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Assessing DSGE Models with Capital Accumulation and Indeterminacy

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Assessing DSGE Models with Capital Accumulation and Indeterminacy

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Abstract

The simulated results of this paper show that New Keynesian DSGE models with capital accumulation can generate substantial persistencies in the dynamics of the main economic variables, due to the stock nature of capital. Empirical estimates on U.S. data from 1960:I to 2008:I show the response of monetary policy to inflation was almost twice lower than traditionally considered, as capital accumulation creates an additional channel of influence through real interest rates in the production sector. Versions of the model with indeterminacy empirically outperform determinate versions. This paper allows for the reconsideration of previous findings and has significant monetary policy implications.

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Introduction

Dynamic stochastic general equilibrium (DSGE) models with nominal rigidities ("New Keynesian" models) became very popular in the analysis of monetary policy in the last decades. Canonical versions of these models are well-studied and estimated. As these models are micro-founded, their estimation using the Bayesian approach, which allows us to specify prior distributions for parameters, became very popular. As multiple equilibria can arise in these models under a wide set of parameters, only is a limited set of econometric estimation methods can be applied.

Traditionally, papers estimated canonical models only under determinacy, ruling out possibilities for indeterminacy. The standard result is that determinacy arises mainly under active monetary policy rules (the nominal interest rate increases more than one-to-one with inflation) in New Keynesian DSGE models, while passive monetary policy (the nominal interest rate increases less than one-to-one with inflation) leads to indeterminacy (see Kerr and King (1996), Rotemberg and Woodford (1998), and Christiano and Gust (1999)). Clarida, Gali and Gertler (2000) estimate monetary policy rules in the U.S. for different periods of time and show that U.S. monetary policy was passive during the pre-Volcker period (1960-1979) and active into the Volcker-Greenspan period (1979-1996). Lubik and Schorfheide (2004) were the first to estimate a standard New Keynesian model allowing for indeterminacy.

In contrast to canonical New Keynesian models, where the interest rate affects output only through the consumption and saving decisions of households, in this paper the investment decisions of firms are added. The no-arbitrage condition between the real return on bonds and the real return on capital implies that the capital rental rate increases when monetary policy responds to higher inflation by increasing the interest rate. This response increases the cost of renting capital, leading to cost-push inflation. Dupor (2001), Carlstrom and Fuerst (2005), Kurozumi and Zandweghe (2008), Kurozumi (2006), Huang and Meng (2007), and Xiao (2008) study properties of models with capital accumulation and show that this crucial feature changes the stability structure and dynamics of the models and makes indeterminacy likely.

This paper studies New Keynesian DSGE models with capital accumulation, different Taylor rules, and the potential for indeterminacy. While the stability properties of models with capital accumulation are well-analyzed[1], there appear to be no papers that simulate and estimate these models allowing indeterminacy to occur. This paper is offered to fill this gap. The analysis is distinguished from the conventional New Keynesian studies in three ways. First, different versions of New Keynesian models with capital accumulation are simulated, and their dynamic properties are discussed. Second, the models are estimated on U.S. data from 1960:I to 2008:I. Following Lubik and Schorfheide (2004), passive monetary policy (consistent with indeterminacy) is assumed *a priori* for the pre-Volcker period and active monetary policy (consistent with

[1] See Dupor (2001), Carlstrom and Fuerst (2005), Kurozumi and Zandweghe (2008), Kurozumi (2006), Huang and Meng (2007), and Xiao (2008).

determinacy) for the post-1982 period. Using state-space decomposition and the Kalman filter, the overall likelihood of the model is maximized taking into account prior distributions of the parameters, and inferences are made with a likelihood-based approach by adopting the Metropolis-Hastings techniques. Third, the estimated models are compared using the Bayesian approach. While some explanations of the results are consistent with the recent findings of Mavroeidis (2010), the empirical estimates of this paper differ from the results of Lubik and Schorfheide (2004) and Clarida, Gali and Gertler (2000).

Though baseline New Keynesian models have become very popular in the analysis of monetary policy, many authors show that these models are unable to generate enough persistence in inflation and output. Fuhrer and Moore (1995) show that a sticky wage model can generate persistence in the price level but not in the inflation rate. Chari, Kehoe and McGrattan (2000) point out that models with nominal rigidities do not generate enough persistence in output following a monetary shock. The simulated results of this paper show that models with capital accumulation can generate substantial persistencies among the major economic variables, as the stock nature of capital adds persistency to the dynamics of all other variables in the models.

Later research attempts to determine whether there were switches in monetary policy or structural changes in the fundamental parameters of the economy. Sims and Zha (2006) use a vector autoregression (VAR) approach to estimate a multivariate regime-switching model for U.S. monetary policy. They find that the main changes were in the monetary policy rules. Smets and Wouters (2003) and Smets and Wouters (2007) find most of the structural parameters are stable over those two periods. The biggest difference concerns the variances of the structural shocks. The main drawback of the VAR approach is that, due to rational expectations, agents can anticipate changes in parameters of the economy, leading to inconsistent estimates. One method to overcome this problem is to estimate a fully-specified DSGE model that can be re-solved for alternative policy rules. Lubik and Schorfheide (2004) exogenously split data into two sets and show that U.S. monetary policy during the post-1982 period was consistent with determinacy, whereas during the pre-Volcker policy was not. Schorfheide (2005) estimates a basic New Keynesian monetary DSGE model, in which monetary policy follows a regime switching process, and confirms the switch in monetary policy between the pre-Volcker and post-Volker periods.

There are identification issues with estimation of forward-looking Markov-switching rational expectations models. Beyer and Farmer (2007) argue that it is not always possible to decide whether the data are generated from determinate or indeterminate models. Farmer, Waggoner and Zha (2008) provide a set of necessary and sufficient conditions for determinacy in a class of forward looking Markov-switching rational expectations models. Mavroeidis's (2010) model shows that policy before Volcker led to indeterminacy, however, the model is not accurately identifiable using data after 1979.

The paper is structured as follows. In Section 1, a New Keynesian DSGE model with capital accumulation and different monetary policy rules is derived. In Section 2, different versions of this model are simulated and their dynamic properties are analyzed. In Section 3, the model is fitted to quarterly U.S. data on output, inflation, nominal interest rates, consumption, and capital from 1960:I to 2008:I and the estimation methodology and prior distributions of the parameters are discussed. The empirical results are presented in Section 4. Estimated models are compared in Section 5. The last section contains concluding remarks.

1. Model

Following Yun (1998), Carlstrom and Fuerst (2005), and Kurozumi and Zandweghe (2008), a New Keynesian DSGE model with sticky prices and capital accumulation in discrete time is constructed. The economy consists of a large number of households, monopolistically competitive firms, and a monetary authority that changes the nominal interest rate in response to inflation and output.

1.1. Households

Households seek to maximize their expected life-time utility function:

$$E_t \sum_{t=0}^{\infty} \beta^t U(C_t, \frac{M_{t+1}}{P_t}, 1 - L_t) \tag{1}$$

where E_t is the conditional expectations operator on the information set available at date t, β is the discount factor, C_t is consumption, M_{t+1} is nominal money holdings and the beginning of the period *(t+1),* P_t is a price level, $\frac{M_{t+1}}{P_t}$ is real money balances[2], and $(1 - L_t)$ is leisure.

The utility function is separable in leisure and takes the following functional form:

$$U(C_t, \frac{M_{t+1}}{P_t}, 1 - L_t) = \frac{C_t^{1-\sigma}}{1-\sigma} + \theta \ln \frac{M_{t+1}}{P_t} + \phi L_t \tag{2}$$

At the beginning of each period t, a household has M_t cash balances and B_{t-1} nominal bonds. A household starts period t by trading bonds and receiving a lump-sum monetary transfer T_t from the government. A household receives interest payments on bonds B_{t-1} with gross interest rate R_{t-1} and spends money on new bonds B_t. A household also receives real factor payments from the labor market $w_t L_t$ and capital market $[r_t + (1-\delta)]K_t$, receives firm's profits Π_t, and spends money on next period capital K_{t+1} and current consumption C_t at current prices P_t. Each household chooses C_t, M_{t+1}, and L_t to maximize (1) subject to the sequence of intertemporal budget constraints:

[2] As in recent papers we introduce end-of-period money holdings to be consistent with the Dupor continuous-time analysis (for discussion, see Carlstrom and Fuerst, 2005)

$$M_{t+1} + B_t + P_t C_t + P_t K_{t+1} = M_t + T_t + B_{t-1} R_{t-1} + P_t \{w_t L_t + [r_t + (1-\delta)] K_t\} + \Pi_t \tag{3}$$

The first order conditions for the household's maximization problem are the following:

$$\frac{U_C}{U_L} = -\frac{1}{w} \tag{4}$$

$$U_C(t) = \beta E_t \{U_C(t+1)[r_{t+1} + (1-\delta)]\} \tag{5}$$

$$\frac{U_C(t)}{P_t} = \beta R_t E_t \left(\frac{U_C(t+1)}{P_{t+1}} \right) \tag{6}$$

$$\frac{U_m(t)}{U_C(t)} = \frac{R_t - 1}{R_t} \tag{7}$$

Equation (4) is a standard consumption-labor condition. Equation (5) is the Euler equation of consumption dynamics. Equation (6) is the Fisher equation that connects inflation and interest rates. Equation (7) is a money demand equation.

1.2. Firms

Firms are monopolistic competitors in the intermediate good market. The final output Y_t is produced from intermediate goods $y_t(i)$ with Dixit-Stiglitz (1977) technology:

$$Y_t = \{\int_0^1 [y_t(i)^{\frac{\eta-1}{\eta}}] di\}^{\frac{\eta}{\eta-1}} \tag{8}$$

The corresponding demand for an intermediate good *i* possesses constant price elasticity η :

$$y_t(i) = Y_t \left(\frac{P_t(i)}{P_t} \right)^{-\eta} \tag{9}$$

where $P_t(i)$ is the price of the intermediate good and P_t is the price of the final good.

