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Learning, Monetary Policy and Asset Prices

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Abstract

We explore the stability properties of interest rate rules granting an explicit response to stock prices in a New-Keynesian DSGE model populated by Blanchard-Yaari non-Ricardian households. The constant turnover between long-time stock holders and asset-poor newcomers generates a financial wealth channel where the wedge between current and expected future aggregate consumption is affected by the market value of financial wealth, making stock prices non-redundant for the business cycle. We find that if the financial wealth channel is sufficiently strong, responding to stock prices enlarges the policy space for which the rational expectations equilibrium is both determinate and learnable (in the E-stability sense of Evans and Honkapohja, 2001). In particular, the Taylor principle ceases to be necessary and also mildly passive policy responses to inflation lead to determinacy and E-stability. Our results appear to be more prominent in economies characterized by a lower elasticity of substitution across differentiated products and/or more rigid labor markets.

JEL Classification Numbers: E4, E5

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I. Introduction

Since the burst of the dot.com bubble in early 2000, the interaction between financial markets and aggregate fluctuations has become one of the main topics of research in macroeconomics. The strong shift of private savings towards the stock market and real estate witnessed over the last fifteen years, both in the U.S. and other developed economies, seems to suggest that the boom in consumption which occurred first in the mid-to-late nineties and then in the mid 2000s was financed by heavily relying on unprecedented stock market and housing market performances.[1] Since then—and even more after the 2007-8 global financial crisis—policy-makers and academic economists have lively debated whether central banks should move the policy rate in response to asset/stock price fluctuations.[2]

From the point of view of macroeconomic stability, and particularly of equilibrium determinacy, the *conventional wisdom* appears to be that an explicit response to stock prices is a bad idea. Using the benchmark New Keynesian model, seminal works by Bullard and Schaling (2002) and Carlstrom and Fuerst (2007) show that including a positive response to stock prices in interest rate rules restricts the policy space where the rational expectations equilibrium (REE) is determinate and, therefore, can be a source of sunspot-driven self-fulfilling expectations fluctuations.[3] As a result, rules that respond to stock prices also require a *sufficiently* active response to inflation—a reinforced Taylor principle—to ensure determinacy. To some extent, this result is not surprising. The benchmark New Keynesian model, which is typically based on an infinitely-lived representative agent, does not foresee any structural linkage between financial markets and real activity, making stock prices completely redundant for consumption decisions and, consequently, the business cycle. Hence, there is no specific rationale for why the central bank should move the interest rate in response to stock price changes.

In this paper, we revisit the important issue of whether interest rate rules should respond to stock prices, in a New Keynesian DSGE model where, because of the presence of non-Ricardian households, stock prices are non-redundant for business cycle fluctuations. Our structural model, which builds on Nisticò (2012), is a discrete-time stochastic version of the Blanchard (1985) and Yaari (1965) perpetual-youth model, adapted to a New-Keynesian framework where monetary policy is non-neutral. Due to the Blanchard-Yaari structure, the turnover in financial markets between long-time traders (holding assets) and newcomers (entering the market with no assets) implies a non-degenerate distribution in both financial wealth and consumption across agents. By breaking the identity between current and future traders in the economy, this heterogeneity makes financial markets intertemporally incomplete, and hence weakens the typical consumption smoothing motive. In particular, the wedge between the current and the expected level of aggregate consumption is driven not only by the ex ante real interest rate, as in the standard New Keynesian model, but also by the market value of financial wealth, since the latter is responsible for the difference between the consumption level of long-time traders and that

[1] Case et al. (2005) and Carrol et al. (2011) provide statistical evidence for housing and financial wealth effects on consumption.

[2] The debate between Bernanke and Gertler (1999, 2001) and Cecchetti et al. (2000, 2002) is the most prominent. Dupor (2005) discusses optimal monetary policy in a New-Keynesian model subject to exuberance shocks. For a recent discussion on asset price bubbles, including the policy debate, see Evanoff et al. (2012).

[3] For a general discussion on determinacy versus indeterminacy in the becnhmark New Keynesian model see, for instance, Bullard and Mitra (2002).

of newcomers. Through this mechanism, which we refer to as the *financial wealth channel*, stock price fluctuations feedback then into real activity via their wealth effects on consumption.[4]

One appealing feature of our model is its tractability. As the turnover rate goes to zero, only infinitely-lived traders operate in the market, and our model collapses to the benchmark New Keynesian model studied by Bullard and Schaling and by Carlstrom and Fuerst. In the extreme case of a turnover rate equal to one, agents are instead one-period-lived. They do not save but simply consume out of their labor income, behaving as the "non-Ricardian" rule-of-thumb consumers studied by Gali et al. (2004). By varying the turnover rate in the market, we are therefore able to assess the role of the degree of "non-Ricardianness" in the economy for the appropriate response of monetary policy to stock prices.

We assess whether the existence of the financial wealth channel (henceforth, FWC) justifies augmenting interest rate rules with an explicit response to stock prices.[5] Our evaluation criteria are equilibrium determinacy and learnability; that is, monetary policy should induce a determinate and learnable REE. Under determinacy, a REE is exclusively driven by fundamental disturbances, ruling out the effects of extrinsic uncertainty such as noise, market sentiment, and all other factors often referred to as "sunspots." Learnability of an equilibrium—in the E-stability sense of Evans and Honkapohja (2001)—requires that such equilibrium can be attained by agents who do not possess rational expectations at the outset, but instead make forecasts using simple adaptive rules, like least squares learning.[6] Both determinacy and learnability are therefore clearly desirable from a policy-making perspective.[7]

Our main result is that the presence of the FWC, which makes stock prices non-redundant, can overturn the conventional wisdom. Under suitable conditions, a sufficiently positive turnover in financial markets — the FWC is sufficiently strong — implies that adding a positive response to stock prices in the interest rate rule *enlarges* the policy space for which the equilibrium is both determinate and learnable, as long as such response is not excessive. The Taylor principle is no longer necessary as sunspot equilibria can also be ruled out by rules granting a mildly passive response to inflation. This result appears to be more prominent in economies featuring the following: (i) lower elasticities of substitution across differentiated products of a size comparable to the post-1990 U.S. estimates by Broda and Weinstein (2006) and/or (ii) more sluggish adjustments of real wages to market conditions of a size comparable to the parametrization used by Blanchard and Gali (2007) and Uhlig (2007). Our results are qualitatively robust across specifications of policy rules differing in the timing of their arguments (forward-looking versus contemporaneous) and in the presence or not of an explicit response to

[4] To some extent, stock market wealth imposes real effects on consumption the same way government bonds do in Blanchard (1985). In that set-up, Ricardian equivalence fails, and an increase in the current real value of a bond portfolio affects consumption.

[5] In this paper, we focus on the response to asset prices relative to fundamentals (risk premiums).

[6] As argued by McCallum (2009a), "...for any RE solution to be considered plausible, and thereby relevant for policy analysis, it should be learnable," in the sense that economic agents should be able to learn the quantitative relevance of all fundamental disturbances for the equilibrium dynamics. McCallum (2009a,b) and Cochrane (2009) provide an interesting discussion about equilibrium determinacy and learnability as equilibrium selection criteria.

[7] In this paper, we refrain from studying issues related to the design and implementation of *optimal* monetary policy, including the optimal response to stock prices.

output. Our analysis also highlights the possibility of learnable sunspot equilibria, when the response to stock prices is so large that makes the rule induce indeterminacy.

This paper relates to previous works in the macroeconomic literature. First of all, it extends the analysis of Bullard and Schaling (2002) and Carlstrom and Fuerst (2007) to the case of forward-looking interest rate rules, and to the learnability of both fundamental-driven and sunspot-driven equilibria. Both of these works do in fact restrict the analysis to equilibrium determinacy under contemporaneous rules. As extensively discussed in Evans and Honkapohja (2001), the learnability of a REE is crucial for the design of policies since, in the real world, economic agents do not possess rational expectations at the outset. We make extensive use of some of their results for the learnability of fundamentals solution, as well as more recent ones by Evans and McGough (2005a,b) on the learnability of sunspot solutions.[8]

Our work also contributes to the extensive literature trying to identify alternative transmission channels of financial shocks to real activity. Among the most prominent channels discussed in this literature are the *borrower's balance sheet channel* — which builds on the agency costs/financial accelerator framework of Bernanke et al. (1999) — and the *bank liquidity/intermediation channel,* as discussed in Brunnermeier (2009) and Boz and Mendoza (2014) — where balance sheets' maturity mismatches and limited enforcement in banking propagate adverse asset price shocks in a Fisherian deflationary spiral fashion.[9] Both channels emphasize the supply-side effect of asset price fluctuations and its indirect impact on households' consumption-saving decisions. The transmission channel we analyze in this paper, the FWC, is instead direct and entirely demand-side. In our view, it well describes the stock-market-driven over-consumption typically observed during episodes of fast growth, like the mid-to-late 1990s. A FWC could indeed be built into a New Keynesian model alongside a financial accelerator à la Bernanke et al. (1999) or other forms of financial/credit frictions, thereby providing a framework with both demand-side and supply-side transmission channels of financial shocks. Airaudo et al. (2013) make some progress in this direction showing that a credit channel, on top of the FWC, makes responding to stock prices even more beneficial for equilibrium determinacy.[10]

Although the model structure is completely different, our work is also related to Farmer (2010, 2012a,b). In these papers, Farmer proposes a new theory for the existence of a structural linkage between the stock market and real activity (in particular, unemployment) based on the old-keynesian view. He solves the equilibrium indeterminacy problem coming from labor market inefficiencies by postulating a stock price belief-function which allows agents to select the equilibrium. His analysis shows that self-fulfilling waves of optimism and pessimism can generate booms and recessions in

[8] In this regard, our paper investigates the implications of relaxing the assumption of rational expectations for the design of monetary policy. It abstracts from analyzing other policy issues associated with relaxing the assumptions of complete information among agents and homogeneous beliefs and preferences.

[9] The Basel Committee on Banking Supervision (2011) gives an extensive survey of the theoretical and empirical literature on these channels.

[10] Singh et al. (2012) pursue the equilibrium determinacy analysis for rules responding to stock prices (in their case, Tobin's q) in the benchmark financial accelerator model. Airaudo et al. (2013) also show that, under indeterminacy (which occurs without an explicit response to stock prices in the Taylor rule), their model can generate relative volatilities of key financial variables (e.g. the price-dividend ratio) which are very close to what is observed in post-1990 U.S. data. Their result hints to the possibility that the financial instability witnessed since the mid-to-late 1990s was the result of waves of (rational) exuberance and pessimism in financial markets.

economic activity.

The rest of the paper is organized as follows. Section II presents the model and discusses extensively its key elements of departure from the benchmark New-Keynesian framework. Section III presents the log-linearized equilibrium conditions describing the aggregate dynamics of the economy. Section IV presents our main results on equilibrium determinacy and the learnability of both fundamental-driven and sunspot-driven REE. Section V discusses the robustness of our results under several extensions. Finally, section VI concludes and provides avenues for future research.

II. The Model

A. Households

The demand-side of the economy is a discrete-time stochastic version of the perpetual youth model introduced by Blanchard (1985) and Yaari (1965), along the lines of Nisticò (2012).[11] The economy is populated by an indefinite number of cohorts of Non-Ricardian agents who survive between any two subsequent periods with constant probability $1-\gamma$. We interpret the concepts of "living" and "dying" in the economic sense of being or not being operative in markets, therefore affecting economic activity through the individual decision-making process. This interpretation is similar to that in Farmer (2002), which derives interesting asset pricing implications from introducing Blanchard-Yaari consumers in an otherwise standard real-business-cycle model. As he argues, if γ were strictly interpreted as the probability of dying, the model's quantitative implications would be essentially identical to those in an infinitely-lived representative agent environment.

Assuming that entry and exit rates are equal, and that total population has size one, in each period a fraction γ of the population leaves the market and a new cohort of exact equal size enters. In this sense, our economy is characterized by a constant turn-over, at rate γ, between newcomers (holding no assets) and long-time traders (holding assets) in financial markets.

Lifetime utility for the representative agent of the cohort which entered the market at time $j \leq t$ (from now on, the $j-$th cohort representative agent) is

$$E_t \sum_{k=0}^{\infty} \beta^k (1-\gamma)^k \Big[\ln C_{j,t+k} + \delta \ln(1 - N_{j,t+k})\Big], \tag{1}$$

where $\beta \in (0,1)\,, \gamma \in [0,1)$ and $\delta > 0$.[12] Future utility is discounted because of impatience (by the intertemporal discount factor β) and uncertain lifetime in the market (by the probability of remaining active in the market between any two subsequent periods, $1-\gamma$).

