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A Risk-Based Debt Sustainability Framework: Incorporating Balance Sheets and Uncertainty

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Abstract

This paper proposes a new framework for the analysis of public sector debt sustainability. The framework uses concepts and methods from modern practice of contingent claims to develop a quantitative risk-based model of sovereign credit risk. The motivation in developing this framework is to provide a clear and workable complement to traditional debt sustainability analysis which—although it has many useful applications—suffers from the inability to measure risk exposures, default probabilities and credit spreads. Importantly, this new framework can be adapted for policy analysis, including debt and reserve management.

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" The right answer for crisis avoidance is controlling risk...The appropriate conceptual framework is value at risk – a model driven estimate of the maximum risk for a particular balance sheet situation over a specified horizon. There are genuine issues of modeling, but there is no issue whatsoever in recognizing that this approach is the right one. If authorities everywhere enforced a culture of risk-oriented evaluation of balance sheets, extreme situations such as those of Asia would just disappear or, at worst, become a rare species."

Rudiger Dornbusch (1998)

I. Introduction

This paper proposes a new framework for the analysis of public sector debt sustainability, based on contingent claims analysis (CCA). The framework uses concepts and methods from modern finance and from the analysis of corporate debt to develop a quantitative model of sovereign credit risk. By incorporating uncertainty into sovereign risk-adjusted balance sheets, this framework ties together debt sustainability analysis (DSA) with early indicators of vulnerabilities. The motivation in developing this framework is to provide a clear and workable complement to traditional debt sustainability analysis, which—although it has many useful applications—suffers from several important shortcomings.

Typically, traditional debt sustainability is predicated on finding the fiscal adjustment that is required to keep the public sector debt ratio stable or declining. The rationale stems from the expectation that if the debt ratio is stable or declining, the government's intertemporal budget constraint is met. Using this logic, various indicators have been developed to check whether certain policies stabilize or increase the debt ratio (Buiter, 1993). In general, policies that increase the debt ratio are eschewed on the basis that they lead to unsustainable debt dynamics. In the IMF, emphasis is also given to the level at which the debt ratio is stabilized, where a comfort level is set at about 40 percent of GDP (IMF 2005) for emerging market countries.[1]

While appealingly simple, this approach can provide misleading assessments of debt sustainability. As explained below, traditional debt sustainability analysis suffers from several important deficiencies. The most obvious is the excessive focus on stabilizing a debt to GDP ratio that is highly aggregated and cannot properly account for changes in risk appetite, or the impact of changes in the maturity structure and currency composition of debt. It is not surprising, therefore, that debt-to-GDP ratios have been found to have poor correlation with market based measures of risk, such a spreads from credit default swaps or bond spreads. This problem is particularly acute during periods of stress where vulnerabilities may build or subside rapidly, but the accounting ratio remains largely invariant to default risk.

[1] Although the proposed risk-based DSA is applicable for all countries, it is more useful for the analysis of debt sustainability in countries with a substantial portion of commercial debt. It can also be applied for low-income countries, if they have non-concessional external debt (with appropriate debt sustainability thresholds).

This paper proposes a new framework that attempts to address the most critical shortcomings of traditional debt sustainability analysis. The purpose in introducing a new framework is not to replace traditional debt sustainability analysis—since it is useful as a quick and rough indicator of debt vulnerabilities—but to provide policymakers with a tool to assess debt sustainability in a more considered manner. Among the key elements that distinguish the new approach are:

- It incorporates information about the balance sheet structure of the sovereign, which includes key assets of the government and monetary authorities, and contingent liabilities to other sectors. This information is relevant to assess the ability of the sovereign to make payments on its debt obligations;

- It includes uncertainty in a realistic and explicit way by allowing sovereign assets to evolve with uncertainty relative to the promised payments on debt;

- It assesses debt sustainability using forward-looking risk indicators. The risk measures capture non-linear changes in credit risk that reflect the probability of default or distress over a specific horizon along with the premium required to compensate for the risk. Importantly, the risk indicators capture the idiosyncratic nature of the different types of debt, particularly foreign currency versus local currency debt;

- It provides a multisector setting which allows linkages between key sectors of the economy—such as the non-financial corporate, financial and sovereign sectors—to be examined and their potential feedback effects estimated and valued;

- It is amenable to scenario and simulation analysis and, thus, provides a useful tool to analyze the impact of macroeconomic shocks on debt sustainability and possible risk-mitigating policy measures. With simulation, a confidence level can be expressed with respect to the risk indicator of debt sustainability.

The approach advanced in this paper builds on the work of Gray, Merton, and Bodie (2003), as well as Gapen et al (2004, 2005), which uses contingent claims methodology to derive estimates of credit risk. This approach was pioneered by Merton (1974) in pricing risky corporate debt.

The structure of the paper is organized as follows. Section II provides a critique of traditional debt sustainability analysis. Section III introduces the authors' concept of debt sustainability. Section IV reviews the contingent claims approach (CCA) and shows how it can be applied to perform more robust analyses of debt sustainability. Section V discusses how macroeconomic policy can be adjusted in response to changes in debt sustainability conditions, with Section VI providing an illustrative example. Section VII concludes.

II. What Are the Shortcomings of Traditional Debt Sustainability Analysis?

The traditional debt sustainability analysis (DSA),[2] which aims to keep the public sector debt ratio stable or declining suffers from a number of shortcomings that can result in gaps in assessing debt sustainability.[3]

- First, a rising debt to GDP ratio does not necessarily imply unsustainable debt dynamics. Countries may have to run large deficits to smooth consumption, or increase expenditure in investment activities and structural reforms to enhance future growth prospects. This may lead to an increase in the debt ratio, but should not, in and of itself, imply that countries are pursuing fiscal policies that are unsustainable. In fact, the theory underpinning debt sustainability does not require a bounded debt ratio; it only requires that future primary surpluses are sufficient to satisfy the government's intertemporal budget constraint.

