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Macrofinance Model of the Czech Economy: Asset Allocation Perspective

Miroslav Kollar

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Macrofinance Model of the Czech Economy: Asset Allocation Perspective[1]

Prepared by Miroslav Kollar[2]

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Abstract

The paper developes a VAR macrofinance model of the Czech economy. It shows that yield misalignments from the yields implied by the macrofinance model partially determine subsequent yield changes over three to nine months. These yield misalignments tend to persist for a number of months. This persistence of the misalignments was explained by (a) the fact that the macro-economy influences asset markets only at lower frequencies, (b) the liquidity effect particularly during the times of capital inflows to Czech Republic, and (c) the fact that not all misalignments were greater than their historical one standard deviation.

JEL Classification Numbers: C58, E43, G11

Keywords: yield curve modeling, macrofinance modeling, vector auto-regression model

Author's E-Mail Address: mkollar@imf.org

[1] This paper benefited from comments by Gunes Asik (London School of Economics), prof. Robert Holman (University of Economics in Prague), prof. Josef Jilek (Czech National Bank), Kamil Kladivko (Norwegian School of Economics), Martin Kollar (Axa Asset Management), Lubos Komarek (Czech National Bank), and Christian Schmieder (International Monetary Fund). All errors are of the author.

[2] Senior Advisor to Executive Director, Office of Executive Director for Austria, Belarus, Belgium, Czech Republic, Hungary, Kosovo, Luxembourg, Slovak Republic, Slovenia and Turkey.

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I. INTRODUCTION

The interaction between the science of macroeconomics and the science of finance has been very vague. Macroeconomics has traditionally focused on the description of economic growth, inflation developments and the business cycle behavior, without sufficient attention to financial assets. Finance has been more informed by the achievements of microeconomics rather than macroeconomics. General equilibrium theory and utility theory have all penetrated finance models but little attention has been paid to the behavior of the business cycle, for example.

It is usually at the peak of or shortly after a major economic crisis that paradigm shifts occur. For example, already since the late 1990s and early 2000s, economists tried to merge standard macroeconomic workhorse models with models of the term structure of interest rates. Their motivation was to better understand the interaction between the yield curve and the macro-economy. The result has been the growing literature on macrofinance models.

This paper came out from the conviction of the author that macroeconomic developments were one of the major determinants of medium-to-longer term movements of financial markets. This paper tries to bridge the finance and macroeconomics fields of research. To our knowledge, this paper is one of the few pivotal works that tries to apply macrofinance modeling to Czech data.

We will estimate a macrofinance model of the Czech economy based on the so-called Nelson-Siegel framework. In other words, we will estimate a dynamic term structure of interest rates (i.e. yield curve) with the help of macroeconomic variables. We will claim that the resulting model yield curve is a fair-value yield curve as determined by the state of the macroeconomy. In turn, we will try to investigate whether the actual yield curve indeed tends to revert back to the yield curve determined by the state of the macroeconomy. We will study various properties of such approach and illustrate its usefulness for yield forecasting and for investment purposes.

II. MACROFINANCE MODELING

Macrofinance modeling is a relatively new research agenda that combines models of the yield curve with simple macroeconomic models or with macroeconomic variables.[3] The main motivation of such model structure is to understand both the influence of macroeconomic variables on the yield curve and the information value of the yield curve with respect to the macro-economy.

The joint macrofinance framework can illuminate various macroeconomic relationships, hence a number of questions arise: Do macroeconomic variables influence the yield curve? Do macroeconomic fundamentals help predict the yield curve? Does the yield curve help predict macroeconomic fundamentals?

On top of these questions, the understanding of the macroeconomic determinants of the yield curve is important for central banks and market analysts for a number of other reasons. Firstly, this knowledge can be used in enlarging the current macroeconomic models for a coherent treatment of longer-term interest rates, that have significant impact on the economy

[3] Kim (2007).

(e.g. on consumer-, mortgage- and corporate-loans; and therefore on prices and volumes of financial and physical assets). Secondly, this knowledge can be used also in financial stability analysis of central banks (e.g. stress testing).

The pivotal stream of statistical macrofinance models came out from the works of Nelson, Siegel (1987) and Litterman, Scheinkman (1991), making use of the well-established empirical fact that government bond yield curves can be explained by using only up to three factors. Nelson, Siegel (1987) and later on Svensson (1995) proposed an exponential component statistical approach to modeling yield curve that requires the estimation of only a small number of parameters. Such parsimonious model of the term structure has the desired properties with respect to the shape of the yield curve that are not guaranteed by the polynomial approximation. This statistical approach to yield curve modeling was afterwards given a more dynamic interpretation by Diebold, Li (2006)[4] that will be the subject point of our empirical analysis.

Diebold, Li (2006) regard the Nelson-Siegel factors as time-varying latent factors that could be interpreted as level, slope and curvature factors of the yield curve and they can follow a vector autoregression process. The level factor is usually connected to long-term inflation expectations. The slope factor is supposed to reflect the stance of the monetary policy, henceforth the short-term expectations of growth and inflation. The macroeconomic interpretation of the curvature factor is not straightforward. One might argue that the curvature factor should reflect medium-term expectations of growth and inflation. The terms that multiply these factors are factor loadings. The macrofinance models based on this approach (e.g. Diebold, Rudebusch, Aruoba, 2006; Hoerdahl et al., 2004) enrich the vector autoregression of the Nelson-Siegel latent factors with macroeconomic variables (the reduced form models) or with a model of the macro-economy. Diebold, Rudebusch, Aruoba (2006) find that the level factor is highly correlated with inflation and the slope factor is highly correlated with the real activity.

A complementary approach to macrofinance modeling uses the condition of no-arbitrage for the term structure of interest rates, where the spot rate is modeled as a linear function of the latent factors and the factor loadings are recursively defined starting from the short rate onwards.[5] The absence of arbitrage along the term structure of interest rates assumes the expectations hypothesis of the term structure and the pricing kernel is given by the market price of risk. The term structure is then fitted at a point in time so that no arbitrage possibilities exist along the yield curve.[6] The no-arbitrage macrofinance models add macroeconomic variables to the vector of latent factors (Ang, Piazzesi, 2003) or combine a no-arbitrage term structure model with a Neo-Keynesian macroeconomic model (Rudebusch, Wu, 2004).

The choice of the no-arbitrage approach to macrofinance modeling over the statistical approaches is not straightforward. First, in countries with less developed debt markets or less liquid parts of the yield curve, transitory arbitrage opportunities may persist for some time, hence imposition of the no-arbitrage restriction might lead to model misspecification.[7]

[4] Of course, there are also other statistical approaches, like spline-modeling. On these approaches see Cairns (2004) and James, Webber (2000).

[5] Ang, Piazzesi (2003), Wu (2005).

[6] For an overview of no-arbitrage interest rates models see Cairns (2004) and James, Webber (2000).

[7] Hoffmaister et al. (2009).

Second, the no-arbitrage term structure models need to assume the expectations hypothesis of the term structure, i.e. that long term yields are equal to the average of the expected spot rates.[8] Third, no-arbitrage models of the term structure are less successful in describing the dynamics of yield curve over time as compared to fitting the cross-section of yields at a particular point in time.[9] Fourth, the no-arbitrage macrofinance models require estimating a lot more parameters than the simpler statistical models with macroeconomic underpinnings. This might lead to more estimation errors and more complex interpretation of the models. Fifth, even with best no-arbitrage models, there is a wide diversity in estimates of no-arbitrage values among experts and practitioners.[10]

The no-arbitrage macrofinance models might be theoretically more appropriate and more relevant for pricing of fixed income instruments. On the other hand, policy-makers, forecasters and market analysts might make greater use of less complex statistical macrofinance models – the avenue that we decided to follow in this paper as well.

Let us conclude this section with a note on the related literature in Czech Republic. By now, there have been only a few comprehensive attempts to apply macrofinance models to Czech data.[11] Nevertheless, there have been a number of papers studying the term structure of interest rates in the Czech Republic.[12] There have also been a number of studies dealing with a particular relationship between the macroeconomic variables and some parts of the yield curve, but without any attempt to model the whole yield curve and link it to macroeconomic variables.[13]

III. Macrofinance model based on Nelson-Siegel framework

Dynamic Nelson-Siegel Interpretation

The Nelson-Siegel framework (Nelson, Siegel, 1987)[14] is probably the most frequent statistical method used by practitioners for estimating the zero-coupon yield curve (BIS, 2005).[15] The Nelson-Siegel framework is a parsimonious approach that models the whole yield curve using a single exponential functional form by estimating only a small number of parameters. These parameters are then all that is required to describe the whole yield curve. Such parsimonious model of the term structure has the desired properties with respect to the shape of the yield curve that are not guaranteed by polynomial approximation.

As early as Litterman, Scheinkman (1991) with the aid of the principal components analysis of the yield curve, it has been a well-established empirical fact that most of the behavior in

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[8] After adjusting for risk premium.

[9] Diebold, Rudebusch, Aruoba (2006).

[10] Bhansali (2007, and 2011).

[11] See for example Skop (2009).

[12] See for example Slavik (2001), Malek, Radova, Sterba (2007), Dobias (2008), Kladivko, Zimmermann, Cicha (2007) or Kladivko (2010).

[13] See for example Pikora (2007), Kotlan (1999).

[14] The original Nelson-Siegel framework was later on extended by Svensson (1995).

[15] As will be showed below, the Nelson-Siegel framework can be also used for modeling the dynamic behavior of the yield curve.

the government bond yield curves can be explained by using only up to three factors. These factors tend to be interpreted as the level, slope and curvature factors of the yield curve.[16]

Diebol, Li (2006) introduced a dynamic interpretation of the Nelson-Siegel framework. From their perspective, the Nelson-Siegel framework can be used for modeling the dynamics of the yield curve. This interpretation of the Nelson-Siegel framework will be the cornerstone of our macrofinance modeling in the following text.

The original Nelson-Siegel factors are regarded by Diebold, Li (2006) as time-varying latent factors that could be interpreted, based on Litterman, Scheinkman (1991), as level, slope and curvature factors of the yield curve. The terms that multiply these factors are called factor loadings. The dynamics of the whole yield curve can be therefore captured by describing the dynamics of the three latent factors, which can eventually be translated into the dynamics of individual yields.

Following Diebold, Li (2006) we start with the standard static Nelson-Siegel exponential representation for the spot rate curve, where the spot rate *r(τ)* of a zero-coupon bond maturing in τ periods is the following

$$r(\tau) = \beta_1 + \beta_2 \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_3 \left(\frac{1-e^{\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) \quad (1)$$

where β_1, β_2, β_3 are unknown parameters and can be interpreted as level, slope and curvature of the yield curve and unknown parameter $\lambda > 0$ can be interpreted as the rate of exponential decay.

Changes in factor β_1 result in parallel shifts in the yield curve. The loading of factor β_1 does not change with maturity and therefore the factor affects all the yields by the same amount. As time to maturity approaches infinity, the spot rate converges to β_1. Factor β_1 can be interpreted as the level of the yield curve.

As time to maturity approaches zero, the spot rate converges to $\beta_1 + \beta_2$. Since factor β_2 is the difference between the long rate ("level") and the short spot rate, then the factor β_2 can be interpreted as the slope of the yield curve. An increase in β_2 raises short yields more than long yields, hence changing the slope of the yield curve. The loading of factor β_2, i.e. $\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right)$, equals one at zero maturity and approaches zero with increasing maturities.

Factor β_3 represents the curvature of the yield curve. It converges to zero for the shortest and the longest maturities, thereby creating a hump in the yield curve if the factor β_3>0 or a U-shaped yield curve if the factor β_3<0. For example, for β_3>0, the loading of factor β_3, i.e. $\left(\frac{1-e^{\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)$, starts at zero at the shortest maturities, increases at intermediate maturities, and falls back to zero at the longest maturities.

[16] Nelson, Siegel (1987).

Finally, parameter λ determines the location of the hump (or U-shape) of the yield curve and the exponential decay of the yield curve. The larger the parameter λ is, the faster is the decay of the yield curve's curvature.

The Nelson-Siegel formulation of the yield curve assures desirable properties of the yield curve (Diebold, Li, 2006). Forward rates are positive along the whole yield curve and the discount factor is approaching zero with increasing maturity. Also, the yield curve has only a limited number of shapes, which is desirable.

The dynamic interpretation of the Nelson-Siegel approach treats the level ($\beta_{1,t}$), slope ($\beta_{2,t}$) and curvature ($\beta_{3,t}$) latent factors as time-varying and therefore allows us to describe the whole yield curve at any point in time using only these three factors. The dynamic version of equation (1) with time-varying latent factors is therefore the following

$$r_t(\tau) = \beta_{1,t} + \beta_{2,t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3,t}\left(\frac{1-e^{\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right). \tag{2}$$

Diebold, Li (2006) showed that this dynamic interpretation of Nelson-Siegel framework is well suited for interest rate forecasting and dynamic analysis of interest rate behavior.

Data Used in the Model

In the following macrofinance model, we used monthly data from May 2000 to February 2010 on the Czech government zero-coupon bond spot curve for maturities from three months up to ten years, as estimated by Kladivko (2010).

The following monthly data were used by Kladivko (2010) for the computation of the zero-coupon spot curve.[17] For maturities up to six months, the Czech interbank money market rates were used. In particular, one-week, one-month and three-month PRIBOR rates[18]. The reason for using the interbank money market rates for maturities up to six months is that there are no consistent Czech government debt instruments data for short maturities available. For the purpose of zero-coupon spot rate curve estimation, these interbank money market rates were transformed into zero-coupon bond prices. The zero-coupon yields with maturities up to one year are therefore affected by both the interbank money market and the bond market. For maturities from six months up to ten years, the Czech government bond yield data collected by the Prague Stock Exchange were used. In particular, the mid quotes for six months, nine months, and one- to ten-years maturities.

The following bonds were excluded from the estimation by Kladivko (2010). First, all bonds with less than 180 days to maturity because of their relatively low liquidity. Second, for the first tranche of each bond, all bonds before they reach 30 days after the issue date were excluded. This was motivated by the observed odd behavior of the price quotes of some of the new bond issues during the first few weeks of their trading. Third, the 6.08%/2001 bond (issue number 27) was excluded, since this bond appears to be constantly over-priced. Fourth, the 4.85 %/2057 bond (issue number 54) was excluded, because it is not actively traded on the market. Finally, all floating interest rate bonds were excluded, since their use in yield curve estimation is not straightforward.

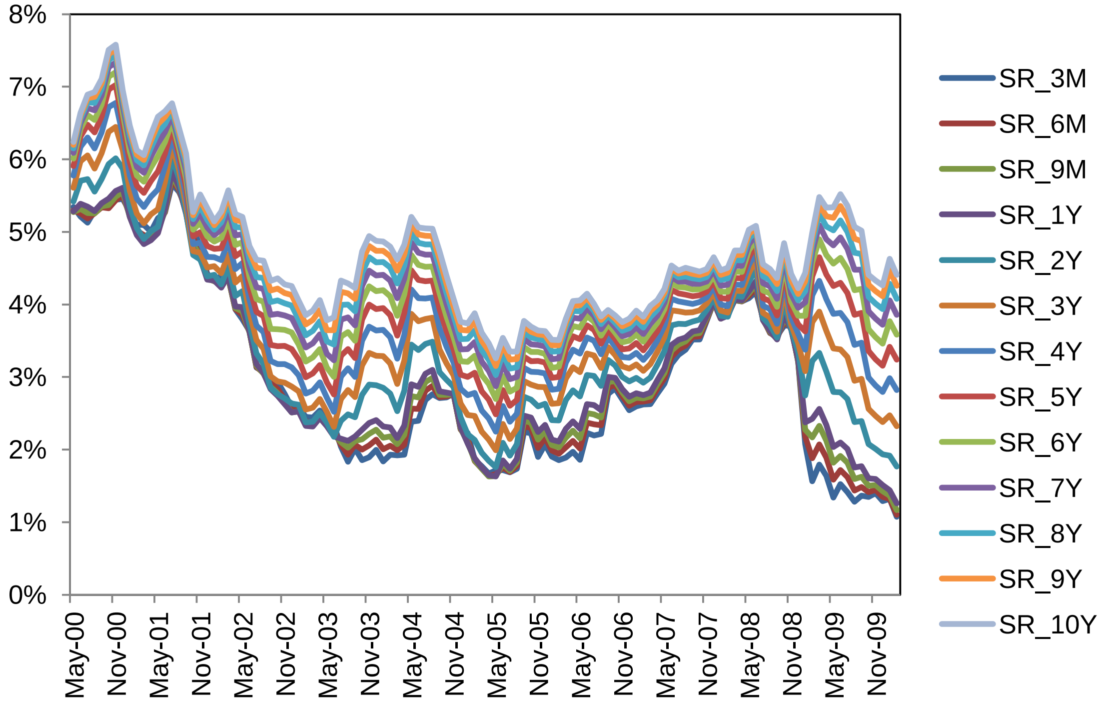
[17] For more on Czech government bond data see Kladivko (2010).

