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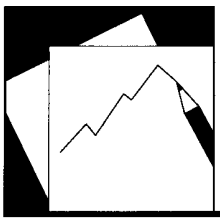

IMF Working Paper

Why Does Bad News Increase Volatility and Decrease Leverage?

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Research Department

Why Does Bad News Increase Volatility and Decrease Leverage?

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Abstract

The literature on leverage until now shows how an increase in volatility reduces leverage. However, in order to explain pro-cyclical leverage it assumes that bad news increases volatility. This paper suggests a reason why bad news is more often than not associated with higher future volatility. We show that, in a model with endogenous leverage and heterogeneous beliefs, agents have the incentive to invest mostly in technologies that become volatile in bad times. Together with the old literature this explains pro-cyclical leverage. The result also gives rationale to the pattern of volatility smiles observed in the stock options since 1987. Finally, the paper presents for the first time a dynamic model in which an asset is endogenously traded simultaneously at different margin requirements in equilibrium.

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I. Introduction

After the recent financial crisis there is almost universal agreement on two stylized facts:

1. Leverage is pro-cyclical, i.e., high during normal times and low during anxious or crisis times. Figures 1 and 2, taken from Geanakoplos (2010), show leverage and asset prices for the housing market and for AAA Securities. They both show that leverage is pro-cyclical: prices rise as leverage increases, and prices fall as leverage decreases. In particular, both leverage and prices collapsed during the recent financial crisis.

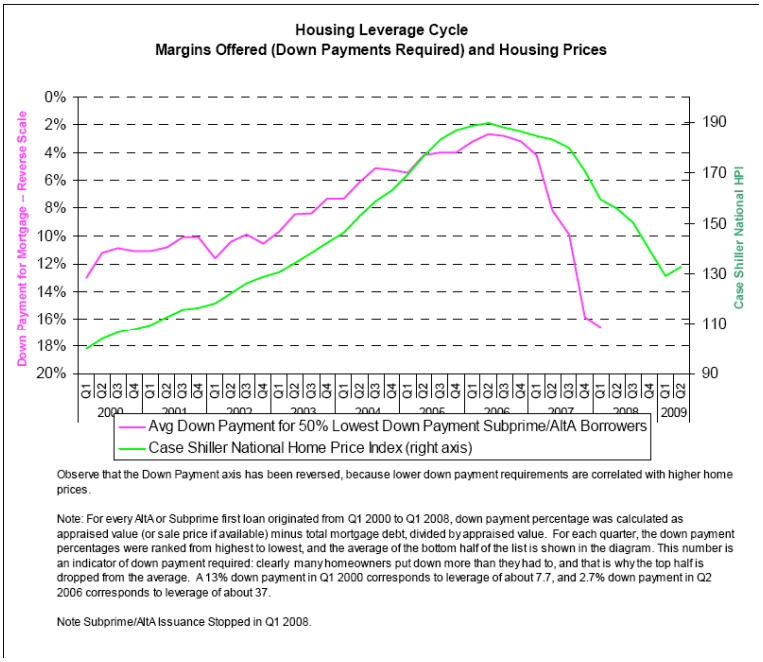

Figure 1: Pro-cyclical leverage: Housing.

2. Bad news, at least very bad news, is associated with very high volatility. Figure 3 shows the VIX index, the Chicago Board Options Exchange Volatility Index, a popular measure of the implied volatility of SP 500 index options. A high value corresponds to a more volatile market and therefore more costly options. Often referred to as the fear index, it represents one measure of the market's expectation of volatility over the next 30 day period. We clearly see that the index was very high during the recent financial crisis implying that bad news indeed came associated with high volatility.

   So, why does bad news increase volatility and decrease leverage? Recent literature has gone

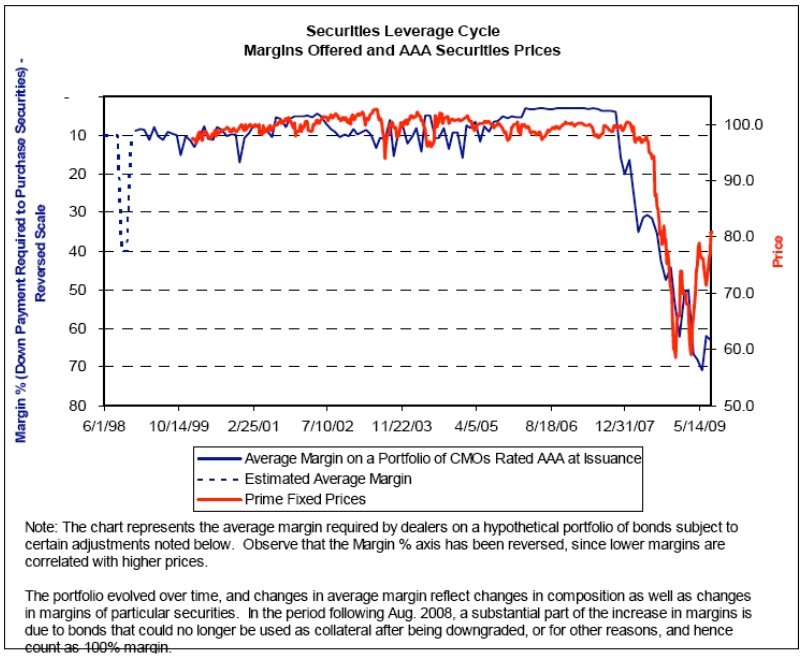

Figure 2: Pro-cyclical leverage: AAA Securities.

quite far in understanding the link between high volatility and low leverage.[2] However, all this work *assumes* that bad news is associated with higher volatility. This lack of explanation is problematic for two reasons. First, because the way information (and bad news) gets revealed in an economy should be endogenous. Second, because if we do not have a theory that explains why bad news induces high volatility we are only half way in explaining the pro-cyclical pattern of leverage observed in the data. The main goal of this paper is to shed light on this missing link and hence fully understand the relationship between news, volatility and leverage.

With this in mind we consider two types of projects (assets) with exactly the same payoff distribution in the last period. In the first project, bad news comes associated with an increase in future payoff volatility. We call this the “Post-Bad News Volatile project" (from now on BV). In the second project good news induces high future payoff volatility. We will call this the “Post-Good News Volatile project" (from now on GV).[3]

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[2]For example, Geanakoplos (1997, 2003, 2009) shows how supply and demand determine equilibrium leverage and why higher volatility reduces leverage. He suggested that big crises occur when bad news is of a particular kind he called “scary bad news", because it raises volatility (as well as decreasing expectations) and hence reduces leverage.

[3]Since these two projects are ultimately identical, in the BV project good news induces low volatility and in the GV project bad news induces low volatility.

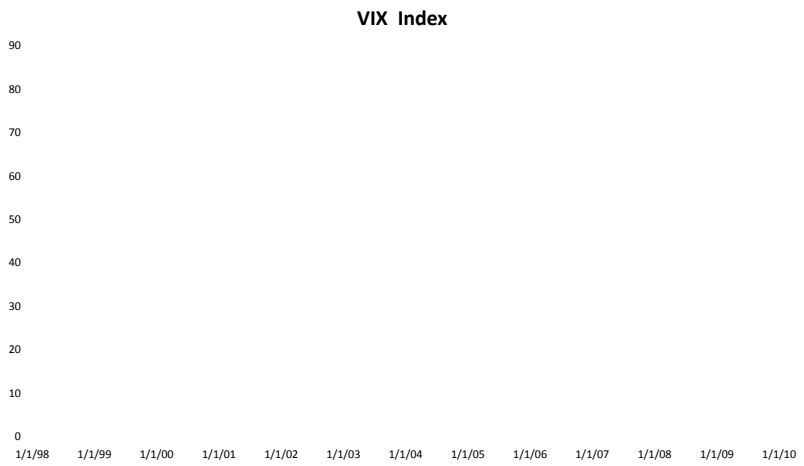

Figure 3: VIX index.

Three BV examples of bad news inducing higher volatility are: i) an airline announces that the plane is now expected to be 10 minutes late, which makes people worry it will be an hour late, ii) a bank announces it has lost $5 billion, which makes investors fear another $20 billion may follow, and iii) subprime delinquencies shoot up from 2% to 5%, which makes people worry they may go up to 30%. A GV example of good news inducing higher volatility might be that after a presidential candidate wins a crucial primary he may become president or be destroyed by a hitherto unknown scandal.

Notice that in the three BV examples each piece of bad news reveals only a little information about expected outcomes but creates a lot of uncertainty, while in the GV example it is the good news that raises expected outcomes a little but creates much more volatility.

In our model agents can use these projects (assets) as collateral to borrow money, and leverage is endogenous. Agents are presented with a menu of one-period non-contingent promises, each collateralized by one unit of asset (or project). Leverage becomes endogenous since in equilibrium not all promises are actively traded. Financial contracts are micro founded by a collateralized loan market. We suppose that agents differ only in their beliefs (heterogenous priors).

We first study prices and leverage of each project when it is the only asset in the economy. As shown in Geanakoplos (2003) and Fostel-Geanakoplos (2010), in this context leverage is

endogenously determined in equilibrium, and corresponds to the “Value at risk equal zero" rule. Agents can promise at most the worst case scenario in the future preventing default from occurring in equilibrium. We study both projects in a three period economy first, and then extend the results to longer horizons.

