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Stochastic Volatilities and Correlations, Extreme Values and Modeling the Macroeconomic Environment, Under Which Brazilian Banks Operate

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Stochastic Volatilities and Correlations, Extreme Values and Modeling the Macroeconomic Environment, Under Which Brazilian Banks Operate

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Abstract

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Using monthly data for a set of variables, we examine the out-of-sample performance of various variance/covariance models and find that no model has consistently outperformed the others. We also show that it is possible to increase the probability mass toward the tails and to match reasonably well the historical evolution of volatilities by changing a decay factor appropriately. Finally, we implement a simple stochastic volatility model and simulate the credit transition matrix for two large Brazilian banks and show that this methodology has the potential to improve simulated transition probabilities as compared to the constant volatility case. In particular, it can shift CTM probabilities towards lower credit risk categories.

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1. Introduction

Forecasting stochastic volatilities has been a topic under intense scrutiny, particularly after the 1987 crash in the U.S. market, with many important applications ranging from portfolio selection to the valuation of derivatives such as options. However, perhaps because of data availability, many studies have focused solely on the U.S. market, while episodes of high volatilities are more frequent in emerging markets. Notable exceptions are studies on the Singaporean market (Tse and Tung (1992)) and on the Turkish market (Balaban (1998))[2]. With respect to the frequency of the data, these studies usually look at weekly and daily time series[3]. From an empirical perspective, many of these studies employ ARCH/GARCH family models (when not solely focusing on these models), which require either a higher frequency or a longer period, in order to assure convergence in the parameters estimation. From a practical standpoint, since the main purpose of these studies was to assess volatility models forecasting performance, forecasting longer term volatilities is much more challenging, as the macroeconomic environment where these markets operate also evolve quite dynamically over time. Furthermore, most of these studies have focused on stock market volatility, with few exceptions (e.g. West and Cho (1995) and Brooks and Burke (1998) on forecasting exchange rate volatility, to cite some).

Empirical evidence from these studies is mixed, and no volatility forecasting model can be singled out as the supreme one. Indeed, Tse (1991) and Tse and Tung (1992) find that EWMA performed better than competing models (including other naïve models and GARCH models), while Dimson and Marsh (1990) recommend forecasting volatility with exponential smoothing and regression models. Similarly, Balaban, Bayar, and Faff (2002) also find that exponential smoothing models produced superior forecasts, while ARCH-type models provided the worst forecasts. Akgiray (1989) find evidence in favor of a GARCH(1,1) model, while Brailsford and Faff (1996) contend that ARCH models perform better (although their results are sensitive to the error statistics employed).

Using this as a basis, the current paper aims to expand the empirical literature on two fronts. First, we employ monthly data on a variety of macroeconomic return time series (interest rates, FX rate, commodity prices, and equity indices), for the U.S. and Brazil. Brazil is currently one of the leading emerging economies, and has experienced a series of recent

[2] Balaban, Bayar, and Faff (2002) perform an extensive analysis of volatility forecasting models, for weekly and monthly volatility in fourteen stock markets: Belgium, Canada, Denmark, Finland, Hong Kong, Italy, Japan, Netherlands, Philippines, Singapore, Thailand, the UK, and the US. Other examples of studies focusing on stock markets other than the US market are: Tse (1991) on Japan, Brailsford and Faff (1993, 1996) on Australia; Adjaoute, Bruand, and Gibson-Asner (1998) on Switzerland; and Franses and Ghijsels (1999) on Italy, Spain, Germany and Netherlands.

[3] Figlieswki (1997) and Balaban, Bayar, and Faff (2002) assess the accuracy of forecasting models over monthly time series, while Andersen and Bollerslev (1998) and Andersen, Bollerslev, and Lange (1999) focus on intra-day data.

shocks[4], which allows us to examine the forecast accuracy of models during highly volatile periods. Further, utilizing several different time series also allows us to capture various nuances of macroeconomic shocks, as these variables share some degree of correlation. For this purpose, we follow Brailsford and Faff (1996) and Balaban, Bayar, and Faff (2004) and compare a number of stochastic volatilities models. Second, we also examine the forecast performance of the models in predicting covariances. Few previous studies have examined correlation forecasts, and are usually constrained to few variables because of the large numbers of parameters that are generally required to estimate corresponding correlation matrices[5] for many stochastic correlations models. An alternative could be the work of Engle (2002), who proposes an extension to the multivariate GARCH estimators (Bollerslev (1990)), in which the correlation matrix containing the conditional correlations is allowed to be time varying (the Dynamic Conditional Correlation – DCC – model). Under the DCC approach, Engle estimates the conditional correlation through an exponential smoother estimator, which is estimated in two steps from univariate GARCH equations, allowing for the estimation of large correlation matrices.

Our results confirm the general finding that there is not a model that can be singled out as the ultimate performer in forecasting volatilities and covariances. Unlike Brailsford and Faff (1996), however, our results are not sensitive to the forecast error measure utilized. In addition, many times forecast errors are very close across different volatilities and covariances models. This result allow us to select a model for volatilities and covariances that would be convenient in terms of computational effort, to be inserted in a more robust simulation framework – the Portfolio Simulation Approach (PSA) – for modeling detailed banks' portfolios and balance sheets. Under the PSA, updating stochastic volatilities and correlations is just a small fraction of a comprehensive Monte Carlo (MC) exercise that integrates market and credit risk components[6]. We chose to use a very simple volatility stochastic model that ressembles the exponentially weighted moving average (EWMA) model. Once the initial variance/covariance is inserted, the updates are easily calculated from one time step to the next, within the MC simulations, without a substantial increase in computational time. Further, as we shall see, the smoothing parameter can be changed so as to produce simulated returns and volatilities distributions that are reasonably close to historical distributions.

Indeed, an important issue addressed in this paper relates to the widely documented non-normality of returns distributions[7]. In particular, historical returns distributions are found to possess heavy mass concentration in the tails, while many standard asset price stochastic

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[4] The Brazilian economy, since it emerged in 1994 from a period of hyperinflation, has been affected by at least four major crises: the Mexican crisis in 1994/95, the Asian Tigers crisis in 1997, the Russian crisis in 1998, and, more recently, the Argentinean crisis in 2002.

[5] See Bollerslev, Engle, and Wolldridge (1988), Bollerslev (1990), Bollerslev, Engle, and Nelson (1994), Din and Engle (2001), and Lopes and Valter (2001) to cite some.

[6] See Barnhill and Maxwell (2002), for more detailed information on the PSA framework.

[7] Fama (1965), Duffie and Pan (1997), Muller et al. (1998), and Cont (2001), to cite some.

processes are based on normal or lognormal shocks. The issue of heavy tailed returns distributions has important implications for risk management and, in particular, for methodologies such as the Value at Risk, designed to measure the likelihood of extreme events. Clearly, a failure in modeling the tails of the distributions appropriately would result in an underestimation of risk measures. Several studies have studied the insertion of volatility stochastic models, as a way of capturing the heavy-tail feature of historical returns distributions (Stein and Stein (1991), Hull and White (1998), Eberlein, Karlsen, and Kristen (2003), to cite some). While these studies advocate modeling prices processes with stochastically varying volatility parameter, they have not looked into correlated stochastic price processes, which is the focus of our exercise. We simulate returns for a set of correlated stochastic variables, including Brazilian and US short-term interest rates, Brazilian and US equity indices, oil and gold prices, and Brazilian foreign exchange rate (relatively to US dollar). This exercise is important because it allows us to look at the stochastic volatilities not only at the individual level but also at an integrated level where stochastic correlations also affect the simulated returns through a Cholesky factorization.

We show that it is possible to obtain simulated return distributions that have heavy tails, as we decrease the decay factor. We are also able to simulate distributions of changes in volatilities that closely match historical distributions, once the smoothing factor is appropriately calibrated. This result suggests that a comprehensive optimization of variance and covariance decay factors may produce accurate distributions for returns, volatilities, change in volatilities, and covariances. It also highlights the fact that minimizing the root means squared error (RMSE), to obtain optimal decay factors, may not be the best approach if one is trying to simulate return distributions with heavier tails. As a matter of fact, different sets of optimal decay factors are obtained, depending upon the criterion that is used for optimization, be it to increase the probability mass toward the tails or match the distribution of changes in volatilities over a particular time-frame, or some other criteria.

Finally, we use the stochastic volatility/covariance updates and simulate a credit transition matrix (CTM) for two large Brazilian banks. Credit transition matrices report the probability that a loan, or bond, or any other financial instrument subjected to credit risk, can move from one credit risk category to the other. This simulation was performed by Barnhill, Souto, and Tabak (2003) for constant volatilities and they found simulated CTM to be very close to the historical CTM that is estimated by a Brazilian Credit Risk Bureau, using comprehensive historical data provided by the banks. One shortcoming of their study, however, was the inability to capture default probabilities for the top 2 credit quality categories, as evidenced by the historical CTM. Even though we do not fully succeed in reproducing historical default rates at the top 2 credit quality categories, our results show that using stochastic volatilities with decay factors estimated in this paper to minimize the absolute difference in changes in volatilities for 12-months time windows[8], between historical and

[8] For both historical and simulated volatilities time series, these differences are calculated as $\Delta\sigma^2 = \sigma_{12}^2 - \sigma_1^2$, where the numerical subscripts indicate the first and the twelfth month of the rolling time window. For historical data, differences are calculated for rolling time windows, covering the period from January 1995 to December (continued)

simulated volatilities, can shift CTM probabilities towards lower credit categories, which is more consistent with stylized facts observed in emerging economies like Brazil.

The remainder of this paper proceeds as follows. In section 2 we present an overview of the Brazilian macroeconomic environment, followed by estimation and assessment of out-of-sample forecast performance of several variance-covariance models. A Monte Carlo (MC) exercise is conducted in section 3, with the purpose of examining the distribution of simulated returns and changes in volatilities and covariances, in contrast to observed historical distributions. We then simulate the credit transition matrix for two large Brazilian banks and contrast the results between the constant and stochastic volatility cases in section 4. Final concluding remarks are brought in section 5.

2. Forecasting Volatilities and Covariances

In testing the forecasting accuracy of volatility and covariances models, we have to keep in mind the future application of the optimal model in the context of our MC simulations of banks' portfolios and balance sheets. One of the simulation steps involves modeling stochastic returns over a time period, for a number of underlying state variables such as interest rates, foreign exchange rates, and equity indices. Each simulated path for price levels over a time period, say 1 year, comprises a number of time steps, which, in theory, could be set at any frequency (daily, weekly, monthly, etc.). However, given the robust structure of the simulation code and the number of MC runs usually performed to produce reasonable Value-at-Risk estimates, setting the time step at a daily or even weekly frequency, would result in a tremendous computational effort, restricting significantly the scope of the simulations[9]. Thus, for practical reasons, each price path is simulated with a monthly time step, usually over a short time period of one to three years.

A further simplifying assumption within MC simulations regards parameters estimates. Ideally, parameters of forecasting models should be updated, as new simulated

2004. For simulated volatilities, differences were estimated between December and January of 2004, for 2000 runs. Then we compare mean, standard deviation and various percentiles between the two distributions.

[9] For example, if we simulate returns over 1-year horizon, using daily time-steps, for each of the MC simulation run, we need to construct the price path for each of the variables. Each price path will then have 365 simulated data points. For 1000 simulation runs (in fact, this will represent 2000 runs, because we also produce antithetic values, to reduce MC error) and 10 variables, this represents 365 x 1000 x 10 x 2 = 7,300,000 data points to be generated, just to construct the price paths, that will later be used to re-price banks' portfolios and balance sheet accounts. If generating each price data point requires some significant computational time (even if significant means a fraction of a second), then it becomes clear that the simulation will consume a lot of computational time, making the exercise almost unfeasible, from a practical standpoint. Another degree of difficulty relates to the fact that the simulation package was built in an Excel platform, restricting it to be used with PC's that are far slower than main-frame computers.

values are added to the time series. While interesting, time-varying parameters would substantially increase computational effort and time, as these parameters would be re-estimated each time step of each simulation run. We keep matters simple and assume in this study that the parameters of the stochastic volatility models will not vary overtime, in the course of the 12 monthly time steps performed in the MC simulations, once they have been optimally estimated using historical data. Yet, we argue that stochastic updating is an improvement over the static case, where volatilities and correlations are estimated once, and fed into the MC simulation as time invariant. Further, it brings into the simulation methodology another degree of uncertainty and captures another facet of the risk faced by financial institutions, as volatilities and covariances are allowed to vary stochastically. Finally, as we shall see, we will be able to reproduce reasonably well some characteristics of returns and volatilities distributions, such as the heavy-tail of returns distributions largely documented in the literature, by using the exponentially weighted moving average model.

2.1. Historical Realized Volatilities and Covariances

For assessing forecasting performance of various volatility and covariances models, we need to identify a benchmark over which we can estimate forecast errors. However, volatility and covariance cannot be directly observed. In the case of volatility, the literature has usually followed two different approaches: (i) estimate implied volatilities, using an option-pricing model (e.g. Jorion (1995) and Figliewski (1997), among others); or (ii) estimate ex-post realized volatilities on historical returns time series (e.g. Akgiray (1989), Brailsford and Faff (1996), among many others). The implied volatility approach has several shortcomings. First one needs to have data on options prices for the underlying variables, and many emerging economies, like Brazil, have illiquid (if any) options market. Second, one has to agree that the assumptions underlying the option-pricing model are realistic enough to provide reasonable estimates for the implied volatilities. Finally, there is no way of recovering 'implied' covariances, making it impossible to use this approach for estimating realized covariances.

Andersen and Bollerslev (1998) proposed to use a measure of volatility that is based on observations within the period. This approach has some pitfalls when used on high frequency data, as it may be subjected to bid-ask bounce and irregular spacing of the price quotes, which will affect volatility estimates. However, these problems are less likely to affect lower frequency data, as in our study. We will thus use daily returns to estimate monthly volatilities and covariances, from January 1995 to December 2004.