[18] More concretely, the "mid" rate calculated as (offer rate + bid rate)/2 was used.

The reasons for starting our estimation from year 2000 are twofold. Firstly, there was insufficient number of bonds available before 1999. Kladivko (2010) needed at least four bonds in order to estimate the yield curve. Secondly, we wanted to avoid structural breaks. For monetary policy, we work within one monetary-policy regime, i.e. inflation targeting that began to be implemented from 1998 onwards. From 1998 to 2000, the Czech National Bank conducted a disinflationary policy of high interest rates accompanied by high volatility of interest rates. From the perspective of fiscal policy, all bonds issued before 1998 were excluded, because they were issued under different taxation regimes. The taxation regime influences the bond price and therefore bonds with different taxation regimes cannot be put together into one yield curve.

Even though the time series for interest rates in Czech Republic are not as long as in the U.S. or U.K., where the term-structure models of interest rates have been mostly developed for, the mean-reverting tendency of interest rates is slightly visible from the following figure in the Czech Republic, too.

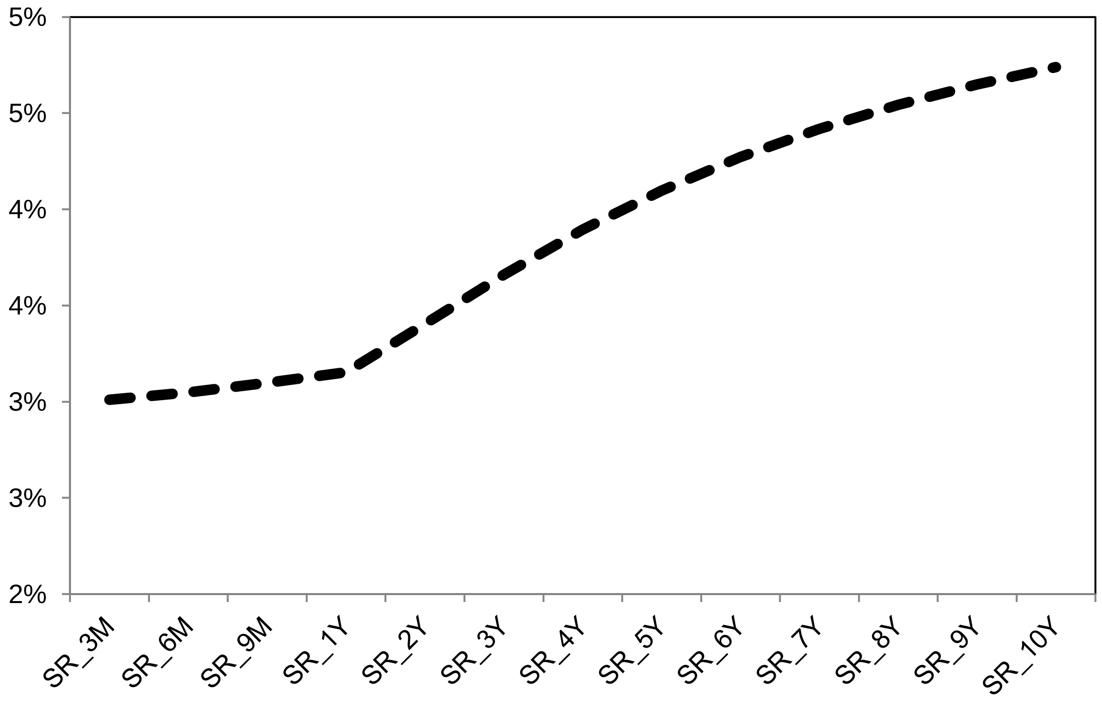
Figure 1: Czech Government Zero-Coupon Bond Yields (Spot Rates)



Source: Prague Stock Exchange; Czech National Bank

The average Czech government zero-coupon yield curve from May 2000 until February 2010 is depicted on the following figure. We can view the average yield curve as the historical long-term equilibrium yield curve.

Figure 2: Average Czech Yield Curve



Source: Prague Stock Exchange; Czech National Bank; own calculations

The following table contains the descriptive statistics, including the autocorrelation functions, for the Czech government zero-coupon yield curve.

Table 1: Descriptive Statistics of Czech Government Zero-Coupon Bond Yields

	Central Moments						Autocorrelations		
Maturity	**Mean**	**StDev**	**Min**	**Max**	**Skew**	**Kurt**	**Lag1**	**Lag2**	**Lag3**
SR_3M	2.972	1.300	0.872	5.564	0.531	2.162	0.960	0.923	0.878
SR_6M	3.016	1.258	0.925	5.634	0.555	2.217	0.960	0.918	0.872
SR_9M	3.068	1.225	0.988	5.700	0.582	2.292	0.959	0.913	0.864
SR_1Y	3.126	1.198	1.066	5.763	0.612	2.378	0.957	0.908	0.857
SR_2Y	3.383	1.130	1.469	6.017	0.749	2.745	0.951	0.893	0.834
SR_3Y	3.641	1.092	1.923	6.449	0.882	3.085	0.947	0.884	0.822
SR_4Y	3.876	1.065	2.241	6.797	0.982	3.348	0.946	0.882	0.817
SR_5Y	4.083	1.042	2.479	7.053	1.044	3.517	0.947	0.882	0.816
SR_6Y	4.261	1.023	2.690	7.242	1.075	3.606	0.949	0.885	0.818
SR_7Y	4.413	1.006	2.870	7.383	1.083	3.637	0.950	0.887	0.820
SR_8Y	4.543	0.992	3.024	7.490	1.075	3.631	0.952	0.890	0.823
SR_9Y	4.654	0.980	3.153	7.575	1.056	3.604	0.954	0.893	0.826
SR_10Y	4.750	0.971	3.262	7.643	1.031	3.567	0.956	0.895	0.828

Source: Prague Stock Exchange; Czech National Bank; own calculations

There is a number of stylized facts of the Czech yield curve that we can infer from the previous tables and figures. First, the average Czech yield curve is increasing and concave. Second, the persistence of yield dynamics is high. Third, the short end of the yield curve is more volatile than the long end of the yield curve. Forth, short rates are relatively more persistent than long yields.[19]

We accompany the yield curve data with the following monthly macroeconomic variables from May 2000 to February 2010. Since we are using monthly data frequency, we cannot use GDP data as they are only available with quarterly frequency. Therefore, in order to capture the activity of the real economy, we use the index of seasonally adjusted real industrial production in Czech Republic and the Euro Area. Real industrial production gap is estimated using the standard Hodrick-Prescott filter procedure (*IPP_GAP, IPP_GAP_EUR*). Next, we use monthly data on the year-on-year changes of domestic and Euro Area consumer price-inflation rate (*INFL, INFL_EUR*). We also use the inflation target set by the Czech National Bank (*INFL_TARGET*). Furthermore, we use the CZK/EUR exchange rate. The exchange rate gap was also estimated using the standard Hodrick-Prescott filter (*EURCZK_GAP*). To fully depict the foreign economic environment, we also use the 3M EURIBOR (*EURIBOR_3M*) and 10Y German government bond yield (*GER_10Y*).

Principal Components Analysis of the Czech Zero-Coupon Yields

Before proceeding further with developing our Nelson-Siegel type macrofinance model of the Czech economy, let us first for illustration purposes conduct a principal components analysis of the zero-coupon yields and contrast its results with macro-economic variables.[20] The principal components analysis reduces the set of original variables to a small number of new variables – principal components – that have the sufficient statistical power to explain the dynamics of the original set of variables. These principal components are linear combinations of the original variables. The principal components are not correlated among each other.[21]

We applied the principal components analysis on the 3-month to 10-year Czech government zero-coupon bond yields from 2000 to 2010. The results are shown in the following table.

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[19] This fourth stylized fact is exactly the opposite from what we see in the U.S. yield curve. (Diebold, Li, 2006)

[20] For a similar approach see also Litterman, Scheinkman (1991), Diebold, Li (2006), or Hoffmaister et al. (2009).

[21] On the technique of principal components analysis, see for example Tsay (2002).

Table 2: Principal Components Analysis of Czech Government Bond Yields

Principal Components Analysis

Sample (adjusted): 2000M05 2010M02
Included observations: 118 after adjustments
Balanced sample (listwise missing value deletion)
Computed using: Ordinary correlations
Extracting 3 of 13 possible components
Maximum number of components: 3

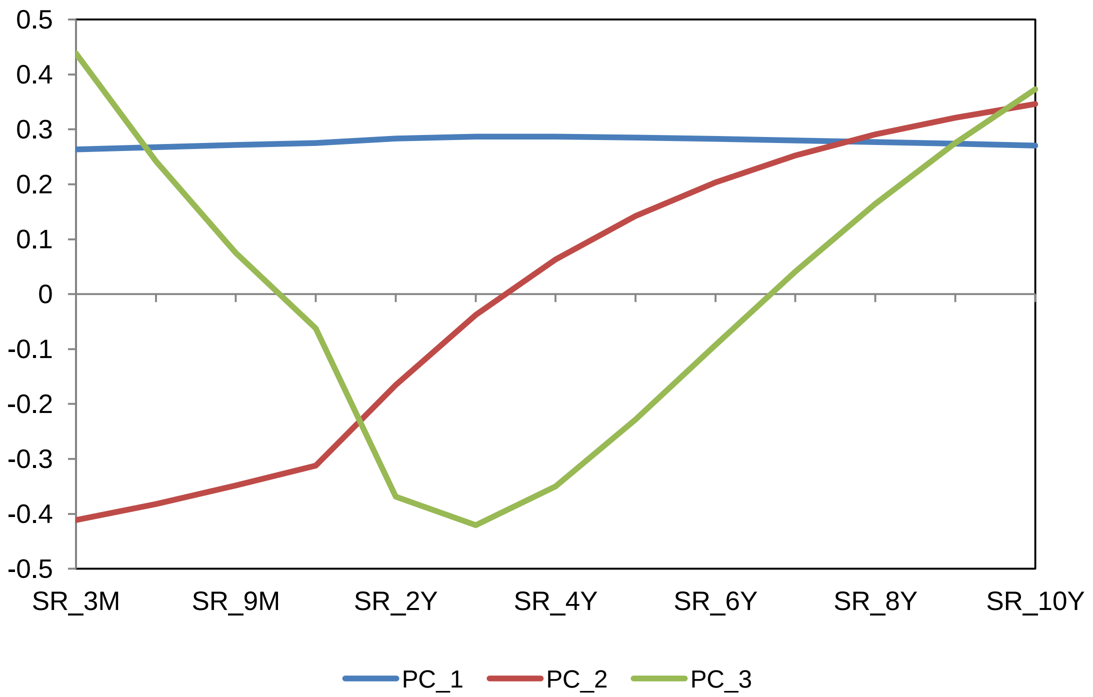
Eigenvalues: (Sum = 13, Average = 1)

Number	Value	Difference	Proportion	Cumulative Value	Cumulative Proportion
1	12.02478	11.10842	0.9250	12.02478	0.9250
2	0.916361	0.859930	0.0705	12.94114	0.9955
3	0.056431	0.054064	0.0043	12.99757	0.9998

Source: own calculations

As Table 2 shows, the first principal component explains as much as 92.5 percent of the variation in the Czech government bond yields. The first three components cumulatively explain up to 99.98 percent of the variation in the Czech government bond yields. This result conforms to findings of a number of similar studies.[22]

Figure 3: Factor Loadings of the Principal Components across Maturities



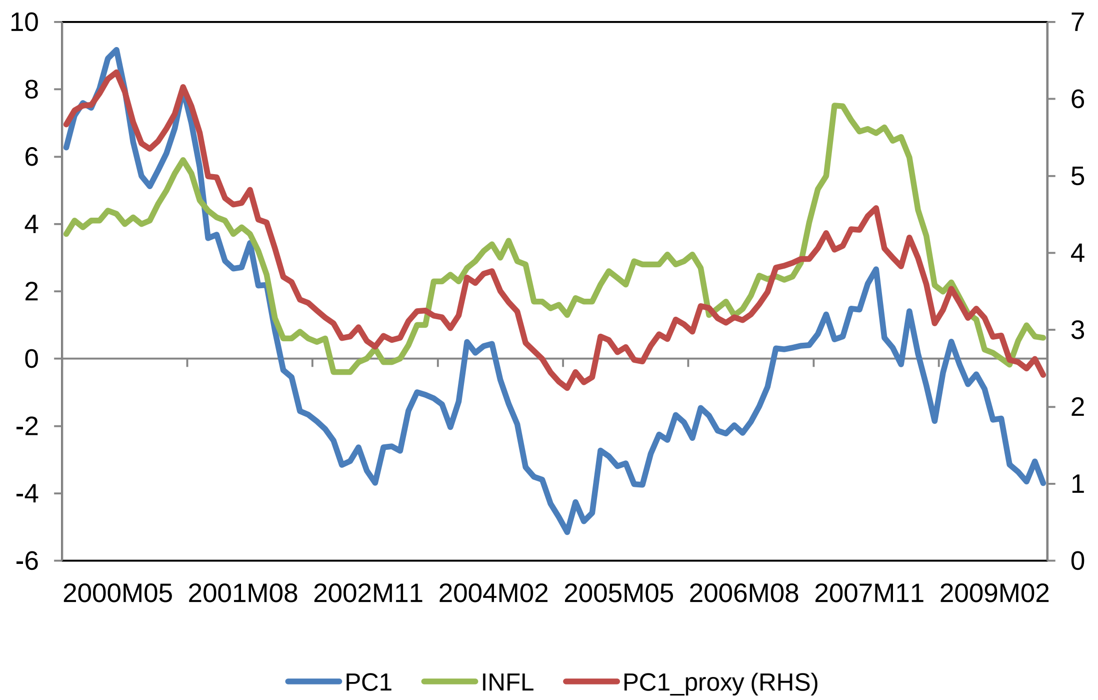
Source: own calculations

[22] Litterman, Scheinkman (1991), Diebold, Li (2006).

Figure 3 portrays the values of factor loadings of the three principal components. In line with the literature, the factor loading of the first component is positive and uniform across the maturity spectrum. An upwards or downwards shift in the first component will therefore affect uniformly all maturities. This corresponds to the interpretation of the first component as the approximation of the level of interest rates. The factor loading of the second principal component is increasing with the maturity of the bonds. The second principal component is therefore a good approximation of the slope of the yield curve. The U shape of the factor loading of the third principal component corresponds to its interpretation as the approximation of the curvature of the yield curve.

The following three figures illustrate the historical development of the first three principal components, their empirical proxies and macroeconomic proxies as explained below.

Figure 4: First Principal Component and its Empirical and Macro Proxies



Source: Prague Stock Exchange; Czech National Bank; own calculations

Figure 4 shows the development of the first principal component together with its suggested empirical approximation and macroeconomic approximation. The common empirical approximation[23] of the level of interest rates is $(r(3M) + r(2Y) + r(10Y))/3$. The suggested macroeconomic approximation of the first principal component is the inflation rate.[24] As we see, the first principal component nicely corresponds to the behavior of inflation rate and the empirical approximation of the level of interest rates.