The main findings are: i) the initial price in the BV project is higher than in the GV project, ii) initial leverage is higher in the BV project than in the GV project and iii) leverage is pro-cyclical in the BV project and counter-cyclical in the GV project.

Why do the projects have different prices and leverage characteristics in equilibrium? First, BV is more valuable than GV at the beginning because it can be leveraged more. A higher borrowing capacity implies that all the assets in the economy can be afforded by fewer and extremely optimistic investors with the highest asset valuation. This naturally raises the project’s price. Second, the BV project can be leveraged more at time zero due to the type of bad news. Given the endogenous leverage rule, the maximum agents can promise is the worse case scenario in the immediate future: the price of the project after bad news. But in the BV project the price does not fall as much precisely because bad news is little informative. On the contrary, bad news in the GV project is very informative, lowering the promise in equilibrium. Finally, the cyclical properties derived in each project are a direct consequence of the difference in volatility after bad news between the projects.

Having understood all the properties of prices and leverage in each individual project we move on to answer the main question. If these projects were considered as part of the same economy, which project would agents choose: one in which volatility goes up after bad news (BV) or one in which volatility goes up after good news (GV)?

We consider an extended version of the previous three-period baseline economy in which both projects co-exist and agents own a technology that can transform labor into a portfolio of different projects. Unlike the previous case, “Value at Risk equal zero" is not the only contract traded in equilibrium. As shown in Fostel-Geanakoplos (2010), two non-contingent promises will be actively traded in equilibrium for each asset: a risk-less promise and a risky one that defaults in the worst state. Each contract has an associated leverage, and *asset leverage* is defined as the average leverage over all the traded contracts that use the asset as collateral. Two new things appear in this extended model (that were not in the baseline model with one asset) which are more in tune with what we observe in the real world. First, there is default in equilibrium and second the same asset is traded simultaneously at different margin requirements by different investors.

We show that all agents choose mainly the BV project. In fact, in the simulated equilibrium all agents choose to invest their labor in a portfolio with a 70% share of the BV project. Or equivalently, 70% of the economy invest in BV projects when given the opportunity to choose. Moreover, both projects present the same leverage characteristics as when considered separately, i.e. the BV project can be leveraged more than GV project and leverage is pro-cyclical in the BV project and counter-cyclical in the GV project. Of course,

the immediate implication of this finding is that, since we assume that both projects are independent, most of the times when we observe bad news we will observe high volatility and low leverage explaining both stylized facts above.

This result also suggests an explanation for the observed “Volatility Smile" in stock options. This refers to the fact that implied volatility has a negative relationship with the strike price, so volatility decreases as the strike price increases. Hence, bad news comes (or are assumed to come) with high volatility. This effect is even larger when considered on indexes as SP500. This pattern has existed for equities only after the stock market crash of 1987. This has led some economist like Bates (2000) and Rubinstein (1995) to explain volatilites smiles by “crashophobia". Traders were concerned about the possibility of another crash and they priced options accordingly. Our result provides a completely different explanation. Our agents are perfectly rational, they endogenously chose projects associated with volatile bad news since they can leverage more with them.

The paper is related to a literature on collateral and credit constraints as in Bernanke, Gertler and Gilchrist (1999), Caballero and Krishnamurthy (2001), Fostel and Geanakoplos (2008a), Holmstrom and Tirole (1997), Kiyotaki and Moore (1997) and Shleifer and Vishny (1992). More closely, our paper is related to a literature on leverage as in Araujo, Kubler and Schommer (2009), Acharya and Viswanathan (2009), Adrian and Shin (2009), Brunnermeier and Pedersen (2009), Cao (2010), Fostel and Geanakoplos (2008b and 2010), Geanakoplos (1997, 2003 and 2010), Gromb and Vayanos (2002) and Simsek (2010). It is also related to work that studies the asset price implications of leverage as Hindy (1994), Hindy and Huang (1995) and Garleanu and Pedersen (2009). Some of these papers focus on *investor-based leverage* as in Acharya and Viswanathan (2009), Adrian and Shin (2009) and Gromb and Vayanos (2002), and others as Brunnermeier and Pedersen (2009), Cao (2010), Fostel and Geanakoplos (2008b and 2010), Geanakoplos (1997, 2003 and 2009) and Simsek (2010) focus on *asset-based leverage*. Not all these models present a theory of endogenous leverage, most of them assume a “VAR=0" rule and study the cyclical properties of leverage as well as its asset pricing implications. In Acharya and Viswanathan (2009) and Adrian and Shin (2009) the endogeneity of leverage relies on asymmetric information and moral hazard problems between lenders and borrowers. In Araujo et. al (2009), Cao (2010), Geanakoplos (1997, 2003, 2009), Fostel-Geanakoplos (2008) and Simsek (2010) endogeneity does not rely on asymmetric information, rather financial contracts are micro founded by a collateralized loan market. However, while all of these papers related low leverage with high volatility, none of them explain or endogenize the type of bad news, but rather *assume* that bad news comes with an increase in volatility. Furthermore, our paper is the first model to solve fully for endogenous leverage in a dynamic economy with a continuum of agents and more than two successor states. Geanakoplos (1997) showed how to make leverage endogenous by defining a contract as an ordered pair (promise, collateral) and requiring that every contract be priced in equilibrium, even if it is not actively traded. In Geanakoplos (1997, 2003, 2009) and Fostel-Geanakoplos (2008) only one contract is traded. Araujo et.al (2009) gives a two period example of an asset which is used as collateral in two different actively traded contract.

The paper is organized as follows. Section 2 presents the general model of endogenous leverage. Section 3 characterizes the equilibrium properties of asset prices and leverage in each project considered as a separate economy. Section 4 considers the two projects as part of the same economy and studies the full equilibrium, which includes the project choice.

II. A General Equilibrium Model of Endogenous Leverage

A. Time and Uncertainty

1. The model is a finite-horizon general equilibrium model, with time $t = 0, \cdots, T$. Uncertainty is represented by a tree of date-events or states $s \in S$, including a root $s = 0$. Each state $s \neq 0$ has an immediate predecessor s^*, and each non-terminal node $s \in S \backslash S_T$ has a set $S(s)$ of immediate successors. Each successor $\tau \in S(s)$ is reached from s via a branch $\sigma \in B(s)$; we write $\tau = s\sigma$. We denote the time of s by the number of nodes $t(s)$ on the path from 0 to s^*.

B. Financial Contracts and Collateral

1. A financial contract (A, C) consists of both a promise, A, and collateral backing it, C. Collateral consists of durable goods, which will be called assets. The lender has the right to seize as much of the collateral as will make him whole once the loan comes due, but no more.

   Suppose there is a single storable consumption good c and $k = 1, ..., K$ assets which pay dividends d_s^k in each state s. We take the consumption good as numeraire and denote the price of asset k in each state as p_s^k. We will focus on one-period non-contingent contracts. Contract j_s^k is of the form $(j \cdot \widetilde{1}_s, 1_k)$, where $\widetilde{1}_s \in R^{S(s)}$ stands for the vector of ones with dimension equal the number of successors of s and 1_k stands for one unit of asset k. Hence, contract j_s^k promises j units of consumption good in each successor state of s and the promise is backed by one unit of asset k. Contract $j_s^k \in J_s^k$ where J_s^k is the set of all contracts at state s that use as collateral one unit of asset k. Finally, $J_s = \bigcup_k J_s^k$ and $J = \bigcup_{s \in S \backslash S_T} J_s$.

   The price of contract j_s^k in state s is π_s^{jk}. An investor can borrow π_s^{jk} today by selling contract j_s^k in exchange for a promise of j tomorrow. Since the maximum a borrower can lose is his collateral if he does not honor his promise, the actual delivery of contract j_s^k in states $\tau \in S(s)$ is $min\{j, p_\tau^k + d_\tau^k\}$. If the collateral is big enough to avoid default, the price of contract j_s^k is given by $\pi_s^{jk} = j/(1 + r_s)$, where r_s is the riskless interest rate (and hence does not depend on the asset used as collateral).

   The margin requirement m_s^{jk} associated to contract j_s^k in state s is given by

$$m_s^{jk} = \frac{p_s^k - \pi_s^{jk}}{p_s^k} \tag{1}$$

Leverage associated to contract j_s^k in state s is the inverse of the margin, $1/m_s^{jk}$ and the *Loan to Value* (LTV) associated to contract j_s^k in state s is $1 - m_s^{jk}$.

We define the *asset margin requirement* for asset k, m_s^k, as the trade-value weighted average of m_s^{jk} across all contracts actively traded in equilibrium that used asset k as collateral.[4]

C. Production

1. Each investor h has an endowment of the consumption good and labor, denoted by $e_s^h \in R_+$ and $l_s^h \in R_+$ in each state $s \in S$. We assume that the consumption good and labor are present at time 0, $\sum_{h\in H} e_0^h > 0, \sum_{h\in H} l_0^h > 0$.

   Every agent has direct access to two types of constant-returns-to-scale production processes in the model: an inter-period and a within-period production. The inter-period production is a simple way to model consumption good durability in the economy. A unit of consumption warehoused in state s yields one unit of consumption in all successors states. There is no depreciation.

   The second type of production, the within-period production, transforms labor, l, into a portfolio of assets to be chosen by the investor in the set
   $Z_s^h = \{(z_s^1, ..., z_s^K) \in R_+^K : z_s^1 + ... + z_s^K \leq l_s^h\}$. Any investor can use his l_s^h units of labor to produce any combination of assets.