2.2. Initial Volatilities and Covariances

Some of the volatility and covariance forecasting models we will be examining require an initial value for volatility and covariance, like the exponentially weighted moving average

(EWMA). In the case of emerging economies that have experienced several shocks with widespread effects, selecting a period for estimating the best initial volatility or covariance is not trivial. In addition, GARCH-type models require a significant number of observations for parameters estimation. With monthly data, these models would require a long period for a proper optimization convergence, which is a limitation in the case of Brazilian data. As discussed in Section 3, Brazil has shifted from a hyperinflation period to a more stable economy in July 1994. That would directly challenge the covariance stationarity assumption that is central to ARCH-GARCH models. Given the fact that the period prior to January of 1995 configures a completely different picture of what is later observed in the Brazilian economy after the implementation of the "Real" plan, as discussed above, we will use data from January 1995 to December 1996 to estimate initial volatilities and covariances and data from January 1997 to December 2002 to estimate models' parameters. For the ARCH/GARCH models we will use 8 years of monthly data, from January 1995 to December 2002 to assure proper convergence of parameters estimates (see Figlewski (1997)). Three different initial volatilities and covariances[10] will be used in this study: (i) using historical data from January 1995 to December 1996 (24 observations); (ii) using historical data from January 1996 to December 1996 (12 observations); and (iii) the realized volatility as of December of 1996. Data from January 2003 to December 2003 will be used for out-of-sample forecast.

In Table 1 we present the estimated initial volatilities for the three cases listed above. While there are some significant differences in volatilities using different historical periods, the biggest differences occur when one contrasts them with realized volatilities. For example, realized volatility for S&P 500 index in December 1996 was 8.79E-03, while using historical returns have yielded a volatility of 5.78E-04 for the period of January 1995 to December 1996, and of 9.52E-04 for the period of January 1996 to December 1996. The results for the covariance matrix[11] are even more sensitive to the choice of the time period for the initial guess. Indeed, in many cases there is a reversal in covariance sign and, as we shall see, with significant impact on the estimation of optimal decay factor for the exponentially weighted moving average process. This underscores the importance of properly chosing the historical period for estimating volatilities and correlations. It is a very interesting research topic that goes beyond the scope of this study.

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[10] It is not obvious which period should be used to estimate a representative initial guess, and this is why we are proposing to use three different proxies for initial volatilities. While these time periods were arbitrarily chosen, we tried to balance what we know from the recent history in Brazil's economy, with the inherent sampling error when estimating volatilities with few observations.

[11] Results can be provided upon request.

2.3. Stochastic Volatilities and Covariances Models

Before we make the case for using a simpler model for forecasting volatility and covariance, we will draw heavily from Brailsford and Faff (1996) and compare various volatility models using a variety of error statistics. As other studies have shown, we will argue that simpler models can perform reasonably well, if not better than more sophisticated models, particularly at a lower frequency (e.g. Dimson and Marsh (1990), Tse (1991), and Tse and Tung (1992)). We will start with very simple naïve models and move towards relatively more sophisticated techniques. For all models, i and j represent the variables time series ($i = j$ are variances and $i \neq j$ are covariances), and t denotes the time period.

a) Random Walk (no drift)

Under a random walk with no drift, the best forecast for current volatility and covariance is the last realized volatility and covariance, as in expression below:

$$\sigma_{ij,t}^{RW} = \sigma_{ij,t-1}^{real.}. \tag{1}$$

b) Historical mean volatility

For the historical mean model, current volatility and covariance is forecasted using an equally weighted average of all past realized volatilities and covariances:

$$\sigma_{ij,t}^{HMV} = \frac{1}{t-1}\sum_{k=1}^{t-1}\sigma_{ij.k}^{real.}, \tag{2}$$

where $k = 1$ corresponds to the January 1995 observation.

One of the disadvantages of this model is that recent shocks in the time series carry the same weight as events that have occurred long ago, while one would expect more recent events to have a greater impact on the expected volatility.

c) Moving average model (MA-α)

A slight variation of the historical mean model would be to forecast current volatilities and covariances using only the past observations on realized volatility and covariance over a particular time window.

$$\sigma_{ij,t}^{MA(\alpha)} = \frac{1}{\alpha}\sum_{k=t-\alpha}^{t-1}\sigma_{ij,k}^{real.}, \tag{3}$$

where α represents the moving time window over which the mean volatility and covariance is calculated. For the sake of this exercise, and considering the data frequency and the stylized facts underlying the recent history of Brazilian economy, described in section 3 above, we consider two different moving time windows: (i) $\alpha = 12$ months; and (ii) $\alpha = 24$ months.

This specification eliminates the influence of 'older' shocks entirely, and may still not be the optimal choice for forecasting volatilities and covariances, depending upon the size of the moving time-window utilized. Some shocks have a long memory, although with a decreasing influence in time.

d) Exponentially weighted moving average model

One way to deal with the memory shortcomings of the historical and of the moving average model, is to consider an exponentially weighted moving average (EWMA) that will attach higher weights to more recent shocks, while still retaining some influence from older events, which has been extensively used by both practitioners and academicians:

$$\sigma_{ij,t}^{EWMA} = \lambda \sigma_{ij,t-1}^{EWMA} + (1-\lambda)\sigma_{ij,t-1}^{real.}, \qquad (4)$$

where λ is the decay factor, to be determined empirically so as to achieve the minimum root mean squared error (RMSE)[12]. In Table2 we present the optimal λ's considering the three different 'guesses' for initial volatility and covariances – Jan/95 to Dec/96, Jan/96 to Dec/96, and realized as of Dec/96. For Ibovespa and Brazilian FX rate, decay factors are not sensitive to the initial guess and remain basically the same (0.97 for FX rate and 0.95 for Ibovespa). For the Brazilian interest rate there is some variation in the decay factor, when using the 1996 year as initial guess, compared to the other two cases. It is worth mentioning that 1996 was a year of declining Brazilian interest rates, while 1995 was a year of strong variation in the Brazilian interest rates, when Brazil was shaken by the Mexican crisis, right after the implementation of the Real Plan. By the end of 1996, Brazil was starting to feel some premature impact from the Asian Crisis, and December was a month of higher volatility in the interest rate. For the other variables (US interest rate, oil, gold, and S&P 500), there is a significant difference between the optimal decay factors, using the realized volatility as of Dec/96, and the other two initial guesses. For example, the decay factor for the US interest rate is 0.85 when using the years of 1995 and 1996 as initial guess, and 0.84 when using the year of 1996 as an initial guess. However, it jumps to 0.99 when using the realized volatility as of Dec/96 is used as initial guess. Similar pattern is found for the other variables. Differences in RMSE are somewhat comparable to the differences found for decay factors. For all variables, however, the smallest RMSE is obtained for the realized volatility as of

[12] Error statistics other than RMSE have yielded very similar results. For the optimization, we have restricted λ in the interval [0.01, 0.99], using 0.01 steps.

Dec/96 as initial guess. Covariances are much more stable and there are fewer differences in the decay factors and corresponding RMSE across different initial guesses[13].

e) Regression model

This very simple model considers that the expected value of current volatility and covariance is a linear function of the lagged-realized volatility and covariance. For estimating the parameters, realized volatilities are regressed on their own first lagged values as in the projection below:

$$\sigma_{ij,t}^{real.} = \beta_0 + \beta_1 \sigma_{ij,t-1}^{real.} + \varepsilon_{t-1}. \tag{5}$$

Then, the estimated parameters $\widehat{\beta}_0$ and $\widehat{\beta}_1$ are used to estimate the forecast for current volatility as:

$$\sigma_{ij,t}^{REG} = \widehat{\beta}_0 + \widehat{\beta}_1 \sigma_{ij,t-1}^{real.}. \tag{6}$$

We estimate these regressions in two ways: (i) using the same parameters for forecasting volatilities and covariances over the entire out-of-sample window; and (ii) updating the parameters with new forecasts, while dropping the oldest observations. As we shall see in the next section, there are not substantial differences in out-of-sample forecast errors between the two cases.

Despite of its simplicity, Brailsford and Faff (1996) and Balaban, Bayar, and Faff (2002) found this regression to perform reasonably well in forecasting volatilities.

f) ARCH (1) model

As in Brailsford and Faff (1996) and Balaban, Bayar, and Faff (2002), and following Engle (1982), we will also estimate an ARCH (1) model as follows:

$$R_t = a + bR_{t-1} + e_t, \tag{7}$$

where the conditional variance is modeled as:

$$h_t = \alpha_0 + \alpha_1 e_{t-1}^2 + u_t, \tag{8}$$

with u_t assumed to be normally[14] distributed with zero mean and variance h_t^2, given the information set Ω_{t-1} (which includes $R_{t-1}, R_{t-2}, \ldots$), or simply $u_t \mid \Omega_{t-1} \sim N(0, h_t^2)$.

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[13] Results can be provided upon request.

g) GARCH (1,1) model

A variation of the ARCH (1) model above would entail not only the squared error term from equation (7), but also the lag volatility h_{t-1} as in the GARCH (1,1) model below:

$$h_t = \alpha_0 + \alpha_1 e_{t-1}^2 + \beta_1 h_{t-1} + u_t , \tag{9}$$

with $u_t \mid \Omega_{t-1} \sim N(0, h_t^2)$.

Brailsford and Faff (1993) found evidence favoring the used of a GARCH(3,1) model, whose forecasting performance was also investigated in this study. We did not found it to outperform the other models.

h) EGARCH (1,1) model

One shortcoming of the GARCH model, as pointed out by Figliewski (1997), is that it restricts the impact of a shock to be independent of its sign, whereas there is evidence of an asymmetric response for some markets, notably the stock market. Stock return volatility increases following a sharp price drop, but a price rise of the same size may even lead to lower volatility. In order to deal with this asymmetry problem, we propose to estimate the following two models: (i) the exponential GARCH model proposed by Nelson (1991), and (ii) a modified GARCH model proposed by Glosten, Jagannathan, and Runkle (1993).

The Nelson's (1991) EGARCH (1,1) formulation is:

$$\ln\left(h_t\right) = \alpha_0 + \gamma\left(\frac{e_{t-1}}{h_{t-1}}\right) + \lambda\left[\left(\frac{\left|e_{t-1}\right|}{h_{t-1}}\right) - \left(\frac{2}{\pi}\right)^{1/2}\right] + \beta_1 \ln\left(h_{t-1}\right) + u_t , \tag{10}$$

again with $u_t \mid \Omega_{t-1} \sim N(0, h_t^2)$.

In the case of covariances, it would be extremely difficult and time consuming to try to fit a multivariate GARCH for modeling covariances among 8 variables. Previous studies have usually focused on 2 or 3 variables at most[15]. For lower frequency data, like ours, the

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[14] Bollerslev (1990) warns that residual distributions may not be normal and he suggested using a t-distribution. Other authors have proposed to use more sophisticated distributions to deal with non-normality in error terms, such as the Gram-Chalier type distribution utilized by Lee and Tse (1991) and Tse (1991). Still, Tse (1991) finds that EWMA have outperformed GARCH-type models.

[15] E.g. Bollerslev, Engle, and Nelson (1994), Engle and Kroner (1995), Ding and Engle (2001), and Lopez and Valter (2001) to cite some.

literature has favored the use of simpler models in comparison to ARCH-GARCH processes. Even the Dynamic Conditional Correlation model proposed by Engle (2002), and which overcomes the problem of estimating multivariate GARCH, has not outperformed other simpler models like the EWMA by a large margin.

2.4. Forecast Errors

Forecast accuracy can be assessed through different error statistics, as discussed in Brailsford and Faff (1996) and Balaban, Bayar, and Faff (2002). In this study we focus on the most popular measure, the root mean squared error (RMSE)[16]:

$$RMSE = \sqrt{\frac{1}{T-t}\sum_{k=t}^{T}\left(\sigma_{ij,k}^{forecast} - \sigma_{ij,k}^{real.}\right)^2}\ ; \tag{11}$$

Results are provided in Table 3 and some comments are in order. First, it is not possible to single out the model that provides superior forecasts. Random walk ranked first in four occasions when forecasting volatilities for Brazilian interest rate, US interest rate, Brazilian FX rate, and Ibovespa, while the moving average with a 12-month time window, linear regression (with parameter update), and EWMA with Dec/96 as initial guess, ranked first once. For covariances[17], the ranking is even more dispersed, with virtually almost all models have ranked first at least once. This adds to the mixed results that have been found in the literature, favoring some models depending upon the variable, the frequency, and the period analyzed[18]. Second, naïve models generally perform better than more sophisticated models. For no variable, did any of the ARCH-GARCH specifications tried here outperform the other models[19]. This result is consistent with the literature findings (e.g. Tse (1991) and Tse and Tung (1992) to cite some) that naïve models perform better than ARCH/GARCH

[16] We have also estimated the mean absolute error (MAE) and the mean absolute percentage error (MAPE), and obtained similar results for forecast performance ranking over various models.

[17] Results can be provided upon request.

[18] For example, Andersen and Bollerslev (1998) and Andersen, Bollerslev, and Lange (1999) find that ARCH models produce significantly accurate forecasts for intra-day data. For daily data, Tse (1991) and Tse and Tung (1992) find that EWMA produces superior forecasts. Brailford and Faff (1996) provide evidence that ARCH-type models and a simple regression model appear to provide superior forecast volatility, although their results are sensitive to the forecast error statistic utilized. Similar results are obtained by Balaban, Bayar, and Faff (2002), for a larger sample of countries (fourteen stock markets), for weekly and monthly volatility.

[19] Gold time series yielded fairly small coefficients for all ARCH and GARCH specifications. Also, GARCH(3,1) coefficients were also very small for all variables. Other ARCH and GARCH specifications have produced reasonable estimates for coefficients.

models. Finally, the forecast out-of-sample performances of various models, as measured by RMSE, have been reasonably close many times.