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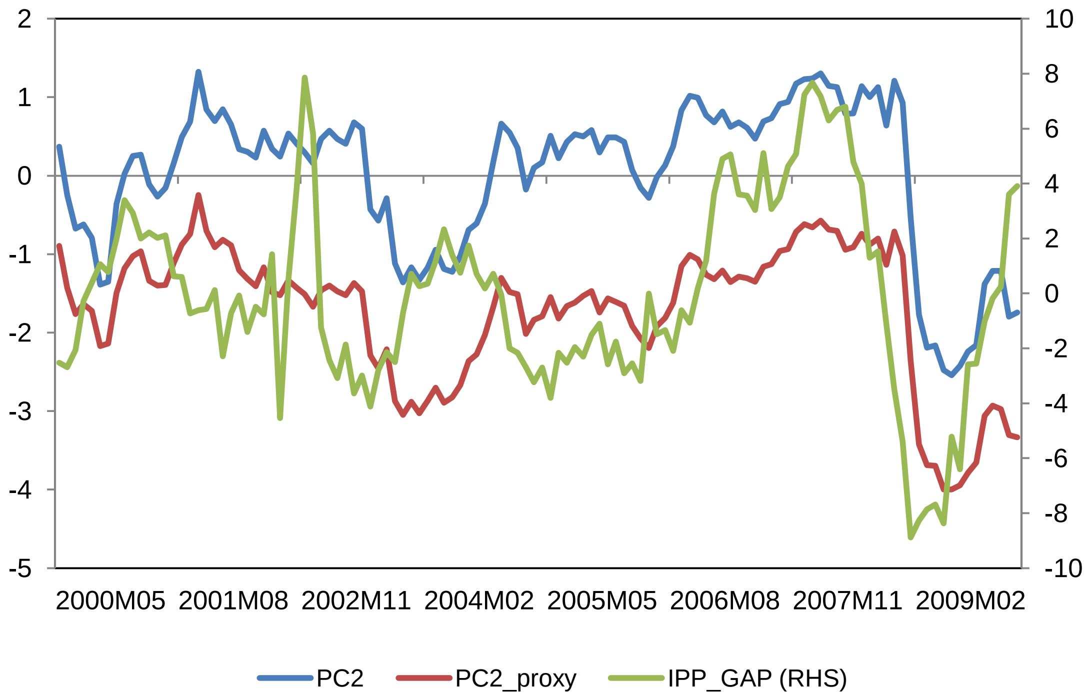
[23] See Diebold, Li (2006) or Diebold, Rudebusch, Aruoba (2005).

[24] See Diebold, Rudebusch, Aruoba (2005) for a similar macroeconomic interpretation.

Figure 5: Second Principal Component and its Empirical and Macro Proxies



Source: Prague Stock Exchange; Czech National Bank; Czech Statistical Office; own calculations

Figure 5 shows the development of the second principal component together with its suggested empirical approximation and macroeconomic approximation. The commonly used empirical approximation of the slope of the yield curve is the following $(r(3M) - r(10Y))$.[25] The suggested macroeconomic approximation of the second principal component is industrial-production gap.[26] The behavior of the second principal component and the slope of the yield curve is very similar. The strength of the co-movement between the second principal component and the industrial-production gap has increased during the recent years of our data set.

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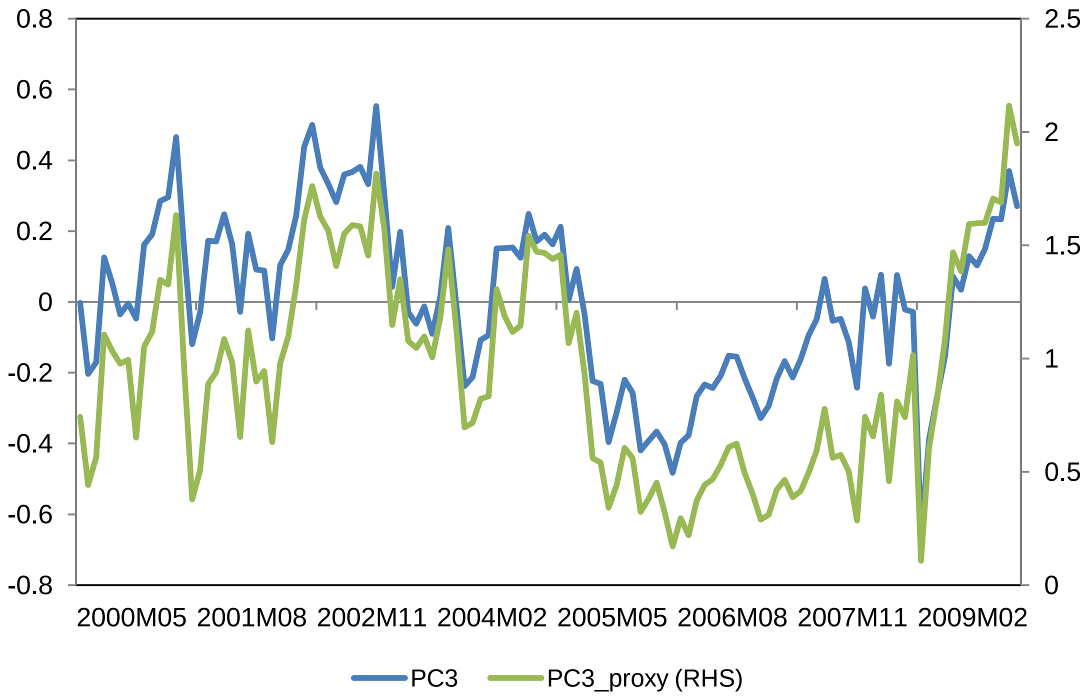
[25] As we see, the slope is inverted for the purpose of this exercise.

[26] See Diebold, Rudebusch, Aruoba (2005) for a similar macroeconomic interpretation. The GDP gap is not directly available in monthly frequency.

Figure 6: Third Principal Component and its Empirical Proxy



Source: Prague Stock Exchange; Czech National Bank; own calculations

Figure 5 shows the development of the third principal component together with its suggested empirical approximation. The commonly used empirical approximation of the curvature of the yield curve is the following $(2 \times r(2Y) - r(3M) - r(10Y))$. There is not straightforward macroeconomic approximation of the third principal component, and henceforth of the curvature of the yield curve. As we see, the empirical approximation of the curvature of the yield curve tracks very well the development of the third principal component.

From a Yields-Only Nelson-Siegel Model to a VAR Macrofinance Nelson-Siegel Model of the Czech Economy

Even though the yields-only Nelson-Siegel model of the yield curve describes relatively well the behavior and dynamics of interest rates, it offers little insight into the underlying economic forces that drive the behavior of interest rates. In an effort to provide such an insight into underlying economic forces driving interest rate movements, we will enrich the Nelson-Siegel framework with macro-economic variables.

We will start with the Diebold, Li (2006) dynamic interpretation of the Nelson-Siegel framework as described above, which is for better traceability reproduced here again

$$r_t(\tau) = \beta_{1,t} + \beta_{2,t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3,t}\left(\frac{1-e^{\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) \quad (3)$$

where the level, slope and curvature factors are time-varying latent factors.

Following Diebold, Li (2006), we will estimate the macrofinance model of the term structure of interest rates in two steps. [27] In the first step, we use the data on the Czech zero-coupon yield curve estimated by Kladivko (2010) and we estimate the Diebold, Li (2006) dynamic version of the Nelson-Siegel specification (3) using ordinary least squares regression on the cross-section of the yields data. As Diebold, Li (2006) we fixed the lambda parameter of the exponential decay, particularly on the value of 0.56 so as the sum of squared residuals of the Nelson-Siegel regression is minimized. After applying this procedure, we arrive at the estimates of the yields-only Nelson-Siegel beta factors for the level, slope and curvature of the yield curve for each period of our data set. The properties of the estimates of these yields-only Nelson-Siegel factors are the following.

Table 3: Descriptive Statistics of the Nelson-Siegel Factors from the Yields-Only Model of the Term Structure of Interest Rates

	Central Moments						Autocorrelations		
Factors	**Mean**	**StDev**	**Min**	**Max**	**Skew**	**Kurt**	**Lag1**	**Lag2**	**Lag3**
beta_1 (level)	0.056	0.010	0.041	0.085	0.704	2.912	0.957	0.902	0.847
empirical level	0.037	0.011	0.022	0.063	0.957	2.897	0.950	0.892	0.836
beta_2 (slope)	-0.026	0.012	-0.057	-0.008	-0.891	3.070	0.930	0.834	0.753
empirical slope	-0.017	0,008	-0.040	-0.002	-0.967	3.185	0.898	0.784	0.666
beta_3 (curvature)	-0.022	0.013	-0.054	0.009	-0.191	2.258	0.817	0.683	0.568
empirical curvature	-0.009	0.005	-0.021	-0.001	-0.289	2.089	0.791	0.652	0.534

Source: Prague Stock Exchange; Czech National Bank; own calculations

As we see from the previous table, both the average empirical slope of the Czech zero-coupon government bond yield curve and the estimated Nelson-Siegel slope factor are positive[28], the Nelson-Siegel slope factor is more volatile that the empirical slope. The level (both empirical and derived from the Nelson-Siegel model) is highly persistent. The slope (both empirical and derived from Nelson-Siegel model) is more persistent than the curvature.

The following figures illustrate the co-movements between Nelson-Siegel factors from the yields-only model, empirical level, slope and curvature, and their macro-economic proxies.

[27] Diebold, Rudebush, Aruoba (2005) used a one-step approach to estimating the Nelson-Siegel-type macrofinance model of the term structure of interest rates using the state-space interpretation of the model. This approach, however, requires estimation of a large number of parameters. We will therefore follow the two-step approach developed by Diebold, Li (2006).

[28] The slope in the Nelson-Siegel representation is the negative of the traditionally defined slope, i.e. as long yield minus short yield. We used the Nelson-Siegel “reverse” slope definition in our work.

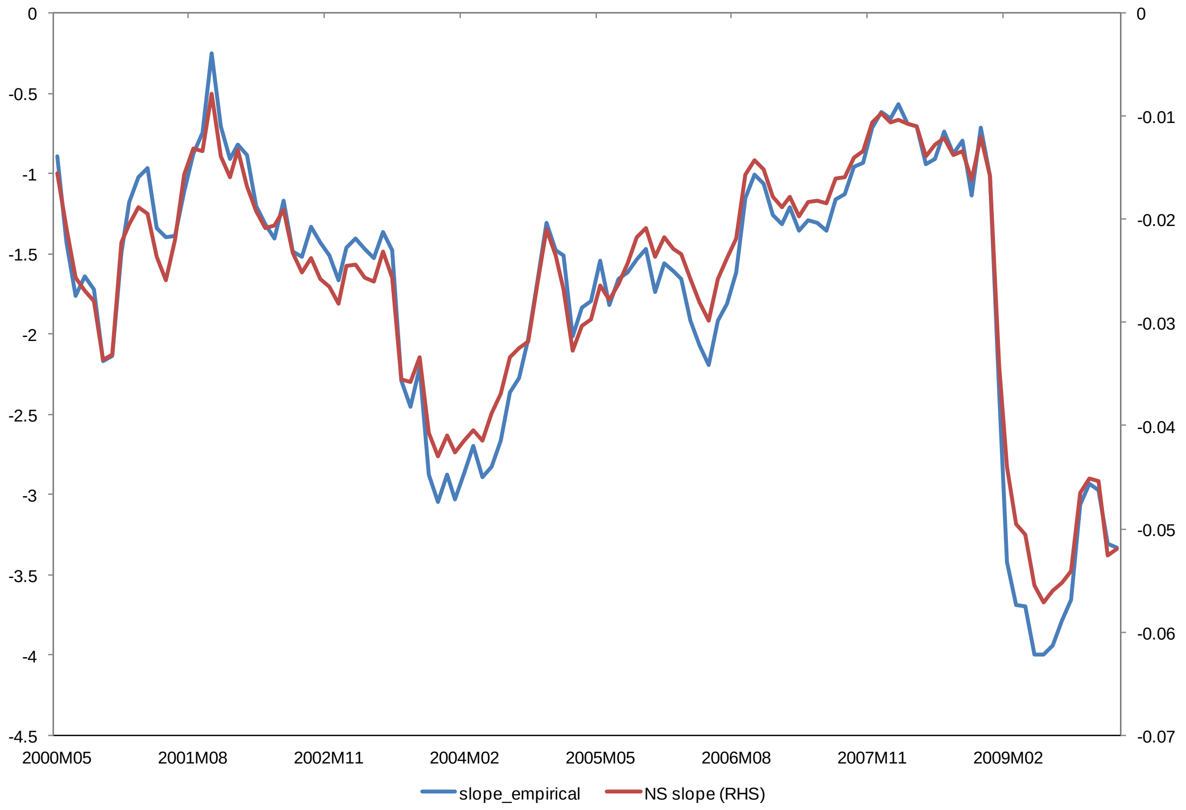
Figure 7: Yields-Only Model – Level Factor and its Empirical and Macro Proxies

Source: Prague Stock Exchange; Czech National Bank; own calculations

As we see from the previous figure, the behavior of the yields-only Nelson-Siegel level factor is closely related to the empirical proxy of the level of interest rates that we defined[29] above in the following way $(r(3M) + r(2Y) + r(10Y))/3$. The behavior of the level factor and its suggested macro-economic approximation of inflation rate are less closely related.

[29] See also Diebold, Rudebusch, Aruoba (2005) for a similar approach.

Figure 8: Yields-Only Model – Slope Factor and its Empirical Proxy

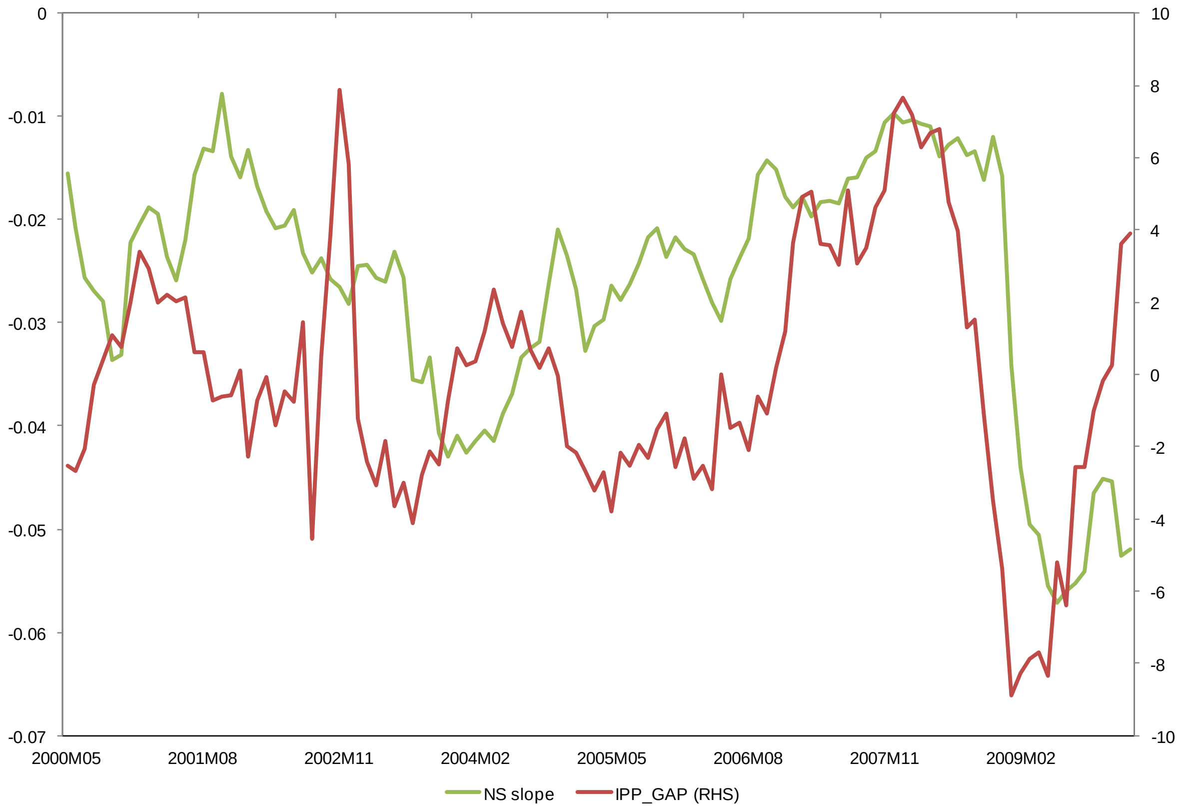


Source: Prague Stock Exchange; Czech National Bank; own calculations

The previous figure shows that the behavior of the yields-only Nelson-Siegel slope factor follows very closely the empirical approximation of the slope factor that we defined[30] above in the following way $\left(r(3M) - r(10Y)\right)$.