D. Utility

1. The von-Neumann-Morgenstern expected utility of each investor $h \in H$ is characterized by a Bernoully utility, u^h, a discounting factor, δ^h and subjective probabilities, q^h. We assume that the Bernoulli utility function for consumption in each state $s \in S$, $u^h : R_+ \to R$, is differentiable, concave, and monotonic. Agent h assigns subjective probability q_s^h to the transition from s^* to s; naturally $q_0 = 1$. Letting $\bar{q}_s^h$ be the product of all $q_{s'}^h$ along the path from 0 to s, we have

$$U^h = \sum_{s\in S} \bar{q}_s^h (\delta^h)^{t(s)} u^h(c_s) \tag{2}$$

E. Budget Set

1. Given asset and contract prices $((p_s^k, \pi_s^{jk}), s \in S, j_s^k \in J_s^k)$, each agent $h \in H$ decides what assets to produce, z_s, consumption, c_s, warehousing, w_s, asset holdings, y_s, and contract

[4] For a detailed description see Fostel-Geanakoplos (2010)

sales (borrowing) and purchases (lending), $\varphi_{j_s^k}$, in order to maximize utility (2) subject to the budget set defined by

$B^h(p,\pi) = \{(z,c,w,y,\varphi) \in R_+^{SK} \times R_+^S \times R_+^S \times R_+^{SK} \times (R^{J_s})_{s\in S\setminus S_T} : \forall s$
$(c_s + w_s - e_s^h - w_{s*}) + \sum_k p_s^k(y_s^k - y_{s^*}^k - z_s^k) \leq$
$\sum_k y_{s^*}^k d_s^k + \sum_{j_s^k \in J_s} \varphi_{j_s^k} \pi_s^{jk} - \sum_{j_{s*}^k \in J_{s*}} \varphi_{j_{s*}^k} min(p_s^k + d_s^k, j);$
$z_s \in Z_s^h;$
$\sum_{j_s^k \in J_s} max(0, \varphi_{j_s^k}) \leq y_s^k, \forall k\}$

In each state s, expenditures on consumption and warehousing minus endowments and storage, plus total expenditures on assets minus asset holdings carried over from the last period and asset output from the within-period technology, can be at most equal to total asset deliveries plus the money borrowed selling contracts, minus the payments due at s from contracts sold in the previous period.[5] Within-period production is feasible. Finally, those agents who borrow must hold the required collateral.

Let us emphasize two important things. First, notice that there is no sign constraint on $\varphi_{j_s^k}$: a positive (negative) $\varphi_{j_s^k}$ indicates the agent is selling (buying) contracts or borrowing (lending) π_s^{jk}. Second, notice that we are assuming that short selling of assets is not possible. This assumption, however, is not crucial for the results in the paper as we discuss in Section 3.8.

F. Collateral Equilibrium

1. A *Collateral Equilibrium* in this economy is a set of asset prices and contract prices, production and consumption decisions, and financial decisions on assets and contract holdings $((p,\pi),(z^h,c^h,w^h,y^h,\varphi^h)_{h\in H}) \in$
$(R_+^K \times R_+^{J_s})_{s\in S\setminus S_T} \times (R_+^{SK} \times R_+^S \times R_+^S \times R_+^{SK} \times (R^{J_s})_{s\in S\setminus S_T})^H$ such that $\forall s$

   (a) $\sum_{h\in H}(c_s^h + w_s^h - e_s^h - w_{s*}^h) = \sum_{h\in H} y_{s^*}^h d_s$

   (b) $\sum_{h\in H}(y_s^h - y_{s^*}^h - z_s^h) = 0$

   (c) $\sum_{h\in H} \varphi_{j_s^k}^h = 0, \forall j_s^k \in J_s$

   (d) $(z^h,c^h,w^h,y^h,\varphi^h) \in B^h(p,\pi), \forall h$
   $(z,c,w,y,\varphi) \in B^h(p,\pi) \Rightarrow U^h(c) \leq U^h(c^h), \forall h$

Markets for consumption, assets and promises clear in equilibrium and agents optimize their utility in their budget set. As shown by Geanakoplos and Zame (1997), equilibrium in this model always exists under the assumptions we have made so far.

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[5] We take $y_{0*}^h = 0$.

III. News and Leverage

A. A One-Asset Baseline Example

1. In this section we assume that there is only one asset. Throughout the paper we consider assets and projects as synonyms.

   Suppose there are three periods, $t = 0, 1, 2$. The single asset, Y, delivers only at the final period. We assume that state 0 has two successors U, for up, and D, for down, representing good and bad news respectively. Each of these states $s \in \{U, D\}$ has at most two successors sU and/or sD, at which the asset pays 1 or $R < 1$, respectively.[6] Figure 4 depicts a tree consistent with this description.

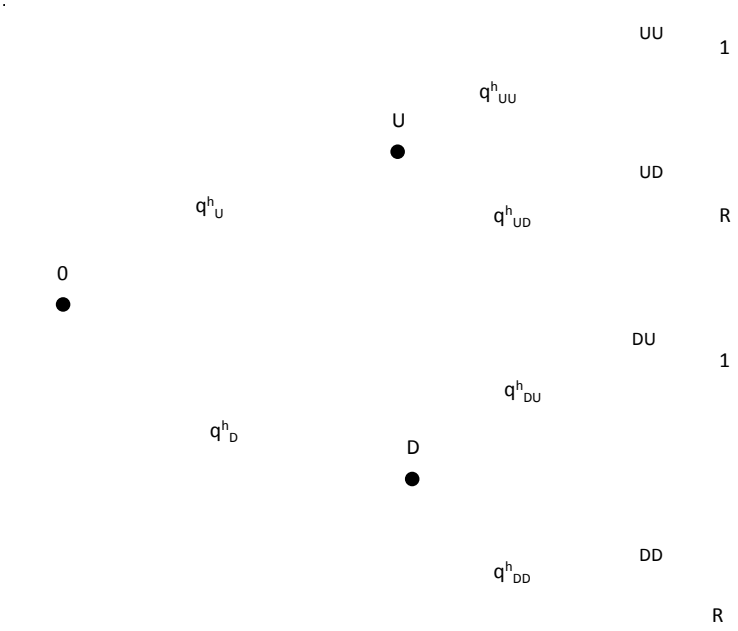

Figure 4: Asset payoff description.

   U can be interpreted as good news since we assume that

$$q^h_{UU} > q^h_{DU}, \forall h \tag{3}$$

   i.e., the probability of full payment after U is higher than after D.

[6] R can be interpreted as a recovery value in case of asset default.

In this example the set of states is $S \subseteq \{0, U, D, UU, UD, DU, DD\}$.

There is a continuum of heterogenous agents indexed by $h \in H = [0, 1]$. The only source of heterogeneity is in subjective probabilities, q_s^h. The higher the h, the more optimistic the agent is about the future. Whenever $h > h'$, $q_U^h > q_U^{h'}$ and $q_{sU}^h > q_{sU}^{h'}$ for $s \in \{U, D\}$, provided s has two successors.

Agents are risk neutral and do not discount the future. They start at $t = 0$ with an endowment of 1 unit of the consumption good and 1 unit of labor. More formally, $U^h = \sum_{s \in S} \bar{q}_s^h c_s$, $e_0^h = 1$ and $e_s^h = 0, s \neq 0$, and $l_0^h = 1$ and $l_s^h = 0, s \neq 0$.

In this baseline economy with one asset it is clear that in equilibrium every investor will transform his labor into one unit of the asset at time 0.

A more subtle conclusion is the following result regarding leverage:

Proposition 1: *In this economy, in which every node has at most two successors states, the only contract j_s traded in equilibrium is the one which promises $j_s = min_{\tau \in S(s)}\{p_\tau + d_\tau\}$.*

Proof: See Geanakoplos (2003), Fostel-Geanakoplos (2010).

In every state, the only contract actively traded is the one promising the minimal payoff in the future. Equilibrium default is endogenously ruled out and the contract will trade at the riskless interest rate r_s. All contracts will be priced in equilibrium, but only one will be actively traded.

As discussed before, *leverage* is endogenously determined in equilibrium. In particular, the proposition *derives* the conclusion that the only contract traded in equilibrium is the one given by the Value at Risk equal zero rule *assumed* by many other papers in the literature.

In equilibrium the risk-less interest rate must be zero: $r_s \leq 0$ because agents do not discount the future, and the presence of the perfect warehousing technology prevents $r_s < 0$.

By proposition 1, buying 1 unit of Y *on margin* at state s means: selling a promise of $\min_{\tau \in S(s)}[p_\tau + d_\tau]$ using that unit of Y as collateral, and paying $(p_s - \min_{\tau \in S(s)}[p_\tau + d_\tau])$ in cash. The Loan to Value (LTV) of Y at s is,

$$LTV_s = \frac{\min_{\tau \in S(s)}[p_\tau + d_\tau]}{p_s} \tag{4}$$

If $s \in \{U, D\}$ has only one successor sU, then s must be good news and so $s = U$. Moreover, every agent will agree on $q_{sU}^h = q_{UU}^h = 1$ and so in equilibrium we must have $p_U = d_{UU} = 1$ and therefore $LTV_U = 1/1 = 100\%$. Analogously, if $s \in \{U, D\}$ has only

one successor sD, then $s = D$, $q^h_{sD} = q^h_{DD} = 1$, $p_D = d_{DD} = R$ and therefore $LTV_U = R/R = 100\%$. If $s \in \{U, D\}$ has two successors then $R < p_s < 1$ and hence $LTV_s = R/p_s < 100\%$. Thus, when volatility post $s \in \{U, D\}$ is zero (because there is only one successor of s), $LTV_s = 100\%$, whereas when volatility post $s \in \{U, D\}$ is positive, $LTV_s < 100\%$.

