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The Role of Productivity, Transportation Costs, and Barriers to Intersectoral Mobility in Structural Transformation

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Abstract

The process of economic development is characterized by substantial reallocations of resources across sectors. In this paper, we construct a multi-sector model in which there are barriers to the movement of labor from low-productivity traditional agriculture to modern sectors. With the barrier in place, we show that improvements in productivity in modern sectors (including agriculture) or reductions in transportation costs may lead to a rise in agricultural employment and through terms-of-trade effects may harm subsistence farmers if the traditional subsistence sector is larger than a critical level. This suggests that policy advice based on the earlier literature needs to be revised. Reducing barriers to mobility (through reductions in the cost of skill acquisition and institutional changes) and improving the productivity of subsistence farmers needs to precede policies designed to increase the productivity of modern sectors or decrease transportation costs.

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1. Introduction

In many developing countries, employment in the agriculture sector is very high relative to that in the developed countries. Globally, the poorest 5% of the countries have about 86% of their labor force in agriculture, whereas the richest 5% have less than 5%. In the process of development, economies experienced significant movements of labor from agriculture into modern sectors.

The notion that economic growth and development has been associated with significant movements of labor out of agriculture and into manufacturing and services (structural transformation) has been put forward starting with Clark (1940), Kuznets (1966), Rostow (1959) and Chenery and Syrquin (1975). More recently economists have focused on structural transformation in the context of multi-sector models in order to present a more nuanced explanation of differences in productivity and growth rates across countries. This literature has emphasized, on the one hand, differences in productivity across agricultural and non-agricultural sectors¹ and, on the other hand, barriers to structural transformation that include costs of transportation, of skill acquisition, and cultural factors. In this paper, we develop a general equilibrium model that incorporates both productivity differences and some empirically prominent barriers to the transformation of the economy from one where subsistence agriculture is dominant to one where modern sectors play a more significant role.

In what follows, we construct a three-sector model with a traditional subsistence agriculture, modern agriculture, and a non-agricultural sector (manufacturing and/or services). Agricultural goods are produced in the rural sector, whereas manufacturing and services are produced in the urban areas. Our setup differs from existing ones along a number of dimensions. Perhaps most importantly, we bring together two strands of the recent literature. The first of these emphasizes the role played by barriers to goods mobility, such as transportation costs for the movement of labor out of agriculture. Herrendorf et al. (2012) study the effect of the construction of railroads in the US during 1840-1860, and find that the associated reduction in transportation costs lead to settlement of the most fertile land in the Midwest, and a reduction in the agricultural labor force. Adamopoulos (2011) shows that transportation costs can lead to low aggregate output per worker, by reducing productivity within sectors and distorting allocation of resources across locations and between sectors. He analyzes the effect of cross country transportation cost disparities and finds that improvements in transportation productivity would have an asymmetric result on the poor and developed countries, with the former gaining more. Gollin and Rogerson (2014) allow for heterogeneity in agriculture through differences in the costs of transportation. They carry out some numerical exercises matching the parameters of their model to a typical sub-Saharan economy and find that transportation costs are quantitatively important in terms of both allocations and welfare. The second strand, exemplified by Caselli and Coleman (2001) and Hayashi and Prescott (2008), focuses on barriers to

¹ Restuccia, Yang and Zhu (2008) look at labor productivity differences in both agriculture and non-agriculture between the richest 5% of the countries and poorest 5%. They report that GDP per worker differences in agriculture is a factor of 78, whereas in non-agriculture it is a factor of 5. Based on these productivity differences, McMillan and Rodrik (2011) argue that as much as a fifth of the productivity gap between developing and advanced countries would be eliminated if the inter-sectoral distribution of employment in the developing countries matched that in the developed countries. Gollin, Lagakos and Waugh (2014) report that in countries in the lowest quartile of income distribution, the value added per worker is about 5.6 higher in non-agriculture than in agriculture, and this factor drops to 2 for the upper quartile.

labor mobility that keep agricultural employment relatively high for extended periods of time in the process of development. These barriers may take a number of different forms. Caselli and Coleman (2001) emphasize the costly acquisition of skills as the barrier that impedes the movement of labor from agriculture into non-agricultural employment. Hayashi and Prescott (2008), on the other hand, argue that both the cultural values of the Japanese extended family and the institutions of prewar Japan acted as barriers to the movement of labor out of agriculture in the 1885-1940 period². In the rest of the paper, we adopt the specific barrier suggested by Caselli and Coleman (2001) and suppose that labor employed in subsistence agriculture is unskilled, whereas both modern agriculture and manufactures employ skilled labor with no possibilities of substitution anywhere between these two types of labor. We also assume that the cost of skill acquisition is too high for unskilled workers. However, this specific barrier could be interpreted in any number of ways and as long as there exists some barrier to the mobility of labor between the traditional and modern sectors, our conclusions will remain valid. In addition our model incorporates productivity differences across sectors as well as transportation costs. We model productivity differences by taking the idea that throughout the development process modern and traditional technologies coexist seriously and incorporate two agricultural sectors that produce a single agricultural good using a traditional technology (in the subsistence sector) and a modern technology³. As for transportation costs, our model incorporates three types: the ones involved in transporting manufactures to rural areas as both a consumption good and an intermediate input in the production in modern agriculture, and those involved in transporting the agricultural good from the rural to the urban areas.

Our model produces a number of novel insights with regard to the interaction of different sectors, technologies and barriers in the process of structural transformation. One of the most important of these insights concerns the effects of productivity improvements in modern agriculture. In a standard model of the type used in the existing literature, this improvement would lead to agriculture shedding labor and manufactures expanding employment and output. This is still the case in our model if the initial subsistence agriculture employment and output are lower than a critical value. However, if the initial size of the subsistence agriculture is large enough, the modern agriculture sector expands in response to a technological shock that makes it more productive. The intuitive reason for this hinges upon the changes in terms of trade faced by both agricultural sectors that produce the same good as the size of subsistence agriculture crucially affects the relative demand changes for both the agricultural and manufacturing goods. We also show that the interactions between the barrier to labor mobility and the simultaneous use of traditional and modern technologies in agriculture give rise to welfare reductions for subsistence farmers who get hurt by the deterioration of their terms of trade when modern agriculture becomes more productive. Similar perverse welfare results arise in cases when there are reductions in transportation costs that lower the terms of trade subsistence farmers face. These results are established analytically in our setup.

²O'Brien (1996) argues that much of the explanation for the persistence of a comparatively large subsistence agriculture sector in France (as opposed to England) prior to the twentieth century lies in the cultural and institutional factors that kept a large fraction of the labor force in rural areas of France.

³Gollin, Parente and Rogerson (2004) also develop a growth model with agricultural sector where agricultural and non-agricultural output can be produced with different technologies contemporaneously, in their case however they focus on market and home production.

We then turn to counterfactual numerical exercises by calibrating our model as far as possible to match the stylized features of a sub-Saharan African economy. Here we are interested in the magnitude of the differential effects of changes in productivity and transportation costs taken either separately or grouped together. Our focus is the consequences of these changes on the economy's potential structural transformation (as measured by the allocation of resources across sectors) and on the changes in welfare of different groups in society. We find that improvements in the productivity of modern agriculture have significant negative welfare consequences on subsistence farmers through terms of trade deteriorations, but benefit workers in the modern sectors. As mentioned before, in contrast to the findings in the existing literature, whether the modern agriculture sector expands or contracts in response depends on the initial size of subsistence agriculture. Improvements in manufacturing productivity improve the welfare of all workers (with subsistence farmers benefiting from a terms of trade improvement), yet their effect on the allocation of resources across sectors and the magnitude of changes in welfare depend again on the initial size of the subsistence agriculture sector. In terms of the direction and magnitude of structural transformation improvements in the productivity of the subsistence sector appear to be the most important as they lead to the most substantial reallocations of skilled labor away from (modern) agriculture to manufactures. Not surprisingly, subsistence farmers gain the most in terms of welfare when it is their productivity that rises. Reductions in transportation costs have comparatively small effects on the allocation of resources and welfare either taken by themselves or together with other changes. Finally, lowering the barrier to the intersectoral mobility of labor in the form of a reduction in the cost of skill acquisition that enables some unskilled to become skilled reduces the share of agricultural output in total output while reducing the labor force engaged in agriculture.

Our model is most closely related to Gollin and Rogerson (2014). We differ most importantly from Gollin and Rogerson (2014) in that we allow for barriers to the mobility of labor between the subsistence sector and the modern sectors. Together with heterogeneity in agriculture through differences in production techniques, including the type of resources employed, our model yields a richer set of results concerning the effects of changes in (both agricultural and manufacturing) productivity and transportation costs. Thus, whether improvements in, say the productivity of modern agriculture, result in a reallocation of labor away from it depends crucially on the size of the subsistence agricultural sector that is modeled to be less productive. Our richer set of results include those of Gollin and Rogerson (2014) as a subset: the smaller is the size of subsistence agriculture the more likely it is for improvements in productivity of modern agriculture to reduce the latter's share of employment. Further, we show that such improvements reduce the welfare of subsistence farmers through their negative impact on the terms of trade such farmers face, and may reduce the overall welfare as well. What these results suggest, in contrast to the existing literature, is that in the poorest developing countries with large subsistence agriculture sectors, policy needs to follow a sequence. Measures that reduce barriers to labor mobility from subsistence farming to modern sectors and that help raise the productivity of subsistence farmers need to precede implementation of policies that are designed to increase productivity in modern sectors or decrease transportation costs.

