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# Risky Bank Lending and Optimal Capital Adequacy Regulation

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**Abstract**



We study the welfare properties of a New Keynesian monetary economy with an essential role for risky bank lending. Banks lend funds deposited by households to a financial-accelerator sector, and face penalties for maintaining insufficient net worth. The loan contract specifies an unconditional lending rate, which implies that banks can make loan losses. Their main response is to raise lending rates to rebuild net worth. Prudential rules that adjust minimum capital adequacy requirements in response to loan losses significantly increase welfare. But the gains from eliminating limited liability and moral hazard would be an order of magnitude larger.

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Keywords: Bank lending; lending risk; financial accelerator; optimal monetary policy; optimal prudential policy; bank capital adequacy; welfare analysis.

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# I. Introduction

Macro-financial linkages have been a major aspect of the financial and real crisis that started to affect the world economy in 2007. The financial sector was involved both in triggering the crisis, and also in affecting the transmission of the initial shocks to the rest of the economy. This has led to a major rethink in economics, which previously tended to downplay the importance of the financial sector for macroeconomic developments. One of the results was a search for appropriate theoretical models. Some building blocks were available, such as the seminal work on corporate balance sheets and the financial accelerator of Bernanke, Gertler and Gilchrist (1999). But there was very little pre-existing work on banks that was suitable for incorporation into conventional macroeconomic models. This has started to change since 2007, but much work remains to be done.[1] This paper is a contribution to that theoretical literature. It develops a model of risky corporate bank lending that closely resembles the way such lending works in real economies. Namely, the loan contract specifies an interest rate on performing loans that is not state-contingent, so that banks can make losses if a larger number of loans defaults than was expected at the time of setting the lending rate. Furthermore, banks face costs of violating minimum capital adequacy regulations, and therefore respond to loan losses by raising their lending rate in order to rebuild their net worth, thereby adversely affecting the real economy. We jointly analyze the macroeconomic effects of prudential or capital adequacy rules and of conventional central bank interest rate rules. The metric for effectiveness is household welfare, which is evaluated by way of grid searches over the coefficients of both monetary and prudential policy rules.

We find that prudential rules can have significant positive welfare effects when a significant share of the shocks affecting the economy are shocks to the creditworthiness of corporate borrowers, which have recently been found to be important in empirical work. Prudential rules lower minimum capital adequacy requirements in the face of contractionary shocks to borrower riskiness. They also reduce the amount of work that conventional interest rate policy needs to do to stabilize the economy. We find that the welfare gains available from prudential rules are large by the standards of this literature, while the welfare gains from using policy interest rates are similar to what has been found elsewhere.

Our work differs from other recent papers on this subject along the following dimensions:

First, *banks have their own net worth*, and are exposed to non-diversifiable aggregate risk determined endogenously on the basis of optimal debt contracts. A number of other authors, such as Christiano, Motto and Rostagno (2010) or Curdia and Woodford (2010), explain interactions between the real and financial sectors by considering how the price of credit affects real factors, while the financial sector exhibits zero net worth (both ex ante and ex post) at all times. This precludes an analysis of macro-prudential capital adequacy regulation of bank balance sheets.

Second, *banks are lenders* rather than holders of risky equity. Gertler and Karadi (2010) and Angeloni and Faia (2009) make the latter assumption, which is appropriate for

[1] See Kiyotaki and Gertler (2010) and Christiano and Ikeda (2010) for surveys of frictions in models of banking. Bernanke, Gertler and Gilchrist (1999) and this paper rely on asymmetric information and costly state verification to model financial frictions. Gertler and Karadi (2010) and Meh and Moran (2010) are models of moral hazard.

investment banks or mutual funds, but not for money center banks. Underperforming equity and underperforming loans have different implications for macroeconomic transmission channels.

Third, *bank lending is endogenously risky.* In most existing models, if lending risk exists, it is only idiosyncratic and fully diversifiable, or introduced through ad-hoc exogenous shocks. To give rise to endogenous non-diversifiable risk in our model, the traditional financial accelerator framework of Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999) is modified, by making some terms of the debt contract non-contingent on future aggregate outcomes. The idea of non-contingent contracts is also found in Zhang (2009). The key difference between our approach and hers is that we assume that the lending rate is non-contingent, while in Zhang (2009) the level of productivity below which bankruptcy occurs is non-contingent.

Fourth, *bank capital is subject to regulation*, and regulation is a critical factor in determining banks' choice of capital. Moreover, the capital regulation is not hard-wired into banks' decision-making as a continuously binding constraint, as in Angeloni and Faia (2009) or van den Heuvel (2008). We rather see regulation as a system of penalties imposed on banks in case they fall below the regulatory minimum. Such penalties then create behavioral incentives for banks to choose endogenous regulatory capital buffers under uncertainty, an idea first advocated by Milne (2002). Capital buffers are an important empirical regularity observed in virtually all banking systems, as documented by Jokipii and Milne (2008). They are also a critical component of Basel capital adequacy regulations. In our model, the buffers are an optimal equilibrium phenomenon resulting from the interaction of optimal debt contracts and regulation. By contrast, Gerali et al. (2010) create time-varying excess capital by using a quadratic cost short-cut. In our framework it becomes possible in principle to interpret the responses of capital buffers to various shocks using value-at-risk (or capital-at-risk) types of conditions, used by, among others, Estrella (2004) or Peura and Jokivuolle (2004). We do not pursue this further in the current paper, but will do so in future work.

Fifth, *acquiring fresh capital is subject to market imperfections.* This is a necessary condition for capital adequacy regulation to have non-trivial effects, and for the capital buffers to exist. This fact is emphasized by van den Heuvel (2002) when describing the bank capital channel of monetary policy, and examples of partial equilibrium models with such imperfections include Estrella (2004) with dynamic quadratic adjustment costs, or Peura and Keppo (2006) with a recapitalization delay. We use the "extended family" approach of Gertler and Karadi (2010), whereby bankers (and also non-financial entrepreneurs) transfer part of their accumulated equity positions to the household budget constraint at an exogenously fixed rate. This is closely related to the original approach of Bernanke, Gertler and Gilchrist (1999), and to the dividend policy function of Aoki, Proudmand and Vlieghe (2004).

The rest of the paper is organized as follows. Section II presents the model, including its calibration and the methodology for computing welfare. Section III discusses the results. Section IV concludes.

# II. The Model

We consider a closed economy that consists of households, capital goods producers, capital investment funds, banks, manufacturers and the government. Full derivations of the optimization problem of each set of agents are contained in a separate Technical Appendix. The economy grows at the constant exogenous growth rate $x = T_t/T_{t-1}$, where $T_t$ is labor augmenting technology. The model's real variables, say $z_t$, therefore have to be rescaled by $T_t$, where we will use the notation $\check{z}_t = z_t/T_t$. The steady state of $\check{z}_t$ is denoted by $\bar{z}$.

## A. Households

The utility of a representative household, indexed by $i$, at time $t$ depends on an external consumption habit $c_t(i) - \nu c_{t-1}$, where $c_t(i)$ is individual consumption and $c_t$ is aggregate per capita consumption, and where consumption is a CES aggregate over varieties supplied by manufacturers, with elasticity of substitution $\theta$. Utility also depends on labor hours $h_t(i)$, and on holdings of real deposit money balances, $D_t(i)/P_t$, where $D_t(i)$ is nominal deposits and $P_t$ is the consumer price index. Lifetime expected utility at time 0 of an individual household is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ S_t^c (1 - \frac{v}{x}) \log(c_t(i) - \nu c_{t-1}) - \psi \frac{h_t(i)^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} + \zeta \log \left( \frac{D_t(i)}{P_t} \right) \right\} , \tag{1}$$

where $\beta$ is the discount factor, $S_t^c$ is a shock to the marginal utility of consumption, $v$ indexes the degree of habit persistence, $\eta$ is the labor supply elasticity, $\psi$ and $\zeta$ fix the utility weights of labor disutility and real deposit money balances, and the scale factor $(1 - v/x)$ ensures that the marginal utility of consumption is independent of the degree of habit persistence in steady state. All households have identical initial endowments and behave identically. The household index $i$ is therefore only required for the distinction between $c_t(i)$ and $c_{t-1}$, and will therefore henceforth be dropped.

Each household represents an extended family that consists of three types of members, workers, entrepreneurs and bankers. Entrepreneurs and bankers enter their occupations for random lengths of time, after which they revert to being workers. There is perfect consumption insurance within each household. Workers supply labor, and their wages are returned to the household each period. Each entrepreneur (banker) manages a capital investment fund (bank) and transfers earnings back to the household at the time when his period as an entrepreneur (banker) ends. Before that time he retains accumulated earnings within the capital investment fund (bank). This means that while the household ultimately owns both capital investment funds and banks, equity cannot be freely injected into or withdrawn from these entities. That in turn means that equity and leverage matter for capital investment funds' and banks' decisions.