B. Equilibrium

1. Let us describe the system of equations that characterizes the equilibrium. Because of linear utilities and the continuity of utility in h and the connectedness of the set of agents $H = [0, 1]$, at each state s there will be a *marginal buyer*, h_s, who will be indifferent between buying or selling Y. All agents $h > h_s$ will buy all they can afford of Y, i.e., they will sell all their endowment of the consumption good and borrow to the max using Y as collateral. On the other hand, agents $h < h_s$ will sell all their endowment of Y and lend to the more optimistic investors. Equating expenditures and revenues provides us with the first three equations in our system.

   At $s = 0$ aggregate revenue from sales of the asset is given by p_0.[7] On the other hand, aggregate expenditure on the asset is given by $(1 - h_0)(1 + p_0) + p_D$. The first term is total income (endowment plus revenues from asset sales) of buyers $h \in [h_0, 1]$. The second term is borrowing, which from proposition 1 is p_D. Equating we have

$$p_0 = (1 - h_0)(1 + p_0) + p_D \tag{5}$$

   Let $s \in \{U, D\}$ have two successors sU and sD. Total revenue from asset sales must equal total expenditure on asset purchases. This gives us

$$p_s = (p_s - p_D) + (h_0 - h_s)(p_0 + 1) + R \tag{6}$$

   The first term on the RHS is the income after debt repayment of those holding the asset from period 0. The second term is the income of the new buyers $h \in [h_s, h_0]$, carried over from period 0. The last term is new borrowing. Notice that because at s the original buyers $h \in [h_0, 1]$ can only borrow R, which is less than the p_D they owe, they will not be able to roll over all their loans without selling some assets. Hence, $h_s < h_0$, i.e. the marginal buyer must go down. If s has just one successor then it does not matter who the marginal buyer is because they all agree and any one agent can buy all the assets since leverage is 100%.

   The next equations state that the price at $s \in \{U, D\}$ is equal to the marginal buyer's valuation of the asset's future payoff.

[7] All asset endowments and production add to 1 and without loss of generality are put up for sale even by those who buy it.

$$p_s = q_{sU}^{h_s} 1 + q_{sD}^{h_s} R \tag{7}$$

The last equation equates the marginal utility to h_0 of one dollar to the marginal utility of using one dollar to purchase Y at $s = 0$:

$$\frac{q_U^{h_0} p_U(q_{UU}^{h_0}/q_{UU}^{h_U}) + q_D^{h_0} p_D(q_{DU}^{h_0}/q_{DU}^{h_D})}{p_0} = \frac{q_U^{h_0} 1(q_{UU}^{h_0}/q_{UU}^{h_U}) + q_D^{h_0} 1(q_{DU}^{h_0}/q_{DU}^{h_D})}{1} \tag{8}$$

This last equation needs further explanation. Notice that payoffs on both sides of the equation are weighted by the ratio $(q_{sU}^{h_0}/q_{sU}^{h_s})$ for $s \in \{U, D\}$. If agent h_0 reaches state $s \in \{U, D\}$ with a dollar he will want to leverage his wealth to the max to purchase Y.[8] This will result in a gain per dollar of
$\frac{q_{sU}^{h_0}(1-R)}{p_s - R} = \frac{q_{sU}^{h_0}(1-R)}{q_{sU}^{h_s} 1 + q_{sD}^{h_s} R - R} = \frac{q_{sU}^{h_0}}{q_{sU}^{h_s}}$

Hence the marginal utility of a dollar at time 0 is given by the probability of reaching U times the dollar times the marginal utility given above plus the analogous expression for reaching D. This explains the RHS of equation (8).[9] The LHS has exactly the same explanation once we realize that the best action for the h_0 at $s \in \{U, D\}$ is to sell the asset and use the cash to buy it on margin. If s has a unique successor, then $(q_{sU}^{h_1}/q_{sU}^{h_s}) = 1$ and the same equations applies.

We have a system of six equations, described by expressions (5)-(8), and six unknowns: marginal buyers and asset prices at $s = 0, U, D$.

C. Projects

1. Suppose there are two different projects, variations of the baseline example discussed above. These projects are exactly the same in terms of final asset payoff distribution. To fix ideas, suppose that the probability of final good output 1 is

$$1 - (1 - h)^2 = q_U^h q_{UU}^h + (1 - q_U^h) q_{DU}^h \tag{9}$$

The only difference between the two projects is in the way information is revealed in the intermediate period. More precisely, projects can differ in the post-volatility induced by

[8] Agents are perfectly rational and foward looking. There are other options at $s = D$, like eating the good, storing it or buying Y unleveraged, but they are all dominated in equilibrium by leveraging to the max.

[9] Another way of understanding the same is to notice that buying Y on margin at s is equivalent to buying the Arrow security that pays only at up (since at down the net payoff is zero). The price of this security is given by $q_{sU}^{h_s}$, the marginal buyer's valuation. Hence, with a dollar, h_0 can buy $1/q_{sU}^{h_s}$ units which are worth $(q_{sU}^{h_0}/q_{sU}^{h_s})$, explaining the ratio.

news in the intermediate period. By post-volatility we mean the final payoff volatility conditioned on reaching a particular node or state.

D. Pro-Cyclical Leverage

1. There is only one project that gives rise to pro-cyclical leverage and we describe it in figure 5.

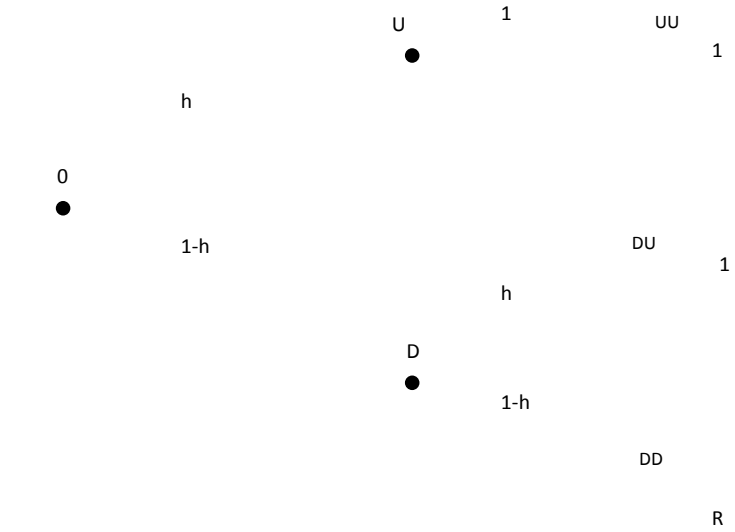

Figure 5: BV Project.

The probabilities in the tree satisfy equations (3) and (9). If state U is reached in the second period, uncertainty is completely resolved since the asset pays for sure 1 at the end. Leverage at U is 100%. However, if D is reached, uncertainty remains. In fact, D is bad news, but of the sort that not only decreases the expected asset payoff compared with U but also increases final payoff volatility. This project represents the situation in which each piece of bad news is not very informative and induces high future volatility. We call it "Post-Bad News Volatility" project, BV.[10]

We solve the system of equations described in section 3.2 to find the equilibrium in this project. Table 1 shows equilibrium prices, marginal buyers and leverage for $R = .2$. It is

[10]This is the example in Geanakoplos (2003, 2009).

easy to check that this is indeed an equilibrium, i.e investors are maximizing and markets clear.

Table 1: BV Equilibrium.

	0	U	D
Price, p_s	0.95	1.00	0.69
Marginal Buyer, h_s	0.87	1.00	0.62
Leverage, LTV_s	0.73	1.00	0.29

The first observation is that the price of Y falls from 0 to D, from $.95$ to $.69$, a fall of 27%. The marginal buyer at $t = 0$, $h = .87,$ thinks at the beginning that there is a probability of 1.69% of reaching the disaster state DD, but once D is reached this probability rises to 13%. This would imply a fall in the price of only 9%. So why is the crash of 27% so much bigger than the bad news of 9%? There are three reasons for the crash.

First, as we just saw, is the presence of bad news. The second reason is that after bad news, the leveraged investors lose all their wealth: the value of the asset at D is exactly equal to their debt, so they go bankrupt. Therefore even the topmost buyer at D is below the marginal buyer at 0. Third, with the arrival of bad news, leverage goes down (margins go up), from $LTV_0 = .73$ to $LTV_D = .3$, so more buyers are needed at D than at 0. Thus the marginal buyer at D is far below the marginal buyer at 0: $h_D = .62 < .87$. The asset falls so far in price at D because every agent values it less and because the marginal buyer is so much lower.

The main result of this exercise is that the BV project endogenously generates *pro-cyclical* leverage. With bad news, leverage goes down and with good news leverage goes up. Why is this? As mentioned before, bad news not only decreases expected asset payoff in the future, but increases future volatility as well and good news reduces the volatility. By equation (4) an increase in volatility increases endogenous margin requirements and lowers leverage in equilibrium. This phenomenon was called the *Leverage Cycle* by Geanakoplos (2003) and extended further to many assets and adverse selection by Fostel-Geanakoplos (2008).