The paper proceeds as follows. Section 2 describes the model. Section 3 presents the calibration and quantitative evaluation. Section 4 concludes.

2. The Model

2.1. Firms

A typical assumption made in the literature on structural transformation is that agricultural sector is the traditional one and manufacturing and services are the modern ones without taking into consideration the heterogeneity of production technologies in the economy. In contrast, we assume that even though the non-agriculture good is produced only with modern technology, there are two available technologies for producing the agricultural good: a subsistence/traditional and a modern one.⁴ Thus modern technology could be used in either modern agriculture or non-agriculture sector, and in what follows, we suppose that education is a barrier to adopting modern technologies. Firms take the technology that they can use as given. Farms in the subsistence sector use unskilled labor as the only factor of production. Letting population be equal to a mass n and educated workers equal to a mass n_e yields the unskilled workers to be equal to $n - n_e$. Assuming a constant input-output coefficient of $1/A_u$, the output, Y_u^A , of “food” by the subsistence agriculture sector is then given by:

$$Y_u = A_u (n - n_e) \quad (1)$$

Competitive firms produce food in the modern agriculture sector by combining intermediate inputs z and a fraction θ of skilled labor in a CRS Cobb-Douglas form:

$$Y_n = A_n (\theta n_e)^\beta z^{1-\beta} \quad (2)$$

Intermediate inputs are produced by competitive firms in the manufacturing sector which operate in the urban area and are subject to iceberg transportation costs η , such that farms in modern agriculture pay effectively $p(1 + \eta)$, where p is the relative price of manufactured good in terms of food.

Firms in modern agriculture choose the optimal amount of intermediate inputs and labor used in modern agriculture to maximize profits $Y_n - w_n n_n - p(1 + \eta)z$ (where $n_n := \theta n_e$). The solution of this problem yields the following first-order conditions

$$p(1 + \eta) = (1 - \beta) A_n k^{-\beta} \quad (3)$$

$$w_n = \beta A_n k^{1-\beta}, \quad (4)$$

where we define the capital-labor ratio k as $k := z/(\theta n_e)$.

⁴One could argue that heterogeneous technologies exist in both the agriculture and non-agriculture sectors and focusing on the different technologies just in the agriculture sector may be misleading. However, we focus on the heterogeneity in the agriculture sector for two reasons. First, as Eberhardt and Teal (2012) show the heterogeneity of production functions (measured by the capital coefficients) in agriculture is much wider than that in the manufacturing sector. Secondly, this heterogeneity is quite substantial in the poorest developing countries on which this paper focuses.

Note that the total amount of labor in agriculture is $n_a = n - n_e + \theta n_e$.

The only input of production in the manufactures sector is skilled labor:

$$Y_m = A_m n_m \quad (5)$$

where subscripts m denote manufactures and $n_m = (1 - \theta)n_e$. Firms in manufacturing choose the optimal amount of skilled labor to maximize profits $pY_m - w_m n_m$. The solution of this problem yields the following first order condition: $w_m = pA_m$.

We also impose the condition that MPL in modern sector is always higher than in traditional agriculture, with the result that skilled workers will choose to work only in the modern sectors. We also note that wages in the modern sector do not have to be necessarily equal.

2.2. Households

All households consume food and manufactured goods according to the same non-homothetic utility function, and differ only in the budget constraint that they face. Their preferences are given by

$$u(c_j^A, c_j^M) = \alpha \ln(c_j^A - \bar{c}_A) + \ln c_j^M \quad (6)$$

where $\bar{c}_A > 0$ denotes the subsistence level of food consumption and $\alpha > 0$ is the relative weight of food in preferences. Finally, c_j^A denotes individual consumption of food and c_j^M denotes individual consumption of manufacturing good ($j = \overline{m, u, n}$). This formulation yields an income elasticity of food demand that is below one. Let m , n , u represent households that live in urban areas and work in the manufacturing sector, that live in rural areas and work in the modern agriculture sector, and that live in rural areas and work in subsistence agriculture, respectively.

Both type of workers supply their labor endowment inelastically.

Consumers in rural areas pay a price inclusive of transportation costs, η_1 , for the manufactured goods, and receive a wage equal to their marginal product of labor:

$$w_i = c_i^A + p(1 + \eta_1)c_i^M \quad (7)$$

where $i = \overline{u, n}$.

Consumers living in the urban areas receive a wage from working in the manufacturing sector and the price they pay for food includes the cost, η_2 , of transporting it from the rural areas:

$$w_m = c_m^A(1 + \eta_2) + pc_m^M \quad (8)$$

We also assume that the level of agricultural productivity is high enough so that our economy operates above the subsistence level of food (i.e. $A_u \geq \bar{c}_A$).

Households maximize (6) subject to their respective budget constraints 7 and 8. The optimality conditions imply that households equate the marginal rate of substitution between the two consumption goods to the relative price, incorporating transportation costs such that

$$c_i^A = \bar{c}_A + \alpha p(1 + \eta_1)c_i^M \quad (9)$$

where $i = \overline{u, n}$ and

$$c_m^A = \bar{c}_A + \frac{\alpha pc_m^M}{1 + \eta_2} \quad (10)$$

Using the first order conditions and budget constraints for the households, and the profit maximization conditions for the firms in the modern sectors we can derive analytically individual consumption levels for both unskilled and skilled workers as function of their wages.

$$c_u^A = \frac{\bar{c}_A + \alpha A_u}{1 + \alpha}, c_u^M = \frac{A_u - \bar{c}_A}{p(1 + \alpha)(1 + \eta_1)} \quad (11a)$$

$$c_n^A = \frac{\bar{c}_A + \alpha w_n}{1 + \alpha}, c_n^M = \frac{w_n - \bar{c}_A}{p(1 + \alpha)(1 + \eta_1)} \quad (11b)$$

$$c_m^A = \frac{\bar{c}_A(1 + \eta_2) + \alpha w_m}{(1 + \alpha)(1 + \eta_2)}, c_m^M = \frac{w_m - (1 + \eta_2)\bar{c}_A}{p(1 + \alpha)} \quad (11c)$$

2.3. Market clearing

Aggregating across sectors and assuming, by Walras' law, that excess demand for food, ED^A , equals zero in equilibrium yields:

$$ED^A := D^A - S^A = (n - n_e)c_u^A + \theta n_e c_n^A + (1 - \theta)n_e c_m^A(1 + \eta_2) - (Y_u + Y_n) = 0. \quad (12)$$

Substituting equation (3) into (12) we can express the excess demand for food, ED^A , as a function of k , θ , and the parameters

$$ED^A := \psi(\theta, k; A_n, A_m, A_u, \eta, \eta_1, \eta_2, n, n_e, \bar{c}_A), \quad (13)$$

with $\psi_i > 0$ $i \in \{4, 8, 10, 11\}$, $\psi_i < 0$ $i \in \{1, 2, 5, 6, 9\}$, $\psi_i \leq 0$ $i = 3$, $\psi_i = 0$ $i = 7$, where D^A and S^A denote the demand for and supply of food production and

$$Y^M = (1 + \eta_1) [(n - n_e)c_u^M + \theta n_e c_n^M] + (1 - \theta)n_e c_m^M + z(1 + \eta) \quad (14)$$

So that aggregate output of food, $Y^A (= S^A)$ can be expressed as

$$Y^A = n\bar{c}_A + \eta_2 (1 - \theta) n_e \bar{c}_A + \alpha p[Y^M - z(1 + \eta)]. \quad (15)$$

Since there are no impediments to the movement of labor across the two modern sectors, in utility terms skilled workers will receive the same pay-off in these two sectors:

$$U^{nm} := U_n - U_m = (1 + \alpha) \left\{ \ln \frac{w_n - \bar{c}_A}{w_m - (1 + \eta_2)\bar{c}_A} \right\} + \alpha \ln(1 + \eta_2) - \ln(1 + \eta) = 0 \quad (16)$$

where U_i ($i = n, m$) denotes the utility pay-off to skilled labor working in sector i and using (3) we have

$$U^{nm} = \varphi(\theta, k; A_n, A_m, A_u, \eta, \eta_1, \eta_2, n, n_e, \bar{c}_A). \quad (17)$$

with $\varphi_i > 0$ $i \in \{2, 6, 8, 11\}$ $\varphi_i < 0$ $i \in \{3, 4, 7\}$, $\varphi_i = 0$ $i \in \{1, 5, 9, 10\}$. All the partial derivatives, ψ_i and φ_i are derived explicitly in the Appendix.

2.4. Equilibrium

An equilibrium for this economy, given transportation costs (η, η_1, η_2) , sectoral productivities (A_n, A_m, A_u) , distribution of skill in the population (n, n_e) , and subsistence level of consumption for the agricultural good \bar{c}_A is characterized by the relative price of manufactured good, capital-labor ratio, as well

as the fraction of skilled labor employed in modern agriculture sector (p, k, θ) together with wages (w_n, w_m) , and allocations of consumption for each type of households that solve their respective constrained optimization problem and clear markets.

