



IMF Working Paper

A General Equilibrium Model of Sovereign Default and Business Cycles

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Research Department

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Abstract

Emerging markets business cycle models treat default risk as part of an exogenous interest rate on working capital, while sovereign default models treat income fluctuations as an exogenous endowment process with ad-noc default costs. We propose instead a general equilibrium model of both sovereign default and business cycles. In the model, some imported inputs require working capital financing; default on public and private obligations occurs simultaneously. The model explains several features of cyclical dynamics around default triggers an efficiency loss as these inputs are replaced by imperfect substitutes; and default on public and private obligations occurs simultaneously. The model explains several features of cyclical dynamics around defaults, countercyclical spreads, high debt ratios, and key business cycle moments.

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1 Introduction

Episodes of sovereign default are characterized by a striking set of empirical regularities. In particular, the event windows plotted in Figure 1 for a cross-country sample of 23 default events in the 1977-2009 period highlight the following three facts:

(1) *Default events are associated with deep recessions.* On average, GDP and consumption fall about 5 percent below trend, and imported inputs and total intermediate goods fall nearly 20 percent below trend. Labor falls to a level about 15 percent lower than in the three years prior to the defaults. Net exports jump about 10 percentage points of GDP in the span of the two quarters before and after default events. These observations are in line with the findings of Levy-Yeyati and Panizza (2006) showing that default events coincide with large GDP drops in an event analysis for 39 developing countries covering the 1970-2005 period. In addition, Tomz and Wright (2007) studied defaults from 1820 to 2004 and found the maximum default frequency when output is at least 7 percent below trend.

(2) *Interest rates on sovereign debt peak at about the same time as output hits its trough and defaults occur, and they are negatively correlated with GDP.* These two empirical regularities are visually evident by comparing the output and interest rate plots in Figure 1. In addition, Neumeyer and Perri (2005) and Uribe and Yue (2006) report cyclical correlations between GDP and country interest rates ranging from zero to -0.8, with averages of -0.55 in Neumeyer and Perri and -0.42 in Uribe and Yue.¹

(3) *External debt as a share of GDP is high on average, and higher when countries default.* The mean debt ratio before the default events in Figure 1 was about 50 percent, and reached about 72 percent at the time of the defaults. Looking at all emerging and developing countries, as defined in IMF (2006), foreign debt was 1/3 of GDP on average over 1998-2005. Highly indebted poor countries had the highest average debt ratio, at about 100 percent of GDP, followed by the Eastern European and Western Hemisphere countries, with averages of about 50 and 40 percent of GDP respectively. Looking at defaults historically, Reinhart et al. (2003) report that the external debt ratio averaged 71 percent of GDP for all developing country defaults in the 1824-1999 period. This is very close to the 72 percent mean estimate for our default events in Figure 1.

¹Neumeyer and Perri used data for Argentina, Brazil, Korea, Mexico and the Philippines. Uribe and Yue added Ecuador, Peru and South Africa, but excluded Korea.

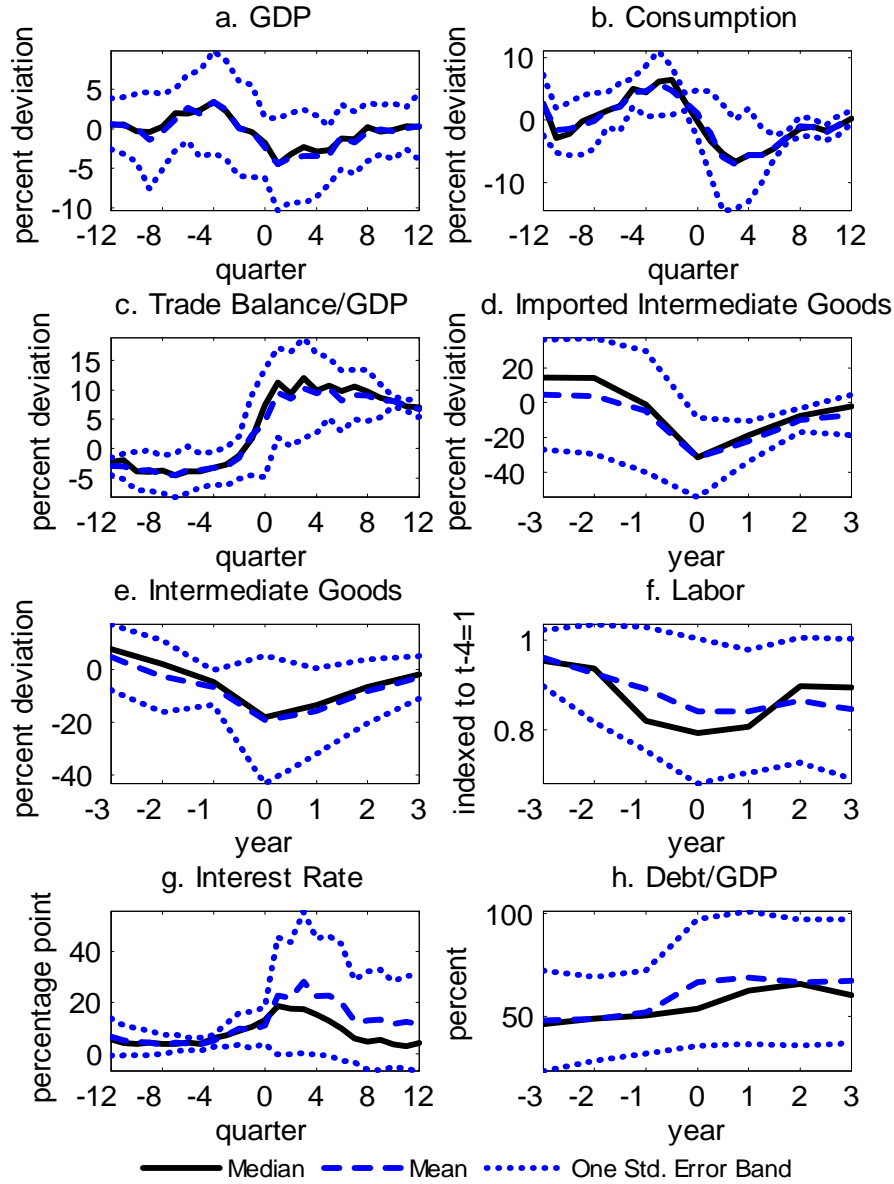


Figure 1: Macroeconomic Dynamics around Sovereign Default Events

Note: GDP, consumption, and trade balance/GDP are H-P detrended. Imported inputs and intermediate goods are log-linearly detrended. Labor data is indexed so that employment 4 years before default equals 1. The event window for GDP is based on data for 23 default events over the 1977- 2009 period. Due to data limitations, the sample period and/or the number of events varies in some of the other windows. Full details are provided in the Data Appendix.

It has proven difficult to provide a joint explanation of these stylized facts in International Macroeconomics, because of a crucial disconnect between two key bodies of theory: On one hand, quantitative models of business cycles in emerging economies explain countercyclical country interest rates by modeling the interest rate on sovereign debt as an exogenous interest rate charged on foreign working capital loans obtained by firms.² In these models, default is exogenous and hence facts (1) and (3) are left unexplained. On the other hand, quantitative models of sovereign default based on the classic setup of Eaton and Gersovitz (1981) generate countercyclical sovereign spreads by assuming that a sovereign borrower faces shocks to an exogenous output endowment with ad-hoc output costs of default.³ Since output is an exogenous endowment, these models cannot address fact (1) and they do poorly at explaining fact (3). In short, business cycle models of emerging economies cannot explain the default risk premia that drive their findings, and sovereign default models cannot explain the cyclical output dynamics that are critical for their results.

This paper proposes an equilibrium model of sovereign default and economic fluctuations that provides a solution to the disconnect between those two classes of models. The model features a transmission mechanism that links endogenous default risk with private economic activity via the financing cost of working capital used to pay for a subset of imported inputs. These subset of imported inputs can be replaced with other imported inputs or with domestic inputs, but these are only imperfect substitutes, and as a result default causes an endogenous efficiency loss in production of final goods.

The contribution of this framework is that it is the first to provide a setup in which the equilibrium dynamics of output and sovereign default are determined jointly, and influence each other via the interaction between foreign lenders, the domestic sovereign borrower, domestic firms, and households. In particular, a fall in productivity increases the likelihood of default and hence sovereign spreads, and this in turn increases the firms' financing costs causing an efficiency loss that amplifies the negative effects of productivity shocks on output. This in turn feeds back into default incentives and sovereign spreads.

Quantitative analysis shows that the model does well at explaining the three key stylized facts of sovereign defaults. Moreover, the model's financial amplification mechanism amplifies the effect of TFP shocks on output by a factor of 2.7 when the economy defaults, and the model matches salient features of emerging markets business cycles such as the high variability of consumption, the countercyclical dynamics of net exports, and the correlation between output and default.

These results hinge on three important features of the model: First, the assumption that producers of final goods require working capital financing to pay for imports of a subset of intermediate goods. Second, the efficiency loss in final goods production that occurs when the country defaults, because the loss of access to credit for some imported inputs forces firms to

²See Neumeyer and Perri (2005), Uribe and Yue (2006) and Oviedo (2005).

³See, for example, Aguiar and Gopinath (2006), Arellano (2008), Bai and Zhang (2005) and Yue (2010).

substitute into other imported and domestic inputs that are imperfect substitutes. Third, the assumption that the government can divert the private firms' repayment when it defaults on its own debt.

The above key features of the model are in line with existing empirical evidence. Amiti and Kronings (2007) and Halpern, Koren and Szeidl (2008) provide firm-level evidence of the imperfect substitutability between foreign and domestic inputs, and the associated TFP effect of changes in relative factor costs. In particular, they study the impact of reducing imported input tariffs on firm-level productivity using data for Indonesia and Hungary, and find that imperfect substitution of inputs accounts for the majority of the effect of tariff cuts on TFP. Gopinath and Neiman (2010) find important evidence of within-firm shifts from imported to domestic inputs in the Argentine debt crisis of 2001-2002. Reinhart and Rogoff (2010) and Reinhart (2010) show that there is a tight connection between banking crises, with widespread defaults in the nonfinancial private sector, and sovereign defaults, and that private debts become public debt after sovereign defaults.

The model's financial transmission mechanism operates as follows: Final goods producers use labor and an Armington aggregator of imported and domestic inputs as factors of production, with the two inputs as imperfect substitutes. Domestic inputs require labor to be produced. Imported inputs come in different varieties described by a Dixit-Stiglitz aggregator, and a subset of them needs to be paid in advance using foreign working capital loans. Under these assumptions, the optimal input mix depends on the country interest rate (inclusive of default risk), which is also the financing cost of working capital, and on TFP. When the country has access to world financial markets, final goods producers use a mix of all varieties of imported inputs and domestic inputs, and fluctuations in default risk affect the cost of working capital and thus induce "regular" fluctuations in factor demands and output. In contrast, when the country defaults, final goods producers substitute away from the imported inputs that require working capital financing, because of the surge in their financing cost. This reduces production efficiency sharply because of the imperfect substitutability across varieties of imported inputs and across domestic and foreign inputs, and because in order to increase the supply of domestic inputs labor reallocates away from final goods production.⁴

When the economy defaults, both the government and firms are excluded from world credit markets for some time, with an exogenous probability of re-entry as is common in quantitative studies of sovereign default. Since the probability of default depends on whether the sovereign's value of default is higher than that of repayment, there is endogenous feedback between the economic fluctuations induced by changes in default probabilities and country risk premia. In particular, rising country risk in the periods leading to a default causes a decline in economic

⁴As a result, part of the output drop that occurs when the economy defaults shows as a fall in the Solow residual (i.e. the fraction of *aggregate* GDP not accounted for by capital and labor). This is consistent with the data from emerging markets crises showing that a large fraction of the observed output collapse is attributed to the Solow residual (Meza and Quintin (2006), Mendoza (2010)). Moreover, Benjamin and Meza (2007) show that in Korea's 1997 crisis, the productivity drop followed in part from a sectoral reallocation of labor.

activity as the firms' financing costs increase. In turn, the expectation of lower output at higher levels of country risk alters repayment incentives for the sovereign, affecting the equilibrium determination of default risk premia.

A key feature of our model is that the efficiency loss caused by sovereign default generates an endogenous output cost that is an increasing, convex function of TFP. This differs sharply from the two approaches followed to model ad-hoc costs of default in the literature. One approach models default costs as a fixed percentage of the realization of an exogenous endowment when a country defaults (e.g. Aguiar and Gopinath (2006), Yue (2010)). In this case, default is just as costly, in percentage terms, in a low-endowment state as in a high-endowment state (i.e. the percent cost is independent of the endowment realization), and hence average debt ratios are low when the models are calibrated to actual default frequencies. The second approach is the asymmetric formulation proposed by Arellano (2008). Below a certain threshold endowment level, there is no cost of default, and above it the sovereign's income is reduced to the same constant level regardless of the endowment realization. Thus, in the latter case the percent cost of default increases linearly with the endowment realization. This formulation makes default more costly in good states, making default more likely in bad states and increasing debt ratios. However, debt ratios in calibrated models are still much lower than in the data, unless features like multiple maturities, dynamic renegotiation or political uncertainty are added.⁵

The default cost in our model is a general equilibrium outcome driven by the effects of sovereign risk on private markets. This endogenous cost adds "state contingency" to the default option, allowing the model to support higher mean debt ratios at observed default frequencies. Our baseline calibration supports a mean debt-output ratio of 23 percent, nearly four times larger than in Arellano (2008). In addition, in our model outputs costs of default are always incurred at equilibrium, whereas with Arellano's formulation defaults occur mostly when the endowment is lower than the threshold endowment value, so actual costs of default are zero at equilibrium. Moreover, in our setup, output itself falls sharply when the economy defaults, because the model's financial transmission mechanism amplifies the effects of TFP shocks on output. In contrast, in existing sovereign default models, large output drops can only result from large, exogenous endowment shocks.

The assumptions that both foreign and domestic inputs *and* the varieties of imported inputs are imperfect substitutes are critical for the above properties of the default cost.⁶ The cost

⁵Arellano (2008) obtained a mean debt-output ratio of 6 percent using her asymmetric cost. Aguiar and Gopinath (2006) obtained a mean debt ratio of 19 percent using the fixed percent cost, but at a default frequency of only 0.23 percent. Yue (2010) used the same cost in a model with renegotiation calibrated to observed default frequencies, and obtained a mean debt ratio of 9.7 percent. Studies that have obtained higher debt ratios with modifications of the Eaton-Gersovitz environment, but still assuming exogenous endowments, include: Cuadra and Sapriza (2008), D'Erasmus (2008), Bi (2008a) and (2008b), Chatterjee and Eyigungor (2008), Benjamin and Wright (2008), and Lizarazo (2005).

⁶If the inputs are perfect substitutes there is no output cost of default, because firms can shift inputs without affecting production and costs. If they are complements, production is either zero (with unitary elasticity of substitution) or not defined (with less-than-unitary elasticity) when the economy defaults and cannot access imported inputs.

is higher and becomes a steeper function of TFP at lower elasticities of substitution, because the inputs become less similar. The elasticity of labor supply also influences the output cost of default. In particular, the cost is larger the higher this elasticity, because default triggers a reduction in total labor usage. However, output costs of default, and the efficiency loss that drives them, are still present even if labor is inelastic. Final goods producers still have to shift from a subset of imported input varieties to other imported inputs and to domestic inputs, and labor still reallocates from final goods to intermediate goods production.

The treatment of the financing cost of working capital in this paper differs from the treatment in Neumeyer and Perri (2005) and Uribe and Yue (2006), who treat this cost as an exogenous variable calibrated to match the interest rate on sovereign debt. In contrast, in our setup both interest rates are driven by endogenous sovereign risk. In addition, in the Neumeyer-Perri and Uribe-Yue models, working capital loans pay the wages bill in full, while in our model firms use working capital to pay only for a subset of imported intermediate goods. This lower working capital requirement is desirable because empirical estimates suggest that working capital is a small fraction of GDP (Schmitt-Grohe and Uribe (2007) estimate 9.3 percent annually for the United States, we estimate 6 percent for Argentina in Section 4).

Our analysis is also related to the literature documenting explicit and implicit sanctions on trade flows and trade credit in response to sovereign defaults. Both are relevant for our analysis because the implications of our model are identical whether default triggers exclusion from trade credit or trade sanctions affecting imports of some intermediate goods. Kaletsky (1985) argued that exclusion from trade credit might be the heaviest penalty that a defaulter faces. He documented the exclusion from trade credit experienced by the countries that defaulted in the 1980s, and showed estimates of short-term private credit nearly as large as unpaid interest in medium-term sovereign debt. More recently, Kohlscheen and O’Connell (2008) showed evidence of sharp declines in trade credit from commercial banks during default episodes. Rose (2005) conducted a cross-country analysis of trade flows and default, and found that default has a large, persistent negative effect on bilateral trade between creditor and debtor countries, and Martinez and Sandleris (2008) provided further empirical evidence on the association between sovereign defaults and the decline in trade.

The rest of the paper proceeds as follows: Section 2 presents the model. Section 3 examines the effects of interest rate changes on production and factor allocations in partial equilibrium. Section 4 explores the full model’s quantitative implications. Section 5 concludes.