[11] For other recent discrete-time versions of the perpetual youth model see, among others, Di Giorgio and Nisticó (2007, 2013), Farmer et al. (2011), Leith and Wren-Lewis (2000), and Smets and Wouters (2002).

[12] The assumption that the utility function is log-separable between consumption $C_{j,t}$ and leisure $(1-N_{j,t})$ is necessary in order to retrieve time-invariant parameters characterizing the equilibrium conditions. See Smets and Wouters (2002) for a non-stochastic framework with CRRA utility.

Consumers have access to two financial assets: state-contingent bonds and risky equity. At the end of period t, the $j-$th cohort representative agent holds a portfolio of contingent claims with one-period ahead stochastic nominal payoff $B_{j,t+1}$ and a continuum of equity shares issued by monopolistically competitive firms operating in the productive sector, i.e., $S_{j,t+1}(i)$ for $i \in [0,1]$. The real price of a share issued by the $i-$th firm is $Q_t(i)$. For the long-time traders (indexed by $j < t$), nominal financial wealth carried over from the previous period is given by:

$$A_{j,t} \equiv \left[B_{j,t} + P_t \int_0^1 \Big(Q_t(i) + D_t(i) \Big) S_{j,t}(i)\, di \right] \text{ for } j < t. \tag{2}$$

This includes the nominal payoffs on the contingent claims, $B_{j,t}$, and the price plus dividend on each share of the equity portfolio, $Q_t(i) + D_t(i)$ for $i \in [0,1]$. As in Blanchard (1985), financial wealth $A_{j,t}$ also pays off the gross return on an insurance contract that redistributes, among the agents that have not been replaced (and in proportion to one's current wealth), the financial wealth of the ones who have left the market. Total personal financial wealth which the $j-$th cohort enters period t with is given by $A_{j,t}$ accrued by a factor of $\frac{1}{1-\gamma}$:[13]

$$\Omega_{j,t} \equiv \frac{A_{j,t}}{1-\gamma} \text{ for } j < t. \tag{3}$$

Newcomers (indexed by $j = t$) enter instead with no financial wealth, that is:

$$A_{j,t} = \Omega_{j,t} = 0 \text{ for } j = t. \tag{4}$$

The difference between (2) and (4) is the key element of heterogeneity in our economy. As in Blanchard (1985), the constant turnover in markets—combined with the absence of bequests and of any other form of wealth-equalizing fiscal transfer—implies a non-degenerate distribution of financial wealth across cohorts. As discussed below, this generates a structural linkage between the stock market and real activity through the demand side.

At time t, the $j-$th cohort representative agent seeks to maximize (1) subject to a sequence of budget constraints of the following form:

$$P_t C_{j,t} + E_t\{\mathcal{F}_{t,t+1} B_{j,t+1}\} + P_t \int_0^1 Q_t(i) S_{j,t+1}(i)\, di \leq W_t N_{j,t} - P_t T_{j,t} + \Omega_{j,t}, \tag{5}$$

where $E_t\{\mathcal{F}_{t,t+1} B_{j,t+1}\}$ is the portfolio of state-contingent claims paying $B_{j,t+1}$ the next period and $\mathcal{F}_{t,t+1}$ is the common stochastic discount factor. The household gets labor income $W_t N_{j,t}$ from working in the productive sector and pays lump-sum taxes $P_t T_{j,t}$ to the government. The optimal plan is subject to a standard non-Ponzi game condition (NPG): $\lim_{k\to\infty} E_t \left[\mathcal{F}_{t,t+k} (1-\gamma)^k \Omega_{j,t+k} \right] = 0$.

From the first-order conditions of the household's problem, we obtain the following relationships:

$$\frac{\delta C_{j,t}}{1 - N_{j,t}} = \frac{W_t}{P_t}, \tag{6}$$

[13] The gross return $\frac{1}{1-\gamma}$ per unit on the insurance contract is the result of perfect competition and free entry into the insurance market. The insurance contract is identical to that in Blanchard (1985).

$$\mathcal{F}_{t,t+1} = \beta \frac{C_{j,t} P_t}{C_{j,t+1} P_{t+1}}, \tag{7}$$

and

$$P_t Q_t(i) = E_t \Big\{ \mathcal{F}_{t,t+1} P_{t+1} \Big[Q_{t+1}(i) + D_{t+1}(i) \Big] \Big\}, \text{ for each } i \in [0,1], \tag{8}$$

which have straightforward interpretations. Equation (6) equates the marginal rate of substitution between consumption and leisure to the real wage; equation (7) defines the stochastic discount factor $\mathcal{F}_{t,t+1}$; and equation (8) is the pricing equation for the equity share issued by the $i-$th firm.

Using the definition of individual wealth in (2)-(3), the individual budget constraint (5) can be written as a stochastic difference equation in $\Omega_{j,t}$:

$$P_t C_{j,t} + (1-\gamma)\, E_t\{\mathcal{F}_{t,t+1} \Omega_{j,t+1}\} \leq W_t N_{j,t} - P_t T_{j,t} + \Omega_{j,t}. \tag{9}$$

Moreover, by forward iteration on $\Omega_{j,t+1}$ and the NPG condition, equation (9) gives us:

$$P_t C_{j,t} = [1 - \beta(1-\gamma)]\, (\Omega_{j,t} + H_{j,t}), \tag{10}$$

which defines individual consumption as a linear function of total financial and non-financial wealth, $H_{j,t}$. The term $[1 - \beta(1-\gamma)]$ represents the constant marginal propensity to consume out of total wealth, while non-financial wealth is the appropriately discounted expected stream of future disposable labor income, i.e., $H_{j,t} \equiv E_t \sum_{k=0}^{\infty} \mathcal{F}_{t,t+k} (1-\gamma)^k (W_{t+k} N_{j,t+k} - P_{t+k} T_{j,t+k})$.

B. Production

The supply-side of the economy has two sectors: a retail sector and a wholesale sector. The retail sector is perfectly competitive and produces the final consumption good Y_t out of a continuum of intermediate goods through the following technology: $Y_t = \left[\int_0^1 Y_t(i)^{(\epsilon-1)/\epsilon} di\right]^{\epsilon/(\epsilon-1)}$, where $\epsilon > 1$ is the elasticity of substitution between any two varieties of intermediate goods. Prices in the retail sector are perfectly flexible. The optimal demand for the intermediate good $Y_t(i)$ is given by $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t$, while $P_t = \left[\int_0^1 P_t(i)^{1-\epsilon}\, di\right]^{1/(1-\epsilon)}$ is the price of the final consumption good.

The wholesale sector is made of a continuum of firms indexed by i, for $i \in [0,1]$. They act under monopolistic competition and are subject to nominal rigidities in price setting. The $i-$th firm hires labor from a competitive labor market to produce the $i-$th variety of a continuum of differentiated intermediate goods which are sold to retailers. Production in turn follows a simple linear technology: $Y_t(i) = Z_t N_t(i)$, where the aggregate total factor productivity Z_t is stochastic. We assume that $z_t \equiv \ln Z_t$ follows a standard AR(1) stationary process: $z_t = \rho_z z_{t-1} + \nu_t$, where $\rho_z \in (0,1)$ and ν_t is an *iid* disturbance.

We introduce nominal rigidities following Calvo's staggered price setting: each firm in the wholesale sector optimally revises its price with probability $1-\theta$ in any given period t. Real marginal costs are equal across firms and given by $MC_t = (1-\tau)\frac{W_t}{Z_t P_t}$, where τ is a labor subsidy set by the government.[14] The $i-$th chooses the optimal price $P_t^*(i)$ to maximize $E_t \sum_{k=0}^{\infty} \theta^k \mathcal{F}_{t,t+k} Y_{t+k}(i)\left(P_t^*(i) - P_{t+k} MC_{t+k}\right)$ subject to the demand constraint $Y_{t+k}(i) = \left(\frac{P_t^*(i)}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}$. A larger (respectively smaller) elasticity of substitution across differentiated goods, ϵ, implies a smaller (respectively larger) market power for the monopolistically competitive wholesale firms. As a result, on average, real dividends distributed by the i-th firm and the related stock price are larger the lower is ϵ.[15]

C. Aggregation

With a fraction γ of each cohort replaced by an equally sized cohort of newcomers each period (such that population size is constant), the time t size of the cohort which entered the market in period $j \leq t$ is given by $\gamma(1-\gamma)^{t-j}$. This allows us to define the aggregator $X_t = \sum_{j=-\infty}^{t} \gamma(1-\gamma)^{t-j} X_{j,t}$ for $X = C, N, B, T, H$, and $S(i)$ with $i \in [0,1]$.

Letting $Q_t \equiv \int_0^1 Q_t(i)\,di$ and $D_t \equiv \int_0^1 D_t(i)\,di$ be, respectively, the aggregate stock price index and aggregate dividends, the aggregation across cohorts allows us to write the aggregate economy counterparts of, respectively, equations (6), (8), and (10):

$$\frac{\delta C_t}{1-N_t} = \frac{W_t}{P_t} \tag{11}$$

$$Q_t = E_t\Big\{\mathcal{F}_{t,t+1}\Pi_{t+1}\Big[Q_{t+1} + D_{t+1}\Big]\Big\}, \tag{12}$$

and

$$P_t C_t = [1-\beta(1-\gamma)]\left(\Omega_t + H_t\right), \tag{13}$$

where aggregate wealth Ω_t is defined as:

$$\Omega_t \equiv \left[B_t + P_t \int_0^1 \Big(Q_t(i) + D_t(i)\Big) S_t(i)\,di\right]. \tag{14}$$

Similar to the individual case, we can write the aggregate budget constraint as a stochastic difference equation in aggregate wealth:

$$P_t C_t + E_t\{\mathcal{F}_{t,t+1}\Omega_{t+1}\} \leq W_t N_t - P_t T_t + \Omega_t. \tag{15}$$

[14] As discussed in details in Appendix A, we choose the labor subsidy τ to equate the real wage to the marginal productivity of labor (which is one). This assumption, which is rather standard in the New Keynesian literature, implies that the steady-state Frisch elasticity of labor does not depend on the elasticity of substitution across goods ϵ. All our results are robust to the elimination of such subsidy.

[15] From Appendix A, we have that steady-state real dividends, D, are equal to $1/\left[(1+\delta)\,\epsilon\right]$, where δ denotes the Frisch elasticity of labor supply. Clearly, dividends are decreasing in both δ and ϵ. That is, a higher labor elasticity implies larger production costs following a given increase in real wages, while a higher demand elasticity lowers the mark-up on marginal costs. Both lead to lower profits.

Combining this equation with (13) and the definition of aggregate non-financial wealth H_t we obtain, after some manipulation, the following expression:

$$\frac{\beta(1-\gamma)}{1-\beta(1-\gamma)}P_tC_t = \gamma E_t\{\mathcal{F}_{t,t+1}\Omega_{t+1}\} + \frac{1-\gamma}{1-\beta(1-\gamma)}E_t\{\mathcal{F}_{t,t+1}P_{t+1}C_{t+1}\}. \tag{16}$$

Moreover, state-contingent bonds are in zero net supply in every period ($B_t = 0$) which together with a constant stock of equity shares issued by wholesale firms allow us to derive that $\Omega_t = P_t(Q_t + D_t)$.[16] Using this we can write the term $E_t\{\mathcal{F}_{t,t+1}\Omega_{t+1}\}$ entering (16) as:

$$E_t\{\mathcal{F}_{t,t+1}\Omega_{t+1}\} = E_t\{\mathcal{F}_{t,t+1}P_{t+1}(Q_{t+1}+D_{t+1})\} = P_tQ_t, \tag{17}$$

where the last equality follows from (12).

By combining equations (16) and (17), we obtain:

$$\frac{\beta(1-\gamma)}{1-\beta(1-\gamma)}C_t = \gamma Q_t + \frac{(1-\gamma)}{1-\beta(1-\gamma)}E_t\{\mathcal{F}_{t,t+1}\Pi_{t+1}C_{t+1}\}, \tag{18}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate and the stochastic discount factor $\mathcal{F}_{t,t+1}$ satisfies the following non-arbitrage condition with the riskless nominal interest rate R_t:

$$E_t\{\mathcal{F}_{t,t+1}\} = \frac{1}{R_t}. \tag{19}$$

The first term γQ_t on the right-hand side of (18) constitutes the linkage between the stock market and the real side of the economy, and we will refer to it as the *financial wealth channel* (henceforth, FWC). A positive turnover ($\gamma > 0$) implies that stock market wealth distorts the intertemporally optimal consumption profile. To grasp this, suppose that agents in the market at time t expect an increase in dividend payments at $t+1$ and, for simplicity, suppose that such increase is temporary—it does not last beyond $t+1$. By the stock price equation (12), the current stock price index Q_t increases. Economic agents who are in the market will immediately increase their current consumption to optimally smooth the anticipated shock. Tomorrow, however, a fraction of these individuals will be replaced by newcomers whose consumption profile is not affected by the temporary shock, since they enter the market without equity shares. Consequently, the increase in financial wealth following the increase in dividends affects current aggregate consumption more than the aggregate level expected for tomorrow. So increases in stock prices affect the optimal consumption profile. On the contrary, with no turnover ($\gamma = 0$), stock price fluctuations do not affect this profile, since (18) reduces $C_t = \beta^{-1}E_t\{\mathcal{F}_{t,t+1}\Pi_{t+1}C_{t+1}\}$.

