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Precautionary Reserves: An Application to Bolivia

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Precautionary Reserves: An Application to Bolivia

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Abstract

Using precautionary savings models we compute levels of optimal reserves for Bolivia. Because of Bolivia's reliance on commodity exports and little integration with capital markets, we focus on current account shocks as the key balance of payments risk. These models generate an optimal level of net foreign assets ranging from 29 to 37 percent of GDP. For comparison purposes, we contrasted these results with standard rule of thumb measures of reserve adequacy, which in the case of Bolivia resulted in substantially lower levels of adequate reserves. These differing results emphasize the need to appropriately account for country-specific risks in order to derive adequate measures of reserve buffers.

JEL Classification Numbers: F31, F32

Keywords: Reserve adequacy, Optimal Reserves, Precautionary Motive, Balance of Payments Crises.

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I. Introduction

Over the last decade, many emerging and developing countries around the world exhibited an important improvement in their net foreign asset position (Lane and Milesi-Ferretti, 2007). Such trend renewed the attention of policy and academic circles who proposed two key explanations for the increase in foreign reserves: one hypothesis argues that the reserve accumulation was the direct result of export-led growth strategies (Dooley, Folkerts-Landau and Garber, 2003), and the other that reserves are used as a self-insurance mechanism against risks of balance of payment crises (Jeanne and Ranciere, 2006 and others).

The self-insurance hypothesis has motivated a wide arrange of studies, focusing on the precautionary demand for assets that arises from the presence of a sudden-stop risk, and along this line of research we find Kim (2008), Durdu, Mendoza, and Terrones (2009), Aisenman and Lee (2007), Alfaro and Kanczuk (2009), Caballero and Panageas (2008), Jeanne and Ranciere (2006) and others. Other studies have focused on risks stemming from the current account such as Dhasmana and Drummond (2008) and Barnichon (2008). The key argument of this hypothesis relies on precautionary savings theory, which corresponds to the amount of extra savings that households decide to accumulate as a response to the knowledge that the future is uncertain (Carroll and Kimball, 2007).

From a policy perspective, deriving benchmarks of reserve adequacy is relevant because they provide a guide for measures such as size and frequency of foreign exchange interventions. Inspired by the growing literature on reserve adequacy, this paper derives estimates of optimal levels of net foreign assets for Bolivia. Over the last five years, Bolivia accumulated unprecedented levels of foreign reserves resulting in a sharp improvement of its international investment position to become a net external creditor in 2008. The main contributor to this development was the also record-high current account surplus that followed from high export prices over 2005–2008.[2] As the cycle turns around and the significant current account surplus declines, obtaining estimates of an optimal level of reserves becomes highly relevant for economic policy design.

Motivated by the particular case of Bolivia, we proceed to calibrate a standard precautionary savings model with borrowing constraints. In this model, we concentrate on current account shocks as the key balance of payment risk. We choose this focus because Bolivia, being a commodity exporter with little reliance on foreign capital inflows, is more exposed to fluctuations in export revenues than to sudden interruptions of capital inflows. However, the inability to tap international capital markets that arises in a sudden stop is embedded in the model by imposing borrowing constraints. We additionally solve a version of the model were

[2] Bolivia also benefited during this time from debt relief, which also contributed substantially to the improvement in the country's net foreign asset position.

agents choose also investment levels. This feature offers additional insights because agents would not only save to shield consumption, but also to finance investment.

While Bolivia's reserves seem to far exceed the thresholds considered as adequate under traditional rules of thumb, such as the ratio to imports and to short-term external debt, the excess is not as large when compared to our model-based measures. Simulations of the models presented in this paper focusing on current account shocks suggest an optimal level of net foreign assets for Bolivia between 29 to 37 percent of GDP, although these results could vary substantially depending on the chosen calibration. However, even under fairly conservative calibrations, Bolivia would be around the optimal. Strictly speaking, the models used in this paper give us estimates of the net foreign asset position for the economy as a whole. However, when compared to the data, we use the foreign reserves of the financial system (including the central bank), because it is the most readily available measure. While these calculations imply a sizable amount of excess reserves, it is worth noticing that there may be other roles reserves play that are not examined here, for instance, as a signaling device for private sector investors. Setting aside other potential reserve accumulation objectives, these results suggest that the recent surge in reserves in Bolivia is consistent with a precautionary motive aiming at shielding the economy from future current account shocks. At a more general level, the distinct results obtained from the different models emphasize the need to properly account for country-specific balance of payments risks when deriving measures of reserve adequacy.

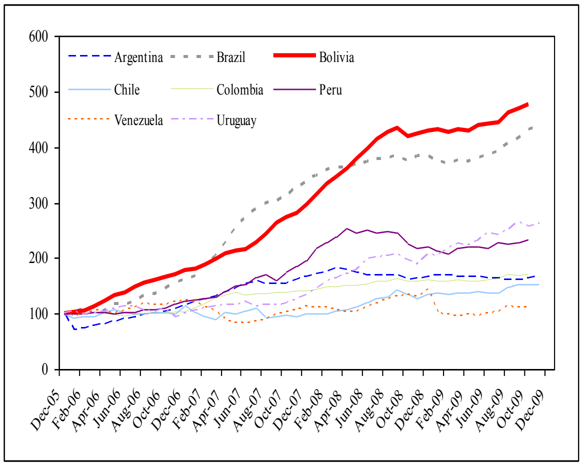
This paper is organized as follows; the next section discusses some key developments in Bolivia's external sector. Section II presents the model. Section III shows the numerical solution. Section IV presents the extension with investment. Section V performs some sensitivity analysis. Section VI presents standard rule of thumb reserve adequacy measures, and Section VII concludes.

II. Bolivia: External Sector Developments

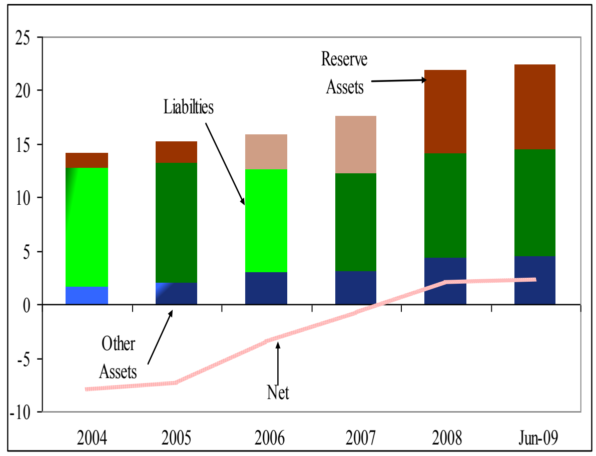
Over the last five years, a number of countries around the world experienced a sharp increase in the stock of foreign reserves held at central banks. In Latin America, the rise was particularly accentuated for Bolivia, Brazil, and Peru (Figure 1). In the case of Bolivia, the increase in its foreign asset position was significant enough that it became a net external creditor in 2008**.** The accumulation of foreign assets stemmed primarily from a major terms of trade improvement that followed from the sharp increase in Bolivia's key export commodity prices during the period 2004–2008, an important expansion in natural gas export volumes, and migrants remittances received mainly from Spain, USA, and Argentina, which increased fivefold between 2003 and 2008 to about 6 percent of GDP.