Specifically, at a given point in time a fraction $(1 - \mathfrak{f})$ of the representative household's members are workers, a fraction $\mathfrak{f}(1 - \mathfrak{b})$ are entrepreneurs, and a fraction $\mathfrak{f}\mathfrak{b}$ are bankers. Entrepreneurs (bankers) stay in their occupations for one further period with unconditional probability $p^e$ $(p^b)$. This means that in each period $(1 - p^e)\mathfrak{f}(1 - \mathfrak{b})$ entrepreneurs, and $(1 - p^b)\mathfrak{f}\mathfrak{b}$ bankers, exit to become workers, and the same number of workers is assumed to randomly become entrepreneurs (bankers). The shares of workers, entrepreneurs and bankers within the representative household therefore remain constant over time. Distribution of net worth by entrepreneurs (bankers), at the time they revert to being workers, ensures that the aggregate net worth of the corporate and banking sectors does not grow to the point where debt financing becomes unnecessary. Finally, the representative household supplies startup funds to its new entrepreneurs and bankers, and we assume that these represent small fractions $\iota^c$ $(\iota^b)$ of the existing stocks of aggregate net worth in the two sectors. As we will show below, each existing entrepreneur (or banker) will make identical decisions that are proportional to his existing stock of accumulated earnings, so that aggregate decision rules for these two sectors are straightforward to derive. Therefore, the parameters that matter for aggregate dynamics are the shares of aggregate corporate net worth $n_t$ and banking sector net worth $e_t$ paid out to households each period, $(1-p^e)\mathfrak{f}\,(1-\mathfrak{b})\,n_t$ and $(1-p^b)\mathfrak{f}\mathfrak{b}e_t$, net of startup funds to new entrepreneurs and bankers, $\iota^c n_t$ and $\iota^b e_t$. As both are proportional to the aggregate stocks of net worth, their net effect can be denoted by $\delta n_t$ and $\tilde{\delta} e_t$, and our calibration is therefore simply in terms of $\delta$ and $\tilde{\delta}$. These parameters can alternatively be thought of as fixed dividend policies of the capital investment fund and banking sectors, and for simplicity we will utilize this terminology in the remainder of the paper.

Households can hold nominal domestic government debt $B_t$ and nominal bank deposits $D_t$, with real debt and deposits given by $b_t = B_t/P_t$ and $d_t = D_t/P_t$, and with the time subscript $t$ denoting financial claims held from period $t$ to period $t+1$. The gross nominal interest rate on government debt held from $t$ to $t+1$ is $i_t$, and the corresponding rate for bank deposits is $i_{d,t}$. We denote gross inflation by $\pi_t = P_t/P_{t-1}$, and gross real interest rates on government bonds and deposits by $r_t = E_t\left(i_t/\pi_{t+1}\right)$ and $r_{d,t} = E_t\left(i_{d,t}/\pi_{t+1}\right)$. In addition to interest income households receive labor income, dividend distributions from manufacturers and capital goods producers, and lump-sum incomes earned by administering corporate bankruptcies $\Upsilon_t^e$. Real labor income equals $w_t h_t$, where $w_t = W_t/P_t$ is the real wage rate. Real dividend distributions equal $\int_0^1 \Pi_t(j)dj + \Pi_t^k$, where $\Pi_t$ is real profits, $j$ indexes different manufacturers, and $k$ denotes capital goods producers. Finally, households pay lump-sum taxes $\tau_t$ to the government. The household's budget constraint in real terms is

$$
\begin{aligned}
b_t + d_t \;&=\; r_{t-1}b_{t-1} + r_{d,t-1}d_{t-1} - c_t - \tau_t \\
&\quad + w_t h_t + \int_0^1 \Pi_t(j)dj + \Pi_t^k + \Upsilon_t^e \,. 
\end{aligned} \tag{2}
$$

The household maximizes (1) subject to (2). Denoting the multiplier of the budget constraint by $\lambda_t$, and normalizing by $T_t$, we obtain the following first-order conditions for $c_t$, $h_t$, $B_t$ and $D_t$:

$$\frac{S_t^c(1-\frac{v}{x})}{\check{c}_t - \frac{\nu}{x}\check{c}_{t-1}} = \check{\lambda}_t \ , \tag{3}$$

$$\psi h_t^{\frac{1}{\eta}} = \check{\lambda}_t \check{w}_t \ , \tag{4}$$

$$\check{\lambda}_t = \frac{\beta}{x} i_t E_t \left(\frac{\check{\lambda}_{t+1}}{\pi_{t+1}}\right) \ , \tag{5}$$

$$\check{\lambda}_t = \frac{\beta}{x} i_{d,t} E_t \left(\frac{\check{\lambda}_{t+1}}{\pi_{t+1}}\right) + \frac{\zeta}{\check{d}_t} \ . \tag{6}$$

## B. Capital Goods Producers

Capital goods producers, which are identical, have unit mass, and are indexed by $j$, produce the capital stock used by capital investment funds. They are competitive price takers, and are owned by households, who receive their dividends as lump-sum transfers. A capital goods producer purchases previously installed capital $\tilde{k}_{t-1}(j)$ from capital investment funds and investment goods $I_t(j)$ from manufacturers to produce new installed capital $\tilde{k}_t(j)$, where $\tilde{k}_t(j) = \tilde{k}_{t-1}(j) + I_t(j)$, subject to investment adjustment costs

$$G_{I,t}(j) = \frac{\phi_I}{2} I_t \left(S_t^i \frac{(I_t(j)/x)}{I_{t-1}(j)} - 1\right)^2 \ , \tag{7}$$

where $I_t$ is aggregate investment and $S_t^i$ is a shock to investment demand. The nominal price level of previously installed capital is denoted by $Q_t$. Since the marginal rate of transformation from previously installed to newly installed capital is one, the price of new capital is also $Q_t$. The optimization problem is to maximize the present discounted value of dividends by choosing the level of new investment $I_t(j)$:[2]

$$\underset{\{I_t(j)\}_{t=0}^{\infty}}{Max} \ E_0 \Sigma_{t=0}^{\infty} \beta^t \lambda_t \Pi_t^k(j) \ , \tag{8}$$

$$\Pi_t^k(j) \ = \ \left[q_t \left(\tilde{k}_{t-1}(j) + I_t(j)\right) - q_t \tilde{k}_{t-1}(j) - I_t(j) - G_{I,t}(j)\right] \ .$$

In equilibrium all capital goods producers behave identically, so that the index $j$ can henceforth be dropped. The solution to the optimization problem is

$$q_t = 1 + \phi_I S_t^i \left(\frac{\check{I}_t}{\check{I}_{t-1}}\right)\left(S_t^i \frac{\check{I}_t}{\check{I}_{t-1}} - 1\right) - E_t \beta \frac{\check{\lambda}_{t+1}}{\check{\lambda}_t} \phi_I S_{t+1}^i \left(\frac{\check{I}_{t+1}}{\check{I}_t}\right)^2 \left(S_{t+1}^i \frac{\check{I}_{t+1}}{\check{I}_t} - 1\right) \tag{9}$$

The stock of physical capital evolves as

$$k_t = (1-\Delta)k_{t-1} + I_t \ , \tag{10}$$

where $\Delta$ is the depreciation rate, $k_t = \tilde{k}_t$ and $(1-\Delta)k_{t-1} = \tilde{k}_{t-1}$.

[2] Any value of capital is profit maximizing.

## C. Capital Investment Funds

Capital investment funds purchase the capital stock from capital goods producers and rent it to manufacturers. Each capital investment fund $j$ finances its end of time $t$ capital holdings (at current market prices) $Q_t k_t(j)$ with a combination of its end of time $t$ net worth $N_t(j)$ and bank loans $L_t(j)$. Its balance sheet constraint in real normalized terms is therefore given by

$$q_t \check{k}_t(j) = \check{n}_t(j) + \check{\ell}_t(j) \; . \tag{11}$$

After the capital purchase each capital investment fund draws an idiosyncratic shock which changes $k_t(j)$ to $\omega_{t+1} k_t(j)$ at the beginning of period $t+1$, where $\omega_{t+1}$ is a unit mean lognormal random variable distributed independently over time and across capital investment funds. The standard deviation of $\ln(\omega_{t+1})$, $\sigma_{t+1}$, is itself a stochastic process that will play a key role in our analysis. We will refer to this as the borrower riskiness shock. The cumulative distribution function of $\omega_{t+1}$ is given by $\Pr(\omega_{t+1} \leq x) = F_{t+1}(x)$.

Defining the real rental rate of capital as $r_{k,t}$, the capital investment fund's real return to utilized capital is given by

$$ret_{k,t} = E_t \frac{r_{k,t+1} + (1 - \Delta)\, q_{t+1}}{q_t} \; , \tag{12}$$

with the corresponding nominal return given by $Ret_{k,t} = E_t \left( ret_{k,t} \pi_{t+1} \right)$.