From the price setting problem in the wholesale sector, we have that $P_t^*(i) = P_t^*$, i.e., all firms able to reset their price will choose a common value P_t^*. Aggregating across firms and using the relative demand $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t$ we obtain aggregate output: $Z_tN_t = Y_t\Xi_t$, where $N_t \equiv \int_0^1 N_t(i)\,di$ is the aggregate level of hours worked and $\Xi_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_t} di$ is an index of price dispersion over the

[16] Here we have assumed, without loss of generality, $S_t(i) = 1$ for all $i \in [0,1]$.

continuum of firms in the wholesale sector. Market clearing in this economy implies that consumption is equal to output, $Y_t = C_t$, and that labor demanded by wholesale firms is equal to labor supplied by the households. Moreover, since dividends are equal to corporate profits in the wholesale sector, we have that $D_t = Y_t (1 - MC_t)$.

III. The Log-linearized Economy and its Equilibrium

Following standard techniques, we log-linearize the equilibrium conditions stated in the previous section around the unique non-stochastic steady state.[17] From now on, lower case letters will denote percentage deviations of the original variable from its respective steady-state value, i.e., $x_t = \log\left(\frac{X_t}{X}\right)$. In a log-linearized form, the model can be reduced to the demand-side, supply-side, and financial-side blocks, which together with the interest rate rule will pin down the equilibrium dynamics of the economy. We proceed to describe in detail these blocks as well as the interest rate rule.

The *demand-side block* is obtained from equations (18), (19), and $C_t = Y_t$, and it reduces to the following aggregate IS curve:

$$y_t = \frac{1}{1+\psi} E_t y_{t+1} + \frac{\psi}{1+\psi} q_t - \frac{1}{1+\psi} (r_t - E_t \pi_{t+1}). \tag{20}$$

The term $\frac{\psi}{1+\psi} q_t$ captures the FWC at work in our model whose strength depends on the composite coefficient ψ:

$$\psi \equiv \gamma \frac{1 - \beta(1-\gamma)}{(1-\gamma)\epsilon} \frac{1+r}{r} > 0, \tag{21}$$

where r denotes the steady-state (real) interest rate. From equation (21) and the fact that r is strictly increasing in γ (see Appendix A), the parameter ψ can be written as a function of the turnover rate γ, i.e., $\psi(\gamma)$, with the following properties:

$$\psi(0) = 0 \quad \text{and} \quad \psi'(\gamma) > 0. \tag{22}$$

From (22), the implications of the turnover rate γ for the aggregate IS curve (20) are clear. With no turnover in markets ($\gamma = 0$), ψ is equal to zero and equation (20) collapses to the aggregate IS curve observed in a benchmark New-Keynesian model, namely, $y_t = E_t y_{t+1} - (r_t - E_t \pi_{t+1})$. In this case, movements in aggregate activity are only driven by the real interest rate, thus making stock price fluctuations completely redundant for the business cycle. A positive turnover in markets ($\gamma > 0$), on the other hand, makes ψ positive, and therefore generates a FWC: stock price fluctuations feedback into real activity (via their wealth effects on consumption) through the term $\frac{\psi}{1+\psi} q_t$ in (20). A higher turnover rate raises ψ, and hence $\frac{\psi}{1+\psi}$, strengthening the FWC.

It is also possible to prove that ψ is strictly decreasing in the elasticity ϵ, meaning that the FWC is quantitatively more significant in economies characterized by lower elasticities of substitution (i.e.,

[17] See Appendix A for details about the steady state. The linearization of the equilibrium conditions is rather straightforward. A detailed derivation is available from the authors upon request.

higher market power) in the market for differentiated products. This is has to do with the fact that the parameter ψ can also be written as follows: $\psi = \gamma\left(\frac{1}{1-\gamma} - \beta\right)\frac{\Omega}{PC}$, i.e., it is proportional to the steady-state ratio of real financial wealth $\frac{\Omega}{P}$ (given by $Q + D$) to consumption C. As discussed at the end of sub-section II.B, steady-state dividends, and therefore financial wealth, are negatively related to ϵ. Hence, for a given positive turnover γ, the FWC is stronger in economies where the profitability from financial investments is, on average, larger.

The *supply-side block* comes from the solution of the optimal price setting problem in the intermediate good sector, the definition of real marginal costs $\frac{MC_t}{P_t} = (1-\tau)\frac{W_t}{Z_t P_t}$, and the aggregate labor supply equation (11). It is given by the New Keynesian Phillips curve:

$$\pi_t = \tilde{\beta} E_t \pi_{t+1} + \kappa(1+\chi)\left(y_t - z_t\right), \tag{23}$$

where $\kappa \equiv \frac{(1-\theta)(1-\theta\tilde{\beta})}{\theta}$, $\chi \equiv \frac{N}{1-N}$ is the inverse of the (steady-state) Frisch elasticity of labor supply and $\tilde{\beta} \equiv \frac{\beta}{1+\psi}$.

Regarding the demand-side and supply-side blocks, a positive turnover in markets makes the economy *de facto* less forward-looking at the aggregate level, and therefore captures the possibility of myopia in both consumers' and firms' behavior.[18] To see this note that, by the properties spelled in (22), a positive turnover also diminishes the impact of the real interest rate and expected future output on current real activity in the IS curve (20) and, by diminishing the importance of inflationary expectations in the Phillips curve (23), it increases the relative importance of current marginal costs—i.e., $(1+\chi)(y_t - z_t)$—for inflation determination. The turnover rate γ could then be thought as an index of the degree of "non-Ricardianness" in the economy capturing two extreme cases. For $\gamma = 0$, all agents are Ricardian: they make consumption and saving decisions taking advantage of all trading opportunities available in the market, as in the benchmark New-Keynesian model. On the other hand, for $\gamma \to 1$, all agents become non-Ricardian: they do not save but simply consume out of current net income, as in the rule-of-thumb consumers model of Gali et al. (2004).

The *financial-side block* describes the stock price dynamics. After linearizing equation (12), and combining the linearized versions of equations (11), (18), (19) with the market clearing condition for goods, aggregate technology, the definition of real marginal costs and of real dividends, we obtain the following stock price equation:

$$q_t = \tilde{\beta} E_t q_{t+1} - \lambda E_t y_{t+1} - \left(r_t - E_t \pi_{t+1}\right) + \varphi z_t, \tag{24}$$

where $\varphi \equiv \left(1 - \tilde{\beta}\right)(\epsilon - 1)(1+\chi)\rho_z$ and

$$\lambda \equiv \left(1 - \tilde{\beta}\right)\left[(\epsilon - 1)(1+\chi) - 1\right]. \tag{25}$$

According to (24), the current stock price q_t depends positively on its future expectation $E_t q_{t+1}$ and the technology shock z_t, but negatively on the real interest rate $r_t - E_t \pi_{t+1}$ and future expected activity

[18] On these grounds, Freedman et al. (2010) motivate the introduction of Blanchard-Yaari consumers in the IMF's Global Integrated Monetary and Fiscal Model. Jappelli and Pistaferri (2010) provide an extensive review of the literature testing for the existence of liquidity constraint and myopia in individual consumption.

$E_t y_{t+1}$. The negative relationship between future output and the current stock price is due to the counter-cyclicality of dividends (current and expected) which arises in any New-keynesian framework under flexible wages.[19]

To close the model and solve for the equilibrium of this economy, we describe the monetary policy block as an instrumental forward-looking interest rate rule (Taylor rule), whereby the (log-linearized) short-term interest rate is set in response to expected future inflation and stock prices:

$$r_t = \phi_\pi E_t \pi_{t+1} + \phi_q E_t q_{t+1}, \tag{26}$$

with $\phi_\pi > 0$ and $\phi_q > 0$.[20] Empirical support for this type of rules can be found in Clarida et al. (2000), as well as in the more recent study by Boivin and Giannoni (2006). We also adopt a standard terminology and refer to a rule with $\phi_\pi > 1$ ($\phi_\pi < 1$) as an *active* (respectively, *passive*) rule. An active rule is also said to satisfy the *simple* Taylor principle.[21]

To conclude this section, note that combining the different blocks (20), (23), (24) and (26) allows us to reduce further the model and write it as the following stochastic linear system:

$$x_t = \Gamma E_t x_{t+1} + \Theta z_t, \tag{27}$$

where $x_t = [y_t, \pi_t, q_t]'$, and Γ and Θ are conformable matrices, whose entries depend on structural and policy parameters.

IV. Equilibrium Determinacy and Learnability

This section examines the conditions for the (local) determinacy and learnability of the Rational Expectations Equilibrium (REE) with respect to the policy parameters ϕ_π and ϕ_q entering the instrumental rule (26). The determinacy analysis relies on the results of the seminal work by Blanchard and Kahn (1980), while the learnability analysis focuses on the Expectational Stability (E-stability) concept of Evans and Honkapohja (2001). Under the learnability analysis, we will focus on two types of representations of REE: (i) the Minimal State Variable representation of a fundamental equilibrium (MSV-FE) and (ii) the Common Factor representation of stationary sunspot equilibria (CF-SSE). As in McCallum (2003), the MSV-FE of the system (27) can be described as $x_t = \mathcal{N} z_t$, where $\mathcal{N}$ is a conformable matrix. The MSV-FE may exist under both determinacy or indeterminacy. On the other hand, if the equilibrium is indeterminate (and the roots are real), the CF-SSE corresponds to $x_t = \mathcal{N} z_t + \mathcal{G} \zeta_t$, where ζ_t is a sunspot shock following an autoregressive process, and $\mathcal{N}$ and $\mathcal{G}$ are conformable matrices, as in Evans and McGough (2005a,b). Appendix B provides a detailed description of the methodologies, including the determinacy and learnability conditions.

[19] The same mechanism is present in Carlstrom and Fuerst (2007). We will relax the wage flexibility assumption in Section V.

[20] The model also has a fiscal side, where the government always levies lump-sum taxes to pay the labor subsidy to firms, balancing period-by-period its budget constraint. So we can abstract from the accumulation of public debt and somehow ignore the government budget constraint in the determinacy and learnability analyses.

[21] Section V presents results for rules responding to expected future output, as well as for contemporaneous rules responding to current-period inflation and stock price levels.

We proceed to show that, under suitable conditions, the FWC can overturn the *conventional wisdom*, according to which an explicit response to stock prices facilitates equilibrium indeterminacy and, as a consequence, ignites non-fundamental aggregate instability. To grasp the role of the FWC, we consider first the benchmark New Keynesian model, which is obtained by setting $\gamma = 0$ in the reduced form system described by equations (20)-(25) above. In this case, the FWC is absent and we can use the resulting model to show how, from the perspective of determinacy and learnability, responding to stock prices ($\phi_q > 0$) can be destabilizing. The results are summarized in the following Proposition.

Proposition 1 *Assume no turnover in financial markets:* $\gamma = 0$.

Determinacy. *The REE is locally determinate if and only if:*

$$\phi_q < \frac{1+\beta}{1+\lambda} \qquad \text{and} \qquad 1 + \frac{\lambda}{\kappa}\phi_q < \phi_\pi < 1 + \frac{2(1+\beta)}{\kappa} - \frac{2+\lambda}{\kappa}\phi_q \tag{28}$$

E-Stability of MSV-FE. *The MSV-REE is E-stable if and only if*

$$\phi_\pi > 1 + \frac{\lambda}{\kappa}\phi_q \tag{29}$$

E-Stability of CF-SSE. *There exist E-stable CF representations of SSE if and only if*

$$\phi_\pi > 1 + \frac{2(1+\beta)}{\kappa} - \frac{2+\lambda}{\kappa}\phi_q \quad \text{for} \quad \phi_q \leq \frac{1+\beta}{1+\lambda} \tag{30}$$

$$\phi_\pi > 1 + \frac{\lambda}{\kappa}\phi_q \quad \text{for} \quad \phi_q > \frac{1+\beta}{1+\lambda} \tag{31}$$

Proof. See Appendix C.1. ■

The proposition extends the results of the *conventional wisdom,* as those in Carlstrom and Fuerst (2007), in at least two dimensions. First, it considers the case of a *forward-looking* specification, which is, in general, more prone to indeterminacy.[22] As a matter of fact, the result in (28) implies the existence of an upper bound $\phi_q^* \equiv \frac{1+\beta}{1+\lambda}$ on the response to stock prices above which the equilibrium is always indeterminate, for any active (and passive) policy rule. Second, it assesses the learnability of both fundamental and non-fundamental solutions. In particular, it shows that there exist policy parametrization for which the equilibrium is indeterminate but *still* the MSV-FE is learnable—e.g., for $\phi_q > \frac{1+\beta}{1+\lambda}$ and $\phi_\pi > 1 + \frac{\lambda}{\kappa}\phi_q$.[23] This may question the relevance of some of the indeterminacy results by previous works. After all, despite multiple equilibria, all the fundamental solutions are

[22] Carlstrom and Fuerst's analysis is restricted to a *contemporaneous* specification, under which a determinate equilibrium is always attainable, for any $\phi_q > 0$, as long as the response to inflation is sufficiently aggressive.

