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Sudden Stops and Optimal Self-Insurance

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Abstract

This paper presents a simple model of optimal reserves that can be easily calibrated to
compute optimal reserves as well as the implied probability of a sudden stop for given
reserves. The model builds upon the global games framework of Morris and Shin to establish
a unique relationship between the probability of a sudden stop and the level of reserves. The
calibration results for 15 selected emerging market countries in Latin America, Asia and
other regions over the sample period of 1993-2006 suggest that the risk of sudden stops may
have declined to a low level in recent years in all countries in the sample. The results also
suggest that Asia and Russia may have been significantly over insured since early 2000s with
estimated excess reserves of US$ 1 trillion in total at end-2006.

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I. INTRODUCTION

The recent large increase in foreign reserves, especially in Asia, has received renewed attention in various policy fora and academic circles. Two competing views have been put forth regarding this recent trend. The "self-insurance" view suggests that the rapid accumulation of reserves may reflect the increased risk of a balance-of-payments crisis or the increased cost of a crisis, or both—a view broadly in line with many conventional models of optimal reserves. According to this view, the Asian crises in the late 1990s may have changed significantly the perception of national authorities and international investors of the risk of highly disruptive financial crises. The "mercantilist" view, put forth for example by Dooley, Folkerts-Landau and Garber (2003), argues that the rapid and seemingly excessive accumulation of reserves is a by-product of the export-driven growth strategy that these countries have pursued by maintaining an undervalued exchange rate.

Several recent studies have attempted to shed light on these two opposing views.[2] Edison (2003) has taken up this issue by estimating the optimal reserves, and comparing the out-of-sample projections with actual reserves. Aizenman and Lee (2006) have shown that the mercantilistic motive for reserves accumulation has little explanatory power for actual reserves, and based on this result, developed a simple model that can generate large precautionary demand for reserves.

Jeanne and Ranciere (2006) have departed from the regression analyses by developing a simple model of absorption smoothing to derive the optimal level of reserves, and calibrating it to examine whether the calibration results match the behavior of actual reserves for the sample of middle-income countries over the period 1975-2003. In a similar context, Garcia and Soto (2004) develop a model in which the crisis probability depends on the ratio of reserves to short-term debt, and the country seeks to minimize the expected cost of a crisis and holding reserves. Their estimates of the optimal reserve levels for East Asian countries over the period of 2000-2003 turn out to be highly sensitive to the assumed cost of a crisis, however.

These studies have several limitations. First, the reduced-form regression analyses, albeit useful to explain the determining factors of reserve levels, have little to say about the optimal level of reserves. For this reason, judging the extent of excess reserves or reserve shortfalls based on out-of-sample forecasts—as was done by Edison (2003)—has little theoretical basis. Second, the existing calibration models of optimal reserves often require the estimation of the probability (and the size) of a sudden stop from actual data. In most cases, sudden stops are identified by the incidence of net capital outflows above certain threshold (e.g., 5 percent of GDP), which is inevitably *ad hoc*. More important is that the estimated

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[2] There has been vast literature on optimal precautionary reserves since the early 1960s. More recently, Frenkel and Jovanovic (1981) developed a stochastic model of optimal reserves. Ben-Bessat and Gottlieb (1992) provided a prototype model that takes an explicit account of the risk and output cost of a crisis in the determination of the optimal reserve level.

probability of a sudden stop (or crisis) explains relatively little variation in actual data (with very low pseudo-R^2) and, in some cases, is assumed to be exogenous to reserves.

In this paper, we develop a simple model of optimal precautionary reserves by using the global games framework of Morris and Shin (2006). Unlike in other studies, the probability and the size of sudden stop is endogenously determined as an equilibrium outcome of the creditor coordination problem. This feature of the model is particularly useful to explain how a sudden stop can be triggered by liquidity shortage in emerging market countries. Assuming that a country seeks to minimize the expected cost of a crisis taking account of the opportunity cost of holding reserves, the model generates an analytically tractable closed-form solution for the optimal level of precautionary reserves. Consequently, the model can be directly calibrated with no need for ancillary estimation of the probability of a crisis from data. Moreover, the model can be calibrated to compute the ex ante implied probability of a sudden stop for both optimal and actual reserve levels.

The model is calibrated for selected emerging market countries over the sample period of 1993-2006. According to the calibration results, both Latin America and Asia, except for a few countries, have been under insured in the first half of the sample period, but far better insured in the latter half. Not surprisingly, the results also suggest that most Asian countries in the sample may have been increasingly over insured since the early 2000s. The calibrated probabilities of a sudden stop for actual reserves often increase noticeably in the run-up to a crisis among countries that experienced a crisis at some point in time during the sample period, rendering some support for the use of the model for diagnosing the vulnerability to the risk of sudden stops and informing reserve management policy.

The rest of the paper is organized as follows. Section II sets up a global games model of the creditor coordination, and derives the optimal reserve formula assuming that emerging market countries minimize the expected output cost of a sudden stop while taking account of the cost of holding reserves. Section III discusses the results of the model calibration undertaken for selected emerging market countries. Section IV concludes the paper.

II. THE MODEL

Consider an emerging market country that faces the risk of sudden stops given the limited access to capital markets, particularly with regard to short-term debt obligations. For simplicity we assume that the country has no external lending so that gross debt equals net debt. We also assume that the country's total external debt—which is composed of long-term debt and short-term debt—is sustainable as long as all short-term debt is rolled over. Despite this assumption, the country may default because of illiquidity, which is the focus of the model. Since long-term creditors play no role in determining whether the country defaults or not, we focus hereafter on the behavior of the short-term creditors.

To model the limited market access of emerging market countries, we make a distinction between the demand (by private creditor) for and the supply (by the country) of short-term borrowing, which are denoted by ΔD^d and ΔD^s respectively where D stands for short-term debt. Specifically, we assume that

$$\Delta D^d \leq \Delta D^s \quad \text{if } \Delta D^s \geq 0$$
$$\Delta D^d = \Delta D^s \quad \text{if } \Delta D^s < 0$$

This assumption indicates that the country's market access could be limited only if there is a balance of payments need: otherwise, the country faces no risk of sudden stops. Accordingly, ΔD is demand-determined since $\Delta D \equiv \min[\Delta D^d, \Delta D^s] = \Delta D^d$.

Using the balance of payment identity, the evolution of the stock of reserves is characterized by

$$R_{t+1} = R_t + (CA_{t+1} + KA_{t+1} - \Delta D_{t+1}) + \Delta D_{t+1} = R_t + \theta_{t+1} + \Delta D_{t+1}$$

where R is the stock of reserves at the end of the period, CA and KA stand for the current and capital/financial account balance, respectively, and θ is the change in reserves net of short-term borrowing. Given the assumption that ΔD is demand-determined, the country would remain liquid and avoid default at each point in time only if:

(1) $$R_t + \theta_{t+1} \geq -\Delta D_{t+1}^d$$

This liquidity condition simply requires that the country's total liquid assets should be large enough to meet net capital outflows.

Creditor Coordination Problem

The liquidity condition (1) clearly indicates that the occurrence of sudden stops (and liquidity crises) depends on the behavior of creditors represented by ΔD^d. In order to model the behavior of short-term creditors, we build upon the global game framework developed by Morris and Shin (2006). As is well known, multiple equilibria are common in many models of currency or banking crisis in which the outcome of creditor or depositor coordination is not uniquely pinned down. By assuming noisy information for creditors/depositors, however, the global games framework generates a unique equilibrium solution to the coordination problem. As such, the framework is particularly useful in our setting in that it allows for systematically linking the probability of a sudden stop to the level of reserves and, hence, deriving an analytically tractable solution for the optimal level of precautionary reserves.

We suppose that there is a continuum of risk-neutral short-term creditors whose total mass is normalized to 1. Denoting by $x \in [0, 1]$ the share of short-term creditors who roll over, gross short-term borrowing can be expressed as

(2) $$(\Delta D_{t+1}^d + D_t) = x \cdot (\Delta D_{t+1}^s + D_t) \quad \Rightarrow \quad \Delta D_{t+1}^d = x \Delta D_{t+1}^s - (1-x) D_t$$

With this specification, the country retains full market access for $x = 1$ while being completely shut out from external borrowing if $x = 0$. Therefore, the country's market access

is limited whenever $x < 1$. By using (2) and dropping time subscripts for simplicity, the liquidity condition (1) can accordingly be reformulated to yield,

$$(3) \qquad\qquad R + \theta + \Delta D^s \ \geq \ (1-x)(D + \Delta D^s)$$

The inspection of (3) suggests that certain restrictions should be imposed on the value of ΔD^s since, otherwise, the inequality would hold trivially by increasing the supply of short-term debt indefinitely.[3] An explicit modeling of the determination of ΔD^s as an outcome of the country's optimization is beyond the scope of this paper. Instead, we simply assume $\Delta D^s = 0$ in what follows so that creditors do no more than rolling over the existing claims. This restriction is quite extreme for normal periods, although it may not be unrealistic for emerging market countries facing heightened liquidity pressure.

We assume that θ is stochastic and normally distributed with mean μ and variance σ^2, both of which could be time varying. We also assume that θ is not directly observable but each creditor i observes noisy private signal $q_i = \theta + \varepsilon_i$ in each period where $\varepsilon_i \sim N(0, \omega^2)$ is uncorrelated with θ. By using the private signal, each creditor updates her belief on θ, and formulates a strategy to determine whether to roll over their claims on the country by taking R as given.

Private creditors face different payoff depending on whether to roll over or exit. The creditor who rolls over faces uncertain payoff. If no default occurs, her payoff is $1 + r_D$ where r_D is the interest rate that the country pays on short-term debt. Without loss of generality, we assume that $r_D = r + s$ where r is the risk-free interest rate and s is the country risk premium. If the country defaults, the creditor recovers a fraction ϕ of her claim where $0 \leq \phi < 1$. The creditor who exits gets risk-free payoff of $1 + r$.

Given this setup of the model, the Appendix solves the creditor coordination problem assuming that creditors use a switching strategy and that the private signal is arbitrarily precise (i.e., $\omega \to 0$) so that creditors face effectively no informational uncertainty regarding θ.[4] Focusing on the limiting case with $\omega \to 0$ may look extreme at first glance, but it would not be unrealistic for emerging market countries with good quality of statistical data and timely data dissemination.

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[3] In fact, this problem arises because of the specification of the model that restricts the value of x between 0 and 1 in order for it to represent the share of creditors who roll over.