Figure 1. Bolivia: Selected External Sector Variables.

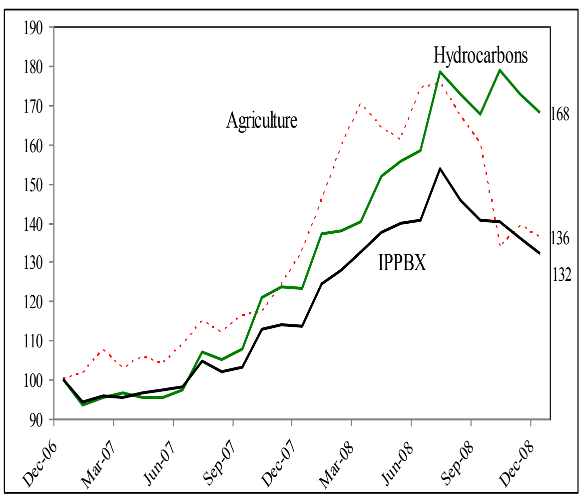
Central Bank Foreign Reserves,
December 2005=100



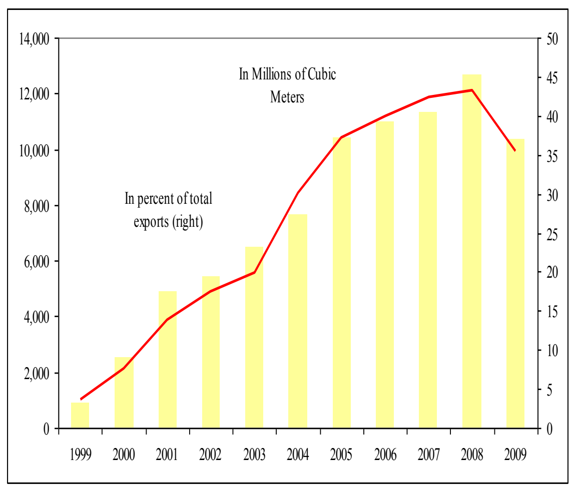
Bolivia's Net External Position, US$ billions



Export Prices
December 2006=100



Bolivian Natural Gas Exports



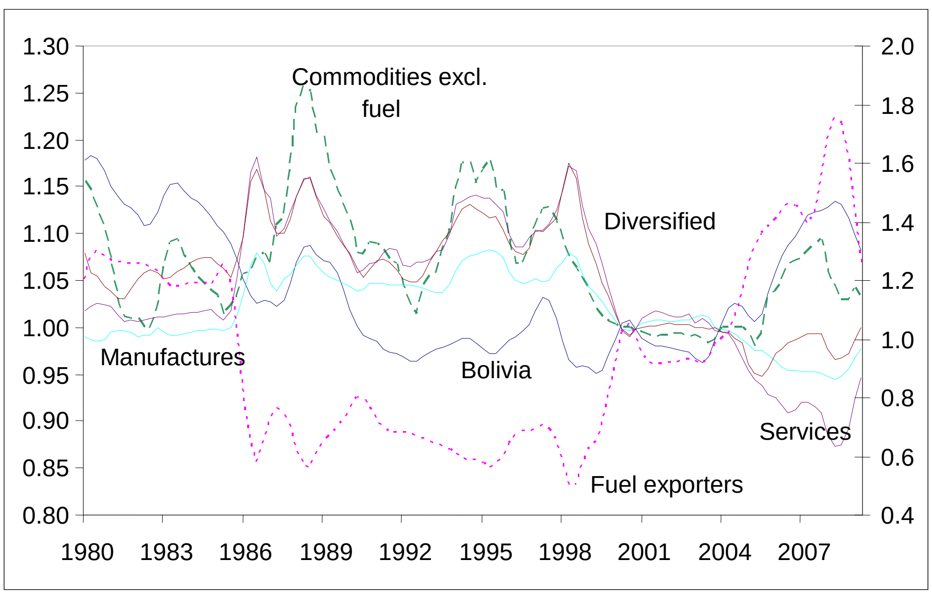
Source: Central Banks of Bolivia, and Central Banks of respective countries.
Note: IPPBX = Commodities Price Index Export.

For Bolivia, most of the adjustment to the external shock came through accumulation of reserves, given Bolivia's crawling peg exchange rate regime and the absence of an institutional arrangement to manage hydrocarbons wealth.[3]

The sharp increase in Bolivian export prices translated into a significant improvement in terms of trade (Figure 2). From looking at Figure 2, one can notice that when Bolivian terms of trade are compared against different groups of countries—according to World Economic Outlook's classification—their evolution resembles more the trajectory of energy exporters only recently, as natural gas exports gained importance.[4]

Characterizing the behavior of terms of trades for Bolivia is important for calibration purposes, given that we will have to make an assumption regarding how the distribution of future shocks will look like. The discussion so far suggests that an appropriate assumption is that it will resemble that of energy exporters.

Figure 2. Terms of Trade



Source: World Economic Outlook.

[3] During the period 2005–2007 the public sector regained controlling ownership over the hydrocarbons sector and became the recipient of large natural gas export receipts, resulting in substantial fiscal savings. However, due to the lack of a dedicated institutional arrangement to manage this wealth, these resources were simply deposited at the central bank. See Cerutti and Mansilla (2006) for a review of the evolution of the hydrocarbons sector in Bolivia.

[4] Bolivian gas export prices are linked by contract to the evolution of fuel oil prices and are adjusted quarterly. Therefore, as natural gas exports gained importance, Bolivia's terms of trade behavior became closer to the evolution of terms of trade series of energy exporters.

III. The Model

We begin by laying out the key underlying assumptions on consumer's behavior. The model is a standard precautionary savings model, developed in partial equilibrium in the sense that the consumer faces an exogenously given interest rate and income process (a variant of Carroll, 2004). Households are assumed to consume only tradable goods and make consumption decisions as to maximize the expected present discounted value of the utility derived from consumption,

$$Max_{\{C_t\}_0^\infty} E_t \sum_{t}^{\infty} \beta^t u(C_t) \tag{1}$$

where β denotes the discount factor, *u(.)* the utility function, assumed to be of the constant relative risk aversion type, and *E* the expectations operator. The exclusion of non-tradable goods would likely yield higher estimates of the optimal level of net foreign assets because in reality, they also provide utility to consumers. Moreover, income may also have a component that is not directly related to exports. However, any resulting difference between this model and a version including non-tradables would be highly sensitive to the elasticity of substitution between the two types of goods. In a highly industrialized country, one could expect this difference to be larger because consumers have a wide array of domestically-produced goods that can substitute for foreign goods, at the same time that a fraction of income is not directly affected by the external shock. However, in an economy with less diversified sources of income together with fewer options for substitution—as one would expect to be the case in an export commodity-oriented economy—this may not be the case.