We assume that the capital investment fund receives a standard debt contract from the bank. This specifies a nominal loan amount $L_t(j)$ and a gross nominal retail rate of interest $i_{r,t}$ to be paid if $\omega_{t+1}$ is high enough. The critical difference between our model and those of Bernanke, Gertler and Gilchrist (1999) and Christiano, Motto and Rostagno (2010) is that the interest rate $i_{r,t}$ is assumed to be pre-committed in period $t$, rather than being determined in period $t+1$ after the realization of $t+1$ aggregate shocks. The latter, conventional assumption insures zero ex-post profits for banks at all times, while under our debt contract banks make zero expected profits, but realized ex-post profits generally differ from zero. Capital investment funds who draw $\omega_{t+1}$ below a cutoff level $\bar{\omega}_{t+1}$ cannot pay this interest rate and go bankrupt. They must hand over everything they have to the bank, but the bank can only recover a fraction $(1 - \xi)$ of the value of such capital investment funds. The remaining fraction represents a remuneration for monitoring work performed, which is assumed to be paid out to households in a lump-sum fashion. The cutoff productivity level is determined by equating, at $\omega_{t+1} = \bar{\omega}_{t+1}$, the gross interest charges due in the event of continuing operations to the gross idiosyncratic return on the capital investment fund's capital stock. Denoting the wholesale real lending rate that banks would charge to notional zero-risk borrowers by $r_{\ell,t}$, banks' ex-ante zero profit constraint, in real terms, is therefore given by

$$r_{\ell,t} \check{\ell}_t(j) = E_t \left\{ \left(1 - F(\bar{\omega}_{t+1})\right) r_{r,t} \check{\ell}_t(j) + (1 - \xi) \int_0^{\bar{\omega}_{t+1}} q_t k_t(j) ret_{k,t} \omega f(\omega) d\omega \right\} \tag{13}$$

This states that the payoff to lending on the right-hand side must equal the wholesale interest charges on the left-hand side. The first term on the right is the real interest income on loans to borrowers whose idiosyncratic shock exceeds the cutoff level, $\omega_{t+1} \geq \bar{\omega}_{t+1}$. The second term is the amount collected by the bank in case of the borrower's bankruptcy, where $\omega_{t+1} < \bar{\omega}_{t+1}$. This cash flow is based on the return $ret_{k,t}\omega$ on capital investment $q_t k_t(j)$, but multiplied by the factor $(1-\xi)$ to reflect a proportional bankruptcy cost $\xi$.

We adopt a number of definitions that simplify the following derivations. First, the lender's gross share in nominal capital earnings $Ret_{k,t} Q_t k_t(j)$ is given by

$$\Gamma_{t+1} = \Gamma(\bar{\omega}_{t+1}) \equiv \int_0^{\bar{\omega}_{t+1}} \omega_{t+1} f(\omega_{t+1}) d\omega_{t+1} + \bar{\omega}_{t+1} \int_{\bar{\omega}_{t+1}}^{\infty} f(\omega_{t+1}) d\omega_{t+1} \ ,$$

while the lender's monitoring costs share in capital earnings is

$$\xi G_{t+1} = \xi G(\bar{\omega}_{t+1}) = \xi \int_0^{\bar{\omega}_{t+1}} \omega_{t+1} f(\omega_{t+1}) d\omega_{t+1} \ .$$

Then the capital investment fund's share in capital earnings is

$$1 - \Gamma_{t+1} = \int_{\bar{\omega}_{t+1}}^{\infty} \left(\omega_{t+1} - \bar{\omega}_{t+1}\right) f(\omega_{t+1}) d\omega_{t+1} \ .$$

The parameters of the capital investment fund's debt contract are chosen to maximize its profits, subject to zero expected bank profits. Denoting the multiplier of the participation constraint by $\tilde{\lambda}_t$, the capital investment fund's optimization problem can be written as

$$\underset{\check{k}_t(j), \bar{\omega}_{t+1}}{Max} \ E_t \left\{ (1-\Gamma_{t+1}) \frac{ret_{k,t}}{r_{\ell,t}} \frac{q_t \check{k}_t(j)}{\check{n}_t(j)} \right. \tag{14}$$
$$\left. + \tilde{\lambda}_t \left[ (\Gamma_{t+1} - \xi G_{t+1}) \frac{ret_{k,t}}{r_{\ell,t}} \frac{q_t \check{k}_t(j)}{\check{n}_t(j)} - \frac{q_t \check{k}_t(j)}{\check{n}_t(j)} + 1 \right] \right\} \ .$$

The condition for the optimal loan contract is identical to Bernanke, Gertler and Gilchrist (1999),

$$E_t \left\{ (1-\Gamma_{t+1}) \frac{ret_{k,t}}{r_{\ell,t}} + \frac{\Gamma^{\omega}_{t+1}}{\Gamma^{\omega}_{t+1} - \xi G^{\omega}_{t+1}} \left[ \frac{ret_{k,t}}{r_{\ell,t}} (\Gamma_{t+1} - \xi G_{t+1}) - 1 \right] \right\} = 0 \ , \tag{15}$$

where $\Gamma^{\omega}$ and $G^{\omega}$ are the partial derivatives of $\Gamma$ and $G$ with respect to $\bar{\omega}_{t+1}$. Notice that each capital investment fund faces the same returns $ret_{k,t}$, $r_{\ell,t}$ and $r_{r,t}$, and the same risk environment characterizing the functions $\Gamma$ and $G$. Aggregation of the model over capital investment funds is then trivial because both borrowing and capital purchases are proportional to the capital investment fund's level of net worth. Indices $j$ can therefore be dropped.

For welfare analysis it is critical that the model should retain the full stochastic structure of the underlying optimization problem derived above, rather than taking a shortcut by replacing the financial accelerator block with a reduced form equation for the external finance premium. The functions $\Gamma$ and $G$ and their derivatives involve cumulative distribution functions of the standard normal distribution. In MATLAB and DYNARE/DYNARE++ this can be represented by using either the cumulative normal function (normcdf), or by using the complementary error function (erfc).

Net worth represents an additional state variable, whose evolution in real terms is given by

$$\check{n}_t = r_{\ell,t-1}\frac{\check{n}_{t-1}}{x} + \frac{q_{t-1}\check{k}_{t-1}}{x}\left(ret_{k,t-1}\left(1-\xi G_t\right) - r_{\ell,t-1}\right) - \delta\check{n}_t + \check{\Lambda}_t^{\ell} \ , \tag{16}$$

where all but the last term is identical to Bernanke, Gertler and Gilchrist (1999) and Christiano, Motto and Rostagno (2010), while $\Lambda_t^{\ell}$ represents ex-post loan losses by banks, which are given by

$$\check{\Lambda}_t^{\ell} x = r_{\ell,t-1}\check{\ell}_{t-1} - q_{t-1}\check{k}_{t-1} ret_{k,t-1}\left(\Gamma_t - \xi G_t\right) \ . \tag{17}$$

Losses are therefore positive if wholesale interest expenses, which are the opportunity cost of banks' retail lending funds, exceed banks' net (of monitoring costs) share in capital investment funds' gross capital earnings. This will be the case if a larger than anticipated number of capital investment funds defaults, so that, ex-post, banks find that they have set their pre-committed retail lending rate at an insufficient level to compensate for lending losses. Of course, relative to the case of zero ex-post loan losses in Bernanke, Gertler and Gilchrist (1999), banks' losses $\check{\Lambda}_t^{\ell}$ are entrepreneurs' gains, which explains why $\check{\Lambda}_t^{\ell}$ enters with a positive sign in (16).

## D. Banks

Each bank $j$ intermediates funds between households and capital investment funds, and operates under limited liability for its shareholders. It holds equity to protect itself against the penalties that become due to the government/regulator if it violates official minimum capital adequacy requirements. Its total equity exceeds the minimum requirements, in order to provide a buffer against adverse shocks that cause loan losses and a destruction of equity.

The rationale for imposing minimum capital adequacy regulations on banks arises out of moral hazard due to their shareholders' limited liability, which creates an incentive for banks to not protect themselves against negative shocks to profits that are larger than their existing equity base. Banks therefore have an incentive to take on large amounts of lending risk and to minimize their own equity base. As this would mean that depositors are exposed to significant risks of capital losses, one solution is for deposit contracts to reflect that risk, and to thereby discipline bankers. But this solution is often held to be impractical, as it requires depositors to engage in costly monitoring, and also because it may leave the financial system prone to bank runs when adverse information about individual banks is revealed. The policy solution has therefore generally been some form of deposit insurance that obviates the need for complicated deposit contracts, and that minimizes the probability of bank runs. But in that case, given that deposit insurance schemes are generally not sufficiently funded to insure against systemic crises, the risks of large capital losses simply accrue to taxpayers rather than depositors. Deposit insurance therefore has to be accompanied by direct capital adequacy regulations that penalize banks for maintaining an insufficient equity buffer, and thereby exposing taxpayers to the risk of capital losses. That is the main case investigated in this paper, and the calibration of these regulations will be such that the probability of banks becoming insolvent and having to call on deposit insurance is vanishingly small. But we also briefly consider, in

Section III.D, another alternative to capital adequacy regulations that is generally treated as a benchmark in the literature, the case of banks operating under unlimited liability.

Loans $L_t$ are banks' only asset[3], while the liability side of their balance sheet consists of deposits $D_t$ and equity $E_t$. In real normalized form their balance sheet is therefore given by

$$\check{\ell}_t(j) = \check{d}_t(j) + \check{e}_t(j) \ . \tag{18}$$

Our analysis focuses on bank solvency considerations and ignores liquidity management problems. Banks are therefore modeled as having no incentive, either regulatory or precautionary, to maintain cash reserves at the central bank. Because, furthermore, for households cash is dominated in return by bank deposits, in this economy there is no demand for government-provided real cash balances.

Banks are assumed to face costs of falling short of official capital adequacy regulations. The regulatory framework we assume introduces a discontinuity in outcomes for banks. In any given period, a bank either remains sufficiently well capitalized, or it falls short of capital requirements and must pay a penalty to the government. In the latter case, bank net worth suddenly drops further. The cost of such an event, weighted by the appropriate probability, is incorporated into the bank's optimal capital choice. Modeling this regulatory framework under the assumption of homogenous banks would lead to outcomes where all banks simultaneously either pay or do not pay the penalty. A more realistic specification therefore requires a continuum of banks, each of which is exposed to idiosyncratic shocks, so that there is a continuum of capital adequacy ratios across banks, and a time-varying small fraction of banks has to pay penalties in each period.

