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## Bank Capitalization as a Signal

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Monetary and Capital Markets Department

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**Abstract**

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The level of a bank's capitalization can effectively transmit information about its riskiness and therefore support market discipline, but asymmetry information may induce exaggerated or distortionary behavior: banks may vie with one another to signal confidence in their prospects by keeping capitalization low, and banks' creditors often cannot distinguish among them —tendencies that can be seen across banks and across time. Prudential policy is warranted to help offset these tendencies.

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“Today’s approval to repay the TARP capital reaffirms the value and strength of U.S. Bancorp’s diversified business mix and on-going commitment to prudent risk management.”

Richard K. Davis,  
Chairman, President and Chief Executive Officer of U.S. Bancorp,  
June 9, 2009

“Repayment [of government stakes] from recapitalized banks would normally signal an improved financial position for banks, with a potentially positive effect”

*Global Financial Stability Report*, p. 141  
International Monetary Fund  
October 2009

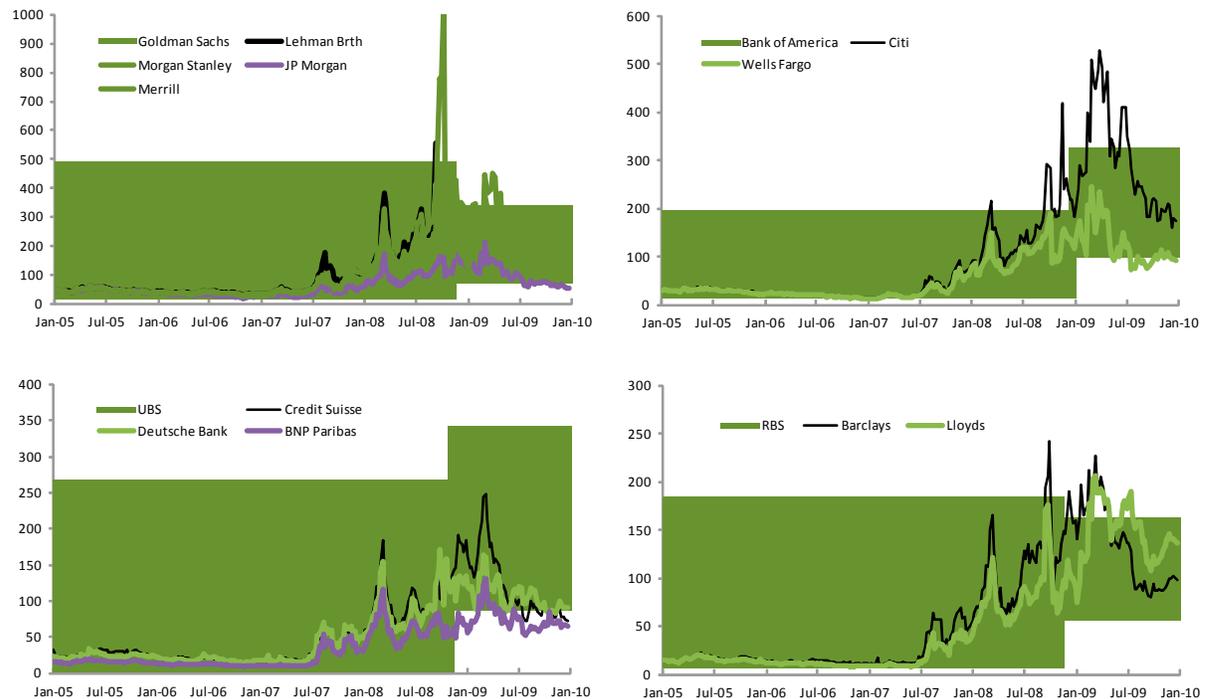
## I. INTRODUCTION

A remarkable phenomenon observed during the recent global financial crisis was the eagerness of financial institutions to reduce their capitalization by repaying government support as soon as possible, even as the crisis was far from over. While they had several motives—notably the desire to extract themselves from government influence over their management and especially their compensation practices—lowering capitalization in this way was often also intended to demonstrate financial strength and confidence, and was interpreted as such. Another phenomenon was that the banks that got into most trouble had not been identified in advance as being much more vulnerable than others; at least during good times, markets and analysts did not differentiate greatly. As shown below, the credit default swap (CDS) rates for major U.S. and European banks—which are meant to reflect market perceptions of probability of default—were low and very close to one another until the on-set of the crisis in July 2007, and even then the common market movement was dominant for some time.<sup>2</sup> A third, and possibly related observation is that banks generally did not build up large reserves during good times, but rather used the opportunity of favorable macroeconomic conditions and ready availability of financing to expand their balance sheets. In most countries, neither regulators nor market discipline pushed back strongly against this tendency.

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<sup>2</sup> Haldane (2011) shows that the reported Tier 1 capital ratios of banks that received government support following the Lehman collapse did not differ from those that did not. However, he presents illustrative evidence that the ratio of a bank’s market capitalization to its assets or debt was more indicative of its vulnerability from about a year in advance of the onset of the crisis.

### Bank 10-year CDS rates (Basis points)



Source: Bloomberg.

One explanation for all three phenomena is based on the role of capitalization as a signal—the notion behind the quotations given above. For a given institution, having low capitalization increases the marginal cost of funding, because there is a greater risk of (costly) bankruptcy, but the marginal cost will typically be highest for an institution that is anyway risky. Hence, even when there is no question of asymmetric information (or moral hazard or adverse selection), one would normally expect riskier institutions to have higher capitalization than do low-risk institutions, such that risk-adjusted returns are equalized.<sup>3</sup>

Yet, financial institutions and especially banks are typically highly complex, and one of their core functions is to overcome the large asymmetries of information that exist between savers and borrowers. Hence, a bank's riskiness is not fully observable to outsiders, including in particular depositors and others providing non-equity financing. Given the commercial sensitivity and complexity of financial operations, a bank cannot reveal this information directly with full credibility, yet its cost of financing depends on its perceived riskiness. It

<sup>3</sup> This relationship holds across sectors: insurance companies, which are typically incur relatively low default risk, normally display more leverage than do banks. The Basel capital accords are based on the notion of a minimum risk-adjusted capital ratio, which implies that unweighted capital ratios may be lower for low-risk banks.

therefore has an incentive to use credible signals (such as capitalization) to convey to those providers of financing that it offers high and safe returns. A credible, revealing signal is one that only a bank of a certain type would send because sending the signal is least costly (on the margin) for that particular type of bank. Specifically, a low-risk financial institution may optimally choose to be less well capitalized than a high-risk institution because low capitalization carries for the former a relatively low cost in terms of increase probability of failure. However, the signal is imperfect, and therefore banks may attempt to pretend to be less risky than they truly are. Indeed, those with riskier business models may wish to pretend to be safer than they know themselves to be by keeping capitalization low, which may in turn induce the safer banks to decrease capitalization still further in order to maintain the distinction. As demonstrated here, these tendencies may arise across a range of banks and over time. It follows that market discipline is of limited effectiveness, and there is substantial scope for prudential policy.

The notion that differences in marginal cost enable economic agents to convey credible signals has found wide application in economics.<sup>4</sup> A market can display a “pooling” equilibrium, where participants behave identically and so cannot be distinguished; a “separating” equilibrium, where participants behave in distinct ways in order to send credible signals; or a “mixed” equilibrium, where some pool and some separate. In the area of corporate finance, signaling has been evoked to explain, for example, dividend behavior and the relation between internal and external finance.<sup>5</sup> In a related paper, Hardy and Tieman (2008) start from a recognition that the volume of credit extended by a bank can be an informative signal of its abilities in loan selection and management. Hence, under asymmetric information, banks may rationally lend more than they would otherwise in order to demonstrate their quality, thus negatively affecting financial system soundness. However, their paper does not include a role for bank capitalization.