The sequence of events is as follows: at the beginning of the period, consumers have net foreign assets X_t , conditional on which they make consumption decisions (C_t), in the middle of the period with the remainder X_t - C_t invested in a risk-free security that yields *R*. At the end of the period—after decisions have been made—income is realized, which determines with how much assets the consumer arrives in period *t+1*. This sequence of events translates into the following law of motion for net foreign assets

$$X_{t+1} = R(X_t - C_t) + \tau_{t+1} Y_{t+1} + A_t \tag{2}$$

Where *Y* reflects the level of permanent income, τ denotes transitory shocks to income, assumed to be mean-one, i.i.d., and distributed over a non-negative support, and finally *A* denotes all other non-export net current receipts. In order to capture the limited ability to tap international credit markets, we introduce a borrowing constraint that requires consumption to be as high as the current level of net foreign assets.[5]

$$C_t \leq X_t \qquad (3)$$

One simplifying step that can be taken at this point is to assume that $Y_{t+1}=Y_t$. With this assumption we rule out permanent shocks to income and focus only on transitory fluctuations, the only source of income variation. We normalize the problem by the level of permanent income and obtain the following optimization problem, expressed in Bellman's equation form[6]

$$V(x_t) = Max_{\{C_t\}_0^\infty} \{u(c_t) + \beta E_t V(x_{t+1})\} \qquad (4)$$

Subject to

$$x_{t+1} = R(x_t - c_t) + \tau_{t+1} + a \qquad (5)$$

$$c_t \leq x_t \qquad (6)$$

The problem's first order condition is given by

$$u'(c_t) = R\beta E_t V'(x_{t+1}) \qquad (7)$$

which sets the marginal utility of consumption equal to the present discounted value of the marginal utility of savings, or of holding higher net foreign assets—denoted by $V'(x_{t+1})$. Using the envelope theorem, we can derive the standard Euler equation for consumption, which relates the marginal utility of current consumption with the marginal utility of future consumption.

$$u'(c_t) = R\beta E_t u'(c_{t+1}) \qquad (8)$$

Consumption is chosen as to equate the marginal utility of current consumption with the expected value of the discounted marginal utility of future consumption. A necessary condition for the model to have a well-defined solution is to impose a degree of impatience

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[5] It is worth mentioning that borrowing constraints are not a necessary condition to generate a precautionary motive. Provided there is a large enough probability that a really bad event happens (i.e. zero income, or business failure), consumers would optimally choose not to borrow (Carroll, 2004 and Sandri, 2009).

[6] Small letters therefore denote the normalized version of capital letters, that is $x=X/Y$. We also make the assumption that the ratio $A/Y=a$ is time-invariant. The derivation of the normalized problem is trivial, although the interested reader can find a more stylized version in Carroll (2004).

on consumers (Carroll 2004).[7] Impatience will induce consumers to favor current consumption against future consumption and hence rules out consumer's wealth growing perpetually.

While prices play an important role in estimating balance of payment risks, volumes also matter. Therefore, we introduce an additional assumption, that τ--the transitory shocks to income—includes two i.i.d. components, that is $\tau = \varsigma\gamma$. These two independent components in turn represent transitory shocks to export volumes, γ, and terms of trade shocks, ς.[8] We make the potentially restrictive assumption that they are independent, when in reality they may be correlated due to fluctuations in demand and supply and the corresponding movement of the equilibrium price. However, we choose to model them as independent variables for simplicity.

A. The Optimal Level of Net Foreign Assets

Before we proceed to the numerical solution of the model, we make some simplifying assumptions, only for illustrative purposes[9] with the goal of gaining further insights on the relation between the optimal level of reserves and uncertainty. The model we have laid out has two fundamental ingredients: prudence,[10] which induces the consumer to save in order to minimize the impact of future shocks on consumption, and impatience, which induces the consumer to spend today instead of tomorrow. The optimal level of reserves is the value of assets such that these two forces are exactly balanced. In order to obtain an analytical expression we assume that $\varsigma_{t+1}=1$ for all t, that is, there are no terms of trade shocks. Therefore, the only remaining source of uncertainty in this simplified version is volume shocks. Second, we assume that γ can only take two values, 1 or 0, with probability $1-\Theta$ and Θ respectively. This last assumption could be interpreted as a sudden depletion of resources to export, so, once the economy faces a bad realization of the shock, it no longer has goods to export and would live thereafter of the resources accumulated up to that point. The solution for the economy that has exhausted its resources is given by the standard perfect foresight solution to the above consumption problem, given by[11],[12]

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[7] Impatience in this case requires that $1/\beta>R$.

[8] The correlation between real growth in Bolivian exports with World's real imports growth is 0.41 for the period 1980-2008.

[9] The discussion follows Carroll (2009) closely.

[10] Prudence is a direct implication of the convexity of marginal utility of consumption, which in this case is guaranteed by the assumption of a constant relative risk aversion utility function.

[11] This is a standard result in consumption theory, therefore, derivations are omitted but essentially it is the result of equating the present discounted value of consumption with that of income, using the fact that equation (8) tells us the rate of growth of consumption over time. Notice that the derivation is here simplified by the fact

(continued…)

$$c_t^0 = \underbrace{(1 - R^{-1}(R\beta)^{1/\rho})}_{k} x_t \tag{9}$$

this implies that the optimal solution is to consume a constant fraction of wealth in every period. Turning to the case of an economy that has not exhausted its resources, the optimal consumption decision is determined by equation (8), which with the simplifying assumptions made here becomes

$$\begin{aligned} 1 &= R\beta E_t\left(c_{t+1}/c_t\right)^{-\rho} \\ 1 &= R\beta\left[(1-\theta)\left(c_{t+1}^1/c_t^1\right)^{-\rho} + \theta\left(c_{t+1}^0/c_t^1\right)^{-\rho}\right] \\ \left(c_{t+1}^1/c_t^1\right)^{\rho} &= R\beta\left[(1-\theta) + \theta\left(c_{t+1}^1/c_{t+1}^0\right)^{\rho}\right] \end{aligned} \tag{10}$$

where the last equation indicates that in the absence of any risk—that is, if $\Theta=0$—consumption growth would simply equal the growth that would arise under perfect foresight $(R\beta)^{1/\rho}$. For $\Theta>0$, consumption growth becomes affected by the difference in the consumption level that would occur in absence of resources, c^0_{t+1}, and what consumption would be if resources are still available c^1_{t+1}. Because consumption is clearly higher in the case where resources are still available, the expression in brackets is a number strictly greater than one, which implies that consumption growth—for an economy that has still resources available—is boosted by the presence of uncertainty. Conditional on the present discounted value of consumption being equal to the present discounted value of income,[13] higher consumption growth implies a lower initial level of consumption, which in turn suggests that savings must be higher as the risk of depleting resources increases, resulting precisely in the precautionary motive that arises in this type of problems.

