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Optimal maturity structure of sovereign debt in situation of near default

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Optimal maturity structure of sovereign debt in situation of near default

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Abstract

We study the relationship between default and the maturity structure of the debt portfolio of a Sovereign, under uncertainty. The Sovereign faces a trade-off between a future costly default and a high current fiscal effort. This results into a debt crisis in case a large initial issuance of long term debt is followed by a sequence of negative macro shocks. Prior uncertainty about future fundamentals is then a source of default through its effect on long term interest rates and the optimal debt issuance. Intuitively, the Sovereign chooses a portfolio implying a risk of default because this risk generates a correlation between the future value of long term debt and future fundamentals. Long term debt serves as a hedging instrument against the risk on fundamentals. When expected fundamentals are high, the Sovereign issues a large amount of long term debt, the expected default probability increases, and so does the long term interest rate.

JEL Classification Numbers: D84, F34, H63
Keywords: Long Term Debt; Maturity Structure; Optimal Default; Rational Expectations; Sovereign Debt Crisis; Uncertainty

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CONTENTS

I. INTRODUCTION

We study the maturity structure of the portfolio of debt of a Sovereign which may find itself in a situation of near-default. The Sovereign faces a trade-off between the default cost and the fiscal effort (positive primary balances) necessary to meet its financial obligations. In particular, after a negative macroeconomic shock, the Sovereign may find too costly to exert a high fiscal effort and may prefer to let the debt burden increase. The Sovereign may then be in situation of near-default or financial fragility: An additional negative macroeconomic shock in the following period will make it impossible for the Sovereign to reimburse the debt that comes due, and therefore a necessity to default on its debt.

We analyze how the maturity structure of the portfolio of debt of the Sovereign is related with the occurrence of such a situation of financial fragility: What choice of maturity structure does lead to financial fragility? How is the choice of a maturity structure and the perspective of default influenced by incoming market news and information produced by financial analysts, as summarized by ex-ante prior uncertainty about future fundamentals? In particular, do optimistic news always lead market participants to be more confident in the ability of the Sovereign to repay its debt?

We consider a model focussing on the financing decisions of the Sovereign, namely a 3 period model where the Sovereign (*i*) issues both short term (ST) and long term (LT) debt in the initial period, (*ii*) exerts a fiscal effort and/or rolls over ST debt in the intermediate period, and (*iii*) reimburses its debt if macroeconomic fundamentals are good enough, or default on its debt in the last period. Financial markets are perfectly competitive, investors (who buy the debt issued by the Sovereign) are risk neutral, and information is symmetric. Hence, high interest rates and debt crisis in our model do not follow from a risk premium required by investors, a lack of liquidity, or a coordination issue among investors (like it is the case in models of self-fulfilling debt crisis). High interest rates reflect a high expected default probability (Arellano and Ramanarayanan (2012) provide empirical evidence that the movements in the expected default probability are the main determinants of interest rate changes).

Default can occur in the last period only. It is not a direct decision of the Sovereign, but the consequence of previous financing decisions made by the Sovereign prior to the date of default: the Sovereign chooses to take the risk of a default in the final period when the financing need in the intermediate period is so large that a fiscal effort is too costly to cover this need and the Sovereign prefers to rollover ST debt. In summary, the Sovereign decides (or not) to "potentially default", *i.e.*, to be in a position that will lead to a default in the final period in case of a negative macroeconomic shock.

In the model, the question of the maturity structure corresponds to a choice between two kinds of debt (in the initial period). Short term (ST) debt corresponds to a very short term horizon, where there is no default risk, while long term (LT) debt corresponds to a longer horizon where the default is possible if the fundamentals are bad and the fiscal effort of the Sovereign is insufficient. Hence the default risk is related to LT debt only. Issuing LT debt can be useful for the Sovereign because it provides insurance against future bad shocks (which will raise interest rates in the next period), but it is more costly than ST debt because of the risk of default.

A. RESULTS

A first result is that the debt structure (together with the fundamentals) influences the level of fiscal efforts exerted by the Sovereign and the occurrence of default. The debt mix initially issued by the Sovereign modifies its incentive to exert a fiscal effort in the intermediate period: A large amount of LT debt and poor macroeconomics fundamentals lead to potential default due to an insufficient fiscal effort (*i.e.*, default in the final period if the fundamentals in that period are low). This result is consistent with the idea (Jeanne 2009) that LT debt creates an incentive to default: ST financing disciplines the Sovereign in the intermediate period better than LT financing does. Intuitively, when the initial choice of the maturity structure mainly involves the issuance of LT debt, the need for rolling over ST debt in the intermediate period is small. In the intermediate period, an increase in the expected probability of default, which increases the interest rate, is therefore not very costly (it bears only on debt issued in that period). Thus there is a high incentive for the Sovereign to decrease the effort in the intermediate period at the cost of issuing more debt at a high interest rate (and to be subsequently exposed to default).

We qualify this incentive effect of LT debt by showing that there is a partition of the fundamentals in 3 regions. In the two extreme regions (low and high), the occurrence of default does not depend on the maturity structure of the debt portfolio (whatever the amount of LT debt issued, the effort level always determines no default/potential default when the fundamentals are in the high/low region). Only in the region of moderate values is the occurrence of default a function of the maturity structure.

Our main result is that prior uncertainty about fundamentals is a source of default: Insolvency of the Sovereign is not solely determined by the sequence of fundamentals. Indeed, prior uncertainty about fundamentals affects interest rates, which affects the choice of the optimal portfolio. This in turn determines the choice of default in the following period. Notably, when it is initially known that fundamentals will be moderate, the Sovereign chooses a portfolio which provides the incentives not to default. With uncertainty, the optimal portfolio sometimes leads to the choice of potential default for these same fundamentals.

Uncertainty about macroeconomic fundamentals generates a need for the Sovereign to hedge its portfolio against this risk. LT debt sometimes plays the role of a hedging tool. This is possible only if the price of LT debt (the interest rate) is correlated with fundamentals. Since the interest rate is determined by the expected probability of default, the correlation between interest rates and fundamentals requires that the Sovereign be exposed to default when fundamentals are weak (interest rates are high/low when the default probability is high/low, which corresponds to weak/strong fundamentals).

The cost of default is then the price to pay by the Sovereign to get access to a hedging tool. Usually a hedging strategy is meant to avoid default. Yet, in this paper, the hedging strategy leads to default under certain fundamentals: The choice of default for some fundamentals is what allows the Sovereign to gain access to a hedging strategy against the risk on future fundamentals. Market incompleteness (in particular, the non existence of indexed bonds) is what constrains the Sovereign to make such a choice.

A striking result of comparative statics states that a more optimistic initial belief about future fundamentals leads to higher interest rates. Indeed, default occurs after a sequence of negative macroeconomic shocks in the case when a large amount of LT debt has been issued. Consequently, a large amount of LT debt can be issued only at a LT interest rate reflecting the associated positive expected probability of default. When the probability of a sequence of negative macroeconomic shocks is low (optimism), the Sovereign chooses to issue an amount of LT debt generating default: the LT interest rate is larger than the risk free rate. Conversely, when the probability of a sequence of negative macroeconomic shocks is high (pessimism), a large amount of LT debt could only be issued at a very high interest rate reflecting the very high expected probability of default. Then, the Sovereign issues a limited amount of LT debt that provides, at the interim stage, incentives to exert a fiscal effort sufficient to avoid default, and the LT interest rate remains low (it is the risk free rate).

B. IMPLICATIONS FOR POLICY ISSUES

This model is one of rational debt crises, which shows that the markets are not able to manage extreme risks. An international institution can perform this function, by offering contracts distinct from the contracts offered by the markets. Indeed, contracts that involve a sequence of ST debt, conditional on the fiscal effort of the Sovereign, may provide appropriate incentives to the Sovereign to exert a sufficient fiscal effort and meet its obligations.

A contract involving a commitment by the international financial institution to roll over ST debt, conditionally on the Sovereign exerting a given effort level, is a contract whose value is correlated with fundamentals (through the effort cost, that depends on the fundamentals). Indeed, a difference between such a contract and LT debt as defined in this paper is that international financial institutions offer these contracts at no additional costs (other than the cost of exerting the effort level required by the contract), while the issuance of LT debt is associated with a risk of costly default reflected in the interest rate. This suggests a theoretical underpinning for the role played by international financial institutions: International financial institutions, by offering contracts involving conditionality, make the exposure to default risk less attractive to the Sovereign.

C. LITERATURE

Our paper belongs to a set of theoretical papers on LT debt and debt crisis with an endogenous maturity structure associated with the idea that LT debt is issued in normal times, while ST debt is only issued in times of financial distress (Arellano 2008, Arellano and Ramanarayanan 2012, Broner, Lorenzoni and Schmukler 2007). The closest papers to ours are Arellano (2008) and Conesa and Kehoe (2012, 2014). With respect to the former, the main distinction is that we focus on the optimal portfolio choice problem of the Sovereign and in particular the hedging role of the LT debt instrument. Conesa and Kehoe (2012, 2014) develop a model of self-fulfilling crises in which, under certain conditions, the government chooses to "gamble for redemption" (lowering its fiscal efforts and increasing its debt which increases its vulnerability to crises). This "gamble" is analogous to potential default in our paper (choice of being exposed to a crisis with positive probability) but the timing and the structure of the model is significantly different.

Another closely related paper is Cole and Kehoe (1996). They show that when fundamentals are moderate, the occurrence of a debt crisis depends on uncertainty. This uncertainty is a sunspot, not correlated with the expectations about fundamentals, and the crisis results from a coordination problem: A crisis occurs when investors stop rolling over Sovereign debt (while no default occurs if debt is rolled over). In our paper, the uncertainty driving the conditions of default is uncertainty about fundamentals, and we abstract from coordination issues: A debt crisis occurs only when the Sovereign is not solvent.

In Broner, Lorenzoni and Schmukler (2013), the optimal maturity structure is linked to the risk premium required by risk averse investors to hold the debt: there is an arbitrage between paying the risk premium to get LT financing and paying the fiscal adjustment required at the intermediate period by ST financing (in order to rollover ST financing). Our model does not require risk averse investors.

Jeanne (2009) provides a theoretical understanding of the incentive role played by LT debt. ST debt disciplines the Sovereign but creates the risk of a roll-over crisis due to the large number of uncoordinated investors. Jeanne *et al.* (2008) provides insights about the role of the IMF. One distinguishing feature of our results is that default in our paper is related to optimistic expectations about fundamentals.

Buera and Nicolini (2004) analyze the maturity structure as a substitute to state contingent bonds, but do not include default. It is related to issues of market incompleteness considered in our paper. Grossman and Van Huyck (1988) are an early reference on defaultable Sovereign debt as contingent claims.

A number of recent papers have dealt with coordination issues. Our paper does not: there is always a unique equilibrium and default is not due to coordination failure among creditors but to insolvency of the Sovereign. Morris and Shin (2004) study the correlation between macroeconomic fundamentals and default of a Sovereign due to the inability of the Sovereign to rollover existing debt (because investors expect not to be reimbursed in the future). This is a global game where the crisis is due to a coordination issue among investors triggered by informational asymmetries (which implies no common knowledge of actions). See also Chamley (2004) and Morris and Shin (2006) for coordination issues under incomplete information.

A number of empirical papers have studied the link between interest rates and the supply of sovereign bonds (Challe *et al.* (2012), Laubach (2009), Longstaff (2004)). Others have looked at the relationship between a Sovereign's fiscal situation and the slope of the yield curve (Reinhard and Sack (2000), Dai and Philippon (2006)). Other papers (Hatchondo *et al.*

(2009, 2010, 2012)) look respectively at the relationship between bond duration and sovereign default, debt dilution and interest rate spreads paid by the sovereigns, and sovereign debt duration, rollover risk and debt dilution.

Restructuring sovereign debt is the subject of a number of papers (Ghosal and Miller (2003), Jeanne *et al.* (2008), Bolton and Jeanne (2009), and the references therein among others). The former papers show that a bankruptcy regime for sovereigns may complete the set of incomplete sovereign debt contracts, while the latter paper addresses the question of seniority among creditors.

The paper is organized as follows. Section II presents the model and the equilibrium debt prices as a function of investors' expectations. The model is then solved backward: It is first solved at the intermediate period (Section III), then solved at the initial period. Section IV states the main results (conditions such that prior uncertainty leads to default), and Section V describes the optimal portfolio. Section VI concludes and discusses the role played by international financial institutions. The proofs are gathered in an Appendix.

II. THE MODEL

Consider a model with 3 periods $t = 0, 1, 2$ and two types of agents: investors and a Sovereign.

The Sovereign has a financing need at $t = 0$ (it carries over a stock D of debt). This need is covered in two different ways:

- issuing debt: It issues both ST debt (lasting 1 period) at $t = 0$ and $t = 1$ and LT debt (lasting 2 periods) at $t = 0$,
- exerting a fiscal effort (primary balances) at $t = 1$ and $t = 2$.

There is a continuum of competitive risk neutral investors. This is a simplifying assumption (detailed below) so that the price of debt is its discounted expected future value.

Information structure. Information about macroeconomic fundamentals at $t = 1$ and $t = 2$ is incomplete but symmetric (among investors and Sovereign). Uncertainty about fundamentals at $t = 1, 2$ is summarized by a real variable θ_t. The values of θ_1 are described in Section IV. θ_2 can take 2 values θ_2^L and θ_2^H (with $0 < \theta_2^L < \theta_2^H$). θ_1 and θ_2 are not correlated. In the initial period 0, all the agents (investors and the Sovereign) assign a prior probability π_t^s to the event $\theta_t = \theta_t^s$ for $t = 1, 2$ and every state s. At the beginning of $t = 1$, θ_1 is made public and agents have no further information about θ_2 (and hence they do not revise their prior probabilities on θ_2). At the beginning of $t = 2$, θ_2 is made public. In addition, θ_t affects the fiscal efforts as described below.

The objective of the Sovereign. The Sovereign minimizes the cost of the fiscal efforts $e_t \geq 0$ at $t = 1, 2$ subject to budget constraints (detailed below). We assume a quadratic cost function:

$$\min E\left(\frac{e_1^2}{\theta_1}+e_2^2\right).$$

This objective calls for 2 comments:

- A quadratic cost is a simple example of a cost function c with the following convexity properties: $\frac{\partial^2 c}{\partial\theta_1\partial e_1}<0$ so that the optimal e_1 is increasing in θ_1. Hence, high θ_1 is such that the cost is low and it is easy to exert a high effort.
- One may prefer an objective symmetric in θ_t (namely $E\left(\frac{e_1^2}{\theta_1}+\frac{e_2^2}{\theta_2}\right)$). We have verified that this corresponds to the same model but for the interpretation of θ_2 in relation with the conditions for default (see below). We prefer the first presentation.

Definition of default. Assume that the Sovereign reimburses the debt whenever it can. Assume there is no default at $t=1$ (the fiscal effort e_1 always covers the financing need). Default occurs at $t=2$ whenever the fiscal effort e_2 is not large enough to repay the debt (e_2 is exogenously bounded, see below).
The definition of default relies on the following interpretation of ST and LT debts. The horizon that corresponds to period $t=1$ is short enough that agents at $t=0$ face no uncertainty regarding the Sovereign's ability to repay its financial obligations that come due at $t=1$.[2] The horizon that corresponds to period $t=2$ is such that agents at $t=0$ (and $t=1$) consider default as an event that cannot be excluded.
In case of default, no debt is reimbursed, the Sovereign must pay an exogenous penalty cost. There is no renegotiation, roll-over or restructuring activated as default takes place.

