

# Transparency and Monetary Policy with Imperfect Common Knowledge

Mauro Roca

# **IMF Working Paper**

#### Research

# Transparency and Monetary Policy with Imperfect Common Knowledge

# Prepared by Mauro Roca<sup>1</sup>

Authorized for distribution by Atish Ghosh

April 2010

#### **Abstract**

# This Working Paper should not be reported as representing the views of the IMF.

The views expressed in this Working Paper are those of the author(s) and do not necessarily represent those of the IMF or IMF policy. Working Papers describe research in progress by the author(s) and are published to elicit comments and to further debate.

Is it desirable that central banks be more transparent in the communication of sensible information when agents have diverse private information? In practice, there exists some consensus about the benefits of acting in this way. However, other studies warn that increasing the precision of public information may raise the volatility of some aggregate variables - in particular, the price level - due to the disproportionate influence that it exerts on agents' decisions, and that this, in turn, will have negative effects on welfare. This paper studies the welfare effects of varying levels of transparency in a model of price-setting under monopolistic competition and imperfect common knowledge. Our results indicate that more precise public information never leads to a reduction of welfare in this framework. We find that the beneficial effects of decreased imperfect common knowledge due to a more precise common signal always compensates the potential rise in aggregate volatility. Moreover, we show that, in contrast to what has previously been assumed, the variability of the aggregate price level has no detrimental welfare effects in this model.

JEL Classification Numbers: E0, E20, E52, D82.

Keywords: Social Value of Public Information, Monetary Policy, Transparency.

Author's E-Mail Address: mroca@imf.org

\_

<sup>&</sup>lt;sup>1</sup> I am grateful to Jean Boivin, Marc Giannoni, Bruce Preston, and especially Michael Woodford for helpful discussions and invaluable guidance. I also thank Rajeev Cherukupalli, Atish Ghosh, Christian Hellwig, Guido Sandleris and seminar participants at Columbia University for useful comments. First version: January 2005.

	Contents	Page
I.	Introduction	3
II.	Model	5
	A. Households	6
	B. Firms	
	C. Monetary policy	8
III.	Equilibrium under imperfect common knowledge	9
IV.	Welfare effects of transparency	12
	A. An appropriate measure of welfare	
V. VI.	Objectives of monetary policy  Conclusions	
٧1.	Colletasions	10
App	pendices	
I.	Household maximization	19
II.	Optimal pricing rule	21
III.	Welfare	23
Ref	erences	25

## I. Introduction

In the last few years, major central banks of developed countries have made constant improvements in the way they communicate with the public. We can easily find some examples of this behavior both among those banks that target the interest rate, like the Federal Reserve, and among those that target inflation, like the Bank of England. The evident effort of these banks in conveying high quality information to private agents in a timely manner seems to be guided by the widespread consensus on the beneficial stabilizing effects of conducting monetary policy under a high degree of transparency.

There are, however, some theoretical warnings regarding the potential negative welfare effects that could be attributed to a more transparent disclosure of public information. One of these derives from the implications of the influential work of Morris and Shin (2002). Their model depicts a strategic game among a continuum of agents that need to choose an action in order to maximize a payoff function composed of two terms: one that increases as the action gets closer to some unknown fundamental of the economy, and the other, the coordination element, that increases as the action gets closer to other agents' actions. Agents choose their most preferred action after observing two noisy signals about the fundamental state of the economy, one that is common knowledge (public signal), and another that is idiosyncratic to each agent and not known by others (private signal). The authors show that an increase in the precision of the private signal is always welfare improving, but an increase in the precision of the public one could, under certain circumstances, reduce social welfare. Their conclusions have led to the interpretation that more transparency in the communication of sensitive information, for example by a central bank, could have detrimental effects on collective welfare.

This paper contributes to this debate by analyzing the welfare effects of varying levels of transparency in the disclosure of public information, and throwing light on the desirable approach to monetary policy in a price-setting model of monopolistic competition with imperfect common knowledge. We assume that there is a continuum of households and a continuum of monopolistically competitive, imperfectly informed firms that base their pricing decisions in the observation of noisy signals, both public and private, regarding the level of the natural rate of interest — a hidden fundamental that represents a composite of all real disturbances in the model. There is also a perfectly informed monetary authority that conducts policy in order to maximize social welfare and communicates a public signal to firms. Transparency in this model is represented by the precision of this public signal.

Using a model with the same features, Amato and Shin (2003) argue that more transparency could have negative welfare effects because it might increase macroeconomic volatility, especially of the aggregate price level. They conclude that these findings show that the main implications of Morris and Shin (2002) are still applicable to the dynamic general equilibrium setting. According to them, public information is a "double-edged instrument for public policy:" while it is efficient in guiding the actions of strategically related agents, the problem is that they overreact to public information in detriment of their own private signals. Thus, any mistake in the disclosure of centralized information could be greatly damaging.

In contrast, we find that those warnings are not applicable to this framework. First, when we consider plausible assumptions about certain key parameters, we show that aggregate volatility might only increase in rare cases. The increment in volatility due to more transparency is the exception rather than the rule. Second, even when this happens, increased transparency creates a trade-off between a negative welfare effect due to increased aggregate volatility, and a positive one due to decreased dispersion of individual prices. Evaluating an appropriate measure of social welfare, we show that the latter always dominates, and consequently that welfare always improves with more transparency.

Contrary to previous arguments, we observe that the underweighting of private information is beneficial because it solves the main inefficiency created by the imperfect common knowledge of strategic agents. Moreover, we show that the monetary authority should conduct its policy in order to homogenize agents' beliefs, discouraging any use of private information regardless of the relative precision of that information with respect to public announcements. Intuitively, it is easier to tune policy instruments when agents have homogeneous, potentially highly biased, beliefs, than when they have heterogeneous, possibly rather precise ones.

This paper is related to the literature on the effects of imperfect common knowledge that flourished after the reconsideration made by Woodford (2003a) of the Phelps (1970)-Lucas (1972, 1973) hypothesis that attributed temporary real effects of monetary disturbances to imperfect information about these shocks. Woodford shows that the interaction of imperfect common knowledge and strategic complementarities among price setters produces the desirable persistent real effects of monetary disturbances, a missing feature of the Lucas (1972). The key mechanism behind that result confirms the insight in Phelps (1983) that the infinite hierarchy of higher-order expectations is relevant for setting prices and that the higher their order the more sluggish their adjustment to changing conditions.

Following Sims (2003)'s rational inattention hypothesis, firms in Woodford (2003a) model observe the world from their own windows not because they do not have access to timely public information but rather because they have limited capacity to process it. While different in some respects, as discussed by Sims (2003), the perspective that firms adopt in processing information can alternatively be modeled as a signal extraction problem in which agents do not observe past aggregate variables. This is the path followed by Hellwig (2002), Amato and Shin (2003), and this paper in order to consider the aggregate effects of private and public information.<sup>2</sup> Our paper is thus in line with work that show that the provision of public information accelerates the adjustment of higher-order expectations to disturbances, but that this is at the cost of introducing informational noise that can increase economic volatility, and hence result in a decrease in social welfare.