3. Monte Carlo and Distribution of Simulated Returns

As mentioned before, there is extensive empirical evidence that historical return distributions for a number of macroeconomic variables such as interest rates, foreign exchange rate, and commodity and equity prices present properties that are not compatible with normal distributions. Indeed, in the wake of many booms and crashes, return distributions on these variables appear to have significant probability mass in the tails, characteristic of heavy tailed distributions. This fact has important implications for risk management and in particular for methodologies such as VaR, designed to capture the risk of extreme events.

The issue of extreme events becomes even more important when assessing the risk financial institutions might face when operating in a macroeconomic environments characteristic of volatile emerging economies, as is the case of Brazil. Not accounting properly for extreme events probability could result on underestimation of bank default risk, should Brazil face another crisis, for example. We want to argue that, even though simple, the proposed volatility process can capture most of the historical distributions features, when we select appropriate decay factors. Indeed, as we shall see, decreasing the decay factor does increase the mass concentration at the tails and also permits matching historical distribution of changes in volatilities closer. This is a very important point. This criterion is widely used both by academics and practitioners. We will contend that if the objective is to match closer the historical distribution of returns or the historical evolution of volatilities, than optimal decay factors for volatily process may no longer coincide with the ones minimizing RMSE.

3.1. Historical Returns: Fat-Tail Distributions

In Table 4 we present probability mass at different standard deviation cut-offs from the mean, over different overlapping time windows, and using daily returns from 07/1/1994 to 12/31/2003[20]. The standard deviation cut-offs correspond to the main percentiles in the normal distribution, for one tail: 3.08σ, 2.335σ, 1.63σ, and 1.28σ for 99.9, 99, 95, and 90 normal percentiles respectively. Thus, for these cut-offs, corresponding mass probabilities at one of the tails would be 0.001, 0.01, 0.05, and 0.1, for a normal distribution. If historical distributions have heavy tails, then estimated mass probabilities would be larger at some (or all) these cut-offs. As we can see in Table 8, this is the case for the 3.08σ and 2.335σ cut-

[20] For the FX rate, data goes from 1/18/1999 to 12/31/03, corresponding to the period when Brazil abandoned the fixed rate regime.

offs for daily returns for all variables, while the mass in the 1.63σ, and 1.28σ cut-offs are generally smaller than for normal distributions, highlighting the fat-tail feature of these historical series, coming at the expense of smaller mass in the center of the distribution. But these tail probabilities, by themselves, do not constitute a final proof that the distribution is fat-tailed. Indeed, as we shall see below, a simple normality test statistic fails to reject the null of normality. As the time-window for estimating returns increases, the mass in the tails decreases, although it is still heavy for the interest rates. Another interesting feature of these distributions is its mass asymmetry between two tails. For daily returns for the Brazilian interest rate, for example, the mass at the 99.9 right percentile is 0.009, while it is 0.006 at the left percentile. For oil, the right tail mass at 99.9% level is 0.005, while it is 0.009 at the left tail. This highlights the asymmetric response volatility has shown when facing positive or negative shocks. In general, the literature has pointed to a more acute response for the volatilities in the presence of negative shocks as compared to positive shocks. Here, however, we find that it can go either way, depending upon the variable. Indeed, for US rate and S&P 500, it can even be symmetric, with equal mass distributions at both tails. Finally, it is worth mentioning that almost none of the tail probabilities appear to be consistent with those of a normal distribution.

Another interesting test to be performed, which complements the information provided by the tail probability mass, is a test of normality. We employ here the well- known Wilk-Shapiro (W-S) test[21]. W-S statistics is a value between 0 and 1, with numbers close to 1 indicating normality. For historical returns, over different time windows, we present W-S statistics and the probability of being smaller than W-S in Table 5. For all returns time series and time windows, the W-S statistics are well below 1, ranging from 0.189 (Brazilian interest rate, daily returns in Panel A) to 0.025 (Oil, 6-months time window in Panel C). The associated probabilities of falling below W-S values are fairly small (≤ 0.01 for all cases). Still, because the W-S statistics are so much smaller than one, we interpret these results as evidence of non-normality. However, non-normality here comes from peaked distributions, not fat tails. This feature is also apparent in Table 4, when the tail probability corresponding to the 1.63σ and 1.28σ cut-offs are generally bigger than for 95 and 90 normal percentiles, indicating that there is a larger mass concentrations towards the center of the simulated distribution.

We introduce the simulation methodology in more details in the next section. Then we contrast simulation results with historical distributions, with different sets of decay factors.

[21] In spite of being widely used, the W-S statistics should be taken only as one piece of evidence and not a final word on whether the distribution is normally distributed or not.

3.2. Simulation Methodology

3.2.1. Simulating Interest Rates

Risk-free stochastic interest rate is simulated as an Orstein-Ühlenbeck process, via the Hull and White extended Vasicek model (Hull and White (1990)):

$$\Delta r = a\left(\frac{\theta(t)}{a} - r\right)\Delta t + \sigma\Delta z\,, \tag{12}$$

where Δr is the risk-neutral process by which r changes; a is the rate at which r reverts to its long term mean; r is the instantaneous continuously compounded short-term interest rate; $\theta(t)$ is an unknown function of time that is chosen so that the model is consistent with the initial term structure and is calculated from the initial term structure; Δt is a small increment to time; σ is the instantaneous standard deviation of r, which is assumed to be constant; and Δz follows a Wiener process driving term structure movements with Δz being related to Δt by the function $\Delta z = \varepsilon\sqrt{\Delta t}$.

The function $\theta(t)$ is estimated as:

$$\theta(t) = F_t(0,t) + aF(0,t) + \frac{\sigma^2}{2a}(1 - e^{-2at}), \tag{13}$$

where $F(0,t)$ is the instantaneous forward rate for a maturity t, as seen at time zero, and the subscript t denotes a partial derivative with respect to t.

For the purpose of model validation, parameters for the interest rate processes are estimated using historical monthly data from July 94 to December 2003.

3.2.2. Simulating Asset Returns and Prices

The value of equity market indices, FX rate, and commodity prices are assumed to follow a geometric Brownian motion (GBM), with constant expected growth rate and volatility. The expected growth rate is estimated as the expected return on the asset minus its dividend yield. For a discrete time step, Δt, the GBM can be written as:

$$S + \Delta S = S\exp\left[\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma\varepsilon\sqrt{\Delta t}\right], \tag{14}$$

where: S is the value of the asset at time t; μ is the expected growth rate; σ is the volatility, ε is a random shock from a standardized normal distribution; and Δt is a small increment to time.

The return on the asset is then estimated as

$$K_m = ln((S + \Delta S)/S)\ , \tag{15}$$

As for the interest rate models, we use historical data over the same period (Jul/94 to Dec/03) to estimate the parameters for expression (14).

3.2.3. Cholesky Decomposition

For drawing returns for multiple shocks of correlated stochastic processes, we input a correlation matrix of historical returns for the variables in the following sequence: domestic interest rate, foreign interest rate, FX rate, commodities prices, domestic equity index, and foreign equity index. We then draw independent samples $x_1, x_2, ..., x_n$ from univariate standardized normal distributions. The shocks to the variables will then be estimated sequentially so that:

$$\varepsilon_1 = x_1\ ,$$

$$\varepsilon_2 = \rho_{12} x_1 + x_2 \sqrt{1 - \rho_{12}^2}\ ,$$

$$\varepsilon_3 = \rho_{13} x_1 + \rho_{23} x_2 + x_3 \sqrt{1 - \rho_{13}^2 - \rho_{23}^2}\ \text{, etc.,} \tag{16}$$

where ρ_{ij} is the correlation between variables x_i and x_j. This procedure is known as the Cholesky Decomposition.

3.2.4. Stochastic Volatilities and Covariances

Variances and covariances are updated each time step via the following model:

$$\sigma_{i,t}^2 = \lambda \sigma_{i,t-1}^2 + (1-\lambda)(\sigma_{i,t-1} u_t)^2\ , \tag{17a}$$

for variances and:

$$\sigma_{ij,t} = \lambda \sigma_{ij,t-1} + (1-\lambda) \sigma_{ij,t-1} u_t\ , \tag{17b}$$

for covariances, where $u_t \sim N(0,1)$. Initial variances and covariances for starting up the updates for each simulation run are estimated as the average realized variances and covariances over the entire Jul/94 to Dec/03 period. For the first set of simulations we use the lambdas that minimize the RMSE between EWMA forecasts and realized volatilities (third row, Table 4) and covariances (Panel C, Table 5). Later, as we shall see, we will vary lambda so as to obtain distributions for simulated returns, volatilities, and changes in volatilities that are similar to observed historical distributions.

3.3. Simulation Results

For the purpose of model validation, we have performed simulations for two cases: (i) constant volatility; and (ii) stochastic volatilities and covariances. For constant volatility both volatilities and correlations are estimated using historical data and will be kept time invariant. For the stochastic updates, we utilize the same input as for the constant volatility and decay factors that were estimated using historical data on realized variances and covariances, so as to minimize the root mean squared error. Estimated decay factors are in the range of .96-.99 (Tables 4 and 5), which is also consistent with what Risk Metrics reports as their estimates of decay factors for several indices in emerging economies, at a monthly frequency[22].

3.3.1. Constant Volatility and Covariances

The base case for comparison is the one with constant volatilities and covariances. If shocks are normally distributed, then mass probability for simulated returns should be very close to the normal cut-offs. Further, since the underlying stochastic processes do not explicitly discriminate between positive and negative shocks, there shouldn't be any significant difference in the probability mass between right and left tails. We have run 2000 simulations using the stochastic processes described in Section 3.2 above, but we kept volatilities and covariances constant (for this, we just set $\lambda = 1$, yielding $\sigma^2_{ij,t} = \sigma^2_{ij,t-1}$). In Table 6, we present probabilities for falling above (right tail) or below (left tail) different tail cut-offs, for 1-month, 6-months, and 12-months time windows. Results are clear: for all variables, the probability mass at different tail cut-offs are very close to those corresponding normal distribution cut-offs, over all time windows. For example, for 1-month time window (panel A), at 99.9% level, tail probabilities are in the range of 0 to 0.002 (for normal distribution it is expected to be 0.001); at 99% level they are in the range of 0.006 to 0.014, against 0.01 for normal distribution. At 95% level we found tail probabilities to be in the range of 0.043 to 0.055, when normal distributions have 0.05 probability mass; and at the 90% level they are in the range of 0.087 to 0.108 (for normal distribution it is expected to be 0.10). Minor departures from the normal distribution are probably due to sampling error. Further, probability mass are very symmetric between left and right tails. Wilk-Shapiro statistics (Table 7) show that simulated returns are normally distributed. W-S values are fairly close to one for all variables and all returns time windows, ranging from 0.984 (Brazilian interest rate, 1-month returns, and Brazilian FX rate, 6-months returns to 0.991 (Brazilian and US interest rates, 12-months returns), although the probability of being less than W-S is somewhat significant. But these probabilities need to be interpreted with caution. Considering the high W-S values, being likely to be smaller than 0.991 may only mean that values can still

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[22] Risk Metrics recommends $\lambda = 0.97$ for monthly data.

be reasonably close to 1. This is one limitation of W-S statistics and the reason why it should not be taken as a final word on whether or not a variable is normally distributed or not.

While not surprising at all, these results are a nice (and important) check that the underlying stochastic framework is working as expected for the constant volatility case. In what follows, we will allow λ to have a value different than one and see what happens to the distribution of returns and changes in volatilities.

3.3.2. Stochastic Volatility with RMSE Lambdas

The second case to be analyzed corresponds to stochastic volatilities, with decay factors that were estimated as minimizing the RMSE for historical returns data, which are in the range of 0.96-0.99 for both volatilities and covariances. For Tail probabilities, results presented in Table 8 are fairly close to the static case. Probability mass is practically the same as for the static case and very similar as for the normal distribution, with no significant difference between left and right tails. W-S statistics (Table 9) are very similar to the constant volatility case: they are very close to one for all variables and all returns time-windows, with some significant probability of being less than W-S. Even with such a high smooth factor, we do capture some volatility variation, although changes in volatilities are small (Table 10), much smaller than for historical distributions, and much less volatile. For example, while the standard deviation of historical Brazilian interest rate volatilities is 0.0236 (panel A), simulated volatilities have a standard deviation of 0.0015. Percentiles also show the difference between historical and simulated values, for λ close to 1: while historical distribution possesses percentiles cut-offs going from -0.0574 (1% percentile) to 0.0791 (99% percentile), simulated values stay in the range of -0.0043(1% percentile) to 0.0045 (99% percentile). Similar patterns can be observed on other variables and for different time-windows (panels B and C). Interestingly, changes in covariances[23] are generally of same magnitude as for historical simulations. These results are important to show that decay factors that minimize RMSE, do not succeed in reproducing the heavy tail or the volatility evolution that we observe historically.

3.3.3. Changing Lambda to Increase Tail Mass

Following the results in the previous section, a natural conjecture is whether or not it is possible to increase the probability mass in the tails by changing the decay factor. We present in Table 11 simulation results for 12-months returns using different values for the decay factor and some comments are in order. First, in general, as we decrease λ, we increase the mass at the tails. For example, for Brazilian interest rate, probability mass at the right tail at the 99.9% level moved from 0.004 ($\lambda = 0.90$) to 0.015 ($\lambda = 0.50$), for Brazilian FX rate,

[23] Results can be provided upon request.

left tail probability at the 99.9% level moved from 0.003 ($\lambda = 0.90$) to 0.011 ($\lambda = 0.50$). However, as we keep decreasing λ, tail probabilities also decrease. For example, right tail probability for Brazilian interest rate at 99.9% level moves back to 0.005. Similar retractions can be observed on all other variables. The reason for that relies on two counterbalancing effects that are inherent to our model. The first one, as we decrease lambda we decrease the smoothing effect, putting more weight on the recent shocks and thus allowing for some significant spikes in volatilities, which can subsequently produce high simulated returns. The counterbalancing effect is a volatility trap. Remember that the model we are implementing is $\sigma_{ij,t} = \lambda\sigma_{ij,t-1} + (1-\lambda)\sigma_{ij,t-1}\nu_t$, where $\nu_t \sim N(0,1)$. So, 66.7% of the time, the drawn shock components will be between 0 and 1, which will reduce the value of $\sigma_{ij,t}$ compared to $\sigma_{ij,t-1}$, because the term $\sigma_{ij,t-1}\nu_t$ will be smaller than $\sigma_{ij,t-1}$. Since this will happen quite frequently (2/3 of the time), once the volatility reaches a certain level, even if the simulation draw a high shock component, it will not be enough to bring volatility back to (or above) its initial value. Likewise, a high volatility trap can also occur, but at a lower frequency because we draw $\nu_t \sim N(0,1)$, more frequently at the 0 – 1 range. The lower the decay factor, the faster the volatility can reach the trap level. This result highlights the limitation of our model in capturing the fat tail feature of historical returns distributions described in the literature. Still, as long as λ is not set close to 0, it is possible to achieve a higher probability mass in the tails. This result also provides incentive for testing volatility models that would not fall into volatility traps. In this regard, mean reverting models can be quite interesting[24]. Not only they capture explicitly the mean reverting feature of volatilities, as extensively documented in the literature, but they also avoid the problem of volatility trap, as volatility will always be dragged back to a long-term mean. This is the topic of another ongoing research.

