \delta \, NAIRU_t$ and the model can be transformed into a model with a time varying "constant." Under these assumptions the two models become:

Nonlinear model: $$\pi_t = \alpha(L)\,\pi_t + Z_t\,UNR_t^{-1} - \gamma + \epsilon_t^{\pi} \tag{16}$$

Linear model: $$\pi_t = \alpha(L)\,\pi_t + X_t - \delta\,UNR_t + \epsilon_t^{\pi} \tag{17}$$

Estimates of the time-varying parameters Z_t and X_t can be obtained from standard Kalman filter routines once users specify a prior for the noise-to-signal ratio. Specifying a weak prior for the noise-to-signal ratio will imply volatile estimates for the underlying NAIRU because the Z_t and X_t parameters will be allowed to vary considerably to explain variation in the observed inflation series. In the limit, a very loose prior will imply extremely volatile NAIRU estimates as noise from the true residual of the inflation equation (ϵ_t^{π}) is effectively transferred into the NAIRU and its disturbance term η^t. We refer to the resulting NAIRU series from both models as model-consistent estimates because once a prior for the underlying variance of the NAIRU has in effect been specified, the Phillips curve model is used explicitly as a measurement equation to construct the actual NAIRU series.

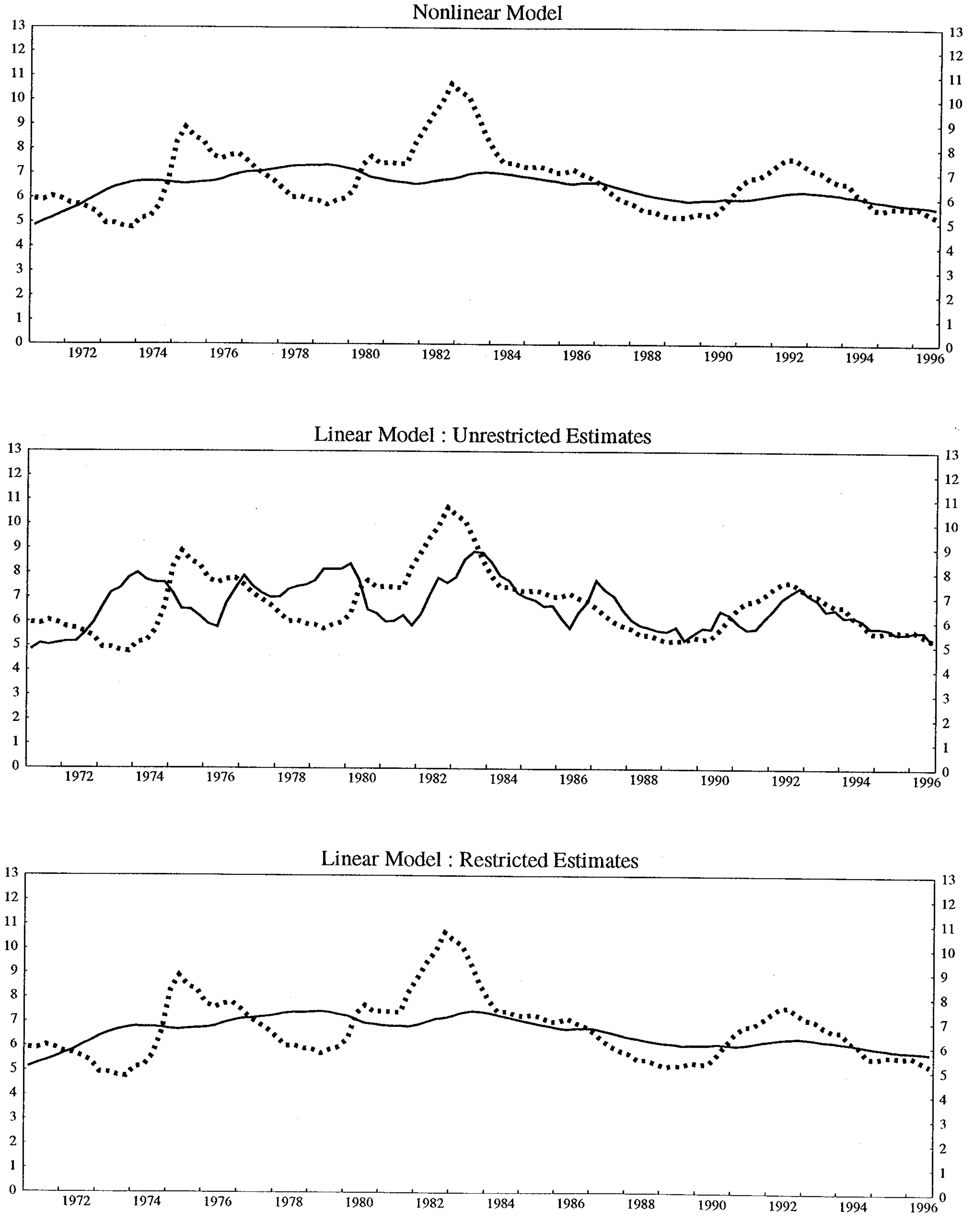
For empirical studies concerned principally with statistical inference, this approach avoids the asymptotic prefilter bias that is associated with using measures of central tendency because the constructed gaps from such techniques are fundamentally inconsistent with the nonlinear Phillips curve model. In effect, because such techniques impose the condition that α is equal to zero—recall Figure 2—they effectively eliminate the possibility of finding evidence in favor of the nonlinear model. For this reason, a better interpretation of these empirical studies is that researchers have not found evidence against nonlinearity, but instead have simply constructed gaps that are inconsistent with nonlinearities. Based on pooled data from the G-7 industrial countries, Laxton, Meredith and Rose (1995) suggest a simple empirical strategy that attempted to correct for this prefilter bias by estimating the α parameter directly. While this approach will work in large samples, some recent work with Monte Carlo experiments suggests that this technique will still tend to falsely reject the nonlinear model in the types of sample sizes that have been used in most empirical work.

E. Estimation Results

To estimate Equations (16) and (17) we included 8 lags of past inflation to proxy for inertia in the inflation process that arises because of contracts and expectational dynamics. This assumption about the appropriate lag length was set to be consistent with the work by Braun and Chen (1996). The estimation results, where inflation is measured by the U.S. consumer price index, are reported in Table 1 and the resulting NAIRU estimates are plotted in Figure 3. In both cases these estimates were obtained by using the default option of unity for the noise-to-signal ratio in TSP. As can be seen in Figure 3, this estimation strategy produces fairly smooth estimates of the NAIRU from the nonlinear model but considerably

Figure 3: Unemployment and NAIRU Estimates From 3 Models: 1971Q1 to 1996Q3

Unemployment Rate (dashed), NAIRU (solid thick line)



more volatile estimates from the linear model. In both models the estimated parameters on the excess demand pressure terms involving the unemployment rate are highly significant.

Without imposing priors on the plausible variability of the NAIRU, a purely statistical interpretation of these two models suggests that the data prefer the linear model. For example, if we examine a raw measure of fit that does not penalize variation in the estimated NAIRU series, we would conclude that the data strongly prefer the linear model: the variance of the estimated residuals is 2.57 for the nonlinear model and 1.10 for the linear model. However, the better fit of the linear model requires implausible estimates of the underlying NAIRU series. The standard deviation of the first-difference of the NAIRU is 0.08 in the nonlinear model and 0.38 in the linear model.[13] However, over this same sample period the standard deviation in the first-difference of the actual unemployment rate was 0.36. Thus, the model-consistent estimates from the linear model suggest that the underlying NAIRU is actually more volatile than the observed unemployment rate. This result explains why users of this technique, e.g. Gordon (1996), have been quick to impose explicit priors on variability in the NAIRU.

Table 1 also includes estimates of the sample average value for the NAIRU series. The linear model produces an average NAIRU of 6.7 percent, which is fairly close to the actual value of the average unemployment rate. As expected, the nonlinear model produces a lower estimate of the NAIRU of 6.4 percent. Both models suggest that the U.S. unemployment rate in 1996:Q3 was below the NAIRU. The nonlinear model implies that the NAIRU was 5.6 percent in 1996:Q3 and the linear model indicates that it was 5.2 percent.

The nonlinear model produces a fairly traditional characterization of historical U.S. business cycles. According to this model there were two periods of significant overheating in the 1970s and one in the late 1980s. The nonlinear model also suggests that the unemployment rate has been less than the NAIRU since 1994:Q3 but the extent of overheating has been substantially less than in past episodes. In retrospect, the estimates from the nonlinear model provide some support for the preemptive tightening in monetary policy in 1994 that was designed to prevent a recurrence of serious overheating—see Greenspan (1994). The nonlinear model also produces significantly larger estimates of excess supply during the major disinflationary episode that commenced in the early 1980s.