In works more closely related to ours, Angeletos and Pavan (2004), using a model of investment

While there might be some correlation in the interpretation of public information by private agents, it is harder to justify, under the rational inattention hypothesis, that the common effect can be isolated from the idiosyncratic mistakes in order to analyze the consequences of different levels of precision in the public signal. This feature is easier to accommodate in the signal extraction approach.

complementarities, and Hellwig (2005), using a cash-in-advance model with monopolistic competition, show that more transparency is not necessarily detrimental to welfare. The main difference between our work and the latter is that we attempt to stress the monetary policy implications derived from the analysis of the framework used by Amato and Shin (2003), in which the nominal short-run interest rate is the main policy instrument — rather than a monetary aggregate — utilized by the monetary authority to counteract the effects of real disturbances represented by the natural rate of interest. We consider this neo-Wicksellian framework, thoroughly explained in Woodford (2003b), a better approximation of how policy is currently conducted in major central banks, and hence a more appropriate structure to analyze our problem of interest.

Finally, it is worth mentioning that despite the fact that imperfect common knowledge could be the result of either rational inattention or idiosyncratic noisy signals, the predictions that are derived from these different assumptions could be markedly different. One important respect in which these differ is in the conduct of monetary policy. Adam (2004) analyzes optimal policy under the rational inattention hypothesis using a model similar to the one proposed by Woodford (2003a). Adam's paper derives policy recommendations under the assumption that the central bank has control over nominal demand and that its objective is to minimize a loss function — derived from the model — that increases with variability of both the output gap and the aggregate price level. In contrast, we find that under the signal extraction hypothesis, price stabilization is not desirable according to a welfare maximizing criterion. Moreover, in our framework the policymaker should not counteract aggregate price variability in order to buffer real variables from exogenous shocks.

The remainder of the paper is organized as follows. Section 2 lays out a simple general equilibrium model with monopolistic competition and imperfect common knowledge. Section 3 derives the equilibrium conditions. Section 4 discusses the volatility and welfare effects of transparency. Section 5 analyzes the design of monetary policy. Section 6 concludes.

## II. MODEL

To analyze the effects of public and private information we use the baseline dynamic stochastic general equilibrium model of Woodford (2003b), with the modifications proposed by Amato and Shin (2003). This is basically a flexible price model composed of a continuum of households, monopolistic competitive firms, and a monetary authority.

One of the main features of this model is that the strategically related firms set prices, in order to maximize profits, based on their perception of current economic conditions. We only depart from the assumption of perfectly informed agents that have rational expectations by allowing firms to have heterogeneous information about the state of the economy. However, we assume that households and the monetary authority have full information, and consequently, common knowledge about the state variables. The rationale behind this asymmetric assumption will be made clear later, but it can be anticipated that is the combination of strategic behavior and imperfect common knowledge that creates the more interesting features in this model; since

households do not make any strategic decision, assuming that they have imperfect common knowledge will add little to the analysis at the expense of great complexity.<sup>3</sup>

#### A. Households

The representative household maximizes an expected discounted sum of utilities

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U_t \right\} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ u\left(C_t, \xi_t\right) - \int_0^1 v(H_t(i), \xi_t) di \right] \right\}, \tag{1}$$

where  $0<\beta<1$  is the discount factor,  $C_t$  is an index of consumption ,  $\xi_t$  is a vector of disturbances, and  $H_t(i)$  is the quantity of labor supplied to the production of one of the differentiated goods indexed by i in the unit interval. We assume that the representative household supplies all types of labor. For each value of  $\xi_t$ , the period felicity function  $u\left(.,\xi_t\right)$  is increasing and concave, and the period disutility of labor  $v(.,\xi_t)$  is increasing and convex.

Households form expectations in any period, represented by  $E_t$ , conditional on a complete information set that is, consequently, homogeneous among them.

The representative household is constrained by a single intertemporal budget restriction due to the existence of complete financial markets. Alternatively, this restriction can be represented by a period flow budget constraint

$$M_t + B_t \le W_t + P_t Y_t - T_t - P_t C_t, \tag{2}$$

and the usual transversality conditions. Assets are divided in two categories,  $M_t$ , the end-of-period balances in the financial asset that represents the unit of account — that we call money — and  $B_t$ , the nominal value of the end-of-period portfolio of the remaining financial assets;  $W_t$  represents the beginning-of-period financial wealth. The consumption basket  $C_t$  is given by the Dixit and Stiglitz (1977) constant-elasticity aggregator

$$C_t \equiv \left[ \int_0^1 C_t(i)^{\frac{\theta - 1}{\theta}} di \right]^{\frac{\theta}{\theta - 1}},\tag{3}$$

where  $\theta > 1$  is the elasticity of substitution, and  $C_t(i)$  is the consumption of good of type i. The corresponding price index

$$P_t \equiv \left[ \int_0^1 P_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}},\tag{4}$$

is the minimum cost of one unit the composite good given the individual prices  $P_t(i)$ . Each household owns an equal share of each firm's profits, denoted by  $\Pi_t(i)$ ; hence, total nominal

<sup>&</sup>lt;sup>3</sup> A model including this assumption could display other interesting features, like more persistence of consumption and output, but it would add little to the problem of our interest.

income is given by

$$P_t Y_t = \int_0^1 [w_t(i) H_t(i) + \Pi_t(i)] di,$$

where  $w_t(i)$  is the nominal wage paid to labor of type i.

Finally, we assume that fiscal policy is Ricardian, government collects (net) taxes  $T_t$ , and issue liabilities in accordance with a zero debt policy.

In appendix A.1, we show that after log-linearizing the intertemporal first order conditions for the household around the (full information) deterministic steady state, and using the equilibrium condition that aggregate output must equal aggregate demand, we obtain the following intertemporal aggregate relation

$$x_t = E_t x_{t+1} - \sigma(\hat{\imath}_t - E_t \pi_{t+1} - r_t^n), \tag{5}$$

where  $x_t$  is the output gap defined as the log-deviation of aggregate output from its natural rate,  $\sigma$  is the intertemporal elasticity of substitution,  $\hat{\imath}_t$  is the deviation of the gross short-run nominal interest rate — the monetary policy instrument — from its steady state,  $\pi_t$  is the inflation rate, and  $r_t^n$  is the "natural rate of interest." This composite disturbance includes all the exogenous real shocks in the model, and we assume that it follows an independent-identically distributed stochastic process characterized by

$$r_t^n \stackrel{iid}{\sim} N(0, \sigma_{\varepsilon}^2)$$
 (6)

#### B. Firms

Each monopolistic competitive firm indexed by  $i \in [0, 1]$ , produces one of the differentiated goods using a production function of the form

$$Y_t(i) = A_t f(H(i)), \tag{7}$$

where  $A_t$  represents a technological coefficient, and f(.) is an increasing, concave function that satisfies the Inada conditions. This can be interpreted as implicitly assuming that the capital stock is fixed and homogeneously distributed among firms.