The precautionary motive is better appreciated by making a further simplifying assumption of setting the coefficient of relative risk aversion equal to 1, $\rho=1$. The result is a fairly simple formula for the optimal level of net foreign assets,[14] given by (11).

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that future income is 0 because resources have been exhausted. Therefore, we only need to equate the present value of consumption with current wealth.

[12] In order to distinguish the case of an economy with available resources and that in which they have been depleted, we include a superscript to the relevant variables that takes the value of 1 (resources available) and 0 (resources depleted).

[13] This is an implication of the assumption that the consumer cannot die in debt.

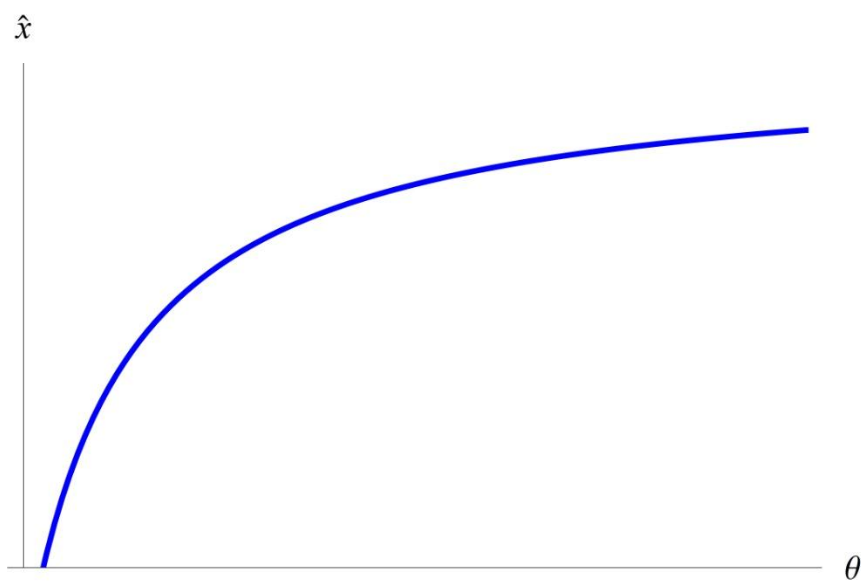
[14] Details about the derivation are provided in the appendix.

$$x = \frac{R}{R\left[R\left(\frac{(R\beta)^{-1}-1}{\theta}+1\right)(1-\beta)-1\right]+1}+1 \qquad (11)$$

Notice that an increase in Θ, which is our measure of risk in this simplified version, causes the denominator in (11) to shrink,[15] which in turn increases the optimal level of reserves. This argument is better appreciated in Figure 3, where we are plotting Equation (11) for a range of values for Θ[16]. There is a clear positive relationship between the optimal level of reserves and the risk of resources depletion, with the biggest marginal impact at initially low levels of risk.

Having established the theoretical grounds for the relationship between the demand for foreign assets and risk, one important remaining policy issue is that of who holds these reserves: the central bank or private consumers. However, this is not a critical issue, as long as central bank's objectives are in line with those of private consumers with regards to smoothing demand fluctuations. In that case, the central bank is providing insurance to private agents against income fluctuations.

Figure 3. Optimal Reserves and Risk



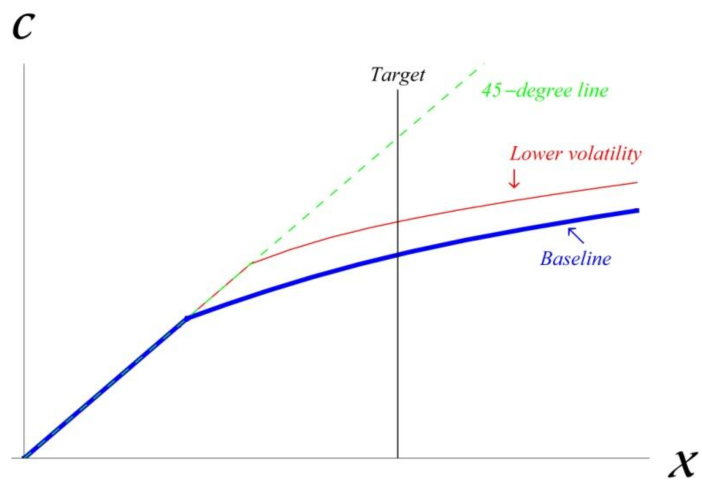
[15] Recall that the impatience assumption implies that (Rβ)<1, hence the expression inside the brackets is a positive number and strictly larger than 1. Moreover, *k<1*, which together with the impatience assumption imply that the whole denominator in (11) is positive.

[16] Values for β and R are shown in the calibration section.

IV. Numerical Solution

We now return to the baseline version of the model, with terms of trade and export volume shocks. The problem is a conventional dynamic programming problem and is solved by backwards induction using Carroll (2006)' endogenous gridpoints method. The calibration section in the appendix discusses the chosen values for the parameters of the model. The solution of the model boils down to finding a time-invariant consumption rule as a function of the level of current assets *c(x)* that determines consumers' decisions. Such a rule and the corresponding optimal level of reserves are depicted in Figure 4.

Figure 4. Consumption Rule



The thick solid line depicts the baseline optimal consumption rule as a function of the current level of net foreign assets. The baseline scenario corresponds to the case where the parameters of the distribution of terms of trade shocks are calibrated to match the volatility of the terms of trade series for energy exporters. For comparison purposes, we also solved the model calibrating the distribution of terms of trade shocks as to match Bolivia's historical terms of trade volatility (thin solid line), which turn out to be less volatile than the terms for trade for energy exporters. In both cases, the variance of export volume shocks is set equal to the baseline calibration. The 45 degree line illustrates the instances where the borrowing constraint is binding. In those circumstances, consumption of tradable goods can at most equal the level of net foreign assets. The vertical line denotes the optimal level of foreign assets consumers hold under the baseline scenario, which lies to the right of the point at which the liquidity constraint begins to bind. The buffer stock of net foreign assets minimizes the likelihood of consumers falling in the constrained region.