Timing.

- At $t=0$, the Sovereign issues ST_0 and LT to cover an exogenous debt D:
$$D=p_0^{'}ST_0+p_0LT, \tag{1}$$
where ST_0 and LT are the face values of ST and LT debt respectively (p_0' and p_0 are the market prices of one unit of ST and LT debt respectively).
- At $t=1$, the Sovereign reimburses ST_0 by issuing ST_1 and exerting a fiscal effort e_1:
$$ST_0=e_1+p_1ST_1,$$
where ST_1 is the face value of ST debt issued at $t=1$ (p_1 is the market price of one unit of debt at $t=1$).
- At $t=2$, the Sovereign reimburses $LT+ST_1$ whenever possible:

[2] We may relax this assumption by considering default at $t=1$ (the conditions of default being analogously defined as in $t=2$). We may then consider an equilibrium where parameters do not lead to default at $t=1$. We choose the simpler model presented here.

- if $LT + ST_1 \leq \theta_2$, there is no default: $e_2 = LT + ST_1$,
- if $LT + ST_1 > \theta_2$, there is default: $e_2 = \sqrt{K}$ where $K > 0$ is the exogenous penalty cost (the cost at $t = 2$ is $e_2^2 = K$) and θ_2 can be interpreted as the maximum fiscal effort.

Comments. The model is kept as simple as possible: it is a finite horizon model, with default possible in the last period only; the cost function is quadratic (so that it has all the desirable convexity properties).
The model focuses on the trade-off for the Sovereign between the current fiscal effort and the future default cost:

- At $t = 2$, the Sovereign does not decide to default or not (the debt is always repaid when possible)
- The Sovereign strategically chooses at $t = 0$ and $t = 1$ (not) to be in a position that leads to default at $t = 2$ (typically when θ_2 takes the low value θ_2^L)

The investors are "fictitious" agents in the model. This is a way to model a supply of funds that is infinitely elastic at a price corresponding to the discounted expected value of the debt. The model calls for the following additional comments:

- Two parameters are related to default: θ_2 determines if there is default or not, while K is the cost of default (and may include "non pecuniary" costs of various kinds: reputational, political,...).
- The Sovereign is not concerned with the value of LT at $t = 1$. One may consider that there is a secondary market for LT debt at $t = 1$. The associated price is p_1 since at $t = 1$ the LT debt and the newly issued ST debt ST_1 are equivalent assets (there is no seniority consideration). The Sovereign does not participate in this market.
- The finite horizon assumption is a shortcut for the following feature: perpetual rollover is not possible. There is one period where all the debt that is due must be repaid by means of positive primary balances. The debt cannot be financed by extra rollover (which corresponds to the idea that access to financial markets is limited).

Debt Pricing. Assume that there is a (positive) exogenous risk free interest rate r (over one period, r is constant over time). The investors have an unlimited access to borrowing and lending opportunities at rate r. Given the assumption of risk neutrality, the price of the debt issued by the Sovereign is always equal to its discounted expected future value.

At $t = 1$, denote π^D the probability (assessed at $t = 1$)[3] of a default at $t = 2$. Given that nothing is reimbursed in case of default, the price of one unit of debt (either ST_1 or LT) at $t = 1$ is:

$$p_1 = \frac{1 - \pi^D}{1 + r}. \tag{2}$$

[3]Under the assumptions of Rational Expectations and symmetric information, all the agents have the same expectation π^D.

There is only one price because there is no arbitrage and no seniority considerations apply. The price p_1 is endogenous: it depends on θ_1 through the default probability π^D: in equilibrium, the occurrence of default at $t = 2$ depends on the accumulated debt burden and in particular on the debt ST_1 issued in $t = 1$, while the decisions of the Sovereign at $t = 1$ (both the amount ST_1 issued and the effort e_1) depend on the macroeconomic fundamentals θ_1.

The two kinds of debt issued at $t = 0$ are priced as follows.

- The price of one unit of ST debt ST_0 is:

$$p_0' = \frac{1}{1+r},$$

  (as its $t = 1$ face value is exogenous, equal to 1, with no risk of default).

- The price of one unit of LT debt LT is:

$$p_0 = \frac{E(p_1)}{1+r} = \frac{1 - E(\pi^D)}{(1+r)^2}, \tag{3}$$

  where $E(\pi^D)$ is the expectation at $t = 0$ of the default probability. This expectation is computed as the mean of the values of π^D using the common prior about θ_1 (π^D is the expectation at $t = 1$ of the default probability).

At $t = 0$, the yield curve consists of the ST rate r and the one period expected return of LT debt (*i.e.*, $E(p_1)/p_0 = 1 + r$). It is flat because investors are risk neutral (there is no risk premium on the volatility of p_1) and we assume no default in period 1.[4]

The budget constraint (1) faced by the Sovereign at $t = 0$ writes:

$$D = \frac{1}{1+r} ST_0 + \frac{1 - E(\pi^D)}{(1+r)^2} LT. \tag{4}$$

III. THE CHOICE OF POTENTIAL DEFAULT IN THE INTERMEDIATE PERIOD

The trade-off between the current fiscal effort and the future default cost is realized in equilibrium as follows: the debt portfolio chosen at $t = 0$ provides incentives to exert a fiscal effort e_1 at $t = 1$ that leads (or not) to default at $t = 2$ in case of a sequence of negative shocks. The equilibrium describes how the maturity structure of the debt disciplines (or not) the efforts of the Sovereign.

[4] Under the assumption that investors are risk averse, in the case of a positive expected default probability (and only in this case), a risk premium would appear and p_0 would decrease. The price differential between this case and the no default case would increase. This should not affect the intuition of the results.

For this purpose, we solve backward for the behavior of the Sovereign. In this section, we solve for the optimal choice of the Sovereign at $t = 1$. The next section considers $t = 0$.

At $t = 1$, (ST_0, LT) and θ_1 are given. In equilibrium, the amount of debt ST_1 issued at $t = 1$ influences the probability π^D of default (occurring at $t = 2$) assessed at $t = 1$ as follows:

$$\begin{aligned}
\pi^D &= 0 \text{ if } ST_1 + LT \leq \theta_2^L, \\
\pi^D &= \pi_2^L \text{ if } \theta_2^L < ST_1 + LT \leq \theta_2^H, \\
\pi^D &= 1 \text{ if } ST_1 + LT > \theta_2^H.
\end{aligned}$$

Since $p_1 = \frac{1-\pi^D}{1+r}$, these values of π^D determine the supply curve faced by the Sovereign at $t = 1$ when issuing ST_1:

$$p_1 = \frac{1}{1+r} \text{ if } ST_1 \leq \theta_2^L - LT, \tag{5}$$

$$p_1 = \frac{\pi_2^H}{1+r} \text{ if } \theta_2^L - LT < ST_1 \leq \theta_2^H - LT, \tag{6}$$

$$p_1 = 0 \text{ if } ST_1 > \theta_2^H - LT. \tag{7}$$

This supply curve corresponds to the fact that investors are competitive: the Sovereign gets the lowest possible interest rate consistent with its repayment capacity and the investors supply any amount of funds at an interest rate consistent with the Sovereign's repayment capacity. In other words, we abstract from any coordination problem between investors and the Sovereign.[5]

The optimization problem of the Sovereign at $t = 1$ is to choose (ST_1, e_1) to minimize the cost function V, where

$$V = \frac{{e_1}^2}{\theta_1} + \pi_2^L e_2 \left(\theta_2^L\right)^2 + \pi_2^H e_2 \left(\theta_2^H\right)^2, \tag{8}$$

subject to

$$ST_0 = e_1 + p_1 ST_1, \tag{9}$$

$$e_1 \geq 0, \tag{10}$$

$$ST_1 + LT \geq 0, \tag{11}$$

$$e_2\left(\theta_2^L\right) = \begin{cases} ST_1 + LT & \text{if } ST_1 + LT \leq \theta_2^L, \\ \sqrt{K} & \text{otherwise,} \end{cases} \tag{12}$$

$$e_2\left(\theta_2^H\right) = \begin{cases} ST_1 + LT & \text{if } ST_1 + LT \leq \theta_2^H, \\ \sqrt{K} & \text{otherwise.} \end{cases} \tag{13}$$

$e_2\left(\theta_2^L\right)$ and $e_2\left(\theta_2^H\right)$ are the values of the fiscal effort at $t = 2$ subject to the realization of $\theta_2 = \theta_2^L, \theta_2^H$. $\sqrt{K}$ may be strictly larger than θ_2^H (see Assumption 1 below), implying a discontinuity in the effort when $ST_1 + LT$ increases up to the default. We do not exclude $ST_1 \leq 0$ (but we exclude $LT + ST_1 \leq 0$ which makes no sense with a quadratic cost function).

[5]The analysis of the coordination problems raised by issuance of LT debt is part of a companion paper.

The decision of the Sovereign at $t = 1$ can *a priori* result in three different outcomes at $t = 2$: no default, default when $\theta_2 = \theta_2^L$ only, default when either $\theta_2 = \theta_2^L$ or $\theta_2 = \theta_2^H$ (the case "default when $\theta_2 = \theta_2^H$ only" is impossible by construction). We show in Appendix that the third outcome never occurs (see Claim 12). It follows that the decision of the Sovereign at $t = 1$ is to choose between either "no default" or "default when $\theta_2 = \theta_2^L$ only".

In general, the decision of the Sovereign at $t = 1$ depends on θ_1 and (ST_0, LT). The intuition can be summarized as follows:

- Potential Default (PD):
  If the Sovereign faces a large debt burden (ST_0, LT) and poor fundamentals θ_1, then avoiding default at $t = 2$ requires a large effort e_1 which is very costly. The Sovereign prefers then to exert a low effort which leads to default at $t = 2$ when $\theta_2 = \theta_2^L$ only (no default occurs when $\theta_2 = \theta_2^H$). This choice entails the payment of the default penalty K but this is accompanied by a relaxation of the debt burden (*i.e.*, the present discounted value of the debt burden at $t = 1$ decreases through the increase in the interest rate). This choice will be referred to as "potential default" throughout the paper since the Sovereign's decision at $t = 1$ leads to default conditional on the realization of $\theta_2 = \theta_2^L$ (and the associated solution will be called the PD solution).

- No Default (ND):
  If the Sovereign faces a moderate debt burden (ST_0, LT) and/or good fundamentals θ_1, and it is not very costly to exert an effort e_1 that excludes the default at $t = 2$, then the Sovereign chooses to exert such an effort. This choice will be referred to as "no default" throughout the paper (and the associated solution will be called the ND solution).

The cost function V defined in (8) takes two different forms as a function of ST_1. Using the budget constraint (9) and the values of p_1 (see (5) and (6)), we have:

- in case of no default at $t = 2$

$$V^{ND} = \frac{\left(ST_0 - \frac{1}{1+r} ST_1\right)^2}{\theta_1} + \left(LT + ST_1\right)^2 \tag{14}$$

- in case of potential default (default at $t = 2$ when $\theta_2 = \theta_2^L$ only)

$$V^{PD} = \frac{\left(ST_0 - \frac{\pi_2^H}{1+r} ST_1\right)^2}{\theta_1} + \pi_2^H \left(LT + ST_1\right)^2 + \pi_2^L K \tag{15}$$

For given θ_1 and (ST_0, LT), the value of the cost in case of no default is

$$\min_{ST_1 \leq \theta_2^L - LT} V^{ND}. \tag{16}$$

$V^{ND}_{\min}$ denotes the value of this minimum when the solution is interior ($ST_1 < \theta_2^L - LT$) while $V^{ND}_{\min C}$ denotes the value of the minimum when the solution is on the boundary: $ST_1 = \theta_2^L - LT$. The value of the cost in case of potential default is

$$\min_{ST_1 > \theta_2^L - LT} V^{PD}. \tag{17}$$

$V^{PD}_{\min}$ denotes the value function of this minimization program.

Before we state the results, we make the following assumptions:

Assumption 1 *Assume:*

- $0 \leq LT \leq \theta_2^L$,
- $\pi^D \in \left[0, \pi^D_{\max}\right]$ *with* $\pi^D_{\max} = \pi_2^L$,
- $D > \frac{\theta_2^L}{(1+r)^2}$,
- $K \geq \left(\theta_2^H\right)^2$,
- $\pi_2^L K > \left(\theta_2^L\right)^2$.

The first assumption means that the investors do not buy more LT debt than can be repaid at $t = 2$ (the risk of default will follow from issuing new ST debt at $t = 1$). The second assumption means that default follows from some sequence of fundamentals θ_1, θ_2, but default requires $\theta_2 = \theta_2^L$ (an event with prior probability π_2^L). This corresponds to θ_2^H large enough to avoid default. The third assumption means that the initial debt burden is larger than the (discounted) maximum effort at $t = 2$ if θ_2^L occurs (debt reimbursement then requires a sufficient effort e_1 at $t = 1$, which creates the trade-off between effort and default - otherwise, the model would be meaningless). The fourth assumption means that default is always more costly than paying back the debt. The fifth assumption means that the penalty cost of default is large with respect to the probability and value of the low fundamental θ_2^L at $t = 2$. This is a technical assumption required for algebraic simplicity.

The next Lemma fully describes the optimal choice of the Sovereign at $t = 1$, depending on its default choice (characterized below in Proposition 4).

Lemma 2 *Let* $E\left(\pi^D\right) \in \left[0, \pi^D_{\max}\right]$ *be a given expectation at* $t = 0$ *of the default probability and* (ST_0, LT) *a debt portfolio satisfying the* $t = 0$ *budget constraint (where* $p_0 = \frac{1-E\left(\pi^D\right)}{(1+r)^2}$*). The debt portfolio is characterized by* LT *and* $E\left(\pi^D\right)$ *(with* $ST_0 = (D - p_0 LT)(1 + r)$*).*

- *When the Sovereign at* $t = 1$ *chooses not to default,*

– *if in addition:*

$$\theta_1 \geq \frac{D}{\theta_2^L} + \frac{E\left(\pi^D\right)}{(1+r)^2}\frac{LT}{\theta_2^L} - \frac{1}{(1+r)^2}, \tag{18}$$

then the optimal choice of the Sovereign at $t=1$ is the interior ND solution. In that case, the amount of ST debt issued is:

$$ST_{1,\min}^{ND} = \frac{\frac{1}{1+r}ST_0 - \theta_1 LT}{\frac{1}{(1+r)^2} + \theta_1}. \tag{19}$$

The associated cost V is:

$$V_{\min}^{ND}\left(\theta_1, LT, E\left(\pi^D\right)\right) = \frac{\left(D(1+r) + \frac{E\left(\pi^D\right)}{1+r}LT\right)^2}{\frac{1}{(1+r)^2} + \theta_1}. \tag{20}$$

– *Otherwise, the optimal choice is the constrained ND solution $ST_1 = \theta_2^L - LT$ (the maximum amount ST_1 consistent with no default) and the associated cost V is:*

$$V_{\min C}^{ND}\left(\theta_1, LT, E\left(\pi^D\right)\right) = \frac{\left(D(1+r) + \frac{E\left(\pi^D\right)}{1+r}LT - \frac{1}{1+r}\theta_2^L\right)^2}{\theta_1} + \left(\theta_2^L\right)^2. \tag{21}$$

- *When the Sovereign at $t=1$ chooses to potentially default, the optimal choice is the PD solution:*

$$ST_{1,\min}^{PD} = \frac{\frac{1}{1+r}ST_0 - \theta_1 LT}{\frac{\pi_2^H}{(1+r)^2} + \theta_1}, \tag{22}$$

and the associated cost V is:

$$V_{\min}^{PD}\left(\theta_1, LT, E\left(\pi^D\right)\right) = \frac{\left(D(1+r) + \frac{E\left(\pi^D\right)-\pi_2^L}{1+r}LT\right)^2}{\frac{\pi_2^H}{(1+r)^2} + \theta_1} + \pi_2^L K. \tag{23}$$

The PD solution exists only if

$$\theta_1 < \frac{D}{\theta_2^L} + \frac{E\left(\pi^D\right)}{(1+r)^2}\frac{LT}{\theta_2^L} - \frac{1}{(1+r)^2}\left(\pi_2^L\frac{LT}{\theta_2^L} + \pi_2^H\right). \tag{24}$$

The proof (in Appendix) is purely computational.