- Second, the main focus of the approach is on stabilizing the debt ratio, with very little attention paid to whether the level at which the debt stabilizes might be too "high" (unsustainable) or sufficiently "low" (sustainable). Most studies have attempted to fill this gap by mapping the debt ratios to a "safe" threshold, derived by examining the level of external debt at which defaults occur.[4] Not surprisingly, the studies produced estimates that are quite far apart, ranging from as low as 15-20 percent of GDP to 50-60 percent of GDP for emerging market countries. Estimates for mature countries are higher but equally far apart, with the sustainable debt to GDP ratio reaching as high as 350 percent and with an average value of 85 percent (Mendoza, 2003).

- Third, since the debt ratio is highly aggregated—short-term, long-term, foreign and local currency debt are usually lumped together—and released on low frequency, it does not properly account for the impact of changes in the maturity structure or currency composition of debt on debt sustainability. For example, the debt-to-GDP ratio does not change if a decrease in short-term foreign currency debt is matched by an equal book value increase in long-term foreign currency debt, even though there has been a decrease in sovereign risk due to the extended maturity profile. Similarly,

[2] The IMF and the World Bank use two types of DSA framework to analyze sovereign debt sustainability, one – for low-income countries, and the other – for emerging market economies and advanced countries. To assess debt sustainability, the traditional DSA uses ratios of public debt to GDP, fiscal balance, and exports on an annual basis.

[3] As pointed out most recently by Alvardo et al. (2004) and Chalk and Hemming (2000). See also Appendix I for details.

[4] Recent studies on external debt crises in emerging markets found varying thresholds for the sustainable level of public debt. They range from as low as 15-20 percent of GDP (Reinhart, Rogoff, and Savastano (2003)) to 50 percent of GDP (Manasse, Roubini, and Schimmelpfening (2003)).

it is hard to adapt an accounting based debt ratio to analyzing roll-over risk, where higher frequency cash flow considerations are more relevant.

- Fourth, the approach does not fully take into account the level and changes in the assets and liabilities of the public sector which affect debt sustainability. It frequently fails to incorporate some important public sector assets which are relevant to the ability to pay debt, such as foreign currency reserves of the monetary authorities and seignorage revenues. Also, certain contingent liabilities and other expenditures which can affect debt sustainability are usually excluded. Since changes in assets and liabilities may shift over time what is regarded as "safe thresholds" of debt-to-GDP, a debt sustainability assessment derived in the absence of a full understanding of balance sheet changes misses an important component of risk analysis.

- Fifth, the approach does not distinguish between the different nature of credit risk for sovereign foreign currency debt as compared to the risks associated with sovereign local currency debt. The risk premium for foreign currency denominated debt is a default (or forced restructuring) risk premium and there is an active market with frequent quotes on credit default swaps on emerging market foreign currency debt that can be seen as setting a "benchmark". The risk premium for local currency debt, on the other hand, is a combination of the risk of inflation/dilution and default risk (or forced restructuring). There are no publicly available credit default swap quotes on emerging market local currency debt. Clearly, this distinction is important since a sovereign cannot print foreign currency and, in the event of distress, may be forced to default on foreign currency obligations. However, sovereigns do have control over the issue of local currency debt and can print money to make promised payments on local currency debt. Since defaults are costly,[5] countries may choose to first inflate or dilute their local currency debt before contemplating a possible forced restructuring of public debt. Thus, in addition to default risk, the risk of inflation/dilution is embedded in the risk premium on local currency debt.

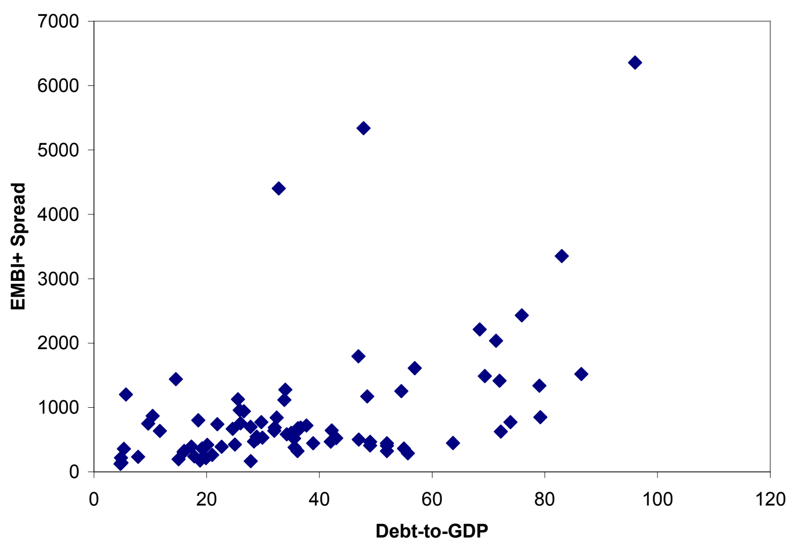
- Finally, the approach does not incorporate uncertainty or volatility in defining the underlying macroeconomic parameters that determine debt sustainability, relying instead on steady state assumptions on growth, real interest rates and the exchange rate. Volatility can be driven both by policy and by exogenous shocks, including political risk. Emerging market countries tend to experience greater volatility than mature markets, in part because of their limited capacity to raise taxes and a volatile revenue base. As Catao and Kapur (2006) have shown, external volatility in the terms of trade has a significant impact on the probability that countries will default, and differences in external volatility and macroeconomic volatility account for much of the differences in the default histories of countries on their external debt during the past several decades. Malone (2005) has shown that external volatility, in particular the volatility of the terms-of-trade, has a significant and positive impact on sovereign

[5] IMF, 2003.

spreads. If the exposure to external shocks for a country changes, then this would change what should be regarded as a "sustainable" debt level for the country. Macroeconomic volatility greatly increases countries' susceptibility to sudden stops and should be a factor that is taken into account in the assessment of debt sustainability. Abstracting from volatility means that the traditional debt sustainability analysis is not able to capture the nonlinear nature of credit events. To the extent that stress tests are conducted on the projected debt ratio—by making less optimistic assumptions on the underlying macroeconomic parameters than those assumed in the baseline scenario—they are conducted on an ad hoc basis and do not take into account correlations among the relevant macroeconomic parameters.