It may perhaps be surprising that the structure imposed in the nonlinear model produces considerably smoother estimates of the NAIRU than what is obtained from the linear model for the same prior on the noise-to-signal ratio. Recall that the linear model attempts to explain variation in inflation with a coefficient δ times a labor market pressure term ($NAIRU_t - UNR_t$). Note that any systematic upward (downward) bias in the estimate of δ that

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[13]The estimation period is 1968:Q1 to 1996:Q3 but it takes several quarters to build up estimates of the NAIRU. The estimates of volatility in the NAIRU were computed from 1971:Q1 to 1996:Q3.

is caused by misspecification errors can be compensated for by smaller (larger) estimates of the gaps.[14] The last line in Table 1 presents an estimate of the linear model where the standard deviation of the first-difference of the NAIRU is constrained to be the same as in the estimated nonlinear model. With this restriction there is a significant deterioration in the fit of the linear model, such that the restricted linear model now has about the same fit as the nonlinear model. Not surprisingly, because this model produces larger estimates of the gap, it results in a smaller estimate of δ.

While it may be easy to understand why linear models have difficulty in tying down estimates of the NAIRU, it will perhaps be even more surprising why imposing moderate convexity in the Phillips curve is sufficient to do the job. Because the nonlinear model predicts that significant overheating will result in enormous inflationary pressures, it tends to place a high penalty on extremely large excess demand gaps. In addition, the nonlinear model predicts that we should observe persistent and larger excess supply gaps. It is interesting that this structure is sufficient to tie down estimates of the NAIRU not only in this dataset but in several other datasets that we have investigated.

V. Some Illustrative Estimates Of The Unemployment Costs Of Overheating and Rapid Disinflation

As discussed in the preceding section, the conclusions that can be drawn from the empirical evidence presented in this paper regarding the degree of linearity in the Phillips curve depend on which hypothesis is put on the pedestal: if the null hypothesis is that the Phillips relationship is linear, it cannot be rejected by the data, and similarly, a nonlinear specification cannot be rejected if it is the null hypothesis. Consequently, the strategy adopted here is to impose moderate convexity in the Phillips curve and explore the policy implications of this specification in comparison with a straight-line Phillips curve. In particular, we wish to contrast the macroeconomic effects of a true Phillips "curve" with those implied by the standard linear version employed by many economists.

As pointed out by Summers (1986), this specification can be characterized as an "integral gap" model because if the current inflation rate is specified as a linear function of the output gap and of the lagged inflation rate, then over any given time period the change in the rate of inflation depends on the cumulative value of the gap over this same period. Consequently, if there is no change in inflation, the average value of the output gap must be zero. Summers therefore concludes that the natural rate embodied in the integral gap Phillips relationship implies that stabilization policy cannot affect the average level of output, as

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[14]In other words, the linear model would have the identical fit if we doubled δ and halved all of the gaps. In this sense, the linear model does not provide very strong identification restrictions for the measured NAIRU.

raising output in one period necessitates an equivalent reduction in output in another period to avoid accelerating inflation.

We share Summers' concerns that the integral gap model implies only a limited role for stabilization policy. As shown in the simulations reported below, the costs in higher unemployment to reduce inflation are no greater than the gains in employment associated with overheating, so that there is no clear benefit from stabilizing output. In addition, the integral gap model is seriously deficient because it takes no account of forward-looking expectations and the importance of the credibility of monetary policy. The linear feature of this model implies that the inflationary impact of unemployment rates significantly below the NAIRU will be underestimated, and the feedback of actual inflation on inflation expectations will take considerable time. Thus the integral gap model implies that there are relatively limited inflationary consequences of allowing an economy to overheat, which is in direct contrast to the implications of a convex Phillips relationship.

Estimates of the potential gains from stabilization policy under the assumption of convexity in the output-inflation process have been presented in Clark, Laxton and Rose (1995) and (1996). Here we provide some illustrative calculations of the unemployment costs of overheating which highlight the importance of the assumption made about the degree of convexity in the Phillips curve, namely, that the assumption of a linear Phillips relationship will lead to an underestimation of these costs. We also examine the implications of a convex Phillips curve for the sensitivity of the costs of disinflation to the speed at which disinflation takes place. A nonlinear Phillips curve implies that there is a declining marginal effectiveness of a contraction in output in reducing inflation, the larger the magnitude of the deflation. This is in contrast with a linear Phillips curve, where the effectiveness of a contractionary policy in reducing inflation (for given inflation expectations and inertia in wage and price setting) is independent of the speed of disinflation. Thus a convex Phillips curve provides an argument in favor of a gradual disinflation as opposed to a "cold shower" approach to reducing inflation.[15]

A. Small Model of the U.S. Inflation-Unemployment Tradeoff

The calculations used to derive these results are made using a small model, summarized in Table 2, which is similar to that in the two papers by Clark, Laxton and Rose cited above. The model consists of the equations for the integral gap (linear restricted) Phillips curve and the nonlinear Phillips curve that are reported in Table 1. It also includes an equation explaining the gap between the NAIRU and the unemployment rate, which is a function of the lagged gap and the real interest rate, defined as the nominal short-term interest rate minus the

[15] As pointed out below, this result has been derived analytically by Bean (1996).

expected inflation rate.[16] This equation, which corresponds to the output gap equation in Clark, Laxton and Rose (1996) and in Fuhrer and Moore (1995), is needed to link aggregate demand—proxied by the unemployment rate—and the nominal short-term interest rate controlled by the monetary authority.[17]

As in Clark, Laxton, and Rose, it is assumed that this interest rate is adjusted by the monetary authority by means of a forward-looking policy reaction function in response to deviations of the current unemployment rate from the NAIRU and to deviations of the inflation rate expected three periods ahead from the target inflation rate.[18] Inflation expectations on the part of both the private and the public sectors are assumed to be generated by the model, which is the eight-quarter distributed lag on past inflation described in the previous section. The nominal interest rate is also assumed to be adjusted fully to inflation expectations, i.e., the authorities control the nominal interest rate to achieve the desired real rate. Finally, in equilibrium, the short-term interest rate is equal to the long-run real rate, rr^*, plus expected inflation.

This monetary policy reaction function has in addition three features that need to be mentioned. First, the weight on the inflation gap is three times that on the unemployment gap. The larger weight was chosen in order to produce smooth business cycle properties. As will be seen below, such a reaction function leads to a gradual adjustment in unemployment and inflation back to their target levels without overshooting when there are moderate excess demand and disinflation shocks. Second, in certain preliminary simulations the reaction function resulted in negative nominal interest rates. In order to preclude this possibility, the interest rates generated by the reaction function are bounded from below at zero. Third, in some of the simulation experiments described below it is assumed that there is a delay in adjusting the interest rate in response to a shock.

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[16]As the simulations of these models are deterministic, there is no difference between the NAT and the NAIRU and they are treated as being the same. For a discussion of the algorithm and its properties see Armstrong et al. (1995).

[17]The parameters in the gap equation were taken from an estimated equation that regressed the unemployment gaps from the nonlinear Phillips curve on a measure of the real federal funds rate. The latter was defined by subtracting the measure of inflation expectations in the Phillips curve from the federal funds rate.

[18]This monetary policy reaction function is similar to a Taylor (1993) rule, the major difference being that here the monetary authority adjusts the nominal interest rate in response to the projected, not the current, inflation rate. For a recent empirical analysis of this type of Taylor rule, see Clarida and Gertler (1996). Recently, there has been work on deriving optimal policy feedback rules in models related to that used here. This has been done by Bean (1996) using extrapolative expectations and a nonlinear output-inflation tradeoff, whereas Clark, Goodhart and Huang (1996) use rational expectations and a linear Phillips curve.

In the simulations of the model described in Table 2, the NAIRU was assumed to be 5.5 percent, as this appears to be close to recent estimates for the United States: it is slightly below the level of the NAIRU recently estimated by Gordon (1994), it is near the U.S. Administration's current estimate of the NAIRU, and is essentially the same as the estimate of 5.6 percent for 1996 in Braun and Chen (1996).[19] Moreover, a 5.5 percent figure is in line with the estimates of the NAIRU for 1996:Q3 implied by the empirical results in the preceding section. For a linear model, the choice of the NAIRU is immaterial, but not for the Phillips curve considered below.

B. Unemployment Costs of Overheating

In the first simulation experiment there is overheating resulting from a demand shock that reduces the unemployment rate by ½ percentage point below the 5.5 percent NAIRU, i.e., to 5 percent, in the first quarter. Figure 4a shows the response of the economy with the integral gap model, i.e., the linear Phillips curve. Reflecting its own intrinsic dynamics, the unemployment rate falls to about 4¾ percent in the second and third quarters, but then starts to increase in response to the rise in nominal and real interest rates. There is assumed to be a two-quarter delay in the monetary policy response to the negative unemployment shock in order to permit a build-up in inflationary pressure. The rise in the nominal interest rate, which occurs in the third quarter, may be unrealistically high, as no attempt has been made to fine-tune the estimated interest sensitivity of employment, i.e., demand, and because other channels through which the interest rate affects demand, e.g.,the exchange rate, have been ignored. Given the specification of the monetary policy reaction function, both the unemployment rate and the inflation rate return to their target values, which are 5.5 percent and 5 percent, respectively. The equilibrium real interest is assumed to be 4 percent, so that the long-run nominal interest rate is 9 percent. As there is only an inflation rate but not a price level target, the latter rises by about 1.7 percent. The lower left-hand panel shows the property of the integral gap model, namely, that the cumulative effect of the demand shock on the unemployment rate is zero in the long run, i.e., the reduction in unemployment below the NAIRU is exactly matched by a rise in unemployment above the NAIRU.