For interpretation purposes, we express the resulting optimal level of reserves[17] as a ratio to the long-run consumption level to obtain an optimal reserves/imports level and then take it to the data. The resulting ratio corresponds to 1.02 and 1.20 for the lower volatility and baseline cases respectively. Because this ratio is measured against the long-run level of imports, we approximate the latter by applying a filter to the observed imports-to-GDP ratio[18] and then back out the level of reserves—expressed as percentage points of GDP. The corresponding optimal levels for the calibrations shown in Figure 4 yield 25 and 29 percent of GDP as of 2009, as illustrated in Figure 5. We compare this figure with the reported level of reserves by the consolidated financial system (central bank and financial institutions). We take this broad measure because as suggested earlier, the model provides an estimate of the economy-wide optimal level of net foreign assets. We also provide an additional measure excluding gold holdings and dollar-deposits from the public sector in the central bank. We use this additional measure of reserves because, first, it is unlikely that reserves would decline to zero, and therefore we assume that the amount of gold is the minimum level of reserves.[19] Second, in this additional measure we subtract the dollar deposits of the public sector in the Central Bank because it is likely that these resources would not be part of reserves if the government had a dedicated arrangement to manage hydrocarbons wealth.[20] The comparison yields that the actual level exceeds the suggested optimal—under the baseline scenario—by about 25 percentage points of GDP (15 percent under the alternative definition of reserves). It is worth emphasizing that the model-based optimal levels are not sensitive to the business cycle, so the time-variation observed in the graph is a consequence of our approach to measure the long-run level of imports.[21]

[17] The optimal level of reserves can be found by means of simulations until the level of assets stops changing.

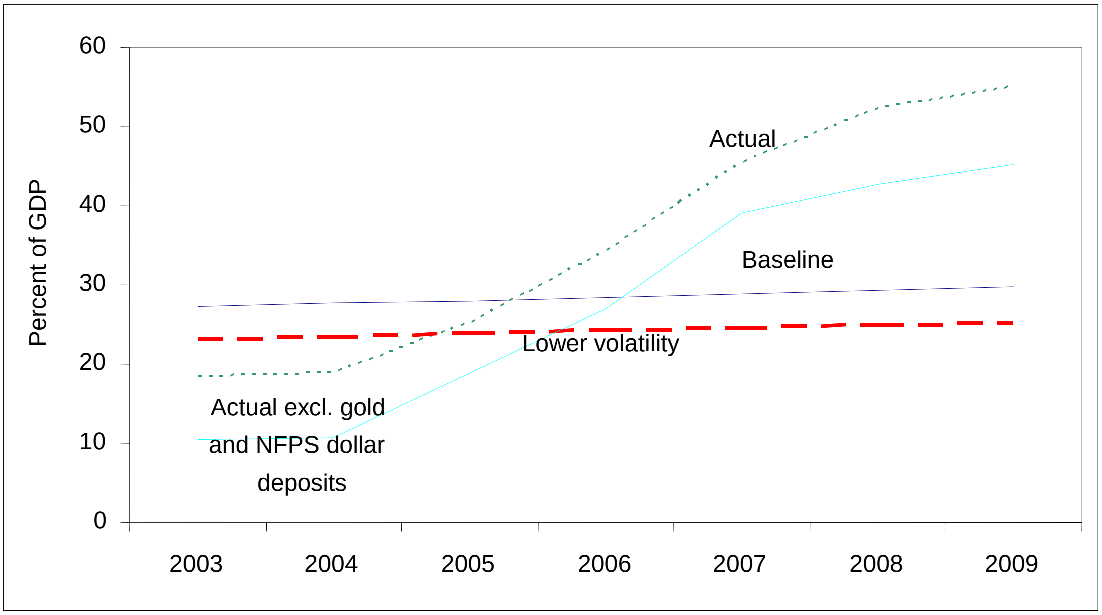
[18] The smoothing is implemented by using an HP-filter setting λ=1600 over the period 1980–2008.

[19] In fact, during the severe crisis of 19821985 the gold was not used.

[20] We deduct only the dollar-denominated deposits because the local currency component is assumed to be destined to be used in non-tradable goods.

[21] With a simple modification to the model, one could introduce the state of the economy as a state variable. To the extent that the volatility of terms of trade shocks and volume shocks vary with the business cycle, one could derive a state-dependent optimal level of reserves.

Figure 5. Optimal and Actual Financial System Net Foreign Assets



Source: Banco Central de Bolivia and author's calculations.

V. Adding Investment

In this section we extend the previous model to include investment decisions adopting a structure similar to Sandri (2009). Because now income is endogenous, consumers save not only for precautionary purposes, but also to finance the stock of capital. We can think now of an economy that is populated by workers, whose salaries are flexible and accommodate the shocks that hit their employers' revenues, and by entrepreneurs who own projects which produced export-related goods. Entrepreneurship is in fact important in Bolivia as shown in Zoltan and Szerb (2009), where Bolivia exhibited the highest entrepreneurship rate in the sample.[22] In this version of the model, investment takes the form of capital goods that are used in the production of export goods. The modified model can now be written as:

$$Max_{\{c_t,k_t\}_0^\infty} E_t \sum_t^\infty \beta^t u(c_t) \tag{12}$$

Subject to

$$x_{t+1} = R(x_t - c_t - k_t) + \tau_{t+1} k_t^\alpha + (1-\delta)k_t + a \tag{13}$$

$$c_t + k_t \leq x_t \tag{14}$$

[22] In the same study, an important obstacle that entrepreneurs find is the limited access to credit, which provides further support to our assumption of borrowing constraints.

Where the key difference with the previous version is that the budget constraint now includes an endogenous source of income and investment decisions. In Equation (13) $\tau_{t+1} k_t^{\alpha}$ denotes income from exports, which imposes decreasing marginal returns on capital.[23] At the end of production, a fraction δ of capital depreciates. As before, we also include a constant term "a" to account for net non-export current receipts. Notice that capital enters the budget constraint contemporaneously. This simplification avoids the need to keep track of the capital stock. As before, Bellman's equation for the optimization problem can be written as

$$V(x_t) = Max_{\{c_t, k_t\}_0^{\infty}} \{u(c_t) + \beta E_t V(x_{t+1})\} \tag{15}$$

Subject to

$$x_{t+1} = R(x_t - c_t - k_t) + \tau_{t+1} k_t^{\alpha} + (1-\delta) k_t + a \tag{16}$$

$$c_t + k_t \leq x_t \tag{17}$$

with first order conditions

$$F.O.C.(c): u'(c_t) = BRE_t V'(x_{t+1}) \tag{18}$$

$$F.O.C.(k): BRE_t V'(x_{t+1}) = BE_t V'(x_{t+1})(\alpha \tau_{t+1} k_t^{\alpha-1} + (1-\delta)) \tag{19}$$

Equation (18) is the same as in the earlier model. Equation (19) is the first order condition for investment, which equates the marginal value of net foreign assets holdings to the marginal value of investing in capital to produce exporting goods. After some algebra, we can derive the corresponding Euler equations.

$$u'(c_t) = \beta R E_t u'(c_{t+1}) \tag{20}$$

$$u'(c_t) = \beta E_t u'(c_{t+1})(\alpha \tau_{t+1} k_t^{\alpha-1} + (1-\delta)) \tag{21}$$

Equation (20) is the same as in the previous version of the model. Equation (21) relates the marginal utility of current consumption with the expected marginal utility of investing in the risky production project, with rate of return $\alpha \tau_{t+1} k_t^{\alpha-1} + (1-\delta)$.