[4] It is crucial for the uniqueness result of the global game that informational uncertainty—no matter how small it is—exists regarding θ. If θ is public information known to all creditors with certainty, multiple equilibria cannot be ruled out.

In the limiting case with $\omega \to 0$ and assuming $\Delta D^s = 0$, the unique equilibrium solution of the creditor coordination problem is characterized by:[5]

$$(4) \qquad x = \begin{cases} 0 & \text{if } \theta \le \theta^* = \gamma D - R \\ 1 & \text{otherwise} \end{cases}$$

where $\gamma = (1 + r - \phi)/(1 + r - \phi + s) < 1$. γ reflects the ex ante expected value of x, which depends on the risk premium as well as the recovery ratio under default. The default threshold θ^* is the critical state of θ for which the financing gap after reserves equals the expected shortfall in net short-term borrowing.[6] According to (4), the unique equilibrium solution is of binary nature: either no one exits, or all creditors exit. In the latter case, the country defaults falling victim to a sudden stop.

Since the country defaults whenever $\theta \le \theta^*$ in equilibrium, the probability of a crisis triggered by a sudden stop, denoted by P, is given by

$$(5) \qquad P = \Pr[\theta \le \theta^*] = \Phi\left(\sigma^{-1}\{\gamma D - R - \mu\}\right)$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal. The monotonocity of Φ ensures a unique relationship between the probability of a sudden stop and the level of reserves.

The inspection of (5) suggests that the country may be able to lower the probability of a crisis by reducing both reserves and short-term debt by an equal amount if the return on foreign reserves is less than what the country pays for short-term debt.[7] Specifically, suppose foreign reserves earn the risk free return of r, implying that net interest payment increases—and the current account deteriorates—when short-term debt and foreign reserves increases by the same amount. Keeping this in mind, totally differentiating P while imposing $\Delta D = \Delta R$ yields,

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[5] The binary nature of the equilibrium solution (i.e., $x = 0$ or 1) suggests that the herd behavior of creditors could be justified as an optimal behavior. In a more general case with $\omega > 0$, x will remain between 0 and 1 in equilibrium.

[6] If the expression for θ^* is rearranged as $\theta^* + R - D = -(1 - \gamma)D$, the left hand side is the financing gap while the right hand side is the expected shortfall in net external borrowing since $E(\Delta D^d - \Delta D^s) = -(1 - \gamma)(D + \Delta D^s) = -(1 - \gamma)D$ for $\Delta D^s = 0$ as can be seen from (2).

[7] See Dani Rodrik (2006) for further discussion on the liquidity-preserving debt reduction. He argues that the apparently large and rapid buildup of foreign reserves in emerging market countries since the early 1990s, together with no noticeable reduction in short-term debt, is difficult to reconcile with rationality unless one is assuming a prohibitly high cost of short-term debt reduction.

$$\Delta P \Big|_{\Delta D = \Delta R} = \frac{\partial P}{\partial D} \Delta D + \frac{\partial P}{\partial R} \Delta R = (\frac{\partial P}{\partial D} + \frac{\partial P}{\partial R}) \Delta D = f(\theta^*)(\gamma + s - 1) \Delta D$$

where $f(\theta^*)$ is the normal density evaluated at $\theta = \theta^*$, and the last equality exploits the fact that $\partial \mu / \partial D = -(r + s)$ and $\partial \mu / \partial R = -r$. Since $(\gamma + s - 1) > 0$, reducing short-term debt and reserves by the same amount ($\Delta D = \Delta R < 0$) leads to a lower crisis probability.[8]

Emerging Market Country's Optimization Problem

Establishing the unique relationship between the probability of a sudden stop and the level of reserves, we now turn to the optimization problem of the country. The time line of the model is assumed as follows. The country sets at the end of each period the optimal level of reserves R for the next period. After R is set, creditors receive private signal on θ and formulate their trigger strategy to exit or roll over their claims.

The objective of the country is specified as

$$(6) \qquad \underset{\{R_s\}}{Min} \; Z_t = E_t \sum_{s=t+1}^{\infty} \psi^{s-t} z_s = E_t \sum_{s=t+1}^{\infty} \psi^{s-t} [P_s C_s + \rho_s R_s], \qquad 0 < \psi < 1$$

where the expectation is taken at the end of period t. C refers to the output cost of a sudden stop, and ρ is the unit cost of holding reserves measured by the differential between the country's long-term borrowing rate and the short-term yield on foreign reserves. The first term in the bracket is the expected cost of a sudden stop. The second term reflects the opportunity cost of holding reserves.

Implicit in the specification of the country's objective function is the assumption that in each period, foreign reserves is the only instrument that the country depends on to insure itself against the risk of a sudden stop. This assumption greatly simplifies the analysis but rules out at the outset any effort by the country to adjust and/or obtain extra liquidity from private or official sources within each period.

For tractability of the model, several simplifying assumptions are introduced. First, the underlying shock θ is assumed to be independent over time. Second, a sudden stop, if occurs, lasts only one period and the country regains access to the capital market to accumulate reserves—through new long-term borrowing if necessary—to a desired level before the beginning of the next period. Finally, we assume that there is no informational asymmetry between short-term creditors and the country. Given the time line of the model, the first two assumptions effectively reduce the dynamic optimization problem in (6) into a single-period optimization problem.

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[8] If the risk premium s is endogenously determined, γ may not be invariant to changes in short-term debt and reserves. Nevertheless, the liquidity-preserving debt reduction would likely reduce the crisis probability. See Section III for further discussion.

Optimal Reserve Formula

Since the optimization problem of the country is reduced to a single-period optimization, the first-order condition for the country's cost minimization problem is given by

$$\frac{\partial z}{\partial R} = 0 \quad \Rightarrow \quad \frac{\partial P}{\partial R} \cdot C + \rho = 0$$

Solving for R yields the closed-form formula for the optimal level of reserves as given by,

(7) $$R^* = \gamma D - \mu + \sigma G(\rho / C, \sigma)$$

where $G(\rho / C, \sigma) = \{-2 \ln (\sqrt{2\pi}\sigma\rho / C)\}^{1/2} > 0$. The first two terms on the right hand side represent the financing need predicted when the country sets the optimal level of reserves, suggesting that the optimal level of reserves fully covers the predictable risk of sudden stops. The last term on the right hand side captures the precautionary demand for reserves arising from the need to guard against the unpredictable risk of a sudden stop.

The optimal reserve formula deviates in several respects from the Greenspan-Guidotti rule of full coverage of short-term external debt. The first term suggests less than full coverage of short-term debt since $\gamma < 1$. Depending on the values of the second and third terms, however, optimal reserves may more than fully cover short-term debt. Indeed, the formula implies that other things being equal, optimal reserves would increase by more than one-for-one with an increase in short-term debt because of increased interest payments. Specifically, $\partial R^* / \partial D = \gamma + (r + s) > 1$ where the term $(r + s)$ captures the increased interest payments on short-term debt which is fully predictable and thus included in μ.

As expected, for a given level of short-term debt, the demand for reserves is higher for higher output cost of a sudden stop and for smaller opportunity cost of holding reserves. A rise in the risk-free interest rate—embedded in γ—increases, ceteris paribus, the optimal level of reserves by making the exit less costly to creditors while the opposite is true for the increases in the risk premium and the recovery ratio. Finally, the marginal impact on the level of optimal reserves of the volatility of reserve changes is positive if $\sigma \le (2\pi e)^{-1/2} (C / \rho)$.

Substituting (7) into (5) yields the probability of a sudden stop when the country is optimally insured, which is given by

(8) $$P^* = \Phi(-G(\rho / C, \sigma))$$

The probability of a sudden stop under optimal self-insurance depends negatively on the output cost of a sudden stop but is positively related to the opportunity cost of holding reserves, as well as the volatility of the current account. Intuitively, the country would attempt to reduce further the risk of a sudden stop if it is costly relative to holding reserves.

Conversely, the country would tolerate higher risk of a sudden stop if holding reserves is costly relative to the output cost of a sudden stop.

Finally, the volatility of θ, which is the fundamental source of uncertainty in the model, affects negatively the (expected) marginal benefit of holding reserves but not the marginal cost. Consequently, the country would insure itself only partially against the risk of a sudden stop stemming from unpredictable developments in θ. This intuition also explains why P^* depends on neither the level of short-term debt nor the expected value of θ, both of which are predetermined when the country determines the optimal level of reserves and, hence, constitute the predictable risk of a sudden stop.

III. MODEL CALIBRATION

In this section, the optimal reserve formula is calibrated for 15 selected emerging market countries with significant market access for the period of 1993-2006.[9] The calibration exercise also computes the ex ante *implied* probability of a sudden stop for both actual and optimal reserve levels, the former of which could be considered as a crisis indicator.

Before taking the formula to calibration, it is worth noting the timing convention used for calibration. Recall that according to the model the optimal level of reserves refers to the level at the end of period, and that the model treats short-term debt as given when the country determines the optimal level of reserves. To be consistent with this timing aspect of the model, the optimal level of reserves for year t is calibrated by using short-term debt at the end of year t, and compared with actual reserves at the end of year t. In contrast, the implied probability of a sudden stop for given reserves at time t refers to the probability for time t+1.

For ease of cross-country comparison, we calibrate the model to obtain the optimal level of reserves expressed in percent of GDP, rather than in dollar amount. To that end, we normalize all variables in the model—except for the interest rates and the risk premium—relative to GDP. Scaling by GDP is consistent with the assumed time line of the model because GDP is known by the time (i.e., at the end of each period) the country determines the optimal level of reserves.

Specifically, the optimal level of reserves relative to GDP is calibrated by using the modified formula given by

$$(7a) \qquad\qquad R_t^* / GDP_t = \gamma (D_t / GDP_t) - \hat{\mu} + \hat{\sigma} G(\rho / \overline{C}, \hat{\sigma})$$

where $\hat{\mu} = E_t(\theta_{t+1} / GDP_t)$, $\hat{\sigma}^2 = Var(\theta_{t+1} / GDP_t)$ and $\overline{C} = C / GDP_t$. Likewise, the implied probability of a sudden stop is computed by using the formulas given by

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[9] Emerging market countries in the sample include: Argentina, Brazil, Chile, China, Colombia, Indonesia, Korea, Malaysia, Mexico, Peru, Philippines, Russia, Thailand, Turkey, and Venezuela. They are selected largely based on their economic size.