Figure 4b shows the effects of nonlinearity in the Phillips curve on the response of inflation and unemployment to excess demand. As all features of the simulation experiment are the same, including inflation expectations, in the linear and nonlinear models, a comparison of Figures 4a and 4b reveals the impact of nonlinearity alone. The effect of the demand shock has significantly larger effects on inflation, the price level and interest rates. Particularly noteworthy is that the deflation needed to return the inflation rate to its target level is now

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[19]The U.S. Administration's estimate is implied by the following quotation from the 1996 *Economic Report of the President*, p. 53: ". . . recent evidence strongly argues that the sustainable rate of unemployment has fallen below 6 percent, perhaps to the range of 5.5 to 5.7 percent. The Administration's forecast falls on the conservative end of this range by projecting the unemployment rate at 5.7 percent over the near term."

Figure 4a: Integral Gap Model Responses to Overheating (Negative Unemployment Shock)

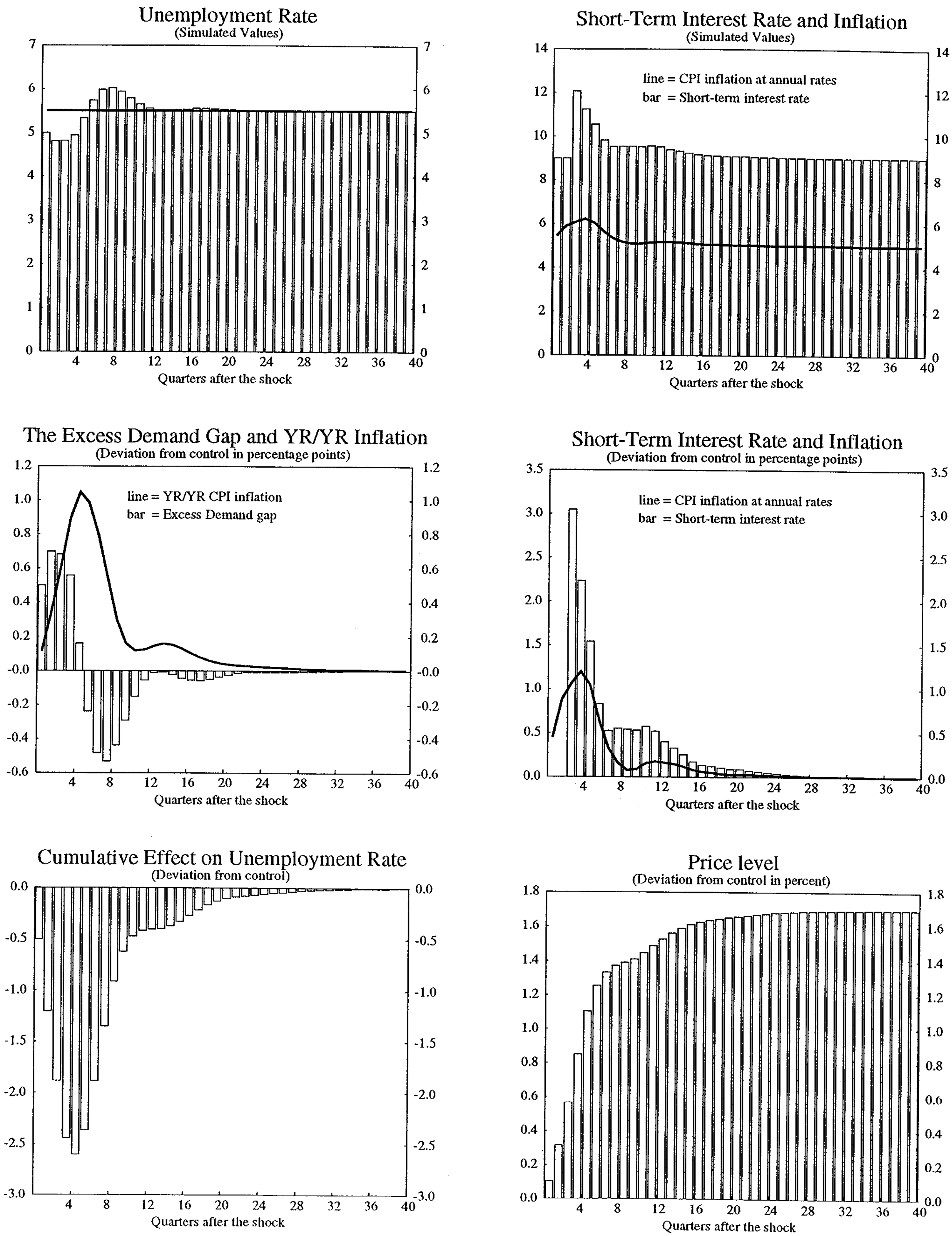
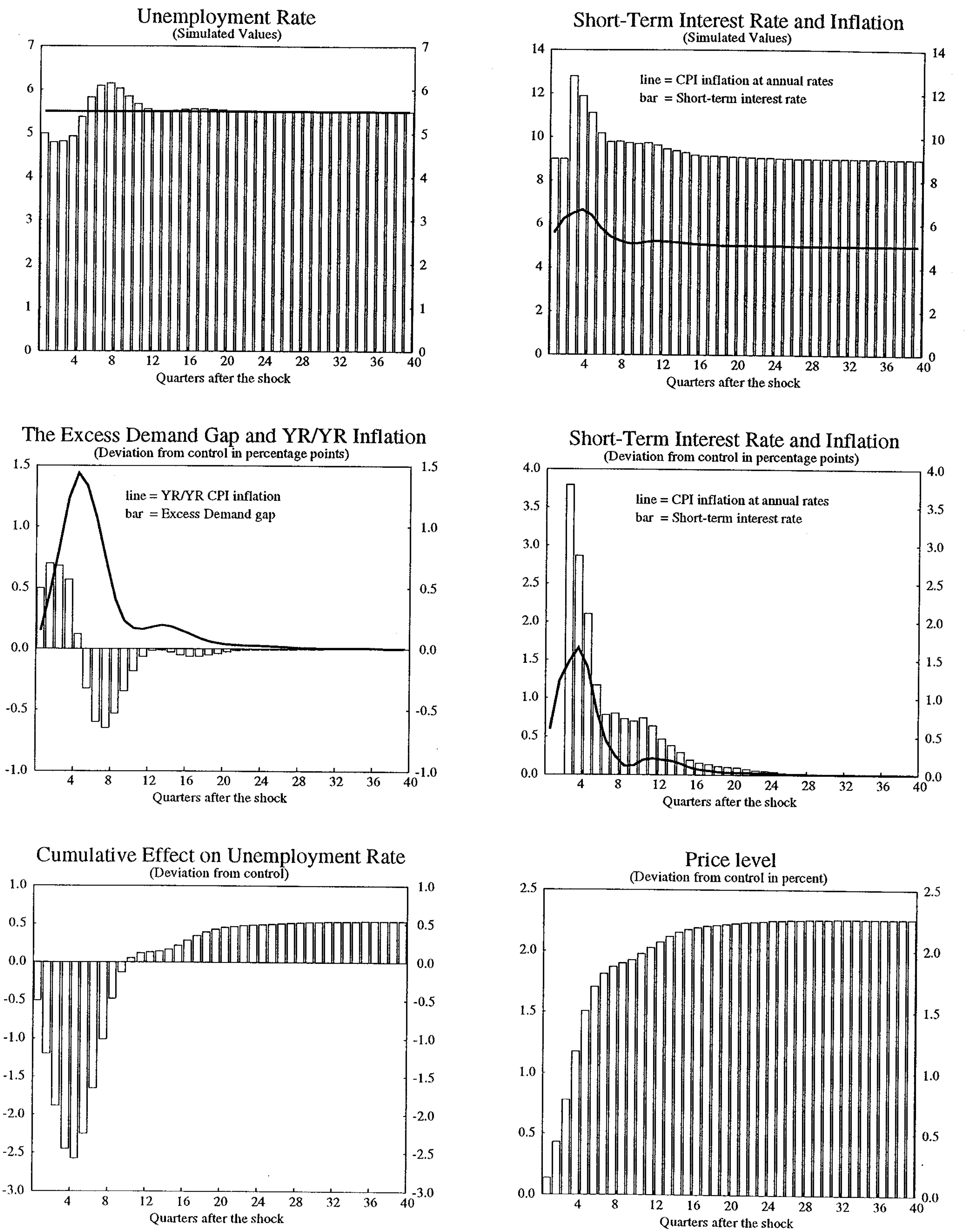


Figure 4b: Nonlinear Model Responses to Overheating (Negative Unemployment Shock)



more severe than in the linear model. As shown in the lower left-hand panel, the cumulative effect of the demand shock on the unemployment rate is significantly positive i.e., the employment losses exceed the initial employment gains.[20] This shows that there are clear costs of overheating when the model is nonlinear; these costs will be larger, the greater the degree of nonlinearity and the larger the shock.