The production function of each firm exhibits constant returns to scale:

$$f(K, L) = K^{\alpha} L^{1-\alpha} \tag{10}$$

The first order conditions for the cost minimization problem are the following:

$$r_t = z_t f_K(K_t, L_t) \tag{11}$$

$$w_t = z_t f_L(K_t, L_t) \tag{12}$$

where z_t is the marginal cost of production (see Appendix 1 for details).

With the Cobb-Douglas production function (10) the first order conditions take the form:

$$r_t = \alpha z_t Y_t / K_t \tag{13}$$

$$w_t = (1-\alpha) z_t (Y_t / K_t)^{\frac{-\alpha}{(1-\alpha)}} \tag{14}$$

The Calvo (1983) staggered pricing model is used, assuming that each period a fraction $(1-\nu)$ of firms gets a signal to set a new price. Therefore, each firm maximizes the sum of discounted profits taking into account the probability of changing its price. The optimization problem of a firm takes the form:

$$E_t \sum_{j=t}^{\infty} \left(\frac{\nu}{\prod_{i=0}^{j} R_i} \right)^j \left[\left(\frac{P_t(i)}{P_t} \right)^{-\eta} Y_t \left(\frac{P_t(i)}{P_t} - z_t \right) \right] \to \max_{P_t(i)} \tag{15}$$

The profit maximization conditions give a log-linearized New Keynesian Phillips Curve[3] of the form:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \lambda \hat{z}_t \tag{16}$$

where $\hat{\pi}_t$ is inflation and $\lambda = \frac{(1-\upsilon)(1-\beta\upsilon)}{\upsilon}$ is the real marginal cost elasticity of inflation.

1.3. Monetary policy rules

Monetary policy reacts to inflation and output with interest rate smoothing:

$$R_t = (R_{t-1})^{\rho_R} \left[R \left(\frac{E_t \pi_{t+k}}{\pi} \right)^{\psi_\pi} \left(\frac{Y_t}{Y} \right)^{\psi_Y} \right]^{(1-\rho_R)} \tag{17}$$

where R, π, Y are the steady-state values of the interest rate, inflation, and output, respectively. Parameters ψ_π and ψ_Y are the elasticities of the interest rate with respect to inflation and output, respectively. Two basic specifications of the monetary policy rule are considered: with response to current inflation (k=0) and future expected inflation (k=1). Interest rate smoothing is introduced with the autocorrelation coefficient ρ_R. In this framework, the monetary policy is active if the nominal interest rate increases more than one-to-one with

[3] See Gali and Gertler (1998) and Clarida, Gali and Gertler (1999) for details.

inflation ($\psi_\pi > 1$), otherwise, it is passive ($\psi_\pi < 1$). Also, a model with $\psi_Y = 0$ would give a standard model with capital in discrete time as in Carlstrom and Fuerst (2005) — an analog of the Dupor (2001) continuous time model.

1.4. Dynamics of the model

The dynamics of the model are represented by a system of first order conditions log-linearized around the steady state for households and firms (18-23), the monetary policy rule (24), shocks of preferences and marginal cost (25-26) (see Appendix 2 for details):

$$\hat{R}_t - E_t\hat{\pi}_{t+1} = \sigma(E_t\hat{C}_{t+1} - \hat{C}_t - \varepsilon_{g,t}) \tag{18}$$

$$\hat{R}_t - E_t\hat{\pi}_{t+1} = [1-\beta(1-\delta)](E_t\hat{z}_{t+1} + E_t\hat{Y}_{t+1} - \hat{K}_{t+1}) \tag{19}$$

$$\sigma\hat{C}_t = \hat{z}_t + \frac{\alpha}{1-\alpha}(\hat{K}_t - \hat{Y}_t) \tag{20}$$

$$\hat{K}_{t+1} = (1-\delta)\hat{K}_t + \delta\hat{I}_t \tag{21}$$

$$\hat{Y}_t = s_C\hat{C}_t + s_I\hat{I}_t \tag{22}$$

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \lambda\hat{z}_t + \varepsilon_{z,t} \tag{23}$$

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)(\psi_\pi E_t\hat{\pi}_{t+k} + \psi_Y\hat{Y}_t) + \varepsilon_{R,t} \tag{24}$$

$$\varepsilon_{g,t} = \rho_g\varepsilon_{g,t-1} + \upsilon_{g,t}, \quad \upsilon_{g,t} \text{ is } iid\ (0,\ \sigma_g^2) \tag{25}$$

$$\varepsilon_{z,t} = \rho_z\varepsilon_{z,t-1} + \upsilon_{z,t}, \quad \upsilon_{z,t} \text{ is } iid\ (0,\ \sigma_z^2) \tag{26}$$

Equation (18) is the Euler equation for the household's dynamic optimization problem with a preference shock $\varepsilon_{g,t}$, which follows an AR(1) process with an autocorrelation coefficient of ρ_g (Equation 25). Equation (19) is the Fisher relation between the nominal interest rate, expected future inflation, and real interest rate, where the latter is determined in the production sector. Equation (20) is the wage-equilibrium relation of the log-linearized equations (4) and (14). Equation (21) is the capital accumulation relation with a depreciation rate δ. Equation (22) is the division of the steady-state output between consumption and investment with shares s_C and s_I, respectively. Equation (23) is a New Keynesian Phillips Curve derived from the Calvo staggered-pricing model with a marginal cost shock $\varepsilon_{z,t}$, which follows an AR(1) process with an autocorrelation coefficient of ρ_z (Equation 26). Equation (24) is the log-linearized monetary policy rule (17) with an interest rate shock $\varepsilon_{R,t}$. As the money supply is endogenous and the Ricardian equivalence holds in this model, the hidden government budget constraint and the equation for the evolution of government debt are implicitly satisfied.

Straightforward re-arrangements of the variables $\hat{z}_t$ and $\hat{I}_t$ in the model give a system of variables $\hat{C}_t, \hat{R}_t, \hat{\pi}_t, \hat{Y}_t, \hat{K}_t, \varepsilon_{g,t}, \varepsilon_{z,t}$:

(27) $$\hat{C}_t = E_t \hat{C}_{t+1} - \frac{1}{\sigma}\left[\hat{R}_t - E_t \hat{\pi}_{t+1}\right] + \varepsilon_{g,t}$$

(28) $$\hat{R}_t - E_t \hat{\pi}_{t+1} = [1-\beta(1-\delta)](\sigma E_t(\hat{C}_{t+1}) + \frac{1}{1-\alpha}\left[E_t \hat{Y}_{t+1} - E_t \hat{K}_{t+1}\right])$$

(29) $$\hat{K}_{t+1} = (1-\delta)\hat{K}_t + \frac{\delta}{s_I}(\hat{Y}_t - (1-s_I))\hat{C}_t)$$

(30) $$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \lambda(\sigma \hat{C}_t - \frac{\alpha}{1-\alpha}(\hat{K}_t - \hat{Y}_t)) + \varepsilon_{z,t}$$

(31) $$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)(\psi_\pi E_t \hat{\pi}_{t+k} + \psi_Y \hat{Y}_t) + \varepsilon_{R,t}, \quad \varepsilon_{R,t} \text{ is } iid\,(0,\ \sigma_R^2)$$

(32) $$\varepsilon_{g,t} = \rho_g \varepsilon_{g,t-1} + \upsilon_{g,t}, \quad \upsilon_{g,t} \text{ is } iid\,(0,\ \sigma_g^2)$$

(33) $$\varepsilon_{z,t} = \rho_z \varepsilon_{z,t-1} + \upsilon_{z,t}, \quad \upsilon_{z,t} \text{ is } iid\,(0,\ \sigma_z^2)$$

Adding capital accumulation to the model makes real interest rate connected to the marginal product of capital, this is contained in the Fisher equation (28). Also, output does not equal consumption in the absence of capital, as in Lubik and Schorfheide (2004), but is split between consumption and investment in this model. This is incorporated in the New Keynesian Phillips curve equation (30) through the output equation (22). By including investment, this model has the capital accumulation equation (29), which influences interest rates through the equations (26) and (28).

To compare, the canonical New Keynesian model can be presented by a system of three equations: the IS equation, the Phillips curve, and a monetary policy rule, similar to equations (27), (30), and (31), respectively. In that way, the interest rate affects output only through the consumption-savings decision of the household and not through the production sector.

2. Model simulations

The model (27)-(33) exhibits different types of dynamics depending on its parameter values (see Carlstrom and Fuerst (2005), Sosunov and Khramov (2008), Kurozumi and Zandweghe (2008), Kurozumi (2006), Huang and Meng (2007), and Xiao (2008)). Under a wide set of parameters, the model is determinate if the monetary authority implements an active monetary policy ($\psi_\pi > 1$), and the model is indeterminate if the monetary policy is passive ($\psi_{\pi} < 1$). Therefore, two major versions of the model (27)-(33), with active and passive monetary policies, are simulated.[4] For the baseline calibration most of the parameter values are the same as prior means used by Lubik and Schorfheide (2004); the rest of the parameters are calibrated according to stylized facts (Table 6). The Matlab-based computer package *Dynare* was used to calculate theoretical moments for the endogenous variables of the model. The simulation results of the two versions of the model with current-looking passive and active monetary policy rules and corresponding moments of consumption, interest rate, inflation, output, and capital are presented in Tables 1-4.

The version of the model with passive monetary policy demonstrates substantially higher volatility of interest rate and inflation compared to the version with active monetary policy (Table 1). This can be explained by the existence of indeterminate equilibria. As long as the monetary policy authority is unable to respond sufficiently to changes in inflation by raising interest rates substantially, the volatility of inflation and, therefore, nominal interest rates, is higher. Both models reproduce a similar volatility of capital to that of U.S. data with lower volatilities of consumption, interest rate, and inflation (Table 5).