3.3.4. Matching the Historical Distribution for Changes in Volatilities

In addition to adding mass in to the tails, changing the decay factor also allows us to match the historical distribution of change in volatilities more closely. Being able to simulate more accurately the volatilities is of particular interest for valuing financial instruments that are directly linked to volatility as, for example, via the well-known Black-Scholes option pricing formula. This would be very important when undertaking risk assessments for portfolios having substantial option exposure.

[24] The idea of modeling volatility through a mean reverting process is not new. In fact, Stein and Stein (1991) have done that to derive an explicit formula for pricing options, also with the intention of capturing the heavy tail feature of returns distribution. More recently, some authors have mixed the mean reverting processes with a poison component, to increase the mass in the tails.

We have manually changed the decay factors and compared simulated distributions (2000 runs) of changes in volatilities, at 12-month time window, with historical distributions, so as to minimize the absolute difference[25]. The decay factors that were found to produce the best simulated distribution of changes in volatilities as presented in Table 12, and they are fairly different than the ones obtained so as to minimize RMSE. For example, for interest rates now, the optimal decay factors are 0.83 and 0.81 (for Brazilian and U.S. interest rates respectively), while for minimum RMSE they were 0.99 for both. Similarly, we obtained 0.82, 0.92, 0.85, 0.91, and 0.94 for Brazilian FX rate, Oil, Gold, Ibovespa, and S&P 500, while for minimizing RMSE they were 0.97, 0.99, 0.99, 0.96, and 0.99 respectively. The latter numbers, as mentioned before, are also consistent with what Risk Metrics generally recommend for monthly time series. Once again, these results highlight the inadequacy of using the minimum RMSE for obtaining decay factors, if the objective is to obtain, for example, distribution of changes in volatilities that better resemble historical values. In Table 13 we present the results for the tail probabilities, corresponding to the new decay factors. Tail probability mass are not significantly differently from the normal levels. At 99.9% percentile, for example, for all variables and all returns time-windows, simulated returns have tail mass are in the range of 0 to 0.005. It seems that for the stochastic volatility model to be able to produce heavy tail distributions, we need to use smaller $\lambda's$. W-S statistics presented in Table 14 just reinforce these results: W-S values are all close to 1, although the probability of being less than W-S is somewhat big (exceptions are the Brazilian and US interest rates, for which prob.<W-S is 0.01).

On the distribution of changes in volatilities[26] (Table 15), however, we can observe a significant improvement on simulated values, as we were able to reproduce the evolution in volatilities more closely. Take, for example, the distribution for 1-month returns for Brazilian interest rate. The 1% percentile simulated level is -0.00617 while the historical level is -0.00574. For other percentiles the values depart slightly from historical values. Also, generally, the dispersion increases a bit as the time window increases, For the Brazilian interest rate, the same simulated percentile stands at -0.1398 and -0.1487, for 6- and 12-months changes in volatilities, while historical values are –0.1060 and –0.0931 respectively[27]. These results have important implications, because they show the capability of our model in

[25] Even though we do not optimize decay factors so as to match the evolution of covariances, it is theoretically possible to do that. It is, however, more challenging, as changes in the decay factor for one variable will have direct and perhaps off-setting impact on other covariances.

[26] Distributions of changes in covariances were fairly similar to the case where decay factors were estimated so as to minimize RMSE. Considering that we are using exactly the same decay factors here (we are just updating decay factors for volatilities), this result is not surprising at all. Tables are available upon request.

[27] We also found sampling error to increase as λ decreases, which derives directly from our model formulation: the smaller the λ, the heavier the weight of the shock component $\sigma_{t-1}^2 u_t$, magnifying the sampling error.

reproducing evolution of volatilities that are comparable to observed historical values. This is, as already highlighted above, central for risk assessment of portfolios of options-alike instruments, for which volatility play a crucial role. It is important to reemphasize that we have not been able to accomplish that with $\lambda's$ determined as minimizing RMSE: $\lambda's$ were very close to one and simulated changes in volatilities were fairly flat.

4. Simulating Credit Transition Matrix for Two Large Brazilian Banks

In the previous section we provided evidence that the our model produces reasonable forecasts for volatilities and covariances, several times outperforming more elaborate models. We also showed that it is possible to simulate return distributions, with appropriate decay factors, that are comparable to fat-tailed distribution, as largely documented in the empirical literature. It would be interesting to examine the implications of utilizing stochastic volatilities and covariances, in some applications. For this purpose, we revisit a simulation exercise performed by Barnhill, Souto, and Tabak (2003), for estimating credit transition matrix for 2 large Brazilian banks. Credit transition matrices represent probabilities that loans in one particular credit risk category migrates to another category.

Modeling credit transition probabilities is central to fixed income portfolio risk assessments. Recently, the Central Bank of Brazil has established a Credit Risk Bureau, which collects information on bank credit rating for borrowers and credit transition probabilities. Barnhill, Souto, and Tabak (2003) estimate the parameters for and implement a credit risk model developed by Barnhill and Maxwell (2002) – the Portfolio Simulation Approach (PSA) – to simulate the credit transition matrix (CTM) for two Brazilian banks loans portfolio. Crucial to obtaining the CTM is a debt-to-value ratio (D/V) distributional analysis for different credit risk categories, using historical data for Brazilian companies that are borrowing from these banks. While data on D/V are usually publicly available, the ratings banks assign to the companies is considered confidential in Brazil, and protected by law. The D/V distributional analysis for 2 large Brazilian banks has been carried out by the Central Bank of Brazil, as of December of 2002, and published in Barnhill, Souto, and Tabak (2003). The basic idea is that firms' will migrate from one credit risk category to the other, as their simulated D/V ratio[28] falls below or above the maximum/minimum D/V thresholds, in the same spirit of Merton's (1974) model. Barnhill, Souto, and Tabak (2003) have obtained simulated CTM that was very close to the historical CTM estimated by the Brazilian Credit Bureau (Table 16), although they have not succeeded in obtaining any simulated default rates for the top two credit quality loans at all.

[28] Firms' equities are estimated via the CAPM one-factor model.

Historical and simulated credit transition matrices for these two banks are very similar to one another. The most important difference being that the simulated default rates on AA and A rated loans is zero or close to zero, while the historical default rates have a small positive value. In order to provide a more precise measure of how close one transition matrix is to the other, we use the metrics proposed by Jafry and Shuermann (2004), defined as:

$$M_{SVD}(P) \triangleq \frac{\sum_{i=1}^{N} \sqrt{\lambda_i\left(\tilde{P}'\tilde{P}\right)}}{N}, \tag{18}$$

where P is the transition matrix, $\tilde{P} \triangleq P - I$, I is the identity matrix, λ_i is the i^{th} eigenvalue of P and N is the order of the matrix P.

For the historical CTM, $M_{SVD} = 0.3294$, while for the simulated CTM, $M_{SVD} = 0.3306$, resulting in $\Delta M_{SVD} = 0.0012$. As compared to values provide in Jafry and Shuermann (2004) for bootstrapped SVDs, this value is a strong indicator that the two CTMs are indeed very similar to one another[29].

We repeat the same exercise, with stochastic volatilities and covariances, in an effort to better match the CTM, particularly with respect to default rates. For this purpose, we use the lambdas that have been obtained so as to more closely match the distribution of changes in volatilities, over a 12-month time-window. As discussed before, different optimization criteria could have been used, yielding different sets of optimal decay factors. Results are presented in Tables 17 and 18 and few comments are in order. First, calibrated target D/V ratios are higher, which can cause simulated Brazilian companies to move more to lower credit quality categories. Indeed, this is exactly what we observe in Table 27, as transition probabilities towards lower credit ratings are higher when using smaller $\lambda's$. We expect this dispersion to increase for even smaller $\lambda's$. This is an interesting feature of our simulation, particularly as applied to emerging economies like Brazil, where companies seem to be more prone to negative shocks than to positive shocks, and where negative shocks have occurred more frequently in the recent history. Second, even though we still do not succeed in achieving significant default rates for top 2 credit risk categories, we do capture some probability rate for A loans (0.01) and we expect to be able to obtain a slightly higher default rate, with smaller $\lambda's$. Finally, it is important to stress that these results are consistent with the results we obtained for simulated returns distributions. Remember that we have not been able to capture heavy tail in the distribution, for the lambdas used so as to match closer historical

[29] In Jafry and Shuermann (2004), average ΔM_{SVD} ranged from -0.03491 to 0.01071, for transition probability matrices estimated using different methods, but over the same dataset of S&P ratings histories.

changes in volatilities, although we have showed that smaller $\lambda's$ can make the probability mass tail to increase slightly. In this respect, volatility models that also incorporate the unsystematic volatility factor, like models that are combined with a Poison process, have great potential in inducing higher probability mass tails, which we expect to result in higher default rates at the lower credit risk levels.

5. Concluding Remarks

We have examined the performance of stochastic volatility processes, through different dimensions. First, from the perspective of out-of-sample forecasting performance, none of the models we have estimated consistently outperforms the others. Further, forecasting errors were generally close across different models and these results are in line with what the literature has found. For us they had a practical implication, because we could then choose the model that was more convenient to be implemented computationally, considering that it was inserted in a robust Monte Carlo exercise – the Portfolio Simulation Approach (PSA) – for modeling detailed banks' portfolios and balance sheets. Under this approach, updating stochastic volatilities and correlations is just a small fraction of a comprehensive Monte Carlo exercise that integrates market and credit risk components.

Another important dimension refers to the non-normality of price returns that are widely documented in the literature, particularly that price returns have fat-tailed distributions. In this respect, several studies have advocated the use of stochastic volatilities as a way to increase the probability mass toward the tails. We have followed this line of research and updated volatilities using very simple model that ressemble the EWMA. Our results show that it is possible to increase the probability mass in the tails and to more closely match distributions of change in volatilities over different time windows, when selecting appropriately decay factors. And selected decay factors in these cases were different than the decay factors that are obtained by minimizing the root mean squared errors (RMSE). If one wants to use our model, so as to obtain heavier mass tails, then the RMSE approach is not the best one for obtaining the appropriate decay factors.

The final application, simulating credit transition matrices, still highlights the limitations of including a stochastic volatility process within the Monte Carlo simulation. We do not fully succeed in obtaining historical default rates at the top 2 credit quality categories, although we have achieved a higher degree of dispersion. This result is important when simulating banks' portfolios that operate in volatile economies, as is the case of Brazilian banks.

In this regard, it is important to note that we have modeled only systematic volatilities[30]. One potential venue for future research is to model unsystematic volatilities,

[30] We have modeled the volatilities of stochastic processes that affect directly the systematic component of risk, in the one factor asset pricing model used to revalue firms' equity.

which will likely increase the simulated default rates. That can be accomplished by modeling the volatility that impacts the firm-specific risk component in the CAPM formula, for calculating borrowers' simulated equity value. Further, it is well known and documented that volatilities have a mean reversion pattern and that volatilities react differently to negative and positive shocks. These features are not captured through our model. Another potential future venue of research would be to investigate the performance of different volatility models, so as to be able to capture all these nuances. Using mean reverting models for volatilities, combined with a poison process is a line of research that is being currently pursued. While some of the recent studies only look at the volatility dimension, perhaps to keep the matters simple, we stress the importance of considering the correlation between stochastic processes.

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Table 1
Initial Volatilities for EWMA Stochastic Updates

This table presents the three cases for initial volatilities utilized to estimate EWMA optimal decay factor and out-of-sample EWMA forecast errors. For the historical data cases, volatilities were estimated as the variance of returns over the periods indicated below. Monthly realized volatilities were estimated using within month returns.

	BR rate	US rate	FX rate	Oil	Gold	Ibovespa	S&P 500
Historical data (Jan./95 - Dec./96)	7.79E-03	1.17E-06	1.22E-04	3.38E-03	2.86E-04	8.93E-03	5.78E-04
Historical data (Jan./96 - Dec./96)	1.40E-03	1.21E-06	1.15E-06	4.70E-03	3.71E-04	3.10E-03	9.52E-04
Realized volatilities in Dec./96	9.46E-03	4.04E-04	1.07E-03	2.52E-02	3.45E-03	8.00E-03	8.97E-03

Table 2
Optimal Decay Factors (Volatilities), Minimizing RMSE

Optimal decay factors obtained for different volatility time series and corresponding root mean squared errors (RMSE). Optimal decay factors were selected as the ones which minimized the RMSE.