More precisely, the two equations (3) and (16) jointly solve for p and k . Given the solution for k , equation (12) can then be used to solve for θ . Equation (4) and $w_m = pA_m$ then solve for wages, while (11a)-(11c) do so for the consumption levels and (6) for utilities. It is helpful to use Figure 1 to visualize the solution of the model. The left-hand side panel in the figure displays the determination of k and p . The curve zz depicting the equation (3) slopes downward as an increase in the relative price of manufactures, p , reduces the demand for manufactures as an input and lowers $k = z/(\theta n_e)$. The curve labeled $U^{nm} = 0$ slopes upward as a rise in p increases w_m and, thus the utility pay-off to working in manufactures. Free mobility of labor would then lead to skilled workers moving to the manufacturing sector, reducing θ and, thus, increasing k . As (3) and (16) depend only on the two endogenous variables p and k , the left-hand panel of Figure 1 determines these two variables. On the right-hand panel, the curve $ED^A = 0$ depicting equation (12) is drawn downward sloping as excess demand for food is decreasing in both k and θ .

As it will play an important role later on in our analysis of productivity shocks, it is useful to employ Figure 1 to visualize the effects of an increase in the number of skilled workers, n_e . First, note that the change in n_e does not affect either k or p (see equations (3) and (16)). It does however, reduce the supply of food produced by subsistence farmers and increase its demand by raising the number of skilled workers who receive relatively higher wages. The excess demand these changes generate is eliminated by an increase in the number of skilled workers employed by modern agriculture. In the figure, this is depicted by the rightward shift of the $ED = 0^A$ curve and the rise in θ .

2.5. Productivity shocks

We now turn to a discussion of some counterfactual experiments starting with productivity shocks in the three sectors of the economy.

2.5.1. In the Manufacturing Sector

The first productivity shock we consider is an increase in the productivity A_m of the manufacturing sector. The main result we obtain is summarized in the following Lemma. All the proofs are in the Appendix.

Lemma 1. A rise in the productivity, A_m , of the manufacturing sector reduces (increases) employment in manufactures (agriculture) if initially subsistence farmers can meet the subsistence food needs of the economy.

To see why, note that the increase in the productivity of the manufacturing sector leads to a decrease in the price of the manufactured good, p , and an increase in wages in the modern sector ($\hat{w}_m = (1 - \beta_n)\hat{A}_m$ where for any x we have $\hat{x} = dx/x$). Consequently, consumption of the manufactured final good for both skilled and unskilled workers increases, so does the consumption of food for skilled workers. In addition, the fall in p leads to a rise in the amount of intermediate goods used, z , in modern agriculture. The increase in the productivity of modern agriculture raises only the wages of skilled workers and, thus, increases income inequality. However, as unskilled workers can consume more of the manufactured good, their welfare increases as well. The output of food also rises in the modern agriculture sector, but the effect on labor flows is ambiguous and depends on the size of the subsistence agriculture sector and, thus, the amount of labor, θn_e , that needs to be allocated to modern agriculture to satisfy the subsistence needs of the population. The clearest way to see this algebraically is to consider the case where there are no transportation costs for final goods. In that case $d\theta/dA_m = \Omega(Y_u - n\bar{c}_A)$ with $\Omega = (1 - \beta)\theta/[(1 + \alpha)A_m Y_n] > 0$. If subsistence farming cannot meet subsistence food requirements ($Y_u < n\bar{c}_A$) so that a larger share θ of the skilled labor is in modern agriculture, a rise in A_m reduces θ moving labor out of agriculture to manufactures. However, if initially $Y_u > n\bar{c}_A$, manufactures lose labor to agriculture. This last result is interesting as it goes against the conventional result that improvements in productivity always lead to the flow of labor out of the agricultural sector.

To gain a better understanding of the effect of an increase in manufacturing productivity, A_m , on the allocation of skilled labor between manufactures and the modern agricultural sector, it is useful to observe Figure 2. Here by raising w_m the increase in A_m shifts the $U^{nm} = 0$ curve up, reducing p and raising k . On the right-hand panel, ceteris paribus, the rise in k will lead to a decline in θ . However, the increase in productivity A_m and, thus, in the wages of skilled workers in manufacturing, w_m , increases the demand for food and shifts the $ED^A = 0$ curve up, with the shift being larger the higher is the initial number of skilled workers in manufacturing (i.e., the higher is $1 - \theta$ or the lower is θ given by θ_L in the figure). As the figure shows, when the shift in $ED^A = 0$ is small (as is the case when θ is initially high at θ_H), the net effect of the change in k and the shift in $ED^A = 0$ is a fall in θ . Again, this is the conventional result. However, when the shift in $ED^A = 0$ is large (as is the case when θ is initially low at θ_L , i.e., when the subsistence sector is able to cover the subsistence food needs), the net effect of the change in k and the shift in $ED^A = 0$ is a rise in θ and, thus, a decline in the manufactures employment.

2.5.2. In Modern Agriculture

We consider next an increase in the productivity A_n of the modern agriculture sector. The main result we obtain is summarized in the following Lemma.

Lemma 2. A rise in the productivity, A_n , of the modern agriculture sector reduces (increases) employment in manufactures (agriculture) if initially subsistence farmers can meet the subsistence food needs of the economy.

To see why, note that the increase in the productivity of the modern agriculture sector leads to an increase in the supply of food, a decline in its relative price, and, thus, a rise in the relative price of manufactured good, p , and increase in the wages of skilled workers who are employed solely in the modern sectors. As a result, skilled workers are able to consume both more agriculture and manufactured goods. As the increase in productivity of modern agriculture has no effect on the wages of unskilled workers, it increases income inequality. Consumption of the manufactured good decreases for unskilled workers, reducing their welfare. Production of food increases in the modern agriculture sector. However, the effect on labor flows out of agriculture and the amount of intermediate inputs used in modern agriculture depends as before on the size of subsistence agriculture. To see the effects of the increase in A_n on θ and k , it is useful to refer to Figure 3, where two initial labor allocations are indicated as above by θ_L and θ_H . The rise in A_n shifts the zz curve upwards and the $U^{nm} = 0$ curve downwards, the net effect being a rise in both k and p . Ceteris paribus, this would lead to an outflow of skilled labor from modern agriculture to manufactures. However, the rise in A_n also leads to a shift of the $ED^A = 0$ curve. Given θ , whether excess demand for food rises or falls depends on the initial size of the subsistence agriculture sector. If it is small so that we have θ_H , the $ED^A = 0$ curve shifts to the left, θ falls and the manufacturing sector expands. This is the case emphasized by the existing literature which typically assumes away subsistence agriculture. However, if subsistence farming is large enough initially with labor allocation given by θ_L , the improvement in the productivity, A_n , of modern agriculture leads to a rightward shift of the $ED^A = 0$ curve, with the result that θ rises and the manufacturing sector shrinks.

2.5.3. In Subsistence Agriculture

We next turn to the analysis of an increase in the productivity, A_u , of the subsistence agriculture sector. The main result we obtain is summarized in the following Lemma.

Lemma 3. A rise in the productivity, A_u , of the subsistence agriculture sector increases (reduces) employment in manufactures (agriculture).

The mechanism behind this result is straightforward to see: A rise in the productivity, A_u , of the subsistence agriculture, raises the quantity of food produced and increases the incomes of the farmers there, leading to higher levels of consumption of both food and manufactures. Given that the income elasticity of food demand is below one, demand for food produced in modern agriculture decreases, such that skilled labor moves away from the modern agriculture sector to manufactures. The decline in the amount of labor in the former reduces the marginal productivity of the intermediate input and its use and leads to a decrease in the price of manufactures. The increase in the supply of food coupled with the higher demand for non-agricultural final good due to increase income of subsistence workers raises the relative price, p , of the latter. The net effect of these changes is an unchanged p , higher wages in the subsistence sector, and unchanged wages and consumption levels for skilled workers.

2.6. Transportation costs of intermediate inputs

Finally, we study a decrease in the cost, η , of transporting the intermediate inputs to modern agriculture. Our main result is as follows.

Lemma 4. A decrease in the cost, η , of transporting the intermediate inputs to modern agriculture reduces (increases) employment in manufactures (agriculture) if initially subsistence farmers can meet the subsistence food needs of the economy.

A reasoning similar to our previous results holds here. The decrease in η reduces the cost of producing food in the modern agriculture sector and raises the relative price, p , of manufactured goods, as well as leading to a rise of wages in the modern sector. Consumption of both agricultural and non-agricultural goods increases for skilled workers, whose welfare improves. As the terms of trade they face deteriorates with the rise in p , consumption of the final manufactured good and welfare decreases for unskilled workers. The effect of the fall of η on labor flows out of agriculture, the overall amount of intermediate inputs designed to be used in modern agriculture, as well as the total production of manufactured goods depends on the size of subsistence agriculture relative to the subsistence food needs of the population.