This is shown in Figures 4c and 4d. Recall that we have imposed only moderate convexity on the data by assuming that the short-run Phillips curve only becomes vertical as the unemployment rate approaches zero. This is an extreme assumption which biases the results toward underestimating the unemployment costs of overheating. A more realistic assumption is that the absolute minimum unemployment rate is probably some small positive number. In order to explore the implications of greater convexity in the Phillips curve we have therefore assumed that this minimum level is 2 percent and have simulated the effect of moderate overheating with (ϕ) equal to 2 in Equation (11). The effects of overheating using this alternative nonlinear model are shown in Figure 4c, where the aggregate demand shock is the same as before. The inflationary effect of the same excess demand shock is now much larger, almost twice that in the case of modest convexity and nearly three times larger than in the case of a linear Phillips curve. As a consequence, the deflation needed to undo the inflationary shock is also much larger, with the cumulative effect on the unemployment rate rising to 1 percent, double that in Figure 4b. This result shows that the costs of disinflation are quite sensitive to the degree of convexity in the Phillips curve.

The costs of disinflation with a nonlinear Phillips curve are also dependent on the magnitude of the shock. This is shown in Figure 4d, where (ϕ) is again set equal to 2 percent but where the shock is twice as large, i.e., the initial reduction in the unemployment rate is now 1 percentage point. The peak effect on inflation is over twice that in Figure 4c. What is particularly noteworthy is that the costs of disinflation, as measured by the cumulative effect on the unemployment rate, increase six-fold to about 6 percent. This is twelve times larger than in the linear case. Both Figures 4c and 4d therefore show that the costs of disinflation with a nonlinear Phillips curve can indeed be substantial.

C. Unemployment Costs of a Gradual Disinflation

We now turn to the implications of convexity in the Phillips curve for the costs of disinflation. We do this by comparing the effects on the economy of the two different Phillips curves to a disinflation shock which is calibrated as reduction in the target rate of inflation, π^*, equal to 5 percentage points. As the target inflation rate in the baseline has been set arbitrarily

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[20]The unemployment rate returns to the unchanged NAIRU, but the cumulative difference between the unemployment rate (UNR) and the NAIRU is not zero, as in the integral gap model. Rather, that amount by which UNR lies above the NAIRU is greater than the amount by which UNR is initially below the NAIRU.

Figure 4c: Alternative Nonlinear Model Responses to Overheating (Negative Unemployment Shock)

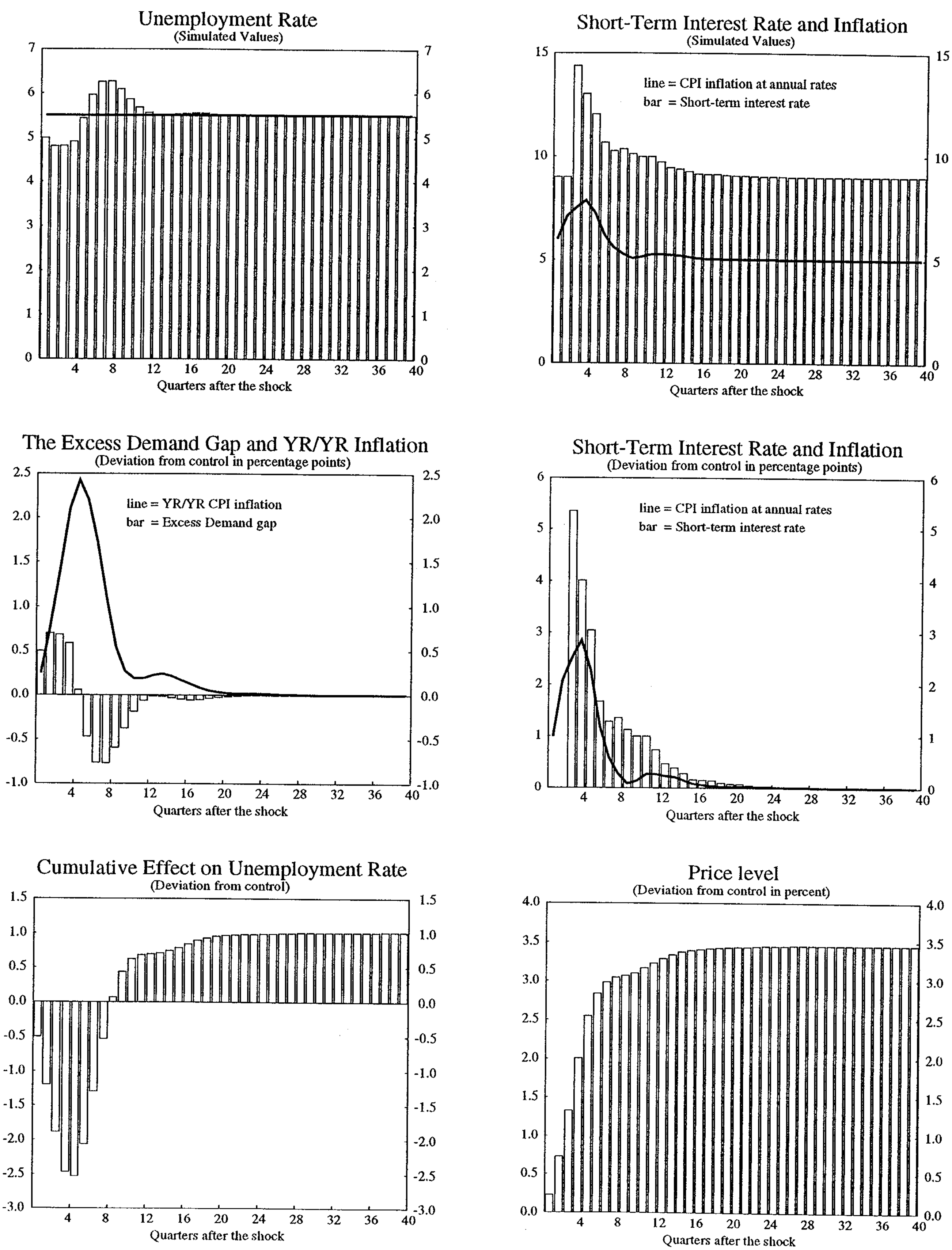
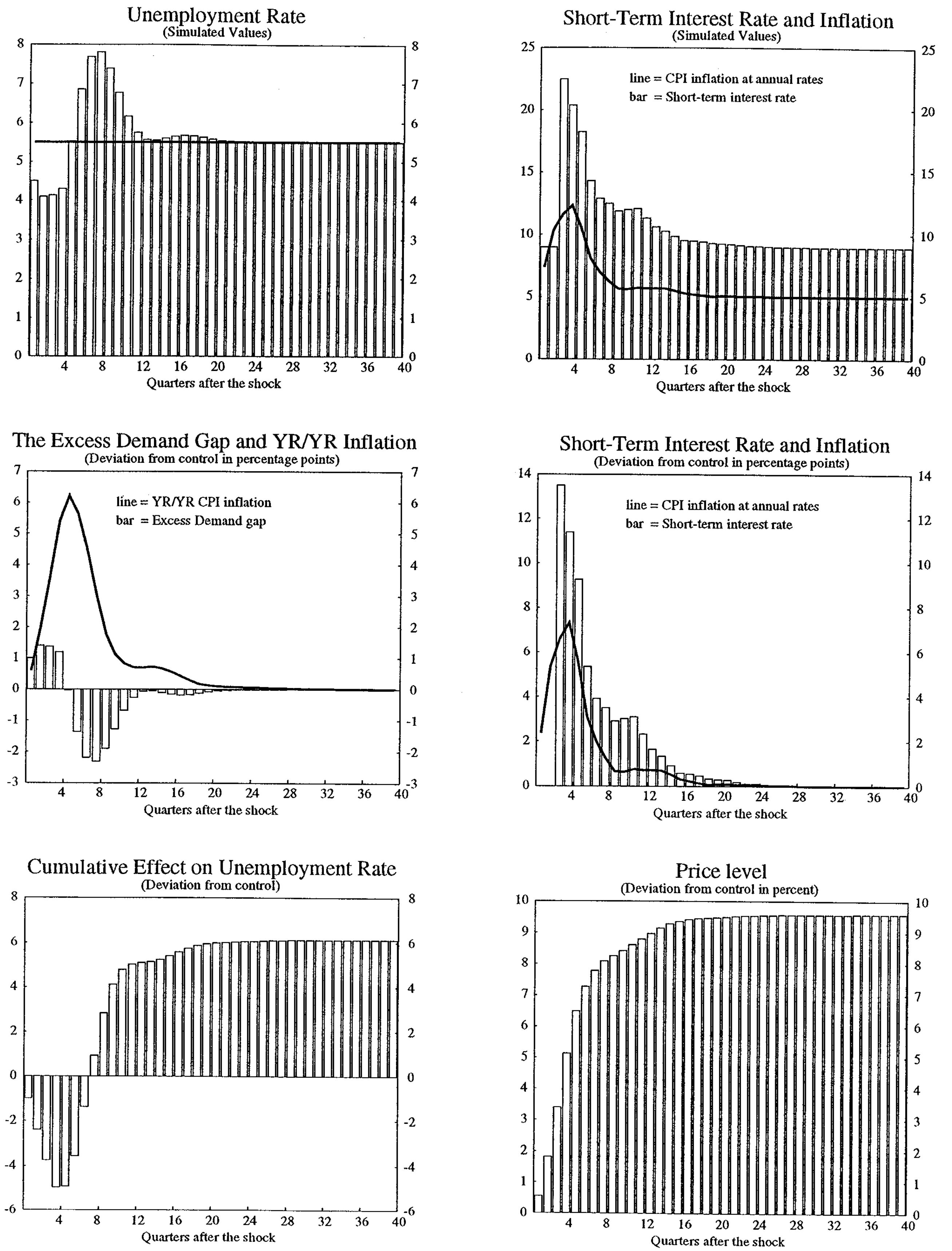


Figure 4d: Alternative Nonlinear Model Responses to Serious Overheating (Larger Negative Unemployment Shock)



at 5 percent, the disinflation shock involves a decline in the target to zero.[21] This disinflation shock is described as "gradual" because the adjustment of actual inflation to the new target is smooth and there is no overshooting or induced cyclical behavior in the unemployment rate. We consider below a "cold shower" disinflation shock in which the same desired reduction in inflation is achieved much more quickly.