[30] See also Diebold, Rudebusch, Aruoba (2005) for a similar approach.

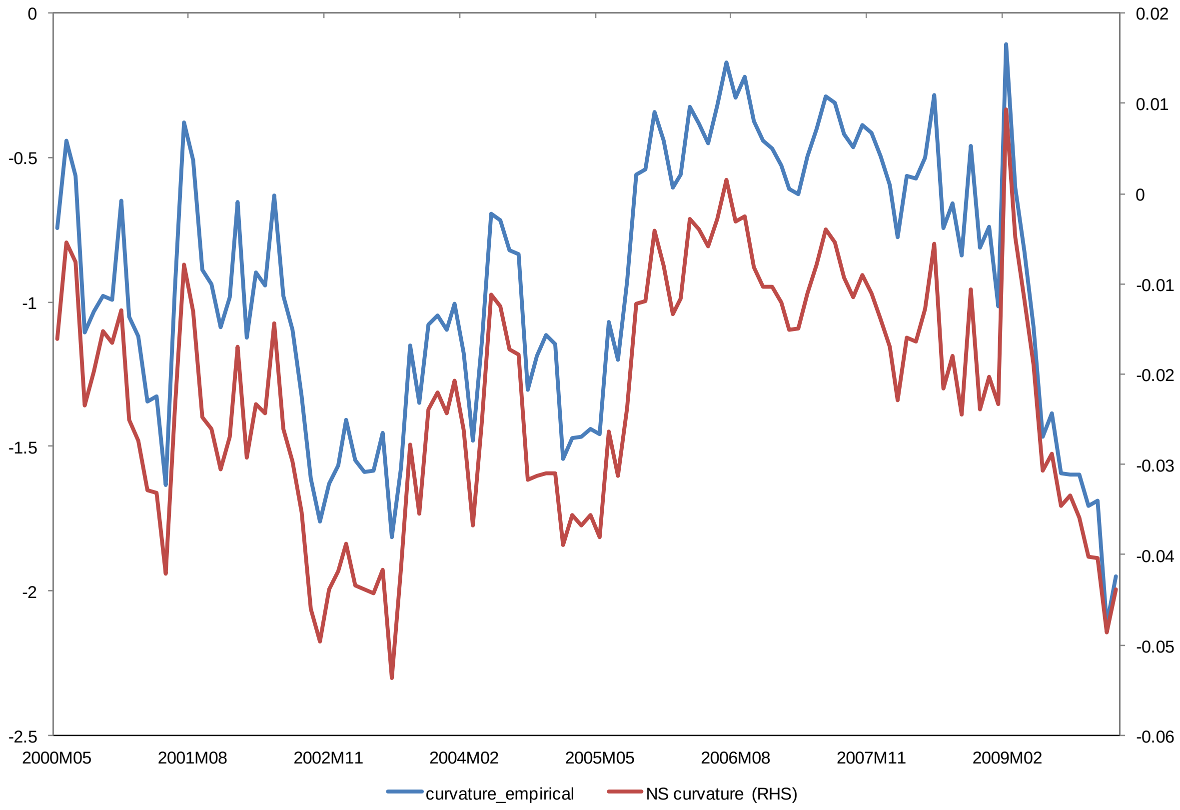
Figure 9: Yields-Only Model – Slope Factor and its Macro Proxy



Source: Prague Stock Exchange; Czech National Bank; Czech Statistical, Office; own calculations

As we see from the previous figure, the behavior of the slope factor is also closely related to the behavior of its macro-economic approximations in terms of industrial production gap as defined above. Nevertheless, the co-movement between these two series is less strong than between the slope factor and its empirical approximation.

Figure 10: Yields-Only Model – Curvature Factor and its Empirical Proxy



Source: Prague Stock Exchange; Czech National Bank; own calculations

The last figure in these series shows that the yields-only Nelson-Siegel curvature factor also closely follows its empirical approximation that we defined[31] above in the following way $\left(2 \times r(2Y) - r(3M) - r(10Y)\right)$. There exists no meaningful macro-economic approximation of the curvature of the yield curve.

After extracting the yields-only Nelson-Siegel factors from the Diebold, Li (2006) dynamic interpretation of the Nelson-Siegel model, we can now proceed to the cornerstone of our analysis and develop a macrofinance model along the lines of the dynamic interpretation of the Nelson-Siegel framework. This time we will assume that the extracted yields-only Nelson-Siegel factors follow a VAR process together with a number of other macro-economic variables, which allows us to incorporate the behavior of the macro-economy into the dynamics of the yield curve.

We will postulate the following non-structural VAR(1) model of the latent factors and macro-economic variables

$$F_t = c + \rho F_{t-1} + \epsilon_t \tag{4}$$

where $F_t = (\beta_{1t}, \beta_{2t}, \beta_{3t}, IPP\_GAP_t, INFL_t, EURCZK\_GAP_t, IPP\_GAP\_EUR_t, INFL\_TARGET_t, INFL\_EUR_t, GER\_10Y_t, EURIBOR\_3M_t)'$ is a (11x1) vector of model's variables, ρ is the matrix of autoregressive coefficients of order (11x11), c is a (11x1) vector

[31] See also Diebold, Rudebusch, Aruoba (2005) for a similar approach.

of intercept terms, and ε_t is a (11x1) vector of the IID error term. The macro-economic variables used in this model are considered to be the minimum set of macro-economic fundamentals required to capture the basic dynamics of the macro-economy.[32, 33] In our estimation procedure, we treat the latent Nelson-Siegel factors from the yields-only model, the industrial production gap, domestic inflation rate, and EURCZK exchange rate gap as endogenous variables. We treat the intercept, the EUR industrial production gap, the domestic inflation target, the EUR inflation rate, the German 10Y government bond yield, and the 3M EURIBOR rate as exogenous variables. The exogenous variables are explaining the development of macro-economic and financial variables abroad. We also regard the domestic inflation target of the Czech National Bank as exogenous. For a small and open economy, one can be assured that the economy is not able to a large extent influence the behavior of macro-economic and financial variables abroad and therefore the abroad environment can be with confidence treated as exogenous.

The following table provides the VAR(1) Nelson-Siegel based macrofinance model parameter estimates. Bold letters indicate those parameters that are significant on 10 percent level of significance.

[32] See Diebold, Rudebusch, Auroba (2006), and Gasha et al. (2010). A smaller set of macroeconomic variables would risk insufficient number of explanatory variables, i.e. the omitted variables problem.

[33] Despite the fact that econometric models of the long yield of the government bond yield curve are usually using in their model specification, among others, fiscal variables, the standard models of the term structure of interest rates as well as the macrofinance models quoted in this paper do not usually use fiscal variables in their model specification. A recent work by Afonso and Martins (2010) tried to integrate fiscal variables into a standard yield curve modeling framework. We did not use fiscal variables in our model specification for the following reasons: (1) there are no reliable fiscal data with monthly frequency; (2) we tried to avoid regime change, particularly with respect to changes in taxation regimes; (3) tests of the validity of the expectations hypothesis of the term structure of interest rates show that the hypothesis holds in the long run, which implies that the central bank's influence on the short end of yield curve has effects on the long end of yield curve in the long run; (4) finally, as Afonso and Martins (2010) pointed out, yield-curve's sensitivity to fiscal variables has been smaller in European economies than in the United States, and the yield-curve's sensitivity to fiscal variables is smaller when the level of government debt is small, which was the case for Czech Republic in the period that we analyzed.

Table 4: VAR(1) Estimation Output – Nelson-Siegel Macrofinance Model

	BETA1	BETA2	BETA3	IPP_GAP	INFL	EURCZK_GAP
BETA1(-1)	**0.702332**	0.083869	**-0.534034**	33.25381	**35.51626**	-27.89325
BETA2(-1)	**-0.148055**	**0.898208**	**-0.405933**	-11.98930	4.669369	-17.25395
BETA3(-1)	**-0.061912**	**0.095832**	**0.579669**	4.589846	**10.64592**	-18.25013
IPP_GAP(-1)	**-0.000328**	**0.000463**	-9.69E-05	**0.525097**	-0.002829	0.049510
INFL(-1)	**0.000728**	-0.000471	**0.001735**	-0.251310	**0.759299**	-0.061757
EURCZK_GAP(-1)	**0.000262**	-0.000175	**0.000696**	0.087540	**0.060948**	**0.859264**
C	-0.002687	-0.003059	**-0.021894**	-2.092292	-0.396184	1.519523
IPP_GAP_EUR	0.000169	-1.21E-05	**-0.000896**	**0.562117**	0.048839	-0.108772
INFL_TARGET	0.000959	0.000564	-0.000936	-0.745851	-0.163710	0.483007
INFL_EUR	-0.001106	0.001256	0.001161	-0.179125	**0.560488**	-0.263273
GER_10Y	**0.001857**	-0.001220	0.003915	0.622309	**-0.349132**	-0.779259
EURIBOR_3M	**0.001078**	-0.000231	**0.003738**	0.382345	0.077909	0.449278
Adj. R-squared	0.933425	0.917258	0.710352	0.829275	0.954243	0.801448

Source: own calculations

Our estimation results were mostly as expected by standard economic relationships and showed a number of interesting findings. The level and slope parameters are highly persistent.

The level factor is explained by its own lag and by the lagged slope factor. It is also explained by the lagged curvature factor, by lagged domestic industrial production gap (we would have however expected opposite sign of the parameters), lagged domestic inflation rate (the level of interest rates goes up when inflation rate goes up), lagged EURCZK exchange rate gap (the level of interest rates goes up when the domestic currency depreciates), and foreign long-term and short term interest rates (the level of domestic interest rates goes up when foreign long and short rates increase).

For the slope factor, the statistically significant explanatory variables were the lag in the slope factor and the lagged curvature factor. They were followed by the lagged domestic industrial production gap (the short end of the yield curve increases when domestic industrial production gap increases, pointing to the stabilization role of the central bank with respect to economic growth). The domestic inflation target was not a statistically significant determinant of the slope of the interest rates.

The curvature factor was statistically determined by its own lag, by the lagged level and slope factors. Statistically significant were also the lagged domestic inflation rate, lagged EURCZK exchange rate gap, foreign industrial production gap, and foreign short term interest rates.[34]

The domestic industrial production gap is explained by its own lag and by foreign industrial production gap, pointing to the openness of the Czech economy and its dependence on the industrial development in the Euro Area. On the other hand, the lagged slope of the interest rates was not a statistically significant determinant of domestic industrial production gap.

The domestic inflation rate was primarily explained by the lagged level factor and the curvature factors, followed by the lag in domestic inflation rate (pointing to high persistence of domestic inflation rate), by the lagged EURCZK exchange rate gap (pointing to a pass-through effect from currency depreciation), by foreign inflation rate (echoing the role of the Euro Area development in determining domestic prices), and by foreign long-term interest rates.

The EURCZK exchange rate proved to be highly persistent.

A Note on Diagnostic Tests

The VAR lag order selection criteria showed that the lag of order one was an appropriate choice for our model. All of the residuals from our macrofinance model were stationary. The results of the VAR Residual Serial Correlation LM test showed that there was no residual serial correlation after the lag of the order one on a five-percent significance level, which illustrates that our VAR model specification with one lag seems to be correct. The results of the residual autocorrelation functions for each of the residuals attributable to the endogenous variables of our VAR macrofinance model showed that the residual autocorrelations for *beta1, beta3, domestic industrial production gap, domestic inflation rate, and EURCZK exchange rate gap* were not statistically significant at any lag. The residual autocorrelations for *beta2* were statistically significant up to the third lag.

The results of the Augmented Dickey-Fuller unit-root tests for stationarity of the variables of our macrofinance model showed that *beta3*, *domestic industrial production gap*, *EURCZK exchange rate gap*, *foreign inflation rate*, and *foreign industrial production gap* were all stationary time series. The *beta1* and *beta2* coefficients as well as the *domestic inflation rate*, and the exogenous variables *EURIBOR 3M rate*, *German 10Y government bond yield* and the *domestic inflation target* were non-stationary and seem to be integrated of order one.

Even though the presence of non-stationary variables risks running into spurious regressions and making the associated coefficients lack their standard limit distributions, Sims et al. (1990) show that consistent estimates of VAR coefficients are obtained with classical methods even when unit roots (i.e. non-stationarity) are present. We can even argue that by differencing or de-trending the non-stationary variables a significant amount of information is lost from the time series. If we are interested in the inter-linkages between variables, the

[34] The level, slope and curvature factors in table 4 are the outcomes from the macrofinance model.

suggested procedure will be running the VAR model together with both stationary and non-stationary data as long as the model specification is theoretically reasonable. We therefore follow this procedure and are confident that most of our key endogenous macro-economic variables are indeed stationary.

Further residual tests showed slight problem with the normality of residuals with respect to the kurtosis. This problem might be given by the use of financial markets data in our estimation. The White Heteroskedasticity test pointed to possible slight heteroskedasticity problems, in particular with respect to residuals form the exchange rate gap equation. Nevertheless, the VAR model withstood the stability condition check.

Impulse Responses and Variance Decomposition

The one standard-deviation impulse responses will provide us with valuable information on how the individual endogenous variables from our macrofinance model respond to shocks in remaining endogenous variables from our macrofinance model. We will study the interactions between the yield-curve variables and the macro-economy. We will also contemplate whether the individual macroeconomic variables respond to shocks in remaining macroeconomic variables in the traditional manner. Following Gasha et al. (2010) we will differentiate between four groups of responses.

Responses of the macroeconomic variables to macroeconomic shocks: Shock to the domestic industrial production gap (i.e. more positive output gap) slightly increases inflation in the first five periods, and has negligible effect on the exchange rate gap. The effect on the inflation rate is consistent with demand pressures stemming from positive output gap. Shock to inflation rate has negligible effect on industrial production gap, and slightly appreciates the domestic exchange rate throughout all periods, which is consistent with FX market expectations of a monetary policy tightening. Shock to exchange rate gap (i.e. more depreciating domestic currency) slightly shifts the industrial production gap into positive territory, which is consistent with a response expected for an export-oriented small economy such as the Czech economy. Exchange rate shock significantly increases domestic inflation, which illustrates strong exchange rate pass-through effect.