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[23] We maintain our small letter notation because of the assumption of constant permanent income. Moreover, any fixed component in the production function is assumed to be absorbed by τ.

A. Solution

The model is solved with the same numerical solution method employed earlier and the corresponding optimal decision rules are shown in Figure 6. For comparison purposes we have also included in the figure the consumption rule for the problem without investment, solved under the baseline calibration. Notice how the consumption function ($c^e(x)$) lies below the one without investment, which follows from the fact that the consumer now finds profitable to cut consumption and finance the stock of capital, denoted by $k(x)$. Savings now fulfill the precautionary motive function and to finance the capital stock. Therefore, the ratio of optimal reserves to long-run consumption is now higher at 1.52 (before 1.20). As before, we back out the corresponding level in terms of GDP to obtain 37 percent, as shown in Figure 7. For illustrative purposes, we also show the baseline optimal level derived in the previous section. The analysis from both versions of the model yields an optimal level between 29 and 37 percent of GDP. Therefore, the larger the importance of entrepreneurs, the closer the optimal level would be to 37 percent.

Figure 6. Optimal Decision Rules with Investment

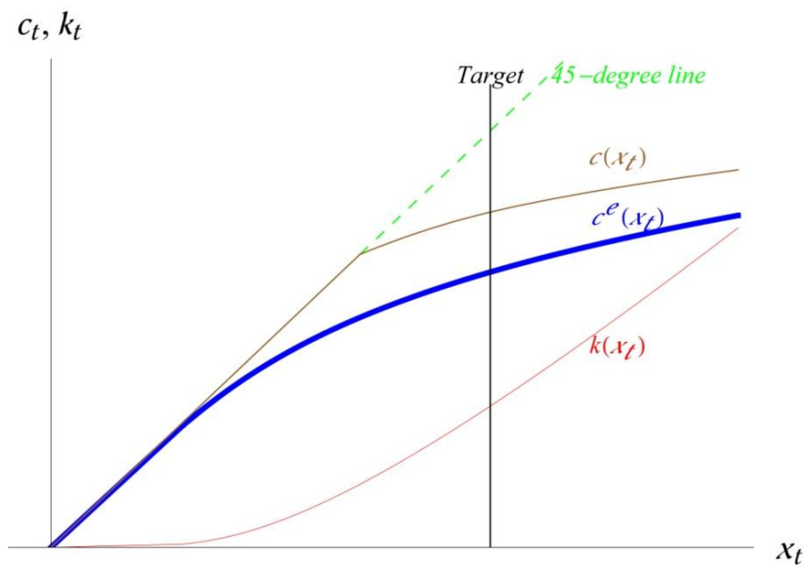
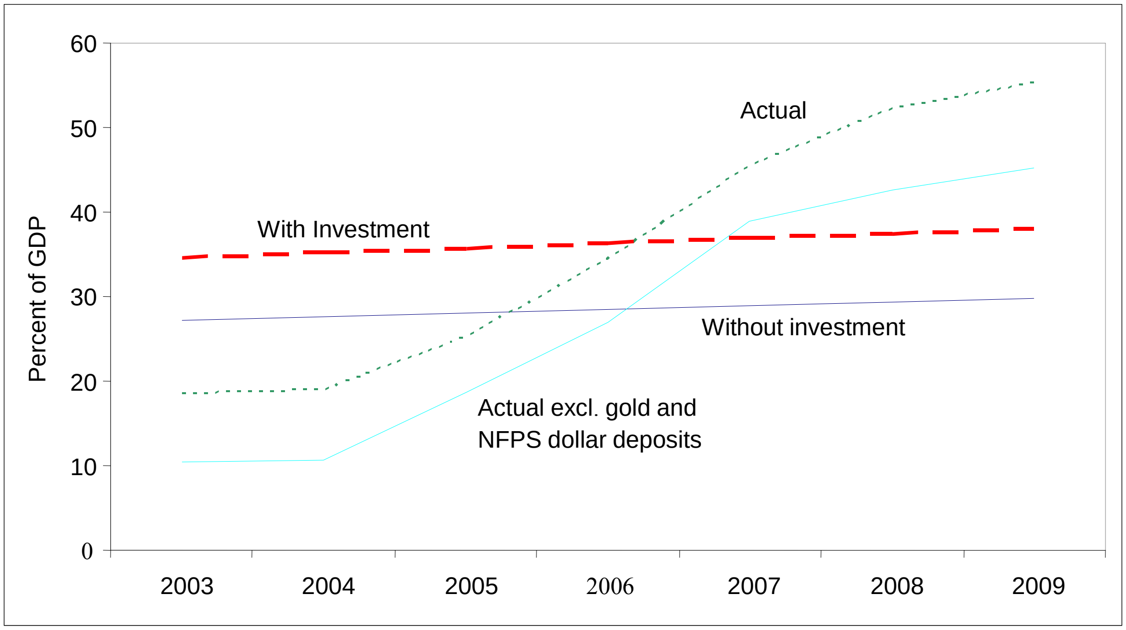


Figure 7. Optimal and Actual NFA with and without Investment



Source: Banco Central de Bolivia and author's calculations.

VI. Sensitivity Analysis

We perform now some sensitivity experiments to gauge how the optimal level of net foreign assets changes when parameters change. Some sensitivity analysis was already performed with the distribution of shocks in the previous sections; therefore, we will focus now on variations in the discount factor and degree of risk aversion, the two key parameters that determine the intensity of the two key ingredients of the model: prudence and impatience. Table 1 shows the resulting optimal level of net foreign assets under different values of these two parameters for both versions of the model presented in this paper.

Table 1. Sensitivity Analysis: Optimal Net Foreign Assets, In percent of GDP

		Without Investment			With Investment		
				B			
		0.92	**0.94 ***	0.96	0.92	**0.94 ***	0.96
	1	25.4	26.6	32.2	25.7	28.4	35.7
ρ =	**2***	27.9	**29.4**	37.9	32.5	**37.4**	50.6
	3	29.8	31.8	42.8	39.0	46.1	63.9

* Denotes baseline calibrations.