E. Counter-Cyclical Leverage

1. Every other project gives rise to counter-cyclical leverage because $p_U > p_D$ and hence $LTV_U = R/p_U < R/p_D = LTV_D$. We concentrate on the simplest example, which we call "Post-Good News Volatility" project, GV, defined by the following tree depicted in figure 6.

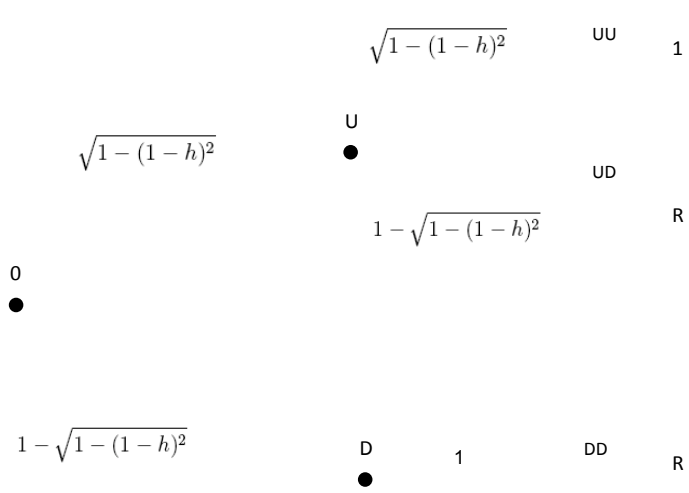

Figure 6: GV Project.

These probabilities also satisfy equations (3) and (9), that is, every agent h thinks the terminal probabilities of 1 and R are the same for GV as for BV. If D is reached, all uncertainty is resolved given that the asset pays for sure the low dividend R, and leverage is 100%. However, if U is reached uncertainty remains and leverage falls: investors can still borrow R but the price is higher. This GV project represents the situation in which each piece of good news, as opposed to bad news as in the BV project, is not very informative and induces high future volatility.

We solve the system of equations described in section 3.2 to find the equilibrium in this project. Table 2 shows equilibrium prices, marginal buyers and leverage for $R = .2$. It is easy to check that this is indeed an equilibrium, i.e investors are maximizing and markets clear.

In equilibrium, the asset price collapses from $.89$ all the way to $.2$ given the imminent nature of the disaster once D has been reached. It goes up at U to $.94$. The marginal buyer at $t = 0$ and $t = U$ is the same, so optimists roll-over their debt once they reach U.

Table 2: GV Equilibrium.

	0	U	D
Price, p_s	0.89	0.94	0.20
Marginal Buyer, h_s	0.63	0.63	0.63
Leverage, LTV_s	0.22	0.21	1.00

F. Why BV is So Different from GV?

1. The main findings from Sections 3.4 and 3.5 are the following:

   (a) The initial price is higher in the BV project $(.95)$ than in the GV project $(.89)$.

   (b) Initial leverage is higher in the BV project $(LTV = .73)$ than in the GV project $(LTV = .22)$.

   (c) Leverage is pro-cyclical in the BV project and counter-cyclical in the GV project.

   Why is this the case?

   First, the reason why the BV project is more valuable than the GV project is because it can be leveraged more at the beginning. A higher borrowing capacity implies that all the assets in the economy can be afforded by fewer investors, so that the marginal buyer is more optimistic. This naturally raises the project's price.

   Second, BV can be leveraged more at time zero due to the type of bad news. By proposition 1 the maximum agents can promise is the worse case scenario in the immediate future, i.e, the price of the project after bad news. But in the BV project the price does not fall as much precisely because bad news is less informative and volatile. By contrast, bad news in the GV project is very informative, lowering the promise in equilibrium.

   Third, as explained before, the cyclical properties derived in each project are a direct consequence of the difference in volatility between the projects. In BV bad news induces

future volatility, lowering leverage, while in GV good news induces volatility, lowering leverage.

G. BV vs GV: Long Run Analysis

1. Having completely characterized the equilibrium in the two projects, considered as separate economies, we wonder if these results hold when we consider longer horizons. With this in mind, we extend our previous examples for an N horizon economy. We maintain the same terminal probabilities for outcomes 1 and R, independent of N, with constant probabilities of up throughout each tree. The BV and GV projects are described in figure 7. In the BV project, as before, the imminent occurrence of the bad final outcome R is pushed until the very end, and bad news comes in small drops with an associated higher future volatility. On the other hand, in the GV project, good news, instead of bad news, has the property of revealing little information and inducing high volatility. We calculate the equilibrium for each project separately. The complete system of equations that characterizes the equilibrium in each project is described in detailed in Appendix 1. They are the natural (though not obvious) extension of the three period case. The prices and leverage are noted at some of the nodes for $N = 10$ in figure 7, complete equilibrium information is presented in Appendix 1.

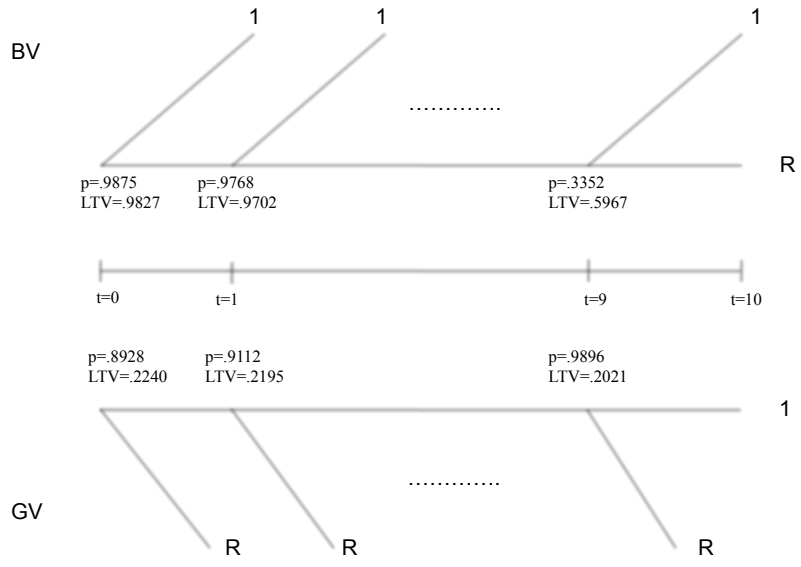

Figure 7: Prices and leverage for BV and GV projects, N=10 periods.

Figure 7 shows that the results of previous sections hold even in longer horizon economies. The price of the BV project is higher than the GV project and leverage is pro-cyclical in the

BV project and counter-cyclical in the GV project. In fact, the longer the horizon the bigger the gap in initial prices.

H. Arrow-Debreu Equilibrium

1. In order to help understand why BV is more valuable than GV we also calculate the Arrow-Debreu equilibrium for each project. It is evident that every agent will wait until the last period to consume. In each case there are three terminal states. The difference is that in the BV project the good event (where the dividend is 1) is partitioned into two states, UU and DU, whereas in the GV project the bad event (in which the dividend is .2) is partitioned into two states, UD and DD. See figure 8.

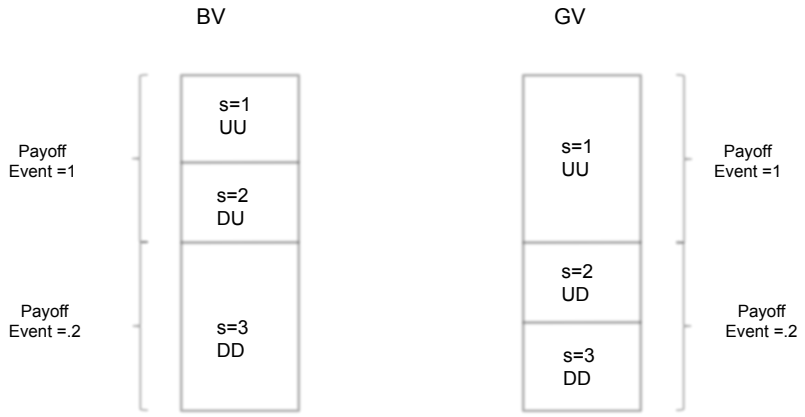

Figure 8: Projects and payoff event partitions.

To compute the Arrow-Debreu equilibrium, we guess that agents above h_1 buy only the Arrow security for state 1, agents $h_1 > h > h_2$ will buy only the Arrow security 2 and agents below h_2 will buy the Arrow security 3. Endowments in each state are the cash plus the asset dividends in each state.

As we can see in table 3 the price of the BV project is $.55$, higher than the price of the GV project, $.48$. Asset prices are given by the sum of the Arrow prices weighted by the asset

Table 3: Arrow-Debreu equilibrium for BV and GB projects.

	BV Project	GV Project
p_1	0.2848	0.3598
p_2	0.1484	0.173
p_3	0.5668	0.4672
Asset Price	0.5465	0.4878
h_1	0.4789	0.2624
h_2	0.2074	0.1915

dividend in each state.[11] The price of the good event is given by the sum of the first two Arrow prices, a total of $.4332$ in the BV project. In contrast, the price of the good event is given only by the first arrow price in the GV project, $.3598$. Of course, this difference makes the asset price higher in the BV project.

