WP/07/122

IMF Working Paper

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Mario Catalán, Jaime Guajardo, and
Alexander W. Hoffmaister

INTERNATIONAL MONETARY FUND

 WP/07/122

IMF Working Paper

European Department

Coping with Spain's Aging: Retirement Rules and Incentives

Prepared by Mario Catalán, Jaime Guajardo, and Alexander Hoffmaister*

Authorized for distribution by James Daniel

May 2007

Abstract

This paper evaluates the macroeconomic and welfare effects of extending the averaging period used to calculate pension benefits in a pay-as-you-go system. It also examines the complementarities between reforms extending the averaging period and those increasing the retirement age under alternative tax policies. The analysis is based on a model in the Auerbach-Kotlikoff tradition applied to the Spanish economy. Without reforms, the simulations suggest that aging-related spending as a share of output will increase 16 percentage points by 2050, which are twice as much as in European Commission (2006) projections due to general equilibrium effects. Also, reforms extending the averaging period to the entire work life limit expenditure pressures at the peak of the demographic shock as much as increasing the retirement age in line with life expectancy (4 percentage points of GDP). These reforms and prefunding the demographic shock mitigate the adverse macroeconomic effects of aging and improve welfare.

JEL Classification Numbers: E1, H2, and J22

Keywords: pension reform, dynamic general equilibrium model, and Spanish pension system

Authors' E-Mail Addresses: mcatalan@imf.org, jguajardo@imf.org, ahoffmaister@imf.org

* We are grateful for useful comments received on earlier versions of this paper from seminar and conference participants at IMF Institute (2007); NBER Summer Institute (2006); Johns Hopkins University, SAIS (2006); National University of Tucumán, Argentina (2006); IMF European Department (2004); and Spanish Ministry of Finance (2004).

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I. INTRODUCTION

The standard view in the pension literature is that pay-as-you-go (PAYG) systems distort labor market incentives as returns on pension contributions are lower than those on other forms of savings (Samuelson (1958)). Thus, parametric reforms discussions of PAYG systems focus prominently on tightening the contribution-benefit linkage, and thereby increasing actuarial fairness (Lindbeck and Persson (2003) and references therein). In this connection, extending the averaging period of contributions used to calculate pension benefits has received a fair amount of attention in the literature but not in the context of quantitative assessments using applied dynamic general equilibrium (DGE) models.

This paper evaluates the macroeconomic and welfare effects of extending the averaging period of contributions used to calculate pension benefits in a PAYG system. It also examines the complementarities between reforms extending the averaging period and those increasing the retirement age under alternative tax scenarios. The evaluation is based on a DGE framework in the Auerbach-Kotlikoff (1987) tradition applied to the Spanish economy.

Spain serves as a valuable case study due to its pension rules, social agreements, and demographic profile. First, the Spanish public PAYG system calculates pension benefits based on average, inflation-indexed, gross wage earnings in the corresponding averaging period. Moreover, reforms implemented in 1997 doubled the averaging period to the last 15 years of an individual's work life with the view of enhancing the contribution-benefit linkage. Second, Spain reached a broad political and social consensus—known as the *Pacto de Toledo*—on the need to preserve the public PAYG system through reforms geared to ensuring its sustainability, including by improving the alignment of benefits and contributions; the *Pacto* ruled out privatization and reforms toward compulsory fully funded schemes.[1] Third, even considering Spain's remarkable immigration phenomenon, the demographic shock is expected to be larger and thus pose a more substantial challenge for long-run fiscal sustainability than elsewhere in Europe.

The simulations of the baseline scenario highlight the fiscal challenge: pension expenditures increase as a share of output by 16 percentage points, and the consumption tax rate rises by more than 30 percentage points by 2050. These estimates reflect an unchanged retirement age and averaging period, with aging-related expenditures financed by consumption tax rate adjustments (tax-as-you-go). The projected fiscal pressures reflect increases in the dependency ratio and wage rates—as labor becomes scarcer—and the effect of these increases on individual pension benefits. Also, pension expenditure pressures are

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[1] The *Pacto* was established through a broad political agreement—ratified by the Spanish Congress by virtual unanimity in April, 1995—seeking to preserve the public PAYG nature of the old-age pension system through parametric reforms; labor unions joined the call for reform a year later. The *Pacto* was updated in 2003 and continues to stress the need to align pension benefits and contributions.

9 percentage points and 3 percentage points higher than those projected, respectively, by the European Commission (EC, 2006) and Rojas (2005).[2]

Addressing the demographic shock through parametric reforms to the PAYG system delivers sizeable macroeconomic and welfare benefits. Two pension reform scenarios under tax-as-you-go policies highlight these benefits.

First, a partial pension reform scenario—gradually increasing the retirement age[3] and holding constant the averaging period—attenuates expenditure pressures and the needed increase in taxes, while boosting the aggregate capital stock and output. This reform will reduce the increase in pension expenditure as a share of ouput by 4 percentage points, and the consumption tax rate increase by 7 percentage points over the next 40 years. Furthermore, it induces a "bust-boom" cycle in the aggregate labor supply: aggregate effective labor declines over the next two decades, but increases thereafter, reflecting grandfathering clauses and households' intertemporal labor substitution effects.

Second, a full pension reform scenario—that in addition extends gradually the averaging period to the entire work life—will further reduce the increase in pension expenditure as a share of output by 4 percentage points, and the consumption tax rate increase by 7 percentage points over the next 40 years. Thus, extending the averaging period is as important in mitigating the aging-related spending and tax pressures as increasing the retirement age in 2008–50. The labor supply response to the reform involves a trade-off: extending the averaging period increases households' labor effort, particularly at high-skill ages by removing intertemporal distortions, but it discourages labor effort by reducing pension benefits as real wages—driven by technological progress—increase over time. The analysis illustrates that, in contrast to increasing the retirement age, extending the averaging period—conditional on raising the retirement age—triggers a "boom-bust" cycle in aggregate effective labor, making it scarcer as the demographic storm intensifies.

Overall, both reforms deliver significant welfare gains. These gains arise from the lower and flatter paths of consumption tax rates, which reduce distortions in households' consumption-saving decisions, and the extension of the averaging period, which removes labor market distortions.

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[2] Along with other general equilibrium effects, the effect of rising wage rates on individual pension benefits accounts to a large extent for the difference between this paper's simulated pension expenditure pressures and the corresponding EC projections. In the baseline scenario discussed below, individual pension benefits rise faster than output per capita at the peak of the demographic shock. In contrast, the EC projections assume that individual pension benefits rise in line with output per capita.

[3] Specifically, the retirement age of generations retiring in the 2050s increases by two years, and for later generations, it increases two years every decade up to a maximum increase of eight years. On average, the retirement age in the population increases in line with life expectancy during the demographic transition.

Prefunding the fiscal impact of the demographic shock, in either pension reform scenario, by "tax smoothing"—a once-and-for-all increase in the consumption tax rate in 2008—can further attenuate the adverse macroeconomic effects in the demographic transition. However, prefunding shifts the tax burden from generations that are active when the dependency ratio peaks to current and future generations. The combined effects of pension reforms and prefunding create net welfare losses for some current generations and net gains for all future generations (relative to the baseline scenario). Thus, additional compensating mechanisms are needed to ensure a Pareto improving package of reforms. These may include delaying the increase in the consumption tax rate or targeting transfers to net losers financed with debt.

The simulations also reveal two broader implications of extending the averaging period. First, extending the averaging period limits the increase in individual pension benefits at the peak of the demographic transition—when wage rates rise faster—more than in the long run.[4] In other words, with a tax-as-you-go policy, the extension's contribution to limit tax rate increases varies over time. Second, in the long run, extending the averaging period to the entire work life results in the highest welfare gains when technological progress is high. With no technological progress, long-run welfare gains are largest when the averaging period is shorter than the entire work life.

While this paper is more comprehensive in assessing the effects of extending the averaging period, previous papers have also studied these effects. Diamond (2001) discussed the labor market distortions arising from the short averaging period and advocated extending it to the entire work life but did not quantify this reform. Without employing a DGE approach, Jimeno (2000 and 2003) concluded that extending the averaging period results in pension expenditure reductions between 1 and 2 percentage points of output, significantly smaller than those reported in this paper. Using DGE approaches, Sánchez Martín (2001) found that extending the averaging period reduces pension expenditures as a share of output by about 1.8 percentage points by 2050, while Díaz-Saavedra (2005) assesses those reductions to amount to 2.5 percentage points. In the latter, however, the (endogenous) retirement age decreases when the averaging period is extended, and thus, those results are not comparable to those discussed below.

The rest of the paper is organized as follows. Section II discusses the model and its calibration. Section III discusses the baseline results. Section IV assesses the effects of pension reforms. Section V concludes.

[4] At the peak of the demographic transition, extending the averaging period accounts for half of the tax rate reduction obtained from a full pension reform in Spain. In the long run, however, extending the averaging period accounts for a tenth of the tax rate reduction.

II. The Model

A. Model Overview

The model follows the Auerbach-Kotlikoff tradition.[5] The (closed) economy is populated by overlapping generations of finitely lived households, atomistic firms, and an infinitely lived government. Households consume and accumulate assets during their lifetime, work during their youth, and retire when old. Firms produce the single good in the model using labor and capital, and the government collects income, consumption and payroll taxes to finance government expenditures and pension benefits, and redeem the initial government debt.

Although the general equilibrium structure is standard, the model incorporates specific features of the Spanish pension system. In particular, it incorporates a stylized version of the Spanish pension rule whereby the old-age benefit is calculated based on average gross wage earnings in the corresponding averaging period; in the baseline scenario below, the averaging period is the last 15 years of the individual's work life.[6]

In addition, to capture the effects on household behavior of demographic aging and pension reforms, the model includes the following elements. Households retire at an exogenously given age, but labor supply is endogenous as households choose the amounts of labor and leisure time during their work life. Households' labor skills (productivity) vary exogenously with age to account for the observed hump-shape in wage rates over years of employment. Labor-augmenting productivity growth causes real wage growth over time, thus allowing the model to capture the trade-off noted above: extending the averaging period discourages labor effort by reducing average wage earnings and pension benefits, but it encourages labor effort by eliminating intertemporal distortions, particularly when skills are high. Life expectancy is exogenous but increases over time to match demographic projections. Finally, the model explicitly accounts for the effects of population aging on public health-related expenditures. These features allow the model to meaningfully quantify the effects of reforming the pension rule on labor market incentives and macroeconomic outcomes, including complementarities with alternative tax policies to finance old-age related spending. The model is presented in stationary form and, for the reader's convenience, its notation is summarized in Table 1.

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[5] A survey of the literature—extending back to Auerbach and Kotlikoff (1987)—can be found in Kotlikoff (2000). The numerical solution methods involved are described in Heer and Maußner (2005), and Judd (1999).

[6] The model is real, that is, money and inflation play no explicit role. This is consistent with the paper's focus on the long-term effects of aging. Also, the inflation indexation of pension contributions and benefits implies that inflation is neutral with respect to the pension system.

B. Household

The lifetime utility of a household born at time t is determined by its lifetime consumption (c) and leisure (l), and is given by

$$U_t = \sum_{s=1}^{T_t+T_t^R} \beta^{s-1} \cdot \left\{ \log(c_{t+s-1}^s) + \gamma \cdot \log(l_{t+s-1}^s) \right\}, \tag{1}$$

where the household's life is characterized by two distinct phases: a work life lasting T_t periods or years ($s = 1,...,T_t$) and a mandatory retirement lasting T_t^R years ($s = T_t + 1,...,T_t + T_t^R$). Note that the model allows the household's life expectancy and retirement age to vary across generations, and henceforth, these are assumed to be nondecreasing over time. The household is endowed with a fixed number of hours per year, which is normalized so that work (n) and leisure (l) add up to one:

$$l_{t+s-1}^s = \begin{cases} 1 - n_{t+s-1}^s & s = 1,...,T_t \\ 1 & s = T_t + 1,...T_t + T_t^R \end{cases}. \tag{2}$$

A household accumulates assets (A) during its work life according to the following budget constraint:

$$(1+\xi) \cdot A_{t+s}^{s+1} = [1 + r_{t+s-1} \cdot (1 - \tau_{t+s-1}^I)] \cdot A_{t+s-1}^s + (1 - \tau_{t+s-1} - \tau_{t+s-1}^I) \cdot W_{t+s-1} \cdot e^s \cdot n_{t+s-1}^s - (1 + \tau_{t+s-1}^c) \cdot c_{t+s-1}^s, \tag{3}$$

where next year's assets are determined by adding to this year's assets the household's savings, which are obtained by adding net return on assets to net wage income and subtracting gross consumption. As noted above, the household's labor productivity per hour varies with age according to a skill premium (e^s). The premium is defined as the relative productivity of an s-year old household to that of a one-year old (unskilled) household; the latter is normalized to 1, and thus, W denotes the wage per unit of labor time of an unskilled worker. In equation (3), the household takes as given the payroll (τ), income (τ^I), and consumption (τ^c) tax rates, and the interest (r) and wage (W) rates.[7]

[7] Income taxes are levied on labor income and asset earnings; for simplicity, these tax rates are assumed to be the same.

During retirement, the household's wage income is replaced by an old-age pension (*b*) in the budget constraint, as follows:

$$(1+\xi)\cdot A_{t+s}^{s+1} = [1+r_{t+s-1}\cdot(1-\tau_{t+s-1}^{I})]\cdot A_{t+s-1}^{s} + \frac{b_{t+T_t}^{T_t+1}}{(1+\xi)^{s-T_t-1}} - (1+\tau_{t+s-1}^{c})\cdot c_{t+s-1}^{s}\,. \tag{4}$$

Note that the old-age pension for a household born at time *t* and retiring at time $t+T_t$ can be expressed as follows:

$$b_{t+T_t}^{T_t+1} = \psi\cdot\frac{1}{\mu}\cdot\sum_{j=T_t+1-\mu}^{T_t}\frac{W_{t+j-1}}{(1+\xi)^{T_t+1-j}}\cdot e^{j}\cdot n_{t+j-1}^{j}, \tag{5}$$

where the average (gross) wage in the averaging period (covering the last μ years before retirement) is "scaled down" by the replacement ratio (ψ).[8] Note that pension benefits and real wage earnings are discounted by labor augmenting productivity growth (ξ) in the stationary-transformed equations (4) and (5). This discounting reflects the fact that the household's pension benefits (equation (4)) and the past nominal wage earnings used to compute the initial pension benefit (equation (5)) are adjusted by inflation, but not by productivity growth.[9]

The model assumes that there are no intergenerational bequests or inheritances: the household is born (enters the labor force) with zero assets at age $s=1$, and dies without assets at age $s=T_t+T_t^R+1$,

$$A_t^1 = A_t^{T_t+T_t^R+1} = 0. \tag{6}$$

The household's problem is to choose the paths of consumption, leisure and asset holdings $\left\{c_{t+s-1}^{s}, l_{t+s-1}^{s}, A_{t+s-1}^{s}\right\}_{s=1}^{T_t+T_t^R}$ to maximize its lifetime utility (1) subject to constraints (2)–(6).

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[8] The pension benefit formula, before applying stationary transformations is given by: $\hat{b}_{t+T_t}^{T_t+1} = \psi\cdot\frac{1}{\mu}\cdot\sum_{j=T_t+1-\mu}^{T_t}\hat{W}_{t+j-1}\cdot e^{j}\cdot n_{t+j-1}^{j}$. Note that the pension benefit, once determined, remains constant in real terms throughout retirement, that is $\hat{b}_{t+T_t}^{T_t+1} = \hat{b}_{t+s-1}^{s}$ for $s=T_t+2,\ldots,T_t+T_t^R$.

[9] Consistent with the majority of old-age pensions in Spain, pensions are taken as not taxed.

The household's problem can be expressed as a sequence of two dynamic optimization problems, as follows:

$$\underset{\left\{c_{t+s-1}^{s}, l_{t+s-1}^{s}, A_{t+s}^{s+1}\right\}_{s=1}^{T_t}}{Max} \sum_{s=1}^{T_t} \beta^{s-1} \cdot \left\{\log(c_{t+s-1}^{s}) + \gamma \cdot \log(l_{t+s-1}^{s})\right\} + \beta^{T_t} \cdot V(A_{t+T_t}^{T_t+1}, b_{t+T_t}^{T_t+1})$$

subject to (2), (3), (5) and (6).

where $V(A_{t+T_t}^{T_t+1}, b_{t+T_t}^{T_t+1})$ is the household's value function or discounted indirect utility when it retires at time $t+T_t$ having reached the age of T_t+1 years. Upon retirement, the household's optimization problem can be expressed recursively, and a closed-form solution for the value function (V) follows from the log utility assumption.[10]

Two sets of conditions solve the household's problem under standard dynamic optimization techniques; Table 2 contains both sets of first order conditions where $V_A(.)$ and $V_b(.)$ denote the partial derivatives of $V(.)$ with respect to $A_{t+T_t}^{T_t+1}$ and $b_{t+T_t}^{T_t+1}$. The first set—equations (7), (9) and (11)—refers to a household's consumption-leisure choice at specific ages (intratemporal first order conditions). In each period, the household equates the marginal utility of consumption (scaled by wages) to the marginal utility of leisure. The second set—equations (8), (10), (12) and (13)—governs the household's consumption-saving decisions over time (intertemporal first order conditions or Euler equations).[11] In this case, households equate the marginal utility of current consumption to the discounted marginal utility of future consumption (scaled by the net return on savings).

These equations reflect the peculiarities of the Spanish pension rule—averaging period and inflation indexation of contributions and benefits—and whether a household is working or retired. Specifically, while the household is in the labor force the pension rule introduces three subperiods in the household's optimization problem. The first comprises the initial years in the labor force prior to the averaging period (μ) $(s=1,\ldots,T_t-\mu)$, when the household's annual wage earnings do not affect its future pension benefits. The second corresponds to the first *μ-1* years of the averaging period $(s=T_t-\mu+1,\ldots,T_t-1)$, when the consumption-leisure choice

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[10] Note that the household's value function $V(.)$ depends not only on its stock of assets at retirement ($A_{t+T_t}^{T_t+1}$) and its future annual pension benefits ($b_{t+T_t}^{T_t+1}$) but also on future interest rates and income tax rates. A detailed derivation of the value function (V) can be found in Appendix I.