(5a)
$$P_{t+1} = \Phi\left(\hat{\sigma}^{-1}\{\gamma(D_t/GDP_t) - R_t/GDP_t - \hat{\mu}\}\right)$$

(8a)
$$P_{t+1}^* = \Phi(-G(\rho/\overline{C}, \hat{\sigma}))$$

Given the timing convention noted above, (7a)-(8a) and (5a) can be calibrated for each year by using information only up to that year. This feature is desirable since the calibrated probability of a sudden stop for actual reserves could be considered as a leading indicator of the risk of a sudden stop or a crisis.[10] To be more specific, suppose that a crisis or a sudden stop occurs in period t. Given the timing convention, the implied probability of a sudden stop for period t is determined by using information up to period t-1. Thus, if the probability tends to rise significantly in the crisis year or in the year prior to a crisis, it would have good potential as a leading indicator of a crisis.

Finally, the potential endogeneity of the risk premium may need separate attention in calibrating the model. The optimal reserve formula and the expression for the probability of a sudden stop are derived by treating as given the risk premium and the recovery value under default. In reality, however, the risk premium would be determined endogenously in the market taking into account the probability of a crisis and the recovery value. In this light, the default probability specified in (5a) would be regarded only as an interim solution since it depends on γ, which in turn depends on the default probability if the risk premium is endogenous.

In the calibration exercise, this simultaneity problem is addressed by requiring the indifference condition (A.2) in the Appendix to hold ex ante. Accordingly, we first obtain the equilibrium value of γ by numerically solving the following equality given by

(9)
$$P(\gamma) = 1 - \gamma$$

where $P(\gamma)$ is as specified in (5a).[11] Since $P(\gamma)$ is monotonically increasing in γ while the opposite is true for the right hand side, and since both are confined between 0 and 1 by construction, there is a unique solution $\hat{\gamma}$ for (9). As discussed in section II, liquidity-preserving debt reduction leads to a lower default probability (i.e., higher $\hat{\gamma}$) since for given γ, it lowers $P(\gamma)$ but does not affect $1 - \gamma$.

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[10] As discussed below, ρ is estimated from data over the entire sample period in the calibration exercise and thus reflects contemporaneous and future information. Nonetheless, this is less likely to undermine the validity of the calibrated probabilities of a sudden stop as a leading indicator of a crisis since the underlying subcomponents of ρ are assumed to be either time invariant or exogenous in the calibration.

[11] It is important to distinguish between (9) and (A.2) although both are identical in expression. The condition (9) is an ex ante condition for the endogenous risk premium to be consistent with the default probability in equilibrium, while (A.2) is an indifference condition that holds for a marginal creditor whose private signal coincides with the switching point.

Since the probability of a sudden stop under optimal self-insurance P^* is uniquely determined by (5a), the value of $\hat{\gamma}$ consistent with the optimal level of reserves, denoted by $\hat{\gamma}^*$, is simply given by $\hat{\gamma}^* = 1 - P^*$ according to (9). For the implied probability of a sudden stop for actual reserves, $\hat{\gamma}$ is obtained numerically by using (9). Note that the calibration requires no information on the recovery value ϕ, which is unobservable from data anyway.

Data

External debt and macroeconomic data used for calibration are taken from the WEO and IFS database of the IMF. Short-term external debt D is defined on an *original* maturity basis, in order to be consistent with the assumption of the model that long-term creditors do not play a role in triggering a liquidity crisis. The stock of reserves R is measured inclusive of IMF credit in the calibration exercise since what matters for the country is the total stock of reserves, not the composition between borrowed and unborrowed reserves.

The output cost of sudden stop is fixed at 10 percent of GDP (i.e., $\overline{C} = 0.1$) for all countries in the calibration, which is consistent with the estimate suggested by Hutchison and Noy (2002) and used in Jeanne and Ranciere (2006).[12] The opportunity cost of holding reserves ρ is measured by the sum of the (average) normal risk premium and the term premium. For the normal risk premium, we use the average normal EMBI spread taken from Table 8 of Jeanne and Ranciere. For countries that are not covered in their study, we construct the average normal EMBI spread excluding observations during crisis periods. The term premium is measured by the difference between the three-year U.S. Treasury bond yield and the US T-bill rate.

Finally, in the calibration the stochastic component θ for year t is constructed as follows:

$$\theta_t = \Delta R_t - \Delta D_t - NFD_t$$

where *NFD* represents net disbursements by the IMF. Note that in contrast to the stock of reserves, θ is defined net of IMF disbursements given the focus of the model on self-insurance. Measured in this way, θ would better reflect the uncertainty faced by the country that seeks to insure against the risk of a sudden stop relying solely on its own foreign reserves. In the calibration, $\hat{\mu}$ and $\hat{\sigma}$ are estimated by simple historical averages and standard deviations of the ratio $\hat{\theta} = \theta / GDP_{-1}$ taken over the 10-year period prior to each year (i.e., between t-10 and t-1).

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[12] Two alternatives of $\overline{C} = 0.05$ and $\overline{C} = 0.15$ are also considered for sensitivity check, but their results are not reported because they are broadly similar. The calibration results for $\overline{C} = 0.05$ and $\overline{C} = 0.15$ are available upon request.

Actual and Calibrated Optimal Reserves

Table 1 reports both actual and calibrated optimal reserves (end-of-period) for 15 selected emerging market countries. Over the full sample period, calibrated optimal reserves average between about 10 and 30 percent of GDP for most countries in the sample. At the regional level, the optimal reserve levels are higher for Asia than Latin America by 5½ percent of GDP in both full sample and two subsamples, reflecting in part Asia's higher short-term debt ratio and lower opportunity cost of holding reserves.[13] The optimal reserve levels decline between the two sub periods in 11 countries, largely as a result of a decline in short-term debt.

The calibrated optimal reserve levels are far higher than implied by the Greenspan-Guidotti rule of full coverage of short-term debt. This is not surprising not only because the Greenspan-Guidotti rule does not take into account the cost of holding reserves but also the optimal reserve formula is derived under quite extreme assumptions. Recall that the optimal reserve formula, at the outset, rules out any new external borrowing or adjustment efforts that emerging market countries might undertake to avoid sudden stops. Since the calibration exercises are undertaken by using annual data, these restrictions are tantamount to ruling out borrowing and/or policy adjustment over the time span of a full year.[14] Accordingly, calibrated optimal reserves should be viewed to constitute an upper bound of precautionary reserves.

Actual reserves also exhibit substantial cross-country variation during the sample period, ranging from 6 percent (Brazil) to 38 percent (Malaysia). In fact, regional difference in actual reserves between Latin America and Asia turns out to be larger than observed for optimal reserves. Viewed across two subsample periods, all countries but Chile and Venezuela have increased reserves relative to GDP during the latter half. Asia is particularly pronounced in this regard with a steep average increase of 10 percent of GDP between the two subsample periods, in comparison to a corresponding increase of 2 percent of GDP in Latin America. Russia and Turkey also increased reserves significantly during the latter half of the sample period. Comparing actual reserves with calibrated optimal reserves indicates that 11 countries in our sample have been under insured during the first half of the sample period with average reserve shortfall of 8 percent of GDP. In contrast, 11 countries have been over insured during the second half with average excess reserves of 7 percent of GDP.

For comparison, we calibrate the model to construct a reserve level that is consistent with the implied probability of a sudden stop of 5 percent. This exercise, albeit crude and simple, provides some useful benchmark against which the model-specific level of optimal reserves

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[13] During the period of 1993-2006, the ratio of short-term debt to GDP is on average 10 percent for Asia (excluding China) and 6 percent for Latin America.

[14] They would become less onerous from the practical point of view if the calibration were carried out by using higher frequency data (e.g., quarterly or monthly data).

can be compared.[15] The results are presented in the last three columns of Table 1. The benchmark reserve levels for the full sample period are lower than the optimal levels by 2½ percent of GDP for Latin America and 4 percent of GDP for Asia. The benchmark level is broadly comparable to the level of actual and optimal reserves for Latin America, while it is significantly lower than either actual or optimal reserves in Asia.

The regional and period averages mask important variation across countries and over time. For this reason, Figure 1 plots actual and calibrated optimal reserves for individual countries. In the figure, both actual and optimal reserves for year t are the end-of period figures. According to the timing convention used in the model, they constitute actual and optimal reserves for the next year (i.e. beginning of year t+1).

Latin American countries, except Chile and Colombia, appear to have been persistently under insured except for the later years in the sample period, albeit at varying degree across countries and over time (Panel A). During the first half of the sample period, the average extent of under-insurance is highest for Peru (9 percent) followed by Argentine (7 percent) and Mexico (6½ percent). All Latin American countries in the sample have achieved or exceeded the optimal levels toward the end of the sample period. Chile and Colombia, neither of which was inflicted by full-blown capital account crises during the sample period, have been over insured for most of the sample period.

All Asian countries but China and Malaysia also appear to have been persistently under insured prior to the Asian crisis. Reserve shortfalls are substantial for Korea and Thailand, averaging at 18 and 12 percent of GDP, respectively, during 1993-99, followed by Philippines and Indonesia (Panel B). As noted earlier, most Asian countries in the sample increased their reserves significantly during the second half and, as a result, appear to have been over insured at least since 2002 except for Philippines where actual reserves have been tightly tracking the optimal levels during the second half of the sample period. Excluding Philippines, the extent of over-insurance in Asia ranges from 4 percent of GDP (Indonesia) to 33 percent of GDP (Malaysia) at end-2006. If these numbers are taken literally, the combined total excess reserves of the six Asian countries in the sample amounts to US$ 850 billion at end-2006, about 80 percent of which is accounted for by China. Finally, Russia is more similar to Asian countries as it has been over insured since 2004 thanks to large oil surpluses, while Turkey remained persistently under insured for most of the time before 2003.

Implied Probability of a Sudden Stop

The mirror image of the reserve levels is the implied probability of a sudden stop. Table 2 reports the implied probability of a sudden stop for actual and optimal reserve levels. Note that the timing convention differs from that used for Table 1 since the implied probabilities of a sudden stop for year t is determined by the level of reserves at the end of year t-1. For

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[15] Note that the calibrated optimal level of reserves depends on the particular specification of the country's objective function shown in (6) while the benchmark reserve level does not.

this reason, the sample period for Table 2 runs from 1994 to 2007. Thus, for example, the implied probability for 2002 is associated with reserves at end-2001.