To this end, we have investigated two alternative approaches. The first is to assume a second layer of idiosyncratic productivity shocks in the specification of capital investment funds' problem. Each bank is specialized in lending to a particular sector of the economy, and each of these sectors comprises a continuum of capital investment funds subject to idiosyncratic productivity shocks. Furthermore, there is a continuum of such sectors that are themselves each exposed to sector-specific productivity shocks. The second approach is to assume an idiosyncratic component in the return to loans in the specification of banks' problem. This can reflect a number of individual bank characteristics, such as differing loan recovery rates, and differing success at raising non-interest income and minimizing non-interest expenses, where the sum of the last two categories would have to sum to zero over all banks.

The former approach turned out to be considerably more complex, because it requires the solution of a fixed-point problem in parameterizing and solving the model. To compute the distribution of each bank's ex-ante return on loans, one needs to know the distribution of both the aggregate and the sector-specific returns to capital. While one can assume the sector-specific returns to be independent of the aggregate outcomes, the distribution of the aggregate return to capital is determined by the optimal choices of all of its agents, including its banks. Banks' optimal behavior, however, in turn depends on the distribution of the aggregate return. It is therefore necessary to iterate until a fixed point

[3] Future versions of this model will also allow for bank holdings of government securities. In the present model these are assumed to remain in zero net supply (see below).

in the aggregate return to capital and banks' optimal choices is found. This imposes prohibitive costs in repeatedly evaluating the model for welfare purposes. Fortunately the second approach is much more tractable, and we thus follow it in this paper.

Specifically, banks are assumed to be heterogeneous in that the return on their loan book is subject to an idiosyncratic shock $\tilde{\omega}_{t+1}$ that is lognormally distributed, with $E(\tilde{\omega}_{t+1}) = 1$ and $Var(\tilde{\omega}_{t+1}) = \tilde{\sigma}^2_{t+1}$ and with the density function and cumulative density functions of $\tilde{\omega}_{t+1}$ denoted by $f(\tilde{\omega}_{t+1})$ and $F(\tilde{\omega}_{t+1})$.

The regulatory framework stipulates that banks have to pay a real penalty of $\chi\check{\ell}_t(j)$ at time $t+1$, as a lump-sum payment to the government/regulator, if the sum of the gross returns on their loan book, net of gross deposit interest expenses and loan losses, is less than a fraction $\gamma_t$ of the gross returns on their loan book:

$$r_{\ell,t}\check{\ell}_t(j)\tilde{\omega}_{t+1} - r_{d,t}\check{d}_t(j) - \check{\Lambda}^{\ell}_{t+1}(j) < \gamma_t r_{\ell,t}\check{\ell}_t(j)\tilde{\omega}_{t+1} \; . \tag{19}$$

Because the left-hand side equals pre-dividend (and pre-penalty) net worth in $t+1$, while the term multiplying $\gamma_t$ equals the value of assets in $t+1$, $\gamma_t$ represents the minimum capital adequacy ratio. We will henceforth refer to to the capital adequacy ratio as the CAR or the Basel ratio. We denote the cut-off idiosyncratic shock to loan returns below which the minimum CAR is breached by $\overline{\tilde{\omega}}_{t+1}$, and note that $E_t(\check{\Lambda}^{\ell}_{t+1}(j)) = 0$. Then we have the following conditions for the ex-ante and ex-post cutoff loan return shock:

$$E_t\left(\overline{\tilde{\omega}}_{t+1}\right) \equiv \frac{r_{d,t}\check{d}_t}{\left(1-\gamma_t\right) r_{\ell,t}\check{\ell}_t} \; , \tag{20}$$

$$\overline{\tilde{\omega}}_t \equiv \frac{r_{d,t-1}\check{d}_{t-1} + \check{\Lambda}^{\ell}_t}{\left(1-\gamma_{t-1}\right) r_{\ell,t-1}\check{\ell}_{t-1}} \; . \tag{21}$$

Banks choose loans and deposits to maximize their pre-dividend net worth, which equals the sum of gross returns on the loan book minus gross interest charges on deposits, loan losses, and penalties:

$$\underset{\check{\ell}_t(j),\check{d}_t(j)}{Max} \; E_t\left[r_{\ell,t}\check{\ell}_t(j)\tilde{\omega}_{t+1} - r_{d,t}\check{d}_t(j) - \check{\Lambda}^{\ell}_{t+1}(j) - \chi\check{\ell}_t(j)F(\overline{\tilde{\omega}}_{t+1})\right] \; .$$

Using the balance sheet identity, and letting $\mathfrak{L}_t(j) = \check{\ell}_t(j)/\check{e}_t(j)$, this can be rewritten as

$$\underset{\mathfrak{L}_t(j)}{Max} \; E_t\left[\left(r_{\ell,t} - r_{d,t}\right)\mathfrak{L}_t(j) + r_{d,t} - \chi\mathfrak{L}_t(j)F\left(\frac{r_{d,t}\left(1-\frac{1}{\mathfrak{L}_t(j)}\right) + \check{\Lambda}^{\ell}_{t+1}}{\left(1-\gamma_t\right) r_{\ell,t}}\right)\right] \; , \tag{22}$$

with first-order necessary condition

$$E_t\left[r_{\ell,t} - r_{d,t} - \chi\left(F\left(\overline{\tilde{\omega}}_{t+1}\right) + f\left(\overline{\tilde{\omega}}_{t+1}\right)\left(\frac{r_{d,t}}{\left(1-\gamma_t\right) r_{\ell,t}\frac{\check{\ell}_t}{\check{e}_t}}\right)\right)\right] = 0 \; . \tag{23}$$

Because balance sheet items can be easily aggregated over all banks, for the same reasons as in the case of capital investment funds, we have dropped bank-specific indices and replaced $\check{\ell}_t(j)/\check{e}_t(j)$ by $\check{\ell}_t/\check{e}_t$. The optimality condition states that banks' wholesale lending rate is equal to their deposit rate plus a term that depends on the penalty for breaching the minimum CAR. That term includes the penalty coefficient $\chi$ and expressions that determine the likelihood of a breach. Banks will therefore set their wholesale lending rate $r_{\ell,t}$ at a premium over their deposit rate $r_{d,t}$. Their retail rate $r_{r,t}$ on the other hand is at another premium over $r_{\ell,t}$, to compensate for the bankruptcy risk of capital investment funds. A sensible interpretation of the wholesale rate is therefore as the rate a bank would charge to a hypothetical capital investment fund (not present in the model) with zero default risk.

Banks' net worth represents an additional state variable of the model, and in real terms is given by

$$\check{e}_t = \frac{1}{x}\left(r_{\ell,t-1}\check{\ell}_{t-1} - r_{d,t-1}\check{d}_{t-1} - \check{\Lambda}_t^{\ell}x - \chi\check{\ell}_{t-1}F(\overline{\omega}_t)\right) - \tilde{\delta}\check{e}_t \ . \tag{24}$$

## E. Manufacturers

The technology of each manufacturer $j$ is given by

$$y_t(j) = \left(S_t^a T_t h_t(j)\right)^{1-\alpha} k_{t-1}(j)^{\alpha} \ , \tag{25}$$

where $S_t^a$ is a transitory shock to labor-augmenting technology. Cost minimization implies standard input demands for labor and capital

$$h_t = (1-\alpha)\frac{mc_t}{\check{w}_t}\check{y}_t \ , \tag{26}$$

$$\frac{\check{k}_{t-1}}{x} = \alpha\frac{mc_t}{r_t^k}\check{y}_t \ , \tag{27}$$

where

$$mc_t = A\left(\frac{\check{w}_t}{S_t^a}\right)^{1-\alpha}\left(r_t^k\right)^{\alpha} \ , \tag{28}$$

$A = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}$, and where manufacturer-specific indices have been dropped because in equilibrium all manufacturers behave identically. We denote the price of product variety $j$ by $P_t(j)$, and the aggregate price level by $P_t$. Manufacturers maximize the present discounted value of future revenue $(P_t(j)/P_t)\, y_t(j)$ minus expenditures. The latter includes marginal costs $mc_t y_t(j)$, a Rotemberg (1982)-style quadratic price adjustment cost that allows for a nonzero central bank inflation target $\bar{\pi}$, and a fixed cost $T_t\Phi$ that will be used to calibrate the model's steady state income shares. The optimal price setting problem is

$$\underset{\{P_t(j)\}_{t=0}^{\infty}}{Max} \quad E_0\sum_{t=0}^{\infty}\beta^t\lambda_t\left[\frac{P_t(j)}{P_t}y_t(j) - mc_t y_t(j) - \frac{\phi_p}{2}y_t\left(\frac{P_t(j)}{P_{t-1}(j)} - \bar{\pi}\right)^2 - T_t\Phi\right] , \tag{29}$$

subject to

$$y_t(j) = y_t\left(\frac{P_t(j)}{P_t}\right)^{-\theta} , \tag{30}$$

where the latter is the standard demand function for varieties derived from Dixit-Stiglitz demands for aggregate output. We assume that households, investors and the government demand an identical aggregate over varieties. Letting $\mu = \theta/(\theta - 1)$, the optimality condition, again after dropping manufacturer-specific indices, is given by

$$\mu mc_t - 1 = \phi_p(\pi_t - \bar{\pi})\pi_t - \beta\frac{\check{\lambda}_{t+1}}{\check{\lambda}_t}\frac{\check{y}_{t+1}}{\check{y}_t}\phi_p(\pi_{t+1} - \bar{\pi})\pi_{t+1}\ . \tag{31}$$