The crucial differences between the two versions of the model arise from the variance decomposition of shocks, correlation matrices of endogenous variables, and impulse response functions (IRFs). First, the preferences (demand) shock is the main drivers of volatility in the version of the model with passive monetary policy, explaining more than 90 percent of the volatility of endogenous variables (Table 2). In contrast, the marginal cost (supply) shock explains more than 50 percent of variance in the model with active monetary policy. These results are similar to the findings of Smets and Wouters (2007), who show that “demand” shocks can explain a substantial share of the variance in output in the general version of a New Keynesian model.

The two versions of the model demonstrate different correlations among the major variables (Table 3). In the version of the model with active monetary policy, the nominal interest rate plays the role of the active monetary policy instrument and is negatively correlated with output and capital. These theoretical results are consistent with the stabilizing role of a nominal interest rate in the economy. In contrast, in the version of the model with passive monetary policy, the nominal interest rate is positively correlated with output, capital, and consumption. Passive monetary policy is unable to respond sufficiently to shocks, indeterminacy and additional shock propagation. A response of the monetary authority to supply and demand shocks leads to co-movements in the dynamics of the interest rate and real variables as changes in the nominal interest rate are not enough to diminish the effect of shocks and reverse the dynamics of the economy. Therefore, the version of the model with passive monetary policy demonstrates substantially higher volatility among the economic variables, which is consistent with U.S. data for the pre-Volcker period (1960:I to 1979:II). Theoretical IRFs support this intuition (Appendixes 3-4).

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[4] Dynamics of the versions of the model with forward-looking monetary policy rules are similar to versions with current-looking monetary policy rules and, therefore, is not discussed in this paper.

As stated in the Introduction, while the baseline "New Keynesian" models became very popular in the analysis of monetary policy, many papers show that these models are unable to generate enough persistencies in inflation and output (see Chari, Kehoe, and McGrattan (2000), Fuhrer and Moore (1995), Fernandez-Villaverde and Rubio-Ramirez (2004)). First-order autocorrelation coefficients for consumption, interest rate, output, and capital are more than 0.85 in the U.S. data (Appendix 6). Most of the New Keynesian models fail to replicate even half of these correlation levels (see Rubio-Ramirez and Rabanal (2005)). In contrast, the simulated results of this paper show that models with capital accumulation can, in fact, generate substantial persistencies (Table 4). The autocorrelation coefficients for consumption and capital are more than 0.9 in both active and passive monetary policy rule versions of the model. The autocorrelations of nominal variables, such as inflation and the nominal interest rate, are also very high due to the stock nature of capital, which adds persistency to the dynamics of all other variables. The autocorrelation coefficients for consumption and capital are very high, representing substantial consumption smoothing and slow adjustment of capital stock. In the version with passive monetary policy, the interest rate and output autocorrelation coefficients are higher than in the version with active monetary policy, again, due to the fact that passive monetary policy is unable to adjust the interest rate sufficiently to control shock propagation.

	Version with current-looking passive monetary policy.		Version with current-looking active monetary policy.	
Variable	Mean	St. dev.	Mean	St. dev.
Consumption	0	0.36	0	0.30
Interest rate	0	0.40	0	0.16
Inflation	0	0.50	0	0.30
Output	0	1	0	1
Capital	0	0.46	0	0.53

Table 1. Simulation results of the model. Theoretical moments.
Note: all variables are in log deviations from their steady-state values.

	Version with current-looking passive monetary policy.				Version with current-looking active monetary policy.		
	Interest rate shock	Preference (demand) shock	Marginal cost (supply) shock	Sunspot shock	Interest rate shock	Preference (demand) shock	Marginal cost (supply) shock
Consumption	0.33	92.75	6.35	0.57	0.21	44.95	54.84
Interest rate	0.72	95.31	3.3	0.67	0	34.74	65.25
Inflation	0.62	96.61	2.03	0.74	0.46	32.89	66.65
Output	0.18	95.5	3.65	0.67	1.49	31.25	67.26
Capital	0.31	93.14	6.01	0.54	0.23	40.38	59.4

Table 2. Simulation results of the model. Variance decomposition (in percent).
Note: all variables are in log deviations from their steady-state values.

Version with current-looking passive monetary policy					
Variables	Consumption	Interest rate	Inflation	Output	Capital
Consumption	1	0.6	0.54	0.30	0.98
Interest rate	0.6	1	0.98	0.81	0.73
Inflation	0.54	0.98	1	0.89	0.69
Output	0.30	0.81	0.89	1	0.45
Capital	0.98	0.73	0.69	0.45	1
Version with current-looking active monetary policy					
Variables	Consumption	Interest rate	Inflation	Output	Capital
Consumption	1	-0.09	0.01	0.04	0.81
Interest rate	-0.09	1	0.98	-0.96	-0.46
Inflation	0.01	0.98	1	-0.92	-0.35
Output	0.04	-0.96	-0.92	1	0.39
Capital	0.81	-0.46	-0.35	0.39	1

Table 3. Simulation results of the model. Matrix of correlations.
Note: All variables are in log deviations from their steady states.

Version with current-looking passive monetary policy					
Order	1	2	3	4	5
Consumption	0.995	0.983	0.965	0.943	0.918
Interest rate	0.885	0.782	0.690	0.609	0.537
Inflation	0.779	0.692	0.614	0.544	0.482
Output	0.384	0.330	0.286	0.248	0.216
Capital	0.991	0.975	0.956	0.933	0.908
Version with current-looking active monetary policy					
Order	1	2	3	4	5
Consumption	0.896	0.812	0.745	0.689	0.641
Interest rate	0.723	0.527	0.389	0.292	0.223
Inflation	0.603	0.423	0.297	0.208	0.147
Output	0.799	0.549	0.375	0.253	0.168
Capital	0.992	0.972	0.943	0.910	0.874

Table 4. Simulation results of the model. Coefficients of autocorrelation.

3. Empirical Approach

3.1. Data

The system of equations (27)-(33) is fitted to quarterly postwar U.S. data on output, inflation, nominal interest rates, consumption and capital from 1960:I to 2008:I. As in Lubik and Schorfheide (2004), output is a log of real per capita GDP (GDPQ), inflation is the annualized percentage change of CPI (CPI-U), and the Federal Funds Rate (FYFF) in percent is used as the nominal interest rate. Real Personal Consumption Expenditures (PCECC96) is used for consumption from the St. Louis Fed database. The time series for capital is constructed using Real Gross Private Domestic Investment (GPDIC96) starting from 1947, taking the initial amount of capital consistent with the steady state level of capital and iterating it forward with a depreciation rate of 2 percent.

The Hodrick-Prescott filter is used to remove trends from the consumption, output, and capital series to make the analysis comparable with Lubik and Schorfheide (2004) (see the sample moments in Table 5, Appendices 5-6, and graphs in Figure 1). Consistent with earlier papers, the data sample 1960:I to 2008:I can be analyzed according to the following sub-samples:

1. the pre-Volcker period (1960:I to 1979:II) — the period of passive monetary policy is used in Lubik and Schorfheide (2004);
2. the post-1978 period (1978:III to 1997:IV) — the Volcker disinflation period (commonly excluded from estimates);
3. the post-1982 period (1982:IV to 1997:IV) — the period of active monetary policy analyzed in Lubik and Schorfheide (2004);
4. the post-1982 period (1982:IV to 2008:I) — the period of active monetary policy, before the financial crisis, included in this paper.

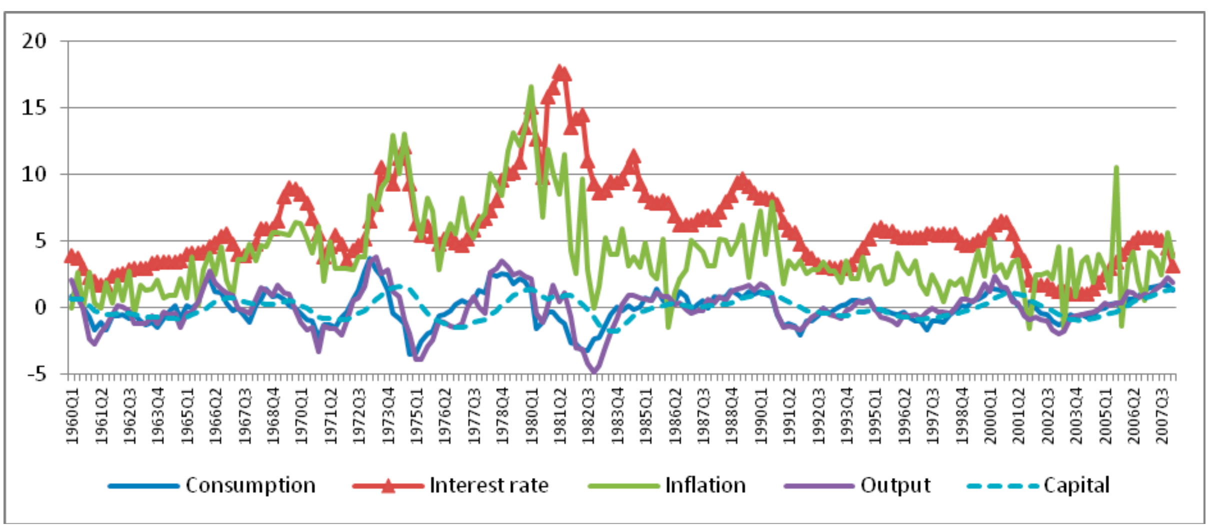

Figure 1. Dynamics of U.S. output, inflation, nominal interest rate, consumption, and capital (in log deviations from the Hodrick-Prescott filtered trend), 1960:I -2008:I.