The next section lays out the general model. The subsequent section describes the various conditions under which possible signaling equilibria can obtain, followed by a worked example. Extensions of the model are then reviewed. The last section concludes.

## II. THE MODEL

Consider a bank with certain investment opportunities that yield (one plus) the rate of return  $\alpha + \beta\varepsilon$ , where  $\varepsilon \in [\underline{\varepsilon}, \bar{\varepsilon}]$  is a random variable with a differentiable distribution function  $g$  and

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<sup>4</sup> The need to send credible signals can explain a range of behaviors. Famous examples include Spence (1973) on signaling in education choices, Rothschild and Stiglitz (1976) on insurance, and Milgrom and Roberts (1982) on limit pricing to deter entry. There is a voluminous literature on the theoretical basis of signaling games, including Riley (1979) and Mailath (1987).

<sup>5</sup> Early examples include Ross (1977), Leland and Pyle (1977), Bhattacharya (1979), Myers and Majluf (1984), and Gertner, Gibbons and Scharfstein (1988). Tirole (2006) Chapter 6 provides an overview.

cumulative distribution function  $G$ . The parameters  $\alpha$  and  $\beta$  describe the bank's investment opportunities; it is assumed that  $\beta > 0$ . Without loss of generality, the total balance sheet size of the bank is normalized to unity. The bank chooses a proportion  $F \in [0, 1]$  of its balance sheet to be financed through borrowing, perhaps in the form of deposits or securities; its capitalization ratio is therefore  $(1-F)$ .<sup>6</sup> Each unit of financing costs  $i$ . The realized profit is thus

$$\alpha + \beta\varepsilon - iF.$$

It is assumed that  $\alpha + \beta\hat{\varepsilon} > 0$ , so it is at least possible that the bank is profitable. There is a certain level of the random component of the return on the investments, denoted by  $\hat{\varepsilon}$ , below which the bank loses money and is liquidated. The break-even point is given by

$$\alpha + \beta\hat{\varepsilon} - iF = 0. \quad (1)$$

For simplicity, it is assumed that investments become worthless if the bank is liquidated. Since bankruptcy is costly, the Modigliani-Miller theorem does not apply; the form of financing has real effects.

The bank is risk-neutral and its objective is to maximize the expected one-period rate of return on its capital  $(1-F)$ . Since the bank is bankrupt if the return is less than the costs of financing, the expected return on capital equals

$$\begin{aligned} V &= \frac{1}{1-F} E(\alpha + \beta\varepsilon - iF | \alpha + \beta\varepsilon - iF > 0) \\ &= \frac{1}{1-F} \int_{\hat{\varepsilon}}^{\bar{\varepsilon}} (\alpha + \beta\varepsilon - iF) g(\varepsilon). \end{aligned}$$

The providers of financing (for convenience, call them depositors) are also risk neutral and have an alternative safe investment that yields  $s$ . The rate  $i$  paid by the bank is such that depositors are just indifferent between financing the bank and making the safe investment, given their information set. Depositors can observe  $F$  but have only an estimate  $\tilde{\alpha}$  of the true  $\alpha$ . On this basis they make an estimate of the break-even level of the random return:

$$\tilde{\alpha} + \beta\tilde{\varepsilon} - iF = 0. \quad (2)$$

The no-arbitrage condition is that

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<sup>6</sup> The leverage ratio is the inverse thereof. All assets have the same risk weight, so the risk-weighted and non-weighted capitalization ratios are proportional to one another.

$$s = E(i|\alpha + \beta\varepsilon - iF > 0, \alpha = \tilde{\alpha})$$

$$= \int_{\hat{\varepsilon}}^{\bar{\varepsilon}} i g(\varepsilon),$$

from which it follows that

$$i = \frac{s}{1 - G(\hat{\varepsilon})}. \quad (3)$$

Hence, the bank's value function can be written

$$V(\alpha, \tilde{\alpha}, F) = \frac{1}{1 - F} \int_{\hat{\varepsilon}}^{\bar{\varepsilon}} (\alpha + \beta\varepsilon - i(\tilde{\alpha}, F)F) g(\varepsilon). \quad (4)$$

The core problem facing the bank is to choose  $F$  to maximize (4) subject to (1), (2) and (3). It is convenient to use substitutions among the various equations to simply them as

$$\alpha + \beta\hat{\varepsilon} - \frac{sF}{1 - G(\hat{\varepsilon})} = 0 \quad (5)$$

$$\tilde{\alpha} + \beta\hat{\varepsilon} - \frac{sF}{1 - G(\hat{\varepsilon})} = 0 \quad (6)$$

$$V(\alpha, \tilde{\alpha}, F) = \frac{1}{1 - F} \int_{\hat{\varepsilon}}^{\bar{\varepsilon}} \beta(\varepsilon - \hat{\varepsilon}) g(\varepsilon). \quad (7)$$

$V_n$  will be used to denote the partial derivative of  $V$  with respect to the  $n$ -th argument, taking into account the constraints, and  $V_{nm}$  will be used to denote the second partial derivative of  $V$  with respect to the  $n$ -th and  $m$ -th arguments, taking into account the constraints. Thus,

$$V_1 = \frac{1}{1 - F} \int_{\hat{\varepsilon}}^{\bar{\varepsilon}} -\beta \frac{\partial \hat{\varepsilon}}{\partial \alpha} g(\varepsilon), \quad (8)$$

where  $\frac{\partial \hat{\varepsilon}}{\partial \alpha} = -1/\beta$ , so  $V_1 = (1 - G(\hat{\varepsilon}))/ (1 - F)$  is necessarily positive; a higher expected return increases the value of the bank. Also,

$$V_2 = \frac{1}{1 - F} \int_{\hat{\varepsilon}}^{\bar{\varepsilon}} -\beta \frac{\partial \hat{\varepsilon}}{\partial \tilde{\alpha}} g(\varepsilon), \quad (9)$$

where, from (5) and (6),

$$\frac{\partial \hat{\varepsilon}}{\partial \tilde{\alpha}} = \frac{sFg(\tilde{\varepsilon})}{\beta(1-G(\tilde{\varepsilon}))^2} \frac{\partial \tilde{\varepsilon}}{\partial \tilde{\alpha}} = -\frac{sFg(\tilde{\varepsilon})}{(1-G(\tilde{\varepsilon}))^2}. \quad (10)$$

Since this term is always negative,  $V_2$  is also positive; if depositors believe that a bank has higher expected returns on its investments and therefore a lower probability of default, they will accept a lower return on the financing that they provide. Hence, the value of the bank is higher; it is advantageous to owners when depositors believe the bank to have high expected returns. This positive relationship between depositors' beliefs and the value of the bank gives the bank an incentive to signal that it has high expected returns. It is easy to derive from (8) that

$$V_{12} = \frac{-g(\hat{\varepsilon})}{1-F} \frac{\partial \hat{\varepsilon}}{\partial \tilde{\alpha}},$$

which, using (10), can be seen to be always positive: the marginal benefit of being thought to have high expected returns increases with actual expected returns, and vice versa. Finally,

$$V_3 = \frac{1}{1-F} \left[ V - \int_{\hat{\varepsilon}}^{\tilde{\varepsilon}} \left( \beta \frac{\partial \hat{\varepsilon}}{\partial F} \right) g(\varepsilon) \right]. \quad (11)$$

Hence, using (8)

$$\begin{aligned} V_{13} &= \frac{1-G(\hat{\varepsilon})}{(1-F)^2} - \frac{g(\hat{\varepsilon})}{(1-F)} \frac{\partial \hat{\varepsilon}}{\partial F} \\ &= \frac{1-G(\hat{\varepsilon})}{(1-F)^2} - \frac{g(\hat{\varepsilon})}{(1-F)} \left[ \frac{\frac{s}{(1-G(\tilde{\varepsilon}))}}{\beta - \frac{sFg(\tilde{\varepsilon})}{(1-G(\tilde{\varepsilon}))^2}} \right], \end{aligned} \quad (12)$$

which in general can be positive or negative. Greater financing leverages higher expected returns on investment. However, if perceived returns on investment are unchanged, proportionately more financing results in higher financing costs, which both reduces profits and increases the probability of failure.