Figure 5 shows the effects of a gradual disinflation strategy with the integral gap, i.e., linear model. A large initial increase in nominal and real interest rates is needed to open up a deflationary gap—unemployment above the NAIRU—to reduce the inflation rate from 5 percent to zero. As wages and prices, while sticky in the short run, are assumed in the long run to be as flexible in the downward direction as in the upward direction, the rise in the unemployment rate above the NAIRU is temporary and there is no change in the NAIRU itself.[22] As the effects of the gradual disinflation shock are very similar for the nonlinear model, they are not shown in a separate figure. However, this is not the case, as shown below, with a gradual inflation shock.

We next compare the model responses to a gradual disinflation shock with the responses to a gradual inflation shock of the same magnitude. These are shown in Figures 6a and 6b, where the target inflation rate has been raised from 5 percent to 10 percent. The responses are essentially the mirror images of those in Figure 5. The main difference between the two models can be seen by comparing the cumulative effect on the unemployment rate. With the integral gap model, the magnitude of the effect is the same but opposite sign. By contrast, with the nonlinear Phillips curve the cumulative reduction in the unemployment rate is only about 9 percentage points with the inflation shock, whereas the cumulative increase in the unemployment rate is about 14 percentage points with deflation. This is a direct implication of nonlinearity and therefore asymmetry: unemployment below the NAIRU generates inflation at an increasing rate, whereas unemployment above the NAIRU reduces inflation at a decreasing rate. In other words, there is diminishing marginal effectiveness of the unemployment gap in reducing inflation, the larger the gap. Thus the "benefits" of a surge in excess demand that reduces unemployment and raises the inflation rate is more than offset by the rise in unemployment required to return inflation to its target level. By contrast, with the integral gap model, there is no net unemployment cost arising from a bout of inflation generated by excess demand.

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[21]It should be noted that the choice for the baseline inflation rate in our model is arbitrary, as the level of the inflation rate does not affect the properties of the model.

[22]We discuss below the analysis of Akerlof, Dickens and Perry (1996) who argue that because of downward wage and price rigidity, achieving a target of zero inflation will raise the NAIRU.