	Consumption *	Interest rate	Inflation	Output*	Capital*
Full data sample (1960:I to 2008:I)					
Mean	-0.004	6.056	4.142	-0.001	-0.014
Std	1.238	3.259	3.171	1.526	0.742
StD/Output	0.811	2.136	2.078	1.000	0.486
Pre-Volcker period (1960:I to 1979:II)**					
Mean	0.030	5.473	4.646	-0.012	-0.106
Std	1.435	2.425	3.359	1.759	0.753
StD/Output	0.816	1.379	1.910	1.000	0.428
Post-1978 period (1978:III to 1997:IV)					
Mean	-0.203	7.941	4.398	-0.163	0.001
Std	1.163	3.605	3.261	1.457	0.737
StD/Output	0.798	2.474	2.238	1.000	0.506
Post-1982 period (1982:IV to 1997:IV)**					
Mean	-0.056	6.630	3.325	-0.169	-0.176
Std	0.917	2.144	1.626	1.268	0.669
StD/Output	0.723	1.691	1.282	1.000	0.528
Post-1982 period (1982:IV to 2008:I)					
Mean	0.083	5.478	3.081	0.024	-0.051
Std	0.911	2.458	1.849	1.219	0.704
StD/Output	0.747	2.016	1.517	1.000	0.578

Table 5. Sample moments for quarterly postwar U.S. data on output, inflation, nominal interest rates, consumption, and capital.
** In log deviations from the Hodrick-Prescott filtered trend.*
*** Data sample used in Lubik and Schorfheide (2004).*

3.2. Estimation approach

The Bayesian approach is used to estimate the model by constructing prior distributions of the parameters and maximizing the likelihood of the model. The Kalman filter with state and measurement equations is used to fit the data to the model. The Bayesian approach takes advantage of the general equilibrium approach and outperforms GMM and ML in small samples. Furthermore, it does not rely on the identification scheme of the VAR, though does follow the likelihood principle.

The model (27)-(33) is a system of the variables $\hat{C}_t, \hat{R}_t, \hat{\pi}_t, \hat{Y}_t, \hat{K}_t, \varepsilon_{g,t}$, and $\varepsilon_{R,t}$, with a vector of parameters presented in Table 6. The observed capital, consumption, and output deviations from the trends, along with inflation and interest rate are stacked in the vector $y_t = \left[\hat{C}_t, \hat{R}_t, \hat{\pi}_t, \hat{Y}_t, \hat{K}_t\right]^T$, such that the measurement equation is of the form:

$$y_t = \begin{bmatrix} 0 \\ r^* + \pi^* \\ \pi^* \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} s_t = y^* + S s_t$$

The state equation is:

(34) $$s_t = Fs_{t-1} + Q\varepsilon_t$$

where r^* and π^* are the steady-state inflation and real interest rate, respectively, $s_t = \left[\hat{C}_t, \hat{R}_t, \hat{\pi}_t, \hat{Y}_t, \hat{K}_t, \varepsilon_{g,t}, \varepsilon_{z,t}\right]^T$ is a vector of system variables, r^* is determined from $\beta = (1 + r^*)^{1/4}$, F and Q are the system matrixes, and ε_t is a vector of shocks.

As the posterior distribution of the estimated model is proportional to the product of the likelihood function and the prior, the overall likelihood of the model is maximized taking into account the prior distributions of the parameters and using the state-space decomposition with the Kalman filter. The inference is made with a likelihood-based approach by adopting the random walk Metropolis-Hastings algorithm to obtain 100,000 draws and estimate the moments of the parameter distributions.

3.3. Prior Distributions

The specification of the prior distributions is summarized in Table 6. Most of the priors are the same as in Lubik and Schorfheide (2004). The model is estimated separately for the pre-Volcker period from 1960:I to 1979:II, assuming *a priori* passive monetary policy (consistent with indeterminacy), and for the post-1982 period, assuming *a priori* active monetary policy (consistent with determinacy). The beta distribution is used as a prior for the response of the monetary policy rule to the inflation parameter (ψ_π) centered around 0.5 and 1.5 for the pre-Volcker and post-1982 periods, respectively.

The response of the monetary policy rule to the output parameter (ψ_Y) is centered around 0.25, which is consistent with empirical findings in the range of 0.06 to 0.43. Persistency of the interest rate parameter in the monetary policy rule (ρ_R) is centered around 0.5 and bounded by the beta distribution to be in the interval (0,1). The steady state inflation (π^*) and the interest rate (r^*) are centered around 4 and 2 percent per annum, respectively. The real marginal cost elasticity of inflation (λ) is centered around 0.3, assuming that firms reset optimal prices once every three or five quarters, on average. The prior for household risk aversion parameter (σ) is centered around 2, which makes households more risk averse than in the case of logarithmic utility. Shocks of preferences and technology are assumed to follow an AR(1) process with autocorrelation parameters centered around 0.7 and to have a zero prior correlation. Variances of shocks are considered to have inverse gamma distributions.

Adding capital and investment activity to the model makes it necessary to specify parameters related to capital accumulation activity and production sector. The priors for capital share in output (α) and investment share in output (s_I) are centered around 0.3 with the standard deviation of 0.1 and are bounded by the beta distribution to be in the interval (0,1).

	Parameter	Prior mean	Prior std	Distribution
Monetary policy rule parameters				
ψ_π	Response of monetary policy rule to inflation (version of the model with passive monetary policy)	0.5	0.1	Gamma
ψ_π	Response of monetary policy rule to inflation (version of the model with active monetary policy)	1.5	0.1	Gamma
ψ_Y	Response of monetary policy rule to output	0.25	0.15	Gamma
ρ_R	Persistency of interest rate in monetary policy rule	0.5	0.2	Beta
Steady-state inflation and real interest rate values				
π *	Steady-state inflation	4	2	Gamma
r*	Steady-state interest rate	2	1	Gamma
Standard model parameters				
λ	Real marginal most elasticity of inflation in Calvo model	0.3	0.1	Beta
σ	Inverse elasticity of intertemporal substitution of consumption	2	0.5	Gamma
ρ_g	Persistence of preference shock	0.7	0.1	Beta
ρ_z	Persistence of technology (marginal cost) shock	0.7	0.1	Beta
Capital-related parameters				
α	Share of capital income	0.3	0.1	Beta
s_I	Share of investment in output	0.3	0.1	Beta
ρ_{gz}	Correlation between technology (marginal cost) and preference shocks	0	0.4	Normal
Variances of measurement errors				
σ_R	Standard deviation of the interest rate shock	0.31	0.16	Inverse gamma
σ_g	Standard deviation of the preference shock	0.38	0.2	Inverse gamma
σ_z	Standard deviation of the marginal cost shock	1	0.52	Inverse gamma
Other shock parameters				
σ_{Cm}	Standard deviation of measurement error in the consumption equation	0.1	0.01	Inverse gamma
σ_{Rm}	Standard deviation of measurement error in the interest rate equation	0.1	0.01	Inverse gamma
$\sigma_{\pi m}$	Standard deviation of measurement error in the inflation equation	0.1	0.01	Inverse gamma
σ_{Ym}	Standard deviation of measurement error in the output equation	0.1	0.01	Inverse gamma
σ_{Km}	Standard deviation of measurement error in the capital equation	0.1	0.01	Inverse gamma
σ_s	Standard deviation of the sunspot shock	0.1	0.01	Inverse gamma

Table 6. Baseline calibration and prior distributions of the parameters of the model.

4. Estimation Results

The model is estimated separately for the pre-Volcker and post-1982 periods, assuming *a priori* passive monetary policy ($\psi_\pi < 1$), consistent with indeterminacy during the pre-Volcker period, and active monetary policy ($\psi_\pi > 1$), consistent with determinacy during the post-1982 period. For both data samples the versions of the model with current and forward looking monetary policy rules (29) are estimated:

- current-looking monetary policy rule (response to current inflation)

(35) $$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)(\psi_\pi \hat{\pi}_t + \psi_Y \hat{Y}_t) + \varepsilon_t$$

- forward-looking monetary policy rule (response to expected inflation)

(36) $$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)(\psi_\pi E_t \hat{\pi}_{t+1} + \psi_Y \hat{Y}_t) + \varepsilon_{R,t}$$

The priors, the posterior parameter estimates, and the confidence intervals for the pre-Volcker period (1960:I to 1979:II) and for the post-1982 period are presented in Tables 7-8. The inference is made with a likelihood-based approach by adopting the Metropolis-Hastings techniques[5]. The pre-Volcker period posteriors are conditional on indeterminacy and passive monetary policy rules, the post-1982 posteriors are conditional on determinacy and active monetary policy rules. For comparison, the estimates of Lubik and Schorfheide (2004) for a standard New Keynesian model without capital are included.

For the pre-Volcker period the estimates of the response of monetary policy to inflation (ψ_π) are 0.385 in current-looking version and 0.644 in the forward-looking version. For the post-1982 period these estimates are about 1.1 in all cases. These estimates are almost twice smaller than those in Lubik and Schorfheide (2004) or in Clarida, Gali and Gertler (2000), explained by the fact that in the model with capital accumulation there is an additional channel of monetary policy influence through the real interest rate in the production sector. Therefore, the monetary policy can respond less aggressively to changes in inflation to obtain the same goals.

While the response of the monetary policy rule to output (ψ_Y) is about 0.5 for the pre-Volcker and the earlier post-1982 (1982:IV to 1997:IV) periods in the case of current-looking monetary policy rule, it is substantially higher for the 1982:IV to 2008:I period, where the pre-crisis years are added. The estimates of the response of the monetary policy rule to output are much higher for the pre-Volcker period than in Lubik and Schorfheide (2004).

Adding capital accumulation activity into the model makes returns in production sector dependent on the amount of output and capital in the economy. As the capital dynamics is persistent over time, it translates additional persistency into real interest rates. Therefore, monetary policy becomes more focused on smoothing

[5] The Markov Chain Monte Carlo (MCMC) Metropolis-Hastings algorithm with five blocks was used with 100,000 simulations to obtain the inference.

interest rates by increasing the value of ρ_R, estimated to be about 0.8 in the model. While the steady-state inflation and real interest rate estimates are very close to those in Lubik and Schorfheide (2004) for the pre-Volcker period, the steady-state inflation rate is substantially lower and the real interest rate is a marginally lower the for post-1982 period, depending on the model specifications.