Initial Volatility Cases:		BR rate	US rate	FX rate	Oil	Gold	Ibovespa	S&P 500
Variance for Jan/95 to	λ	0.99	0.85	0.97	0.86	0.87	0.95	0.85
Dec/96 period	RMSE	9.63E-03	5.30E-04	7.28E-03	1.76E-02	7.28E-03	8.81E-03	1.01E-02
Variance for Jan/96 to	λ	0.93	0.84	0.97	0.87	0.87	0.95	0.86
Dec/96 period	RMSE	1.28E-02	5.30E-04	7.28E-03	1.74E-02	7.28E-03	9.04E-03	1.01E-02
Realized Variances as of	λ	0.99	0.99	0.97	0.99	0.99	0.96	0.99
December of 96	RMSE	7.94E-03	3.29E-04	7.26E-03	1.56E-02	6.58E-03	8.28E-03	8.08E-03

Table 3
Out-of-Sample Forecast Performance for Volatility Models

This Table reports out-of-sample ranking and performance based on the RMSE for various volatility forecast models. First column for each time series is the corresponding RMSE for each model and the second column brings the ranking of the model (best performer for each time series are highlighted). MA-12 and MA-24 are the moving average models over 12- and 24-months rolling time windows. EWMA (i) uses the volatility estimated for the period of Jan/95 to Dec/96 as the initial guess, while EWMA (ii) uses the volatility estimated for the period of Jan/96 to Dec/96, and EWMA (iii) uses the volatility estimated as of Dec/96 as initial guess.

	BR rate		US rate		FX rate		Oil	
	Error	Ranking	Error	Ranking	Error	Ranking	Error	Ranking
Random Walk	0.230	1	9.827E-03	1	0.390	1	1.540	7
Historical Mean	0.446	6	2.809E-02	11	0.396	2	1.150	2
MA-12	0.378	5	9.398E-03	2	0.641	10	1.302	6
MA-24	0.347	4	1.612E-02	3	0.563	9	1.168	3
EWMA (i)	0.286	3	1.882E-02	8	0.835	12	2.201	12
EWMA (ii)	0.789	13	1.882E-02	9	0.844	13	2.158	10
EWMA (iii)	0.267	2	2.207E-02	10	0.764	11	1.129	1
Linear Regression (no update)	0.648	12	5.813E-02	12	0.541	7	1.186	5
Linear Regression (updating dynam.)	0.583	8	5.794E-02	11	0.549	8	1.170	4
ARCH(1)	0.639	11	1.759E-02	4	0.509	4	2.120	8
GARCH(1,1)	0.601	9	1.788E-02	5	0.527	6	2.168	11
GARCH(3,1)	0.569	7	1.847E-02	6	0.463	3	2.121	9
EGARCH(1,1)	0.628	10	1.850E-02	7	0.520	5	n.a.	-

	Gold		Ibovespa		S&P 500	
	Error	Ranking	Error	Ranking	Error	Ranking
Random Walk	0.417	4	0.402	1	0.323	2
Historical Mean	0.381	3	0.613	7	0.391	5
MA-12	0.369	1	0.625	9	0.494	7
MA-24	0.377	2	0.587	6	0.453	6
EWMA (i)	1.002	13	0.557	5	0.980	12
EWMA (ii)	0.998	11	0.926	11	0.964	11
EWMA (iii)	0.789	7	0.624	8	0.351	4
Linear Regression (no update)	0.438	5	0.577	4	0.339	3
Linear Regression (updating dynam.)	0.447	6	0.517	3	0.303	1
ARCH(1)	0.996	9	1.701	13	0.847	9
GARCH(1,1)	0.996	8	1.428	12	0.807	8
GARCH(3,1)	0.996	10	0.412	2	0.847	10
EGARCH(1,1)	0.998	12	0.890	10	n.a.	-

Table 4
Tail Probabilities – Historical Returns

This table presents probability mass at the tails, for historical returns over different time windows, estimated using different cut-offs correspond to the main percentiles, for one tail, in the normal distribution: 3.08σ, 2.335σ, 1.63σ, and 1.28σ for 99.9, 99, 95, and 90 normal percentiles respectively. Thus, for these cut-offs, corresponding mass probabilities at one of the tails should be 0.001, 0.01, 0.05, and 0.1, for a normal distribution (values highlighted in the last column). Also highlighted are the cases were the tail probability mass were significantly bigger than the normal cut-offs.

Panel A: 1-Day Returns.

Probability	BR rate	US rate	FX rate	Oil	Gold	Ibovespa	S&P 500	Normal
N. Obs.	2350	2357	1296	2365	2346	2458	2458	
Right Tail:								
99.9% Perc.	0.009	0.008	0.010	0.005	0.008	0.006	0.007	0.001
99% Perc.	0.020	0.017	0.018	0.011	0.019	0.016	0.016	0.010
95% Perc.	0.040	0.035	0.030	0.038	0.044	0.043	0.046	0.050
90% Perc.	0.065	0.057	0.053	0.074	0.072	0.071	0.076	0.100
Left Tail:								
99.9% Perc.	0.006	0.008	0.007	0.009	0.006	0.010	0.005	0.001
99% Perc.	0.015	0.015	0.014	0.017	0.015	0.020	0.015	0.010
95% Perc.	0.038	0.032	0.034	0.044	0.042	0.045	0.052	0.050
90% Perc.	0.063	0.057	0.056	0.085	0.067	0.075	0.092	0.100

Panel B: 1-Month Returns (Daily Data).

Probability	BR rate	US rate	FX rate	Oil	Gold	Ibovespa	S&P 500	Normal
N. Obs.	2341	2337	1276	2345	2326	2438	2438	
Right Tail:								
99.9% Perc.	0.017	0.001	0.013	0.000	0.009	0.001	0.002	0.001
99% Perc.	0.027	0.006	0.022	0.007	0.015	0.005	0.008	0.010
95% Perc.	0.040	0.028	0.037	0.040	0.045	0.045	0.032	0.050
90% Perc.	0.049	0.054	0.056	0.090	0.086	0.081	0.069	0.100
Left Tail:								
99.9% Perc.	0.003	0.025	0.002	0.003	0.001	0.009	0.007	0.001
99% Perc.	0.020	0.037	0.011	0.020	0.005	0.023	0.024	0.010
95% Perc.	0.041	0.057	0.049	0.052	0.036	0.066	0.062	0.050
90% Perc.	0.067	0.073	0.081	0.103	0.066	0.097	0.105	0.100

Table 4 – Cont.
Tail Probabilities – Historical Returns

Panel C: 6-Month Returns (Daily Data).

Probability	BR rate	US rate	FX rate	Oil	Gold	Ibovespa	S&P 500	Normal
N. Obs.	2234	2230	1169	2238	2219	2331	2331	
Right Tail:								
99.9% Perc.	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
99% Perc.	0.017	0.000	0.023	0.012	0.009	0.002	0.000	0.010
95% Perc.	0.049	0.030	0.061	0.055	0.071	0.037	0.015	0.050
90% Perc.	0.079	0.067	0.087	0.105	0.132	0.089	0.051	0.100
Left Tail:								
99.9% Perc.	0.005	0.009	0.000	0.000	0.000	0.000	0.005	0.001
99% Perc.	0.024	0.034	0.000	0.004	0.004	0.018	0.023	0.010
95% Perc.	0.078	0.109	0.036	0.039	0.034	0.058	0.066	0.050
90% Perc.	0.110	0.122	0.117	0.110	0.101	0.108	0.125	0.100

Panel D: 12-Month Returns (Daily Data).

Probability	BR rate	US rate	FX rate	Oil	Gold	Ibovespa	S&P 500	Normal
N. Obs.	2104	2100	1039	2108	2089	2201	2201	
Right Tail:								
99.9% Perc.	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
99% Perc.	0.000	0.000	0.000	0.023	0.004	0.008	0.000	0.010
95% Perc.	0.022	0.009	0.018	0.054	0.057	0.058	0.001	0.050
90% Perc.	0.052	0.109	0.116	0.107	0.133	0.105	0.035	0.100
Left Tail:								
99.9% Perc.	0.012	0.009	0.000	0.000	0.000	0.000	0.000	0.001
99% Perc.	0.036	0.045	0.012	0.000	0.000	0.001	0.002	0.010
95% Perc.	0.078	0.079	0.076	0.037	0.031	0.040	0.087	0.050
90% Perc.	0.109	0.111	0.089	0.118	0.095	0.084	0.184	0.100

Table 5
Wilk-Shapiro Normality Test – Historical Returns

This table presents Wilk-Shapiro statistics, for historical returns, over 1-day, 1-month, 6-, and 12-months time windows (Panels A, B, C, and D respectively). First line in each panel brings the Wilk-Shapiro statistics. Values close to one indicate normality. In parenthesis are placed the probability of being less than the W-S value is placed in parenthesis.

Panel A: Daily returns.

	BR rate	US rate	FX rate	Oil	Gold	Ibovespa	S&P 500
Wilk-Shapiro Statistics	0.189	0.160	0.188	0.061	0.093	0.074	0.062
	(0.010)	(0.010)	(0.010)	(0.010)	(0.010)	(0.010)	(0.010)

Panel B: 1-month returns.

	BR rate	US rate	FX rate	Oil	Gold	Ibovespa	S&P 500
Wilk-Shapiro Statistics	0.188	0.127	0.135	0.032	0.072	0.053	0.059
	(0.010)	(0.010)	(0.000)	(0.010)	(0.010)	(0.010)	(0.010)

Panel C: 6-months returns.

	BR rate	US rate	FX rate	Oil	Gold	Ibovespa	S&P 500
Wilk-Shapiro Statistics	0.13	0.159	0.168	0.025	0.035	0.031	0.098
	(0.010)	(0.010)	(0.000)	(0.010)	(0.010)	(0.010)	(0.010)

Panel D: 12-months returns.

	BR rate	US rate	FX rate	Oil	Gold	Ibovespa	S&P 500
Wilk-Shapiro Statistics	0.153	0.186	0.157	0.034	0.053	0.044	0.168
	(0.010)	(0.010)	(0.000)	(0.010)	(0.010)	(0.010)	(0.010)

Table 6
Tail Probabilities: Simulated Constant Volatility vs. Historical Returns

This table presents probabilities at the tails, for simulated returns using constant volatilities, estimated using different cut-offs correspond to the main percentiles, <u>for one tail</u>, in the normal distribution: 3.08σ , 2.335σ , 1.63σ , and 1.28σ for 99.9, 99, 95, and 90 normal percentiles respectively. Thus, for these cut-offs, corresponding mass probabilities at one of the tails should be 0.001, 0.01, 0.05, and 0.1, for a normal distribution (values highlighted in the last column).

<u>Panel A</u>: 1-Month Time Window.

	BR rate		US rate		FX rate		Oil		Gold		Ibovespa		S&P 500		
Probability	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Normal
N. Obs.	2341	2000	2337	2000	1276	2000	2345	2000	2326	2000	2438	2000	2438	2000	
<u>Right Tail</u>:															
99.9% Perc.	0.017	0.000	0.001	0.000	0.013	0.001	0.000	0.001	0.009	0.000	0.001	0.000	0.002	0.001	0.001
99% Perc.	0.027	0.013	0.006	0.013	0.022	0.008	0.007	0.010	0.015	0.007	0.005	0.011	0.008	0.010	0.010
95% Perc.	0.040	0.052	0.028	0.044	0.037	0.049	0.040	0.053	0.045	0.050	0.045	0.062	0.032	0.045	0.050
90% Perc.	0.049	0.095	0.054	0.100	0.056	0.100	0.090	0.106	0.086	0.099	0.081	0.105	0.069	0.093	0.100
<u>Left Tail</u>:															
99.9% Perc.	0.003	0.000	0.025	0.002	0.002	0.001	0.003	0.001	0.001	0.002	0.009	0.003	0.007	0.003	0.001
99% Perc.	0.020	0.010	0.037	0.013	0.011	0.011	0.020	0.012	0.005	0.017	0.023	0.009	0.024	0.008	0.010
95% Perc.	0.041	0.051	0.057	0.047	0.049	0.050	0.052	0.046	0.036	0.047	0.066	0.049	0.062	0.050	0.050
90% Perc.	0.067	0.110	0.073	0.101	0.081	0.101	0.103	0.093	0.066	0.098	0.097	0.096	0.105	0.101	0.100

<u>Panel B</u>: 6-Months Time Window

	BR rate		US rate		FX rate		Oil		Gold		Ibovespa		S&P 500		
Probability	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Normal
N. Obs.	2234	2000	2230	2000	1169	2000	2238	2000	2219	2000	2331	2000	2331	2000	
<u>Right Tail</u>:															
99.9% Perc.	0.000	0.001	0.000	0.002	0.000	0.000	0.000	0.001	0.000	0.002	0.000	0.000	0.000	0.002	0.001
99% Perc.	0.017	0.012	0.000	0.008	0.023	0.009	0.012	0.014	0.009	0.008	0.002	0.014	0.000	0.008	0.010
95% Perc.	0.049	0.045	0.030	0.049	0.061	0.056	0.055	0.045	0.071	0.043	0.037	0.050	0.015	0.055	0.050
90% Perc.	0.079	0.089	0.067	0.090	0.087	0.099	0.105	0.096	0.132	0.088	0.089	0.101	0.051	0.108	0.100
<u>Left Tail</u>:															
99.9% Perc.	0.005	0.003	0.009	0.001	0.000	0.000	0.000	0.002	0.000	0.002	0.000	0.001	0.005	0.001	0.001
99% Perc.	0.024	0.013	0.034	0.013	0.000	0.009	0.004	0.008	0.004	0.010	0.018	0.007	0.023	0.013	0.010
95% Perc.	0.078	0.057	0.109	0.061	0.036	0.052	0.039	0.046	0.034	0.055	0.058	0.044	0.066	0.045	0.050
90% Perc.	0.110	0.099	0.122	0.100	0.117	0.104	0.110	0.106	0.101	0.104	0.108	0.101	0.125	0.095	0.100

Table 6 – Cont.
Tail Probabilities: Simulated Constant Volatility vs. Historical Returns

Panel C: 12-Months Time Window.