The probability of a sudden stop implied by optimal reserves turns out to be quite low remaining at less than 1 percent in most countries and never exceeding 5 percent. Consistent with optimal and actual reserves as shown in Table 1, the probability of a sudden stop is far higher for actual reserves in both regions (except for Asia during the second half). The probability of a sudden stop for actual reserves is broadly similar at about 8 percent between Latin America and Asia over the full sample period, although both regions exhibit substantial time variation across two subsample periods. Not surprisingly, the regional average is slightly higher for Asia (16 percent) than Latin America (13 percent) during the first half which includes a particularly turbulent period for Asia, while the opposite holds in the second half. Reflecting trend increases in reserves in both regions during the second half, the implied probability of a sudden stop declines sharply to less than 4 percent in Latin America and near zero in Asia.

As a benchmark, Table 2 also presents the implied probability of a sudden stop under the assumption that the country would defend against a sudden stop by not only holding reserves but also undertaking adjustments if necessary. Specifically, we assume that faced with liquidity pressure, the country undertakes current account adjustment in the order of 3 percent of GDP, which is broadly consistent with the median adjustment observed in the run-up to a crisis among capital account crisis countries.[16]

Augmenting reserves by current account adjustments in the run-up to a heightened market pressure would help lower the likelihood of a sudden stop, particularly when the probability of a sudden stop is high in the first place. For instance, a 3 percent of GDP adjustment lowers the probability of a sudden stop by 6–7 percentage points in Latin America and Asia in the first half. But the adjustment of the same size leads to a smaller decline (2½ percentage points for Latin America and almost zero for Asia) in the second half. At the individual country level, the effect of augmented reserves is pronounced in Argentina, Brazil, Mexico, Korea, Russia and Turkey, all of which have been persistently and significantly under insured during the first half of the sample period and experienced a crisis at some point in time during the sample period.

Figure 2 shows the time profile of the implied probability of a sudden stop associated with actual and augmented reserve levels for individual countries. The corresponding probabilities associated with optimal reserves are not reported as they mostly remain at less than 1 percent and never exceeds 5 percent. As expected, the implied probability of a sudden stop associated with actual reserves either declines over time despite ups and downs or remains low throughout the sample period. In addition, for countries that experienced capital account crises at some point in time during the sample period it often peaks (locally) in the crisis year or the year immediately after a crisis, and tends to increase noticeably in the crisis year or the year prior to a crisis. Local peaks at the crisis year or immediately after are easily explained

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[16] See Ramakrishnan and Zalduendo (2006).

by a steep increase in the external debt-to-GDP ratio during or immediately after a crisis triggered by large exchange rate depreciation.

More interesting is the tendency to rise noticeably in the run-up to a crisis. For instance, it increases by 16 percentage points in 2001 for Turkey, 9 percentage points in 2001 for Argentina, 7 percentage in 1997 for Korea (starting from an already high level), and 5 percentage points in 1998 for Brazil. For Thailand, the implied probability of a sudden stop actually falls in 1997 but still remains at 30 percent. These results suggest that the model might have some predictive power with regard to financial crises or sudden stops.

One disturbing feature of the calibrated probabilities of a sudden stop is that for several countries they seem too high to be easily reconciled with actual frequency of sudden stops observed from data, particularly during the first half of the sample. For instance, assuming temporal independence of the calibrated probabilities, the probability of no sudden stop for Argentina and Turkey between 1994 and 2000 is less than 12 percent and 7 percent, respectively. The model's extreme assumptions are clearly one suspect for too high probabilities. Another suspect would be found from the possibility that key parameters of the model do poor job in the calibration. For example, if θ is trending upward or subject to positive structural breaks, its calibrated mean and standard deviation ($\hat{\mu}$ and $\hat{\sigma}$) by using 10-year moving averages and standard deviations would likely overstate the uncertainty faced by private creditors and thus unduly raise the implied probability of a sudden stop.

Yet another and more notable reconciliation may be found from the potential role of the IMF in crisis prevention. If IMF-supported programs help reduce the risk of a sudden stop beyond the pure liquidity enhancing effect of financial assistance, the high levels of the probability of a sudden stop—calibrated from the pure liquidity point of view—would not necessarily be inconsistent with a much lower likelihood of a crisis supported by data.[17] Indeed, Table 3 shows that IMF-supported programs were in place for countries and years in which the implied probability of a sudden stop either remains or reaches high levels but no capital account crisis has occurred.

Ex-post θ and Ex-ante θ^*

The implied probability of a sudden stop for each year is the ex ante probability viewed at the end of the previous year (or at beginning of the same year), assuming neither additional external borrowing nor policy adjustment by the country over the span of a full year. In contrast, actual frequency of sudden stops observed from data is an ex post outcome that is influenced by various economic and financial developments within each year, which are not captured in the calibration.

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[17] Kim (2006) provides analytical underpinnings for the role of IMF-supported programs in crisis prevention. Ramakrishnan and Zalduendo (2006) find that IMF disbursements are a significant factor in crisis prevention, and that the benefit of IMF support goes beyond pure liquidity effects of unconditional resources such as the country's own reserves.

Bearing this point in mind, we examine how well the model performs in explaining actual sudden stops. According to the model, the country would face a sudden stop if actual θ falls below the default threshold θ^*. We evaluate the performance of the model by using this metric. To that end, we focus on sudden stops that have turned into full blown capital account crises as identified by Ramakrishnan and Zalduendo (2006).[18] We augment their list of capital account crises by adding Mexico (1994-95), which was classified as a borderline case in their study. This results in 9 capital account crisis years in total in our sample, or equivalently the unconditional probability of a capital account crisis of 4.3 percent.[19]

Figure 3 compares actual θ with the default threshold θ^* calibrated by using actual reserves. The model appears to perform relatively well to explain capital account crises. Specifically, the model correctly identifies the capital account crises of Argentina (2001), Mexico (1994), Korea (1997), Russia (1998), Thailand (1997), and Turkey (2000). It nearly misses Brazil (1998) by small margin, but fails to capture Indonesia (1997), and Malaysia (1997). If near misses are counted as a correct signal, type I error (false negative) of the model is 22 percent.

The performance of the model also seems satisfactory in terms of type II error (false positive). Specifically, the model falsely identifies sudden stops for Argentina (1995), Brazil (1999), Mexico (1995), Indonesia (1995), Korea (1996), Thailand (1995), and Turkey (1993). Accordingly, type II error of the model remains low at about 3½ percent.

Reserve Accumulation in Latin America and Asia

An important conclusion that emerges from the analysis thus far is that both Latin America and Asia except for a few countries have been under insured in the 1990s but far better insured or over insured in recent years. A subtle difference remains, however, between Latin America and Asia in that underlying the improved reserve position was largely the combined effect of declining optimal reserves and moderate increases in actual reserves in the former while it was mainly the steep increase in actual reserves in the latter.

To further examine the underlying factors of such difference between the two regions, we examine the sources of reserve accumulation by using the balance of payment accounting identity $CA + KA = \Delta R$ where KA also includes errors and omissions. Figure 4 illustrates the decomposition of reserve accumulation into cumulative current and capital/financial account balances over the period of 1993-2006 (taking 1992 as the base year).

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[18] They employ a two-pronged data-driven approach to identify capital account crises for the sample of 27 emerging market countries over the period of 1994-2004. They identify "high market pressure" events by using cluster analysis in the first stage, and then classify those events as capital account crises that are accompanied by large and persistent capital outflows. Their classification of capital account crises include most widely accepted lists of capital account crises, except for Mexico (1994) which is a borderline case in their study.

[19] Although the duration of a capital account crisis is typically longer than one year in our sample, we only count the year in which a capital account crisis occurred.

Not surprisingly, the two regions are starkly different in the development of the current account, particularly in the latter half of the sample period. In all Latin American countries but Venezuela, the contribution of the current account to reserve accumulation in cumulative terms has been negative during the entire sample period while net capital inflows have been more than financing reserve accumulation (Panel A). In contrast, it is the dramatic post-crisis turnaround in the current account that has financed reserve accumulation of Asian crisis countries since 1998 (Panel B).[20] Korea stands out as its capital account has resumed surpluses shortly after the crisis while others have sustained net outflows (retiring external debt) during the post-crisis period. Russia is broadly similar to Asian countries in that reserve accumulation has been largely driven by persistent current account surpluses during the second half of the sample. Turkey resembles Latin American countries with consistently negative contribution of the current account to reserve accumulation.

The sharp contrast between Latin America and Asia in terms of the development in the current account suggests that Latin America may have been limited in generating or sustaining current account surpluses because of low trade openness and small export base. During the period of 1993-2006, the average GDP share of exports of goods and services remains at less than 20 percent for Latin American countries, compared to near 50 percent for Asian countries in the sample. Given such low trade openness, the scope for a significant post-crisis turnaround in the current account may have been limited despite large exchange rate adjustment and severe import compression.

IV. CONCLUDING REMARKS

The model developed in the paper has several advantages compared to other studies in the literature on optimal self insurance. Most important is that the probability of a sudden stop has a sound theoretical basis as it is explicitly modeled as a unique equilibrium outcome of the creditor coordination problem, rather than assumed to be exogenously given or estimated from data as in the existing models. The model explicitly links the probability of a sudden stop to not only local (country-specific) factors such as the level of short-term debt and the volatility in the balance of payments but also global factors such as U.S. interest rates. This feature of the model is particularly desirable for assessing the risk of sudden stops.

The calibration results are broadly consistent with market perceptions: i) the optimal level of reserves itself may have declined in Latin America, reflecting in part improved fundamentals and low global interest rates; ii) emerging market countries may have been better insured in recent years than in the 1990s; and iii) Asia, particularly China and Malaysia, may have been significantly over insured since early 2000s. The calibration results suggest large excess reserves in Asia and Russia with combined total of US$ 1 trillion at end-2006. Assuming an opportunity cost of 5 percent per year, the net cost of holding excess reserves would amount US$ 50 billion a year or, equivalently, 1 percent of GDP. These estimates, albeit speculative,

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[20] The mercantilistic view on reserve management put forth by Dooley, Folkerts-Landau and Garber (2003) may have its own merit if applied to China. For other Asian countries, the fact that the timing of the turnaround in the current account coincides with the Asian crisis casts some doubt on the mercantilistic view.

suggest significant potential for welfare gain from diversifying excess reserves by investing them into higher-yield assets or from retiring short-term external debt by running down reserves or both.

Several caveats are in order, however. First, the model assumes that neither the cost of a sudden stop nor the recovery value depends on the level of reserves. This simplifying assumption could be relaxed but only at the expense of greater complexity of the model with no analytically tractable solution. Second, given the focus on a liquidity crisis, the model does not explicitly address the issue of solvency. Last but not least, the model abstracts from policy adjustment, capital controls or emergency lending by multilateral institutions that the country may seek to defend against sudden stops, although some experiments are undertaken in the model calibration to capture such elements. Incorporating these alternative defense measures into the model would tend to lower the level of optimal reserves. In this regard, the formula should be viewed as the upper bound of optimal reserves.