Figure 5: Integral Gap Model Responses to a Gradual Disinflation Shock

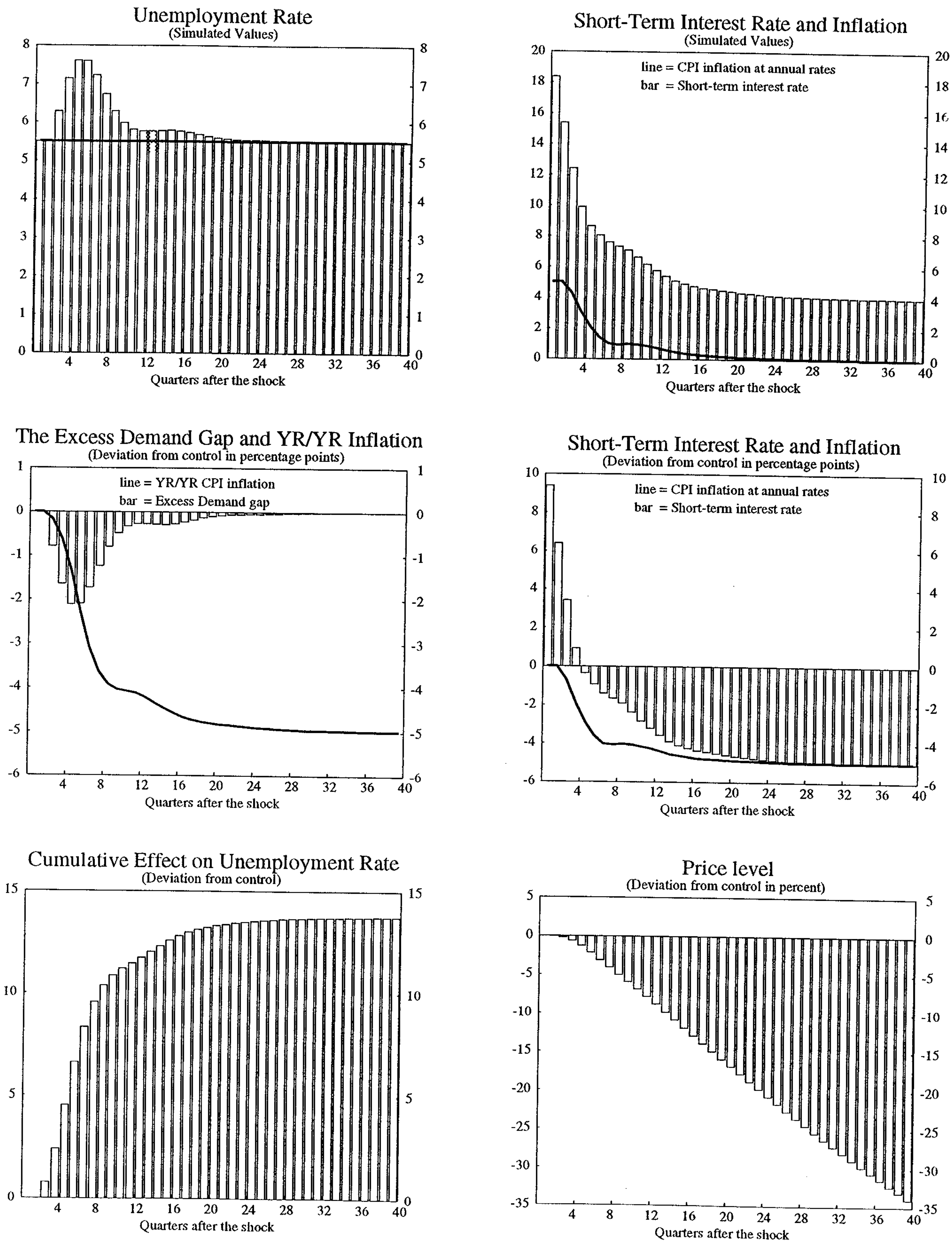


Figure 6a: Integral Gap Model Responses to a Gradual Inflation Shock

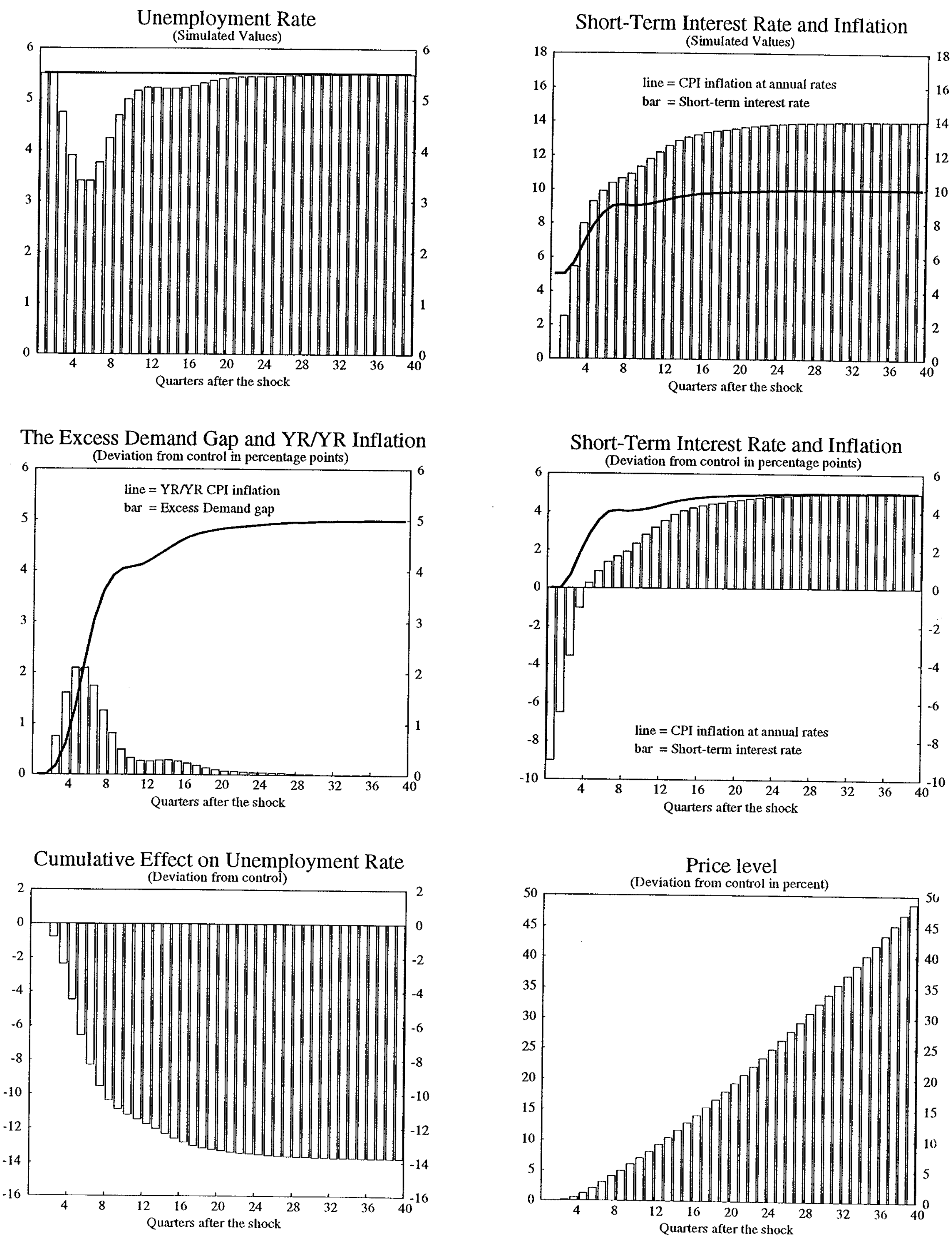
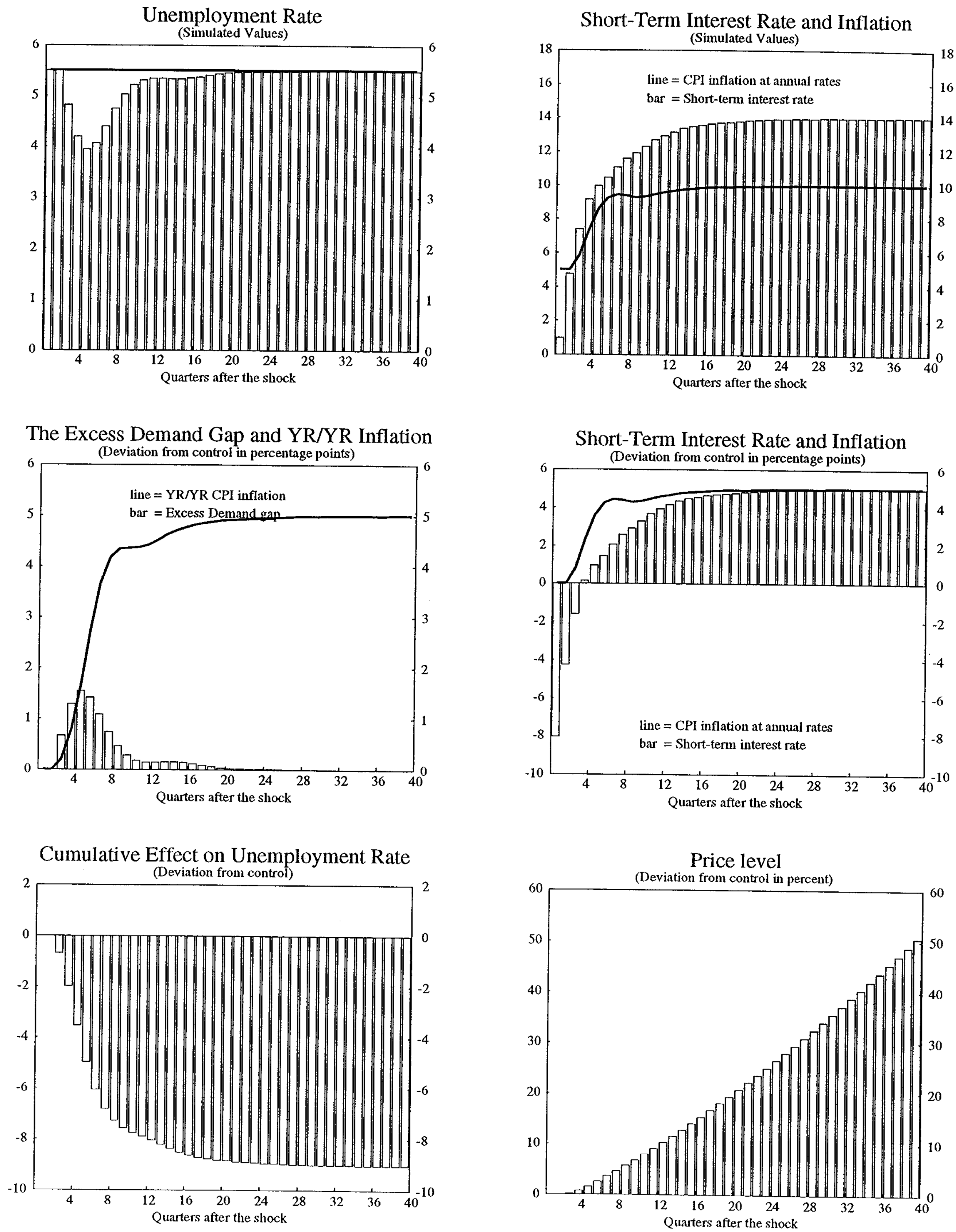


Figure 6b: Nonlinear Model Responses to a Gradual Inflation Shock



D. Unemployment Costs of a "Cold Shower" Disinflation Shock

Finally, we examine the implications of convexity in the Phillips curve for the costs of a "cold shower" disinflation shock, shown in Figures 7a and 7b, in order to compare these costs with those arising from the gradual disinflation shock, shown in Figure 5. The difference between the two shocks is the speed in achieving the reduction in the inflation target from 5 percent to zero. In the cold shower scenario, the nominal short-term interest rate is raised by 10 percentage points to 19 percent and held there for four quarters. This adjustment is very close to the increase in the interest rate in the first quarter with the gradual response, but where the interest rate then declines rapidly in the following three quarters. With the cold shower approach the interest rate is determined by the monetary policy reaction function starting in the fifth quarter. As can be seen in Figures 7a and 7b, this disinflation policy is so severe that the nominal interest rate reaches its lower bound of zero by the seventh quarter.

The cold shower disinflation policy achieves the zero inflation target much sooner than gradual disinflation, but at the cost of inducing cycles in the economy and overshooting the target, which is particularly noticeable in the integral gap model. This disinflationary scenario is not designed to be particularly realistic, but rather to bring out clearly the implications of convexity for the costs of disinflation. Nevertheless, it is worth noting that the change in interest rates in Figures 7a and 7b is comparable to the rise in short-term rates over the period 1978-81 in Canada and the United States when the central banks in both countries tightened monetary policy aggressively to engineer a reduction in inflation of a similar magnitude.

The implications of convexity are again most visible in the cumulative effect of the shock on the unemployment rate. With a linear Phillips curve in Figure 7b, the long-run cumulative change in the unemployment rate is about 14 percentage points, the same as with gradual disinflation shown in Figure 5. Of course, in the short run the cost of disinflation is considerably higher—reaching a peak of over 20 percentage points—because of the much stronger policy stance. By contrast, with the nonlinear model the costs of disinflation are larger in two respects. First, a comparison of Figures 7b with 7a shows that for the same shock the cumulative effect on the unemployment rate is higher as a result of nonlinearity: 19 percentage points versus 14 points. Second, a comparison of Figure 7b with Figure 5 shows that with a nonlinear Phillips curve, the long-run costs of disinflation are lower, the slower the pace at which the disinflation takes place: 14 percentage points with a gradual response compared to 19 percentage points with a cold shower policy. This result illustrates the key feature of a convex Phillips curve whereby the unemployment costs of deflation depend on the speed of adjustment because of the diminishing marginal effectiveness in reducing inflation of an additional percentage point increase in the unemployment rate. This