For these reasons it is not surprising, therefore, that the debt ratio has a very poor correlation with market based measures of credit risk, such as a spreads from credit default swaps or bond spreads, which are direct market measures of the risk premium needed to compensate for the risk default on (foreign currency) debt. Indeed, a plot of debt-to-GDP ratios versus credit spreads for a selected number of emerging market countries indicates at best a weak relationship between debt-to-GDP ratios and default risk (Figure 1).[6]

Figure 1. Correlation of Debt-to-GDP Ratio with Spreads: Panel Data for 19 Emerging Market Countries, 1996-2002.



[6] While the empirical literature on sovereign spreads and default find that debt-to-GDP is certainly an important determinant of both (Rowland and Torres (2004), Ades et al. (2000), Min (1998)), its value as a stand-alone predictor of either spreads or default is mediocre at best. Of course, this is usually true of any variable included in spread regressions or logit/probit analyses of default events; to get a good explanatory or predictive model it is important to utilize information on an array of relevant variables affecting the default probability.

III. A NEW RISK-BASED FRAMEWORK

This paper proposes a new risk-based framework that introduces the concept of distress (default) into public debt sustainability analysis (DSA). Distress is defined as the risk that the sovereign borrower does not have sufficient resources to meet outstanding debt service obligations on time, because its assets have fallen below promised payments on liabilities. This may lead to a debt restructuring that reduces the net present value of the debt. Sovereign borrowers pay creditors a spread for assuming the risk of loss of value either through non-payment, inflation, or dilution of local currency debt. Thus, the new framework explicitly takes into account the uncertain evolution of sovereign assets and liabilities, to provide for a risk adjusted sovereign balance sheet.[7]

A. Concepts of Sustainability with Credit Risk

The proposed conceptual framework makes a distinction in the treatment of foreign and local currency debt in pricing risk. This distinction is important because, in principle, to cover expected losses on local currency debt, the government could print more currency (causing inflation) or force a dilution of local currency debt by issuing more bonds. The option to print money is not available for foreign currency debt, and the ability to tap foreign currency is constrained by the amount held in reserves.

In this framework, debt sustainability means that the market value of debt remains above a specific threshold for a given confidence level over a specific time horizon. In other words, the expected loss and associated risk premium should lie below their corresponding specific thresholds for a given confidence level.[8]

- *Foreign currency* debt can be defined as sustainable if the probability of default or forced restructuring over a horizon period does not exceed a specific "low" threshold, for example, it does not exceed 0.50 percent over one year, which is a benchmark default probability recommended by the Basel Committee for risk management of financial institutions (Van Deventer and Imai, 2003). While other benchmarks of "low" or "sustainable" default probabilities could be made, a practical approach is to adopt the Basel recommendation as a benchmark for sovereign risk. A default probability of 0.5 percent over one year corresponds approximately to the BBB rating by sovereign rating agencies (Table 1). Hence, another way to define sustainability of foreign currency debt is to use average sovereign cumulative default probabilities—calculated by sovereign rating agencies for each rating grade and maturity—as thresholds. While spreads vary according to the recovery rate and liquidity, a 0.5 percent cumulative default probability usually corresponds to credit

[7] Sovereign balance sheet was first introduced in the contingent claims approach (CCA), see Gapen et al (2004 and 2005), Gray (2002), Gray, Merton, and Bodie (2002), and Gray and Malone (2008), for a full description of the CCA approch.

[8] The value of debt is the present value of promised payments less the present value of expected losses.

spreads in the range of 50 to 1000 basis points. This quantifies the impact that sovereign spreads on foreign currency debt have on the cost of new borrowing or refinancing, and spreads beyond a certain threshold are likely to signal a significantly reduced ability to borrow (roll-over risk). There are active market quotes pricing default risk on sovereign foreign currency debt, such as credit default swaps (CDS) spreads and Emerging Markets Bond Index Global (EMBIG) spreads (Table 1).

Table 1. Approximate Sovereign Cumulative Default Probabilities*
(percent)

Rating	1-year	3-year	5-year	7-year
AAA	0.0	0.0	0.0	0.0
AA	0.1	0.3	0.6	0.8
A	0.3	0.9	1.4	1.8
BBB	0.5	1.5	2.7	3.6
BB	1.4	5.6	9.0	16.7
B	7.0	16.0	22.0	32.0
CCC/CC	19.0	39.0	53.3	60.5

* The default frequencies presented here are only indicative and were obtained by averaging the sovereign data for the Standard&Poor's with the sovereign data for Moody's for the same rating categories, when available. When neither were available, the corporate data from rating agencies and Crouhy, Mark, and Galai (2000) were used.

- *Local currency debt* can be defined as sustainable if expected losses and associated risk premiums are kept below specific thresholds for a specific time period. However, it is difficult to define sustainability thresholds for the local currency debt, as (i) there are no market quotes on CDS spreads for emerging market countries;[9] (ii) there are no default probability matrices produced by rating agencies; (iii) the debt is not necessarily actively traded on the secondary market; (iv) there is no extensive history of defaults on local currency debt in the majority of countries; and (v) the government could print more currency (causing inflation) or force a mandatory restructuring of this type of debt to cover expected losses on local currency debt.[10] Thus, thresholds for the local currency debt should be derived using risk analysis, and the thresholds could differ for countries under specific circumstances and time horizons, taking into account feedback from macroeconomic policies and exogenous shocks. Deriving specific thresholds for local currency debt is beyond the scope of this paper and more research is needed to identify appropriate methods for calculating such debt sustainability thresholds.

An important goal of the DSA should be to provide policymakers with a framework that offers alternatives for policy decisions and is not restricted to a quantitative assessment of the

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[9] The absence of CDSs reinforces the notion that the risks associated with local currency debt are different from those for foreign currency debt.

[10] Risk premiums for those possibilities are built-in in sovereign indexed debt or floating rate debt.

future prospects for the level of public indebtedness. In particular, within the framework proposed in this paper, sovereign borrowers could evaluate the impact of various policies—alternative debt structures; changes in fiscal policy and reserve holdings; and use of hedging instruments to mitigate potential risks.