Table 1. Actual and Calibrated Optimal Reserves for Selected EM Countries: 1993-2006

Country/Region	Actual R/GDP 1/			Optimal R/GDP 1/			R/GDP for P(SS)=5% 1/		
	1993-06	1993-99	2000-06	1993-06	1993-99	2000-06	1993-06	1993-99	2000-06
All Countries	15.6	12.6	18.3	15.4	17.1	13.7	12.2	14.0	10.6
Latin America	12.0	11.4	12.6	12.8	14.8	10.9	10.4	12.3	8.5
Argentina	9.1	7.0	11.2	15.1	13.9	16.4	12.5	11.4	13.6
Brazil	6.4	5.7	7.1	8.3	9.1	7.4	6.4	7.2	5.7
Chile	19.5	20.8	18.3	15.1	15.2	15.0	11.1	10.9	11.3
Colombia	11.1	9.8	12.4	8.4	9.6	7.1	6.5	7.3	5.6
Mexico	7.1	5.8	8.3	9.1	12.2	6.1	6.6	9.3	4.0
Peru	16.4	16.1	16.7	17.2	25.1	9.3	14.7	22.5	6.9
Venezuela	14.3	14.5	14.1	16.5	18.3	14.7	14.8	17.2	12.4
Asia	21.2	16.2	25.9	18.2	20.1	16.5	14.1	16.1	12.3
Excluding China	21.7	17.1	25.8	19.8	22.3	17.6	15.6	18.2	13.3
China	18.9	11.5	26.3	10.2	9.1	11.2	6.4	5.5	7.2
Indonesia	12.4	10.3	14.5	10.3	12.7	7.9	7.6	10.0	5.2
Korea 2/	18.1	9.6	24.2	22.9	27.1	20.0	18.0	22.0	15.1
Malaysia	37.8	32.0	43.7	20.9	20.1	21.8	15.9	15.2	16.5
Philippines	14.4	11.5	17.4	17.4	18.1	16.7	14.3	15.0	13.6
Thailand	25.5	22.0	29.0	27.6	33.6	21.6	22.3	28.6	16.1
Other	11.7	6.0	15.5	15.7	16.4	15.4	13.1	13.8	12.8
Russia 2/	12.5	3.6	17.6	14.8	16.7	13.6	12.0	13.9	10.9
Turkey	11.0	8.5	13.5	16.6	16.0	17.1	14.2	13.7	14.7

1/ R and P(SS) refer to the stock of foreign reserves at end-year and the implied probability of a sudden stop, respectively.
2/ For Korea and Russia, averages are taken over the period of 1995-2006 and 1996-2006, respectively.

Table 2. Implied Probability of a Sudden Stop for Selected EM Countries: 1994-2007

Country/Region	Actual P(SS) 1/			Optimal P(SS) 1/			Augmented P(SS) 1/		
	1994-07	1994-00	2001-07	1994-07	1994-00	2001-07	1994-07	1994-00	2001-07
All Countries	9.5	17.4	2.9	0.9	1.0	0.7	5.0	9.9	1.0
Latin America	8.2	12.6	3.8	1.0	1.4	0.6	3.6	5.9	1.3
Argentina	20.9	24.7	17.1	0.9	0.8	1.0	10.1	12.3	7.9
Brazil	8.9	14.1	3.8	0.6	0.6	0.5	1.1	2.1	0.2
Chile	0.1	0.0	0.1	0.3	0.4	0.3	0.0	0.0	0.0
Colombia	0.5	1.0	0.0	0.4	0.6	0.3	0.0	0.1	0.0
Mexico	12.2	23.0	1.3	0.5	0.6	0.3	5.3	10.5	0.1
Peru	8.0	15.9	0.0	1.7	2.8	0.6	5.4	10.8	0.0
Venezuela	7.1	9.8	4.4	2.5	3.8	1.2	3.2	5.5	0.9
Asia	7.7	16.3	0.3	0.6	0.6	0.6	4.7	10.2	0.1
Excluding China	8.9	18.9	0.3	0.6	0.6	0.7	5.6	12.2	0.1
China	1.5	3.0	0.0	0.2	0.2	0.2	0.2	0.4	0.0
Indonesia	6.2	12.5	0.0	0.4	0.4	0.4	1.7	3.5	0.0
Korea 2/	19.4	45.0	1.1	0.6	0.7	0.6	14.8	35.1	0.4
Malaysia	0.2	0.5	0.0	0.9	0.8	1.0	0.1	0.2	0.0
Philippines	8.8	17.2	0.5	0.7	0.8	0.7	4.2	8.3	0.0
Thailand	9.9	19.6	0.2	0.5	0.4	0.6	6.9	13.9	0.0
Other	19.7	37.8	7.2	1.3	1.3	1.2	10.7	22.7	2.7
Russia 2/	19.0	45.4	3.9	1.7	1.8	1.7	11.9	29.9	1.6
Turkey	20.3	30.1	10.5	0.8	0.8	0.8	9.6	15.4	3.7

1/ P(SS) refers to the implied probability of a sudden stop. Augmented P(SS) assumes current account adjustment of 3 percent of GDP.

2/ For Korea and Russia, averages are taken over the period of 1996-2007 and 1997-2007, respectively.

Table 3. IMF Arrangements for Selected EM Countries: 1993-2006

| Country	Start date	Expiration date 1/	Arrangement type 2/	Country	Start date	Expiration date 1/	Arrangement type 2/
Latin America				**Asia**			
Argentina	3/31/92	3/30/96	EFF	Indonesia	11/5/97	8/25/98	SBA
Argentina	4/12/96	1/11/98	SBA	Indonesia	8/25/98	2/4/00	EFF
Argentina	2/4/98	3/10/00	EFF(P)	Indonesia	2/4/00	12/31/03	EFF
Argentina	3/10/00	1/23/03	SBA(P)				
Argentina	1/24/03	8/31/03	SBA	Korea	12/4/97	12/3/00	SBA
Argentina	9/20/03	9/19/06	SBA				
				Philippines	6/24/94	3/31/98	EFF
Brazil	1/29/92	8/31/93	SBA	Philippines	4/1/98	12/31/00	SBA(P)
Brazil	12/2/98	9/14/01	SBA				
Brazil	9/14/01	9/5/02	SBA(P)	Thailand	8/20/97	6/19/00	SBA
Brazil	9/6/02	3/31/05	SBA				
				Other			
Colombia 3/	12/20/99	12/19/02	EFF(P)				
Colombia	1/15/03	5/14/05	SBA(P)	Russia	4/11/95	3/26/96	SBA
Colombia	4/29/05	10/29/06	SBA(P)	Russia	3/26/96	3/26/99	EFF
				Russia	7/28/99	12/27/00	SBA
Mexico	2/1/95	2/15/97	SBA				
Mexico	7/7/99	11/30/00	SBA	Turkey	7/8/94	3/7/96	SBA
				Turkey	12/22/99	2/4/02	SBA
Peru	3/18/93	3/17/96	EFF	Turkey	2/4/02	2/3/05	SBA
Peru	7/1/96	3/31/99	EFF(P)	Turkey	5/11/05	5/10/08	SBA
Peru	6/24/99	2/8/01	EFF(P)				
Peru	3/12/01	2/1/02	SBA(P)				
Peru	2/1/02	2/29/04	SBA(P)				
Peru	6/9/04	8/8/06	SBA(P)				
Venezuela	7/12/96	7/11/97	SBA				

Source: IMF MONA database.

1/ Some of these expiration dates were extended, and some refer to programs that stopped early.

2/ P in the parenthesis refers to precautionary arrangements.

3/ Treated as precautionary because the country did not draw IMF resources, although it did not indicate as much at the outset of the arrangement.