	BR rate		US rate		FX rate		Oil		Gold		Ibovespa		S&P 500		
Probability	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Normal
N. Obs.	2104	2000	2100	2000	1039	2000	2108	2000	2089	2000	2201	2000	2201	2000	
Right Tail:															
99.9% Perc.	0.000	0.003	0.000	0.002	0.000	0.001	0.000	0.001	0.000	0.002	0.000	0.002	0.000	0.000	0.001
99% Perc.	0.000	0.014	0.000	0.012	0.000	0.011	0.023	0.009	0.004	0.012	0.008	0.012	0.000	0.014	0.010
95% Perc.	0.022	0.046	0.009	0.051	0.018	0.051	0.054	0.052	0.057	0.052	0.058	0.047	0.001	0.054	0.050
90% Perc.	0.052	0.097	0.109	0.088	0.116	0.099	0.107	0.107	0.133	0.098	0.105	0.096	0.035	0.094	0.100
Left Tail:															
99.9% Perc.	0.012	0.003	0.009	0.002	0.000	0.001	0.000	0.000	0.000	0.001	0.000	0.002	0.000	0.002	0.001
99% Perc.	0.036	0.013	0.045	0.017	0.012	0.010	0.000	0.009	0.000	0.009	0.001	0.011	0.002	0.009	0.010
95% Perc.	0.078	0.050	0.079	0.050	0.076	0.056	0.037	0.058	0.031	0.055	0.040	0.055	0.087	0.045	0.050
90% Perc.	0.109	0.087	0.111	0.095	0.089	0.104	0.118	0.101	0.095	0.102	0.084	0.103	0.184	0.103	0.100

Table 7
Wilk-Shapiro Normality Test – Constant Volatility Simulated Returns

This table presents Wilk-Shapiro statistics, for simulated returns using constant volatilities, over 1-, 6-, and 12-months time windows (Panels A, B, and C, respectively). First line in each panel brings the Wilk-Shapiro statistics. Values close to one indicate normality. In parenthesis are placed the probability of being less than the W-S value is placed in parenthesis.

Panel A: 1-month time window.

	BR rate	US rate	FX rate	Gold	Oil	Ibovespa	S&P 500
Wilks-Shapiro Statistic	0.984	0.988	0.989	0.987	0.986	0.989	0.988
	(0.135)	(0.792)	(0.888)	(0.637)	(0.421)	(0.847)	(0.659)

Panel B: 6-month time window.

	BR rate	US rate	FX rate	Gold	Oil	Ibovespa	S&P 500
Wilks-Shapiro Statistic	0.988	0.990	0.984	0.987	0.990	0.985	0.986
	(0.785)	(0.941)	(0.113)	(0.508)	(0.911)	(0.292)	(0.466)

Panel C: 12-month time window.

	BR rate	US rate	FX rate	Gold	Oil	Ibovespa	S&P 500
Wilks-Shapiro Statistic	0.991	0.991	0.986	0.988	0.989	0.988	0.987
	(0.981)	(0.964)	(0.440)	(0.790)	(0.877)	(0.713)	(0.524)

Table 8
Tail Probabilities: Historical Returns Simulated vs. Stochastic Volatility (λ Minimizing RMSE)

This table presents probabilities at the tails over 1-, 6-, and 12- month time window, for simulated returns using decay factors that minimized RMSE (Tables 4 and 5), estimated using different cut-offs correspond to the main percentiles, for one tail, in the normal distribution: 3.08σ, 2.335σ, 1.63σ, and 1.28σ for 99.9, 99, 95, and 90 normal percentiles respectively. Thus, for these cut-offs, corresponding mass probabilities at one of the tails should be 0.001, 0.01, 0.05, and 0.1, for a normal distribution (values highlighted in blues in the last column).

Panel A: 1-Month Time Window.

	BR rate		US rate		FX rate		Oil		Gold		Ibovespa		S&P 500		
Probability	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Normal
N. Obs.	2341	2000	2337	2000	1276	2000	2345	2000	2326	2000	2438	2000	2438	2000	
Right Tail:															
99.9% Perc.	0.017	0.001	0.001	0.000	0.013	0.001	0.000	0.001	0.009	0.000	0.001	0.001	0.002	0.000	0.001
99% Perc.	0.027	0.009	0.006	0.007	0.022	0.013	0.007	0.009	0.015	0.009	0.005	0.007	0.008	0.006	0.010
95% Perc.	0.040	0.051	0.028	0.045	0.037	0.045	0.040	0.059	0.045	0.051	0.045	0.050	0.032	0.051	0.050
90% Perc.	0.049	0.096	0.054	0.087	0.056	0.097	0.090	0.098	0.086	0.112	0.081	0.102	0.069	0.102	0.100
Left Tail:															
99.9% Perc.	0.003	0.001	0.025	0.001	0.002	0.001	0.003	0.000	0.001	0.000	0.009	0.003	0.007	0.000	0.001
99% Perc.	0.020	0.009	0.037	0.009	0.011	0.011	0.020	0.010	0.005	0.007	0.023	0.009	0.024	0.012	0.010
95% Perc.	0.041	0.055	0.057	0.062	0.049	0.061	0.052	0.052	0.036	0.050	0.066	0.050	0.062	0.058	0.050
90% Perc.	0.067	0.099	0.073	0.105	0.081	0.103	0.103	0.107	0.066	0.097	0.097	0.099	0.105	0.098	0.100

Panel B: 6-Months Time Window

	BR rate		US rate		FX rate		Oil		Gold		Ibovespa		S&P 500		
Probability	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Normal
N. Obs.	2234	2000	2230	2000	1169	2000	2238	2000	2219	2000	2331	2000	2331	2000	
Right Tail:															
99.9% Perc.	0.000	0.001	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.002	0.001
99% Perc.	0.017	0.008	0.000	0.009	0.023	0.009	0.012	0.009	0.009	0.009	0.002	0.011	0.000	0.011	0.010
95% Perc.	0.049	0.041	0.030	0.043	0.061	0.043	0.055	0.049	0.071	0.053	0.037	0.050	0.015	0.054	0.050
90% Perc.	0.079	0.088	0.067	0.093	0.087	0.089	0.105	0.098	0.132	0.089	0.089	0.095	0.051	0.105	0.100
Left Tail:															
99.9% Perc.	0.005	0.004	0.009	0.003	0.000	0.001	0.000	0.002	0.000	0.003	0.000	0.001	0.005	0.001	0.001
99% Perc.	0.024	0.018	0.034	0.013	0.000	0.011	0.004	0.010	0.004	0.008	0.018	0.008	0.023	0.006	0.010
95% Perc.	0.078	0.055	0.109	0.058	0.036	0.059	0.039	0.054	0.034	0.054	0.058	0.056	0.066	0.047	0.050
90% Perc.	0.110	0.095	0.122	0.095	0.117	0.112	0.110	0.108	0.101	0.097	0.108	0.101	0.125	0.096	0.100

Table 8 – Cont.
Tail Probabilities: Historical Returns Simulated vs. Stochastic Volatility (λ Minimizing RMSE)

Panel C: 12-Months Time Window.

	BR rate		US rate		FX rate		Oil		Gold		Ibovespa		S&P 500		
Probability	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Normal
N. Obs.	2104	2000	2100	2000	1039	2000	2108	2000	2089	2000	2201	2000	2201	2000	
Right Tail:															
99.9% Perc.	0.000	0.000	0.000	0.002	0.000	0.001	0.000	0.002	0.000	0.002	0.000	0.000	0.000	0.002	0.001
99% Perc.	0.000	0.009	0.000	0.008	0.000	0.020	0.023	0.008	0.004	0.009	0.008	0.011	0.000	0.008	0.010
95% Perc.	0.022	0.047	0.009	0.044	0.018	0.052	0.054	0.052	0.057	0.044	0.058	0.046	0.001	0.053	0.050
90% Perc.	0.052	0.091	0.109	0.091	0.116	0.086	0.107	0.096	0.133	0.102	0.105	0.090	0.035	0.105	0.100
Left Tail:															
99.9% Perc.	0.012	0.002	0.009	0.003	0.000	0.000	0.000	0.001	0.000	0.002	0.000	0.004	0.000	0.003	0.001
99% Perc.	0.036	0.013	0.045	0.012	0.012	0.010	0.000	0.012	0.000	0.011	0.001	0.012	0.002	0.013	0.010
95% Perc.	0.078	0.061	0.079	0.055	0.076	0.044	0.037	0.049	0.031	0.052	0.040	0.054	0.087	0.053	0.050
90% Perc.	0.109	0.105	0.111	0.092	0.089	0.098	0.118	0.100	0.095	0.100	0.084	0.093	0.184	0.103	0.100

Table 9
Wilk-Shapiro Normality Test – Stochastic Volatility Simulated Returns
(λ Minimizing RMSE)

This table presents Wilk-Shapiro statistics, for 2000 simulated returns using decay factors that minimized RMSE (Tables 4 and 5) and, over 1-, 6-, and 12-months time windows (Panels A, B, and C, respectively). First line in each panel brings the Wilk-Shapiro statistics. Values close to one indicate normality. In parenthesis are placed the probability of being less than the W-S value is placed in parenthesis.

Panel A: 1-month time window.

	BR rate	US rate	FX rate	Gold	Oil	Ibovespa	S&P 500
Wilks-Shapiro Statistic	0.990	0.985	0.988	0.984	0.986	0.989	0.981
	(0.927)	(0.277)	(0.732)	(0.135)	(0.471)	(0.867)	(0.654)

Panel B: 6-month time window.

	BR rate	US rate	FX rate	Gold	Oil	Ibovespa	S&P 500
Wilks-Shapiro Statistic	0.987	0.993	0.985	0.987	0.987	0.988	0.990
	(0.649)	(0.999)	(0.169)	(0.500)	(0.594)	(0.836)	(0.940)

Panel C: 12-month time window.

	BR rate	US rate	FX rate	Gold	Oil	Ibovespa	S&P 500
Wilks-Shapiro Statistic	0.989	0.989	0.985	0.988	0.987	0.986	0.987
	(0.803)	(0.879)	(0.196)	(0.681)	(0.564)	(0.412)	(0.557)

Table 10
Changes in Volatilities: Historical vs. Simulated Stochastic Volatility (λ Minimizing RMSE)

This table presents statistics for changes in volatilities over 1-, 6-, and 12- month time window, for simulated returns using decay factors that minimized RMSE (Tables 4 and 5), and compared to historical values. For each time series, the first column brings statistics for historical returns and the second column is for corresponding simulated values.

Panel A: 1-Month Time Window.

	BR rate		US rate		FX rate		Oil		Gold		Ibovespa		S&P 500	
	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.
N. Obs.	112	2000	112	2000	112	2000	112	2000	112	2000	112	2000	112	2000
Mean	-0.0010	0.0000	0.0000	0.0000	-0.0018	0.0000	-0.0001	0.0000	-0.0004	0.0000	-0.0011	-0.0001	-0.0007	0.0000
St. Dev.	0.0236	0.0015	0.0013	0.0001	0.0197	0.0050	0.0399	0.0031	0.0165	0.0010	0.0291	0.0109	0.0154	0.0024
1% Perc.	-0.0574	-0.0043	-0.0032	-0.0002	-0.0572	-0.0160	-0.0867	-0.0085	-0.0368	-0.0035	-0.0607	-0.0301	-0.0287	-0.0063
5% Perc.	-0.0252	-0.0026	-0.0018	-0.0001	-0.0339	-0.0077	-0.0548	-0.0053	-0.0212	-0.0017	-0.0396	-0.0197	-0.0237	-0.0050
10% Perc.	-0.0188	-0.0017	-0.0009	-0.0001	-0.0210	-0.0061	-0.0464	-0.0034	-0.0183	-0.0009	-0.0321	-0.0119	-0.0178	-0.0028
Median	-0.0014	0.0000	-0.0001	0.0000	-0.0008	0.0001	-0.0062	0.0000	-0.0012	0.0000	-0.0044	0.0000	-0.0004	0.0000
90% Perc.	0.0147	0.0019	0.0011	0.0001	0.0202	0.0061	0.0479	0.0038	0.0162	0.0008	0.0323	0.0115	0.0154	0.0023
95% Perc.	0.0264	0.0023	0.0018	0.0001	0.0287	0.0078	0.0583	0.0049	0.0257	0.0015	0.0537	0.0185	0.0314	0.0048
99% Perc.	0.0791	0.0045	0.0044	0.0001	0.0406	0.0146	0.0948	0.0073	0.0534	0.0030	0.0764	0.0233	0.0380	0.0063

Panel B: 6-Months Time Window.

	BR rate		US rate		FX rate		Oil		Gold		Ibovespa		S&P 500	
	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.
N. Obs.	107	2000	107	2000	107	2000	107	2000	107	2000	107	2000	107	2000
Mean	-0.0013	-0.0001	-0.0001	0.0000	-0.0006	-0.0004	0.0009	0.0000	0.0002	0.0000	-0.0019	-0.0001	-0.0011	0.0000
St. Dev.	0.0361	0.0045	0.0015	0.0001	0.0363	0.0113	0.0482	0.0084	0.0182	0.0028	0.0411	0.0253	0.0215	0.0050
1% Perc.	-0.1060	-0.0102	-0.0047	-0.0003	-0.0875	-0.0237	-0.1135	-0.0179	-0.0434	-0.0056	-0.0937	-0.0565	-0.0489	-0.0124
5% Perc.	-0.0425	-0.0071	-0.0022	-0.0002	-0.0503	-0.0196	-0.0686	-0.0148	-0.0294	-0.0047	-0.0719	-0.0414	-0.0361	-0.0087
10% Perc.	-0.0281	-0.0054	-0.0011	-0.0001	-0.0312	-0.0170	-0.0486	-0.0128	-0.0177	-0.0035	-0.0492	-0.0300	-0.0261	-0.0052
Median	-0.0011	-0.0003	0.0000	0.0000	-0.0070	-0.0009	0.0028	0.0003	0.0003	0.0000	0.0003	-0.0024	0.0004	0.0002
90% Perc.	0.0303	0.0066	0.0013	0.0002	0.0396	0.0152	0.0504	0.0107	0.0192	0.0034	0.0417	0.0349	0.0293	0.0051
95% Perc.	0.0588	0.0079	0.0021	0.0002	0.0640	0.0210	0.0694	0.0145	0.0289	0.0047	0.0917	0.0393	0.0398	0.0088
99% Perc.	0.1054	0.0109	0.0044	0.0004	0.1041	0.0249	0.1260	0.0184	0.0457	0.0068	0.1021	0.0510	0.0481	0.0135

Table 10 – Cont.