A signaling equilibrium is possible when  $V_{13} > 0$  because this condition ensures that decreasing capitalization is less expensive for a bank with good investment opportunities than for a bank with worse investment opportunities. Here it will be assumed that  $V_{13} > 0$ , which is plausible because the first term contains in the denominator  $(1-F)^2$ , which is on the order of 0.01 if the capitalization ratio is 10 percent: there is always a if the capitalization ratio for which  $V_{13} > 0$ . Likewise, if capitalization is very high, the bank eventually has a zero

probability of failure, implying that  $G=g=0$ , so again  $V_{13}>0$ . The condition is possibly not met at some intermediate point where  $g$  becomes very large.

### III. MODEL ANALYSIS

#### A. Full Information

A baseline for what follows is provided by the situation where there are no information asymmetries, that is, where depositors know all the parameters of bank's investment opportunities. Thus,  $\alpha = \tilde{\alpha}$ ,  $\hat{e} = \tilde{e}$ , and the constraint (6) coincides with (5). Then the first order condition (FOC) for the maximum of the bank's value function is

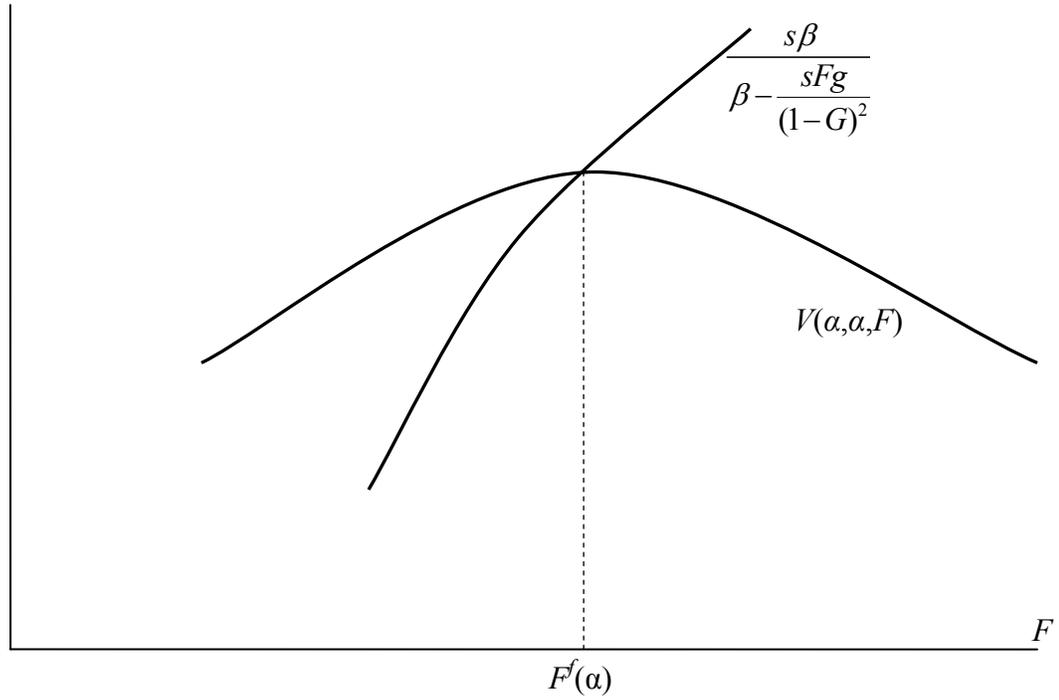
$$\frac{1}{1-F} \left[ V - \int_{\hat{e}}^{\tilde{e}} \left( \beta \frac{\partial \hat{e}}{\partial F} \right) g(\varepsilon) \right] = \frac{1}{1-F} \left[ V - \frac{s\beta}{\beta - \frac{sFg}{(1-G)^2}} \right] = 0. \quad (13)$$

The first term in square brackets captures the benefit to shareholders of lower capitalization, and the second term captures the cost of higher financing that is provoked by the higher probability of bankruptcy. Since  $V$  is assumed to be positive, for the FOC can be met the second term in brackets must be negative. Since also  $s$  and  $\beta$  are positive, a necessary condition for the existence of a maximum with a positive level of financing is that

$$\beta - \frac{sFg}{(1-G)^2} > 0. \quad (14)$$

Condition (14) is necessary to ensure that  $\frac{\partial \hat{e}}{\partial \alpha} < 0$  (implying that a bank with higher expected returns can break even with a worse realized return because financing costs are lower) and will be assumed to hold throughout the relevant range. The optimally-chosen  $F$  under symmetric information, denoted by  $F^f(\alpha)$ , will serve as a benchmark.  $V^*(\alpha, \alpha, F^f(\alpha))$  denotes the maximized value function. The choice of  $F^f(\alpha)$  can be represented in the following diagram: the FOC is met where the curve  $V$  intersects the curve  $s\beta / (\beta - sFg / (1-G)^2)$ .

### Optimal Capitalization Under Full Information



It is assumed that the second order condition (SOC)

$$V_{33}(\alpha, \alpha, F^f(\alpha)) \equiv \frac{s\beta}{1-F} \left[ \beta - \frac{sFg}{(1-G)^2} \right]^{-2} \frac{sF(g'(1-G) + g^2)}{(1-G)^3} < 0 \quad (15)$$

for a maximum is met. Given (14), a sufficient condition for the SOC to be met is that  $g' \geq 0$ ; it will be assumed that  $g' \geq 0$  throughout the relevant range. Then the implicit function theorem can be used with equation (13) and condition (14) to show that  $F^f(\alpha)$  depends positively on  $\alpha$  and negatively on  $s$ , so higher expected returns leads to more financing, and higher cost of funding reduces financing.

### B. Equilibria with Partial Information and Two Bank Types

In some situations, banks may plausibly fall into one of two categories, namely, those that are relatively strong because they have high-yielding investment opportunities, and those that are weaker with lower-yielding investment opportunities. (Subsequently we will look at the case of a continuum of types.) Perhaps some banks are long established and have established relationships with the most bankable clients, or they have better techniques and databases for loan evaluation, or they have more conservative management controls and governance. Specifically, we assume that banks can be of only two types, with respectively a high ( $\alpha_H$ ) and low ( $\alpha_L$ ) expected yield on investments ( $\alpha_H > \alpha_L$ ). A proportion  $\theta$  of banks are strong

with high yields available, while the remaining  $1 - \theta$  are weaker and have lower yielding investments.

The equilibrium behavior of banks may involve either pooling or separating. In a pooling equilibrium, all banks have the same capitalization, so depositors cannot infer anything from leverage; they have to rely on their ex ante estimate of the average expected investment yield. The “good” banks get to choose the credit volume, but their cost of financing is raised by the depositors’ fear of the intermingled low-return banks. In a separating equilibrium, banks with high expected returns have such a low capital ratio that low-return banks prefer not to imitate; imitation would expose them to an unacceptable risk of bankruptcy. Hence, depositors can distinguish between banks, so costs of financing reflect bank type-specific riskiness, but the high-yield banks are constrained to be less well capitalized than they would under full information in order to discourage imitation.