$V_{\min}^{ND}$ and $V_{\min C}^{ND}$ are increasing in LT (the result for $V_{\min C}^{ND}$ follows from Assumption 1), while $V_{\min}^{PD}$ is decreasing in LT (since $E\left(\pi^D\right) \leq \pi_{\max}^D$ and $D(1+r) + \frac{E\left(\pi^D\right)-\pi_2^L}{1+r}LT \geq 0$, by Assumption 1). Indeed, in the case of ND, the $t=1$ discounted debt burden is $ST_0 + \frac{1}{1+r}LT$. An increase in LT (with the small associated decrease in ST_0 along the $t=0$ budget constraint) increases this $t=1$ discounted debt burden, which results into a cost increase. Similarly, in the case of PD, the $t=1$ discounted debt burden is $ST_0+\frac{\pi_2^H}{1+r}LT$, an increase in LT (with the large associated decrease in ST_0 along the $t=0$ budget constraint) decreases this $t=1$ discounted debt burden, which results into a cost decrease. The effect of a portfolio rebalancing on the $t=1$ debt burdens in both the PD and ND cases is a consequence of the $t=0$ price $p_0=\frac{1-E(\pi^D)}{(1+r)^2}$ of LT being a discounted average of the $t=1$ prices of LT, $\frac{1}{1+r}$ and $\frac{\pi_2^H}{1+r}$ (*i.e.*, $\frac{\pi_2^H}{1+r}\leq(1+r)\,p_0\leq\frac{1}{1+r}$).

Condition (24) is a necessary condition for existence of the PD case which rewrites as $ST_{1,\min}^{PD}+LT>\theta_2^L$ (a bound on debt capacity).

In order to state Proposition 4, we define four thresholds on the fundamentals θ_1.

Definition 3 *Let θ_1^- be the unique positive solution of:*

$$\frac{\left(D\,(1+r)-\frac{1}{1+r}\theta_2^L\right)^2}{\theta_1}+\left(\theta_2^L\right)^2=\frac{\left(D\,(1+r)\right)^2}{\frac{\pi_2^H}{(1+r)^2}+\theta_1}+\pi_2^LK. \tag{25}$$

Let θ_1^+ be the unique positive solution of:

$$\frac{\left(D\,(1+r)+\frac{E\left(\pi^D\right)-1}{1+r}\theta_2^L\right)^2}{\theta_1}+\left(\theta_2^L\right)^2=\frac{\left(D\,(1+r)+\frac{E\left(\pi^D\right)-\pi_2^L}{1+r}\theta_2^L\right)^2}{\frac{\pi_2^H}{(1+r)^2}+\theta_1}+\pi_2^LK. \tag{26}$$

θ_1^+ depends on $E\left(\pi^D\right)$. Let $\hat{\theta}_1$ be the value of θ_1^+ for $E\left(\pi^D\right)=0$, and $\check{\theta}_1$ be the value of θ_1^+ for $E\left(\pi^D\right)=\pi_{\max}^D$. The values θ_1^-, θ_1^+, $\hat{\theta}_1$ and $\check{\theta}_1$ do not depend on the decisions (ST_0,LT) of the Sovereign at $t=0$ and θ_1^-, $\hat{\theta}_1$ and $\check{\theta}_1$ do not depend on the $t=0$ belief $E\left(\pi^D\right)$. We have

$$0<\theta_1^-<\hat{\theta}_1\leq\theta_1^+\leq\check{\theta}_1. \tag{27}$$

We show in Appendix that θ_1^-, θ_1^+, $\check{\theta}_1$ and $\hat{\theta}_1$ are well defined and the set of above inequalities holds true.[6] These inequalities provide bounds on θ_1^+ independent of $E\left(\pi^D\right)$.

Condition (25) defining θ_1^- rewrites $V_{\min C}^{ND}\left(\theta_1,0,E\left(\pi^D\right)\right)=V_{\min}^{PD}\left(\theta_1,0,E\left(\pi^D\right)\right)$, which corresponds to the equality between the cost of no default and the cost of potential default in the case where no LT debt has been issued at $t=0$ ($LT=0$). This equality is independent of the value of $E\left(\pi^D\right)$. Condition (26) defining θ_1^+ rewrites $V_{\min C}^{ND}\left(\theta_1,\theta_2^L,E\left(\pi^D\right)\right)=V_{\min}^{PD}\left(\theta_1,\theta_2^L,E\left(\pi^D\right)\right)$, which corresponds to the equality between the cost of no default and the cost of potential default in the case where the maximum amount of LT debt has been issued at $t=0$ ($LT=\theta_2^L$,

[6] In particular, the interval $\left[0,\theta_1^-\right)$ is not empty (θ_1^- tends to 0 when K goes to infinity).

see Assumption 1) at price $p_0 = \frac{1-E(\pi^D)}{(1+r)^2}$. The value $\hat{\theta}_1$ is the θ_1^+ at the best possible price (characterized by the lowest possible probability of default). The value $\check{\theta}_1$ is the θ_1^+ at the worst possible price (characterized by the highest possible probability of default $\pi^D_{\max}$). All four definitions correspond to values of θ_1 where the Sovereign is indifferent between potential default and no default. In all four cases, the ND solution considered is the constrained solution.

Proposition 4 presents 3 regions of fundamental values for θ_1: a low region displaying potential default, a high region displaying no default and an intermediate region where incentives matter (the debt portfolio issued at $t = 0$ determines the choice of the Sovereign at $t = 1$).

Whenever $\theta_1 \leq \theta_1^-$, the Sovereign chooses at $t = 1$ potential default whatever the amount of LT debt issued. As more LT debt favors the choice of potential default and larger θ_1 favors no default, then the maximum value of θ_1 leading to potential default is obtained for $LT = 0$: this is exactly θ_1^-. Along the same lines, the Sovereign chooses not to default as long as θ_1 is large enough, the smallest value of θ_1 consistent with no default is obtained for the largest value of LT ($LT = \theta_2^L$): this is exactly θ_1^+. In the intermediate region, $\theta_1^- \leq \theta_1 < \theta_1^+$, the amount of LT debt determines potential default or no default.

Proposition 4 *For a given expected default probability $E\left(\pi^D\right) \in \left[0, \pi^D_{\max}\right]$, the debt portfolio (ST_0, LT) satisfying the $t = 0$ budget constraint is characterized by LT (with $ST_0 = (D - p_0 LT)(1 + r)$ and $p_0 = \frac{1-E(\pi^D)}{(1+r)^2}$). Three cases describe the optimal choice of the Sovereign at $t = 1$:*

- *If $\theta_1 < \theta_1^-$, then the Sovereign chooses to potentially default (independently of the debt portfolio).*
- *If $\theta_1^- \leq \theta_1 < \theta_1^+$, then the choice of the Sovereign depends on the debt portfolio: there is a threshold $LT^*\left(\theta_1, E\left(\pi^D\right)\right)$ with $0 < LT^*\left(\theta_1, E\left(\pi^D\right)\right) < \theta_2^L$ such that the Sovereign chooses not to default iff $LT < LT^*\left(\theta_1, E\left(\pi^D\right)\right)$. $LT^*\left(\theta_1, E\left(\pi^D\right)\right)$ decreases in $E\left(\pi^D\right)$ and increases in θ_1.*
- *If $\theta_1 \geq \theta_1^+$, then the Sovereign chooses not to default (independently of the debt portfolio).*

θ_1^+ increases from $\hat{\theta}_1$ to $\check{\theta}_1$ when $E\left(\pi^D\right)$ increases from 0 to $\pi^D_{\max}$.

The formal proof is in the Appendix.

Proposition 4 shows that the Sovereign chooses not to default for large values of θ_1, irrespective of the accumulated debt (ST_0, LT). The existence of the LT debt instrument increases the potential for default: When the Sovereign does not have access to LT debt (*i.e.*, $LT = 0$), the optimal choice of the Sovereign at $t = 1$ in a state $\theta_1 > \theta_1^-$ is not to default. By contrast, the next section shows that, when LT debt is available, the Sovereign sometimes issues an amount of LT debt implying potential default at $t = 1$ when $\theta_1 > \theta_1^-$.

As already stressed, the ND solution can be either interior or constrained (*i.e.*, the accumulated debt burden at $t = 1$ is interior, $LT{+}ST_1 < \theta_2^L$, or on the boundary: $LT{+}ST_1 = \theta_2^L$). Corollary 5 shows that for large enough values of θ_1, the ND solution is interior. Focussing on this region simplifies the analysis in the next section.

Corollary 5 *Let*

$$\theta_1^* = \frac{D}{\theta_2^L} + \frac{\pi_{\max}^D}{(1+r)^2} - \frac{1}{(1+r)^2} > \hat{\theta}_1.$$

If $\theta_1 \geq \theta_1^*$, then the Sovereign chooses the interior ND solution, independent of the debt portfolio and the $t = 0$ belief $E\left(\pi^D\right)$.

The proof is in Appendix.

IV. THE ROLE OF INITIAL UNCERTAINTY

We show how the expectations of agents about future macroeconomic fundamentals influence the maturity structure of the Sovereign's debt and then its conditions of default. More precisely, we solve for the optimal choice of the Sovereign as of period 0, and we show how the prior belief about the fundamentals θ_1 influences the optimal portfolio allocation (ST_0, LT) and the probability of default at $t = 2$. Building on the previous section, the default behavior does not depend on the debt portfolio (ST_0, LT) for θ_1 in the low region $\left[0, \theta_1^-\right)$ or the high region $\left[\check{\theta}_1, +\infty\right)$. Thus, the problem reduces to the analysis of the conditions determining the choice between ND and PD at $t = 1$ for θ_1 in the median region $\left[\theta_1^-, \check{\theta}_1\right)$.

At $t = 0$, the Sovereign faces an infinitely elastic supply of ST debt at the price $\frac{1}{1+r}$ (since the ST debt is default-free and investors are risk neutral). In contrast, the price at which the LT debt is supplied is determined by the investors' expected probability of Sovereign default $E\left(\pi^D\right)$: the assumption of risk neutrality implies that the debt mix issued by the Sovereign is priced at the discounted value of $E\left(\pi^D\right)$. In equilibrium, the investors rationally expect the probability of default (occurring at $t = 2$) to depend on the portfolio (ST_0, LT) issued by the Sovereign. To compute the true expected probability of default $E\left(\pi^D\right)$, investors rationally anticipate what will happen at $t = 1$ (as presented in the previous Section):

- If (ST_0, LT) issued by the Sovereign results into choices by the Sovereign at $t = 1$ leading to ND, then the investors buy the LT debt at price $p_0 = \frac{1}{(1+r)^2}$ ($E\left(\pi^D\right) = 0$ in this case).
- If (ST_0, LT) issued by the Sovereign results into choices by the Sovereign at $t = 1$ leading to PD (*i.e.*, default after some values of θ_1 and $\theta_2 = \theta_2^L$, and no default otherwise), then the investors buy the LT debt at price $p_0 = \frac{1-E\left(\pi^D\right)}{(1+r)^2}$ with $E\left(\pi^D\right) > 0$.

The supply curve corresponds to the fact that investors are competitive and have rational expectations: the Sovereign gets the lowest possible interest rate consistent with its repayment capacity and the investors supply any amount of funds at an interest rate consistent with the Sovereign's repayment capacity. There is no coordination issue here.

As a benchmark case, we show that in the absence of uncertainty at $t = 0$ about future fundamentals θ_1, the Sovereign prefers not to default when θ_1 is expected to be large enough and it chooses a debt portfolio accordingly.

Proposition 6 *In the absence of uncertainty about θ_1 (there is only one value of θ_1), the optimal portfolio chosen by the Sovereign at $t = 0$ leads to no default iff $\theta_1 \geq \theta_1^-$. The optimal portfolio is:*

- *any portfolio if $\theta_1 \geq \hat{\theta}_1$,*
- *any portfolio with $LT \leq LT^*\left(\theta_1, E\left(\pi^D\right)\right)$ and $E\left(\pi^D\right) = 0$ if $\hat{\theta}_1 \geq \theta_1 \geq \theta_1^-$,*
- *any portfolio with $LT > LT^*\left(\theta_1, E\left(\pi^D\right)\right)$ and $E\left(\pi^D\right) = \pi_2^L$ if $\theta_1 < \theta_1^-$.*

Proposition 6 states that when there is no prior uncertainty about fundamentals at the intermediate period, the optimal portfolio leads to ND when the unique possible value of θ_1 is expected to be in the median or high region, and to PD when it is expected to be in the low region. In any case, the choice of the optimal portfolio itself is largely indeterminate. Indeed, with no uncertainty at $t = 0$ about the default behavior at $t = 1$, the $t = 1$ price of LT debt is perfectly foresighted (this is either $p_1 = \frac{1}{1+r}$ in case ND is chosen at $t = 1$ or $p_1 = \frac{\pi_2^L}{1+r}$ in case PD is chosen at $t = 1$). Consequently, the use of LT debt is no different from rolling over ST debt: the maturity structure of the debt portfolio is irrelevant, only the incentive imposed by LT debt matters (the incentive constraint consists of the constraint on LT debt in Proposition 6).

We now turn attention to the case where there is some uncertainty at $t = 0$ regarding θ_1. We show that PD may occur for θ_1 in the median region (Propositions 7 and 8). In other words, Proposition 6 does not extend to the case of several values of θ_1: With uncertainty about θ_1, the knowledge at $t = 0$ that $\theta_1 \geq \theta_1^-$ does not imply ND with probability 1.

Indeed, as soon as there is some $t = 0$ uncertainty about the fundamentals θ_1, the Sovereign wants to hedge against this risk. But asset markets at $t = 0$ are incomplete: only ST and LT debt are available. The ex-post return of ST debt is independent of θ_1 (ST debt is default free). However, the return on LT debt is correlated with θ_1 when the Sovereign chooses at $t = 1$ PD for some values of θ_1 and ND for other values of θ_1. LT debt can then serve as a hedging tool to transfer risk (beyond the role of transferring debt burdens over time). At $t = 0$, the Sovereign faces a trade-off between the use of LT debt as a hedging tool and the wish to not default: the Sovereign at $t = 0$ prefers not to default for θ_1 in the median region (Proposition 6) but it sometimes chooses a portfolio giving incentives to default for θ_1 in the median region in order to hedge against the risk on θ_1 (see below). In summary, prior uncertainty about future fundamentals is a source of default. Proposition 7 formally states this result.

Proposition 7 involves 2 values of θ_1: θ_1^M in the median region $\left[\theta_1^-, \check{\theta}_1\right)$ and θ_1^H in the high region. For analytical simplicity, we assume $\theta_1^H \geq \theta_1^*$ so that the optimal $t=1$ decision of the Sovereign is an interior ND solution (see Corollary 5). The $t=0$ prior probabilities on θ_1^M and θ_1^H are π_1^M and π_1^H respectively.