Every period, each producer sets a price in order to maximize expected nominal profits knowing that she faces the relative demand function

$$Y_t(i) = Y_t \left(\frac{P_t(i)}{P_t}\right)^{-\theta}.$$
 (8)

Consequently, the objective function can be represented as

$$E_t^i \Pi^i(P_t(i), P_t, Y_t, A_t) = E_t^i \left\{ m(Y_t) \left[ P_t^{\theta} Y_t P_t(i)^{1-\theta} - w_t(i) f^{-1} (P_t^{\theta} Y_t P_t(i)^{-\theta} / A_t) \right] \right\}, \quad (9)$$

where  $m(Y_t)$  is the stochastic discount factor that the firm uses to weight profits in each state.

The details of the maximization problem are explained in appendix A.2. There we show that the optimal pricing rule of firm i is given by

$$p_t^i = E_t^i p_t + \bar{\zeta} E_t^i x_t, \tag{10}$$

where  $p_t^i = \log P_t(i)$ ,  $p_t = \log P_t$ , and  $\bar{\zeta} \in (0,1)$  is a parameter describing the degree of strategic complementarities experienced by firms.

In this case,  $E_t^i$  represents the expectation of the firm conditional on its private information set at time t. Hence, we can observe that firms find optimal to set a price equal to a linear combination — weighted by the degree of strategic complementarities — of their own expectation of the aggregate price level and of the output gap.

In order to fully characterize firms' optimal decisions we need to impose some structure to their information sets. Following Amato and Shin (2003), we assume that they are composed of private and public signals about current level of the natural rate of interest. In particular, firm i's information set can be represented as

$$\begin{split} I_t^i &= \left\{s_\tau^i\right\}_{\tau=0}^t, \\ s_t^i &= \begin{bmatrix} r_t^i \\ r_t \end{bmatrix}, \end{split}$$

where  $r_t^i$  and  $r_t$  are respectively a private and a public signal characterized by the following stochastic processes

$$\begin{aligned} r_t^i &= r_t^n + \nu_t^i & \nu_t^i \stackrel{iid}{\sim} N(0, \sigma_{\nu}^2), \\ r_t &= r_t^n + \eta_t & \eta_t \stackrel{iid}{\sim} N(0, \sigma_{\eta}^2). \end{aligned}$$

Finally, averaging (10) over all firms we obtain an expression for the aggregate price level

$$p_t = \bar{E}_t p_t + \bar{\zeta} \bar{E}_t x_t,$$

where  $\bar{E}_t = \int_0^1 E_t^i(.) di$  simply represents the average expectation of the individual firms. The aggregate price level is then a weighted average of the average expectation of the price level itself, and of the average expectation of the output gap. If firms had common knowledge, the average expectation would be equivalent to the usual first order expectation, but the assumption of imperfect common knowledge, given the existence of strategic complementarities among price setters, force them to estimate the average estimate of others in order to set its own price. Moreover, since the firm knows that every other firm faces the same problem, it also has to estimate the average estimate of that average estimate, and so on in an infinite recursion. In Section 3, we formally demonstrate this claim that the entire infinite hierarchy of higher order expectations will matter for the price setting decision.

# C. Monetary policy

We close the model by assuming that the monetary authority follows a targeting rule of the following type

$$p_t + \lambda x_t = r_t^n, \tag{11}$$

where  $\lambda$  is a policy coefficient that characterizes the monetary regime.

This assumption is restrictive in the sense that the optimal target could potentially include other variables that are present in the model, but on the other hand, it is flexible enough to nest the policy assumptions that have been used in the related literature. Woodford (2003a)'s assumption of monetary policy can be interpreted as nominal GDP targeting, Adam (2004) explicitly assumes this type of targeting, and Amato and Shin (2003) assumes a more general targeting rule of the type discussed here. As pointed by Svensson (2003), assigning different values to the policy coefficient  $\lambda$  produces different types of targeting rules. When  $\lambda=1$ , we can interpret (11) as a nominal GDP targeting (adjusted for the natural rate of output),  $\lambda=0$  could be interpreted as strict price-level targeting, and  $\lambda\to\infty$  could be interpreted as strict output gap targeting.

Finally, we assume that the monetary authority, like households, has complete information about the state of the economy.

#### III. EQUILIBRIUM UNDER IMPERFECT COMMON KNOWLEDGE

In this section we characterize the equilibrium under imperfect common knowledge by additionally assuming that firms enter each period with only a diffuse prior about the natural rate of interest. This assumption allow us to isolate the problem of optimal use of private and public information that is of our interest.

In order to solve for the equilibrium price, we begin by substituting (11) in (10), and after averaging over all firms we obtain an alternative expression for the aggregate price level<sup>4</sup>

$$p_t = (1 - \zeta)\bar{E}_t p_t + \zeta \bar{E}_t r_t^n. \tag{12}$$

By successively substituting (12) in (10), averaging over i, and introducing the notation  $\bar{E}_t^{(k)}$  for the average expectation of order k, we make explicit the dependence of the aggregate price level on higher order expectations

$$p_t = \sum_{k=1}^{\infty} \zeta (1 - \zeta)^{(k-1)} \bar{E}_t^{(k)} r_t^n, \tag{13}$$

where  $\zeta = \lambda^{-1}\bar{\zeta}$ . This expression tells us that the aggregate price is going to depend in a linear

<sup>&</sup>lt;sup>4</sup> In doing this we need to assume that  $\lambda \neq 0$ , but we can still interpret  $\lambda \to 0$  as strict price level targetting. From the analysis below it will be clear that it would not be optimal to follow this kind of policy.

combination of expectations of different orders with weights that are determined by the value of  $\zeta$ , a coefficient that depends both on policy and on the strategic complementarities faced by firms; from now on, we are going to refer to it as the "perceived" degree of strategic complementarities. Intuitively, for a given policy regime, a greater degree of strategic complementarities would imply that firms have to give more weight to the beliefs of others in order to set their optimal prices. In the limiting case that  $\zeta = 1$ , the strategic behavior disappears and firms only need to consider the (first order) expectation of the shock.

In order to solve the infinite sum in (13) we exploit the Gaussian distribution of the signals.<sup>5</sup> Assume that there is a finite number n of firms. Then, there exists a row vector  $a^i$  such that

$$E_t^i r_t^n = a^i s_t^i, (14)$$

and a square matrix  $A^{ij}$  such that

$$E_t^i s_t^j = A^{ij} s_t^i. (15)$$

These matrices are obtained from the covariance matrix of the shock and the n vectors of signals about it. Since we assume symmetry, these matrices are the same for all i, and their elements are given by

$$a^{i} = a = \begin{bmatrix} \frac{\sigma_{\eta}^{2}}{\sigma_{\nu}^{2} + \sigma_{\eta}^{2}} & \frac{\sigma_{\nu}^{2}}{\sigma_{\nu}^{2} + \sigma_{\eta}^{2}} \end{bmatrix} \quad \forall i,$$
 (16)

$$a^{i} = a = \begin{bmatrix} \frac{\sigma_{\eta}^{2}}{\sigma_{\nu}^{2} + \sigma_{\eta}^{2}} & \frac{\sigma_{\nu}^{2}}{\sigma_{\nu}^{2} + \sigma_{\eta}^{2}} \end{bmatrix} \quad \forall i,$$

$$A^{ij} = A = \begin{bmatrix} \frac{\sigma_{\eta}^{2}}{\sigma_{\nu}^{2} + \sigma_{\eta}^{2}} & \frac{\sigma_{\nu}^{2}}{\sigma_{\nu}^{2} + \sigma_{\eta}^{2}} \\ 0 & 1 \end{bmatrix} \quad \forall i, j.$$

$$(16)$$

Let the average signal be given by

$$\bar{s}_{n,t} = n^{-1} \sum_{i=1}^{n} s_t^i,$$

then, we can express the expectation of firm i about that average signal as

$$E_t^i \bar{s}_{n,t} = n^{-1}(n-1)As_t^i + n^{-1}s_t^i = A_n s_t^i.$$
(18)