Figure 4 shows the effects of the decrease of η on the allocation of skilled labor. Here the decrease in costs in modern agriculture leads to an upward shift of the zz curve. The increase in k would ceteris paribus reduce θ , however, the fall in η increases excess demand for food and shifts the $ED^A = 0$ rightward. As before, if initially the subsistence sector is relatively small (θ is relatively high at θ_H) the shift in $ED^A = 0$ is small and the agriculture sector shrinks as in the existing literature. Otherwise, the shift in $ED^A = 0$ (with an initial θ at θ_L) is larger, with the result that θ rises, and, thus, the manufacturing sector loses workers and shrinks.

3. Quantitative analysis

In this section we complement the analytical results obtained above with a quantitative analysis designed to illustrate the relative magnitudes of the effects of the counterfactual experiments described previously. We calibrate our model to be consistent with the most salient features of sub-Saharan economies relevant for our purposes. This numerical analysis also allows us consider the relative welfare consequences of changes in productivity, transportation costs, and the share of skilled workers in the labor force. We also incorporate the transportation costs of the final goods.

We normalize the size of the population n , to be equal to one. We set labor income share in the modern agriculture sector $\beta = 0.4$, implying a share for intermediate inputs of $1 - \beta = 0.6$. The preference parameter for food is $\alpha = 0.2$. We normalize $A_u = 1$ and set $\bar{c}_A = 0.6$. Since our results depend critically on whether initially the subsistence agriculture sector produces enough to meet

the subsistence needs, $n\bar{c}_A + \eta_2(1 - \theta)n_e\bar{c}_A$, of the population, we need to generate two cases one in which these needs are met and another where they are not. A parsimonious way of doing this is to consider two levels of n_e . A high enough n_e would yield a level of output, $Y_u = A_u(n - n_e)$, for subsistence farming such that subsistence food needs of the population cannot be met by this means alone. For this purpose we choose $n_e = 0.4$. Conversely, if $n_e = 0.3$, Y_u is high enough to meet the subsistence needs through subsistence farming alone.

We set transportation costs $\eta = \eta_1 = \eta_2 = 0.3$. This number is close to that used in Adam, Bevan and Gollin (2013), who consider transportation costs of about 20% of total final consumption to be a reasonable estimate in their analysis of Tanzania, as well as within the range of 0.1 to 0.6, which are respectively the lower and upper limits for transportation costs considered by Gollin and Rogerson (2014) when discussing close and remote rural regions in Uganda.

We choose the technology parameters $A_n = A_m = 2.8$ to obtain a skill premium $w/w_u = (\theta w_n + (1 - \theta)w_m)/w_u$ of 2.2, and $A_n = A_m = 4.5$ to obtain a wage premium of w/w_u of 4.65.⁵

Table 1: Base parameters

Preference parameters	$\bar{c}_A = 0.6;$	$\alpha = 0.2$
Production parameters	$\beta = 0.4;$	$n_e = 0.3, 0.4$
	$A_u = 1$	$A_m = A_n = 2.8, 4.5$
Transportation costs	$\eta = \eta_1 = \eta_2 = 0.3$	

The benchmark equilibrium values are reported in Table 2 and Table 3.

Table 2: Base equilibria $A_n = A_m = 2.8$

	p	n_a	θ	c_u^A	c_m^A	c_n^A	c_u^M	c_m^M	c_n^M	Y^A/Y	w_m	w_n	w/w_u
$n_e = 0.3$	0.79	0.71	0.04	0.67	0.79	0.89	0.32	1.52	1.39	0.55	2.23	2.32	2.23
$n_e = 0.4$	0.79	0.64	0.09	0.67	0.79	0.89	0.32	1.52	1.39	0.50	2.23	2.32	2.23

⁵Gollin, Lagakos and Waugh (2014) report that in countries in the lower quartiles of income distribution, Q3(Q4) the value added per worker is on average 3.4 (5.6) higher in non-agriculture than in agriculture. Taking into consideration the sector differences in hours worked and human capital, they derive an “adjusted” non-agricultural productivity gap of 1.9 (3). Since there is a small share of labor in modern agriculture in our model, we use the non-agriculture/agriculture gap as a good approximation for the wage premium that we consider. These numbers are also consistent with Young (2013) who focuses on consumption, not income and finds a urban-rural gap of around four for a set of developing countries.

Table 3: Base equilibria $A_n = A_m = 4.5$

	p	n_a	θ	c_u^A	c_m^A	c_n^A	c_u^M	c_m^M	c_n^M	Y^A/Y	w_m	w_n	w/w_u
$n_e = 0.3$	1.03	0.71	0.05	0.67	1.09	1.36	0.25	3.12	2.86	0.40	4.62	5.18	4.65
$n_e = 0.4$	1.03	0.63	0.07	0.67	1.09	1.36	0.25	3.12	2.86	0.36	4.62	5.18	4.66

Given the calibration of parameters discussed above, the benchmark equilibria we obtain have between 63% and 71% of population employed in agriculture, depending on n_e and the skill premium (or wage inequality arising from duality), w/w_u . Farmers in the subsistence sector spend about 33 percent of their income on manufactured goods. Workers in the modern sectors have the same welfare, but due to transportation costs which lead to different prices for consumption goods in the two locations, this level of welfare is achieved with different wages and consumption patterns. Thus, to achieve the same utility workers in the modern agricultural sector are compensated at slightly higher wages. They spend about 60-73 percent on their income on manufactured goods, whereas workers in manufacturing spend about 54 to 70 percent of their income on manufactured goods. Skilled workers in the rural areas consume more of the agriculture good $c_n^A > c_m^A$, whereas urban workers consume more of the manufactured good $c_n^M < c_m^M$. The higher their wages are relative to subsistence wages, the more pronounced these differences are in terms of consumption of food. c_n^M is 9% lower than c_m^M regardless of the level of wage inequality, whereas c_n^A is 13% (25%) higher than c_m^A for $w/w_u = 2.23$ (4.65). One last point to note is that if the skill premium is higher, the relative price of manufactured goods is higher and thus welfare of workers in subsistence agriculture is lower. Inequality in our model has a direct impact on welfare through its impact on relative prices because of the non-homotheticity built into the model through subsistence level of food consumption.

3.1. Changes in Productivity and Transportation Costs

We focus on the four potential driving forces in the allocation of labor in agriculture: productivity shocks in manufacturing, subsistence and modern agriculture, and transportation costs. More specifically we consider an increase of 10% in productivity of modern sectors, i.e. $A_n = 1.1 * A_{n0}$, $A_m = 1.1 * A_{m0}$, $A_u = 1.1 * A_{u0}$, and a reduction in transportation costs of 10%, i.e. $\eta = 0.9 * \eta_0$, together with different combinations of these shocks. Table 4 presents an abbreviated version of the results of our simulations with respect to individual shocks, focusing on changes in the allocation of labor in agriculture, as well as the relative price of the manufactured goods, with their implications on welfare when $A_n = A_m = 2.8$. In the Appendix: Table 5 and Table 6 we report all the results of the calibration exercises that we perform.

Table 4: Productivity shocks

		A_n	A_m	η, η_1, η_2	η	A_u
	$\Delta\%p$	9.47	-4.03	1.43	1.34	0.00
	$\Delta\%c_u^A$	0.00	0.00	0.00	0.00	2.50
	$\Delta\%c_m^A$	3.44	2.02	1.39	0.49	0.00
	$\Delta\%c_n^A$	4.71	2.77	0.60	0.67	0.00
	$\Delta\%c_u^M$	-8.65	4.20	0.92	-1.32	25.00
	$\Delta\%c_m^M$	4.67	13.13	1.99	0.71	0.00
	$\Delta\%c_n^M$	4.67	13.13	2.79	0.71	0.00
	$\Delta\%Welf_u$	-2.72	1.24	0.27	-0.40	8.05
	$\Delta\%Welf_n$	1.43	2.75	0.61	0.22	0.00
$n_e = 0.4$	$\Delta\%\theta$	-3.36	-2.06	-2.92	-0.52	-24.00
	$\Delta\%Welf$	-0.62	2.00	0.45	-0.09	3.99
$n_e = 0.3$	$\Delta\%\theta$	3.61	2.22	-5.02	0.55	-79.95
	$\Delta\%Welf$	-1.08	1.84	0.41	-0.15	4.87

The first point to note about the simulation results concerns the changes in the relative price of manufactures, p , as these affect the terms of trade producers face, and thus, their welfare. Changes in p , consumption levels, and thus welfare for the different categories of workers are invariant to the size of subsistence agriculture. However, higher inequality, which makes the manufactured goods relatively more expensive, has the effect of reducing the quantity of manufactured goods that the workers can consume.

All the shocks we analyze are positive shocks to productivity and thus they raise the *total* amount of food and manufactured good that is consumed throughout the economy. However they lead to the reallocation of consumption among the different types of workers, and thus can potentially reduce overall welfare. Increases in A_n and reductions in the cost of transportation of intermediate goods, η , through their negative effect on the terms of trade subsistence farmers face have negative effects on these farmers' consumption of manufactures, c_u^M . However if the reduction in transportation costs is not confined to that affecting intermediate good prices, but also affects the prices of the final goods, the relative price (including iceberg costs) of manufactured goods, $p(1 + \eta_2)$, decreases and c_u^M increases slightly.