The restriction to 2 values of θ_1 only is not a strong assumption because within the high (or low) region, it is equivalent to studying an average of values in that region.[7] For the median region, a similar argument applies with some limitation linked to the changing behavior of the Sovereign w.r.t. default, within this region.

Lemma 2 and Corollary 5 show that at $t=0$, the expected cost is either:

$$\pi_1^M V_{\min C}^{ND}\left(\theta_1^M, LT, 0\right) + \pi_1^H V_{\min}^{ND}\left(\theta_1^H, LT, 0\right), \tag{28}$$

when the choice of (ST_0, LT) leads to ND ($i.e.$, $E\left(\pi^D\right)=0$ in this case)[8] or:

$$\pi_1^M V_{\min}^{PD}\left(\theta_1^M, LT, \pi_1^M \pi_2^L\right) + \pi_1^H V_{\min}^{ND}\left(\theta_1^H, LT, \pi_1^M \pi_2^L\right), \tag{29}$$

when the choice of (ST_0, LT) leads to PD ($i.e.$, $E\left(\pi^D\right) = \pi_1^M \pi_2^L$ in this case: default occurs after θ_1^M and θ_2^L).

We compute the value of LT minimizing the expected cost at $t=0$ of the Sovereign in the ND and PD cases. Proposition 7 below gives conditions under which the Sovereign chooses to default or not.

Case No Default. The solution in this case is the value of LT minimizing the expected cost (28) under the constraint that this solution is consistent with ND at $t=1$ under $\theta_1 = \theta_1^M$, $i.e.$, $LT \leq LT^*\left(\theta_1^M, 0\right)$ (where $LT^*\left(\theta_1^M, 0\right)$ is the threshold defined in Proposition 4 for θ_1^M and $E\left(\pi^D\right)=0$; the Sovereign chooses ND if $\theta_1^H > \theta_1^*$ whatever the portfolio, see Corollary 5). Every debt portfolio (ST_0, LT) on the budget constraint that leads to ND at $t=1$ under θ_1^M is optimal. Intuitively, issuing LT debt is equivalent to rolling over ST debt (both debts are risk-free): The value functions $V_{\min C}^{ND}$ and $V_{\min}^{ND}$ do not depend on the portfolio choice, they are determined by the $t=1$ discounted value of the portfolio $ST_0 + \frac{1}{1+r}LT$, which equals $D(1+r)$. Lemma 2 implies that the value (28) of the objective is:

$$\pi_1^L\left(\frac{\left(D(1+r) - \frac{1}{1+r}\theta_2^L\right)^2}{\theta_1^L} + \left(\theta_2^L\right)^2\right) + \pi_1^H \frac{\left(D(1+r)\right)^2}{\frac{1}{(1+r)^2} + \theta_1^H}. \tag{30}$$

[7] A starting point for this argument is that, for a given (ST_0, LT) the expected value $E\left(V_{\min}^{ND} | \theta_1 \geq \theta_1^*\right)$ is the value $V_{\min}^{ND}$ for an average value $\theta_{avg} \geq \theta_1^*$ defined by (using obvious notation):

$$\left(\sum_n \pi_1^n\right) \frac{1}{\pi_2^H\left(\frac{1}{1+r}\right)^2 + \theta_{avg}} = \sum_n \left(\pi_1^n \frac{1}{\pi_2^H\left(\frac{1}{1+r}\right)^2 + \theta_1^n}\right).$$

[8] As written above, the probability of default is correctly expected, conditionnal on (ST_0, LT).

Case Potential Default. The solution in this case is the value of LT minimizing the expected cost (29) under the constraint that this solution is consistent with the choice of potential default at $t = 1$ under $\theta_1 = \theta_1^M$,*i.e.*, $LT > LT^* \left(\theta_1^M, \pi_1^M \pi_2^L\right)$ (where $LT^* \left(\theta_1^M, \pi_1^M \pi_2^L\right)$ is the threshold defined in Proposition 4 for θ_1^M and $E\left(\pi^D\right) = \pi_1^M \pi_2^L$). Computations (see proof of Proposition 7 in Appendix) show that the minimum value of the expected cost is:

$$\frac{\left(D\left(1+r\right)\right)^2}{\frac{1-\pi_1^M \pi_2^L}{(1+r)^2} + \bar{\theta}_1} + \pi_1^M \pi_2^L K, \tag{31}$$

where $\bar{\theta}_1 = \pi_1^M \theta_1^M + \pi_1^H \theta_1^H$.

To determine which case the Sovereign chooses, we need to compare the expected costs (30) and (31). Proposition 7 describes this choice.

Proposition 7 *Assume that θ_1 takes two values $\theta_1^M \in \left[\theta_1^-, \hat{\theta}_1\right)$ and $\theta_1^H \geq \theta_1^*$ with probabilities π_1^M and π_1^H respectively. Assume that $D > \frac{6}{(1+r)^2}\theta_2^L$. There is a threshold $\hat{\pi}_1^H$ $(0 < \hat{\pi}_1^H < 1)$ such that:*

- *For $\pi_1^H > \hat{\pi}_1^H$, the Sovereign chooses a debt portfolio that leads to potential default (*i.e.*, default occurs after $\theta_1 = \theta_1^M$ and $\theta_2 = \theta_2^L$).*
- *For $\pi_1^H \leq \hat{\pi}_1^H$, the Sovereign chooses a debt portfolio that leads to no default.*

The proof is in Appendix, where the expression of $\hat{\pi}_1^H$ is provided (Equation (57)). The assumption on D stated in the proposition is more demanding than in Assumption 1. If this stronger assumption is not met, then the threshold $\hat{\pi}_1^H$ still exists for θ_1^M close enough to θ_1^- or θ_1^H large enough (see the proof). The proof also shows that default occurs for moderate values of the penalty cost K (Condition (58) characterizing default is an upper bound on K).

A surprising feature of Proposition 7 is that more optimistic expectations about fundamentals (a large π_1^H) imply a positive expected probability of default. Potential default allows the Sovereign to relax its effort e_1 under θ_1^M (see Corollary 11) but it is costly: Interest rates on LT debt are higher at $t = 0$ (since default is rationally expected) and the Sovereign suffers a penalty cost K (larger than any effort level e_2). When π_1^H is high, the increase in the interest rate due to this choice is small (the price of LT debt is $p_0 = \frac{1-E\left(\pi^D\right)}{(1+r)^2}$ with $E\left(\pi^D\right) = 0$ in case of ND and $E\left(\pi^D\right) = \pi_1^M \pi_2^L$ in case of PD under θ_1^M), and this leads to the choice of PD.

The choice of PD for a median value θ_1^M is related to the effort costs being very different between the 2 states θ_1^M and θ_1^H: The Sovereign has an incentive to distinguish efforts across states θ_1^M and θ_1^H, which amounts to allocating the debt burden across states. This is achieved through the portfolio decision. As explained above, this "hedging" strategy can lead to PD under θ_1^M (while ND occurs under θ_1^H). We now give an interpretation in terms of the (in)completeness of debt markets at $t = 0$:

- At $t = 0$, the Sovereign chooses (ST_0, LT) to minimize $\pi_1^M V\left(\theta_1^M\right) + \pi_1^H V\left(\theta_1^H\right)$, where $V(\theta_1)$ is the expected cost at $t = 1$ in state θ_1 ($V\left(\theta_1^H\right)$ is $V_{\min}^{ND}$, see (20), and $V\left(\theta_1^M\right)$ is either $V_{\min C}^{ND}$ or $V_{\min}^{PD}$, see (21) and (23)). $V(\theta_1)$ is increasing in the discounted debt burden $B_1(\theta_1) = ST_0 + p_1(\theta_1) LT$ at $t = 1$. Rewriting the $t = 0$ budget constraint (4) as:[9]
$$\pi_1^M B_1\left(\theta_1^M\right) + \pi_1^H B_1\left(\theta_1^H\right) = (1 + r) D, \tag{32}$$
shows that the problem of the Sovereign is to allocate the initial debt burden D across states θ_1^M and θ_1^H. In general, the solution of this optimization problem involves $B_1\left(\theta_1^M\right) \neq B_1\left(\theta_1^H\right)$ (The FOC implies that the marginal costs $\frac{dV\left(\theta_1^H\right)}{dB_1\left(\theta_1^H\right)}$ and $\frac{dV\left(\theta_1^M\right)}{dB_1\left(\theta_1^M\right)}$ are equal).

- If (ST_0, LT) implies ND, then $p_1(\theta_1) = \frac{1}{1+r}$ and every portfolio (ST_0, LT) leads to the same value $B_1\left(\theta_1^M\right) = B_1\left(\theta_1^H\right) = (1 + r) D$ (because of the $t = 0$ budget constraint (4)). The Sovereign can choose no other $B_1\left(\theta_1^M\right)$ and $B_1\left(\theta_1^H\right)$ along the constraint (32). Markets are incomplete: the 2 assets are identical (issuing LT debt is equivalent to rolling over ST debt).

- If (ST_0, LT) implies default when θ_1^M and θ_2^L occur, then $p_1\left(\theta_1^M\right) < p_1\left(\theta_1^H\right)$ and markets are complete: The Sovereign can choose any values $B_1\left(\theta_1^M\right)$ and $B_1\left(\theta_1^H\right)$ along the constraint (32) using an appropriate portfolio, and it achieves the minimum cost in this case. When the Sovereign substitutes LT for ST_0 (under the $t = 0$ budget constraint (4)), $B_1\left(\theta_1^M\right)$ decreases and $B_1\left(\theta_1^H\right)$ increases (because $p_1\left(\theta_1^M\right) < (1 + r) p_0 < p_1\left(\theta_1^H\right)$): The debt burden is transferred from θ_1^M to θ_1^H.

In summary, the cost of default is the price to pay by the Sovereign to gain access to complete debt markets.

Proposition 7 shows that the uncertainty about θ_1 sometimes leads to PD when θ_1 is in the median region. The choice of PD in the median region creates some correlation between θ_1 and the return on LT debt (since θ_1 belongs either to the median or high region, and ND is the only solution in the high region). Proposition 8 complements this result: When θ_1 is expected to belong either to the median or low region (where PD is the only solution), the same "hedging" argument implies the choice of ND for θ_1 in the median region, and p_1 varies with θ_1 ($p_1 = \frac{1}{1+r}$ under θ_1^M and $p_1 = \frac{\pi_2^H}{1+r}$ under θ_1^L).

Proposition 8 *Assume that θ_1 takes two values $\theta_1^L \in \left(0, \theta_1^-\right)$ and $\theta_1^M \in \left[\theta_1^-, \hat{\theta}_1\right)$. The Sovereign always chooses a debt portfolio that leads to no default if θ_1^M occurs, and to potential default if θ_1^L occurs.*

The proof is in Appendix. The argument is the same as for Proposition 7.

[9]For $s = M, H$, π_1^s is the "Arrow price" of one unit of the debt burden $B_1\left(\theta_1^s\right)$.

V. THE OPTIMAL PORTFOLIO

This section expands on the results of the previous section by describing the portfolio and effort choices.

The next proposition investigates the effect of the $t = 0$ uncertainty on the debt portfolio. The debt portfolio is determinate only if the default behavior at $t = 1$ varies in θ_1 (see above). For simplicity, we restrict attention to the case of 2 values of θ_1, one value θ_1^M leading to the choice of PD at $t = 1$ and the other value θ_1^H leading to the choice of ND. The value θ_1^M belongs either to the low region (in which case the decision of the Sovereign is necessarily PD) or the median region (in which case we further assume that the decision of the Sovereign is PD, as in Proposition 7). For simplicity, we choose $\theta_1^H \geq \theta_1^*$ so that the ND solution is interior (the value of the cost is $V_{\min}^{ND}$).

Recall that the debt portfolio (ST_0, LT) issued by the Sovereign is the one minimizing:

$$\pi_1^M V_{\min}^{PD}\left(\theta_1^M, LT, E\left(\pi^D\right)\right) + \pi_1^H V_{\min}^{ND}\left(\theta_1^H, LT, E\left(\pi^D\right)\right), \tag{33}$$

under the $t = 0$ budget constraint (4):

$$D = \frac{1}{1+r} ST_0 + \frac{1 - E\left(\pi^D\right)}{(1+r)^2} LT, \tag{34}$$

with $E\left(\pi^D\right) = \pi_1^M \pi_2^L$ (the probability of default is correctly expected at $t = 0$). The optimal portfolio is the solution of the FOC:

$$\frac{d}{dLT}\left(\pi_1^M V_{\min}^{PD} + \pi_1^H V_{\min}^{ND}\right) = 0.$$

Proposition 9 *Assume that either $\theta_1^M \in \left[0, \theta_1^-\right)$ or $\theta_1^M \in \left[\theta_1^-, \check{\theta}_1\right)$ with the Sovereign choosing PD under θ_1^M. Assume $\theta_1^H \geq \theta_1^*$. The optimal portfolio (ST_0, LT) is given by:*

$$ST_0 = \frac{\theta_1^H - \frac{1}{\pi_2^L}\left(\theta_1^H - \theta_1^M\right)}{\frac{1-\pi_1^M\pi_2^L}{(1+r)^2} + \bar{\theta}_1} D(1+r), \tag{35}$$

$$LT = \frac{\frac{1}{1+r} + \frac{1+r}{\pi_2^L}\left(\theta_1^H - \theta_1^M\right)}{\frac{1-\pi_1^M\pi_2^L}{(1+r)^2} + \bar{\theta}_1} D(1+r), \tag{36}$$

where $\bar{\theta}_1 = \pi_1^M \theta_1^M + \pi_1^H \theta_1^H$. LT is decreasing in π_1^H. If $\theta_1^M > \pi_2^H \theta_1^H$, then $ST_0 > 0$ and ST_0 is decreasing in π_1^H. The $t = 0$ price of LT debt, $p_0 = \frac{1-\pi_1^M\pi_2^L}{(1+r)^2}$, is increasing in π_1^H.

If the condition $\theta_1^M > \pi_2^H \theta_1^H$ does not hold, then $ST_0 \leq 0$: The Sovereign lends ST to increase its LT borrowing. We disregard this case. If the ND solution under θ_1^H is constrained (the value of the cost is $V_{\min C}^{ND}$), then the exact expression of the optimal portfolio is different, but the monotonicity properties remain the same. The proof in Appendix presents the mathematical counterpart of the intuition below.

In the optimization program of the Sovereign, the belief about fundamentals θ_1 appears both in the objective (33) and the budget constraint (34). This leads to two effects of an increased optimism: one due to the change in the price of LT (this effect is related to the uncertainty of the investors) and another due to the change in the weights on the value functions in the objective (this effect is related to the uncertainty of the Sovereign).

The first effect is easily described as a standard price effect. An increase in the prior belief π_1^H of θ_1^H decreases the expected probability $E\left(\pi^D\right)$, decreases the LT interest rate, and this impacts (ST_0, LT) as follows: a substitution effect (the change in the "relative price" leads to an increased LT and a decreased ST_0), and an income effect (both debts decrease: the lower LT interest rate allows for the financing of D with smaller amounts of debts). The proof of Proposition 9 shows that each of the 2 effects can dominate.