## F. Government

Government spending is assumed to be exogenous and equal to a fixed fraction $s_g$ of steady state GDP times a shock $S_t^g$:

$$\check{g}_t = S_t^g s_g \bar{y}\ . \tag{32}$$

The government also receives the penalty payments of banks that violate the minimum CAR, $\check{\Upsilon}_t^b = \frac{\chi}{x}\check{\ell}_{t-1}F(\bar{\bar{\omega}}_t)$. The specification of tax and debt policy is redundant because taxes are lump-sum and households are Ricardian. We therefore assume for simplicity that initial government debt is zero, and that the government balances its budget in each period:

$$\check{g}_t = \check{\tau}_t + \check{\Upsilon}_t^b\ . \tag{33}$$

Monetary policy is given by a forward-looking interest rate rule:

$$i_t = (i_{t-1})^{m_i}\left(\frac{x}{\beta}\bar{\pi}\right)^{(1-m_i)}\left(\frac{\pi_{4,t+3}}{(\bar{\pi})^4}\right)^{(1-m_i)m_\pi}\left(\frac{\check{y}_t}{\bar{y}}\right)^{(1-m_i)m_y}\left(\frac{\check{\ell}_t}{\bar{\ell}}\right)^{(1-m_i)m_\ell}\left(\frac{\check{\ell}_t/\check{y}_t}{\bar{\ell}/\bar{y}}\right)^{(1-m_i)m_d},$$

$$\pi_{4,t} = \pi_t\pi_{t-1}\pi_{t-2}\pi_{t-3}\ . \tag{34}$$

The first three coefficients of this rule are the conventional interest rate smoothing coefficient and the feedback coefficients on inflation and the output gap. The last two represent deviations of the loan stock or the loans-to-output ratio from their trends. We have verified that changing the inflation forecast horizon in this rule makes only a small difference to our main results. Optimizing the coefficients $m_i$, $m_\pi$, $m_y$, $m_\ell$ and $m_d$ is the first component of our welfare analysis.

Prudential policy varies the minimum capital adequacy coefficient $\gamma_t$ systematically in response to the business cycle. We posit the rule

$$\gamma_t = (\gamma_{t-1})^{p_\gamma}(\bar{\gamma})^{1-p_\gamma}\left(\frac{\check{y}_t}{\bar{y}}\right)^{(1-p_\gamma)p_y}\left(\frac{\check{\ell}_t}{\bar{\ell}}\right)^{(1-p_\gamma)p_\ell}\left(\frac{\check{\ell}_t/\check{y}_t}{\bar{\ell}/\bar{y}}\right)^{(1-p_\gamma)p_d}, \tag{35}$$

where we only allow for for one of the three gap-coefficients $p_y$, $p_\ell$ and $p_d$ to be nonzero at one time. This rule allows the minimum CAR to change with deviations of output, the loan stock or the loans-to-output ratio from their trends. Optimizing the coefficients $p_\gamma$, $p_y$, $p_\ell$ and $p_d$ is the second component of our welfare analysis.

## G. Equilibrium

In equilibrium all agents maximize their objective functions, and the goods market clears:

$$\check{y}_t = \check{c}_t + \check{I}_t + \check{g}_t\ . \tag{36}$$

The model's five shock processes are given by

$$\log(z_t) = (1-\rho_z)\log(\bar{z}) + \rho_z \log(z_{t-1}) + \varepsilon_t^z \ , \tag{37}$$

where $z_t \in \left\{S_t^a, S_t^c, S_t^i, S_t^g, \sigma_t\right\}$.

## H. Calibration

For a quantitative illustration, we calibrate our model at the quarterly frequency for the United States. We use data for the period 1990Q1 - 2010Q2 for some parameters, and we rely on the literature for a number of other parameters.

The real growth rate is calibrated at 2% per annum, the steady state real interest rate at 3% per annum, and the steady-state inflation rate at 2% per annum. Following Smets and Wouters (2003), the habit parameter $v$ is set to 0.7. The labor supply elasticity $\eta$ is fixed at 1, a common assumption in the monetary business cycle literature. The investment adjustment cost parameter, at $\phi_I = 2.5$, is close to Christiano, Eichenbaum and Evans (2005). The price adjustment cost parameter is set to $\phi_p = 200$. Together with the assumption that the gross markup equals $\mu = 1.1$, this is equivalent to assuming that the average duration of price contracts equals roughly 5 quarters in a model with Calvo (1983) pricing and Yun (1996) indexation. The cost share of private capital $\alpha$ and the fixed cost $\Phi$ are calibrated to obtain a capital income share (including markups net of fixed costs) of 40% and a private investment to GDP ratio of 19%. The steady state government spending to GDP ratio is fixed at 18%.

The coefficients $\delta$, $\tilde{\delta}$, $\bar{\sigma}$, $\tilde{\sigma}$, $\xi$, $\chi$, $\bar{\gamma}$ and $\zeta$ in capital investment funds', banks' and households' equilibrium conditions are endogenized by fixing a number of steady state balance sheet ratios and interest rate margins. Capital investment funds' steady state leverage ratio, meaning their ratio of debt to equity, is 100%, which is well supported by the data for non-financial corporate leverage. Banks' steady state CAR is 10.5%, and $\bar{\gamma}$ is fixed so that in steady state penalties start to apply to banks that drop below a 8% minimum CAR. This is in line with current Basel III proposals, which are for a 8% minimum CAR, a 2.5% capital conservation buffer that takes the steady state CAR to 10.5%, and an additional countercyclical buffer at the discretion of national authorities. Our paper can be understood as an investigation into the optimal design of this countercyclical buffer.

The steady state real deposit rate equals 2.75%, with the 25 basis points discount to the real policy rate due to a positive utility weight of deposits in the utility function $\zeta > 0$. The steady state wholesale real lending rate equals 3.3%, with the 55 basis points margin over the real deposit rate due to positive regulatory penalties $\chi > 0$, and where for a given $\chi$ that margin is increasing in the riskiness of banks $\tilde{\sigma}$. Regulatory penalties $\chi$ are calibrated at one third of one percent of the outstanding loan volume, but we will perform sensitivity analysis with respect to this parameter. The steady state retail real lending rate equals 4.3%, with the 100 basis points margin over the wholesale rate due to positive bankruptcy monitoring costs $\xi > 0$, and where for a given $\xi$ that margin is increasing in the riskiness of capital investment funds $\sigma$. These 100 basis points represent the model's external finance premium. The traditional Bernanke, Gertler and Gilchrist (1999) measure of the external finance premium averages the retail lending rate with the lower loan recoveries from defaulting borrowers. In our steady state this premium equals 65 basis points. The share of capital investment funds going bankrupt in each quarter is calibrated at one percent, while the share of banks hitting the minimum CAR in each quarter is calibrated at two percent.

To illustrate the behavior of the model, the autocorrelation coefficients and standard deviations of the model's five shocks are calibrated to generate standard deviations and autocorrelations that are similar to those of U.S. macroeconomic variables, given the standard calibration of the U.S. monetary policy reaction function used by the Federal Reserve Board's SIGMA model.[4] The specification includes an assumption of correlated shocks, as without that feature our model would have difficulty generating the empirically observed positive correlation between consumption and investment, for two main reasons. First, the absence of an open economy dimension makes the mutual crowding-out effects of investment and consumption shocks much stronger than they would be if higher investment or consumption demand could be satisfied in part by drawing in imports. Second, the absence of a lending channel between banks and households implies that negative shocks to bank equity only depress investment-related lending but not consumption-related lending, while in practice both of these tend to happen simultaneously. Introducing consumption-related lending into the structural model, for example as mortgage loans, would help with this aspect. But it would go beyond the more limited purpose of this exercise, which is to analyze the roles of corporate bank lending, and of the effects of prudential regulation on corporate bank lending, in as simple and transparent a model as possible. We therefore mimic the fact that negative shocks to bank equity simultaneously depress investment and consumption, by assuming that contractionary shocks to borrower riskiness $\sigma$ spill over to consumption demand as contractionary shocks to $S^c$.

It turns out that for the purpose of this paper the details of the shock calibration, including their contribution to the variance of GDP, only matter in one single respect, the share of macroeconomic volatility explained by shocks to borrower riskiness, which are key for our welfare analysis. In our illustrative calibration these shocks account for a 16% share in the volatility of GDP, which is fairly close to what other papers in the literature have found. For example, Christiano, Motto and Rostagno (2010) estimate that share at 16% for the euro area and 19% for the United States (including signalling effects), and Christiano, Trabandt and Walentin (2010) estimate it at 25% for Sweden.