Averaging (14) over i, we obtain an expression for the average expectation of the natural rate of interest

$$\bar{E}_t r_t^n = n^{-1} \sum_{i=1}^n a s_t^i = a \bar{s}_{n,t},$$

and by successively applying (14) and averaging over all firms we obtain an expression for the

We apply to this framework the procedure used by Ui (2003) in solving the Lucas (1972) model.

average expectation of order k

$$E_{t}^{i}\bar{E}_{t}r_{t}^{n} = E^{i}a\bar{s}_{n,t} = aA_{n}s_{t}^{i}$$

$$\bar{E}_{t}^{(2)}r_{t}^{n} = \bar{E}_{t}\bar{E}_{t}r_{t}^{n} = aA_{n}\bar{s}_{n,t}$$

$$\bar{E}_{t}^{(k)}r_{t}^{n} = aA_{n}^{k-1}\bar{s}_{n,t}.$$
(19)

Substituting (19) in (13) we obtain a new expression for the aggregate price level

$$p_{t} = \sum_{k=1}^{\infty} \zeta (1-\zeta)^{(k-1)} a A_{n}^{k-1} \bar{s}_{n,t}$$

$$p_{t} = \zeta a (I - (1-\zeta)A_{n})^{-1} \bar{s}_{n,t}.$$

Using (18) we can express the expectations of firm i of the aggregate variables as

$$E_t^i p_t = c_n s_t^i,$$
  
$$E_t^i x_t = \lambda^{-1} (a - c_n) s_t^i,$$

where  $c_n = \zeta a (I - (1 - \zeta)A_n)^{-1} A_n$ .

Substituting these last expressions in (10) we can rewrite the optimal pricing rule as

$$p_t^i = [(1 - \zeta)c_n + \zeta a] s_t^i,$$
 (20)

and averaging over i we can express the aggregate price level as

$$p_t = \left[ (1 - \zeta)c_n + \zeta a \right] \bar{s}_{n,t}.$$

Using the Law of Large Numbers,

$$\lim_{n \to \infty} \bar{s}_{n,t} = \bar{s}_t = \begin{bmatrix} r_t^n \\ r_t \end{bmatrix}$$

$$\lim_{n \to \infty} A_n = A$$

$$\lim_{n \to \infty} c_n = c = \zeta a (I - (1 - \zeta)A)^{-1} A,$$

we obtain the following solutions for the aggregate variables

$$p_t = [(1 - \zeta)c + \zeta a] \begin{bmatrix} r_t^n \\ r_t \end{bmatrix},$$

$$x_t = \lambda^{-1} \begin{bmatrix} 1 & 0 \end{bmatrix} - [(1 - \zeta)c + \zeta a] \begin{bmatrix} r_t^n \\ r_t \end{bmatrix}.$$

Finally, substituting according to (16) and (17), these expressions can alternatively be written as

$$p_t = \psi r_t^n + (1 - \psi) r_t, \tag{21}$$

$$x_t = \lambda^{-1} \left[ (1 - \psi)r_t^n + (\psi - 1)r_t \right], \tag{22}$$

where 
$$\psi = \frac{\zeta \sigma_{\eta}^2}{\sigma_{\nu}^2 + \zeta \sigma_{\eta}^2}$$
.

These expressions indicate that the equilibrium values of the aggregate variables are determined by a linear combination of the exogenous shock and the public signal about it, where the weights are given by the relative precision of the signals, and the perceived degree of strategic complementarities. The influence of  $\zeta$  is what differentiates this case from a typical signal extraction problem, since firms take into account the strategic interrelation among them, and try to put more weight in the commonly known public signal than otherwise would do in absence of any complementarities. Intuitively, more weight is going to be given to the private signal, the more precise is this signal relative to the public one, and the lesser is the perceived degree of strategic complementarities.

#### IV. WELFARE EFFECTS OF TRANSPARENCY

In these kind of models with imperfect information about some fundamental state of the economy, and in which some agents have access to public and private signals, transparency has been usually associated with the precision of a public signal communicated by a centralized institution. Additionally, it has implicitly been assumed that the policy maker can decide about what is the appropriate level of transparency to communicate this information to the private sector. In this subsection we analyze how changes in the relative precision of the public signal affect the volatility of aggregate variables, and the dispersion of individual prices around the aggregate price level. Then, explicitly considering a measure of welfare we can arrive to an objective assessment of the welfare implications of different degrees of transparency in the communication of public information.

From (20), (21), (22) we can easily obtain the following expressions for the dispersion of individual prices and the volatility of aggregate variables

$$V_i(p^i) = \psi^2 \sigma_{\nu}^2$$
,  $V(p) = \sigma_{\varepsilon}^2 + (1 - \psi)^2 \sigma_{\eta}^2$ ,  $V(x) = \lambda^{-2} (1 - \psi)^2 \sigma_{\eta}^2$ . (23)

Let's abstract for now from changes in the monetary policy regime assuming that the monetary authority follows a nominal output gap targeting as it has been previously assumed in the related literature; in terms of the policy coefficient, we initially assume that  $\lambda=1$ .

If the weighting of the signals in the price setting rule were to remain constant with changes in their relative precision, we would observe that price dispersion will decrease with more precise private signals — or less imperfect common knowledge — and that the volatility of the aggregate

variables will diminish with more transparency. But this analysis would be misleading because it is not taking into consideration that the weighting itself changes with the relative precision of the signals, and with the degree of strategic complementarities among firms.

When transparency is increased, firms will give a greater weight to the public signal so it immediately follows that price dispersion will be diminished, but it is less clear how the variability of aggregate variables will be affected. Using simple calculus it can be observed that volatility will be increased only when more weight is being given to the private signal, that is when

$$\psi > \frac{1}{2}.\tag{24}$$

Amato and Shin (2003) conclude that more transparency could lead to increased volatility, specially in the aggregate price level and that it would be welfare decreasing, corroborating with this model the findings of Morris and Shin (2002). However, the occurrence of this scenario does not appear to be very likely in the current model. First, since the condition under which volatility is increased depends inversely in the level of transparency, at some point of increased transparency, and considering that the precision of the private signals remain constant, more weight will relatively be given to the public signal, and volatility will decrease — that is, more weight will be given to an increasingly precise signal. Second, even if transparency could only be marginally affected, condition (24) is not expected to be met under plausible assumptions. This condition can alternatively be expressed as  $\frac{1/\sigma_{\eta}^2}{1/\sigma_{\nu}^2} < \bar{\zeta}$ . Hence, under the assumption of strategic complementarities, we can only obtain the undesired result if the public signal is less precise than the private one. Moreover, under the usually estimated values of  $\bar{\zeta}$  — around 0.10 to 0.15 —<sup>6</sup> that requirement will also imply that the noise in the public signal should be at least approximately seven times bigger than the noise in the private signal, while it would be natural to think that the idiosyncratic signals are the ones with the greater variances.<sup>7</sup>

The intuition behind these results can be traced back to the well know Keynesian "beauty contest." In this particular case, firms will be willing to give more weight to the public signal in presence of strategic complementarities because that signal is common knowledge, and they want to "hedge" against the risk of their own private signals. That risk is increasing in the degree of strategic complementarities because in that case the pricing decision of other firms will have more incidence on their own profits. Consequently, under a considerable degree of strategic complementarities, firms will only rely more in their own private signals if the public signal is sufficiently imprecise.