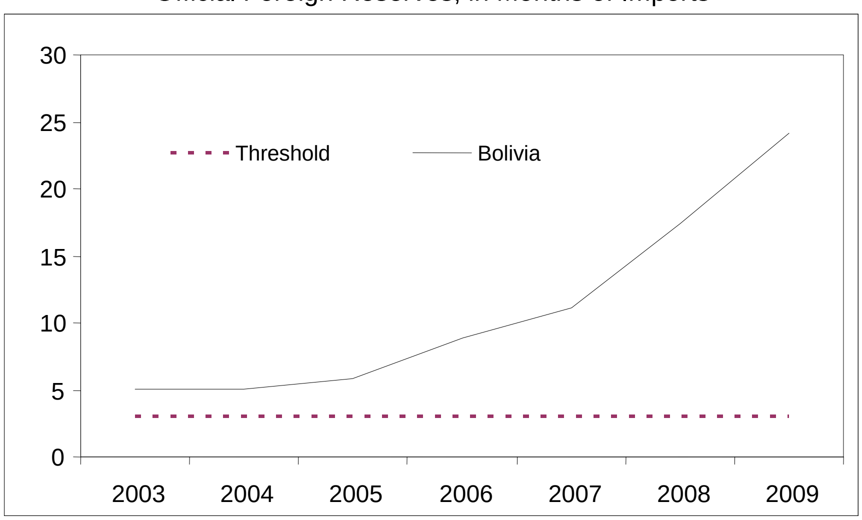
The sensitivity analysis yields substantial variation in the level of optimal net foreign assets as we move around the two key parameters. Notice how as we intensify risk aversion and reduce impatience, the excess amount of reserves virtually disappears, when compared to the current actual level of slightly below 60 percent. However, this implies that the opportunity cost of holding reserves would fall below 4 percent, which is unrealistic given the average interest rate paid on existing foreign debt or open market operations instruments. Therefore, if we were to interpret as the baseline calibration for impatience as the upper bound, and accept a higher degree of risk aversion, say 3, then according to the model with investment, Bolivia would be just at the optimum when we strip gold and dollar-deposits of the public sector in the central bank. This analysis would imply that even under fairly conservative calibrations, the model yields that Bolivia is around the optimum.

VII. Other Benchmarks

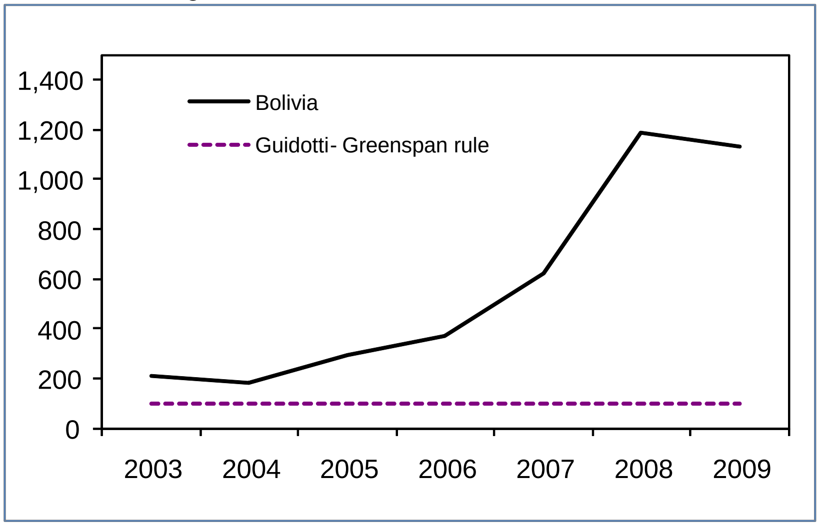
Finally, we appeal to popular rules of thumb measures of reserve adequacy. The first one is the relation of official reserves to imports, usually expressed in months of imports. Here, the conventional wisdom is that reserves should offer coverage for at least three months. Figure 8 shows the observed level of reserves and the corresponding threshold. Bolivia far exceeds the recommended coverage according to this traditional rule of thumb. Another similarly popular measure is the denominated Guidotti-Greenspan rule, which recommends a level of reserves that at least covers 100 percent of short-term foreign debt. Given the little integration of Bolivia with international capital markets, and the fact that most public debt is long-term, short-term foreign debt in Bolivia is very low, even under remaining maturity basis (plotted in Figure 8), resulting in a very large indicator of reserve adequacy under this definition, as shown in Figure 8.

Figure 8. Rule of Thumb Reserve Adequacy Indicators

Official Foreign Reserves, in Months of Imports



Official Foreign Reserves to Short-Term External Debt, Percent



Source: Central Bank of Bolivia.

VIII. Conclusions

Following a surge in commodity prices and important increases in export volumes, Bolivia's net foreign asset position improved markedly to become a net external creditor in 2008. Similar trends have been documented for other developing and emerging economies. A conventional wisdom to explain this increase in reserve asset holdings has been to appeal to precautionary savings theory. Under this view, reserve accumulation follows a precautionary motive to build buffers and shield domestic demand from balance of payments crises. Inspired by the growing literature on precautionary reserve holdings, we proceeded to calibrate a standard precautionary savings model with borrowing constraints with data on Bolivia. The model was then extended to include also investment decisions. Estimates of the optimal level of net foreign assets from both models resulted in the range 29 and 37 percent of GDP. Alternatively, we also contrasted these results with standard rule of thumb measures of reserve adequacy, which suggested a level of excess reserves substantially above what is suggested by our model-based calculations. While the results were shown to be sensitive to the calibration of parameters and to the definition of net foreign assets, even under conservative calibrations Bolivia would be close to the optimum. However, because these model-based measures give us measures of economy-wide optimal net foreign asset holdings, comparing them with foreign reserves of the financial system is a conservative approach since we are leaving out the non-financial private sector. These results suggest the need to properly account for country-specific balance of payment risks when deriving reserve adequacy indicators.

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Appendix

A. Derivation of the Optimal Level of Reserves

Recall that the solution for consumption when resources have been exhausted is given by

$$c_t^0 = \underbrace{(1 - R^{-1}(R\beta)^{1/\rho})}_{k} x_t$$

For the case where resources are still available, the Euler equation is given by

$$\begin{aligned}
u'(c_t) &= R\beta E_t u'(c_{t+1}) \\
c_t^{-\rho} &= R\beta E_t c_{t+1}{}^{-\rho} \\
1 &= R\beta E_t \left(\frac{c_{t+1}}{c_t}\right)^{-\rho} \\
1 &= R\beta \left[(1-\theta)\left(\frac{c_{t+1}^1}{c_t^1}\right)^{-\rho} + \theta\left(\frac{c_{t+1}^0}{c_t^1}\right)^{-\rho} \right] \\
1 &= R\beta \left(\frac{c_{t+1}^1}{c_t^1}\right)^{-\rho} \left[(1-\theta) + \theta\left(\frac{c_{t+1}^1}{c_t^1}\right)^{\rho}\left(\frac{c_{t+1}^0}{c_t^1}\right)^{-\rho} \right] \\
\left(\frac{c_{t+1}^1}{c_t^1}\right)^{\rho} &= R\beta \left[(1-\theta) + \theta\left(\frac{c_{t+1}^1}{c_{t+1}^0}\right)^{\rho} \right] \\
\frac{c_{t+1}^1}{c_t^1} &= (R\beta)^{1/\rho} \left[\theta\left\{\left(\frac{c_{t+1}^1}{c_{t+1}^0}\right)^{\rho} - 1\right\} + 1 \right]^{1/\rho} \\
\frac{c_{t+1}^1}{c_t^1} &= (R\beta)^{1/\rho} \left[\theta\left\{\left(\frac{c_{t+1}^1}{kx_{t+1}^0}\right)^{\rho} - 1\right\} + 1 \right]^{1/\rho}
\end{aligned}$$