[11] When the household retires, it faces only the inter-temporal first order condition as it no longer supplies labor.

(intratemporal first-order conditions) also reflects the fact that wage earnings accrued in this subperiod provide additional utility during retirement because of their effect on the pension benefit;[12] however, the consumption-saving decision remains unchanged. In the final year of the averaging period $(s=T_t)$, the consumption-saving decision reflects, nonetheless, the retirement of the individual in the following period (V_A). When the household retires $(s=T_t+1,...,T_t+T_t^R-1)$, however, there is no labor supply choice and only the consumption-saving decision remains.[13]

Aggregate household consumption (C_t^h), effective labor supply (N_t^h), and assets (A_t^h) are obtained by aggregating individual household's variables at each point in time, as follows:[14]

$$N_t^h = \sum_{s=1}^{T_t} e^s \cdot n_t^s \cdot \frac{P_t^s}{P_t}, \qquad A_t^h = \sum_{s=1}^{T_t+T_t^R} A_t^s \cdot \frac{P_t^s}{P_t}, \qquad C_t^h = \sum_{s=1}^{T_t+T_t^R} c_t^s \cdot \frac{P_t^s}{P_t}.$$

C. Firms

Firms maximize a (stationary-transformed) profit function net of capital depreciation Π_t^f. They do so subject to a constant-returns-to-scale Cobb-Douglas production function with labor-augmenting technological progress,

$$\Pi_t^f = \mathrm{Z} \cdot \left(K_t^f\right)^{\alpha} \cdot \left(N_t^f\right)^{1-\alpha} - (r_t + \delta) \cdot K_t^f - W_t \cdot N_t^f,$$

where δ is the rate of capital depreciation. Both output and factor markets are perfectly competitive, and therefore, individual firms face given wages (W_t) and rental rates (r_t). The

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[12] All else equal, the household increases the supply of labor during the reference period because of this added "benefit" to work.

[13] The analysis starts with a full set of generations, $T_0 + T_0^R$, at time $t = 0$. Thus, during the first $T_0 + T_0^R$ years a number of "truncated" optimization problems are associated with those households of ages $\tilde{s} = 2,...,T_0 + T_0^R$ that were born before $t = 0$. Note that we are assuming that all "truncated" generations have the same work life and retirement periods.

[14] Note the contrast between the definitions of the *aggregate effective labor supply* (N_t^h) and the *aggregate labor effort*. The *aggregate effective labor supply* is the sum of the time devoted to work by all the generations in the labor force in a given year, where each generation's working time is weighted by its skills and population size. By contrast, the *aggregate labor effort* (n_t^h) is the weighted (by population size) sum of the time devoted to work by all generations in the labor force, without accounting for skill differences: $n_t^h = \sum_{s=1}^{T_t} n_t^s \cdot \frac{P_t^s}{P_t}$.

first order conditions require that W_t and $r_t + \delta$ equal, respectively, the marginal product of labor and capital:

$$W_t = Z \cdot (1-\alpha) \cdot \left(\frac{K_t^f}{N_t^t} \right)^{\alpha}, \qquad r_t + \delta = Z \cdot \alpha \cdot \left(\frac{K_t^f}{N_t^t} \right)^{-(1-\alpha)}.$$

D. The Government

The government sets taxes to ensure long-run fiscal sustainability. As noted before, the government collects payroll, income, and consumption taxes from households. Tax revenues are used to finance public consumption (*G*), pension benefits, and redeem government debt (*D*). Public consumption has two components: health-related public consumption which is driven by changes in the population's age structure (see Appendix II for details); and non health-related public consumption which remains constant as a share of aggregate output. Thus, the government's budget constraint is as follows:[15]

$$D_{t+1} \cdot (1+\xi) \cdot \frac{P_{t+1}}{P_t} = (1+r_t) \cdot D_t + [G_t - \tau_t^I \cdot (r_t \cdot A_t^h + W_t \cdot N_t^h) - \tau_t^c \cdot C_t^h] + \sum_{s=T_t+1}^{T_t+T_t^R} \frac{b_{t+T_t+1-s}^{T_t+1}}{(1+\xi)^{s-T_t-1}} \cdot \frac{P_t^s}{P_t} - \tau_t \cdot W_t \cdot N_t^h,$$

where the (nonsocial security) primary deficit (term in brackets), and the social security deficit (last two terms) are shown separately.

E. Equilibrium

An equilibrium simultaneously places all households and firms on their maximizing paths, establishes the solvency of the government, and clears markets. Consider an initial

[15] The budget constraint, before stationary transformations, is given by

$\widehat{D}_{t+1} = (1+r_t) \cdot \widehat{D}_t + [\widehat{G}_t - \tau_t^I \cdot (r_t \cdot \widehat{A}_t^h + \widehat{W}_t \cdot \widehat{N}_t^h) - \tau_t^c \cdot \widehat{C}_t^h] + \sum_{s=T_t+1}^{T_t+T_t^R} \hat{b}_t^s \cdot P_t^s - \tau_t \cdot \widehat{W}_t \cdot \widehat{N}_t^h$. Notice that the (stationary-transformed) old-age pension for a household born at time $t - T_t$ and retiring at time t is given by $b_t^{T_t+1} = \frac{\psi}{\mu} \cdot \sum_{j=T_t+1-\mu}^{T_t} \frac{W_{t-T_t+j-1}}{(1+\xi)^{T_t+1-j}} \cdot n_{t-T_t+j-1}^j$. Using the equality $\hat{b}_t^s = \hat{b}_{t+T_t+1-s}^{T_t+1}$ for $s = T_t+1, \ldots, T_t + T_t^R$, it can be shown that $\frac{\hat{b}_t^s}{(1+\xi)^t} = \frac{b_{t+T_t+1-s}^{T_t+1}}{(1+\xi)^{s-T_t-1}}$. The discount of the stationary-transformed pension benefit by productivity growth reflects the fact that nominal benefits are adjusted only by inflation.

population of size P_0 with age structure $\left\{P_0^s\right\}_{s=1}^{T_0+T_0^R}$, a given sequence of newborn cohorts $\left\{P_t^1\right\}_{t=1}^{\infty}$ with work lives $\left\{T_t\right\}_{t=1}^{\infty}$ and life expectancies $\left\{T_t+T_t^R\right\}_{t=1}^{\infty}$, initial government debt $D_0 \geq 0$, capital stock $K_0 > 0$, and distribution of assets $\left\{A_0^s\right\}_{s=1}^{T_0+T_0^R}$, such that $D_0 + K_0 = A_0^h = \sum_{s=1}^{T_0+T_0^R} A_0^s \cdot \frac{P_0^s}{P_0}$. The equilibrium is a collection of lifetime plans for both, households born during the period of analysis ($t \geq 0$),

$$\left\{c_{t+s-1}^s, l_{t+s-1}^s, A_{t+s}^{s+1}\right\}_{s=1}^{T_t+T_t^R}, \text{ for } t = 0,1,...,\infty,$$

and those born before then ($t < 0$)—households of ages 2 through $T_0 + T_0^R$ at $t = 0$—that face "truncated" lifetime plans

$$\left\{c_{s-\tilde{s}}^s, l_{s-\tilde{s}}^s, A_{1+s-\tilde{s}}^{s+1}\right\}_{s=\tilde{s}}^{T_0+T_0^R}, \text{ for } \tilde{s} = 2,...,T_0+T_0^R,$$

a sequence of allocations for the firms $\left\{K_t^f, N_t^f\right\}_{t=0}^{\infty}$, a sequence of relative prices of labor and capital $\left\{W_t, r_t\right\}_{t=0}^{\infty}$, and a sequence of government policy variables including payroll, income, and consumption tax rates, and government consumption and debt, $\left\{\tau_t, \tau_t^I, \tau_t^c, G_t, D_t\right\}_{t=0}^{\infty}$, such that:

- the sequence of allocations $\left\{K_t^f, N_t^f\right\}_{t=0}^{\infty}$ solves the firm's optimization problem;
- the lifetime plans for households born during the period of analysis $\left\{c_{t+s-1}^s, l_{t+s-1}^s, A_{t+s}^{s+1}\right\}_{s=1}^{T_t+T_t^R}$, for $t \geq 0$, solve their optimization problems; and the lifetime plans for households of ages $\tilde{s} = 2,...,T_0+T_0^R$ at time $t = 0$ $\left\{c_{s-\tilde{s}}^s, l_{s-\tilde{s}}^s, A_{1+s-\tilde{s}}^{s+1}\right\}_{s=\tilde{s}}^{T_0+T_0^R}$ solve their truncated optimization problems;
- the government budget constraint is satisfied for $t \geq 0$;
- the labor market clears, $N_t = N_t^f = N_t^h$, for $t \geq 0$; the asset market clears, $A_t = D_t + K_t^f = A_t^h$, for $t \geq 0$; and the output market clears,

$$K_{t+1} \cdot (1+\xi) \cdot \frac{P_{t+1}}{P_t} = (1-\delta) \cdot K_t + Y_t - C_t - G_t$$, for $t \geq 0$, where $Y_t = Y_t^f$ and $C_t = C_t^h$ are the equilibrium aggregate output and consumption levels.[16]

F. Balanced Growth Equilibrium and Calibration

To calibrate the model, a balanced growth equilibrium is defined. The economy exhibits a balanced-growth equilibrium—assuming constant population growth rate (*p*), work life ($T_t = T$), and retirement period ($T_t^R = T^R$)—when the government implements a fiscal policy characterized by a constant government expenditure-to-output ratio, constant tax rates, and a constant debt-to-output ratio.[17] Along the balanced growth equilibrium path, all endogenous variables grow at constant rates (Table 3). The balanced-growth equilibrium can be expressed as a steady state in *detrended* variables in the stationary-transformed model.

The model is calibrated to match some stylized facts and relevant features of the Spanish economy as follows:

- Standard parameter values in the real business cycle literature are used to set the household's discount factor (β) and the depreciation rate (δ). The share of capital in production (α) is obtained from previous Spain-specific calibrations and econometric studies. The total factor productivity parameter (Z) is set so that the capital-output ratio in the initial steady state is consistent with the Spanish data. The rate of labor-augmenting technological progress is set to be consistent with long-term output per capita growth. The value of (*p*) matches the average population growth rate for 1900–70. The leisure parameter is calibrated so that the fraction of time worked by a representative household in the population is 0.274.[18]

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[16] The economy's aggregate constraint $K_{t+1} \cdot (1+\xi) \cdot \frac{P_{t+1}}{P_t} = (1-\delta) \cdot K_t + Y_t - C_t - G_t$ is obtained from the aggregate constraint of the household sector, the first-order conditions of firms, the market equilibrium conditions, and the government budget constraint. The aggregate constraint of the household sector at time t is given by

$$A_{t+1}^h \cdot (1+\xi) \cdot \frac{P_{t+1}}{P_t} = [1 + r_t \cdot (1-\tau_t^I)] \cdot A_t^h + (1-\tau_t^I - \tau_t) \cdot W_t \cdot N_t^h + \sum_{s=T_t+1}^{T_t+T_t^R} \frac{b_{t+T_t+1-s}^{T_t+1}}{(1+\xi)^{s-T_t-1}} \cdot \frac{P_t^s}{P_t} - (1+\tau_t^c) \cdot C_t^h .$$

[17] The age structure of the population remains invariant over time, and thus, both components of public consumption (health-related and non health-related) are constant as a share of output.

[18] Assuming that a household or individual sleeps 8 hours per day, the leisure-work decision is made for the remaining 16 hours. This translates into a total of 112 (=7x16) hours per week. Assuming 40 working hours per week, the individual works 35.7 (=40/112) percent of the non-sleep time. Adjusting the fraction of time worked by labor force participation—about 77 percent for those between the ages of 16-64—yields 0.274.

- Tax rates are calibrated to match effective rates observed in 1994–2004. Specifically, the payroll tax rate (τ) matches the observed ratio of social security contributions to wage income, and the consumption (τ^c) and income tax rates (τ^I) match the ratios of indirect tax revenues to private consumption and direct tax revenues to GDP.

- The pension replacement ratio value is set to 0.65. Because of recent changes to the social security system in Spain, the replacement ratios for new pensioners differ from those of pensioners retired under previous regimes. The parameter choice matches ratios for the most recent cohorts of pensioners. The values of the work life and retirement periods are set so that households enter the labor force when they are on average 22 years old, retire at age 62 and live 80 years with certainty, which is the implicit "life expectancy" at birth.

- The household's labor skills profile by age (e^s) was calibrated to match the Spanish workers' average hourly wage profile by age (Appendix II). This profile is similar to that of US workers—as reported by Hansen (1993)—in that skills are low at the beginning of the work life, peak at about 50 years of age, and decline to intermediate levels by the end of the work life.

Given these values, the calibration exercise verifies that the resulting values of the endogenous variables in the initial steady state and the fiscal ratios match closely those of the Spanish data (Table 4).

G. Household's Labor, Consumption, and Asset Holdings in the Initial Steady State

As anticipated, the 15-year averaging period introduces a discrete jump in the households' labor effort profile precisely 15 years before retirement (Figure 1). Note that at the beginning of work life, households are relatively unskilled and thus work few hours; however, as they age, and labor skills improve, time devoted to work increases. Still, households' labor effort peaks before the beginning of the averaging period even though skills are still increasing.[19] This is because households reduce labor effort *before* the averaging period to compensate for the higher labor effort they will exert *during* the averaging period. Intuitively, the desire to anticipate leisure before the averaging period dominates the incentive to increase labor effort provided by the gains in skills. Also, upon entering the averaging period the number of hours worked jumps, and remains high until retirement, because households internalize the effect of their labor effort on the future pension.

Still, household's labor effort implies increasing wage earnings throughout their worklife. Accordingly, households incur debt at the beginning of their lives to *partially* smooth consumption—which still increases over time because the households' rate of time preference is lower than the net rate of return on assets. During the averaging period,

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[19] Note in Appendix II that the household's skills peak at about 25 years of employment, whereas in Figure 1, the labor effort peaks after 12 years of employment.

households intensify their asset accumulation to supplement their pension income and boost consumption during retirement. In retirement, consumption is highest and assets are depleted.

III. Baseline Simulations

The time line for the simulations corresponds to a 370-year period divided into three unequal subperiods. In the first century, the economy is in the steady state described in Section II. The middle segment covers the demographic transition—from a high to a low fertility rate, time-varying immigration, and increasing life expectancy—that takes 170 years to work itself out. In the final 100 years, the economy is in a new steady state characterized by lower population growth and higher life expectancy than in the first century.[20] The beginning of the three subperiods corresponds, respectively, to the years 1857, 1957, and 2127.

A. Demographic Transition

Among the exogenous elements in the simulations, the demographic shock and immigration merit specific attention. The onset of the demographic transition—specifically, to higher life expectancy—is taken to be 1957, with households (22 years of age) entering the labor force that year and dying 59 years later (81 years old) in 2015 (Table 5). Life expectancy is assumed to increase one year per decade starting in 1957 so that households entering the labor force nine decades later (in 2047) die at 90 years of age (in 2114). From 2047, the life expectancy of households entering the labor force remains fixed at 90 years.

The number of labor force entrants reflects the combined effects of fertility and immigration and is set so that the endogenous trajectory of the model's dependency ratio—the ratio of population aged 62 years and older over population between 22 and 61 years of age—matches that of the available official projections through 2060 (Figure 2).[21] Accordingly, the annual growth rate of labor force entrants in the period 1980–2006 is higher than in the initial steady state, declines between 2006 and 2059, and is constant afterward. Note that the generation entering the labor force in 2060 dies in 2127, the year marking the end of the demographic transition and the beginning of the final steady state.

Following the Spanish National Statistics Institute (INE), a low- and a high-immigration scenarios are considered. In the low immigration scenario, the dependency ratio is stable at about 35 percent until 1985, declines slowly for the next twenty years due to immigration and other temporary factors, bottoms out at 34 percent by 2004 and then rises slowly to reach about 35.5 percent by 2010. From 2010, and for the next forty years, however, the ratio increases sharply to over 85 percent in 2050. In the high immigration scenario, the dependency ratio also rises sharply, but is still similar to the low immigration scenario.

[20] Specifically, the annual rate of population growth in the last century is equal to 0.5 percent—the average observed in the decade ending in 2001—reflecting a moderate rebound from the minimum (0.3 percent per annum) observed in the decade ending in 1991.

[21] Note that if the growth rate of labor force entrants and life expectancy are constant—as in the steady states—the growth rate of the total population is equal to that of the labor force entrants.

Note that in the model after peaking, the dependency ratio declines for the next three decades (2060–90) as labor force entrance recovers. However, the ratio increases again when the growth of labor force entrance stabilizes, reflecting the continued improvements in life expectancy. Only when both labor force entrance and life expectancy stabilize does the dependency ratio gradually converge (from above) to its steady state value of 59 percent. Although the exact numbers vary, the evolution of the dependency ratio is qualitatively the same across the immigration scenarios.

B. Baseline Macroeconomic Scenario

Several assumptions are made regarding government policies and immigration. First, the parameters of the social security system—retirement age and averaging period—remain unchanged over the 370 years of analysis. Second, as fiscal pressures arise during the demographic transition, the government implements a "tax-as-you-go" policy: consumption tax rates are adjusted so that the government's budget constraint holds while other tax rates and the government nonhealth expenditure-to-output and debt-to-output ratios remain constant.[22] Third, as noted above, since the low- and high-immigration scenarios result in similar trajectories of the dependency ratio, the simulations discussed below are for the low-immigration scenario.