2 A Model of Sovereign Default and Business Cycles

There are four groups of agents in the model, three in the “domestic” small open economy (households, firms, and the sovereign government) and one abroad (foreign lenders). There are also two production sectors in the domestic economy, a sector f of final goods producers and a

sector m of intermediate goods producers.

2.1 Households

Households choose consumption and labor supply so as to maximize a standard time-separable utility function $E \left[\sum_{t=0}^{\infty} \beta^t u(c_t - g(L_t)) \right]$, where $0 < \beta < 1$ is the discount factor, and c_t and L_t denote consumption and labor supplied in period t respectively. $u(\cdot)$ is the period utility function, which is continuous, strictly increasing, strictly concave, and satisfies the Inada conditions. Following Greenwood, Hercowitz and Huffman (1988), we remove the wealth effect on labor supply by specifying period utility as a function of consumption net of the disutility of labor $g(L_t)$, where $g(\cdot)$ is increasing, continuously differentiable and convex. This formulation of preferences plays an important role in allowing international real business cycle models to explain observed business cycle facts, and it also simplifies the “supply side” of the model.⁷

Households take as given the wage rate w_t , profits paid by firms in the f and m sectors (π_t^f, π_t^m) and government transfers (T_t) . Households do not borrow directly from abroad, but the government borrows, pays transfers, and makes default decisions internalizing their utility.⁸ Consequently, the households’ optimization problem reduces to:

$$\max_{c_t, L_t} E \left[\sum \beta^t u(c_t - g(L_t)) \right], \quad (1)$$

$$s.t. \quad c_t = w_t L_t + \pi_t^f + \pi_t^m + T_t. \quad (2)$$

The optimality condition for labor supply is:

$$g'(L_t) = w_t. \quad (3)$$

For purposes of the quantitative analysis, we define $g(L) = \frac{L^\omega}{\omega}$ with $\omega > 1$. Hence, the Frisch elasticity of labor supply is given by $1/(\omega - 1)$. The period utility function takes the standard constant-relative-risk-aversion form $u(c, L) = \frac{(c - L^\omega/\omega)^{1-\sigma} - 1}{1-\sigma}$ with $\sigma > 0$.

2.2 Final Goods Producers

Firms in the f sector produce using labor L_t^f and intermediate goods M_t , and a time-invariant capital stock k .⁹ They face Markov TFP shocks ε_t with a transition probability distribution

⁷Removing the wealth effect on labor supply is useful because otherwise the wealth effect pushes labor to display a counterfactual rise when TFP falls or when consumption drops sharply, as is the case in default episodes.

⁸This assumption is very common in the Eaton-Gersovitz class of models but it is not innocuous, because whether private foreign debt contracts are allowed, and whether they are enforceable vis-a-vis government external debt, affects the efficiency of the credit market equilibrium (see Wright (2006)).

⁹Sovereign debt models generally abstract from capital accumulation for simplicity. Adding capital makes the recursive contract with default option significantly harder to solve because it adds an additional endogenous state variable. Moreover, changes in the capital stock have been estimated to play a small role in output dynamics around financial crises (see Meza and Quintin (2006) and Mendoza (2007)).

function $\mu(\varepsilon_t|\varepsilon_{t-1})$. The production function is Cobb-Douglas:

$$y_t = \varepsilon_t \left(M \left(m_t^d, m_t^* \right) \right)^{\alpha_M} (L_t^f)^{\alpha_L} k^{\alpha_k} \quad (4)$$

with $0 < \alpha_L, \alpha_M, \alpha_k < 1$ and $\alpha_L + \alpha_M + \alpha_k = 1$.

The mix of intermediate goods is determined by a standard CES Armington aggregator that combines domestic inputs m_t^d and imported inputs m_t^* , with the latter represented by a Dixit-Stiglitz aggregator that combines a continuum of differentiated imported inputs m_j^* for $j \in [0, 1]$:

$$M_t = \left[\lambda \left(m_t^d \right)^\mu + (1 - \lambda) \left(m_t^* \right)^\mu \right]^{\frac{1}{\mu}}, \quad m_t^* \equiv \left[\int_{j \in [0,1]} \left(m_{jt}^* \right)^\nu dj \right]^{1/\nu} \quad (5)$$

The firms' purchases of variety j of imported inputs are denoted by m_{jt}^* . The "within" elasticity of substitution across all varieties is given by $\eta_{m_j^*} = |1/(v - 1)|$. The Armington elasticity of substitution between m_t^* and m_t^d is defined as $\eta_{m^d, m^*} = |1/(\mu - 1)|$ and λ is the Armington weight of domestic inputs.¹⁰ The following parameter restrictions are assumed to hold: $0 < \nu, \mu < 1$ and $0 \leq \lambda < 1$. $\lambda < 1$ is necessary because without use of imported inputs default would be costless. In addition, foreign and domestic inputs and the varieties of imported inputs need to be imperfect substitutes (i.e. $0 < \nu, \mu < 1$) in order for the output cost of default to increase with ε , as we show later.

Imported inputs are sold in world markets at exogenous time-invariant prices p_j^* for $j \in [0, 1]$ defined in terms of the price of final goods, which is the numeraire. The relative price of domestic inputs p_t^m is an endogenous equilibrium price.

A subset Ω of the imported input varieties defined by the interval $[0, \theta]$, for $0 < \theta < 1$, needs to be paid in advance using working capital financing.¹¹ The rationale for splitting imported inputs this way is to provide for a flexible treatment of imported inputs, so that in default episodes, when access to the set Ω of imported inputs is hampered by exclusion from credit markets, imported inputs do not vanish, even though they adjust sharply, as observed in the data.

We model working capital following the classic pay-in-advance setup of Fuerst (1992) and Christiano and Eichenbaum (1992), which is also widely used in business cycle models of emerging economies (e.g. Neumeyer and Perri (2005), Uribe and Yue (2006), Mendoza (2010)). Working capital loans κ_t are intraperiod loans repaid at the end of the period that are obtained from

¹⁰This structure of aggregation of imported and domestic inputs is similar to those used in the empirical work of Gopinath and Neiman (2010) and Halpern, Koren and Szeidl (2009).

¹¹We assume that the entire cost of purchasing the varieties in Ω needs to be paid in advance. Hence, θ determines the "intensity" of the working capital friction in a similar way as the standard working capital models use θ to define the fraction of the cost of a single input that is paid in advance (e.g. Neumeyer and Perri (2005) and Uribe and Yue (2006)). We could also introduce an extra parameter so that the varieties in Ω require that only a fraction of their cost be paid in advance, but lowering this fraction would have similar effects as keeping the fraction at 100 percent and lowering Ω instead.

foreign creditors at the interest rate r_t . This interest rate is linked to the sovereign interest rate at equilibrium, as shown in the next section.

In our model, the standard pay-in-advance condition driving the demand for working capital is:

$$\frac{\kappa_t}{1+r_t} \geq \int_0^\theta p_j^* m_j^* dj. \quad (6)$$

Profit-maximizing producers of final goods choose κ_t so that this condition holds with equality. Domestic inputs and the varieties of imported inputs in the $[\theta, 1]$ interval do not require working capital, but this assumption is just for simplicity, the key element is that at high levels of country risk (including periods without access to foreign credit markets) the financing cost of the set Ω of foreign inputs is higher than that of other inputs. Moreover, it is reasonable to assume that trade in domestic inputs is largely intra-firm trade and is at least partially collateralized by the goods themselves, whereas this mechanism may not work as well for imported inputs because of government interference with payments via confiscation or capital controls, which are common during default episodes—as was clearly evident in Argentina’s 2001 default.

Final goods producers choose factor demands in order to maximize date- t profits taking w_t , r_t , p_j^* , and p_t^m as given. Date- t Profits are:

$$\pi_t^f = \varepsilon_t (M_t)^{\alpha_M} (L_t^f)^{\alpha_L} k^{\alpha_k} - \int_0^1 p_j^* m_{jt}^* dj - r_t \int_0^\theta p_j^* m_{jt}^* dj - p_t^m m_t^d - w_t L_t^f. \quad (7)$$

Following Uribe and Yue (2006), we show in Appendix 1 that the above static profit maximization problem follows from a standard problem maximizing the present value of dividends subject to the working capital constraint. Moreover, Appendix 1 also establishes two features of the final goods producers’ optimal plans that are important for our model: First, the interest rate determining the cost of working capital is the same as the between-period rate on one-period loans. Second, since the firms’ payoff function is linear and factor demands are characterized by standard conditions equating marginal products to marginal costs (see below), firms do not have an incentive to build precautionary savings to self insure against changes in factor costs. Furthermore, even if this incentive were at play, building up a stock of *foreign* deposits to provide self-finance of working capital to pay foreign suppliers is ruled out by the standard assumption of the Eaton-Gersovitz setup that countries cannot build deposits abroad, otherwise debt exposed to default risk cannot exist at equilibrium (as shown by Bulow and Rogoff (1989)).

The price of m_t^* is the standard CES price index $\int_{j \in [0,1]} (p_j^*)^{\frac{\nu}{\nu-1}} dj$. Because some imported inputs carry the financing cost of working capital, we can express this price index as follows:

$$P^*(r_t) = \left[\int_\theta^1 (p_j^*)^{\frac{\nu}{\nu-1}} dj + \int_0^\theta (p_j^* (1+r_t))^{\frac{\nu}{\nu-1}} dj \right]^{\frac{\nu-1}{\nu}}.$$

As we show in the next Section, the set Ω of imported inputs is not used when a country defaults

because the financing cost becomes prohibitive (or equivalently, we could assume this is the case because part of the punishment for default is exclusion from the Ω set of world input markets), and hence when a country is in financial autarky the price index of imported inputs is:

$$P^* = \left[\int_{\theta}^1 (p_j^*)^{\frac{\nu}{\nu-1}} dj \right]^{\frac{\nu-1}{\nu}} \text{ when } r_t \rightarrow \infty.$$

We use a standard two-stage budgeting approach to characterize the solution of the final goods producers' optimization problem. In the first stage, firms choose L_t^f, m_t^d and m_t^* , given the factor prices w_t, p_t^m and $P^*(r_t)$, to maximize date-t profits:

$$\pi_t^f = \varepsilon_t \left(M(m_t^d, m_t^*) \right)^{\alpha_M} \left(L_t^f \right)^{\alpha_L} k^{\alpha_k} - P^*(r_t) m_t^* - p_t^m m_t^d - w_t L_t^f, \quad (8)$$

where $M(m_t^d, m_t^*) = (\lambda (m_t^d)^\mu + (1 - \lambda) (m_t^*)^\mu)^{\frac{1}{\mu}}$. Then, in the second stage they choose their demand for each variety of imported inputs.

The first-order conditions of the first stage are:

$$\alpha_M \varepsilon_t k^{\alpha_k} \left(M(m_t^d, m_t^*) \right)^{\alpha_M - \mu} \left(L_t^f \right)^{\alpha_L} (1 - \lambda) (m_t^*)^{\mu-1} = P^*(r_t) \quad (9)$$

$$\alpha_M \varepsilon_t k^{\alpha_k} \left(M(m_t^d, m_t^*) \right)^{\alpha_M - \mu} \left(L_t^f \right)^{\alpha_L} \lambda (m_t^d)^{\mu-1} = p_t^m \quad (10)$$

$$\alpha_L \varepsilon_t k^{\alpha_k} M_t^{\alpha_M} \left(L_t^f \right)^{\alpha_L - 1} = w_t. \quad (11)$$

Given m_t^* , the second stage yields a standard CES system of demand functions for imported inputs that can be split into a subset for varieties that do not require working capital and the subset in Ω :

$$m_{jt}^* = \left(\frac{p_j^*}{P^*(r_t)} \right)^{-\frac{1}{1-\nu}} M^*, \text{ for } j \in [\theta, 1],$$

$$m_{jt}^* = \left(\frac{p_j^*(1+r_t)}{P^*(r_t)} \right)^{-\frac{1}{1-\nu}} M^*, \text{ for } j \in [0, \theta].$$

where $P^*(r_t) = \left[\int_{\theta}^1 (p_j^*)^{\frac{\nu}{\nu-1}} dj + \int_0^{\theta} (p_j^*(1+r_t))^{\frac{\nu}{\nu-1}} dj \right]^{\frac{\nu-1}{\nu}}$.

When the country is in default, and thus final goods producers cannot access working capital financing, the demand function system becomes the limit of the above system as $r_t \rightarrow \infty$:

$$m_{jt}^* = \left(\frac{P^*}{p_j^*} \right)^{-\frac{1}{1-\nu}} M^*, \text{ for } j \in [\theta, 1],$$

$$m_{jt}^* = 0, \text{ for } j \in [0, \theta],$$

where $P^* = \left[\int_{\theta}^1 \left(p_j^* \right)^{\frac{\nu}{\nu-1}} dj \right]^{\frac{\nu-1}{\nu}}$.

2.3 Intermediate Goods Producers

Producers in the m^d sector use labor L_t^m and operate with a production function given by $A(L_t^m)^\gamma$, with $0 \leq \gamma \leq 1$ and $A > 0$. A represents both the role of a fixed factor and an invariant state of TFP in the m^d sector. Given p_t^m and w_t , the profit maximization problem of intermediate goods firms is:

$$\max_{L_t^m} \pi_t^m = p_t^m A(L_t^m)^\gamma - w_t L_t^m. \quad (12)$$

Their labor demand satisfies this standard optimality condition:

$$\gamma p_t^m A(L_t^m)^{\gamma-1} = w_t. \quad (13)$$

2.4 Equilibrium in Factor Markets and Production

Take as given a finite interest rate r_t , which means that sector f has access to credit markets, and a TFP realization ε_t . The corresponding (partial) equilibrium factor allocations and prices are given by the values of $[m_t^*, m_t^d, L_t^f, L_t^m, L_t]$ and $[p_t^m, w_t]$ that solve the following nonlinear system:

$$\alpha_M \varepsilon_t k^{\alpha_k} \left(M \left(m_t^d, m_t^* \right) \right)^{\alpha_M - \mu} \left(L_t^f \right)^{\alpha_L} (1 - \lambda) \left(m_t^* \right)^{\mu - 1} = P^* (r_t) \quad (14)$$

$$\alpha_M \varepsilon_t k^{\alpha_k} \left(M \left(m_t^d, m_t^* \right) \right)^{\alpha_M - \mu} \left(L_t^f \right)^{\alpha_L} \lambda \left(m_t^d \right)^{\mu - 1} = p_t^m \quad (15)$$

$$\alpha_L \varepsilon_t k^{\alpha_k} \left(M \left(m_t^d, m_t^* \right) \right)^{\alpha_M} \left(L_t^f \right)^{\alpha_L - 1} = w_t \quad (16)$$

$$\gamma p_t^m A(L_t^m)^{\gamma-1} = w_t \quad (17)$$

$$g'(L_t) = w_t \quad (18)$$

$$L_t^f + L_t^m = L_t \quad (19)$$

$$A(L_t^m)^\gamma = m_t^d \quad (20)$$

Conditions (14)-(20) drive the effects of fluctuations in TFP and interest rates on production and factor allocations. We study these effects in detail in Section 3. Note also that during periods of exclusion from world credit markets, the factor allocations and prices are determined as the limiting case of the above nonlinear system as $r \rightarrow \infty$. The sector f does not have access to foreign working capital financing and hence to the set Ω of imported inputs.

Using the above optimality conditions, it follows that total value added valued at equilibrium

relative prices is given by $(1 - \alpha_M)\varepsilon_t(M_t)^{\alpha_M}(L_t^f)^{\alpha_L}k^{\alpha_k} + p_t^m A(L_t^m)^\gamma$. Moreover, given the CES formulation of M_t , the value of imported inputs satisfies $P^*(r_t)m_t^* = \alpha_M\varepsilon_t(M_t)^{\alpha_M}(L_t^f)^{\alpha_L}k^{\alpha_k} - p_t^m m_t^d$. Given these results, we can calculate GDP as gross production of final goods minus the cost of imported inputs, adjusting for the fact that in most emerging economies GDP at constant prices is computed fixing prices as of a base year using Laspeyres indexes (while in the model $P^*(r_t)$ varies over time because of fluctuations in the rate of interest). Hence we define GDP as $gdp_t \equiv y_t - P^*m_t^*$, using a time-invariant price index of imported inputs.¹²

2.5 The Sovereign Government

The sovereign government trades with foreign lenders one-period, zero-coupon discount bonds, so markets of contingent claims are incomplete. The face value of these bonds specifies the amount to be repaid next period, b_{t+1} . When the country purchases bonds $b_{t+1} > 0$, and when it borrows $b_{t+1} < 0$. The set of bond face values is $B = [b_{\min}, b_{\max}] \subset R$, where $b_{\min} \leq 0 \leq b_{\max}$. We set the lower bound $b_{\min} > -\frac{\bar{y}}{r}$, which is the largest debt that the country could repay with full commitment. The upper bound b_{\max} is the highest level of assets that the country may accumulate.¹³

The sovereign cannot commit to repay its debt. As in the Eaton-Gersovitz model, when the country defaults it does not repay at date t and the punishment is exclusion from the world credit market in the same period. The country re-enters the credit market with an exogenous probability ϕ , and when it does it starts with a fresh record and zero debt.¹⁴

We add to the Eaton-Gersovitz setup an explicit link between default risk and private financing costs. This is done by assuming that a defaulting sovereign can divert the repayment of the firms' working capital loans to foreign lenders.¹⁵ Hence, both firms and government default together. As explained in the introduction, this is in line with the historical evidence documented by Reinhart and Rogoff (2010) and Reinhart (2010). We also provide empirical evidence later in this Section showing a tight link between private and public borrowing costs.