The second effect is that an increase in the prior belief π_1^H in the objective (33) implies a decrease in LT. The intuition for this second effect relies on the hedging role played by LT. As written above, a decrease in LT (with the associated increase in ST_0 along the $t = 0$ budget constraint (34)) decreases the debt burden under θ_1^H and increases the debt burden under θ_1^M (as the $t = 0$ price $p_0 = \frac{1-E\left(\pi^D\right)}{(1+r)^2}$ of LT is a discounted average of the $t = 1$ prices of LT, $\frac{1}{1+r}$ and $\frac{\pi_2^H}{1+r}$). Since the value functions $V_{\min}^{PD}$ and $V_{\min}^{ND}$ are convex and increasing functions of the discounted debt burden, the increase in π_1^H is compensated by a decrease in LT (to satisfy the FOC $\frac{d}{dLT}\left(\pi_1^M V_{\min}^{PD} + \pi_1^H V_{\min}^{ND}\right) = 0$).

Proposition 9 shows that the income effect always dominates: The lower LT interest rate (following the increase in π_1^H) implies a decrease of both ST and LT debts. The Sovereign needs to issue less debt to cover its financing need D. Corollary 10 shows that this results in decreased $t = 1$ debt burdens in both states θ_1^M and θ_1^H, which in turn implies lower fiscal efforts in both periods: The Sovereign smooths over time and states the benefits of the lower LT interest rate (notice that default still occurs after θ_1^M and θ_2^L since θ_1^M is assumed to always create the incentive to default).

Corollary 10 *Assume that either $\theta_1^M \in \left[0, \theta_1^-\right)$ or $\theta_1^M \in \left[\theta_1^-, \check{\theta}_1\right)$ with the Sovereign choosing PD under θ_1^M. Assume $\theta_1^H \geq \theta_1^*$. The fiscal efforts at $t = 1$ are:*

$$e_1\left(\theta_1^M\right) = \frac{\theta_1^M}{\frac{\pi_2^H}{(1+r)^2} + \theta_1^M} B_1\left(\theta_1^M\right), \tag{37}$$

$$e_1\left(\theta_1^H\right) = \frac{\theta_1^H}{\frac{1}{(1+r)^2} + \theta_1^H} B_1\left(\theta_1^H\right), \tag{38}$$

and the fiscal efforts at $t = 2$ are:

$$e_2\left(\theta_1^M, \theta_2^H\right) = \frac{\frac{1}{1+r}}{\frac{\pi_2^H}{(1+r)^2} + \theta_1^M} B_1\left(\theta_1^M\right), \qquad e_2\left(\theta_1^M, \theta_2^L\right) = \sqrt{K}, \tag{39}$$

$$e_2\left(\theta_1^H, \theta_2^H\right) = e_2\left(\theta_1^H, \theta_2^L\right) = \frac{\frac{1}{1+r}}{\frac{1}{(1+r)^2} + \theta_1^H} B_1\left(\theta_1^H\right), \tag{40}$$

where the $t = 1$ discounted debt burdens are $B_1(\theta_1) = (ST_0 + p_1(\theta_1) LT)$ where $p_1(\theta_1)$ is $\frac{1}{1+r}$ or $\frac{\pi_2^H}{1+r}$ depending on ND or PD:

$$B_1\left(\theta_1^M\right) = \frac{\frac{\pi_2^H}{(1+r)^2} + \theta_1^M}{\frac{1-\pi_1^M \pi_2^L}{(1+r)^2} + \bar{\theta}_1} D(1+r), \qquad B_1\left(\theta_1^H\right) = \frac{\frac{1}{(1+r)^2} + \theta_1^H}{\frac{1-\pi_1^M \pi_2^L}{(1+r)^2} + \bar{\theta}_1} D(1+r).$$

The debt burdens $B_1\left(\theta_1^M\right)$, $B_1\left(\theta_1^H\right)$, and the effort levels $e_1\left(\theta_1^M\right)$, $e_1\left(\theta_1^H\right)$, $e_2\left(\theta_1^H, \theta_2^L\right)$, $e_2\left(\theta_1^H, \theta_2^H\right)$ and $e_2\left(\theta_1^M, \theta_2^H\right)$ are all decreasing in π_1^H ($e_2\left(\theta_1^M, \theta_2^L\right)$ is constant, equal to the default value).

The proof is omitted. It is included in the proof of Lemma 2 (Corollary 10 and Lemma 2 solve the same optimization problem).

In summary, for given default behavior (one PD state θ_1^M, one ND state θ_1^H - again, the ND solution under state θ_1^H can be interior or constrained without changing the monotonicity properties of the result), an increased prior optimism about future fundamentals ($t = 0$ belief about fundamentals at $t = 1$) unambiguously improves the situation of the Sovereign: The debt burden decreases in each state at $t = 1$, efforts decrease in each state and each period. This improvement is implemented through a change in the portfolio that consists of a decrease in both ST_0 and LT. In particular, more optimism implies a lower LT interest rate and a smaller issuance of LT.

This result contrasts with the effect of a change in the default behavior (Proposition 7). Everyone understands that a large issuance of LT debt results in PD in case of a median θ_1. Therefore, a large issuance of LT debt is accompanied by a jump in the interest rate, this jump is large/small in case the expected probability of default is high/small (the probability π_1^H of the good state is low/high). Hence, as long as π_1^H is low, the Sovereign does not issue a large amount of LT debt, and the portfolio gives the incentives at $t = 1$ to exert the fiscal effort necessary to avoid default. When π_1^H is high, at $t = 0$, the Sovereign issues large quantities of LT debt that leads everyone to expect PD, and the interest rate makes a (small) jump. This discourages the Sovereign from exerting a high enough effort e_1 at $t = 1$, and self-fulfills PD. This generates the transfer of debt burden between states explained above. More optimism implies a larger issuance of LT debt and a higher LT interest rate. The next corollary completes Proposition 7. It formally describes the ND and PD cases in terms of LT, the debt burden and the efforts.

Corollary 11 *Assume that $\theta_1^M \in \left[\theta_1^-, \check{\theta}_1\right)$ and $\theta_1^H \geq \theta_1^*$. At $t=1$, the Sovereign chooses the interior ND solution under θ_1^H. If the Sovereign chooses the (constrained) ND solution under θ_1^M at $t=1$, then the LT debt issued satisfies:*

$$LT \leq LT^*\left(\theta_1^M, 0\right),$$

the two debt burdens are equal:

$$B_1\left(\theta_1^M\right) = D\left(1+r\right) = B_1\left(\theta_1^H\right),$$

and the effort is:

$$e_1\left(\theta_1^M\right) = D\left(1+r\right) - \frac{\theta_2^L}{1+r}.$$

If the Sovereign chooses the PD solution under θ_1^M at $t=1$, then the LT debt issued is given by (36) and satisfies:

$$LT > LT^*\left(\theta_1^M, 0\right),$$

the two debt burdens satisfy:

$$B_1\left(\theta_1^M\right) < D\left(1+r\right) < B_1\left(\theta_1^H\right),$$

and the effort is:

$$e_1\left(\theta_1^M\right) < D\left(1+r\right) - \frac{\theta_2^L}{1+r}.$$

The proof of the corollary follows from gathering previous results. In case of ND under θ_1^M, the threshold on LT is defined using $E\left(\pi^D\right)=0$, the debt burdens follow from the $t=0$ budget constraint (34), the effort is associated with the constrained ND solution. In case of PD under θ_1^M, LT satisfies $LT > LT^*\left(\theta_1^M, \pi_1^M \pi_2^L\right)$ and the result follows from $LT^*\left(\theta_1^M, E\left(\pi^D\right)\right)$ being decreasing in $E\left(\pi^D\right)$. Corollary 10 proves the results w.r.t. the debt burdens. The effort satisfies the $t=1$ budget constraint (see (9) and (13)):

$$e_1\left(\theta_1^M\right) = B_1\left(\theta_1^M\right) - \frac{\pi_2^H e_2\left(\theta_1^M, \theta_2^H\right)}{1+r},$$

and $e_2\left(\theta_1^M, \theta_2^H\right) > \theta_2^L$ (PD case) implies the result.

More computations would show that a similar result holds true when θ_1 belongs to the low or median regions (refer to Proposition 8). In particular, $B_1\left(\theta_1^L\right) < D\left(1+r\right) < B_1\left(\theta_1^M\right)$ in case where the PD solution is chosen under θ_1^L and the ND solution is chosen under θ_1^M.

VI. CONCLUDING REMARKS: THE ROLE OF INTERNATIONAL FINANCIAL INSTITUTIONS

The trade-off faced by the Sovereign and the structure of its portfolio of debt have been analyzed here. We have seen how the prior uncertainty on future fundamentals affects the issuance of LT debt and thus the occurrence of default. We have shown that uncertainty is a source of default: uncertainty about future fundamentals sometimes leads to a debt portfolio choice that gives incentives to default, while another debt portfolio (optimal under another prior belief) would have avoided default.

If the Sovereign issues only ST debt, this is accompanied with an important fiscal effort to be able to meet the financial obligations when they come due at $t = 1$. In order not to support a high fiscal effort, and smooth its fiscal efforts, the Sovereign sometimes prefers to issue a mixture of short and LT debt, but by so doing may expose itself to a risk of default under a large issuance of LT debt. The deterioration of the financing conditions of the Sovereign is self-fulfilling: A large issuance of LT debt at a high LT interest rate creates an incentive for the Sovereign to choose at $t = 1$ to be exposed to default. This increased default risk then justifies the high interest rates.

For the Sovereign, issuing LT debt at $t = 0$ amounts to selling the risk on future fundamentals θ_1 (a low θ_1 implies a low effort e_1 which may increase the expected probability of default π^D and the LT interest rate). The Sovereign (through endogenous default) creates risk on the value of LT debt, correlated to the exogenous risk on θ_1, in order to "pass on" the risk to investors.

Default is sensitive to the portfolio of the Sovereign only if the fundamentals θ_1 belongs to the region of median values. Our results exhibit two kinds of situations where the markets give opposite incentives to the Sovereign. When the prior belief about θ_1 is very pessimistic (θ_1 expected in the low/median regions), the markets provide incentives to get out of the default region whenever possible (θ_1 occurs in the median region). When the prior belief about θ_1 is rather optimistic (θ_1 expected in the median/high regions with a large probability on the high region), the markets provide incentives to default whenever possible (θ_1 occurs in the median region): The markets rationally disregard the risk associated with the unlikely occurrence of a median θ_1 with default.

The markets being unable to monitor closely tail risks (understood as in the previous paragraph), there is a role for international financial institutions to offer debt contracts that involve conditionality, typically on the level of fiscal efforts exerted by the Sovereign. The theoretical insight here is related with the completeness of the debt market, as explained in the previous section, because one positive role played by international financial institutions is to give to the Sovereign access to a broader set of financial contracts. In particular, at $t = 0$, an international financial institution can buy ST debt and simultaneously commit to rollover this ST debt at $t = 1$ under conditions contingent on the fiscal effort e_1. The value of this financial contract for the Sovereign depends on the cost of the effort e_1 required by the international financial institution. Since the effort cost depends on the fundamentals θ_1, the value of this financial contract at $t = 1$ is correlated with θ_1.

To be more precise, this contract would be described in our model as follows. At $t = 0$, the institution stands ready to offer LT financing at a lower interest rate than the market. Since the aim of such contracts is to avoid default (after a value θ_1 in the median region in our model) and LT debt may lead to default, this contract would be offered in limited amount (the Sovereign would complement its financing needs by issuing ST debt given that the LT interest rate is too high in the markets). The contract is also conditional: It depends on the effort level at $t = 1$. In case this effort is insufficient, the contract terms are revisited (in the sense of a deterioration of the debt burden of the Sovereign). The only theoretical ingredient that would be added to the model is then a new LT "asset" whose intermediate value is contingent on the effort level e_1 (and then on the fundamental θ_1 through the optimization problem of the Sovereign). We do not otherwise depart from the structure of the model (including rational expectations, competitive markets, optimal behavior of investors and Sovereign). In particular, it is optimal for the Sovereign to take advantage of this new LT asset (the market for LT debt would be inactive as a consequence of the optimal choice by the Sovereign) and to avoid default after a median θ_1 through the choice of an appropriate effort level e_1 (since the debt burden is reduced in this case at $t = 1$).

In summary, international financial institutions provide the Sovereign a debt contract whose future value is correlated with future fundamentals. Such debt contracts allow the Sovereign to hedge against the risk on future fundamentals. One role played by international financial institutions is then to provide the Sovereign with hedging instruments. This role is important since the Sovereign can otherwise hedge its position on competitive financial markets only if it bears the cost of being exposed to default in some state (namely, the Sovereign issues risky debt at high interest rates). This role needs to be further studied in light of the usual issues of moral hazard associated with the perspective of intervention of international financial institutions.

APPENDIX

Proof of Lemma 2. We first solve the ND case. V^{ND} (defined in (14)) is a parabola with a minimum at $ST_{1,\min}^{ND}$ and the value of V^{ND} associated with this minimum is:

$$V_{\min}^{ND} = \frac{\left(\frac{1}{1+r}LT + ST_0\right)^2}{\frac{1}{(1+r)^2} + \theta_1}, \tag{41}$$

which rewrites as (20), using the $t = 0$ budget constraint. Whenever $ST_{1,\min}^{ND} \leq \theta_2^L - LT$ (*i.e.*, (18)), this defines the ND solution. Otherwise, the ND solution is constrained and given by:

$$V_{\min C}^{ND} = \frac{\left(ST_0 + \frac{1}{1+r}LT - \frac{1}{1+r}\theta_2^L\right)^2}{\theta_1} + \left(\theta_2^L\right)^2. \tag{42}$$

which rewrites as (21), using the $t = 0$ budget constraint.

We now solve the PD case. V^{PD} is a parabola with a minimum at $ST_{1,\min}^{PD}$ equal to:

$$V_{\min}^{PD} = \frac{\left(ST_0 + \frac{\pi_2^H}{1+r}LT\right)^2}{\pi_2^H\left(\frac{1}{1+r}\right)^2 + \theta_1} + \pi_2^L K,$$

which rewrites as (23), using the $t = 0$ budget constraint. If $ST_{1,\min}^{PD} > \theta_2^L - LT$ (equivalent to (24)), then $ST_{1,\min}^{PD}$ is obviously the solution of the minimization program. If Condition (24) does not hold, $ST_{1,\min}^{PD} \leq \theta_2^L - LT$, then the fact that the domain of the minimization program is an open interval in $\mathbb{R}$ implies that the program admits no solution (the solution "should be" the boundary solution $ST_1 = \theta_2^L - LT$, but this value is not in the domain). **End of the proof.**

Claim 12 (Full Default) *Assume Assumption 1. At $t = 1$, the Sovereign never chooses ST_1 leading to default with probability 1 at $t = 2$ (i.e., default when either $\theta_2 = \theta_2^L$ or $\theta_2 = \theta_2^H$).*

Proof of Claim 12. If the Sovereign chooses ST_1 leading to default with probability 1 at $t = 2$, then $p_1 = 0$ (see (7)), the budget constraint at $t = 1$ is $ST_0 = e_1$ and the cost at $t = 1$ is:

$$V^{FD} = \frac{ST_0^2}{\theta_1} + K. \tag{43}$$

Since the cost in the ND case is either $V_{\min}^{ND}$ or $V_{\min C}^{ND}$ (with $V_{\min}^{ND} \leq V_{\min C}^{ND}$), we check that $V^{FD} > V_{\min C}^{ND}$. This rewrites (using (42)):

$$\frac{(ST_0)^2}{\theta_1} + K > \frac{\left(ST_0 + \frac{1}{1+r}LT - \frac{1}{1+r}\theta_2^L\right)^2}{\theta_1} + \left(\theta_2^L\right)^2.$$

Assumption 1 implies $K > \left(\theta_2^L\right)^2$ and $ST_0 > ST_0 + \frac{1}{1+r}LT - \frac{1}{1+r}\theta_2^L$ and $ST_0 + \frac{1}{1+r}LT \geq (1+r)\,D > \frac{1}{1+r}\theta_2^L$ (using the budget constraint). Hence, the above inequality is true. **End of the proof.**

Claim 13 *Let $P^-[\theta_1]$ and $P^+[\theta_1]$ be two polynomials of degree 2 in θ_1:*

$$\begin{aligned} P^-[\theta_1] &= \left(\pi_2^L K - \left(\theta_2^L\right)^2\right)\theta_1\left(\frac{\pi_2^H}{(1+r)^2}+\theta_1\right) + \left(D(1+r)\right)^2\theta_1 \\ &\quad - \left(D(1+r) - \frac{1}{1+r}\theta_2^L\right)^2\left(\frac{\pi_2^H}{(1+r)^2}+\theta_1\right), \end{aligned}$$

and:

$$\begin{aligned} P^+[\theta_1] &= \left(\pi_2^L K - \left(\theta_2^L\right)^2\right)\theta_1\left(\frac{\pi_2^H}{(1+r)^2}+\theta_1\right) + \left(D(1+r) + \frac{E\left(\pi^D\right)-\pi_2^L}{1+r}\theta_2^L\right)^2\theta_1 \\ &\quad - \left(D(1+r) + \frac{E\left(\pi^D\right)-1}{1+r}\theta_2^L\right)^2\left(\frac{\pi_2^H}{(1+r)^2}+\theta_1\right). \end{aligned}$$

Assume $\theta_1 \geq 0$. $\theta_1 \geq \theta_1^-$ is equivalent to $P^-[\theta_1] \geq 0$. $\theta_1 \geq \theta_1^+$ is equivalent to $P^+[\theta_1] \geq 0$.