## I. Welfare

Expected welfare is given by

$$\mathcal{W}_t = u_t + \beta E_t \mathcal{W}_{t+1} , \tag{38}$$

where $u_t$ is the period utility of a representative household at time $t$. We define the Lucas (1987) compensating consumption variation $\eta < 0$ (in percent) of a particular combination of monetary and prudential rule coefficients as the percentage reduction in average

[4] The coefficients are $m_i = 0.7$, $m_\pi = 2.0$ and $m_{ygr} = 0.25$, where the latter is a coefficient on output growth rather than on the output gap.

consumption that households experiencing the best possible combination of coefficients, with associated welfare $E\mathcal{W}^{opt}$, would be willing to tolerate in order to remain indifferent between their expectation of welfare and the expectation of welfare $E\mathcal{W}^{rule}$ under the particular combination of monetary and prudential rule coefficients. The first step is to evaluate welfare under both assumptions relative to steady state welfare $E\mathcal{W}^{ss}$. We thereby obtain $\eta^{rule}$ and $\eta^{opt}$, where the formula for $\eta^{rule}$ is

$$\eta^{rule} = 100 \left( 1 - \exp \left( \frac{\left( E\mathcal{W}^{rule} - E\mathcal{W}^{ss} \right) (1-\beta)}{\left( 1 - \frac{v}{x} \right)} \right) \right) > 0 \; , \tag{39}$$

and similarly for $\eta^{opt}$. Finally, we obtain $\eta = \eta^{rule} - \eta^{opt}$. We use DYNARE++ to compute unconditional welfare and compensating consumption variations. We perform a multi-dimensional grid search over all monetary and prudential rule coefficients.

As a benchmark, we also evaluate welfare for a particular specification of the optimal policy under the timeless perspective. That specification retains the baseline model's frictions in equity markets and in corporate lending, but it removes frictions in the banking sector. Details are explained in Section III.C.

# III. Results

In Section III.A we present numerical results for the optimal overall combination of monetary and prudential rule coefficients. In Section III.B we inspect the impulse response function for shocks to borrower riskiness, to build intuition for the more detailed discussion of welfare results in the remainder of the paper. In Section III.C we quantify the overall welfare gains associated with different monetary and prudential rule combinations as a function of key rule coefficients. We combine this with an analysis of the implied volatilities of the two policy instruments, the nominal interest rate and the minimum CAR. In Section III.D we discuss the optimal policy under the timeless perspective when banks operate under unlimited shareholders' liability.

## A. Optimal Coefficient Combination

To obtain a baseline for both the impulse response simulation and welfare comparisons, we first determine the joint overall welfare optimum across all coefficients by way of grid searches. We find that the optimal smoothing parameter for the prudential rule, $p_\gamma$, is always very close to zero, and we therefore simplify the further analysis by setting $p_\gamma = 0$. We also find that introducing a loan gap or a loan-to-output gap into the monetary rule does not have welfare benefits once all remaining coefficients are set to their overall optimum values, and we therefore set $m_\ell = m_d = 0$. This leaves the monetary coefficients $m_\pi$, $m_i$ and $m_y$ to be optimized jointly with one of the prudential rule coefficients. We do so by way of four-dimensional grid-searches. In doing so we limit the search over inflation feedback coefficients to a plausible range of $m_\pi \in [1.5, 3.0]$.

We find that when prudential policy responds to the output gap, welfare gains are significantly smaller than when it responds to the loan gap or the loans-to-output gap.

The reason is that the latter directly capture a key aspect of bank balance sheets, and in response adjust a tool that moves bank balance sheets in the desired direction, while output gaps are subject to many influences that have little connection with the state of banks. For the same reason, loan gaps are slightly superior to loans-to-output gaps. We will therefore from now on concentrate only on the case of loan gaps. The overall optimal coefficient combination for that case is $m_i = 0$, $m_\pi = 3$, $m_y = 0.1$, and $p_\ell = 6.0$. Here we have restricted the prudential coefficient to be no larger than 6, for reasons that will be explained below. The precise values of $m_i$ and $m_y$ do not have large effects on welfare outcomes, and in the subsequent analysis we therefore hold them at their overall optimum values.

## B. Impulse Response Function

Figure 1 shows impulse responses for a one standard deviation shock to borrower riskiness that illustrate the effects of different assumptions concerning the countercyclicality of the prudential rule.[5] Specifically, we present results for $p_\ell \in \{0, 3, 6\}$, while keeping the monetary rule coefficients at the optimal values $m_i = 0$, $m_\pi = 3$ and $m_y = 0.1$.

We observe that countercyclical capital adequacy requirements - lowering the CAR in the face of a contractionary shock - reduce the volatility of output, hours, consumption and investment, as well as reducing the required fluctuations in policy interest rates. The shock, as can be seen in the third row of the figure, impairs corporate asset values. The effect on net worth is twice as large as that on asset values because corporate leverage equals 100 percent. Corporate leverage for a one standard deviation shock increases by around 2.75 percentage points. But in the period of the shock banks are locked into their old lending rates, and as a result they suffer lending losses that reduce their net worth by over 3 percent. Given high steady state bank leverage, this is enough to reduce their CAR by around 0.3 percentage points, which leads them to raise the interest rate they charge to capital investment funds. The initial interest rate increase, in the absence of a prudential response, is due to a roughly 100 basis points increase in the retail rate over the wholesale rate as banks compensate for higher lending risk. This, as well as the accompanying reduction in lending volumes, serves to further reduce economic activity. The wholesale rate starts to decrease immediately as monetary policy aggressively lowers the policy interest rate, which by arbitrage reduces bank funding costs and thus, by (23), lending rates. But the wholesale rate decreases more, and loan volumes drop less, when prudential policy is also aggressive. An aggressive prudential rule, with $p_\ell = 6$, responds to the reduction in bank loans by reducing capital adequacy requirements temporarily (but very persistently given the persistent effects of the shock). This reduces the need for banks to quickly rebuild their equity base in order to escape further penalties, so that their wholesale rate now rises by less on impact, and is also lower in the medium term. This in turn has positive feedback effects on the corporate sector, reducing borrower riskiness endogenously, which means that the margin of retail rates over wholesale rates declines slightly. From the responses of output, hours and consumption it is clear that this policy has positive payoffs in terms of welfare. In this context it is important to point out that the lognormal distribution functions which determine interest rate premia in the capital

[5] The figure refers to capital investment funds as corporates.

investment fund and banking sectors are highly nonlinear, with disproportionately large effects on volatility in the case of the largest shocks. This means that rules which dampen the effects of such shocks have significant welfare benefits.

### C. Overall Welfare and Policy Instrument Volatility

Figure 2 shows our main results for welfare and for policy instrument volatility. The left column shows results for an economy where all five shocks are present, while the right column shows results when only the shocks to borrower riskiness are present. The top row shows welfare outcomes as a function of the most important monetary and prudential rule coefficients $m_\pi$ and $p_\ell$, holding the monetary coefficients $m_i$ and $m_y$ at their overall optimum values. The middle and bottom rows show the volatilities in policy instruments associated with the welfare results in the first row. We have included the latter because, from a policymaker's perspective, policies need to not only be welfare-enhancing, but also practically feasible. This would not be the case if large welfare gains were to require extremely volatile nominal interest rates, or extremely large changes in minimum CAR. Presumably this is because such volatility has a high cost that should properly be part of the objective function, but it is not obvious how to incorporate this into the welfare computations. We therefore opt instead to present these measures side by side. Welfare gains are shown relative to the best policy rule available over the range that we consider, namely a prudential rule with $p_\ell = 6$ and a monetary rule with $m_\pi = 3$. We first discuss results for the case where all shocks are present, and then compare to the case where only borrower riskiness shocks are present.

As is common in this type of analysis, an aggressive response to inflation is desirable, and we find gains of an order of magnitude that are typical for, or perhaps a little larger than, found in this literature. Specifically, when the remaining coefficients are at their overall optimum values, increasing $m_\pi$ from 1.5 to 3.0 results in a welfare gain of around 0.04% when all shocks are present. Prudential targeting of loan gaps on the other hand, specifically raising $p_\ell$ from 0 to 6, leads to a larger welfare gain of around 0.20% of steady state consumption. It should however be added that the welfare gains from optimizing monetary policy in this model understate the overall gains from credibly stabilizing inflation, as those gains should properly include the avoidance of infrequent but large recessions that occur when inflation expectations become unanchored, so that policy is forced to stabilize inflation through a deep recession.

A further benefit of prudential rules, as seen in the middle row of Figure 2, is that they lower (by around 30 basis points) the volatility of nominal interest rates for any given inflation coefficient in the monetary rule. This is because in the presence of a banking sector capital adequacy requirements can substitute for some of the work that policy interest rates are expected to do. For example, if a strong response to borrower riskiness shocks comes through prudential policy that reduces the increase in bank lending rates, this limits the required reduction in the policy rate.

Finally, of course, more aggressive prudential rules increase the volatility of the minimum CAR. For quite aggressive prudential rules this volatility becomes large, with standard errors of minimum CAR at $p_\ell = 6$ that equal over 3 percentage points. This may be at the limit, or beyond, of what policymakers would consider acceptable, but welfare gains at somewhat less aggressive prudential rules are still significant. The reason why we have limited our grid search for prudential rule coefficients to a maximum of $p_\ell = 6$ is to avoid regions where the volatility of minimum CAR becomes unrealistically large.