The baseline simulations suggest that pension expenditures, as a share of output, increase by 16 percentage points by 2050 with severe macroeconomic consequences (Figure 3 and Table 7). The consumption tax rate peaks at 51 percent in 2050—more than 30 percentage points higher than in 2007—to finance the social security deficit. As a result, output and consumption per capita (detrended by technological progress) are 18 percent lower than in the initial steady state.[23]

Moreover, as taxes start rising in 2010, output per capita deteriorates long before the peak of the demographic shock in 2050, but remains unscathed for a couple of decades (through 2025). This reflects the fact that capital per capita increases due to the rising share in the population of old working households whose asset holdings peak at retirement. In addition, aggregate effective labor is also sustained by the rising share of old high-skilled working households and a higher marginal productivity of labor. The change in the population's age structure also accounts for the increase in consumption per capita, which is reinforced by the anticipation of consumption by young generations foreseeing tax rate increases. After 2025, however, capital, labor, output and consumption per capita fall sharply until about 2050. This

[22] Auerbach and Kotlikoff (1987) and De Nardi and others (1999) show that financing the demographic shock in the US economy with consumption taxes is less distortionary than financing it with other taxes—payroll and income taxes—as the tax burden is shared by more generations. Catalán and others (2005) confirms this result for the Spanish economy. Note that the ratio of government consumption-to-output varies over time according to the evolution of the population's age structure, reflecting the provision of health-care services, as described in Appendix II.

[23] Note that in Figure 3 and Table 7, output (consumption) is stationary-transformed as indicated in Table 1—adjusted by technological progress and population growth. Accordingly, these variables can also be interpreted as output (consumption) per capita deviations from their long term trend.

turnaround comes about as the share of newer generations—less able to accumulate assets because they have been taxed heavily since birth—increases relative to previous, more affluent, generations. In addition, the recently retired affluent generations start depleting their sizeable asset holdings, which reinforces the downturn in output and the capital stock. Note that factor prices track the evolution of the dependency and the capital-output ratios: the return on capital falls, and the wage rate (detrended by technological progress) increases until about 2050. Also, the wage rate increases imply that the (detrended) average pension rises by 14 percent and exacerbates the expenditure pressures.

The general equilibrium effects (average pension and output) account for about half of the pension expenditure pressures and also explain the difference with the EC's assessment of the fiscal impact of aging. Decomposing pension expenditure increases through 2050—16 percentage points as a share of output—shows that: the average pension increase accounts for 3 percentage points, the decline in output per capita accounts for 4.5 percentage points, and the change in the population's age structure accounts for the remaining 8.6 percentage points (Table 8).

Further insights into the baseline simulation can be gleaned from households' behavior (Figure 1). In this connection, it is useful to consider two generations of households: one entering the labor force in 1990 and the other entering in 2010. The former generation is among the most heavily taxed generations as it dies in 2053, when taxes peak. The latter generation is among those living through the widest tax rate swings as it dies in 2074, and thus faces both the sharp tax increase associated with the demographic shock and the subsequent decline as the shock dissipates.

Consider first the hypothetical case when factor prices are constant so that these households only face changes in consumption taxes. The path of consumption tax rates noted above—a sharp increase during 2010–50, and subsequent decrease through 2080—generates wealth and substitution effects affecting households' lifetime plans. In particular, an intertemporal consumption substitution effect arises from households foreseeing higher (lower) future consumption tax rates, which thus anticipate (delay) consumption.[24] Note that both generations shift consumption away from the high tax rate period, but their consumption profiles differ. The 1990 generation anticipates consumption in response to heavy tax rates at the end of its life (around 2050), reinforcing the aggregate consumption boom prior to 2025. The 2010 generation, however, tends to postpone consumption until the end of its life, when tax rates have already declined. Their behavior, thus, reinforces the collapse in aggregate consumption around 2050.

Changes in factor prices provide an additional source of intertemporal substitution. Specifically, the rising wage rates before 2050 induce an intertemporal delay of work effort in the lifetimes of the 1990 and 2010 generations—whose work lives end in the years 2030

[24] An increasing path of consumption tax rates would also induce households to anticipate labor effort if wage rates remained constant, as the consumption-leisure ratio in each period is inversely related to the consumption tax rate (equation (7) in Table 2). However, wage rates vary over time in the simulations.

and 2050—relative to generations born in the initial steady state.[25] Both elements, higher wage rates and higher households' labor effort in the averaging period, increase individual pension benefits, and thus—in addition to the increased dependency ratio—put further pressure on public finances.

In the final steady state, the dependency ratio and the consumption tax rate are higher than in the initial steady state due to a longer retirement period—as life expectancy increases and the working life remains fixed—and a smaller rate of population growth (Figure 4). The negative wealth effect associated with higher taxes implies lower lifetime consumption, and higher lifetime work effort (for the representative household), because of the need to accumulate more assets and enhance the pension income to finance a longer retirement period.[26] Indeed, asset holdings continue increasing during the initial years of retirement, in contrast with the initial steady state.

IV. Simulations of Pension Reforms

As discussed above, two reform scenarios are considered. A "partial pension reform" increases households' retirement age but leaves the averaging period unchanged. Additionally, a "full pension reform" extends the averaging period used to compute the pension benefit to the entire work life. Before discussing the demographic transition, it is useful to first understand the long run impact of these reforms.

A. Effects of Pension Reform in the Final Steady State

In a nutshell, reforming the pension system improves welfare in the final steady state. In other words, the benefits associated with lower taxes—including lower intertemporal distortions in consumption and labor—more than offset the welfare costs of pension benefits cuts in the final steady state. A detailed discussion of this result follows.

Partial pension reform

Increasing the retirement age from 47 to 49 years—and the work life from 46 to 48 years—lowers the dependency ratio and boosts the aggregate effective labor supply by increasing the

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[25] Rising wages increase pension benefits for a given labor effort—a positive wealth effect that provides an incentive to consume more but work and save less. However, this effect is small compared to the opposite negative wealth effect induced by high consumption taxes.

[26] Note that the longer life expectancy fully accounts for the higher work effort. Our utility assumptions imply that the wealth and substitution effects on leisure of higher consumption tax rates cancel out; thus, lifetime work effort would not change if population growth were lower but life expectancy remained constant.

number of cohorts working each period (Figure 5).[27,28] The greater lifetime labor income results in a higher capital accumulation, which in turn increases the capital-labor ratio and the wage rate. Even though the wage rate is higher, individual pensions decline because the hump-shaped skills imply a reduction in households' average labor skills (and wage incomes earned) in the averaging period. Still, welfare improves as the lower individual pensions and dependency ratio imply a reduction in consumption tax rates and an increase in consumption.

Full pension reform

Extending the averaging period to the entire work life increases welfare monotonically despite the decline in pension benefits due to the concomitant decline in consumption tax rates (Figure 5). The interaction of the hump-shaped skills and technological progress plays a central role in explaining why aggregate effective labor and output per capita increase when the averaging period is shorter than 15 years, but decrease when it is longer. Specifically,

- When the averaging period is shorter than 15 years—as it was prior to the 1997 reform—extending it results in a higher aggregate effective labor supply, which reflects the higher average skills of households in the averaging period and enhanced labor effort during high-skill ages.[29] Pension benefits decrease, however, as this effect is more than offset by the impact of technological progress, as high wage earning years are more heavily discounted by growth in productivity.[30]

- When the averaging period is 15 years—as it is currently—or longer, extending it decreases the aggregate effective labor supply as it results in lower average skills in the averaging period. This and technological progress reduce the pension benefit.

In either case, the decline in the pension benefit allows a reduction in the consumption tax rate that boosts household's consumption and welfare.

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[27] With a work life of 48 years and a retirement period of 20 years, the model's dependency ratio in the final steady state is the same as in the initial steady state (0.35). Also, with a work life of 48 years, the ratio of working-to-retirement years for a given household in the final steady state is slightly higher (2.4) than in the initial steady state (2.2)—an unchanged ratio would result from a work life of 47 years and a retirement period of 21 years.

[28] Since households enter the labor force with 22 years of age, this corresponds to increasing the retirement age from 68 to 70 years. Compared to the initial steady state, increasing the model's retirement age from 40 to 48 years corresponds to increasing the natural retirement age from 62 to 70 years.

[29] Figure 4 does not show labor profiles by age for this particular reform; however, it is still useful to illustrate why extending the averaging period induces households to exert more effort when they are highly skilled, thus increasing aggregate effective labor.

[30] Labor-augmenting technological progress increases real wages over time, whereas the pension calculation is based on inflation-adjusted wage earnings during the household's averaging period. In equation (5), an increase in ξ reduces, *ceteris paribus*, the pension benefit $b_{t+T_t}^{T_t+1}$.

These simulations also suggest that a full pension reform increases welfare in the final steady state. As noted above, increasing the retirement age while holding constant the averaging period (partial reform) increases welfare, and so does extending the averaging period while holding constant the retirement age (full reform). Combining these partial effects improves welfare unambiguously.

A word of caution regarding the welfare effects of extending the averaging period: the monotonic improvement depends on the rate of technological progress.[31] Specifically, when there is no technological progress, welfare is maximized when the averaging period is shorter than the entire work life (see Appendix III). This is because when the averaging period is already long and is extended further, the pension benefit rises as the decline in effective labor induces higher wages that, in contrast to the discussion above, are not deflated by technological progress. Consumption tax rates, in turn, increase to finance the higher pension benefits, reducing household's welfare.

B. Effects of Pension Reforms in the Demographic Transition

Before turning to the simulation results in the demographic transition, it is important to clarify the assumptions made regarding household's expectations of reforms, and the announcement and implementation of the reforms. Specifically, reforms are unanticipated: before they are announced, households envisage the baseline scenario to unfold. And reforms are simultaneously announced and implemented at the *beginning* of 2008; the announcement is credible and includes the partial grandfathering clauses for transitional generations as detailed below.

Grandfathering clauses

The principle guiding the design of grandfathering clauses in this study is to provide more grandfathering to households nearing retirement—with less time to adjust to the new pension rules—and gradually less grandfathering to households further away from retirement (Table 6).[32] Households that are 48 years of age or older in 2008—including those already retired and those that have entered the averaging period—are fully grandfathered.

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[31] Also, in these steady state simulations, households' welfare improves when pension benefits and the consumption tax rate fall. This is because households use tax reductions to save on their own for retirement—earning a market rate of return. We do not model myopia or self-control problems that could prevent households from saving in this rational, forward looking way. If these additional distortions were present, the welfare analysis and the policy implications could change significantly—for instance, in order to replicate the welfare implications of our analysis, as part of the reform strategy, the government could place the tax reduction in individual retirement accounts to deal with the self-control problem.

[32] Only a partial grandfathering is considered as a complete grandfathering would not yield pension expenditure savings until the late 2040s—when the first generations of the reformed system retire, too late to mitigate the adverse macroeconomic effects of aging.

Younger households face, to a varying degree, the parametric reforms starting in 2008. Specifically,

- The *retirement age* of households with 34–47 years of age in 2008 is increased by one year (and their work lives extended to $T = 41$ years) relative to the baseline. The retirement age of generations aged 22–33 years in 2008 and of those entering the labor force in 2009–16 is increased by two years (and their work lives extended to $T = 42$ years). Starting in 2017, the retirement age increases two years every decade until work lives reach 48 years—generations born in 2017–26 have work lives lasting 44 years, and all generations born after 2046 have work lives lasting 48 years.

- The *averaging period* of each generation 47 years of age, or younger, in 2008 is extended to the whole period between 2008 and retirement. The averaging period of households born in 2008 and later is extended to their entire work lives.[33]

Partial pension reform

A partial pension reform attenuates the expenditure pressures and consumption tax rate increases that are needed, while boosting the aggregate capital stock and output. It partially offsets the adverse effects of aging and limits the increase in pension expenditure and tax rates in the period 2008–50.

Comparing the paths of aggregate variables in the partial pension reform—holding the averaging period at 15 years—with those in the baseline scenario reveals the effects of increasing the retirement age (Figure 3 and Table 7):

- Pension expenditures, as a share of output, increase by about 12 percentage points between 2008 and 2050—4 percentage points less than in the baseline. The consumption tax rate peaks in 2053 at 44 percent—up 25 percentage points from 2007. Still, this is about 7 percentage points lower than in the baseline. In the final steady state the consumption tax rate is 19 percent, 18 percentage points lower than in the baseline.

- The capital stock per capita is consistently higher than in the baseline, albeit significantly so after 2015. The aggregate labor supply per capita is significantly higher than in the baseline after 2023, but slightly lower than in the baseline before 2023, reflecting the grandfathering rules and households' behavior. Output per capita is significantly higher than in the baseline only since the early 2020s. By 2050, when the dependency ratio peaks, output per capita is 4 percent higher than in the baseline—14 percent lower than in the initial steady state.

[33] Note that the generation aged 47 years old—26 years old in the model—enters the averaging period in 2008. Before the pension reforms (in the baseline), the generation aged 47 in 2008—with 26 years in the workforce—retires in 2023 and has a 15 year averaging period. After the pension reforms, this generation retires in 2024 and has a 16 year averaging period.

- Capital-labor ratios and wage rates are consistently higher than in the baseline; rates of return on assets are lower than in the baseline, except for a brief period (2038–44).

- Consumption per capita is considerably higher than in the baseline after 2025.

- Pension benefits are lower than in the baseline, particularly after 2023. As in the baseline simulations, however, the (detrended) average pension increases—by 9 percent—until 2055, and falls sharply thereafter. Also, the reform reduces the retired-to-total population ratio from 43 percent (baseline) to 26 percent.

- The decomposition of the pension expenditure pressures through 2050 show that the average pension, the population's age structure, and output per capita have similar contributions to limiting expenditure increases relative to the baseline—between 1.3 and 1.5 percentage points of output (Table 8).

A "bust-boom" cycle in aggregate effective labor is associated with the partial reform scenario: aggregate effective labor declines before the 2020s, but increases thereafter, reflecting the grandfathering clauses and households' intertemporal labor substitution effects. The small decline in the effective labor supply in 2008–22 (Figure 8) can be explained by households' behavior. As households work longer (with an unchanged averaging period) they substitute labor intertemporally: working more at older ages, when their skills have already declined, while exerting less effort during their middle work lives, when their skills are highest. At the outset of the reform, this substitution results in a small reduction in the aggregate labor supply. As time goes by, however, more households enter the upper age ranges and exert high labor effort, which together with a larger number of working cohorts, results in aggregate labor gains.

Full pension reform

A full pension reform delivers more significant macroeconomic improvements: it further limits the increase in pension expenditures, as a share of output, between 2008 and 2050 to 8 percentage points of output, that is, 8 percentage points less than in the baseline; tax rate increases are also further limited to 37 percent or about 14 percentage points lower than in the baseline. Thus, extending the averaging period—conditional on increasing the retirement age—is as important in mitigating the macroeconomic effects of aging as increasing the retirement age at the peak of the demographic shock, 2050.

By comparing the full and partial pension reform scenarios (Figure 3 and Table 7), the "pure" effects of extending the averaging period from 15 years to the entire work life can be assessed. Specifically, extending the averaging period:

- Reduces the pension expenditure pressures: pension expenditures increase, as a share of ouput, 8 percentage points between 2008 and 2050—4 percentage points lower than in the partial reform. The consumption tax rate peaks in 2050 at 37 percent—7 percentage points lower compared with the partial reform, but still 18 percentage points higher than in 2007. In the final steady state the consumption tax rate is 17 percent, 2 percentage points lower than in the partial reform scenario.[34]

- Leads to consistently higher capital stock levels. Aggregate effective labor per capita is higher than in the partial reform before 2020, but lower thereafter. Output per capita is higher before 2025, but lower afterwards, as the higher capital stock does not offset the lower labor input. By 2050, output per capita is 1 percent lower compared to the partial reform (15 percent lower than in the baseline).

- Increases capital-labor ratios after 2013, implying higher wage rates and lower rates of return on capital. Before then, however, the higher effective labor—caused by the extension of the averaging period—reduces the capital-labor ratio.

- Boosts consumption per capita before 2023, but reduces it afterwards.

- Reduces the (detrended) average pension benefit by 11 percent between 2007 and 2050. Until the 2020s, however, the grandfathering clauses prevent sharp reductions in pension benefits. Also, the reduced average pension accounts for the whole reduction in pension expenditure increases—the contributions of the population’s age structure and output per capita are unchanged—relative to a partial reform (Table 8).

Intuitively, extending the averaging period causes an intertemporal labor substitution at the household level that is reflected at the aggregate level. Specifically, households intensify labor effort during the middle of their work lives, when skills are highest, and exert less effort when they are close to retirement. In contrast with the delayed aggregate labor gains in the partial reform, the intertemporal labor substitution effect in the full pension reform leads to immediate aggregate labor gains. But the costs of anticipating the aggregate labor gains show up later. As a larger share of the population approaches retirement, aggregate effective labor declines, and thus reduces aggregate labor precisely when labor is most scarce, that is, when the dependency ratio starts rising sharply. Note further that compared with the partial reform, a “boom-bust” cycle in aggregate effective labor emerges, which makes labor more scarce as the demographic storm intensifies.

The relative contributions of the pension reforms—increasing the retirement age and extending the averaging period—to limiting the consumption tax rate increase vary over time. At the peak of the demographic transition, extending the averaging period accounts for half of the tax rate reduction obtained from a full pension reform (relative to the baseline). In the final steady state, however, extending the averaging period accounts for just a tenth of the tax rate reduction. Intuitively, extending the averaging period lowers pension benefits more when (detrended) wage rates rise over time, as is the case in 2010–50. When (detrended) wage rates are constant, as is the case in the final steady state, the relative contribution of extending the averaging period is much smaller.

[34] Compared to the baseline: pension expenditure pressures are 8 percentage points lower, the peak tax rate in 2050 is 14 percentage points lower—but still 18 percentage points higher than in 2007—and in the final steady state, the consumption tax rate is 20 percentage points lower.

C. Effects of Tax-Smoothing Policies

Prefunding the fiscal costs associated with aging—with or without pension reforms—is simulated by a once-and-for-all increase in the consumption tax rate in 2008. This tax-smoothing policy seeks to avoid the distortions and adverse macroeconomic effects resulting from sharp changes in tax rates in the tax-as-you-go policy discussed so far. Also, tax smoothing reduces the tax burden imposed on the younger households during the toughest years of the demographic transition, but at the cost of increasing the burden on older and future generations.