## A. An appropriate measure of welfare

On a purely theoretical basis, the increased volatility of the aggregate variables due to more

<sup>&</sup>lt;sup>6</sup> See Woodford (2003b,ch.3) for a detailed discussion of the degree of strategic complementarity under alternative assumptions.

 $<sup>^7</sup>$  Svensson (2006) finds a similar result for the model of Morris and Shin (2002); in that case, even when this ratio is affected by parametric assumptions, he finds a minimum of 8.

transparency is still possible, but even if this occurs, it would be incorrect to jump to the conclusion that this will be detrimental to welfare. One of the main advantages of working with models based in micro foundations is that we can use the assumed utility function to rank alternative policies, or scenarios, according with their effects on welfare. This is the path we follow next.

We show in appendix A.3 that the expected value of the period contribution to utility can be expressed to second order as

$$E_t U_t \approx \Xi \left[ \bar{\zeta} V(x) + \theta V_i(p^i) \right],$$
 (25)

where  $\Xi$  is a negative constant. From this expression we observe that expected utility will be negatively affected by output gap variation and individual price dispersion. We could identify the first component of the social welfare function with a measure of aggregate volatility, and the second with a measure of dispersion of individual decisions, or lack of coordination among private agents due to their imperfect common knowledge. It could appear surprising that aggregate price volatility — or inflation — does not affect welfare as would be the case in a model with competitive monopolistic price setters that face price rigidities *a la* Calvo (1983). The difference is that in the present flexible price model, in contrast with the typical sticky-price model, stability of the aggregate price level is neither sufficient nor necessary to eliminate individual price dispersion, that is mainly determined by the imperfect common knowledge of the firms. Since both type of models share the competitive monopolistic assumption, it is the dispersion of output levels across sectors, originated in turn in individual price differences, that is relevant for welfare; the difference is that in the current analysis we cannot link price stability to elimination of this dispersion.

Substituting in (25) according to (23) and differentiating with respect to the variance of the public signal, we observe that more transparency is undoubtedly welfare improving

$$\frac{\partial (E_t U_t)}{\partial \sigma_{\eta}^2} \approx \Xi \frac{\bar{\zeta}(\sigma_{\nu}^2)^2}{\left(\sigma_{\nu}^2 + \bar{\zeta}\sigma_{\eta}^2\right)^3} \left[\sigma_{\nu}^2 + (2\theta - 1)\bar{\zeta}\sigma_{\eta}^2\right] < 0.$$

The interpretation of this result is straightforward. Even in the empirically implausible case that condition (24) is met, more transparency creates a trade-off between increased volatility of the output gap and reduction in price dispersion. We find that for any theoretical value of the elasticity of substitution, the latter effect dominates, and consequently a more precise public signal produces a positive effect in expected welfare. Of course, when condition (24) is not met, the reduction in output gap volatility reinforces the reduction in price dispersion and expected welfare improves with transparency.

Even if the model is highly stylized, and the analysis has been simplified in many respects, we have shown that there is no ground to conclude that increased transparency in the disclosure of public information could be damaging to welfare. Moreover, policy recommendations based on anti-transparency arguments could be insufficiently justified.

In summary, the general results stated in Morris and Shin (2002) are not applicable to this

framework. The origin of the discrepancy resides in what is considered an appropriate measure of social welfare. While the Morris and Shin findings are based on an aggregated loss function that penalizes only the deviation of individual actions from the fundamental — the volatility component — but not the discrepancy among their actions — the lack of coordination component — we have shown that an appropriate measure of welfare in our model must contain both components. Moreover, we find that the beneficial effects of increased transparency in the second component always dominates the pernicious effects on the former, and hence our conclusions are different from theirs.

## V. OBJECTIVES OF MONETARY POLICY

We have found that the monetary authority must always pursue the higher degree of accuracy in the transmission of critical information. Evidently, there are limits to the precision of public information, or it could be too costly to achieve. However, if the central bank can use other policy instruments to make agents coordinate their decisions on the public signal, the degree of transparency, at least understood in that way, ceases to be important in terms of welfare.

From (25) it is clear that the policy problem is to decrease price dispersion and achieve output gap stability. Until now, we have assumed that the central bank follows a nominal output gap targeting, but if we allow the monetary authority to freely choose the policy coefficient in (11) we find a rather simple solution to this problem.

The main inefficiency that the monetary authority must overcome is the imperfect common knowledge of firms, that is, the informational noise created by their private signals, that produces the undesirable price dispersion. We have seen that firms assign a weight to this signal that is decreasing both in the increasing relative precision of the public signal and in the perceived degree of strategic complementarities. Then, for any level of transparency, the central bank can increase the weight that firms optimally assign to the public signal, and thereby increase the coordination among them, by pursuing an strict output gap targeting, that is, letting  $\lambda \to \infty$  in (11). Since the policy regime is common knowledge, all firms will expect that the monetary authority will use its policy instruments to maintain the output gap close to zero. Hence, and according to (10), they will find optimal to equalize their individuals prices to their expectations of the aggregate price level. Since all firms face the same problem, and this is common knowledge, they find optimal to disregard their private signals and give an increasing weight to the public signal, or equivalently,  $\psi \to 0$ . It can be checked from (21) and (22) that these strategies constitute an equilibrium.

In summary, if the central bank follows a strict output gap targeting, both output variability and price dispersion can be eliminated, and welfare will be maximized according to (25). From (23) we observe that the volatility of the aggregate price level, on the other hand, will be increased since it will follow the public signal. Its variability, then, will be composed by the volatility of the natural rate of interest, and by the noise introduced by the public signal. However, this increased

<sup>&</sup>lt;sup>8</sup> We do not discuss the inefficiency arising from monopolistic competition because it is not originated in the informational problem in which we focus, and it can be easily solved with a subsidy.

volatility does not have any pernicious effect on welfare. On the contrary, it is the variability of the aggregate price level that buffers the real economy against real disturbances and allows the maximization of welfare.

Under these assumptions, more transparency in the disclosure of public information could decrease price variability but it will not have any effect on real allocations, and consequently on welfare. If the noise introduced by the public signal is totally eliminated, the price level would track the natural rate of interest, as in the complete information case.