[23] Notice that, by setting $\phi_q = 0$, the results in (28) and (29) of Proposition 1 reduce to the equilibrium determinacy and E-stability results stated, respectively, in Propositions 4 and 5 in Bullard and Mitra (2002). In their benchmark New Keynesian model, the upper bound on the coefficient ϕ_π is simply $1 + 2\frac{1+\beta}{\kappa}$, which never binds for any realistic calibration of structural parameters. This makes the Taylor principle *de facto* necessary and sufficient for equilibrium determinacy in this model.

learnable for these policy parametrizations. However, the previous proposition also shows that these policy parametrizations under which there is indeterminacy *and* learnable fundamental-driven REE can also induce learnable sunspot equilibria (E-stable CF-SSE).[24] So macroeconomic stability can be at stake because of extrinsic uncertainty.

The analytical results presented in the proposition are displayed graphically in the left panel of Figure 1. Taking one period in the model to correspond to a quarter, we set the discount factor β equal to 0.99. The inverse Frisch labor elasticity, χ, is equal to 0.25—i.e., an elasticity equal to 4—as common in the macro-labor literature.[25] We choose the Calvo probability of price rigidity to make the elasticity of current inflation to marginal costs in the Phillips curve, κ, equal to 0.019, as in Carlstrom and Fuerst (2007). The parameterization of the elasticity of substitution across differentiated goods is based on the micro-evidence by Broda and Weinstein (2006). They report median elasticity values equal to, respectively, 2.5 and 2.1. for their pre-1990 and post-1990 samples on sectoral U.S. data. We have therefore set ϵ equal to 2.3, their mid-point estimate.[26] This implies that monopolistically competitive firms enjoy a considerable degree of market power, which, in our set-up, is necessary to generate sufficiently large profits/dividends and, as a result, sufficiently large gains from equity holdings. In section V we will discuss how a significant FWC can also obtain for much higher elasticities if combined with real wage rigidities.

The left panel of Figure 1 shows that an increase in the policy coefficient ϕ_q diminishes the size of policy space where the equilibrium is determinate (white area) and where the MSV-FE is E-stable (white and light gray areas labeled by ES-MSV-FE). This is a direct consequence of the fact that $1 + \frac{\lambda}{\kappa}\phi_q$ is strictly increasing in ϕ_q. At the same time, raising ϕ_q enlarges the possibility of having an indeterminate equilibrium (dark gray area) and a E-unstable MSV-FE (dark gray area labeled EU-MSV-FE). Within the regions of indeterminacy, the reduced form system's eigenvalues are all real. It is therefore possible to apply the results by Evans and McGough (2005a,b) regarding the existence of E-stable CF-SSE. Learnable CF-SSE exist within the light gray area (where the MSV-FE is also learnable) but not in the dark gray area (where the MSV-FE is not learnable).

Can the FWC overturn these (extended) results of the *conventional wisdom*? To answer this, look at the right panel in Figure 1. It corresponds to the case of a positive turnover in financial markets, for which we choose $\gamma = 0.05$. This value is somewhat intermediate between the low-end estimates based on consumers' expected working lifetime (ranging between 0.005 and 0.015) and the high-end estimates based on observed portfolio turnover in financial markets.[27] With respect to the latter, Kozora (2010) reports an average investment horizon for U.S. institutional investors ranging between 19 and 25 months, for the period 1990-2007. Cella et al. (2013) report similar results, with an average portfolio turnover rate of about 17% at quarterly frequency, which implies an investment horizon of

[24] Given the lack of endogenous lagged variables in the system, determinacy implies learnability of the MSV-FE. By close inspection of the proposition, one can notice that, within the indeterminacy region, learnable CF representations of SSE exists when the MSV-FE is also learnable. See Appendix B for a detailed discussion on this. We defer a discussion on the policy implications of this finding to the end of this section.

[25] See, for instance, Imai and Keane (2004).

[26] Nakamura and Steinsson (2010), Midrigran (2011), and Ravn et al. (2010) are notable examples of recent quantitative macro-models using a similar parametrization for this elasticity.

[27] See Leith and Wren-Lewis (2000) and Smets and Wouters (2002) for a calibration based on expected lifetime.

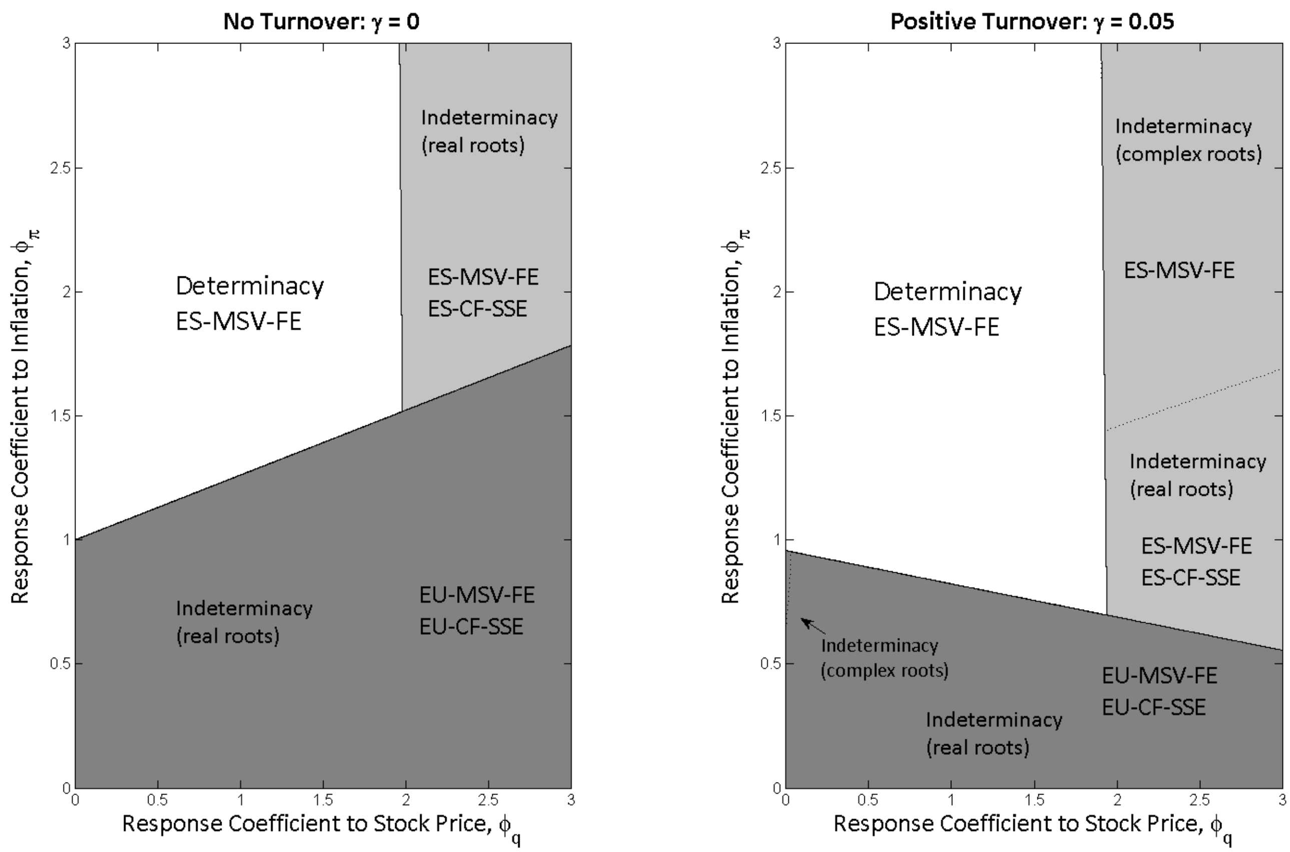

Figure 1: Determinacy and E-stability Analysis under a Forward-Looking Rule. Legend: ES = E-stable; EU = E-unstable; MSV-FE = Minimal State Variable representation of a Fundamental Equilibrium; CF-SSE = Common Factor representation of a Stationary Sunspot Equilibrium.

about 6 quarters. Using Bayesian methods, Castelnuovo and Nisticò (2010) estimate a version of our model on U.S. data. Their posterior mean for γ is equal to 0.13, giving a planning horizon slightly shorter than 2.5 years, which is rather close to the estimates in the empirical finance literature.

Figure 1 reveals that the most notable difference with respect to the no-turnover case (left panel) is the fact that with the FWC ($\gamma = 0.05$), a positive response to stock prices *enlarges* the policy space where the equilibrium is determinate (white area) and where the MSV-FE is learnable (white and light gray areas). Note that the lower determinacy/E-stability frontier is now downward-sloping in ϕ_q. Therefore a rather mild FWC can overturn the *conventional wisdom*.[28] In particular, as long as ϕ_q is not excessive (in the specific case, lower than 2), responding to stock prices does not interfere with the *simple* Taylor principle. Actually, it improves upon it: not only an *active* but also a mildly *passive* response to inflation guarantee a unique REE and a learnable MSV-FE. On the other hand, similarly to the no-turnover case, we find that there exist policy parametrizations for which the MSV-FE is learnable despite the occurrence of multiple sunspot equilibria (light gray area again) and that, at least for the case of real roots, those sunspot equilibria have representations that are learnable (E-stable CF-SSE).[29]

To build some economic intuition for why the FWC can overturn the *conventional wisdom*, we focus on the learnability of the MSV-FE. In this case, the white and light gray areas of Figure 1 show that a positive ϕ_q enlarges the policy space where the MSV-FE is learnable in the model with FWC, while it reduces this space in the model with no FWC. One can prove this analytically by deriving and comparing the conditions for the response coefficients ϕ_π and ϕ_q under which the MSV-FE is E-stable. Proposition 1 already provides the sufficient and necessary condition (29) when the FWC is absent and, naturally, one can derive similar conditions when the FWC is present. For the purpose of providing an economic intuition of our results, the following Proposition 2 presents a sufficient condition for the learnability of the MSV-FE.[30]

Proposition 2 *Recall the definitions of ψ and λ from, respectively, equations (21) and (25). Let $\gamma^* \in (0,1)$ be the unique solution to $\psi = \lambda$, such that $\psi > \lambda$ for $\gamma > \gamma^*$.*

Then, if the turnover rate γ is larger than γ^* (i.e., the FWC is sufficiently strong), a sufficient

[28] We have set $\gamma = 0.05$, but a downward-sloping line obtains for γ as low as 0.02.

[29] The methodology proposed by Evans and McGough (2005a,b) works only for the case of real roots. As they point out, complex roots pose the problem for the common factor representation of sunspot equilibria. To the best of our knowledge, general results extending to the case of complex roots are not publicly available in the literature.

[30] As the proof of Proposition 2 in the Appendix shows, the following condition in (32) is slightly stricter than the necessary and sufficient condition:

$$\phi_\pi > 1 - \psi \frac{\left(1-\tilde{\beta}\right)\left(1-\tilde{\beta}+\lambda\right)}{\kappa\left(1-\tilde{\beta}+\psi\right)} + \frac{\left(1-\tilde{\beta}\right)(\lambda-\psi)}{\kappa\left(1-\tilde{\beta}+\psi\right)}\phi_q \equiv \Phi_\gamma.$$

However, the sufficient condition in (32) has a more straightfoward economic interpretation. Note that the right-hand side of (32) corresponds to an upward parallel shift of the right-hand side of the necessary and sufficient condition in this footnote. From the right panel of Figure 1 it is easy to infer that such shift is quite small, such that both conditions almost coincide.

condition for the learnability of the MSV-FE is

$$\phi_\pi > \Phi_\gamma \equiv 1 + \frac{\left(1-\tilde{\beta}\right)(\lambda-\psi)}{\kappa\left(1-\tilde{\beta}+\psi\right)}\phi_q, \tag{32}$$

where, since $\psi > \lambda$, the threshold Φ_γ is strictly decreasing in ϕ_q.