We start with a positive productivity shock in modern agriculture that increases A_n by 10 percent. This leads to an increase in p of 9.47 (9.75%) depending on the initial level of wage inequality. As we saw, whether skilled labor moves into manufactures or not depends on the ability of the subsistence agriculture sector to meet the subsistence level food needs of the economy. The magnitude of the

reallocation of resources depends on the initial wage premium. As Tables 5-6 show, the decline (rise) in the share, θ , of skilled labor in modern agriculture is more pronounced if the wage premium is smaller. The increase in A_n , even when it is combined with a decrease in labor in agriculture leads to an increase in overall consumption of food that varies between 1.18% to 2.82% depending on the initial conditions. This increase is quite small, due to the fact that consumption of food by farmers in subsistence agriculture is not affected and they represent 60-70% of consumers. Consumption of food increases by 3.44% (5.29%) for people in urban areas, and 4.71% (6.57%) for skilled workers living in rural areas. As incomes of workers in modern sectors increase, their consumption of manufactured goods increases by 4.67% (1.8%). The terms of trade deterioration for subsistence farmers, on the other hand, implies a 8.65% (8.89%) fall in their consumption of manufactures and a corresponding decline in their welfare. As subsistence farmers account for such a large share of the population, overall welfare for the economy falls⁶. The lowest overall welfare decrease is associated with the smallest size of subsistence agriculture and the lowest wage inequality. This result has important policy consequences: it highlights the fact that improvements in technology used in modern agriculture does have redistributive effects that are welfare improving for some (workers in modern agriculture and manufactures) but welfare reducing for others (subsistence farmers) in dual economies that have significant barriers to mobility among sectors.

Second, consider an increase of 10 percent in productivity, A_m , in manufactures. The increase in the supply of manufactures reduces their relative price by 4.03% (3.87%). Whether manufactures gains workers or not depends on the output of the subsistence agriculture sector relative to the subsistence level food needs of the economy. The magnitude of the reallocation of resources again depends on the initial wage premium. As Tables 5-6 show, the decline (rise) in the share, θ , of skilled labor in modern agriculture is more pronounced if the wage premium is smaller. The rise in the wages of the workers in the two modern sectors that follows the productivity shock increases their consumption of both goods, with the fall in the relative price of manufactures inducing a sharp rise in its consumption of about 13.13% (11.21%), but a small increase in food consumption varying between 2.02% to 3.86% as expected. For subsistence farmers, the improvement in their terms of trade allows higher consumption of manufactures of 4.2% (4.03%). These increases in consumption imply improvements in the welfare of all agents in the economy.

As for a 10 percent productivity improvement in subsistence farming, we observed above that this does not affect the terms of trade subsistence farmers face. Consequently, the rise in the output of these farmers increases their income, welfare, and consumption of both goods. As the output of modern agriculture suffers a decline, labor is allocated from this sector to manufactures. With wages of skilled labor employed in the modern sectors as well as the relative prices remaining unchanged, the consumption and welfare of these workers remain constant. As a result, we observe a decline in the wage premium of about 9%. As subsistence farmers comprise 60-70 percent of the labor force, the welfare gains they enjoy translate into the sharpest increase in overall welfare (ranging from 3.79 to 4.87 percent depending on the specification) that we see in response to any of the counterfactual experiments of an isolated productivity improvement that we conduct.

⁶As subsistence farmers' share of population decreases, increases in A_n might lead to increases in overall welfare.

We now turn to two different combinations of productivity improvements. First, consider the case of a productivity increase in the two modern sectors that takes the form of a 10 percent rise in both A_n and A_m . Given what we observed when these productivity improvements are taken separately, the results here are mostly what we expected. As the effect of A_n on p dominates that of A_m , terms of trade move against agriculture, with the result that subsistence farmers suffer welfare losses as before. The reallocation of skilled labor across the two modern sectors follows the same logic observed before and depends on the initial size of subsistence agriculture. However, the share of agricultural output in total output declines regardless of the direction of the reallocation of labor. Skilled labor having become more productive, it benefits from the improvements and experiences relatively large welfare gains that outweigh the losses suffered by subsistence farmers. The second combined productivity improvement we experiment with is a 10 percent rise in A_u , A_n , and A_m . This uniform productivity improvement results in price movements and reallocations of resources one typically associates with structural transformation: the relative price of the agricultural good declines and labor moves out of (modern) agriculture into manufactures. Consumption levels of both goods by all workers goes up. The increases in welfare that are registered are the highest (ranging from 4.5 to 5.6 percent depending on the parameter values) among all counterfactual experiments that we conduct.

Finally, we consider different combinations of reductions in the transportation costs. Starting with a fall of 10 percent in the cost, η , of transporting manufactured intermediates, we see in Tables 5-6 that the relative price of manufactures rises. This has the, by-now-familiar, negative effect on the consumption and welfare of subsistence farmers. Skilled workers, on the other hand, benefit from this fall in transportation costs, increasing their consumption of both goods. Again the allocation of skilled labor across the two modern sectors depends on the initial size of the subsistence agriculture sector as before. If all transportation costs (η , η_1 , and η_2) fall simultaneously, though p rises, changing relative prices inclusive of transportation costs benefit subsistence farmers as well, who increase their consumption of manufactures. Another difference with the previous case is that now the reallocation of labor is independent of the initial size of subsistence agriculture, and skilled labor moves away from modern agriculture to manufactures regardless. The effects of productivity improvements combined with decreases in transportation costs are also reported in Tables 3-4. In these cases, as the effects of productivity improvements uniformly dominate those of declines in transportation costs of the same magnitude, the results shown are similar in sign to those obtained for the productivity changes.

What our analysis suggests so far in terms of policy analysis is that policies that encourage improvements in modern sector productivity or reductions in transportation costs can have perverse effects on the allocation of labor between agriculture and manufactures and may turn out to be welfare reducing in those economies where the initial size of the subsistence agriculture sector exceeds a critical value. On the other hand, improvements in the productivity of subsistence agriculture always improve welfare and allocate labor away from agriculture. These considerations suggest the following sequence for policy. Measures that reduce the barriers to mobility between subsistence and modern sectors should be adopted first to decrease employment in subsistence farming. These

can be implemented simultaneously with policies that encourage adoption of improved technologies in subsistence farming. It is only after the share of employment in the subsistence sector has declined that policies that incentivize agents to adopt more productive technologies in the modern sectors or that lower transportation costs should be implemented.

4. Conclusion

Many poor developing countries have relatively large parts of their labor force employed in agriculture. Though some of these workers are employed by farms using modern agricultural equipment and technology, a significantly high share of agricultural employment remains in subsistence or quasi-subsistence agriculture, using traditional technologies with low productivity. Why these countries have such large fractions of their workers in agriculture remains an open question.

Our paper suggests that a number of mechanisms may help answer this question. First, barriers to labor mobility from the subsistence farming hinterland to the modern sectors (taking perhaps a number of forms including high costs of skill acquisition, as well as cultural and institutional restrictions) would help explain why employment in the subsistence sector remains high. Second, productivity differences and transportation costs play an important role but not necessarily in the direction suggested by the existing literature. We show, among others, that productivity improvements in modern agriculture may actually increase the employment share of agriculture in those countries where subsistence agriculture is initially large. Similarly reductions in certain transportation costs may increase the employment share of agriculture in these economies. Intuitively, these results that run counter to the findings in the previous literature arise from the barriers to the movement of labor out of subsistence agriculture as well as the terms of trade effects of changes in productivity and transportation costs. Such terms of trade effects also have rather significant negative consequences for subsistence farmers. Further, where such farmers comprise a sizable enough majority of the population, the welfare losses they suffer outweigh the gains that accrue to the rest of the population such that winners cannot compensate the losers. These results suggest that in economies with significant distortions, second-best welfare paradoxes may be important and policy recommendations concerning public investment in infrastructure designed to improve productivity in the modern sectors or lower transportation costs need to be reconsidered. Our model instead would provide basis for policies designed to improve productivity in subsistence agriculture and lower the cost of education of subsistence farmers should come first. Once the barriers to labor mobility have been reduced and the subsistence sector becomes smaller, policies directed at the modern sectors and transportation costs would have the desired effects.