This observation should not be interpreted as an anti-transparency message. It rather tries to convey that if the central bank can use other instruments to make agents coordinate in the use of the public signal; it would matter little the precision of this coordinating signal, it would only matters that all agents use the common — potentially rather imprecise — information. What is important is the elimination of the inefficiency created by the imperfect common knowledge of the firms. Hence, and contrary to what has previously been argued, the underweighting of private information is beneficial in this case.

Finally, it is important to note that the policy implications derived from this model are markedly different from those obtained by Adam (2004), who analyzes optimal monetary policy under imperfect common knowledge originated in the rational inattention hypothesis. This author assumes that the central bank controls nominal demand in order to minimize a period loss function that increases with both variability of the output gap, and variability of the aggregate price level. The different results are the consequence of different mechanisms that can be used in order to achieve the same objectives. In both cases, and under the monopolistic competitive assumption, what is relevant for welfare — apart from output gap stabilization — is the reduction of output variability among firms. This in turn, will be the result of individual price dispersion created by the imperfect common knowledge of firms. While in the rational inattention framework the dispersion in private beliefs is proportionally related to the variability of fundamentals that the agents are trying to observe due to a constant capacity constraint to process information, in the signal extraction problem, that link could be altered by varying the relative precision of the different signals, or the degree of strategic complementarities among firms. Hence, under the rational inattention hypothesis there are gains from price level stabilization because it can only be achieved through the reduction in the volatility of fundamentals, that in turn induce a reduction in individual price dispersion. In contrast, we have shown that this is not necessarily true under the signal extraction assumption.

#### VI. CONCLUSIONS

In situations characterized by imperfectly informed agents that base their decisions on stochastic signals about the true fundamentals that they are trying to know, transparency in the disclosure of public information has been associated with a higher precision of a commonly observed signal. Also, it has been implicitly assumed that the policy maker controls that level of accuracy. An accepted result in this literature is that more transparency could be prejudicial to welfare because

it induces agents to give more weight in their decisions to the public announcement in detriment of their own private information. Then, welfare could be damaged if the public signal is not sufficiently accurate. This result has been used to warn about the potential damaging effects of excessive transparency in the conduct of monetary policy.

Using a general equilibrium model characterized by monopolistic competition and imperfect common knowledge, we show that those results are not robust enough to constitute a valid criterion for policy recommendations. On the contrary, we observe that welfare never decreases with increasing levels of transparency. Our results are driven by the fact that the use of the public signal is beneficial to counteract the inefficiency created by the imperfect common knowledge of strategically related agents. Moreover, even when a highly imprecise common signal might increase macroeconomic volatility under rare parametric assumptions, its associated negative consequences on welfare are more than compensated by the former effect.

Most of our analysis has concentrated in analyzing transparency as it has been traditionally interpreted in the related literature — more precision in the disclosure of public information about certain relevant macroeconomic variables — but we also show that the policymaker could obtain greater benefits from being clear, and committed, about the objectives pursued by policy. The issue of the precision about the communication of sensible information is of a lesser importance when the policy objectives are incontrovertible. The intuition behind these findings is that if the only inefficiency arises from some agents' imperfect common knowledge, and there exist some public signal that could act as a coordinating device, other policy instruments could be used to make agents homogenize their beliefs around that signal, discouraging the use of private information and hence eliminating that inefficiency. In this scenario, the precision of the coordinating signal ceases to be of any importance.

We additionally show that the correct approach to policy in this flexible price model is to follow a strict output gap targeting, without any consideration to price level stabilization. Movements in the aggregate price level isolates real variables from exogenous shocks, and since only the latter are relevant for welfare, monetary policy should never be aimed to reduce price variability.

Hence, from this stylized model it can be inferred that monetary policy should not consider inflation stabilization among its objectives. This cast doubts about the usefulness of the baseline version of the model for further policy recommendations, and set a research agenda to analyze potential extensions.

An immediate extension would be to abandon the "cashless limit" by including monetary frictions. A valid shortcut would be to introduce real balances in the utility function, that in turn would make explicit the welfare effects derived from price level volatility. Even when the elasticity of utility with respect to real balances is accepted to be quantitatively not important, it still would be useful to check the robustness of our conclusions in this alternative setup.

Another improvement would be to adopt a more general targeting rule that includes other variables present in the model, i.e. lagged price level. In this case, it would be interesting to observe if the

flexible price nature of the model would still determine that prices should not be prevented from adjusting under an optimal policy design.

Introducing some staggering mechanism in price adjustment *a la* Calvo (1983) would give a rationale for price stabilization, and even if imperfect common knowledge will not be the cause for inflation stabilization it could be appealing to analyze the interaction between these assumptions. A possible outcome is that imperfect common knowledge could produce the persistence of inflation missing in the forward-looking Calvo (1983) model. Additionally, in a model with price frictions the desirability of full transparency might not hold.<sup>9</sup>

Finally, a more promising road to follow would be to assume that the monetary authority also has imperfect information about the state of the economy. Under this assumption not only the limits to the precision of the public signal would be explicitly determined, but also the optimal policy problem would become more challenging.

<sup>9</sup> Ghosh (2002) shows that in a model with price stickeness and forward-looking exchange rate determination, it is optimal for the central bank to retain some secrecy regarding foreign exchange operations.

- 19 - Appendix I

## **APPENDICES**

#### I. HOUSEHOLD MAXIMIZATION

Each period, the representative household chooses to optimally smooth intertemporal consumption according to the Euler relation

$$\Lambda_t = \beta i_t E_t \left( \Lambda_{t+1} \right), \tag{A1}$$

where  $\Lambda_t = \frac{u_c(C_t, \xi_t)}{P_t}$  is the marginal utility of income, and  $i_t$  is the gross nominal risk-free interest rate. Since this is a model lacking monetary frictions, and there is a positive money supply, this rate will coincide in equilibrium with the interest rate paid on money balances by the monetary authority; this is the policy instrument. Given the intertemporal decision about the appropriate quantity of the consumption basket to consume each period, the representative household intratemporally allocate their expenditure across differentiated goods according to

$$C_t(i) = C_t \left(\frac{P_t(i)}{P_t}\right)^{-\theta}.$$

The representative household also supplies each type of labor according to

$$\frac{v_h(H_t(i), \xi_t)}{\Lambda_t} = w_t(i). \tag{A2}$$

Market clearing requires that  $C_t(i) = Y_t(i)$  for all i, and integrating over all goods we must have

$$C_t = Y_t. (A3)$$

Hence, aggregate output, like aggregate consumption, is a Dixit-Stiglitz aggregate of the differentiated goods

$$Y_t \equiv \left[ \int_0^1 Y_t(i)^{\frac{\theta - 1}{\theta}} di \right]^{\frac{\theta}{\theta - 1}} \tag{A4}$$

Using these relations we also obtain (8), the relative demand function faced by each individual supplier.