Proof. See Appendix C.2. ■

Comparing the sufficient condition (32) with its counterpart in the non-FWC case ($\phi_\pi > 1 + \frac{\lambda}{\kappa}\phi_q$) reveals how the FWC, through the turnover rate γ, affects the threshold on the response coefficient to inflation above which MSV-FE are learnable. Under no FWC, this threshold is defined by $1+\frac{\lambda}{\kappa}\phi_q$ and is always increasing in ϕ_q; whereas under FWC and for $\gamma > \gamma^*$, this threshold is $\Phi_\gamma \equiv 1+\frac{\left(1-\tilde{\beta}\right)(\lambda-\psi)}{\kappa\left(1-\tilde{\beta}+\psi\right)}\phi_q$, and is strictly decreasing in ϕ_q. This is because, as the proof of Proposition 2 shows, the coefficient $\frac{\left(1-\tilde{\beta}\right)(\lambda-\psi)}{\kappa\left(1-\tilde{\beta}+\psi\right)}$ is negative for a strong enough FWC, i.e., if $\gamma > \gamma^*$. Note also that $\Phi_\gamma < 1$, such that condition (32) holds for any active response to inflation, and stock price targeting is not a threat for aggregate stability.

The sufficient condition of Proposition 2 can now help us develop some simple economic intuition of how the FWC can overturn the *conventional wisdom.* To do so, we re-write the inequality in (32) as follows:

$$\phi_\pi + \frac{\left(1-\tilde{\beta}\right)(\psi-\lambda)}{\kappa\left(1-\tilde{\beta}+\psi\right)}\phi_q - 1 > 0. \tag{33}$$

This inequality can be related to the *long-run* Taylor principle discussed by Bullard and Mitra (2002). From equations (20), (23) and (24), one can see that the left-hand side of (33) corresponds to the long-run change in the real interest rate following a permanent one percent increase in inflation under the interest rate rule (26). The term $\frac{\left(1-\tilde{\beta}\right)(\psi-\lambda)}{\kappa\left(1-\tilde{\beta}+\psi\right)}$ represents in fact the permanent change in the stock price index following a one percent permanent increase in inflation. When the inequality in (33) holds, the interest rate rule (26) responds to higher inflation by inducing a real interest rate increase. Then any (permanent) increase in inflation (expectations) — driven, for instance, by a sunspot shock — will be corrected over time, since the higher real interest rate will push down output and then inflation. As a result, the initial sunspot-driven increase in inflation will not be self-fulfilled. When the FWC is sufficiently strong (namely, $\gamma > \gamma^*$), a one percent *increase* in inflation implies a permanent $\frac{\left(1-\tilde{\beta}\right)(\psi-\lambda)}{\kappa\left(1-\tilde{\beta}+\psi\right)}$ percent *increase* in the stock price index. Therefore, a positive response ϕ_q implies that condition (33) can hold—i.e., a long-run increase in the real interest rate is obtained—even if the policy rule grants a *passive* response to inflation, $\phi_\pi < 1$. On the contrary, when $\gamma = 0$ (absent FWC), we have that $\frac{\left(1-\tilde{\beta}\right)(\psi-\lambda)}{\kappa\left(1-\tilde{\beta}+\psi\right)} = -\frac{\lambda}{\kappa}$, and the inequality in (33) becomes $\phi_\pi - \frac{\lambda}{\kappa}\phi_q - 1 > 0$. In this case, a one percent increase in inflation drives down the stock price index by $\frac{\lambda}{\kappa}$ percent. Thus, a positive response ϕ_q implies that, in order to generate a real interest rate increase, the central bank's response to inflation must be *sufficiently active.*

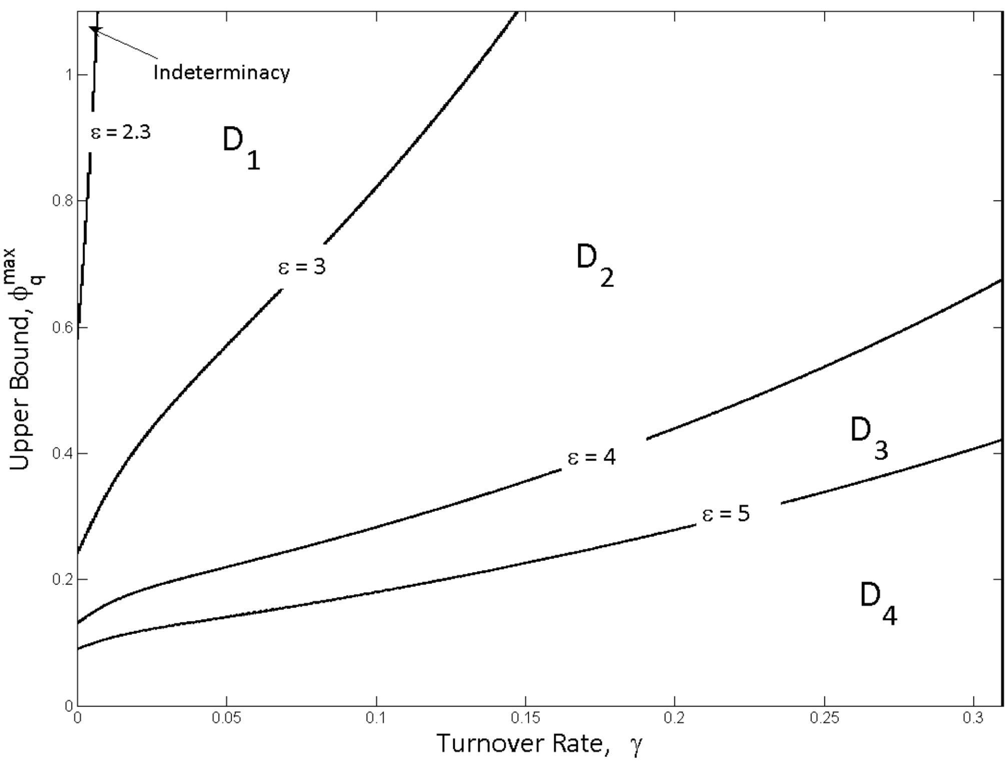

Figure 2: Upper Bound on the Response to Stock Prices ϕ_q for a Given Active Response to Inflation. Plot of $\phi_q^{\max}$ defined in (34) with respect to γ, for alternative parametrizations of ϵ. For given ϵ, equilibrium determinacy occurs below the upward-sloping curves. Example: the REE is determinate within the area $D_1+D_2+D_3+D_4$ for $\epsilon = 2.3$, within the area $D_2+D_3+D_4$ for $\epsilon = 3$, and so on.

The economic intuition behind the long-run positive comovement between stock prices and inflation when γ is sufficiently large is the following. Suppose the stock price increases permanently. By the stock price equation, this implies a long-run increase in real dividends, and hence a decrease in real marginal costs. In turn, by the Phillips curve, permanently lower marginal costs lead to a permanent decrease in inflation. This is the standard channel at work in the benchmark New-Keynesian model. The FWC works in the other direction. The higher stock price generates an increase in aggregate activity via the financial wealth effect entering the Euler equation. In turn, this implies an increase in labor demand by firms, which results in higher real wages. With marginal costs of production rising, by the Phillips curve, inflation will increase. When γ is sufficiently large, the FWC prevails on the standard New-Keynasian channel, thus creating a positive long-run comovement between stock prices and inflation.

Note that the FWC continues to have implications for determinacy/E-stability for the case of $\gamma < \gamma^*$. To see why, consider the condition in (32) again. If $\gamma < \gamma^*$ (and hence $\psi < \lambda$), the inequality holds for:

$$\phi_q < \phi_q^{\max} \equiv \frac{\kappa\left(1-\tilde{\beta}+\psi\right)}{\left(1-\tilde{\beta}\right)(\lambda-\psi)}\left(\phi_\pi - 1\right); \tag{34}$$

that is, for a given *active* response to inflation ($\phi_\pi > 1$), there exists an upper bound $\phi_q^{\max}$ on the policy response to the stock price.[31] Figure 2 displays $\phi_q^{\max}$ as a function of the turnover rate γ, for alternative parametrizations of the elasticity ϵ. Note that $\phi_q^{\max}$ is monotonically increasing in γ. This implies that, even if the turnover rate is not high enough to overturn the conventional wisdom, the upper bound on the response to stock prices is less tight for a larger γ. Hence, a stronger FWC enlarges the value range of ϕ_q consistent with determinacy/E-stability. Consider the benchmark parameterization: $\epsilon = 2.3$. While $\phi_q^{\max}$ is positive (equal to 0.6) if $\gamma = 0$, its value increases very rapidly as γ becomes positive, and tends to infinity as γ approaches γ^* (which, in this case, is about 0.02). Although the quantitative effect is smaller, the upper bound $\phi_q^{\max}$ is strictly increasing in γ also for higher elasticities.

V. Extensions and Robustness

We discuss the robustness of our results to alternative monetary policy specifications and to a simple modification to the structural framework. For what concerns alternative policies, we consider the case of forward-looking rules also responding to output, and the case of contemporaneous policy rules. We then show how the introduction of real rigidities enhances the scope for our previous results.

Responding to Output The top two panels in Figure 3 display the results for a forward-looking rule also responding to expected future output, i.e., $r_t = \phi_\pi E_t\pi_{t+1} + \phi_q E_t q_{t+1} + \phi_y E_t y_{t+1}$, with $\phi_y = 0.5/4$, as in the baseline parameterization of Taylor (1993). Comparing these panels with Figure 1 reveals that a positive response to output improves determinacy, with or without the FWC. However, it does

[31] Carlstrom and Fuerst (2007) express their determinacy condition also in these terms.

not affect qualitatively our previous conclusions: for a positive response to stock prices to enlarge determinacy, we still need a sufficiently positive turnover rate γ.

Contemporaneous Rules The bottom two panels in Figure 3 refer to the case of a contemporaneous interest rate rule, i.e., $r_t = \phi_\pi \pi_t + \phi_q q_t$. The main difference with respect to the forward-looking specification's results displayed in Figure 1 is the disappearance of policy parametrizations for which there exist multiple sunspot equilibria but the MSV-FE is learnable (no light gray areas). Under a contemporaneous rule, there are only two possible outcomes: either a determinate REE with a learnable MSV-FE representation; or, an indeterminate REE, with MSV-FE and CF-SSE which are not learnable. Moreover, for the case of $\gamma > \gamma^*$ (right panel), there is no upper bound on ϕ_q: for every active policy rule, the REE is determinate and the MSV-FE is E-stable for any $\phi_q \geq 0$. Despite the differences, the main message remains: a sufficiently positive turnover rate γ overturns the conventional wisdom by allowing stock price targeting to have some benefits for determinacy/E-stability.

Real Wage Rigidity The FWC, we propose in this paper, has a higher potential to overturn the conventional wisdom if combined with any form of real rigidity, dampening the sensitivity of the real wage to market conditions. This is because, by increasing the response of real dividends to output, a sluggish wage adjustment makes equity holdings relatively more important for consumption decisions, thus boosting the FWC. We briefly consider the consequences of introducing a *real* wage rigidity by assuming that the *real* wage does not fully respond to labor market conditions, as a result of not modeled imperfections, similar to Blanchard and Gali (2007).[32]

More specifically, assume that the real wage paid to workers is a weighted average of a notional wage (with weight $1-\xi$) and, using the terminology of Hall (2005), of a wage norm (with weight ξ). We set the notional wage equal to the real wage occurring in a perfectly flexible labor market, i.e., the marginal rate of substitution between consumption and leisure as given by equation (11). As the wage norm, we consider instead the fully efficient real wage occurring in steady state.[33] From (11), simple algebra shows that the (log) real wage now satisfies $w_t - p_t = (1-\xi)\left[(1+\chi)\, y_t - \chi z_t\right]$, where the parameter $\xi \in [0,1]$ is the index of real wage rigidity.[34] With this slight modification, the parameter λ defined in (25) is replaced by the following expression: $\lambda \equiv \left(1-\tilde{\beta}\right)\left[(\epsilon-1)(1+\chi)(1-\xi)-1\right]$. Then a higher degree of real wage rigidity (higher ξ) lowers the value of λ. In turn, this lowers the threshold turnover rate γ^* above which ψ is larger than λ, a condition needed to overturn the conventional wisdom, as discussed in Proposition 2.

[32] A similar result would obtain if we instead assumed *nominal* wages to be sticky. However, this would complicate the set-up as it would require the introduction of a wage Phillips curve. See Castelnuovo and Nisticó (2010) for a model along these lines.