In the absence of pension reform, the consumption tax rate must increase to 25.4 percent—a hike of 6.4 percentage points compared with 2007—to prefinance the demographic shock (Figure 6 and Table 7). From that level, a partial pension reform reduces the tax rate by 1.2 percentage points to 24.2 percent, and a full pension reform reduces it further by 0.8 percentage points to 23.4 percent. Regardless of the pension reform scenario, the government debt-to-output ratio declines rapidly before the peak of the dependency ratio, and the government becomes a net creditor.[35]

Compared with tax-as-you-go, these simulations suggest that:

- Labor, output, and consumption per capita all increase during the worst period of the demographic transition (2025–55), but decline before 2025 and after 2055.
- Capital per capita and capital-labor ratios are higher before 2060, and lower thereafter.
- Pension expenditure-to-output ratios do no vary significantly.

Intuitively, tax smoothing reduces the intertemporal distortions on asset accumulation relative to the tax-as-you-go scenario, but also affects the intergenerational welfare distribution. Pension reforms combined with tax-smoothing policies would result in a Pareto improvement if the negative welfare effects of tax smoothing on some generations could be avoided by some compensating mechanism.

D. Welfare Analysis

To complete the discussion of the simulation results, cross generational welfare is computed for the baseline and reform scenarios noted above, under both tax policies.

[35] Note that the resulting changes in debt-to-output ratios—between the 2008 and minimum levels—are very large—about 70 percentage points under a full pension reform.

Tax-as-you-go

In a nutshell, under a tax-as-you-go policy both the partial and full pension reforms reduce the welfare of generations entering the labor force in 1983–2002; losses for these generations are larger with a full pension reform. All generations entering the labor force before 1983, and after 2002, are either unaffected or benefit from the reforms; gains for these generations are larger under a full pension reform (Figure 7, upper panel). This is because the generations that entered the labor force before 1983 have already retired or have entered the averaging period before 2008, and thus, are fully grandfathered (Table 6). Although their pension rule does not change, they benefit from the lower consumption tax rates and higher returns on asset holdings made possible by the reform.

However, generations that entered the labor force between 1983 and 2002—the first generations not fully grandfathered—are adversely affected by the reforms. Specifically, their retirement age increases by one year if they entered the labor force between 1983 and 1994, and by two years if they entered the labor force after 1994; also, their averaging period is extended to the whole period between 2008 and retirement. Thus, decisions made by these generations before the reform are no longer optimal. Since these generations must reassess their plans, their resulting labor effort, consumption and asset accumulation profiles are not as smooth as if the reforms had been anticipated. Moreover, these generations gain welfare from reduced tax rates at the end of their lives, but these gains do not fully offset the welfare losses resulting from a shorter retirement period and lower pension benefits.

Generations that entered the labor force between 2002 and 2007 benefit from the reform as the reduction in tax rates in the second half of their lives more than offsets the suboptimal allocations made before the reforms are implemented. Likewise, those generations entering the labor force on and after 2008 are net winners as they have not made suboptimal choices and benefit from lower tax rates during most of their lives.

Tax smoothing

Conditional on full pension reform, tax-smoothing policies deliver welfare gains to generations that enter the labor force in 1992–2055, and cause welfare losses to generations that enter the labor force in 1951–1991, and after 2055 (Figure 7, middle panel).

This reflects the fact that the tax burden shifts from generations that are alive during the peak of the demographic shock to previous and future generations. The generations that entered the labor force before 1992 (40 years of age and older) did not face any major tax rate increase under the tax-as-you-go policy, and thus, are worse off when taxes are increased in 2008 with the tax-smoothing policy. Those generations entering the labor force between 1992 and 2064 would have faced large tax hikes in the tax-as-you-go policy—well beyond those in the tax smoothing policy, and thus benefit from a lower and constant consumption tax rate. The lower tax rates thus increase their welfare and eliminate distortions in their saving decisions. Finally, those generations entering the labor force after 2064 are worse off as they face a higher tax rate than with a tax-as-you-go policy.

Compared to the tax-as-you-go baseline, a full pension reform with tax smoothing reduces the welfare of generations entering the labor force in 1951–98, and improves the welfare of all other generations. Also, in a partial pension reform fewer generations—those entering the labor force in 1951–92—lose welfare, but the welfare gains of other generations are smaller than in a full pension reform (Figure 7, lower panel).

Although this paper does not attempt to find mechanisms to achieve a Pareto improvement—such that none of the existing, nor future generations, lose relative to the baseline—those mechanisms may involve delaying the increase in the consumption tax rate or targeting transfers to net losers financed by public debt.

V. Conclusion

When considering parametric reforms of PAYG pension systems, academic and policy discussions alike have focused prominently on tightening the link between contributions and benefits. Among the policies proposed to strengthen this link is extending the averaging period used in computing pension benefits. This reform, however, has received scant attention in the quantitative DGE context.

This paper seeks to fill this gap in the literature. Specifically, it evaluates the macroeconomic and welfare effects of extending the averaging period in a PAYG system using a DGE model in the Auerbach-Kotlikoff tradition. The complementarities between reforms extending the averaging period and those increasing the retirement age under alternative tax policies are also examined. The analysis is applied to the Spanish economy where the averaging period has taken center stage in pension reform discussions.

In the absence of reforms, pension expenditures will increase by 16 percentage points as a share of output by 2050. General equilibrium effects explain why these increases are more than twice those in EC (2006). While the EC's assessment assumes that pension benefits increase broadly in line with output per capita, the results discussed in this paper capture the fact that household's pension benefits increase sharply relative to output per capita during the peak of the demographic transition. This marked increase is driven by rising wages as labor becomes scarce. Moreover, this general equilibrium effect also accounts for the more severe macroeconomic consequences as consumption tax rates must increase to finance the additional aging-related spending.

Extending the averaging period can be an effective reform to limit the adverse macroeconomic consequences of aging. Specifically, this extension can mitigate aging-related spending and tax pressures during the demographic transition as much as increasing the retirement age in line with life expectancy. In Spain, a full pension reform—increasing the retirement age and extending the averaging period to the entire work life—reduces the pension expenditure increase, as a share of output, by 8 percentage points by 2050; the extension of the averaging period accounts for half of the reduction. Also, extending the averaging period can deliver welfare gains despite reducing individual pension benefits. These gains arise from a lower and flatter consumption tax rate path—which reduces distortions in consumption-saving decisions—and the extended averaging period—which removes labor market distortions. Tax smoothing can also limit the adverse macroeconomic consequences of aging by further reducing consumption-saving distortions.

Although the quantitative results are Spain-specific, these point to broader results in extending the averaging period. First, the extension limits the increase in individual pension benefits at the peak of the demographic transition more than in the long run. In other words, with a tax-as-you-go policy, the extension's contribution to limit tax rate increases varies over time. Second, in the long run, extending the averaging period to the entire work life may be suboptimal if technological progress is insufficient. With no technological progress, long-run welfare gains are largest when the averaging period is shorter than the entire work life.

Appendix I. Value Function at Retirement

The value function $V(A_{t+T_t}^{T_t+1}, b_{t+T_t}^{T_t+1})$ is the solution of the following problem:

$$V(A_{t+T_t}^{T_t+1}, b_{t+T_t}^{T_t+1}) = \underset{\left\{c_{t+s-1}^{s}, A_{t+s}^{s+1}\right\}_{s=T_t+1}^{T_t+T_t^R}}{Max} \sum_{s=T_t+1}^{T_t+T_t^R} \beta^{s-1} \cdot \log(c_{t+s-1}^{s})$$

subject to (4), (6) and given $A_{t+T_t}^{T_t+1}$ and $b_{t+T_t}^{T_t+1}$.

Notice that the household's asset holdings at retirement ($A_{t+T_t}^{T_t+1}$), and the annual pension benefit ($b_{t+T_t}^{T_t+1}$) are given, as they are determined by household's past decisions.

Let $\tilde{r}_t$ denote the year t rate of return on assets holdings net of the income tax, $\tilde{r}_t = r_t \cdot (1-\tau_t^I)$. We use the budget constraint (4) to solve for c_{t+s-1}^{s} and to express the value function recursively, in a Bellman's equation form (for $s = T_t+1, ..., T_t+T_t^R$), as follows:

$$V\left(A_{t+s-1}^{s}, b_{t+T_t}^{T_t+1}\right) = \underset{A_{t+s}^{s+1}}{Max} \ \log\left\{(1+\tilde{r}_{t+s-1}) \cdot A_{t+s-1}^{s} + \frac{b_{t+T_t}^{T_t+1}}{\left(1+\xi\right)^{s-T_t-1}} - \left(1+\xi\right) \cdot A_{t+s}^{s+1}\right\} + \beta \cdot V\left(A_{t+s}^{s+1}, b_{t+T_t}^{T_t+1}\right).$$

We obtain the value function by backward induction, that is, we start with the household's problem in its last year of life, and proceed backwards.

The household's problem at date $t+T_t+T_t^R-1$ (household's age is $s = T_t+T_t^R$) is given by

$$V\left(A_{t+T_t+T_t^R-1}^{T_t+T_t^R}, b_{t+T_t}^{T_t+1}\right) = \underset{A_{t+T_t+T_t^R}^{T_t+T_t^R+1}}{Max} \ \log\left\{\left(1+\tilde{r}_{t+T_t+T_t^R-1}\right) \cdot A_{t+T_t+T_t^R-1}^{T_t+T_t^R} + \frac{b_{t+T_t}^{T_t+1}}{\left(1+\xi\right)^{T_t^R-1}} - \left(1+\xi\right) \cdot A_{t+T_t+T_t^R}^{T_t+T_t^R+1}\right\}.$$

The household consumes all its remaining assets in its last period of life, as it leaves no bequests and the no-Ponzi condition ($A_{t+T_t+T_t^R}^{T_t+T_t^R+1} = 0$) is satisfied. Thus, the solution is given by

$$V\left(A_{t+T_t+T_t^R-1}^{T_t+T_t^R}, b_{t+T_t}^{T_t+1}\right) = \log\left\{\left(1+\tilde{r}_{t+T_t+T_t^R-1}\right) \cdot A_{t+T_t+T_t^R-1}^{T_t+T_t^R} + \frac{b_{t+T_t}^{T_t+1}}{\left(1+\xi\right)^{T_t^R-1}}\right\}.$$

1. The household's problem at date $t+T_t+T_t^R-2$ (household's age is $T_t+T_t^R-1$) is given by

$$V\left(A_{t+T_t+T_t^R-1}^{T_t+T_t^R}, b_{t+T_t}^{T_t+1}\right)=\underset{A_{t+T_t+T_t^R}^{T_t+T_t^R+1}}{Max}\ \log\left\{\left(1+\tilde{r}_{t+T_t+T_t^R-1}\right)\cdot A_{t+T_t+T_t^R-1}^{T_t+T_t^R}+\frac{b_{t+T_t}^{T_t+1}}{(1+\xi)^{T_t^R-1}}-(1+\xi)\cdot A_{t+T_t+T_t^R}^{T_t+T_t^R+1}\right\}.$$

The household consumes all its remaining assets in its last period of life, as it leaves no bequests and the no-Ponzi condition ($A_{t+T_t+T_t^R}^{T_t+T_t^R+1}=0$) is satisfied. Thus, the solution is given by

$$V\left(A_{t+T_t+T_t^R-1}^{T_t+T_t^R}, b_{t+T_t}^{T_t+1}\right)=\log\left\{\left(1+\tilde{r}_{t+T_t+T_t^R-1}\right)\cdot A_{t+T_t+T_t^R-1}^{T_t+T_t^R}+\frac{b_{t+T_t}^{T_t+1}}{(1+\xi)^{T_t^R-1}}\right\}.$$

2. The household's problem at date $t+T_t+T_t^R-2$ (household's age is $T_t+T_t^R-1$) is given by

$$V\left(A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}, b_{t+T_t}^{T_t+1}\right)=\underset{A_{t+T_t+T_t^R-1}^{T_t+T_t^R}}{Max}\ \log\left\{\left(1+\tilde{r}_{t+T_t+T_t^R-2}\right)\cdot A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}+\frac{b_{t+T_t}^{T_t+1}}{(1+\xi)^{T_t^R-2}}-(1+\xi)\cdot A_{t+T_t+T_t^R-1}^{T_t+T_t^R}\right\}$$
$$+\beta\cdot V\left(A_{t+T_t+T_t^R-1}^{T_t+T_t^R}, b_{t+T_t}^{T_t+1}\right).$$

Plug the solution of $V\left(A_{t+T_t+T_t^R-1}^{T_t+T_t^R}, b_{t+T_t}^{T_t+1}\right)$ found in 1 to obtain the following expression:

$$V\left(A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}, b_{t+T_t}^{T_t+1}\right)=\underset{A_{t+T_t+T_t^R-1}^{T_t+T_t^R}}{Max}\ \log\left\{\left(1+\tilde{r}_{t+T_t+T_t^R-2}\right)\cdot A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}+\frac{b_{t+T_t}^{T_t+1}}{(1+\xi)^{T_t^R-2}}-(1+\xi)\cdot A_{t+T_t+T_t^R-1}^{T_t+T_t^R}\right\}$$
$$+\beta\cdot\log\left\{\left(1+\tilde{r}_{t+T_t+T_t^R-1}\right)\cdot A_{t+T_t+T_t^R-1}^{T_t+T_t^R}+\frac{b_{t+T_t}^{T_t+1}}{(1+\xi)^{T_t^R-1}}\right\}.$$

Find the first order condition of this optimization problem and solve for $A_{t+T_t+T_t^R-1}^{T_t+T_t^R}$,

$$A_{t+T_t+T_t^R-1}^{T_t+T_t^R}=\frac{\beta\cdot\prod_{i=1}^{2}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)\cdot A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}}{(1+\xi)\cdot(1+\beta)\cdot\left(1+\tilde{r}_{t+T_t+T_t^R-1}\right)}-\frac{\left[1-\beta\cdot\left(1+\tilde{r}_{t+T_t+T_t^R-1}\right)\right]\cdot\frac{b_{t+T_t}^{T_t+1}}{(1+\xi)^{T_t^R-2}}}{(1+\xi)\cdot(1+\beta)\cdot\left(1+\tilde{r}_{t+T_t+T_t^R-1}\right)};$$

plug this expression into the value function $V\left(A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}, b_{t+T_t}^{T_t+1}\right)$ and solve, as follows:

$$V\left(A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}, b_{t+T_t}^{T_t+1}\right) = (1+\beta)\cdot\log\left\{\prod_{i=1}^{2}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)\cdot A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1} + \left(2+\tilde{r}_{t+T_t+T_t^R-1}\right)\cdot\frac{b_{t+T_t}^{T_t+1}}{(1+\xi)^{T_t^R-2}}\right\} - \Omega_1,$$

where Ω_1 is a constant: $\Omega_1 = \log\left(1+\tilde{r}_{t+T_t+T_t^R-1}\right) + (1+\beta)\cdot\log(1+\beta) + \beta\cdot\log(1+\xi) - \beta\cdot\log(\beta)$.

3. The household's problem at date $t+T_t+T_t^R-3$ (household's age is $s = T_t + T_t^R - 2$):

$$V\left(A_{t+T_t+T_t^R-3}^{T_t+T_t^R-2}, b_{t+T_t}^{T_t+1}\right) = \underset{A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}}{Max}\ \log\left\{\left(1+\tilde{r}_{t+T_t+T_t^R-3}\right)\cdot A_{t+T_t+T_t^R-3}^{T_t+T_t^R-2} + \frac{b_{t+T_t}^{T_t+1}}{(1+\xi)^{T_t^R-3}} - (1+\xi)\,A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}\right\} + \beta\cdot V\left(A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}, b_{t+T_t}^{T_t+1}\right).$$

Replacing $V\left(A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}, b_{t+T_t}^{T_t+1}\right)$ from 2, we can write the previous expression as follows:

$$V\left(A_{t+T_t+T_t^R-3}^{T_t+T_t^R-2}, b_{t+T_t}^{T_t+1}\right) = \underset{A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}}{Max}\ \log\left\{\left(1+\tilde{r}_{t+T_t+T_t^R-3}\right)\cdot A_{t+T_t+T_t^R-3}^{T_t+T_t^R-2} + \frac{b_{t+T_t}^{T_t+1}}{(1+\xi)^{T_t^R-3}} - (1+\xi)\cdot A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}\right\}$$
$$+ \beta\cdot(1+\beta)\cdot\log\left\{\prod_{i=1}^{2}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)\cdot A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1} + \left(2+\tilde{r}_{t+T_t+T_t^R-1}\right)\cdot\frac{b_{t+T_t}^{T_t+1}}{(1+\xi)^{T_t^R-2}}\right\} - \beta\cdot\Omega_1.$$

Find the first order condition and solve for $A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1}$,

$$A_{t+T_t+T_t^R-2}^{T_t+T_t^R-1} = \frac{\beta\cdot(1+\beta)\cdot\prod_{i=1}^{3}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)\cdot A_{t+T_t+T_t^R-3}^{T_t+T_t^R-2}}{(1+\xi)\cdot\left(1+\beta+\beta^2\right)\cdot\prod_{i=1}^{2}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)} + \frac{\left[\beta\cdot(1+\beta)\cdot\prod_{i=1}^{2}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right) - \left(2+\tilde{r}_{t+T_t+T_t^R-1}\right)\right]\cdot\frac{b_{t+T_t}^{T_t+1}}{(1+\xi)^{T_t^R-3}}}{(1+\xi)\cdot\left(1+\beta+\beta^2\right)\cdot\prod_{i=1}^{2}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)}.$$

Plug this previous expression into the value function $V\left(A_{t+T_t+T_t^R-3}^{T_t+T_t^R-2}, b_{t+T_t}^{T_t+1}\right)$ and solve,

$$V\left(A_{t+T_t+T_t^R-3}^{T_t+T_t^R-2}, b_{t+T_t}^{T_t+1}\right) = \left(1+\beta+\beta^2\right)\cdot\log\left\{\prod_{i=1}^{3}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)\cdot A_{t+T_t+T_t^R-3}^{T_t+T_t^R-2} + \left[1+\left(1+\tilde{r}_{t+T_t+T_t^R-1}\right)+\prod_{i=1}^{2}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)\right]\cdot\frac{b_{t+T_t}^{T_t+1}}{(1+\xi)^{T_t^R-3}}\right\} - \Omega_2,$$

where Ω_2 is a constant: $\Omega_2 = \log\left(1+\tilde{r}_{t+T_t+T_t^R-1}\right) + \log\left(1+\tilde{r}_{t+T_t+T_t^R-2}\right) + \left(1+\beta+\beta^2\right)\cdot\log\left(1+\beta+\beta^2\right)$

$$+ \beta\cdot(1+\beta)\cdot\log(1+\xi) - \beta\cdot(1+\beta)\cdot\log\left[\beta\cdot(1+\beta)\right].$$