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Appendix

Using (13) and (17) we can derive explicitly and sign the following partial derivatives:

$$\psi_1 = \frac{-k^{-\beta}n_e}{(1+\alpha)(1+\eta)} [\alpha A_n(1-\beta) [A_m + (1+\eta)k] + (1+\eta)(A_n k^{1-\beta} + \eta_2 \bar{c}_A)k^\beta] < 0 \quad (18)$$

$$\psi_2 = \frac{-\alpha k^{-1}n_e}{(1+\alpha)} \{(1-\theta)\beta w_m + (1-\beta)(\theta/\alpha\beta)w_n [1+\alpha(1-\beta)]\} < 0 \quad (19)$$

$$\psi_3 = (1+\alpha)^{-1} [Y_u - n\bar{c}_A - \eta_2 n_e(1-\theta)\bar{c}_A] \leq 0 \quad (20)$$

$$\psi_4 = \frac{\alpha p n_e(1-\theta)}{(1+\alpha)} > 0 \quad (21)$$

$$\psi_5 = \frac{-(n-n_e)}{(1+\alpha)} < 0 \quad (22)$$

$$\psi_6 = \frac{-\alpha w_m n_e(1-\theta)}{(1+\alpha)(1+\eta)} < 0 \quad (23)$$

$$\psi_8 = \frac{(1-\theta)n_e \bar{c}_A}{(1+\alpha)} > 0 \quad (24)$$

$$\psi_9 = \frac{-(A_u - \bar{c}_A)}{(1+\alpha)} < 0 \quad (25)$$

$$\psi_{10} = \frac{(A_u - \bar{c}_A)n}{(1+\alpha)n_e} > 0 \quad (26)$$

$$\psi_{11} = \frac{n + \eta_2 n_e(1-\theta)}{(1+\alpha)} > 0 \quad (27)$$

$$\varphi_2 = k^{-1} \left(\frac{(1-\beta)w_n}{w_n - \bar{c}_A} + \frac{\beta w_m}{w_m - \bar{c}_A(1+\eta_2)} \right) > 0 \quad (28)$$

$$\varphi_3 = \frac{[1 - (1+\eta_2)^{1/(1+\alpha)}]\bar{c}_A}{w_m} < 0 \quad (29)$$

$$\varphi_4 = \frac{-(1+\alpha)p}{w_m - \bar{c}_A(1+\eta_2)} < 0 \quad (30)$$

$$\varphi_6 = \frac{(1+\alpha)p}{[w_m - \bar{c}_A(1+\eta_2)](1+\eta)} > 0 \quad (31)$$

$$\varphi_7 = -1/(+\eta_1) < 0 \quad (32)$$

$$\varphi_8 = \frac{\bar{c}_A}{w_m - \bar{c}_A(1+\eta_2)} + \frac{\alpha}{1+\eta_2} > 0 \quad (33)$$

$$\varphi_{11} = \frac{(1 + \alpha)}{w_m - \bar{c}_A(1 + \eta_2)} \left[(1 + \eta_2) - \frac{w_m - \bar{c}_A(1 + \eta_2)}{w_n - \bar{c}_a} \right] > 0 \text{ if } \eta_1 = \eta_2 \quad (34)$$

Analytics of Comparative Statics

General results for the effects of changes in parameters on k and θ are given by

$$\frac{dk}{dx_j} = -\frac{\varphi_j \psi_1}{\Delta}, \quad \frac{d\theta}{dx_j} = \frac{-\varphi_2 \psi_j + \varphi_j \psi_2}{\Delta}, \quad \Delta := \varphi_2 \psi_1 < 0 \quad (35)$$

for parameter x_j ($j \in \{3, \dots, 11\}$) in equations (18) to (34).

Analytically clean results can be obtained by setting initial values of transportation costs η_1 and η_2 equal to zero. In this case using (12)-(17) we derive

$$Y_u - n\bar{c}_A = Y_n \left(\alpha \frac{\beta - \theta}{\theta} - 1 \right) \quad (36)$$

which is $\geq (<)0$ depending on the size of subsistence agriculture $Y_u \geq (<)n\bar{c}_A$.

We can then obtain:

$$w_n = w_m \quad (37)$$

$$Y_n = \left(\frac{1 - \beta}{\beta} \right)^{1 - \beta} \left(\frac{A_m}{1 + \eta} \right)^{1 - \beta} A_n \theta n_e \quad (38)$$

$$z = \theta n_e A_m \frac{1 - \beta}{\beta(1 + \eta)} \quad (39)$$

$$p = \left(\frac{1 - \beta}{1 + \eta} \right)^{1 - \beta} \left(\frac{\beta_n}{A_m} \right)^{\beta} A_n \quad (40)$$

Thus, we can derive the comparative static results concerning the effects of changes in A_m , A_n , A_u , η , and n_e on the variables of interest θ , z , and p .

Proof of Lemma 1:

$$\frac{\partial \theta}{\partial A_m} = \frac{1 - \beta}{1 + \alpha} \frac{\theta}{A_m} \left(\alpha \frac{\beta - \theta}{\theta} - 1 \right) \geq (<)0 \quad (41)$$

if $Y_u \geq (<)n\bar{c}_A$.

The effect of a productivity shock in the manufacturing sector on z and p does not depend on the size of subsistence agriculture:

$$\frac{\partial z}{\partial A_m} = \frac{1-\beta}{\beta} \frac{\theta n_e}{1+\eta} \left(\frac{1-\beta}{1+\alpha} \left(\alpha \frac{\beta-\theta}{\theta} - 1 \right) + 1 \right) > 0 \quad (42)$$

$$\frac{\partial p}{\partial A_m} = -\beta \left(\frac{1-\beta}{1+\eta} \right)^{1-\beta} \beta^\beta A_m^{-\beta-1} A_n < 0 \quad (43)$$

Proof of Lemma 2:

$$\frac{\partial \theta}{\partial A_n} = \frac{1}{1+\alpha} \frac{\theta}{A_n} \left(\alpha \frac{\beta-\theta}{\theta} - 1 \right) \geq (<) 0 \quad (44)$$

if $Y_u \geq (<) n \overline{c_A}$.

$$\frac{\partial z}{\partial A_n} = \frac{\partial \theta}{\partial A_n} A_m n_e \frac{1-\beta}{\beta(1+\eta)} \geq (<) 0 \quad (45)$$

$$\frac{\partial p}{\partial A_n} = \left(\frac{1-\beta}{1+\eta} \right)^{1-\beta} \left(\frac{\beta}{A_m} \right)^\beta > 0 \quad (46)$$

if $Y_u \geq (<) n \overline{c_A}$.

Proof of Lemma 3:

$$\frac{\partial \theta}{\partial A_u} = -\frac{(n-n_e)}{1+\alpha} \frac{\theta}{Y_n} < 0 \quad (47)$$

$$\frac{\partial p}{\partial A_u} = 0 \quad (48)$$

Proof of Lemma 4:

$$\frac{\partial \theta}{\partial \eta} = -\frac{1-\beta}{1+\alpha} \frac{\theta}{1+\eta} \left(\alpha \frac{\beta-\theta}{\theta} - 1 \right) \leq (>) 0 \quad (49)$$

if $Y_u \geq (<) n \overline{c_A}$.

$$\frac{\partial z}{\partial \eta} = -\frac{z}{1+\eta} \left(\frac{1-\beta}{1+\alpha} \left(\alpha \frac{\beta-\theta}{\theta} - 1 \right) + 1 \right) < 0 \quad (50)$$

$$\frac{\partial z(1+\eta)}{\partial \eta} = \frac{\partial \theta}{\partial \eta} n_e A_m \frac{1-\beta}{\beta} \leq (>) 0 \quad (51)$$

if $Y_u \geq (<) n \overline{c_A}$.

$$\frac{\partial p}{\partial \eta} = -(1-\beta)(1-\beta)^{1-\beta} \left(\frac{\beta(1+\eta)}{A_m} \right)^\beta A_n < 0 \quad (52)$$

Effects of change in the size of skilled labor force:

$$\frac{\partial \theta}{\partial n_e} = \frac{\theta}{1+\alpha} \left(\frac{A_u}{Y_n} + \frac{\alpha(\beta-\theta)}{\theta n_e} - \frac{1}{n_e} \right) > 0 \quad (53)$$

$$\frac{\partial z}{\partial n_e} = \frac{1-\beta}{\beta} \frac{A_m}{1+\eta} \frac{\partial \theta n_e}{\partial n_e} > 0 \quad (54)$$

$$\frac{\partial n_a}{\partial n_e} = \frac{\theta n_e}{1+\alpha} \left(\frac{A_u}{Y_n} + \frac{\alpha\beta}{\theta n_e} \right) + \theta - 1 \leq 0 \quad (55)$$

if $w_u \leq w_n$.