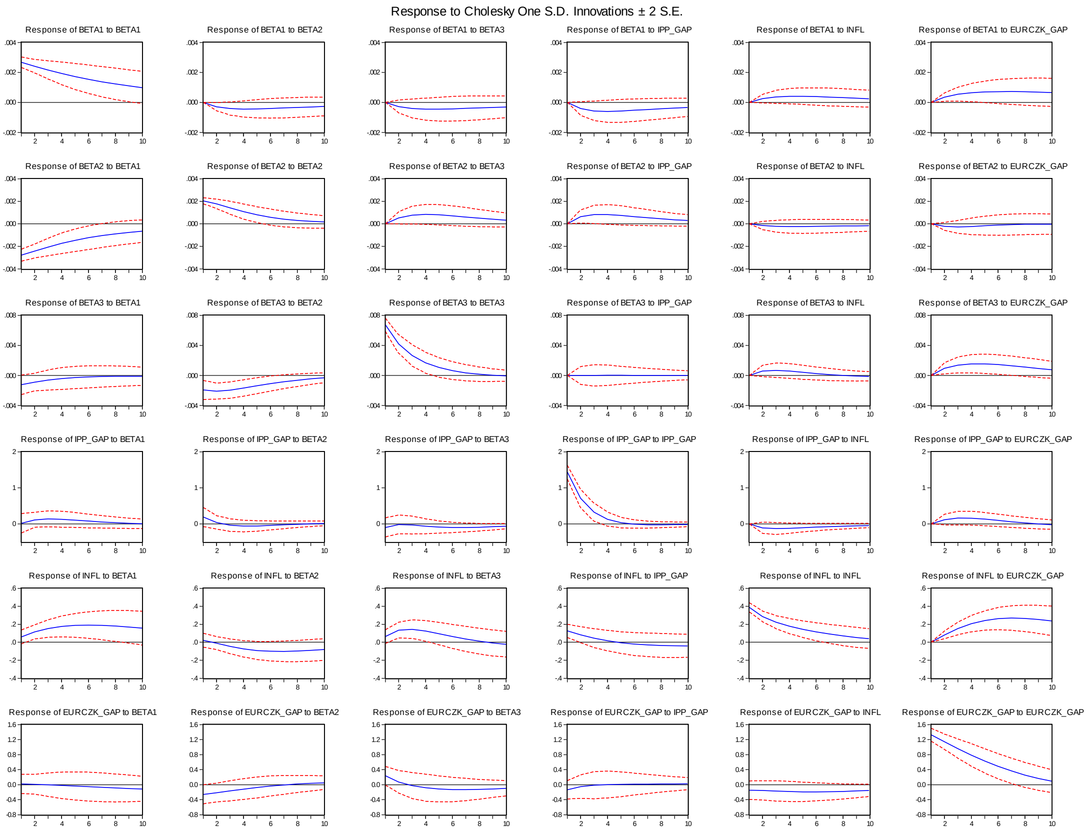
Responses of the macroeconomic variables to yield curve shocks: Shock to the level factor (i.e. increase in the level of interest rates) slightly turns the industrial production gap into positive territory, has negligible effect on the exchange rate gap, and increases the inflation rate. Interestingly, the level factor is usually identified with long-term inflation expectations, and henceforth possibly the positive response of inflation rate to a shock to the level factor. Shock to the slope factor (i.e. in our interpretation of the slope factor, it means a less steeper yield curve) at the beginning turns the industrial production gap into positive territory but eventually has an expected small effect on turning the industrial production gap slightly into negative territory. Shock to the slope factor decreases the inflation rate and leads to appreciating the domestic currency, reflecting tighter monetary policy. Shock to the curvature factor does not have straightforward economic interpretation.

Responses of the yield curve to macroeconomic shocks: Positive shock to the domestic industrial production gap surprisingly slightly decreases the level of interest rates along all ten periods. The shock makes the yield curve less steeper on the whole horizon (i.e. the shorter rates decrease by less than the longer rates), which points to possible stabilization role of monetary policy or to the fact that output gaps have played an important role in inflation-targeting monetary policy regimes. Shock to the inflation rate increases the level of interest rates on the whole horizon, again pointing to a relationship between inflation expectations and the level of interest rates. This reaction might also indicate that long-term inflation expectations are not firmly anchored, where a surprise increase in inflation feeds through to higher long-term inflation expectations, i.e. raising the level factor. Unfortunately, the reaction of the slope factor to inflation is negligible. Shock to the exchange rate gap (i.e. more depreciating domestic currency) increases the level of interest rates on the whole horizon and has negligible effect on the slope factor. The effect of the exchange-rate gap shock on the level factor might illustrate the market participants' understanding of a potential interest rate hike after an episode of currency depreciation in the environment of strong exchange rate pass-through effect to inflation.

Responses of the yield curve to yield curve shocks: Shock to the level factor increases the slope of the yield curve, i.e. the longer end of the yield curve increases to a larger extent after an increase in the level of interest rates. This is consistent with the interpretation of the level factor as long-term inflation expectations, which eventually induces monetary policy tightening. Shock to the slope factor (i.e. decrease in the slope of the yield curve) decreases the level of interest rates. All three yield curve factors are highly persistent.

The following figure shows the impulse responses discussed above.

Figure 11: Impulse Responses from the Macrofinance Model



Source: own calculations

Another interesting perspective to the dynamics between the yield curve and the macro-economy is provided by the variance decomposition output from our VAR macrofinance model.

The first table of the variance decomposition output provides the results for the yield curve factors. At a one-month horizon, the variation in the yield curve is entirely driven by the yield curve itself, unrelated to macroeconomic variables. Only in the later periods the macro-economy adds additional drive to the variation in the yield curve. The variation in the level factor is driven primarily by its own variation, to a much smaller extent also the by variation in the slope and curvature factors. Nevertheless, starting at a five-month horizon, the combined effect of the exchange rate gap and the industrial production gap accounts for 10 to 15 percent of the variation in the level factor, suggesting that the macro-economy contributes as much as 15 percent to the variation in the level of interest rates at horizons longer than five months[35]. Interestingly, the variation in the slope factor is driven by its own variation only to approximately 35 to 25 percent. The variation in the level factor plays a more important role (around 60 percent) in explaining the variation in the slope factor. The macro-economy kicks in at a four-month horizon, and the industrial production gap accounts for 5 to 7 percent of the variation in the slope factor. The variation in the curvature factor is explained by its own variation to as much as 89 percent at short horizons, but in the longer horizons the slope factor plays an important role (up to 17 percent). The exchange rate gap enters the scene at the four-month horizon, accounting for 5 to 12 percent of the variation in the curvature factor within the ten periods-horizon analyzed.

In summary, the variation in the macroeconomy influences the variation in the yield curve significantly only after approximately five months, and at the ten-period horizon, the variation in the macroeconomic variables explains almost 40 percent of the variation in the yield curve factors.

[35] The same logic applies below.

Table 5: Variance Decomposition – Yield Curve Factors

Variance Decomposition of BETA1:

Period	BETA1	BETA2	BETA3	IPP_GAP	INFL	EURCZK_GAP
1	100.0000	0.000000	0.000000	0.000000	0.000000	0.000000
2	96.17831	0.644781	0.565258	1.275302	0.442138	0.894213
3	91.65966	1.343245	1.267111	2.583572	1.000655	2.145760
4	87.84332	1.876654	1.860399	3.528308	1.480847	3.410477
5	84.86376	2.248344	2.305724	4.145620	1.843713	4.592840
6	82.56427	2.501566	2.625788	4.534586	2.100288	5.673499
7	80.77008	2.674580	2.853671	4.775991	2.273055	6.652623
8	79.34580	2.794239	3.017717	4.924554	2.383740	7.533949
9	78.19643	2.878124	3.138804	5.015243	2.450147	8.321248
10	77.25710	2.937539	3.231278	5.069988	2.485937	9.018158

Variance Decomposition of BETA2:

Period	BETA1	BETA2	BETA3	IPP_GAP	INFL	EURCZK_GAP
1	65.07486	34.92514	0.000000	0.000000	0.000000	0.000000
2	62.86656	33.65239	1.286973	1.829643	0.113794	0.250636
3	60.92349	31.72071	2.964828	3.689515	0.256902	0.444548
4	59.65992	29.92485	4.454481	5.043561	0.384522	0.532673
5	58.94736	28.46636	5.603015	5.938474	0.489865	0.554923
6	58.59741	27.35090	6.418665	6.507439	0.576613	0.548970
7	58.46395	26.52480	6.963084	6.862600	0.649727	0.535837
8	58.45044	25.92323	7.307112	7.082250	0.713029	0.523944
9	58.49759	25.48766	7.512991	7.217457	0.768941	0.515363
10	58.57061	25.17115	7.628816	7.300449	0.818796	0.510178

Variance Decomposition of BETA3:

Period	BETA1	BETA2	BETA3	IPP_GAP	INFL	EURCZK_GAP
1	3.084574	7.528247	89.38718	0.000000	0.000000	0.000000
2	3.202990	11.13543	84.07904	0.000350	0.437091	1.145102
3	3.174594	13.78921	79.07070	0.000335	0.872790	3.092369
4	3.093995	15.51434	75.01836	0.000309	1.125443	5.247553
5	3.011119	16.54306	71.97728	0.000299	1.220865	7.247376
6	2.944768	17.10945	69.79587	0.000288	1.230290	8.919339
7	2.898358	17.39172	68.28536	0.000294	1.210187	10.21408
8	2.869643	17.51086	67.27465	0.000317	1.192147	11.15238
9	2.855291	17.54349	66.62282	0.000332	1.188701	11.78937
10	2.852483	17.53539	66.21879	0.000331	1.201227	12.19178

Source: own calculations

The following table of the variance decomposition output provides the results for the macroeconomic variables. The variation in the industrial production gap is mainly determined by its own variation, and above the six-month horizon also partially by the variation in inflation and exchange rate gap (combined effect of around 5 percent) as well as the yield curve factors' variation (combined effect of around 5 percent).

The variance of inflation rate is in the first two periods determined by its own variation (86 to 77 percent) but this effect quickly fades away and the variation in exchange rate gap (26 percent at the six-month horizon) and the level of the interest rate (17 percent at six-month horizon) begin to play significant roles. The influence of exchange rate on inflation is expected for a small and open economy. Also, the strong link between inflation and the level factor (i.e. the proxy for inflation expectations) is expected. The effect of the industrial production gap on inflation's variance also fades away from 9 percent in the first period to less than 3 percent at the seven-month horizon. The combined contribution of the yield curve factors to the variance of inflation rate at ten-month horizon amounts to approximately 30 percent.

The variance of the exchange rate gap is to a large extent determined by its own variation and to a small effect by the variation of inflation rate (4 percent at ten-month horizon) and the slope and curvature factors (2 to 3 percent). The combined effect of the yield curve factors on the variance of exchange rate is at the ten-month horizon approximately 6 percent.

In summary, we can conclude that, especially for inflation rate, there is a statistically significant interaction between the yield curve factors and the macro-economy, and that the macro-economy enters this interaction only after some time, i.e. not immediately.[36]

[36] We should be also mindful of the fact that the variation of yield curve factors is significantly explained by their own autocorrelations. We can, however, meaningfully assume that also these autocorrelations of yield curve factors include some information about the macro-economy.

Table 6: Variance Decomposition – Macroeconomic Variables

Variance Decomposition of IPP_GAP:						
Period	BETA1	BETA2	BETA3	IPP_GAP	INFL	EURCZK_GAP
1	0.010616	1.685220	0.463589	97.84057	0.000000	0.000000
2	0.476626	1.391647	0.378490	96.77831	0.463567	0.511358
3	1.100097	1.357173	0.393284	94.72476	1.042928	1.381759
4	1.620380	1.452991	0.533606	92.67506	1.522918	2.195049
5	1.956378	1.549039	0.804156	91.07901	1.872851	2.738561
6	2.135114	1.599476	1.140518	89.99224	2.121512	3.011144
7	2.211820	1.611923	1.469511	89.30427	2.299621	3.102860
8	2.234059	1.607587	1.744711	88.87622	2.427885	3.109540
9	2.233557	1.602502	1.949569	88.59592	2.519217	3.099238
10	2.228181	1.603914	2.087693	88.38979	2.582333	3.108092
Variance Decomposition of INFL:						
Period	BETA1	BETA2	BETA3	IPP_GAP	INFL	EURCZK_GAP
1	1.991394	0.267732	2.301584	9.058361	86.38093	0.000000
2	5.513484	0.196782	7.322385	7.547012	77.24799	2.172349
3	9.412245	0.652740	10.11514	5.890891	66.91840	7.010585
4	12.84048	1.508685	10.62588	4.586378	57.31738	13.12120
5	15.50122	2.463342	9.901441	3.701785	49.24990	19.18231
6	17.43721	3.309270	8.788686	3.147722	42.83474	24.48237
7	18.80607	3.966915	7.725268	2.816890	37.87936	28.80549
8	19.76589	4.435031	6.872454	2.626796	34.10733	32.19251
9	20.44029	4.744881	6.254930	2.522551	31.25812	34.77922
10	20.91725	4.934916	5.843199	2.469671	29.11705	36.71791
Variance Decomposition of EURCZK_GAP:						
Period	BETA1	BETA2	BETA3	IPP_GAP	INFL	EURCZK_GAP
1	0.017198	3.499733	2.867031	0.979076	1.144046	91.49292
2	0.012421	3.434781	1.821388	0.661659	1.361928	92.70782
3	0.009781	3.292294	1.422503	0.516990	1.710241	93.04819
4	0.015046	3.127266	1.368242	0.446705	2.152848	92.88989
5	0.037475	2.971524	1.495758	0.410707	2.651719	92.43282
6	0.087458	2.843503	1.708045	0.392152	3.169439	91.79940
7	0.173723	2.752609	1.943606	0.383498	3.672834	91.07373
8	0.301034	2.701341	2.163891	0.381120	4.135580	90.31703
9	0.469007	2.686866	2.347324	0.383156	4.539654	89.57399
10	0.672236	2.702699	2.485284	0.388562	4.875614	88.87560

Source: own calculations

IV. Making the Macrofinance Model Operational for Asset-Allocation Purposes

We can now proceed with an application of our macrofinance framework to asset-allocation purposes, starting with a few important definitions.

Let us call the *beta* factors extracted from the VAR macrofinance model the macro-implied *betas*. After inserting these macro-implied beta factors back into the original Nelson-Siegel equation (2) we arrive at a model zero-coupon yield curve that will be called the macro-implied zero-coupon yield curve. We claim here that the macro-implied zero-coupon yield curve is indeed the fair-value zero-coupon yield curve given by macroeconomic fundamentals. We will use the macro-implied zero-coupon yield curve and the actual zero-coupon yield curve to evaluate as to what extent the actual zero-coupon yield curve is over-valued or under-valued with respect to the macro-implied zero-coupon yield curve, i.e. with respect to macroeconomic fundamentals. The difference between the fair-value yield curve and the actual yield curve will be called the misalignment.[37] In further steps, we will evaluate to what extent the yield-curve misalignments predict the subsequent yield movements.[38] In other words, we will measure the predictive power of the misalignments in explaining the changes in bond yields. This exercise will help us convey the message that is present throughout this paper – that movements in financial markets in general (and in the yield curve in particular) are a reflection of the macro-economy; or the other way round, that macro-economy is one of the key drivers of financial-market fluctuations.

The following figure compares the selected actual Czech zero-coupon yields[39] with their macro-implied counterparts. At the first glance, we can observe that the actual zero-coupon yields track the macro-implied yields rather well.[40] Nevertheless, we can also observe that misalignments exist and they persist for some time. The persistence of bond-overvaluations is particularly large for longer-term bond yields in periods between 2001 and 2004, and between 2005 and 2008. This might have coincided with the periods of rapid capital inflows into Czech Republic (invested substantially in Czech government bond securities) that might have driven the bond yields far too low for extended periods of time.