Another striking result of this paper is the shift in preferences of households. Lubik and Schorfheide (2004) find that households increased the degree of risk aversion (inverse elasticity of intertemporal substitution of consumption) between the pre-Volcker and post-1982 periods only slightly. It is shown in this paper that risk aversion (σ) increased substantially between these periods from about 0.4 for the pre-Volcker period to about 2.5 for the post-1982 period. This result arises from the fact that investment activity permits breaking the direct connection between interest rate and consumption dynamics in the Euler equation, due to the no-arbitrage condition between bonds and real sector returns. This allows for the explaination of consumption dynamics not only in terms of changing interest rate but changing preferences as well. The decrease in the levels of elasticity of intertemporal substitution of consumption can be connected with the development of financial markets and the decreasing tightness of borrowing constraints in the U.S. over time.

In the model with capital, the inflation dynamics described by the expectational Phillips curve is more complicated than in the standard model due to a direct connection between output and marginal cost through capital markets. Therefore, the estimates of real marginal cost elasticity of inflation (λ) are different from those in Lubik and Schorfheide (2004). First, they are substantially lower than in Lubik and Schorfheide (2004) for the pre-Volcker period and higher for the post-1982 period. Second, in the model with capital they increased over time, while in Lubik and Schorfheide (2004) they fell.

As neither government expenditures nor net exports are included in the model directly, some fluctuations in output are not explained by changes in investment and consumption. This influences the estimates of capital share in output (α), which are about 0.4 and 0.65 for the pre-Volcker and post-1982 periods, respectively. Also, the estimate of the share of investment in output (S_I) is lower than expected — about 0.07 in the baseline specification of the model.

The standard deviations estimates (σ_g, σ_z, and σ_R) as well as those for the degree of persistence of shocks(ρ_g and ρ_z) are consistent with other empirical findings. The correlation between shocks (ρ_{gz}) is slightly positive for the pre-Volcker period and very negative (about -0.98) for the post-1982 period.

Parameter	Prior: Mean	Prior: Std	Prior: Distribution	Indeterminacy with current-looking monetary policy rule (1960:I to 1979:II): **Posterior mean**	Confidence interval		Indeterminacy with forward-looking monetary policy rule (1960:I to 1979:II): **Posterior mean**	Confidence interval		Lubik and Schorfheide (2004) (1960:I to 1979:II): **Posterior mean**
Monetary policy rule parameters										
ψ_π	0.50	0.20	Gamma	**0.385**	0.206	0.557	**0.644**	0.425	0.874	**0.770**
ψ_Y	0.25	0.15	Gamma	**0.574**	0.361	0.797	**0.416**	0.194	0.628	**0.170**
ρ_R	0.50	0.20	Beta	**0.840**	0.782	0.898	**0.780**	0.683	0.882	**0.600**
Steady state inflation and real interest rate values										
π *	4.00	2.00	Gamma	**4.258**	2.673	5.880	**4.305**	2.583	5.987	**4.280**
r*	2.00	1.00	Gamma	**1.041**	0.576	1.532	**1.012**	0.573	1.470	**1.130**
Standard model parameters										
σ	2.00	0.50	Gamma	**0.391**	0.229	0.525	**0.354**	0.224	0.475	**1.450**
λ	0.30	0.10	Beta	**0.257**	0.119	0.396	**0.248**	0.117	0.381	**0.770**
ρ_g	0.70	0.10	Beta	**0.935**	0.903	0.968	**0.938**	0.907	0.969	**0.680**
ρ_z	0.70	0.10	Beta	**0.686**	0.547	0.826	**0.670**	0.532	0.812	**0.820**
Capital-related parameters										
α	0.33	0.05	Beta	**0.397**	0.319	0.476	**0.394**	0.311	0.474	-
s_I	0.30	0.10	Beta	**0.078**	0.072	0.083	**0.078**	0.072	0.083	-
Variances of shocks										
σ_R	0.31	0.16	Inv Gamma	**0.176**	0.150	0.200	**0.160**	0.132	0.187	**0.230**
σ_g	0.38	0.20	Inv Gamma	**0.125**	0.104	0.144	**0.124**	0.104	0.144	**0.270**
σ_z	1.00	0.52	Inv Gamma	**0.402**	0.331	0.468	**0.396**	0.331	0.457	**1.130**
Variances of measurement errors										
σ_{Cm}	0.10	0.01	Inv Gamma	**0.311**	0.305	0.317	**0.311**	0.304	0.317	-
σ_{Rm}	0.10	0.01	Inv Gamma	**0.104**	0.077	0.134	**0.100**	0.079	0.122	-
$\sigma_{\pi m}$	0.10	0.01	Inv Gamma	**0.099**	0.076	0.119	**0.100**	0.079	0.119	-
σ_{Ym}	0.10	0.01	Inv Gamma	**0.311**	0.303	0.317	**0.311**	0.304	0.317	-
σ_{Km}	0.10	0.01	Inv Gamma	**0.076**	0.064	0.088	**0.076**	0.064	0.087	-
Other shock parameters										
σ_s	0.10	0.01	Inv Gamma	**0.314**	0.312	0.317	**0.314**	0.312	0.317	**0.200**
ρ_{gz}	0.00	0.40	Normal	**0.114**	-0.186	0.413	**0.082**	-0.223	0.376	**0.140**

Table 7. Priors and posterior estimation results of the model with indeterminacy for the pre-Volcker period (1960:I to 1979:II).

Notes: Two specifications of the monetary policy rule (17) — with a response to current inflation (k=0) and to future expected inflation (k=1) — are considered. The estimates of Lubik and Schorfheide (2004) are of the standard New Keynesian model without capital.

Parameter	Prior			Determinacy with current-looking monetary policy rule (1982:IV to 1997:IV)			Determinacy with current-looking monetary policy rule (1982:IV to 2008:I)			Determinacy with forward-looking monetary policy rule (1982:IV to 2008:I)			Lubik and Schorfheide (2004) (1982:IV to 1997:IV)
	Mean	Std	Distribution	**Posterior mean**	Confidence interval		**Posterior mean**	Confidence interval		**Posterior mean**	Confidence interval		**Posterior mean**
Monetary policy rule parameters													
ψ_π	1.5	0.2	Gamma	**1.110**	1.100	1.122	**1.107**	1.100	1.115	**1.109**	1.100	1.121	**2.190**
ψ_Y	0.25	0.15	Gamma	**0.576**	0.366	0.760	**0.795**	0.573	1.026	**0.961**	0.613	1.369	**0.300**
ρ_R	0.5	0.2	Beta	**0.818**	0.757	0.888	**0.846**	0.803	0.892	**0.888**	0.846	0.933	**0.840**
Steady state inflation and real interest rate values													
π^*	4	2	Gamma	**2.256**	1.128	3.332	**3.113**	2.177	4.038	**2.602**	0.989	4.013	**3.430**
r^*	2	1	Gamma	**3.053**	2.605	3.489	**1.830**	1.459	2.220	**2.816**	2.345	3.350	**3.010**
Standard model parameters													
σ	2	0.5	Gamma	**2.514**	1.920	3.094	**2.405**	1.943	2.867	**2.231**	1.675	2.792	**1.860**
λ	0.3	0.1	Beta	**0.694**	0.599	0.792	**0.787**	0.708	0.867	**0.615**	0.513	0.718	**0.580**
ρ_g	0.7	0.1	Beta	**0.933**	0.915	0.951	**0.908**	0.890	0.927	**0.925**	0.906	0.944	**0.830**
ρ_z	0.7	0.1	Beta	**0.776**	0.741	0.813	**0.691**	0.650	0.733	**0.766**	0.729	0.805	**0.850**
Capital-related parameters													
α	0.33	0.05	Beta	**0.646**	0.626	0.664	**0.657**	0.650	0.664	**0.646**	0.625	0.664	-
s_I	0.3	0.1	Beta	**0.080**	0.071	0.089	**0.074**	0.069	0.079	**0.075**	0.066	0.084	-
Variances of shocks													
σ_R	0.31	0.16	Inv Gamma	**0.407**	0.317	0.494	**0.391**	0.333	0.452	**0.373**	0.305	0.439	**0.180**
σ_g	0.38	0.2	Inv Gamma	**0.102**	0.085	0.117	**0.092**	0.083	0.101	**0.105**	0.088	0.122	**0.180**
σ_z	1	0.52	Inv Gamma	**2.330**	1.822	2.844	**2.337**	1.993	2.686	**1.935**	1.459	2.394	**0.640**
Variances of measurement errors													
σ_{Cm}	0.1	0.01	Inv Gamma	**0.314**	0.311	0.317	**0.314**	0.311	0.317	**0.314**	0.311	0.317	-
σ_{Rm}	0.1	0.01	Inv Gamma	**0.094**	0.076	0.110	**0.095**	0.078	0.111	**0.100**	0.077	0.121	-
$\sigma_{\pi m}$	0.1	0.01	Inv Gamma	**0.102**	0.078	0.124	**0.100**	0.077	0.123	**0.100**	0.078	0.123	-
σ_{Ym}	0.1	0.01	Inv Gamma	**0.307**	0.295	0.317	**0.140**	0.098	0.196	**0.306**	0.293	0.317	-
σ_{Km}	0.1	0.01	Inv Gamma	**0.088**	0.071	0.104	**0.122**	0.095	0.145	**0.096**	0.077	0.116	-
Other shock parameters													
σ_s	-	-	-	-	-	-	-	-	-	-	-	-	-
ρ_{gz}	0	0.4	Normal	**-0.985**	-0.993	-0.976	**-0.977**	-0.987	-0.967	**-0.984**	-0.994	-0.975	**0.360**

Table 8. Priors and posterior estimation results of the model with indeterminacy for the post-1982 period.

Notes: Two specifications of the monetary policy rule (17) — with a response to current inflation (k=0) and to future expected inflation (k=1) — are considered. The estimates of Lubik and Schorfheide (2004) are of the standard New Keynesian model without capital.