4. Repeating the procedure backwards, the value function at date $t+T_t$ (household's age is T_t+1) is given by

$$V\left(A_{t+T_t}^{T_t+1}, b_{t+T_t}^{T_t+1}\right)=\left(\sum_{j=1}^{T_t^R}\beta^{j-1}\right)\cdot\log\left\{\prod_{i=1}^{T_t^R}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)\cdot A_{t+T_t}^{T_t+1}+\left\{1+\sum_{j=1}^{T_t^R-1}\left[\prod_{i=1}^{j}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)\right]\right\}\cdot b_{t+T_t}^{T_t+1}\right\}-\Omega,$$

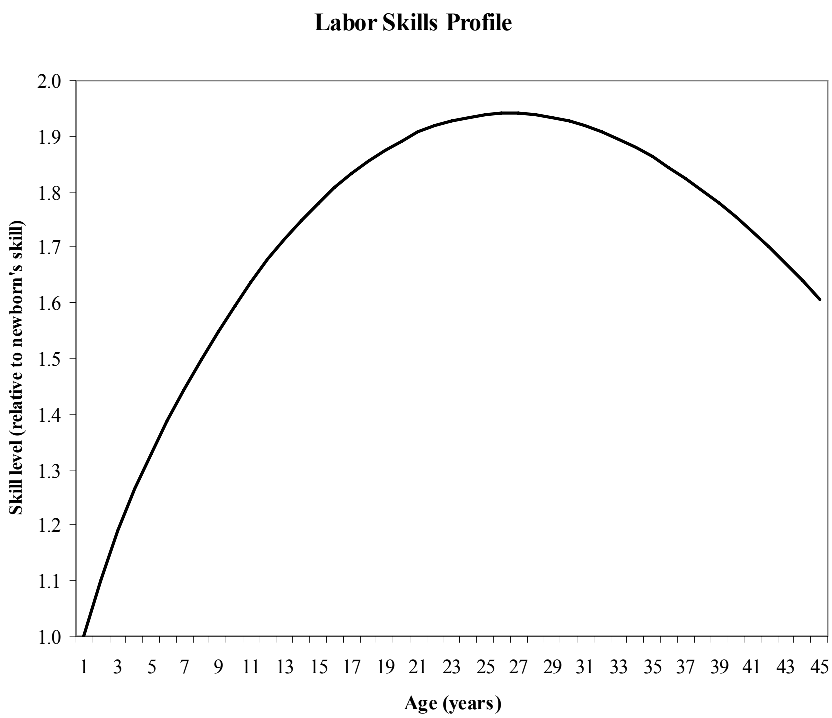
where Ω is a constant. The derivatives of the value function with respect to changes in asset holdings $\left(V_A\right)$ and pension benefits $\left(V_b\right)$ are given by

$$V_A(.)=\frac{\left(\sum_{j=1}^{T_t^R}\beta^{j-1}\right)\cdot\prod_{i=1}^{T_t^R}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)}{\prod_{i=1}^{T_t^R}\left(1+r_{t+T_t+T_t^R-i}\right)\cdot A_{t+T_t}^{T_t+1}+\left\{1+\sum_{j=1}^{T_t^R-1}\left[\prod_{i=1}^{j}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)\right]\right\}\cdot b_{t+T_t}^{T_t+1}},$$

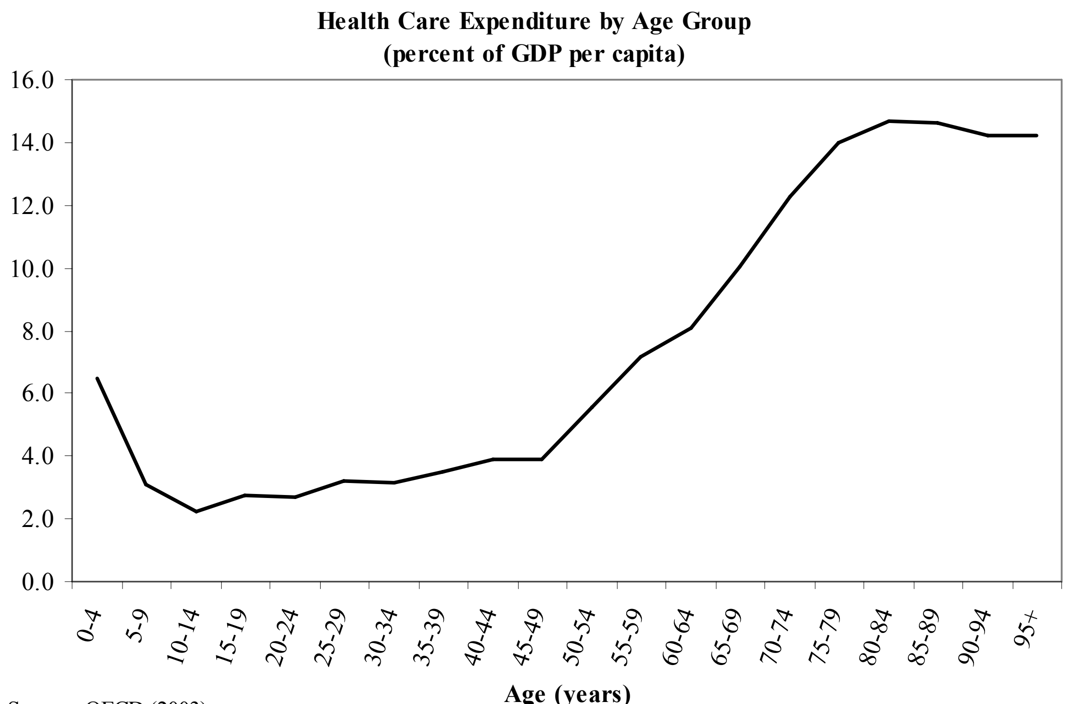
$$V_b(.)=\frac{\left(\sum_{j=1}^{T_t^R}\beta^{j-1}\right)\cdot\left\{1+\sum_{j=1}^{T_t^R-1}\left[\prod_{i=1}^{j}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)\right]\right\}}{\prod_{i=1}^{T_t^R}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)\cdot A_{t+T_t}^{T_t+1}+\left\{1+\sum_{j=1}^{T_t^R-1}\left[\prod_{i=1}^{j}\left(1+\tilde{r}_{t+T_t+T_t^R-i}\right)\right]\right\}\cdot b_{t+T_t}^{T_t+1}}.$$

Appendix II. Calibration Data Sources

Household's labor skills by age (figure below): the labor skills profile by age was calibrated to match the relative wage rates (per hour) earned by households in different age groups in Spain, according to data from the Spanish National Statistics Institute (INE). The calibration of skills for households with more than 30 years in the workforce is based on Hansen (1993).



Government health expenditure profile by age: the figure below shows the private and public health-related expenditure by age group as a share of GDP per capita in Spain in the year 1998. Dividing by the total population in that year, we obtained the sum of the private and public health-related expenditures by age group as a share of GDP. Using the fact that the sum of public health-related consumption over all age groups was 5.5 percent of GDP in 1998, and assuming that private and public health-related expenditures exhibit the same age profiles, we obtained the public health-related expenditure profile by age group as a share of output. Dividing by the population of each age group and multiplying by output, we obtained the expenditure per individual of a given age group and assumed that it grows over time at the rate of technological progress.[36] Having the public expenditure per individual of each age group, it was straightforward to track the total public health-related expenditure over time.



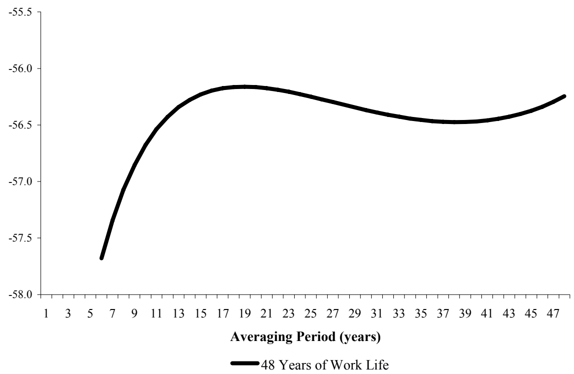
Source: OECD (2003).

[36] In recent years, health related expenditures per individual have grown faster than output. Therefore, our assumptions may be underestimating future health-related expenditure pressures arising from population aging.

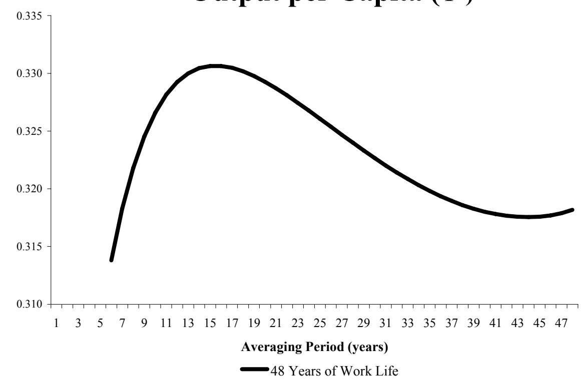
Appendix III. Comparative Statics in the Final Steady State

We evaluate the welfare and macroeconomic effects of extending the averaging period when the rate of labor-augmenting technological progress is zero ($\xi = 0$). We find that welfare is maximized for an averaging period of 19 years.[37]

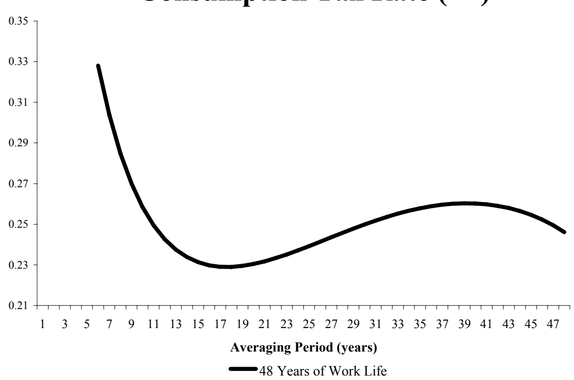
Welfare



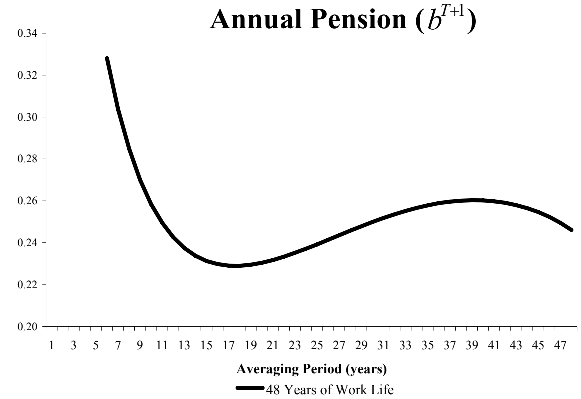
Output per Capita (Y)



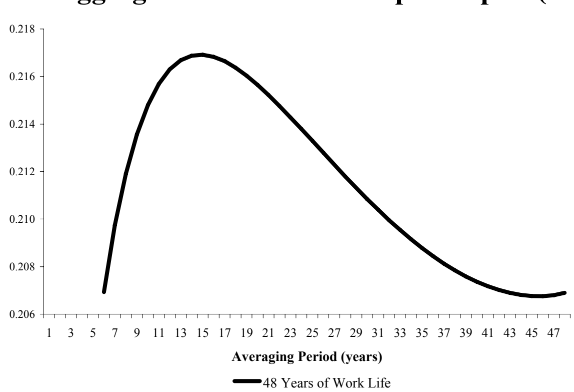
Consumption Tax Rate (τ^c)



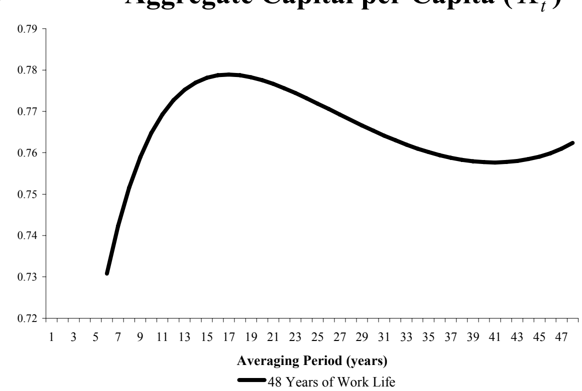
Annual Pension (b^{T+1})



Aggregate Effective Labor per Capita (N_t)



Aggregate Capital per Capita (K_t)



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[37] With the exception of the consumption tax rate, all variables are expressed as deviations from trend.

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Table 1. Variable Definition and Notation

Variable	Notation	Stationary Transformation	Variable	Notation	Stationary Transformation
Parameters					
Discount factor (utility)	β		Rate of labor augmenting technological progress	ξ	
Leisure preference (utility)	γ		Replacement ratio (pension rule)	ψ	
Capital share (production)	α		Averaging period (pension rule)	μ	
Capital depreciation rate	δ		Constant rate of population growth 1/	p	
Labor skill	e^s		Total factor productivity	Z	
Population					
s -year old population	P_t^s		Total population	P_t	
Households					
Labor effort	n_t^s		Aggregate effective labor supply	$\hat{N}_t^h$	$N_t^h = \frac{\hat{N}_t^h}{P_t}$
Leisure	l_t^s		Aggregate labor effort	$\hat{n}_t^h$	$n_t^h = \frac{\hat{n}_t^h}{P_t}$
Consumption	$\hat{c}_t^s$	$c_t^s = \frac{\hat{c}_t^s}{(1+\xi)^t}$	Aggregate consumption	$\hat{C}_t^h$	$C_t^h = \frac{\hat{C}_t^h}{(1+\xi)^t \cdot P_t}$
Asset holdings	$\hat{A}_t^s$	$A_t^s = \frac{\hat{A}_t^s}{(1+\xi)^t}$	Aggregate asset holdings	$\hat{A}_t^h$	$A_t^h = \frac{\hat{A}_t^h}{(1+\xi)^t \cdot P_t}$
Annual pension	$\hat{b}_{t+T_t}^{T_t+1}$	$b_{t+T_t}^{T_t+1} = \frac{\hat{b}_{t+T_t}^{T_t+1}}{(1+\xi)^{t+T_t}}$			
Firms					
Aggregate capital demand	$\hat{K}_t^f$	$K_t^f = \frac{\hat{K}_t^f}{(1+\xi)^t \cdot P_t}$	Aggregate labor demand	$\hat{N}_t^f$	$N_t^f = \frac{\hat{N}_t^f}{P_t}$
Aggregate output	$\hat{Y}_t^f$	$Y_t^f = \frac{\hat{Y}_t^f}{(1+\xi)^t \cdot P_t}$	Profits (net) 2/	$\hat{\Pi}_t^f$	$\Pi_t^f = \frac{\hat{\Pi}_t^f}{(1+\xi)^t \cdot P_t}$
Factor Prices					
Gross rate of return on assets	r_t		Wage rate (unskilled labor)	$\hat{W}_t$	$W_t = \frac{\hat{W}_t}{(1+\xi)^t}$
Tax Rates					
Social security contribution	τ_t		Consumption tax	τ_t^c	
Income tax	τ_t^I				
Government					
Debt	$\hat{D}_t$	$D_t = \frac{\hat{D}_t}{(1+\xi)^t \cdot P_t}$	Expenditure	$\hat{G}_t$	$G_t = \frac{\hat{G}_t}{(1+\xi)^t \cdot P_t}$

Note: Superscripts (subscripts) indicate the age of the household (time period); stock variables are dated at the beginning of the corresponding year. 1/ Population growth rates are constant only along balanced growth equilibrium paths. 2/ Profits are net of capital depreciation.