### Pooling

In the pooling equilibrium, where both types of banks disburse the same volume of credit, the objective functions for high and low return banks are respectively

$$V(\alpha_H, \underline{\alpha}, F) = \frac{1}{1-F} \int_{\hat{e}_H}^{\bar{\varepsilon}} \beta(\varepsilon - \hat{e}_H)g(\varepsilon) \text{ such that } \alpha_H + \beta\hat{e}_H - \frac{sF}{1-G(\underline{e})} = 0, \quad (16)$$

$$V(\alpha_L, \underline{\alpha}, F) = \frac{1}{1-F} \int_{\hat{e}_L}^{\bar{\varepsilon}} \beta(\varepsilon - \hat{e}_L)g(\varepsilon) \text{ such that } \alpha_L + \beta\hat{e}_L - \frac{sF}{1-G(\underline{e})} = 0, \quad (17)$$

and

$$\underline{\alpha} + \beta\underline{e} - \frac{sF}{1-G(\underline{e})} = 0 \quad (18)$$

where  $\underline{\alpha} = \theta\alpha_H + (1-\theta)\alpha_L$  is the average parameter; the presence of  $\underline{\alpha}$  rather than  $\alpha_H$  or  $\alpha_L$  as the second argument of the value functions indicates that these are value functions under pooling. The high-yield bank chooses  $F$  to maximize (16) subject to (18), while the low-yield banks imitates. The pooling level of financing is denoted by  $F^P(\alpha_H)$ . To make the problem interesting, it is assumed that the low-yield bank would have an incentive to imitate the high-yield banks, if the latter acted as if there were full information; otherwise the high-yield banks would be unconstrained. The relevant condition is:

$$V(\alpha_L, \underline{\alpha}, F^f(\alpha_H)) > V^*(\alpha_L, \alpha_L, F^f(\alpha_L)). \quad (19)$$

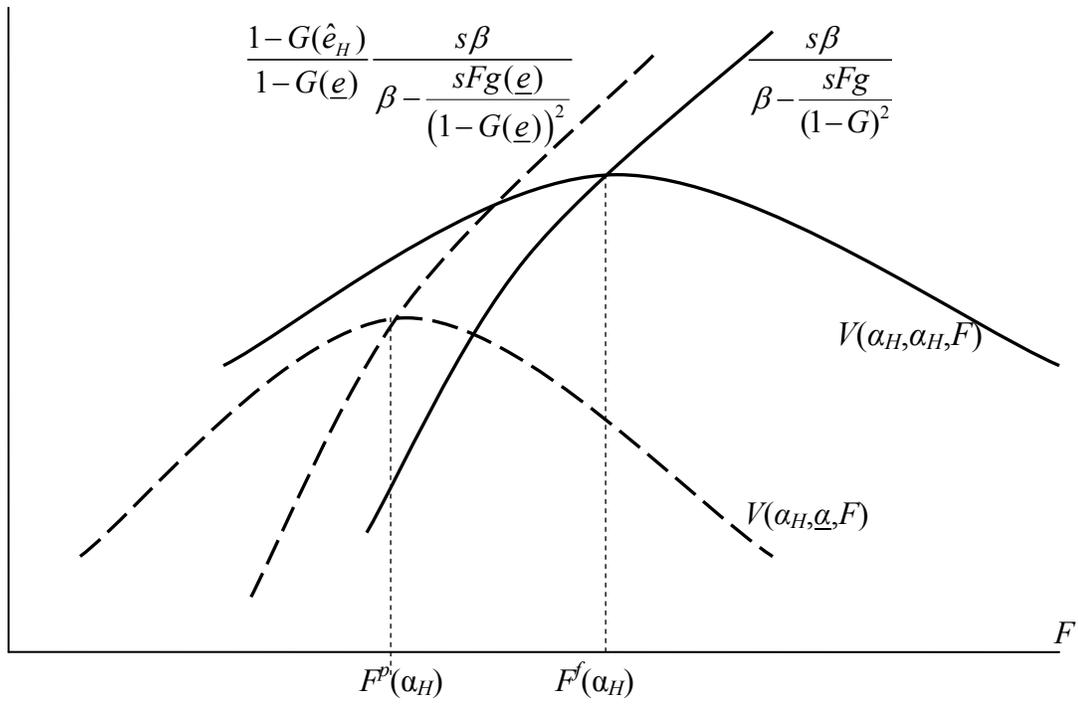
It can be shown that, in a pooling equilibrium, banks of type  $\alpha_H$  have higher capitalization than in the full information case, while banks of type  $\alpha_L$  have lower capitalization than under full information. Examine the FOC derived from (16) and (18):

$$\frac{1}{1-F} \left[ V(\alpha_H, \underline{\alpha}, F) - \int_{\hat{e}_H}^{\bar{e}} \left( \beta \frac{\partial \hat{e}_H(\underline{e})}{\partial F} \right) g(\underline{e}) \right] =$$

$$\frac{1}{1-F} \left[ V - \frac{1-G(\hat{e}_H)}{1-G(\underline{e})} \frac{s\beta}{\beta - \frac{sFg(\underline{e})}{(1-G(\underline{e}))^2}} \right] = 0. \quad (20)$$

Since  $\underline{\alpha} < \alpha_H$ ,  $V(\alpha_H, \underline{\alpha}, F) < V(\alpha_H, \alpha_H, F)$ . It follows also that  $\hat{e}_H < \underline{e}$ , and hence  $1-G(\hat{e}_H) > 1-G(\underline{e})$ . By the assumption that  $g' > 0$ ,  $g(\hat{e}_H) < g(\underline{e})$ . Therefore, the first term in square brackets in (20) is smaller than the analogous term in (13) with  $\alpha = \alpha_H$ , which is the FOC for optimal capitalization under full information for the high-yield bank. Furthermore, the second term in (20) is larger in absolute value than the analogous term in (13). It follows that the optimum  $F^p(\alpha_H) < F^f(\alpha_H)$ . The two FOCs can be shown graphically as follows:

#### Optimal Capitalization Under the Pooling Equilibrium



The differences between (20) and the full information FOC for the low-yield bank (given by (13) with  $\alpha = \alpha_L$ ) are the converse. Hence, (20) implies higher credit than would be optimal with low technology under full information. The low-yield bank does not choose the volume of credit under pooling (it imitates the high-yield bank), so  $F^p(\alpha_H) = F^p(\alpha_L) > F^f(\alpha_L)$ .

Thus, in a pooling equilibrium, the bank with the better investment opportunities is more capitalized than in the full information case, while the banks with the inferior investment

opportunities is less well capitalized. Whether the total average capitalization ratio would be higher or lower than under full information depends on the specific functional forms and cannot be determined in general.

### Separating

Under some circumstances, the high-type bank might benefit from credibly signaling its type to depositors, thus creating a separating equilibrium, rather than let the low-yield bank imitate it and suffer higher funding costs. In such an equilibrium, a high-yield bank's net worth is given by (7) subject to (5) and (6) with  $\alpha = \tilde{\alpha} = \alpha_H$ ; the low-yield bank's net worth in analogous but with  $\alpha = \tilde{\alpha} = \alpha_L$ .