IV. Contingent Claims Approach for Risk-based Debt Sustainability Analysis

Risk-based public debt sustainability can be assessed using the contingent claims approach (CCA). The CCA defines the fundamental relationships between the value of the assets and the value of the claims.[11]

Three elements in the sovereign balance sheet are important in defining distress risk: (i) the value of sovereign assets; (ii) the volatility of sovereign assets; and (iii) the distress barrier derived from the schedule of promised payments on debt. The distress barrier typically takes the form of the book value of short- and long-term debt obligations, including the interest payments and amortized principal. More specifically, the distress barrier is defined as the sum of (i) the principal payments discounted to the present at the risk-free rate and (ii) the present value of the interest payments up to maturity, t. [12]

The value of risky debt can be expressed as[13]

$$D = Be^{-rt} - P = Be^{-rt} - \left(Be^{-rt}N\left(-d_2\right) - A_0 N\left(-d_1\right)\right) \tag{1}$$

where r is the risk-free interest rate; B is the default-free value of debt at time zero; A_0 is the asset level at time zero; and the formulas for the d_i-s are described in the Appendix. The risk premium—or credit spread—to compensate for expected loss is defined as

$$s = y_t - r = -\frac{1}{t}\ln\left(1 - \frac{P}{Be^{-rt}}\right) \tag{2}$$

where y_t is the yield-to-maturity for the risky debt, derived from $D = Be^{-y_t t}$.

For a framework with both senior and subordinated debt with distress barriers B_{Sr} and B_{Sub} correspondingly, the spreads can be expressed as

[11] See Gray, Merton, and Bodie (2002, 2006) and Gapen et al. (2005), Gray (2007) for detailed description of the CCA approach. Also see Gray, Merton, and Bodie (2007).

[12] The distress barrier can be approximated as the sum of the present value of principal payments, discounted at the risk-free rate and the present value of interest payments up to horizon t.

[13] See Appendix for the details.

$$s_{SrDebt} = -\frac{1}{t}\ln\left(1 - \frac{P_{Sr}}{B_{Sr}e^{-rt}}\right), \text{ and} \tag{3}$$

$$s_{SubDebt} = -\frac{1}{t}\ln\left(1 - \frac{P_{Sub}}{B_{Sub}e^{-rt}}\right). \tag{4}$$

A. CCA and Asset Value and Volatility

The distress barrier can be measured using balance sheet information; however, the value and volatility of assets cannot be measured directly. Although the market value of sovereign assets is not directly observable, it can be estimated using the "contingent claims" relationship between sovereign assets and liabilities.[14] Therefore, the value and volatility of sovereign assets can be derived from observable prices on the value and volatility of liabilities. Market participants incorporate their collective view in observable market prices of liabilities, which means that the implied asset value and volatility reflect the best available forecast of the expectations of market participants.[15]

Using the consolidated balance sheet of the government and the central bank, the CCA allows one to observe the foreign currency market value of base money and local currency debt and the distress barrier of foreign currency debt. The base money and local currency debt converted into foreign currency have similarities to the "market capitalization" of the sovereign, and they are junior liabilities, since they can be issued or repurchased similar to "shares." Thus, the CCA can use the market value of the junior liabilities of the sovereign, similar to the way Moody's KMV uses the market prices of a company's market capitalization (equity) to infer asset value and risk indicators to assess the sustainability of the company's debt.[16]

B. Extended Model for Debt with Different Priorities

Countries set different priorities for foreign and local currency debt, as in many cases the sovereigns find it easier to inflate or dilute local currency debt in a distress situation before

[14] Gapen et al., 2005 and Gray and Jones, 2006.

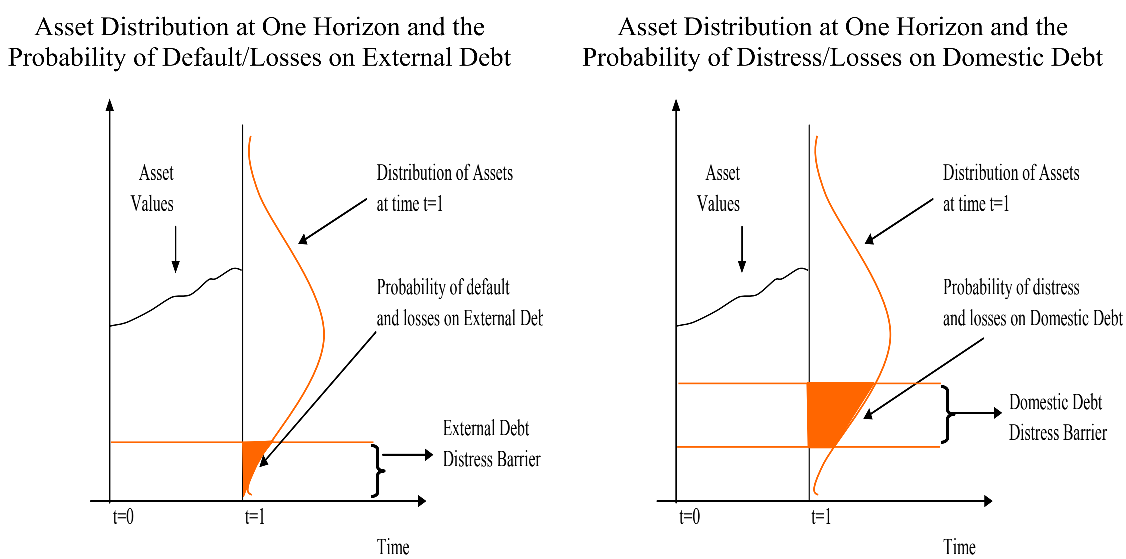
[15] CCA has already been applied to over 200,000 firms and financial institutions around the world by Moody's KMV and others, and it is used by over 2,500 banks and financial institutions.

[16] Sims (1999) treats local currency debt as "equity." See also, Gray, Merton, and Bodie (2006) and Keller, Kunzel, and Souto (2007). Base money and local currency debt have value in international financial markets similar to "sovereign equity" .In the event of a sovereign default, creditors would typically received restructured debt of a lower value (with a principal haircut). Having said that, sovereigns prefer not to default but restructure their distressed debt in a similar manner as corporates to preserve their sovereign ratings and reputation for future borrowing at the lowest possible interest charges.

defaulting on foreign currency debt.[17] In this case, foreign currency debt becomes senior debt, while local currency debt becomes subordinated debt.[18] Similarly, the debt priorities can vary with maturity for the debt of the same currency.