The sovereign government chooses a debt policy (amounts and default or repayment) along with private consumption and factor allocations so as to solve a recursive social planner's problem.¹⁶ The state variables are the bond position and TFP, denoted by the pair (b_t, ε_t) , and the

¹²We use $P^* = 1$, which follows from the fact that $p_j^* = 1$ for all $j \in [0, 1]$ and assuming a zero real interest rate in the base year. Note, however, that changes in our quantitative results are negligible if we use the equilibrium price index $P^*(r_t)$ instead, because default is a low frequency event, and outside default episodes interest rates display very small fluctuations.

¹³ b_{\max} exists when the interest rates on a country's saving are sufficiently small compared to the discount factor, which is satisfied in our paper since $(1 + r^*)\beta < 1$.

¹⁴We abstract from debt renegotiation. See Yue (2010), Bi (2008b) and Benjamin and Wright (2008) for quantitative studies of sovereign default with renegotiation.

¹⁵Notice that existing models of emerging markets business cycles with working capital (e.g. Neumeyer and Perri (2005) and Uribe and Yue (2006)) already assume that the sovereign interest rates and private financing costs are equal. Here we endogenize interest rates and the two rates are equalized as an equilibrium outcome.

¹⁶We show in Appendix 2 that this planner's problem yields the same equilibrium as a decentralized Ramsey-like equilibrium in which the government maximizes households' utility subject to the resource constraints, the

planner takes as given the bond pricing function $q_t(b_{t+1}, \varepsilon_t)$. Since at equilibrium the default risk premium on sovereign debt will be the same as on working capital loans, the net interest rate on working capital satisfies $r_t = 1/q_t(b_{t+1}, \varepsilon_t) - 1$. The planner's payoff is given by:

$$V(b_t, \varepsilon_t) = \max \left\{ v^{nd}(b_t, \varepsilon_t), v^d(\varepsilon_t) \right\}, \quad (21)$$

where $v^{nd}(b_t, \varepsilon_t)$ is the value of continuing in the credit relationship with foreign lenders (i.e., “no default”), and $v^d(\varepsilon_t)$ is the value of default. If $b_t \geq 0$, the value function is simply $v^{nd}(b_t, \varepsilon_t)$ because in this case the economy uses the credit market to save, receiving a return equal to the world's risk free rate r^* .

The continuation value is given by the choice of $[c_t, m_t^d, m_t^*, L_t^f, L_t^m, L_t, b_{t+1}]$ that solves this constrained maximization problem:

$$v^{nd}(b_t, \varepsilon_t) = \max_{c_t, m_t^d, m_t^*, L_t^f, L_t^m, L_t, b_{t+1}} \left\{ \begin{array}{l} u(c_t - g(L_t)) \\ + \beta E[V(b_{t+1}, \varepsilon_{t+1})] \end{array} \right\}, \quad (22)$$

subject to:

$$c_t + q_t(b_{t+1}, \varepsilon_t) b_{t+1} - b_t \leq \varepsilon_t f \left(M(m_t^d, m_t^*), L_t^f, k \right) - m_t^* P^* \left(\frac{1}{q_t(b_{t+1}, \varepsilon_t)} - 1 \right), \quad (23)$$

$$L_t^f + L_t^m = L_t$$

$$A(L_t^m)^\gamma = m_t^d$$

where $f(\cdot) = M^{\alpha_M} (L_t^f)^{\alpha_L} k^{\alpha_k}$. The first constraint is the resource constraint of the economy. The last two constraints are the resource constraints in the markets for labor and domestic inputs respectively.

Notice that the planner faces the same effects of interest rates on output and factor allocations operating via the working capital channel that affects the private sector. In particular, for a given bond pricing function $q_t(b_{t+1}, \varepsilon_t)$ and any pair $(b_{t+1}, b_t) \in B$, including the optimal choice of b_{t+1} , the optimal factor allocations chosen by the planner $[m_t^*, m_t^d, L_t^f, L_t^m, L_t]$ satisfy the conditions (14)-(20) that characterize equilibrium in factor markets, with w_t and p_t^m matching the shadow prices given by the Lagrange multipliers of the resource constraints for labor and domestic inputs. In addition, the planner internalizes the households' desire to smooth consumption, and hence transfers to them an amount equal to the negative of the balance of trade (i.e. the flow of resources private agents need to finance the gap between GDP and consumption).

government budget constraint, the constraint that factor allocations need to be consistent with private equilibrium conditions, and the assumption that the diverted working capital repayments are not rebated to households (i.e. they are an extra default cost or a tax used to finance unproductive government purchases). We also discuss in Appendix 2 the decentralized equilibrium in the alternative case in which these repayments are rebated.

The value of default is:

$$v^d(\varepsilon_t) = \max_{c_t, m_t^d, m_t^*, L_t^f, L_t^m, L_t} \left\{ \begin{array}{l} u(c_t - g(L)) \\ + \beta(1 - \phi) E v^d(\varepsilon_{t+1}) + \beta \phi E V(0, \varepsilon_{t+1}) \end{array} \right\}, \quad (24)$$

subject to:

$$\begin{aligned} c_t &= \varepsilon_t f \left(M \left(m_t^d, m_t^* \right), L_t^f, k \right) - m_t^* P^* \\ L_t^f + L_t^m &= L_t \\ A(L_t^m)^\gamma &= m_t^d \end{aligned} \quad (25)$$

Note that $v^d(\varepsilon_t)$ takes into account the fact that in case of default at date t , the country has no access to financial markets that period, and hence the country consumes the total income given by the resource constraint in the default scenario. In this case, since firms cannot borrow to finance the subset Ω of imported inputs, the equilibrium allocations for $[m_t^d, m_t^*, L_t^f, L_t^m, L_t]$ and the price index P^* are those that solve system (14)-(20) as $r \rightarrow \infty$. The value of default at t also takes into account that at $t+1$ the economy may re-enter world capital markets with probability ϕ and associated value $V(0, \varepsilon_{t+1})$, or remain in financial autarky with probability $1 - \phi$ and associated value $v^d(\varepsilon_{t+1})$.

The definitions of the default set and the probability of default are standard from Eaton-Gersovitz models (see Arellano (2008)). For a debt position $b_t < 0$, default is optimal for the set of realizations of ε_t for which $v^d(\varepsilon_t)$ is at least as high as $v^{nd}(b_t, \varepsilon_t)$:

$$D(b_t) = \left\{ \varepsilon_t : v^{nd}(b_t, \varepsilon_t) \leq v^d(\varepsilon_t) \right\}. \quad (26)$$

The probability of default at $t+1$ perceived as of date t , $p_t(b_{t+1}, \varepsilon_t)$, can be induced from the default set and the transition probability function of productivity shocks $\mu(\varepsilon_{t+1}|\varepsilon_t)$ as follows:

$$p_t(b_{t+1}, \varepsilon_t) = \int_{D(b_{t+1})} d\mu(\varepsilon_{t+1}|\varepsilon_t). \quad (27)$$

The economy is considered to be in financial autarky when it has been in default for at least one period and remains without access to world credit markets. The optimization problem of the sovereign is the same as the problem in the default period. This is the case because, since the Bulow-Rogoff result requires the economy not to be able to access funds saved abroad during periods of financial autarky, before defaulting the economy could not have built up a stock of savings abroad to provide working capital financing to firms to purchase imported inputs. Alternatively, we can assume that the default punishment includes exclusion from both world capital markets and the subset Ω of world markets of intermediate goods.

The model preserves these standard features of the Eaton-Gersovitz model: Given ε_t , the value of defaulting is independent of the level of debt, while the value of not defaulting increases

with b_{t+1} , and consequently the default set and the equilibrium default probability grow with the country's debt. The following theorem formalizes these results:

Theorem 1 *Given a productivity shock ε and a pair of bond positions $b^0 < b^1 \leq 0$, if default is optimal for b^1 , then default is also optimal for b^0 and the probability of default at equilibrium satisfies $p^*(b^0, \varepsilon) \geq p^*(b^1, \varepsilon)$.*

Proof. See Appendix 3. ■

2.6 Foreign Lenders

International creditors are risk-neutral and have complete information. They invest in sovereign bonds and in private working capital loans. Foreign lenders behave competitively and face an opportunity cost of funds equal to r^* . Competition implies that they expect zero profits at equilibrium, and that the returns on sovereign debt and the world's risk-free asset are fully arbitrated:

$$q_t(b_{t+1}, \varepsilon_t) = \begin{cases} \frac{1}{1+r^*} & \text{if } b_{t+1} \geq 0 \\ \frac{[1-p_t(b_{t+1}, \varepsilon_t)]}{1+r^*} & \text{if } b_{t+1} < 0 \end{cases} . \quad (28)$$

This condition implies that at equilibrium bond prices depend on the risk of default. For a high level of debt, the default probability is higher. Therefore, equilibrium bond prices decrease with indebtedness. This result, formalized in Theorem 2 below, is again in line with the Eaton-Gersovitz model and is also consistent with the empirical evidence documented by Edwards (1984).

Theorem 2 *Given a productivity shock ε and bond positions $b^0 < b^1 \leq 0$, the equilibrium bond prices satisfy $q^*(b^0, \varepsilon) \leq q^*(b^1, \varepsilon)$.*

Proof. See Appendix 3. ■

The returns on sovereign bonds and working capital loans are also fully arbitrated. Because the sovereign government diverts the repayment of working capital loans when it defaults, foreign lenders assign the same risk of default to private working capital loans as to sovereign debt, and hence the no-arbitrage condition between sovereign lending and working capital loans implies:

$$r_t(b_{t+1}, \varepsilon_t) = \frac{1}{q_t(b_{t+1}, \varepsilon_t)} - 1, \text{ if } \kappa_t > 0. \quad (29)$$

This arbitrage result raises a key empirical question: Are the interest rates faced by sovereign governments and private firms closely related? Answering this question in full is beyond the scope of this paper, but we do provide evidence suggesting that corporate and sovereign interest rates tend to move together. To study this issue, we constructed estimates of firm-level effective interest rates as the ratio of a firm's total debt service divided by its total debt obligations using the *Worldscope* database. We then aggregated for each country by computing the median across

firms. Table 1 reports these estimates of corporate interest rates together with the standard EMBI+ measure of interest rates on sovereign debt, both as time-series averages over 1994 to 2005, as well as the correlations between the two over the same period.

Table 1 shows that the two interest rates are positively correlated in most countries, with a median correlation of 0.7, and in some countries the relationship is very strong (see Figure 2).¹⁷ The Table also shows that, with the exceptions of Argentina, China and Russia, the effective financing cost of firms is higher on average than the sovereign interest rates.

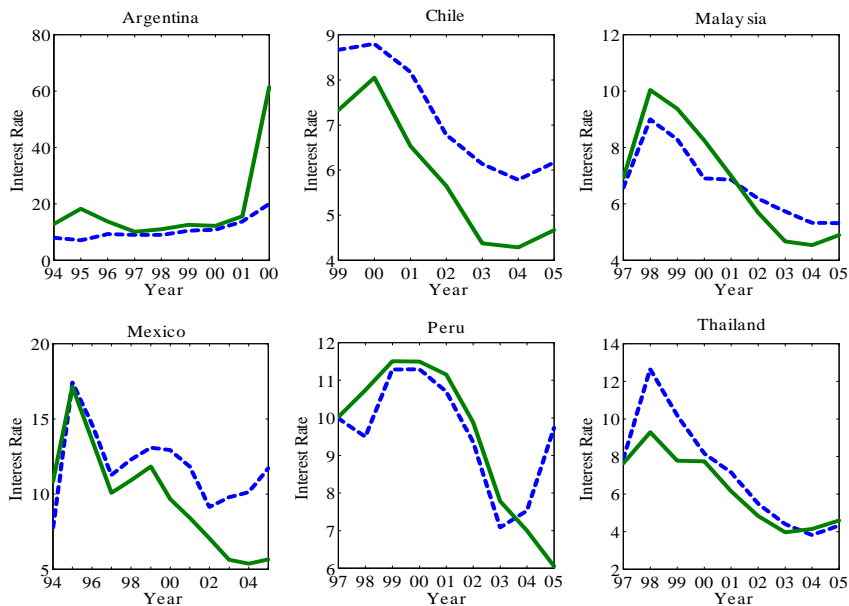
Table 1: Sovereign and Corporate Interest Rates

Country	Sovereign Interest Rates	Median Firm Interest Rates	Correlation
Argentina	13.32	10.66	0.87
Brazil	12.67	24.60	0.14
Chile	5.81	7.95	0.72
China	6.11	5.89	0.52
Colombia	9.48	19.27	0.86
Egypt	5.94	8.62	0.58
Malaysia	5.16	6.56	0.96
Mexico	9.40	11.84	0.74
Morocco	9.78	13.66	0.32
Pakistan	9.71	12.13	0.84
Peru	9.23	11.42	0.72
Philippines	8.78	9.27	0.34
Poland	7.10	24.27	0.62
Russia	15.69	11.86	-0.21
South Africa	5.34	15.19	0.68
Thailand	6.15	7.30	0.94
Turkey	9.80	29.26	0.88
Venezuela	14.05	19.64	0.16

There is also strong historical evidence in favor of the assumption driving the arbitrage of private and government interest rates in the model, namely that the government diverts the repayment of the firms' foreign obligations. This is documented in the comprehensive studies by Reinhart and Rogoff (2010) and Reinhart (2010) and in Boughton's (2001) historical account of the IMF's handling of the 1980s debt crisis (see in particular Chapter 9). These studies show that it is common for governments to take over the foreign obligations of the corporate sector in actual default episodes, particularly when a domestic banking crisis occurs in tandem with sovereign default, which is a frequent occurrence. In addition, Arteta and Hale (2007)

¹⁷Arellano and Kocherlakota (2007) and Agca and Celasun (2009) provide further empirical evidence of the positive relationship between private domestic lending rates and sovereign spreads. Corsetti, Kuester, Meier and Muller (2010) show that this feature is also present in the data of OECD countries.

and Kohlscheen and O’Connell (2008) provide evidence of significant adverse effects of sovereign default on private access to foreign credit. Arteta and Hale show that there are strong negative effects on private corporate bond issuance during and after default episodes. Kohlscheen and O’Connell document that the volume of trade credit provided by commercial banks falls sharply when countries default. The median drops in trade credit are about 35 and 51 percent two and four years after default events respectively.



———— Sovereign Bond Interest Rates - - - Median Firm Financing Cost

Figure 2: Sovereign Bond Interest Rates and Median Firm Financing Costs

2.7 Recursive equilibrium

Definition 1 *The model’s recursive equilibrium is given by (i) a decision rule $b_{t+1}(b_t, \varepsilon_t)$ for the sovereign government with associated value function $V(b_t, \varepsilon_t)$, consumption and transfers rules $c(b_t, \varepsilon_t)$ and $T(b_t, \varepsilon_t)$, default set $D(b_t)$ and default probabilities $p^*(b_{t+1}, \varepsilon_t)$; and (ii) an equilibrium pricing function for sovereign bonds $q^*(b_{t+1}, \varepsilon_t)$ such that:*

1. *Given $q^*(b_{t+1}, \varepsilon_t)$, the decision rule $b_{t+1}(b_t, \varepsilon_t)$ solves the social planner’s recursive maximization problem (21).*
2. *The consumption plan $c(b_t, \varepsilon_t)$ satisfies the resource constraint of the economy*
3. *The transfers policy $T(b_t, \varepsilon_t)$ satisfies the government budget constraint.*
4. *Given $D(b_t)$ and $p^*(b_{t+1}, \varepsilon_t)$, the bond pricing function $q^*(b_{t+1}, \varepsilon_t)$ satisfies the arbitrage condition of foreign lenders (28).*

Condition 1 requires that the government’s default and borrowing decisions be optimal given the interest rates on sovereign debt. Condition 2 requires that the private consumption and factor

allocations implied by these optimal borrowing and default choices be feasible.¹⁸ Condition 3 requires that the decision rule for government transfers shifts the appropriate amount of resources between the government and the private sector (i.e. an amount equivalent to net exports when the country has access to world credit markets, or zero when the economy is in financial autarky). Notice also that given conditions 2 and 3, the consumption plan satisfies the households' budget constraint. Finally, Condition 4 requires the equilibrium bond prices that determine country risk premia to be consistent with optimal lender behavior.

A solution to the above recursive equilibrium includes solutions for sectoral factor allocations and production with and without credit market access. A solution for equilibrium interest rates on working capital as a function of b_{t+1} and ε_t follows from (29). Solutions for equilibrium wages, profits and the price of domestic inputs follow then from the firms' optimality conditions and the definitions of profits described earlier.