Proof. The coefficient of degree 2 of $P^-[\theta_1]$ is $\left(\pi_2^L K - \left(\theta_2^L\right)^2\right) \geq 0$ (Assumption 1), and the coefficient of degree 0 is $P^-[0] < 0$. Hence, $P^-[\theta_1]$ admits a unique positive real root. This real root is θ_1^- (since Condition (25) rewrites $P^-[\theta_1] = 0$), and $\theta_1 \geq \theta_1^-$ is equivalent to $P^-[\theta_1] \geq 0$. The same argument holds true for $P^+[\theta_1]$ and θ_1^+. **End of the proof.**

Computations for Definition 3. Claim 13 shows that θ_1^- and θ_1^+ are well defined. Since $\check{\theta}_1$ and $\hat{\theta}_1$ are the values θ_1^+ for $E\left(\pi^D\right) = \pi_{\max}^D$ and $E\left(\pi^D\right) = 0$ respectively, $\check{\theta}_1$ and $\hat{\theta}_1$ are well defined.

$\theta_1^- < \hat{\theta}_1$ is equivalent to $P^+\left[\theta_1^-\right] < 0$ for $E\left(\pi^D\right) = 0$ (Claim 13). Using definition (25) of θ_1^-, we have:

$$\pi_2^L K - \left(\theta_2^L\right)^2 = \frac{\left(D(1+r) - \frac{1}{1+r}\theta_2^L\right)^2}{\theta_1^-} - \frac{\left(D(1+r)\right)^2}{\frac{\pi_2^H}{(1+r)^2}+\theta_1^-}, \tag{44}$$

and we find that $P^+\left[\theta_1^-\right] < 0$ writes:

$$\left(D(1+r) - \frac{\pi_2^L}{1+r}\theta_2^L\right)^2 < \left(D(1+r)\right)^2,$$

which holds true (Assumption 1).

For any value $E\left(\pi^D\right) \in \left[0, \pi_{\max}^D\right]$, θ_1^+ is implicitly defined by $P^+\left[\theta_1^+\right] = 0$ so that (by implicit functions theorem):

$$\frac{d\theta_1^+}{dE\left(\pi^D\right)} = -\frac{\frac{\partial P^+}{\partial E\left(\pi^D\right)}\left[\theta_1^+\right]}{\frac{\partial P^+}{\partial \theta_1}\left[\theta_1^+\right]}.$$

Since θ_1^+ is the unique positive root of the polynomial $P^+\left[\theta_1^+\right]$, then $\frac{\partial P^+}{\partial \theta_1}\left[\theta_1^+\right] > 0$. In addition, $\frac{\partial P^+}{\partial E(\pi^D)}\left[\theta_1^+\right] \leq 0$ is equivalent to:

$$\theta_1^+ \leq \frac{D}{\theta_2^L} + \frac{E\left(\pi^D\right) - 1}{\left(1+r\right)^2},$$

Claim 13 implies that this rewrites $P^+\left[\theta_1\right] \geq 0$ for $\theta_1 = \frac{D}{\theta_2^L} + \frac{E\left(\pi^D\right)-1}{(1+r)^2}$, which holds true (with $\left(\theta_2^L\right)^2 \leq K$, Assumption 1). Hence, $\frac{d\theta_1^+}{dE(\pi^D)} \geq 0$, and θ_1^+ increases from $\hat{\theta}_1$ to $\check{\theta}_1$ when $E\left(\pi^D\right)$ increases from 0 to $\pi^D_{\max}$. In particular, $\hat{\theta}_1 \leq \theta_1^+ \leq \check{\theta}_1$. **End of the computations.**

A straightforward consequence of Lemma 2 is:

Claim 14 *For a given $E\left(\pi^D\right)$ and debt portfolio (ST_0, LT) satisfying the $t = 0$ budget constraint, three intervals of θ_1 are defined:*

- *Interval 1:*
$$\theta_1 \in I_1 = \left[0, \frac{D}{\theta_2^L} + \frac{E\left(\pi^D\right)}{\left(1+r\right)^2}\frac{LT}{\theta_2^L} - \frac{1}{\left(1+r\right)^2}\right). \tag{45}$$
The PD solution exists and the ND solution is constrained.

- *Interval 2:*
$$\theta_1 \in I_2 = \left[\frac{D}{\theta_2^L} + \frac{E\left(\pi^D\right)}{\left(1+r\right)^2}\frac{LT}{\theta_2^L} - \frac{1}{\left(1+r\right)^2}, \frac{D}{\theta_2^L} + \frac{E\left(\pi^D\right)}{\left(1+r\right)^2}\frac{LT}{\theta_2^L} - \frac{1}{\left(1+r\right)^2}\left(\pi_2^L\frac{LT}{\theta_2^L} + \pi_2^H\right)\right).$$
The PD solution exists and the ND solution is interior.

- *Interval 3:*
$$\theta_1 \in I_3 = \left[\frac{D}{\theta_2^L} + \frac{E\left(\pi^D\right)}{\left(1+r\right)^2}\frac{LT}{\theta_2^L} - \frac{1}{\left(1+r\right)^2}\left(\pi_2^L\frac{LT}{\theta_2^L} + \pi_2^H\right), +\infty\right).$$
The PD solution does not exist and the ND solution is interior.

Claims 15 to 18 are used in the proof of Proposition 4 below.

Claim 15 *For a given $E\left(\pi^D\right)$ and debt portfolio (ST_0, LT) satisfying the $t = 0$ budget constraint, if θ_1 belongs to I_2 and I_3 defined in Claim 14, then the Sovereign chooses the interior ND solution at $t = 1$.*

Proof. For $\theta_1 \in I_3$, the interior ND solution is the only solution available. For $\theta_1 \in I_2$, the choice of no default (*i.e.*, $V_{\min}^{ND} \leq V_{\min}^{PD}$) writes, according to (20) and (23):

$$\frac{\left(D\left(1+r\right)+\frac{E\left(\pi^D\right)}{1+r}LT\right)^2}{\frac{1}{(1+r)^2}+\theta_1} \leq \frac{\left(D\left(1+r\right)+\frac{E\left(\pi^D\right)-\pi_2^L}{1+r}LT\right)^2}{\frac{\pi_2^H}{(1+r)^2}+\theta_1}+\pi_2^L K. \tag{46}$$

The condition defining the lower bound of I_2 rewrites:

$$\frac{D\left(1+r\right)+\frac{E\left(\pi^D\right)}{1+r}LT}{\frac{1}{(1+r)^2}+\theta_1} \leq \theta_2^L\left(1+r\right),$$

so that an upper bound for the LHS in (46) is:

$$\left(\frac{1}{\left(1+r\right)^2}+\theta_1\right)\left(\theta_2^L\right)^2\left(1+r\right)^2.$$

The condition defining the upper bound of I_2 rewrites:

$$\frac{D\left(1+r\right)+\frac{E\left(\pi^D\right)-\pi_2^L}{1+r}LT}{\frac{\pi_2^H}{(1+r)^2}+\theta_1} > \theta_2^L\left(1+r\right),$$

so that a lower bound for the RHS in (46) is:

$$\left(\theta_2^L\right)^2\left(1+r\right)^2\left(\frac{\pi_2^H}{\left(1+r\right)^2}+\theta_1\right)+\pi_2^L K.$$

Hence a sufficient condition for (46) is:

$$\left(\frac{1}{\left(1+r\right)^2}+\theta_1\right)\left(\theta_2^L\right)^2\left(1+r\right)^2 < \left(\theta_2^L\right)^2\left(1+r\right)^2\left(\frac{\pi_2^H}{\left(1+r\right)^2}+\theta_1\right)+\pi_2^L K,$$

which holds true since $\left(\theta_2^L\right)^2 < K$ (Assumption 1). **End of the proof.**

Claim 16 *For a given $E\left(\pi^D\right)$, if θ_1 belongs to I_1, then there exists a threshold LT^* (function of $E\left(\pi^D\right)$ and θ_1) such that for any debt portfolio (ST_0, LT) satisfying the $t=0$ budget constraint:*

- *If $LT \leq LT^*$, then the Sovereign chooses at $t=1$ a value ST_1 leading to no default,*
- *If $LT > LT^*$, then the Sovereign chooses at $t=1$ a value ST_1 leading to potential default.*

$LT^* \in \left[0, \theta_2^L\right]$ decreases in $E\left(\pi^D\right)$ and increases in θ_1.

Proof. The Sovereign chooses not to default iff:

$$V_{\min C}^{ND}\left(\theta_1, LT, E\left(\pi^D\right)\right) \leq V_{\min}^{PD}\left(\theta_1, LT, E\left(\pi^D\right)\right),$$

where the two values are defined in Equations (21) and (23). $V_{\min C}^{ND}\left(\theta_1, LT, E\left(\pi^D\right)\right)$ is increasing in LT since Assumption 1 implies that

$$D(1+r) + \frac{E\left(\pi^D\right)}{1+r}LT - \frac{1}{1+r}\theta_2^L \geq D(1+r) - \frac{1}{1+r}\theta_2^L > 0.$$

$V_{\min}^{PD}\left(\theta_1, LT, E\left(\pi^D\right)\right)$ is decreasing in LT since $\left(E\left(\pi^D\right) - \pi_2^L\right) \leq 0$ (Assumption 1) and:

$$\begin{aligned} D(1+r) + \frac{E\left(\pi^D\right) - \pi_2^L}{1+r}LT &\geq D(1+r) - \frac{\pi_2^L}{1+r}LT, \\ &\geq D(1+r) - \frac{\pi_2^L}{1+r}\theta_2^L > 0. \end{aligned}$$

This shows existence of the threshold LT^*. Assumption 1 implies $LT^* \in \left[0, \theta_2^L\right]$.

The threshold LT^* is implicitly characterized by:

$$V_{\min C}^{ND}\left(\theta_1, LT^*, E\left(\pi^D\right)\right) = V_{\min}^{PD}\left(\theta_1, LT^*, E\left(\pi^D\right)\right). \tag{47}$$

Differentiating this condition implies:

$$\frac{dLT^*}{dE\left(\pi^D\right)} = -\frac{\frac{\partial}{\partial E\left(\pi^D\right)}\left(V_{\min C}^{ND} - V_{\min}^{PD}\right)}{\frac{\partial}{\partial LT}\left(V_{\min C}^{ND} - V_{\min}^{PD}\right)}.$$

We compute this derivative to show that it is non positive. First, we have:

$$\begin{aligned} \frac{\partial}{\partial LT}\left(V_{\min C}^{ND} - V_{\min}^{PD}\right) &= \frac{E\left(\pi^D\right)}{1+r}\frac{2\left(D(1+r) + \frac{E\left(\pi^D\right)}{1+r}LT - \frac{1}{1+r}\theta_2^L\right)}{\theta_1} \\ &\quad - \frac{\left(E\left(\pi^D\right) - \pi_2^L\right)}{1+r}\frac{2\left(D(1+r) + \frac{E\left(\pi^D\right) - \pi_2^L}{1+r}LT\right)}{\frac{\pi_2^H}{(1+r)^2} + \theta_1}, \end{aligned}$$

and $\frac{\partial}{\partial LT}\left(V_{\min C}^{ND} - V_{\min}^{PD}\right) > 0$ is equivalent to:

$$\begin{aligned} & E\left(\pi^D\right)\left(D(1+r) + \frac{E\left(\pi^D\right)}{1+r}LT - \frac{1}{1+r}\theta_2^L\right)\left(\frac{\pi_2^H}{(1+r)^2} + \theta_1\right) \\ > \ & \theta_1\left(E\left(\pi^D\right) - \pi_2^L\right)\left(D(1+r) + \frac{E\left(\pi^D\right) - \pi_2^L}{1+r}LT\right). \end{aligned}$$

Since $D(1+r) > \frac{1}{1+r}\theta_2^L \geq \frac{\pi_2^L}{1+r}LT$ (Assumption 1), the LHS of this inequality is positive and the RHS is negative (using $\left(E\left(\pi^D\right)-\pi_2^L\right) \leq 0$, Assumption 1 again). Hence, $\frac{\partial}{\partial LT}\left(V_{\min C}^{ND}-V_{\min}^{PD}\right) > 0$. The partial derivative $\frac{\partial\left(V_{\min C}^{ND}-V_{\min}^{PD}\right)}{\partial E(\pi^D)}$ is:

$$\frac{2LT}{1+r}\frac{\left(D(1+r)+\frac{E\left(\pi^D\right)}{1+r}LT-\frac{1}{1+r}\theta_2^L\right)}{\theta_1}-\frac{2LT}{1+r}\frac{\left(D(1+r)+\frac{E\left(\pi^D\right)-\pi_2^L}{1+r}LT\right)}{\frac{\pi_2^H}{(1+r)^2}+\theta_1},$$

and $\frac{\partial}{\partial E(\pi^D)}\left(V_{\min C}^{ND}-V_{\min}^{PD}\right) \geq 0$ is equivalent to:

$$\frac{\left(D(1+r)+\frac{E\left(\pi^D\right)}{1+r}LT-\frac{1}{1+r}\theta_2^L\right)^2}{\theta_1^2} \geq \frac{\left(D(1+r)+\frac{E\left(\pi^D\right)-\pi_2^L}{1+r}LT\right)^2}{\left(\frac{\pi_2^H}{(1+r)^2}+\theta_1\right)^2}, \tag{48}$$

since $\left(D(1+r)+\frac{E\left(\pi^D\right)}{1+r}LT-\frac{1}{1+r}\theta_2^L\right) > 0$ and $\left(D(1+r)+\frac{E\left(\pi^D\right)-\pi_2^L}{1+r}LT\right) > 0$ (Assumption 1: $D(1+r) > \frac{1}{1+r}\theta_2^L \geq \frac{\pi_2^L}{1+r}LT$). Condition (47) implies:

$$\frac{\left(D(1+r)+\frac{E\left(\pi^D\right)-\pi_2^L}{1+r}LT^*\right)^2}{\frac{\pi_2^H}{(1+r)^2}+\theta_1} \leq \frac{\left(D(1+r)+\frac{E\left(\pi^D\right)}{1+r}LT^*-\frac{1}{1+r}\theta_2^L\right)^2}{\theta_1},$$

as $\left(\pi_2^L K-\left(\theta_2^L\right)^2\right) > 0$ (Assumption 1). Given $\frac{1}{\frac{\pi_2^H}{(1+r)^2}+\theta_1} < \frac{1}{\theta_1}$, Condition (48) holds true at LT^*, and $\frac{\partial}{\partial E(\pi^D)}\left(V_{\min C}^{ND}-V_{\min}^{PD}\right) \geq 0$ follows at LT^*. This shows that:

$$\frac{dLT^*}{dE\left(\pi^D\right)} \leq 0.$$

We now turn attention to:

$$\frac{dLT^*}{d\theta_1} = -\frac{\frac{\partial}{\partial\theta_1}\left(V_{\min C}^{ND}-V_{\min}^{PD}\right)}{\frac{\partial}{\partial LT}\left(V_{\min C}^{ND}-V_{\min}^{PD}\right)}.$$

We compute:

$$\frac{\partial}{\partial\theta_1}\left(V_{\min C}^{ND}-V_{\min}^{PD}\right) = -\frac{\left(D(1+r)+\frac{E\left(\pi^D\right)}{1+r}LT-\frac{1}{1+r}\theta_2^L\right)^2}{\theta_1^2}+\frac{\left(D(1+r)+\frac{E\left(\pi^D\right)-\pi_2^L}{1+r}LT\right)^2}{\left(\pi_2^H\left(\frac{1}{1+r}\right)^2+\theta_1\right)^2},$$

and $\frac{\partial}{\partial\theta_1}\left(V_{\min C}^{ND}-V_{\min}^{PD}\right) \leq 0$ is equivalent to Condition (48). Hence $\frac{\partial}{\partial\theta_1}\left(V_{\min C}^{ND}-V_{\min}^{PD}\right) \leq 0$ at LT^* and:

$$\frac{dLT^*}{d\theta_1} \geq 0.$$

End of the proof.