Figure 7a: Integral Gap Model Responses to a Cold Shower Disinflation Shock

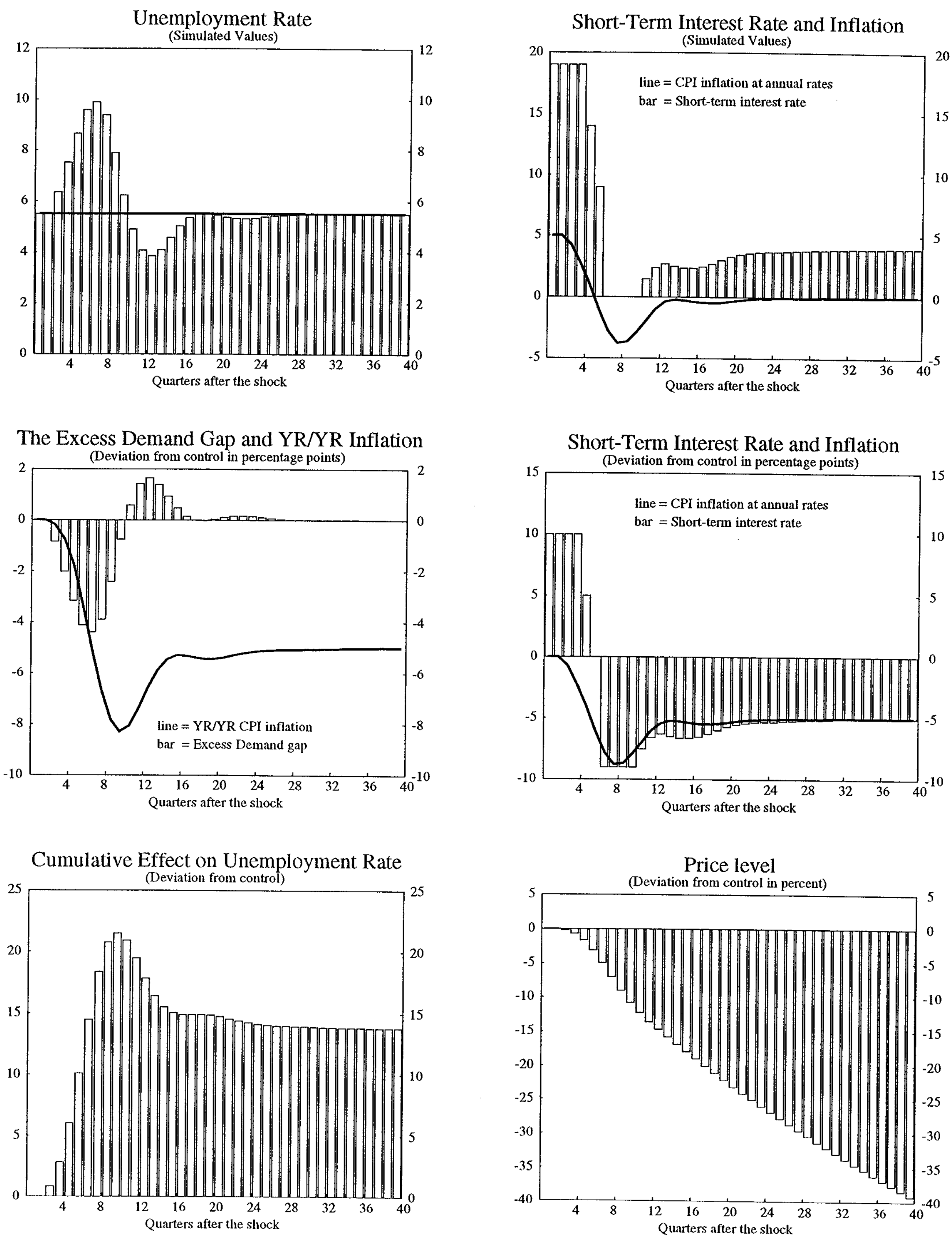
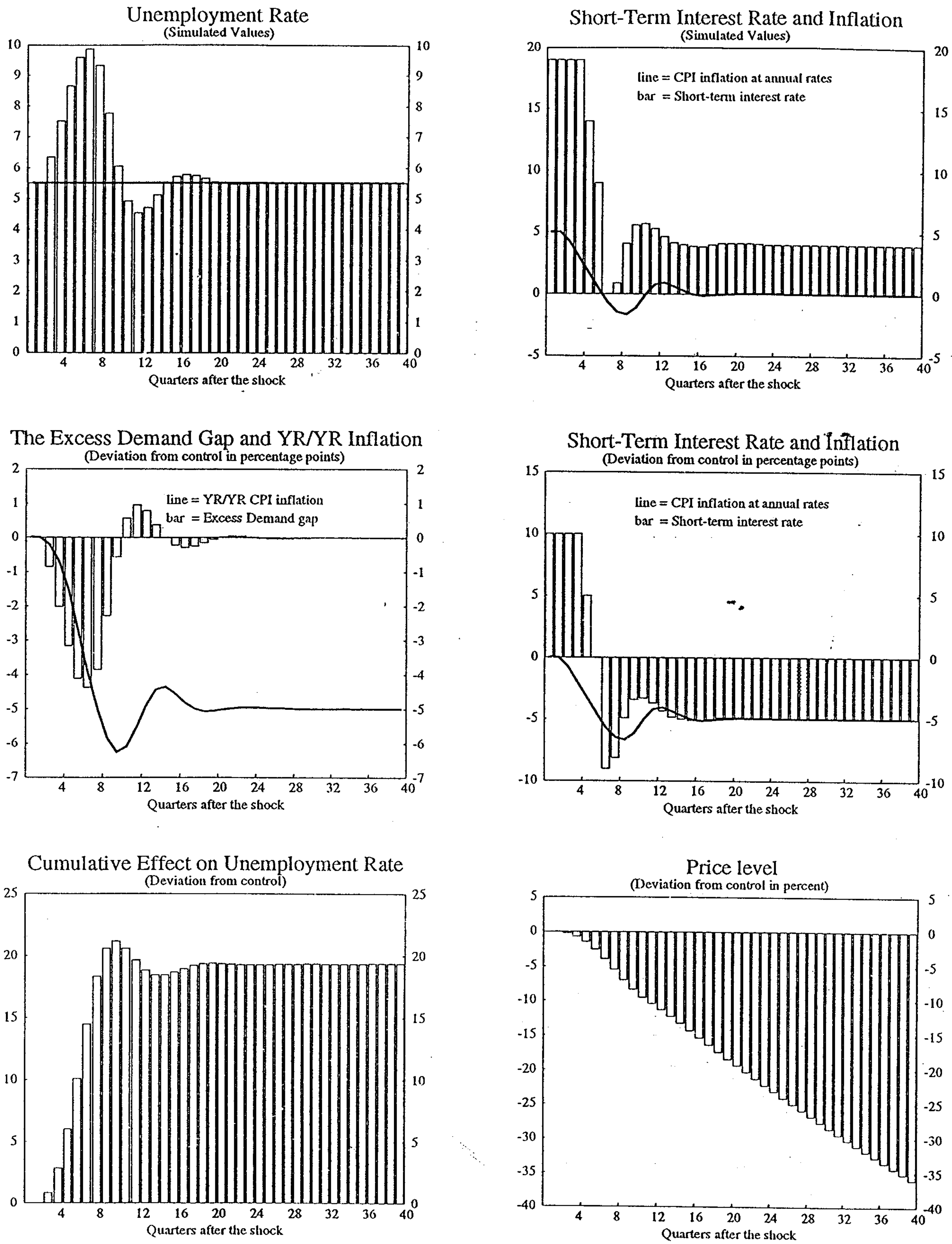


Figure 7b: Nonlinear Model Responses to a Cold Shower Disinflation Shock



implication of convexity means that gradual disinflation is less costly in terms of foregone output and employment than a more rapid "cold shower" approach to reducing inflation.[23]

E. Costs of Disinflation, Inflation Expectations, and Credibility

The scenarios described above have been designed to illustrate the connection between the convexity of the Phillips curve and the costs of disinflation. However, it needs to be stressed that the actual costs of disinflation in any particular situation depend on a number of other factors as well, especially the extent to which expectations are forward looking and the degree of credibility in government policies. We have deliberately chosen to hold these factors constant in order to focus exclusively on a comparison between linear and nonlinear models of the Phillips curve. Taking into account these other factors will clearly affect the absolute costs of disinflation, but doing so is unlikely to affect the qualitative result that relative costs depend importantly on the degree of convexity in the Phillips curve.

As our analysis embodies the extreme assumption that there is no forward-looking component of inflation expectations, our absolute measure of the sacrifice ratio—the change in unemployment required to achieve a given reduction in inflation—is probably biased upward to the extent that expectations have some forward-looking element. In our scenarios this is given by the ratio of the long-run cumulative effect on the unemployment rate divided by the long-run change in inflation. With the linear model this is equal to 14/5 = 2.8 and is invariant, as noted above, to the speed of adjustment. By contrast, with nonlinearity the sacrifice ratio is not a fixed number but depends on the speed at which deflation takes place; in our illustrative scenarios, it is equal to 14/5 = 2.8 with gradual deflation and 19/5 = 3.8 with rapid deflation. These figures are higher than most of the estimates contained in Ball (1994), which vary from 0.0 to 3.6 for a wide range of industrial countries, with the average ratio for the United States equal to 2.35.[24] Our calculated ratios are above the recent estimate of 2.0 contained in Braun and Chen (1996). They are also higher than the recent estimates for Spain

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[23]In a model similar to that used here, Bean (1996) derives closed-form solutions for optimal monetary policy rules using a dynamic programing approach. In the case of no uncertainty he finds that with a linear Phillips curve there is no advantage to smoothing the disinflation response over time and therefore what he calls a "cold turkey" strategy is optimal. By contrast, with a convex Phillips curve the optimal rule involves spreading out the disinflation because of the declining marginal effectiveness of a large output gap in reducing inflation.