3 Country Risk and Default Costs in Partial Equilibrium

3.1 Interest Rate Changes and Factor Allocations

The effects of interest rate changes on factor allocations play a central role in our model because they are a key determinant of both output dynamics and the output cost of default. We illustrate these effects by means of a partial-equilibrium numerical example in which the interest rate is exogenous. We use the parameter values set in the calibration described in Section 4, and solve for factor allocations and prices using conditions (14)-(20) for different values of r .

Figure 3 shows six charts with the allocations of L , L^f , L^m , M , m^d , and m^* for values of r ranging from 0 to 80 percent. Each chart includes results for the baseline calibration, in which we set $\mu=0.65$, which corresponds to $\eta_{m^d, m^*} = 2.86$, and $\nu = 0.59$, which implies $\eta_{m_j^*} = 2.44$. In addition, we show four alternative scenarios in which all but one of the baseline parameter values are changed. Two of the scenarios consider lower values of η_{m^d, m^*} (1.96, which is the threshold below which m^d and m^* switch from gross substitutes to gross complements, and the Cobb-Douglas case of unitary elasticity).¹⁹ The other two scenarios assume a high within elasticity of substitution across imported input varieties ($\eta_{m_j^*} = 10$) and inelastic labor supply. To facilitate comparisons across these scenarios, the results are plotted as ratios relative to the allocations when $r = 0$.

The charts in Figure 3 illustrate three effects by which the rate of interest affects equilibrium factor allocations. First, as chart 3b shows, an increase in r reduces the aggregate demand for m^* because of the direct effect by which the hike in r increases the marginal cost of the subset of Ω of imported inputs, which is in turn reflected in an increase in $P^*(r)$. This is the case for

¹⁸In addition, since factor allocations satisfy conditions (14)-(20), these allocations are also consistent with a competitive equilibrium in factor markets.

¹⁹Note that the threshold would be at the unitary elasticity of substitution if labor supply were inelastic.

any $0 < \mu < 1$. Second, an increase in r has indirect effects that lower the demand for total intermediate goods (M) and labor in the final goods sector (L^f), because of the Cobb-Douglas structure of the production function of final goods (see charts 3a and 3e). The direction of these effects is also the same for any $0 < \mu < 1$. Third, an increase in r has effects on the output and labor allocations in the intermediate goods sector, but the direction of these do depend on the value of μ (within the $(0, 1)$ range). In particular, higher r leads to an increase or a decline in m^d and L^m depending on whether the value of μ makes imported and domestic inputs gross substitutes or complements (see charts 3c and 3f). If μ is high (low) enough for the two inputs to be gross substitutes (complements), both m^d and L^m increase (fall) as r and $P_m^*(r)$ rise, so m^d and L^m rise (fall) as m^* falls. As a result, the decline in M and L produced by an increase in the rate of interest is larger when domestic and foreign inputs are gross complements than when they are gross substitutes (see charts 3a and 3d).

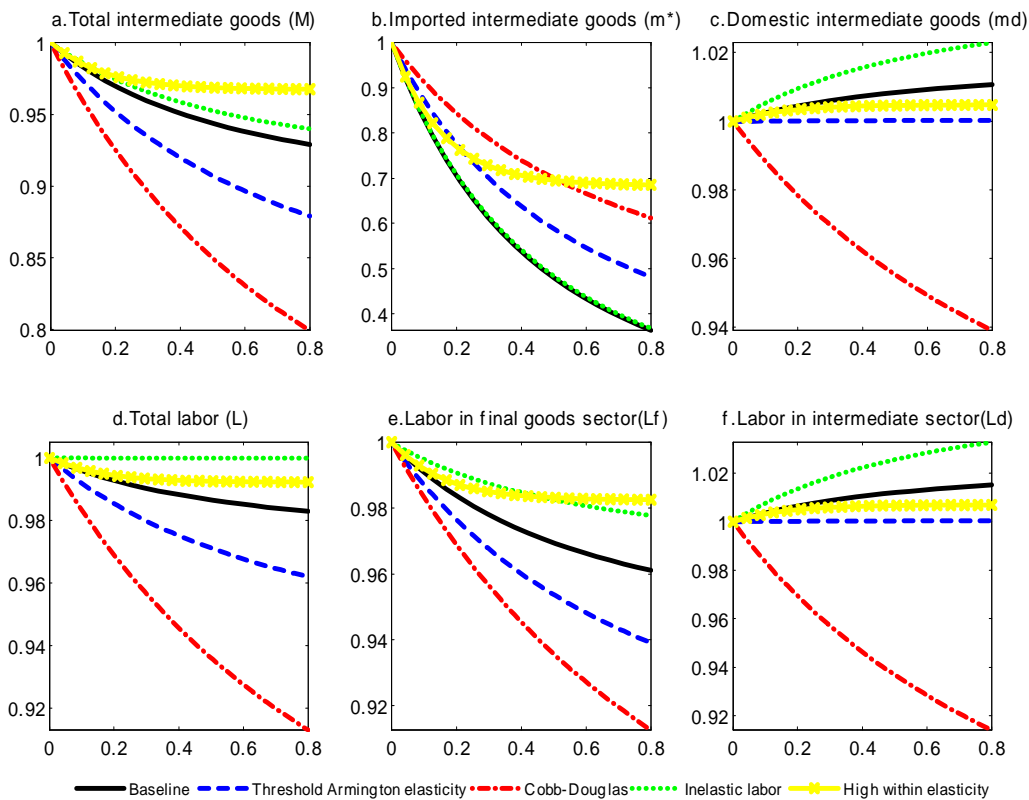


Figure 3: Effects of interest rate shocks on intermediate goods and labor allocations

Compare now the baseline case with the inelastic labor scenario. The effect on m^* is nearly unchanged. M falls less with inelastic labor, however, because m^d rises more, and this is possible

because with inelastic labor supply L cannot fall in response to interest rate hikes, and this results in a larger increase in L^m and a smaller decline in L^f . Thus, even with inelastic labor supply, increases in r affect the efficiency of production by inducing a shift from foreign to domestic inputs, and by reallocating a given total labor endowment from production of final goods to production of intermediate goods.

Finally, compare the interest rate effects of the baseline case with the scenario with high within elasticity, in which $\eta_{m_j^*} = 10$ ($v = 0.9$). As imported inputs become better substitutes, the effect of higher interest rates increasing the marginal cost of the Ω set of imported inputs weakens, since imported inputs that do not require payment in advance are closer substitutes of those that do. As a result, aggregate m^* falls less as the interest rate rises, causing a smaller decline (rise) in M (m^d), and less reallocation of labor from sector f to sector m .

Notice that qualitatively the effects of increasing η_{m^d, m^*} or $\eta_{m_j^*}$ are similar. The higher either of these two elasticities are, the weaker the working capital channel, and hence the weaker the effects of interest rate fluctuations on production and factor allocations. The two are not equivalent, however, because when imported input varieties become better substitutes, final goods producers can substitute into varieties outside the Ω set facing exogenous prices p_j^* , whereas when domestic inputs become better substitutes, substituting away into domestic inputs is done facing an endogenous price p^m . In our baseline calibration, we found that (again in the partial equilibrium setup used in Figure 3) changing η_{m^d, m^*} alters the size of the interest rate effects on factor allocations much more than changing $\eta_{m_j^*}$. In particular, consider that in Figure 3, as we move from the threshold Armington elasticity case ($\eta_{m^d, m^*} = 1.96$) to the baseline ($\eta_{m^d, m^*} = 2.86$) we are increasing η_{m^d, m^*} by a factor of about 1.5, while moving from the baseline to the $\eta_{m_j^*} = 10$ scenario we are increasing $\eta_{m_j^*}$ by a factor of 5, yet the amount by which interest rate effects differ from the baseline is similar in both cases. Moreover, we also computed scenarios with values of $\eta_{m_j^*}$ lower than the baseline and obtained negligible changes in interest rate effects relative to the baseline.

3.2 Output costs of default

Using the same numerical example, we can now examine how the output cost of default varies with ε , and how this relationship depends on μ , v and ω . Figure 4 shows two plots of the output cost of default as a function of ε . The plot on the left compares the case with baseline parameters with a scenario in which we lower μ from the 0.65 baseline value to 0.4 (η_{m^d, m^*} falls from 2.86 to 1.66). The plot on the right compares the baseline scenario with a scenario in which we lower ν from 0.59 to 0.3 ($\eta_{m_j^*}$ falls from 2.43 to 1.43). For each value of ε in the horizontal axis, the output cost of default is measured as the percent fall in output that occurs when the government defaults, which is computed as the value of output implied by the factor allocations that result from conditions(14)-(20) as $r \rightarrow \infty$ relative to the level implied by the same conditions when $r = 0.01$.

Figure 4 illustrates three key properties of the model: First, the output cost of default is increasing and convex in ε . This is the case because, with Cobb-Douglas technologies and competitive markets, the negative effect of increases in marginal costs on factor demands is larger at higher TFP levels.²⁰ Second, the cost of default is higher the lower is η_{m^d, m^*} . This is an implication of the previous results showing that the negative effects of interest rate shocks on factor allocations are larger when domestic inputs are poorer substitutes of imported inputs. Third, the cost of default is also higher the lower is $\eta_{m_j^*}$, although the effect of varying ν on the output cost of default is considerably smaller than the effect of varying μ , in line with what we found earlier.

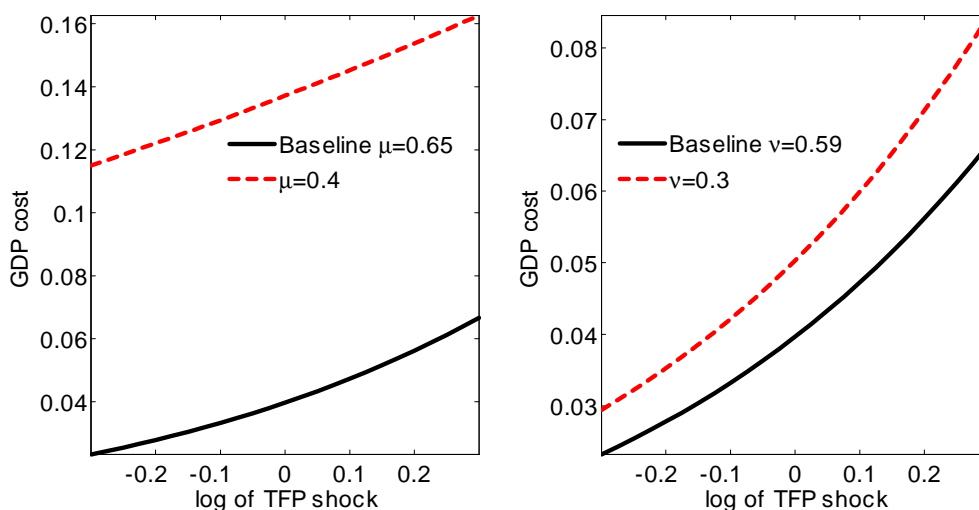


Figure 4: Output Costs of Default as a Function of TFP Shock

The fact that the output cost of default increases with ε implies that default is more painful at higher TFP levels. This result is critical for the model’s ability to support high debt levels at the observed default frequencies, and producing defaults in “bad” times, because it makes default more attractive at lower states of productivity. In this way, default works as a desirable implicit hedging mechanism given the incompleteness of asset markets.

Figure 5 illustrates further how the cost of default declines as η_{m^d, m^*} rises. This Figure plots the output cost of default for a constant value of TFP ($\varepsilon = 1$) at different values of η_{m^d, m^*} . Again, the cost of default becomes smaller at higher Armington elasticities because the inputs are closer substitutes, and hence the efficiency loss when firms shift to use fewer foreign inputs is smaller. Quantitatively, the Figure shows that already for $\eta_{m^d, m^*} > 4$, the mechanism driving

²⁰This is the case in turn because of the "strong" convexity of Cobb-Douglas marginal products. Consider for simplicity the case in which production $\varepsilon F(m)$ requires a single input m . In this case, “strong convexity” means that $F(m)$ satisfies $F'''(m) > (F''(m))^2 / F'(m)$, which holds in the Cobb-Douglas case.

efficiency losses in the model becomes very weak and is effectively the same as if the inputs were perfect substitutes.

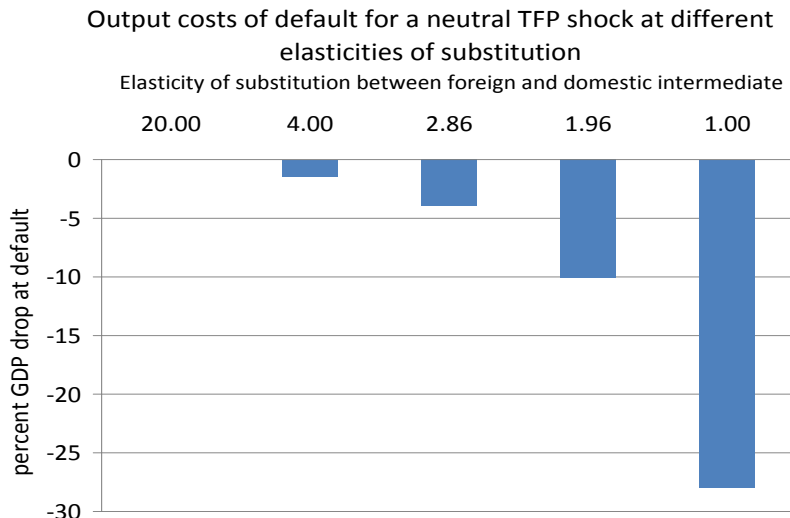


Figure 5: Output Costs of Default at a Neutral TFP Shock

A similar analysis of the output costs of default as the one illustrated in Figures 4 and 5 but for different values of ω (instead of μ and ν) shows that a higher labor supply elasticity (i.e. lower ω) increases the cost of default, converging to about 11.5 percent for infinitely elastic labor supply. The output cost of default is increasing in TFP for any value of ω , but, in contrast with what we found for μ , the slope of the relationship does not change as ω changes.²¹

The labor market equilibrium illustrated in Figure 6 provides the intuition behind the result that higher labor supply elasticity produces larger output costs of default. For simplicity, we plot labor demands and supply as linear functions. The labor demand functions are given by the marginal products in the left-hand-side of (11) and (13), and the labor supply is given by the marginal disutility of labor in the left-hand-side of (3). Since labor is homogenous across sectors, total labor demand is just the sum of sectoral demands. The initial labor market equilibrium is at point A with wage w^* , total labor L^* and sectoral allocations L_m^* and L_f^* .

Consider now a positive interest rate shock. This leads to a reduction in labor demand in final goods from L_f^D to \tilde{L}_f^D . This occurs because, as explained earlier, higher r causes a reduction in M and the marginal product of L_f is a negative function of M (since the production function is Cobb-Douglas). As a result, total labor demand shifts from L^D to \tilde{L}^D .²² The new labor market equilibrium is at point \tilde{A} . The wage rate, the total labor allocation, and the labor allocated to

²¹We also found that adjusting A has qualitatively similar effects as changing ω .

²²In Figure 6, we hold constant p_m for simplicity. At equilibrium, the relative price of domestic inputs changes, and this alters the value of the marginal product of L_d , and hence labor demand by the m sector. The results

final goods are lower than before, while labor allocated to production of domestic inputs rises (assuming that foreign and domestic inputs are gross substitutes). In contrast, assuming that labor is infinitely elastic would make L^s an horizontal line at the level of w^* and the interest rate hike would leave w unchanged. As a result, L falls more, L_m is unchanged instead of rising, and L_f falls less.²³ Hence, the adverse effect on output is stronger. At the other extreme, with inelastic labor L^s is a vertical line at the level of L^* . Now L cannot change, but w falls more than in Figure 6, L_m rises more, and L_f falls more. Hence, the decline in output is smaller.

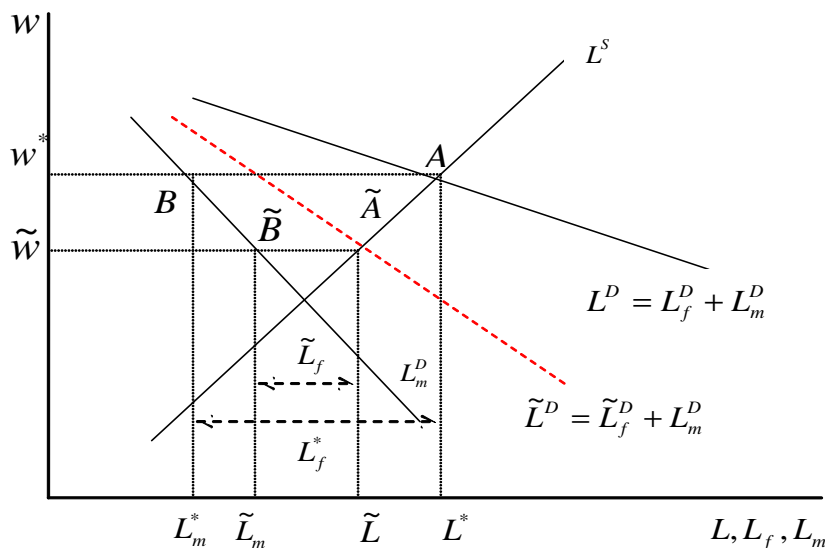


Figure 6: Interest Rate Shocks and the Labor Market Equilibrium

4 Quantitative Analysis

4.1 Baseline Calibration

We study the quantitative implications of the model by conducting numerical simulations setting the model to a quarterly frequency and using a baseline calibration based largely on data for Argentina, as is standard practice in quantitative studies of sovereign default. Table 2 shows the calibrated parameter values.

of our numerical analysis do take this into account and still are roughly in line with the intuition derived from Figure 6.