A comparison between the left and right columns of Figure 2 shows that the welfare gains from a more aggressively countercyclical prudential policy are attributable almost exclusively to borrower riskiness shocks, while the gains from a more active monetary policy seen in the left column arise mostly under traditional demand and supply shocks. The reduction in policy interest rate volatility under a more aggressive prudential policy is also mostly due to borrower riskiness shocks. So is the increase in minimum CAR, but here a substantial residual is explained by other shocks.

Figure 3 shows how welfare gains change when we change aspects of the banking and regulatory technology. The top left panel shows the baseline from Figure 2. The top right panel shows the consequences of banks becoming riskier, in the sense that the steady state of $\tilde{\sigma}$ rises to the point where the steady state share of banks reaching the minimum CAR in each quarter increases from 2 percent to 3 percent. We observe that the welfare difference between the worst and best rules considered increases from 0.24% to 0.32%. The reason is that when banks face a higher risk of having to pay penalties, they will raise lending rates more aggressively in response to negative borrower riskiness shocks, so that relaxation of capital adequacy requirements becomes a more powerful tool. The bottom right panel shows the consequences of penalty rates equalling 0.5% instead of 0.33% of the value of loans. The effects on the welfare difference between the worst and best rules are very similar to the previous case, and for similar reasons.

The bottom left panel of Figure 3 shows the consequences of a different prudential rule. Figure 2 showed that while the welfare gains of aggressive prudential rules are significant, the associated volatility of minimum CAR can also become quite large. This however is due partly to the fact that other shocks generate a significant part of the volatility of the minimum CAR. The question therefore arises whether a direct response of prudential policy to borrower riskiness shocks alone, which would eliminate volatility of minimum CAR due to other shocks, could still produce large welfare gains.[6] The prudential rule we consider for this case is

$$\gamma_t = \bar{\gamma} - p_b \frac{\ln(\sigma_t/\bar{\sigma})}{100} . \tag{40}$$

Figure 3 shows that this rule produces much smaller welfare gains that reach a maximum of 0.08% around $m_\pi = 3$ and $p_b = 6$, with welfare in fact declining (not shown) beyond $p_b = 6$. The reason is that the shock to borrower riskiness itself dies out comparatively quickly while its effects on corporate and bank balance sheets are much more long-lived. Policy should optimally focus on minimizing the persistent effects of impaired balance sheets, through higher lending rates, on the rest of the economy, and a policy response to loans accomplishes that objective much better than rule (40).

[6]This analysis is done purely as a thought experiment that helps us understand the nature of optimal prudential rules. It is hard to think how, in practice, a usable empirical counterpart of $\sigma_t$ could be identified.

### D. Moral Hazard and Optimal Policy

As discussed above, in the absence of either deposit contracts that force banks to maintain a sizeable equity buffer, or of minimum CAR that accomplish the same objective, limited liability would give rise to a moral hazard problem for banks. With a prudential rule as calibrated in this paper, this problem is effectively eliminated and can be disregarded in the computational solution of the model. Banks build up regulatory capital buffers well in excess of the minimum CAR, and even the few banks that do violate the minimum CAR due to unfavorable idiosyncratic shocks are still very far away from losing all of their equity. Depositors therefore never have to worry about the safety of their returns.[7] Capital adequacy regulations thus protect against the negative implications of limited liability and moral hazard. But this raises the question of how much better the economy could do in the absence of a moral hazard problem, specifically under unlimited liability, where capital adequacy regulations could be eliminated altogether. We answer this question by computing a specific theoretically optimal policy that takes some frictions of the baseline model as given, but that removes two key frictions in the banking sector. Specifically, the policymaker takes as given the equity market frictions associated with the extended family specification of households, and he also takes as given the asymmetric information friction between banks and capital investment funds. But the policymaker is able to eliminate all capital adequacy regulations on banks by setting $\chi = 0$, after imposing unlimited liability in the banking sector. This means that households which become bankers can be asked to supply whatever equity is needed, out of the funds of the extended family, to make up any shortfall. Given this guarantee, the actual amount of equity in the banking system becomes irrelevant, as any shortfalls are covered by this contingent claim on households. The policymaker can then freely choose the remaining policy variables $i_t$ and $\gamma_t$ instead of being limited to policy rules (34) and (35). With $\chi = 0$, the minimum CAR $\gamma_t$ of course becomes irrelevant. More importantly, optimal steady state bank equity is exactly equal to zero, or bank leverage is infinite. Furthermore, bank equity exhibits a unit root. The welfare gain of this policy over the best of our simple rules equals 2.52%, which is roughly ten times larger than the gain from pursuing an aggressively countercyclical prudential rule instead of a rule with rigidly fixed minimum CAR. In other words, the gains from eliminating the effects of moral hazard in banking are an order of magnitude larger than the gains from optimizing simple prudential rules that prevent banks from exploiting moral hazard.

## IV. Conclusion

We have presented a theoretical model where risky corporate bank lending is an essential part of the macroeconomic transmission mechanism. Lending is risky because banks lock in lending rates before they know the final performance of the underlying projects. As a consequence their lending rates, which on average compensate them for the risks of corporate loans at the time they are set, turn out to be too low when borrowers' creditworthiness is impaired through a negative shock. When banks face regulatory costs

[7] Technically, the risk of a bank going bankrupt is not zero but extremely close to zero. Formally, for depositors to perceive their returns as completely safe one would have to assume implicit government deposit insurance for the residual risk.

of maintaining insufficient net worth, they respond to loan losses by trying to rebuild their net worth through higher lending rates, especially when regulatory costs are inflexible in the sense that CAR do not respond to the state of the economy. Higher lending rates further aggravate and prolong the effects of loan losses on the aggregate economy. We use this feature to model the effects of more flexible, countercyclical prudential capital adequacy rules on macroeconomic performance.

We find that prudential rules can have very large effects on macroeconomic volatility and welfare when a significant share of the shocks hitting the economy are shocks to the creditworthiness of corporate borrowers. As several authors have recently found such shocks to be empirically important, this result is of considerable relevance. Prudential rules then work by lowering the CAR in the face of shocks that raise borrower riskiness and loan losses, thereby allowing banks to reduce the interest rates they charge to already distressed borrowers. This results in reductions in the volatilities of output, hours, consumption and investment, and furthermore it reduces the amount of work that conventional interest rate policy has to perform, thereby contributing to less volatile policy interest rates. The downside is that for the most aggressively countercyclical prudential rules the volatility of minimum CAR can become quite large, partly because it may not be possible to selectively respond only to variations in loan volumes associated with changes in the creditworthiness of borrowers.

## References

Angeloni, I. and Faia, E. (2009), "A Tale of Two Policies: Prudential Regulation and Monetary Policy with Fragile Banks", The Kiel Institute for the World Economy Working Paper Series, No. 1569.

Aoki, K., Proudman, J. and Vlieghe, G. (2004), "House Prices, Consumption, and Monetary Policy: A Financial Accelerator Approach", *Journal of Financial Intermediation*, **13(4)**, 414–435.

Benes, J. and Kumhof, M. (2011), "Risky Bank Lending and Optimal Macro-Prudential Regulation", IMF Working Paper (forthcoming).

Bernanke, B., Gertler, M. and Gilchrist, S. (1999), "The Financial Accelerator in a Quantitative Business Cycle Framework", in: Taylor, J.B., Woodford, M. (eds.), Handbook of Macroeconomics, Volume 1C. Elsevier, Amsterdam, pp. 1341-1393.

Calvo, G.A. (1983), "Staggered Prices in a Utility-Maximizing Framework", *Journal of Monetary Economics*, **12**, 383-398.

Carlstrom, C. and Fuerst, T. (1997), "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis", *American Economic Review*, **87(5)**, 893–910.

Christiano, L. J., Eichenbaum, M. and Evans, C. L. (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, **113(1)**, 1–45.

Christiano, L. and Ikeda, D. (2010), "Government Policy, Credit Markets and Economic Activity", Working Paper, Northwestern University.

Christiano, L., Motto, R. and Rostagno, M. (2010), "Financial Factors in Economic Fluctuations", ECB Working Paper Series, No. 1192.

Christiano, L., Trabandt, M. and Walentin, K. (2010), "Introducing Financial Frictions and Unemployment into a Small Open Economy Model", Sveriges Riksbank Working Paper Series, No. 214.

Curdia, V. and Woodford, M. (2010), "Credit Spreads and Monetary Policy", *Journal of Money, Credit and Banking*, **24(6)**, 3–35.

Estrella, A. (2004), "The Cyclical Behavior of Optimal Bank Capital", *Journal of Banking and Finance*, **28(6)**, 1469-1498.

Gerali, A., Neri, S., Sessa, L. and Signoretti, F. (2010), "Credit and Banking in a DSGE Model of the Euro Area", Bank of Italy Working Paper Series, No. 740.

Gertler, M. and Karadi, P. (2010), "A Model of Unconventional Monetary Policy", Working Paper, New York University.

Jokipii, T. and Milne, A. (2008), "The Cyclical Behaviour of European Bank Capital Buffers", *Journal of Banking and Finance*, **32(8)**, 1140–1451.

Kiyotaki, N. and Gertler, M. (2010), "Financial Intermediation and Credit Policy in Business Cycle Analysis", Handbook of Monetary Economics (forthcoming).

Lucas, R.E. Jr. (1987), "Models of Business Cycles", *Oxford, New York: Basil Blackwell.*

Meh, C. and Moran, K. (2010), "The Role of Bank Capital in the Propagation of Shocks", *Journal of Economic Dynamics and Control*, **34**, 555-576.