Why is the Arrow price of the good event worth more in the BV economy than in the GV economy, even though every agent attaches the same probability? Due to heterogenous priors, a finer partition of the good event allows agents to bet, increasing the Arrow price of the good event.

From the Arrow-Debreu equilibria we conclude that the gap in initial asset prices between BV and GV obtained in sections 3.4 and 3.5 does not rely on market incompleteness or the assumed short-selling constraints. The Arrow-Debreu equilibrium helps us understand why in collateral equilibrium leverage makes BV more valuable than GV. In the BV collateral economy, one can bet on a payoff of 1 by in effect buying the UU Arrow security via leverage at $s = 0$, or by warehousing at $s = 0$ and then leveraging at $s = D$, thus in effect buying a combination of UU and DU Arrow securities. In the GV collateral economy, one can only bet on the payoff of 1 via the UU Arrow security.

Though the gap in price between the complete markets BV and GV economies is just as big as in the collateral BV and GV economies, the absolute price level of the complete market

[11] Note that the sum of the Arrow prices is equal to 1 due to the presence of an inventory technology with zero profit.

economies is much lower. In the complete market economies pessimists can bet on the bad $.2$ outcome, whereas in the collateral equilibrium they cannot because of the short sale constraint.

IV. Volatile Bad News or Volatile Good News?

1. The main question we want to answer in this section is: if agents have the opportunity to use their labor to produce either of the two type of projects, BV and GV, which project would they choose in equilibrium?

   It is very tempting to jump to the conclusion that all agents will choose the BV project since it has a higher price at the beginning in the separate economies. Unfortunately this answer is incorrect. Further inspection reveals that once everyone else has chosen the BV project, it becomes profitable for any one agent to produce the GV project. So we need to appeal to the full force of the model in section 2 to solve the problem.

   Suppose there are two assets, X and Y, with independent payoffs. Asset X corresponds to the BV project and asset Y to the GV project. The joint tree of payoffs is described in figure 9. Note that state $s = 0$ now has four successors. For example, the state (U, U) in the intermediate period corresponds to the situation in which X (BV) and Y (GV) receive good news. The probability of such event for agent h is $h\sqrt{1-(1-h)^2}$.

   Agents are as in the baseline example in section 3. They can transform their unit of labor into a portfolio of different projects at $t = 0$. The within-period technology is given by $Z_0^h = \{(z_0^X, z_0^Y) \in R_+^2 : z_0^X + z_0^Y = 1\}$, where z_0^X is the share of X (BV project) and z_0^Y the share of Y (GV project).

   Figure 9 shows the equilibrium prices at each node for both assets, BV and GV, respectively for $R = .2$. At equilibrium, all agents choose to produce the same mix $z_0^X = .7$ and $z_0^Y = .3$. But how did we find equilibrium?

A. Equilibrium Leverage

1. Before moving on to solve the model, let us go back to the question of endogenous leverage. Proposition 1 holds for the intermediate states $s \in \{UU, UD, DU, DD\}$, since for each asset there are at most two distinct successor payoff values. Hence, the only contract traded in all intermediate states is the one that prevents default in equilibrium as in section 3.

   However, the situation is different at time 0 since there are four successor states in $S(0)$ with three distinct successor payoff values for each asset[12], and therefore it is not possible to appeal to the result anymore. In fact, the following holds

[12] X's price is 1 at UU and UD and Y's price is R at UD and DD.

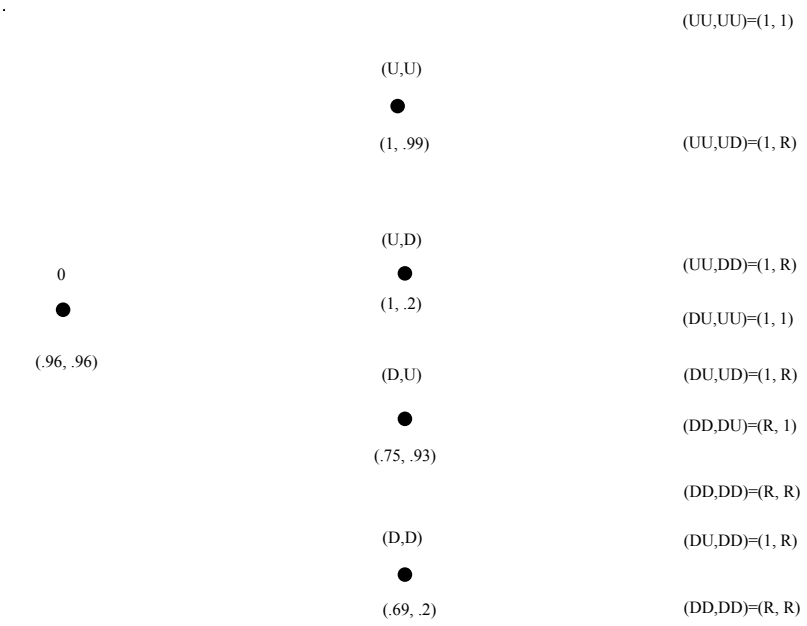

Figure 9: Joint BV and GV economy

Proposition 2: *In this economy, two contracts are traded in equilibrium at time* 0 *for each asset: the one which promises* $j_s^k = p_{DD}^k$ *and the one that promises* $j_s^k = p_{DU}^k$.

Proof: Fostel-Geanakoplos (2010).

For each asset two types of contracts will be traded: one that promises the worst-case scenario and another that promises the middle-case scenario. While the first one is risk-less as before, the second one is not since it defaults in the worst state. In this model, not only is there default in equilibrium, but also the same asset is traded simultaneously with different margin requirements by different investors. Araujo et.al. (2009) and Fostel-Geanakoplos (2010) show this in a two period model. We show in the following section that this is an equilibrium also in a dynamic setting for the first time. The dynamic setting is more difficult because the payoffs of the risky bonds are endogenous.

B. Procedure to Find the Equilibrium

B.1 Variables

1. Finding an equilibrium in this economy seems a daunting task. The first thing we will do is to find an equilibrium for any fixed $z_0^X, z_0^Y = 1 - z_0^X$. Then using the fact that the two asset

prices at the beginning ought to be equal in a genuine equilibrium, we will find the z_0^X that precisely accomplishes that.[13]

Notice that some prices are obvious, X's price equals 1 for sure at UU and UD, whereas Y's price is R at UD and DD. It is also clear that at UD all uncertainty is resolved and there is no more trade.

Buying an asset on margin using a financial contract defines a down-payment at time 0 and a profile of net payoffs in the future. In this sense, we can think of nine securities in total at time 0, six risky and three risk-less: i) buying X on margin using the risky bond (the one that promises p_{DU}^X), ii) buying X on margin using the risk-less bond (which promises p_{DD}^X), iii) buying Y on margin using the risky bond (the one that promises p_{DU}^Y), iv) buying Y on margin using the risk-less bond (which promises $p_{UD}^Y = p_{DD}^Y$), v) the risky bond that promises p_{DU}^X, vi) the risky bond that promises p_{DU}^Y, vii) the risk-less bond that promises p_{DD}^X, viii) the risk-less bond that promises $p_{UD}^Y = p_{DD}^Y$ and ix) warehousing.

In equilibrium the riskless interest rate will be zero, as before, hence all the riskless bonds will be priced equal to their respective promise. In addition to z_0^X and z_0^Y we still need to find the value of 20 variables:

- Asset prices: $p_0^X, p_0^Y, p_{UU}^Y, p_{DU}^X, p_{DU}^Y, p_{DD}^X$.
- Risky bond prices at $s = 0$: π^X, π^Y, where π^k is the price of the bond that promises p_{DU}^k in all successors states in the future.
- Asset marginal buyers: $h_M^X, h_M^Y, h_m^X, h_m^Y, h_{UU}^Y, h_{DU}^X, h_{DU}^Y, h_{DD}^X$, where $h_M^k(h_m^k)$ corresponds to the marginal buyer of the k asset leveraging with the risky (risk-less) bond.
- Risky bond marginal buyers: h^{BX}, h^{BY}.
- Asset purchases at $s = 0$ leveraging with the risky bond: y^X, y^Y.

B.2 Regimes

Next, we will guess a regime, consisting of a ranking of the marginal buyers and a description of what each agent buys in each node, in order to be able to define a system of equations. Once we get a solution we need to check: first, that $p_{DU}^X > p_{DD}^X$, so that prices are consistent with our guess about which bonds are risky and riskless on X, second, that $p_{UU}^Y > p_{DU}^Y$, so that prices are consistent with with our guess about which bonds are risky and riskless on Y, and finally, that each regime is genuine, i.e. all agents are maximizing with those choices.

1. We next describe the regimes at each node. Figure 10 shows a graphical illustration of them and of the equilibrium values of all marginal buyers.

[13] Hopefully if we start with a good guess of z_0^X near the true value we will be able to shift z_0^X until prices are equal without changing the equilibrium regime by continuity.