²³The last effect hinges on the fact that the gap between L_m^D and L^D widens as the wage falls. This is a property of factor demands with Cobb-Douglas production.

Table 2: Baseline Calibration

<i>Calibrated Parameters</i>		<i>Value</i>	<i>Target statistics</i>	
Int. goods share in gross output of final goods	α_m	0.43	Argentina's national accounts	
Capital share in gross output of final goods	α_k	0.17	Standard capital share in GDP (0.3)	
Labor share in gross output of final goods	α_L	0.40	Standard labor share in GDP (0.7)	
Labor share in GDP of int. goods	γ	0.7	Standard labor share in GDP (0.7)	
Coefficient of relative risk aversion	σ	2	Standard RBC value	
Risk-free interest rate	r^*	1%	Standard RBC value	
Curvature parameter of labor supply	ω	1.455	Frisch wage elasticity (2.2)	
Re-entry probability	ϕ	0.083	Dias and Richmond (2007)	
Armington weight of domestic inputs	λ	0.62	Regression estimate	
Armington curvature parameter	μ	0.65	Regression estimate	
Dixit-Stiglitz curvature parameter	ν	0.59	Gopinath and Neiman (2010)	
<i>Parameters set with SMM</i>		<i>Value</i>	<i>Targets from data</i>	
Autocorrelation of TFP shocks	ρ_ϵ	0.95	GDP autocorrelation	0.82
Standard deviation of TFP shocks	σ_ϵ	1.7%	GDP std. deviation	4.7%
Intermediate goods TFP coefficient	A	0.31	Output drop in default	13%
Subjective discount factor	β	0.88	Default frequency	0.69%
Upper bound of imported inputs with working capital	θ	0.7	Working capital to GDP ratio estimate	6%
TFP semi-elasticity of exogenous capital flows	ξ	-0.67	Net exports/GDP at default	10%

The share of intermediate goods in gross output α_M is set to 0.43, which corresponds to the average ratio of intermediate goods to gross production calculated using annual data for Argentina for the period 1993-2005 from the United Nation's *UNData*.²⁴ Given α_M , we set $\alpha_k = 0.17$ so that the capital income share *in value added* of the f sector ($\alpha_k/(1 - \alpha_M)$) matches the standard 30 percent ($0.17/(1 - 0.43) = 0.3$). These factor shares imply a labor share in gross output of final goods of $\alpha_L = 1 - \alpha_M - \alpha_k = 0.40$, which yields a labor share *in value added* of $\alpha_L/(1 - \alpha_M) = 0.7$ in line with the standard 70 percent labor share. The labor share in intermediate goods production γ is also set to 0.7.

The risk aversion parameter σ is set to 2 and the quarterly world risk-free interest rate r^* is set to 1 percent, which are standard values in quantitative business cycle and sovereign default studies. The curvature of labor disutility in the utility function is set to $\omega = 1.455$, which implies a Frisch wage elasticity of labor supply of $1/(\omega - 1) = 2.2$. This is the value typically used in RBC models of the small open economy (e.g. Mendoza (1991) and Neumeyer and Perri (2005)), and is based on estimates for the U.S. quoted by Greenwood, Hercowitz and Huffman (1988).

²⁴Mendoza (2010) reports a very similar share for Mexico, and Gopinath, Itskhoki, and Rigobon (2010) show shares in the 40-45 percent range for several countries.

The probability of re-entry after default is 0.083, which implies that the country stays in exclusion for three years after default on average. This is the estimate obtained by Dias and Richmond (2007) for the median duration of exclusion using a partial access definition of re-entry. A three-year exclusion period is also in the range of the estimates reported by Gelos et al. (2003).²⁵

The values of μ and λ are set using data on the ratio of imported to domestic intermediate goods at constant prices and the associated relative prices, together with the condition equating the marginal rate of technical substitution between m^* and m^d with the corresponding price ratio (which follows from conditions (14) and (15)). National Accounts data for Argentina, however, do not provide a breakdown of intermediate goods into domestic and imported, so we obtained them instead from Mexican data for the period 1988-2004 and assumed that the ratios are similar.²⁶ A nonlinear regression of the optimality condition implied by (14) and (15) produced estimates of $\mu = 0.65$ and $\lambda = 0.62$, both statistically significant (with standard errors of 0.11 and 0.12 respectively). These two estimates also allow the model to match the average ratios of imported to domestic inputs at current and constant prices in the Mexican data, which are 18 and 15.7 percent respectively.

The values $\mu = 0.65$ and $\lambda = 0.62$ imply that $\eta_{m^d, m^*} = 2.9$ and that there is a small bias in favor of domestic inputs. This Armington elasticity is in the range of existing empirical estimates for several countries, but the estimates vary widely. McDaniel and Balistreri (2002) review the literature and quote estimates ranging from 0.14 to 6.9. They explain that elasticities tend to be higher when estimated with disaggregated data, in cross-sectional instead of time-series samples, or when using long-run instead of short-run tests. In the next Section we conduct sensitivity analysis to study the effects of changing this and other key parameters on our main quantitative findings.

The value of v is difficult to set because it requires analysis of disaggregated data on imported intermediate goods. Gopinath and Neiman (2010) examined a large firm-level dataset for Argentina that included the disaggregation of import varieties. They focused in particular on the dynamics of trade adjustment at the firm level around the 2002 default event. We set $\eta_{m_j^*} = 2.44$ ($v = 0.59$) in line with the elasticity across varieties that they reported. They also concluded, in line with our argument, that trade adjustment via the extensive margin at the firm level, with firms shifting from imported to domestic inputs, was very significant in the aftermath of Argentina's default. Interestingly, they also set $\eta_{m^d, m^*} = 2.08$ ($\mu = 0.519$), which is close to

²⁵The two studies use different definitions of re-entry. Gelos et al. use actual external bond issuance of public debt. Dias and Richmond define reentry when either the private or public sectors can borrow again, and they also distinguish partial reaccess from full reaccess (with the latter defined as positive net debt flows larger than 1.5 percent of GDP). Gelos et al. estimate an average exclusion of 5.4 years in the 1980s and nearly 1 year in the 1990s.

²⁶Several countries have input expenditure ratios similar to Mexico's, but the ratios can vary widely. Goldberg and Campa (2008) report ratios of imported inputs to total intermediate goods for 17 countries that vary from 14 to 49 percent, with a median of 23 percent. This implies ratios of imported to domestic inputs in the 16-94 percent range, with a median of 30 percent.

the value in our calibration.

Calibrating the model to the data also requires accounting for the fact that, contrary to what the model predicts, international capital flows, and the trade imbalances they finance, do not vanish completely when sovereigns default on private lenders—trade balances actually rise into surpluses, as shown in Figure 1.²⁷ An important component of these continuing capital flows are those vis-a-vis international organizations, on which countries very rarely default.²⁸ In the case of Argentina, the country made repayments to international organizations for about 2.7 percent of GDP in 2002 and as large as 5 percent of GDP by 2006. To adjust our quantitative analysis accordingly, we introduce an amount x_t of exogenous capital flows that are independent of the borrowing and default decisions. For simplicity, and to prevent these capital flows from altering default incentives significantly, we assume that x_t is perfectly correlated with TFP and given by $x_t = \xi \ln \varepsilon_t$, and calibrate the semi-elasticity parameter ξ to data for Argentina as described below. In addition, we will show later that the key features of our quantitative results, except for surge in net exports during the exclusion period, are invariant to removing x_t .

We calibrate the remaining six parameters (σ_ε^2 , ρ_ε , A , β , θ , and ξ) using the simulated method of moments (SMM) to target a set of moment conditions from the data. Productivity shocks in final goods production follow an AR(1) process:

$$\log \varepsilon_t = \rho_\varepsilon \log \varepsilon_{t-1} + \epsilon_t, \quad (30)$$

with $\epsilon_t \stackrel{iid}{\sim} N(0, \sigma_\varepsilon^2)$. We use Tauchen’s (1986) quadrature method to construct a Markov approximation to this process with 25 realizations. Data limitations prevent us from estimating (30) directly using actual TFP data, so we set σ_ε^2 and ρ_ε in the SMM procedure to target the standard deviation and first-order autocorrelation of quarterly H-P detrended GDP. We use seasonally-adjusted quarterly real GDP from Argentina’s Ministry of Economy and Finance (MECON) for the period 1980Q1 to 2005Q4. The standard deviation and autocorrelation of the cyclical component of GDP are 4.7 percent and 0.79 respectively. The TFP process obtained using SMM features $\rho_\varepsilon = 0.95$ and $\sigma_\varepsilon = 1.7$ percent.

The targets for setting A , β , θ and ξ are, respectively, the decline in output at default, the frequency of default, the share of working capital financing in GDP, and the rise in the trade balance-GDP ratio at default.²⁹ The default frequency is 0.69 percent, since Argentina

²⁷The prediction that net exports go to zero when the country is excluded from credit markets is not particular to our model. All existing quantitative models of sovereign default in the Eaton-Gersovitz class have the same feature, because the only way to finance a trade imbalance in this class of models is with foreign credit.

²⁸Only 23 countries have defaulted on the IMF since it was created in 1945 (see Aylward and Thorned (1998)), and these are low income countries or countries in armed conflicts without access to private lenders (e.g. Liberia, Somalia, Congo, Sudan, Afghanistan, Iraq). In all the sovereign defaults included in the event analysis of Figure 1, payments to the IMF continued even after countries defaulted on private lenders, except in the case of Peru in the 1980s.

²⁹ A is useful for targeting the output drop at default because, as mentioned in Section 2, changes in A have similar effects as changes in ω . In particular, lower values of A yield larger output drops at default without altering the slope of the relationship between TFP and these output drops.

has defaulted five times since 1824 (the average default frequency is 2.78 percent annually or 0.69 percent quarterly). Output in the first quarter of 2002 was 13 percent below trend.³⁰ The trade-balance output ratio rose by 10 percentage points in the quarter when Argentina defaulted. Lacking working capital data, we follow Schmitt-Grohe and Uribe’s (2007) strategy to proxy working capital as the fraction of M1 held by firms, using an estimate for the U.S. showing that firms own about two-thirds of M1. Using Argentina’s M1 data and the same two-thirds of firm ownership, we estimate Argentina’s working capital at about 6 percent of GDP. Given all these targets, the SMM procedure yields $A = 0.31$, $\beta = 0.88$, $\theta = 0.7$, and $\xi = -0.67$.³¹

4.2 Cyclical Co-movements in the Baseline Simulation

We evaluate the quantitative performance of the model by comparing moments from the data with moments from the model’s stochastic stationary state. In order to compute the latter, we feed the TFP process to the model and conduct 2000 simulations, each with 500 periods and truncating the first 100 observations.

Table 3 compares the moments from Argentine data (Column (1)) with those produced by the model (Column (2)). The data for National Accounts aggregates is from the sources noted in the calibration. The debt data are from the World Bank’s *GDF* dataset for the 1980-2005 period. The bond spreads data are quarterly EMBI+ spreads on Argentine foreign currency denominated bonds from 1994Q2 to 2001Q4, taken from J.P. Morgan’s EMBI+ dataset. Labor is measured using the employment rate from Argentina’s *INDEC* Permanent Survey of Households. These data yield a similar correlation between labor and the country real interest rate as the one in Neumeyer and Perri (2005) (-0.49 in our data v. -0.45 in their paper). Note also that in Table 3 we follow the sovereign default literature in listing data correlations relative to bond spreads, rather than relative to country interest rates, as in Neumeyer and Perri, but the correlations are similar using either variable (e.g. the correlation between GDP and the interest rate is -0.57 and with the spread is -0.62).

Table 3 shows that the model produces a debt-to-GDP ratio of almost 23 percent on average. This is mainly the result of the effects on default incentives of the large output drop that occurs when the country defaults, and the increasing output cost of default as a function of TFP. Although a 23 percent average debt ratio is still below Argentina’s 35 percent, it is much larger than the mean debt ratios typically obtained in quantitative models of sovereign default with exogenous output costs already targeted to improve the models’ quantitative performance. For

³⁰Argentina declared default in the last week of December in 2001, but it is reasonable to assume that, in quarterly data, the brunt of the real effects of the debt crisis were felt in the first quarter of 2002. Arellano (2008) also follows this convention to date the default as of the first quarter of 2002. She estimated the output cost at 14 percent, measured as a deviation from a linear trend.

³¹Note that β is relatively low compared to typical RBC calibrations, but is in the range of values used in sovereign default models (e.g. β in Aguiar and Gopinath (2006), Arellano (2008), and Yue (2010) ranges from 0.8 to 0.953). These lower discount factors are often justified by arguing that political economy incentives lead government decision-makers to display higher rates of time preference.

instance, Arellano (2008) obtained a mean debt ratio of 6 percent of GDP calibrating her asymmetric cost of default so that income when the economy defaults is the maximum of the actual endowment realization or 97 percent of the average endowment. Yue's (2010) model with renegotiation and an exogenous two-percent proportional output cost of default produced an average debt ratio of 9.7 percent. Using the same default cost, Aguiar and Gopinath (2006) obtained a higher mean debt ratio (27 percent), but with a default frequency of only 0.02 percent. In contrast, our baseline model produces a 23 percent debt ratio at the 0.7 percent default frequency observed for Argentina.

Table 3: Statistical Moments in the Baseline Model and in the Data

	(1)	(2)	(3)
Statistics	Data	Model	Model w/o x_t
Average debt/GDP ratio	35%	22.89%	21.52%
Average bond spreads	1.86%	0.71%	1.02%
Std. dev. of bond spreads	0.78%	1.20%	1.43%
Consumption std. dev./GDP std. dev.	1.44	1.04	1.07
Correlations with GDP			
bond spreads	-0.62	-0.21	-0.25
trade balance	-0.87	-0.53	-0.32
labor ¹	0.39	0.52	0.53
intermediate goods ¹	0.90	0.99	0.99
Correlations with bond spreads			
trade balances	0.82	0.19	0.11
labor ¹	-0.42	-0.24	-0.34
intermediate goods ¹	-0.39	-0.23	-0.24
Historical default-output co-movements			
correlation between default and GDP ¹	-0.11 ²	-0.11	-0.11
fraction of defaults with GDP below trend ¹	61.5% ²	86%	84%
fraction of defaults with large recessions ¹	32.0% ²	37%	47%

Note 1: Statistical moment computed at annual frequency.

Note 2: Cross-country historical estimate for 1820-2004 from Tomz and Wright (2007). Large recessions in Tomz and Wright are deviations from trend in the range -11 to -7 percent, and in the model are recessions with GDP at least two standard deviations below trend (i.e. deviations from trend of at least -9.2 percent).

The variability of the spreads is higher in the model than in the data, while the mean spread in the model is smaller than in the data. The mean spread of 0.71 percent in the model is close to the default frequency, because we assume a zero recovery rate on defaulted debt and risk-neutral creditors, which imply that bond spreads are linked one-to-one with default probabilities (see

eq. (28)). Thus, the model can only generate an average bond spread of a similar magnitude as the default frequency.

The model yields a negative correlation between spreads and GDP, albeit less negative than in the data, because sovereign bonds have higher default risk in bad states. As we noted in the Introduction, both quantitative models of sovereign default and of business cycles in emerging economies also produce countercyclical interest rates, but in the former output is an exogenous endowment and in the latter country risk is exogenous. In contrast, our model produces countercyclical country risk in a setting in which both output and country risk are endogenous, and influence each other.

The model is also consistent with two key stylized facts of emerging markets business cycles: countercyclical net exports and consumption variability that exceeds output variability (although the model underestimates both moments relative to the data). The first result follows from the fact that, when the country is in a bad TFP state, it faces higher interest rates and tends to borrow less from abroad. The country's trade balance thus increases, leading to a negative correlation between net exports and output. The second result occurs because the ability to use external debt to smooth consumption is negatively affected by the higher interest rates induced by increased default probabilities. The sovereign borrows less when the economy faces an adverse TFP shock, and thus households adjust consumption by more than in the absence of default risk. On the other hand, because agents are impatient, the benevolent government borrows more to increase private consumption when the TFP shock is good. Hence, the variability of consumption rises. As with the countercyclical spreads, existing models of sovereign default and emerging markets business cycles can also account for countercyclical net exports and consumption variability in excess of GDP variability, but working under the assumption that either country risk or output fluctuations are exogenous.

The model predicts correlations of labor and intermediate goods with bond spreads and with GDP in line with those observed in the data. The correlation of GDP with intermediate goods is 0.90 in the data, compared with 0.99 in the model, and with spreads (before the 2002 default) is -0.39, compared with -0.23 in the model.³² The correlation of GDP with labor is 0.39 in the data v. 0.52 in the model, and with spreads is -0.42 v. -0.24 in the model. The negative correlations of labor and intermediate goods with the interest rate are due to the credit transmission mechanism that operates via working capital (as explained in Section 2). In turn, intermediate goods and labor are positively correlated with GDP because of the standard real-business-cycle effects of TFP shocks, and because of the reinforcing effect of the countercyclical spreads.