Table 2. First Order Conditions—Household's Optimization Problem

	Consumption-Leisure Decision (Intratemporal condition)	Consumption-Saving Decision (Intertemporal condition)
Working Age 1) Before averaging period $(s=1,...,T_t-\mu)$	$\frac{\gamma}{l^s_{t+s-1}} = \frac{W_{t+s-1}\cdot e^s \cdot (1-\tau_{t+s-1}-\tau^I_{t+s-1})}{c^s_{t+s-1}\cdot(1+\tau^c_{t+s-1})}$ (7)	$\frac{(1+\xi)}{c^s_{t+s-1}\cdot(1+\tau^c_{t+s-1})} = \beta\cdot\frac{[1+r_{t+s}\cdot(1-\tau^I_{t+s})]}{c^{s+1}_{t+s}\cdot(1+\tau^c_{t+s})}$ (8)
2) During averaging period $(s=T_t-\mu+1,...,T_t-1)$	$\frac{\gamma}{l^s_{t+s-1}} = \frac{W_{t+s-1}\cdot e^s\cdot(1-\tau_{t+s-1}-\tau^I_{t+s-1})}{c^s_{t+s-1}\cdot(1+\tau^c_{t+s-1})} + W_{t+s-1}\cdot e^s\cdot\frac{\psi}{\mu}\cdot\frac{\beta^{T_t+1-s}}{(1+\xi)^{T_t+1-s}}\cdot V_b(A^{T_t+1}_{t+T_t}, b^{T_t+1}_{t+T_t})$ (9)	$\frac{(1+\xi)}{c^s_{t+s-1}\cdot(1+\tau^c_{t+s-1})} = \beta\cdot\frac{[1+r_{t+s}\cdot(1-\tau^I_{t+s})]}{c^{s+1}_{t+s}\cdot(1+\tau^c_{t+s})}$ (10)
3) Last year before retirement $(s=T_t)$	$\frac{\gamma}{l^{T_t}_{t+T_t-1}} = \frac{W_{t+T_t-1}\cdot e^s\cdot(1-\tau_{t+T_t-1}-\tau^I_{t+T_t-1})}{c^{T_t}_{t+T_t-1}\cdot(1+\tau^c_{t+T_t-1})} + W_{t+T_t-1}\cdot e^s\cdot\frac{\psi}{\mu}\cdot\frac{\beta}{(1+\xi)}\cdot V_b(A^{T_t+1}_{t+T_t}, b^{T_t+1}_{t+T_t})$ (11)	$\frac{(1+\xi)}{c^{T_t}_{t+T_t-1}\cdot(1+\tau^c_{t+T_t-1})} = \beta\cdot V_A(A^{T_t+1}_{t+T_t}, b^{T_t+1}_{t+T_t})$ (12)
Retirement $(s=T_t+1,...,T_t+T^R_t-1)$		$\frac{(1+\xi)}{c^s_{t+s-1}\cdot(1+\tau^c_{t+s-1})} = \frac{\beta\cdot[1+r_{t+s}\cdot(1-\tau^I_{t+s})]}{c^{s+1}_{t+s}\cdot(1+\tau^c_{t+s})}$ (13)

Table 3. Balanced Growth Path

Variable	Growth Rate
$\hat{Y}_t,\ \hat{C}_t,\ \hat{K}_t,\ \hat{D}_t,\ \hat{G}_t,\ \sum_{s=T+1}^{T+T^R}\hat{b}^s_t\cdot P^s_t$	$p+\xi+p\cdot\xi$
$\widehat{W}_t,\ \hat{b}^{T+1}_{t+T},\ (\hat{c}^1_t,...,\hat{c}^{T+T^R}_t),\ (\hat{A}^2_t,...,\hat{A}^{T+T^R}_t)$	ξ
$\widehat{N}_t$	p
$r_t,\ (n^1_t,...,n^T_t)$	0

Table 4. Calibration of the Baseline Model (Initial Steady State)

Symbol	Definition	Value	Source
α	Share of capital	0.3300	Fernandez de Cordoba and Kehoe (2000) and Estrada et al. (2004). 1/
γ	Leisure preference	1.8700	Value set so that the fraction of working time for the representative household is 0.274.
β	Discount factor	0.9500	From the real business cycle literature.
δ	Depreciation rate	0.0600	From the real business cycle literature.
Z	Total factor productivity	0.6100	Value set so that the capital-output ratio is 2.01. 2/
τ	Social security payroll tax rate	0.1950	Social security contributions over wage income. 3/
τ^c	Consumption tax rate	0.1890	Indirect tax revenues as percentage of private consumption (average 1994-2004). 4/
τ^I	Capital-income tax rate	0.1360	Direct tax revenues and other current revenues as percentage of GDP (average 1994-2004).
p	Rate of population growth	0.0085	Average 1900-1970.
G/Y	Government consumption (fraction of total output)	0.2280	Average 1994-2004. 5/
D/Y	Government debt (fraction of total output)	0.4100	General government (includes regional governments), 2004.
ψ	Replacement ratio	0.5438	Value set so that the pension at retirement over the (net) average wage income for the working population is 0.65. 6/
μ	Averaging period	15.0000	15 years is the reference period since the reform of 1997.
ξ	Rate of labor-augmenting technological progress	0.0150	Set to result in a 1.5 percent annual rate of output per capita growth (average).
T	Work life (years)	40.0000	Set to match individuals' entry to the labor force at age 22 and retirement at age 62.
T^R	Retirement life (years)	18.0000	Households live 80 years with certainty.

Notes: 1/ Fernandez de Cordoba and Kehoe (2004) calibrate their model using a share-of-capital parameter value of 0.302, whereas Estrada et al. (2004) estimate the parameter value at 0.36 using econometrics.
2/ Fernandez de Cordoba and Kehoe (2004) set the capital-output ratio value at 2.03.
3/ Social security contributions are 13 percent of GDP, and the labor share is 0.67.
4/ Indirect tax revenues are 11.3 percent of GDP, and the share of consumption in GDP is 0.59.
5/ Government consumption (0.228) = Current expenditures (0.359) - Social transfers (0.128) - Interest payments (0.037) + Capital expenditures (0.034). Also, Government consumption (0.228) = Health-related public expenditures (0.055) + Non-health public expenditures (0.173).
6/ The wage income is net of payroll taxes. According to Serrano et al. (2004), the average replacement ratio (average pension income over the net average wage income) in 2002 was 0.625 (0.517) for new (old) pensioners, increasing about 1.25 percentage points per year.

Sources: National Accounts and Labor Statistics: Instituto Nacional de Estadistica (INE) and AMICO; Fiscal Accounts: Ministerio de Economia y Hacienda, IGAE; Population: INE; Life expectancy: World Health Organization and World Development Indicators (World Bank).

Table 5. Demographic Transition: Model's Time Line[1/]

	Period			
	First Century	Transitional		Last Century
Calendar year	1857-1956	1957-2059	2060-2126	2127-2227
Growth of labor force entrants	0.85	variable	0.5	0.5
Life expectancy of labor force entrants 2/	80 (58)	increases one year per decade	90 (68)	90 (68)
Population growth	0.85	variable	variable	0.5

1/ Annual percentage rates of growth unless otherwise indicated.

2/ Natural life expectancy at birth of the cohort entering the labor force in a given year. Numbers in parentheses indicate remaining life time upon entry to the labor force in the model. Strictly, life expectancy increases one year per decade between 1957 and 2047, and is constant thereafter.

Table 6. Decomposition of the Change in Pension Expenditure (2007-2050) 1/
(Percentage points of output)

	Pension expenditure	Output per capita	Retired-to-total population ratio	Average pension
Baseline	16.1	4.5	8.6	3.0
Partial reform	11.9	3.0	7.2	1.7
Full reform	7.9	2.9	7.2	-2.2

1/ Expenditures correspond to a tax-as-you-go policy. The decomposition is obtained by re-expressing the pension expenditure to output ratio as (Avg. pension)•(retired-to-total population)/(output per capita), where (Avg. pension) = $\sum_{s=T_t+1}^{T_t+T_t^R} \left(b_{t+T_t+1-s}^{T_t+1} \big/ (1+\xi)^{s-T_t-1} \right) \cdot \left(P_t^s \Big/ \sum_{s=T_t+1}^{T_t+T_t^R} P_t^s \right)$ and (retired-to-total population)= $\sum_{s=T_t+1}^{T_t+T_t^R} P_t^s \Big/ P_t$.

Table 7. Macroeconomic Effects of Pension Reforms and Fiscal Policies During the Demographic Transition

Tax-as-you-go

Period	Aggregate Capital	Aggregate Effective Labor	Aggregate Capital-Labor Ratio	Aggregate Output	Output Growth	Net Wage Rate	Net Rate of Return on Assets	Consumption Tax Rate	Aggregate Consumption	Pension Benefit	Government Consumption-Output Ratio	Pension Expenditure-Output Ratio	Government Debt-Output Ratio
							No pension reform						
2001-07	100.0	100.0	100.0	100.0	0.022	100.0	0.091	0.186	100.0	100.0	0.228	0.108	0.410
2008-10	100.8	100.4	100.4	100.5	0.020	100.1	0.090	0.190	100.9	100.5	0.227	0.109	0.410
2011-15	102.2	100.8	101.4	101.3	0.018	100.5	0.089	0.198	101.8	100.7	0.225	0.113	0.410
2016-20	102.7	100.1	102.6	101.0	0.013	100.8	0.088	0.218	102.6	102.4	0.226	0.122	0.410
2021-25	103.8	99.8	104.0	101.1	0.011	101.3	0.087	0.237	102.6	104.6	0.225	0.131	0.410
2026-30	102.6	97.2	105.6	98.9	0.007	101.8	0.086	0.276	101.3	108.2	0.230	0.151	0.410
2031-35	101.2	94.4	107.2	96.6	0.005	102.3	0.084	0.318	98.2	112.0	0.236	0.172	0.410
2036-40	97.4	89.1	109.3	91.7	0.002	103.0	0.082	0.383	93.2	116.8	0.248	0.205	0.410
2041-45	92.5	84.3	109.6	86.9	0.003	103.1	0.082	0.445	87.4	121.1	0.262	0.238	0.410
2046-50	88.5	81.4	108.7	83.7	0.007	102.8	0.083	0.493	83.3	124.8	0.272	0.262	0.410
2051-55	86.1	81.2	106.0	82.8	0.009	102.0	0.085	0.508	81.7	126.3	0.275	0.268	0.410
2056-60	86.8	83.1	104.5	84.3	0.012	101.5	0.087	0.490	82.6	125.9	0.270	0.257	0.410
2061-65	88.4	86.0	102.9	86.8	0.013	100.9	0.088	0.448	85.1	123.6	0.263	0.236	0.410
2066-70	92.6	90.1	102.8	90.9	0.017	100.9	0.088	0.392	88.5	120.8	0.250	0.207	0.410
2071-75	95.2	92.0	103.4	93.1	0.016	101.1	0.088	0.361	91.2	118.1	0.245	0.191	0.410
2076-80	98.1	93.3	105.1	94.8	0.016	101.7	0.086	0.344	92.3	116.7	0.240	0.182	0.410
							Partial pension reform						
2001-07	100.0	100.0	100.0	100.0	0.022	100.0	0.091	0.186	100.0	100.0	0.228	0.108	0.410
2008-10	100.9	100.3	100.6	100.5	0.020	100.2	0.090	0.191	100.8	100.5	0.227	0.109	0.410
2011-15	102.3	100.6	101.7	101.1	0.018	100.6	0.089	0.199	101.5	100.8	0.225	0.113	0.410
2016-20	103.0	99.8	103.2	100.9	0.013	101.0	0.088	0.219	102.2	102.6	0.226	0.122	0.410
2021-25	104.6	100.1	104.5	101.6	0.013	101.5	0.087	0.227	102.7	103.1	0.224	0.125	0.410
2026-30	103.9	97.9	106.1	99.8	0.008	102.0	0.085	0.256	102.0	104.6	0.228	0.139	0.410
2031-35	103.0	95.4	108.0	97.9	0.006	102.6	0.084	0.294	99.0	108.4	0.233	0.157	0.410
2036-40	100.3	92.1	108.9	94.7	0.008	102.9	0.083	0.331	95.7	110.1	0.241	0.176	0.410
2041-45	96.5	88.3	109.2	91.0	0.004	103.0	0.083	0.374	91.4	112.0	0.250	0.197	0.410
2046-50	93.0	84.9	109.5	87.5	0.007	103.0	0.082	0.419	87.4	115.7	0.260	0.220	0.410
2051-55	90.7	84.2	107.7	86.3	0.008	102.5	0.084	0.438	85.3	118.0	0.264	0.228	0.410
2056-60	91.9	85.9	106.9	87.9	0.015	102.2	0.084	0.413	86.0	115.4	0.259	0.215	0.410
2061-65	93.9	89.5	105.0	90.9	0.012	101.6	0.086	0.357	89.3	108.0	0.250	0.184	0.410
2066-70	98.8	92.5	106.8	94.5	0.016	102.2	0.085	0.308	91.5	106.5	0.241	0.159	0.410
2071-75	102.0	95.5	106.8	97.6	0.019	102.2	0.085	0.250	95.3	95.3	0.233	0.128	0.410
2076-80	105.8	97.8	108.2	100.3	0.019	102.6	0.083	0.215	97.1	93.7	0.227	0.109	0.410
							Full pension reform						
2001-07	100.0	100.0	100.0	100.0	0.022	100.0	0.091	0.186	100.0	100.0	0.228	0.108	0.410
2008-10	101.0	102.7	98.3	102.2	0.025	99.4	0.092	0.189	101.5	100.4	0.223	0.108	0.410
2011-15	103.9	102.4	101.5	102.9	0.018	100.5	0.089	0.196	102.7	100.4	0.221	0.111	0.410
2016-20	105.4	100.5	104.8	102.1	0.012	101.6	0.086	0.216	103.1	102.4	0.223	0.121	0.410
2021-25	107.1	99.5	107.6	101.9	0.011	102.4	0.084	0.225	102.7	102.2	0.223	0.125	0.410
2026-30	106.1	96.1	110.5	99.3	0.006	103.3	0.081	0.254	100.9	97.3	0.229	0.137	0.410
2031-35	104.9	92.7	113.2	96.6	0.005	104.2	0.079	0.285	97.0	95.4	0.236	0.152	0.410
2036-40	101.8	88.8	114.7	92.9	0.007	104.6	0.078	0.314	93.1	93.1	0.245	0.165	0.410
2041-45	97.8	85.3	114.6	89.3	0.005	104.6	0.078	0.340	88.8	89.9	0.255	0.176	0.410
2046-50	94.4	82.8	113.9	86.5	0.008	104.4	0.079	0.362	85.4	85.9	0.263	0.185	0.410
2051-55	92.4	82.7	111.7	85.8	0.009	103.7	0.080	0.360	84.2	84.7	0.265	0.182	0.410
2056-60	93.5	85.1	109.9	87.8	0.015	103.2	0.082	0.327	85.5	85.0	0.260	0.164	0.410
2061-65	95.6	88.4	108.2	90.8	0.012	102.6	0.083	0.277	88.7	81.4	0.251	0.137	0.410
2066-70	100.3	91.7	109.4	94.4	0.016	103.0	0.082	0.235	91.2	80.7	0.241	0.117	0.410
2071-75	103.4	94.1	110.0	97.1	0.017	103.2	0.082	0.195	94.3	76.9	0.235	0.096	0.410
2076-80	107.0	95.7	111.8	99.3	0.018	103.7	0.080	0.172	95.6	74.7	0.229	0.085	0.410

Tax-smoothing

Period	Aggregate Capital	Aggregate Effective Labor	Aggregate Capital-Labor Ratio	Aggregate Output	Output Growth	Net Wage Rate	Net Rate of Return on Assets	Consumption Tax Rate	Aggregate Consumption	Pension Benefit	Government Consumption-Output Ratio	Pension Expenditure-Output Ratio	Government Debt-Output Ratio
							No pension reform						
2001-07	100.0	100.0	100.0	100.0	0.022	100.0	0.091	0.186	100.0	100.0	0.228	0.108	0.410
2008-10	101.1	97.1	104.1	98.4	0.013	101.3	0.087	0.254	98.4	100.7	0.231	0.112	0.379
2011-15	102.4	98.2	104.3	99.5	0.020	101.4	0.087	0.254	99.1	101.6	0.229	0.115	0.240
2016-20	103.7	98.4	105.3	100.1	0.015	101.7	0.086	0.254	100.5	103.9	0.227	0.124	0.038
2021-25	106.5	99.4	107.1	101.7	0.015	102.3	0.084	0.254	101.0	106.8	0.224	0.132	-0.203
2026-30	108.0	98.3	109.8	101.4	0.011	103.1	0.082	0.254	101.6	111.9	0.225	0.149	-0.476
2031-35	109.7	97.1	112.9	101.1	0.009	104.1	0.080	0.254	100.2	117.9	0.225	0.169	-0.761
2036-40	108.7	93.1	116.7	98.0	0.006	105.2	0.077	0.254	97.9	125.5	0.232	0.200	-1.017
2041-45	104.6	88.9	117.7	93.8	0.005	105.5	0.076	0.254	94.0	133.4	0.243	0.234	-1.182
2046-50	99.9	86.0	116.2	90.4	0.006	105.1	0.077	0.254	90.9	140.5	0.252	0.262	-1.217
2051-55	95.2	85.3	111.6	88.5	0.006	103.7	0.081	0.254	89.1	143.2	0.257	0.275	-1.132
2056-60	93.6	86.6	108.1	88.9	0.009	102.6	0.083	0.254	88.8	142.3	0.256	0.271	-0.983
2061-65	92.9	88.5	104.9	89.9	0.010	101.6	0.086	0.254	89.7	138.4	0.253	0.255	-0.816
2066-70	96.0	91.9	104.4	93.2	0.016	101.4	0.087	0.254	91.0	133.2	0.244	0.227	-0.705
2071-75	98.7	93.5	105.6	95.2	0.016	101.8	0.086	0.254	92.7	128.3	0.239	0.209	-0.670
2076-80	103.0	94.9	108.5	97.5	0.018	102.7	0.083	0.254	93.4	125.5	0.234	0.196	-0.712
							Partial pension reform						
2001-07	100.0	100.0	100.0	100.0	0.022	100.0	0.091	0.186	100.0	100.0	0.228	0.108	0.410
2008-10	101.1	98.0	103.2	99.0	0.015	101.0	0.088	0.241	98.8	100.7	0.230	0.111	0.386
2011-15	102.8	98.8	104.0	100.1	0.020	101.3	0.087	0.241	99.6	101.4	0.228	0.114	0.281
2016-20	104.1	98.8	105.3	100.5	0.015	101.7	0.086	0.241	101.0	103.7	0.227	0.123	0.138
2021-25	106.6	99.9	106.7	102.1	0.015	102.2	0.085	0.241	101.6	104.7	0.223	0.126	-0.021
2026-30	108.2	99.0	109.3	102.0	0.011	103.0	0.082	0.241	102.3	107.5	0.223	0.138	-0.219
2031-35	109.6	97.8	112.0	101.5	0.009	103.8	0.080	0.241	101.1	112.8	0.224	0.155	-0.411
2036-40	108.4	95.4	113.6	99.5	0.010	104.3	0.079	0.241	99.3	116.5	0.229	0.173	-0.555
2041-45	105.9	92.4	114.7	96.7	0.005	104.6	0.078	0.241	96.6	120.8	0.236	0.194	-0.654
2046-50	102.2	89.2	114.6	93.3	0.006	104.6	0.078	0.241	94.0	126.8	0.244	0.220	-0.647
2051-55	97.8	88.1	111.0	91.2	0.006	103.5	0.081	0.241	92.3	130.1	0.250	0.233	-0.501
2056-60	95.8	89.0	107.6	91.2	0.011	102.5	0.084	0.241	91.8	126.7	0.250	0.225	-0.249
2061-65	95.3	91.5	104.2	92.7	0.008	101.4	0.087	0.241	92.8	117.9	0.246	0.197	0.023
2066-70	98.1	93.2	105.3	94.8	0.012	101.7	0.086	0.241	93.4	113.2	0.240	0.173	0.266
2071-75	99.6	94.9	105.0	96.4	0.016	101.6	0.086	0.241	94.7	99.4	0.236	0.141	0.467
2076-80	103.0	96.4	106.9	98.5	0.018	102.2	0.084	0.241	95.2	94.8	0.231	0.119	0.578
							Full pension reform						
2001-07	100.0	100.0	100.0	100.0	0.022	100.0	0.091	0.186	100.0	100.0	0.228	0.108	0.410
2008-10	101.3	100.9	100.4	101.0	0.022	100.1	0.090	0.234	99.8	100.5	0.225	0.109	0.390
2011-15	104.6	101.1	103.5	102.2	0.020	101.1	0.087	0.234	101.1	100.9	0.223	0.112	0.298
2016-20	106.9	100.0	106.9	102.2	0.013	102.2	0.084	0.234	102.4	103.4	0.223	0.121	0.174
2021-25	109.6	99.8	109.8	102.9	0.013	103.1	0.082	0.234	102.2	103.7	0.221	0.124	0.048
2026-30	110.6	97.5	113.5	101.6	0.009	104.3	0.079	0.234	101.9	99.0	0.224	0.136	-0.094
2031-35	111.0	95.1	116.8	100.1	0.007	105.3	0.077	0.234	99.5	97.5	0.228	0.149	-0.207
2036-40	108.6	91.8	118.3	97.0	0.008	105.7	0.075	0.234	96.8	95.5	0.235	0.161	-0.263
2041-45	104.8	88.6	118.3	93.7	0.005	105.7	0.076	0.234	93.4	92.8	0.243	0.171	-0.264
2046-50	100.6	86.1	116.9	90.6	0.007	105.3	0.077	0.234	90.8	89.3	0.251	0.182	-0.183
2051-55	96.4	85.4	112.9	88.9	0.006	104.1	0.080	0.234	89.3	88.1	0.256	0.182	-0.009
2056-60	94.8	86.8	109.2	89.3	0.011	103.0	0.083	0.234	89.1	88.1	0.255	0.167	0.214
2061-65	94.7	89.0	106.4	90.9	0.009	102.1	0.085	0.234	90.2	84.1	0.251	0.142	0.429
2066-70	97.8	91.1	107.4	93.2	0.014	102.4	0.084	0.234	91.1	82.5	0.244	0.122	0.602
2071-75	99.8	92.4	108.0	94.8	0.015	102.6	0.084	0.234	92.4	77.8	0.240	0.101	0.724
2076-80	103.1	93.5	110.2	96.6	0.017	103.3	0.082	0.234	92.8	74.6	0.236	0.089	0.778