The difference between probability distributions and distress barriers of senior and subordinated debt shows that spreads are smaller (larger) than the risk premium on local currency (foreign currency) debt (Figure 2).[19] This situation seems to reflect debt structures in countries with relatively high levels of both foreign and local currency debt,[20] such as Brazil and Turkey.

Figure 2. Asset Distribution for Debt of Different Priorities



Source: MfRisk

The model points out several important differences. First, the model indicates low spreads on foreign currency debt—in this example is around 40 basis points for one year— that may be observed in the CDS markets. Second, while there are no CDS spreads for local currency debt or other directly observable market measures, the estimated premiums on floating rate local currency debt are high—about 200-400 basis points. High premiums are likely due to

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[17] Sims (1999) and Gapen et al. (2005) have observed that sovereigns frequently treat their foreign debt as a senior claim on their assets.

[18] Foreign currency debt is not always homogeneous, for example, claims held by some creditors—such as multilateral international financial institutions, Paris Club, etc.—are effectively treated as senior to private creditors. Also, some foreign currency claims are collateralized.

[19] Since the foreign currency debt barrier is below that of the local currency debt, the area underneath the distress barrier for foreign currency debt is smaller than that for local currency debt.

[20] Most of such debt has relatively short maturity of one to four years, and there is a high proportion of floating and inflation linked debt.

credit risk on top of a “typical” inflation premium, which depends on factors unrelated to debt management operations. The risk premium wedge between term structures of foreign real, foreign nominal, and local currency nominal interest rates is a combination of two factors, which are difficult to separate: (i) expected and unexpected inflation (unrelated to debt risk premium) and (ii) credit risk premium on local currency debt.[21]

These types of debt structures seem to be vulnerable to a potential vicious circle that a CCA model can capture. The risk premium on the local currency debt increases, if the value of sovereign assets declines, and the volatility increases—due to a lower fiscal revenue, a more depreciated local currency, and a higher exchange rate volatility. This feeds into a higher forward exchange rate, increasing spreads on foreign currency debt. Thus, a vicious circle can lead to a spiral of more depreciated and more volatile spot and forward exchange rates as well as significantly higher risk premiums on local and foreign currency debt (Gray and Malone, 2008).

While the model proposed in this paper is approximate, it can be used to test what factors—such as higher fiscal revenues, longer maturity of local currency debt, a higher share of fixed interest debt in the debt portfolio, as opposed to floating rate or inflation linked debt—may reduce the severity and likelihood of such a vicious circle. This model can also be useful in quantifying debt structures and assessing what kind of debt structures and currency compositions have lower risk.

If foreign and local currency types of debt have the same priority, the model calculates the spreads and risk premiums by combining the two distress barriers into one. The spreads then become the same for foreign and local currency debt. For example with the same priority, one-year spreads on both types of debt could be 120bps. However, if the foreign currency debt is considered “senior,” the foreign currency debt spread would be 40bps and the local currency risk premium would be 300 bps (200-400 bps).[22]

V. Model Applications: Scenario Analysis

The main difference between the traditional and risk-based DSA is that in the traditional DSA framework, the risk indicator (state variable) is the future debt-to-GDP ratio, whereas in the modified CCA framework, the risk indicators are the credit spread of foreign currency debt, its default probability, and risk premiums on local currency debt. Using the debt-to-GDP ratio as the metric for ranking alternative policies may create conceptual and practical difficulties and limit its usefulness. The modified CCA framework defines debt sustainability in terms of keeping risk indicators (state variables) below certain threshold values, which is

[21] See Keller, Kunzel, and Souto (2007) for an application of these ideas to the case of Turkey.

[22] For example, Eastern European countries, which are planning to join the Euro area may have lower exchange rate volatility than other emerging market countries and expect to benefit from contingent financial support from the EU in the future. They also may be expected to treat the priority of different types of debt equally since they will be likely operating in the legal environment of the EU.

what countries and investors care most about. The modified CCA model delivers predictions for market spreads, a default probability, and a distance to distress for a time horizon of one to five years ahead.

The model introduced in the previous sections can be used for analyzing the impacts of the changes in sovereign balance sheet components and macroeconomic policy variables on debt sustainability. Moreover, the model allows one to analyze the effect of changes in model parameters on the risk indicators and the level of sovereign assets.

To incorporate macroeconomic variables into the CCA framework, one needs to connect them to the process that controls the value of sovereign assets. The sovereign asset value can be viewed as a portfolio of the following components: (i) international reserves (R), (ii) present value of the primary fiscal surplus (PVPS), (iii) contingent liabilities (CL), and (iv) other remaining items (Other):

$$Asset_{Sov\$,0} = R + PVRS_{\$} - CL_{\$} + Other_{\$} \tag{5}$$

The value of the foreign currency reserves can be observed and the guarantee can be estimated from the banking and corporate sector CCA model. Subtracting these from the implied sovereign asset, one can calculate the residual which includes the primary fiscal surplus. Provided that the expected present value of the primary fiscal surplus is estimated, "Other" can be expressed as[23]

$$Other_{\$} = Asset_{Sov\$,0} - R_{\$} + CL_{\$} - E\left[PVRS_{\$}\right]. \tag{6}$$

Formula (5) is used to evaluate the effect of changes in key macroeconomic variables on sovereign assets, and consequently, on risk indicators, spreads and distress probabilities.

The change in for spreads on foreign currency debt due to changes in key macroeconomic variables can be written as

$$\Delta s_{ForDebt} = -\frac{1}{t}\left(1 - \frac{P_{ForDebt} + \Delta P_{ForDebt}}{B_{ForDebt} + \Delta B_{ForDebt}}\right), \tag{7}$$

where the changes in the implicit put option, P, come from changes in key macroeconomic parameters:

$$\Delta P_{ForDebt} = f\left(A_{Base} + \Delta PVRS_{\$} + \Delta R_{\$} - LC_{\$}, B_{ForDebt} + \Delta B_{ForDebt}, B_{LCDebt} + \Delta B_{LCDebt}\right). \tag{8}$$

Correspondingly, the change in spreads on local currency debt due to changes in key macroeconomic variables can be written as

[23] "Other" may be due to various factors, including contingent financial support from other governments or multilaterals.

$$\Delta s_{LCDebt} = -\frac{1}{t}\left(1 - \frac{P_{LCDebt} + \Delta P_{LCDebt}}{\left(B_{LCDebt} + \Delta B_{LCDebt}\right) - \left(B_{ForDebt} + \Delta B_{ForDebt}\right)}\right), \tag{9}$$

where the changes in the implicit put option come from changes in key macroeconomic parameters

$$\Delta P_{LCDebt} = f\left(A_{Base} + \Delta PVRS_{\$} + \Delta R_S - CL_{\$}, \sigma_{Base} + \Delta\sigma_{Base}, B_{ForDebt} + \Delta B_{ForDebt}, B_{LCDebt} + \Delta B_{LCDebt}\right), \tag{10}$$

The method developed for conducting policy analysis under the modified CCA framework is similar to that under the traditional DSA framework. In particular, after estimating a baseline scenario, the model allows one to evaluate scenarios with specific policy changes. Table 2 summarizes the effects of changing key variables in the CCA framework for DSA.