Milne, A. (2002), "Bank Capital Regulation as an Incentive Mechanism: Implications for Portfolio Choice", *Journal of Banking and Finance*, **26(1)**, 1–23.

Peura, S. and Jokivuolle, E. (2004), Simulation Based Stress Tests of Banks' Regulatory Capital Adequacy", *Journal of Banking and Finance*, **28(8)**, 1801–1824.

Peura, S. and Keppo, J. (2006), "Optimal Bank Capital with Costly Recapitalisation", *Journal of Business*, **79(4)**, 2163–2201.

Rotemberg, J.J., (1982), "Monopolistic Price Adjustment and Aggregate Output", *Review of Economic Studies*, **49(4)**, 517-31.

Smets, F. and Wouters, R. (2003), "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area", *Journal of the European Economic Association*, **1(5)**.

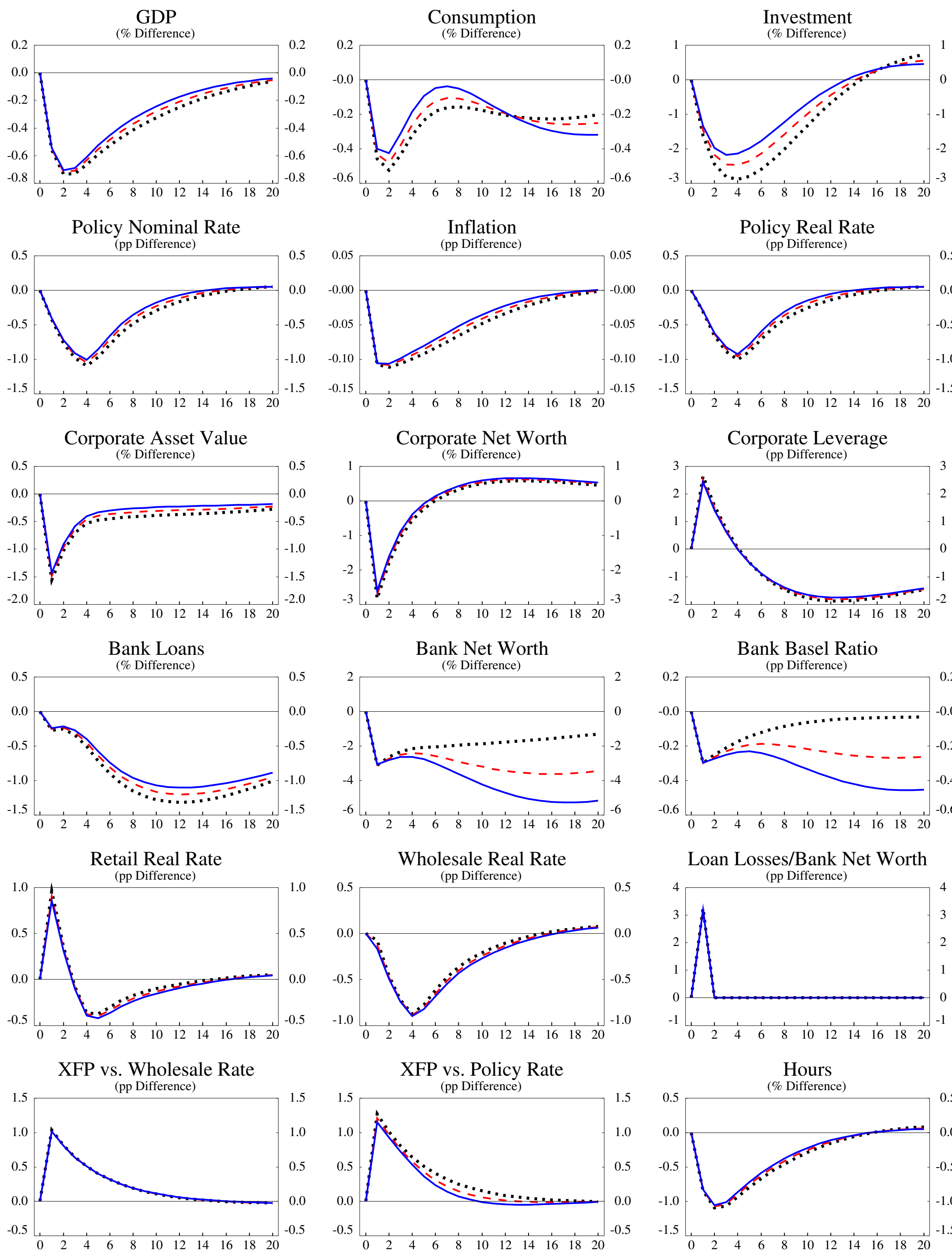
Van den Heuvel, S. (2002), "Does Bank Capital Matter for Monetary Transmission?", Federal Reserve Bank of New York Economic Policy Review, 259–265.

Van den Heuvel, S. (2008), "The Welfare Cost of Bank Capital Requirements", *Journal of Monetary Economics*, **55(2)**, 298-320.

Yun, T. (1996), "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles", *Journal of Monetary Economics*, **37**, 345-370.

Zhang, L. (2009), "Bank Capital Regulation, the Lending Channel and Business Cycles", Deutsche Bundesbank Discussion Paper Series, 33/2009.

Figure 1. Firm Riskiness Shock - Impulse Responses




Prudential Rule Feedback Coefficients on Loans: ... = 0, - - = 3, — = 6

Figure 2. Welfare and Instrument Volatility

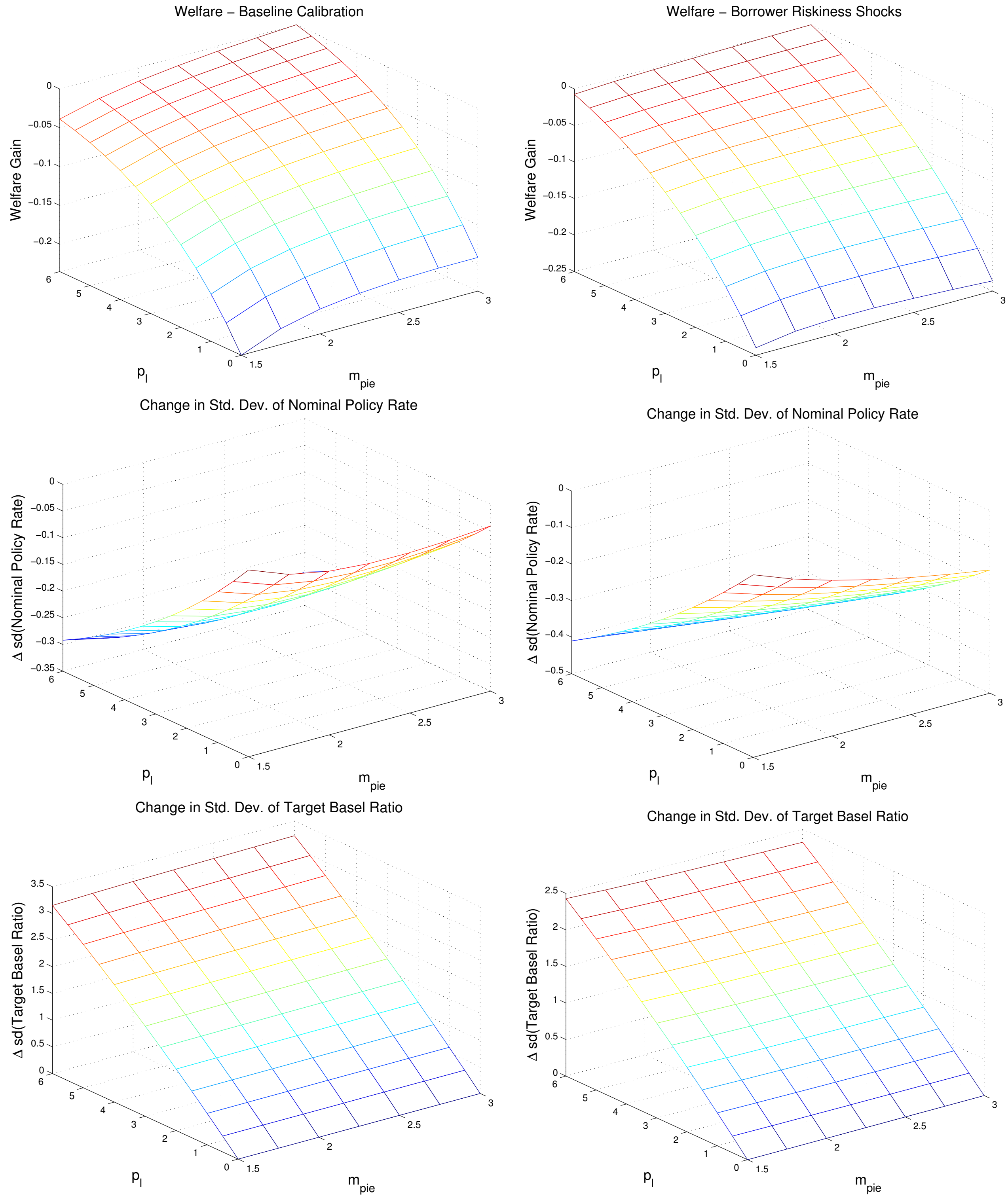


Figure 3. Welfare - Different Model Parameterizations