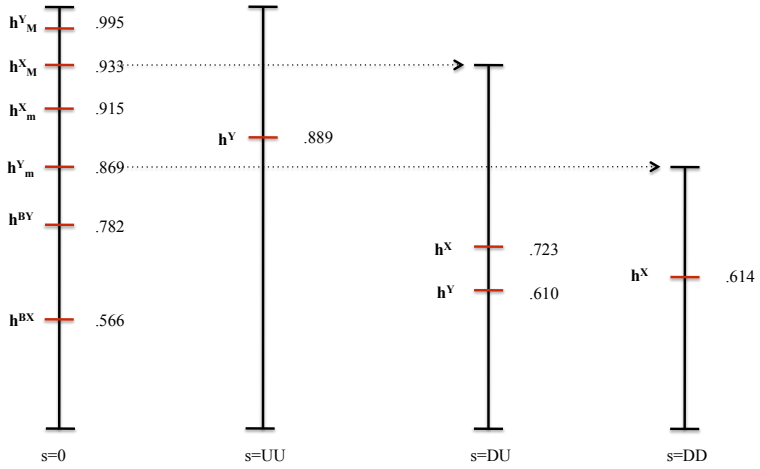

Figure 10: Equilibrium Regimes.

- At $s = 0$

  $h > h^Y_M$ buy Y, sell X and promise p^Y_{DU}. $h^Y_M > h > h^X_M$ buy X, sell Y and promise p^X_{DU}. $h^X_M > h > h^X_m$ buy X, sell Y and promise p^X_{DD}. $h^X_m > h > h^Y_m$ buy Y, sell X and promise R. $h^Y_m > h > h^{BY}$ sell both assets and buy the BY bond (so lend in the risky market collateralized by Y). $h^{BY} > h > h^{BX}$ sell all assets and buy the BX bond (so lend in the risky market collateralized by X). Finally, $h < h^{BX}$ sell everything, hold risk-less securities (so lend in the risk-less markets).

- At $s = UU$

  $h > h^Y_{UU}$ buy Y and promise R. Below lend and buy X. $h^X_m > h^Y_{UU} > h^Y_m$.

- At $s = DU$

  All $h > h^X_M$ go bankrupt since they promise exactly what they own. $h > h^X_{DU}$ buy X and promise R. $h^X_{DU} > h > h^Y_{DU}$ buy Y and promise R. All $h < h^Y_{DU}$ lend. Finally, $h^{BY} > h^X_{DU} > h^Y_{DU} > h^{BX}$.

- At $s = DD$

  All $h > h^Y_m$ are out of business either because they default or they have no money left. $h > h^X_{DD}$ buy X and promise R. $h < h^X_{DD}$ lend. Finally, $h^{BY} > h^X_{DD} > h^{BX}$.

The system of equations is conceptually an extension of the system in section 3. In every state supply equals demand for all the securities. Also marginal buyers are determined by an indifference condition between investing in two different securities. As before, all marginal utility of a dollar invested in any security is weighted by the marginal utility of future actions in each state. The system is presented in Appendix 2.

C. Agents Prefer the BV Project

1. All the values listed in figures 9 and 10 are consistent with the assumed regimes and prices as discussed in section 4.2.2. It turns out also that this equilibrium is genuine in the sense that all agents' decisions are optimal.[14]

   The most important thing to observe is that $z_0^X = .7$, this is, all agents choose to invest their labor in a portfolio with a 70% share of the BV project. Or equivalently, 70% of the economy invests in BV projects when given the opportunity to choose. The consequence of this is that, since we assumed that the two projects were independent, 70% of the time when bad news occurs they will be of the volatile type, and we will observe pro-cyclical leverage.

D. Leverage Reconsidered

When the asset could take on at most two immediate successor values, equilibrium determines a unique actively traded promise and hence leverage. With three or more successor values, we cannot expect a simple promise. But equilibrium still determines the average leverage used to buy each asset.

1. Equilibrium leverage is presented in table 4. There are eight securities in total, six risky securities and two risk-less securities (without considering warehousing). Columns 2 and 3 show the holdings and value of such holdings for each of the securities. Most importantly, column 4 shows the LTV of each of the four traded contracts. As was expected, LTV is higher for the risky contracts (they have a higher promise) for both assets. Finally, column 5 shows the LTV for each asset. Whereas the LTV for BV is $.76$, it is only $.6$ for GV. As defined in section 2, asset LTV is a weighted average. For example the LTV for BV is obtained from the total amount borrowed using all contracts, $.423 + .091$ divided by the total value of collateral, $.966 \times .695$.

   As in section 3, BV can be leveraged more than the GV. Second, also as before, leverage in BV is pro-cyclical while it is counter-cyclical in GV. Third, notice that even though both projects have the same initial price in equilibrium, for both assets the price is higher than in section 3 ($.966$ versus $.95$ for BV and $.89$ for GV). The main reason for this difference is that

[14] The risky bond prices at date 0 are $\pi^X = .7521$ on a promise of $.7548$, corresponding to an interest rate of $.36\%$ and $\pi^Y = .9156$ on a promise of $.9366$, corresponding to an interest rate of 2.3%. The most leveraged asset purchases at date 0 are $y^X = .520$ and $y^Y = .184$. The verification that each agent is indeed maximizing is available upon request.

now with a different tree, more contracts are traded in equilibrium, not only the risk-less one. Both assets can be leveraged more now using risky contracts which promise more (and hence default as well). Whereas there is not so much difference between the minimum promise and the medium promise for BV (.691 and .754) this difference is significant for GV (.2 and .936). For a precise discussion between leverage and asset prices see FG (2010).

Leverage at s=0

Security	Holdings	Holdings Value	Contract LTV	Asset	Asset LTV
Y lev Medium	0.186	0.180	0.947	**X (SBN)**	0.766
X lev Medium	0.563	0.544	0.778		
X lev Min	0.132	0.128	0.715	**Y (CBN)**	0.660
Y lev Min	0.119	0.115	0.207		
Y risky bond	0.186	0.171			
X risky bond	0.563	0.423			
Y riskless bond	0.119	0.024			
X riskless bond	0.132	0.091			

Leverage at intermediate nodes

	UU	UD	DU	DD
X (SBN)	1.000	1.000	0.184	0.201
Y (CBN)	0.061	1.000	0.065	1.000

Table 4: Contract and Asset Leverage.

So, why did agents choose BV more? The simple reason is that BV can be leveraged more at the beginning. So the most optimistic agents will choose BV. However, as soon as less optimistic people opt for volatile bad news projects, its price will start to decline and the GV project will start to become attractive to other investors. This process will continue until prices are equal in equilibrium.

E. Volatility Smiles

Our main result also suggests an explanation for the observed “Volatility Smile" in stock options. This refers to the fact that implied volatility has a negative relationship with the strike price, so volatility decreases as the strike price increases. Hence, bad news comes (or are assumed to come) with high volatility. This effect is even larger when considered on indexes as S&P500.

1. The pattern has existed for equities only after the stock market crash of 1987. This has led some economist like Bates (2000) and Rubinstein (1995) to explain volatilites smiles by “crashophobia". Traders are concerned about the possibility of another crash and they price options accordingly. Our result provides a completely different explanation. Our agents are perfectly rational, they endogenously choose projects associated with volatile bad news since they can leverage more with them.

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Appendix I: BV and GV Projects: Long Run Analysis

1. Notice that since the final probability of disaster is constant (regardless of N), the probability of bad news in period k is given by $(1-h_k)^{2/k}$.

   - $p_{N+1} = R$
   - $p_N = (1-(1-h_N)^{2/N}) + (1-h_N)^{2/N} R$
   - $h_{N-1} = \frac{h_N(1+p_N)}{1+p_{N+1}}$
   - $p_{N-1} = \frac{(1-(1-h_{N-1})^{2/N}) + (1-h_{N-1})^{2/N} \frac{(1-(1-h_{N-1})^{2/N})}{(1-(1-h_N)^{2/N})} p_N}{(1-(1-h_{N-1})^{2/N}) + (1-h_{N-1})^{2/N} \frac{(1-(1-h_{N-1})^{2/N})}{(1-(1-h_N)^{2/N})}}$
   - $h_{N-2} = \frac{h_{N-1}(1+p_{N-1})}{1+p_N}$

     $\vdots$
   - $p_1 = \frac{(1-(1-h_1)^{2/N}) + (1-h_1)^{2/N} \frac{(1-(1-h_1)^{2/N})}{(1-(1-h_2)^{2/N})} p_2}{(1-(1-h_1)^{2/N}) + (1-h_1)^{2/N} \frac{(1-(1-h_1)^{2/N})}{(1-(1-h_2)^{2/N})}}$
   - $h_0 = \frac{h_1(1+p_1)}{1+p_2} = 1$

   We use the fact that the marginal buyer rollover his debt at every node to build up the system and then verify that the guess is correct. Notice that the probability of good news in period k is given by $(1-(1-h_k)^2)^{1/N}$.

   - $p_1 = ((1-(1-h_k)^2)^{1/N})^N + (1-((1-(1-h_k)^2)^{1/N})^N) R$
   - $p_1 = \frac{(1-h_1)+R}{h_1}$

     $\vdots$
   - $p_k = ((1-(1-h_k)^2)^{1/N})^{N-k} + (1-((1-(1-h_k)^2)^{1/N})^{N-k}) R$

   Tables 5 and 6 present all the equilibrium values.

Table 5: BV equilibrium N=10.