Figure 1. Actual and Calibrated Optimal Reserves for Selected EM Countries: 1993-2006
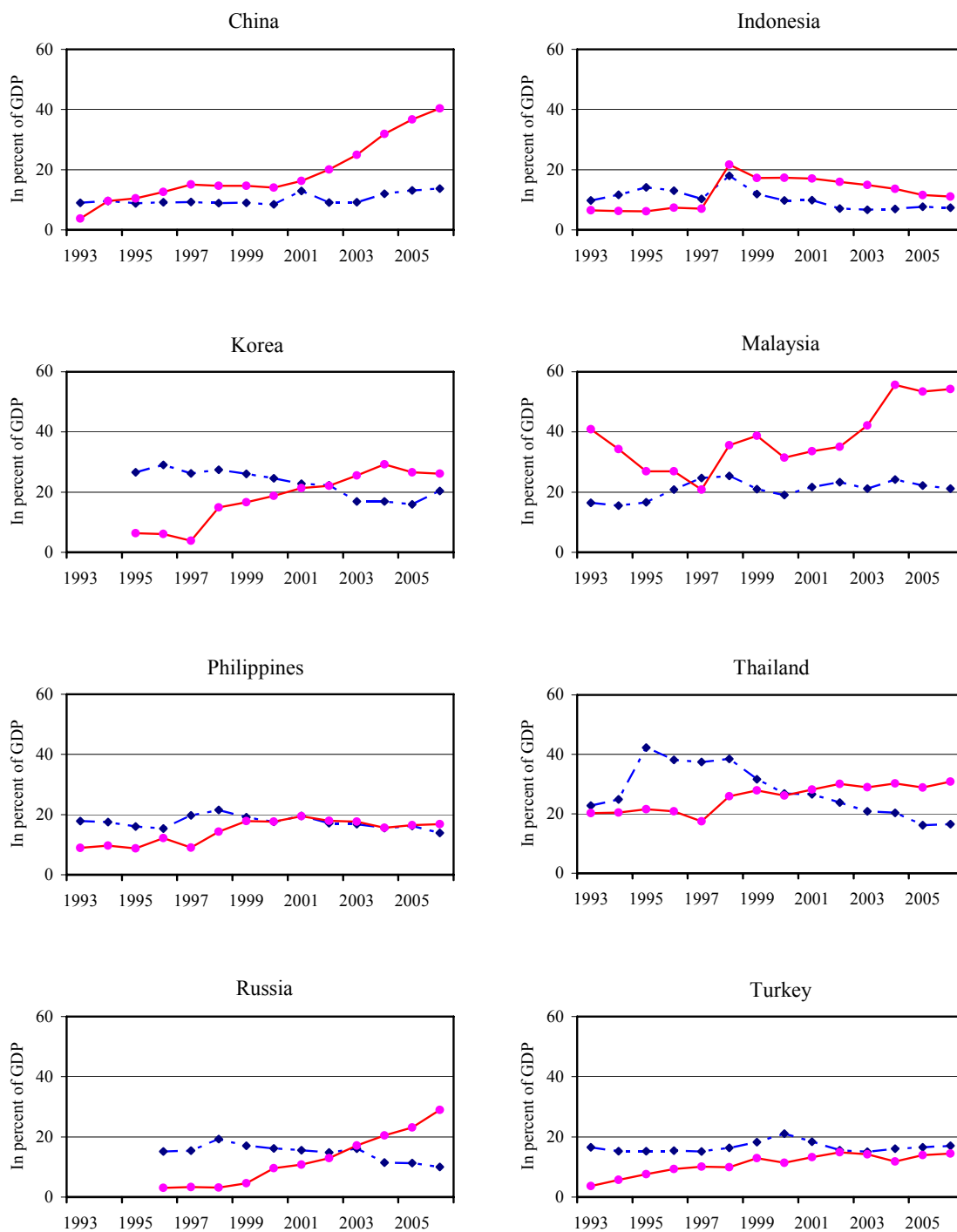
A. Latin America
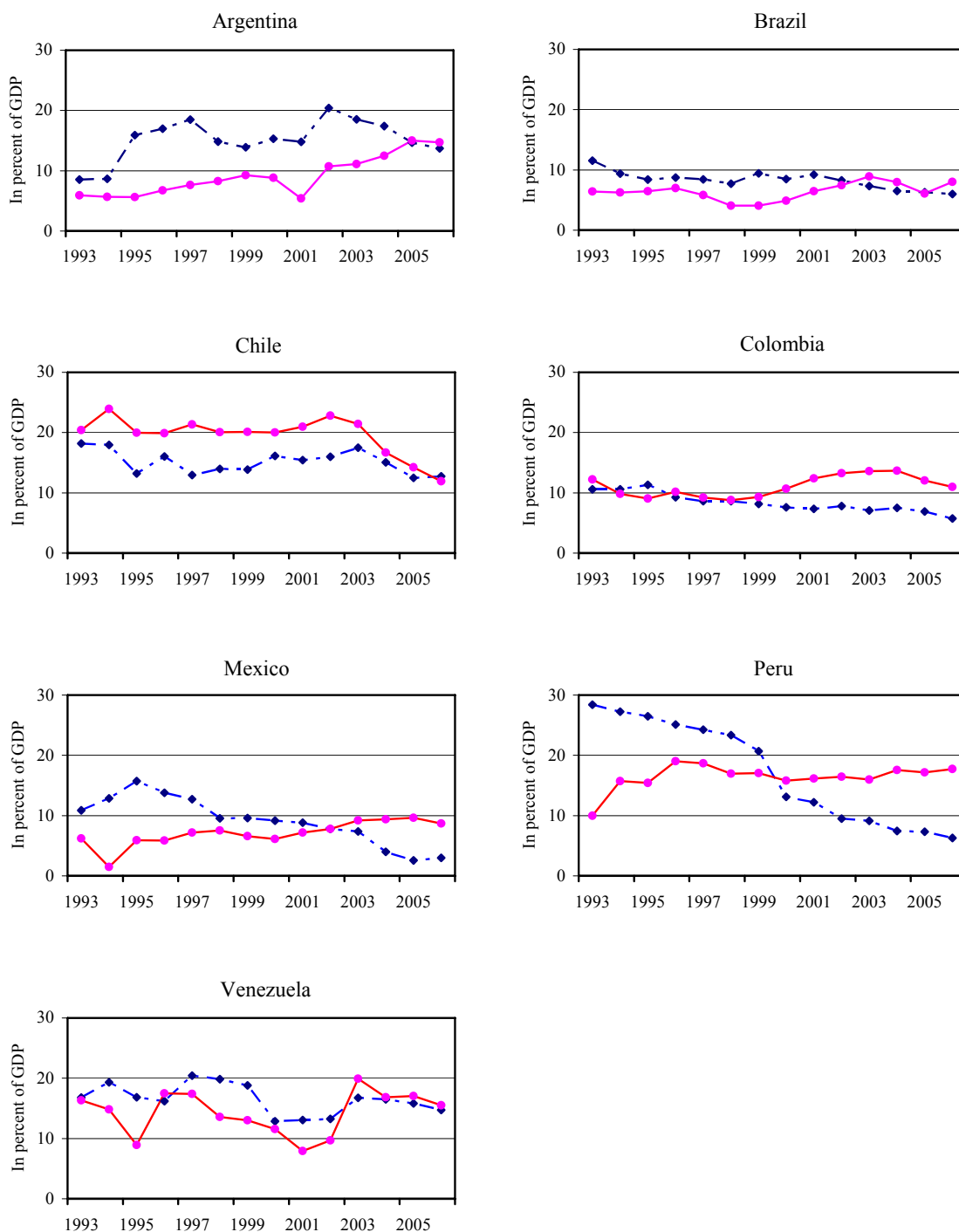
Note: Solid and broken lines refer to actual and calibrated optimal reserves, respectively.

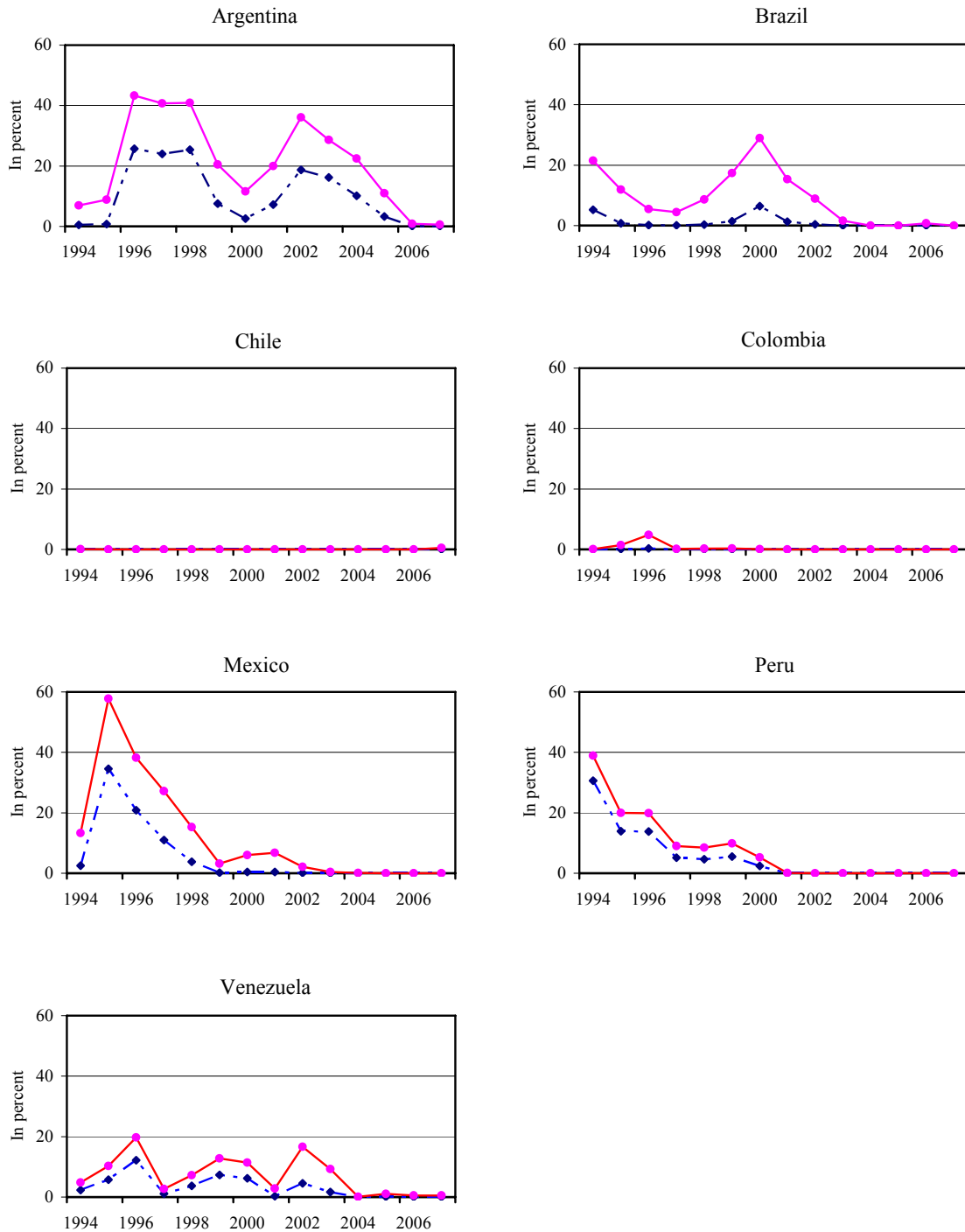Figure 1. Actual and Calibrated Optimal Reserves for Selected EM Countries: 1993-2006

B. Asia, Russia and Turkey

Note: Solid and broken lines refer to actual and calibrated optimal reserves, respectively.

Figure 2. Implied Probability of a Sudden Stop for Selected EM Countries: 1994-2007

A. Latin America

Note: Solid and broken lines refer to the probability of a sudden stop implied by actual reserves and augmented reserves with a 3 percent of GDP adjustment in the current account, respectively.