Log-linearizing (A1) around the deterministic steady state in which  $\xi_t = 0$ ,  $\frac{P_{t+1}}{P_t} = 1$ ,  $i_t = \bar{\imath} = \beta^{-1}$ ,  $Y_t = \bar{Y}$ , using (A3), and introducing the notation  $\hat{z}_t \equiv \ln\left(\frac{z_t}{\bar{z}}\right)$  to represent the log-deviation of a variable from its steady state  $\bar{z}$ , we obtain

$$\hat{Y}_t = g_t + E_t \left( \hat{Y}_{t+1} - g_{t+1} \right) - \sigma \left( \hat{\imath}_t - E_t \pi_{t+1} \right),$$

where  $\sigma \equiv -\frac{u_c}{u_{cc}\bar{Y}} > 0$  is the intertemporal elasticity of substitution of aggregate expenditure, and

 $g_t \equiv -\frac{u_{c\xi}}{u_{cc}Y}\xi_t$  indicates the shift in the relation between real income and the marginal utility of real income due to preference shocks.

Finally, introducing the notation  $Y_t^n$  to describe the "natural rate of output" (to be defined in appendix A.2) we obtain (5)

$$x_t = E_t x_{t+1} - \sigma(\hat{i}_t - E_t \pi_{t+1} - r_t^n),$$

where

$$x_t \equiv \hat{Y}_t - \hat{Y}_t^n,$$

is the output gap, and

$$r_t^n \equiv \sigma^{-1} \left[ E_t \left( \hat{Y}_{t+1}^n - g_{t+1} \right) - \left( \hat{Y}_t^n - g_t \right) \right],$$

is the "natural rate of interest."

- 21 - Appendix II

## II. OPTIMAL PRICING RULE

The first order condition to the problem of maximizing (9) shows that each supplier wishes to set a price such that its expected relative price is a desired constant mark-up  $\mu = \frac{\theta}{\theta - 1}$  over expected real marginal costs

$$E_t^i \left[ m(Y_t)(1-\theta)Y_t \left( \frac{P_t(i)}{P_t} \right)^{-(1+\theta)} \left( \frac{P_t(i)}{P_t} - \mu s \left( Y_t(i), Y_t, \bar{\xi}_t \right) \right) \right] = 0, \tag{A5}$$

where  $\bar{\xi}_t = [\xi_t; A_t]$  includes both the preference and technological shocks, and

$$s\left(Y_{t}(i), Y_{t}, \bar{\xi}_{t}\right) = \frac{v_{h}(f^{-1}(Y(i)/A_{t}), \xi_{t})}{u_{c}\left(Y_{t}, \xi_{t}\right) A_{t}} \frac{1}{f'\left(f^{-1}(Y(i)/A_{t})\right)},\tag{A6}$$

is the real marginal cost that has been derived considering the corresponding labor supply (A2).

Under common knowledge, all firms would set the same price and the same quantity of each good would be supplied. This common quantity is the "natural rate of output" and it is implicitly defined by

$$s\left(Y_t^n, Y_t^n, \bar{\xi}_t\right) = \mu^{-1}.$$

From this expression, we can also observe that -to first order- the natural rate of output is just a linear combination of the real disturbances

$$\hat{Y}_t^n = \frac{\sigma^{-1}}{\omega + \sigma^{-1}} g_t + \frac{\omega}{\omega + \sigma^{-1}} q_t, \tag{A7}$$

where we redefine  $g_t \equiv -\frac{u_{c\xi}}{u_{cc}Y}\bar{\xi}_t$ ,  $q_t \equiv -\frac{v_{y\xi}}{v_{yy}Y}\bar{\xi}_t$ , and  $\omega > 0$  is the elasticity of the real marginal cost with respect to own supply.

If additionally we consider the deterministic case  $\bar{\xi}_t = 0$ , we observe that the steady state real marginal cost is equal to the inverse of the desired mark-up, and that the steady state level of aggregate output is implicitly defined by

$$s\left(\bar{Y},\bar{Y},0\right) = \mu^{-1}.$$

Log-linearizing (A5) around this steady state,  $\bar{\xi}_t = 0$ ,  $\frac{P_t(i)}{P_t} = 1$ ,  $\frac{P_{t+1}}{P_t} = 1$ ,  $Y_t = Y_t^n = \bar{Y}$ , we obtain the following pricing rule

$$p_t^i = E_t^i p_t + E_t^i \hat{s}_t^i, \tag{A8}$$

where  $p_t^i = \log P_t(i)$ ,  $p_t = \log P_t$ , and

$$\hat{s}_t^i = \omega \hat{Y}_t^i + \sigma^{-1} \hat{Y}_t - (\omega + \sigma^{-1}) \hat{Y}_t^n,$$

is the log-deviation of real marginal cost from its steady state.

Finally, using the relative demand function (8), from (A8) we can obtain (10)

$$p_t^i = E_t^i p_t + \bar{\zeta} E_t^i x_t,$$

where  $\bar{\zeta}=\frac{\omega+\sigma^{-1}}{1+\omega\theta}\in(0,1)$  is a parameter describing the degree of strategic complementarities between the suppliers of monopolistic competitive goods.

- 23 - Appendix III

## III. WELFARE

From (1), we have the following expected contribution to utility

$$E_t U_t = E_t \left[ u \left( C_t, \xi_t \right) - \int_0^1 v(H_t(i), \xi_t) di \right],$$

where we can substitute consumption and labor using (A3) and (7) to obtain

$$E_t U_t = E_t \left[ \tilde{u} \left( Y_t, \bar{\xi}_t \right) - \int_0^1 \tilde{v}(Y_t(i), \bar{\xi}_t) di \right], \tag{A9}$$

using the definitions

$$\tilde{u}\left(Y_{t}, \bar{\xi}_{t}\right) \equiv u\left(Y_{t}, \xi_{t}\right),\tag{A10}$$

and

$$\tilde{v}(Y_t(i), \bar{\xi}_t) \equiv v(f^{-1}(Y_t(i)/A_t), \xi_t). \tag{A11}$$

Using this last expression, we can also express (A6) as

$$s\left(Y_{t}(i), Y_{t}, \bar{\xi}_{t}\right) = \frac{\tilde{v}_{y}(Y_{t}(i), \bar{\xi}_{t})}{u_{c}\left(Y_{t}, \xi_{t}\right)},\tag{A12}$$

Since it is not relevant for our analysis, and since it will not change our conclusions, we are going to abstract from the inefficiencies created by market power assuming that the steady state value of output is close to the efficient level, or that there is a subsidy that mitigates this inefficiency. That is, the steady state value of (A12) could be considered sufficiently close to one.