$$\frac{\partial p}{\partial n_e} = 0 \quad (56)$$

Tables

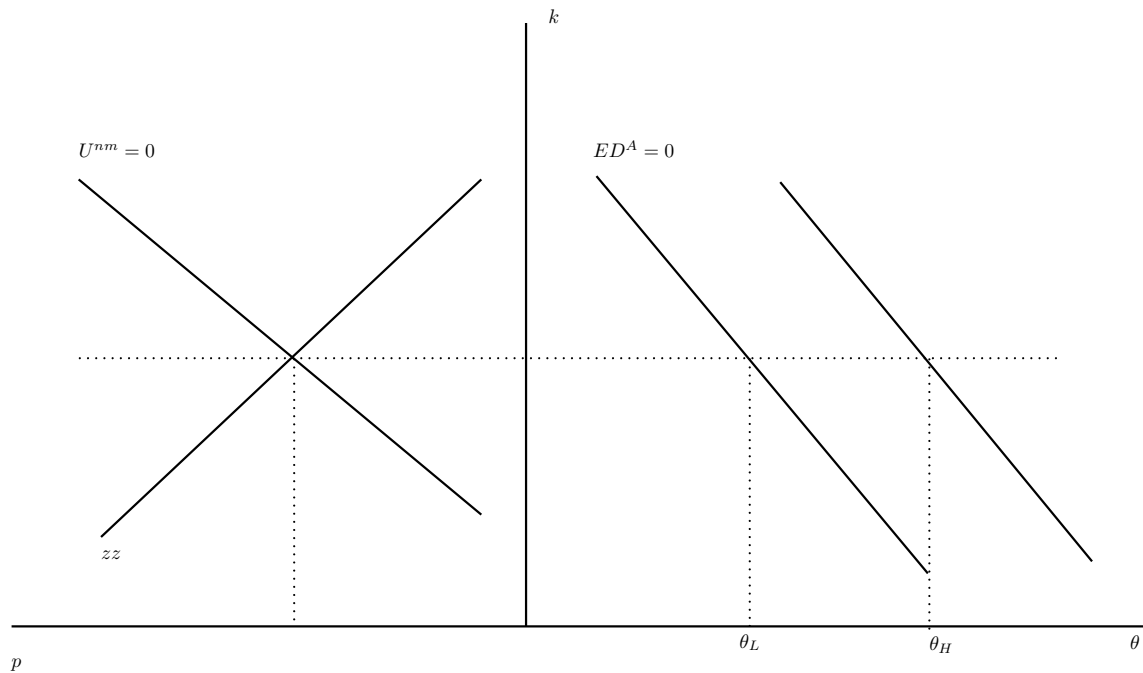
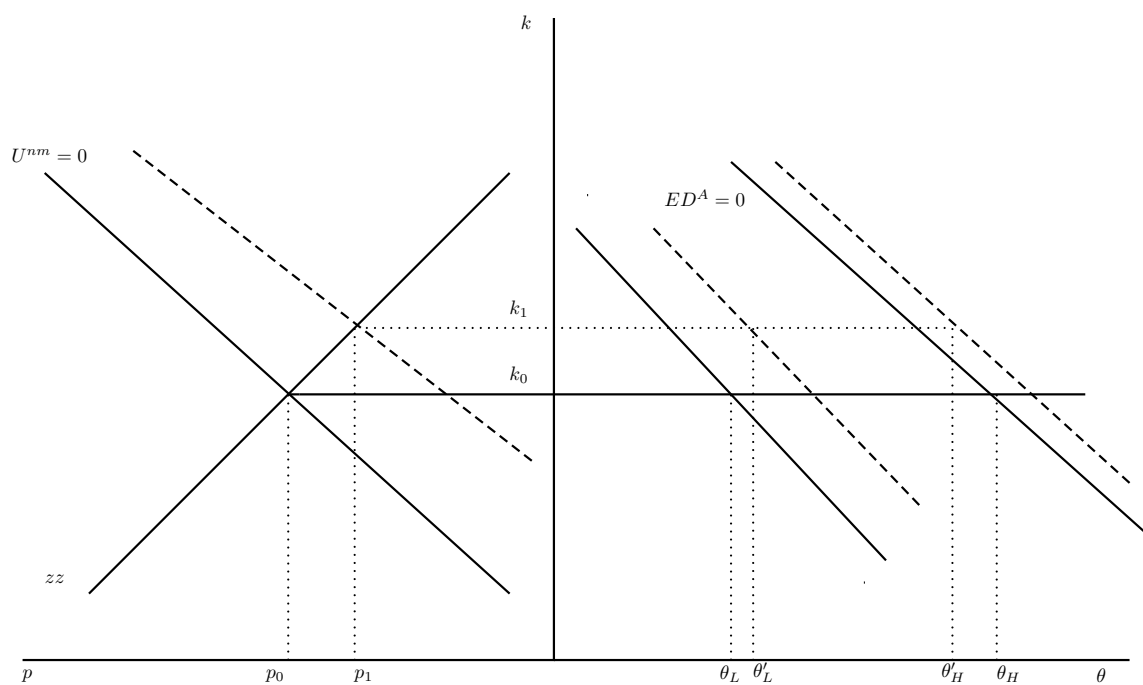
Table 5: Comparative static results when differences in productivity between modern and subsistence sector are small ($A_n = A_m = 2.8$)

		A_n	A_m	η, η_1, η_2	η	A_u	A_n, η_i	A_m, η_i	A_n, A_m	A_u, η_i	A_n, A_m, η_i	A_n, A_m, A_u
	$\Delta\%p$	9.47	-4.03	1.43	1.34	0.00	11.10	-2.62	5.09	1.43	6.69	5.09
	$\Delta\%c_u^A$	0.00	0.00	0.00	0.00	2.50	0.00	0.00	0.00	2.50	0.00	2.50
	$\Delta\%c_m^A$	3.44	2.02	1.39	0.49	0.00	4.99	3.51	5.67	1.39	7.31	5.67
	$\Delta\%c_n^A$	4.71	2.77	0.60	0.67	0.00	5.34	3.39	7.77	0.60	8.41	7.77
	$\Delta\%c_u^M$	-8.65	4.20	0.92	-1.32	25.00	-7.86	5.11	-4.84	26.14	-4.05	18.95
	$\Delta\%c_m^M$	4.67	13.13	1.99	0.71	0.00	6.51	15.22	18.01	1.99	19.94	18.01
	$\Delta\%c_n^M$	4.67	13.13	2.79	0.71	0.00	7.34	16.12	18.01	2.79	20.88	18.01
	$\Delta\%Welf_u$	-2.72	1.24	0.27	-0.40	8.05	-2.46	1.50	-1.49	8.32	-1.24	6.56
	$\Delta\%Welf_n$	1.43	2.75	0.61	0.22	0.00	2.00	3.34	4.11	0.61	4.64	4.11
$n_e = 0.4$	$\Delta\%n_a$	-0.19	-0.11	-0.16	-0.03	-1.33	-0.33	-0.26	-0.29	-1.47	-0.42	-1.42
	$\Delta\%\theta$	-3.36	-2.06	-2.92	-0.52	-24.00	-5.90	-4.74	-5.22	-26.65	-7.54	-25.69
	$\Delta\%z$	-2.17	8.54	-0.68	2.02	-24.00	-2.69	7.91	6.25	-24.97	5.80	-16.69
	$\Delta\%k$	1.23	10.82	2.30	2.55	0.00	3.41	13.28	12.10	2.30	14.44	12.10
	$\Delta\%Y^A/Y$	-3.81	-2.29	-1.06	-0.56	-0.46	-4.86	-3.35	-6.06	-1.50	-7.10	-6.38
	$\Delta\%c^M$	1.45	10.97	1.79	0.22	6.22	3.10	12.85	12.48	8.06	14.21	18.41
	$\Delta\%c^A$	1.56	0.92	0.57	0.22	1.27	2.19	1.52	2.57	1.85	3.24	3.83
	$\Delta\%w/w_u$	9.57	5.63	1.42	1.35	-9.17	11.17	7.16	15.76	-7.88	17.48	5.12
	$\Delta\%Welf$	-0.62	2.00	0.45	-0.09	3.99	-0.21	2.43	1.33	4.43	1.73	5.32
$n_e = 0.3$	$\Delta\%n_a$	0.06	0.04	-0.09	0.01	-1.38	-0.01	-0.04	0.10	-1.45	0.03	-1.08
	$\Delta\%\theta$	3.61	2.22	-5.02	0.55	-79.95	-0.76	-2.35	5.61	-84.12	1.60	-62.62
	$\Delta\%z$	4.88	13.28	-2.83	3.12	-79.95	2.63	10.61	18.39	-83.76	16.27	-58.09
	$\Delta\%k$	1.23	10.82	2.30	2.55	0.00	3.41	13.28	12.10	2.30	14.44	12.10
	$\Delta\%Y^A/Y$	-3.43	-2.06	-0.90	-0.50	-0.77	-4.33	-2.95	-5.48	-1.65	-6.38	-6.06
	$\Delta\%c^M$	0.23	10.15	1.66	0.04	8.50	1.75	11.89	10.39	10.22	11.98	18.49
	$\Delta\%c^A$	1.18	0.70	0.45	0.17	1.52	1.68	1.17	1.95	1.97	2.48	3.46
	$\Delta\%w/w_u$	9.53	5.61	1.42	1.34	-9.22	11.14	7.14	15.71	-7.92	17.44	5.00
	$\Delta\%Welf$	-1.08	1.84	0.41	-0.15	4.87	-0.7	2.22	0.72	5.27	1.08	5.59

Table 6: Comparative static results when differences in productivity between modern and subsistence sector are big ($A_n = A_m = 4.5$)