[33] Other formulations of *real* wage rigidities assume the wage norm to be equal to the past wage $\frac{W_{t-1}}{P_{t-1}}$, such that the (log) real wage corresponds to an exponentially-decaying weighted average of the infinite stream of past flexible real wages. See, for instance, Uhlig (2007). Our simpler specification retains the same logic—namely, the current real wage does not fully respond to current labor market conditions—without requiring any major modification to the reduced form linear system.

[34] For $\xi = 0$, the real wage is fully flexible, as in the case studied in the previous section. For $\xi = 1$ instead, $w_t = 0$, i.e., the real wage is constant, as in the canonical model of Hall (2005).

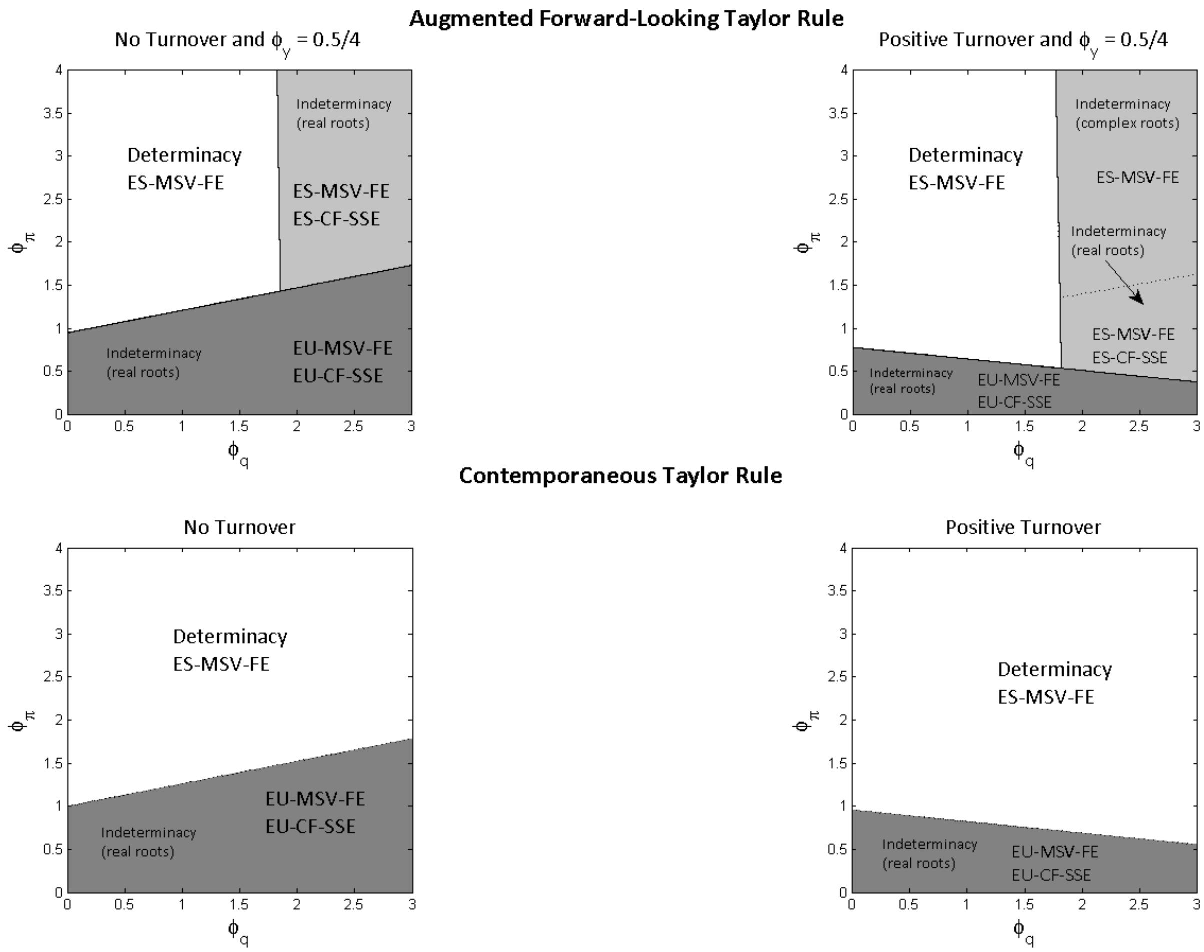

Figure 3: Determinacy and E-stability Analysis for Forward-Looking Rules Responding to Output (Top Panels) and Contemporaneous Rules (Bottom Panels). Legend: ES = E-stable; EU = E-unstable; MSV-FE = Minimal State Variable representation of a Fundamental Equilibrium; CF-SSE = Common Factor representation of a Stationary Sunspot Equilibrium.

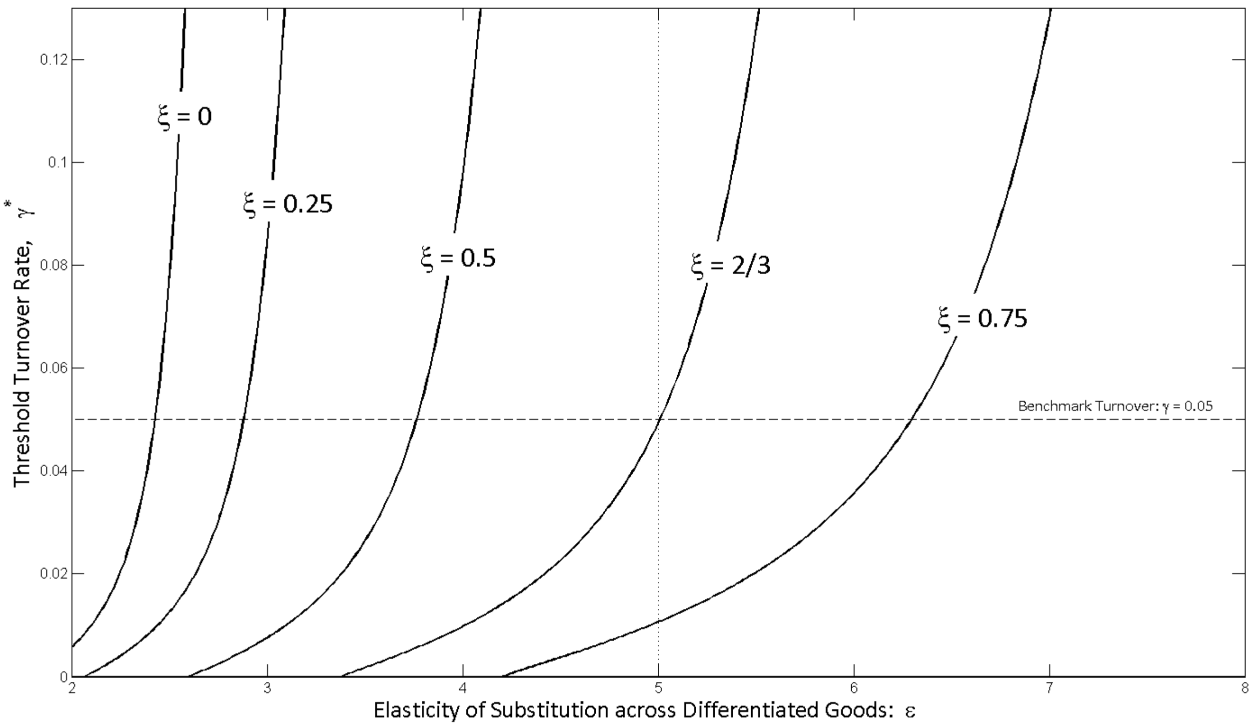

Figure 4: The Importance of Demand Elasticity and Real Wage Rigidities for the Threshold Turnover Rate γ^*.

An important consequence of introducing real wage rigidity is that our previous results can also be obtained for larger elasticities, including those coming from aggregate calibrations. To see this more clearly, we construct Figure 4, where we plot the threshold γ^* defined in Proposition 2 as a function of ϵ for alternative degrees of real wage rigidity ξ. We restrict the plot to the range $[0,\ 0.13]$, where the upper-bound corresponds to the high-end estimate of the turnover rate, as in Castelnuovo and Nisticó (2010) and the empirical finance literature discussed in section IV. The horizontal dashed line corresponds to our benchmark parameterization: $\gamma = 0.05$. As it clearly appears, the threshold γ^* is a strictly increasing function of ϵ, for any degree of real wage rigidity. An increase in ξ shifts the threshold towards the lower-right corner, thus making it more likely for $\gamma > \gamma^*$ to hold. For instance, consider the case of $\xi = 0.5$. The range of plausible turnover rates above γ^*—i.e., the interval above γ^* for a specific ϵ—is non-empty for $2 < \epsilon < 4$ when $\xi = 0.5$, for $2 < \epsilon < 5.5$ (roughly) when $\xi = 2/3$, and for $2 < \epsilon < 7$ when $\xi = 0.75$.[35] Looking at the same picture from a different perspective, consider the case of $\epsilon = 5$, which is a common parameterization in the macro literature. Then, the threshold γ^* is equal to the benchmark value 0.05 if $\xi = 2/3$, and drops to below 0.01 if $\xi = 0.75$.

[35] In their benchmark calibration, Blanchard and Gali (2007) set the wage rigidity parameter (which is their case is the weight on past real wages) equal to 0.9. Using a similar specification, Uhlig (2007) shows that a high degree of real wage rigidity is needed to generate both asset pricing and macroeconomic facts with a baseline real business cycle model.

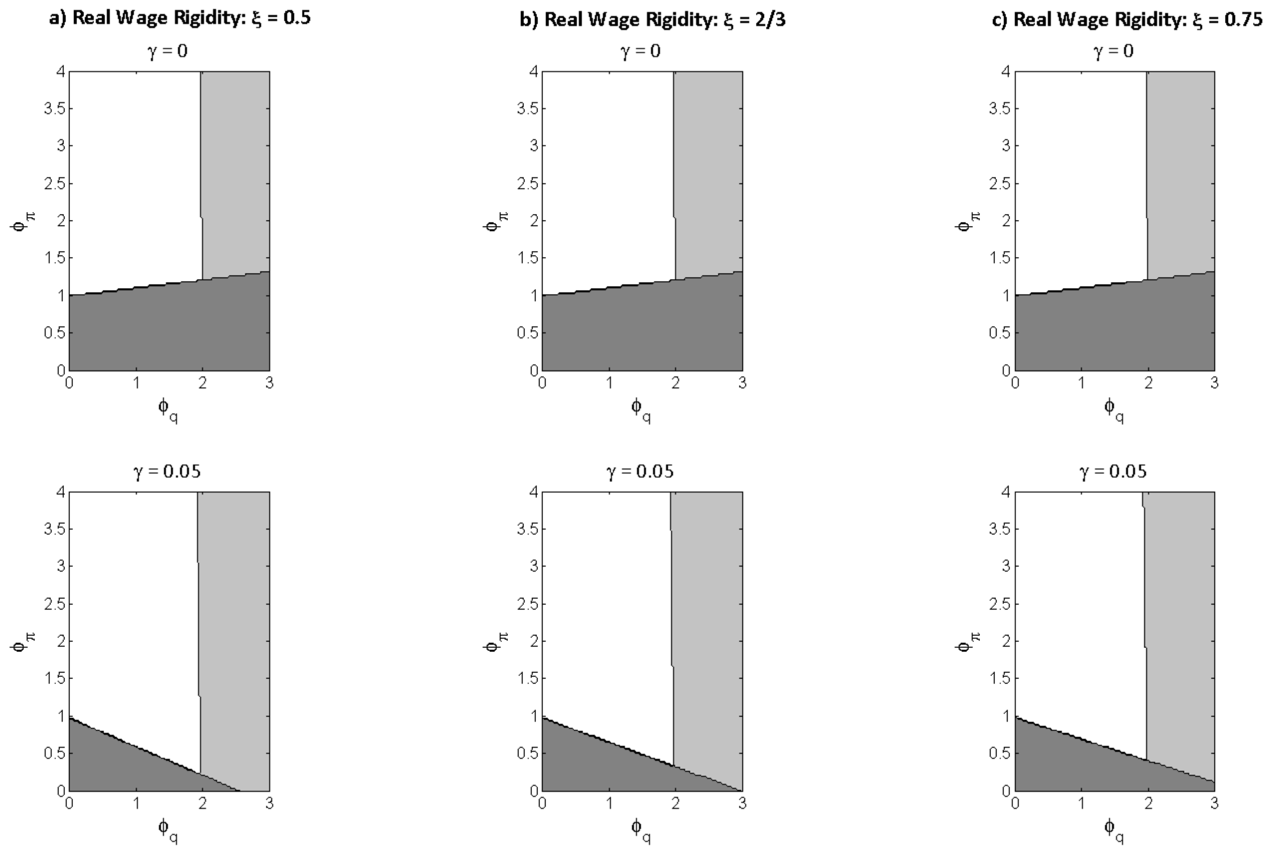

Figure 5: Determinacy and E-stability Analysis with Real Wage Rigidity. White area: determinacy and ES-MSV-FE. Light gray area: indeterminacy and ES-MSV-FE. Dark gray area: indeterminacy and EU-MSV-FE.