The Claim below shows that, for $\theta_1 < \theta_1^-$, the Sovereign chooses to potentially default independently of $E\left(\pi^D\right)$.

Claim 17 *For a given $E\left(\pi^D\right)$ and debt portfolio (ST_0, LT) satisfying the $t = 0$ budget constraint, if θ_1 belongs to I_1, then we have: the Sovereign chooses the PD solution (i.e., $LT^* = 0$)[10] iff $\theta_1 < \theta_1^-$. In addition, $\theta_1^- \in I_1$.*

Proof. Given Claim 16, it is enough to show that, for $LT = 0$, the Sovereign chooses not to default iff $\theta_1 \geq \theta_1^-$. The optimal choice of no default at $t = 1$ writes $V_{\min C}^{ND} \leq V_{\min}^{PD}$. Using (21) and (23), this rewrites (for $LT = 0$):

$$P^- [\theta_1] \geq 0.$$

This is equivalent to $\theta_1 \geq \theta_1^-$ (Claim 13). In addition, $\theta_1^- \leq \theta_1^+$ (see Definition 3) and $\theta_1^+ \in I_1$ (Claim 18) so that $\theta_1^- \in I_1$. **End of the proof.**

The Claim below shows that, for $\theta_1 \geq \theta_1^+$, the Sovereign chooses not to default.

Claim 18 *For a given $E\left(\pi^D\right)$ and debt portfolio (ST_0, LT) satisfying the $t = 0$ budget constraint, if $\theta_1 \in I_1$, then the Sovereign chooses the constrained ND solution (i.e., $LT^* = \theta_2^L$) for $\theta_1 \geq \theta_1^+$, and the threshold LT^* is strictly smaller than θ_2^L for $\theta_1 < \theta_1^+$. In addition, $\theta_1^+ \in I_1$.*

Proof. The threshold LT^* is θ_2^L iff $V_{\min C}^{ND} \leq V_{\min}^{PD}$ for $LT = \theta_2^L$. This writes:

$$\frac{\left(D\left(1+r\right) + \frac{E\left(\pi^D\right)-1}{1+r}\theta_2^L\right)^2}{\theta_1} + \left(\theta_2^L\right)^2 \leq \frac{\left(D\left(1+r\right) + \frac{E\left(\pi^D\right)-\pi_2^L}{1+r}\theta_2^L\right)^2}{\pi_2^H\left(\frac{1}{1+r}\right)^2 + \theta_1} + \pi_2^L K.$$

This latter inequality rewrites $P^+ [\theta_1] \geq 0$, which is equivalent to $\theta_1 \geq \theta_1^+$ (Claim 13). Analogously, $\theta_1^+ \in I_1$, *i.e.*, $\theta_1^+ \leq \frac{D}{\theta_2^L} + \frac{E\left(\pi^D\right)}{(1+r)^2}\frac{LT}{\theta_2^L} - \frac{1}{(1+r)^2}$, is equivalent to:

$$P^+ \left[\frac{D}{\theta_2^L} + \frac{E\left(\pi^D\right)}{\left(1+r\right)^2}\frac{LT}{\theta_2^L} - \frac{1}{\left(1+r\right)^2}\right] \geq 0.$$

This holds true (with $\left(\theta_2^L\right)^2 \leq K$, Assumption 1). **End of the proof.**

Proof of Proposition 4. The computations for Definition 3 show that θ_1^+ increases from $\hat{\theta}_1$ to $\check{\theta}_1$ when $E\left(\pi^D\right)$ increases from 0 to $\pi_{\max}^D$. Combining Claims 14 to 18 proves the proposition. **End of the proof.**

[10] This is a slight abuse of notation since Claim 16 states that the PD solution is chosen for $LT > LT^*$ only.

Proof of Corollary 5. Claim 14 shows that $\theta_1 \in I_2 \cup I_3$ iff:

$$\theta_1 \geq \frac{D}{\theta_2^L} + \frac{E\left(\pi^D\right)}{(1+r)^2}\frac{LT}{\theta_2^L} - \frac{1}{(1+r)^2}. \tag{49}$$

A sufficient condition is then:

$$\theta_1 \geq \frac{D}{\theta_2^L} + \frac{\pi_{\max}^D}{(1+r)^2} - \frac{1}{(1+r)^2}.$$

Claim 15 shows that the Sovereign chooses the interior ND solution when this condition holds. **End of the proof.**

Proof of Proposition 6. If the Sovereign issues a portfolio leading to ND, then the expected cost is either $V_{\min}^{ND}\left(\theta_1, LT, E\left(\pi^D\right)\right)$ or $V_{\min C}^{ND}\left(\theta_1, LT, E\left(\pi^D\right)\right)$ with $E\left(\pi^D\right) = 0$. Using (20) and (21), these values of the costs do not depend on LT:

$$\begin{aligned} V_{\min}^{ND}\left(\theta_1, LT, 0\right) &= \frac{\left(D\left(1+r\right)\right)^2}{\frac{1}{(1+r)^2} + \theta_1}, & (50)\\ V_{\min C}^{ND}\left(\theta_1, LT, 0\right) &= \frac{\left(D\left(1+r\right) - \frac{1}{1+r}\theta_2^L\right)^2}{\theta_1} + \left(\theta_2^L\right)^2. & (51)\end{aligned}$$

If the Sovereign issues a portfolio leading to PD, then the expected cost is $V_{\min}^{PD}\left(\theta_1, LT, E\left(\pi^D\right)\right)$ with $E\left(\pi^D\right) = \pi_2^L$. Using (23), $V_{\min}^{PD}\left(\theta_1, LT, \pi_2^L\right)$ does not depend on LT:

$$V_{\min}^{PD}\left(\theta_1, LT, \pi_2^L\right) = \frac{\left(D\left(1+r\right)\right)^2}{\frac{\pi_2^H}{(1+r)^2} + \theta_1} + \pi_2^L K. \tag{52}$$

$V_{\min}^{PD}\left(\theta_1, LT, E\left(\pi^D\right)\right) < V_{\min C}^{ND}\left(\theta_1, LT, 0\right)$ rewrites $P^-\left[\theta_1\right] < 0$. This is equivalent to $\theta_1 < \theta_1^-$ (Claim 13). Given that the ND solution is constrained for $\theta_1 < \theta_1^-$ ($\theta_1^- \in I_1$, see Claims 14 and 17), this shows that the Sovereign prefers the PD solution at $t = 0$ for $\theta_1 < \theta_1^-$. For $\theta_1 \geq \theta_1^-$, either the ND solution is constrained and we have $V_{\min}^{PD}\left(\theta_1, LT, E\left(\pi^D\right)\right) \geq V_{\min C}^{ND}\left(\theta_1, LT, 0\right)$, or the ND solution is interior and $V_{\min}^{ND}\left(\theta_1, LT, 0\right) \leq V_{\min}^{PD}\left(\theta_1, LT, \pi_2^L\right)$. In both cases, the Sovereign prefers the ND solution at $t = 0$. Proposition 4 provides the restrictions needed on the portfolio to ensure that the solution that is preferred at $t = 0$ is chosen at $t = 1$. **End of the proof.**

Proof of Proposition 7. We first compute the optimal LT and the value of the expected cost at the optimum in the PD case. The FOC of the minimization of (29) is:

$$\frac{d}{dLT}\left(\pi_1^M V_{\min}^{PD} + \pi_1^H V_{\min}^{ND}\right) = 0. \tag{53}$$

This rewrites:

$$\pi_1^M \frac{E\left(\pi^D\right) - \pi_2^L}{1+r} \frac{\left(D\left(1+r\right) + \frac{E\left(\pi^D\right) - \pi_2^L}{1+r}LT\right)}{\pi_2^H\left(\frac{1}{1+r}\right)^2 + \theta_1^M} + \pi_1^H \frac{E\left(\pi^D\right)}{1+r} \frac{\left(D\left(1+r\right) + \frac{E\left(\pi^D\right)}{1+r}LT\right)}{\frac{1}{(1+r)^2} + \theta_1^H} = 0, \tag{54}$$

with $E\left(\pi^D\right)=\pi_1^M\pi_2^L$, and:

$$LT=\frac{\frac{\pi_1^M\pi_2^L-\left(\pi_1^M+\pi_1^H\pi_2^H\right)E\left(\pi^D\right)}{(1+r)^2}+\pi_1^M\left(\pi_2^L-E\left(\pi^D\right)\right)\theta_1^H-\pi_1^H E\left(\pi^D\right)\theta_1^M}{\pi_1^M\left(\frac{\pi_2^L-E\left(\pi^D\right)}{1+r}\right)^2\left(\frac{1}{(1+r)^2}+\theta_1^H\right)+\pi_1^H\left(\frac{\pi_2^H}{(1+r)^2}+\theta_1^M\right)\left(\frac{E\left(\pi^D\right)}{1+r}\right)^2}D.$$

LT is positive (the denominator is positive, the numerator as well since $E\left(\pi^D\right)=\pi_1^M\pi_2^L$ and $\pi_1^M\left(\pi_2^L-E\left(\pi^D\right)\right)\theta_1^H\geq\pi_1^M\left(\pi_2^L-E\left(\pi^D\right)\right)\theta_1^M\geq\pi_1^H E\left(\pi^D\right)\theta_1^M$). The numerator is decreasing in $E\left(\pi^D\right)$, the denominator is decreasing as well. Straightforward algebra leads to (31) (and the associated optimal portfolio, which is the one stated in Proposition 9, see (35) and (36)).

This value of LT satisfies:

$$V_{\min}^{PD}\left(\theta_1^M,LT,\pi_1^M\pi_2^L\right)<V_{\min C}^{ND}\left(\theta_1^M,LT,0\right)<V_{\min C}^{ND}\left(\theta_1^M,LT,\pi_1^M\pi_2^L\right),$$

where the second inequality follows from $V_{\min C}^{ND}$ being increasing in $E\left(\pi^D\right)$ (the first one states that the PD solution is preferred to the constrained ND solution). This shows that this value of LT satisfies the incentive constraint for PD and therefore determines a PD solution.

The Sovereign chooses a portfolio (ST_0,LT) implying no default iff the value (30) of the expected cost is smaller than the value (31) of the expected cost. This rewrites as:

$$Q\left[\pi_1^H\right]\leq 0,$$

where $Q\left[\pi_1^H\right]$ is a polynomial of degree 2 in π_1^H:

$$\left(\left(1-\pi_1^H\right)\left(\frac{\left(D\left(1+r\right)-\frac{1}{1+r}\theta_2^L\right)^2}{\theta_1^M}+\left(\theta_2^L\right)^2-\pi_2^L K\right)+\pi_1^H\frac{\left(D\left(1+r\right)\right)^2}{\frac{1}{(1+r)^2}+\theta_1^H}\right)\times \tag{55}$$

$$\left(\frac{1-\left(1-\pi_1^H\right)\pi_2^L}{\left(1+r\right)^2}+\theta_1^M+\pi_1^H\left(\theta_1^H-\theta_1^M\right)\right)-\left(D\left(1+r\right)\right)^2. \tag{56}$$

One of the roots of $Q\left[\pi_1^H\right]$ is $\pi_1^H=1$. For $\pi_1^H=0$, we have:

$$Q\left[0\right]=\left(\frac{\left(D\left(1+r\right)-\frac{1}{1+r}\theta_2^L\right)^2}{\theta_1^M}+\left(\theta_2^L\right)^2-\pi_2^L K\right)\left(\frac{1-\pi_2^L}{\left(1+r\right)^2}+\theta_1^M\right)-\left(D\left(1+r\right)\right)^2,$$

and $Q\left[0\right]\leq 0$ since it rewrites exactly as $P^-\left[\theta_1^M\right]\geq 0$, which is equivalent to $\theta_1^M\geq\theta_1^-$ (Claim 13). The other root is:

$$\hat{\pi}_1^H=\frac{Q\left[0\right]}{C}, \tag{57}$$

that is the ratio between the coefficient $Q\left[0\right]$ of degree 0 and the coefficient C of degree 2 where:

$$C=\left(\frac{\left(D\left(1+r\right)\right)^2}{\frac{1}{(1+r)^2}+\theta_1^H}+\pi_2^L K-\frac{\left(D\left(1+r\right)-\frac{1}{1+r}\theta_2^L\right)^2}{\theta_1^M}-\left(\theta_2^L\right)^2\right)\left(\frac{\pi_2^L}{\left(1+r\right)^2}+\theta_1^H-\theta_1^M\right).$$

We distinguish 2 subcases:

- If $C > 0$, then $\hat{\pi}_1^H < 0$ and $Q\left[\pi_1^H\right] \leq 0$ "between the roots" (in particular, $Q\left[\pi_1^H\right] \leq 0$ for every $\pi_1^H \in [0,1]$).

- If $C < 0$, then $\hat{\pi}_1^H > 0$ and $Q\left[\pi_1^H\right] \leq 0$ "outside the roots". Either $\hat{\pi}_1^H > 1$ (and $Q\left[\pi_1^H\right] \leq 0$ for every $\pi_1^H \in [0,1]$), or $\hat{\pi}_1^H < 1$ (and $Q\left[\pi_1^H\right] \leq 0$ for $0 \leq \pi_1^H \leq \hat{\pi}_1^H$ and $Q\left[\pi_1^H\right] \geq 0$ for $\hat{\pi}_1^H \leq \pi_1^H \leq 1$).