Figure 2. Implied Probability of a Sudden Stop for Selected EM Countries: 1994-2007
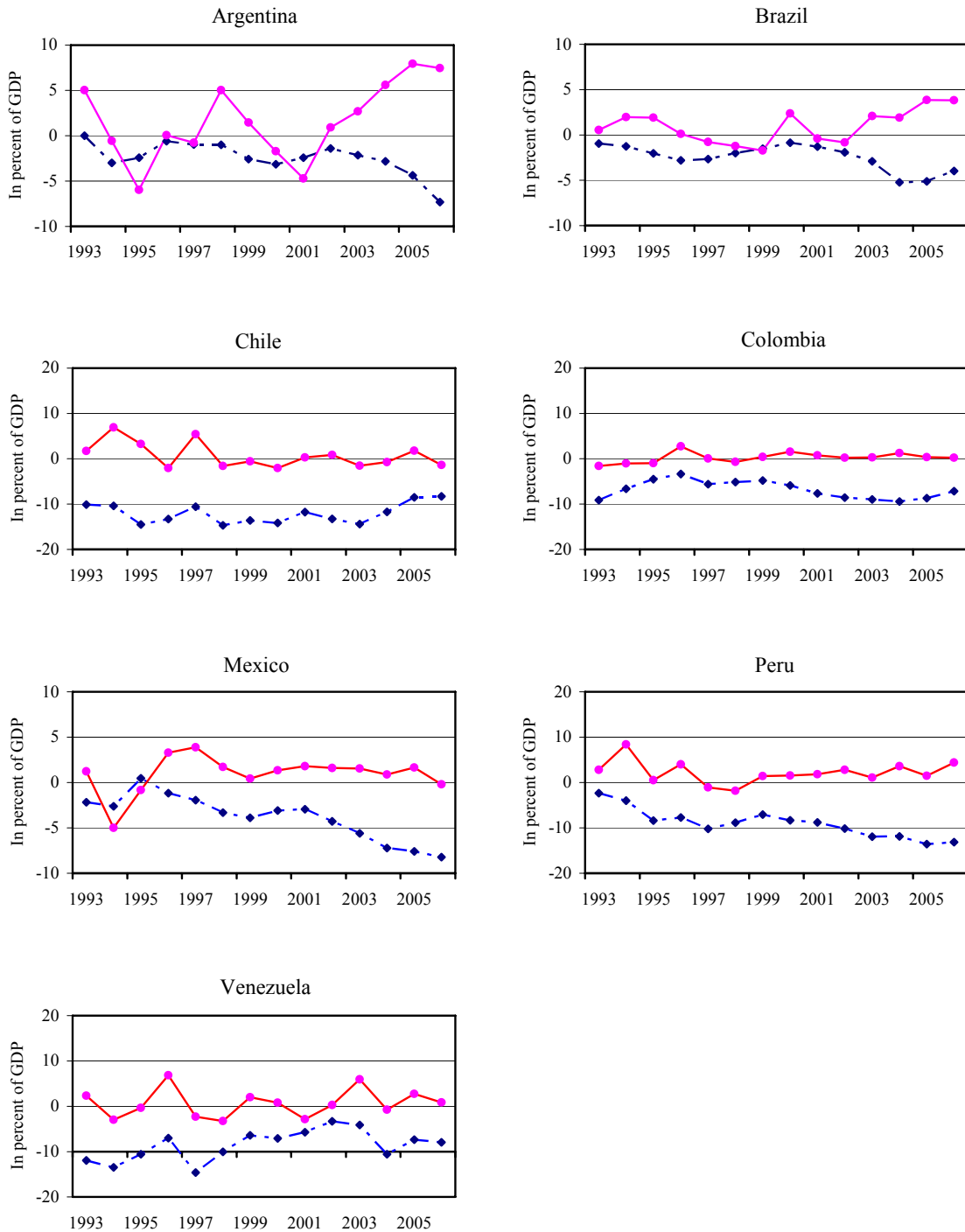
B. Asia, Russia and Turkey

Note: Solid and broken lines refer to the probability of a sudden stop implied by actual reserves and augmented reserves with a 3 percent of GDP adjustment in the current account, respectively.

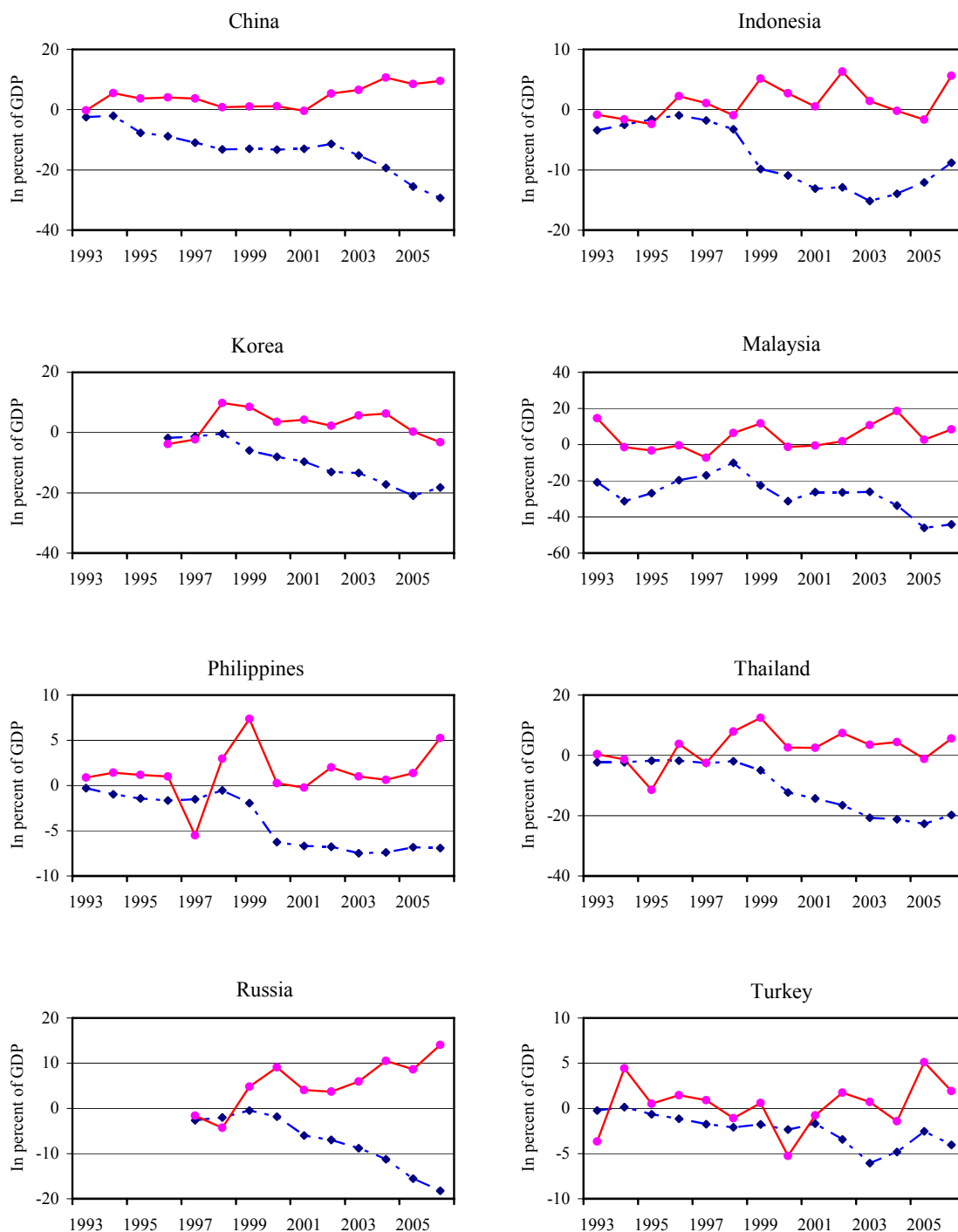Figure 3. Actual θ and Default Threshold θ[*] for Selected EM Countries: 1993-2006

A. Latin America

Note: Solid and broken lines refer to actual θ and default threshold θ[*] implied by actual reserves, respectively.

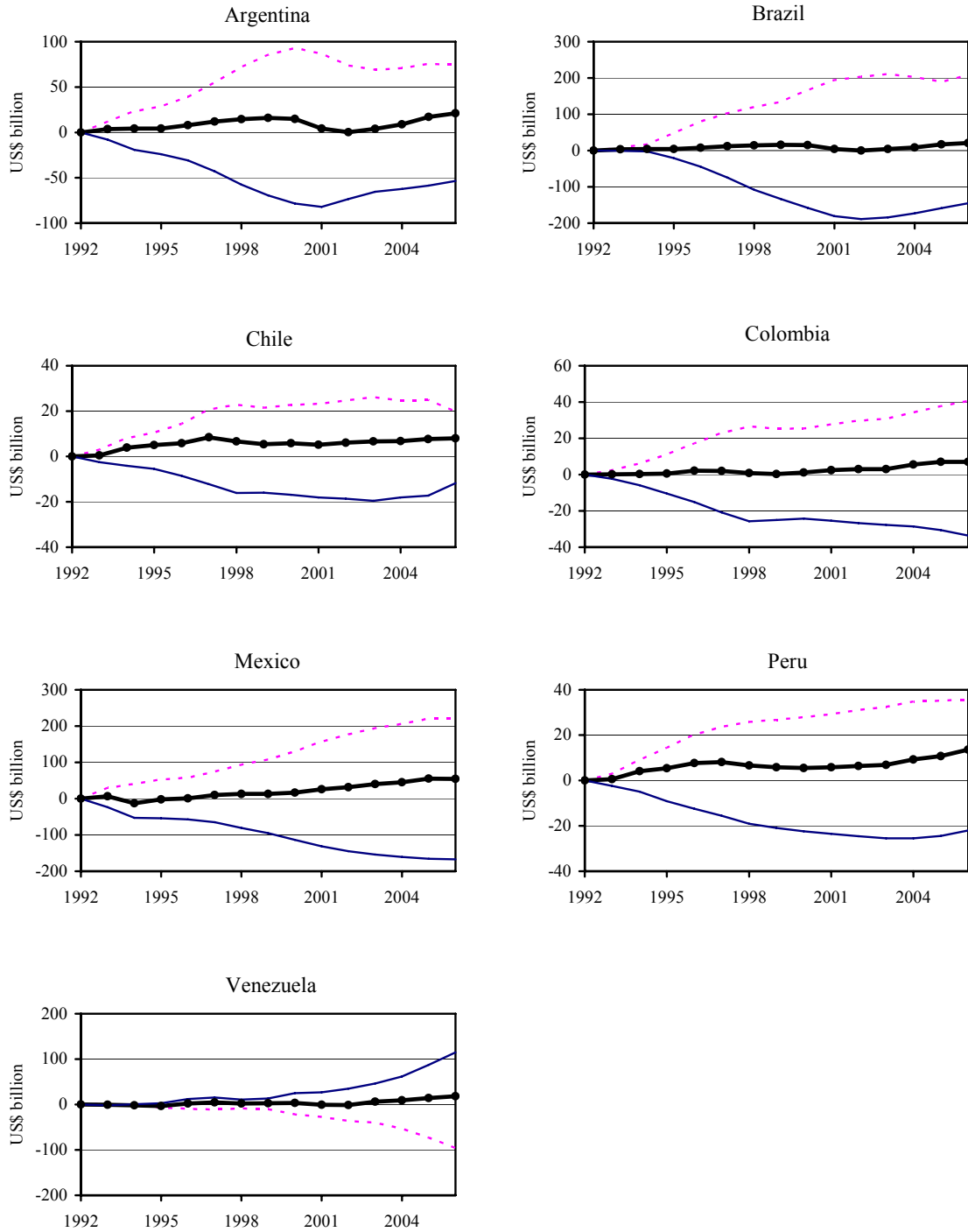Figure 3. Actual θ and Default Threshold θ[*] for Selected EM Countries: 1993-2006

B. Asia, Russia and Turkey

Note: Solid and broken lines refer to actual θ and default threshold θ[*] implied by actual reserves, respectively.