Interestingly, interest rate fluctuations and working capital have important effects in our model even though average sovereign spreads, and hence the average interest rate on working

³²We exclude financial autarky periods in computing correlations with spreads because in the model the default spread goes to infinity when the economy defaults, and hence correlations with the country interest rate are undefined.

capital, do not differ much from the one-percent risk free rate. In contrast, Neumeyer and Perri (2005) and Uribe and Yue (2006) found working capital to be important for emerging markets business cycles using average interest rates around 7 percent and assuming that 100 percent of the wages bill is paid with credit (i.e. the share of working capital in GDP is about 2/3rds). Thus, as Oviedo (2005) also showed, in their models significant effects of working capital require high interest rates on average and a large ratio of working capital credit, while our model requires an average interest rate of about 1.7 percent and a working capital credit requirement calibrated to an Argentine estimate of only 6 percent of GDP.

We also report in Table 3 three moments that pertain to the relationship between default and output in historical cross-country data for the period 1820-2004 based on the work of Tomz and Wright (2007): The correlation between defaults and GDP and the fractions of defaults that occur when GDP is below trend and when recessions are unusually large. Because these are moments based on annual data, we show comparable annual-equivalent moments from the model. The correlation between defaults and GDP in the model is a close match to the actual correlation. The model can produce defaults that occur in good times (defined as GDP above trend) at the annual frequency, but this happens only with 14 percent of defaults. Hence, 86 percent of defaults occur with GDP below trend, which is roughly 25 percentage points more than in the data. On the other hand, the model is close to the data in terms of the fraction of defaults that occur with deep recessions. In the model, 37 percent of defaults occur with GDP two standard deviations or more below trend (which amounts to -9.2 percent below trend), while Tomz and Wright found that 32 percent of defaults occur with GDP in the lowest quintile of the distribution of GDP deviations from trend (which in their data corresponds to the range of -11 to -7 percent below trend).

Column (3) of Table 2 shows the effects of removing the exogenous component of capital flows x_t . This has small effects on most of the moments, except the correlation between net exports and GDP and the mean spreads. Since x_t can be viewed as an exogenous trade deficit that is perfectly correlated with TFP, removing it makes the trade balance less countercyclical. The mean spreads change because we do not change any of the other parameters, and hence without x_t the probability of default increases to about 1 percent, and the mean spreads rise accordingly because of the risk neutral lenders' arbitrage condition. Thus, the exogenous component of capital flows plays a small role in our results, except for enabling the model to produce the trade surplus regularly observed after default events.

4.3 Macroeconomic Dynamics around Default Events

We study the model's ability to match output dynamics around default episodes by applying event study techniques to simulated time series data. The left panel in Figure 7 plots the model's average path of output around default events together with the data for Argentina's HP detrended GDP around the 2002 default (1999Q1 to 2005Q3). The event window covers 12

quarters before and after debt defaults, with the default events normalized to date 0. We plot the average for output in the model at each date $t = -12, \dots, 12$ around default events in the simulations. Hence, the simulated GDP line represents the average behavior of output around defaults in the model's stochastic stationary state. Since Argentina's data is for a single default event, instead of a long-run average, we add dashed lines with one-standard-error bands around the model simulation averages.

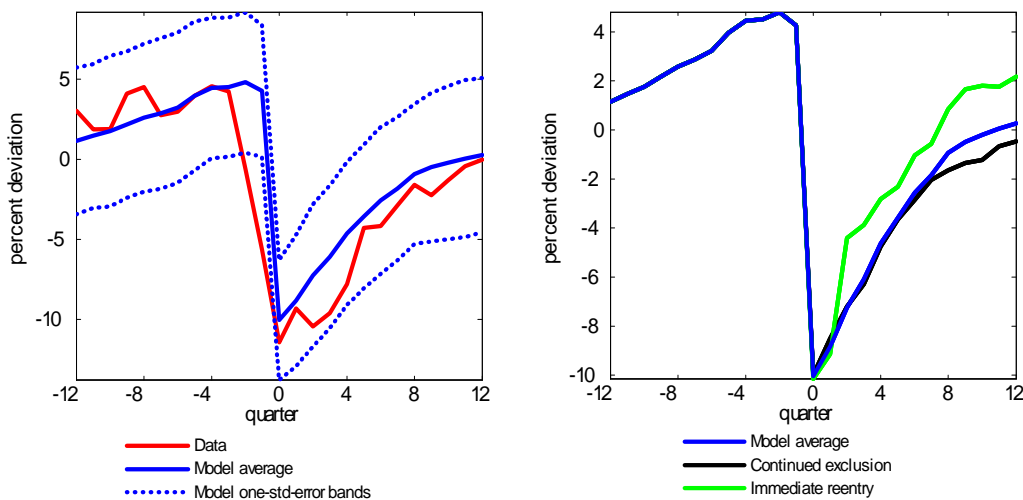


Figure 7: Output around Default Events

The model's mean output dynamics are a good match of those observed in the data. The size of the output drop in the date of the default is matched by calibrating A , but the model also tracks closely Argentina's output dynamics before and after the default. Except for the quarter just before the default, for which the model predicts higher output than in the data, output in the data remains inside the model's one-standard-error bands throughout the twenty-five quarter event window.

Defaults in the model are triggered by adverse TFP shocks, but these shocks are not unusually large. On average, TFP declines by 7.95 percent in the model's default events, which is roughly 1.5 times of the 5.8 percent standard deviation of the calibrated TFP process (σ_ε). Thus, defaults occur with a TFP shock of just about 1.5 standard deviation in size, which suggests that the model's financial transmission mechanism amplifies significantly the real effects of TFP shocks when these shocks trigger default.

The amplification effect can be quantified by computing the average output drop that the model produces in response to a 7.95 percent TFP shock when there is no default, and comparing it with the 13 percent mean output drop that the same shock produces when default occurs.

Without default, a 7.95 percent TFP shock produces a mean output drop of about 4.75 percent. Thus, the amplification coefficient due to default is $13/4.75 = 2.73$.

The recovery of output after defaults is driven by two effects: First, since ε is mean-reverting and defaults occur with TFP below trend, TFP improves on average after defaults. Therefore, even though the country remains in financial autarky on average for three years after a default, the economy recovers because TFP improves. The second effect is the surge in output that occurs if the country re-enters credit markets (as final goods producers switch back to a more efficient mix of imported and domestic inputs). This can happen with 8.3 percent probability every quarter.

These two effects are illustrated in the right panel of Figure 7, which shows the simulated paths of GDP with continued exclusion for 12 quarters after default and with immediate re-entry one period after default. In the first scenario, the recovery reflects only the effect of the mean reversion of ε . Since the probability of re-entry is low (at 8.3 percent), GDP in this exclusion scenario is about the same as in the model average for about 7 quarters after default, but then it moves below the model average because 8 quarters and beyond after a default the probability of re-entry, with the associated efficiency gain, starts to weigh more on the model average. In contrast, the scenario with immediate re-entry shows a big rebound in GDP at $t = 1$, because of the efficiency gain that occurs in that period. The model average lies uniformly below the immediate re-entry scenario because it always assigns some weight to the continued exclusion scenario, in which output is lower, and it generally weighs more the effect of TFP recovery than the effect of credit market re-entry because of the low re-entry probability.

The model's V-shaped output dynamics are qualitatively consistent with the data of emerging markets that experienced Sudden Stops. Calvo, Izquierdo and Talvi (2006) conducted a cross-country empirical analysis of the recovery of emerging economies from Sudden Stops, and found that most recoveries are not associated with improvements in credit market access. In our model as well, recovery occurs (on average) even though the economy continues to be excluded from world credit markets. Notice, however, that the recovery without credit is always modest relative to what occurs if credit market access is regained.

Figure 8 shows event windows that compare the actual default event dynamics from our cross-country dataset (as shown in Figure 1) with the average of the model simulations, the corresponding one-standard-error bands and also the data from Argentina's 2002 default.³³ The plots show event windows for GDP, consumption, the trade balance-GDP ratio, imported inputs, total intermediate goods, labor, the country interest rate and the debt-GDP ratio.

³³We provide both the cross-country medians and the observations for Argentina's default because we aim to illustrate how well the model can match both the behavior across the default events in our cross-country dataset and the data for Argentina. The former is harder because the model's calibration is based on Argentine data, and thus misses cross-country variation in the model's key parameters.

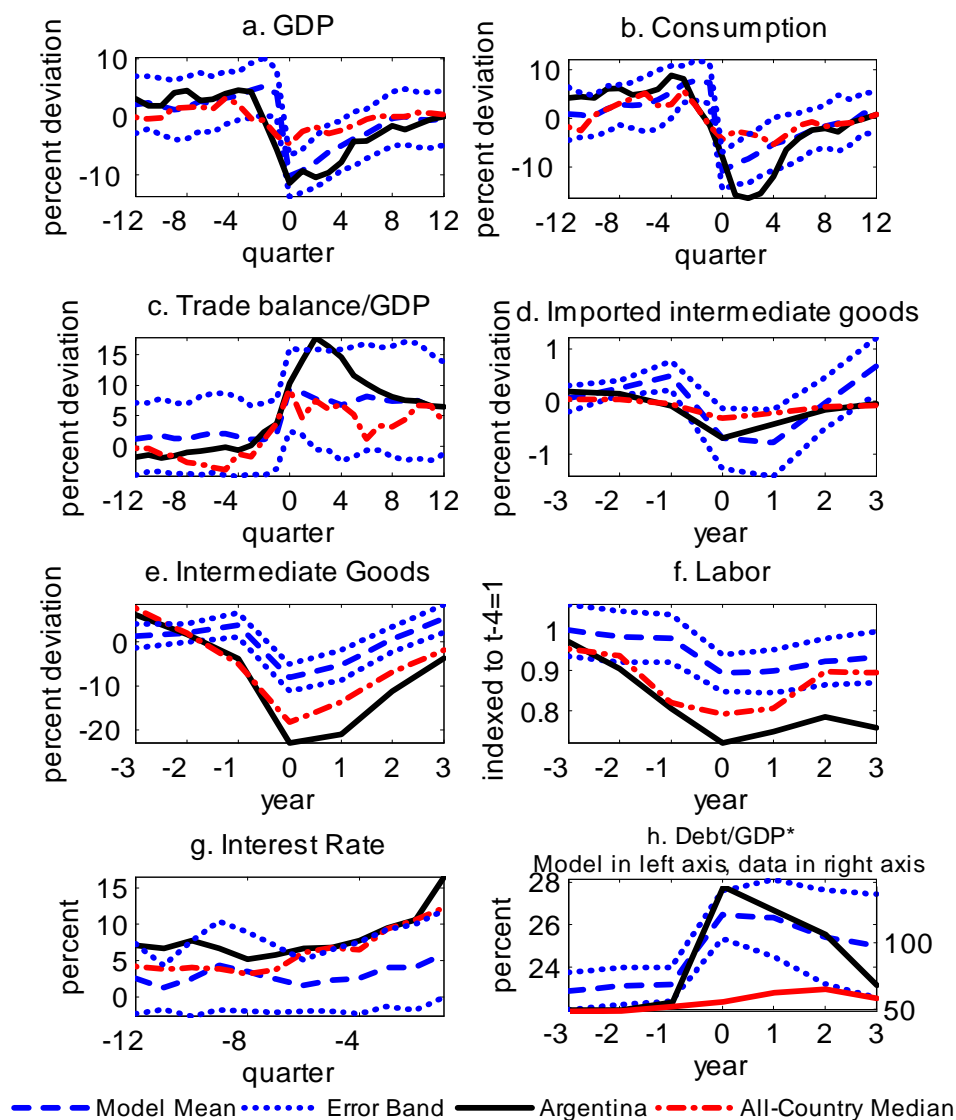


Figure 8: Macro Dynamics around Default Events

Note: Consumption is H-P detrended. Trade balance and intermediate goods are computed as the ratio of GDP and H-P detrended. For labor, imported inputs, total intermediate goods, and debt, we aggregate the simulated data into annual frequency. Imported inputs and total intermediate goods are log-linearly detrended. Labor is rescaled by the level of the corresponding measure at 3 years before default. We take the annual data from Argentina and apply the same procedure.

**The scale for Argentine and cross-country median debt/GDP is drawn on the right axis.*

The model does well at replicating the observed behavior of GDP, consumption and net exports, in the sense that the actual paths of these variables are mostly within the model’s standard error bands and close to the model averages. This is true for Argentina 2002 and for the cross-country medians. The GDP and consumption drops that occur at $t = 0$ are larger in the model than in the cross-country medians, but this is because Argentina’s recession was larger than the median and we calibrated the model to match Argentina’s output collapse.

The model is consistent with the data in predicting a relatively stable path for net exports before the default, a sudden increase around the default date and a sustained surplus after that. The adjustment to include the exogenous component of capital flows is important for the sudden increase, but not for the stable trade balance before default. The rise in net exports when default occurs is triggered by a low TFP realization ε_0 , which is associated with a surge in x_0 . Without this, net exports would go to zero at $t = 0$ and a small deficit on average during the remainder of the exclusion period, because during exclusion there would be no capital flows to finance a trade imbalance (the average deficit during exclusion would follow from averaging a zero trade balance under financial autarky with a trade deficit in case of re-entry to credit markets).

The qualitative features of the dynamics of imported inputs, total intermediate goods and labor are also in line with the data. Quantitatively, however, the model predicts smaller declines in labor and intermediate goods when defaults occur than in the data—with the caveat that nontrivial data limitations in terms of country coverage and sample periods for these variables makes them a weaker benchmark against which to compare the model’s dynamics. The drop in imported inputs in the model is similar to the one observed in Argentina, but larger than the cross-country median, reflecting again the fact that the model is calibrated to match the recession in Argentina, which was larger than the cross-country median.

The model also does well at capturing the qualitative features of the dynamics of the interest rate (inclusive of country risk) and the debt ratio, but quantitatively it tends to underestimate both. The model produces a steady increase in the interest rate for the six quarters before default.³⁴ The debt ratio is relatively stable in the years before default, then surges with the default, and drops in the years that follow.³⁵ As explained earlier, interest rates in the model are relatively low because they are pinned down by the default frequency, given the arbitrage condition of risk neutral lenders, and the observed default frequency is low. The low debt ratio is also in line with the previous result showing that while this model can support significantly higher debt ratios than existing models of sovereign default, it still underestimates actual debt ratios.

³⁴As explained earlier, we do not show interest rates for $t \geq 0$ because in the default state the default risk is infinite.

³⁵To make debt ratios comparable across data and model during periods of exclusion, we adjusted the model’s measure to match the practice followed in the World Bank dataset, which includes defaulted debt and the corresponding interest in arrears in the debt estimates. Thus, the mean debt ratio for the model after default in the event plot is the average of the pre-default debt ratio and the debt ratio chosen in the case of re-entry.

The sharp declines in GDP, consumption, labor and intermediate goods that occur when access to credit markets is lost indicate that the model yields predictions consistent with the Sudden Stops observed in emerging economies. In most of the Sudden Stops literature, however, the loss of credit market access is modeled as the result of an exogenous shock, whereas in this model the exclusion from credit markets and the economic collapse are endogenous and influence each other.³⁶

4.4 Sensitivity Analysis

In this Section we conduct a sensitivity analysis to evaluate the robustness of the model’s key quantitative predictions to changes in the range of imported inputs that require working capital θ , the parameters of the Armington aggregator μ and λ , the within-variety elasticity parameter ν , the labor elasticity parameter ω , and the re-entry probability ϕ . The results are summarized in Table 4.³⁷ This Table shows the main statistical moments we used to evaluate the performance of the model for each alternative scenario, and also reproduces the statistics from the Argentine data and the baseline model (see rows (1) and (2)).

(a) *Working capital*

Rows (3)-(5) report results varying the value of θ . Row (3) removes working capital altogether by setting $\theta = 0$, while rows (4) and (5) set θ smaller ($\theta = 0.6$) and larger ($\theta = 0.8$) than the 0.7 baseline.

Without working capital, there is no output cost of default and no financial amplification effect. Hence, to keep the results comparable with the other results in Table 4, in which default is costly, and with those reported in quantitative studies of sovereign debt that use exogenous proportional costs of default (e.g. Aguiar and Gopinath (2006), Yue (2010)), we introduce an exogenous proportional cost of default when $\theta = 0$. This cost is set so that TFP falls by as much as needed to produce an output drop of 13 percent when default occurs, which is the same size as the drop in our baseline calibration. The other parameters are kept unchanged.

The model without working capital performs much worse than the baseline. In particular, the frequency of defaults (proxied by the mean spread because of the lender’s risk neutrality) falls from 0.7 to 0.05 percent, the variability of spreads also nearly vanishes, spreads become procyclical, and the mean debt ratio drops to 9 percent. This marked worsening in the performance of the model follows from the fact that, without working capital, bond spreads no longer affect factor demands and production, and thus the cost of default becomes independent of TFP. In short, this scenario shows that if we reduce our model to a variant of the standard

³⁶Mendoza (2010) proposed an alternative model of endogenous Sudden Stops based on collateral constraints and Irving Fisher’s debt-deflation mechanism instead of sovereign default risk.