5. Model comparison

The four versions of the model are estimated and compared on the three data samples in order to evaluate the odds of each model for a certain period of time (Table 10). The Bayesian approach is used to evaluate the probability of each model. In a simple two-model case, the ratio of the posterior probabilities of the two models is calculated as:

$$\frac{P(A_1 \mid Y_T)}{P(A_2 \mid Y_T)} = \frac{P(A_1)}{P(A_2)} \frac{P(Y_T \mid A_1)}{P(Y_T \mid A_2)}$$

where $\frac{P(A_1 \mid Y_T)}{P(A_2 \mid Y_T)}$ is the posterior odds ratio, $\frac{P(A_1)}{P(A_2)}$ is the prior odds ratio, and $\frac{P(Y_T \mid A_1)}{P(Y_T \mid A_2)}$ is the Bayes factor that uses the models' estimates.

In the case of more than two models, the posterior probability of a model A_i is calculated as:

$$P(A_i \mid Y_T) = \frac{P(A_i) P(Y_T \mid A_i)}{\sum_j P(A_j) P(Y_T \mid A_j)}$$

where $\sum_j P(A_j) P(Y_T \mid A_j)$ is the sum across all models. Equal prior probabilities are assumed for each model and Bayes factor probabilities are calculated using empirical distributions of the estimated parameters:

$$P(Y_T, A) = \int_{\theta_A} p(\theta_A \mid Y_T, A) p(\theta_A \mid A) d\theta_A$$
$$\hat{p}(Y_T, A) = 2\pi^{k/2} \mid \Sigma_{\theta^M} \mid^{-1/2} \; p(\theta_A^M \mid Y_T, A) p(\theta_A^M \mid A)$$

The probability $p(\theta_A \mid Y_T, A)$ is integrated over the set θ_A of k estimated parameters, assuming a normal distribution for the estimation of $\hat{p}(Y_T, A)$.

A comparison of the versions of the model with indeterminacy and determinacy under current-looking monetary policy rules is presented in Table 9. In contrast to Lubik and Schorfheide (2004), the model with indeterminacy dominates the determinate model with a posterior probability of 1.000 for all sample periods. In the case of comparison of all four versions of the model, the version with indeterminacy and a forward-looking monetary policy rule dominates all other models (Table 10). For the post-1982 period (1982:IV to 1997:IV) only, the probability of the model with indeterminacy and current-looking monetary policy rule is about 0.2.

	Indeterminacy with current-looking monetary policy rule	Determinacy with current-looking monetary policy rule
Pre-Volcker period (1960:I to 1979:II)		
Priors	0.50	0.50
Log Marginal Density	-710.713	-870.123
Posterior probability	1.000	0.000
Post-1982 period (1982:IV to 1997:IV)		
Priors	0.50	0.50
Log Marginal Density	-434.684	-481.719
Posterior probability	1.000	0.000
Post-1982 period (1982:IV to 2008:I)		
Priors	0.50	0.50
Log Marginal Density	-567.100	-599.605
Posterior probability	1.000	0.000

Table 9. Bayesian comparison of versions of the model with determinacy and indeterminacy.

	Indeterminacy with current-looking monetary policy rule	Indeterminacy with forward-looking monetary policy rule	Determinacy with current-looking monetary policy rule	Determinacy with forward-looking monetary policy rule
Pre-Volcker period (1960:I to 1979:II)				
Priors	0.250	0.250	0.250	0.250
Log Marginal Density	-710.713	-704.941	-870.123	-884.576
Posterior probability	0.003	0.997	0.000	0.000
Post-1982 period (1982:IV to 1997:IV)				
Priors	0.250	0.250	0.250	0.250
Log Marginal Density	-434.684	-433.305	-481.719	-520.444
Posterior probability	0.201	0.799	0.000	0.000
Post-1982 period (1982:IV to 2008:I)				
Priors	0.250	0.250	0.250	0.250
Log Marginal Density	-567.100	-433.305	-599.605	-631.436
Posterior probability	0.000	1.000	0.000	0.000

Table 10. Bayesian comparison of the four versions of the model.

Conclusions

Different versions of the New Keynesian model with capital accumulation, different Taylor rules, and the possibility of indeterminacy are simulated and estimated in this paper. Capital accumulation activity introduces new channels of influence for monetary policy on the economy through the no-arbitrage condition between bonds and real sector returns. In canonical models, interest rates affect output solely through the consumption-savings decision of the household in the absence of investment. It is shown in this paper that investment activity changes the monetary transmission mechanisms and allows for the reconsideration and re-estimation of monetary policy.

The approach of the paper is threefold. First, different versions of a New Keynesian model with capital accumulation are simulated and their dynamic properties are discussed. While most of the canonical Keynesian models cannot replicate high autocorrelation levels among the main economic variables, the simulation results of this paper show a model with capital accumulation can generate substantial persistencies in major economic variables. The stock nature of capital adds persistency to the dynamics of all other variables in a model. In the simulated versions of the model the autocorrelation coefficients for consumption and capital are more than 0.9, and the autocorrelations of inflation and nominal interest rate are also very high. Also, interest rate and output autocorrelation coefficients are higher in the model with passive as opposed to active monetary policy due to the fact that passive monetary policy is unable to adjust the interest rate sufficiently to phase out shock propagations.

Simulated dynamics show that preferences (demand) shocks are the main drivers of volatility in the version of the model with passive monetary policy, explaining more than 90 percent of volatility in endogenous variables. In contrast, marginal cost (supply) shocks explain more than 50 percent of variance in the version with active monetary policy. Also, the versions with passive monetary policy demonstrate substantially higher interest rate and inflation volatility than the version with active monetary policy, which can be explained by the existence of indeterminate equilibria in the version with passive monetary policy.

Second, different versions of the model with capital accumulation were fitted to the quarterly postwar U.S. data on output, inflation, nominal interest rates, consumption, and capital from 1960:I to 2008:I. The versions were estimated separately for the pre-Volcker and post-1982 periods assuming, *a priori*, passive monetary policy, consistent with indeterminacy during the pre-Volcker period, and active monetary policy, consistent with determinacy during the post-1982 period.

For the pre-Volcker period the estimates of the response of monetary policy to inflation are almost twice lower than in Lubik and Schorfheide (2004) and in Clarida, Gali and Gertler (2000). The main argument for this lower response is that in models with capital accumulation there is an additional channel of monetary policy influence on the economy through the real interest rate in the production sector. For the pre-Volcker period the estimates of the response of monetary policy to inflation are 0.385 for the current-looking and 0.644 for the forward-looking monetary policy rules. For the post-1982 period, these estimates are about 1.1 for various model specifications.

The estimated response of the monetary policy rule to output is about 0.5 for the pre-Volcker and post-1982 (1982:IV to 1997:IV) periods in the case of the current-looking monetary policy rule. However, it is substantially higher for the 1982:IV to 2008:I period, when the pre-crisis years were included in the sample. Also, it was found that the steady-state inflation and real interest rate values are very close to those in Lubik and Schorfheide (2004) for the pre-Volcker period, while the steady inflation rate is substantially lower and the real interest rate is a marginally lower than in Lubik and Schorfheide (2004) for the post-1982 period depending on model specifications.

A striking finding of the paper is that risk aversion increased substantially over time in the U.S. from about 0.4 for the pre-Volcker period to about 2.5 for the post-1982 period. This differs from the findings of Lubik and Schorfheide (2004), who found that households increased the degree of risk aversion between the pre-Volcker and post-1982 periods only slightly. This result arises from the fact that the investment activity allows us to break the direct connection between interest rate and consumption dynamics in the Euler equation due to the no-arbitrage condition between bonds and real sector returns. This explains the consumption dynamic not only in terms of interest rate changes but in terms of preferences as well. The decrease in the levels of elasticity of intertemporal substitution of consumption can be connected with the development of financial markets and loosening of borrowing constraints in the U.S. over time.

Finally, in contrast to Lubik and Schorfheide (2004), it was found that the version of the model with indeterminacy dominates the determinate version for all sample periods. Comparing the version with indeterminacy and determinacy with current- and forward-looking monetary policy rules, the version with indeterminacy and forward-looking monetary policy rule dominates all other versions.

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Appendix 1. Firm's problem.

The first order conditions for the cost minimization problem are:

$$r_t = z_t f_K(K_t, L_t)$$
$$w_t = z_t f_L(K_t, L_t)$$

where z_t is the marginal cost of production.

Proof:

min: $w_t L + r_t K_t$

s.t.: $f(K, L) = \overline{f}$

$£ = w_t L + r_t K_t - \lambda(f(K, L) - \overline{f})$

FOC:

$w_t = \lambda_t f_L(K_t, L_t)$

$r_t = \lambda_t f_K(K_t, L_t)$

Dual problem:

$£ = c(f(K, L)) - \lambda(f(K_t, L_t) - \overline{f})$

FOC:

$c'(f(K_t, L_t)) = \lambda_t = z_t$

Combining:

$$r_t = z_t f_K(K_t, L_t)$$
$$w_t = z_t f_L(K_t, L_t)$$

Appendix 2. Log-linearization of the model.