However, in order for a separating equilibrium to be feasible, two incentive compatibility constraints must be met. One constraint is that the low-yield bank prefers not to imitate the signal of the high-yield bank, that is, that imitating is not advantageous relative to the best obtainable outcome when depositors can infer the low-yield bank to be of the low type. First the high-yield bank must choose its financial ratio  $F^s(\alpha_H)$  such that the low-yield bank does not want to imitate:

$$V^*(\alpha_L, \alpha_L, F) \geq V(\alpha_L, \underline{\alpha}, F^s(\alpha_H)) \quad (21)$$

Condition (21) defines  $F^s(\alpha_H)$ , the high-yield bank's financing ratio that achieves separation. Thus, the high-yield bank discourages imitation by increasing its financing; since the marginal net benefit of financing is always higher for the high-yield bank than for the low-yield bank, it will stop being worthwhile for the latter to imitate before it becomes too costly for the high-yield bank.

Second, the incentive compatibility constraint comes into play: the profits of the high-yield bank must be higher at  $F^s(\alpha_H)$  than they would be under a pooling equilibrium:

$$V(\alpha_H, \alpha_H, F^s(\alpha_H)) \geq V^*(\alpha_H, \underline{\alpha}, F) . \quad (22)$$

This second condition hence defines the boundary between the separating and the pooling equilibrium. Assuming (21) is satisfied, (22) ensures that the high-yield type prefers separating over pooling. Consequently, in case (22) is not satisfied, the high type would prefer pooling, as the additional benefit from signaling its type to financiers does not outweigh the extra signaling costs of having to supply more credit than in the pooling case.

A separating equilibrium obtains when both incentive compatibility constraints (21) and (22) are satisfied. Banks of type  $\alpha_L$  have the same financing ratio as under full information; as the low-yield bank's type will be revealed to financiers, this bank has no incentive to deviate from its full information supply of credit, i.e.,  $F^s(\alpha_L) = F^f(\alpha_L)$ . Typically banks of type  $\alpha_H$  have

more financing (lower capitalization) than in the full information case.<sup>7</sup> Heuristically, one can imagine starting from a situation in which  $\alpha_H - \alpha_L$  is so large that imitation is not worthwhile for the weak bank, and then allowing that difference to decrease. As the difference between decreases, the capitalization levels converge, and eventually the strong bank is threatened with imitation. To distinguish itself, it needs to decrease its capitalization level so that it stays well below that of the weak bank. Similarly, starting from a pooling equilibrium and allowing the difference  $\alpha_H - \alpha_L$  to increase, one reaches a transition point where the stronger bank wishes to separate itself by much lower capitalization.

Because, under separating, the high-yield bank has lower capitalization and higher funding costs than at the full-information optimum, it incurs a greater probability of failure. The probability of default of the low-yield bank is unaffected. The average is therefore higher. The average bank value certainly declines relative to the full-information level: the value of the high-yield bank is less, because the need to signal has driven its capitalization level below the optimal level under full information, while that of the low-yield bank is unaffected.

It is noteworthy that the behavior of average capitalization differs depending on the equilibrium that obtains. Under full information, how a bank reacts to changes in the cost of financing or its investment opportunities depends just on that bank's own characteristics. Under pooling, the reaction of every bank's capitalization to a shock is determined by how the high-yield bank is affected. The low-yield bank will imitate how the high-yield bank reacts to a change in  $\alpha_H$ , which affects both its investment opportunities and, through  $\underline{\alpha}$ , its financing costs. The parameters  $\theta$  and  $\alpha_L$  still have an influence on the capitalization, but only by affecting the high-yield bank through changes in  $\underline{\alpha}$ . Under separating, the incentive compatibility constraint (which relates to the profitability of low-yield banks) dictates how high-yield banks will react.

There is a range of combinations of parameters defining the boundary between the separating and the pool equilibrium. This boundary is defined implicitly by the condition that the high-yield bank is indifferent between pooling and separating, where the separating level of credit is such that the low-yield bank is also indifferent. Thus:

$$V^*(\alpha_H, \underline{\alpha}, F) = V(\alpha_H, \alpha_H, F^s), \quad (23)$$

such that (21) is met with equality. A small shift in the parameters can make the banking sector switch from one equilibrium to another. Such a shift might be caused, for example, by macroeconomic developments that affect expected returns on investments or the safe rate of interest. For example, a small shift that leads high-yield banks to attempt to separate could generate a large and abrupt increase in financing.

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<sup>7</sup> There may be several other equilibria, for example, where the strong bank has especially high capitalization, but they involve a reversal of the order of capitalization levels.

### A parameterized example

To illustrate these possibilities and demonstrate the possible existence of these equilibria, a parameterized example is presented. In particular, it is assumed that the random variable  $e$  follows a uniform distribution over the unit interval. With this assumption, the model can be solved explicitly. It is then easy to show that

$$V = \frac{1}{1-F} \frac{\beta}{2} (1-\hat{e})^2, \quad F \in [0,1), \quad (24)$$

and the equations for  $\hat{e}$  and  $\tilde{e}$  introduce other quadratic and hyperbolic functional forms.

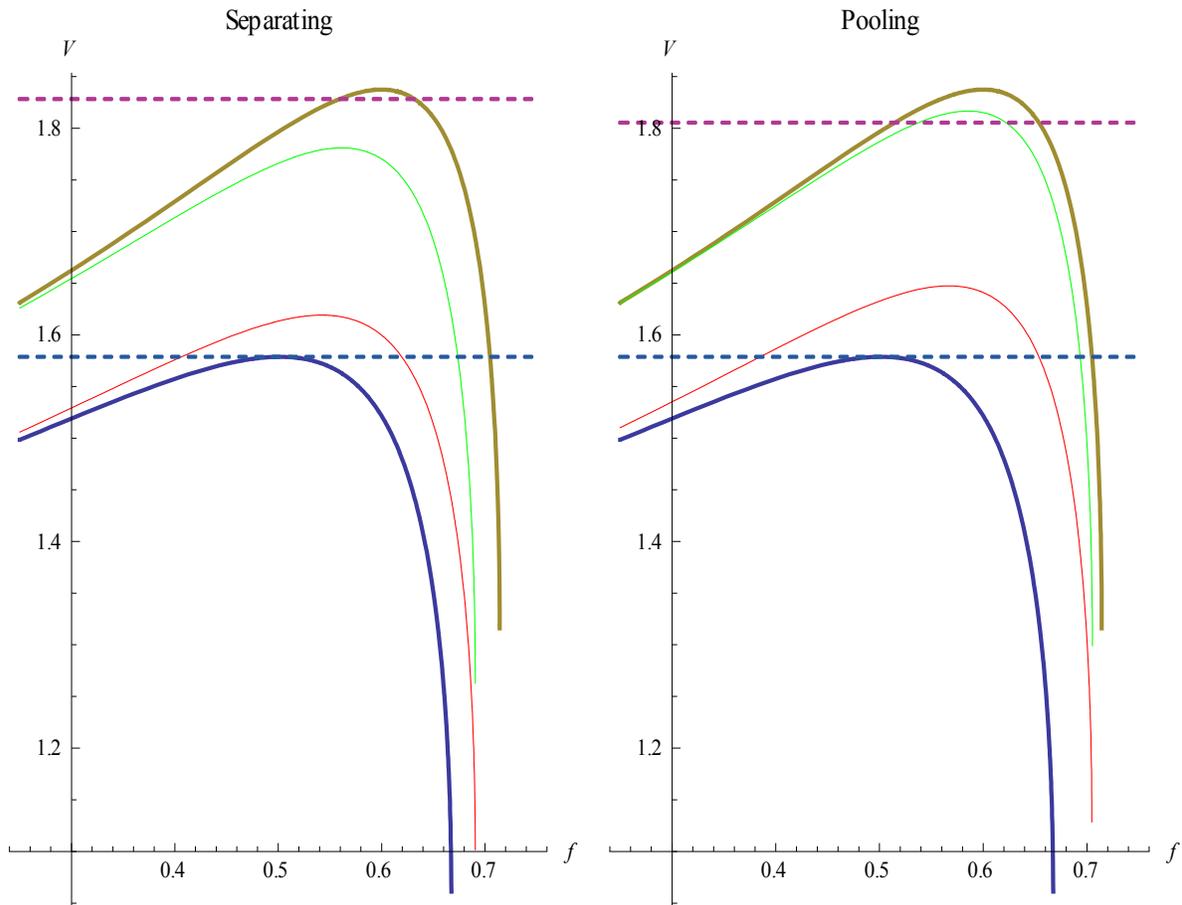
A particular parameterization that illustrates the possible equilibria is as follows:  $\alpha_L = 0.7$ ;  $\alpha_H = 0.75$ ;  $\beta = 0.75$ ; and  $s=1.05$ . As  $\theta$  varies from 0 to 1, the equilibrium shifts: at  $\theta = 0.5$ , for example, there is separating, but at  $\theta = 0.8$  there is pooling.<sup>8</sup> The values of principal variables are presented below: when  $\theta = 0.5$ , the high-return bank can achieve expected profitability of 1.828 by separating, with financing of 0.632, but only 1.778 in pooling, with financing of 0.561. When  $\theta = 0.8$ , achieving separation yields 1.805 for the strong bank; accepting pooling yields 1.812, with financing of 0.584, while the profitability of the low-yield bank rises from 1.579 to 1.646 and its financing rises from 0.502 to 0.584.