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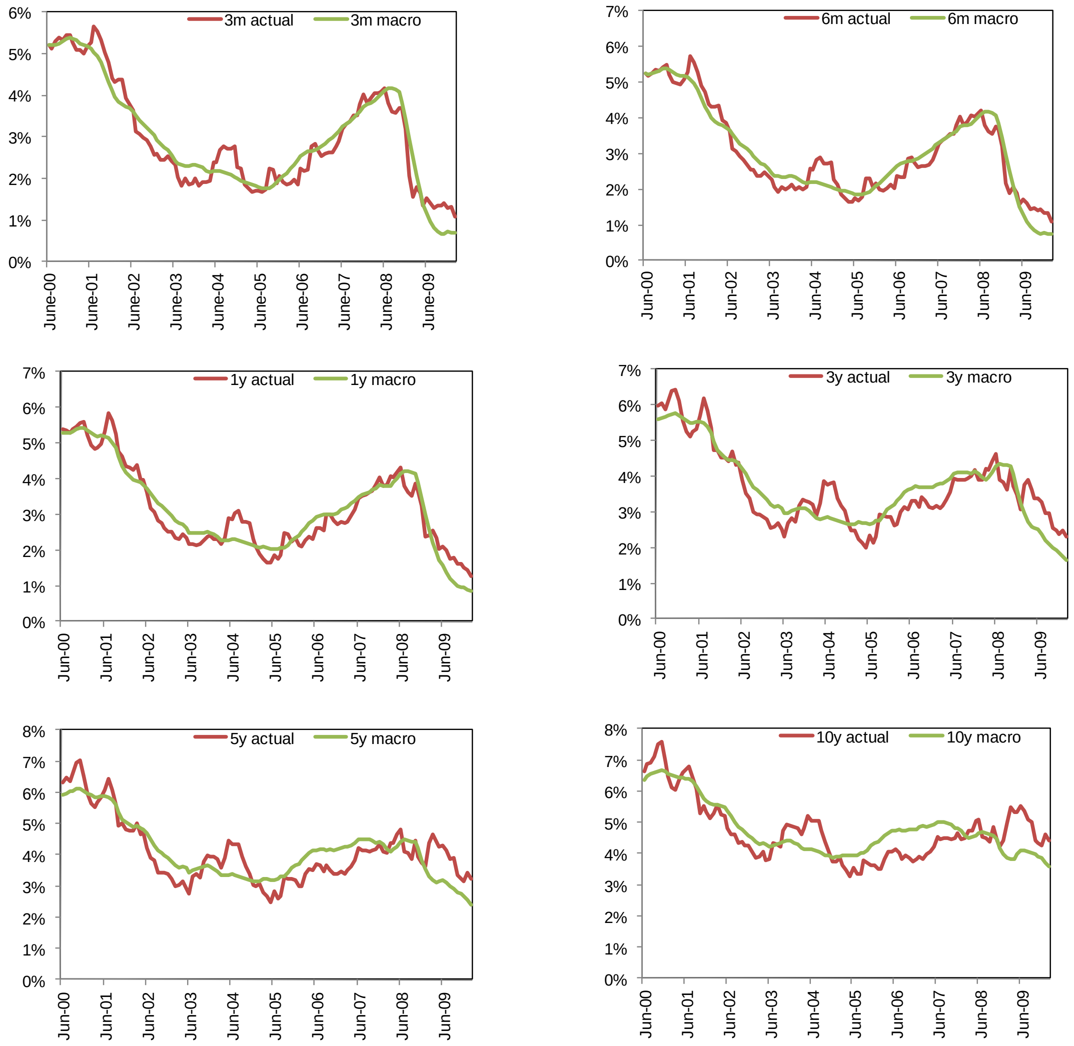
[37] In statistical terms, our term “misalignment” equals the forecasting error of the model yield versus the observed yield.

[38] Similar approach for U.S., German, Japanese and U.K. yield curves was used by Verstyuk (2007) and Verstyuk (2008), though with a slightly different formulation of the macroeconomic part of the macrofinance model.

[39] As noted above, the Czech zero-coupon data set is provided by Kladivko (2010).

[40] We also tried the fit of the yields-only model of the yield curve and it was tracking the actual yield curve very smoothly. For more see Kladivko (2010).

Figure 12: Czech Zero-Coupon Bond Yields and Their Fair Values at Different Maturities



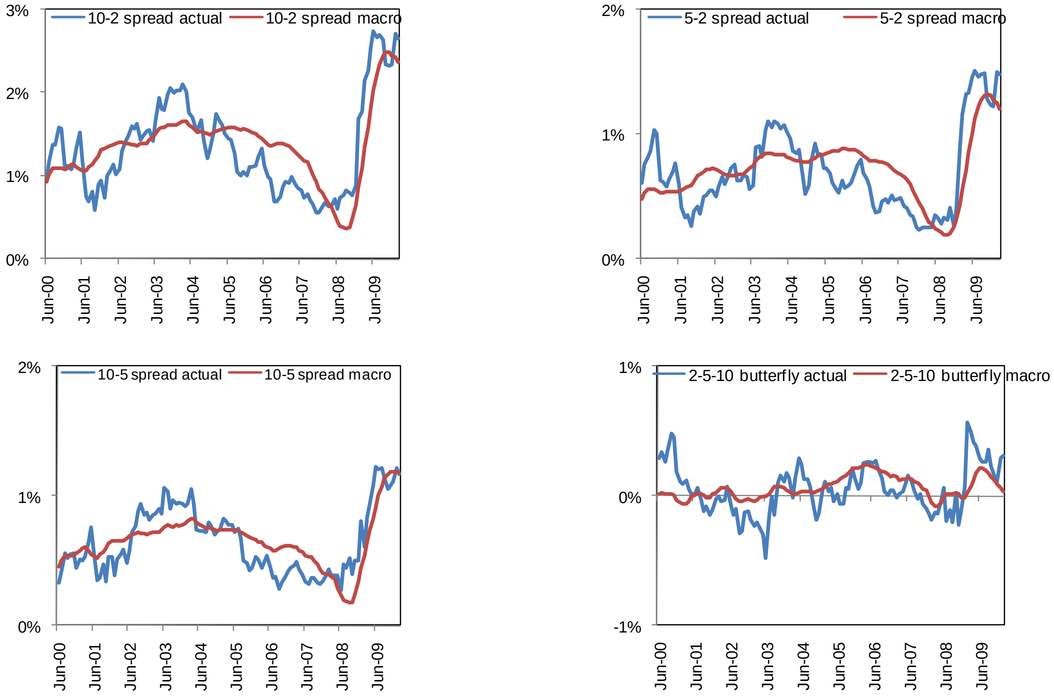
Source: Prague Stock Exchange; Czech National Bank; own calculations

The following figure presents the comparison of selected actual yield-curve spreads and butterfly spreads[41] and their respective macro-implied counterparts. Here, the misalignments persisted for longer periods of time, too. The time periods are the same as with bond-yield misalignments and they coincided with periods of rapid capital inflows into Czech Republic that might have driven the bond-yield spreads far too low for extended periods of time.

[41] The "2-5-10 butterfly" spread is equal to:

$2 \times (5year\ bond\ yield) - 2year\ bond\ yield - 10year\ bond\ yield.$

Figure 13: Czech Yield-Curve Spreads, Butterflies and Their Fair Values



Source: Prague Stock Exchange; Czech National Bank; own calculations

Identifying Misalignments and Timing Subsequent Yield Movements

In what follows, we try to investigate to what extent the current yield curve misalignments from its fair value help us predict the subsequent movements of the yield curve.[42] We will claim that the macro-implied yield curve is indeed the fair value yield curve given by macroeconomic fundamentals towards which the actual yield curve naturally needs to tend. The current yield curve misalignment from its fair value can therefore be regarded as the fundamentally implied (or required) subsequent yield change in order for the actual yields to converge to their fair values. It measures the predictive power of the misalignments in explaining subsequent bond yield changes.[43]

Let us therefore, in the following tables and figures, show how our macrofinance model can be used for practical purposes. We will judge the ability of the macro-implied yield curve (or the misalignments from the macro-implied fair value yield curve, respectively) to "predict" actual Czech zero-coupon yield changes during the period from 2000 to 2010.[44]

[42] See also Verstyuk (2007) and Verstyuk (2008) for a similar approach.

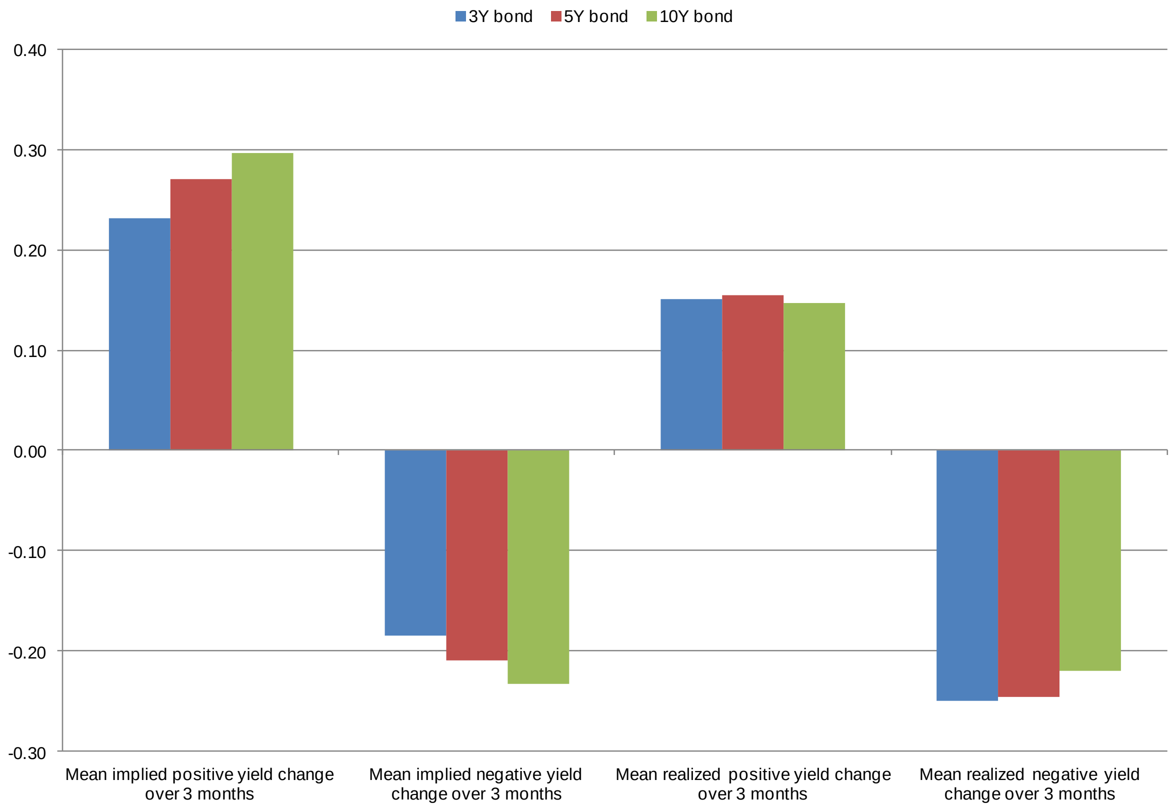
[43] In a different context and in a more formalized way, a similar exercise (in its *logic*) is being conducted by the tests of the validity of the pure expectations hypothesis of interest rates. In these tests, the implied interest rate change given by the pure expectations hypothesis (i.e. implied by the forward rate) is contrasted to the realized interest rate change. On empirical evidence on the pure expectations hypothesis see Campbell, Lo, MacKinlay (1997), Campbell, Shiller (1991), Cook, Hahn (1990), Ilmanen, Iwanowski (1996) and Ilmanen (1995).

[44] At this stage, this is not a statistical exercise and not an out-of-sample forecasting test. Such out-of-sample forecasting test will be conducted bellow.

We constructed the misalignments for three benchmark bonds, three-year, five-year, and ten-year bond. In order to capture the forecasting ability of the misalignments, we lagged the subsequent changes in yields of these three bonds by three, six and nine months, and we compared them with the current misalignments for these bonds. This means that we were postulating that current misalignments in these three bonds' yields are capable of predicting the change of their yields over three, six or nine months. Interestingly, the one-month lag did not produce favorable results implying that the misalignments are not corrected within very short periods of time. This is however consistent with our macrofinance VAR results produced above that showed that macroeconomic variables started to play role in the variation of the yield curve after no less than five months.

The following three figures provide the first glance at our results.[45] We can see that the current misalignments produce rather good "forecasts" of yield changes over six to nine months. Nevertheless, over the nine-month period the positive and negative yield forecasts are rather asymmetric.

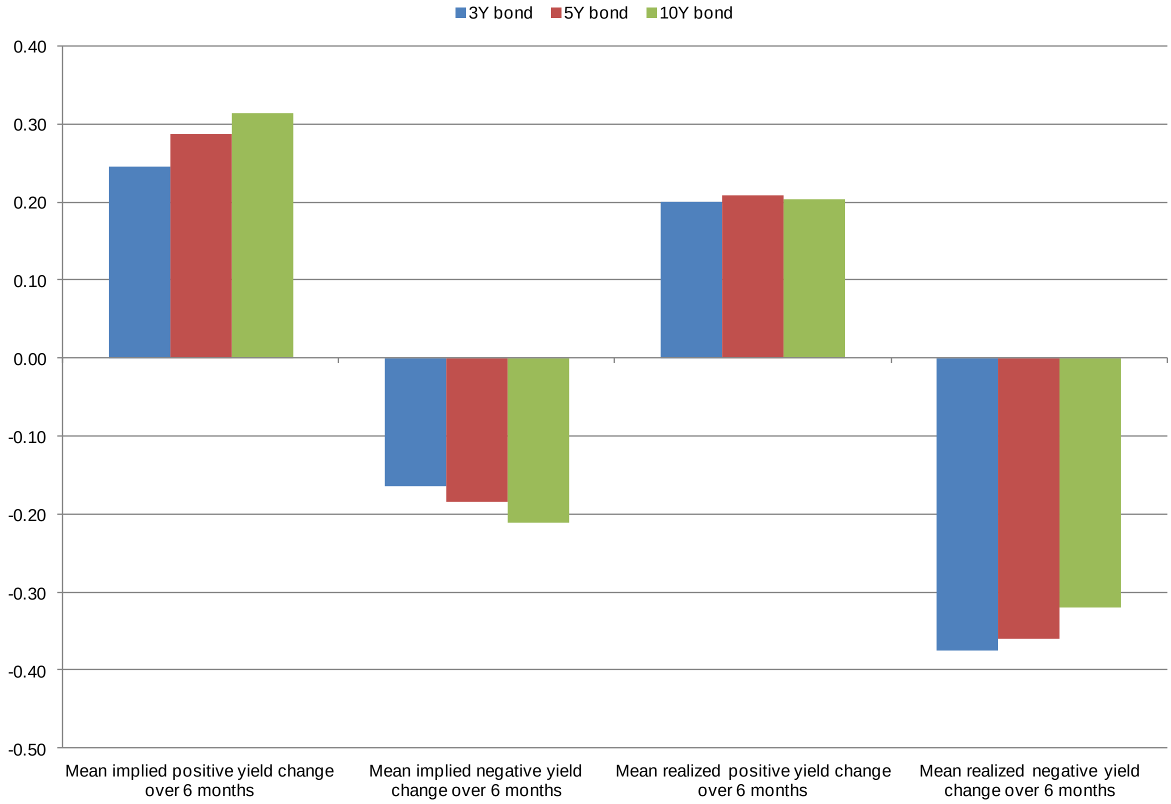
Figure 14: Implied vs. Realized Yield Change over 3 Months



Source: own calculations

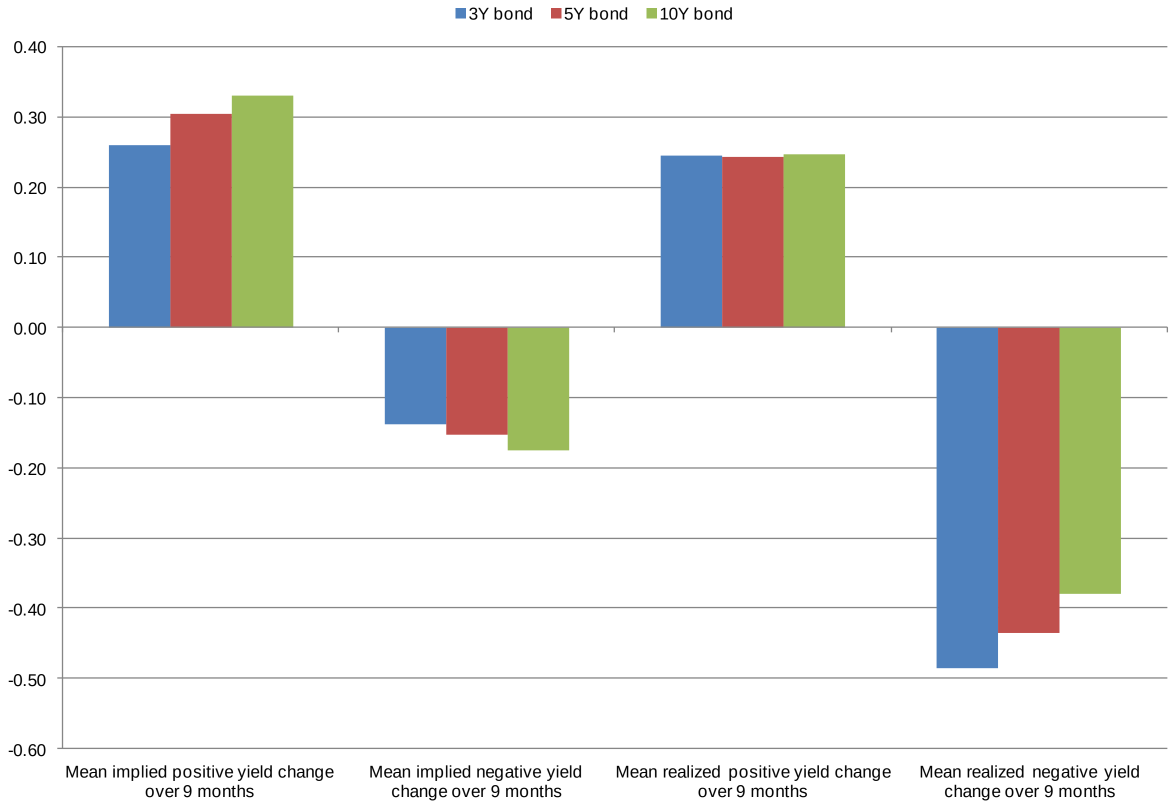
[45] The graph is constructed as follows. The mean implied changes represent the average of positive and negative current misalignments in 3Y, 5Y and 10Y bonds. The mean realized changes represent the average of positive and negative subsequent yield changes over three, six and nine months (lagged three, six and nine months to make them "current" from the perspective of the misalignments) in 3Y, 5Y and 10Y bonds. In fact, the mean implied changes (i.e. the misalignments) are by definition always the same on figures 14, 15 and 16; they only start on different dates due to different lags of yield changes, which cause slight differences in the mean implied changes in these three figures. The correlation exercise in table 7 then in turn determines that our preferred choice of horizon for the subsequent changes that best matches current misalignments is six months.