Notes: Numbers indicate period averages and all variables are defined as in Table 1 and expressed as stationary-transformations. Capital, effective labor, the capital-labor ratio, output, the net wage rate, consumption, and the pension benefit are normalized to 100 in the period 2001-2007. The pension benefit is the annual pension received by the generation that retires in the indicated year.

Table 8. Decomposition of Pension Expenditure Increases—Tax-as-you-go and Low Immigration 1/
(Changes in the period 2007-2050, in percentage points of output)

	Pension expenditure	Output per capita	Retired-to-total population ratio	Average pension
Baseline	16.1	4.5	8.6	3.0
Partial reform	11.9	3.0	7.2	1.7
Full reform	7.9	2.9	7.2	-2.2

1/ The decomposition is given by $\frac{\text{Pension expenditure}}{\text{Output}} = \frac{(\text{Avg. Pension})\cdot(\text{Retired-to-total population ratio})}{(\text{Output per capita})}$, where

Avg. Pension $= \sum_{s=T_t+1}^{T_t+T_t^R} \frac{b_{t+T_t+1-s}^{T_t+1}}{(1+\xi)^{s-T_t-1}} \cdot \frac{P_t^s}{\sum_{s=T_t+1}^{T_t+T_t^R} P_t^s}$, and Retired-to-total population ratio $= \frac{\sum_{s=T_t+1}^{T_t+T_t^R} P_t^s}{P_t}$.

Figure 1. Household's Asset Holdings (A^s), Labor Effort (n^s), and Consumption (c^s)
Profiles by Age (Initial Steady State and Selected Generations)

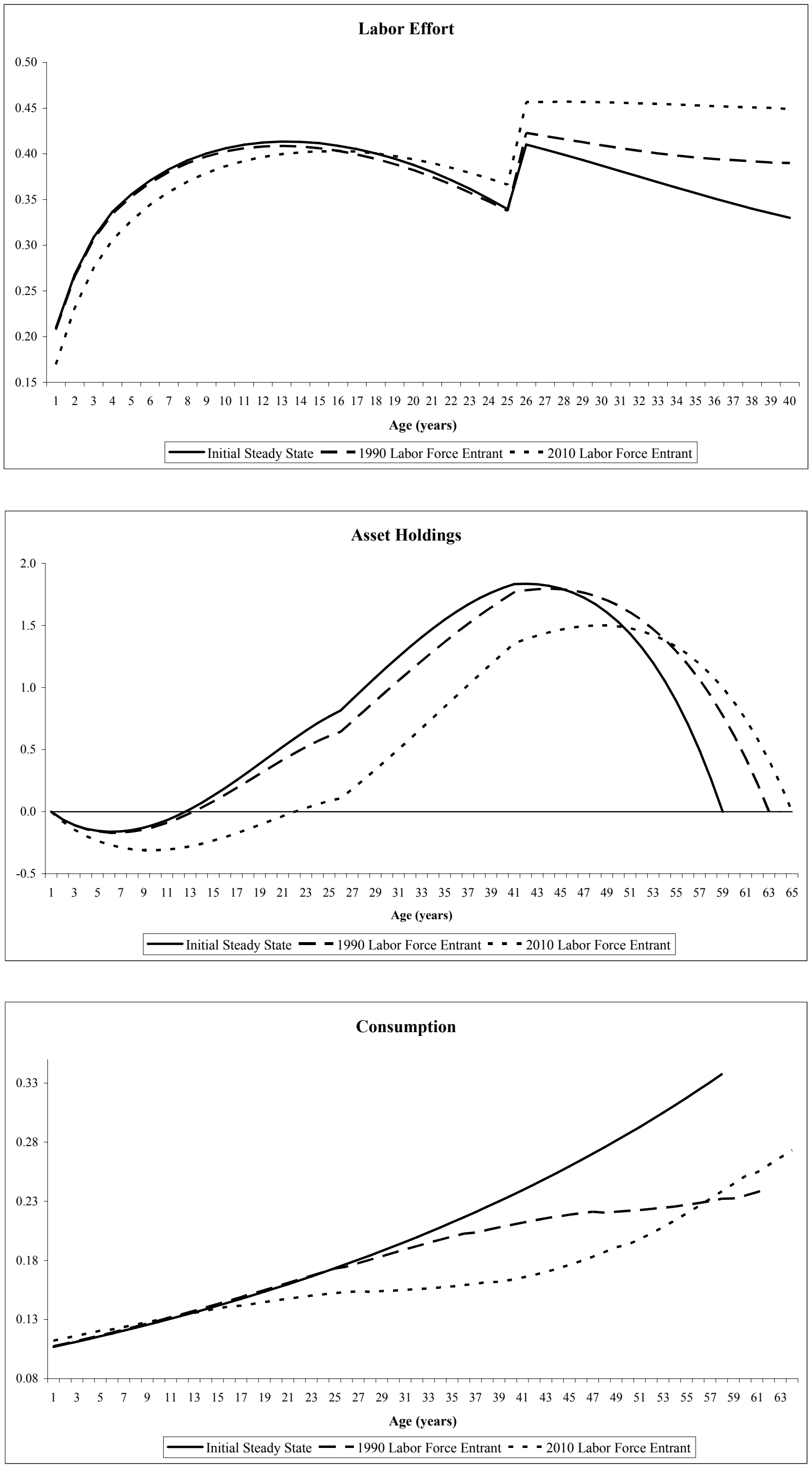


Figure 2. Model's Dependency Ratio

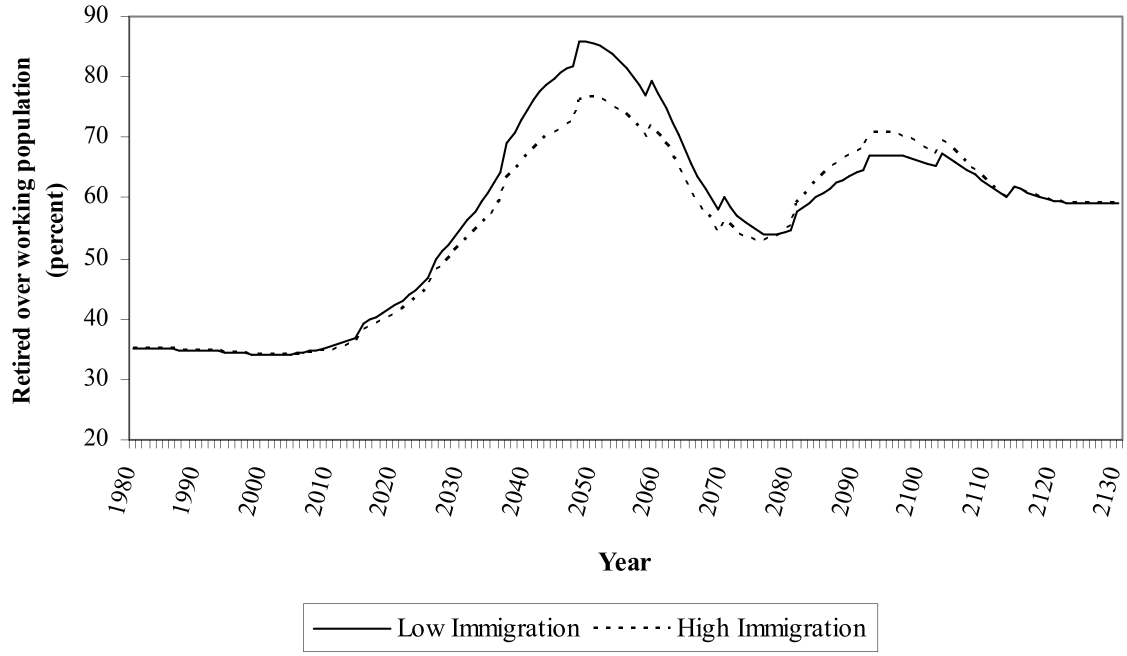


Figure 3. Baseline and Pension Reforms Macroeconomic Scenarios—
Tax-as-you-go and Low Immigration
(Unless otherwise indicated, variables are expressed as deviations from trend)

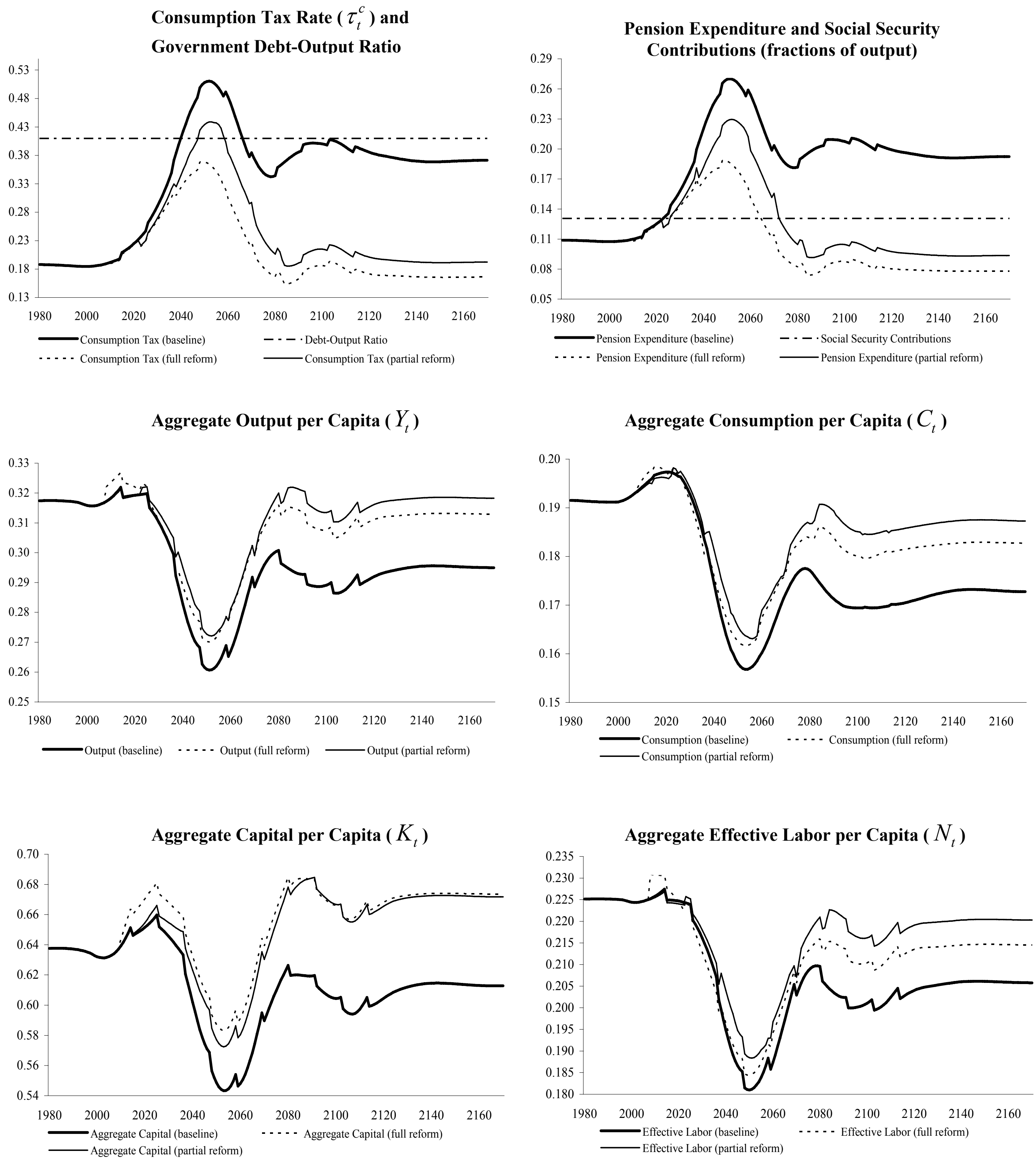
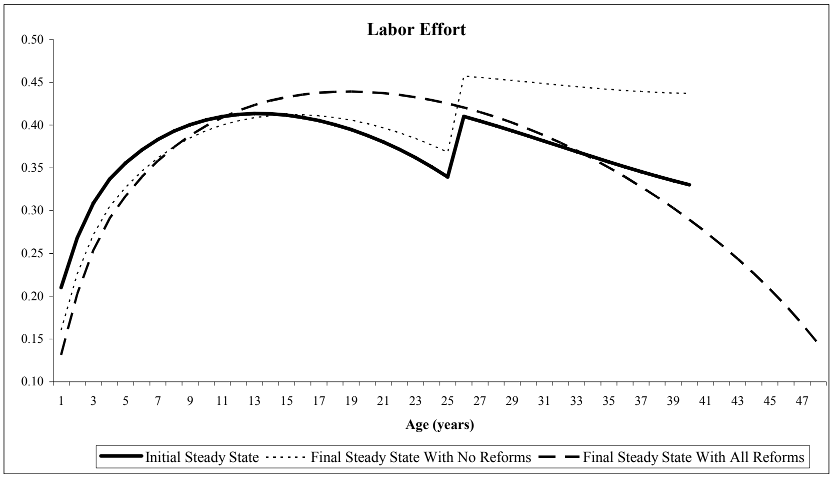
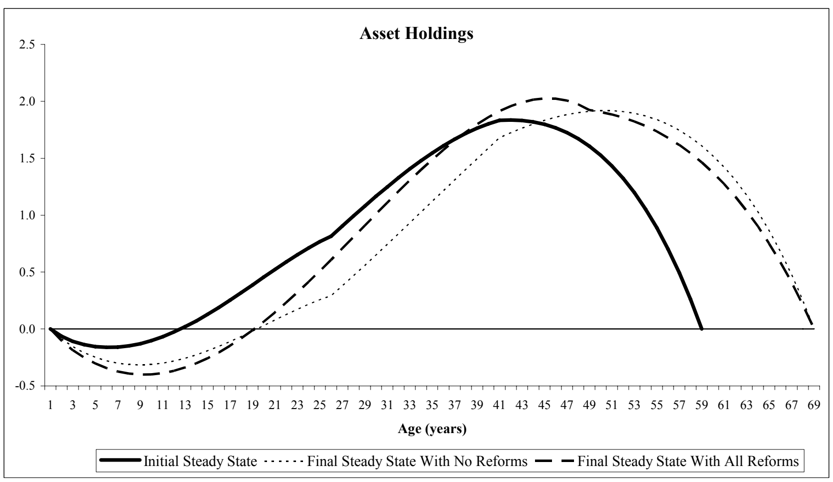


Figure 4. Household's Asset Holdings (A^s), Labor Effort (n^s), and Consumption (c^s) Profiles by Age (Final Steady State Under Alternative Reform Scenarios)