		$\theta = 0.5$	$\theta = 0.8$
Full information			
$V^*(\alpha_L, \alpha_L, F^f(\alpha_L))$	1.579	1.579	1.579
$F^f(\alpha_L)$	0.502	0.502	0.502
$V^*(\alpha_H, \alpha_H, F^f(\alpha_H))$	1.838	1.838	1.838
$F^f(\alpha_H)$	0.600	0.600	0.600
Pooling			
$V(\alpha_L, \underline{\alpha}, F^p(\alpha_H))$		1.618	1.646
$F^p(\alpha_L)$		0.561	0.584
$V^*(\alpha_H, \underline{\alpha}, F^p(\alpha_H))$		1.778	1.812
$F^p(\alpha_H)$		0.561	0.584
Separating			
$V(\alpha_H, \alpha_H, F^s(\alpha_H))$		1.828	1.805
$F^s(\alpha_H)$		0.632	0.654

<sup>8</sup> Calculations were performed in Mathematica.

The two equilibria are illustrated below. The lower dashed line is the maximum of the weak bank's value function under separating; the weak bank will imitate the strong bank if so doing yields more than this level. The upper dashed line represents the value to the strong bank when the separating condition is met. In the left-hand chart, the upper dashed line is above the strong bank's value function under pooling, so it has an incentive to separate. In the right-hand chart, the upper dashed line intersects the strong bank's value function under pooling, so pooling is superior for the strong bank.

### Parameterized Examples of Separating and Pooling Equilibria



### C. Separating Equilibrium with Partial Information and a Continuum of Bank Types

While some markets might be characterized as including distinct groups of strong and weak banks, in other cases it is better to think of banks as occupying locations across a continuous range of investment opportunities. Depositors will not need to identify whether a bank falls into one or the other group, but rather, where in the range it is located. Each bank will need to

determine its optimal behavior when there are always near neighbors with slightly better or slightly worse investment opportunities.

If a fully separating equilibrium exists, almost all banks disburse are less well capitalized than they would under full symmetric information. Each bank wants to distinguish itself from those with worse investment opportunities by reducing capitalization from what it would be under full information; a weaker bank would find imitation too costly in terms of increasing risk of failure (the incentive compatibility constraint). Only the bank with the worst investment opportunities does not need to distinguish itself from yet worse banks. Instead, it would like to be conflated with a bank with higher expected yields, but the incentive compatibility constraint is respected, so emulating the better banks is too costly in terms of additional risk. Hence, the financing ratio of that worst bank is the same as under full symmetric information, and that of all others is higher. It follows that asymmetry of information about the riskiness of banks' portfolios yields lower capitalization and corresponding higher probabilities of bank failure.

The results of Mailath (1987) provide a means of determining whether such a fully separating equilibrium could exist in such a market, and some of its characteristics. In a "fully" or "pure" separating equilibrium, there is no pooling anywhere in the range. In general, however, there may be additional equilibria, with pooling among some sub-set of banks.

In particular, assume that banks are characterized by a range of values of  $\alpha$  such that  $\alpha \in [\tilde{\alpha}, \hat{\alpha}]$ . Equation (7) is the objective function for a bank with a particular  $\alpha$ , and (5) and (6) to implicitly define  $\hat{e}$  and  $\tilde{e}$ . Define  $F = \varphi(\alpha)$  as the increasing function relating optimal financing to bank type under full, symmetric information. Under asymmetric information, Mailath shows that, if certain regularity conditions are met and a single crossing condition obtains, then there exists a strictly monotonic strategy  $F = \tau(\alpha)$  that satisfies the incentive compatibility constraint, i.e., that a fully separating equilibrium exists. The regularity conditions is presented in the appendix.

Strict incentive compatibility requires that

$$\{\tau(\alpha)\} = \arg \max_{y \in \tau([\tilde{\alpha}, \hat{\alpha}])} V(\alpha, \tau^{-1}(y), y), \quad \forall \alpha \in [\tilde{\alpha}, \hat{\alpha}]. \quad (25)$$

Some stronger results are obtained when a certain initial value condition obtains; in the case here, the initial value condition is that, when subject to the incentive compatibility constraint, the bank with the worst investment opportunities behaves as if there were full information.

The single crossing condition is that  $V_3/V_2$  is a strictly monotonic function of  $\alpha$ . In this model

$$\begin{aligned}
\frac{V_3(\alpha, \tilde{\alpha}, C)}{V_2(\alpha, \tilde{\alpha}, C)} &= \frac{\frac{1}{1-F} \left[ V - \int_{\hat{\varepsilon}}^{\bar{\varepsilon}} \left( \beta \frac{\partial \hat{\varepsilon}}{\partial F} \right) g(\varepsilon) \right]}{\frac{1}{1-F} \int_{\hat{\varepsilon}}^{\bar{\varepsilon}} \left( -\beta \frac{\partial \hat{\varepsilon}}{\partial \tilde{\alpha}} \right) g(\varepsilon)} \\
&= \frac{V - (\partial \hat{\varepsilon} / \partial F)(1 - G(\hat{\varepsilon}))}{-\beta (\partial \hat{\varepsilon} / \partial \tilde{\alpha})(1 - G(\hat{\varepsilon}))} = \frac{V - (\partial \hat{\varepsilon} / \partial F)}{-\beta (\partial \hat{\varepsilon} / \partial \tilde{\alpha})}, \tag{26}
\end{aligned}$$