Figure 15: Implied vs. Realized Yield Change over 6 Months



Source: own calculations

Figure 16: Implied vs. Realized Yield Change over 9 Months



Source: own calculations

The following table summarizes the previous three figures in a more formal way. Even though the overall correlations between the macro-implied yield changes (i.e. the misalignments) and the actual (lagged) yield changes are best over the period of nine months, the results for the nine-month forecasts are a little asymmetric and tilted towards positive yield changes, as we saw on the figures above. Therefore, our preferred choice of forecast horizon of the misalignments would be six months, which is also consistent with the variance decomposition results from our macrofinance VAR model that suggested that macro-economy starts influencing the development of the yield curve only after approximately six months. The highest correlation coefficients were generally obtained for the three-year bond and the five-year bond in contrast to the ten-year bond.

Table 7: Evaluating the Czech Macro-Implied Yield Curve's Ability to Predict Actual Bond Yield Changes, 2000-2010, (Absolute Changes in Yields in Percentage Points).

	3Y	5Y	10Y
Mean implied positive yield change over 3 months	0.232	0.271	0.296
Mean implied negative yield change over 3 months	-0.185	-0.209	-0.232
Mean realized positive yield change over 3 months	0.150	0.155	0.147
Mean realized negative yield change over 3 months	-0.250	-0.246	-0.220
Correlation between implied and realized yield changes	0.495	0.461	0.369
Correlation between positive implied and and realized yield changes	0.222	0.171	0.049
Correlation between negative implied and realized yield changes	0.479	0.449	0.380
Mean implied positive yield change over 6 months	0.245	0.286	0.313
Mean implied negative yield change over 6 months	-0.165	-0.185	-0.212
Mean realized positive yield change over 6 months	0.199	0.209	0.203
Mean realized negative yield change over 6 months	-0.375	-0.360	-0.321
Correlation between implied and realized yield changes	0.627	0.612	0.508
Correlation between positive implied and and realized yield changes	0.450	0.467	0.204
Correlation between negative implied and realized yield changes	0.613	0.575	0.503
Mean implied positive yield change over 9 months	0.259	0.303	0.331
Mean implied negative yield change over 9 months	-0.139	-0.153	-0.175
Mean realized positive yield change over 9 months	0.246	0.244	0.247
Mean realized negative yield change over 9 months	-0.485	-0.436	-0.380
Correlation between implied and realized yield changes	0.651	0.651	0.545
Correlation between positive implied and and realized yield changes	0.492	0.565	0.313
Correlation between negative implied and realized yield changes	0.591	0.558	0.504

Source: own calculations

As table 8 shows, for the slope of the yield curve, the correlation between the misalignment and the subsequent change in the slope was very small. For the butterflies, the correlation between the macro-implied and the subsequent realized butterfly changes is presented in the table below.

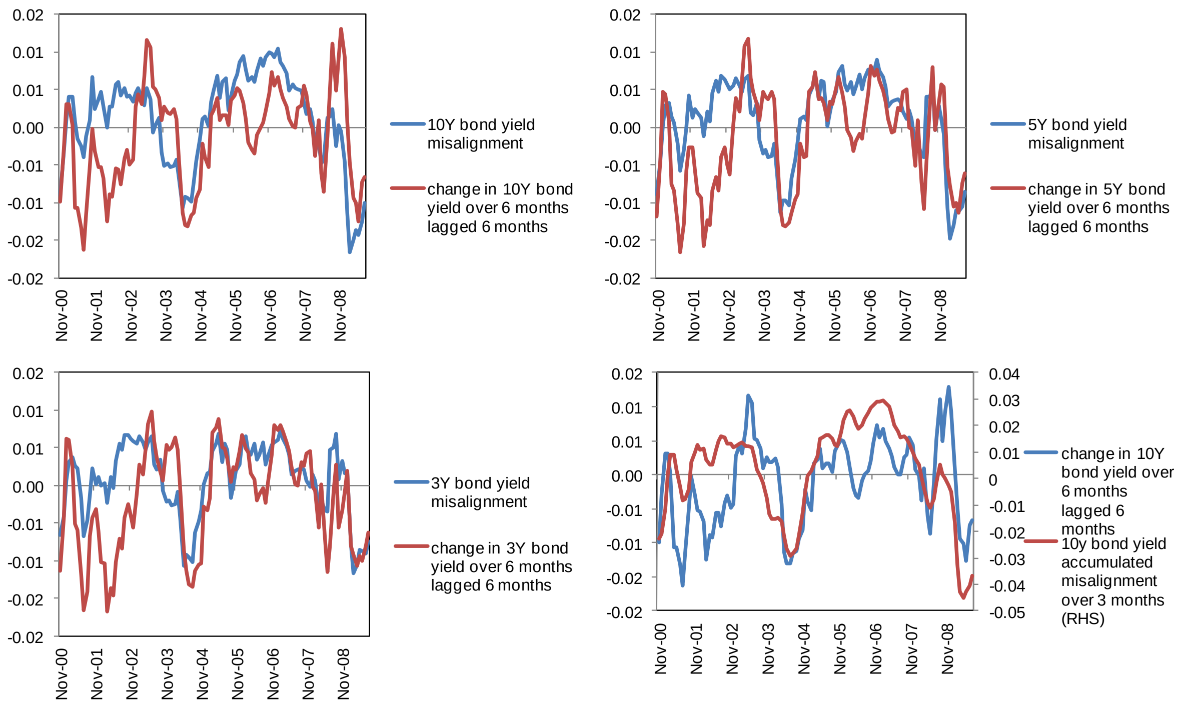
Table 8: Correlation between Macro-Implied and Subsequent Realized Butterfly Changes

	2-5-10 butterfly
Correlation between implied and realized butterfly changes over 3 months	0.536
Correlation between implied and realized butterfly changes over 6 months	0.636
Correlation between implied and realized butterfly changes over 9 months	0.574

Source: own calculations

For illustration purposes, let us present in the following figure the graphs for current bond yield misalignments and the subsequent bond yield changes over six months lagged six months. We can see that the lagged changes in the bond yield closely track the current bond yield misalignment produced by our model. An interesting picture is also provided by the last figure in the bottom right corner, which plots the accumulated misalignment for ten-year bond yield over three months together with the change in ten-year bond yield over six months lagged six months. We see that as the misalignment accumulates over certain period of time, it contributes strongly to the subsequent change in bond yields.

Figure 17: Bond Yield Misalignments and Changes in Bond Yields over Six Months Lagged Six Months



Source: own calculations

As our next illustration of the practical use of our framework, we chose a random day in our time series and produced figures for the actual Czech zero-coupon, the macro-implied (fair value) yield curve at the same day, and the subsequent actual Czech zero-coupon yield curve after nine months[46]. We did the same for yield spreads and butterflies. The following table summarizes the misalignment for the randomly chosen day for yields, spreads and butterflies and the change in benchmark yields, spreads and butterflies over the randomly chosen nine-month period.

Table 9: Example of Yield Curve Misalignment and Subsequent Yield Curve Change

	3y	5y	10y
Misalignment on May 31 2009	-0.009	-0.011	-0.014
Yield change from May 31 2009 to February 28 2010	-0.011	-0.010	-0.009

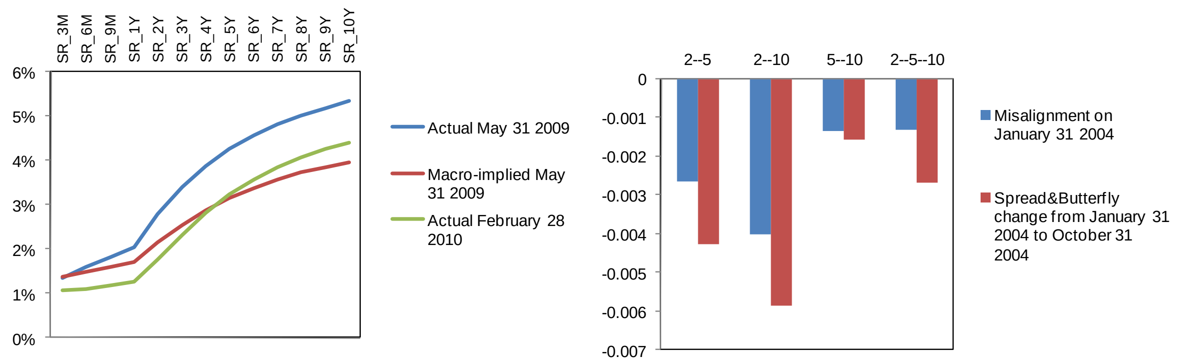
Spreads & Butterflies: Misalignmet and Subsequent Movement

	2–5	2–10	5–10	2–5–10
Misalignment on January 31 2004	-0.00267	-0.00402	-0.00135	-0.00132
Spread&Butterfly change from January 31 2004 to October 31 2004	-0.00428	-0.00586	-0.00158	-0.0027

Source: own calculations

The following figure illustrates the same exercise in a graphical form. We see how nicely the yield curve changes even after nine months towards the predicted misalignment nine months ago. On May 31, 2010, the actual Czech zero-coupon yield curve was undervalued with respect to the macro-implied fair value zero-coupon yield curve. After nine months time, this misalignment (undervaluation) was almost entirely corrected and the actual Czech zero-coupon yield curve approached the macro-implied fair value zero-coupon yield curve from nine months ago. The same is more or less true for the yield spreads and butterfly for the chosen period.

Figure 18: An Example of Yield Curve Misalignment and its Subsequent Correction



Source: Prague Stock Exchange; Czech National Bank; own calculations

Does Macro Make Sense, and If Yes, Why and to What Extent?

In the following paragraphs, we are going to undertake two simple exercises to show the forecasting relevance of the macrofinance model of the term structure of interest rates versus

[46] To show also other than six-months lag and to show that these relationships hold for the lags between three to nine months that we investigated above.

the yields-only model of the term structure of interest rates, and to dig deeper into underlying drivers of yield changes.

In the first exercise, we will compare the out-of-sample forecasting ability of our VAR macrofinance model versus a yields-only Nelson-Siegel model, in which the latent factors follow a VAR process, too. Following Diebold, Li (2006) as well as Kladivko (2010), the comparative criterion will be the root mean squared errors (RMSE) defined as follows

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2} \tag{5}$$

where n is the number of bonds used in the estimation, y_i is the observed zero-coupon bond yield[47], and $\hat{y}_i$ is the estimated zero-coupon bond yield. The larger the RMSE, the greater the forecast error; hence results with the smallest RMSE indicate the most superior model.

We first estimated our macrofinance VAR(1) model of the latent factors (equation 4) until December 2006, produced forecast from January 2007 until September 2007, and inserted the estimated latent factors back to the Nelson-Siegel equation (2) to obtain the macro-implied forecast of the zero-coupon yield curve for the period from January 2007 until September 2007. We deliberately did not produce the forecast during the period of the 2008 global financial crisis to avoid extreme market conditions. We chose the nine-month forecast horizon since this was the maximum horizon used above for analyzing the forecasting ability of current yield misalignments for subsequent yield changes. We then estimated the VAR(2) model of the original yields-only latent factors until December 2006, produced a forecast from January 2007 until September 2007, and inserted the estimated latent factors back to the Nelson-Siegel equation (2) to obtain the forecasted yields-only zero-coupon yield curve for the period from January 2007 until September 2007.[48]

In the following table, we compared the RMSE for both of the models over the nine-month forecasting period. As we see, the results are supportive for the macrofinance VAR model, as its RMSE is lower than that of the yields-only model. This suggests that the inclusion of macroeconomic variables into the yield-curve modeling framework indeed improved the forecasting ability of the model.

[47] In our case the Kladivko (2010) zero-coupon yields.

[48] For the yields-only model, we chose the VAR(2) model specification as it performed better with the lag-length selection criteria. If we were to chose VAR(1) also for the yields-only model, the dominance of the macrofinance model as shown below would be even stronger. However, we wanted the yields-only model to be properly specified and therefore we chose the VAR(2) specification.

Table 10: The Out-of-Sample Forecasting Performance of the Macrofinance Model versus Yields-Only Model

Macro VAR(1)	
	RMSE
1 year	1.05562
2 years	1.01955
3 years	0.96725
4 years	0.92036
5 years	0.88574
6 years	0.86381
7 years	0.8525
8 years	0.8492
9 years	0.85152
10 years	0.85751
AVERAGE	**0.91231**

Yields-Only VAR(2)	
	RMSE
1 year	1.60775
2 years	1.50998
3 years	1.38004
4 years	1.25551
5 years	1.14998
6 years	1.06607
7 years	1.00177
8 years	0.95359
9 years	0.91801
10 years	0.89194
AVERAGE	**1.17346**

Source: own calculations

The following exercise provides a different perspective. We tried to investigate what role the yield misalignments play along other variables in determining the behavior of the yield changes. We ran a multiple ordinary least squares regression where the dependent variable was a change in three-year Czech government bond yield over three months[49], and the independent explanatory variables were the three-year bond yield misalignment from its macro-implied fair value lagged three months, total traded volume on the secondary market with Czech government bonds as collected by the Prague Stock Exchange, the domestic inflation rate, the domestic industrial production gap lagged one month, the EURCZK exchange rate gap lagged three months, the EURIBOR 3M interest rate lagged three months, the ten-year German government bond yield, and the 2003 year dummy variable multiplied by the traded volume variable.[50] Other year-dummy variables multiplied by the traded volume were not statistically significant. Also, we found out that the other contemplated variables, such as the foreign industrial production gap, the foreign inflation rate, the Czech stock market index or the stock market volatility index, were not statistically significant. We incorporated the traded volume into the regression explanatory variables in an attempt to explain the rather long persistence of yield misalignments. We ran the regression from August 2000 until December 2007 so as to avoid the potentially disturbing period of the post-2008 global financial crisis. The regression's results are produced in the following table.

[49] Again, as pointed out in footnote above, we are now using the three-month lag to show also other than six-month and nine-month lags and to show that these relationships are relevant for the lags between three to nine months that we investigated above.

[50] The chosen lags in the particular explanatory variables provided best statistical results and were intuitively most suitable. We also conducted the basic diagnostic tests for the regression specification. The model's specification seemed appropriate, only a slight serial correlation might be a problem of the specified regression model.