Consumption Euler equation

$$\frac{U_C(t)}{P_t} = \beta R_t \left(\frac{U_C(t+1)}{P_{t+1}} \right)$$

as $U_C(t) = C^{-\sigma}$

$$\frac{C_t^{-\sigma}}{E_t C_{t+1}^{-\sigma}} = \beta R_t \left(\frac{P_t}{P_{t+1}} \right)$$

In the log-linearized form:

$$(E_t \hat{C}_{t+1} - \hat{C}_t) = \tfrac{1}{\sigma}\left[\hat{R}_t - E_t \hat{\pi}_{t+1}\right]$$

$$\hat{C}_t = E_t \hat{C}_{t+1} - \frac{1}{\sigma}\left[\hat{R}_t - E_t \hat{\pi}_{t+1}\right]$$

Fisher equation
From

$$U_C(t) = \beta\{U_C(t+1)[r_{t+1} + (1-\delta)]\}$$

$$\frac{U_C(t)}{P_t} = \beta R_t \left(\frac{U_C(t+1)}{P_{t+1}} \right)$$

we have

$$\frac{U_C(t)}{U_C(t+1)} = \beta R_t \left(\frac{P_t}{P_{t+1}} \right) = \beta[r_{t+1} + (1-\delta)]$$

as $r_{t+1} = z_{t+1}\alpha \frac{Y_{t+1}}{K_{t+1}}$

$$R_t \left(\frac{P_t}{P_{t+1}} \right) = r_{t+1} + (1-\delta)$$

As in the steady state: $r_{ss} = \frac{1}{\beta} - 1 + \delta$, the log-linearized form is:

$$\hat{R}_t - E_t \hat{\pi}_{t+1} = \frac{\frac{1}{\beta} - 1 + \delta}{\frac{1}{\beta} - 1 + \delta + (1-\delta)}[E_t \hat{z}_{t+1} + E_t \hat{Y}_{t+1} - \hat{K}_{t+1}] = [1 + \beta(1-\delta)]E_t \hat{z}_{t+1} + E_t \hat{Y}_{t+1} - \hat{K}_{t+1}$$

$$\hat{R}_t - E_t \hat{\pi}_{t+1} = [1 - \beta(1-\delta)](E_t \hat{z}_{t+1} + E_t \hat{Y}_{t+1} - \hat{K}_{t+1})$$

Consumption-labor condition

From $\frac{U_C}{U_L} = -\frac{1}{w}$
As U linear in L:

$$U_C = \frac{1}{w}$$

$$C_t^{\sigma} = z_t(1-\alpha)\frac{Y_t}{L_1}$$

In the log-linearized form:

$$\sigma \hat{C}_t = \hat{z}_t + (\hat{Y}_t - \hat{L}_t)$$

$$\hat{C}_t = \frac{1}{\sigma}\hat{z}_t + \frac{1}{\sigma}\frac{\alpha}{1-\alpha}(\hat{K}_t - \hat{Y}_t)$$

Capital accumulation equation

$$K_{t+1} = (1-\delta)K_t + I_t$$
$$\hat{K}_{t+1} = \frac{(1-\delta)K_{ss}}{K_{ss}}\hat{K}_t + \frac{I_{ss}}{K_{ss}}\hat{I}_t$$
$$\frac{I_{ss}}{K_{ss}} = \delta$$
$$\hat{K}_{t+1} = (1-\delta)\hat{K}_t + \delta\hat{I}_t$$

Output

$$Y = C + I$$
$$\hat{Y}_t = s_C\hat{C}_t + (1-s_C)\hat{I}_t$$

where $s_C = \frac{C_{ss}}{Y_{ss}}$ is the share of consumption in output

New-Keynesian Phillips curve
From the Calvo model:

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \lambda\hat{z}_t$$

Monetary policy rule

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)(\psi_\pi E_t\hat{\pi}_{t+k} + \psi_Y\hat{Y}_t) + \varepsilon_{R,t}$$

Model

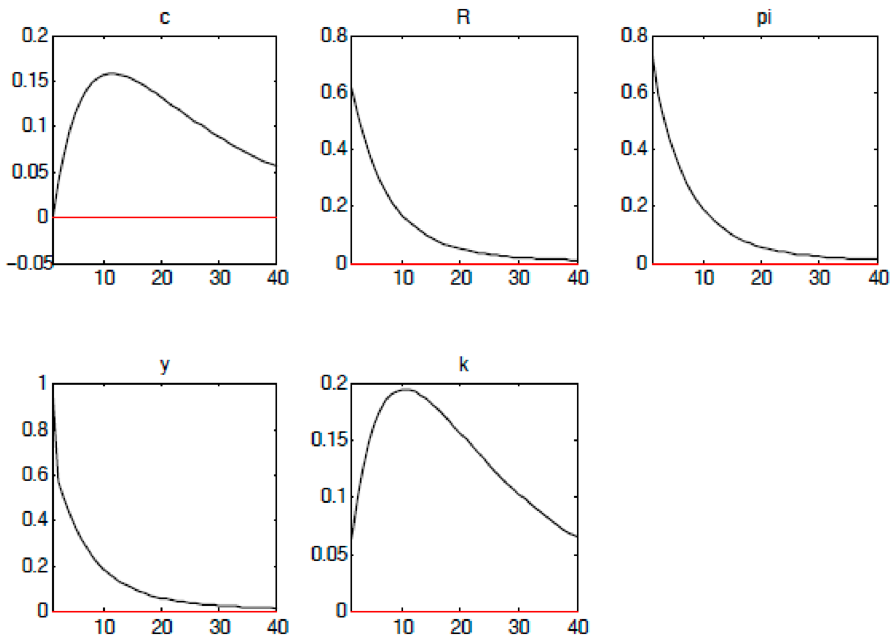
$$\hat{C}_t = E_t\hat{C}_{t+1} - \frac{1}{\sigma}\left[\hat{R}_t - E_t\hat{\pi}_{t+1}\right]$$
$$\hat{R}_t - E_t\hat{\pi}_{t+1} = [1-\beta(1-\delta)](E_t\hat{z}_{t+1} + E_t\hat{Y}_{t+1} - \hat{K}_{t+1})$$
$$\hat{C}_t = \frac{1}{\sigma}\hat{z}_t + \frac{1}{\sigma}\frac{\alpha}{1-\alpha}(\hat{K}_t - \hat{Y}_t)$$
$$\hat{K}_{t+1} = (1-\delta)\hat{K}_t + \delta\hat{I}_t$$
$$\hat{Y}_t = s_C\hat{C}_t + s_I\hat{I}_t$$
$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \lambda\hat{z}_t$$
$$\hat{R}_t = \varphi_R\hat{R}_{t-1} + (1-\varphi_R)(\varphi_\pi E_t\hat{\pi}_{t+j} + \varphi_Y E_t\hat{Y}_{t+j})$$

substituting $\hat{z}_t$ and $\hat{I}_t$
from (3) $\hat{z}_t = \sigma\hat{C}_t - \frac{\alpha}{1-\alpha}(\hat{K}_t - \hat{Y}_t)$
and (5) $(\hat{Y}_t - s_C\hat{C}_t)/s_I = \hat{I}_t$

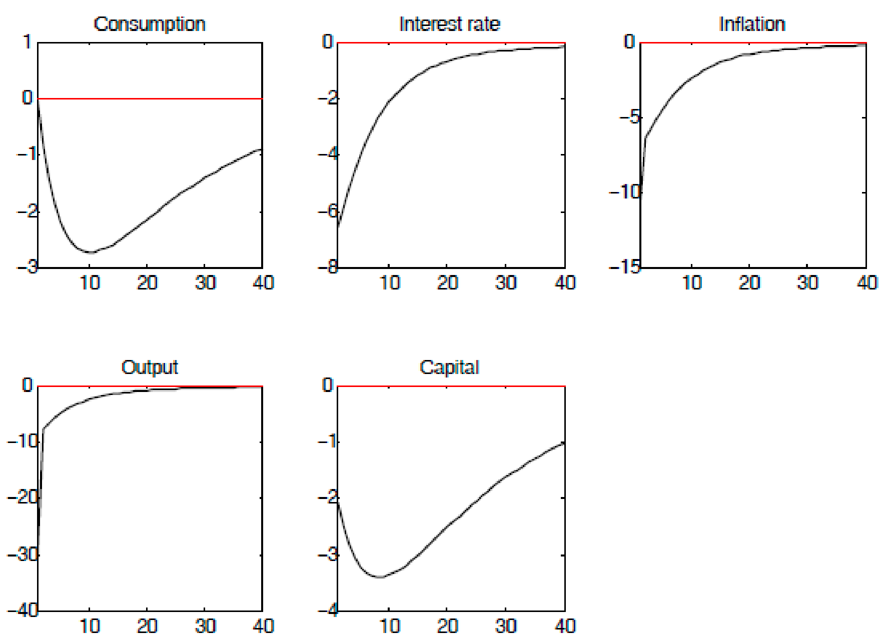
$$\hat{C}_t = E_t\hat{C}_{t+1} - \frac{1}{\sigma}\left[\hat{R}_t - E_t\hat{\pi}_{t+1}\right]$$
$$\hat{R}_t - E_t\hat{\pi}_{t+1} = [1-\beta(1-\delta)](\sigma E_t(\hat{C}_{t+1}) + \frac{1}{1-\alpha}\left[E_t\hat{Y}_{t+1} - E_t\hat{K}_{t+1}\right])$$
$$\hat{K}_{t+1} = (1-\delta)\hat{K}_t + \frac{\delta}{s_I}(\hat{Y}_t - (1-s_I))\hat{C}_t)$$
$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \lambda(\sigma\hat{C}_t - \frac{\alpha}{1-\alpha}(\hat{K}_t - \hat{Y}_t))$$
$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)(\varphi_\pi E_t\hat{\pi}_{t+k} + \varphi_Y E_t\hat{Y}_{t+j}) + \varepsilon_{R,t}$$

Appendix 3. Theoretical IRFs, model with indeterminacy.