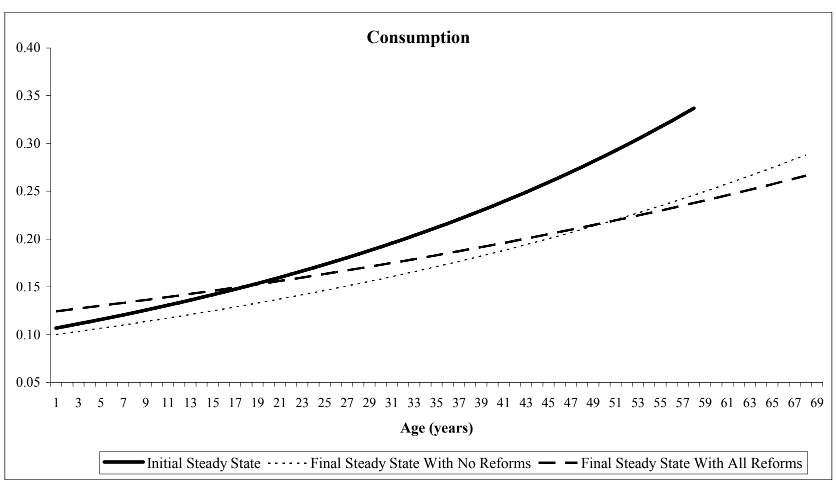
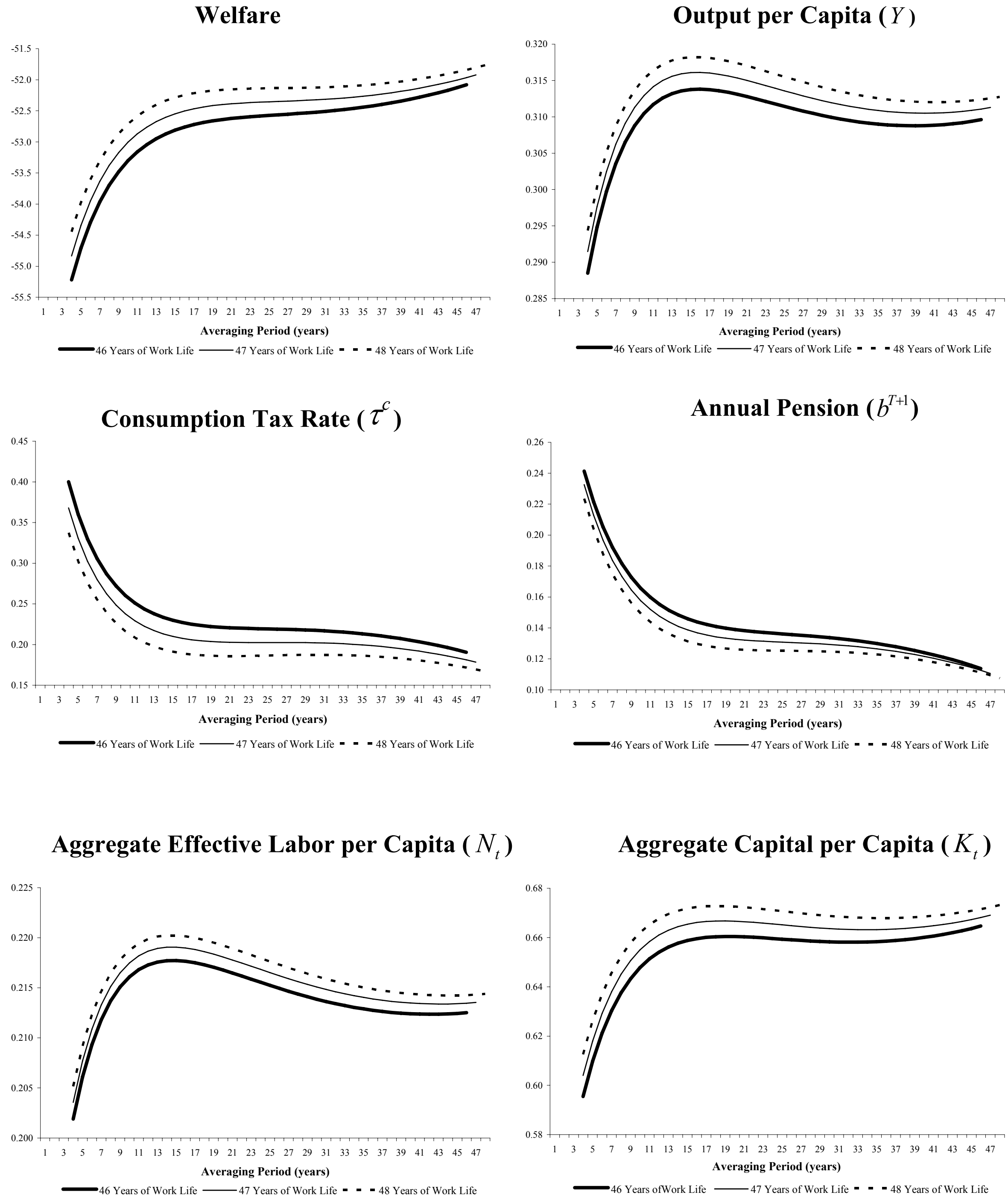


Figure 5. Welfare and Macroeconomic Effects of Pension Reforms in the Final Steady State[1/]

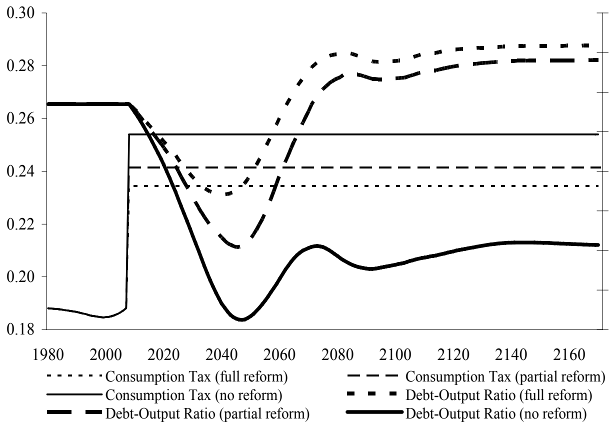


1/ With the exception of the consumption tax rate, all variables are expressed as deviations from trend. Also, welfare is calculated as in equation (1), for $T = 68$ and $T^R = 46, 47, 48$ respectively.

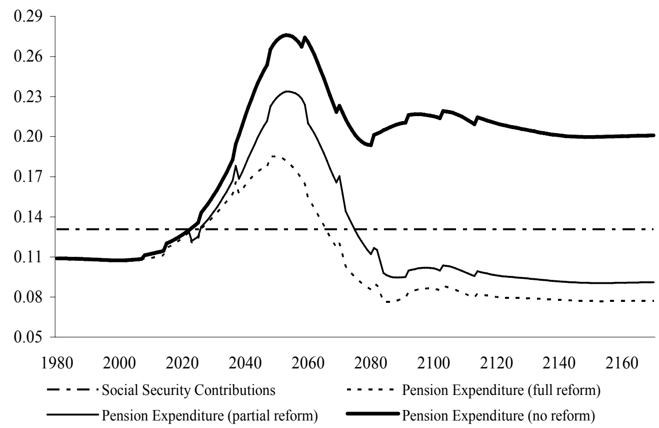
Figure 6. Baseline and Pension Reforms Macroeconomic Scenarios—Tax-Smoothing and Low Immigration

(Unless otherwise indicated, variables are expressed as deviations from trend)

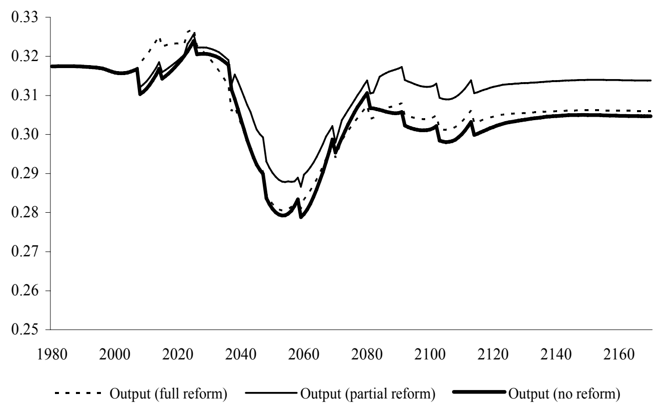
Consumption Tax Rate (τ_t^c) (left) and Government Debt-Output Ratio (right)



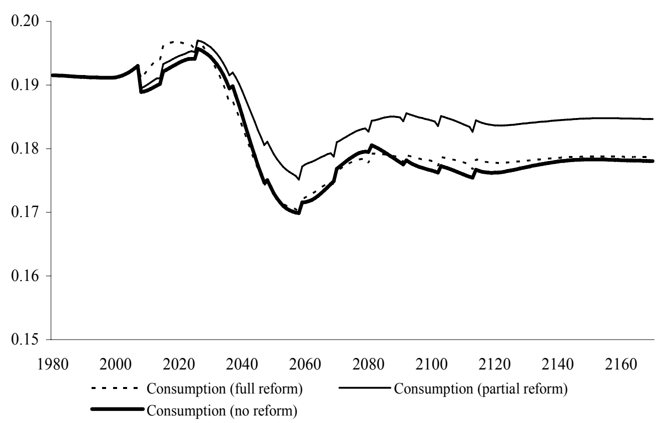
Pension Expenditure and Social Security Contributions (fractions of output)



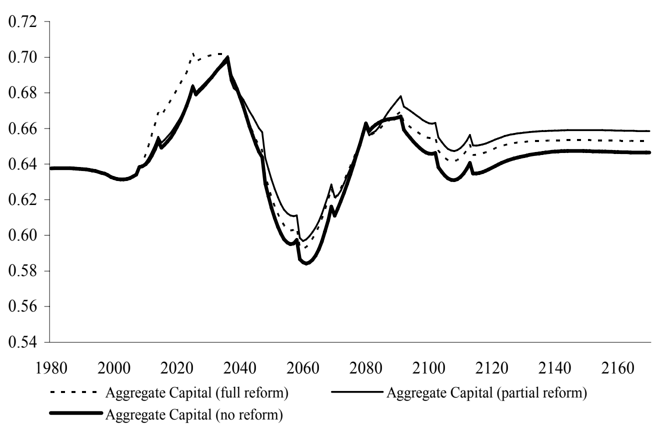
Aggregate Output per Capita (Y_t)



Aggregate Consumption per Capita (C_t)



Aggregate Capital per Capita (K_t)



Aggregate Effective Labor per Capita (N_t)

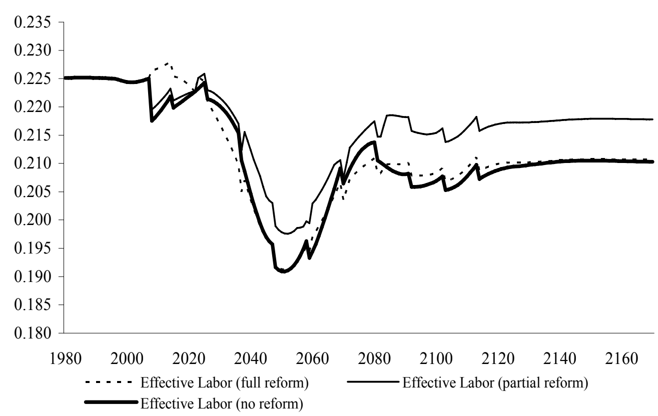


Figure 7. Welfare Effects of Pension Reforms and Fiscal Policies During the Demographic Transition

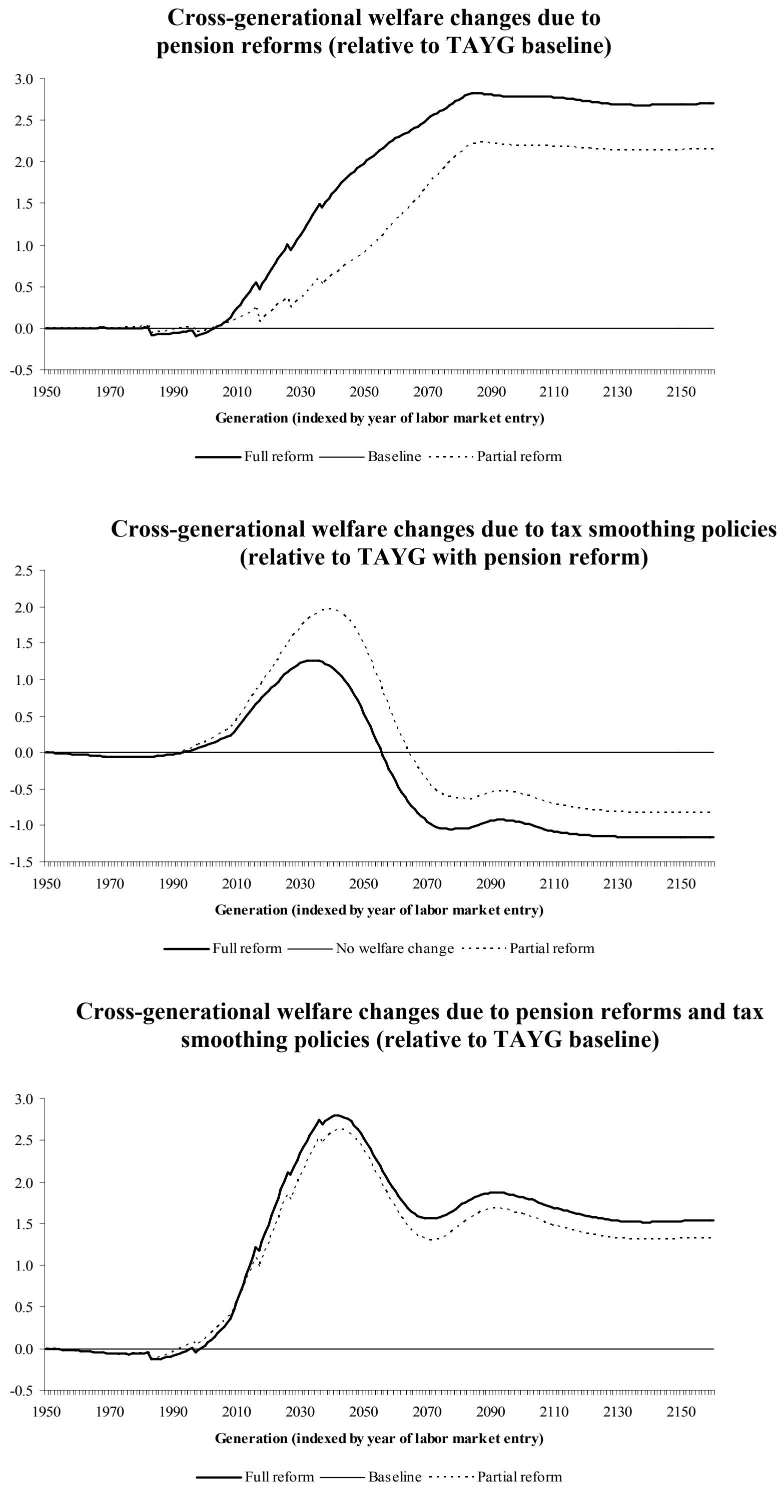
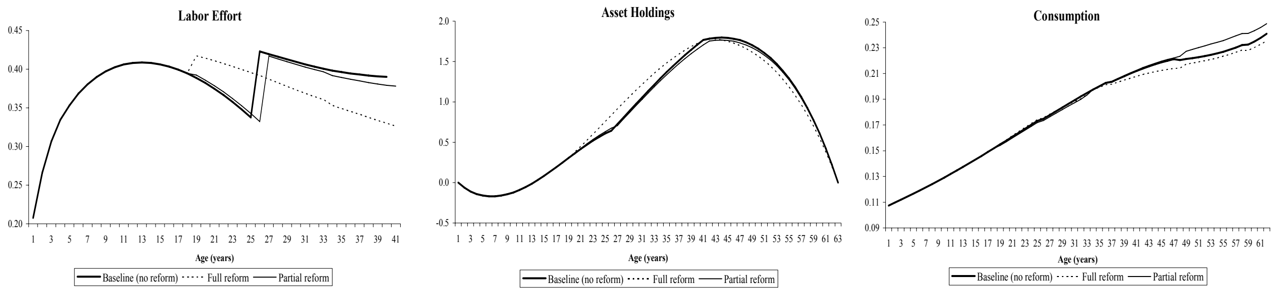
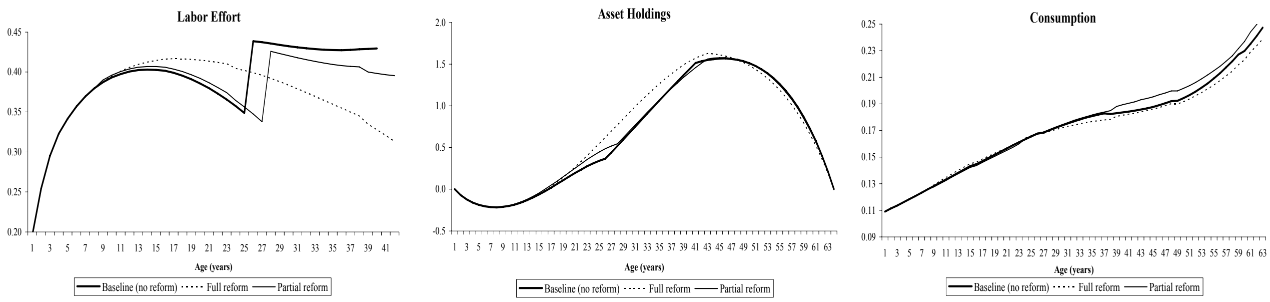


Figure 8. Household's Asset Holdings (A^s), Labor Effort (n^s), and Consumption (c^s) Profiles by Age (During Demographic Transition Under Tax-as-you-go)

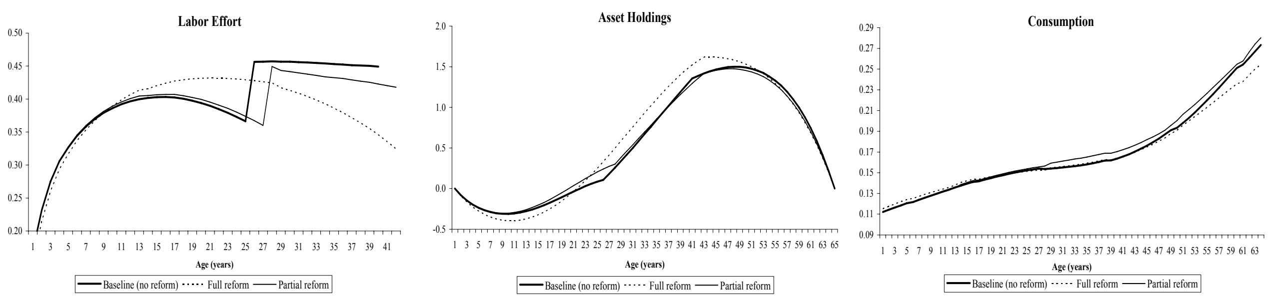
1990 Labor Force Entrant



2000 Labor Force Entrant



2010 Labor Force Entrant



2020 Labor Force Entrant