We begin by taking a second order approximation to (A10) around the deterministic steady state  $\bar{\xi}_t = 0, Y_t = \bar{Y}$ 

$$\tilde{u}\left(Y_{t}, \bar{\xi}_{t}\right) = u(\bar{Y}, 0) + u_{c}(Y_{t} - \bar{Y}) + u_{\xi}\bar{\xi}_{t} + \frac{u_{cc}}{2}(Y_{t} - \bar{Y})^{2} + \bar{\xi}_{t}'\frac{u_{\xi\xi}}{2}\bar{\xi}_{t} + u_{c\xi}(Y_{t} - \bar{Y})\bar{\xi}_{t} + \vartheta(\|\hat{Y}, \bar{\xi}\|^{3}),$$
(A13)

where  $\vartheta(\|\hat{Y}, \bar{\xi}\|^3)$  represents the third order residual. Then, we use the second order expansion of output around its steady state

$$Y_t = \bar{Y} + \bar{Y}\hat{Y}_t + \frac{\bar{Y}}{2}\hat{Y}_t^2 + \vartheta(\|\hat{Y}\|^3),$$

to substitute in (A13) and obtain

$$\tilde{u}\left(Y_{t}, \bar{\xi}_{t}\right) = \bar{Y}u_{c}\left[(1 + \sigma^{-1}g_{t})\hat{Y}_{t} + \frac{1 - \sigma^{-1}}{2}\hat{Y}_{t}^{2}\right] + t.i.p. + \vartheta(\|\hat{Y}, \bar{\xi}\|^{3}),\tag{A14}$$

where t.i.p. collects terms that are independent of policy.

Using similar steps we can approximate to second order (A11) around the deterministic steady state as

$$\tilde{v}(Y_t(i), \bar{\xi}_t) = \bar{Y}u_c \left[ (1 - \omega q_t)\hat{Y}_t^i + \frac{1 + \omega}{2} \left(\hat{Y}_t^i\right)^2 \right] + t.i.p. + \vartheta(\|\hat{Y}, \bar{\xi}\|^3),$$

Integrating this expression over all goods

$$\int_{0}^{1} \tilde{v}(Y_{t}(i), \bar{\xi}_{t}) = \bar{Y}u_{c}\left\{(1 - \omega q_{t})E_{i}(\hat{Y}_{t}^{i}) + \frac{1 + \omega}{2}\left[\left(E_{i}(\hat{Y}_{t}^{i})\right)^{2} + V_{i}(\hat{Y}_{t}^{i})\right]\right\} + t.i.p. + \vartheta(\|\hat{Y}, \bar{\xi}\|^{3}),$$

where  $E_i(\hat{Y}_t^i)$  and  $V_i(\hat{Y}_t^i)$  are respectively the mean and variance across all differentiated goods.

We then use a second order approximation to (A4)

$$\hat{Y}_t = E_i(\hat{Y}_t^i) + \frac{1 - \sigma^{-1}}{2} V_i(\hat{Y}_t^i) + \vartheta(\|\hat{Y}\|^3),$$

to substitute  $E_i(\hat{Y}_t^i)$  and obtain

$$\int_{0}^{1} \tilde{v}(Y_{t}(i), \bar{\xi}_{t}) = \bar{Y}u_{c}\left[(1 - \omega q_{t})\hat{Y}_{t} + \frac{1 + \omega}{2}\hat{Y}_{t}^{2} + \frac{\theta^{-1} + \omega}{2}V_{i}(\hat{Y}_{t}^{i})\right] + t.i.p. + \vartheta(\|\hat{Y}, \bar{\xi}\|^{3}).$$
(A15)

Replacing (A14) and (A15) in (A9) we obtain

$$E_t U_t = E_t \left\{ \bar{Y} u_c \left[ (\sigma^{-1} g_t + \omega q_t) \hat{Y}_t - \frac{\sigma^{-1} + \omega}{2} \hat{Y}_t^2 - \frac{\theta^{-1} + \omega}{2} V_i(\hat{Y}_t^i) \right] + t.i.p. + \vartheta(\|\hat{Y}, \bar{\xi}\|^3) \right\}.$$

Next, using (A7) and discarding irrelevant terms, we obtain an expression in terms of the output gap

$$E_t U_t \approx E_t \left\{ -\frac{\bar{Y}u_c}{2} \left[ \left( \sigma^{-1} + \omega \right) x_t^2 + \left( \theta^{-1} + \omega \right) V_i(\hat{Y}_t^i) \right] \right\}.$$

Using (8), we can substitute the variance across differentiated outputs according to.

$$V_i(\hat{Y}_t^i) = \theta^2 V_i(p_t^i),$$

to finally obtain (25)

$$E_t U_t \approx \Xi \left[ \bar{\zeta} V(x) + \theta V_i(p^i) \right],$$

where 
$$\Xi = -\frac{\bar{Y}u_c(1+\omega\theta)}{2} < 0$$
.

#### REFERENCES

- Adam, K. (2004): "Optimal Monetary Policy with Imperfect Common Knowledge," C.E.P.R. Discussion Paper No. 4594.
- Amato, J., and H. Shin (2003): "Public and Private Information in Monetary Policy Models," *BIS Working Paper*, 138, Monetary and Economics Department, Bank for International Settlements.
- Angeletos, G. M., and A. Pavan (2004): "Transparency of Information and Coordination in Economies with Investment Complementarities," *American Economic Review*, 94(2), 91–98.
- Calvo, G. A. (1983): "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12(3), 383–398.
- Dixit, A. K., and J. E. Stiglitz (1977): "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, 67(3), 297–308.
- Ghosh, A. R. (2002): "Central Bank Secrecy in the Foreign Exhcange Market," *European Economic Review*, (46), 253–272.
- Hellwig, C. (2002): "Public Announcements, Adjustment Delays and the Business Cycle," unpublished manuscript, UCLA.
- \_\_\_\_\_\_, (2005): "Heterogeneous Information and the Benefits of Transparency," unpublished manuscript, UCLA.
- Lucas, R. E. (1972): "Expectations and the Neutrality of Money," *Journal of Economic Theory*, 4(2), 103–124.
- \_\_\_\_\_\_, (1973): "Some International Evidence on Output-Inflation Tradeoffs," *American Economic Review*, 63(3), 326–334.
- Morris, S., and H. S. Shin (2002): "Social Value of Public Information," *American Economic Review*, 92(5), 1521–1534.
- Phelps, E. S. (1970): *Microeconomic Foundations of Employment and Inflation Theory*. Norton, New York.
- \_\_\_\_\_\_, (1983): "The Trouble with 'Rational Expectations' and the Problem of Inflation Stabilization," in *Individual Forecasting and Aggregate Outcomes*, ed. by R. Frydman, and E. S. Phelps, pp. 31–41. Cambridge University Press, New York.
- Sims, C. A. (2003): "Implications of Rational Inattention," *Journal of Monetary Economics*, 50(3), 665–690.

- Svensson, L. E. (2003): "Comment on 'Public and Private Information in Monetary Policy Models' by J.D. Amato and H.S. Shin," presented at the conference 'Monetary Stability, Financial Stability and the Business Cycle,' Bank for International Settlements, Basel, 28-29th. March.
- \_\_\_\_\_\_, (2006): "Social Value of Public Information: Morris and Shin (2002) Is Actually Pro-transparency, Not Con," *American Economic Review*, 96(1), 448–452.
- Ui, T. (2003): "A Note on the Lucas Model: Iterated Expectations and the Neutrality of Money," unpublished manuscript, Yokohama National University.
- Woodford, M. (2003a): "Imperfect Common Knowledge and the Effects of Monetary Policy," in *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, ed. by P. Aghion, R. Frydman, J. Stiglitz, and M. Woodford, pp. 25–58. Princeton University Press, Princeton.
- \_\_\_\_\_\_, (2003b): *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press, Princeton.