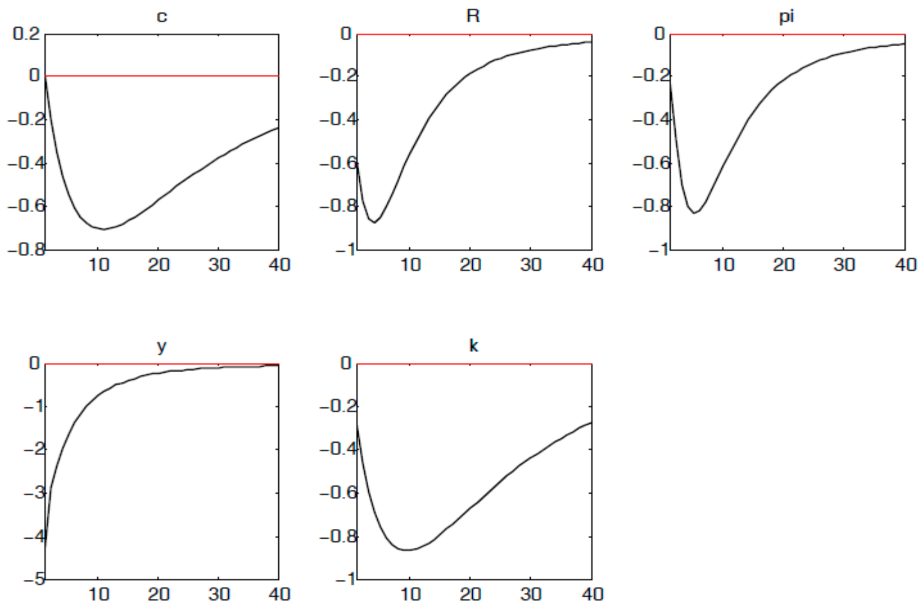
Theoretical IRFs to the interest rate shock.



Theoretical IRFs to the preference shock.

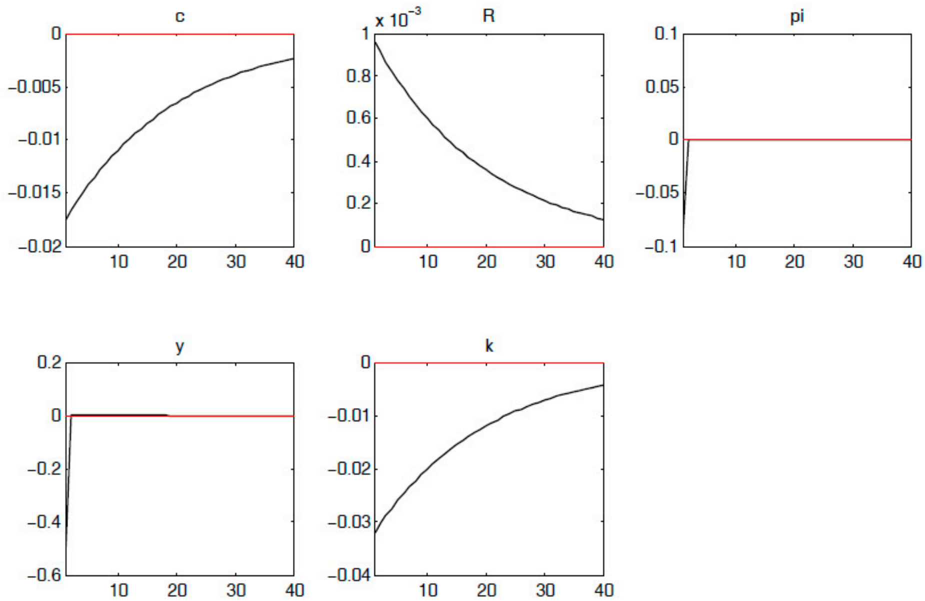


Theoretical IRFs to the marginal cost shock.

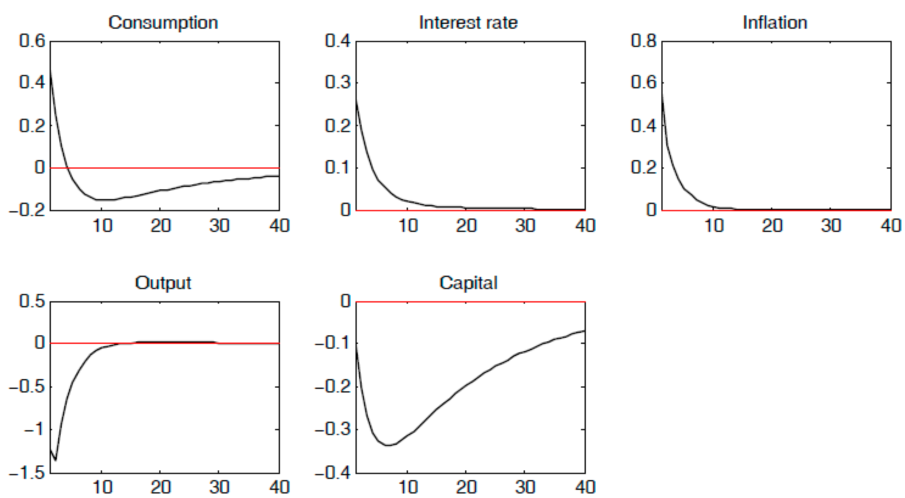


Appendix 4. Theoretical IRFs, model with determinacy.

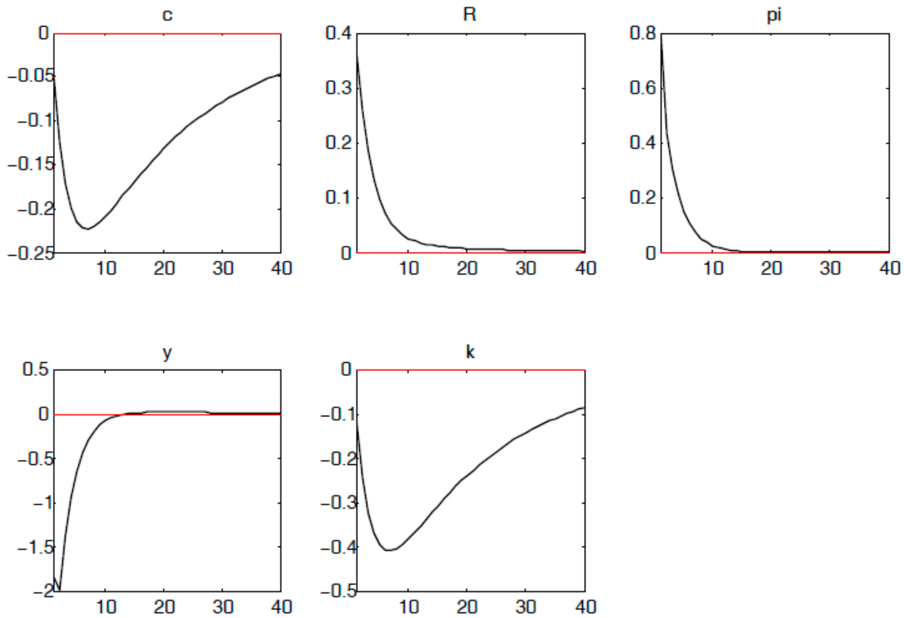
Theoretical IRFs to the interest rate shock.



Theoretical IRFs to the preference shock.



Theoretical IRFs to the marginal cost shock.



Appendix 5. Empirical correlation matrices.

Full sample (1960:I to 2008:IV)

	Consumption	Interest rate	Inflation	Output	Capital
Consumption	1.000	0.031	0.185	0.869	0.272
Interest rate	0.031	1.000	0.652	0.202	0.413
Inflation	0.185	0.652	1.000	0.288	0.380
Output	0.869	0.202	0.288	1.000	0.461
Capital	0.272	0.413	0.380	0.461	1.000

Pre-Volcker period (1960:I to 1979:II)

	Consumption	Interest rate	Inflation	Output	Capital
Consumption	1.000	0.284	0.243	0.888	0.127
Interest rate	0.284	1.000	0.867	0.397	0.569
Inflation	0.243	0.867	1.000	0.288	0.410
Output	0.888	0.397	0.288	1.000	0.403
Capital	0.127	0.569	0.410	0.403	1.000

Post-1978 period (1978:III to 1997:IV)

	Consumption	Interest rate	Inflation	Output	Capital
Consumption	1.000	-0.075	0.312	0.860	0.271
Interest rate	-0.075	1.000	0.667	0.168	0.466
Inflation	0.312	0.667	1.000	0.481	0.559
Output	0.860	0.168	0.481	1.000	0.464
Capital	0.271	0.466	0.559	0.464	1.000

Post-1982 period (1982:IV to 1997:IV)

	Consumption	Interest rate	Inflation	Output	Capital
Consumption	1.000	0.224	0.352	0.836	0.488
Interest rate	0.224	1.000	0.324	0.195	0.084
Inflation	0.352	0.324	1.000	0.337	0.261
Output	0.836	0.195	0.337	1.000	0.538
Capital	0.488	0.084	0.261	0.538	1.000

Post-1982 period (1982:IV to 2008:I)

	Consumption	Interest rate	Inflation	Output	Capital
Consumption	1.000	0.205	0.273	0.864	0.631
Interest rate	0.205	1.000	0.265	0.193	0.091
Inflation	0.273	0.265	1.000	0.268	0.145
Output	0.864	0.193	0.268	1.000	0.594
Capital	0.631	0.091	0.145	0.594	1.000

Appendix 6. Empirical autocorrelations.

Full sample (1960:I to 2008:IV)

Lag	Consumption	Interest rate	Inflation	Output	Capital
1	0.871	0.952	0.669	0.858	0.949
2	0.711	0.885	0.671	0.670	0.836
3	0.527	0.830	0.691	0.449	0.677
4	0.317	0.769	0.574	0.253	0.492
5	0.125	0.701	0.520	0.066	0.297

Pre-Volcker period (1960:I to 1979:II)

Lag	Consumption	Interest rate	Inflation	Output	Capital
1	0.859	0.906	0.765	0.831	0.931
2	0.667	0.757	0.728	0.617	0.791
3	0.431	0.624	0.650	0.360	0.600
4	0.168	0.500	0.579	0.148	0.390
5	-0.067	0.353	0.483	-0.061	0.175

Post-1982 period (1982:IV to 2008:I)

Lag	Consumption	Interest rate	Inflation	Output	Capital
1	0.841	0.960	-0.003	0.825	0.946
2	0.677	0.904	0.168	0.618	0.841
3	0.532	0.837	0.280	0.403	0.703
4	0.358	0.762	-0.026	0.229	0.545
5	0.224	0.683	0.032	0.084	0.385