Table 2. The Effects of Changing Key Variables in the CCA Framework for Debt Sustainability

Change in Input:	Effect in Model:	Change in Output:
Primary Surplus $\uparrow$	$A_S \uparrow$	D2D $\uparrow$, SP$\downarrow$, RP$\downarrow$
Reserves $\uparrow$	$A_S \uparrow$	D2D $\uparrow$, SP$\downarrow$, RP$\downarrow$
DB (For.) $\downarrow$	DB $\downarrow$	D2D $\uparrow$, SP$\downarrow$, RP$\downarrow$
DB (Domestic) $\downarrow$	DB $\downarrow$	D2D $\uparrow$, SP$\downarrow$, RP$\downarrow$
Ex. Rate Vol. $\uparrow$	$\sigma_A \uparrow$	D2D $\downarrow$, SP $\uparrow$, RP$\uparrow$
Other Vol. $\uparrow$	$\sigma_A \uparrow$	D2D $\downarrow$, SP $\uparrow$, RP $\uparrow$
Stock Mkt. $\downarrow$	$A_{Corp} \downarrow \Rightarrow A_{Banks} \downarrow \Rightarrow Guarantee \uparrow \Rightarrow A_S \downarrow$	D2D $\downarrow$, SP $\uparrow$, RP $\uparrow$

Here D2D is distance-to-distress, SP is credit spread on foreign currency debt, and RP is risk premium on local currency debt.

The model can also be used for simulation analysis, taking the distributions of key parameters as inputs. Then, Monte Carlo simulations can be run to construct output distributions for the default probability and the sovereign spreads. The inputs can be shocked to obtain the resulting impact on the distributions of the sovereign spreads and default probability.

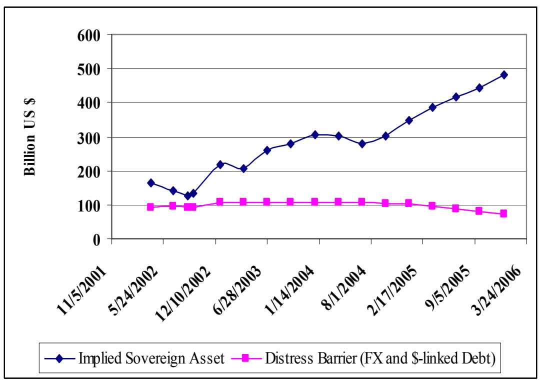
VI. An Illustrative Example

This section provides an example of the application of the model developed in this paper for debt sustainability exercise. Implied sovereign asset value and volatilities are calculated, using information from the book values of sovereign local currency liabilities (base money and local currency debt), market prices on the level and volatility of the forward exchange rate, and the book value of foreign currency obligations. The debt is considered sustainable, if at each point in time, asset value is higher than a pre-defined distress barrier.

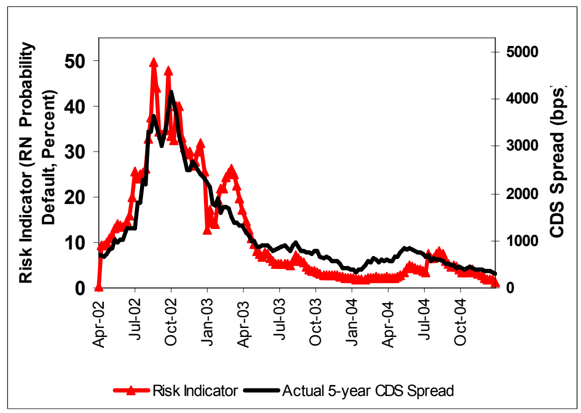
The following illustrative example demonstrates that for a country X, implied sovereign assets were close to the distress barrier in 2002 and have since risen above it (Figure 3a). The model then allows one to calculate risk indicators using the sovereign asset value distribution. The same example shows that the risk indicators for foreign currency debt are highly correlated with the actual five-year CDS spreads on foreign currency debt (Figure 3b).[24]

Figure 3. An Illustration of Model Application

(a) Sovereign Asset Value vs. Distress Barrier (External and $-Linked Debt)



(b) Risk Indicator and Actual 5-yr Spread on Sovereign Foreign Currency Debt



Source: Staff simuations and calculations.

In addition to the analysis of the level of sovereign assets and risk indicators, the model can be used as an integrated framework for debt management, fiscal sustainability, and reserve management. The risk-based DSA framework described in this paper allows one to rank alternative policy choices by the probability of debt sustainability for each time horizon. The probability of debt sustainability gives the debt manager a tool to quantify the effects of a given policy on debt sustainability and to determine which combination of policies produces higher sustainability probabilities and, thus, lower spreads.

The next exercise illustrates how debt management and fiscal policies can improve the sovereign's risk profile and reduce spreads in different states of the world. An example of an external shock for a hypothetical country for a "bad state of the world" includes: (i) 30 percent depreciation of the spot and forward exchange rates and associated increase in exchange rate volatility; (ii) US$8 billion decline in net international reserves; and (iii) 2.5 percentage point increase in interest rates. After applying these changes, the calibrated CCA

[24] Sovereign spreads are not used as inputs to calculate these risk indicators.

model suggests that this shock would increase spreads on five-year debt instruments by about 170bps.