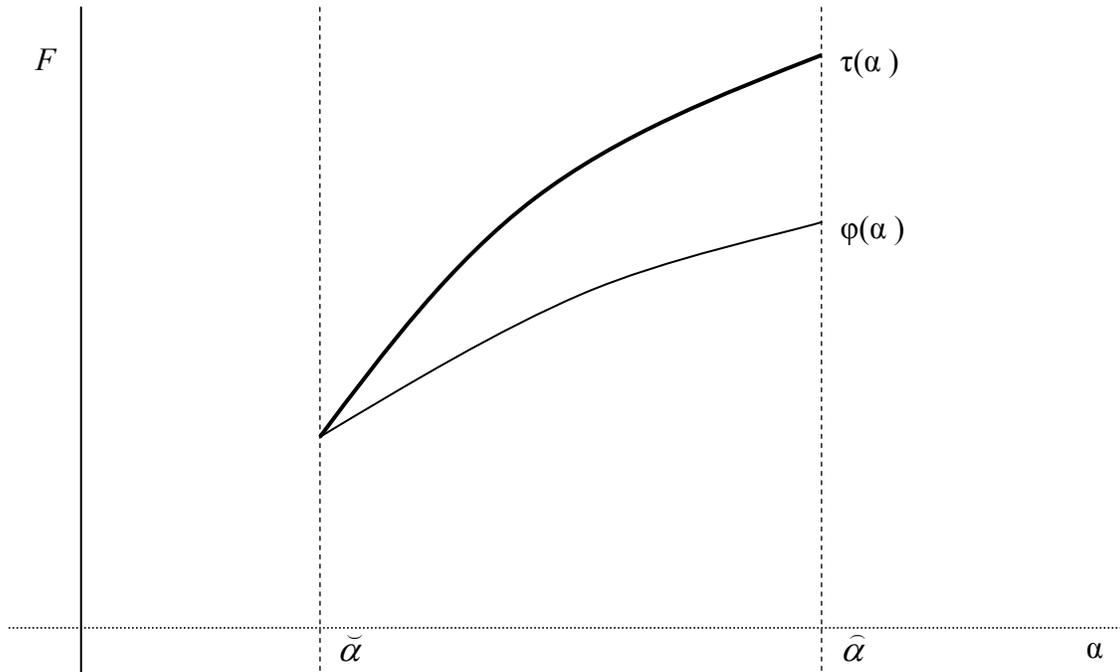
using (9) and (11). The partial derivatives that appear in the denominator and numerator are both independent of  $\alpha$ . Hence,  $V_3/V_2$  is a strictly monotonic function of  $\alpha$  because  $V$  is a strictly monotonic function of  $\alpha$ . The single crossing condition is met. Intuitively, a bank with better investment opportunities has higher marginal profits from obtaining more financing than does a worse bank ( $V_3$  is increasing in  $\alpha$ ), while the margin value of maintaining a good reputation among depositors is less sensitive to actual  $\alpha$ . Hence, the better bank can afford to decrease capitalization until a worse bank does not wish to keep up.

The initial value condition is also met. Intuitively, the worst bank does not need to decrease capitalization because it does not need to distinguish itself from an even worse bank, but it is not worthwhile to imitate a stronger bank, so that bank behaves as under the full information equilibrium. More formally, because  $V_2 > 0$ , we consider the bank with the worst investment opportunities, that is, with  $\alpha = \tilde{\alpha}$ , and use *reduction ad absurdum*. Suppose first that, although separating obtains,  $\tau(\tilde{\alpha}) > \varphi(\tilde{\alpha})$ , that is, the incentive compatible level of financing for this bank is above the optimal full information level. Yet, the bank could then do better by reducing the level of its financing to  $\varphi(\tilde{\alpha})$ , without any deterioration in depositors' estimation of its investment opportunities (which is already revealed) and thus its probability of default; by definition  $V(\tilde{\alpha}, \tilde{\alpha}, \varphi(\tilde{\alpha})) \geq V(\tilde{\alpha}, \tilde{\alpha}, C)$ ,  $C \neq \varphi(\tilde{\alpha})$ . Also, since this bank is at the extreme of the range of values of  $\alpha$ , there is no worse bank that will imitate it. Hence, this level of financing is not incentive compatible. Suppose instead that  $\tau(\tilde{\alpha}) < \varphi(\tilde{\alpha})$ . Then once more the bank with the worst investment opportunities could do at least as well by increasing the level of its credit to  $\varphi(\tilde{\alpha})$ : if depositors still believe that it has the worst investment opportunities, that level of financing yields higher expected profits. Hence, the lower level of financing is not consistent with the incentive compatible constraint. Furthermore, if  $\tau$  is continuous, higher financing from the worst bank would involve pooling with a somewhat better bank, and further increase the former's expected profits. Hence the only incentive compatible level of financing for the bank with the worst investment opportunities is that which is optimal under full, symmetric information.

Therefore, by Mailath's results and under certain conditions, a unique separating equilibrium exists. Furthermore, under this equilibrium, all banks obtain more financing than under full information, with the exception of the bank with the worst investment opportunities, which obtains the same amount. Hence, average capitalization and the share of banks that fail is

higher than under symmetric information, that is, when financiers are familiar with banks' investment opportunities. The relation between  $F$  and  $\alpha$  under symmetric information ( $\varphi(\alpha)$ ) and the separating equilibrium ( $\tau(\alpha)$ ) is shown below.

Financing and Bank Characteristics for a Continuum of Types



These results do not exclude the possibility of pooling or partially pooling equilibria, but they are not more comforting. If there is pooling across some range of banks, some of the banks with better investment opportunities will seek higher capitalization than under full information, but banks with lower expected yields will obtain lower capitalization and take on more risks. Furthermore, capitalization ratios will not be informative about the riskiness of the banks in the pool.

#### IV. EXTENSIONS

The model can be extended in various directions to yield plausible explanations of various other phenomena related to capitalization over time, a richer set of preferences and instruments, and the introduction of new loan technology.

##### A. Signaling over the Cycle

So far consideration has focused on a signaling game played by a range of banks at a point in time. One can also envisage that the game is played by one or more banks over time, where in some periods they receive information that economic conditions will be favorable, and at other times they will receive adverse information. Under some conditions, banks may wish to

signal positive news by lowering capitalization, or to mask unfavorable information by reducing capitalization as they would when good times are expected.

Suppose that time is discrete, and at the start of each period  $t$  the bank learns the value of the parameter  $\alpha_t$ , which follows a Markov process, that characterizes returns in the coming period. The value of  $\alpha_t$  has a continuous distribution function  $h$  on a support  $(\underline{\alpha}, \bar{\alpha})$ .<sup>9</sup> The bank then decides its level of capitalization. The choice of capitalization needs to take into account the value of staying in business to enjoy future profits.

Let  $W_t$  denote the expected net present value of the bank before a bank knows the realization of  $\alpha_t$ , and let  $\rho$  be the discount factor ( $\rho < 1$ ). The expected net present value will comprise the returns in period  $t$ , summed across values of  $\alpha_t$ , and conditional on the bank surviving, plus the discounted net present value next period. Thus,

$$W_t = \int_{\underline{\alpha}}^{\bar{\alpha}} \int_{\hat{e}_t}^{\bar{e}} \left( \frac{(\alpha_t + \beta e_t - iF_t)}{1 - F_t} + \rho W_{t+1} \right) g(e) h(\alpha_t). \quad (27)$$

Likewise, the break-even constraints take the bank's net present value into account:

$$\alpha_t + \beta \hat{e}_t - iF_t + \rho W_t = 0, \quad (28)$$

$$\tilde{\alpha}_t + \beta \tilde{e}_t - iF_t + \rho W_t = 0. \quad (29)$$

Once the bank learns the value of  $\alpha_t$ , its objective function becomes:

$$W_t = \int_{\hat{e}_t}^{\bar{e}} \left( \frac{(\alpha_t + \beta e_t - iF_t)}{1 - F_t} + \rho W_{t+1} \right) g(e), \quad (30)$$

which can be simplified to (7) using (28) and (29). Since  $W_{t+1}$  does not depend on any the value of any variable in period  $t$ , all the conditions for a signaling equilibrium can still be met. The expected value of future profits will affect the actual break-even point  $\hat{e}$  and the break-even point  $\tilde{e}$  estimated by depositors, but not the forms of the various conditions on derivatives such as equation (12).