Period	Mrg buyer	Price bad state	Price good state	Leverage bad state	Leverage good state
0	0.9914	0.9875		0.9827	
1	0.9768	0.9704	1.0000	0.9702	1.0000
2	0.9547	0.9415	1.0000	0.9534	1.0000
3	0.9244	0.8976	1.0000	0.9327	1.0000
4	0.8856	0.8372	1.0000	0.9081	1.0000
5	0.8394	0.7603	1.0000	0.8791	1.0000
6	0.7870	0.6684	1.0000	0.8441	1.0000
7	0.7301	0.5642	1.0000	0.7995	1.0000
8	0.6718	0.4511	1.0000	0.7431	1.0000
9	0.6038	0.3352	1.0000	0.5967	1.0000
10		0.2000	1.0000		

Table 6: GV equilibrium N=10.

Period	Mrg buyer	Price good state	Price bad state	Leverage good state	Leverage bad state
0	0.6340	0.8928		0.2240	
1	0.6340	0.9112	0.2000	0.2195	1.0000
2	0.6340	0.9205	0.2000	0.2173	1.0000
3	0.6340	0.9300	0.2000	0.2151	1.0000
4	0.6340	0.9396	0.2000	0.2129	1.0000
5	0.6340	0.9494	0.2000	0.2107	1.0000
6	0.6340	0.9592	0.2000	0.2085	1.0000
7	0.6340	0.9692	0.2000	0.2064	1.0000
8	0.6340	0.9793	0.2000	0.2042	1.0000
9	0.6340	0.9896	0.2000	0.2021	1.0000
10		1.0000	0.2000		

Appendix 2: System of Equations for the Joint-Project Economy in Section 4

Notation: q_s^h is the probability of state s by buyer h.

1.
- $y^Y = \frac{(1-h_M^Y)+\alpha p_1^X(1-h_M^Y)+(1-\alpha)p_1^Y(1-h_M^Y)}{p_1^Y-\pi^Y}$

$y^X = \frac{(h_M^Y-h_M^X)+(1-\alpha)p_1^Y(h_M^Y-h_M^X)+\alpha p_1^X(h_M^Y-h_M^X)}{p_1^X-\pi^X}$

$(\alpha h_m^X + \alpha(1-h_M^Y) - y^X) = \frac{(h_M^X-h_m^X)+(1-\alpha)p_1^Y(h_M^X-h_m^X)+\alpha p_1^X(h_M^X-h_m^X)}{p_1^X-p_{DD}^X}$

$((1-\alpha)h_m^Y + (1-\alpha)(h_M^Y-h_m^X) - y^Y) = \frac{(h_m^X-h_m^Y)+\alpha p_1^X(h_m^X-h_m^Y)+(1-\alpha)p_1^Y(h_m^X-h_m^Y)}{p_1^Y-R}$

$((1-\alpha)(1-h_M^Y) + y^Y) = \frac{(h_m^Y-h^{BY})(1+\alpha p_1^X+(1-\alpha)p_1^Y)}{\pi^Y}$

$(\alpha(h_M^Y-h_M^X) + y^X) = \frac{(h^{BY}-h^{BX})(1+\alpha p_1^X+(1-\alpha)p_1^Y)}{\pi^X}$

$\frac{q_{UU}^{h_M^Y}(p_{UU}^Y-p_{DU}^Y)}{p_1^Y-\pi^Y}\frac{\sqrt{1-(1-h_M^Y)^2}(1-R)}{p_{UU}^Y-R} = \frac{q_{UU}^{h_M^Y}(1-p_{DU}^X)}{p_1^X-\pi^X}\frac{\sqrt{1-(1-h_M^Y)^2}(1-R)}{p_{UU}^Y-R} + \frac{q_{UD}^{h_M^Y}(1-p_{DU}^X)}{p_1^X-\pi^X}$

- $\frac{q_{UU}^{h_M^X}(1-p_{DU}^X)}{p_1^X-\pi^X}\frac{\sqrt{1-(1-h_M^X)^2}(1-R)}{p_{UU}^Y-R} + \frac{q_{UD}^{h_M^X}(1-p_{DU}^X)}{p_1^X-\pi^X} =$

$= \frac{q_{UU}^{h_M^X}(1-p_{DD}^X)}{p_1^X-p_{DD}^X}\frac{\sqrt{1-(1-h_M^X)^2}(1-R)}{p_{UU}^Y-R} + \frac{q_{UD}^{h_M^X}(1-p_{DD}^X)}{p_1^X-p_{DD}^X} + \frac{q_{DU}^{h_M^X}(p_{DU}^X-p_{DD}^X)}{p_1^X-p_{DD}^X}\frac{h_M^X(1-R)}{p_{DU}^X-R}$

$\frac{q_{UU}^{h_m^X}(1-p_{DD}^X)}{p_1^X-p_{DD}^X}\frac{\sqrt{1-(1-h_m^X)^2}(1-R)}{p_{UU}^Y-R} + \frac{q_{UD}^{h_m^X}(1-p_{DD}^X)}{p_1^X-p_{DD}^X} + \frac{q_{DU}^{h_m^X}(p_{DU}^X-p_{DD}^X)}{p_1^X-p_{DD}^X}\frac{h_m^X(1-R)}{p_{DU}^X-R} =$

$= \frac{q_{UU}^{h_m^X}(p_{UU}^Y-R)}{p_1^Y-R}\frac{\sqrt{1-(1-h_m^X)^2}(1-R)}{p_{UU}^Y-R} + \frac{q_{DU}^{h_m^X}(p_{DU}^Y-R)}{p_1^Y-R}\frac{h_m^X(1-R)}{p_{DU}^X-R}$

$\frac{q_{UU}^{h_m^Y}(p_{UU}^Y-R)}{p_1^Y-R} + \frac{q_{DU}^{h_m^Y}(p_{DU}^Y-R)}{p_1^Y-R}\frac{h_m^Y(1-R)}{p_{DU}^X-R} =$

$= \frac{q_{UU}^{h_m^Y}p_{DU}^Y+q_{UD}^{h_m^Y}R+q_{DU}^{h_m^Y}p_{DU}^Y\frac{h_m^Y(1-R)}{p_{DU}^X-R}+q_{DD}^{h_m^Y}R\frac{h_m^Y(1-R)}{p_{DD}^X-R}}{\pi^Y}$

$\frac{q_{UU}^{h^{BY}}p_{DU}^Y+q_{UD}^{h^{BY}}R+q_{DU}^{h^{BY}}p_{DU}^Y\frac{h^{BY}(1-R)}{p_{DU}^X-R}+q_{DD}^{h^{BY}}R\frac{h^{BY}(1-R)}{p_{DD}^X-R}}{\pi^Y} =$

$= \frac{q_{UU}^{h^{BY}}p_{DU}^X+q_{UD}^{h^{BY}}p_{DU}^X+q_{DU}^{h^{BY}}p_{DU}^X\frac{h^{BY}(1-R)}{p_{DU}^X-R}+q_{DD}^{h^{BY}}p_{DD}^X\frac{h^{BY}(1-R)}{p_{DD}^X-R}}{\pi^X}$

$\frac{q_{UU}^{h^{BX}}p_{DU}^X+q_{UD}^{h^{BX}}p_{DU}^X+q_{DU}^{h^{BX}}p_{DU}^X+q_{DD}^{h^{BX}}p_{DD}^X}{\pi^X} = 1$

$\frac{\sqrt{1-(1-h_{UU}^Y)^2}(1-R)}{p_{UU}^Y-R} = 1$

$(1-\alpha) = \frac{(p_{UU}^Y-p_{DU}^Y)((1-\alpha)(1-h_M^Y)+y^Y)+(1-p_{DU}^X)(\alpha(h_M^Y-h_M^X)+y^X)}{p_{UU}^Y-R} +$

$\frac{(1-p_{DD}^X)(\alpha(h_M^X-h_m^X)+(\alpha h_m^X+\alpha(1-h_M^Y)-y^X))(h_M^X-h_{UU}^Y)/(h_M^X-h_m^X)}{p_{UU}^Y-R}$

$\frac{h_{DU}^X(1-R)}{p_{DU}^X-R} = \frac{\sqrt{1-(1-h_{DU}^X)^2}(1-R)}{p_{DU}^Y-R}$

$\frac{\sqrt{1-(1-h_{DU}^Y)^2}(1-R)}{p_{DU}^Y-R} = 1$

$\alpha = \frac{(p_{DU}^X-p_{DD}^X)(\alpha(h_M^X-h_m^X)+(\alpha h_m^X+\alpha(1-h_M^Y)-y^X))+(p_{DU}^Y-R)((1-\alpha)(h_m^X-h_m^Y)}{p_{DU}^X-R} +$

$\frac{((1-\alpha)h_m^Y+(1-\alpha)(h_M^Y-h_m^X)-y^Y))}{p_{DU}^X-R} +$

$$\frac{p^Y_{DU}((1-\alpha)(1-h^Y_M)+y^Y)(h^{BY}-h^X_{DU})/(h^{BY}-h^{BX})}{p^X_{DU}-R}$$

$$(1-\alpha) = \frac{(h^X_{DU}-h^Y_{DU})/(h^{BY}-h^{BX})p^Y_{DU}((1-\alpha)(1-h^Y_M)+y^Y)}{p^Y_{DU}-R}$$

$$\frac{h^X_{DD}(1-R)}{p^X_{DD}-R} = 1$$

$$\alpha = \frac{R((1-k=\alpha)(1-h^Y_M)+y^Y)+\frac{h^{BY}-h^X_{DD}}{h^{BY}-h^{BX}}p^X_{DD}(\alpha(h^Y_M-h^X_M)+y^X)}{p^X_{DD}-R}$$