However, the potential increase in spreads can be mitigated by a combination of debt management and fiscal policy adjustments. For the above shock, the policy responses include: (i) an increase in primary fiscal surplus from 4.25 percent to 5 percent for each of the next 10 years (which increases assets and lowers the spreads); (ii) use of reserves to buyback USD20 billion of the external debt (which reduces the distress barrier and leads to slightly lower spreads); (iii) an extension of the maturity of foreign currency denominated debt by one year (which also reduces the distress barrier and results in slightly lower spreads); and (iv) an extension of the maturity of local currency denominated debt by two years (which lowers the risk premium for local currency debt). All policies together would reduce five-year spreads from over 225bps to 145bps and local currency risk premiums from 5.5 percent to 3.5 percent (Table 3).

Table 3. Risk Indicators in Scenario Analysis

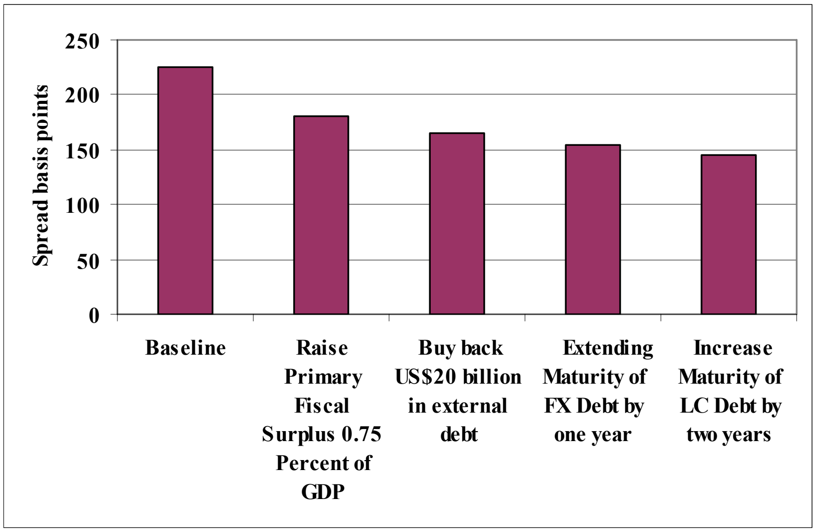
	Risk indicators	
	5-year spread	Local-currency risk premium
Historical scenario - 2004	350	7.0
Baseline	225	5.5
External shock scenario		
Policy scenarios		
Primary fiscal surplus increase by 0.75 percent of GDP	-45	-1.5
Buyback of USD20 billion of external debt	-15	-0.03
Extension of maturity of foreign currency debt by one year	-10	0.0
Extension of maturity of local currency debt by two years	-10	-0.5
Combined impact of policy changes	-80	-2.03

Source: Staff simulations.

An improvement in the primary fiscal surplus, as well as external and domestic debt maturity extensions and external debt buybacks, could plausibly be utilized to reduce five-year spreads on foreign currency debt by 75 bps in the good state of the world, with a 200 bps reduction on local currency debt. The expected spreads on foreign currency debt would progressively decline as the policy options can be subsequently added (Figure 4).[25]

[25] The risk-based DSA allows one to add horizons of specified frequency (such as annual, quarterly, etc.) to create macro scenarios and future debt profiles.

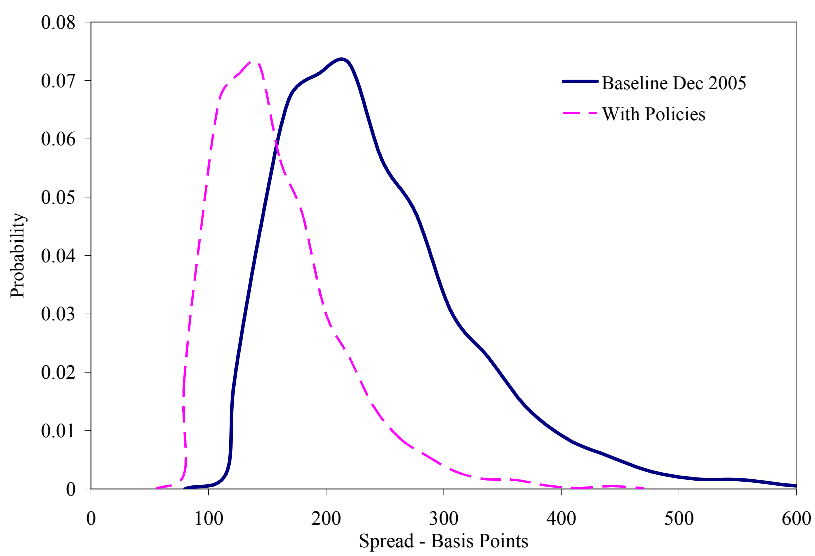
Figure 4. Changes in Spreads Caused by Policy Changes



Source: Staff calculations.

Simulations of the probability distribution of five-year spreads show that policy changes shift the distribution to the left (Figure 5), thus reducing the mean spread from 225 to 145 basis points. The leftward shift of the distribution has lowered the area in the distribution that corresponds to extremely high spreads. In this example, there is a 10 percent probability that spreads will exceed 400bps in the baseline scenario, but more than 10 percent of the tail of the distribution, i.e., the worst spreads, lies to the right of the 250 bps mark after the implementation of favorable policies.

Figure 5. Probability Distribution of 5-year Spread with and without Policy Changes



Source: Staff simulations.

VII. CONCLUSION

Monitoring the financial fragility of the public sector balance sheet and understanding the feedback process between sovereign borrowers and capital markets are crucial steps in identifying the risks of capital account crises and their potential for contagion. A new framework for assessing public debt sustainability presented in this paper incorporates the sovereign balance sheet and the effects of uncertainty. The CCA-based framework for DSA takes into consideration many of the shortcomings of the traditional DSA in order to provide a more complete assessment of debt sustainability. Instead of mapping fiscal balance, growth, and exchange rate dynamics to the future debt-to-GDP ratio, the CCA approach maps these and other relevant macroeconomic variables and components of the sovereign balance sheet to the default probabilities and the sovereign spreads on public debt. It also augments the CCA-based framework to include a "bottom-up" analysis of the evolution of sovereign asset value based on underlying processes for key macroeconomic variables, such as GDP, exchange rate, primary fiscal balance, and others.[26]

The current framework is designed to allow debt managers and policy makers to conduct scenario analysis by changing the level, growth rate, and volatility of key parameters to assess the impact on spreads and real default probabilities. In addition, several types of alternative policies can be evaluated and ranked according to their effectiveness at reducing the probability of distress and the cost of borrowing. Examples of such policies include changes in reserve management, changes in maturity structure of debt, more conservative (liberal) fiscal policy, a different exchange rate regime, and hedging of commodity price fluctuations. This framework is able to handle a variety of policies that could not be incorporated easily into the traditional DSA framework. It is also able to evaluate the impact of a wider variety of shocks, including the effects of supply-side and external shocks, such as changes in the risk-free yield curve in the U.S. and increased volatility or spreads in the U.S. high yield debt markets.