³⁷We also generated plots with the default event dynamics of output comparable to Figure 7. The quantitative differences are small and qualitatively they all have the same pattern, so we decided not to put them in the paper. We also conducted the full sensitivity analysis without the exogenous component of capital flows x_t . Most of the results are similar to those reported in Table 4, which again verifies that x_t plays a minor role, except for enabling the model to produce the trade surplus after default.

Eaton-Gersovitz model with an exogenous proportional cost of default, we obtain results very similar to what other authors have found.³⁸

Table 4: Sensitivity Analysis

	Output drop at default	Mean Debt/GDP ratio	Mean spread	Std. dev of spread	GDP corr. with spread default		frequency of default w. GDP below trend
(1) Data	13%	35%	1.86%	0.78%	-0.62	-0.11	62%
(2) Baseline	13%	22.89%	0.71%	1.20%	-0.21	-0.10	86%
Working capital							
(3) $\theta = 0$	13%	8.99%	0.05%	0.08%	0.24	-0.02	75%
(4) $\theta = 0.6$	13.6%	20.38%	0.44%	0.88%	-0.12	-0.07	83%
(5) $\theta = 0.8$	14.44%	27.03%	0.39%	0.89%	-0.08	-0.07	96%
Armington elasticity							
(6) 2.63 ($\mu = 0.62$)	15.46%	31.23%	0.53%	0.94%	-0.16	-0.09	88%
(7) 3.10 ($\mu = 0.68$)	12.20%	16.15%	0.91%	1.36%	-0.15	-0.10	80%
Armington share							
(8) $\lambda = 0.58$	16.30%	39.07%	0.28%	0.80%	-0.09	-0.07	77%
(9) $\lambda = 0.66$	12.75%	14.19%	0.74%	1.21%	-0.18	-0.08	88%
Within-variety elasticity							
(10) 2.22 ($\nu = 0.55$)	14.46%	25.94%	0.36%	0.84%	-0.07	-0.07	87%
(11) 2.89 ($\nu = 0.65$)	12.43%	19.77%	0.73%	1.17%	-0.12	-0.1	83%
Frisch elasticity of labor supply							
(12) 1.67 ($\omega = 1.6$)	12.6%	22.34%	0.81%	1.22%	-0.16	-0.12	84%
(13) 2.5 ($\omega = 1.4$)	15.2%	24.35%	0.46%	1.06%	-0.03	-0.06	70%
Probability of re-entry							
(14) $\phi = 0.05$	15.29%	37.23%	0.26%	0.71%	-0.12	-0.04	77%
(15) $\phi = 0.1$	13.85%	19.73%	0.61%	1.18%	-0.11	-0.07	89%

Row (5) shows that widening the range of imported input varieties that require working capital to $\theta = 0.8$ (i.e. tightening the working capital constraint), instead of 0.7 as in the baseline, increases the mean debt ratio by about 4.5 percentage points of GDP and generates a larger output cost of default. The probability of default and the variability of spreads fall by about 30 basis points, and the correlation between GDP and spreads increases to -0.08. If instead we assume a narrower range of imported varieties that require working capital ($\theta = 0.6$), Row (4) shows that we obtain a lower mean debt ratio than in the baseline, but the cost of

³⁸Note that the calibrated proportional drop in TFP is less than 1 percent in this experiment. If we set a 2 percent drop as in the literature, the output drop in default becomes 15 percent, the mean debt ratio is 24 percent, and the average spread falls to 0.01 percent, which are results in line with the literature (e.g. Aguiar and Gopinath (2006)).

default and the correlation between GDP and spreads still rise, and the frequency of default and the variability of spreads still fall, as in the case with $\theta = 0.8$. Hence, while rising θ seems to have a monotonic effect increasing the mean debt ratio, the effects on the other moments are non-monotonic. This is because higher θ has potentially opposing effects on default incentives and production plans. On one hand, since a larger fraction of imported inputs requires foreign financing, changes in interest rates have a larger impact on production. This amplifies the response of output to productivity shocks, making output more volatile. On the other hand, default can still lead to a higher output cost even with lower θ if the TFP shock that triggers default is larger with $\theta = 0.6$ than with $\theta = 0.7$. At the same time, the higher output cost of default and GDP variability make it optimal for the sovereign to exercise the default option less often, lowering the default probability and the volatility of bond spreads, and increasing the mean debt/GDP ratio.

The distribution of defaults across “bad times” and “good times” also changes with the value of θ . In particular, higher θ shifts the distribution toward states with GDP below trend. At the annual frequency, $\theta = 0.8$ implies that about 96 percent of defaults occur with output below trend (compared with 86 percent in the baseline and 83 percent with $\theta = 0.6$). The correlation between GDP and default changes slightly to -0.07 with either $\theta = 0.8$ or 0.6.

(b) Parameters of the Armington Aggregator of Domestic and Imported Inputs

Rows (6) and (7) report results for Armington elasticities lower ($\eta_{m^d, m^*} = 2.63$) and higher ($\eta_{m^d, m^*} = 3.1$) than the baseline ($\eta_{m^d, m^*} = 2.9$). Comparing the three scenarios, we find predictable results from making domestic and imported inputs better substitutes, and in this case the effects are monotonic. As inputs become better substitutes, the output cost at default and the mean debt ratio fall, and the frequency of default and the variability of the spreads increase. The effects of lowering the Armington share of domestic inputs to $\lambda = 0.58$ (Row (8)) or increasing it to $\lambda = 0.66$ (Row (9)), relative to the 0.62 baseline are similar: Increasing λ makes the working capital channel operating via imported inputs less relevant, and as a result the output cost at default and the mean debt ratio fall, and the frequency of default and the variability of the spreads increase. Changes in η_{m^d, m^*} and λ have small effects on business cycle comovements. The correlations of GDP with spreads are slightly higher than in the baseline case, but remain weakly negative.

It is worth noting that these variations η_{m^d, m^*} and λ result in expenditure ratios of imported to domestic inputs that differ from those in the baseline and in the Mexican data used for calibration. However, these expenditure ratios are still in the range of those for the countries included in the study by Goldberg and Campa (2006), so scenarios like those in Rows (6)-(9) should be regarded as plausible. Moreover, the range of expenditure ratios they documented would also support a wider range of values of η_{m^d, m^*} and λ than those shown in Table 4, including scenarios that can support very high debt ratios, albeit at lower default probabilities. For example, using $\lambda = 0.5$ we found a mean debt ratio of about 93 percent at a 0.2 percent

default frequency. Similarly, if we lower η_{m^d, m^*} to the threshold value at which imported and domestic inputs become gross complements (1.96), the mean debt ratio rises to 80 percent at a negligible default frequency, and the output cost of default climbs to 22.6 percent.

Changing η_{m^d, m^*} and λ also affects the distribution of default events across output realizations, but in this case the two parameters have effects that go in opposite directions. Aggregating again to annual frequency, the fraction of defaults that occurs with output below trend falls from 88 to 80 percent as we increase η_{m^d, m^*} from 2.63 to 3.1, while it rises from 77 to 88 percent as we increase λ from 0.58 to 0.66. The correlations between GDP and default events change only slightly.

(c) Within Elasticity of Imported Input Varieties

Rows (10) and (11) show results for lower (2.22) and higher (2.89) values of $\eta_{m_j^*}$ than the 2.44 baseline. Making imported input varieties better substitutes also weakens the working capital channel, because it implies that the efficiency loss of shifting away from the subset Ω of imported inputs to its complement is smaller. Hence, as $\eta_{m_j^*}$ increases, the output drop at default and the mean debt ratio fall, while the mean and standard deviation of spreads increase. The fraction of defaults that occur with output below trend falls from 87 to 83 percent as $\eta_{m_j^*}$ rises from 2.22 to 2.89.

These effects are again similar to those we obtained by increasing η_{m^d, m^*} , but quantitatively they are weaker. In particular, we increased $\eta_{m_j^*}$ by 62 basis points in Rows (10) and (11) v. 50 basis points for η_{m^d, m^*} in Rows (6) and (7), and yet the changes in the cost of default, mean debt ratio, mean and variability of spreads, and fraction of defaults with GDP below trend were *smaller* in absolute value. This is in line with our previous findings from the partial equilibrium analysis of the working capital mechanism, showing that changes in η_{m^d, m^*} have larger effects on the responses of production and factor allocations to interest rate fluctuations than changes in $\eta_{m_j^*}$. Thus, for parameterizations around the baseline calibration, the Armington elasticity of substitution across domestic and imported inputs affects more the model's quantitative performance than the within elasticity across imported input varieties.

(d) Frisch Elasticity of Labor Supply

Rows (12) and (13) show results for lower (1.67) and higher (2.5) labor supply elasticities than in the baseline (2.2). As we showed in Section 3, the efficiency loss triggered by default is larger the higher the elasticity of labor supply, because it implies that the labor market adjusts to interest rate hikes with a smaller wage decline and a larger decline in total labor, which in turn produce a smaller increase in L^m and a larger fall L^f . Hence, the output drop at default and the mean debt ratio are bigger as we increase the elasticity of labor supply. At the same time, the default probability (or the mean spread) and the variability of spreads fall with a bigger labor elasticity. The change in labor elasticity also affects the distribution of default events across output realizations. The fraction of defaults with GDP below trend falls to 70 percent when the Frisch elasticity of labor supply is 2.5. The correlation between GDP again

increases slightly but remains weakly negative.

(e) Re-entry Probability (Mean Length of the Exclusion Period)

In Rows (14) and (15) we change the probability of re-entry to 5 and 10 percent respectively, which imply exclusion periods of 5 and 2.5 years respectively (v. 3 years in the baseline). Here we encounter again non-monotonic effects. The output drop at default is higher, and the frequency of default and the variability of spreads lower, when the mean exclusion period is either shorter or longer than in the baseline. In contrast, the mean debt ratio does decline monotonically as we lower the length of the exclusion period, from 37 percent with 5 years of exclusion to about 20 percent with 2.5 years. These results follow from the fact that increasing ϕ has ambiguous effects on the value of default at date t (eq. (24)), by increasing the weigh of the re-entry option at $t + 1$ but lowering the value of continuing in autarky. Intuitively, taking the default option today becomes more attractive because credit market re-entry in the future is more likely (i.e. "easier") but the expected value of staying in autarky is itself lower as the probability of continuing in this state is lower.

The distribution of defaults also changes with the re-entry probability. The higher re-entry probability shifts the distribution toward states with GDP below trend. At the annual frequency, $\phi = 0.1$ implies that about 89 percent of defaults occur with output below trend (compared with 86 percent in the baseline case). Yet for $\phi = 0.05$, the corresponding fraction is 77 percent. As with the other scenarios, the correlations of GDP with defaults and with spreads increase slightly, but remain weakly negative.

Summing up, this sensitivity analysis shows that while the model's statistical moments vary somewhat as we change key parameters, the main quantitative findings are robust to these changes: The model produces large endogenous output drops at default as a result of the working capital channel, it produces higher mean debt ratios at higher default frequencies than is generally the case in the quantitative sovereign debt literature, and generates weakly counter-cyclical spreads. The correlation between GDP and defaults remains slightly negative, and the frequency of defaults with GDP below trend remains mostly in the 73-88 percent range. The exception is the case in which the working capital channel is removed ($\theta = 0$ in Row (3)), which removes the endogenous output cost of default. Replacing it with an exogenous proportional cost calibrated to match the observed output drop, the model falls apart and can only generate a small debt ratio at negligible levels of the default frequency and the variability of spreads, and with pro-cyclical spreads.

5 Conclusions

This paper proposed an equilibrium model of sovereign default and business cycles and showed that its quantitative predictions are broadly in line with observed empirical regularities. The model features a financial amplification mechanism that links default risk with production plans

and factor allocations via an efficiency loss, because increases in default risk increase the firms' financing cost of working capital on a subset of imported inputs. This mechanism produces a novel feedback loop between default risk, business cycles and the output cost of default.

In the model, producers of final goods choose an optimal mix of imported and domestic inputs that are imperfect substitutes in an Armington aggregator, and varieties of imported inputs that are also imperfect substitutes in a Dixit-Stiglitz aggregator. Some imported input varieties require foreign working capital financing, and production of domestic inputs requires reallocation of labor away from final goods production. In this setup, strategic default causes an efficiency loss because final goods producers cannot operate with the imported input varieties that require credit, substituting them for other imported inputs and domestic inputs, and because labor reallocates from the final goods sector to the sector producing domestic inputs. Lenders charge the same default risk premium on working capital loans as on sovereign debt because the sovereign diverts the repayment of working capital loans when the country defaults. These characteristics of the model are in line with empirical evidence on the substitution of inputs around default episodes (e.g. Gopinath and Neiman (2010)), the strong correlation between corporate and sovereign credit conditions (e.g. Arellano and Kocherlakota (2007)), and the governments' assumption of foreign obligations of the private sector when sovereigns default (e.g. Reinhart (2010)).

The model is broadly consistent with three key stylized facts of sovereign debt: (1) the dynamics of macroeconomic aggregates around default events, (2) the negative correlation between interest rates on sovereign debt and output, and (3) high debt-output ratios on average and when defaults take place. The model also explains key emerging markets business cycle moments such as countercyclical net exports, high variability of private consumption, weakly negative correlations between defaults and GDP, and the correlations of intermediate goods and labor with spreads and GDP. Moreover, the results show that default occurs with adverse TFP shocks that are not unusually large on average (about 1.5 times the standard deviation of TFP), and that the model embodies a powerful financial amplification mechanism that amplifies the effect of TFP shocks on GDP by a factor of 2.7 in default events.

The model produces an endogenous output cost of default that is increasing in the state of productivity. This is a key feature of the model that follows from the efficiency loss induced by the working capital channel, and from the convexity of marginal products with Cobb-Douglas production. In this way, our model provides a foundation for the ad-hoc default cost (increasing in endowment income above a threshold) that Arellano (2008) identified as important in order to induce default incentives that trigger default in bad states, at non-negligible debt ratios, and at realistic default frequencies. In addition, the endogenous feedback between production and default in our model produces a mean debt ratio four times larger than in Arellano's model.

The model also provides a solution to the disconnect between sovereign debt models (which rely on exogenous endowment dynamics with ad-hoc costs) and models of emerging markets

business cycles (which assume an exogenous financing cost of working capital calibrated to match the interest rate on sovereign debt). In addition, the model offers an interesting new perspective on how structural features of the private sector and industrial and trade policies may interact with sovereign debt ratios and default risk. The cost of default is lower in economies where intermediate goods are better substitutes, where imported inputs are a smaller share of total inputs (either because of the nature of technology or because of trade protection), or where labor supply has a lower Frisch elasticity. As a result, economies with these characteristics can support lower mean debt ratios at higher default frequencies. This may help explain why the 1980s cluster of debt crises in Latin America affected countries that engaged in import substitution policies in the previous two decades.

We acknowledge, however, that the linkages between sovereign default and private sector credit, and the mechanisms by which default induces efficiency losses, should be the subject of further research. For instance, introducing elements of political uncertainty, debt maturity, secondary debt markets, dynamic renegotiation, and risk averse lenders, all of which have been shown to add significant elements to the analysis of sovereign default in models with exogenous output dynamics, can be very promising lines of research (see, for example, Amador (2003), Benjamin and Wright (2008), Bi (2008a and 2008b), Broner, Martin and Ventura (2008), Chatterjee and Eyigungor (2008), Cuadra and Saprizza (2008), D’Erasmus (2008), and Lizarazo (2005)).

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Appendix 1: The Firms' Dynamic Optimization Problem with Working Capital

We review here the optimization problem faced by a representative firm that needs working capital to pay for a fraction of the cost of imported inputs. The setup is based on a similar derivation in Uribe and Yue (2006). For simplicity, we characterize the problem in partial equilibrium, assuming that all intermediate goods are imported, and that there is a single homogeneous foreign input. We are interested in particular in the following two results:

1. The interest rate on within-period working capital loans is determined by the same rate as the between-period rate of interest on one-period debt.
2. If firms solve a standard problem to maximize the present value of profits, they do not accumulate precautionary asset holdings. In particular, if they start with zero liabilities, they maintain zero liabilities at the end of each period in perpetuity.

Consider a representative firm in a small open economy that produces output by means of a production function that uses imported intermediate goods and labor as inputs,

$$y_t = F(m_t, L_t), \quad (31)$$

where the function F is homogeneous of degree one, increasing in both arguments, and concave. Firms buy their inputs from perfectly competitive markets.

Production is subject to a working capital constraint that requires firms to pay in advance for a fraction θ of the cost of imported inputs, which have a world-determined relative price p . The working capital constraint is:

$$\frac{\kappa_t}{R_t} \geq \theta p m_t; \quad \theta \geq 0,$$

where κ_t denotes the amount of working capital held by the representative firm in period t .