Figure 4 also shows that there exists a continuum of (ϵ, ξ) pairs for which the threshold γ^* can be made equal to our benchmark parameterization of 0.05, but, more generally, to any value included in the plausible range $[0,\ 0.13]$. Equivalently, for any given elasticity $\epsilon \in [2, 8]$ we can identify a degree of wage rigidity ξ such that we obtain an effect similar to what displayed in Figure 1. For instance, let's consider the following three possibilities: a) $\epsilon = 3$ and $\xi = 0.5$; b) $\epsilon = 4$ and $\xi = 2/3$; and c) $\epsilon = 5$ and $\xi = 0.75$. The top panels in Figure 5 display the determinacy/E-stability regions for these three cases under the forward-looking rule (26) assuming no turnover in markets, while the bottom ones assume $\gamma = 0.05$. Similar to Figure 1, a positive turnover makes stock price targeting beneficial for determinacy/E-stability. This result combined with those reported in section IV allow us to conclude the following: the FWC presented in this paper has the potential to overturn the conventional wisdom in economies characterized by lower demand elasticities (hence, higher market power) but more flexible labor markets, and/or higher demand elasticities but more rigid labor markets.[36]

[36] A similar outcome could be obtained by introducing other forms of real rigidities, such as, for instance, consumption externalities/habits or labor indivisibilities.

VI. Conclusions

In the benchmark New-Keynesian model, an explicit response to stock prices in the interest rate rule increases the scope for equilibrium indeterminacy. More specifically, the larger the response to stock prices the larger should be the response to inflation for the equilibrium to be locally unique. This policy trade-off has been clearly highlighted by Bullard and Schaling (2002) and Carlstrom and Fuerst (2007) supporting the conventional wisdom that monetary policy should *not* respond to stock prices. However, the benchmark model does not include any structural linkage between the stock market and real activity, and hence no specific reason for why the central bank should respond to endogenous variables other than inflation and output.

This paper evaluates whether the conventional wisdom carries over to a New-Keynesian DSGE model in which the turnover in financial markets among non-Ricardian agents holding heterogeneous portfolios creates a financial wealth channel (FWC). As a result of this FWC, stock-price fluctuations affect the dynamics of aggregate consumption. The evaluation is performed through extensive analysis on the determinacy and learnability of the rational expectations equilibrium implied by an interest-rate rule that includes an explicit positive response to stock prices.

Our main results can be summarized as follows. Under suitable conditions, the FWC implies that a positive response to stock prices in a forward-looking interest rate rule enlarges the policy space where the equilibrium is determinate, and the fundamental minimal state variable representation is learnable. In this sense, the FWC can overturn the conventional wisdom discussed above. However, if the response is too strong, such policy can lead to a continuum of stationary sunspot equilibria with a learnable common factor representations. These results are robust to alternative specifications of the monetary policy rule, including contemporaneous rules or augmented forward-looking rules that respond to output. They also appear to be quantitatively more prominent in economies characterized by less competitive goods markets and/or more rigid labor markets.

This paper can be extended in different directions. For instance, one could introduce the Blanchard-Yaari structure in a New Keynesian model subject to credit frictions and a financial accelerator, and therefore evaluate the interaction between demand-side and supply-side asset price fluctuations. Airaudo et al. (2013) make some progress in this direction by adding a cost channel of monetary policy transmission and endogenous credit spreads to the model we have presented in this paper. In their environment, responding to stock prices also alleviates the equilibrium indeterminacy problem arising from the cost channel.

Another important issue is the design and implementation of an optimal monetary policy. In this paper, we have intentionally restricted our attention to instrumental interest-rate rules. A recent contribution by Nisticò (2011) shows that a second-order approximation to the welfare-relevant objective for a benevolent government implies a specific concern for financial stability, in addition to the traditional concern for output and inflation stabilization. In Airaudo et al. (2014), we use Nisticò's approximation to analyze the determinacy and learning properties of optimal policy rules.

A Appendix

A. Steady State

We focus on a steady-state equilibrium with zero inflation. We eliminate uncertainty by setting $Z_t = Z = 1$. As in the benchmark New-Keynesian model, at the steady state, real marginal costs satisfy $\frac{MC}{P} = (1-\tau)\frac{W}{P} = \frac{\epsilon-1}{\epsilon}$. We assume that the labor subsidy τ is set to make the real wage equal to the marginal productivity of labor: $\frac{W}{P} = 1$. This implies $\tau = \epsilon^{-1}$. Combining this with $\delta C = \frac{W}{P}(1-N)$ and $Y = N$, we obtain steady state output and hours worked: $Y = N = \frac{1}{1+\delta}$. This implies that that the inverse of the steady state Frisch elasticity of labor is equal to $\chi = \frac{1}{\delta}$. From the market clearing condition $Y = C$, we then have $C = \frac{1}{1+\delta}$. Given $D = Y(1-MC)$ and $\frac{MC}{P} = \frac{\epsilon-1}{\epsilon}$, we find the steady-state dividends: $D = \epsilon^{-1}Y$.

From the aggregate Euler equation (16) and the non-arbitrage condition (19), we obtain:

$$\beta(1+r) = 1 + \gamma\left(\frac{1}{1-\gamma} - \beta\right)\frac{\Omega}{PC}, \tag{A.1}$$

where r is the net real interest rate and $\frac{\Omega}{PC}$ is the steady-state financial wealth to consumption ratio. From equations (12), the steady-state definition of financial wealth $\Omega = P(Q+D)$, the expression for dividends $D = \epsilon^{-1}Y$, and the market-clearing condition $Y = C$, simple algebra gives that $\frac{\Omega}{PC} = \frac{1+r}{\epsilon r}$, which in turn can be substituted into (A.1) to obtain $\beta(1+r) = 1+\gamma\left(\frac{1}{1-\gamma} - \beta\right)\frac{1+r}{\epsilon r}$. This corresponds to a quadratic equation on r which has the following *unique* positive solution:

$$r = \frac{(1-\gamma)(1-\beta) + \frac{\gamma[1-\beta(1-\gamma)]}{\epsilon} + \sqrt{\Psi}}{2\beta(1-\gamma)} \tag{A.2}$$

with

$$\Psi \equiv \left[(1-\gamma)(1-\beta) + \frac{\gamma[1-\beta(1-\gamma)]}{\epsilon}\right]^2 + 4\frac{\gamma[1-\beta(1-\gamma)]\beta(1-\gamma)}{\epsilon} > 0.$$

Given r we can retrieve the remaining steady state values: $Q = \frac{Y}{r\epsilon}$ and $\frac{\Omega}{P} = \frac{Y}{\epsilon}\left(\frac{1+r}{r}\right)$. By straightforward calculus, we can also establish that r is strictly increasing in the turnover rate γ, but strictly decreasing in ϵ.

B. Methodology

Considering the system in (27) with $x_t = [y_t, \pi_t, q_t]'$, the determinacy of equilibrium analysis employs the standard procedure of Blanchard and Khan (1980). Since none of the three endogenous variables is predetermined, the Rational Expectations Equilibrium (REE) is locally determinate if and only if all eigenvalues of the Jacobian Γ lie inside the unit circle in the complex plane.

The learnability analysis follows Evans and Honkapohja (2001). Agents are no longer endowed with rational expectations and are assumed for make forecasts based on simple adaptive learning rules. We adopt the *Euler equation (EE)* learning approach proposed by these authors: agents' decision rules are based on the first order conditions obtained from solving their dynamic optimization problem.[37] We focus, in particular, on Expectational Stability (E-stability) as a learning criterion: a representation of an equilibrium is learnable if it is E-stable. Under E-stability, recursive least-squares learning is in fact locally convergent to the REE, under general conditions. We analyze the E-stability of (i) the Minimal State Variable representation of fundamental equilibria (MSV-FE) and (ii) the Common Factor representations of stationary sunspot equilibria (CF-SSE).

E-Stability of MSV-FE Assume agents follow a Perceived Law of Motion (PLM) which has the same functional form of the true rational expectations solution: $x_t = \mathcal{N} z_t$, where $\mathcal{N}$ is unknown. Iterating forward the PLM and using it to eliminate all the forecasts in the model (27), we obtain the implied Actual Law of Motion (ALM): $x_t = \Gamma\mathcal{A} + (\Gamma\mathcal{N}\rho_z + \Theta)z_t$. Using standard notation, we have a T-mapping $T(\mathcal{A},\mathcal{N}) = (\mathcal{A}^A, \mathcal{N}^A)$, where $\mathcal{A}^A = \Gamma\mathcal{A}$ and $\mathcal{N}^A = (\Gamma\mathcal{N}\rho_z + \Theta)$. The fixed points of this mapping are the REE of the economy. The MSV-FE is E-stable if all the eigenvalues of the matrices $DT_{\mathcal{A}} = \Gamma$ and $DT_{\mathcal{N}} = \rho_z\Gamma$, evaluated at the REE fixed point, have real parts less than one; otherwise, the MSV-FE is E-unstable. Since $|\rho_z| < 1$, the MSV-FE is E-stable if the matrix Γ has all eigenvalues with real parts less than one. Or, equivalently, if the matrix $M \equiv \Gamma - I$ has all roots with negative real parts.

E-Stability of CF-SSE For policy parameterizations leading to indeterminacy, we study the learnability of CF-SSE. We restrict to the case of real roots. In this case, the PLM is $x_t = \mathcal{N} z_t + \mathcal{G}\zeta_t$, where ζ_t is a sunspot shock, and $\mathcal{N}$, and $\mathcal{G}$ are unknown. Suppose indeterminacy is of order one—i.e., in our model that means that Ω possesses one real root outside the unit circle, say ϱ_1—then ζ_t is a univariate stationary "sunspot" following the process $\zeta_t = \rho_\zeta\zeta_{t-1} + \varpi_t$ where $\rho_\zeta \equiv \frac{1}{\varrho_1}$ and ϖ_t is an arbitrary martingale difference sequence. As for the MSV-FE, we can use the CF-related PLM and the system in (27) to derive the T-mapping $T(\mathcal{A},\mathcal{N},\mathcal{G}) = (\mathcal{A}^A, \mathcal{N}^A, \mathcal{G}^A)$, where $\mathcal{A}^A = \Gamma\mathcal{A}$, $\mathcal{N}^A = (\Gamma\mathcal{N}\rho_z + \Theta)$ and $\mathcal{G}^A = \Gamma\mathcal{G}\rho_\zeta$. E-stability of the CF representations require all the eigenvalues of the matrices $DT_{\mathcal{A}} = \Gamma$, $DT_{\mathcal{N}} = \rho_z\Gamma$ and $DT_{\mathcal{G}} = \rho_\zeta\Gamma$ to have real parts less than one. Since both ρ_ζ and ρ_z are inside the unit circle, the CF-SSE is E-stable if the matrix Γ in (27) has all eigenvalues with real parts less than one. Because of the lack of lagged endogenous variables in the reduced form system, the E-stability conditions for the CF-SSE coincide with the E-stability condition for the MSV-FE.[38] This result also holds when indeterminacy is of order higher than one. However, in that case, the sunspot ζ_t would follows a VAR process.

[37] Preston (2006) proposes the alternative *Infinite Horizon (IH)* approach, which does not necessarily lead to the same conclusions one would get under the EE approach. We plan to explore the consequences of having IH learning in our model in a separate project.

[38] This is clearly not the case in linear systems with lagged variables. See Evans and McGough (2005b) for a general discussion.

C. Proofs

C.1. Proof of Proposition 1

Consider the reduced-form equilibrium system (27) where by setting the turnover rate $\gamma = 0$—such that, $\psi = 0$ and $\tilde{\beta} = \beta$—we obtain that

$$\Gamma \equiv \begin{bmatrix} 1 & 1-\phi_\pi & -\phi_q \\ \kappa & \beta+\kappa(1-\phi_\pi) & -\kappa\phi_q \\ -\lambda & 1-\phi_\pi & \beta-\phi_q \end{bmatrix}, \tag{A.3}$$

while Θ is a conformable matrix whose specification is not needed for the analysis.