Changes in Volatilities: Historical vs. Simulated Stochastic Volatility (λ Minimizing RMSE)

Panel C: 12-Months Time Window.

	BR rate		US rate		FX rate		Oil		Gold		Ibovespa		S&P 500	
	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.
N. Obs.	101	2000	101	2000	101	2000	101	2000	101	2000	101	2000	101	2000
Mean	-0.0011	-0.0002	-0.0001	0.0000	0.0028	-0.0001	0.0038	-0.0006	0.0020	0.0000	-0.0002	-0.0005	-0.0005	0.0000
St. Dev.	0.0359	0.0059	0.0014	0.0002	0.0397	0.0165	0.0453	0.0111	0.0198	0.0047	0.0413	0.0421	0.0211	0.0077
1% Perc.	-0.0931	-0.0125	-0.0042	-0.0005	-0.1041	-0.0449	-0.1080	-0.0280	-0.0590	-0.0107	-0.1149	-0.1164	-0.0479	-0.0206
5% Perc.	-0.0711	-0.0106	-0.0023	-0.0004	-0.0618	-0.0300	-0.0778	-0.0205	-0.0309	-0.0090	-0.0585	-0.0494	-0.0400	-0.0133
10% Perc.	-0.0372	-0.0076	-0.0014	-0.0003	-0.0451	-0.0195	-0.0493	-0.0141	-0.0162	-0.0052	-0.0409	-0.0449	-0.0290	-0.0095
Median	-0.0011	-0.0007	-0.0002	0.0000	0.0096	0.0000	0.0059	0.0003	0.0037	0.0004	0.0013	0.0013	0.0016	0.0004
90% Perc.	0.0336	0.0072	0.0010	0.0003	0.0459	0.0175	0.0478	0.0131	0.0202	0.0066	0.0393	0.0439	0.0220	0.0090
95% Perc.	0.0501	0.0092	0.0019	0.0003	0.0558	0.0302	0.0706	0.0170	0.0362	0.0073	0.0730	0.0581	0.0284	0.0128
99% Perc.	0.0840	0.0126	0.0042	0.0005	0.0915	0.0375	0.1074	0.0284	0.0542	0.0094	0.1117	0.0921	0.0478	0.0162

Table 11
Tail Probabilities for Different Decay Factors

This table presents probabilities at the tails over 12- month time window, for 2000 simulated returns using different decay factors for volatilities, estimated using different cut-offs correspond to the main percentiles, for one tail, in the normal distribution: 3.08σ , 2.335σ , 1.63σ , and 1.28σ for 99.9, 99, 95, and 90 normal percentiles respectively. Thus, for these cut-offs, corresponding mass probabilities at one of the tails should be 0.001, 0.01, 0.05, and 0.1, for a normal distribution (values highlighted in blues in the last column).

Panel A: Lambda = 0.90.

	BR rate	US rate	FX rate	Gold	Oil	Ibovespa	S&P 500	Normal
Right Tail:								
99.9% Perc.	0.004	0.004	0.002	0.003	0.002	0.003	0.002	0.001
99% Perc.	0.014	0.014	0.014	0.010	0.015	0.012	0.014	0.010
95% Perc.	0.052	0.050	0.052	0.053	0.047	0.048	0.053	0.050
90% Perc.	0.093	0.083	0.096	0.096	0.093	0.092	0.094	0.100
Left Tail:								
99.9% Perc.	0.004	0.005	0.003	0.004	0.002	0.004	0.002	0.001
99% Perc.	0.012	0.016	0.016	0.012	0.016	0.012	0.016	0.010
95% Perc.	0.050	0.050	0.054	0.056	0.051	0.052	0.056	0.050
90% Perc.	0.099	0.085	0.098	0.097	0.096	0.096	0.097	0.100

Panel B: Lambda = 0.50.

	BR rate	US rate	FX rate	Gold	Oil	Ibovespa	S&P 500	Normal
Right Tail:								
99.9% Perc.	0.015	0.009	0.006	0.002	0.005	0.003	0.006	0.001
99% Perc.	0.021	0.010	0.012	0.003	0.016	0.006	0.014	0.010
95% Perc.	0.039	0.019	0.030	0.010	0.034	0.029	0.031	0.050
90% Perc.	0.052	0.029	0.056	0.027	0.064	0.056	0.049	0.100
Left Tail:								
99.9% Perc.	0.006	0.006	0.011	0.008	0.009	0.011	0.012	0.001
99% Perc.	0.014	0.018	0.016	0.012	0.017	0.021	0.020	0.010
95% Perc.	0.040	0.031	0.039	0.027	0.039	0.046	0.043	0.050
90% Perc.	0.064	0.038	0.064	0.045	0.070	0.065	0.059	0.100

Panel C: Lambda = 0.10.

	BR rate	US rate	FX rate	Gold	Oil	Ibovespa	S&P 500	Normal
Right Tail:								
99.9% Perc.	0.005	0.006	0.004	0.001	0.004	0.001	0.001	0.001
99% Perc.	0.007	0.007	0.008	0.001	0.006	0.003	0.002	0.010
95% Perc.	0.011	0.010	0.017	0.004	0.014	0.006	0.005	0.050
90% Perc.	0.014	0.013	0.029	0.010	0.022	0.008	0.009	0.100
Left Tail:								
99.9% Perc.	0.002	0.006	0.009	0.007	0.007	0.008	0.008	0.001
99% Perc.	0.003	0.009	0.014	0.011	0.011	0.010	0.010	0.010
95% Perc.	0.008	0.013	0.031	0.017	0.021	0.014	0.016	0.050
90% Perc.	0.029	0.017	0.042	0.023	0.030	0.020	0.022	0.100

Table 12
Optimal Decay Factors: Matching 12-Month Historical Changes in Volatilities

Optimal decay factors were obtained so as to minimize the difference in changes in volatilities over a 12-month time window, between historical and simulated stochastic distributions.

	BR rate	US rate	FX rate	Oil	Gold	Ibovespa	S&P 500
λ	0.83	0.81	0.82	0.92	0.85	0.91	0.94

Table 13
Tail Probabilities: Historical Returns vs. Simulated Stochastic Volatility
(λ for Matching 12-Month Historical Changes in Volatilities)

This table presents probabilities at the tails over 1-, 6-, and 12- month time window, for 2000 simulated returns using decay factors, so as to minimize the absolute difference in 12-month changes in volatilities, between historical and simulated volatilities (Table 14), estimated using different cut-offs correspond to the main percentiles, for one tail, in the normal distribution: 3.08σ, 2.335σ, 1.63σ, and 1.28σ for 99.9, 99, 95, and 90 normal percentiles respectively. Thus, for these cut-offs, corresponding mass probabilities at one of the tails should be 0.001, 0.01, 0.05, and 0.1, for a normal distribution (values highlighted in blues in the last column).

Panel A: 1-Month Time Window.

	BR rate		US rate		FX rate		Oil		Gold		Ibovespa		S&P 500		
Probability	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Normal
Right Tail:															
99.9% Perc.	0.017	0.000	0.001	0.003	0.013	0.003	0.000	0.000	0.009	0.002	0.001	0.002	0.002	0.002	0.001
99% Perc.	0.027	0.014	0.006	0.008	0.022	0.012	0.007	0.009	0.015	0.014	0.005	0.014	0.008	0.017	0.010
95% Perc.	0.040	0.059	0.028	0.052	0.037	0.053	0.040	0.060	0.045	0.050	0.045	0.046	0.032	0.047	0.050
90% Perc.	0.049	0.101	0.054	0.101	0.056	0.096	0.090	0.105	0.086	0.089	0.081	0.095	0.069	0.102	0.100
Left Tail:															
99.9% Perc.	0.003	0.001	0.025	0.001	0.002	0.002	0.003	0.001	0.001	0.007	0.009	0.000	0.007	0.000	0.001
99% Perc.	0.020	0.007	0.037	0.013	0.011	0.012	0.020	0.011	0.005	0.016	0.023	0.009	0.024	0.007	0.010
95% Perc.	0.041	0.041	0.057	0.046	0.049	0.044	0.052	0.047	0.036	0.040	0.066	0.050	0.062	0.048	0.050
90% Perc.	0.067	0.093	0.073	0.102	0.081	0.096	0.103	0.095	0.066	0.089	0.097	0.100	0.105	0.105	0.100

Panel B: 6-Months Time Window

	BR rate		US rate		FX rate		Oil		Gold		Ibovespa		S&P 500		
Probability	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Normal
Right Tail:															
99.9% Perc.	0.000	0.003	0.000	0.004	0.000	0.003	0.000	0.001	0.000	0.002	0.000	0.001	0.000	0.003	0.001
99% Perc.	0.017	0.014	0.000	0.008	0.023	0.016	0.012	0.009	0.009	0.018	0.002	0.012	0.000	0.013	0.010
95% Perc.	0.049	0.037	0.030	0.037	0.061	0.057	0.055	0.043	0.071	0.057	0.037	0.054	0.015	0.043	0.050
90% Perc.	0.079	0.081	0.067	0.087	0.087	0.095	0.105	0.098	0.132	0.094	0.089	0.091	0.051	0.081	0.100
Left Tail:															
99.9% Perc.	0.005	0.004	0.009	0.005	0.000	0.005	0.000	0.001	0.000	0.002	0.000	0.003	0.005	0.001	0.001
99% Perc.	0.024	0.016	0.034	0.025	0.000	0.013	0.004	0.014	0.004	0.007	0.018	0.013	0.023	0.008	0.010
95% Perc.	0.078	0.043	0.109	0.056	0.036	0.046	0.039	0.053	0.034	0.045	0.058	0.043	0.066	0.055	0.050
90% Perc.	0.110	0.091	0.122	0.092	0.117	0.088	0.110	0.102	0.101	0.092	0.108	0.095	0.125	0.100	0.100

Table 13 – Cont.
Tail Probabilities: Historical Returns vs. Simulated Stochastic Volatility
(λ for Matching 12-Month Historical Changes in Volatilities)

Panel C: 12-Months Time Window.

	BR rate		US rate		FX rate		Oil		Gold		Ibovespa		S&P 500		
Probability	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Normal
Right Tail:															
99.9% Perc.	0.000	0.004	0.000	0.009	0.000	0.003	0.000	0.000	0.000	0.004	0.000	0.002	0.000	0.003	0.001
99% Perc.	0.000	0.015	0.000	0.016	0.000	0.016	0.023	0.009	0.004	0.014	0.008	0.012	0.000	0.007	0.010
95% Perc.	0.022	0.053	0.009	0.040	0.018	0.049	0.054	0.049	0.057	0.044	0.058	0.047	0.001	0.046	0.050
90% Perc.	0.052	0.088	0.109	0.071	0.116	0.084	0.107	0.097	0.133	0.088	0.105	0.091	0.035	0.091	0.100
Left Tail:															
99.9% Perc.	0.012	0.005	0.009	0.007	0.000	0.004	0.000	0.003	0.000	0.004	0.000	0.005	0.000	0.003	0.001
99% Perc.	0.036	0.019	0.045	0.016	0.012	0.016	0.000	0.012	0.000	0.015	0.001	0.014	0.002	0.013	0.010
95% Perc.	0.078	0.043	0.079	0.037	0.076	0.043	0.037	0.061	0.031	0.053	0.040	0.044	0.087	0.054	0.050
90% Perc.	0.109	0.076	0.111	0.072	0.089	0.080	0.118	0.099	0.095	0.087	0.084	0.094	0.184	0.091	0.100

Table 14
Wilk-Shapiro Normality Test – Stochastic Volatility Simulated Returns
(λ for Matching 12-Month Historical Changes in Volatilities)

This table presents Wilk-Shapiro statistics, for 2000 simulated returns using decay factors, so as to minimize the absolute difference in 12-month changes in volatilities, between historical and simulated volatilities (Table 14), over 1-, 6-, and 12-months time windows (Panels A, B, and C, respectively). First line in each panel brings the Wilk-Shapiro statistics. Values close to one indicate normality. In parenthesis are placed the probability of being less than the W-S value.

Panel A: 1-month time window.

	BR rate	US rate	FX rate	Gold	Oil	Ibovespa	S&P 500
Wilks-Shapiro Statistic	0.984	0.988	0.989	0.987	0.986	0.989	0.988
	(0.135)	(0.792)	(0.888)	(0.637)	(0.421)	(0.847)	(0.659)

Panel B: 6-month time window.

	BR rate	US rate	FX rate	Gold	Oil	Ibovespa	S&P 500
Wilks-Shapiro Statistic	0.989	0.983	0.990	0.987	0.993	0.992	0.991
	(0.865)	(0.049)	(0.904)	(0.629)	(0.999)	(0.990)	(0.983)

Panel C: 12-month time window.

	BR rate	US rate	FX rate	Gold	Oil	Ibovespa	S&P 500
Wilks-Shapiro Statistic	0.980	0.959	0.988	0.985	0.987	0.989	0.993
	(0.001)	(0.000)	(0.685)	(0.295)	(0.609)	(0.857)	(0.995)

Table 15
Changes in Volatilities: Historical vs. Simulated Stochastic Volatility
(λ for Matching 12-Month Historical Changes in Volatilities)

This table presents statistics for changes in volatilities over 1-, 6-, and 12- month time window, for 2000 simulated returns using decay factors, so as to minimize the absolute difference in 12-month changes in volatilities, between historical and simulated volatilities (Table 14), and compared to historical values. For each time series, the first column brings statistics for historical returns and the second column is for corresponding simulated values.

Panel A: 1-Month Time Window.