The above formulation of the working capital constraint corresponds to a timing of transactions akin to a "cash-in-advance constraint," by which firms *must* hold non-interest-bearing *foreign* assets by an amount equal to the fraction θ of the cost of imported inputs. There is also an alternative formulation known as the "shopping time" formulation, according to which firms need to have the working capital $\theta p m_t$ at the end of the period (see Uribe and Yue 2006 and Oviedo 2005). The two differ only in that the former increases the cost of inputs by $\theta (R_t - 1)$, as we show below, and the latter by $\theta (R_t - 1) / R_t$, but in both cases the relevant interest rate is determined by the same interest rate as for one-period debt R_t .

The debt position of the firm, denoted by d_t , evolves according to the following period-by-period budget constraint:

$$d_t = R_{t-1}d_{t-1} - F(m_t, L_t) + w_t L_t + pm_t + \pi_t - \kappa_{t-1} + \kappa_t,$$

where π_t denotes profits in period t , and R_t is the interest rate in one-period bonds. Thus, there is no assumption requiring that the interest rate on between period debt be the same as that on working capital loans.

Define the firm's total net liabilities at the end of period t as $a_t \equiv R_t d_t - \kappa_t$. Then, we can rewrite the budget constraint as:

$$\frac{a_t}{R_t} = a_{t-1} - F(m_t, L_t) + w_t L_t + pm_t + \pi_t + \left(\frac{R_t - 1}{R_t} \right) \kappa_t.$$

Assume the interest rate is positive at all times. This implies that the working capital constraint always binds, or otherwise the firm would incur in unnecessary financial costs, which would be suboptimal. Since the working capital constraint holds with equality, we can eliminate κ_t from the above expression to get:

$$\frac{a_t}{R_t} = a_{t-1} - F(m_t, L_t) + pm_t [1 + \theta (R_t - 1)] + w_t L_t + \pi_t. \quad (32)$$

The first result we highlighted earlier is evident in this expression: The working capital constraint increases the unit cost of imported inputs by the amount $\theta(R_t - 1)$, which is increasing in the interest rate R_t , where R_t is the same interest rate as the one charged on one-period debt.

The firm's problem is to maximize the present discounted value of the stream of profits. In the paper the owners are domestic residents, so firms discount at the households' stochastic discount factor $\beta^t \frac{\lambda_t}{\lambda_0}$, where $\lambda_t = \beta R_t E_t \lambda_{t+1}$ is the Euler equation for bond holdings. Alternatively, firms can be assumed to discount profits at the world interest rates R_t . For the results we show here this does not matter. Under the first alternative, the firm's problem is,

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \pi_t.$$

Using constraint (32) to eliminate π_t from the firm's objective function, the firm's problem can be stated as choosing a_t , m_t , and L_t so as to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left\{ \frac{a_t}{R_t} - a_{t-1} + F(m_t, L_t) - pm_t [1 + \theta (R_t - 1)] - w_t L_t \right\},$$

subject to the no-Ponzi-game constraint $\lim_{j \rightarrow \infty} E_t \frac{a_{t+j}}{\prod_{s=0}^j R_{t+s}} \leq 0$.

The first-order conditions associated with this problem are the Euler equation for net liabilities, $\lambda_t = \beta R_t E_t \lambda_{t+1}$, the no-Ponzi-game constraint holding with equality, and

$$F_m(m_t, L_t) = p[1 + \theta(R_t - 1)] \quad (33)$$

$$F_L(m_t, L_t) = w_t. \quad (34)$$

It is clear from condition (33) that the working capital requirement drives a wedge between the marginal product of imported inputs and their world relative price p . This wedge is larger the larger the financing cost of working capital, $(R_t - 1)$, or the higher the fraction of the cost of imported inputs that needs to be paid with credit, θ .

It is critical to note that, since total net liabilities are irrelevant for the optimal choices of labor and imported inputs and the payoff function of the firm is linear with respect to net liabilities (and all the terms in $\lambda_t = \beta R_t E_t \lambda_{t+1}$ are exogenous to the firm's choices), any process a_t satisfying equation (32) and the firm's no-Ponzi-game constraint is optimal. Hence, if firms start out with zero net liabilities, then an optimal plan consists in holding no liabilities at all times ($a_t = 0$ for all $t \geq 0$), with distributed profits given by

$$\pi_t = F(k_t, h_t) - pm_t[1 + \theta(R_t - 1)] - w_t L_t$$

This implies that firms do not accumulate precautionary holdings of assets, regardless of the input prices they face. Their choices of labor and imported inputs follow from the optimality conditions (33)-(34), which depend only on current values of factor prices, the rate of interest and TFP. These are not assumptions attached to the working capital requirement, but a result that follows from the linear nature of the firm's payoff: If firms maximize the present value of profits, with discount rates independent of the firm's choices and net liabilities entering linearly in profits, there is no incentive for precautionary asset holdings by firms. One can of course propose alternative formulations that deviate from these conditions and would produce precautionary asset demand by firms, but these are not conditions assumed in the setup of the model.

Appendix 2: Decentralized Equilibrium

The social planner's problem studied in Section 2 can be decentralized by formulating the problem of the sovereign as an optimal policy problem akin to a Ramsey problem.³⁹ The government chooses a debt policy (amounts and default or repayment) that maximizes the households' welfare given a bond pricing function $q_t(b_{t+1}, \varepsilon_t)$ and subject to the constraints that: (a) the private sector allocations must be a competitive equilibrium; and (b) the government budget constraint

³⁹See Cuadra and Sapriza (2007) for an analysis of optimal fiscal policy as a Ramsey problem in the presence of sovereign default in an endowment economy.

must hold. The first constraint is dealt with by using conditions (14)-(20) to write down recursive functions that represent the competitive equilibria of factor allocations and factor prices as functions of bond prices and TFP: $\kappa(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$, $M(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$, $m^*(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$, $m^d(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$, $L^f(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$, $L^m(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$, and $L(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t)$, recalling that there is a one-to-one mapping between $q_t(b_{t+1}, \varepsilon_t)$ and r_t . These functions are then entered into the government's optimization problem below.

This decentralization also requires an assumption about how the government deals with the diverted repayment of working capital loans. This diverted repayment can be treated as a cost of default, in which case we obtain the same allocations and prices in the decentralized equilibrium as the planner's problem of Section 2. Alternatively, if the government rebates the repayment as a lump-sum transfer to households, the stock of working capital becomes an extra state variable, because of the income effect on households resulting from the transfer of the working capital repayment when the country defaults. To consider both cases, in what follows we define the state variables of the government's problem as $(b_t, \kappa_{t-1}, \varepsilon_t)$, but κ_{t-1} is only a relevant state if we assume that diverted repayments of working capital are rebated to households. Note, however, that since κ_{t-1} is small in the calibration, our quantitative results change very little regardless of whether we assume that these repayments are rebated or not.

The recursive optimization problem of the government is:

$$V(b_t, \kappa_{t-1}, \varepsilon_t) = \max \left\{ v^{nd}(b_t, \varepsilon_t), v^d(\kappa_{t-1}, \varepsilon_t) \right\}. \quad (35)$$

The continuation value is defined as follows:

$$v^{nd}(b_t, \varepsilon_t) = \max_{c_t, b_{t+1}} \left\{ \begin{array}{l} u(c_t - g(L(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t))) \\ + \beta E[V(b_{t+1}, \kappa(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t), \varepsilon_{t+1})] \end{array} \right\}, \quad (36)$$

subject to

$$c_t + q_t(b_{t+1}, \varepsilon_t) b_{t+1} - b_t \leq \varepsilon_t f \left(M(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t), L^f(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t), k) \right. \\ \left. - m^*(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t) P_m^* \left(\frac{1}{q_t(b_{t+1}, \varepsilon_t)} - 1 \right) \right), \quad (37)$$

where $f(\cdot) = M^{\alpha_M} (L_t^f)^{\alpha_L} k^{\alpha_k}$. Note that the constraint of this problem is again the resource constraint of the economy at a competitive equilibrium.

The working capital loans κ_{t-1} and κ_t do not enter explicitly in the continuation value or in the resource constraint but they are relevant state variables, because the amount of working capital loans taken by final goods producers at date t affects the sovereign's incentive to default

at $t + 1$. In particular, the value of default is:

$$v^d(\kappa_{t-1}, \varepsilon_t) = \max_{c_t} \left\{ \begin{array}{l} u(c_t - g(\tilde{L}(\varepsilon_t))) \\ +\beta(1-\phi)Ev^d(0, \varepsilon_{t+1}) + \beta\phi EV(0, 0, \varepsilon_{t+1}) \end{array} \right\}, \quad (38)$$

subject to:

$$c_t = \varepsilon_t f(\tilde{M}(\varepsilon_t), \tilde{L}^f(\varepsilon_t), k) - m^*(\varepsilon_t) \tilde{P}^* + \kappa_{t-1}. \quad (39)$$

Note that $v^d(\kappa_{t-1}, \varepsilon_t)$ takes into account the fact that in case of default at date t , the country has no access to financial markets that period, and hence the country consumes the total income given by the resource constraint in the default scenario. In this case, since firms cannot borrow to finance the subset Ω of imported inputs, $\tilde{M}(\varepsilon)$, $\tilde{L}(\varepsilon)$ and $\tilde{L}^f(\varepsilon)$ are competitive equilibrium allocations that correspond to the case when the f sector operates without those inputs, and $\tilde{P}^* = \left[\int_{\theta}^1 (p_j^*)^{\frac{\nu}{\nu-1}} dj \right]^{\frac{\nu-1}{\nu}}$. Moreover, because the defaulting government diverts the repayment of last period's working capital loans, household income includes government transfers equal to the appropriated repayment κ_{t-1} (i.e., on the date of default, the government budget constraint is $T_t = \kappa_{t-1}$).

The default set is now defined as:

$$D(b_t, \kappa_{t-1}) = \left\{ \varepsilon_t : v^{nd}(b_t, \varepsilon_t) \leq v^d(\kappa_{t-1}, \varepsilon_t) \right\}. \quad (40)$$

This default set has a different specification than in the typical Eaton-Gersovitz model (see Arellano (2008)), because the state of working capital affects the gap between the values of default and repayment. This results in a two-dimensional default set that depends on b_t and κ_{t-1} , instead of just b_t . Despite of this, however, the probability of default remains a function of b_{t+1} and ε_t only. This is because the f sector's optimality conditions imply that the next period's working capital loan κ_t depends on ε_t and the interest rate, which is a function of b_{t+1} and ε_t . Thus the probability of default at $t + 1$ perceived as of date t for a country with a productivity ε_t and debt b_{t+1} , $p_t(b_{t+1}, \varepsilon_t)$, can be induced from the default set, the decision rule for working capital, and the transition probability function of productivity shocks $\mu(\varepsilon_{t+1}|\varepsilon_t)$ as follows:

$$p_t(b_{t+1}, \varepsilon_t) = \int_{D(b_{t+1}, \kappa_t)} d\mu(\varepsilon_{t+1}|\varepsilon_t), \text{ where } \kappa_t = \kappa(q_t(b_{t+1}, \varepsilon_t), \varepsilon_t). \quad (41)$$

To show the equivalence between the decentralized equilibrium when diverted working capital repayments are not rebated and the planner's problem of Section 2, consider first that, without that rebate, κ_{t-1} is removed from the set of state variables in the government's problem, and from the right-hand-side of the resource constraint when the country defaults. Next, consider the fact that the recursive functions defined above to characterize factor allocations and prices as functions of $q_t(b_{t+1}, \varepsilon_t)$, ε_t imply that conditions (14)-(20) hold. These are identical to the

first-order conditions that set factor allocations for the social planner in Section 2, and the corresponding shadow prices of that planner's problem determine the same wage rate and price of domestic inputs. From these results it follows that, for given $q_t(b_{t+1}, \varepsilon_t)$, b_{t+1} and b_t , the resource constraints in the continuation and default branches are also identical in the two problems, which therefore means that the value functions and optimal bond decision rules, assuming these are well-defined, are also identical.

Appendix 3: Theorem Proofs

PROOF of THEOREM 1

Given a productivity shock ε , the utility from defaulting $v^d(\varepsilon')$ is independent of b . We can also show that the utility from not defaulting $v^{nd}(b, \varepsilon')$ is increasing in b_{t+1} . Therefore, if $V(b^1, \varepsilon') = v^d(\varepsilon')$, then it must be the case that $V(b^0, \varepsilon') = v^d(\varepsilon')$. Hence, any ε' that belongs in $D(b^1, \varepsilon)$ must also belong in $D(b^0, \varepsilon)$.

Let $d^*(b, \varepsilon')$ be the equilibrium default decision rule. The equilibrium default probability is then given by

$$p(b, \varepsilon) = \int d^*(b, \varepsilon') d\mu(\varepsilon'|\varepsilon).$$

From $D(b^1, \varepsilon') \subseteq D(b^0, \varepsilon')$, if $d^*(b^1, \varepsilon') = 1$, then $d^*(b^0, \varepsilon') = 1$. Therefore,

$$p(b^0, \varepsilon) \geq p(b^1, \varepsilon).$$

PROOF of THEOREM 2

From Theorem 1, given a productivity shock ε and $b^0 < b^1 \leq 0$, $p^*(b^0, \varepsilon) \geq p^*(b^1, \varepsilon)$. The equilibrium bond price is given by

$$q(b', \varepsilon) = \frac{1 - p(b', \varepsilon)}{1 + r}.$$

Hence, using Theorem 1, we obtain that:

$$q(b^0, \varepsilon) \leq q(b^1, \varepsilon).$$

Appendix 4: Data Definition and Data Source

Table A1 describes the variables and data sources of our data set for cross-country event studies. Table A2 summarizes the list of countries, default episodes, and the available variables in the analysis.

The default episodes are based on Yeyati and Panizza (2011), Benjamin and Wright (2010), and Standard and Poors report. Because we need the measure of economic variables which typically reflect the impact of default with some lag, we use the quarter after the default an-

nouncement date in the event analysis. This treatment is the same as Yeyati and Panizza who study the drop of GDP in the post-default quarters.

GDP, consumption, and trade balance/GDP are from International Financial Statistics and Yeyati and Panizza(2011). Yeyati and Panizza(2011) compiled the real GDP for countries from national sources for periods when IFS does not record their GDP. GDP, consumption, and trade balance/GDP are H-P detrended.

The imported intermediate inputs are sum of categories for intermediate goods based on the classification of Broad Economic Categories (BEC). The categories for intermediate goods are: (111*) Food and beverages, primary, mainly for industry, (121*) Food and beverages, processed, mainly for industry, (21*) Industrial supplies not elsewhere specified, primary, (22*) Industrial supplies not elsewhere specified, processed, (31*) Fuels and lubricants, primary, (322*) Fuels and lubricants, processed (other than motor spirit), (42*) Parts and accessories of capital goods (except transport equipment), (53*) Parts and accessories of transport equipment.

Intermediate goods are from United Nation, National Accounts Official Country Data. The data is taken from Table 4.1 Total Economy (S.1), I. Production account - Uses Intermediate consumption, at purchaser's prices.

Labor data is the total paid employment data from LABORSTA dataset collected by International Labor Organization.

Because of the serious data limitation for imported inputs, total intermediate goods, and labor, we cannot use HP filter. Imported inputs and total intermediate goods are log-linearly detrended. Labor data is indexed so that the employment 4 years before default is 1.

Table A1: Variables and Sources

Variable	Definition	Frequency	Sources
GDP	Real GDP	quarterly	International Financial Statistics and Yeyati and Panizza(2011)
CON	Real consumption	quarterly	International Financial Statistics
MS	Imported Intermediates (in dollars, deflated by US PPI)	annual	United Nation comtrade (BEC code)
TB	Trade Balance/GDP	quarterly	International Financial Statistics
IM	Intermediate goods (deflated by PPI)	annual	United Nation
L	Labor (paid employment)	annual	International Labor Organization
D	External debt/GNI	annual	Global Development Finance
S	Sovereign bond spreads	quarterly	EMBI+ and EMBI global (J.P. Morgan)

Table A2: List of countries and variables included in the event analysis.

Sovereign Default	Available series
Argentina 1982Q2	GDP, L, D
Argentina 2002Q1	GDP, CON, MS, TB, IM, L, D, S
Chile 1983Q1	GDP, D
Croatia 1992Q2	GDP, L
Domin. Rep. 1993Q1	GDP, D
Ecuador 1999Q3	GDP, CON, MS, TB, IM, D, S
Indonesia 1998Q3	GDP, CON, TB, D
Mexico 1982Q4	GDP, L, D
Moldova 2002Q1	GDP, CON, MS, L, TB, D
Nigeria 1983Q1	GDP, D
Nigeria 1986Q4	GDP, D
Pakistan 1998Q3	GDP, D
Peru 1983Q2	GDP, D
Philippines 1983Q4	GDP, CON, TB, D
Russia 1998Q4	GDP, CON, TB, IM, L, D, S
South Africa 1985Q4	GDP, CON, TB, L, D
South Africa 1993Q1	GDP, CON, TB, IM, L, D
Thailand 1998Q1	GDP, CON, MS, TB, D, S
Ukraine 1998Q4	GDP, MS, IM, L, D, S
Uruguay 1990Q2	GDP, D
Uruguay 2003Q2	GDP, MS, IM, L, D, S
Venezuela 1995Q3	GDP, IM, D, S
Venezuela 1998Q3	GDP, IM, D, S