Figure 4. Sources of Reserve Accumulation for Selected EM Countries: 1993-2006
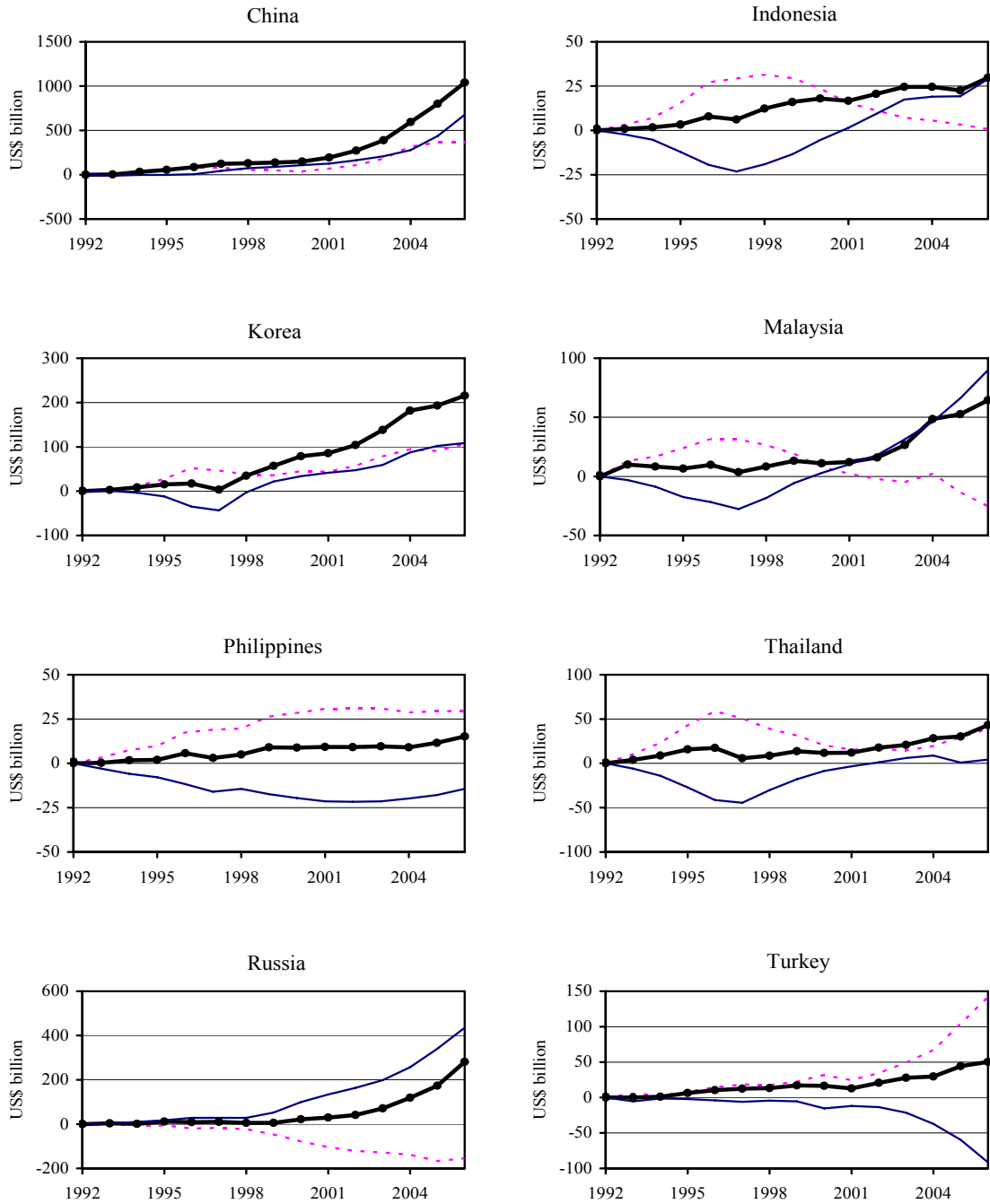
A. Latin America

Note: Bold line with marker refers to cumulative changes in reserves. Solid and dotted lines represent cumulative current and capital account balances, respectively.

Figure 4. Sources of Reserve Accumulation for Selected EM Countries: 1993-2006

B. Asia, Russia and Turkey

Note: Bold line with marker refers to cumulative changes in reserves. Solid and dotted lines represent cumulative current and capital account balances, respectively.

APPENDIX: Creditor Coordination Problem

This Appendix derives a unique equilibrium solution for the creditor coordination problem discussed in Section II.

Given the noisy signal, creditors update their belief on θ in a Bayesian fashion. Upon the realization of private signal $q_i = \theta + \varepsilon_i$, the posterior distribution of θ is normal with mean $\hat{\theta}_i$ and variance $(\sigma\omega)^2/(\sigma^2 + \omega^2)$, where $\hat{\theta}_i$ is given by:

$$\hat{\theta}_i = \left(\frac{\omega^2}{\sigma^2 + \omega^2}\right) \cdot \mu + \left(\frac{\sigma^2}{\sigma^2 + \omega^2}\right) \cdot q_i$$

When creditors use a switching strategy, they have a threshold level ξ (the switching point) such that they roll over if and only if their updated estimate $\hat{\theta}_i$ is greater than ξ. This threshold ξ translates into another threshold for the observed signal q_i given by,

$$\hat{\theta}_i \geq \xi \quad \Leftrightarrow \quad q_i \geq \overline{q} \equiv \left(1 + \frac{\omega^2}{\sigma^2}\right) \cdot \xi \; - \; \frac{\omega^2}{\sigma^2} \cdot \mu$$

Therefore, the incidence of withdrawal by short-term creditors $1 - x$ is determined by the mass of creditors who have received a signal below the threshold $\overline{q}$, which is given by

$$1 - x = \Pr\left[q_i < \overline{q}\right] = \Pr\left[\varepsilon_i < (\overline{q} - \theta)\right]$$

From the liquidity condition (3) with $\Delta D^s = 0$, the critical state of θ at which the country is on the margin of default is given by $\theta^* = (1-x)D - R$ so that the country defaults for all $\theta \leq \theta^*$. Accordingly, the incidence of withdrawal by short-term creditors at $\theta = \theta^*$ is given by

$$1 - x = \Pr\left[\varepsilon_i < (\overline{q} - \theta^*)\right] = \Phi\left(\omega^{-1}(\overline{q} - \theta^*)\right)$$

where $\Phi(\cdot)$ stands for the cumulative distribution function for the standard normal.

Given the assumed normalization of model parameters, this leads to the first equilibrium condition in terms of ξ and θ^* given by,

(A.1)
$$\begin{aligned}
R + \theta^* &= \Phi\left(\omega^{-1}(\overline{q} - \theta^*)\right)D \\
&= \Phi\left(\frac{\omega}{\sigma^2} \cdot (\xi - \mu) + \frac{1}{\omega} \cdot (\xi - \theta^*)\right)D
\end{aligned}$$

Note that the left hand side of (A.1) is increasing in θ^* while the opposite is true for the right hand side so that the liquidity condition given by (3) is satisfied and, hence, no default occurs if $\theta \geq \theta^*$.

The second equilibrium condition states that, *at the switching point*, short-term creditors should be indifferent between rolling over and withdrawing their claims, which requires the following condition given by

(A.2)
$$(1-P)(1+r_D) + P\phi = 1+r \quad \Rightarrow \quad P = 1-\gamma$$

where $\gamma = (1+r-\phi)/(1+r_D-\phi) < 1$. Since $P = \Pr[\theta \leq \theta^*]$ and the posterior distribution of θ at the switching point is normal with mean ξ and variance $(\sigma\omega)^2/(\sigma^2+\omega^2)$, the indifference condition (A.2) can be reformulated as follows:

(A.3)
$$P = \Phi\left(\frac{\sqrt{\sigma^2+\omega^2}}{\sigma\omega} \cdot (\theta^* - \xi)\right) = 1-\gamma \quad \Rightarrow \quad \theta^* - \xi = \frac{\sigma\omega}{\sqrt{\sigma^2+\omega^2}} \cdot \Phi^{-1}(1-\gamma)$$

Plugging (A.3) into (A.1) yields,

(A.4)
$$R + \theta^* = \Phi\left(\frac{\omega}{\sigma^2}(\theta^* - \mu) + \frac{\sqrt{\sigma^2+\omega^2}}{\sigma}\Phi^{-1}(\gamma)\right)D$$

There is a unique solution to (A.4) as long as $\omega D/(\sqrt{2\pi}\sigma^2) \leq 1$.[21] We assume that this condition holds.

For tractability of the model, the analysis hereafter focuses on the limiting case in which $\omega \to 0$ so that the private signal becomes arbitrarily precise. Therefore, creditors face in effect little informational uncertainty regarding θ. In this limiting case, the equilibrium value of θ^* satisfies

$$R + \theta^* \to \Phi\left(\Phi^{-1}(\gamma)\right) \cdot D = \gamma D$$

so that, in the limit,

(A.5)
$$\theta^* = \gamma D - R$$

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[21] This condition simply states that, for all θ^*, the slope of the right hand side of (A.4) should be less than or equal to the slope of the left hand side, which equals 1.

It should be noted that the switching point $\bar{q}$ (and ξ) converges to the default threshold θ^* when $\omega \to 0$ as can be seen from (A.3). This is because the private signal is virtually, albeit not exactly, identical to θ when $\omega \to 0$. Therefore, $q_i < \bar{q}$ for all i whenever $\theta < \theta^*$, suggesting that no creditor rolls over. By the same token, all creditors roll over if $\theta > \theta^*$. Consequently, the equilibrium value of x is either 0 or 1 when $\omega \to 0$. In a more general case with $\omega > 0$, x will remain between 0 and 1 in equilibrium.

It immediately follows from (A.5) that, for given R, the probability of a sudden stop is given by

(A.6)
$$P = \Pr[\theta \le \theta^*] = \Phi\left(\sigma^{-1}\{\gamma D - R - \mu\}\right)$$

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