It follows that there may be a separating equilibrium, where the bank receives information that the coming period will be favorable, but it exaggerates lower capitalization in order to send a credible signal that they do not expect a weak period. Under other parameter values,

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<sup>9</sup> Alternately,  $\alpha_t$  could take on a high or low level with probability  $\theta$  and  $1-\theta$ , respectively.

there may be a pooling equilibrium, where banks expect difficult trading conditions but do not raise capitalization in order to encourage cheaper financing.

It is possible that the type of equilibrium that obtains varies depending on conjunctural conditions. For example, it may be that imitation is relatively cheap for weak banks during good times, and the stronger banks have little incentive to differentiate themselves. Hence, a pooling equilibrium can be seen during the peak of a boom. However, the incentives for a stronger bank to signal its strength may be much greater during a downturn or the initial phases of a recovery, so separating is seen—although, in these conditions, the weaker banks have more incentive to imitate.<sup>10</sup> With multiple equilibria, it is possible that observed behavior switches abruptly. It is indeed suggestive that bank CDS rates went from uniformly low and stable to high and variable once financial strains became more obvious in mid-2007.

### **B. Risk Aversion**

The introduction of risk aversion complicates the analysis considerably, and it is difficult to draw conclusions without imposing restrictive assumptions, for example, about how the coefficient of relative risk aversion varies. Intuitively, if the owners of weak banks are risk averse, they would be less inclined to reduce capitalization in order to imitate the strong banks, because they incur a greater risk of bankruptcy. However, if the owners of strong banks are risk averse, they would be less inclined to take on the extra risks associated with a separating strategy.

Risk aversion on the part of depositors and other outside financiers is likely to increase the incentives for pooling, because banks perceived to be riskier will have to pay higher risk premia. However, pooling with weak banks will increase the funding costs for strong banks more, the more risk averse are depositors, so the strong banks will have more incentive to attempt a separating strategy.

### **C. Investment in Loan Technology**

In the model of Hardy and Tieman (2008), the prevalence of signaling implies that an investment in better loan technology does not merely yield a direct pay-off in terms of higher returns to lending, but also reduces the cost of signaling: a high-yield bank that invests will find it cheaper to differentiate itself from a low-yield bank, and the latter will find it cheaper to imitate the former. Whether there is pooling or separating, investment may be greater than it would be under full information. Similar reasoning applies here if banks can invest in

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<sup>10</sup> The U.S. government forced all major banks to accept support under the TARP program, thus imposing a form of pooling. Banks' efforts to replay support rapidly represent can be interpreted as each one's efforts to separate itself.

developing better investment opportunities: there are incentives for strong banks to differentiate themselves, and for weak banks to catch up.

## V. SUMMARY AND CONCLUSIONS

It is demonstrated in this paper that, under certain conditions, the level of bank capital can effectively transmit information about the riskiness of a bank's assets and therefore support market discipline. Capitalization can be a useful and effective signal when the marginal cost of lower capitalization—in terms of a higher probability of default—is less, the better is the bank's loan book. Depositors and other outside investors will reward the banks that they perceive to be more sound with cheaper financing. However, the asymmetry in information between insiders and outsiders that makes capitalization a valuable signal may also induce exaggerated or distortionary behavior. Thus, in order to distinguish itself from the signal a weak bank might produce, an otherwise strong bank might need to hold its capitalization level below where it would be in the full-information situation. As marginal costs for a weak bank are always higher than for a strong bank, a separating point may exist. In case the profits beyond this point remain higher than the profits the strong bank could reap when the market cannot distinguish it from the other bank (as is the case in a pooling equilibrium), the strong bank will rationally lower capitalization and take on more risk over and above its full-information level. In a pooling equilibrium, it is the weak bank that decreases its capitalization.

The banks that were eager to repay government capital assistance as soon as possible were on good grounds to believe that this action would be taken as a signal of strength and contribute to lower funding costs. Nonetheless, some weaker banks may have felt pressured to repay the government assistance at the same time as others in order not to reveal weaknesses in their balance sheets. The same signaling motives and asymmetric information may help explain why there was not more differentiation across banks before the crisis; few banks would wish to draw attention to themselves by accumulating capital, when such action could be taken as a sign of exposure to large risks. Likewise, even if a bank saw difficult conditions approaching, it may wish to keep its funding costs down by not revealing this information through increasing capitalization in good times.

These possible phenomena suggest that market discipline is unlikely to be enough to achieve an optimal level of bank capitalization, even before taking into account systemic inter-linkages or moral hazard effects associated with government guarantees and bail-outs. It is true that banks typically disclose to the market a great deal of financial information, and allow themselves to be rated. It is true also that banks raise funding in many forms, with many contingent features that blend equity and debt characteristics. However, asymmetric information is inherent in the banking business, so such complications are unlikely to eliminate the relevance of the signaling mechanisms discussed here.

It follows that supervisors can and should play an important role in mitigating these effects of asymmetric information on financial system soundness and the tendency to under-capitalization. Supervisors can compel banks to disclose information about their portfolios, and check that this information is accurate. Even imperfect information should reduce the scope for pooling and thus the incentive to signal through over-leveraging. Furthermore, supervisors have the power to investigate and assess the quality of a bank's portfolio in detail, and can impose higher capital requirements under the so-called Pillar 2 provisions of the Basel II and Basel III capital accords. In this connection, it may occur that a supervisor discovers that a certain bank has a weak portfolio and therefore mandates it to increase capitalization. One benefit of such action is that the incentive for other banks to differentiate themselves by being less well capitalized is reduced. Finally, the tendency to send over-optimistic signals through excessively low capitalization even in good times is one justification for current efforts to require banks to take a more forward-looking approach to capital raising and to require the accumulation of greater reserves during favorable periods.

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#### APPENDIX I: REGULARITY CONDITIONS ON THE OBJECTIVE FUNCTION WITH A CONTINUUM OF BANK TYPES

Mailath (1987, page 1352) lists a number of regularity conditions for his results to hold. They are:

- 1) Smoothness: assuming that  $g$  is at least once continuously differentiable, the integral (4) twice continuously differentiable.
- 2) Belief monotonicity: Equation (9) has been shown to be always positive (and thus never zero).
- 3) Type monotonicity: Equation (12) shows that

$$\begin{aligned} V_{13}(\alpha, \tilde{\alpha}, F) &= \frac{(1-G(\tilde{e}))}{(1-F)^2} - \frac{g(\tilde{e})}{1-F} \frac{\partial \hat{e}}{\partial F} \\ &= \frac{1}{(1-F)^2} \left[ 1 - G(\hat{e}) - \frac{g(\hat{e})(1-F)s(1-G(\tilde{e}))}{(1-G(\tilde{e}))^2 \beta - sFg(\tilde{e})} \right]. \end{aligned}$$

In general (12) can be either positive or negative. It is noteworthy that as  $F$  tends to 1,  $G(\hat{e})$  will tend to a level less than unity, while the second term in square brackets will tend to zero, so (12) is positive. In a neighborhood around  $F = 0$ , the bank never fails, so  $G(\hat{e}) = g(\hat{e}) = 0$  and again (12) is positive. The non-fulfillment of this condition can occur only if  $g$  becomes large for an intermediary value of  $F$ .

- 4) “Strict” quasiconcavity: The condition  $V_3(\alpha, \alpha, F) = 0$  is provided in equation (13). Given (14) and the assumption that  $g' \geq 0$ , equation (15) shows that  $V_{33}(\alpha, \alpha, \varphi(\alpha)) < 0$ , so there is a unique solution to (13).
- 5) Boundedness: Mailath (page 1356) shows that, if the space of possible actions is bounded, then the boundedness condition is always met. In this case,  $F$  is bounded on the interval  $[0, 1]$ , so this condition is always fulfilled.