The last step uses the solution for the case where resources have been exhausted. The next step is to find the consumption level that would prevail at the target level of reserves. This is found by setting $c_t^1 = c_{t+1}^1 = \hat{c}$ in the last equation above.

$$1 = (R\beta)^{1/\rho} \left[\theta\left\{\left(\frac{\hat{c}}{kR(x_t^1 - \hat{c})}\right)^{\rho} - 1\right\} + 1 \right]^{1/\rho}$$

$$\left(\frac{(R\beta)^{-1} - 1}{\theta} + 1\right) = \left(\frac{\hat{c}}{kR(x_t^1 - \hat{c})}\right)^{\rho}$$

In the above derivations we used the budget constraint $x_{t+1}^{0} = R(x_t^1 - c_t^1)$, which tells us the amount of net foreign assets available in t+1 for an economy that had resources to export until period t, but exhausted them then. The next step is to solve for $\hat{c}$.

$$\left(\frac{(R\beta)^{-1}-1}{\theta}+1\right)=\left(\frac{\hat{c}}{kR(x_t^1-\hat{c})}\right)^{\rho}$$

$$\left(\frac{(R\beta)^{-1}-1}{\theta}+1\right)^{1/\rho} kR(x_t^1-\hat{c})=\hat{c}$$

$$\left(\frac{(R\beta)^{-1}-1}{\theta}+1\right)^{1/\rho} kRx_t^1=\hat{c}+\hat{c}\left(\frac{(R\beta)^{-1}-1}{\theta}+1\right)^{1/\rho} kR$$

$$\frac{\left(\frac{(R\beta)^{-1}-1}{\theta}+1\right)^{1/\rho} kRx_t^1}{\left(\frac{(R\beta)^{-1}-1}{\theta}+1\right)^{1/\rho} kR+1}=\hat{c}$$

Now consider the budget constraint for an economy that still has resources, setting *Θ=1* $x_{t+1}^1 = R(x_t^1 - c_t^1) + 1$. At the optimal level of net foreign assets, $x_{t+1}^1 = x_t^1 = x$. We use this fact in the budget constraint to obtain

$$x = R(x-\hat{c})+1$$

$$(x-1)R^{-1} = (x-\hat{c})$$

$$R^{-1} - x(R^{-1}-1) = \hat{c}$$

Using this formula with the expression obtained before for $\hat{c}$ we can solve for x,

$$\frac{\left(\frac{(R\beta)^{-1}-1}{\theta}+1\right)^{1/\rho} kRx}{\left(\frac{(R\beta)^{-1}-1}{\theta}+1\right)^{1/\rho} kR+1}=R^{-1}-x(R^{-1}-1)$$

In order to simplify notation, we set $\Psi = \left(\frac{(R\beta)^{-1}-1}{\theta}+1\right)^{1/\rho} kR$ in the above expression and solve for the optimal level of reserves

$$\frac{\Psi x}{\Psi+1} = R^{-1} - x(R^{-1}-1)$$

$$\frac{\Psi x}{\Psi+1} R = 1 - x(1-R)$$

$$0 = 1 + x(R-1) - \frac{\Psi x}{\Psi+1} R$$

$$0 = 1 + x\left[(R-1) - \frac{\Psi}{\Psi+1} R\right]$$

$$1 = x\left[R\left(\frac{\Psi}{\Psi+1} - 1\right) + 1\right]$$

$$1 = x\left[R\left(\frac{\Psi - \Psi - 1}{\Psi+1}\right) + 1\right]$$

$$1 = x\left[\frac{-R + \Psi + 1}{\Psi+1}\right]$$

$$x = \left[\frac{\Psi+1}{-R+\Psi+1}\right]$$

$$x = \left[\frac{+R - R + \Psi + 1}{-R+\Psi+1}\right]$$

$$x = \frac{R}{-R+\Psi+1} + 1$$

With this simpler version on hand, we can substitute for the definition of ψ to obtain the formula for optimal reserves.

$$x = \frac{R}{-R + \left(\frac{(R\beta)^{-1}-1}{\theta} + 1\right)^{1/\rho} kR + 1} + 1$$

$$x = \frac{R}{-R + \left(\frac{(R\beta)^{-1}-1}{\theta} + 1\right)^{1/\rho} (1 - R^{-1}(R\beta)^{1/\rho})R + 1} + 1$$

$$x = \frac{R}{-R + \left(\frac{(R\beta)^{-1}-1}{\theta} + 1\right)^{1/\rho} (R - (R\beta)^{1/\rho}) + 1} + 1$$

In the last step we used the definition of *k*. We take a further simplifying assumption to arrive to a more compact expression, which is to set the coefficient of relative aversion *ρ=1*.

$$x=\frac{R}{-R+\left(\frac{(R\beta)^{-1}-1}{\theta}+1\right)(R-R\beta)+1}+1$$

$$x=\frac{R}{R\left[R\left(\frac{(R\beta)^{-1}-1}{\theta}+1\right)(1-\beta)-1\right]+1}+1$$

B. Calibration

We use the standard CRRA utility function $u(c_t)=\frac{c_t^{1-\rho}}{1-\rho}$, with $\rho = 2$. The real return on net foreign assets is set to 2 percent a year. The discount factor is chosen at 0.94 in order to guarantee the impatience condition. With this parameterization, a period in this model can be roughly interpreted as 1 year. The terms of trade and export volume shocks are assumed to be i.i.d., log-normally distributed with parameters chosen as to match the volatility of terms of trade shocks for energy exporting countries as classified by the WEO for the period 1970−2010. Alternatively, we also compute the parameters that would correspond to the volatility of Bolivia's terms of trade series over the same period. Similarly, for export volume shocks, we compute the volatility of World's import volumes growth over the period 1980-2010. We also ran a regression relating Bolivian export volumes growth to world's import growth and the slope of the regression was not statistically different from one; therefore, we assumed that shocks to World trade are transferred one for one to Bolivian export volumes. For the version of the model with investment, we choose the share of capital in the production function to be equal to 0.2 and the depreciation rate of capital equal to 10 percent a year. Finally, we compute the average of the net current receipts of non-export components of the current account over the period 1998–2008, which yields 11 percent of exports.