Table 11: Multiple Regression Results

Dependent Variable: YIELD_CHANGE
Method: Least Squares

Sample (adjusted): 2000M11 2007M12
Included observations: 86 after adjustments

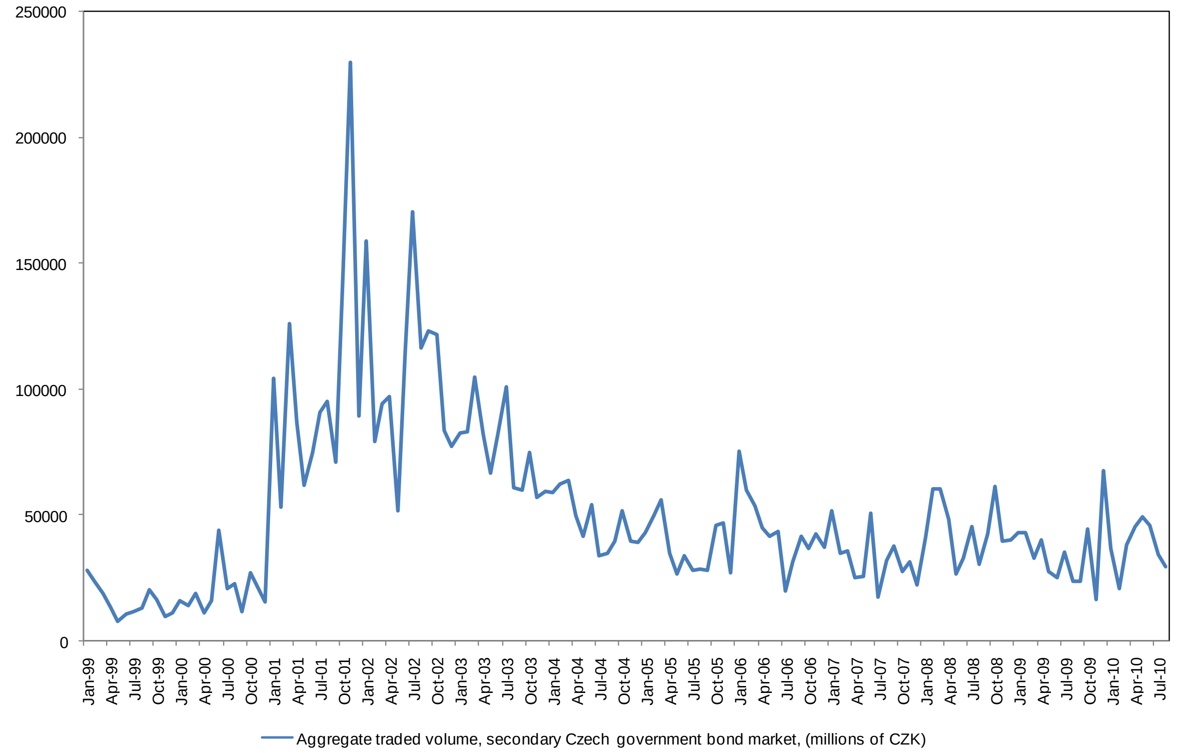
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.011095	0.002564	-4.327860	0.0000
MIS_3Y_BOND(-3)	0.941697	0.077110	12.21232	0.0000
VOLUME	-4.74E-08	9.28E-09	-5.106903	0.0000
INFL	0.001883	0.000365	5.157560	0.0000
IPP_GAP(-1)	0.000237	0.000139	1.696624	0.0938
EURCZK_GAP(-3)	0.000449	0.000157	2.860761	0.0054
EURIBOR_3M(-3)	-0.003890	0.000537	-7.244333	0.0000
GER_10Y	0.004588	0.000779	5.888624	0.0000
DUMMY2003*VOLUME	3.10E-08	1.48E-08	2.086248	0.0403

R-squared	0.792256	Mean dependent var	-0.000726
Adjusted R-squared	0.770673	S.D. dependent var	0.005120
S.E. of regression	0.002452	Akaike info criterion	-9.085002
Sum squared resid	0.000463	Schwarz criterion	-8.828151
Log likelihood	399.6551	F-statistic	36.70616
Durbin-Watson stat	1.194986	Prob(F-statistic)	0.000000

Source: own calculations

As we see from the table above, the regression has relatively high explanatory power (adjusted R-Squared statistic equal to 0.77) and that all the included variables are statistically significant on the ten-percent significance level. The coefficient of the effect of the misalignment on the yield change proved to be the largest of all in absolute terms, and has the expected positive sign. The traded volume variable is also statistically significant, though with rather small absolute value of its coefficient. The negative sign of the volume coefficient is consistent with the expected inverse impact of greater demand on the bond yield. The year-dummy variable multiplied by the traded volume variable for 2003 was statistically significant, implying that the dynamics in the traded volume was in particular significant for the determination of yield changes during 2003, which is consistent with the 2002-2003 period of rather substantial traded volume on the secondary government bond market in the Czech Republic, as shown on the figure below. This period that was marked by significant capital inflows to Czech Republic that to a large extent ended in government bond investments, and might actually have contributed to the prolonged period of too low bond yields relative to their macro-economic counterparts, i.e. to the persistence of the misalignments. This result therefore shows that liquidity in the bond market matters for the duration and magnitude of the bond yield misalignment versus the bond yield's macro-implied fair value.

Figure 19: Aggregate Traded Volume on the Secondary Czech Government Bond Market



Source: Prague Stock Exchange

Both of the previous exercises showed, in their respective capacity, the relevance of the macro-economy for determining or predicting the changes in government bond yields.

V. CONCLUSION OF THE MACROFINANCE MODEL AND SUGGESTIONS FOR FURTHER RESEARCH

We learned that the macro-economy indeed determines a fair value of bond yields towards which the actual yields tend to move. Hence, the macro-economy plays an important role in determining changes in the government bond yields. Misalignments on the Czech government bond market tend to persist for a number of months, but they are eventually always eliminated and the macro-economy always dominates the behavior of the government bond yield curve. Also, as we will see bellow, not all the misalignments are statistically significant and for a prudent investor following the signals from the macro-economy it does not necessarily always make sense to trade on relatively small misalignments.

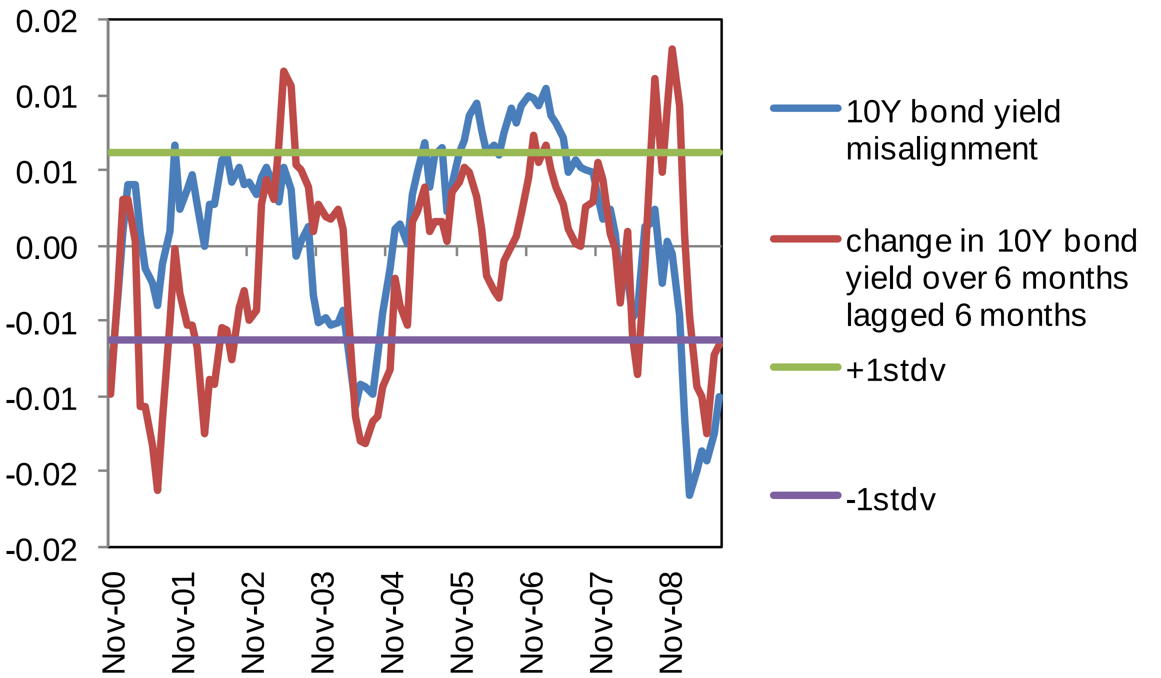
Most of our empirical analysis was conducted using also the data spanning the recent global economic and financial crisis. Nevertheless, as Medeiros, Rodriguez (2011) showed, the Nelson-Siegel framework proved to be sufficiently robust to explain the developments of the term structure of interest rates even during the recent period of stress in financial markets. The authors showed that the main relationships between the yield curve and the macro-economy held rather well during the recent crisis.

Suggestions for Further Research

Most of the macrofinance research focuses on the development of the modeling techniques to grasp the interaction between the yield curve and the macro-economy. Nevertheless, we feel that future research should also try to follow up on the work provided above in this paper and try to test the importance of the macro-implied yield curves for the subsequent yield changes on real-life yield-curve investing strategies. Strategies based on macrofinance signals should be developed and evaluated using real-life market data. These strategies should be then compared to traditional buy-and-hold strategies. The macrofinance signals could be useful in investing across the yield-curve (i.e. buying and selling government bonds of various maturities based on signals from the macro-implied yield curve), or in general asset allocation decisions among various asset classes (e.g. bonds versus equities, etc.).

Let us elaborate on the use of macrofinance modeling for asset allocation purposes a little further. As we see on the next figure, not all the movement in the yield misalignment is outside of one historical standard deviation of these misalignments. That might mean that part of the misalignments might well be due to random (noise) factors not necessarily important for asset allocation decisions. We see that during the period of 2000-2009, the one-standard-deviation rule extracted only a small number of signals from our macrofinance framework. This implies that we are not going to use the macrofinance framework for higher-frequency trading. On the contrary, the investor's allocation will need to be reshuffled based on the behavior of the macro-economy only time to time.

Figure 20: Bond Misalignments, Subsequent Changes in Bond Yields and One Standard Deviation of the Misalignments



Source: Prague Stock Exchange; own calculations

We can illustrate such an investment strategy with an extremely simplified example. We evaluated a hypothetical trading in the 10Y Czech Government bond issue CZECH REPUBLIC 2003 3.7% 16/06/13 S.40 from July 2003 until February 2010. Taking into account one historical standard deviation in the 10Y bond yield misalignment from its macro-implied value, during the period from July 2003 to February 2010, we identified 4 signals representing statistically significant bond yield misalignments. These signals say "buy" when the bond is undervalued according to the macro-implied value, and "sell" when the bond is overvalued according to the macro-implied value; however, the misalignment needs to exceed its one historical standard deviation in order to be traded upon this signal.

We started with an investment of CZK 91.22 (the market price of the bond at the first purchase). The total cash profit from the four buy and sell decisions during the whole trading period was CZK 16.21. Approximate coupon profit during the period the bond was held was CZK 10.18. The total profit from this investment strategy was therefore CZK 26.39. During the whole trading period, the number of years we were not invested with the bond was 3.92 years. During these almost four years, we could have been invested in cash or other bonds (using some across-the-yield-curve misalignment signals), or even in equity, potentially bringing in more profits. Approximate cash coupon profit from an alternative buy-and-hold strategy for the same bond during the same period was 24.3 CZK.

The investment strategy based on the macrofinance signals outperformed the buy-and-hold strategy by at least 8.6 percent during the trading period. If we took into consideration the potential proceeds from the idle cash during the almost 4 un-invested years, the difference might have been even more supportive for the macro-based investment strategy.

This over-simplified example deserves a number of disclaimers. First of all, the proper strategy would have been to trade the ten-year duration not the ten-year bond. Second, our example is simplified as we did not take into consideration transaction costs, taxation, etc. Third, we did not trade the bond until its full maturity as its maturity spanned into the future. Finally, this approach suffers a bias in that it runs the model through the whole time spam and identifies the signals *ex post*, assuming that such results reflect those results that the investor saw at each individual observation (i.e. at each individual decision date).[51] Nevertheless, this example provides a suggestion as to how to implement the information from the macrofinance model into real-life investment decisions.

This example also provided an important complementary explanation to the relatively long persistence of misalignments in the Czech government bond market during the period of 2000-2010. We see that some of the misalignments were within one historical standard deviation of these misalignments, and therefore might have represented only random noise not necessarily important for conducting investment decision directed at eliminating these misalignments.

VI. Conclusion

This paper dealt with the interaction between the macro-economy and financial markets. In the main part, it developed a macrofinance model of the term structure of interest rates and showed the use of such macrofinance model for yield forecasting and investment purposes.

[51] The proper approach would have run the model subsequently at each observation and have evaluated at each observation the existence or non-existence of a signal.

The bottom line of this paper is that the macro-economy matters for the dynamics of financial markets in general, and for the dynamics of the yield curve in particular.

We coined our macrofinance approach in the dynamic version of the Nelson-Siegel model. The Nelson-Siegel framework is a parsimonious approach that models the whole yield curve using a single exponential functional form by estimating only a small number of parameters while guaranteeing the desired properties of the yield curve. Such framework reflects the well-established empirical fact that most of the behavior in the government bond yield curves can be explained by using only up to three factors. The Nelson-Siegel framework is well suited for forecasting and for dynamic analysis of yield curve behavior and is widely used by policy-makers, forecasters and practitioners. Macrofinance modeling is a relatively new research agenda that combines models of the yield curve with simple macroeconomic models or macroeconomic variables. The main motivation of such models is to understand both the influence of the macro-economy on the yield curve as well as the information value of the yield curve with respect to the macro-economy.

We estimated a macrofinance VAR model of the zero-coupon government yield curve for the Czech economy based on the dynamic interpretation of the Nelson-Siegel framework. We named the resulting curve the macro-implied (or fair-value) yield curve, towards which the actual zero-coupon government bond yield curve should naturally tend. Analyzing the properties of our macrofinance model showed that macroeconomic factors tend to have a significant influence on the yield curve after approximately five to six months, which was consistent with other empirical research.[52]

In making our macrofinance model operational, we analyzed to what extent the current misalignments of the actual yields from the macro-implied yields helped us predict subsequent movements in the Czech zero-coupon government bond yields. We maintained that the yield curve misalignments form its macro-implied fair-value can be treated as the macro-implied subsequent yield changes. We measured the predictive power of the misalignments in explaining the changes in bond yields and we found out that current yield misalignments partially determine subsequent yield changes over three to nine months periods.

Finally, we investigated the forecasting performance of the macrofinance VAR model for the Czech economy in comparison to a yields-only VAR model. The incorporation of the macro-economy into the yield curve model improved its forecasting ability. Also, the multiple regression of the determinants of the yield changes confirmed that the current misalignments of actual yields form the macro-implied yields played an important part in explaining the subsequent yield changes. The traded volume proved also significant, suggesting that the liquidity effect might be one of the causes for longer persistence of yield misalignments from the macro-implied yields.[53]

The main part of this paper taught us that the macro-economy played an important role in Czech government bond yield dynamics. Misalignments from the macro-implied fair value on the Czech government bond market tend to persist for a number of months, but they are

[52] These results also correspond to Ilmanen (2011, p. 146) that also shows that the influence of the macro-economy on financial markets is stronger with lower frequencies rather than higher frequencies.

[53] Remember that through the lens of the efficient markets hypothesis, any meaningful misalignment would have to be immediately eliminated by actively trading market participants.

eventually always eliminated and the macro-economy always dominates the behavior of the government bond yield curve. The persistence of these misalignments can be explained by (a) the known empirical fact that the macro-economy kicks into determining the asset markets only at lower frequencies, (b) by the liquidity effect, particularly during the times of capital inflows to Czech Republic that were substantially absorbed by the government bond market, and (c) by the fact that not all misalignments were greater than their historical one standard deviation and therefore might have been representing only random noise that might not have warranted any investment decisions on the part of active market participants.

VII. LITERATURE REFERENCES

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