	BR rate		US rate		FX rate		Oil		Gold		Ibovespa		S&P 500	
	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.
N. Obs.	112	2000	112	2000	112	2000	112	2000	112	2000	112	2000	112	2000
Mean	-0.0010	0.0001	0.0000	0.0000	-0.0018	-0.0001	-0.0001	-0.0002	-0.0004	0.0003	-0.0011	0.0002	-0.0007	0.0001
St. Dev.	0.0236	0.0226	0.0013	0.0007	0.0197	0.0198	0.0399	0.0485	0.0165	0.0191	0.0291	0.0245	0.0154	0.0218
1% Perc.	-0.0574	-0.0617	-0.0032	-0.0020	-0.0572	-0.0763	-0.0867	-0.1382	-0.0368	-0.0464	-0.0607	-0.0745	-0.0287	-0.0465
5% Perc.	-0.0252	-0.0312	-0.0018	-0.0013	-0.0339	-0.0252	-0.0548	-0.1073	-0.0212	-0.0311	-0.0396	-0.0375	-0.0237	-0.0374
10% Perc.	-0.0188	-0.0229	-0.0009	-0.0009	-0.0210	-0.0173	-0.0464	-0.0532	-0.0183	-0.0237	-0.0321	-0.0269	-0.0178	-0.0196
Median	-0.0014	-0.0010	-0.0001	0.0000	-0.0008	-0.0006	-0.0062	-0.0001	-0.0012	0.0003	-0.0044	-0.0002	-0.0004	0.0009
90% Perc.	0.0147	0.0266	0.0011	0.0008	0.0202	0.0186	0.0479	0.0550	0.0162	0.0236	0.0323	0.0255	0.0154	0.0175
95% Perc.	0.0264	0.0441	0.0018	0.0010	0.0287	0.0325	0.0583	0.0853	0.0257	0.0293	0.0537	0.0422	0.0314	0.0318
99% Perc.	0.0791	0.0589	0.0044	0.0018	0.0406	0.0750	0.0948	0.1291	0.0534	0.0584	0.0764	0.0634	0.0380	0.0443

Panel B: 6-Months Time Window.

	BR rate		US rate		FX rate		Oil		Gold		Ibovespa		S&P 500	
	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.
N. Obs.	107	2000	107	2000	107	2000	107	2000	107	2000	107	2000	107	2000
Mean	-0.0013	-0.0008	-0.0001	0.0000	-0.0006	0.0002	0.0009	0.0020	0.0002	-0.0002	-0.0019	-0.0011	-0.0011	-0.0009
St. Dev.	0.0361	0.0541	0.0015	0.0020	0.0363	0.0464	0.0482	0.0925	0.0182	0.0367	0.0411	0.0607	0.0215	0.0450
1% Perc.	-0.1060	-0.1398	-0.0047	-0.0045	-0.0875	-0.0930	-0.1135	-0.2468	-0.0434	-0.0911	-0.0937	-0.1206	-0.0489	-0.1042
5% Perc.	-0.0425	-0.0769	-0.0022	-0.0040	-0.0503	-0.0792	-0.0686	-0.1527	-0.0294	-0.0556	-0.0719	-0.1024	-0.0361	-0.0884
10% Perc.	-0.0281	-0.0615	-0.0011	-0.0023	-0.0312	-0.0613	-0.0486	-0.1172	-0.0177	-0.0430	-0.0492	-0.0864	-0.0261	-0.0490
Median	-0.0011	-0.0060	0.0000	0.0000	-0.0070	-0.0009	0.0028	0.0038	0.0003	0.0005	0.0003	-0.0013	0.0004	-0.0008
90% Perc.	0.0303	0.0633	0.0013	0.0023	0.0396	0.0597	0.0504	0.1002	0.0192	0.0400	0.0417	0.0727	0.0293	0.0508
95% Perc.	0.0588	0.0948	0.0021	0.0029	0.0640	0.0820	0.0694	0.1438	0.0289	0.0548	0.0917	0.1130	0.0398	0.0676
99% Perc.	0.1054	0.1199	0.0044	0.0056	0.1041	0.1041	0.1260	0.2368	0.0457	0.0905	0.1021	0.1328	0.0481	0.1023

Table 15 – Cont.
Changes in Volatilities: Historical Returns vs. Simulated Stochastic Volatility
(λ for Matching 12-Month Historical Changes in Volatilities)

Panel C: 12-Months Time Window.

	BR rate		US rate		FX rate		Oil		Gold		Ibovespa		S&P 500	
	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.	Hist.	Sim.
N. Obs.	101	2000	101	2000	101	2000	101	2000	101	2000	101	2000	101	2000
Mean	-0.0011	0.0027	-0.0001	0.0000	0.0028	0.0003	0.0038	0.0008	0.0020	-0.0008	-0.0002	0.0005	-0.0005	-0.0036
St. Dev.	0.0359	0.0575	0.0014	0.0033	0.0397	0.0497	0.0453	0.1108	0.0198	0.0485	0.0413	0.0954	0.0211	0.0686
1% Perc.	-0.0931	-0.1487	-0.0042	-0.0098	-0.1041	-0.1155	-0.1080	-0.2861	-0.0590	-0.1120	-0.1149	-0.2930	-0.0479	-0.1580
5% Perc.	-0.0711	-0.0791	-0.0023	-0.0044	-0.0618	-0.0892	-0.0778	-0.1884	-0.0309	-0.0844	-0.0585	-0.1469	-0.0400	-0.1013
10% Perc.	-0.0372	-0.0575	-0.0014	-0.0032	-0.0451	-0.0697	-0.0493	-0.1327	-0.0162	-0.0618	-0.0409	-0.1322	-0.0290	-0.0733
Median	-0.0011	0.0057	-0.0002	0.0000	0.0096	0.0065	0.0059	0.0014	0.0037	-0.0004	0.0013	0.0045	0.0016	-0.0070
90% Perc.	0.0336	0.0633	0.0010	0.0030	0.0459	0.0617	0.0478	0.1151	0.0202	0.0630	0.0393	0.1255	0.0220	0.0801
95% Perc.	0.0501	0.0935	0.0019	0.0046	0.0558	0.0791	0.0706	0.1809	0.0362	0.0885	0.0730	0.1531	0.0284	0.1036
99% Perc.	0.0840	0.1643	0.0042	0.0079	0.0915	0.1030	0.1074	0.3080	0.0542	0.1029	0.1117	0.2014	0.0478	0.1586

Table 16
Credit Transition Matrix: Historical vs. Constant Volatility

Credit transition matrix (CTM) for 2 large Brazilian banks as simulated with constant volatility (Panel A) and constructed by the Brazilian Credit Bureau (Panel B). Differences between the two cases are reported in Panel C.

Panel A: Historical CTM for two large Brazilian banks.

	AA	A	B	C	D	E	F	Default
AA	0.901	0.064	0.021	0.005	0.002	0.000	0.000	0.007
A	0.119	0.690	0.102	0.047	0.021	0.003	0.004	0.014
B	0.033	0.110	0.719	0.092	0.020	0.005	0.006	0.016
C	0.033	0.042	0.153	0.674	0.047	0.009	0.013	0.031
D	0.011	0.019	0.040	0.051	0.602	0.039	0.054	0.184
E	0.001	0.078	0.005	0.008	0.041	0.558	0.040	0.268
F	0.008	0.006	0.012	0.023	0.031	0.076	0.568	0.276

Panel B: Simulated CTM for two large Brazilian banks, with constant volatility and covariances.

	AA	A	B	C	D	E	F	Default
AA	0.908	0.092	0.000	0.000	0.000	0.000	0.000	0.000
A	0.120	0.693	0.187	0.001	0.000	0.000	0.000	0.000
B	0.007	0.136	0.664	0.135	0.015	0.016	0.011	0.016
C	0.008	0.107	0.143	0.656	0.023	0.016	0.016	0.033
D	0.002	0.046	0.072	0.069	0.570	0.036	0.027	0.179
E	0.003	0.035	0.068	0.052	0.024	0.496	0.033	0.291
F	0.001	0.030	0.048	0.049	0.025	0.008	0.580	0.260

Panel C: Differences in probability between simulated and historical CTM's.

	AA	A	B	C	D	E	F	Default
AA	-0.007	-0.028	0.021	0.005	0.002	0.000	0.000	0.007
A	-0.001	-0.003	-0.085	0.046	0.021	0.003	0.004	0.014
B	0.026	-0.026	0.055	-0.043	0.006	-0.011	-0.005	0.000
C	0.025	-0.065	0.010	0.019	0.025	-0.007	-0.003	-0.002
D	0.009	-0.027	-0.032	-0.018	0.033	0.003	0.028	0.005
E	-0.002	0.043	-0.063	-0.044	0.017	0.062	0.008	-0.023
F	0.007	-0.024	-0.036	-0.026	0.006	0.068	-0.012	0.016

Source: Barnhill, Souto, and Tabak (2003).

Table 17
Credit Transition Matrix: Historical vs. Stochastic Volatility
(λ for Matching 12-Month Historical Changes in Volatilities)

Credit transition matrix (CTM) for 2 large Brazilian banks as simulated with stochastic volatility using decay factors, so as to minimize the absolute difference in 12-month changes in volatilities, between historical and simulated volatilities, as in Table 14 (Panel A) and constructed by the Brazilian Credit Bureau (Panel B). Simple differences between the two cases are reported in Panel C.

Panel A: Historical CTM for two large Brazilian banks.

	AA	A	B	C	D	E	F	Default
AA	0.901	0.064	0.021	0.005	0.002	0.000	0.000	0.007
A	0.119	0.690	0.102	0.047	0.021	0.003	0.004	0.014
B	0.033	0.110	0.719	0.092	0.020	0.005	0.006	0.016
C	0.033	0.042	0.153	0.674	0.047	0.009	0.013	0.031
D	0.011	0.019	0.040	0.051	0.602	0.039	0.054	0.184
E	0.001	0.078	0.005	0.008	0.041	0.558	0.040	0.268
F	0.008	0.006	0.012	0.023	0.031	0.076	0.568	0.276

Panel B: Simulated CTM for two large Brazilian banks, with stochastic volatility and covariances.

	AA	A	B	C	D	E	F	Default
AA	0.905	0.094	0.001	0.000	0.000	0.000	0.000	0.000
A	0.103	0.698	0.196	0.002	0.000	0.001	0.000	0.001
B	0.007	0.107	0.661	0.190	0.011	0.008	0.003	0.014
C	0.003	0.060	0.145	0.706	0.038	0.012	0.008	0.030
D	0.001	0.020	0.058	0.053	0.579	0.076	0.027	0.187
E	0.001	0.013	0.038	0.045	0.019	0.576	0.042	0.266
F	0.001	0.014	0.047	0.050	0.025	0.007	0.581	0.276

Panel C: Differences in probability between simulated and historical CTM's.

	AA	A	B	C	D	E	F	Default
AA	-0.004	-0.030	0.020	0.005	0.002	0.000	0.000	0.007
A	0.016	-0.008	-0.094	0.046	0.021	0.003	0.004	0.013
B	0.026	0.003	0.058	-0.098	0.009	-0.003	0.004	0.002
C	0.030	-0.018	0.009	-0.032	0.009	-0.003	0.005	0.001
D	0.010	-0.001	-0.018	-0.002	0.023	-0.037	0.027	-0.003
E	0.000	0.066	-0.033	-0.037	0.022	-0.018	-0.002	0.002
F	0.007	-0.008	-0.035	-0.027	0.006	0.070	-0.013	0.001

Table 18
Simulated Credit Transition Matrix Constant vs. Stochastic Volatility
(λ for Matching 12-Month Historical Changes in Volatilities)

Credit transition matrix (CTM) for 2 large Brazilian banks as simulated with constant (Panel A) and stochastic volatility using decay factors, so as to minimize the absolute difference in 12-month changes in volatilities, between historical and simulated volatilities, as in Table 14 (Panel B). Differences between the two cases are reported in Panel C.

Panel A: Simulated CTM for two large Brazilian banks, with constant volatility and covariances.

	AA	A	B	C	D	E	F	Default
AA	0.908	0.092	0.000	0.000	0.000	0.000	0.000	0.000
A	0.120	0.693	0.187	0.001	0.000	0.000	0.000	0.000
B	0.007	0.136	0.664	0.135	0.015	0.016	0.011	0.016
C	0.008	0.107	0.143	0.656	0.023	0.016	0.016	0.033
D	0.002	0.046	0.072	0.069	0.570	0.036	0.027	0.179
E	0.003	0.035	0.068	0.052	0.024	0.496	0.033	0.291
F	0.001	0.030	0.048	0.049	0.025	0.008	0.580	0.260

Panel B: Simulated CTM for two large Brazilian banks, with stochastic volatility and covariances.

	AA	A	B	C	D	E	F	Default
AA	0.905	0.094	0.001	0.000	0.000	0.000	0.000	0.000
A	0.103	0.698	0.196	0.002	0.000	0.001	0.000	0.001
B	0.007	0.107	0.661	0.190	0.011	0.008	0.003	0.014
C	0.003	0.060	0.145	0.706	0.038	0.012	0.008	0.030
D	0.001	0.020	0.058	0.053	0.579	0.076	0.027	0.187
E	0.001	0.013	0.038	0.045	0.019	0.576	0.042	0.266
F	0.001	0.014	0.047	0.050	0.025	0.007	0.581	0.276

Panel C: Differences in probability between simulated constant and stochastic volatilities CTM's.

	AA	A	B	C	D	E	F	Default
AA	0.003	-0.002	-0.001	0.000	0.000	0.000	0.000	0.000
A	0.017	-0.005	-0.009	-0.001	0.000	-0.001	0.000	-0.001
B	0.000	0.029	0.003	-0.054	0.004	0.008	0.009	0.002
C	0.005	0.047	-0.002	-0.050	-0.015	0.005	0.008	0.003
D	0.002	0.026	0.014	0.016	-0.009	-0.040	0.000	-0.008
E	0.002	0.023	0.029	0.006	0.005	-0.080	-0.010	0.025
F	0.001	0.016	0.000	-0.002	0.000	0.001	-0.001	-0.016