The framework developed in this paper does not provide a tool for calculating thresholds for sustainability of local currency debt. The risk associated with local currency debt is different from that related to foreign currency debt, as explained above. In the absence of direct measures of risk for local currency debt, new methods for defining sustainability thresholds should be developed. This is an area for future research.

[26] This framework is similar to the model developed by Xu and Ghezzi, 2002, for the fair value of spreads on emerging market debt.

APPENDIX

A. Uncertainty in Public DSA

Variability of sovereign assets relative to the promised debt payments can be used to measure credit risk. Indeed, a government becomes a more risky creditor, if volatility of the market value of its public assets goes up. Therefore, this government should be charged a higher risk premium than the one with less volatile market value of public assets. The volatility of the assets can be derived from the evolution of assets, which is typically modeled by a diffusion stochastic process that has expected drift and an uncertain component.[27]

Let $A(t)$ be the value of public sector assets (hereafter, "assets") at time t.[28] The asset level depends on changes in international reserves, domestic currency in circulation, present value of the primary fiscal surplus, and other relevant items. The dynamics of the assets can be expressed as

$$\frac{dA}{A} = \mu_A dt + \sigma_A dZ \tag{11}$$

where μ_A is the drift, that is the expected (average) growth rate of the asset value; σ_A is the standard deviation of the asset value; and $dZ, dZ \in \mathrm{N}(0,1)$ is a standard Wiener process.[29]

Thus, the value of assets at time t can be expressed as

$$A_t = A_0 \exp\left[\left(\mu_A - \frac{\sigma_A^2}{2}\right)t + \sigma_A \varepsilon \sqrt{t}\right] \tag{12}$$

Note that the drift is adjusted by $\left(-\sigma_A^2/2\right)$ to get a "true" uncertainty of the evolution of the value of the assets.

B. Distance to Distress in Public DSA

The basic approach to distance to distress is the following. Let B_t be the level of promised payments on the debt—present value of the principle discounted at the risk-free rate plus

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[27] This approach, pioneered by Black and Sholes (1973) and Merton (1974), is commonly used to describe the behavior of a wide variety of assets of all types in finance literature.

[28] Public sector assets are consolidated assets of the government and the monetary authority.

[29] A standard Wiener process is also referred as a geometric Brownian motion, or a random walk, where $dZ = \varepsilon\sqrt{t}$, with $\varepsilon \in N(0,1)$ being normally distributed with zero mean and unit variance.

interest payments up to time t. Since a default occurs when the asset value falls below the distress barrier B_t, the probability of default can be expressed as

$$P\left(A_t \le B_t\right) = P\left(A_o\left[\left(\mu_A - \frac{\sigma_A^2}{2}\right)t + \sigma_A \varepsilon \sqrt{t}\right] \le B_t\right) \tag{13}$$

which can be rewritten as

$$P\left(\varepsilon \le -DD_\mu\right) \tag{14}$$

where $DD_\mu = \dfrac{\ln\left(\dfrac{A_0}{B_t}\right) + \left(\mu_A - \dfrac{\sigma_A^2}{2}\right)t}{\sigma_A\sqrt{t}}$ is the distance to default for assets with distress barrier B_t. Since ε is normally distributed, the probability of default on the debt also follows the standard cumulative normal distribution with the variance equal to minus distance to distress. In other words, DD_μ is the number of standard deviations of the asset from the default barrier.[30]

C. Value and Credit Spread for Risky Debt

The value of risky debt is a claim on uncertain assets (or a contingent claim), $A(t)$, because it is the difference between the default-free value of the debt and the expected loss. The expected loss can be expressed as an implicit put option,

$$P = Be^{-rt}\left(N\left(-d_2\right)\right) - A_0\left(N\left(-d_1\right)\right) \tag{15}$$

where r is risk-free interest rate; B is the default-free value of debt at time zero; A_0 is the asset level at time zero; $d_2 = \dfrac{\ln\left(\dfrac{A_0}{B}\right) + \left(r - \dfrac{\sigma_A^2}{2}\right)t}{\sigma_A\sqrt{t}}$; $d_1 = d_2 + \sigma_A\sqrt{t}$; and $N(\cdot)$ is the cumulative normal distribution function. The CCA model gives a risk neutral or risk adjusted default probability of $N\left(-d_2\right)$. (See Gray, Merton, Bodie forthcoming for more details.)

Thus, the value of risky debt can be expressed as

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[30] The basic approach outlined here can be extended to the case of multiple claims on the sovereign asset, i.e. multiple layers of debt, each with a different distress barrier.

$$D = Be^{-rt} - P = Be^{-rt} - \left(Be^{-rt}N(-d_2) - A_0 N(-d_1)\right) \tag{16}$$

Risk premium—or credit spread—to compensate for the expected loss is defined as

$$s = y_t - r = -\frac{1}{t}\ln\left(1 - \frac{P}{Be^{-rt}}\right) \tag{17}$$

where y_t is the yield-to-maturity for the risky debt, derived from $D = Be^{-y_t t}$.

For a framework with both senior and subordinated debt with distress barriers B_{Sr} and B_{Sub} correspondingly, the spreads can be expressed as

$$s_{SrDebt} = -\frac{1}{t}\ln\left(1 - \frac{P_{Sr}}{B_{Sr}e^{-rt}}\right), \text{ and} \tag{18}$$

$$s_{SubDebt} = -\frac{1}{t}\ln\left(1 - \frac{P_{Sub}}{B_{Sub}e^{-rt}}\right).$$

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