[24]The sacrifice ratio computed by Ball is in terms of output and hence would be expected to be somewhat higher because of labor hoarding over the cycle. While therefore not strictly comparable to our calculated sacrifice ratios, it is noteworthy that his estimate for the United States is nonetheless lower than our calculated values.

in Dolado, Lopez-Salido, and Vega (1996), where the estimated sacrifice ratio ranges between 1.4 and 2.5.[25]

One reason why our estimates are higher is that there is considerable inertia in the inflation process, as current inflation is a function of the current unemployment rate and an eight-quarter distributed lag on past inflation, with no role for forward-looking expectations. As Ball (1994) rightly points out, inflation expectations are an important determinant of the sacrifice ratio. Indeed, he argues that a policy of more rapid disinflation (our "cold shower" disinflation shock) can lower the sacrifice ratio because if the shock therapy affects expectations sufficiently strongly, the change in unemployment required to reduce inflation is reduced. On the basis of his own empirical evidence his conclusion is the opposite of ours, namely, that the faster the pace of disinflation, the lower the sacrifice ratio.

The difference in view is, however, more apparent than real. Our analysis focuses solely on the implications of the shape of the Phillips curve for the relative size of the sacrifice ratio, but this is not necessarily inconsistent with the fact that the absolute magnitude of the sacrifice ratio depends importantly on the nature of inflation expectations and the credibility of policies. Ball may be correct that a more rapid disinflationary policy reduces the sacrifice ratio when it has a strong effect on expectations and the policy action has strong credibility. In terms of Figure 2, a shift in the Phillips curve down and to the left in response to a downward shift in inflation expectations can more than offset a movement along the curve. Our point is simply that the movement along a convex Phillips curve implies that slower adjustment is less costly, in terms of output and employment losses, than a more rapid disinflation policy.

Finally, we should note that while the Phillips curve is asymmetric in that excess demand is more inflationary than excess supply is deflationary, this asymmetry does not arise because of downward rigidity in nominal wages. This contrasts with the recent paper by Akerlof, Dickens, and Perry (1996) which presents evidence that such downward rigidity is an important feature of the U.S. economy. They construct a model of U.S. inflation that has this feature and argue that the sustainable rate of unemployment consistent with steady inflation is not a unique natural rate, but rather depends on the level of inflation. In particular, they find that the lowest possible NAIRU requires a moderately low, nonzero inflation rate, e.g., 3 percent, and that achieving price stability would result in a significantly higher NAIRU. Thus their approach implies that the long-run Phillips curve is convex, whereas our model maintains the standard feature of a vertical Phillips curve in the long run. While there may be some downward rigidity in U.S. nominal wages in the short run, we would argue that such rigidity is likely to disappear over time because the factors generating this rigidity, e.g., perceptions of equity, would become much less important in such a regime.

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[25]Excluded from this range is the estimate using the real business cycle model considered by the authors, as the sacrifice ratio in this model is constrained to be zero by construction.

VI. Concluding Remarks

The objective of this paper has been to return the Phillips curve to its nonlinear roots. We have argued that the relationship between excess demand—as measured by the gap between the unemployment rate and the NAIRU—and inflation adjusted for inflation expectations, is fundamentally a nonlinear one. Thus we want to put the curvature back into the Phillips "curve" and not let the vagaries of limited data sets determine whether it should be regarded as a curve or a straight line.

There are, first of all, strong theoretical reasons for believing that this relationship is not linear. Second, while the small number of episodes of excess demand and supply preclude robust empirical results, there is evidence—some of it presented in this paper—which suggests that the Phillips curve is indeed not a straight line. Moreover, the evidence indicating the level of the real wage is a nonlinear function of the unemployment rate would appear to support the notion of curvature in the Phillips curve. Finally, the derivative of the slope of the short-run inflation-output or inflation-unemployment tradeoff has important policy implications; a nonzero second derivative implies significant costs of economic overheating and corresponding gains from stabilizing the economy around the natural rate of unemployment. Indeed, by moderating fluctuations in demand and employment and thereby limiting movements along a Phillips curve, policymakers can reduce the natural rate.

As discussed in detail in other papers—Laxton, Meredith, and Rose (1995) and Clark, Laxton and Rose (1996)—most existing empirical tests are biased against finding evidence of nonlinearity because they do not take proper account of what convexity entails for the measurement of the output or unemployment gaps. The focus of this paper, by contrast, is not to test for convexity against an alternative hypothesis of linearity, but rather to impose weak convexity on the data and then compare the implications of this specification for the costs of overheating and rapid disinflation with those of an alternative model that imposes strict linearity. The functional form we have chosen imposes quite modest convexity in the neighborhood of zero excess demand and the plausible assumption that the absolute minimum level of unemployment is zero. Given the weak assumption of convexity, it seems likely that our estimates of the costs of overheating are underestimated. We also show that if the minimum unemployment rate is as high as 2 percent, there are significantly larger costs for even modest accounts of overheating.

As was stressed in the preceding section, the estimates of the costs of economic overheating presented here are mainly illustrative. While these costs—as measured by the cumulative effect on the unemployment rate—are significantly larger in the case of the nonlinear model of the Phillips curve, the degree of precision in the quantitative difference between the linear and nonlinear models depends on a host of factors. Among the most important are of course the estimated parameters of the Phillips curves. These depend importantly on both the measures of excess demand and inflation expectations, as one of the key parameters—the impact of the unemployment gap on the difference between actual and expected inflation—is directly affected by these measures. Our approach appears to yield plausible results, but obviously further research is needed to improve the parameter estimates.

Another area for further research relates to certain aspects of the methodology used to calculate the costs of overheating and rapid disinflation. We employed a small simulation model of the U.S. economy for these calculations, and as we used the same model and same assumptions in comparing different specifications of the Phillips curve, our estimates of the costs of overheating may not be that sensitive to alternative models and assumptions. However, one relevant and important issue that is beyond the scope of this paper, but which certainly deserves attention, is the relationship between the shape of the Phillips curve, the credibility of monetary policy, and the degree to which inflation expectations are forward looking. As emphasized in the preceding section, the more inflation expectations are forward looking and the greater the credibility of policy, the lower the costs of overheating. Our analysis in this paper has not explored these connections. An even more ambitious extension of the work described here is the analysis of the extent to which the shape of the Phillips curve could affect the credibility of policy. A highly convex Phillips curve would exacerbate the costs of policy errors and therefore enhance the returns to policy credibility, but the costs themselves might lead agents to be less trusting in the announced policies of the monetary authorities.[26]

[26]For an analysis of this topic, see Isard and Laxton (1996).

Table 1. Estimates of U.S. Phillips Curves with Model Consistent NAIRUS

(T-statistics in parentheses)

Nonlinear model: $\pi_t = \alpha(L)\pi_t + \gamma(u_t^*/u_t - 1) + \epsilon_t^\pi$

Linear model: $\pi_t = \beta(L)\pi_t + \delta(u_t^* - u_t) + \epsilon_t^\pi$

Where:

π	Percent change in the CPI at annual rates
$\alpha(L)\pi_t$	Eight-quarter distributed lag on past inflation
γ	Estimated weight on the unemployment gap in the nonlinear model
δ	Estimated weight on the unemployment gap in the linear model
u	Unemployment rate
u^*	Estimated NAIRU derived from Kalman filter
LLF	Value of the likelihood function in logs
σ^2	Variance of residuals
λ	Standard deviation of quarterly change in the NAIRU (1971:Q1 to 1996:Q3)
η	Average value of the NAIRU (1971:Q1 to 1996:Q3)

Estimation period: 1968:Q1 to 1996:Q3

Model	γ	δ	LLF	σ^2	λ	η
Nonlinear	5.91 (4.45)		-226.70	2.57	0.08	6.4%
Linear		1.83 (5.46)	-221.72	1.10	0.38	6.7%
Linear restricted		0.88 (4.47)	-227.08	2.58	0.08	6.6%

Note: Figures in parentheses are t-statistics.

Table 2. A Small Simulation Model of the U.S. Unemployment-Inflation Process

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Integral Gap Phillips Curve Model (restricted):

$$\pi_t = \pi_t^e + 0.88\,(u_t^* - u_t) + \epsilon_t^\pi$$

Nonlinear Phillips Curve Model:

$$\pi_t = \pi^e + 5.91\,(u_t^*/u_t - 1) + \epsilon_t^\pi$$

Inflation Expectations:

$$\pi_t^e = 0.61\pi_{t-1} - 0.18\pi_{t-2} + 0.51\pi_{t-3} - 0.17\pi_{t-4} + 0.11\pi_{t-5} + 0.08\pi_{t-6} + 0.09\pi_{t-7} - 0.05\pi_{t-8}$$

Labor Market Gap Equation:

$$gap_t = 1.40\,gap_{t-1} - 0.60\,gap_{t-2} - 0.08\,rr_{t-2}$$

Forward-Looking Policy Reaction Function:

$$rs_t = \max\,(\,rr_t^* + \pi^e + 3\,(\pi_{t+3}^e - \pi_{t+3}^*) + gap_t,\ 0\,)$$

Definitions of Real Interest Rate, Labor Market Gap, and CPI Inflation:

$$rr_t = rs_t - \pi_t^e,\quad gap_t = u_t^* - u_t,\quad \pi_t = 100\,[P_t/P_{t-1})^4 - 1]$$

π = CPI inflation at annual rates
u = Unemployment rate
u^* = NAIRU
rr = Real interest rate
rs = Federal funds rate
π^e = Inflation expectations
π^* = Inflation target
P_t = Price level

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