Determinacy Since all variables in $x_t = [y_t, \pi_t, q_t]'$ are non-predetermined, the REE is locally determinate if and only if all eigenvalues of Γ are within the unit circle in the complex plane. The characteristic polynomial of Γ is $P(e) = e^3 - Tr(\Gamma)e^2 + S_2(\Gamma)e - Det(\Gamma) = 0$ where $S_2(\Gamma)$ is the sum of the 2x2 principal minors. By simple algebra we have that $Det(\Gamma) = \beta\left[\beta - \phi_q(1+\lambda)\right]$, $Tr(\Gamma) = 1 + \beta + \kappa(1-\phi_\pi) + \beta - \phi_q$ and $S_2(\Gamma) = \beta(1+\beta) + \beta - \phi_q - \beta\phi_q - \lambda\phi_q + \beta\kappa(1-\phi_\pi)$. Simple algebra shows that one root of $P(e) = 0$ is real and equal to $\beta \in (0,1)$. This allows us to write the characteristic polynomial as $P(e) = (e-\beta)\tilde{P}(e) = 0$ where $\tilde{P}(e) \equiv \left(e^2 + a_1 e + a_2\right) = 0$, with $a_1 = \phi_q - 1 - \beta - \kappa(1-\phi_\pi)$ and $a_2 = \beta - \phi_q(1+\lambda)$. All roots of $\tilde{P}(e) = 0$ are then within the unit circle if and only if *a)* $\tilde{P}(1) = 1 + a_1 + a_2 > 0$, *b)* $\tilde{P}(-1) = 1 - a_1 + a_2 > 0$ and *c)* $|a_2| < 1$. Simple manipulation shows that these conditions are equivalent to those spelled in (28) in the proposition.

E-Stability of MSV-FE. Consider Γ defined in (A.3). From the discussion in Appendix B, for the MSV-FE to be E-stable all eigenvalues of the matrix $M \equiv \Gamma - I$ need to have negative real parts. Simple algebra shows that one of the eigenvalues of M is real and equal to $\beta - 1 < 0$. The characteristic polynomial of M can then be written as $P(e) = (\beta - 1)\tilde{P}(e) = 0$ where $\tilde{P}(e) = e^2 + a_1 e + a_2 = 0$ for $a_1 = (1-\beta) + \kappa(\phi_\pi - 1) + \phi_q$ and $a_2 = \left[\kappa(\phi_\pi - 1) - \lambda\phi_q\right]$. Applying standard results from matrix algebra, all roots of M have then negative real parts if and only if $a_2 > 0$ and $a_1 > 0$. Clearly, $a_2 > 0$ if and only if $\phi_\pi > 1 + \frac{\lambda}{\kappa}\phi_q$. Moreover, it is immediate to see that if $a_2 > 0$ then $a_1 > 0$ as well. Hence, all roots have negative real parts and the MSV-FE is E-stable if and only if $\phi_\pi > 1 + \frac{\lambda}{\kappa}\phi_q$, for any $\phi_q \geq 0$.

E-Stability of CF-SSE From the proof of determinacy, recall the characteristic polynomial of matrix Γ: $P(e) = (e-\beta)\tilde{P}(e) = 0$. Let $\phi_q^* \equiv \frac{(1+\beta)}{1+\lambda}$. From the determinacy conditions in (28), we know that there is indeterminacy for $\phi_\pi > 1 + \frac{2(1+\beta)}{\kappa} - \frac{2+\lambda}{\kappa}\phi_q$ and for $\phi_\pi < 1 + \frac{\lambda}{\kappa}\phi_q$ when $\phi_q < \phi_q^*$, and for any $\phi_\pi \geq 0$ when $\phi_q \geq \phi_q^*$. Moreover, since within these ranges we also have that $\tilde{P}(1) < 0$ or $\tilde{P}(-1) < 0$, then all eigenvalues of Γ are real. From Appendix B, it follows that a E-stable Common Factor representation of a stationary sunspot equilibrium (CF-SSE) exists within the policy space where the MSV-FE is E-stable as well. The conditions stated in the proposition immediately follow after noticing that $1 + \frac{2(1+\beta)}{\kappa} - \frac{2+\lambda}{\kappa}\phi_q \gtreqless 1 + \frac{\lambda}{\kappa}\phi_q$ for $\phi_q \lesseqgtr \phi_q^*$.

C.2. Proof of Proposition 2

Consider the reduced-form equilibrium system (27) where,

$$\Gamma \equiv \begin{bmatrix} \frac{1-\psi\lambda}{1+\psi} & 1-\phi_\pi & \frac{\psi}{1+\psi}\tilde{\beta}-\phi_q \\ \kappa\frac{1-\psi\lambda}{1+\psi} & \tilde{\beta}+\kappa(1-\phi_\pi) & \kappa\left(\frac{\psi}{1+\psi}\tilde{\beta}-\phi_q\right) \\ -\lambda & 1-\phi_\pi & \tilde{\beta}-\phi_q \end{bmatrix}, \tag{A.4}$$

and Θ is a conformable matrix whose forms is not necessary for our analysis. From the discussion in Appendix B, the MSV-REE is learnable (in the E-stability sense) if all eigenvalues of matrix $M \equiv \Gamma - I$ have negative real parts. By the Routh Theorem, all roots of M have negative real parts if and only if the following three conditions are all satisfied: $Det(M) < 0$, $Tr(M) < 0$ and $S_2(M)\,Tr(M) - Det(M) < 0$, where $S_2(M)$ is the sum of the 2x2 principal minors of M. After simple algebra we obtain that:

$$Det(M) = \left(\tilde{\beta}-1\right)\left[\frac{\psi\left(1-\tilde{\beta}+\lambda\right)-\phi_q(\lambda-\psi)}{1+\psi}\right]+\kappa(1-\phi_\pi)\left(1-\frac{\tilde{\beta}}{1+\psi}\right), \tag{A.5}$$

$$Tr(M) = -\frac{\psi}{1+\psi}(1+\lambda)+2\left(\tilde{\beta}-1\right)-\phi_q+\kappa(1-\phi_\pi), \tag{A.6}$$

and

$$\begin{aligned} S_2(M) &= \kappa(\phi_\pi-1)\left(2-\frac{\tilde{\beta}}{1+\psi}\right)+\frac{\psi\left(1-\tilde{\beta}+\lambda\right)-\phi_q(\lambda-\psi)}{1+\psi} \\ &\quad +\left(\tilde{\beta}-1\right)\left[-\frac{\psi}{1+\psi}(1+\lambda)+\tilde{\beta}-1-\phi_q\right]. \end{aligned} \tag{A.7}$$

Using (A.6)-(A.7), after some manipulations, we get

$$\left(\tilde{\beta}-1\right)S_2(M) = \left(\tilde{\beta}-1\right)^2\left[Tr(M)-\left(\tilde{\beta}-1\right)\right]+Det(M)-\kappa(1-\phi_\pi)\tilde{\beta}^2\frac{\psi}{1+\psi},$$

from which the term $S_2(M)\,Tr(M) - Det(M)$ can be written as:

$$\begin{aligned} S_2(M)\,Tr(M)-Det(M) &= S_2(M)\left[Tr(M)-\left(\tilde{\beta}-1\right)\right] \\ &\quad +\left\{\left(\tilde{\beta}-1\right)^2\left[Tr(M)-\left(\tilde{\beta}-1\right)\right]-\kappa(1-\phi_\pi)\tilde{\beta}^2\frac{\psi}{1+\psi}\right\}. \end{aligned} \tag{A.8}$$

Given that $Det(M) < 0$ and $Tr(M) < 0$, it follows that $S_2(M) > 0$ is a necessary condition for $S_2(M)\,Tr(M) - Det(M) < 0$ to hold. From the definitions (A.5)-(A.7), the inequalities $Det(M) < 0, Tr(M) < 0$ and $S_2(M) > 0$ are written as:

$$\phi_\pi > 1-\psi\frac{\left(1-\tilde{\beta}\right)\left(1-\tilde{\beta}+\lambda\right)}{\kappa\left(1-\tilde{\beta}+\psi\right)}+\frac{\left(1-\tilde{\beta}\right)(\lambda-\psi)}{\kappa\left(1+\psi-\tilde{\beta}\right)}\phi_q \equiv \Phi^b\left(\phi_q\right), \tag{A.9}$$

$$\phi_\pi > 1 - \frac{\frac{\psi}{1+\psi}(1+\lambda) + 2\left(1-\tilde{\beta}\right)}{\kappa} - \frac{\phi_q}{\kappa} \equiv \Phi^d\left(\phi_q\right), \tag{A.10}$$

and

$$\phi_\pi > 1 - \frac{\psi\left(1-\tilde{\beta}+\lambda\right) + \left(1-\tilde{\beta}\right)(1+\psi)\left[\psi\frac{1+\lambda}{1+\psi} + 1 - \tilde{\beta}\right]}{\kappa\left[2(1+\psi) - \tilde{\beta}\right]} + \frac{[\lambda - \lambda^e]}{\kappa\left[2(1+\psi) - \tilde{\beta}\right]}\phi_q \equiv \Phi^e\left(\phi_q\right) \tag{A.11}$$

where $\lambda^e \equiv \psi + \left(1-\tilde{\beta}\right)(1+\psi) > \psi$.

Consider (A.9). A sufficient condition for the inequality to hold is that $\phi_\pi > 1 + \frac{\left(1-\tilde{\beta}\right)(\lambda-\psi)}{\kappa\left(1+\psi-\tilde{\beta}\right)}\phi_q \equiv \Phi_\gamma\left(\phi_q\right)$. The slope of $\Phi_\gamma\left(\phi_q\right)$ clearly depends on the sign of $\lambda - \psi$. From the definition of the steady state in Appendix A and its definition in (21), ψ can also be written as $\psi = \beta(1+r) - 1$. By combining the latter with definition of λ in (25) and $\tilde{\beta} \equiv \frac{\beta}{1+\psi}$, we obtain that $\psi > \lambda$ if and only if:

$$[\beta(1+r) - 1]\frac{1+r}{r} \gtreqless [(\epsilon-1)(1+\chi) - 1]. \tag{A.12}$$

From the definition of r in (A.2), simple calculus shows that the left hand side of (A.12) is equal to zero for $\gamma = 0$, it is strictly increasing in γ and tends to infinity as γ goes to unity. As long as the right hand side of (A.12) is positive (which simply requires $\epsilon > \frac{2+\chi}{1+\chi}$, a restriction that, for instance, always holds for $\epsilon > 2$, independently from the value assigned to the inverse Frisch elasticity of labor χ) there exists a unique $\gamma^* \in (0,1)$ such that $\psi > \lambda$ for $\gamma > \gamma^*$.

Let's assume that the turnover rate is larger than the threshold γ^*. Then, since $\lambda^e > \psi > \lambda$, it immediately follows that $\Phi_\gamma\left(\phi_q\right)$, $\Phi^d\left(\phi_q\right)$ and $\Phi^e\left(\phi_q\right)$ are all strictly decreasing in ϕ_q. Moreover, simple but tedious algebra shows that $\Phi_\gamma\left(\phi_q\right) > \max\left\{\Phi^d\left(\phi_q\right), \Phi^e\left(\phi_q\right)\right\}$ for any $\phi_q \geq 0$.[39] This makes conditions (A.10) and (A.11) redundant, such that $\phi_\pi > \Phi_\gamma\left(\phi_q\right)$ is a sufficient condition for $Det(M) < 0$, $Tr(M) < 0$ and $S_2(M) > 0$. It remains to show that $S_2(M)\,Tr(M) - Det(M) < 0$ holds as well. To do that, consider (A.8). Using the definition of $Tr(M)$, after simple algebra, we obtain the following equivalence:

$$Tr(M) - \left(\tilde{\beta} - 1\right) < 0 \Leftrightarrow \phi_\pi > 1 - \frac{\frac{\psi}{1+\psi}(1+\lambda) + 1 - \tilde{\beta}}{\kappa} - \frac{\phi_q}{\kappa} \equiv \Phi^f\left(\phi_q\right),$$

where $\Phi^f\left(\phi_q\right) < \Phi_\gamma\left(\phi_q\right)$ for any $\phi_q \geq 0$. As a consequence, if $\phi_\pi > \Phi_\gamma\left(\phi_q\right)$, then $Tr(M) - \left(\tilde{\beta}-1\right) < 0$, which combined with the previous result $S_2(M) > 0$, implies that $S_2(M)\left[Tr(M) - \left(\tilde{\beta}-1\right)\right] < 0$ and $\left(\tilde{\beta}-1\right)^2\left[Tr(M) - \left(\tilde{\beta}-1\right)\right] < 0$. Extended algebra shows that these last two inequalities are sufficient to imply that $S_2(M)\,Tr(M) - Det(M) < 0$ for any $\phi_\pi > \Phi_\gamma\left(\phi_q\right)$. It then follows that, for $\gamma > \gamma^*$, the condition $\phi_\pi > \Phi_\gamma\left(\phi_q\right)$ is sufficient for all roots of M have negative real parts, and therefore for the MSV-FE to be learnable.

[39] The function $\Phi_\gamma\left(\phi_q\right)$ has the highest intercept and is the flattest among the three.

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