		A_n	A_m	η, η_1, η_2	η	A_u	A_n, η_i	A_m, η_i	A_n, A_m	A_u, η_i	A_n, A_m, η_i	A_n, A_m, A_u
	$\Delta\%p$	9.75	-3.87	1.76	1.38	0.00	11.71	-2.16	5.51	1.76	7.42	5.51
	$\Delta\%c_u^A$	0.00	0.00	0.00	0.00	2.50	0.00	0.00	0.00	2.50	0.00	2.50
	$\Delta\%c_m^A$	5.29	3.11	2.26	0.75	0.00	7.78	5.51	8.71	2.26	11.36	8.71
	$\Delta\%c_n^A$	6.57	3.86	0.56	0.93	0.00	7.16	4.44	10.82	0.56	11.43	10.82
	$\Delta\%c_u^M$	-8.89	4.03	0.59	-1.36	25.00	-8.37	4.63	-5.23	25.74	-4.70	18.47
	$\Delta\%c_m^M$	1.80	11.21	0.81	0.28	0.00	2.55	12.06	13.09	0.81	13.87	13.09
	$\Delta\%c_n^M$	1.80	11.21	1.60	0.28	0.00	3.35	12.94	13.09	1.60	14.76	13.09
	$\Delta\%Welf_u$	-3.03	1.29	0.19	-0.45	8.72	-2.85	1.47	-1.75	8.91	-1.57	6.97
	$\Delta\%Welf_m$	0.67	2.00	0.30	0.10	0.00	0.95	2.28	2.64	0.30	2.92	2.64
$n_e = 0.4$	$\Delta\%n_a$	-0.08	-0.05	-0.04	-0.01	-0.61	-0.12	-0.09	-0.13	-0.65	-0.16	-0.65
	$\Delta\%\theta$	-1.86	-1.14	-0.96	-0.29	-13.58	-2.62	-1.98	-2.90	-14.43	-3.54	-14.52
	$\Delta\%z$	-1.31	9.12	0.51	2.16	-13.58	-0.68	9.76	7.76	-13.17	8.52	-5.14
	$\Delta\%k$	0.57	10.38	1.48	2.45	0.00	1.99	11.97	10.97	1.48	12.50	10.97
	$\Delta\%Y^A/Y$	-4.02	-2.44	-1.18	-0.60	0.01	-5.15	-3.59	-6.34	-1.16	-7.44	-6.27
	$\Delta\%c^M$	0.66	10.44	0.84	0.10	2.77	1.43	11.32	11.13	3.62	11.94	13.76
	$\Delta\%c^A$	2.82	1.66	1.10	0.40	1.06	4.04	2.83	4.65	2.17	5.95	5.71
	$\Delta\%w/w_u$	9.78	5.76	1.69	1.38	-9.19	11.66	7.55	16.12	-7.66	18.11	5.45
	$\Delta\%Welf$	-0.94	1.69	0.25	-0.14	3.79	-0.70	1.93	0.73	4.04	0.97	4.52
$n_e = 0.3$	$\Delta\%n_a$	0.03	0.02	-0.02	0.00	-0.63	0.01	0.00	0.04	-0.64	0.03	-0.49
	$\Delta\%\theta$	1.34	0.82	-1.01	0.21	-30.26	0.56	-0.06	2.08	-31.05	1.44	-23.82
	$\Delta\%z$	1.91	11.28	0.46	2.66	-30.26	2.56	11.91	13.28	-30.02	14.12	-15.47
	$\Delta\%k$	0.57	10.38	1.48	2.45	0.00	1.99	11.97	10.97	1.48	12.50	10.97
	$\Delta\%Y^A/Y$	-3.98	-2.41	-1.09	-0.59	-0.14	-5.03	-3.47	-6.30	-1.22	-7.33	-6.34
	$\Delta\%c^M$	0.11	10.07	0.81	0.02	4.06	0.85	10.92	10.19	4.88	10.96	14.04
	$\Delta\%c^A$	2.24	1.32	0.89	0.32	1.31	3.22	2.26	3.69	2.21	4.73	4.99
	$\Delta\%w_m/w_u$	9.80	5.76	1.71	1.38	-9.25	11.70	7.59	16.14	-7.69	18.16	5.41
	$\Delta\%Welf$	-1.35	1.61	0.24	-0.20	4.75	-1.12	1.84	0.25	4.99	0.47	5.00

Figures

Figure 1: Increase in the number of skilled workers n_e Figure 2: Increase in manufacturing productivity A_m

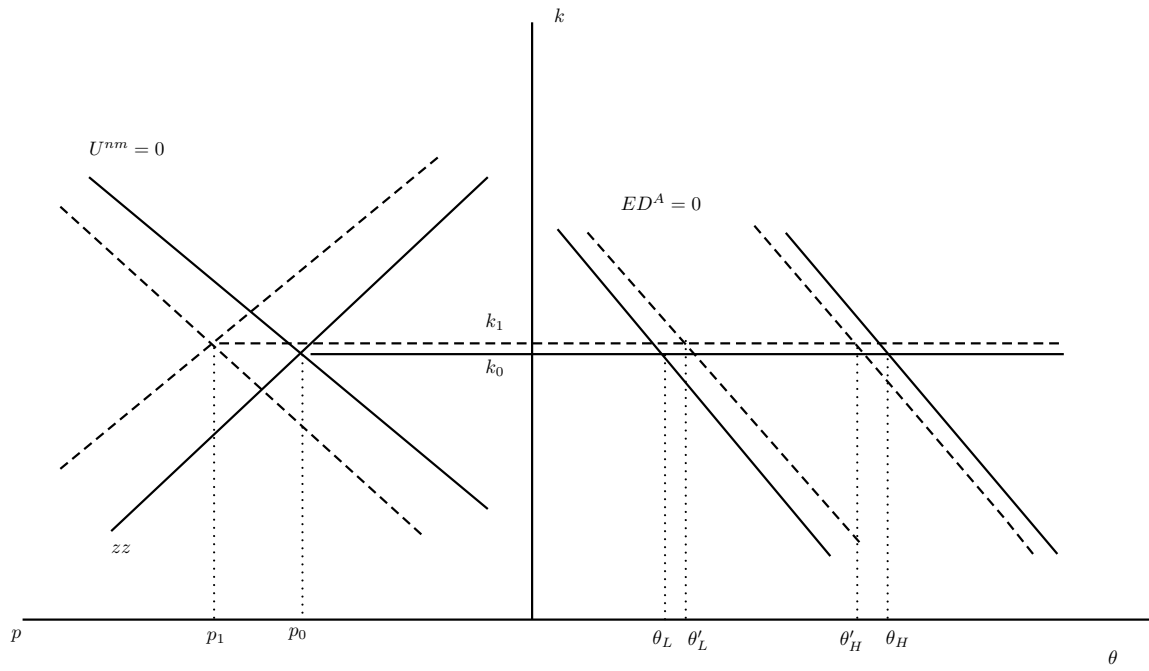


Figure 3: Increase in the productivity of the modern agriculture sector A_n

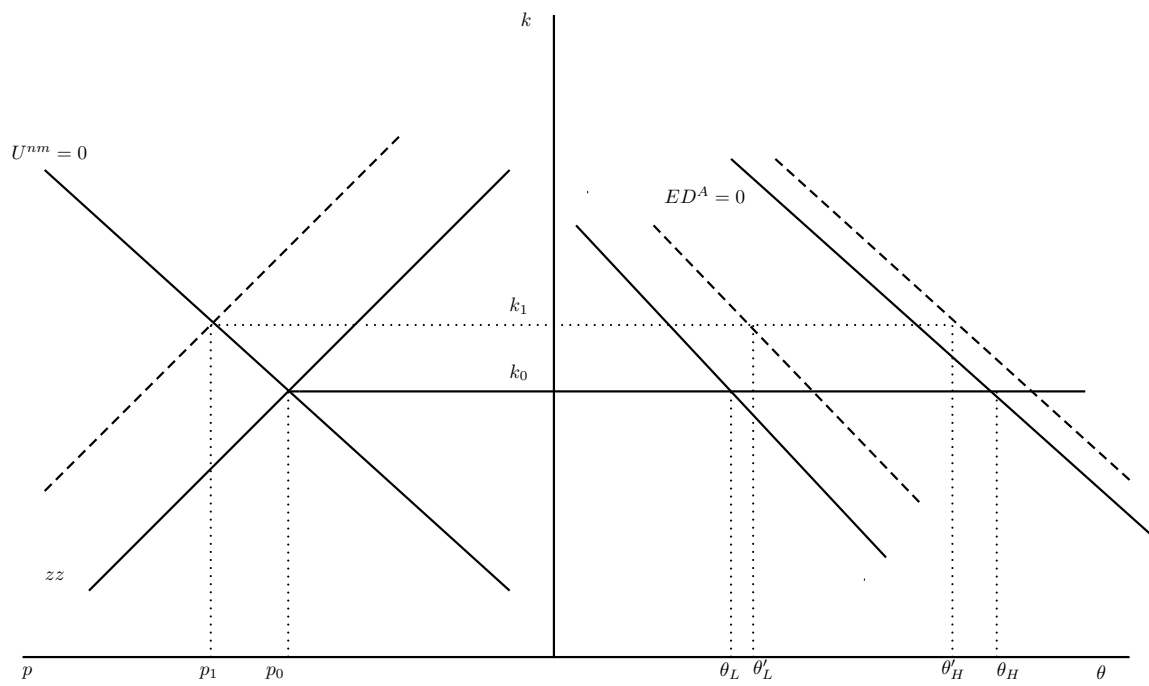


Figure 4: Decrease in the cost of transportation η