Taken together, these cases summarize as follows: If $C > Q[0]$, then $Q\left[\pi_1^H\right] \leq 0$ for every $\pi_1^H \in [0,1]$; If $C < Q[0]$, then $0 < \hat{\pi}_1^H < 1$, $Q\left[\pi_1^H\right] \leq 0$ for $0 \leq \pi_1^H \leq \hat{\pi}_1^H$ and $Q\left[\pi_1^H\right] \geq 0$ for $\hat{\pi}_1^H \leq \pi_1^H \leq 1$. The condition $C < Q[0]$ rewrites as:

$$\left(\pi_2^L K - \left(\theta_2^L\right)^2\right) < \frac{\left(D(1+r) - \frac{1}{1+r}\theta_2^L\right)^2}{\theta_1^M} - \frac{\left(D(1+r)\right)^2}{\frac{1}{(1+r)^2} + \theta_1^H}\left(2 - \frac{\frac{\pi_2^H}{(1+r)^2} + \theta_1^M}{\frac{1}{(1+r)^2} + \theta_1^H}\right), \tag{58}$$

We now show that Condition (58) holds true for $\theta_1^H \geq \theta_1^*$ and $\theta_1^- \leq \theta_1^M \leq \hat{\theta}_1$. First, the RHS of Condition (58) is increasing in θ_1^H (the function $X(a - X)$ is increasing for $0 < X < a/2$, with $X = 1/\left(\frac{1}{(1+r)^2} + \theta_1^H\right)$ and $a = 1/\left(\pi_2^H + (1+r)^2\theta_1^M\right)$). For $\theta_1^H = \theta_1^*$, Condition (58) rewrites:

$$\frac{\pi_2^L K}{\left(\theta_2^L\right)^2} - 1 < \frac{(x-1)^2}{(1+r)^2\theta_1^M} - \frac{x^2}{x + \pi_{\max}^D}\left(2 - \frac{\pi_2^H + (1+r)^2\theta_1^M}{x + \pi_{\max}^D}\right), \tag{59}$$

with $x = D(1+r)^2/\theta_2^L > 1$ (Assumption 1). In order to study (59), notice that $P^+[\theta_1] < 0$ for $E\left(\pi^D\right) = 0$ and $\theta_1 = \frac{\left(x+\pi_{\max}^D\right)(x-1)}{(1+r)^2 x}$ since $P^+[\theta_1] < 0$ rewrites:

$$\frac{(x-1)x}{\left(x + \pi_{\max}^D\right)} < \frac{x\left(x - \pi_2^L\right)^2}{\pi_2^H x + \left(x + \pi_{\max}^D\right)(x-1)} + \frac{\pi_2^L K}{\left(\theta_2^L\right)^2} - 1, \tag{60}$$

which is implied by:

$$\frac{(x-1)x}{\left(x + \pi_{\max}^D\right)} < \frac{x\left(x - \pi_2^L\right)^2}{\pi_2^H x + \left(x + \pi_{\max}^D\right)(x-1)}, \tag{61}$$

which always holds true for $x > 1$. As $P^+[\theta_1] < 0$ implies:

$$(1+r)^2\hat{\theta}_1 < \theta_1 = \frac{\left(x + \pi_{\max}^D\right)(x-1)}{x},$$

we have that the derivative of the RHS of Condition (59) is negative for $\theta_1^M \leq \hat{\theta}_1$: This RHS is decreasing in θ_1^M. Hence, Condition (58) holds true for $\theta_1^H \geq \theta_1^*$ and $\theta_1^- \leq \theta_1^M \leq \hat{\theta}_1$ iff it holds true for $\theta_1^H = \theta_1^*$ and $\theta_1^M = \hat{\theta}_1$, *i.e.*:

$$\frac{\pi_2^L K}{\left(\theta_2^L\right)^2} - 1 < \frac{(x-1)^2}{(1+r)^2\hat{\theta}_1} - \frac{x^2}{x + \pi_{\max}^D}\left(2 - \frac{\pi_2^H + (1+r)^2\hat{\theta}_1}{x + \pi_{\max}^D}\right), \tag{62}$$

Using Condition (26) defining $\hat{\theta}_1$ with $E\left(\pi^D\right) = 0$, Condition (62) rewrites:

$$\left(1 - \frac{\pi_2^L}{x}\right)^2 > Y\left(2 - Y\right), \tag{63}$$

with $Y = \frac{\pi_2^H + (1+r)^2 \hat{\theta}_1}{x + \pi_{\max}^D} > 0$. Before we study (63), notice that $P^+\left[\hat{\theta}_1\right] = 0$ for $E\left(\pi^D\right) = 0$ implies:

$$\frac{(x-1)^2}{(1+r)^2 \hat{\theta}_1} > \frac{\left(x - \pi_2^L\right)^2}{\pi_2^H + (1+r)^2 \hat{\theta}_1},$$

which rewrites as:

$$(1+r)^2 \hat{\theta}_1 < \frac{x}{2} + \frac{2 - \left(3 - \pi_2^L\right) x}{2\left(2x - \pi_2^L - 1\right)},$$

and $x > 1$ implies that

$$(1+r)^2 \hat{\theta}_1 < \frac{x}{2}$$

and $Y < \frac{\pi_2^H + \frac{x}{2}}{x + \pi_{\max}^D} < 1$. Hence, a sufficient condition for (63) is:

$$\left(1 - \frac{\pi_2^L}{x}\right)^2 > \frac{\pi_2^H + \frac{x}{2}}{x + \pi_{\max}^D}\left(2 - \frac{\pi_2^H + \frac{x}{2}}{x + \pi_{\max}^D}\right), \tag{64}$$

Using $\pi_{\max}^D = \pi_2^L$, this rewrites:

$$\frac{1}{4}x^4 - x^3 + x^2 + \pi_2^L\left(\pi_2^L - 4\right)x^2 + \left(\pi_2^L\right)^4 > 0. \tag{65}$$

Given $x > 1$, a lower bound of the LHS is

$$\frac{1}{4}x^4 - x^3 - 2x^2,$$

which is positive for $x > 6$ (this is the only use in the proof of the Assumption $x > 6$). Hence (63) holds true (and so does (62)).

Lastly, note that when θ_1^H tends to $+\infty$, Condition (58) tends to:

$$\left(\pi_2^L K - \left(\theta_2^L\right)^2\right) < \frac{\left(D\left(1+r\right) - \frac{1}{1+r}\theta_2^L\right)^2}{\theta_1^M},$$

which holds true since it follows from $P^+\left[\theta_1^M\right] > 0$ for $E\left(\pi^D\right) = 0$ (*i.e.*, $\theta_1^M < \hat{\theta}_1$). For $\theta_1^M = \theta_1^-$, Condition (58) is:

$$\frac{\left(D\left(1+r\right) - \frac{1}{1+r}\theta_2^L\right)^2}{\theta_1^M} - \frac{\left(D\left(1+r\right)\right)^2}{\frac{\pi_2^H}{(1+r)^2} + \theta_1^M} < \frac{\left(D\left(1+r\right) - \frac{1}{1+r}\theta_2^L\right)^2}{\theta_1^M} - \frac{\left(D\left(1+r\right)\right)^2}{\frac{1}{(1+r)^2} + \theta_1^H}\left(2 - \frac{\frac{\pi_2^H}{(1+r)^2} + \theta_1^M}{\frac{1}{(1+r)^2} + \theta_1^H}\right).$$

This rewrites:

$$\left(\frac{1}{(1+r)^2}+\theta_1^H\right)^2>\left(\frac{\pi_2^H}{(1+r)^2}+\theta_1^M\right)\left(2\left(\frac{1}{(1+r)^2}+\theta_1^H\right)-\left(\frac{\pi_2^H}{(1+r)^2}+\theta_1^M\right)\right),$$

which holds true. **End of the proof.**

Proof of Proposition 8. We compute the expected costs in the two cases where the Sovereign chooses a portfolio leading to PD under θ_1^M and the Sovereign chooses a portfolio leading to ND under θ_1^M. We then compare these two expected costs.

If the Sovereign chooses a portfolio leading to PD under θ_1^M, then the expected cost is (using (23) and $E\left(\pi^D\right)=\pi_2^L$):

$$\begin{aligned}&\pi_1^L V_{\min}^{PD}\left(\theta_1^L, LT, E\left(\pi^D\right)\right)+\pi_1^M V_{\min}^{PD}\left(\theta_1^M, LT, E\left(\pi^D\right)\right)\\ =\;&\left(\frac{\pi_1^L}{\frac{\pi_2^H}{(1+r)^2}+\theta_1^L}+\frac{\pi_1^M}{\frac{\pi_2^H}{(1+r)^2}+\theta_1^M}\right)(D(1+r))^2+\pi_2^L K. \end{aligned}\tag{66}$$

If the Sovereign chooses a portfolio leading to ND under θ_1^M, then the expected cost is (using (21), (23) and $E\left(\pi^D\right)=\pi_1^L\pi_2^L$):

$$\pi_1^L V_{\min}^{PD}\left(\theta_1^L, LT, E\left(\pi^D\right)\right)+\pi_1^M V_{\min C}^{ND}\left(\theta_1^M, LT, E\left(\pi^D\right)\right),$$

and the optimal LT solves the FOC:

$$\frac{d}{dLT}\left(\pi_1^L V_{\min}^{PD}\left(\theta_1^L, LT, E\left(\pi^D\right)\right)+\pi_1^M V_{\min C}^{ND}\left(\theta_1^M, LT, E\left(\pi^D\right)\right)\right)=0.$$

This is:

$$LT=\frac{(1+r)^2}{\pi_2^L}\frac{\left(\theta_1^M-\theta_1^L-\frac{\pi_2^H}{(1+r)^2}\right)D+\left(\frac{\pi_2^H}{(1+r)^2}+\theta_1^L\right)\frac{1}{(1+r)^2}\theta_2^L}{\frac{\pi_1^L\pi_2^H}{(1+r)^2}+\bar{\theta}_1}.\tag{67}$$

The minimum expected cost follows, replacing LT by its value (67):

$$\frac{\left(D(1+r)-\frac{\pi_1^M}{1+r}\theta_2^L\right)^2}{\frac{\pi_1^L\pi_2^H}{(1+r)^2}+\bar{\theta}_1}+\pi_1^L\pi_2^L K+\pi_1^M\left(\theta_2^L\right)^2.\tag{68}$$

The Sovereign prefers the ND solution iff (68) is smaller than (66). Using $P^-\left[\theta_1^M\right]\geq 0$ (*i.e.*, $\theta_1^M\geq\theta_1^-$ and Claim 13), we have:

$$\frac{\left(D(1+r)-\frac{1}{1+r}\theta_2^L\right)^2}{\theta_1^M}\leq\frac{1}{\frac{\pi_2^H}{(1+r)^2}+\theta_1^M}(D(1+r))^2+\pi_2^L K-\left(\theta_2^L\right)^2.$$

A sufficient condition for (68) smaller than (66) is then:

$$\frac{\left(D\left(1+r\right)-\frac{\pi_1^M}{1+r}\theta_2^L\right)^2}{\frac{\pi_1^L\pi_2^H}{(1+r)^2}+\bar{\theta}_1}<\frac{\pi_1^L\left(D\left(1+r\right)\right)^2}{\frac{\pi_2^H}{(1+r)^2}+\theta_1^L}+\frac{\pi_1^M\left(D\left(1+r\right)-\frac{1}{1+r}\theta_2^L\right)^2}{\theta_1^M}.$$

This rewrites as the following polynomial of degree 2 in $\alpha=D\left(1+r\right)^2/\theta_2^L$ being positive:

$$\left(\frac{\pi_1^L}{\frac{\pi_2^H}{(1+r)^2}+\theta_1^L}+\frac{\pi_1^M}{\theta_1^M}-\frac{1}{\frac{\pi_1^L\pi_2^H}{(1+r)^2}+\bar{\theta}_1}\right)\alpha^2-2\pi_1^M\left(\frac{1}{\theta_1^M}-\frac{1}{\frac{\pi_1^L\pi_2^H}{(1+r)^2}+\bar{\theta}_1}\right)\alpha+\pi_1^M\left(\frac{1}{\theta_1^M}-\frac{\pi_1^M}{\frac{\pi_1^L\pi_2^H}{(1+r)^2}+\bar{\theta}_1}\right).$$

The coefficient of degree 2 is positive (convexity of $1/X$). The discriminant is zero since it writes:

$$\pi_1^M\left(\frac{1}{\theta_1^M}-\frac{1}{\frac{\pi_1^L\pi_2^H}{(1+r)^2}+\bar{\theta}_1}\right)^2=\left(\frac{1}{\theta_1^M}-\frac{\pi_1^M}{\frac{\pi_1^L\pi_2^H}{(1+r)^2}+\bar{\theta}_1}\right)\left(\frac{\pi_1^L}{\frac{\pi_2^H}{(1+r)^2}+\theta_1^L}+\frac{\pi_1^M}{\theta_1^M}-\frac{1}{\frac{\pi_1^L\pi_2^H}{(1+r)^2}+\bar{\theta}_1}\right),$$

which simplifies to:

$$\frac{1}{\frac{\pi_1^L\pi_2^H}{(1+r)^2}+\bar{\theta}_1}\left(\frac{\pi_1^M}{\frac{\pi_2^H}{(1+r)^2}+\theta_1^L}+\frac{\pi_1^L}{\theta_1^M}\right)=\frac{1}{\theta_1^M}\frac{1}{\frac{\pi_2^H}{(1+r)^2}+\theta_1^L}.$$

and holds true because:

$$\left(\frac{\pi_1^L\pi_2^H}{\left(1+r\right)^2}+\bar{\theta}_1\right)=\pi_1^L\left(\frac{\pi_2^H}{\left(1+r\right)^2}+\theta_1^L\right)+\pi_1^M\theta_1^M.$$

It follows that the polynomial in α is always positive: the Sovereign always prefers the ND solution. **End of the proof.**

Proof of Proposition 9. The optimal portfolio (35) and (36) is computed in the proof of Proposition 7 above. The variations of LT and ST_0 in π_1^H are obtained through the computation of derivatives of the expressions (35) and (36). The variation of p_0 is obvious.

In the FOC (54), the $E\left(\pi^D\right)$ correspond to the price p_0 (and then to the investors' belief), the π_1^H correspond to the Sovereign's belief in the expected cost $\left(1-\pi_1^H\right)V_{\min}^{PD}\left(\theta_1^M,LT,E\left(\pi^D\right)\right)+\pi_1^HV_{\min}^{ND}\left(\theta_1^H,LT,E\left(\pi^D\right)\right)$. Differentiating (54) w.r.t. LT and π_1^H shows that $\frac{dLT}{d\pi_1^H}<0$ for given $E\left(\pi^D\right)$ (this is the second effect described in the comments below Proposition 9). Differentiating (54) w.r.t. LT and $E\left(\pi^D\right)$ allows the computation of $\frac{dLT}{dE(\pi^D)}$ for given π_1^H (this is the first effect described in the comments below Proposition 9). If $\frac{dLT}{dE(\pi^D)}<0$, then the substitution effect dominates. Otherwise, the income effect dominates. More precisely, $\frac{dLT}{dE(\pi^D)}=-\frac{dFOC}{dE(\pi^D)}/\frac{dFOC}{dLT}$, where FOC is the LHS of (54); $\frac{dFOC}{dLT}>0$ using $\pi_2^L\geq E\left(\pi^D\right)$ (Assumption 1); the sign of $\frac{dFOC}{dE(\pi^D)}$ is shown to depend on the choice of the parameters. **End of the proof.**

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